Microgrid as a Cyber-Physical System:
Dynamics and Control

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Dissertation submitted to the Faculty of
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Electrical Engineering

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March 27th, 2023
Blacksburg, Virginia

Keywords: Microgrid, Resilience, Dynamics, Stability, Cyber-Physical System, Hybrid-DER
Microgrid, Droop Control, Secondary Control, Feedback Control.
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Microgrid as a Cyber-Physical System: Dynamics and Control

Lung-An Lee

(ACADEMIC ABSTRACT)

As a result of climate change, extreme events occur more frequently and at higher severity, causing catastrophic power outages with significant economic losses. Microgrids are deployed as a technology to enhance power system resilience. A microgrid may include one or more distributed energy resources (DERs), including synchronous generators, solar panels, wind turbines, and energy storage systems which are decentralized power sources primarily in a distribution system to enable system recovery from catastrophic events.

Microgrids can be operated in a utility-connected mode or an islanded mode in separation with the hosting transmission or distribution system. As major disasters occur, intentional islanding of a microgrid is a strategy to serve critical loads, within or outside the microgrids, until the utility service is restored. To operate microgrids, dispatch and control capabilities are required that would significantly improve the dynamic performance of the microgrid.

An islanded microgrid can be used to serve critical load as a resiliency source when a severe outage occurs. In an islanded mode, control of a microgrid relies on the communication system significantly. Hence, microgrids are cyber-physical systems and, therefore, the cyber system plays a crucial role in the performance of the cyber-power system. Improper parameters of the cyber system can result in instability of a microgrid system. Simplification of the networked control system model is needed to enhance the computational performance, making the analytical method practical for large-scale power systems.
To reduce the emission of carbon dioxide and alleviate the impact of climate change, the electric power industry has been integrating renewable energy into the power grid. The high penetration of renewable energy at an unprecedented level also raises new issues for the power grid, e.g., low inertia, degraded power quality, and higher uncertainties. Power electronics technology is used for power conversion of renewable energy. As the level of penetration of renewable energy increases, the inverter-based resources (IBRs) are being installed at a fast pace on the power grid. Compared to conventional synchronous generators (SGs), a major technical challenge of IBRs is their low inertia which can lead to system instability.

In this context, the work of this dissertation results in major contributions regarding control algorithms for microgrid resilience, stability, and cyber-physical systems. Specifically, three novel contributions are presented: 1) A coordinated control scheme is proposed to achieve the goals of power dispatch and system regulation for an islanded microgrid. The proposed control scheme improves system dynamics; 2) A method is developed for the determination of critical values for the data reporting period and communication delay. Based on the proposed method, a 2-dimensional stability region of a microgrid in the space of cyber parameters is derived and critical values of cyber parameters are identified based on the stability region; 3) A control scheme is proposed to improve system stability of a hybrid-DER microgrid. The analysis serves to illustrate the stability regions of the hybrid-DER microgrid. A control methodology based on two-time scale decomposition is developed to stabilize the system.
Microgrid as a Cyber-Physical System: Dynamics and Control

Lung-An Lee

(GENERAL AUDIENCE ABSTRACT)

Climate change is causing more frequent and severe weather events, resulting in catastrophic power outages and significant economic losses. To enhance power system resilience, microgrids are proposed as a solution. Microgrids consist of one or more distributed energy resources, such as solar panels, wind turbines, and energy storage systems, which can be operated in a utility-connected or islanded mode. Microgrids can operate in an islanded mode to serve critical loads when an extended outage of the utility grid occurs. Proper dispatch and control capabilities are necessary for the operation and control. However, the performance of a microgrid, especially in an islanded mode, is dependent on the communication system. Excessive cyber latencies can result in system instability of the microgrid.

To reduce carbon dioxide emissions, the power industry is integrating an unprecedented level of renewable energy into the power grid. Power electronics technology is being used for power conversion of renewable energy, and inverter-based resources are being installed at a fast pace into the power grid. One major technical challenge of inverter-based resources is their low inertia, which can lead to system instability.

To address these issues, this dissertation presents three novel contributions: a coordinated control scheme to improve the microgrid dynamics and perform power dispatch and system regulation functions, a method to determine critical values of cyber parameters based on stability regions, and a control scheme to improve system stability of a hybrid-DER microgrid. These contributions provide valuable concepts and methodologies for resilient and stable microgrids that
are critical to meet the operational and control challenges of an electricity infrastructure with a high-level penetration of renewable energy.
Dedication

To my darling wife Claire,

who has always been the most resilient microgrid in my journey,
supplying me with love and support and keeping the lights on even during the darkest of outages.

To my beloved daughter Hannah,

may you continue to grow and thrive in a world full of dynamic challenges,

and may you always have the wisdom and skills to maintain stability in your life.
Acknowledgments

I would like to express my sincere gratitude to my academic advisor, Prof. Chen-Ching Liu, for his guidance, support, and encouragement throughout my graduate studies. His wealth of knowledge and expertise in the field of electric power systems has been invaluable to the completion of this work. Additionally, his patience, wisdom, and constant feedback have been instrumental in helping me navigate the challenges that came with the research. I am truly grateful for his mentorship, which has been instrumental in my personal and professional growth. This work could not have been completed without his invaluable contributions.

I would also like to thank the members of my dissertation committee: Dr. Jaime De La Ree, who provided me with insightful concepts on dynamics, Dr. Vassilis Kekatos, who provided critical comments on distribution systems, Dr. Craig Woolsey, who guided me in control work through his outstanding lectures on Linear Control Theory, and Dr. Kevin Schneider, who gave me the opportunity to conduct microgrid projects for this research. Also, Dr. Virgilio Centeno, who served as a committee member for my qualifying examination. Their expertise and feedback helped me to develop my ideas and refine my work in meaningful ways.

In addition, I would like to thank the project members who helped me sharpen the work, including Dr. Yin Xu and Kefei Mo from Washington State University, Dr. Dushan Boroyevich, Dr. Igor Cvetkovic, Dr. Richard Zhang, and Haris Bin Ashraf from Center for Power Electronics Systems (CPES), Akshay Jain from Power and Energy Center (PEC), Francis Tuffner from Pacific Northwest National Laboratory (PNNL), Dan Ton from Department of Energy (DOE), and our visiting scholars Dr. Hua Ye and Jingyu Wang.
The fellowship and support of my lab partners has been a source of great fulfillment for me. I would like to thank to our group members, Dr. Jing Xie, Dr. Chih-Che (Ryan) Sun, Dr. Juan Carlos Bedoya, Dr. Jennifer Appiah-Kubi, Ruoxi Zhu, Chensen Qi, Nitasha Sahani, Baza Somda Rodriguez, Fahad Alsaeidi, Yousef Akbar, Genesis Alvarez, and Pratigya Shrestha. My appreciation also goes to my Power and Energy Center friends, Xiawen Li, Elliott Colgan, Ardavan Mohammadhassani, Manish Singh, Sangeetha Rajasekeran, Shuchismita Biswas, Sagar Karki, Tapas Barik, Mana Jalali, Sanij Gyawali, Anaga Krishnan, Sherin Ann Abraham, Rounak Meyur, Ikechukwu Dimobi, Nick Skoff, Sina Taheri, and the PEC coordinators Victoria Deal and Lisa Burns.

I would like to express my heartfelt gratitude to my family for their unwavering love, support, and encouragement throughout my academic journey. Their endless sacrifice, understanding, and patience have been instrumental in my success. Their belief in my abilities and strong support have kept me motivated and focused on achieving my goals. I could not have accomplished this without their love.

Finally, I would like to acknowledge the financial support U.S. Department of Energy Office of Electricity through the Pacific Northwest National Laboratory and the Power and Energy Center of Virginia Tech.
Publications from This Dissertation

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Chapter 1

Introduction

The U.S. Department of Energy (DOE) defines the microgrid as “a group of interconnected loads and distributed energy resources within clearly defined electrical boundaries that acts as a single controllable entity with respect to the grid. A microgrid can connect and disconnect from the grid to enable it to operate in both grid-connected and island modes [1].”

As a result of climate change, extreme events occur more frequently and at higher severity, causing catastrophic power outages with significant economic losses. Microgrids are deployed as a technology to enhance power system resilience [2], [3].

DOE and national laboratories have been developing and integrating microgrids to demonstrate the capabilities of microgrids to enhance resilience of the power grid [4], [5]. It is noted that microgrids are integrated with the bulk power grid mostly at the distribution system level as it is where catastrophic outages impact the load.

Microgrids have been developed over decades. The first conceptual microgrid was introduced in 1882 when Thomas Edison constructed his first power plant. It had the features of a microgrid since centralized power generation was not yet developed. Modern microgrids started in the late 1990s when DOE began to study the system’s reliability and resiliency [6]. The concept of microgrids as a resilience source became widely accepted after 2012 when Superstorm Sandy struck the east coast [7]. Extreme events can cause damages to power grids, leading to catastrophic outages on both transmission and distribution systems [8], [9].
As major disasters occur, intentional islanding of a microgrid is an action to sustain service to the critical loads within the microgrid (until the utility service is restored). A microgrid may include one or more distributed energy resources (DERs), including synchronous generators, solar panels, wind turbines, and energy storage systems which are decentralized power sources in a distribution system to enable system recovery from catastrophic events. To continue serving critical loads, microgrids are utilized as resiliency sources in a distribution system when the utility system becomes unavailable [10]–[12]. A resiliency source is provided by distributed energy resources in the local outage area, e.g., a microgrid. Since the capacity of the microgrid is limited, the resiliency source is intended for critical load inside and/or outside the microgrid. A metric of resiliency is proposed, where resiliency is measured by the total electric energy provided to the critical load during the service restoration period [12], [13]. As a result, resiliency can be improved by increasing the amount of available generation resources, flexible distribution grid configuration, and efficiency of system restoration strategy and procedures.

1.1 Microgrid Dynamics and Control

Islanded microgrids sustain service to the microgrid load when the bulk power system is not available. To operate an islanded microgrid, various control strategies and operational scenarios in microgrids have been proposed [14]–[16]. A hierarchical structure is commonly applied for these control schemes [17], [18]. Distribution system restoration algorithms are developed to pick up the maximum critical load with the minimum switching operations [19], [20]. Microgrid sectionalization problems are discussed [21], [22]. Coordination of multiple microgrids with a power sharing scheme of converter-interfaced DERs among multiple microgrids are proposed [23], [24]. With multiple generators at the end of the feeders, the distribution system has a meshed
topology and a bottom-up restoration strategy [25]. The feasibility of such a system needs to be considered, such as resetting protection devices, coordinating the sources and demands, as well as controlling and stabilizing the system. The dynamic performance of microgrids is also an important consideration, which can be categorized into small-signal stability, transient stability, and voltage stability [26]. Small-signal stability has been studied comprehensively, where the focus of the study is on the stability around an equilibrium point based on a linearized model [27]–[29]. On the other hand, transient stability and voltage stability are significant issues, especially for islanded microgrids. Without the utility (swing bus), a large disturbance of the operating condition can lead to instability. The control scheme combining droop control and PI feedback control is validated with good performance in regulation, power dispatch, and transient stability [29]–[31].

1.1.1. Contributions to Microgrid Dynamics and Control

The specific contributions to microgrid dynamics and control approach for stabilizing an islanded microgrid and enhancing system resilience are as follows.

1) A coordinated control scheme is proposed to improve the dynamic performance of the microgrid. As DERs are operated in parallel, the proposed control is designed to regulate the frequency and voltage of the microgrid when DERs have distinct droop characteristics.

2) The proposed control is implemented and tested with a modified IEEE 13-node test system. The restoration process and two transient events are used to validate the control scheme. The control goals (regulation and power dispatch) are met, and transient stability is maintained.
1.2 Cyber-Physical Systems

Under a hierarchical structure, local controllers are used to regulate DERs as the primary control, while systemwide centralized or decentralized microgrid controllers are responsible for coordination as the secondary control. The control strategies and operational scenarios described in section 1.1 indicate that control algorithms are needed to coordinate multiple DERs in a microgrid. To control dispersed DERs, data collection via a communication system is essential. Therefore, microgrids become cyber-physical systems, and the control system and communication system are functionally integrated as a networked control system.

The impact of the cyber system on a physical system is significant. The damage on the physical system caused by failures in the cyber system is discussed [32]. Small-signal stability in a microgrid considering communication delays is analyzed [33], [34]. System stability considering a time delay to model the communication latency has been analyzed and a systematic method to determine the critical cyber parameters is proposed [35]. Root locus analysis shows a long delay time can drive eigenvalues to the right half-plane, causing system instability. A method for determination of the time-delay stability margin for the power system is proposed [36]. Status of cyber components is decided by Sequential Monte Carlo Simulation (SMCS) and their failure/repair rates [37]. Mathematical cyber-physical models are proposed to analyze the impact of cyber-contingency and transient stability in microgrids [38], [39]. PI controllers are used for frequency control considering the communication delays. Stability regions can be obtained to provide stable values of the PI parameters [40], [41]. A distributed secondary voltage control is proposed that incorporates communication delays via a sparse communication network [42]. The analysis shows that system stability is dependent on time delays. A communication delay is included in the small-signal model to study system stability by the eigenvalue analysis [43].
1.2.1. Contributions to Stability of a Cyber-Physical Microgrid

The specific contributions to stability of a cyber-physical microgrid through the determining of critical values of cyber parameters are as follows.

1) An analytical method is proposed to determine the critical values of cyber parameters. The simplified method reduces the computational burden significantly, making it applicable for large-scale power systems.

2) A 2-dimensional stability region is obtained based on the analytical method. The stability region is critical for the design of a communication system supporting a networked control system.

3) The proposed analytical method is implemented and tested with a modified IEEE 13-node test system. A networked control system is used for regulation and power dispatch of the test system. Time-domain simulations are used to validate the analytical method.

1.3 High Penetration of Renewable Energy in Microgrids

The power industry has taken steps to reduce the emission of carbon dioxide to help alleviate the impact of climate change. The reduction primarily results from the integration of renewable energy into the power grid at an unprecedented level. In “RENEWABLES 2022 GLOBAL STATUS REPORT,” the Annual Additions of Renewable Power Capacity indicates that the installation of renewable energy increases by more than 314.5 GW globally in 2021. To achieve International Energy Agency (IEA) Net Zero Scenario, additional 300 GW of renewable energy needs to be installed by 2030, and 825 GW is needed by 2050 for the World Energy Transitions Outlook scenarios from the International Renewable Energy Agency (IRENA) [44].
The high penetration of renewable energy also raises new issues, e.g., low inertia, degraded power quality, and higher uncertainties [45]. The low or no inertia of renewable energy degrades system stability and, furthermore, the intermittency introduces fast dynamics and the inertia difference between DERs can lead to instability as well.

When the bulk grid becomes unavailable, system stability becomes an important control objective for an islanded microgrid. Microgrid stability has been defined and classified, where the electric machine stability and inverter/converter stability are studied individually [26], [46]. However, as the penetration of distributed energy resources, including renewable energy, into the bulk power grid increases, microgrids consist of existing synchronous generators and newly installed renewable energy resources (e.g., solar and/or wind systems). There is a critical need to develop analytical and computational methods for system stability of the hybrid-DER microgrid. While the dynamics of a microgrid in a utility connected mode are dominated by the hosting bulk grid, a microgrid with significant DERs in an islanded mode is by itself a complex nonlinear system that requires new methodologies for stability and control.

1.3.1. Contributions to Stability of a Hybrid-DER Microgrid

The specific contributions to stability of a hybrid-DER microgrid for a systematic stability analysis and implementation of the control loop are as follows.

1) A two-bus system is used to illustrate the stability issue of a hybrid-DER microgrid. A 2-dimensional stability region is developed with respect to DERs’ time scale and level of penetration.

2) A reduced-order algorithm for controller design is proposed to stabilize a hybrid-DER microgrid and relieve the computational burden for a complex microgrid system.
3) Practically, the control loop is implemented among dispersed DERs. Technical issues for implementation of the control loop are considered in the proposed control schemes.

4) The proposed control schemes are applied to Virginia Tech Electric Service (VTES) System. Analysis and time domain simulation are performed to validate the control schemes.

1.4 Organization of the Dissertation

The remainder of this dissertation is organized as follows. Chapter 2 illustrates the concept of system resilience. A metric of system resilience is presented, and resilience enhancement by microgrids using the metric is discussed. Chapter 3 presents dynamics and control of microgrids. A coordinated control scheme is proposed to improve the dynamic performance. To validate the control scheme, the system restoration process and two transient events are simulated on IEEE 13-node system. Chapter 4 is concerned with the stability of a cyber-physical microgrid. An analytical method is proposed to determine the critical values of cyber parameters that is feasible for a large-scale power system. The method is utilized to decide the cyber parameters of IEEE 13-node system, and the result is validated by time-domain simulation. Chapter 5 discusses the stability of a hybrid-DER microgrid. Stability analysis for a two-bus system is performed, and control schemes are proposed to stabilize the hybrid-DER microgrid. The VTES system is used for analysis and time domain simulation. Chapter 6 concludes this dissertation and provides the future works for this research.
Chapter 2

Resilience Enhancement of Power Grid

The traditional index for power system performance is “reliability” which, by DOE, refers to “the ability of the system or its components to withstand instability, uncontrolled events, cascading failures, or unanticipated loss of system components [47].” Reliability is the capability of a power grid to serve the load, i.e., adequacy of the available generation resources to serve the load and to withstand component failures and/or system problems, which is concerned with power system security.

Resilience is different from reliability in that the former refers to the ability of a power system to withstand extreme events and recover from catastrophic outages, while the latter is a cumulative measure of the frequency and duration of power outages [48]. Resilience quantifies the ability to recover a system from a severe outage condition to a functional and operational state [49].

DOE’s definition of resilience is the “ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents [50].” Compared to reliability, resilience is a new concept for power systems. As extreme events occur more frequently, resilience emphasizes the ability of a power system to recover from catastrophic outages.
2.1 Metric of System Resilience

In [12], pre-event steady state ($\Delta T_1$), event progress ($\Delta T_2$), post-event damage assessment state ($\Delta T_3$), restorative state ($\Delta T_4$), post-restoration state ($\Delta T_5$), infrastructure recovery ($\Delta T_6$), and post-event steady state ($\Delta T_7$) are defined as the system states during a catastrophic outage.

Figure 2-1 illustrates the concept of resilience using the system performance curve. The curve starts from a steady state at $t_0$, and then the system performance (MW served by the system) decreases when an extreme event progresses at $t_e$. The post-event damage assessment state is defined as the system performance drops to its lowest value at $t_{pe}$. Then, a microgrid is used to restore the system by serving critical load and the system performance starts to increase at $t_r$. When the critical loads are fully picked-up, the system performance is constant at $t_{pr}$. After the extreme event ends, the recovery of the bulk grid allows non-critical loads to be restored at $t_{ir}$ and the entire system will reach a post-event steady state at $t_{pir}$. Resilience can be quantified by the system performance in the restorative state and post-restoration state, $R_{en}$, where the microgrid provides MWh without the bulk grid.

![Resilience Curve](image_url)

Figure 2-1. Conceptual Resilience Curve
2.2 Resilience Enhancement by Microgrids

As discussed, islanded microgrids can be utilized to serve critical load when the bulk grid becomes unavailable due to extreme events. Also, resilience refers to a power system’s capability to withstand severe events and recover from disruptions. A field test with distributed generators of a campus power system to sustain critical load was performed at Washington State University (WSU). Two DERs were used to form a microgrid at WSU.

The purpose of this field test was to demonstrate that a microgrid can be used as a resilience source by serving critical loads during an outage. Figure 2-2 illustrates a simplified one-line diagram at WSU. Under an islanded mode, the diesel generator was utilized to pick up critical loads on campus. Then, the transformer was energized, and the natural gas generator was connected in parallel to send cranking power to the transmission system through the utility distribution system. The cranking power is used to support starting of non-black-start generating units to accelerate the restoration process. Based on the resilience metric, the simulation results indicated that, for the operating condition of the field test, system resilience is enhanced by an amount of 2.3 MWh by the available microgrid.

![Figure 2-2. One-Line Diagram of the Test System at WSU](image-url)
Chapter 3\(^{(1)}\)

**Dynamics and Control of Microgrids**

The goal of this research is to investigate the dynamic issues arising from restoration and control. Dynamics and control are important issues for the resiliency mode of a microgrid system. During the parallel operation process, uncoordinated terminal voltages or frequencies can cause circulating power among DERs and trip proactive devices. During service restoration of a distribution system under a light load or outage condition, closing an underground cable can cause high voltages. As the critical load is picked up, the frequency and voltage variations of the distribution system should not exceed the acceptable level. Furthermore, frequency and voltage control systems are proposed to enhance the (large-signal) stability of an islanded microgrid. Considering multiple DERs in an islanded microgrid, a control scheme is developed by integrating droop and feedback controls in order to achieve proper power dispatch and stabilize the system. The proposed control method is generic and applicable to microgrids that operate in a resiliency mode.

3.1. Coordinated Microgrid Control Scheme

In this section, an effective method is developed for active and reactive power dispatch. For resiliency consideration, the method is intended for an islanded microgrid without support of the utility system. Considering the maximum restoration capacity in an outage, DERs should output

---

power by the ratio of their rated capacities to avoid overcurrent. Furthermore, the point of common coupling (PCC) between the utility and microgrid must be controlled in order to meet the requirements for reconnection. The capability of regulation is needed so that the system is stable and steered back to the reference values \[51\]. In the research, a droop-control-based mechanism is adopted for power dispatch among DERs with uncoordinated droop coefficients, and a feedback controller is designed to regulate microgrid frequency and the PCC bus voltage to the desired values.

3.1.1. Power Dispatch Mechanism

The droop characteristic of a DER is given by the following equations and illustrated in Figure 3-1. Power dispatch among DERs in proportional to their capacity can be achieved by setting the droop coefficients inversely proportional to their capacity.

However, existing DERs in the distribution systems, such as backup generators, are usually installed over a long-time span and their droop characteristics can be very different. In addition, for most mechanical governors, the droop coefficient is not adjustable. In this study, existing DERs are used as a resiliency source to serve critical load. Therefore, a mechanism for power dispatch among DERs with uncoordinated droop coefficients is needed.

Another way to adjust the amount of active or reactive power provided by DERs is to change the no-load references according to the desired power output values and some certain operation frequency and voltage, i.e.,

\[
\begin{align*}
  f_0^{ref} &= f_{op} + m_p \times P_{DER}^{des} \\
  U_0^{ref} &= U_{op} + m_q \times Q_{DER}^{des}
\end{align*}
\]

(3-1)
where $f_0^{\text{ref}}$ and $U_0^{\text{ref}}$ are no-load frequency and voltage references. $f_{\text{op}}$ and $U_{\text{op}}$ are local operational frequency and voltage references. $m_p$ and $m_q$ are f-P and U-Q droop characteristics.

$P_{\text{DER}}^{\text{des}}$ and $Q_{\text{DER}}^{\text{des}}$ are desired active and reactive power dispatches.

**Figure 3-1. The Droop Characteristics: $f$-$P$ and $U$-$Q$ Curves**

The desired active and reactive power outputs of the DER, $P_{\text{DER}}^{\text{des}}$ and $Q_{\text{DER}}^{\text{des}}$, should vary in real time, following the loading condition. In this research, a mechanism to adjust $P_{\text{DER}}^{\text{des}}$ and $Q_{\text{DER}}^{\text{des}}$ in real time is proposed. The real-time outputs of DERs are sensed and added to calculate the total power generation, including the load demand and line losses. Then the desired output of each DER is determined by assigning a portion of the total power generation to the DER in proportional to its capacity.

Once the desired power of DERs is calculated, the no-load references $f_0^{\text{ref}}$ and $U_0^{\text{ref}}$ will be determined by (3-1). Then each DER will move its droop curve up or down accordingly, as indicated by the red arrows in Figure 3-1. Note that by applying the proposed power dispatch mechanism, the following two objectives are met at the same time: 1) The output power of DERs is proportional to their capacity; and 2) the frequency and terminal voltage at the PCC bus can be regulated locally by setting the values of $f_{\text{op}}$ and $U_{\text{op}}$. 
3.1.2. Frequency and Voltage Regulation

In (3-1), the operation points of DER, $f_{op}$ and $U_{op}$, are set locally, which may not be coordinated and cannot achieve the regulation at PCC. For example, with different settings of $f_{op}$ at different DERs, the system will operate at a frequency equal to none of these settings. Furthermore, the voltage at the PCC bus is not directly controlled but determined indirectly by the terminal voltages of DERs.

To overcome these difficulties, a controller is designed to regulate the frequency and voltage at PCC. A desired frequency and a desired voltage values are set, and measurement signals acquired from PCC bus are compared with the reference values. The error is fed into proportional-integral-derivative (PID) controllers and compensated amounts are determined and added to local settings. Hence, (3-1) is rewritten with the compensated amounts as follows.

$$
\begin{align*}
    f_{0}^{\text{ref}} &= (f_{op} + \delta f_{pcc}) + m_{p} \times P_{\text{DER}}^{\text{des}} = f_{\text{REG}}^{\text{ref}} + f_{\text{DIS}}^{\text{ref}} \\
    U_{0}^{\text{ref}} &= (U_{op} + \delta U_{pcc}) + m_{Q} \times Q_{\text{DER}}^{\text{des}} = U_{\text{REG}}^{\text{ref}} + U_{\text{DIS}}^{\text{ref}}
\end{align*}
$$

(3-2)

where $\delta f_{pcc}$ and $\delta U_{pcc}$ are compensated frequency and voltage at PCC. $f_{\text{REG}}^{\text{ref}}$ and $U_{\text{REG}}^{\text{ref}}$ are adjusted operational frequency and voltage, which are the references for system regulation. $f_{\text{DIS}}^{\text{ref}}$ and $U_{\text{DIS}}^{\text{ref}}$ are defined as the references for power dispatch.

PID controller parameters are designed by the pole/zero placement [52]. With the given governor and excitation systems and their transfer functions, parameters can be determined, depending on selected locations of poles and zeros.
3.2. Implementation of the Control Scheme

Figure 3-2 is an illustration of the control loop. The frequency and voltage at PCC (regulated node) are measured. According to the collected data, the microgrid controller (MGC) computes the desired active and reactive power of DERs and the frequency and voltage deviations at PCC. Then, the power references and deviations are sent to the local controllers (LCs).

![Figure 3-2. Microgrid Control Loop](image)

In LCs, frequency and voltage references are computed by (3-2) and sent to the governor and excitation systems as shown in Figure 3-3. The local operating point, $f_{op}$ and $V_{op}$, will regulate PCC’s frequency and voltage to the desired values.

![Figure 3-3. Implementation of the Control Scheme](image)
3.3. Modified IEEE 13-Node Test System

The modified IEEE 13-node test system is shown in Figure 3-4. The breaker between the utility and node 650 is opened to disconnect the microgrid from the utility system. Three synchronous generators are added at the end of the primary feeder. The rated capacities of these generators are 2200 kVA (Ga), 3000 kVA (Gb), and 1500 kVA (Gc), respectively. The total generation capacity is 6700 kVA. Line 671-684 is modified from 2-phase to 3-phase, with an overhead line configuration 604 in order to connect with a generator (Ga). Two switches are added to the loads under nodes 671 and 675 for service restoration.

Two additional scenarios are developed for the transient stability test in Section 3.4. In the heavy load scenario, an additional load at bus 671 is picked up for evaluation of the transient response following the heavy load pick up. For the fault scenario, a short circuit fault is assumed to occur on the primary side of transformer XFM-1, and then it is cleared by a recloser upstream of Line 632-633.

Figure 3-4. Modified IEEE 13-Node Test System
3.4. Simulation Results – Power Dispatch and Regulation

The proposed control scheme is applied to the modified IEEE 13-node test system. Table 3-1 summarizes the load demands in the IEEE 13-node system. Load on single- or two-phase laterals, distributed load and load on the secondary side of a distribution transformer are fixed. On the other hand, load on the primary three phase feeder is considered switchable loads (load 671 and 675).

Initially, generators serve the fixed loads. Then the distribution system picks up load 671 at 200 seconds and load 675 at 400 seconds, respectively. The ratio of demand to generation is given. Each generator should deliver active power by ratio with their capacity from 22%, 39% to 52%; reactive power from 13%, 23%, to 21%. The frequency and voltage at bus 650 will be regulated to 1 p.u.

<table>
<thead>
<tr>
<th>Load Types</th>
<th>Amounts</th>
<th>Active Power</th>
<th>Reactive Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Load</td>
<td>sectional total</td>
<td>1468</td>
<td>880</td>
</tr>
<tr>
<td></td>
<td>system total</td>
<td>1468</td>
<td>880</td>
</tr>
<tr>
<td></td>
<td>ratio of generation</td>
<td>22%</td>
<td>13%</td>
</tr>
<tr>
<td>Switchable Load 1</td>
<td>sectional total</td>
<td>1155</td>
<td>660</td>
</tr>
<tr>
<td></td>
<td>system total</td>
<td>2623</td>
<td>1540</td>
</tr>
<tr>
<td></td>
<td>ratio of generation</td>
<td>39%</td>
<td>23%</td>
</tr>
<tr>
<td>Switchable Load 2</td>
<td>sectional total</td>
<td>843</td>
<td>-138</td>
</tr>
<tr>
<td></td>
<td>system total</td>
<td>3466</td>
<td>1402</td>
</tr>
<tr>
<td></td>
<td>ratio of generation</td>
<td>52%</td>
<td>21%</td>
</tr>
</tbody>
</table>

Figure 3-5 and Figure 3-6 illustrate the active and reactive outputs of generators during the restoration process. The results illustrate that DERs share power in proportional to their capacity. The active power dispatch is consistent with the expected ratio. The reactive power dispatch, however, slightly deviates from the expected ratio. The reactive power dispatch is swayed by the DER output impedances and unbalanced line impedances that cause unequal voltage drops [53].
Figure 3-6. Reactive Power Dispatch among DERs

Figure 3-7 provides the results of frequency and voltage regulation at bus 650. With the feedback control, they are brought back to the desired values, 1 p.u. The PCC frequency and voltage are controllable, so that the microgrid can be reconnected to the utility smoothly.
The proposed control method addresses two critical issues: 1) Dispatch of power outputs from the DERs, and 2) Regulation of the frequency and voltage of the microgrid in a resiliency mode. Without the regulation function, uncoordinated terminal voltages or frequencies can cause circulating power. Without the power dispatch mechanism, the output power of a single or multiple DER(s) can hit the upper operating limit and change to a constant power mode. Dynamic performance will suffer because only a portion of DERs with remaining capacities can respond to the load variations or transient events.

In the 13-node system, the total load picked up by the DERs represents 56% of DERs’ generation capacity. The upper limit of DERs’ output power is given as 1.1 p.u. Three additional 1000 kVA (0.9 lagging) loads, equivalent to 15% of DERs’ generation capacity, are picked up one by one. Figure 3-8 and Figure 3-9 provide the active power dispatch and the resulting frequency responses with and without the power dispatch mechanism. Without the power dispatch mechanism, Gb contributes too much and, as a result, it has no remaining active power capacity to
react to the frequency variations. Therefore, a longer response time is needed when a heavy load is picked up.

Figure 3-8. Power Dispatch among DERs under a Heavy Load Picking-Up Condition

Figure 3-9. Frequency Responses under a Heavy Load Picking-Up Condition
3.5. Simulation Results – Transient Stability Test

An islanded microgrid works as a resiliency source without support of the utility system. A disturbance can lead the system to a collapse. The proposed control is intended to enhance transient stability of the islanded microgrid.

In this section, the control schemes with and without the control loop during a disturbance are compared. Two disturbance events are tested, i.e., the heavy load pick up scenario and short circuit fault scenario.

3.5.1. Heavy Load Pick Up

The heavy load to be picked up is 3000 kVA (0.9 lagging), representing 45% of generation capacity. Figure 3-10 and Figure 3-11 illustrate the frequency and voltage responses with respect to control scheme with and without the control loop. Without control loop, the responses diverge and collapse eventually. The proposed control scheme is shown to maintain the system stability.

![Figure 3-10. Frequency Responses under 45% Heavy Load Picking-Up Scenario](image-url)
In a small distribution system with a microgrid, picking up a heavy load can be a (relatively) large disturbance to the small generators with limited capacity and low system inertia. Hence, the critical load to be picked up should be limited in size for each switching step. Also, the generators should have the control capability to meet the operating constraints on frequency and voltage. In this study, the control capabilities are provided by governors and exciters of the three distributed generators.

3.5.2. Short Circuit Fault

Under the condition that normal load is served, a short circuit is assumed to occur and then it is cleared by an upstream recloser. The settings of the recloser are shown in Figure 3-12.
Figure 3-12. Recloser Settings

Figure 3-13 and Figure 3-14 illustrate the frequency and voltage responses with respect to the control scheme with and without the feedback control system. Although both control schemes stabilize the system following the fault, the regulation function achieves a faster response and steers the frequency and voltage back to 1 p.u. at bus 650.

Figure 3-13. Frequency Responses under Short Circuit Fault Scenario
In the test case with a fault as shown in Figure 3-13, the frequency response with control loop is faster. However, the second peak of the response under control loop is higher. The control performance should be evaluated in a broader context involving not only speed but also the total cost from an optimal-control point of view.
Chapter 4

Critical Values of Cyber Parameters in a Dynamic Microgrid System

The performance of a cyber system, including data acquisition and exchange, is crucial to an islanded microgrid. The impact of communication delays in data exchange has been analyzed using small-signal analysis based on a state-space model. However, in practice, the reporting period for data acquisition is pre-determined by design, e.g., every 2-10 seconds for data acquisition from the substations to the control center of a power grid. This data “reporting period” of the Supervisory Control And Data Acquisition (SCADA) system is usually much longer than the communication delays that are in the order of milliseconds. The SCADA system is used for transmission and distribution grid control. Standard specifies the requirements of the communication system for SCADA system [54]. For a microgrid, however, the system has a low inertia when it is operated in an islanded mode without support from the utility system. The system can have faster dynamics relative to traditional distribution systems. Therefore, a shorter cyber latency is needed to stabilize the system.

In the existing work, communication delay is the major consideration for system stability. However, the “reporting period” is typical in measurement devices and can be critical for microgrid stability as well. Furthermore, these cyber latencies are dependent. The critical value of communication delay for system stability changes under different reporting periods. The two cyber

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latencies need to be studied concurrently for microgrid stability. The purpose of this research is to
develop an analytical method to determine the critical cyber data reporting and communication
delay to ensure system stability of an islanded microgrid. A control scheme is proposed to regulate
frequency and voltage at the Point of Common Coupling (PCC) node and dispatch power among
DERs. A cyber model is integrated into the control system to evaluate the impact of the reporting
period and communication delay.

This research deals with the use of synchronous generators to serve critical loads in an islanded
microgrid when the utility system is unavailable. The control strategy can fail when the data
reporting period and/or communication delay are long, leading to system instability. In this
research, an analytical method for networked control system is proposed for determination of the
critical values for data reporting period and communication delay in a microgrid [55]. Small-signal
analysis based on the discrete state-space model is conducted and stability regions are identified
considering cyber latencies. A transformation of the transition matrix for networked control system
is derived for reduction of the computational burden for calculation of the stability region. As a
result, the proposed stability region method is applicable for large-scale power systems. The
system stability is assessed based on the networked control method, which is validated by time-
domain simulation using the nonlinear dynamic model.

### 4.1. State-Space Model of Networked Control System

As a cyber-physical system, the system stability of a microgrid is impacted by the cyber
latency. To determine the critical threshold for cyber latency, an analytical method is proposed in
this research.
4.1.1. Networked Control System

A method to analyze the Networked Control System (NCS), considering the data acquisition reporting period \((h)\) and communication delay \((\tau)\) is developed in [55]. The state-space model with a networked control system is presented in Figure 4-1. A continuous state-space model is integrated with a discrete feedback controller. In a continuous state-space model, the control inputs can be fed by plant outputs simultaneously, which might not be true in practice. In a networked control system, the plant outputs are discrete in that the sensor reports the measurements periodically at a specified time interval. Communication delay occurs while the plant and controller are exchanging data across the communication system. Note that the communication delay, \(\tau\), is a lumped value by measurement delay (plant to controller) and command delay (controller to plant). The performance of the control system can be degraded by the cyber latencies.

![Figure 4-1. State-Space Model for a Networked Control System](image)

A continuous plant and a discrete controller are written as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &=Cx(t) \\
\end{align*}
\]

\[
\begin{align*}
u(t^+) &= -Kx(t - \tau) \\
t & \in \{kh + \tau, k = 0, 1, 2, \ldots\}
\end{align*}
\]  

(4-1)
where $A$ is state matrix, $B$ is input matrix, $C$ is output matrix and $K$ is feedback matrix from state variables to input variables. The symbol $h$ is the reporting period, $k$ represents the $k_{th}$ data reporting, and $\tau$ is the communication delay.

The system can be converted to an augmented closed-loop system with a transition matrix ($\Phi$) as (4-2). The transition matrix is used to analyze the cyber-power system stability.

\[
\begin{align*}
\dot{z}((k + 1)h) &= \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & 0 \end{bmatrix} z(kh) \\
&= \begin{bmatrix} \Phi - \Gamma_0(\tau) & \Gamma_1(\tau) \\ -K & 0 \end{bmatrix} z(kh) \\
&= \Phi(k)z(kh)
\end{align*}
\tag{4-2}
\]

where

\[
\Phi = e^{Ah}
\]

\[
\Gamma_0(\tau) = \int_0^{h-\tau} e^{As}Bds
\]

\[
\Gamma_1(\tau) = \int_{h-\tau}^{h} e^{As}Bds
\]

Stability of a discrete time linear time-invariant system is determined by the magnitude of eigenvalues of the transition matrix. The system is (asymptotically) stable if and only if all eigenvalues are located strictly inside the unit circle, i.e., the magnitude is less than 1.

4.1.2. Equivalent Matrix for Eigenvalues

To analyze system stability, the eigenvalues of the transition matrix in (4-2) need to be calculated. However, the computational burden is excessive, making the computation impractical for large-scale power systems. Here, a transformation of the matrix is proposed that preserves non-
zero eigenvalues of the transition matrix. Based on derivation in Appendix A, the equivalent matrix for eigenvalue computation is given by

\[
\Phi'(k) = \begin{bmatrix} e^{Ah} & (e^{A(2h-\tau)} - e^{A(h-\tau)})A^{-1}BK \\ I_n & (I_n - e^{A(h-\tau)})A^{-1}BK \end{bmatrix}
\]  

(4-3)

where \( A \) is non-singular.

With the equivalent matrix, the computational burden is reduced significantly since the complex integration of the exponential of state matrix in (4-2) is removed.

### 4.2. State-Space Model of Proposed Control Scheme

The equivalent matrix is used to determine the stability regions of the networked control system. The proposed control scheme in section 3.1 is used to validate the proposed equivalent matrix. In this section, the derivative control is ignored to simplify the control system model. The proposed control scheme is a hierarchical structure with a centralized controller, which is feasible for the general configuration of microgrids.

In the state-space model (4-1), state variables represent the dynamics of synchronous generators, excitation system (Automatic Voltage Regulator, AVR), and governor system (Turbine Governor, TG). Input variables are the reference values of AVR and TG, which are fed by the controller. The input variables include the feedback loop from state variables, as shown in (4-1). The proposed controller includes two control schemes i.e., regulation and power dispatch. To compute the feedback matrix, \( K \), it is assumed that there exists a non-singular closed-loop state matrix, \( A_c = A - BK \), under continuous time linear time-invariant system.
\[ \dot{x}(t) = Ax(t) + Bu(t) \]
\[ = Ax(t) - BKx(t) \]
\[ = A_c x(t) \]
\[ y(t) = Cx(t) \]

(4-4)

where \( x(t) \) is a \( n \times 1 \) vector, \( u(t) \) is a \( m \times 1 \) vector, \( y(t) \) is a \( p \times 1 \) vector. \( A \) and \( A_c \) are \( n \times n \) matrices. \( B \) is a \( n \times m \) matrix. \( K \) is a \( m \times n \) matrix. \( C \) is a \( p \times m \) matrix.

### 4.2.1. Regulation Control in the State Space

Taking frequency regulation as a control objective, the PI controller in (3-2) can be written as

\[ \dot{q}_1(t) = K_1(f_{op} - f_{PCC}) \]
\[ f_{REG}^{ref}(t) = f_{op} + q_1(t) + K_P(f_{op} - f_{PCC}) \]

(4-5)

where \( f_{REG}^{ref} \) is the frequency reference for regulation control. \( q_1 \) is a new state variable to represent the integral control. \( K_P \) and \( K_1 \) are PI constants. \( f_{op} \) is a constant (1 p.u.) and \( f_{PCC} \) is one of the state variables.

The incremental form of (4-5) is

\[ \Delta \dot{q}_1(t) = -K_1 \times \Delta f_{PCC} \]
\[ \Delta f_{REG}^{ref}(t) = \Delta q_1(t) - K_P \times \Delta f_{PCC} \]

(4-6)

In the microgrid system, \( f_{PCC} \) is a state variable.

\[ \Delta f_{PCC} = R_{1\times n}^{f_{PCC}} \Delta x(t) = R_{1\times n}^{f_{PCC}} A_c^{-1} \Delta \dot{x}(t) \]

(4-7)

where \( R_{1\times n}^{f_{PCC}} \) is a row vector of length \( n \) with all zeros except a 1 at the position of \( f_{PCC} \) in state variables \( (R_{1\times n}^{f_{PCC}} \times x(t) = f_{PCC}) \). Equation (4-6) can be written as
\[
\Delta q_1(t) = -K_1 \times R_{1\times n}^{f_{PCC}} A_c^{-1} \Delta x(t)
\]
\[
\Delta q_1(t) = -K_1 \times R_{1\times n}^{f_{PCC}} A_c^{-1} \Delta x(t)
\]
\[
\Delta f_{\text{REG}}^{\text{ref}}(t) = \Delta q_1(t) - K_p \times \Delta f_{\text{PCC}}
\]
\[
= - (K_p \times R_{1\times n}^{f_{PCC}} + K_1 \times R_{1\times n}^{f_{PCC}} A_c^{-1}) \Delta x(t)
\]
\[
= - K_{\text{REG}} \Delta x(t)
\]

where \( K_{\text{REG}} \) is the feedback matrix for regulation.

## 4.2.2. Dispatch Control in the State Space

Dispatch control signals are the power outputs from DERs that are output variables in the state-space model. Taking active power dispatch as an example, the droop compensation in (3-2) can be linearized and written as

\[
f_{\text{DIS}}^{\text{ref}} = m_p \times P_{\text{DER}}^{\text{des}}
\]
\[
= m_p \times \sum_i W_i P_i
\]
\[
= m_p \times \sum_i W_i R_i^{P_i} Cx(t)
\]

where \( f_{\text{DIS}}^{\text{ref}} \) is the frequency reference for dispatch control. \( m_p \) is the f-P droop characteristics. \( l \) is the number of DERs. \( W_i \) is the ratio of the \( i_{th} \) DER’s rated capacity to the combined generation capacity, respectively. \( P_i \) is the \( i_{th} \) DER’s active power which is an output variable in the state-space model. \( R_i^{P_i} \) is a row vector of length \( p \) with all zeros except a one at the position of \( P_i \) in output variables \( (R_i^{P_i} y(t) = R_i^{P_i} Cx(t) = P_i) \).
After linearizing (4-9), the power dispatch is written as

\[
\Delta f_{\text{DIS}}^{\text{ref}}(t) = m_p \times \Delta P_{\text{DG}}^{\text{des}} \\
= m_p \times \sum_i W_i R_{1xP}^{R_i} C\Delta x(t) \\
= -K_{\text{DIS}}\Delta x(t)
\]  

(4-10)

where \(K_{\text{DIS}}\) is the feedback matrix for power dispatch.

### 4.2.3. Coordinated Control Loop in the State Space

The feedback matrix is formed by the signals of regulation and dispatch controls. Taking frequency control as an example, the reference value of governor system is the combination of (4-8) and (4-10).

\[
\Delta u_{\text{REG}}(t) = \Delta f_{\text{REG}}^{\text{ref}}(t) + \Delta f_{\text{DIS}}^{\text{ref}}(t) \\
= -K_{\text{REG}}\Delta x(t) - K_{\text{DIS}}\Delta x(t) \\
= -(K_{\text{REG}} + K_{\text{DIS}})\Delta x(t) \\
= -K_{\text{TG}}\Delta x(t)
\]

(4-11)

where \(\Delta u_{T G}(t)\) is the frequency reference of governor system. \(K_{\text{TG}}\) is the complete feedback matrix of frequency control.

Following the same process, the feedback matrix for voltage control can be obtained.

\[
\Delta u_{\text{AVR}}(t) = -K_{\text{AVR}}\Delta x(t)
\]

(4-12)

where \(\Delta u_{\text{AVR}}(t)\) is the voltage reference of excitation system. \(K_{\text{AVR}}\) is the complete feedback matrix of voltage control.
The complete feedback matrix of system is written as

\[
\Delta u(t) = \begin{bmatrix}
\Delta u_{TG}(t) \\
\Delta u_{AVR}(t)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-K_{TG} \\
-K_{AVR}
\end{bmatrix} \Delta x(t)
\]

\[
= -K\Delta x(t)
\]  \hspace{1cm} (4-13)

With the state-space model of a microgrid, the feedback matrix of proposed control scheme can be computed with equations (4-5) ~ (4-13). Then, with the given state matrix \(A\), input matrix \(B\), and the feedback matrix \(K\), the equivalent matrix in (4-3) can be formed for the given reporting period and communication delay. The eigenvalues of the equivalent matrix are used to assess system stability.

4.2.4. Stability Region in Cyber System Parameters

The stability region in the 2-dimensional space of cyber system parameters, \(h\) and \(\tau\), is proposed for determination of their critical values for system stability. For different pairs of the cyber parameters, the set of points forming the boundary of the (asymptotic) stability region of the microgrid system is found based on the eigenvalues. It is important to note that the dimension of the stability region is 2, irrespective of the power system size. Therefore, the proposed computational technique for the critical cyber parameters is applicable to large-scale power system models.
4.3. Stability Region of the 13-Node Test System

In this section, the proposed method is applied to the 13-node system in section 3.3. To evaluate the control performance under the influence of a cyber system, a communication link is added to the 13-node test system.

The cyber latency is modeled for the tasks of data exchange, measurements from LCs to MGC, and commands from MGC to LCs, as shown in Figure 4-2. Considering that MGC and LCs can be dispersed in a wide area, cellular communication is a potential choice for the connections between MGC and LCs. At present, 4G/LTE is the most common cellular communication technology. The communication delay of 4G/LTE is a few tens of milliseconds. The benefits of the 4G/LTE technology include good performance in latency (a few tens of milliseconds) and wide availability. Furthermore, there is no need for a proprietary communication network; rather, an existing secure cellular infrastructure can be utilized for microgrid control.

A typical communication delay is much shorter than the time constants of a synchronous generator. Therefore, it should not severely impact system stability. However, with long reporting periods for data acquisition, or under a denial-of-service (DoS) cyberattack, the delay time can be prolonged.

Figure 4-2. Microgrid Test System with an Integrated Communication Channel
4.3.1. Analysis by State-Space Model

The proposed analytical method in Section 4.1 is applied to the modified IEEE 13-node test system. The test system is built with the power system analysis toolbox (PSAT) [56], which only allows a balanced system model. It is assumed that the control only applies to the three-phase portion (primary feeder) of the distribution system and not the single or two-phase lateral(s) branching from the primary feeder.

For an unbalanced system, the structures of the synchronous generator, governor, and exciter remain balanced by design, so the state matrix $A$ and input matrix $B$ do not change. Although the unbalanced output matrix could generate three-phase output variables, the control input takes the average value to calculate the command. To validate the balanced three-phase feeder model for the control system, the balanced steady-state power flow is performed with PSAT and compared to the results in the unbalanced system in Simulink under a continuous mode. Table 4-1 shows the power flow results. Note that the steady-state operating point of the balanced model in PSAT is close to that of the unbalanced model in Simulink.

<table>
<thead>
<tr>
<th>Bus</th>
<th>PSAT (p.u.)</th>
<th>Simulink (p.u.)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>684 (G1)</td>
<td>1.0215</td>
<td>1.0128</td>
<td>0.86</td>
</tr>
<tr>
<td>680 (G2)</td>
<td>1.0267</td>
<td>1.0163</td>
<td>1.02</td>
</tr>
<tr>
<td>675 (G3)</td>
<td>1.0220</td>
<td>1.0133</td>
<td>0.86</td>
</tr>
<tr>
<td>671</td>
<td>1.0201</td>
<td>1.0120</td>
<td>0.80</td>
</tr>
<tr>
<td>632</td>
<td>0.99969</td>
<td>1</td>
<td>0.03</td>
</tr>
<tr>
<td>633</td>
<td>0.99619</td>
<td>0.9972</td>
<td>0.10</td>
</tr>
<tr>
<td>634</td>
<td>0.9762</td>
<td>0.9770</td>
<td>0.08</td>
</tr>
<tr>
<td>650 (PCC)</td>
<td>0.99969</td>
<td>1</td>
<td>0.03</td>
</tr>
</tbody>
</table>

With the data acquisition reporting period and communication delay, the eigenvalues of transition matrix in (4-2) can be computed by the equivalent matrix in (4-3) and used to assess system stability.
For example, with 0.4-second reporting period and zero communication delay, the microgrid system is stable since the dominant eigenvalue, 0.9570, lies within the unit circle. On the other hand, with 0.6-second reporting period and zero communication delay, the system is unstable as the dominant eigenvalue, -1.2676, falls outside the unit circle.

The blue shaded area in Figure 4-3 is the stability region based on the state-space networked control model. The equivalent matrix is used to compute the dominant eigenvalue under the given reporting period and communication delay. Note that the stability region is a two-dimension diagram which is only associated with the two cyber parameters, i.e., reporting period and communication delay.

![Figure 4-3. Stability Regions by Analytical Method and Time-Domine Simulation](image)

4.3.2. Reduced Computational Burden

In this research, an equivalent matrix is proposed to compute the eigenvalues of the transition matrix. The computational burden is reduced remarkably. In the modified IEEE 13-node system, the size of state matrix is 34 by 34 (n = 34), and input matrix is 34 by 6 (m = 6). The computation of eigenvalues is performed in Matlab R2018b with processor Intel(R) Core(TM) i7-10510U CPU.
Using the equivalent matrix (4-3), the computation time is less than one minute for a total of 10201 (101 by 101) matrices in Figure 4-3. On the other hand, due to the integration of exponential of state matrix, the computation is much heavier by using transition matrix (4-2), which requires over two minutes for just one matrix.

4.3.3. Validation by Time-Domain Simulation

Time-domain simulation is performed in Simulink for the purpose of validation for the networked control method. The same models and parameters of DERs, AVR, TGs, and PI controllers are used with the balanced system in PSAT. Various reporting periods and communication delays are applied to analyze system stability. The frequency and voltage at PCC (bus 650) are obtained. Figure 4-4 is the simulation results using 0.4-second and 0.6-second reporting periods.

The proposed control algorithm will regulate the frequency and voltage at the PCC. The simulations start under the continuous mode to reach a stable operating point. Then, the cyber latency is added at 50 seconds to simulate the networked control system. With the shorter reporting period, 0.4 seconds, the system is regulated to 1 p.u. and the system is stable. However, with the larger reporting period, 0.6 seconds, the system voltage starts to oscillate at 120 seconds and diverges eventually, and the system becomes unstable.
To identify the critical values for the reporting period and communication delay, the stability region is obtained by the networked control method with the equivalent matrix. To validate the results, the time-domain simulation is used to find the stability boundary for the modified IEEE 13-node system. The red dotted curves in Figure 4-3 form the stability boundary of the microgrid system. By comparing the results from the networked control method and time-domain simulation, the differences are small. Under certain reporting periods, the maximum differences of communication delays are less than 10 milliseconds for the lower boundary and 35 milliseconds for the upper boundary. Note that the analytical method is based on a linearized system model, so the effect of nonlinearity is not captured. In contrast, time-domain simulation is based on the nonlinear dynamic model. Also, there are limitations in the governor and exciter models that are not incorporated in the linearized model. Moreover, as stated previously, the networked control system analysis is based on a balanced system, while time-domain simulation uses an unbalanced system model.

Figure 4-4. Stability and Unstable System Responses under Time-Domain Simulation
The stability region shows the cyber system performance needs to be coordinated with the physical system. Indeed, in Figure 4-3, when the reporting period (horizontal axis) exceeds 0.49 seconds, stability cannot be achieved by simply minimizing the communication delay. Based on the stability region, the communication delay needs to stay within a calculated range. In Figure 4-3, when the critical reporting period is 0.48 seconds, and the maximum communication delay is 0.3456 seconds (72% of the critical reporting period 0.48 seconds).

Considering the small errors of the state-space model, the critical threshold for communication latency in this test case is set at 0.43-second reporting period and 310-millisecond communication delay (10% margin).

4.3.4. Impact of Controller Parameters on Stability Regions

In this section, the relationship between controller gains and stability margins is analyzed. Variations of the controller gains will result in a change of the 2-dimensional stability region. In the proposed analytical method, the controller is involved in the feedback matrix $K$, which is critical in placing the eigenvalues of the equivalent matrix in (4-3).

The stability of the transition matrix, $\Phi'$, is dependent on the magnitude of the dominant eigenvalue (one with the largest absolute value). In (4-8), the PI parameters, $K_p$ and $K_i$, of the proposed control method are scalar coefficients to the matrix $K$. Increasing the PI parameters leads to a larger magnitude of the dominant eigenvalue, reducing the stability region.

Figure 4-5 shows the trajectories of the dominant eigenvalues as PI parameters increase. Since $K_p$ is much larger then $K_i$ in the base case, $K_p$ dominates the elements of $K$ and the dominant eigenvalue of $\Phi'$. Increasing $K_p$ drives the dominant eigenvalues outside the unit circle quickly, leading to system instability.
Figure 4-5. Trajectories of Dominant Eigenvalues as PI Parameters Increase

Figure 4-6 is the stability regions with different $K_p$ values computed by the analytical method. The red mark x is the determined critical thresholds of cyber latency. In Figure 4-6(a), the base case, the operating point lies inside the stable region but is close to the boundary. This is chosen as a critical operating point with a small margin. In Figure 4-6(b), when the $K_p$ increases to 1.5 times, the stability region shrinks, and the same operating point becomes unstable. With a larger $K_p$, the stability region shrinks further and becomes smaller as shown in Figure 4-6(c).
Figure 4-6. Stability Regions with Different $K_p$ Values
4.4. System Performance under Critical Cyber Latency

In this section, the control scheme and two dynamic scenarios are tested under the critical thresholds for cyber latency determined in section 4.3.3, i.e., $h = 430$ milliseconds and $\tau = 310$ milliseconds. To implement the time-varying communication delays, a normal distribution is used for communication delays with the mean value of 310 milliseconds and the standard deviation of 31 milliseconds in the time-domain simulation.

Two additional scenarios are developed for the transient stability test. In the heavy load scenario, an additional load at bus 671 is picked up for evaluation of the dynamic response. For the fault scenario, a short circuit fault is assumed to occur on the primary side of transformer XFM-1, and the fault is cleared by a recloser upstream of line 632-633.

4.4.1. Restoration Operation under Critical Cyber Latency

The simulation starts with fixed loads. Then, for system restoration two switchable loads under nodes 671 and 675 are picked up at 300 seconds and 600 seconds, respectively. Figure 4-7 and Figure 4-8 show the results of regulation and power dispatch controls. Both control objectives are met under the calculated critical thresholds of cyber latency. Frequency and voltage are steered to 1 p.u. and active and reactive powers are dispatched by the same ratio of power output to the rated capacity.

With three levels of loads, the active power outputs of the three DGs are, respectively, 483.6kW, 660.1kW, and 329.5kW at the initial condition, 872.8kW, 1193.3kW, and 594.5kW when node 671 is restored, and 1135.4kW, 1552.2kW, and 772.8kW when node 675 is restored. Under all three levels, the active powers are dispatched by the same ratio to the respective rated capacity of each DG unit. Similarly, the reactive powers are dispatched by the same ratio.
Figure 4-7. System Frequency and Voltage with Service Restoration Actions

Figure 4-8. Active and Reactive Power Dispatch with Service Restoration Actions
4.4.2. Heavy Load Pick Up under Critical Cyber Latency

The heavy load with 3000 kVA (0.9 lagging) load is to be picked up, representing 45% of the generation capacity. The simulation starts latency, the settling time is longer, and oscillation occurs. The control commands are delayed and updated every 430 milliseconds. Under the specified cyber latency, the system is stable and regulated to 1 p.u. in 20 seconds after this transient event.

![Figure 4-9. System Voltage under Heavy Load Picking-Up Scenario](image)

4.4.3. Short Circuit Fault under Critical Cyber Latency

A short circuit is assumed to occur and then it is cleared by an upstream recloser with two-fast (2 cycles) and two-delayed (5 cycles) operation settings in Figure 3-12. The simulation starts with fixed loads and the short circuit fault occurs at 300 seconds which is cleared by the recloser. Figure 4-10 illustrates the voltage responses without cyber latency and under the specified cyber latency.
A longer settling time and oscillations are observed under the specified cyber latency. The system takes more than 30 seconds to settle and operate at 1 p.u. as the transients diminish.

Figure 4-10. System Voltage under Short Circuit Fault Scenario

The system performance is shown to be stable with the critical threshold for cyber latency. It is important to point out that a distribution system is normally tied to the utility system and hence dynamics are “absorbed” by the utility system. The microgrid system stability scenarios shown in this study clearly demonstrate the importance of system dynamics when the microgrid system operates in a resiliency (islanded) mode. It is also critical to study the interactions between system dynamics and the design and operation of protective devices on the feeder.
Chapter 5\(^{(3)}\)

Stability of a Hybrid-DER Microgrid

To operate islanded microgrids, it is crucial to maintain system stability with respect to various disturbances, large or small. As power electronics technology advances, the penetration of inverter-based resources (IBRs) is increasing at a fast pace. Compared to conventional synchronous generators (SGs), a major technical challenge of IBRs is their low inertia which can lead to system instability [57]–[59]. To improve the stability of low-inertia islanded microgrids, grid-forming inverters (GFM) and Virtual Synchronous Generators (VSG) are proposed to increase damping of the IBRs and emulate the dynamics of a synchronous machine [60]–[62].

Due to the distinct behaviors between IBRs and SGs, the hybrid-DER microgrid can become unstable as well when they are operated without proper coordination [63], [64]. A feedback control system for IBRs and SGs can be used to stabilize the hybrid-DER microgrid [64], [65].

5.1. Technical Issues of Islanded Microgrid Operation

To operate islanded microgrids, it is essential to maintain system stability. Feedback control is needed and, furthermore, the following technical issues need to be considered for implementation of the control loop.

1) A reduced-order algorithm for controller design is needed to relieve the computational burden for a complex microgrid system.

2) For a hybrid-DER microgrid, synchronous generators may have existed for decades and may involve significant costs and time to modify the input ports for feedback control. Instead, IBRs have better flexibility for modification. Therefore, in the proposed approach, feedback control commands are determined as inputs to IBRs. Measurements are acquired from synchronous generators as well.

3) Not all states of the microgrid system are measurable. Full state feedback will need an observer for output feedback control, which is not realistic from the implementation point of view. As an approximation, partial state feedback control will be used that relies only on measurable states. Thus, no observer is needed.

4) Feedback control commands are applied to DERs and, therefore, a communication system and a centralized controller are required. In contrast, decentralized state feedback only takes local states and feeds to local inputs without the need for a communication system.

5.2. Inverter-Based Resources: Following and Forming Control

To integrate renewable energy into a power grid, the inverters are utilized for power conversion and control. The inverter controllers are classified into two types: grid-following (GFL) and grid-forming (GFM). GFL control needs a stiff AC voltage at its terminal and GFL inverters follow the grid voltage and control the output current. Instead, a GFM inverter is designed to control its voltage magnitude and frequency. A GFM inverter can work without an external AC voltage [66]–[68].

Based on the characteristics of GFL and GFM inverters, they are modeled as current and voltage sources, respectively [69]. The voltage source model can be designed to mimic a
synchronous generator [70]. GFM inverters are proposed to add virtual inertia, damping or droop control in order to enhance system stability.

However, the inverter control is a primary control that does not consider the interaction with other DERs. The instability of a hybrid-DER microgrid involves system-wide dynamics. In the proposed approach, a system-wide secondary control scheme is designed to coordinate hybrid DERs.

5.3. Stability of a Two-Bus Hybrid-DER System

A two-bus system with hybrid DERs is used to illustrate the stability analysis and control of a hybrid-DER microgrid. A SG and an IBR serving load is shown in Figure 5-1.

\[ P_g \text{ and } P_i \text{ are the output active powers of SG and IBR, respectively.} \]
\[ E_g \angle \delta_g \text{ and } E_i \angle \delta_i \text{ are the voltage magnitude and phase angle of SG’s and IBR’s terminals, respectively.} \]
\[ Z_{gi} \text{ is the line impedance, and } Z_{g0}, \text{ and } Z_{i0} \text{ are the load impedances.} \]
\[ \delta_{gi} = \delta_g - \delta_i \text{ is defined as the angle difference between SG and IBR.} \]
\[ \text{Admittances are the reciprocals of impedances.} \]
\[
Y_{gl} = \frac{1}{Z_{gl}} = G_{gi} + jB_{gi} = |Y_{gi}| \angle \phi_{gi}
\]
\[
Y_{g0} = \frac{1}{Z_{g0}} = G_{g0} + jB_{g0} = |Y_{g0}| \angle \phi_{g0}
\]
\[
Y_{i0} = \frac{1}{Z_{i0}} = G_{i0} + jB_{i0} = |Y_{i0}| \angle \phi_{i0}
\]

(5-1)

Define \(G_{gg}\) and \(B_{gg}\) as the conductance and susceptance of \(Y_{gg}\). \(G_{ii}\) and \(B_{ii}\) as the conductance and susceptance of \(Y_{ii}\).

\[
Y_{gg} = Y_{gi} + Y_{g0} = (G_{gi} + G_{g0}) + j(B_{gi} + B_{g0}) = G_{gg} + jB_{gg} = |Y_{gg}| \angle \phi_{gg}
\]
\[
Y_{ii} = Y_{gi} + Y_{i0} = (G_{gi} + G_{i0}) + j(B_{gi} + B_{i0}) = G_{ii} + jB_{ii} = |Y_{ii}| \angle \phi_{ii}
\]

(5-2)

Then, the active power of SG can be formulated [64].

\[
P_g = P_{gi} + P_{g0}
\]
\[
= [G_{gi}(E_g^2 - E_g E_i \cos \delta_{gi})] - B_{gi}(E_g E_i \sin \delta_{gi}) + [G_{g0}E_g^2]
\]
\[
= (G_{gi} + G_{g0})E_g^2 + E_g E_i(-G_{gi} \cos \delta_{gi} - B_{gi} \sin \delta_{gi})
\]
\[
= G_{gg}E_g^2 + E_g E_i(-|Y_{gi}| \cos \phi_{gi} \cos \delta_{gi} - |Y_{gi}| \sin \phi_{gi} \sin \delta_{gi})
\]
\[
= G_{gg}E_g^2 - E_g E_i(|Y_{gi}| \cos(-\phi_{gi}) \cos \delta_{gi} - |Y_{gi}| \sin (-\phi_{gi}) \sin \delta_{gi})
\]
\[
= G_{gg}E_g^2 - |Y_{gi}|E_g E_i \cos(\delta_{gi} - \phi_{gi})
\]

(5-3)

Similarly, the active power of IBR can be formulated.

\[
P_i = G_{ii}E_i^2 - |Y_{gi}|E_g E_i \cos(\delta_{gi} - \phi_{gi})
\]
\[
= G_{ii}E_i^2 - |Y_{gi}|E_g E_i \cos(-\delta_{gi} - \phi_{gi})
\]
\[
= G_{ii}E_i^2 - |Y_{gi}|E_g E_i \cos(\delta_{gi} + \phi_{gi})
\]

(5-4)
Assume \(|E_g| \approx |E_i| \equiv 1\ p.u.\) and linearize the system at operating point \(\delta_{gi}^0\).

\[
L(P_g|\delta_{gi}^0) = P_g(\delta_{gi}^0) + P'_g(\delta_{gi}^0) \times (\delta_{gi} - \delta_{gi}^0)
\]
\[
= G_{gg} - |Y_{gi}| \cos(\delta_{gi}^0 - \phi_{gi}) + |Y_{gi}| \sin(\delta_{gi}^0 - \phi_{gi}) \times (\delta_{gi} - \delta_{gi}^0)
\]
\[
L(P_i|\delta_{gi}^0) = P_i(\delta_{gi}^0) + P'_i(\delta_{gi}^0) \times (\delta_{gi} - \delta_{gi}^0)
\]
\[
= G_{ii} - |Y_{gi}| \cos(\delta_{gi}^0 + \phi_{gi}) + |Y_{gi}| \sin(\delta_{gi}^0 + \phi_{gi}) \times (\delta_{gi} - \delta_{gi}^0)
\]
where \(L(f(x)|x^0)\) linearize function \(f(x)\) at \(x^0\).  

\[
\Delta P_g \text{ and } \Delta P_i \text{ are defined as the change of active powers near } \delta_{gi}^0.
\]

\[
\begin{align*}
\Delta P_g &= L(P_g|\delta_{gi}^0) - P_g(\delta_{gi}^0) = |Y_{gi}| \sin(\delta_{gi}^0 - \phi_{gi}) \times (\delta_{gi} - \delta_{gi}^0) = -k_1 \Delta \delta_{gi} \\
\Delta P_i &= L(P_i|\delta_{gi}^0) - P_i(\delta_{gi}^0) = |Y_{gi}| \sin(\delta_{gi}^0 + \phi_{gi}) \times (\delta_{gi} - \delta_{gi}^0) = -k_2 \Delta \delta_{gi}
\end{align*}
\]

where \(k_1 = -|Y_{gi}| \sin(\delta_{gi}^0 - \phi_{gi})\) and \(k_2 = -|Y_{gi}| \sin(\delta_{gi}^0 + \phi_{gi})\).

The aggregate models of SG and the GFM inverter are used for stability analysis. Figure 5-2 gives the transfer function model for the aggregate generator [60].

\[\text{Figure 5-2. Aggregate Generator Model}\]

Ignore the aggregated damping \(D_{g,a} = 0\), then the dynamics of synchronous generator are written in terms of the incremental change of the states.
\[ \Delta \dot{\omega}_g = \frac{1}{M_{g,a}} \times \left( \Delta t_g + \frac{1}{M_{p,gen}} \times \frac{T_B}{T_A} \times (\Delta \omega_s - \Delta \omega_g) - (-k_1 \Delta \delta_{gi}) \right) \]

\[ = \frac{k_1}{M_{g,a}} \Delta \delta_{gi} - \frac{1}{M_{g,a} M_{p,gen}} \frac{T_B}{T_A} \Delta \omega_g + 1 \frac{1}{M_{g,a} M_{p,gen}} \Delta t_g + \frac{1}{M_{g,a} M_{p,gen}} \frac{T_B}{T_A} \Delta \omega_s \]

\[ \Delta t_g = \frac{1 - \frac{T_B}{T_A}}{T_A M_{p,gen}} \times (\Delta \omega_s - \Delta \omega_g) - \frac{1}{T_A} \Delta \omega_s \]

\[ = - \left( 1 - \frac{T_B}{T_A} \right) \Delta \omega_g + \frac{1}{T_A} \Delta t_g + \frac{1 - \frac{T_B}{T_A}}{T_A M_{p,gen}} \Delta \omega_s \]

where \( M_{g,a} \) is aggregated inertia, \( M_{p,gen} \) is governor speed droop, \( T_A \) and \( T_B \) are the parameters of governor’s transfer function, \( \Delta t_g \) is an internal state, \( \Delta \omega_g \) is angular frequency, and \( \Delta \omega_s \) is input reference.

Figure 5-3 shows the GFM inverter model used for the stability analysis [60].

![Figure 5-3. GFM Inverter Model](image)

The relationship between active output power and angular frequency is given as

\[ \Delta \omega_i = M_{p,inv} (\Delta P_{i,set} - \Delta P_f) \]

\[ = M_{p,inv} \Delta P_{i,set} - M_{p,inv} \times \frac{1}{T_m s + 1} \Delta P_i \]

where \( M_{p,inv} \) is droop coefficient, \( T_m \) is time constant of a first order filter, \( \Delta \delta_i \) is electrical angle, \( \Delta \omega_i \) is angular frequency, and \( \Delta P_{i,set} \) is input reference.
The active output power of the GFM model can be rewritten as

\[
\Delta P_i = (T_m s + 1) \Delta P_{i,\text{set}} - \frac{T_m s + 1}{M_{p,\text{inv}}} \Delta \omega_i
\]

\[
= T_m (s \Delta P_{i,\text{set}}) + \Delta P_{i,\text{set}} - \frac{T_m s + 1}{M_{p,\text{inv}}} \Delta \omega_i
\]

(5-9)

Note that, the \(\Delta P_{i,\text{set}}\) is the reference setting point which is a constant. Then, (5-9) can be simplified by eliminating the derivative of constant term, \(s \Delta P_{i,\text{set}}\), [71].

\[
\Delta P_i = \Delta P_{i,\text{set}} - \frac{T_m s + 1}{M_{p,\text{inv}}} \Delta \omega_i
\]

\[
= \Delta P_{i,\text{set}} - \frac{1}{M_{p,\text{inv}}} \Delta \omega_i - \frac{T_m}{M_{p,\text{inv}}} s \Delta \omega_i
\]

(5-10)

The dynamics of the equivalent GFM model are given as

\[
\Delta \dot{\omega}_i = -\frac{M_{p,\text{inv}}}{T_m} \Delta P_i - \frac{1}{T_m} \Delta \omega_i + \frac{M_{p,\text{inv}}}{T_m} \Delta P_{i,\text{set}}
\]

\[
\frac{T_m}{M_{p,\text{inv}}} \Delta \ddot{\delta}_i + \frac{1}{M_{p,\text{inv}}} \Delta \dot{\delta}_i = \Delta P_{i,\text{set}} - \Delta P_i
\]

(5-11)

Compared to conventional swing equation, the coefficients of the second-order derivative and first-order derivative on angle represent inertia and damping, respectively. Therefore, \(M_i \triangleq \frac{T_m}{M_{p,\text{inv}}}\) and \(D_i \triangleq \frac{1}{M_{p,\text{inv}}}\) are defined as the virtual inertia and damping of a GFM model. The inertia equivalence representation of GFM inverter is shown in Figure 5-4.
The dynamics of the inertia equivalence representation of GFM inverter are represented by,

$$\Delta \omega_i = \frac{1}{M_i s + D_i} (\Delta P_{i,\text{set}} - \Delta P_i)$$

$$(M_i s + D_i) \Delta \omega_i = \Delta P_{i,\text{set}} - \Delta P_i$$

$$M_i \Delta \omega_i = \Delta P_{i,\text{set}} - \Delta P_i - D_i \Delta \omega_i$$

$$\Delta \omega_i = \frac{k_2}{M_i} \Delta \delta_{gi} - \frac{D_i}{M_i} \Delta \omega_i + \frac{1}{M_i} \Delta P_{i,\text{set}}$$  \hspace{1cm} (5-12)$$

The state space model of the two-bus hybrid-DER system can be written with four state variables and two input variables as (5-13). It is important to note that the angle difference between the two buses, which is required to compute the coefficients $k_1$ and $k_2$, can be obtained using Phasor Measurement Units (PMUs).

$$\begin{bmatrix} \Delta \delta_{gi} \\ \Delta \omega_g \\ \Delta t_g \\ \Delta \omega_l \end{bmatrix} = \begin{bmatrix} 0 & -T_B/T_A & 0 & -1 \\ \frac{k_1}{M_{g,a}} & -\left(1 - \frac{T_B}{T_A}\right) & 0 & 0 \\ 0 & T_A M_{p,\text{gen}} & 0 & 0 \\ \frac{k_2}{M_i} & 0 & -D_i & \frac{1}{M_i} \end{bmatrix} \begin{bmatrix} \Delta \delta_{gi} \\ \Delta \omega_g \\ \Delta t_g \\ \Delta \omega_l \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{T_B}{T_A} \\ 1 - \frac{T_B}{T_A} \\ 0 \end{bmatrix} \begin{bmatrix} \Delta \omega_{l} \\ \Delta P_{i,\text{set}} \end{bmatrix}$$  \hspace{1cm} (5-13)$$
The system stability will depend on eigenvalues of the state matrix. To study the stability of a hybrid-DER system, the key parameters are the inertia difference between SG and IBR, and their penetration levels. In the GFM model, the time constant is proportional to the virtual inertia, $M_I \equiv \frac{T_m}{M_{p,inv}}$. For the stability criterion, Table 5-1 gives the parameters and their values for the study case.

Table 5-1. Parameters of the Study Case

<table>
<thead>
<tr>
<th>Network Parameters</th>
<th>SG Parameters</th>
<th>GFM Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{gi} = 2.325 + j 1.523 \text{ p.u.}$</td>
<td>$T_A = 0.2 \text{ s}$</td>
<td>$M_{p,inv} = 0.05\omega_s / \text{p.u.}$</td>
</tr>
<tr>
<td>$Z_{g0} = 2.425 + j 0.375 \text{ p.u.}$</td>
<td>$T_B = 0.5 \text{ s}$</td>
<td></td>
</tr>
<tr>
<td>$Z_{i0} = 2.425 + j 0.375 \text{ p.u.}$</td>
<td>$H_{g,a} = 3.5 \text{ s}/\text{p.u.}$</td>
<td></td>
</tr>
<tr>
<td>$\phi_{gi} = 33.23^\circ$</td>
<td>$M_{g,a} \equiv \frac{2H_{g,a}}{\omega_s}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$M_{p,gen} = 0.05\omega_s / \text{p.u.}$</td>
<td></td>
</tr>
</tbody>
</table>

5.3.1. Stability Regions of the Test System

By power flow (5-5) and the parameters in Table 5-1, output powers of SG and GFM inverter at a given operating point can be calculated. Figure 5-5 plots the output powers with respect to the angle difference, $\delta_{gi}^0$. When there is no angle difference between SG and GFM inverter at the operating point, $\delta_{gi}^0 = 0$, the output powers of SG and GFM inverter are equal. When the angle of GFM inverter leads that of SG, $-90^\circ \leq \delta_{gi}^0 < 0$, the output power of GFM inverter is higher than that of SG which means the penetration of GFM inverter is higher than SG, and vice versa.
The bar chart in Figure 5-6 illustrates the level of penetration for SG and GFM inverter, respectively, by percentage. When the $\delta_{gl}^0$ is negative, IBR has a higher penetration level up to 64%. Similarly, SG has a higher penetration level when $\delta_{gl}^0$ is positive.
A two-dimensional stability region concerning operating point, \( \delta_{gil}^0 \), and time constant, \( T_m \), is presented in Figure 5-7. The red curves are the trajectories of zero real-part eigenvalues that are closest to the stable region, and the blue diamonds are the stability boundaries of singularly perturbed systems [72]. The result shows that the system becomes unstable at the lower-left corner when the IBR has a smaller time constant (lower virtual inertia) and at a higher penetration level.

The positive \( \delta_{gil}^0 \) at the upper-right corner indicates that SG has a higher penetration level. The result shows that the system becomes unstable at the upper-right corner when the SG has a smaller time constant (lower inertia) and at a higher penetration level.

To conclude, both unstable corners imply that a hybrid-DER system can be unstable when the inertia difference is large, and the penetration level of lower-inertia DER is high.

![Figure 5-7. Two-Dimensional Stability Regions of the Hybrid-DER System](image)

Figure 5-7 shows two-dimensional stability regions with different SG’s aggregated inertia. When the aggregated inertia increases, the unstable region at the lower-left corner expands, as
shown by the green curve, which means the system is prone to instability. When SG’s inertia increases, the inertia difference between SG and IBR also increases, leading to instability. The upper-right corner becomes more stable, as shown by the green curve, when the SG’s inertia increases because the inertia difference between SG and IBR decreases.

Figure 5-8. Two-Dimensional Stability Regions with Different Aggregated Inertia

Figure 5-9 shows two-dimensional stability regions with different droop coefficients of GFM. Since the droop coefficient is inversely proportional to GFM inverter’s virtual inertia, $M_t \doteq \frac{T_m}{M_{p, inv}}$. When the droop coefficient increases, the unstable region at the lower-left corner expands, shown by the green curve, which means the system is prone to instability. Because the inertia of GFM decreases, the inertia difference also increases which leads to instability, and vice versa for the upper-right corner.
Figure 5-10 shows two-dimensional stability regions with different feeder lengths. It can be observed that when the feeder is shorter, the unstable regions at both corners expand, shown by the green curves, which means the system is prone to instability. Because DERs are highly coupled with a shorter distance, the system stability is compromised. Note that most microgrids are developed at campuses, industrial parks, or military bases in the distribution system which are located within small areas. Therefore, the instability issue in microgrids can be significant due to the high coupling between DERs with shorter feeders.
5.3.2. Numerical Results

An operating condition is chosen to illustrate the stability control design. When the time constant, $T_m$, is 1 millisecond and angle difference, $\delta_{gi}^0$, minus 90 degrees, the system is unstable in Figure 5-7. Then, the state matrix and input matrix are computed using (5-13) and Table 5-1.

$$A = \begin{bmatrix}
0 & 1 & 0 & -1 \\
-16.20 & -7.14 & 53.85 & 0 \\
0 & 0.3978 & -5 & 0 \\
-5672.01 & 0 & 0 & -1000
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 & 0 \\
7.14 & 0 \\
0.3978 & 0 \\
0 & 18849.55
\end{bmatrix}$$

(5-14)

A feedback matrix (discussed later in section 5.4) is designed to feed input variable $\Delta P_{L, set}$ only, which is IBR’s active point setting point.
$$K = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.3553 & -0.0411 & -0.2358 & 0.0004 \end{bmatrix}$$  \hspace{1cm} (5-15)

The eigenvalues of open-loop and closed-loop state matrices can be computed.

$$[\lambda(A), \lambda(A - BK)] = \begin{bmatrix} -1005.64 & -1006.50 \\ -10.22 & -10.24 \\ 3.72 & -2.76 \\ 0.0004 & -0.17 \end{bmatrix}$$  \hspace{1cm} (5-16)

As shown in (5-12), in the open-loop system, there are two unstable eigenvalues on the right half plane. The designed feedback control stabilizes the system by ensuring all eigenvalues are on the left half plane for the closed-loop system.

Figure 5-11 provides the two-dimensional stability regions with the feedback gain. The lower-left corner becomes part of a stable region. This implies that IBR can operate at a small time constant (fast response).

Figure 5-11. Two-Dimensional Stability Regions of the Closed-Loop Hybrid-DER System
5.4. Microgrid Stability Enhancement by Feedback Control

As discussed, the stability of hybrid-DER microgrid is related to the inertia difference among DERs. Mathematically, a DER’s inertia is related to the time constant of its transfer function.

In Figure 5-7, a longer time constant (high virtual inertia) of the IBR can be selected to enhance system stability, e.g., \(0.4737 < T_m < 0.8263\). However, the IBR slows down and, therefore, it is a tradeoff. Furthermore, the analysis uses a two-bus system with simplified dynamic models and power flow equations. With a larger system with complex dynamic models, it is impractical to determine stability regions in higher dimensions that will require a higher computational burden. Instead of setting the time constant of IBRs (primary control), the proposed feedback control (secondary control) is used to coordinate DERs and stabilize the system.

Because SGs and IBRs have distinctly different scales of time constants, time scale separation can be applied on the hybrid-DER microgrid for stability analysis and controller design. The time scale separation can be used as a basis to decompose the system into two subsystems, slow and fast subsystems. The unstable eigenvalues are close to the origin and will be associated with the slow subsystem [73]. Therefore, the design of feedback control can be simplified in the slow subsystem.

5.4.1. Feedback Control Based on Two-Time Scale Decomposition

To apply methods of decomposition for two-time-scale systems, the state variables must be ordered by a sequence, from slowest to fastest. A proper similar transformation of reordering and scaling is needed [73], [74].

The different time scales are identified by eigenvalue analysis based on magnitude of the real parts. However, the physical states might not be classified as “slow” or “fast” state variables, i.e.,
states of the fast system can participate in small eigenvalues and vice versa. For applications, there are no specific rules for classification of the slow or fast states physically and finding a proper similar transformation.

A general N-bus microgrid system is shown in Figure 5-12. The state space model of each DER includes the state, input, and output variables. Some loads have their own dynamics as well, e.g., motors and electric vehicles. Generally, state space dynamic models are developed for every DERs and loads. The variables $x_{\text{DER}_i}$, $u_{\text{DER}_i}$, and $y_{\text{DER}_i}$ are the state, input, and output vectors of $d$ DERs, where $i = 1, 2, 3, \ldots d$. The symbols $x_{\text{LOAD}_j}$, $u_{\text{LOAD}_j}$, and $y_{\text{LOAD}_j}$ are the state, input, and output vectors of $l$ loads, where $j = 1, 2, 3, \ldots l$. Based on the quantities, the system will construct the state, input, and output vectors, i.e.,

$$
x_{(n,1)} = [x_{\text{DER}_1}^T, x_{\text{DER}_2}^T, \ldots x_{\text{DER}_d}^T, x_{\text{LOAD}_1}^T, x_{\text{LOAD}_2}^T \ldots x_{\text{LOAD}_l}^T]^T
$$

$$
u_{(m,1)} = [u_{\text{DER}_1}^T, u_{\text{DER}_2}^T, \ldots u_{\text{DER}_d}^T, u_{\text{LOAD}_1}^T, u_{\text{LOAD}_2}^T \ldots u_{\text{LOAD}_l}^T]^T
$$

$$
y_{(p,1)} = [y_{\text{DER}_1}^T, y_{\text{DER}_2}^T, \ldots y_{\text{DER}_d}^T, y_{\text{LOAD}_1}^T, y_{\text{LOAD}_2}^T \ldots y_{\text{LOAD}_l}^T]^T
$$

(5-17)

where $n$, $m$ and $p$ are the numbers of state, input, and output variables. The subscripts between parentheses are the sizes of matrices and vectors in this section.

Then, the linearization process at a specific operating point will determine the state, input, and output matrices.
The goal of decomposition into two-time-scale systems is to deploy feedback control to maintain system stability considering the technical issues in section 5.1. Therefore, instead of ordering by slow-fast states, the variables are ordered by their availability for acquisition of measurements or control inputs.

Figure 5-13 shows the process of controller design based on the two-time scale decomposition. After constructing a state space model, the priority of controller design is the decentralized partial feedback control using measurable states without the need for a communication system. If the system is still unstable, state feedback among controlled devices is applied, and the communication system must be involved. Finally, observer-based output feedback is implemented when the state feedback control is not able to stabilize the system. All controller designs based on the two-time scale decomposition will be discussed in the remaining of this section.
The system is described in a linearized state space model, and the eigenvalues of state matrix will be used to determine system stability.

\[
\dot{x}_{(n,1)} = A_{(n,n)}x_{(n,1)} + B_{(n,m)}u_{(m,1)} \\
y_{(p,1)} = C_{(p,n)}x_{(n,1)}
\]

(5-18)

where \(x_{(n,1)}\), \(u_{(m,1)}\) and \(y_{(p,1)}\) are the state, input, and output variables. \(A_{(n,n)}\), \(B_{(n,m)}\) and \(C_{(p,n)}\) are the state, input, and output matrices. The subscripts between parentheses are the sizes of matrices and vectors.

A standard similar transformation, real Schur factorization, is used. There exists a similarity transformation of real Schur factorization, \(A_{L(n,n)} = T_{L(n,n)}A_{(n,n)}T_{L(n,n)}^{-1}\), which is a lower quasi-triangular matrix [75]. The eigenvalues of \(A_{(n,n)}\) are identical with those of \(A_{L(n,n)}\) and will appear on the diagonal of \(A_{L(n,n)}\); complex conjugate eigenvalues of a real \(A_{(n,n)}\) correspond to 2-by-
2 blocks on the diagonal of \( A_L(n,n) \). With proper ordering, the eigenvalues of \( A_L(n,n) \) are placed by magnitudes of real parts, from smallest to largest, on the diagonal. Then, the new states of the similar system represent slow and fast states.

\[
A_L(n,n) = T_L(n,n) A(n,n) T_L^{-1}(n,n) = \begin{bmatrix} A_{L11}(n,s,n) & 0_{(nS,nf)} \\ A_{L21}(n,f,n) & A_{L22}(n,f,n) \end{bmatrix}
\]

\[
B_L(n,m) = T_L(n,n) B(n,m)
\]

\[
x_L(n,1) = T_L(n,n) x(1,n) = \begin{bmatrix} x_{LS}(n,s,1) \\ x_{LF}(n,f,1) \end{bmatrix}
\]

where \( T_L(n,n) \) is the orthogonal change of basis matrix of real Schur factorization. \( x_L(n,1) \) is the state variable that can be classified as slow and fast, \( x_{LS}(n,s,1) \) and \( x_{LF}(n,f,1) \). \( A_L(n,n) \) and \( B_L(n,m) \) are the state and input matrices. \( A_{L11}(n,s,n) \), \( A_{L21}(n,f,n) \) and \( A_{L22}(n,f,n) \) are the block matrices of \( A_L(n,n) \).

Since \( A_L(n,n) \) and \( A(n,n) \) are similar, they will have identical eigenvalues. The matrix \( A_L(n,n) \) is a lower triangular block matrix, and its eigenvalues are the eigenvalues of diagonal block matrices. Since the eigenvalues of \( A_L(n,n) \) are ordered in advance, the slow and fast eigenvalues are decomposed into the eigenvalues of \( A_{L11}(n,s,n) \) and \( A_{L22}(n,f,n) \).

\[
\lambda \left( A(n,n) \right) = \lambda \left( A_L(n,n) \right) = \lambda \left( A_{L11}(n,s,n) \right) \cup \lambda \left( A_{L22}(n,f,n) \right)
\]

where \( \lambda \left( A_{L11}(n,s,n) \right) \) are small real part eigenvalues of slow subsystem of \( A_L(n,n) \). \( \lambda \left( A_{L22}(n,f,n) \right) \) are large real part eigenvalues of fast subsystem of \( A_L(n,n) \).
The state space model is rewritten using the similarity system. That is,

\[ \dot{x}_{L(n,1)} = A_{L(n,n)}x_{L(n,1)} + B_{L(n,m)}u_{(m,1)} \]

(5-21)

Then, state feedback control is deployed to place the eigenvalues of the closed-loop state matrix at the desired locations. Figure 5-14 gives the block diagram of state feedback control. The entire power system is a control plant, while the objective is to find a proper feedback gain, \( K \), to close the loop and improve system stability.

The feedback gain takes the states and feeds to the inputs. The state feedback control corresponding to slow states is given by,

\[ u_{(m,1)} = u_{s(m,1)} + u_{f(m,1)} = -K_{s(m,n,s)}x_{LS(n,s,1)} + u_{f(m,1)} \]

(5-22)

where \( u_{s(m,1)} \) and \( u_{f(m,1)} \) are the classified inputs corresponding to slow and fast states. \( K_{s(m,n,s)} \) is the feedback gain corresponding to slow states.
\[
\begin{bmatrix}
\dot{x}_{LS(n,1)} \\
\dot{x}_{Lf(n,f,1)}
\end{bmatrix} =
\begin{bmatrix}
A_{L11(n,s,n)} & 0_{(n,s,n)} \\
A_{L21(n,f,n,s)} & A_{L2}(n,f,n)
\end{bmatrix}
\begin{bmatrix}
x_{LS(n,s,1)} \\
x_{Lf(n,f,1)}
\end{bmatrix}
+ B_L(n,m) \left( -K_{s(m,n,s)} x_{LS(n,s,1)} + u_{f(m,1)} \right)
\]
\[
= \begin{bmatrix}
A_{L11(n,s,n)} & 0_{(n,s,n)} \\
A_{L21(n,f,n,s)} & A_{L22(n,f,n)}
\end{bmatrix}
\begin{bmatrix}
x_{LS(n,s,1)} \\
x_{Lf(n,f,1)}
\end{bmatrix}
+ \begin{bmatrix}
B_{L1(n,s,m)} \\
B_{L2(n,f,m)}
\end{bmatrix} \left( -K_{s(m,n,s)} x_{LS(n,s,1)} + u_{f(m,1)} \right)
\]
\[
= \begin{bmatrix}
A_{L11(n,s,n)} - B_{L1(n,s,m)} K_{s(m,n,s)} & 0_{(n,s,n)} \\
A_{L21(n,f,n,s)} - B_{L2(n,f,m)} K_{s(m,n,s)} & A_{L22(n,f,n)}
\end{bmatrix}
\begin{bmatrix}
x_{LS(n,s,1)} \\
x_{Lf(n,f,1)}
\end{bmatrix}
+ B_L(n,m) u_{f(m,1)}
\]
\]

The closed-loop state matrix with feedback of slow states is given by,
\[
A_{L,_{closed}}(n,n) = \begin{bmatrix}
A_{L11(n,s,n)} - B_{L1(n,s,m)} K_{s(m,n,s)} & 0_{(n,s,n)} \\
A_{L21(n,f,n,s)} - B_{L2(n,f,m)} K_{s(m,n,s)} & A_{L22(n,f,n)}
\end{bmatrix}
\]

where \( \lambda(A_{L,_{closed}}(n,n)) = \lambda(A_{L11(n,s,n)} - B_{L1(n,s,m)} K_{s(m,n,s)}) \cup \lambda(A_{L22(n,f,n)}) \).

The eigenvalues of slow subsystem can be placed using the slow feedback gain. Choose \( K_{s(m,n,s)} \) to make \( \lambda(A_{L11(n,s,n)} - B_{L1(n,s,m)} K_{s(m,n,s)}) \) stable.

To implement the state feedback of fast subsystem, apply real Schur factorization on \( A_{L,_{closed}}(n,n) \) into an upper quasi-triangular matrix.

\[
A_{U(n,n)} = T_{U(n,n)} A_{L,_{closed}}(n,n) T_{U(n,n)}^{-1} = \begin{bmatrix}
A_{U11(n,s,n)} & A_{U12(n,s,n)} \\
0_{(n,f,n)} & A_{U22(n,f,n)}
\end{bmatrix}
\]

\[
B_{U(n,m)} = T_{U(n,n)} B_L(n,m)
\]

\[
x_{U(n,1)} = T_{U(n,n)} x_{L(n,1)} = \begin{bmatrix}
x_{U11(n,1)} \\
x_{Uf(n,f,1)}
\end{bmatrix}
\]
where $T_{U(n,n)}$ is the orthogonal change of basis matrix of real Schur factorization. $x_{U(n,1)}$ is the state variable that can be classified as slow and fast, $x_{US(ns,1)}$ and $x_{UF(nf,1)}$. $A_{U(n,n)}$ and $B_{U(n,m)}$ are the state and input matrices. $A_{LU}$, $A_{U12(ns,nf)}$ and $A_{U22(nf,nf)}$ are the block matrices of $A_{U(n,n)}$.

Since $A_{U(n,n)}$ and $A_{L,close(n,n)}$ are similar, they have identical eigenvalues. $A_{U(n,n)}$ is an upper triangular block matrix, and its eigenvalues are the eigenvalues of diagonal block matrices. Since the eigenvalues of $A_{U(n,n)}$ are ordered in advance, the small and fast eigenvalues are decomposed into the eigenvalues of $A_{U11(ns,ns)}$ and $A_{U22(nf,nf)}$.

$$\lambda\left(A_{L,close(n,n)}\right) = \lambda\left(A_{U(n,n)}\right) = \lambda\left(A_{U11(ns,ns)}\right) \cup \lambda\left(A_{U22(nf,nf)}\right)$$  \hspace{1cm} (5-26)

where $\lambda\left(A_{U1(ns,ns)}\right)$ are small real part eigenvalues of slow subsystem of $A_{L,close(n,n)}$. $\lambda\left(A_{U22(nf,nf)}\right)$ are large real part eigenvalues of fast subsystem of $A_{L,close(n,n)}$.

The state space model is rewritten using the similarity system, i.e.,

$$
\begin{bmatrix}
\dot{x}_{US(ns,1)} \\
\dot{x}_{UF(nf,1)}
\end{bmatrix} =
\begin{bmatrix}
A_{U11(ns,ns)} & A_{U12(ns,nf)} \\
0_{(nf,ns)} & A_{U22(nf,nf)}
\end{bmatrix}
\begin{bmatrix}
x_{US(ns,1)} \\
x_{UF(nf,1)}
\end{bmatrix} + B_{U(n,m)}u_{f(m,1)}
$$

(5-27)

The state feedback control corresponding to fast states is given by

$$u_{f(m,1)} = -K_{f(m,nf)}x_{UF(nf,1)}$$

(5-28)

where $K_{f(m,nf)}$ is the feedback gain corresponding to fast states.
\[
\begin{bmatrix}
\dot{x}_{U_1(n,s,1)} \\
\dot{x}_{U_2(n,f,1)}
\end{bmatrix} =
\begin{bmatrix}
A_{U_11(n,s,n_s)} & A_{U_12(n,s,n_f)} \\
0_{(n_f,n_s)} & A_{U_2(n,f,n_f)}
\end{bmatrix}
\begin{bmatrix}
x_{U_1(n,s,1)} \\
x_{U_2(n,f,1)}
\end{bmatrix} +
B_{U(n,m)} \left(-K_{f(m,n_f)}x_{U_2(n,f,1)}\right)
\]

\[
= \begin{bmatrix}
A_{U_11(n,s,n_s)} & A_{U_12(n,s,n_f)} \\
0_{(n_f,n_s)} & A_{U_2(n,f,n_f)}
\end{bmatrix}
\begin{bmatrix}
x_{U_1(n,s,1)} \\
x_{U_2(n,f,1)}
\end{bmatrix} +
\begin{bmatrix}
B_{U_1(n,s,m)} \\
B_{U_2(n,f,m)}
\end{bmatrix} \left(-K_{f(m,n_f)}x_{U_2(n,f,1)}\right) 
\]

\begin{align*}
\text{(5-29)}
\end{align*}

\[
= \begin{bmatrix}
A_{U_1(n,s,n_s)} & A_{U_12(n,s,n_f)} - B_{U_1(n,s,m)}K_{f(m,n_f)} \\
0_{(n_f,n_s)} & A_{U_2(n,f,n_f)} - B_{U_2(n,f,m)}K_{f(m,n_f)}
\end{bmatrix}
\begin{bmatrix}
x_{U_1(n,s,1)} \\
x_{U_2(n,f,1)}
\end{bmatrix}
\]

The closed-loop state matrix with feedback of fast states is obtained as

\[
A_{U,\text{closed}}(n,n) = \begin{bmatrix}
A_{U_11(n,s,n_s)} & A_{U_12(n,s,n_f)} - B_{U_1(n,s,m)}K_{f(m,n_f)} \\
0_{(n_f,n_s)} & A_{U_2(n,f,n_f)} - B_{U_2(n,f,m)}K_{f(m,n_f)}
\end{bmatrix} 
\]

\begin{align*}
\text{(5-30)}
\end{align*}

where \(\lambda \left(A_{U,\text{closed}}(n,n)\right) = \lambda \left(A_{U_11(n,s,n_s)}\right) \cup \lambda \left(A_{U_2(n,f,n_f)} - B_{U_2(n,f,m)}K_{f(m,n_f)}\right)\).

The eigenvalues of fast subsystem can be placed using the fast feedback gain. Choose \(K_{f(m,n_f)}\) to make \(\lambda \left(A_{U_2(n,f,n_f)} - B_{U_2(n,f,m)}K_{f(m,n_f)}\right)\) stable.

To implement the full state feedback to the physical states. The full feedback of the physical states is given by,
\[ u_{(m,1)} = -K_{s(m,ns)} x_{L_1(n_s,1)} - K_{f(m,n_f)} x_{U(n_f,1)} \]

\[ = \left[ -K_{s(m,ns)} 0_{(m,n_f)} \right] \begin{bmatrix} x_{L_1(n_s,1)} \\ x_{L_f(n_f,1)} \end{bmatrix} + \left[ 0_{(m,ns)} -K_{f(m,n_f)} \right] \begin{bmatrix} x_{U(n_s,1)} \\ x_{U(n_f,1)} \end{bmatrix} \]

\[ = \left[ -K_{s(m,ns)} 0_{(m,n_f)} \right] \begin{bmatrix} x_{L_1(n_s,1)} \\ x_{L_f(n_f,1)} \end{bmatrix} + \left[ 0_{(m,ns)} -K_{f(m,n_f)} \right] T_{U(n,n)} \begin{bmatrix} x_{L_1(n_s,1)} \\ x_{L_f(n_f,1)} \end{bmatrix} \]

\[ = \left[ -K_{s(m,ns)} 0_{(m,n_f)} \right] T_{L_1(n,n)} x_{(n,1)} + \left[ 0_{(m,ns)} -K_{f(m,n_f)} \right] T_{U_L(n,n)} x_{(n,1)} \]

\[ = \left[ -K_{s(m,ns)} 0_{(m,n_f)} \right] \begin{bmatrix} T_{L_1(n,n)} \\ T_{L_2(n,n)} \end{bmatrix} x_{(n,1)} + \left[ 0_{(m,ns)} -K_{f(m,n_f)} \right] \begin{bmatrix} T_{U_L_1(n,n)} \\ T_{U_L_2(n,n)} \end{bmatrix} x_{(n,1)} \]

\[ = \left( -K_{s(m,ns)} T_{L_1(n,n)} - K_{f(m,n_f)} T_{U_L_2(n,n)} \right) x_{(n,1)} \]

\[ = -K_{(m,n)} x_{(n,1)} \]

where \( T_{U_L(n,n)} = T_{U(n,n)} T_{L(n,n)} \)

Normally, the unstable eigenvalues are close to the origin and associated with the slow subsystem. There is no need to apply feedback control on the fast subsystem if it is already stable. Therefore, the entire feedback gain of the fast subsystem can be set to zero, \( K_{f(m,n_f)} = 0_{(m,n_f)} \), and the overall feedback gain can be simplified. That is,

\[ u_{(m,1)} = \left( -K_{s(m,ns)} T_{L_1(n,n)} \right) x_{(n,1)} \]
The two-time-scale system decomposes the system into two subsystems. The controller design is applied only to the slow subsystem which is a reduced-order system. The method is feasible for a large-scale islanded microgrid, and feedback control is shown to enhance system stability.

5.4.2. Availability of Inputs

In the microgrids, the input variables are the references/setting points of DERs. In practice, some of them might not be compatible with others and impractical to modify for feedback signals, e.g., existing large-scale power plants. Therefore, feedback control is only applied to appropriate inputs. Assume there are $m$ inputs, but only $r$ inputs, $r \leq m$, are accessible and available. Order available inputs from 1 to $r$, and unavailable from $r + 1$ to $m$. The feedback gain of unavailable inputs is set to zero which means no control is fed to unavailable inputs.

$$u_{k(1,1)} = 0_{(1,n)}x_{(n,1)} = 0_{(1,1)}, \ k = r + 1, \ldots, m$$ (5-33)

Rewrite the state space model using available inputs to obtain,

$$\dot{x}_{(n,1)} = A_{(n,n)}x_{(n,1)} + B_{1_{(n,1)}}u_{1_{(1,1)}} + \ldots + B_{r_{(n,1)}}u_{r_{(1,1)}}$$

$$= A_{(n,n)}x_{(n,1)} + \begin{bmatrix} B_{1_{(n,1)}} & \cdots & B_{r_{(n,1)}} \end{bmatrix} \begin{bmatrix} u_{1_{(1,1)}} \\ \vdots \\ u_{r_{(1,1)}} \end{bmatrix}$$ (5-34)

$$= A_{(n,n)}x_{(n,1)} + B_{ALB(n,m_{ALB})}u_{ALB(m_{ALB},1)}$$

where $m_{ALB} = r$, $m_{ALB} = m - m_{ALB} = m - r$. 

Equation (5-31) is applied to choose feedback gains \( K_{s(mALB,ns)} \) and \( K_{f(mALB,nf)} \).

\[
\begin{align*}
\mathbf{u}_{(m,1)} &= \begin{bmatrix}
\mathbf{u}_{ALB(mALB,1)} \\
\mathbf{u}_{UALB(mUALB,1)}
\end{bmatrix} \\
&= \begin{bmatrix}
-K_{s(mALB,ns)} T_{L1(ns,n)} - K_{f(mALB,nf)} T_{UL2(nf,n)} \\
0_{(mUALB,n)}
\end{bmatrix} \mathbf{x}_{(n,1)}
\end{align*}
\]

(5-35)

Apply the simplification, (5-32), to make \( K_{f(mALB,nf)} = 0_{(mALB,nf)} \).

\[
\begin{align*}
\mathbf{u}_{(m,1)} &= \begin{bmatrix}
\mathbf{u}_{ALB(mALB,1)} \\
\mathbf{u}_{UALB(mUALB,1)}
\end{bmatrix} \\
&= \begin{bmatrix}
-K_{s(mALB,ns)} T_{L1(ns,n)} \\
0_{(mUALB,n)}
\end{bmatrix} \mathbf{x}_{(n,1)}
\end{align*}
\]

(5-36)

The overall closed-loop system using simplification and available inputs is given by

\[
\begin{align*}
\dot{x}_{(n,1)} &= \mathbf{A}_{(n,n)} \mathbf{x}_{(n,1)} + \mathbf{B}_{(n,m)} \mathbf{u}_{(m,1)} \\
&= \mathbf{A}_{(n,n)} \mathbf{x}_{(n,1)} + \mathbf{B}_{(n,m)} \begin{bmatrix}
-K_{s(mALB,ns)} T_{L1(ns,n)} \\
0_{(mUALB,n)}
\end{bmatrix} \mathbf{x}_{(n,1)}
\end{align*}
\]

(5-37)

In (5-37), the feedback gain to unavailable inputs is zero, i.e., no control is fed to those unavailable inputs.

5.4.3. Feedback Control Using Measurable States

Although the design of controller is order-reduced under the slow-fast decomposition, the application is based on state feedback, where the states may not be measurable. Then, an observer for output feedback would be needed for state estimation. To avoid the usage of an observer, partial state feedback using measurable states is a reasonable tradeoff. Note that if the system is fully
controllable, all eigenvalues can be assigned with full state feedback. Partial state feedback can only assign a few eigenvalues to the desired locations.

Partial state feedback has been proposed [76], [77]; however, the system is small, and the controller is designed for certain states. In the case of a microgrid, the state matrix could be large in size. Fortunately, in the application of two-time-scale system, the purpose is to stabilize the unstable eigenvalues which are normal in the slow subsystem. Furthermore, the system dynamics are dominated by the slow subsystem, and the fast subsystem is expected to decay quickly. Therefore, partial state feedback is considered to stabilize the slow subsystem without degrading stability of the fast subsystem. Start with full state feedback with available inputs and order the state variables as measurable states and unmeasurable states, i.e.,

\[
\begin{align*}
\mathbf{u}_{ALB}(m_{ALB},1) &= \begin{bmatrix}
-\mathbf{K}_s(m_{ALB},ns) & 0(m_{ALB},nf)
\end{bmatrix} \mathbf{T}_{L(n,n)} \mathbf{x}_{(n,1)} \\
+ &\begin{bmatrix}
0(m_{ALB},ns) & -\mathbf{K}_f(m_{ALB},nf)
\end{bmatrix} \mathbf{T}_{U_L(n,n)} \mathbf{x}_{(n,1)} \\
= &\left(\begin{bmatrix}
-\mathbf{K}_s(m_{ALB},ns) & 0(m_{ALB},nf)
\end{bmatrix} \mathbf{T}_{L(n,n)} \\
+ &\begin{bmatrix}
0(m_{ALB},ns) & -\mathbf{K}_f(m_{ALB},nf)
\end{bmatrix} \mathbf{T}_{U_L(n,n)} \right) \begin{bmatrix}
\mathbf{x}_{MES(n_{MES},1)} \\
\mathbf{x}_{UMES(n_{UMES},1)}
\end{bmatrix} \\
= &-\mathbf{K}(m_{ALB},n) \begin{bmatrix}
\mathbf{x}_{MES(n_{MES},1)} \\
\mathbf{x}_{UMES(n_{UMES},1)}
\end{bmatrix}
\end{align*}
\]  

(5-38)

where \( \mathbf{x}_{MES(n_{MES},1)} \) and \( \mathbf{x}_{UMES(n_{UMES},1)} \) are the measurable and unmeasurable states. The symbols \( n_{MES} \) and \( n_{UMES} \) are the numbers of measurable and unmeasurable states, \( n = n_{MES} + n_{UMES} \).

Since the \( \mathbf{x}_{UMES(n_{UMES},1)} \) is unmeasurable, the feedback gain is set to zero, which means they are not participating in the control algorithm. Consequently,
\[ K_{(m_{ALB,n})} = \begin{bmatrix} K_{s_{(m_{ALB,n}s)}} & 0_{(m_{ALB,nf})} \\ 0_{(m_{ALB,nUMES})} & K_{f_{(m_{ALB,nf})}} \end{bmatrix} T_{L(n,n)} + \begin{bmatrix} 0_{(m_{ALB,nf})} \\ K_{f_{(m_{ALB,nUMES})}} \end{bmatrix} T_{UL(n,n)} \]

\[ = \begin{bmatrix} K_{MES_{(m_{ALB,nMES})}} & 0_{(m_{ALB,nUMES})} \end{bmatrix} \]

where \( K_{MES_{(m_{ALB,nMES})}} \) is the feedback gain using measurable states.

Now, the system is decomposed into two subsystems, measurable and unmeasurable subsystems based on practicality. Apply the derivatives in sections 5.4.1 and 5.4.2, but choose \( ns = n_{MES} \) and \( nf = n_{UMES} \) to match the pattern of feedback gain for partial state feedback. It is obtained that,

\[ \begin{bmatrix} K_{MES_{(m_{ALB,nMES})}} & 0_{(m_{ALB,nUMES})} \end{bmatrix} \]

\[ = \begin{bmatrix} K_{s_{(m_{ALB,nMES})}} & 0_{(m_{ALB,nUMES})} \\ 0_{(m_{ALB,nUMES})} & K_{f_{(m_{ALB,nUMES})}} \end{bmatrix} T_{L(n,n)} 
+ \begin{bmatrix} 0_{(m_{ALB,nUMES})} \\ K_{f_{(m_{ALB,nUMES})}} \end{bmatrix} T_{UL(n,n)} \]

\[ = \begin{bmatrix} K_{s_{(m_{ALB,nMES})}} & 0_{(m_{ALB,nUMES})} \end{bmatrix} \begin{bmatrix} T_{L1(n_{MES,n})} \\ T_{L2(n_{UMES,n})} \end{bmatrix} 
+ \begin{bmatrix} 0_{(m_{ALB,nUMES})} \\ K_{f_{(m_{ALB,nUMES})}} \end{bmatrix} \begin{bmatrix} T_{UL1(n_{MES,n})} \\ T_{UL2(n_{UMES,n})} \end{bmatrix} \]

\[ = \begin{bmatrix} K_{s_{(m_{ALB,nMES})}} T_{L1(n_{MES,n})} + K_{f_{(m_{ALB,nUMES})}} T_{UL2(n_{UMES,n})} \end{bmatrix} \]

Note that, the choice of \( K_{f_{(m_{ALB,nUMES})}} \) is dependent on \( K_{s_{(m_{ALB,nMES})}} \) and \( T_{UL(n,n)} \). To stabilize the slow subsystem, design \( K_{s_{(m_{ALB,nMES})}} \) first, then \( K_{f_{(m_{ALB,nUMES})}} \) is chosen to maintain the pattern of feedback gain for partial state feedback.

Since \( T_{U(n,n)} \) and \( T_{UL(n,n)} \) are orthogonal and real, their inverse is equal to the transpose, respectively.
\[
T_{U(n,n)}^{-1} = T_{U(n,n)}^{T} = \begin{bmatrix}
T_{U11}^{T}(n_{MES,n_{MES}}) & T_{U21}^{T}(n_{MES,n_{UMES}}) \\
T_{U12}^{T}(n_{UMES,n_{MES}}) & T_{U22}^{T}(n_{UMES,n_{UMES}})
\end{bmatrix}
\]

\[
T_{UL(n,n)}^{-1} = T_{UL(n,n)}^{T} = \begin{bmatrix}
T_{UL11}^{T}(n_{MES,n_{MES}}) & T_{UL21}^{T}(n_{MES,n_{UMES}}) \\
T_{UL12}^{T}(n_{UMES,n_{MES}}) & T_{UL22}^{T}(n_{UMES,n_{UMES}})
\end{bmatrix}
\]

From (5-40), the following equation must be satisfied, i.e.,

\[
\begin{bmatrix}
0_{(m_{ALB,n_{MES}})} & K_{f}(m_{ALB,n_{UMES}})
\end{bmatrix}
\]

\[
= \begin{bmatrix}
K_{MES}(m_{ALB,n_{MES}}) & 0_{(m_{ALB,n_{UMES}})}
\end{bmatrix}T_{UL(n,n)}^{-1} - \begin{bmatrix}
K_{S}(m_{ALB,n_{MES}}) & 0_{(m_{ALB,n_{UMES}})}
\end{bmatrix}T_{U(n,n)}^{-1}
\]

\[
= \begin{bmatrix}
K_{MES}(m_{ALB,n_{MES}}) & 0_{(m_{ALB,n_{UMES}})}
\end{bmatrix} \times \begin{bmatrix}
T_{UL11}^{T}(n_{MES,n_{MES}}) & T_{UL21}^{T}(n_{MES,n_{UMES}}) \\
T_{UL12}^{T}(n_{UMES,n_{MES}}) & T_{UL22}^{T}(n_{UMES,n_{UMES}})
\end{bmatrix}
\]

\[
- \begin{bmatrix}
K_{S}(m_{ALB,n_{MES}}) & 0_{(m_{ALB,n_{UMES}})}
\end{bmatrix} \times \begin{bmatrix}
T_{UL11}^{T}(n_{MES,n_{MES}}) & T_{UL21}^{T}(n_{MES,n_{UMES}}) \\
T_{UL12}^{T}(n_{UMES,n_{MES}}) & T_{UL22}^{T}(n_{UMES,n_{UMES}})
\end{bmatrix}
\]

\[
= \begin{bmatrix}
K_{MES}(m_{ALB,n_{MES}})T_{UL11}^{T}(n_{MES,n_{MES}}) & K_{MES}(m_{ALB,n_{MES}})T_{UL21}^{T}(n_{MES,n_{UMES}})
\end{bmatrix}
\]

\[
- \begin{bmatrix}
K_{S}(m_{ALB,n_{MES}})T_{UL11}^{T}(n_{MES,n_{MES}}) & K_{S}(m_{ALB,n_{MES}})T_{UL21}^{T}(n_{MES,n_{UMES}})
\end{bmatrix}
\]

To match the pattern of feedback gain for partial state feedback, two equations are given, i.e.,

\[
0_{(m_{ALB,n_{MES}})} = K_{MES}(m_{ALB,n_{MES}})T_{UL11}^{T}(n_{MES,n_{MES}}) - K_{S}(m_{ALB,n_{MES}})T_{UL11}^{T}(n_{MES,n_{MES}})
\]

\[
K_{f}(m_{ALB,n_{UMES}}) = K_{MES}(m_{ALB,n_{MES}})T_{UL21}^{T}(n_{MES,n_{UMES}}) - K_{S}(m_{ALB,n_{MES}})T_{UL21}^{T}(n_{MES,n_{UMES}})
\]

With design of the slow subsystem, the matrices \(K_{S}(m_{ALB,n_{MES}}), T_{L(n,n)}\), and \(T_{UL(n,n)}\) are known. Therefore, \(K_{MES}(m_{ALB,n_{MES}})\) and \(K_{f}(m_{ALB,n_{UMES}})\) can be computed. Assume \(T_{UL11}(n_{MES,n_{MES}})\) is non-singular. It is obtained that,
Note that, the eigenvalues of the fast subsystem cannot be assigned arbitrarily. However, the choice of slow feedback gain and transformation matrices is not unique, and the proper/optimal design can shift the fast eigenvalues slightly and maintain stability of the fast subsystem.

### 5.4.4. Decentralized Feedback Control

In section 5.4.3, the partial feedback controller is designed to avoid the need for an observer. However, in the application of microgrid, the measurable states and available inputs may be associated with different and dispersed DERs and hence a communication system is needed to send the measurements and control commands.

Here, partial feedback control of the entire system is applied to local control of each DER locally. The partial feedback gain can be designed to be a certain pattern as follows. The left part of block matrix, $K_{MES}$, represents the feedback gain of measurable states. The zero columns on the right represent the unmeasurable states. That is,

$$K_{partial} = \begin{bmatrix} K_{MES} & 0 \end{bmatrix}$$

For local feedback control, the states are feedback to the same DER only.

$$u_{i(m,1)} = -K_{li(m,n)} x_{i(n,1)}$$

where $u_{i(m,1)}$ and $x_{i(n,1)}$ are the inputs and states of $i_{th}$ DER. $K_{li(m,n)}$ is the local feedback gain with respect to the $i_{th}$ DER. Consider the available inputs in section 5.4.2 and the feedback gain
of unavailable inputs is set to zero, \( u_{UALB(m_{UALB},1)} = 0_{(m_{UALB},n)}x(n,1) \). The localized nature of overall feedback gain results in diagonal block matrices.

\[
\begin{bmatrix}
  u_{1(m_{1},1)} \\
  \vdots \\
  u_{r(m_{r},1)} \\
  u_{UALB(m_{UALB},1)}
\end{bmatrix}
\]

\[
\begin{bmatrix}
  -K_{11(m_{1},n_{1})}x_{1(n_{1},1)} \\
  \vdots \\
  -K_{rr(m_{r},n_{r})}x_{r(n_{r},1)} \\
  0_{(m_{UALB},1)}
\end{bmatrix}
= \begin{bmatrix}
  K_{11(m_{1},n_{1})} & \cdots & 0_{(m_{1},n_{r})} & 0_{(m_{1},n_{UMES})} \\
  \vdots & \ddots & \vdots & \vdots \\
  0_{(m_{r},n_{1})} & \cdots & K_{rr(m_{r},n_{r})} & 0_{(m_{r},n_{UMES})} \\
  0_{(m_{UALB},n_{1})} & \cdots & 0_{(m_{UALB},n_{r})} & 0_{(m_{UALB},n_{UMES})}
\end{bmatrix} \begin{bmatrix}
  x_{1(n_{1},1)} \\
  \vdots \\
  x_{r(n_{r},1)} \\
  x_{UMES(n_{UMES},1)}
\end{bmatrix}
\]

(5-47)

Use the technique of partial feedback control to design local feedback gains, \( K_{l_{i}(m_{i},n_{i})} \), one by one repeatedly.
**STEP 1.**

Pre-order the states and inputs of a designed DER by measurable states and available inputs.

\[
\dot{x}_{(n,1)}^{order1} = A_{(n,n)}^{order1} x_{(n,1)}^{order1} + B_{(n,m)}^{order1} u_{(m,1)}
\]

\[
= A_{(n,n)}^{order1} \begin{bmatrix} x_{1(n1,1)} \\ \vdots \\ x_{r(nr,1)} \\ x_{UMES(nUMES,1)} \end{bmatrix} + B_{(n,m)}^{ord} \begin{bmatrix} u_{1(m1,1)} \\ \vdots \\ u_{r(mr,1)} \\ u_{UALB(mUALB,1)} \end{bmatrix}
\]

\[
= A_{(n,n)}^{order} \begin{bmatrix} x_{1(n1,1)} \\ \vdots \\ x_{r(nr,1)} \\ x_{UMES(nUMES,1)} \end{bmatrix} + \left[ B_{1}^{order1} \right]_{(n,m1)} B_{remain(n,m-m1)}^{order} \begin{bmatrix} u_{1(m1,1)} \\ \vdots \\ u_{r(mr,1)} \\ u_{UALB(mUALB,1)} \end{bmatrix}
\]

\[
= A_{(n,n)}^{orde} \begin{bmatrix} x_{1(n1,1)} \\ \vdots \\ x_{r(nr,1)} \\ x_{UMES(nUMES,1)} \end{bmatrix} + B_{1}^{orde} \left[ (n,m1) u_{1(m1,1)} \right]
\]

\[
+ B_{remain(n,m-m1)}^{order1} \begin{bmatrix} u_{2(m2,1)} \\ \vdots \\ u_{r(mr,1)} \\ u_{UALB(mUALB,1)} \end{bmatrix}
\]

where \( x_i \) and \( u_i, i = 1,2, ..., r \), are the measurable state and available input of the same DER.
**STEP 2.**

Design $K_{11(m1,n1)}$ and take $x_1$ and $u_1$ as the only measurable state and available input to design the partial feedback gain. The closed-loop system becomes,

$$
\dot{x}^{order1}_{(n,1)} = A^{order1}_{(n,n)} \begin{bmatrix} x_{1\{n1,1\}} \\ \vdots \\ x_{r\{nr,1\}} \\ x_{UMES\{NUMES,1\}} \end{bmatrix} + B^{order1}_{(n,m1)} u_{1\{m1,1\}} + B^{order1}_{remain\{(n,m-m1)\}} \begin{bmatrix} u_{2\{m2,1\}} \\ \vdots \\ u_{r\{mr,1\}} \\ u_{UALB\{mUALB,1\}} \end{bmatrix} = \left(A^{order1}_{(n,n)} - B^{order1}_{(n,m1)} \begin{bmatrix} K_{11\{m1,n1\}} \\ 0\{m1,n1\} \end{bmatrix} \right) x^{order1}_{(n,1)} + B^{order1}_{remain\{(n,m-m1)\}} \begin{bmatrix} u_{2\{m2,1\}} \\ \vdots \\ u_{r\{mr,1\}} \\ u_{UALB\{mUALB,1\}} \end{bmatrix} = A^{order1}_{closed\{(n,n)\}} x^{order1}_{(n,1)} + B^{order1}_{remain\{(n,m-m1)\}} \begin{bmatrix} u_{2\{m2,1\}} \\ \vdots \\ u_{r\{mr,1\}} \\ u_{UALB\{mUALB,1\}} \end{bmatrix}
$$

(5-49)

**STEP 3.**

Now, $A^{order1}_{closed\{(n,n)\}}$ is considered the open-loop state matrix for the next iteration. Repeat STEP 1. & 2. to design $K_{22\{m2,n2\}} \ldots K_{rr\{mr,nr\}}$. 
**STEP 4.**

The entire input variables contain the feedback of local state variables, i.e.,

\[
\mathbf{u}_{(m,1)} = \begin{bmatrix}
\mathbf{u}_{1(m1,1)} \\
\vdots \\
\mathbf{u}_{r(mr,1)} \\
\mathbf{u}_{ULB(mULB,1)}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{K}_{11(m1,n1)} & \cdots & 0_{(m1,nr)} & 0_{(m1,nUMES)} \\
\vdots & \ddots & \vdots & \vdots \\
0_{(mr,n1)} & \cdots & \mathbf{K}_{rr(mr,nr)} & 0_{(mr,nUMES)} \\
0_{(mULB,n1)} & \cdots & 0_{(mULB,nr)} & 0_{(mULB,nUMES)}
\end{bmatrix}
\times
\begin{bmatrix}
\mathbf{x}_{1(n1,1)} \\
\vdots \\
\mathbf{x}_{r(nr,1)} \\
\mathbf{x}_{UMES(nUMES,1)}
\end{bmatrix}
\]

Note that the overall decentralized feedback gain is the combination of all local feedback gains which are designed independently. Therefore, the eigenvalues are placed individually as well. In the design of a local feedback gain, only \( n_l \) states are used, and, as a result, it can only place \( n_l \) eigenvalues.

### 5.4.5. Limitation of the Proposed Method

The proposed method is used to stabilize the system by a reduced-order controller design. If the number of unstable eigenvalues is larger than the size of reduced-order subsystem, a controller design with a larger or full order is needed. Another limitation is that the partial and decentralized feedback controls in sections 5.4.3 and 5.4.4 only design the controller in slow subsystem and shift the eigenvalues in fast subsystem. In a hybrid-DER microgrid, if the eigenvalues corresponding to SGs and IBRs are distinctly different in scale, i.e., far away from each other, the shift of eigenvalues in fast subsystem will be trivial. However, if a system does not have clear groups of
slow and fast subsystems, the partial and decentralized feedback controls can move the eigenvalues in fast subsystem to right half plane.

5.5. Case Study – Virginia Tech Electric Service System

To test and validate the proposed methods in section 5.4, the Virginia Tech Electric Service (VTES) system is modeled. VTES system is owned and operated by Virginia Tech. Generations served from 4 substations (one substation under construction) connected with Appalachian Power, which is part of American Electric Power (AEP), and an existing synchronous generator in the steam plant on campus. Generations serve both the campus and a portion of the town of Blacksburg. The peak demand of VTES is about 60 MW.

An important goal of the VT Climate Action is to achieve 100% renewable electricity by 2030 [78]. To help achieve the goal, 2 MW solar facility and 10 MW battery energy storage system (BESS) are planned for the VTES system as an intermediate future system condition.

To test the hybrid-DER microgrid, the substations are disconnected with the transmission system to form an islanded microgrid, and three DERs are used to serve two critical loads in the islanded microgrid. The test VTES system is given in Figure 5-15. A 5 MW synchronous generator is at substation D, and a 2 MW solar energy and 4 MW BESS are at substations C and B. There are two critical loads with 1.49 MW/0.72 MVAr and 0.63 MW/0.31 MVAr, respectively. In this study case, the loads are constant power loads.
5.5.1. State-Space Model and Stability Analysis of Study System

The study system model is developed in Simulink/Matlab, and the state-space model can be obtained at a steady-state operating point. The parameters of VTES system and load demands are obtained from the historic data. It is observed that the GFM inverter can provide damping but the inertia difference with SG can still lead into system instability, two IBRs are modeled as GFM inverters for the analysis.

The synchronous generator set includes a 7th order SG model [79], a 1st order governor, and a 2nd order exciter. The block diagrams of governor and exciter are given in Figure 5-16 and Figure 5-17. The input variables of generator set are the frequency and voltage references of governor and exciter, $\omega^SG_0$ and $V^SG_{set}$. The frequency is considered a measurable state, $\omega^SG$. 
Each GFM inverter model has 5 states, including three measurements of the low-pass filter (active power, reactive power, and voltage), electrical angle, and voltage PI control [69]. The block diagram of GFM control is given in Figure 5-18.
The input variables of GFM inverter are the frequency and power references, $V_{set}$ and $P_{set}$. The measurable states are output powers and terminal voltage, $P_{inv}$, $Q_{inv}$, and $V_{inv}$. All measurements include a first-order filters with time constant $T$.

\[
P_{inv} = \frac{1}{T_s + 1} (P_a + P_b + P_c)
\]

\[
Q_{inv} = \frac{1}{T_s + 1} (Q_a + Q_b + Q_c)
\]  \hspace{1cm} (5-51)

\[
V_{inv} = \frac{1}{3(T_s + 1)} (V_{ga} + V_{gb} + V_{gc})
\]

where $P_a$, $P_b$ and $P_c$ are three phase active powers while $Q_a$, $Q_b$ and $Q_c$ are three phase reactive powers. $V_{ga}$, $V_{gb}$ and $V_{gc}$ are three phase voltages.

A droop-based GFM inverter model is used without a Phase-Locked Loop (PLL). The GFM inverter can perform its function by regulating terminal voltage and frequency by droop characteristics without the PLL. On the other hand, GFL inverter requires a PLL for its terminal voltage reference [80]. It is recognized that the PLL can provide additional functionality to a GFM inverter, e.g., resynchronization and control model switching. The study case here is used to validate the feedback control between time-scale-separated SGs and GFM inverters. GFL inverters are not considered.

In total, the system has 20 states, and the SG’s angle will be taken as the reference of state space model, which leads to a zero eigenvalue. Therefore, a reduced model is obtained by removing the reference and zero eigenvalue. The reduced system will have 19 states and 6 inputs. Order the variables as measurable/unmeasurable states and available/unavailable inputs to obtain,
\[
X_{(19,1)} = \begin{bmatrix}
P_{\text{inv}}^{\text{BESS}} & Q_{\text{inv}}^{\text{BESS}} & V_{\text{inv}}^{\text{BESS}} & P_{\text{inv}}^{\text{PV}} & Q_{\text{inv}}^{\text{PV}} & V_{\text{inv}}^{\text{PV}} & \omega^{SG} & X_{\text{UMES}(1,12)}
\end{bmatrix}^T
\]

where \(X_{(19,1)}\) is the state vector that includes the measurable states of BESS, solar system, and SG followed by the rest of unmeasurable states. \(u_{(6,1)}\) is the input vector that includes the inputs of BESS, solar system, and SG.

The state space model is linearized at an operating point. Table 5-2 shows the operating point where linearization is performed. The IBRs provide around 63% of total active power generation.

<table>
<thead>
<tr>
<th></th>
<th>Active Power</th>
<th>Reactive Power</th>
<th>Terminal Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG</td>
<td>0.77 MW</td>
<td>0.52 MVAr</td>
<td>0.994 p.u.</td>
</tr>
<tr>
<td>BESS</td>
<td>0.88 MW</td>
<td>0.34 MVAr</td>
<td>0.996 p.u.</td>
</tr>
<tr>
<td>Solar System</td>
<td>0.44 MW</td>
<td>0.16 MVAr</td>
<td>0.996 p.u.</td>
</tr>
<tr>
<td>Total</td>
<td>2.09 MW</td>
<td>1.02 MVAr</td>
<td></td>
</tr>
</tbody>
</table>

In section 5.3, it is verified that the time constant of IBRs will impact the stability of a hybrid-DER microgrid. In this study case, the time constant of the GFM inverter is represented by the time constant of the first order filter in Figure 5-18.

When the time constant of the GFM inverter is 5 ms, the system is stable. All eigenvalues of state matrix are on the open left half plane. If the time constant is set to 1 ms, the system becomes unstable, and a pair of conjugate eigenvalues appears on the right half plane.
5.5.2. Implementation of Feedback Control

In this section, the feedback control in section 5.4 is implemented to stabilize the study system. Four methods are tested:

1) Method 1 uses two-time scale decomposition to reduce the order of feedback controller design.

2) Method 2 considers the input availability and only feeds to IBRs.

3) Method 3 is the partial state feedback control that only takes measurable states for feedback controller.

4) Method 4 is the decentralized state feedback control that only uses local states to determine its input.

Table 5-3 shows the feedback gains of all methods. The first gain, shown in yellow background, uses all states to all inputs which is a full 6x9 matrix. The second gain, in orange background, takes all states to available inputs only. The last two rows mean the inputs to SG which are considered unavailable inputs. The third gain, in green background, only uses the first 7 columns, which are measurable states. The fourth gain, in blue background, takes the first 3 states to the first 2 inputs of BESS and the states 4–6 to inputs 3–4 of solar system. Note that, the decentralized state feedback takes iteration on each DER until all unstable eigenvalues are placed on the left half plane. In this case, the first iteration already stabilizes the system, which is why the local feedback of solar system is zero.
Table 5-3. Feedback Gains of All Methods

<table>
<thead>
<tr>
<th>OPEN LOOP</th>
<th>METHOD 1</th>
<th>METHOD 2</th>
<th>METHOD 3</th>
<th>METHOD 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1114</td>
<td>-0.1114</td>
<td>-0.1114</td>
<td>-0.1114</td>
<td>-0.1114</td>
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<tr>
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<td>-1.1301</td>
<td>-1.1301</td>
<td>-1.1301</td>
<td>-1.1301</td>
</tr>
<tr>
<td>1.4183+j397.639</td>
<td>-1.906</td>
<td>-1.906</td>
<td>-1.4184+j397.639</td>
<td>-1.3238+j398.1509</td>
</tr>
<tr>
<td>1.4183+j397.639</td>
<td>-8.8253</td>
<td>-8.8253</td>
<td>-1.4184+j397.639</td>
<td>-1.3238+j398.1509</td>
</tr>
<tr>
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<td>-18.0732</td>
<td>-18.0732</td>
<td>-1.906</td>
<td>-1.906</td>
</tr>
<tr>
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<td>-64.8246</td>
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<td>-75.1894</td>
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</tr>
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<td>-99.8428</td>
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<td>-66.6697</td>
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<td>-402.1611+j1657.168</td>
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<td>-408.1335+j1615.8426</td>
</tr>
<tr>
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<td>-402.1611+j1657.168</td>
<td>-402.1611+j1657.168</td>
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<td>-6897.5695</td>
<td>-6897.5695</td>
<td>-3704.7564</td>
<td>-6954.4306</td>
</tr>
</tbody>
</table>

Table 5-4 gives the eigenvalues of the open-loop system and closed-loop of all methods. The open-loop system has a pair of conjugate eigenvalues with positive real parts which is an unstable system. All the feedback methods can place the eigenvalues on the left half plane to stabilize the system. The comparison of methods is shown in Table 5-5.

Table 5-4. Eigenvalues of the Study Case

<table>
<thead>
<tr>
<th>OPEN LOOP</th>
<th>METHOD 1</th>
<th>METHOD 2</th>
<th>METHOD 3</th>
<th>METHOD 4</th>
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</thead>
<tbody>
<tr>
<td>-0.1114</td>
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<td>-0.1114</td>
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</tr>
<tr>
<td>-1.1301</td>
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<td>-1.906</td>
<td>-1.906</td>
<td>-1.4184+j397.639</td>
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<td>-1.906</td>
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<tr>
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<td>-75.1894</td>
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<td>-45.8526</td>
</tr>
<tr>
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Table 5-5. Comparison of Proposed Methods

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5.5.3. Analytical Results and Time Domain Simulation

In this section, the analytical results and time domain simulation in Simulink/Matlab are presented. Figure 5-19 gives the impulse response of the study case. Since the open-loop system has unstable eigenvalues, the response diverges eventually. All the feedback methods can bring system to a steady state. Method 3 and method 4 take longer for the ripples to settle.

![Impulse Response](image)

Figure 5-19. Impulse Responses of the Study Case
Figure 5-20, Figure 5-21, and Figure 5-22 display the results of time domain simulations for a transient event in which a heavy load of 1.63 MW and 0.79 MVAr is picked up at 50 seconds. In Figure 5-20, the system becomes unstable and the frequency and active power fail to converge when the time constant of IBRs is small, 1 millisecond. In contrast, Figure 5-21 shows that increasing the time constant to 5 milliseconds stabilizes the system. Finally, in Figure 5-22, the effectiveness of method 4 is tested under a 1 millisecond time constant of IBR without an observer or a communication system. Despite the presence of ripples, the results demonstrate that method 4 can stabilize the system and successfully go through the transient event.

Figure 5-20. Open-Loop System at 1-millisecond IBR Time Constant
5.5.4. Resilience Enhancement by Feedback Control on VTES System

In sections 5.5.2 and 5.5.3, the instability in VTES system is observed, and the proposed feedback is applied to stabilize the system. Figure 5-23 shows enhanced resilience by deploying feedback control on the VTES system. Without feedback control, the system can be unstable.
Therefore, DERs are not able to pick up critical loads and the post-event degraded state ($\Delta T_3$) will be prolonged. The system needs to wait until infrastructure recovery to restore and serve loads. Instead, with feedback control, three DERs are coordinated, and the system is stable. Therefore, the microgrid will serve the critical loads under islanded mode ($\Delta T_4$ and $\Delta T_5$) and enhance system resilience. Resilience enhancement can be approximated by the MWh served during an outage event. The DERs provide 2.09 MW during the outage. Assume the damage assessment takes 30 minutes ($\Delta T_2$ and $\Delta T_3$) before VTES field crew starts to control DERs. The restorative state ($\Delta T_4$) is ignored here as IBRs can initiate and serve loads quickly. The post-restoration state ($\Delta T_5$) is about 208.5 minutes (3.475 hours) [81]. Therefore, the resilience is enhanced by 7.26 MWh (2.09 MW $\times$ 3.475 hours).

Figure 5-23. System Resilience w/o and w/ Feedback Control
Chapter 6

Conclusion and Future Work

In this research, the resilience, dynamics, and stability of islanded microgrids are studied. The microgrid control strategies and the need to enhance system resilience are discussed. System resilience enhancement by enabling microgrids to operate in an islanded mode is investigated. The stability of a cyber-physical microgrid and a hybrid-DER microgrid is analyzed and methods for maintaining system stability are proposed.

6.1. Conclusion

A coordinated microgrid control scheme is developed and validated in this study. The control scheme provides both regulation and dispatch capabilities for the microgrid system. The regulation capability maintains the system frequency and voltage within acceptable ranges as the load or system condition varies. The active and reactive power dispatch mechanism ensures that the output power of DERs is proportional to their capacity. Two transient stability events are used to evaluate dynamic performance on the modified IEEE 13-node system. The simulation results indicate that microgrid system stability is maintained in a resiliency mode.

Also, the impact of cyber latency on microgrid stability is studied. An analytical method based on the networked control system model is proposed. The method integrates the communication and power system models. An equivalent matrix is developed to reduce the computational burden and is feasible for large-scale power systems. Stability regions are obtained in the space of data acquisition reporting period and communication delay. This analytical method provides a
systematic method to determine the stability regions and the critical threshold for cyber latency. The method is applied to the modified IEEE 13-node system, and time-domain simulation is used to validate the results of this analytical method.

Furthermore, the technical challenges of a microgrid with a high penetration of renewable energy are discussed. The operational issues and system stability of islanded microgrids are analyzed. The inverter control is illustrated to analyze the stability of a hybrid-DER microgrid. The results show the stability of islanded microgrids is caused not only by the low inertia but also the inertia difference among DERs.

Feedback control is proposed to improve system stability. Two-time-scale decomposition is used to reduce the order for controller design. Considering the operational limitations, different feedback control laws are developed, i.e., partial state feedback and decentralized/local feedback. The VTES system is used as a study case for validation of the proposed methods. The analysis verifies that the small time constant of IBR can increase the inertia difference between SG and IBRs, leading to instability. The proposed feedback control methods are applied to the VTES system. Time domain simulation shows that the proposed method is able to stabilize the system following disturbances.

6.2. Future Work

The study on system dynamics and stability of a cyber-physical microgrid is used to improve the design of microgrid control and communication systems. It is important to develop a power system with a realistic communication model. In a real-world situation, prolonged cyber latency can occur as a result of a failure of the cyber system or malicious cyberattacks. Future work is to implement the control scheme on a realistic distribution system and transmit data via a complete
communication model to evaluate the behaviors of the cyber layer. A cybersecurity monitoring system for the cyber-physical microgrid will be important. Communication system modeling and cyberattack scenarios need to be developed. A co-simulation between the power system and the communication system can be implemented to observe the interactions between the two layers.

The future work for the hybrid-DER microgrid can include modeling of the two-time-scale system as a singularly perturbed system, which will study the boundary of time constant for system stability and will determine if the time scale separation technique is feasible or not. Then, optimal feedback control design can be developed. For example, Linear Quadratic Regulator (LQR) can be used to design the optimal feedback gain at a minimum cost. Furthermore, a participation factor analysis indicates the participation level of states in unstable eigenvalues that helps to find a proper order for the state matrix and the optimal transformation matrix. The test system can be more comprehensive. In this research, the DERs used in the VTES system include one synchronous generator and two grid-forming inverters. A system includes multiple synchronous generators, grid-forming inverters, and grid-following inverters can be developed. Finally, unlike decentralized feedback control, a centralized control algorithm requires a communication system. The stability of a hybrid-DER microgrid including feedback control loops as a cyber-physical system needs to be studied.
References


Appendix A

Assume the state matrix $A$ is non-singular, and $\lambda$ is non-zero eigenvalue of the transition matrix in (4-2). Then, the following equation is satisfied.

$$
\det[(\Phi(k) - \lambda I_{n+m}] = \det\begin{bmatrix}
\Phi_{11} - \lambda I_n & \Phi_{12} \\
\Phi_{21} & -\lambda I_m
\end{bmatrix}
$$

$$
= 0
$$

(A. 1)

Since $\lambda$ is non-zero eigenvalue, $-\lambda I_m$ is invertible. Hence,

$$
\det\begin{bmatrix}
\Phi_{11} - \lambda I_n & \Phi_{12} \\
\Phi_{21} & -\lambda I_m
\end{bmatrix}
= \det(-\lambda I_m)\det[(\Phi_{11} - \lambda I_n) - \Phi_{12}(-\lambda I_m)^{-1}\Phi_{21}]
$$

$$
= 0
$$

(A. 2)

Since $\det(-\lambda I_m)$ is non-zero,

$$
\det[(\Phi_{11} - \lambda I_n) - \Phi_{12}(-\lambda I_m)^{-1}\Phi_{21}]
= \det\left(\Phi_{11} + \frac{1}{\lambda}\Phi_{12}\Phi_{21} - \lambda I_n\right)
$$

$$
= 0
$$

(A. 3)

Multiplying on both side of (non-zero term, $\det(\lambda A)$, it is obtained that

$$
\det(\lambda^2 A - \lambda A\Phi_{11} - A\Phi_{12} \Phi_{21}) = 0
$$

(A. 4)

To eliminate the integration of exponential of state matrix, the following identity is used.

$$
X\left(\int_0^T e^{xt} dt\right) + I = e^{xT}
$$

(A. 5)

Then, equation (A. 4) can be transformed. That is,
\[ \det(\lambda^2 A - \lambda A\Phi_{11} - A\Phi_{12}\Phi_{21}) \]
\[ = \det \left( \begin{array}{c}
\lambda^2 A \\
-\lambda \left[ A e^{Ah} - A \int_0^{h-\tau} e^{As} \, ds BK \right] \\
+ A \int_0^{h-\tau} e^{As} \, ds BK 
\end{array} \right) \]
\[ = \det \left( \begin{array}{c}
-\lambda A e^{Ah} + \lambda \left[ A \int_0^{h-\tau} e^{As} \, ds + I_n \right] BK - \lambda BK \\
+ \left[ A \int_0^{h-\tau} e^{As} \, ds + I_n \right] BK - \left[ A \int_0^{h-\tau} e^{As} \, ds + I_n \right] BK 
\end{array} \right) \]
\[ = \det \left( \begin{array}{c}
A\lambda^2 \\
+ (e^{A(h-\tau)} BK - BK - A e^{Ah}) \lambda \\
+ (e^{Ah} - e^{A(h-\tau)}) BK 
\end{array} \right) \]
\[ = \det(A) \times \det \left( \begin{array}{c}
I_n \lambda^2 \\
+ A^{-1}(e^{A(h-\tau)} BK - BK - A e^{Ah}) \lambda \\
+ A^{-1}(e^{Ah} - e^{A(h-\tau)}) BK 
\end{array} \right) \]
\[ = 0 \]

Since \( \det(A) \neq 0 \), and \( A^{-1}, e^{Ah}, e^{A(h-\tau)} \) are commutative.

\[ \det \left( \begin{array}{c}
I_n \lambda^2 \\
+ A^{-1}(e^{A(h-\tau)} BK - BK - A e^{Ah}) \lambda \\
+ A^{-1}(e^{Ah} - e^{A(h-\tau)}) BK 
\end{array} \right) \]
\[ = \det \left[ \begin{array}{c}
e^{Ah} - \lambda I_n \\
I_n \\
L_n - e^{A(h-\tau)} A^{-1} BK - \lambda I_n 
\end{array} \right] \]
\[ = 0 \]

Therefore, \( \lambda \) is non-zero eigenvalue of the equivalent matrix given by

\[ \tilde{\Phi}'(k) = \left[ \begin{array}{c}
e^{Ah} \\
I_n \\
I_n - e^{A(h-\tau)} A^{-1} BK 
\end{array} \right] \] (A. 8)