

# Appendix A

## Heat transfer analysis

There are number of studies on the heat transfer of the SMA materials. Few representative examples are the work of Bhattacharyya et al. [69] in which they studied the possibility of using thermoelectric effect in order to improve the cooling response of the SMA actuators. Bhattacharyya et al. [70] also worked on the characterization of the convection heat transfer coefficient for SMA wires. They modeled the coefficient as a linear function of the current of the wire and performed experiments with SMA and non-SMA wires to verify the model. Potapov and da Silva [71] used Liang's kinematics model for SMA elements with constant stress and constant strain in order to find both heating and cooling response times. They assumed the convection heat transfer to be constant and showed the time constant is in agreement with the experiments.

Here we have used the principle of the conservation of energy to model the heat transfer

behavior of the SMA wire. Unlike the works of Bhattacharyya et al. [70] and Potapov and da Silva [71] we have used a convection heat transfer coefficient that is based on the thin cylinder theory. As explained next the convection coefficient is found to be a function of wire temperature.

The heat transfer model of the wire, using the principle of conservation of energy, is written as

$$mC_p \frac{dT}{dt} = I^2 R(\xi) - h(T)A(T - T_\infty) - m\Delta H \dot{\xi} \quad (\text{A.1})$$

where  $C_p$  is the specific heat,  $I$  and  $R$  are electric current and resistance respectively,  $h(T)$  is the convection heat transfer,  $A$  is the circumference area of the wire,  $T$  and  $T_\infty$  are temperature of the wire and ambient respectively,  $\Delta H$  is the latent heat associated with the phase transformation [71]. This equation presents the effect of the Joule heating, convection heat transfer, and latent heat on the internal energy of the wire.

## A.1 Convection heat transfer

Natural heat convection takes place because of Buouyoncy force and the difference between cold and warm air densities. Because of the temperature difference between the vertical surfaces, in this case a cylindrical surface, and the ambient a boundary layer flow develops over the lateral surface. When the boundary layer thickness is smaller than the cylinder diameter, the cylinder can be modeled as a vertical wall ignoring its curvature. In the case of a thin wire, however, the diameter is not necessary larger than the boundary layer thickness

and hence a different Nusselt number and therefore a different convection coefficient should be used. The criteria for considering a cylinder as a vertical wall is

$$\frac{D}{l} > Ra_l^{-\frac{1}{4}} \quad (\text{A.2})$$

where  $D$  and  $l$  are the cylinder's diameter and length respectively and  $Ra_L$  is the Rayleigh number. It can be shown that for the SMA wire this condition is not satisfied for the range of temperature of interest. On the other hand for the vertical thin cylinders the Nusselt number is written as

$$\overline{Nu}_l = \frac{4}{3} \left[ \frac{7Ra_l Pr}{5(20 + 21Pr)} \right]^{\frac{1}{4}} + \frac{4(272 + 315Pr)l}{35(64 + 63Pr)D} \quad (\text{A.3})$$

where  $Pr$  is the Prandtl number,  $\overline{Nu}_l = hl/k$ , and  $Ra_l = g\beta\Delta Tl^3/\alpha\nu$ . In these definitions  $h$  is the length-averaged heat transfer coefficient,  $\Delta T$  is the temperature difference between the surface of the ambient. Also  $\beta$  is the volume expansion coefficient that for the ideal gas can be shown to be  $\beta = \frac{1}{T}$  where  $T$  is the absolute temperature.  $k$ ,  $\alpha$  and  $\nu$  are thermal conductivity, thermal diffusivity and kinematic viscosity of the air calculated at the film temperature  $T_f = \frac{T+T_\infty}{2}$ .

Therefore the convection coefficient is a function of both temperatures of wire and the ambient temperature. We assume that the ambient temperature is constant while the wire's temperatures changes considerably. Table A.1 shows the property of the air over the actuation temperature range [72]. The ambient temperature assumed to be 23°C. Using

Table A.1: Air properties as a function of the SMA wire temperature, the ambient temperature assumed to be  $T_\infty = 23^\circ C$

$T_w(^{\circ}C)$	$T_f(K)$	$k(W/m^{\circ}C)$	$\alpha(m^2/s)$	$\nu(m^2/s)$	$Pr$
31	300	0.0261	2.21e-5	1.57e-5	0.712
51	310	0.0268	2.35e-5	1.67e-5	0.711
71	320	0.0275	2.49e-5	1.77e-5	0.710
91	330	0.0283	2.64e-5	1.86e-5	0.708
111	340	0.0290	2.78e-5	1.96e-5	0.707

these properties the convection coefficient can be calculated. Figure A.1 shows the calculated convection heat transfer coefficient which increases as a function of the temperature of the wire. As shown in the figure, We have approximated it with a linear function as  $h(T) = 0.1558T + 89.33$ , where  $T$  is the temperature of the wire in Centigrade.

It is worth noting that the cooling rate is proportional to the ratio of the surface area to the heat capacity. Thus the ratio of the surface area to the volume of the wire is a indicator of the cooling rate. The generating force on the other hand is proportional to the sectional area of the wire. The surface area/volume is in inverse proportion to the diameter. Therefore, if the diameter is small, the cooling rate is fast. However, the generating force is much smaller.

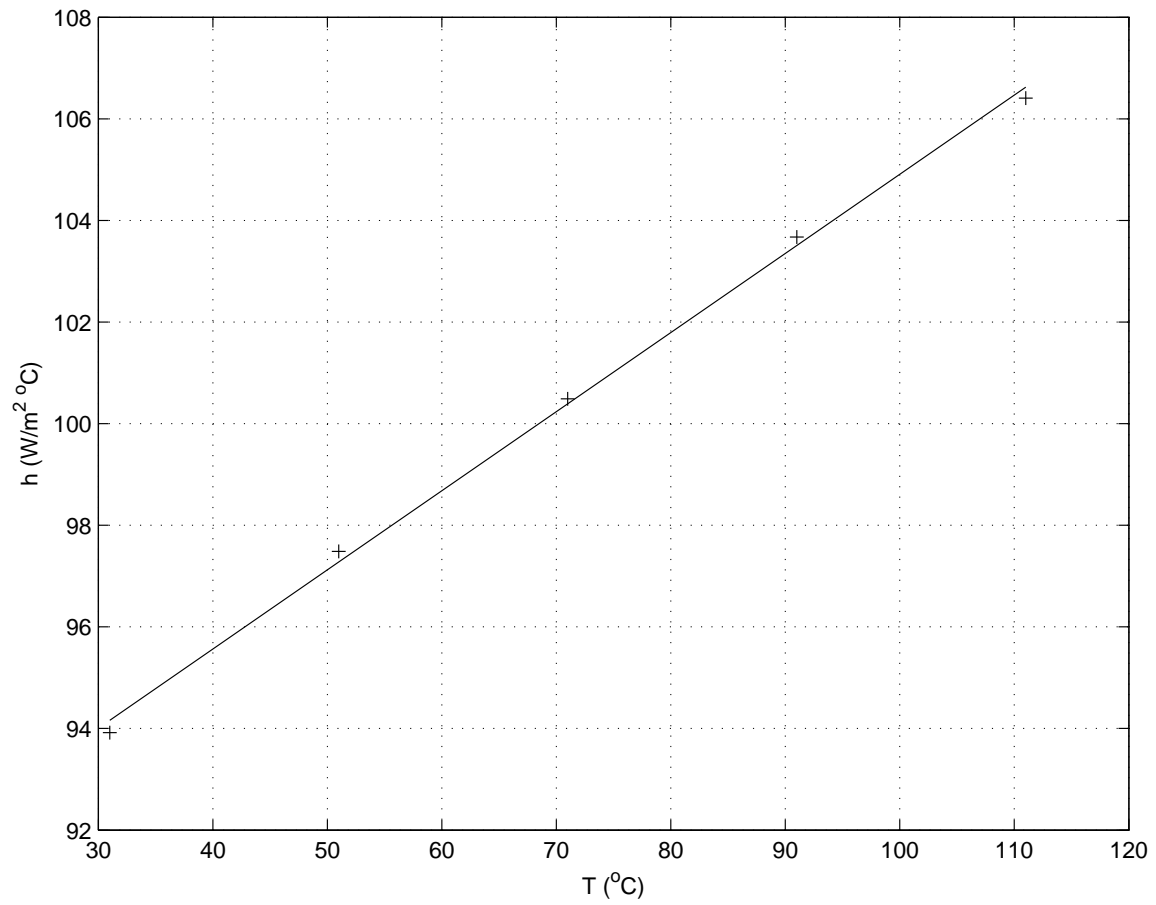


Figure A.1: The convection heat transfer coefficient as the function of the SMA wire temperature

# Appendix B

## Using SMA for Sensing

Although SMAs are mostly used for actuation they also have a good sensing capability. Several properties of an SMA element change as it undergoes martensite phase transformation. Among these properties is the resistivity of SMAs that decreases as the temperature of the wire increases and hence its phase transforms to austenite. Using the SMA rotary actuator we conducted an experiment to evaluate the change in the resistance of the SMA wire.

In the experiment the applied voltage to the wire increased incrementally while recording the current and angular position of the moving arm. Phase transformation is hysteretic therefore the voltage, current and angular position are also recorded while the arm moved down due to the voltage drop. Figures B.1 and B.2 show the hysteretic plot of the voltage and current vs. angular position of the actuator.

While the voltage and current both change with angular position in a hysteretic fashion the

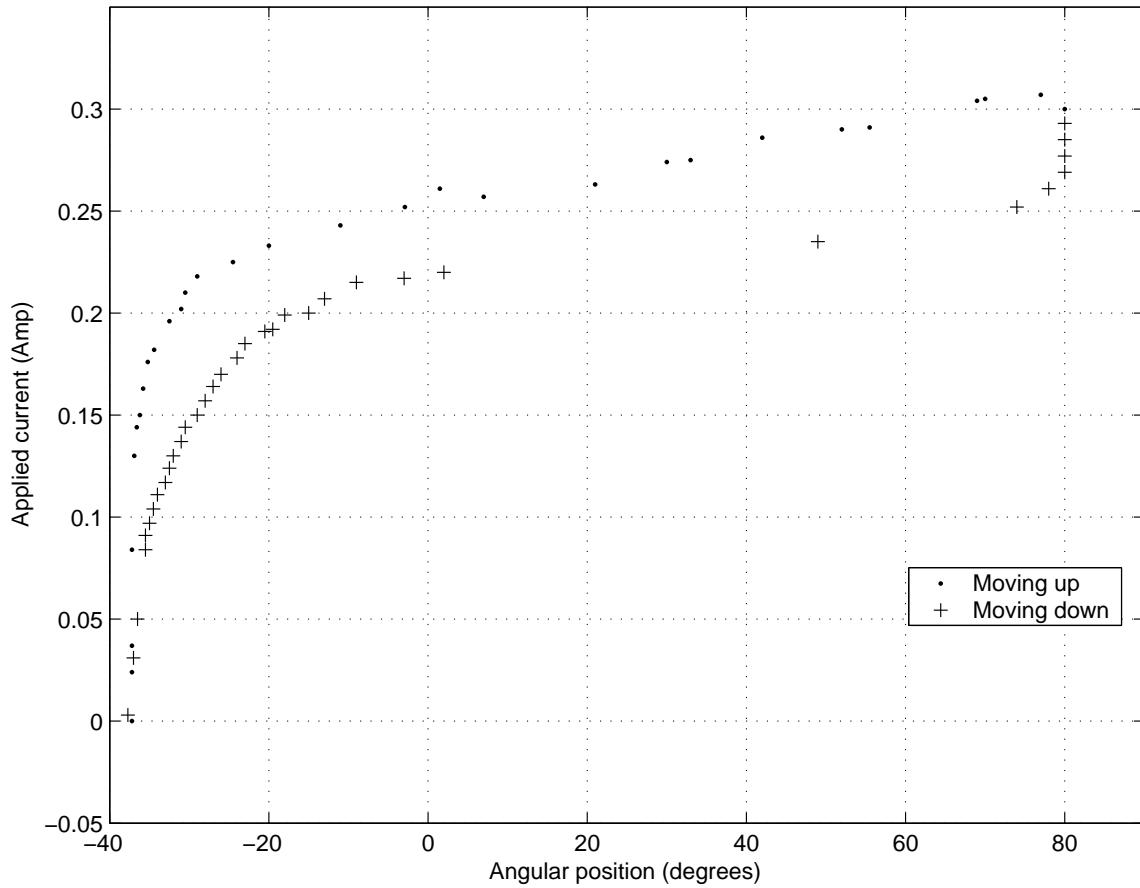


Figure B.1: Current passing through the SMA wire changes as a hysteretic function of the angular position of the actuator

resistance of the wire changes almost linearly with the angle. This is shown in Fig. B.3 along with the linear approximation of resistance as a function of the angular position. The resistance is approximated as

$$R = -2.914\theta + 50.94 \tag{B.1}$$

This experiment is an indication of sensing capabilities of the SMA elements that can be further utilized.

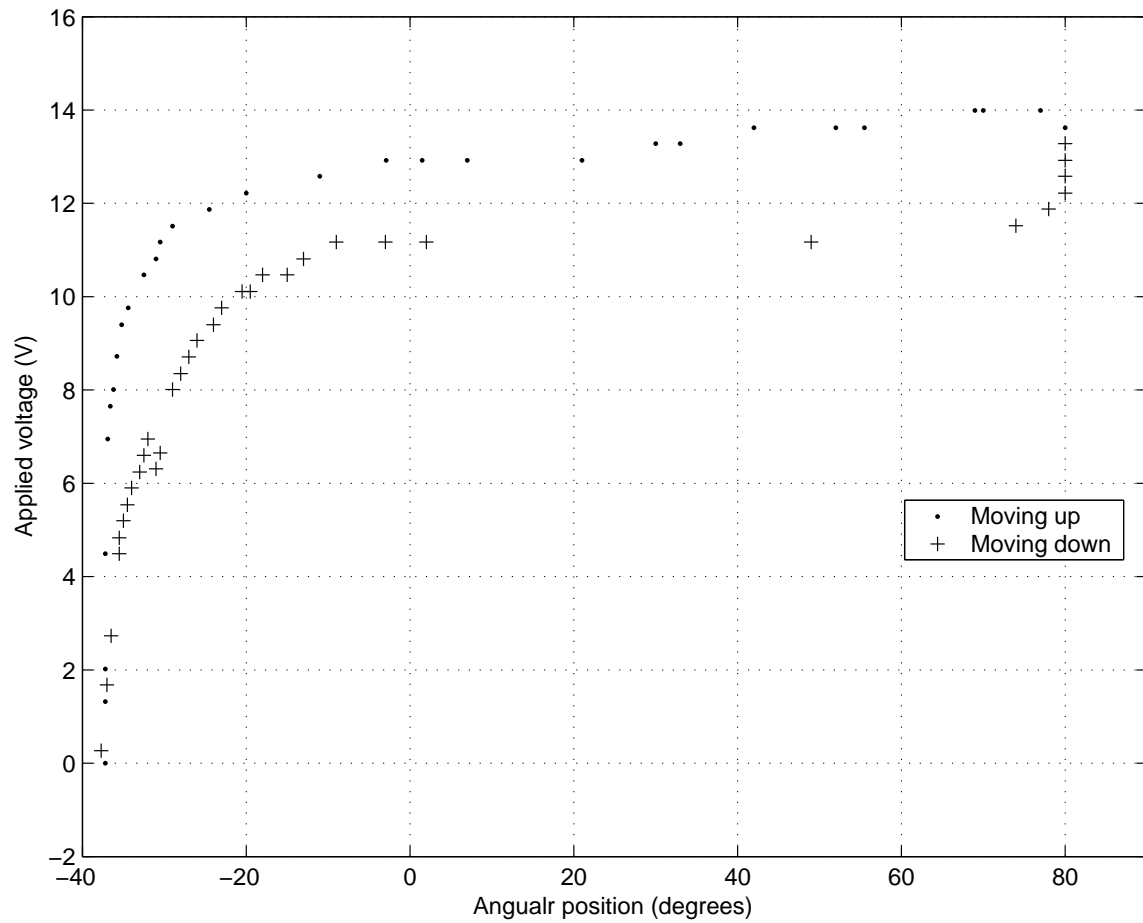


Figure B.2: Voltage of the SMA wire changes as a hysteretic function of the angular position of the actuator



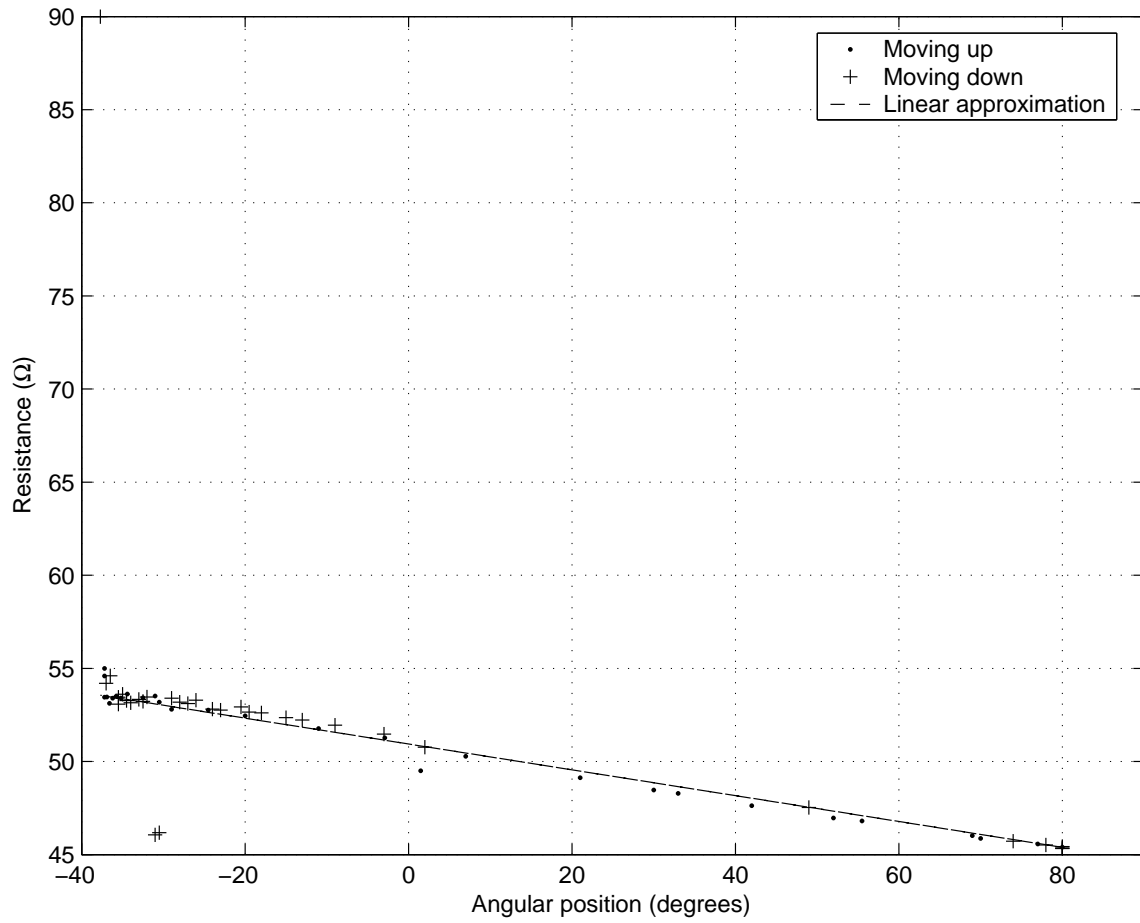


Figure B.3: Resistance of the SMA wire changes almost as a linear function of the angular position of the actuator