Adaptive Control of the Atmospheric Plasma Spray Process for Functionally Graded Thermal Barrier Coatings

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Functionally graded coatings (FGCs) have a material composition continuously varying through the thickness but uniform in the surface parallel to the coated substrate. When used as a thermal barrier on a metallic substrate, the coating composition varies from an almost pure metal near the substrate to a pure ceramic adjacent to the outer surface exposed to a hot environment. Challenging issues in producing high quality FGCs in the presence of external disturbances with an atmospheric plasma spray process (APSP) include controlling the mean temperature, the mean axial velocity, and the positions of the constituent material particles when they arrive at the substrate to be coated. The unavoidable disturbances include fluctuations in the arc voltage and clogging of the powder in the delivery system. For a two-constituent coating, this work proposes using three modified robust model reference adaptive controllers based on the $\sigma$-modified laws and low frequency learning. One controller adjusts the current and flow rates of argon and hydrogen into the torch. The other two controllers adjust the distance of the two powder injector ports from the plasma jet axis and the average injection velocity of each powder. It is shown through numerical experiments that the three controllers implemented in an APSP consistently produce high-quality FGCs.

1. Introduction

Thermal barrier coatings (TBCs) for high temperature applications such as turbine blades in jet engines consist of a ceramic such as zirconia ($\text{ZrO}_2$) on the outer surface and a metallic bond coat of NiCrAlY superalloy on the substrate surface [1]. ZrO$_2$ has low thermal conductivity, excellent chemical stability, and high fracture toughness [2]. NiCrAlY provides a surface texture that bonds well with the substrate, reduces the thermal mismatch between the top coat and the substrate, and enhances resistance to oxidation of the substrate. However, these TBCs prematurely fail due to cracking and delamination during high thermal and mechanical cyclic loading, possibly due to the thermal mismatch and poor bond strength between the metallic coat and the ceramic [3]. A functionally graded coating (FGC) helps reduce the thermal mismatch and improve the bond strength by using either multiple coats of continuously varying composition or a stepwise composition of metallic and ceramic powders [3].

Three approaches to produce FGCs using an atmospheric plasma spray process (APSP) are as follows: (i) single torch–single injector using a premixed powder composition, (ii) single torch–dual injectors, and (iii) dual torches with two independent injection systems [4]. A premixed powder may produce a poor-quality coating due to large differences in the mass densities and melting points of the metallic and the ceramic particles. The in-flight trajectories of the metallic particles are usually located far away from the jet axis as compared to those of the ceramic particles. Dual torches with their independent injection systems allow for the selection of optimal plasma generation and injection parameters for the metallic and the ceramic powders, which may help achieve the desired through-the-thickness
variation in the coating composition. However, there is an increased possibility of external disturbances and fluctuations in each torch and injector system. A single torch-dual injector setup allows for spacing the two injectors to accommodate differences in mass densities and melting temperatures of the metallic and the ceramic particles. The first and the last layers are, respectively, generally coated with pure NiCrAlY powder from the metallic injector and pure ZrO$_2$ powder from the ceramic injector. The intermediate layers are sprayed using both injectors simultaneously with appropriate mass flow rates depending on the NiCrAlY and the ZrO$_2$ compositions. This helps to attain a continuous variation in the volume fractions of particles and their in-flight trajectories for achieving the desired distributions and their spatial locations when they strike the substrate.

In this work, we study a single torch-two injector system depicted in Figure 1. A mixture of argon (Ar) and hydrogen (H$_2$) gases is injected into the gas gun, where they get ionized into plasma while passing over an electric arc between the cathode and the anode. The plasma exits from the gas gun at a high speed and elevated temperature that depend upon the power input, the gun efficiency, and the input flow rates of the two gases. The NiCrAlY and the ZrO$_2$ particles injected through separate ports transversely to the plasma and exchange heat with it before striking the prepared surface of the substrate.

The complex interactions among the plasma, the powder particles and disturbances in the process parameters caused by the arc fluctuations due to erosion of the cathode and the anode, the nozzle wear, the injector wear, the pulsing of powder particles due to leaks and dampness, powder clogging, and variations in the carrier gas flow rate accompanying the powders significantly alter particles’ trajectories through the plasma and hence mean particles’ temperature and axial velocity (collectively called mean particles’ states, MPSs) that affect the coating quality [5–8].

Here, we use controllers based on a model reference adaptive control (MRAC) framework [9] whose robustness has been enhanced by incorporating in it the $\sigma$ modification [10] and the low-frequency learning [11]. We called it the “modified robust MRAC” (MR-MRAC) and illustrated its capability [12] in the presence of bounded external disturbances to adaptively adjust input parameters for a one-material coating to achieve the desired MPSs within small tolerances. Here, we propose the use of three MR-MRACs for producing FGCs comprised of NiCrAlY and ZrO$_2$ particles. One controller adaptively adjusts the flow rates of the Ar and the H$_2$ into the gas gun, and the other two adjust the locations of the injection ports and the average injection velocities of the two powders.

## 2. Methodology

### 2.1. Mathematical and Numerical Models of an APSP

Shang et al. [13], amongst others, have provided a mathematical model (i.e., assumptions made, partial differential equations governing the flow of a plasma by regarding it as a mixture of chemically interacting constituents, initial and boundary conditions) and the associated numerical model. They modified the finite volume method-based software LAVA-P developed at the Idaho National Engineering and Environmental Laboratory to analyze 3-dimensional motions of powder particles within the plasma and considered turbulence modulation. The model considers the heat exchange between the plasma and the powder particles, which are regarded as rigid heat-conducting spheres, and their axisymmetric melting, vaporization, and re-solidification. The drag force between the particles and the plasma drives the particles’ trajectories. Shang et al. [13] demonstrated that for a single constituent powder, the predicted plasma flow and the MPSs of particles agreed well with the corresponding test data. Here, we use LAVA-P to compute the MPSs of a mixture of NiCrAlY and ZrO$_2$ particles.

### 2.2. Finding Input Parameters for Desired $z$-Locations of NiCrAlY and ZrO$_2$ Particles

The three steps involved in addressing this issue are (i) using statistical analyses to identify significant input parameters, (ii) developing response functions relating significant input parameters to the MPSs, and (iii) numerically solving equations of the response functions to get starting values of input parameters [9] for producing an FGC.

We consider eight input parameters for the screening analysis to identify significant parameters that influence the averaged $z$-locations, $C_1(t)$, $C_2(t)$, of NiCrAlY and ZrO$_2$ powder particles in the observation window, where they are measured during the coating process. These input parameters are, $V_{inj}, V_{mfr} = \text{average injection velocities of NiCrAlY and ZrO}_2$; $d_1, d_2 = \text{mean particles’ states, MPSs}$ that affect the coating quality [5–8].

Here, we use controllers based on a model reference adaptive control (MRAC) framework [9] whose robustness has been enhanced by incorporating in it the $\sigma$ modification [10] and the low-frequency learning [11]. We called it the “modified robust MRAC” (MR-MRAC) and illustrated its capability [12] in the presence of bounded external disturbances to adaptively adjust input parameters for a one-material coating to achieve the desired MPSs within small tolerances. Here, we propose the use of three MR-MRACs for producing FGCs comprised of NiCrAlY and ZrO$_2$ particles. One controller adaptively adjusts the flow rates of the Ar and the H$_2$ into the gas gun, and the other two adjust the locations of the injection ports and the average injection velocities of the two powders.

### 2.3. Development of the MR-MRAC

Figure 2 schematically illustrates three different MR-MRACs along with the corresponding measured output variables and the input parameters they adjust. Guduri and Batra [12] have described
the development of the MR-MRAC-1. It entails the following seven steps: (i) specification of the MPSs (output variables) and of the lower and the upper bounds of the significant input parameters selected using the screening analysis, (ii) quantification of disturbances to be considered, (iii) time duration allowed for the process parameters to respond to the disturbances, (iv) a mathematical model of the process linearized around a steady (or an equilibrium) state, (v) system identification, (vi) controller design, and (vii) implementation and testing of the controller. The material in this subsection extends the work described in reference [12] for a single powder port to two powder ports and is included for completeness. The mathematical formulations of the MR-MRAC-1, the MR-MRAC-2, and the MR-MRAC-3 are included in Figure 2 and their implementation in an APSP for generating FGCs are briefly discussed below.

For the FGCs, the screening analysis identified the average injection velocity \( V_{inj}^{(1)} \) and the \( y \)-location \( d_{1y}^{(1)} \) of the injector as significant parameters that influence the average \( z \)-location, \( C_z^{(1)}(t) \), of powder particles in the observation window where they are measured during the coating process.

Limits on the input variables with symbols indicated in parentheses are as follows: air flow rate \( P \), 20 slm \( \leq P \leq 60 \) slm (standard liters per minute); \( H_2 \) flow rate \( Q \), 0 \( \leq Q \leq 20 \) slm; current \( I \), 300 A \( \leq I \leq 600 \) A, average injection velocity of the NiCrAlY powder particles \( V_{inj}^{(1)} \), 5 m/s \( \leq V_{inj}^{(1)} \leq 15 \) m/s; average injection velocity of the ZrO$_2$ particles \( V_{inj}^{(2)} \), 5 m/s \( \leq V_{inj}^{(2)} \leq 15 \) m/s; \( y \)-location of the NiCrAlY injector \( d_{1y}^{(1)} \), 0.2 cm \( \leq d_{1y}^{(1)} \leq 1.5 \) cm; and \( y \)-location of the ZrO$_2$ injector \( d_{2y}^{(2)} \), 0.2 cm \( \leq d_{2y}^{(2)} \leq 1.5 \) cm. It is desired that effects of disturbances die out within 50 ms of their occurrence.

For the MR-MRAC-1 [12], the model relating the three inputs, \( u(t) = \{P(t), Q(t), I(t)\}^T \), and the two outputs, \( \{v(t), T(t)\} \), is

\[
y(t) = \{v(t), T(t)\}^T,
\]

linearized around a steady state is taken:

\[
y(t) = Ay(t) + Bu(t) + d(t), \quad y(0) = y_0. \tag{1}
\]

In (1), elements of matrices are

\[
A = \begin{bmatrix} a_1 & 0 \\ 0 & a_T \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix},
\]

depending upon conditions at time \( t = 0 \), \( y_0 \) is the MPS at time \( t = 0 \) when the steady state has reached and an unknown smooth disturbance \( d(t) \) satisfying \( \|d(t)\|_2 \leq d_{\max} \), \( \|d(t)\|_2 \leq d_{\max} \) with positive bounds \( d_{\max} \) and \( d_{\max} \) is introduced. Here, \( \|d(t)\|_2 = \sqrt{\int_0^T (d(s))^2 \, ds} \). All matrices and vectors in this work are real-valued.
Similarly, for the MR-MRAC-2 and the MR-MRAC-3, we presume the following affine dependence of \( C_z^{(j)}(t) \) upon \( C_z^{(j)}(t), V_{inj}^{(j)} \) and \( d_y^{(j)} \):

\[
C_z^{(j)}(t) = a_m^{(j)} C_z^{(j)}(t) + b_{1m}^{(j)} V_{inj}^{(j)} + b_{2m}^{(j)} d_y^{(j)} + d_z^{(j)}(t), \quad C_z^{(j)}(0) = C_{z0}^{(j)}.
\]  

where \( y_m(t) \) is a reference \( 2 \times 1 \) output vector. Several trials provided the following values for matrices \( A_m \) and \( B_m \):

\[
A_m = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.
\]  

In the absence of a disturbance, i.e., \( d(t) = 0 \) in (1), the asymptotic convergence of the tracking error, \( e(t) \equiv y(t) - y_m(t) \), is achieved using the control law in (4), and the following adaptive law of the MRAC scheme [10]:

\[
\dot{K}(t) = \Lambda \bar{B}_m^T \bar{P}(:,y)^T(t) sgn(l), \quad K(0) = K_0,
\]

\[
\dot{\bar{L}}(t) = -\Lambda \bar{B}_m^T \bar{P}(:,r)^T(t) sgn(l), \quad L(0) = L_0.
\]

Here, \( \Lambda = \Lambda^T \) and \( \bar{P} = \bar{P}^T \) are, respectively, \( 3 \times 3 \) and \( 2 \times 2 \) positive definite matrices.

Similarly, we use the following reference model for updating controller gains to achieve the desired mean normal distributions of the \( z \)-locations on the substrate of the NiCrAlY and the ZrO\(_2\) particles:

\[
\dot{y}_m(t) = A_m y_m(t) + B_m r(t), \quad y(0) = y_{m0}.
\]
In (11), $B_m^{(j)} = \begin{bmatrix} b_m^{(j)} & b_{2m}^{(j)} \end{bmatrix}$, $e^{(j)}(t) = C_z^{(j)}(t) - C_{z,m}^{(j)}(t)$, $r^{(j)}(t) = \begin{bmatrix} C_{z,des}^{(j)}(t) & C_{z,des}^{(j)}(t) \end{bmatrix}^T$, where $\Lambda^{(j)}$ is the $2 \times 2$ real-valued adaptive gain matrix, and $\tilde{K}^{(j)}$ is a positive constant.

Similarly, the adaptive laws for the MR-MRAC scheme for the mean normal distributions of the NiCrAlY and the ZrO$_2$ particles for estimating the gain matrices $K^{(j)}(t)$ and $L^{(j)}(t)$ in terms of the tracking error, $e^{(j)}(t) \equiv C_z^{(j)}(t) - C_z^{(j)}$, are given by (17).
\[ K^{(j)}(t) = \Lambda^{(j)}B^{(j)}_mT^{(j)}e^{(j)}(t)C^{(j)}(t)^T \text{sgn}(I^{(j)}) - \sigma^{(j)}(K^{(j)}(t) - K^{(j)}_{\text{FIG}}(t)), \quad K^{(j)}(0) = K^{(j)}_0, \]
\[ \dot{L}^{(j)}(t) = -\Lambda^{(j)}B^{(j)}_mT^{(j)}e^{(j)}(t)r^{(j)}(t)^T \text{sgn}(I^{(j)}) - \sigma^{(j)}(L^{(j)}(t) - L^{(j)}_{\text{FIG}}(t)), \quad L^{(j)}(0) = L^{(j)}_0, \]
\[ K^{(j)}_{\text{FIG}}(t) = \lambda^{(j)}(K^{(j)}(t) - K^{(j)}_{\text{FIG}}(t)), \quad K^{(j)}_{\text{FIG}}(0) = K^{(j)}_0, \]
\[ \dot{L}^{(j)}_{\text{FIG}}(t) = \lambda^{(j)}(L^{(j)}(t) - L^{(j)}_{\text{FIG}}(t)), \quad L^{(j)}_{\text{FIG}}(0) = L^{(j)}_0. \] (17)

The MR-MRAC and the MRAC identically perform in the absence of external disturbances. The consideration of low-frequency learning in the adaptive laws converts a pure integral type MRAC to a proportional-integral type MRAC.

It is shown in refs. [6, 9, 12, and 11], The MR-MRAC enables fast learning and improves robustness.

The initial estimates of the gain matrices are calculated by using the following expressions:

\[ K_0 = 0, K_0^{(1)} = 0, K_0^{(2)} = 0, L_0 = u_{\text{sol}}(r_0)^{-1}, L_0^{(1)} = u_{\text{sol}}^{(1)}(r_0^{(1)})^{-1} \text{ and } L_0^{(2)} = u_{\text{sol}}^{(2)}(r_0^{(2)})^{-1}, \] (18)
where

\[ u_{\text{sol}} = \begin{bmatrix} P_{\text{sol}} & Q_{\text{sol}} & I_{\text{sol}} \end{bmatrix}^T, \quad u_{\text{sol}}^{(1)} = \begin{bmatrix} V^{(1)}_{\text{inj, sol}} & d^{(1)}_{y, \text{sol}} \end{bmatrix}^T \text{ and } u_{\text{sol}}^{(2)} = \begin{bmatrix} V^{(2)}_{\text{inj, sol}} & d^{(2)}_{y, \text{sol}} \end{bmatrix}^T. \] (19)

Figure 3 shows the scheme for the adaptive control process to get the mean normal distributions of the NiCrAlY and the ZrO2 particles that produce the desired MPSs in the observation window. At the start of the process (t = 0), when only NiCrAlY powder is injected, values of parameters \( V^{(1)}_{inj,0}, d^{(1)}_{y,0}, P_0, Q_0 \) and \( I_0 \) are found from the response functions given by (20) and (21). These are used to determine (i) the initial estimates of the controller gains from the MR-MRACs using (18), (ii) the injection parameters \( V^{(1)}_{inj} \) and \( d^{(1)}_{y} \) from (10), and (iii) the process parameters \( P, Q \) and \( I \) from (4) that are input to the software LAVA-P.

At time \( t = 100 \) ms, the injection of the NiCrAlY particles starts, and its mass injection rate is decreased after every 100 ms interval while that of the ZrO2 increased. The initial estimates of the controller associated with the two injectors are the same as those of the NiCrAlY controller at \( t = 0 \). During the spray process, the averaged z–locations (\( C^{(1)}_{z} \) and \( C^{(2)}_{z} \)) of the NiCrAlY and the ZrO2 powder particles and their combined MPSs in the observation window are fed into the controller. Values of \( V^{(1)}_{inj}, d^{(1)}_{y,0}, P_0, Q_0 \), and \( I_0 \) are adaptively varied by the three MR-MRAC controllers to update the gains and minimize the tracking errors using (16) and (17).

The MR-MRAC controller is tested by assuming that the software LAVA-P represents the actual plant and only the arc voltage is disturbed.

3. Results and Discussion

3.1. Validation of LAVA-P. It is shown in refs. [6, 9, 12, and 13] that the predicted MPSs from the software LAVA-P for the injection of a single material powder agree well with the experimental findings of Williamson et al. [16] and Smith et al. [17]. Here, we demonstrate that the computed MPSs when both the NiCrAlY and the ZrO2 particles are simultaneously injected agree with those computed by Wan et al. [18], who also used LAVA-P. It will ensure that we are correctly using the software.

The two powders are simultaneously injected at 11.7 m/s along with the carrier gas, which flows at 5 slm through injectors located at \( y = 0.6 \) cm, \( z = 0.8 \) cm and \( y = 1.0 \) cm, \( z = 0.8 \) cm, respectively, for the ZrO2 and the NiCrAlY particles. Values of other processing parameters are as follows: \( I = 500 \) A, \( P = 40 \) slm, \( Q = 12 \) slm, voltage = 70 V, and the mass flow rate of NiCrAlY (ZrO2) = 30 (20) g/min.

Figures 4(a) and 4(c) depict the distributions of the NiCrAlY and the ZrO2 particles in the yz–plane at \( t = 10 \) ms. The distributions of these particles in the xz–plane at a distance of 10 cm from the nozzle exit shown in Figures 4(b) and 4(d) suggest that the presently computed in-flight locations of the NiCrAlY and the ZrO2 particles are qualitatively similar to Wan et al.’s results. The quantitative comparison is not feasible since scales and the time when the results are plotted are missing in Wan et al.’s figure.

3.2. Injection of NiCrAlY and ZrO2 Particles through Single vs. Separate Ports. We now investigate differences, if any, in the computed MPSs and particles distributions on the substrate when the two powders are injected through the same port versus different ports. Results are summarized in Table 1.

3.3. Screening of Injection Parameters. We investigate the significance of eight injection parameters, namely, the
average injection velocities of the NiCrAlY and the ZrO2 particles, \( y \)– and \( z \)– locations of their injectors, and their mass flow rates in the observation window averaged in the \( z \)–direction. The process parameters used in this study are as follows: current = 500 A, Ar flow rate = 40 slm, \( H_2 \) flow rate = 10 slm, voltage = 60 V, and particle size = 30–100 \( \mu \)m, the average injection velocity and mass flow rates of the NiCrAlY and the ZrO2 powders (m/s, g/minute) (5, 15), and (15, 40); \( (y, z) \)– locations of the NiCrAlY and ZrO2 injectors (mm) (4, 6), and (12, 10).

Each input parameter is discretized into a \( \varphi \)–level grid \( \{0, (1/\varphi - 1), (2/\varphi - 1), \ldots, 1\} \) where \( \varphi \) is an even integer. For 8 randomly chosen base points requiring 72 simulations, we present in Figure 6 for \( \varphi = 12 \) and \( \varphi = 20 \) the mean and the standard deviations of the elementary effects (EEs) of the injection parameters on the averaged \( z \)–locations of the NiCrAlY and the ZrO2 powder particles. The results for the two values of \( \varphi \) are close to each other. For both powder particles, high values of the mean and of the standard deviations of the elementary effects (EEs) associated with the average injection velocity and the \( y \)–locations of the NiCrAlY and ZrO2 injectors imply that they significantly influence the \( z \)–locations of particles in the observation window.

3.4. Response Functions of Number Averaged \( z \)–Locations of NiCrAlY and ZrO2. The numerical experiments are designed using the Latin hypercube sampling (LHS) approach to generate 100, 200, and 300 samples by taking the following values of the mean and the variance of the normal distribution: current (A) 450, 1000; Ar flow rate (slm) 45, 15; \( H_2 \) flow rate (slm) 8, 4; average injection velocities of NiCrAlY and ZrO2 particles, respectively; \( \nu \) = the mean axial velocity of both particles; \( T \) = the mean temperature of both particles; \( V_{inj}, d_{inj} \) = the average injection velocity of NiCrAlY and ZrO2 particles, respectively; \( d_{(1)}, d_{(2)} = y \)–locations of NiCrAlY and ZrO2 injectors, respectively.

The affine response functions given by (20) for the mean axial velocity (\( \nu_j \)) and the number averaged \( z \)–locations (\( C_{(j)} \)) of the NiCrAlY and ZrO2 particles in terms of the significant input parameters when fitted to the computed data had a regression coefficient, \( R^2 \approx 0.99 \).

\[
y_j = a_0 + a_1 P + a_2 Q + a_3 I + a_{\nu_{inj}} V_{inj} + a_{d_{(j)}} d_{(j)}.
\]  \( (20) \)

Recall that \( j = 1 \) and 2, respectively, for the NiCrAlY and the ZrO2. Values of coefficients \( a_0, a_1, a_2, a_3, a_{\nu_{inj}}, \) and \( a_{d_{(j)}} \) estimated using the regression analysis and the regression coefficient, \( R^2 \), are listed in Table 2 for the NiCrAlY and the ZrO2 powders.

The following polynomial of degree 2 provided \( R^2 \approx 0.97 \) for the mean particles’ temperature \( T \) in the observation window:

\[
T_j = b_0 + \sum_{i=1}^{20} b_i u_i^{(j)}.
\]  \( (21) \)

where \( u_i^{(j)} = \{P, Q, I, V_{inj}, d_{(j)}, P \cdot Q, P \cdot I, P \cdot V_{inj}, P \cdot d_{(j)}, Q \cdot V_{inj}, Q \cdot d_{(j)}, P \cdot Q \cdot V_{inj}, P \cdot Q \cdot d_{(j)}, Q \cdot V_{inj} \cdot d_{(j)}, (P \cdot Q \cdot V_{inj}) \cdot (P \cdot Q \cdot d_{(j)}) \}. \)
Inequations (20) and (21), each variable has been normalized to have a value between 0 and 1. The estimated coefficients of the response functions are listed in Table 3. The effect of the number of samples on these coefficients is negligible. Since the regression coefficients of the variables in (20) and (21) are nearly the same for 100, 200, and 300 samples, we adopt their values for 300 samples.

Values of the process input parameters for the desired outputs are estimated by solving the nonlinear algebraic equations (20) and (21) with an error less than $10^{-6}$ using the

\[
\text{Table 1: Injector locations, particles' injection speed, particles' mean z-location, and the mean axial velocity and temperature of particles when they arrive at the substrate. Values of other parameters are: current = 500 A, voltage = 50 V, Ar flow rate = 40 slm, H}_2\text{ flow rate = 10 slm, particles' diameter between 30 and 100 μm, mass flow rates of NiCrAlY and ZrO}_2\text{ = 20 g/min, and z-locations of powder ports = 8 mm.}
\]

\[
\begin{array}{ccccccccccc}
\text{Injection type} & \text{Case} & \text{y-location of} & \text{Average} & \text{Mean} & \text{Mean} & \text{Averaged z-} \\
\text{} & \text{injector (mm)} & \text{injection} & \text{axial} & \text{temperature (K)} & \text{location of} & \text{location (cm)} \\
& & \text{velocity (m/s)} & \text{velocity (m/s)} & \text{particles (cm)} & \text{NiCrAlY} & \text{ZrO}_2 & \text{NiCrAlY} & \text{ZrO}_2 & \text{NiCrAlY} & \text{ZrO}_2 \\
\hline
\text{Mixed} & \text{AB} & 6 & 6 & 10 & 10 & 75.3 & 105.5 & 2497 & 2784 & -1.2 & -0.86 \\
\text{Separate (ZrO}_2\text{ injector fixed)} & \text{A1} & 4 & 6 & 10 & 10 & 80.9 & 107.7 & 2531 & 2811 & -1.14 & -0.82 \\
& \text{A2} & 8 & 6 & 10 & 10 & 70.2 & 108.5 & 2478 & 2819 & -1.27 & -0.82 \\
& \text{A3} & 4 & 6 & 8 & 10 & 86.4 & 107.8 & 2568 & 2818 & -0.87 & -0.82 \\
& \text{A4} & 8 & 6 & 12 & 10 & 61.4 & 107.6 & 2383 & 2814 & -1.68 & -0.83 \\
\text{Separate (NiCrAlY injector fixed)} & \text{B1} & 6 & 4 & 10 & 10 & 74.8 & 114.1 & 2501 & 2855 & -1.21 & -0.80 \\
& \text{B2} & 6 & 8 & 10 & 10 & 74.8 & 101.1 & 2486 & 2779 & -1.22 & -0.86 \\
& \text{B3} & 6 & 4 & 10 & 8 & 73.6 & 118.1 & 2464 & 2887 & -1.24 & -0.60 \\
& \text{B4} & 6 & 8 & 10 & 12 & 76.1 & 94.6 & 2503 & 2720 & -1.19 & -1.12 \\
\end{array}
\]

Figure 4: Computed instantaneous distributions (at $t = 5$ ms after the start of the injection of particles) of the NiCrAlY (red color) and the ZrO$_2$ (blue color) particles in the yz-plane and the xz-plane: (a) (b) present results, (c) (d) Wan et al.’s results. In both cases, the NiCrAlY and the ZrO$_2$ injectors are located at (10, 8) mm and (6, 8) mm, respectively. Particles injected at the same injection velocity of 11.7 m/s and the carrier gas flow rate of 5 slm and are collected in the observation window, $9.9 \leq y \leq 10.1$ cm.
"ga" toolbox in MATLAB with default values for the parameters and a seed number of 491218382. The error is defined as follows:

\[
\text{Error}^{(j)} = \left[ v^{(j)}_{\text{des}} - v^{(j)}(P, Q, I, V_{\text{inj}}^{(j)}, d_y^{(j)}) \right]^2 + \left[ T^{(j)}_{\text{des}} - T^{(j)}(P, Q, I, V_{\text{inj}}^{(j)}, d_y^{(j)}) \right]^2 + \left[ C^{(j)}_{z,\text{des}} - C^{(j)}_{z}(P, Q, I, V_{\text{inj}}^{(j)}, d_y^{(j)}) \right]^2,
\]

where the subscript "des" represents the desired value of the parameter. The parameters in equation (22) are normalized to have values between 0 and 1.

The solutions for arbitrarily selected values of the desired MPSSs and the number averaged z-locations of the particles are summarized in Table 4. When these values are used as inputs in LAVA-P, the computed values of the MPSSs and the number averaged z-locations of the particles are found to differ by less than 4.5% from their desired values, indicating that (20) and (21) are good representations of the response functions.

3.5. System Identification for Number Averaged Z-Locations of the NiCrAlY and ZrO\(_2\). We find values of constants \(a^{(1)}, a^{(2)}\) and matrices \(b_1^{(1)}, b_1^{(2)}, b_2^{(1)}\) and \(b_2^{(2)}\) in (3) by expressing the disturbance as the sum of 5 sinusoidal variations as in equation (23).

\[
\mathbf{u}(t) = \begin{cases} 
\hat{u}_0, & t \leq 10 \text{ ms}, \\
\hat{u}_0 + u_{a1} \sin (\omega_1 (t - 10)) + u_{a2} \sin (\omega_2 (t - 10)) + u_{a3} \sin (\omega_3 (t - 10)) + u_{a4} \sin (\omega_4 (t - 10)) + u_{a5} \sin (\omega_5 (t - 10)), & t > 10 \text{ ms}.
\end{cases}
\]
Here, \( u_b \) is the base value; \( u_{a1}, u_{a2}, u_{a3}, u_{a4}, \) and \( u_{a5} \) are amplitudes of perturbations; \( \omega_1, \omega_2, \omega_3, \omega_4, \) and \( \omega_5 \) frequencies, and \( t \) the time in ms. The amplitude of each sinusoidal term and its frequency are listed in Tables 5 and 6. These disturbances are sufficiently rich since they contain enough frequencies. The corresponding variations of the inputs and the averaged computed \( z \)-locations in the observation window are listed in Figures 7 and 8, respectively, for samples 1, 2, and 3. To investigate the influence of the number of samples on the variance of the estimated parameters in equation (3), 50 new samples (their input values are omitted here) are randomly generated. The results computed for the 10 and the 50 samples are similar to each other (results for 50 samples are omitted here).

The raw data of the input variations and the corresponding outputs of the averaged \( z \)-locations are processed by subtracting their means from them and then smoothened using a moving average of 15 trailing points. Figure 9 shows the smoothened data used to estimate parameters in (3) for sample 1. The responses of the model fitted with estimated parameters are also depicted in Figure 9 for training and validation. The estimated values of parameters for 10 samples are listed in Table 7. The predictions from models for the averaged \( z \)-location of the NiCrAlY and the ZrO\(_2\) agree well with those found using LAVA-P software, with an average success rate of 74% (79%) for training and 69% (80%) for validation for the NiCrAlY (ZrO\(_2\)). Figure 10 depicts box plots of the estimated parameters of the averaged \( z \)-locations of the NiCrAlY and the ZrO\(_2\) for the 50 samples, which are distributed close to each other with a few outliers enclosed in red circles. Thus, the models in equation (3) satisfactorily provide the averaged \( z \)-locations of the NiCrAlY and the ZrO\(_2\) for variations in the injection variables.

Figure 6: Normalized standard deviation vs. normalized mean EEs for the normalized (\( z \))-locations of (a) NiCrAlY and (b) ZrO\(_2\) particles in the observation window (meanings of other symbols are: \( V_{\text{inj}}^{(1)} \), \( V_{\text{inj}}^{(2)} \) = average injection velocities, respectively, of NiCrAlY and ZrO\(_2\) particles; \( d^{(1)}_y, d^{(2)}_y \) = (\( y \))-locations, respectively, of NiCrAlY and ZrO\(_2\) injectors; \( d^{(1)}_z, d^{(2)}_z \) = (\( z \))-locations, respectively, of NiCrAlY and ZrO\(_2\) injectors; \( MFR^{(1)}, MFR^{(2)} \) = mass flow rate, respectively, of NiCrAlY and ZrO\(_2\) powder particles).
<table>
<thead>
<tr>
<th>Output, $y^{(3)}$</th>
<th>Number of samples</th>
<th>$a_0$</th>
<th>$a_p$</th>
<th>$a_Q$</th>
<th>$a_I$</th>
<th>$a_{v_{m}}^{(1)}$</th>
<th>$a_{d}^{(1)}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single injection using NiCrAlY powder, $j = 1$</td>
<td>100</td>
<td>0.45</td>
<td>0.75</td>
<td>0.13</td>
<td>0.35</td>
<td>-0.59</td>
<td>-0.34</td>
<td>0.99</td>
</tr>
<tr>
<td>Mean axial velocity, $v^{(1)}$</td>
<td>200</td>
<td>0.38</td>
<td>0.66</td>
<td>0.13</td>
<td>0.36</td>
<td>-0.54</td>
<td>-0.31</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.38</td>
<td>0.67</td>
<td>0.15</td>
<td>0.32</td>
<td>-0.54</td>
<td>-0.32</td>
<td>0.99</td>
</tr>
<tr>
<td>Number averaged z-locations of NiCrAlY, $C_x^{(1)}$</td>
<td>100</td>
<td>0.71</td>
<td>0.48</td>
<td>0.06</td>
<td>0.23</td>
<td>-0.94</td>
<td>-0.13</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.84</td>
<td>0.40</td>
<td>0.04</td>
<td>0.20</td>
<td>-0.85</td>
<td>-0.10</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.77</td>
<td>0.42</td>
<td>0.06</td>
<td>0.19</td>
<td>-0.84</td>
<td>-0.10</td>
<td>0.98</td>
</tr>
<tr>
<td>Single injection using ZrO$_2$ powder, $j = 2$</td>
<td>100</td>
<td>0.33</td>
<td>0.79</td>
<td>0.18</td>
<td>0.41</td>
<td>-0.42</td>
<td>-0.37</td>
<td>0.99</td>
</tr>
<tr>
<td>Mean axial velocity, $v^{(2)}$</td>
<td>200</td>
<td>0.28</td>
<td>0.61</td>
<td>0.15</td>
<td>0.34</td>
<td>-0.34</td>
<td>-0.28</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.24</td>
<td>0.78</td>
<td>0.21</td>
<td>0.39</td>
<td>-0.42</td>
<td>-0.37</td>
<td>0.99</td>
</tr>
<tr>
<td>Number averaged z-locations of ZrO$_2$, $C_x^{(2)}$</td>
<td>100</td>
<td>0.72</td>
<td>0.49</td>
<td>0.06</td>
<td>0.24</td>
<td>-0.98</td>
<td>-0.10</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.82</td>
<td>0.41</td>
<td>0.05</td>
<td>0.21</td>
<td>-0.88</td>
<td>-0.06</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.76</td>
<td>0.41</td>
<td>0.05</td>
<td>0.19</td>
<td>-0.87</td>
<td>-0.07</td>
<td>0.98</td>
</tr>
</tbody>
</table>
3.6. Results for the MR-MRAC Process Control System. Recalling that only NiCrAIY powder is injected during the first 100 ms, we choose $P = I_{2d2}$, $I^{(j)} = 1, I^{(j)} = -1$. We select damping parameters as $\sigma = 10$ and $\sigma^{(1)} = \sigma^{(2)} = 0.0001$, filter constants as $\lambda^{(1)} = 1$ and $\lambda^{(2)} = 0.0001$, and the following adaptive gains:

$$\Lambda^{(1)} = \Lambda^{(2)} = \begin{bmatrix} 200 & 0 \\ 0 & 0.2 \end{bmatrix}; \lambda = 10^{-8} \begin{bmatrix} 20 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 5 \end{bmatrix}. \quad (24)$$

For the desired outputs, $r_0 = \begin{bmatrix} v_{des} (0) & T_{des} (0) & 0 \end{bmatrix}^T$, $r_0^{(1)} = \begin{bmatrix} C_{z,des} (0) & C_{x,des} (0) \end{bmatrix}^T$ and $r_{100}^{(2)} = \begin{bmatrix} C_{z,des} (100) & C_{z,des} (100) \end{bmatrix}^T$ with desired values $v_{des} (0) = 75 \text{ m/s}$, $T_{des} (0) = 2400 \text{ K}$, and $C_{z,des} (0) = -1 \text{ cm}$, the input parameters computed from the NiCrAlY response function in Table 4 are $P_{\text{sol}} = 44.69 \text{ slm}$, $Q_{\text{sol}} = 4.51 \text{ slm}$, $I_{\text{sol}} = 482.48 \text{ A, } V_{\text{sol}}^{(1)} = 9.03 \text{ m/s, and } d_{y,\text{sol}}^{(2)} = 0.99 \text{ cm. The estimated initial gains } L_0 \text{ and } L_0^{(1)} \text{ from (18) are as follows:}
Table 4: Comparison of desired outputs for the NiCrAlY and the ZrO\textsubscript{2} powders with those found from LAVA-P using the process parameters obtained as solutions of equations (20) and (21).

<table>
<thead>
<tr>
<th>Desired outputs</th>
<th>Process parameters from equation (20) and (21)</th>
<th>Measured outputs from LAVA-P</th>
<th>Error between desired and measured outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v^{(j)}) (m/s)</td>
<td>(T^{(j)}) (K)</td>
<td>(C^{(j)}) (cm)</td>
<td>(P) (slm)</td>
</tr>
<tr>
<td>Single injection system using the NiCrAlY powder, (j = 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>2400</td>
<td>-1.0</td>
<td>43.06</td>
</tr>
<tr>
<td>60</td>
<td>2500</td>
<td>-1.1</td>
<td>34.44</td>
</tr>
<tr>
<td>80</td>
<td>2100</td>
<td>-1.2</td>
<td>56.37</td>
</tr>
<tr>
<td>85</td>
<td>2200</td>
<td>-1.1</td>
<td>55.00</td>
</tr>
<tr>
<td>75</td>
<td>2400</td>
<td>-1.0</td>
<td>44.69</td>
</tr>
<tr>
<td>65</td>
<td>2300</td>
<td>-1.3</td>
<td>44.06</td>
</tr>
<tr>
<td>Single injection system using ZrO\textsubscript{2} powder, (j = 2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>2600</td>
<td>-1.0</td>
<td>40.77</td>
</tr>
<tr>
<td>90</td>
<td>2700</td>
<td>-0.9</td>
<td>39.34</td>
</tr>
<tr>
<td>95</td>
<td>2800</td>
<td>-0.8</td>
<td>39.16</td>
</tr>
<tr>
<td>100</td>
<td>2750</td>
<td>-1.1</td>
<td>44.03</td>
</tr>
<tr>
<td>105</td>
<td>2500</td>
<td>-1.0</td>
<td>50.84</td>
</tr>
<tr>
<td>110</td>
<td>2650</td>
<td>-1.2</td>
<td>49.75</td>
</tr>
</tbody>
</table>
**Table 5:** Values of variables used for disturbing the injection velocities (cm/s) of the NiCrAlY and the ZrO₂ particles.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$u_b$ (cm/s)</th>
<th>$u_{a1}$ (cm/s)</th>
<th>$u_{a2}$ (cm/s)</th>
<th>$u_{a3}$ (cm/s)</th>
<th>$u_{a4}$ (cm/s)</th>
<th>$u_{a5}$ (cm/s)</th>
<th>$\omega_{1P}$ (rad/ ms)</th>
<th>$\omega_{2P}$ (rad/ ms)</th>
<th>$\omega_{3P}$ (rad/ ms)</th>
<th>$\omega_{4P}$ (rad/ ms)</th>
<th>$\omega_{5P}$ (rad/ ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean, $\mu$</td>
<td>1100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>S.D., $\sigma$</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Single injection system using the NiCrAlY powder**

Sample 1: 1075.56, 1.56, −45.0, 15.0, 0.5, 0.5, 0.5, 0.5
Sample 2: 1072.29, 29.8, 34.2, −21.4, −64.0, 0.1, 0.4, 0.75, 0.36, 0.39
Sample 3: 1098.78, 30.8, −1.0, 0.25, 0.52, 0.67, 0.42, 0.06
Sample 4: 1197.18, 46.8, −32.8, 0.49, 0.59, 0.47, 0.43, 0.59
Sample 5: 1129.4.1, −69.0, 84.7, 30.4, 0.67, 0.49, 0.52, 0.50, 0.53
Sample 6: 1028.62, −10.0, −27.4, 6.3, 0.72, 0.71, 0.43, 0.49, 0.55
Sample 7: 1146.86, 23.0, −12.7, 0.45, 0.44, 0.56, 0.59, 0.53
Sample 8: 1123.5.7, 18.9, −119.0, 2.8, 0.62, 0.36, 0.39, 0.56, 0.079
Sample 9: 1047.71, 51.0, 0.24, 0.56, 0.60, 0.22, 0.67
Sample 10: 1108.−16.4, 68.0, 0.56, 0.66, 0.21, 0.7, 0.44

**Table 6:** Values of variables used to disturb the $y$ - locations (cm) of the NiCrAlY and the ZrO₂ particles.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$u_b$ (cm/s)</th>
<th>$u_{a1}$ (cm/s)</th>
<th>$u_{a2}$ (cm/s)</th>
<th>$u_{a3}$ (cm/s)</th>
<th>$u_{a4}$ (cm/s)</th>
<th>$u_{a5}$ (cm/s)</th>
<th>$\omega_{1P}$ (rad/ ms)</th>
<th>$\omega_{2P}$ (rad/ ms)</th>
<th>$\omega_{3P}$ (rad/ ms)</th>
<th>$\omega_{4P}$ (rad/ ms)</th>
<th>$\omega_{5P}$ (rad/ ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean, $\mu$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>S.D., $\sigma$</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Single injection system using the NiCrAlY powder**

Sample 1: 0.79, 0.031, 0.073, 0.022, −0.058, −0.011, 0.66, 0.39, 0.37, 0.53, 0.31
Sample 2: 0.82, 0.009, 0.003, 0.004, 0.013, 0, 0.42, 0, 0.84, 0.31, 0.65
Sample 3: 0.76, −0.056, −0.031, 0, 0.001, 0.46, 0.76, 0.51, 0.55, 0.57
Sample 4: 0.84, 0.001, 0.014, −0.082, −0.019, −0.041, 0.74, 0.45, 0.62, 0.44, 0.58
Sample 5: 0.85, −0.049, −0.032, 0.042, −0.014, 0.032, 0.5, 0.44, 0.48, 0.45
Sample 6: 0.81, 0.044, −0.008, 0, 0.031, −0.025, 0.33, 0.37, 0.5, 0.34, 0.51
Sample 7: 0.8, 0.002, 0.017, −0.014, −0.033, −0.035, 0.53, 0.31, 0.58, 0.63, 0.47
Sample 8: 0.76, 0.026, −0.001, 0.036, −0.006, 0.015, 0.61, 0.55, 0.56, 0.58, 0.38
Sample 9: 0.79, 0.009, −0.021, −0.018, 0.042, 0.071, 0.57, 0.61, 0.42, 0.43, 0.4
Sample 10: 0.83, −0.022, 0.035, 0.012, 0.025, 0.022, 0.27, 0.64, 0.22, 0.71, 0.73

**Single injection system using the ZrO₂ powder**

Sample 1: 0.74, 0.045, −0.024, 0.011, 0.032, −0.006, 0.71, 0.53, 0.57, 0.67, 0.47
Sample 2: 0.78, −0.006, 0.011, −0.032, −0.029, −0.050, 0.66, 0.61, 0.65, 0.41, 0.80
Sample 3: 0.83, 0.007, −0.049, −0.017, −0.026, 0.031, 0.55, 0.37, 0.72, 0.44, 0.61
Sample 4: 0.82, −0.030, 0.042, −0.001, −0.002, 0.048, 0.58, 0.21, 0.62, 0.71, 0.52
Sample 5: 0.79, −0.010, 0.041, −0.061, 0.019, 0.011, 0.41, 0.55, 0.36, 0.26, 0.45
Sample 6: 0.77, 0.011, 0.022, 0.033, −0.010, −0.013, 0.34, 0.39, 0.43, 0.53, 0.66
Sample 7: 0.78, −0.068, −0.032, 0.007, 0.060, 0.000, 0.22, 0.49, 0.40, 0.59, 0.30
Sample 8: 0.81, 0.035, 0.004, 0.058, 0.016, −0.029, 0.48, 0.77, 0.50, 0.37, 0.54
Sample 9: 0.85, −0.023, −0.008, −0.015, −0.061, −0.022, 0.43, 0.43, 0.28, 0.50, 0.37
Sample 10: 0.81, 0.020, −0.003, 0.025, 0.003, 0.024, 0.53, 0.64, 0.53, 0.57, 0.40
Figure 7: Variations of the injection velocities of the NiCrAlY and the ZrO\textsubscript{2} particles and the \textit{y} – locations of NiCrAlY and ZrO\textsubscript{2} injectors for samples 1, 2, and 3 are listed in Tables 5 and 6.

Figure 8: Variations of averaged \textit{z}-locations of injectors for (a) NiCrAlY and (b) ZrO\textsubscript{2} particles for samples 1, 2, and 3 for which results are shown in Figure 7.
Figure 9: Responses of the models in equation (3) are compared with the smoothened data of (a) the averaged $z$-locations of NiCrAlY particles (fit = 79% (81%) for training (validation)) and (b) the averaged $z$-locations of ZrO$_2$ particles (fit = 74% (83%) for training (validation)).

Table 7: Estimated parameters in equation (3) of the averaged $z$-locations of the NiCrAlY and the ZrO$_2$ particles using inputs listed in Tables 5 and 6.
We investigate the performance of the proposed MR-MRAC control scheme for two example problems. Each problem involves a 5-layered NiCrAlY–ZrO₂ FGC as shown in Figure 11 to get the desired uniformity in the bi-particle distribution and the desired consistency in the MPSs. In the first layer (bond coat), only NiCrAlY powder is injected, whereas the fifth layer (top coat) is injected with only ZrO₂, and the remaining three layers with a combination of both with increasing mass fractions of ZrO₂ particles. Each layer is sprayed for 100 ms. The number of layers in the FGC influences the magnitude of the residual stress between the bond coat (NiCrAlY) and the top coat (ZrO₂). However, by adding a layer with graded composition in between the bond coat and the top coat, the residual stresses dropped significantly (e.g., by 50% [19]). The magnitude of the residual stresses will gradually decrease by adding more layers. In this analysis, we considered only 3 layers between the bond coat and the top coat to get a 5-layered NiCrAlY–ZrO₂ FGC, which is similar to the experimentally sprayed FGC by Khor et al. [20], who employed a single injector and pre-mixed mixtures of NiCoCrAlY and ZrO₂ with the volume fraction of NiCoCrAlY equaling 100, 75, 50, 25, and 0% in the five layers. We could not compare our results with those of Khore et al. because of differences in the number of powder ports used and their not providing MPSs of powder particles when they passed through an observation window. However, they included values of the elastic moduli and the coefficients of thermal expansion for each layer that we have not computed.

The objective of the process control scheme is to attain the desired profiles of the averaged z-locations of both material particles and the MPSs within a settling time of 50 ms despite disturbances in the arc voltage. The desired values in the reference models for the averaged z-locations of NiCrAlY and ZrO₂ particles are arbitrarily chosen from the range of simulated values obtained from the numerical studies reported in this paper. For example, the desired values for the single-particle injection in the 1st layer (NiCrAlY only) and in the 5th layer (ZrO₂ only) are chosen in the range of outputs calculated from the numerical simulations carried out to develop the response function in (20) and (21). For the simultaneous injection of both particles, the values considered are within the range of values listed in Table 1.

In the first example problem, the control algorithm forces the averaged z-locations of the NiCrAlY and the ZrO₂ particles and the MPSs to adaptively track the corresponding outputs of the reference models as shown in Figure 12. Other process parameters, such as the z-location of the injector, the arc voltage, and the particle size distribution, are kept constant. The control responses depicted in Figure 12 confirm that the desired averaged z-locations (1.0 cm below the jet axis) of the NiCrAlY and the ZrO₂ particles in the observation window are achieved. The computed MPSs successfully track the desired MPSs.

In the second example problem, the arc voltage is arbitrarily varied for each layer, and the corresponding control responses of the adaptive control scheme are presented in Figure 13. The change in the arc voltage immediately alters the MPSs. However, the deviations between the measured and desired values of the averaged z-locations of both materials and the MPSs are successfully minimized within the settling time of 50 ms.

3.7. Remarks. Using the proposed robust adaptive control system and injecting each material powder from its own port located in different \( x \)-constant planes may further improve the uniformity in the particle distributions. This
will maintain the necessary axial symmetry with respect to the jet axis. Thermal stresses between the top and the bond coat in traditionally sprayed two-layered TBCs can be reduced by using FGCs. Furthermore, using the developed robust adaptive control system a consistent splat formation of particles upon impact on the substrate can be achieved by maintaining the mean axial velocity at the desired value. The methodology for developing robust adaptive process control for generating FGCs is applicable to other coating methods listed in ref. [21], such as the HVOF spray process, physical vapour deposition, and chemical vapour deposition.

The real-time performance of the proposed robust adaptive control scheme depends on how fast the inputs can be varied, on continuous port movements, on how fast and efficiently the particle distribution and the MPSs can be measured, and on the time lag of the plant to respond to controller-provided inputs.
Figure 12: For the desired graded composition of the coating exhibited in row 1 in column 1 under the disturbance shown in row 1 in column 2, the adaptive tracking performance of the measured outputs to the desired responses of the reference model (in red) is depicted from row 2 to row 5 in column 1 using the adaptive process controller by adjusting the control inputs shown from row 2 to row 6 in column 2.
4. Conclusions

We have employed the model reference adaptive control (MRAC) framework and incorporated in it the $\sigma$-modification and the low frequency learning to propose three modified robust adaptive controllers (MR-MRAC) for consistently producing high quality functionally graded coatings (FGCs) using an atmospheric plasma spray process (APAS). Two controllers adjust the powder port locations and the injection velocities of the powder.
particles to achieve the same mean distributions of the two powder particles, namely, the NiCrAlY and the ZrO₂. The third controller adaptively adjusts the current, the argon flow rate, and the hydrogen flow rate into the gas gun to provide the desired values of the mean velocities and temperatures (collectively called mean particle states, MPSSs) of the two sets of particles.

The physical experiments have been replaced by simulations using the software LAVA-P, whose predictions of the MPSSs for the NiCrAlY and the ZrO₂ particles have been shown to agree well with their experimental findings. The screening analysis identified that the following seven out of eleven process parameters significantly affect the FGC quality: current, Ar flow rate, H₂ flow rate, average injection velocities of the NiCrAlY and the ZrO₂, and locations of their injectors along the plasma jet axis.

For two example problems in which the arc voltage is disturbed, the performance of the MR-MRACs has been established in providing the desired uniformity in the distributions of the two powder particles and their desired MPSSs.

We anticipate that the proposed controller will perform equally well in practical applications and economically enable the production of high-quality FGCs.

Data Availability

All data are included in the plots included in the manuscript. Digital files of the data can be obtained from Dr. Balachandar Guduri (gbalu@vt.edu).

Disclosure

This work is a part of Chapter 4 of the first author’s (Balachandar Guduri) Ph.D. dissertation, which was submitted to Virginia Polytechnic Institute and State University in 2021.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References