New Approach to finding Active-element Patterns for Large Arrays

Alan Larkin O’Donnell

Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science
In
Electrical and Computer Engineering

William Davis
Warren Stutzman
Majid Manteghi

25 April 2013
Blacksburg, Virginia

Keywords: Active-element Patterns, Large Arrays, Floquet
New Approach to finding Active-element Patterns for Large Arrays

Alan O’Donnell

ABSTRACT

In this study a new approach to active-element pattern analysis, for large phased array antennas, was created using Floquet’s theorem. The classic approach to finding active-element patterns uses a full array simulation that can become slow and produce patterns that are specific to certain elements in the array, though basically identical away from the array edge. Instead of producing specific active-element patterns an average active-element pattern could be created and then applied that to the array.

The average active-element pattern can be used for every element in the array with a small margin of error. Using Floquet’s theorem reduces any differences between elements in the array and gives the most accurate active-element pattern within a reasonable time constraint. Floquet average active-element patterns are computed by using an infinite array and a summation is done for the far-field radiation values of a finite array based on the number of elements using typical pattern multiplication techniques. Therefore, accuracy of the Floquet element approach is excellent for arrays on the size of hundreds to thousands of elements.

An active-element pattern is determined by scanning the array and taking the far-field radiation value at each beam scan-angle. Each beam scan-angle value is a summation of the element radiation patterns in that specific direction. These beam scan-angle values are then reduced by the number of elements in the array to form a radiation pattern. This radiation pattern is the average active-element pattern.
DEDICATION

For my parents, Amy and Chris O’Donnell and my fiancé and soon to be wife Britni Brown.
ACKNOWLEDGEMENTS

I would like to thank Dr. William A. Davis for the support and the guidance that he has given during this research. Without his guidance this research and the funding needed for it would have been much more difficult to do and procure.

The research presented in this thesis was funded by NSWC Dahlgren.
# TABLE OF CONTENTS

DEDICATION ................................................................................................................................................................. iii

ACKNOWLEDGEMENTS ................................................................................................................................................ iv

TABLE OF CONTENTS ........................................................................................................................................................ v

LIST OF FIGURES ........................................................................................................................................................... vii

LIST OF TABLES ............................................................................................................................................................. viii

1 INTRODUCTION ............................................................................................................................................................ 1
   1.1 RESEARCH MOTIVATION ........................................................................................................................................ 1
   1.2 THESIS ORGANIZATION .................................................................................................................................. 2

2 BACKGROUND ............................................................................................................................................................... 4
   2.1 IDEAL ARRAY ....................................................................................................................................................... 4
   2.2 NON-IDEAL ARRAY ........................................................................................................................................ 6
   2.3 ACTIVE AND FREE-SPACE ELEMENT PATTERNS ............................................................................................. 7
   2.4 FINDING AN ACTIVE-ELEMENT PATTERN ................................................................................................. 8
   2.5 COMPUTATION METHOD OF SOLVING USUAL APPROACH ................................................................................. 9
   2.6 REDUCTIONS IN COMPUTATIONS FOR LARGE ARRAYS .................................................................................. 10
   2.7 FLOQUET THEOREM FOR REDUCTIONS IN COMPUTATIONS ........................................................................ 11

3 METHOD AND RESOURCES .......................................................................................................................................... 12
   3.1 METHOD OVERVIEW ...................................................................................................................................... 12
   3.2 USED FEKO OVER HFSS .............................................................................................................................. 15
   3.3 CREATE FLOQUET CELL AND FAR-FIELD POINT ...................................................................................... 17
   3.4 MODIFY DATA IN MATLAB ......................................................................................................................... 19
   3.5 RETURN MODIFIED DATA TO FEKO AS AVERAGE ACTIVE-ELEMENT PATTERN ............................................. 20

4 PROCEDURE ................................................................................................................................................................. 21
   4.1 SIMULATION COMPARISON SETUP ............................................................................................................... 21
   4.2 SIMULATION OUTPUT SETUP .......................................................................................................................... 22

5 RESULTS ....................................................................................................................................................................... 24
   5.1 DIFFERENCE IN COMPUTATION TIME ......................................................................................................... 24
   5.2 ACTIVE-ELEMENT PATTERN OVERLAYS ..................................................................................................... 26
      5.2.1 PHI CUT OF 0 DEGREES .................................................................................................................. 26

V
LIST OF FIGURES

Figure 2:1 Free-space pattern for a half wave dipole antenna and Active-element pattern for a 5x5 array of half wave dipole antennas ............................................................................................................. 8
Figure 2:2 Basic set up for finding the active-element pattern of an array element in a linear array ........................................................................................................................................................................... 9
Figure 3:1 Flow chart of the procedure used to find the active-element pattern of a large array. 15
Figure 3:2. Three dimensional radiation pattern for a half wave dipole created from FEKO, HFSS, and Matlab. ....................................................................................................................................................... 16
Figure 3:3. Two dimensional radiation pattern cut at a phi angle of zero degrees using FEKO and HFSS ............................................................................................................................................................................ 16
Figure 3:4 Images of a Floquet array (left) and a 25 element array (right) ........................................ 18
Figure 5:1. Average active-element patterns from each of the array types and sizes and half wave dipole free-space pattern overlayed for a phi angle cut of zero degrees. ........................................ 28
Figure 5:2. Average active-element patterns from each of the array types and sizes and half wave dipole free-space pattern overlayed for a phi angle cut of 90 degrees. .............................................................................................................. 30
Figure 5:3. Plot of the difference between the average active-element patterns from the 121 full element array and the Floquet array when looking at the phi angle cut of zero degrees.................. 32
Figure 5:4. Plot of the difference between the average active-element patterns from the 121 full element array and the Floquet array when looking at the phi angle cut of 90 degrees. ............... 33
LIST OF TABLES

Table 5-1. Computation power and times needed to gather the far-field values for all the array types and sizes needed for the research using a 2 GHz processor .......................................................... 25
1 INTRODUCTION

1.1 RESEARCH MOTIVATION

When analyzing large arrays there are many factors that go into determining how the far-field pattern of the array will be created. The two main factors that go into determining the far-field pattern of the array are the array factor and the element pattern. The array factor is the most noticeable feature in the far-field pattern of an array but the element pattern modifies the pattern in ways that cause the main beam to not be in the correct beam scan-angle direction and nulls to show at angles that are unexpected. Therefore, finding the element pattern when analyzing an array is important.

The issue with finding the element pattern in a large array is that the element pattern is not limited to the single isolated element pattern. The element pattern has to take into account how the energy emitting from a single element reacts with the adjacent elements and reradiates affecting the pattern of the single element. This single element pattern that takes into account the reradiating effect, mutual coupling, is called an active-element pattern.

The issue with finding this active-element pattern by the usual approach to finding the pattern for large arrays is time consuming and requires computation power that cannot be achieved by most computers. Computation power for most simulations of large arrays is extreme due the large structure that has to be virtually created and processed each time there is a simulation done. Therefore, when looking at arrays on the order of hundreds to thousands of elements the simulation process will require each one of the those elements to be placed, the current density evaluated, and the radiation pattern computed for each element, taking into account the mutual coupling for all elements. The computation power is lower for simplistic element arrays, such as dipoles, but becomes more apparent on elements that require larger surface area and complex structures such as the Vivaldi or open-ended waveguide antennas and can only be achieved through the use of super computers, it at all. This extreme computation power is also linked to a very time consuming process. For smaller arrays on the order of hundreds it can take several hours, but for arrays of thousands of element the computation time could be days, depending on the computer system.
This research was focused on finding a new technique to find active-element patterns for large arrays that would reduce the computation time and power problem without sacrificing accuracy in the production of an active-element pattern. The new technique would have to be something that could be implemented on a personal computer and not stress the system. The new technique would also allow the simulation to finish in a reasonable amount of time so that the pattern could be used in first order design work of a large array or to allow the implementation of the pattern to save time when used for other projects.

1.2 THESIS ORGANIZATION

This thesis will discuss the new technique to finding an average active-element pattern for large arrays. This technique will focus on the use of Floquet theory to create a large array without having to simulate each element. This Floquet theory will is already implemented in the FEKO antenna modeling package and will used for all computation in this research.

The second chapter will discuss the needed background knowledge to allow for the development of the technique. This background knowledge includes the computation of array patterns and how free-space element patterns are different from active-element patterns. Chapter 2 will also discuss how Floquet theorem can be implemented into an array scenario and what the limitations are. Chapter 3 will provide information on methods that will be used during this research. These methods include the overall procedure flow, which antenna modeling package will be used for the array calculations, how the antenna model will be setup and how the resulting model data will be used to find the wanted active-element pattern. Chapter 4 presents how the array simulations will be setup specifically for this project. This chapter focuses on what type of array structure will be used, the element type in the array, number of elements, and how the simulation outputs are setup. Chapter 5 compares the results of the simulations and gives explanation to possible difference and where the best comparisons are between a full element array and a Floquet array. In this chapter there is also a comparison done on the computation power and time needed to complete the simulation for the full element and Floquet arrays. Finally, chapter 6 will summarize the important aspects of the thesis, which are the reductions to the computation time and power and what is the least number of elements that can be in an array for the Floquet technique to be used. The last chapter will also discuss possible research for the
future that is linked to this thesis and where precautions should be taken when using this technique.
2 BACKGROUND

This chapter introduces the background need to understand the new active-element pattern technique and why this new technique is able to be applied to large arrays. There will be a discussion on what makes up a far-field pattern for an ideal array and how that pattern changes when the array is non-ideal. The difference between an active-element pattern and a free-space element pattern is shown and the usual method for finding active-element patterns is described. Last, there is an explanation of the assumptions and reductions in the amount of computation needed for large arrays and how those reductions allow Floquet theory to be applied.

2.1 IDEAL ARRAY

When solving for the radiation pattern of any current-based antenna the first equation that needs to be solved or created is the magnetic vector potential function. This magnetic vector potential function is the basis for finding the electric and magnetic far-field radiation equations. The magnetic vector potential \( \overline{A}(\vec{r}) \) for a single antenna with an arbitrary structure is (Stutzman, pg. 15-20, 1998)

\[
\overline{A}(\vec{r}) = \iiint_{v'} \mu \overline{J} e^{-j \beta R} \frac{4 \pi}{d v'}
\]

(1.1)

were \( \overline{J} \) is the electric current density on the antenna. For an array of arbitrary antennas the only difference from equation 2.1 is that the magnetic vector potential is a summation of all the elements that are radiating in the array. Therefore, \( \overline{A}(R) \) for a two dimensional planar array of arbitrary shaped antennas is

\[
\overline{A}(\vec{r}) = \iiint_{v'} \sum_{m=1}^{M} \sum_{n=1}^{N} \mu \overline{J}_{mn} e^{-j \beta R} \frac{4 \pi}{d v'}
\]

(1.2)

This thesis is focused on finding that far-field active-element pattern of an array and in the far-field of any antenna certain approximations can be applied to reduce the calculations. The variable \( R = r - r^{'} \cdot \hat{r} \) when looking at the entire electromagnetic environment starting from the
area outside the structure of the antenna, but in the far-field of the antenna structure \( R \approx r \) in the
denominator due to the fact that the difference between the observation point \( r \) and the source
point \( r' \) is negligible. This approximation makes the array magnetic vector potential
\[
\overline{A}(r) = \sum_{m=1}^{M} \sum_{n=1}^{N} \mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{V'} J_{mn}(r') e^{-j\beta_{mn} r'} dv',
\]
(1.3)

The array magnetic vector potential is not enough to understand the far-field pattern as
mentioned before Equation 2.1 but only the stepping stone to finding it. The far-field pattern will
need to be analyzed from the electric far-field of the array. The electric field for the array of
antennas with arbitrary structures is
\[
\overline{E} = \frac{1}{\mu} \left( \nabla \times \nabla \times \overline{A}(r) \right) - \overline{J}(r)
\]
(1.4)

In the far-field of the array the electric current density \( \overline{J}(r) \) can be ignored since the far-field is
in the source free region of the array. This simplification makes the electric far-field equation
\[
\overline{E} = \frac{1}{j\omega\varepsilon} \nabla \times \nabla \times \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{e^{-j\beta r}}{4\pi r} \iiint_{V'} J_{mn}(r') e^{j\beta_{mn} r'} dv',
\]
(1.5)

The current density \( \overline{J}_{mn}(r') \) can be simplified if it is assumed that the current density on the
elements is the same for each element. This assumption of identical current densities makes
\( \overline{J}_{mn}(r') = I_{mn} \overline{J}_0(r') \). This change in the electric far-field equation simplify to
\[
\overline{E} = \frac{e^{-j\beta r}}{j\omega\varepsilon 4\pi r} \left( \nabla \times \nabla \times \iiint_{V'} J_0(r') e^{j\beta_{mn} r'} dv' \right) \left( \sum_{m=1}^{M} \sum_{n=1}^{N} I_{mn} e^{j\beta_{mn} r'} \right)
\]
(1.6)

The electric far-field can be factored into two separate equations. The first equation is the
element pattern for an isolated element in the array. The isolated element pattern is completely
governed by the element structure and the current density on the element. This isolated element
pattern is
\[
g(\theta, \varphi) = \nabla \times \nabla \times \iiint_{V'} J_0(r') e^{j\beta_{mn} r'} dv',
\]
(1.7)

The second equation that can be factored from the electric far-field equation is the array factor
and this equation is governed by number of elements in the array. This array factor is
Using the two new equations that were factored from equation 2.6 one can create an un-normalized far-field radiation pattern for an array. The un-normalized radiation pattern is

\[ F(\theta, \varphi) = g(\theta, \varphi) AF(\theta, \varphi) \]  \hspace{1cm} (1.9)

The ability of an array far-field to be factored into this form allows the element pattern and the array factor to be found separately. Therefore, if one was to analysis the far-field pattern of any array the array factor could be removed and the resulting pattern would be the element pattern of the antennas that make up the array, and vice versa.

2.2 NON-IDEAL ARRAY

In the last section the there is an assumption made about arrays that cannot be used when analyzing non-theoretical arrays. This incorrect assumption is that the radiation pattern produced by the array does not have any other effects besides the array factor and the element pattern. Realistically there are several environmental effects that have to be taken into account when designing an array. The most important effect to be aware of and take into account when finding the far-field pattern of an array is the mutual coupling effects between radiating elements.

For non-ideal arrays the elements cannot be considered isolated from each other or the array structure. When the radiating elements are placed in close vicinity to each other there is always going to be coupling of energy between the elements and any structure that has the ability to conduct electricity. The coupling of energy between elements is called mutual coupling and affects the overall current density on each element. By affecting the current density the element pattern, Equation 2.7, is changed and then the overall array far-field pattern is changed as well due to Equation 2.9.

With this knowledge that mutual coupling will affect the far-field pattern of the array it would be advantages to use array calculation methods that would include this effect. The mutual coupling of an array can be characterized in two ways (Oliner, 1966). The first way is to use a mutual impedance matrix. However, this mutual impedance matrix does not give the user an active systems representation, which is what, is wanted for this research. The second method is an array characterized by a scattering matrix. This scattering matrix gives the user an “active”
view of the array because it includes the re-radiation effects of all the other elements in the array on the single live or “active” element. This active-element pattern can be found in other ways besides using a scattering matrix though. The scattering matrix does a good job a determining the effects from an entire array on a single element but it is slow to calculate. The faster method is to simple measure the three-dimensional, far-field radiation pattern of a single fed element with all the other elements terminated in a matched generator load. This measured radiation pattern is called an active-element pattern and is the true radiation pattern that the live elements in an array have.

2.3 ACTIVE AND FREE-SPACE ELEMENT PATTERNS

Antenna array far-field patterns have two parts that make up the pattern that is seen. The first part is the array factor which is determined by the number of elements and their spacing. The second is the element pattern and has a large effect on the overall far-field pattern.

When first taught about arrays, the element pattern is the simple free-space pattern of the specific antenna type. However, once an element is placed near other radiating elements it is no longer in free-space and the pattern does not give an accurate representation pattern that should be used with the array factor. The pattern that should be used is one the takes into the account the effects of the adjacent elements around the specific antenna and uses that to modify its far-field pattern. This modified single element far-field pattern is called an active-element pattern and takes into account all the mutual coupling of the elements and gives the array far-field pattern a more accurate representation.

Figure 2:1 below shows active and free-space element patterns (right and left respectively) produced by half wavelength dipoles but the active-element pattern is achieved from a 5x5 array of the antennas placed about a half wavelength away from each other and terminated at each element by a generator load. The free-space element pattern is the expected donut pattern that all dipole antennas have. The active-element pattern does have some of the same features of the dipole pattern, but also has mutual coupling effects that make the pattern no longer a rounded donut shape. Due to the mutual coupling from the surrounding elements the active-element
pattern has more peaks and null areas, mainly in the Y-axis direction (horizontal) due to the larger surface area that the elements have to couple onto in that direction.

![Figure 2:1 Free-space pattern for a half wave dipole antenna and Active-element pattern for a 5x5 array of half wave dipole antennas](image)

Now that the difference has been seen between free-space and active-element patterns, one has to think about the consequences of using the free-space pattern instead of the active-element pattern in simulations and design work of arrays. The two patterns are clearly different and have different peaks and nulls but the overall dB difference is not that much, usually only about 1 dB or less. This may not seem very much at all; but when the pattern is summed several hundred times in a specific beam point-angle direction to find the far-field radiation level, the difference is also several hundred times larger. The difference in the nulls and peaks can also cause the main beam pointing angle to not be exactly where you want it to be due to the pattern not summing in the way it theoretically should using a free-space element.

### 2.4 FINDING AN ACTIVE-ELEMENT PATTERN

Active-element patterns have been defined in different ways and it is important to define what is meant by active-element pattern when stated in this paper. An active-element pattern is the radiation pattern of a single live element when all elements are terminated at the generator load, which is typically determined from an infinite array excitation. The active-element pattern emerges from the single live-element radiation and the mutual-coupling effects of the adjacent elements on that radiation. Figure 2:2 gives the basic setup for finding the active-element pattern.
of a single element in a linear array. The Z is the generator load and there is only one element that is attached to a voltage source. Once this setup is created one has to do a three-dimensional radiation, far-field pattern to find the active-element pattern, and then repeat the process for each element in the array.

![Diagram of linear array](image)

**Figure 2.2** Basic set up for finding the active-element pattern of an array element in a linear array

The method of finding an active-element pattern described above is the usual approach to finding an active-element pattern. There are other approaches to finding the active-element pattern to an array which are less time consuming than testing each antenna separately. Instead of looking at a single element at a time one can assume that the element pattern that is used with the array factor to create the array far-field pattern includes the mutual-coupling effects and the pattern is called an average active-element pattern. This average active-element pattern is a pattern that can be applied to most of the elements in the array if certain precautions are taken and reduces the need to find the active-element pattern for all the elements. Instead of measuring each element all that needs to be done is look at the far-field of the array, remove the array factor, and one is left with the average active-element pattern.

### 2.5 COMPUTATION METHOD OF SOLVING USUAL APPROACH

The usual way of computing active-element patterns is to do a simulation for each element following the first approach mentioned in Section 2.2. This approach requires that the entire array be built in a computer assisted design (CAD) package. Once the array has been built in a CAD package the entire structure will have to meshed with triangles or wire segments, that are
electrically small compared to the operating frequencies wavelengths. These triangles are needed so that the currents over the elements can be solved using a method of moments approach (Harrington, 1968). After the current has been obtained, the far-field pattern can be solved using those solved currents. When looking at the results, a complete spherical far-field pattern is needed. This computation process is then done for each element in the array, which could be over a 1000 times for large arrays.

As mentioned before there are two problems with this approach to finding active-element patterns for elements in a large array. The first problem is that the computation power needed to mesh and calculate the current for a large array is only really achievable by computers that have massive computation power in the form of RAM or processing power. This processing power makes it unrealistic to do first-order design or analysis work of antenna arrays. The second problem is the time it takes to complete all the calculations is quite extreme, once again making the computation method unwanted when designing or analyzing large arrays.

2.6 REDUCTIONS IN COMPUTATIONS FOR LARGE ARRAYS

Large arrays with elements numbering in the hundreds have properties that allow for some reductions in the number of calculations needed to achieve an accurate far-field and active-element pattern. There are a few precautions that have to be taken for these reductions to work properly. First, the elements in the array have to be identical; there cannot be difference in polarization or structure. Second, all the elements have to be equally spaced, making the structure of the array periodic.

With precautions taken in the large array the first reduction in calculating an active-element pattern is that one active-element pattern can be found and applied to all elements except those at the edges. This pattern that can be applied to all elements in a periodic large array is called an average active-element pattern (Stutzman, pg. 125-128, 1998). This pattern can be applied to all elements because, due to the large number of elements in the array, the average environment that each antenna is exposed to is the same. Therefore, by looking in the far-field pattern and removing the array factor, an average active-element pattern will emerge that will be faster to calculate and apply than finding all active-element patterns individually. The average active-
element pattern does have some issues with being applied to the elements that are placed towards the outside of the array.

Another reduction in computation, when finding the active-element pattern, is that large arrays do not have to take into account the edge effects of the array. Edge effects for arrays consist of reflected surface waves and the un-uniformity of the electromagnetic environment around the edge elements that causes the edge element patterns to be different. However, since large arrays have so many elements the number of elements that are actually affected by these edge effects is low and can be ignored (Stutzman, 1998). The edge effects can also be mitigated by adding loading elements and using amplitude tampering in the power applied to elements.

2.7 FLOQUET THEOREM FOR REDUCTIONS IN COMPUTATIONS

The reductions in the computation above make the large array structure periodic. Due to the periodicity, the Floquet theorem can be applied to compute the far-field pattern of the array in a much quicker and less computationally heavy method. The Floquet theorem is a mathematical technique that allows an infinite, periodic structure to be created using a single identical section of that structure. Therefore, by using this Floquet theorem the calculations needed to solve a large array are reduced greatly because the numbers of elements that need to be virtually created has been reduced to one.

The Floquet theorem can also be applied to this large array calculation due to the fact that, when the theorem is applied to arrays the resulting far-field pattern already contains the mutual coupling of the system. Due to this simplification of the mutual coupling, the only hurdle that needs to be accomplished to extract active-element pattern is remove the array factor. Removing the array factor is done by looking at the far-field pattern at only the beam scan-angle direction for the array. This beam scan-angle direction is the summation of the all the element patterns in that direction. By collecting the far-field value of all the possible beam scan-angles and then reducing the value by a power factor proportional to the number of elements in the array, one is left with an active-element pattern.
3 METHOD AND RESOURCES

Chapter 3 covers the method and resources that will be used to conduct the new Floquet active-element pattern technique. The presentation provides a discussion on the flow of the overall technique and how it is done. The flow will be characterized from the creation of the elements needed for the Floquet cell to the modifying of the far-field pattern results to obtain an average active-element pattern. There will be a comparison between two prominent antenna modeling packages (FEKO and HFSS) to find out if one package was better than another when using the Floquet theorem application in each. In this comparison there is a decision to go with the FEKO (FEKO, 2012) antenna modeling package due to its familiarity to the student and the results being almost identical. The last section of this chapter is an explanation of how the MATLAB coding software will be used to modify the FEKO output results and convert the data to an active-element pattern that can be sent back to FEKO to be used as a radiation pattern for further analysis of the large array.

3.1 METHOD OVERVIEW

The new method used to find the average active-element pattern for a large array consists of five basic steps. The steps start from creating the large array in FEKO to how the data is returned to FEKO so that the average active-element pattern can be used in place of physical elements. The entire method is a combination of FEKO and Matlab to gain the wanted results. This method should work with another antenna modeling package (that includes Floquet and some of the generation techniques used in the reconstruction of an infinite array) and coding package to modify the data, but this was the how the method was created for this research.

The first step is to create the large array in the FEKO antenna modeling software, why FEKO was chosen over HFSS will be discussed in Section 3.2. As mentioned in the last chapter, the large array is simulated by a Floquet cell approach to reduce the number of the elements modeled and therefore computation power needed. FEKO has a function named periodic boundaries which allows the user to set up a two dimensional Floquet cell around the array element for testing. Therefore, for the first step one would create one of the elements in the large
array and then create a periodic boundary around the cell that would fit the periodic distance between elements in a two dimensional array.

Once the large array has been created using the periodic boundary in FEKO, step two of finding the far-field values at each beam scan-angle of the array can be started. In the background section it was mentioned that the far-field radiation of an array is the summation of all the active-element patterns in a specific beam scan-angle direction. This is to say that if one was to look at the polar coordinates of $\theta = 45^\circ$, $\varphi = 45^\circ$ in the far-field pattern of the array then the result would be the summation of each active-element pattern in the array at its far-field radiation pattern at $\theta = 90^\circ$, $\varphi = 90^\circ$. Therefore, in FEKO the far-field pattern request was set to find only the far-field radiation values at a specified beam scan-angle. Creating a full, average active-element pattern would require values at enough points to create an accurate three-dimensional pattern. This was done by setting up a grid search in FEKO which would change two variables $(\theta, \varphi)$ that controlled the beam pointing angle of the large array. After each change in the variables the results would be saved to a separate output file that had a numerical call number based on were its was in the grid search. Due to wanting the calculation to be as speedy as possible, the far-field values were only found for phi angles of zero to 90 and theta angles of zero to 90. This quadrant of the pattern is all that is need to create a full active-element pattern for a dipole antenna.

The third step is to collect the data from the FEKO output files and import them into MATLAB so that values can be reduced by the required power factor. This step technically has two functions, but both are run by the same code making it a single step. The MATLAB code is a large loop that has a counter that incrementally goes through all the FEKO output files created and finds the wanted data in the output file. The output file has several results, but for this research the only values that were taken was the phi and theta angles for the single far-field point and the total directivity found. As the data is being pulled from each individual output file the values are put into a large two dimensional matrix that represents a three-dimensional far-field. This code can be used for a single FEKO array at a time but it was much simpler to the code up to read the outputs from all six array cases. When the matrix was completed for a certain array output, the values in the matrix were reduced by dB values of the number of elements that were used or simulated in the array. If the array had 121 elements that were used or simulated in the
use of creating the output value, all the values in the MATLAB matrix were reduced by about 20.8 dB. This matrix of reduced far-field values lead to the average active-element pattern for that array.

After the far-field values are in the two dimensional matrixes and all the values have been reduced by the correct power factor the matrix can be turned into a radiation pattern. The matrix is already a good representation of the average active-element pattern but it can be used to a better extent if placed back into FEKO. This task is not very hard to do, but the matrix does need some manipulation to get into a FEKO ready file. This FEKO ready file has to be a fixed into a .dat file that FEKO can read in the original format that the output file gave far-field data. This is all handled by one program and does not take much time.

The last step in the method overview is to replace the physical elements in the array with the average active-element pattern to reduce the computation power and time even further. Replacing the physical element with the average active-element pattern requires the use of the radiation pattern equivalent source tool in FEKO. This tool calls a .dat file, one that was made in step four, and creates the three dimensional pattern at the location designated. This radiation pattern equivalent source is technically a point source with the pattern of values in the the .dat file. This replacement of the physical element saves computation power due to the reduce structure that needs to calculated in further study of a large array.

A flow chart of the process described above is placed in Figure 3:1 to helpfully clear up any confusion that might have come from reading the overview.
3.2 USED FEKO OVER HFSS

When this research was started it was decided that the computations of the array and the electromagnetic environment should be handled by software that was designed to do exactly that. The two programs considered were HFSS and FEKO, which are two commercial antenna simulation packages. Both programs have a Floquet theorem simulation tool, but which one was more accurate was not known. An ideal dipole was used to check both programs due to the simplicity of the ideal dipole and ability to be calculated in another program, Matlab. The test setup was an ideal dipole located at the origin aligned with the X-axis with a Floquet cell size of $\lambda/2$ for both the X and Y axis of the cell. FEKO and HFSS were used to produce a 5x5 array with the pointing angle of the array to be at zero degrees theta and zero degrees phi or bore sight for the array. The results from FEKO and HFSS would be compared to patterns created in Matlab using theoretical ideal dipole characteristics. For the test the directivity pattern and peak magnitude were chosen as a comparison point so that any differences in feeding between the two programs would be negligible. The results from the test are shown in Figures 3:2 and 3:3.
Figure 3:2. Three dimensional radiation pattern for a half wave dipole created from FEKO, HFSS, and Matlab.

Figure 3:3. Two dimensional radiation pattern cut at a phi angle of zero degrees using FEKO (right) and HFSS (left).

The 3D radiation patterns of all three programs have the same shape and dynamic range which would indicate that HFSS (top left 3D plot) and FEKO(top right 3D plot) are using the Floquet theorem correctly. FEKO and HFSS also have a directivity peak magnitude difference of only .1 dBi. The difference from the peak in the Matlab (bottom 3D plot) results is .3 dBi but
that could be due to the difference in the Matlab code using ideal dipole values and FEKO and HFSS doing full current density calculations on the antennas. The main difference between all the results comes when looking and the dips in the patterns. As an example looking at the polar plots at 90 degrees is a major difference, HFSS (left polar plot) does not go to origin where FEKO (right polar plot) does. This difference may be an difference in how the programs deal with the fact that with the Floquet theorem going to an angle of 90 is technically impossible. However, FEKO took the literal mathematical term and took the graph to the origin and HFSS moved the point to a point that they have determined fits the graph.

Due to the similarities between the programs, and the fact that the only difference is the value of the nulls in the pattern, there is no clear answer to which simulation program is more accurate, or even if such a conclusion could ever be drawn. Therefore, it was opted to use FEKO because it is a more familiar program and it has the ability to easily implement radiation patterns for a .dat file.

### 3.3 CREATE FLOQUET CELL AND FAR-FIELD POINT

As mentioned before, the purpose of this research is to find a different way of calculating a large array without using the hefty amount of computation power needed to artificially create the array. Therefore, the FEKO’s periodic boundary function solves this problem since it is an implementation of Floquet’s theorem, when applied to arrays. The FEKO periodic boundary function only requires that a single element be simulated and then a boundary is setup around the single element. This boundary represents the periodic separation between array elements in the computation and determines the phasing required between elements so that the summation in the far-field is accurately produced. Once the boundary is setup the user defines the number of elements that are to be simulated in either a linear or two-dimensional array. Therefore, FEKO only has to mesh one antenna instead of the entire array. For a better understanding of the difference between a Floquet array setup and the full element setup, images of both models are shown in Figure 3:4.
Figure 3:4 Images of a Floquet array (left) and a 25 element array (right)

As mentioned in the method overview, after the array has been created the far-field is computed in FEKO for the specific beam-scan angle location using the far-field calculation request. When FEKO calculates the far-field value using the Floquet method, the value is based on the number of elements in the array but also that of an infinite array excitation, which fits with the Floquet theory. Due the fact that Floquet theorem cannot be truncated to a finite array it can only calculate results for an infinite array. The infinite array excitation finds the current density that would be present on each element. This current density includes the mutual coupling of all the elements, as mentioned in the Floquet background section. Due to this inclusion of the mutual coupling into the current density, the program can now use simple pattern multiplication to include an array factor that is based on the number of elements that are designated for a large array. Therefore, due to the fact that the current density is calculated using an infinite excitation, the size of the array designated should not make a difference once the power factor has been removed from the far-field results.

The far-field results are not a very difficult thing to get in FEKO, however certain aspects have to be taken into account before running the grid search to produce all the wanted far-field outputs. First, two variables that control the beam scan-angle in phi and theta have to be created in FEKO that are linked to the grid search and then linked into the squint angle designation array of the periodic boundary tool in FEKO. By linking the variables from the grid search to the
periodic boundary tool the loop has been created that will find all the wanted beam scan-angles. Second the variables that are controlling the squint angle of the periodic boundary phasing calculation have to be converted and placed in the far-field request location. This placement of the variables makes sure that when the beam point angle changes so does the far-field request point that the output data always reads the wanted beam scan angel location.

### 3.4 MODIFY DATA IN MATLAB

After FEKO has finished its computation of all the wanted far-field values at specified beam pointing angles the data is ready to be collected and processed by MATLAB. As mentioned in the overview the far-field results are only produced for the quadrant from 0 to 90 degrees in both phi and theta. Measuring this quadrant alone works due to the symmetrical properties of the pattern of a dipole. This ability to measure only a quadrant may change depending on the symmetrical properties of the array element under analysis. In the FEKO output file group, the file that is needed to get the data to produce the average active-element pattern has the suffix .out. Inside this file there is a section that contains the far-field pattern data that is collected and there are several values that can be chosen. The required far-field values for the production of the average active-element pattern are the phi and theta angle used for the effective beam scan angle and the total directivity. These three values represent the location of the beam scan-angle and the far-field radiation value at that point in space.

With the knowledge of what data is produced and where the data is located, the MATLAB matrix can be formed. The MATLAB code has a loop that reads in the file name for a specific grid search output and then finds the means that as the MATLAB code calls each grid search file separately and pulls the needed far-field values. Once the values are pulled they are stored in separate data arrays for later modification. After the data is place in the array a new grid search file name is loaded and the process is repeated. This loop is done until all the grid search output files have been read in and the data collected. The data-field array that contains the directivity values that go through a series of modifications to copy the data from one quadrant into others so that the data now covers all the phi and theta angles and is a two-dimensional matrix that has phi and theta call locations. The newly created matrix is the desired average active-element pattern and is complete and ready to send back to FEKO for further use.
3.5 RETURN MODIFIED DATA TO FEKO AS AVERAGE ACTIVE-ELEMENT PATTERN

The matrix of modified far-field values in MATLAB is the wanted average active-element pattern, as mentioned in the last section. The average active-element pattern can be a created into a radiation pattern that can be entered into FEKO instead of an element, which reduces the amount of computation needed in further analysis of the large array. Implementing the new found average active-element pattern requires modifying the matrix in MATLAB into a listing that can be read by the FEKO radiation pattern feature.

As mentioned in the method overview FEKO has a radiation-pattern excitation source that recreates a far-field pattern given a .dat file. This .dat file needs to have a data setup that is similar to the FEKO output that the grid search values are read from. However, instead of having separate files for each far-field point all the values are now going to be part of the same file. The creation of this file is not difficult with some simple MATLAB coding to turn the matrix back into a single file array and then placing the corrected average active-element value with the correct beam-scan angles to create a matrix three wide. This matrix is then saved with the suffix .dat so that FEKO can read the file and create a three dimensional pattern with it.

The radiation-pattern excitation source, as mentioned in the overview, is a point source that has the radiation-pattern designated in the .dat file. This reduction in the needed computation power leads to faster design work for large arrays. There is one downside to replacing a physical element with the radiation-pattern excitation source. The process does not take into account other electromagnetic sources when calculating its pattern. Due to the inability for power to couple onto a point source the only coupling that will be seen in further models will be the average active-element pattern on structures or other antennas around it. This is not a deal breaker for the replacement of the physical array since all the mutual coupling of the array and its structure is already account for, if the structure is placed in the periodic boundary.
4 PROCEDURE

In this chapter the specifics of how the research was tested. The first section will discuss the structural setup for each array that is being considered and how these setups are realistic for modern arrays. This section will also discuss how the elements are loaded and feed for both the Floquet array and the full element array so that there is no difference between the arrays besides the way the mutual coupling is being calculated and integrated in the far-field patterns. The first section will also explain why there are several sizes of array that are created and included in the results for this research. The second section is an explanation on how the results of the each array will be compared and why those comparisons matter.

4.1 SIMULATION COMPARISON SETUP

This new Floquet method for finding average active-element patterns will have to be tested against some standard or baseline setup up for validation. For this research the baseline setup would be a full element array of the same number of elements that are being simulated using the Floquet method. This test setup up would allow for the comparison of time of computation, computation power needed, and the accuracy of the results. The full-element array and the Floquet array would both go through the same method described in the last chapter to find the active-element array, except the full-element array would have all the elements built into the model.

Arrays have several different types of elements types. The most common array element types are open ended waveguides, Vivaldi, and path antennas. All these elements could be tested with the new Floquet method but due to limitations of the testing computer and the desire to stick to specs for a normal PC, a simpler element would be needed for the creation and analysis of the full-element arrays. With these constants in mind the array element chosen for this research is the half wave dipole with no backing. This element choice has two major benefits for this research. First, the half-wave dipole is a very simply structure and easy to calculate in the hundreds while sticking to wanted computer constraints. Second, the half-wave dipole has a larger mutual coupling effect between parallel elements, which will make half-wave dipoles a good element to
use in the research of finding an active-element pattern and demonstrating the effect of mutual coupling.

The array will also have to be setup in a way that would be practical to use in a non-simulation case. This means that an array and all the elements would have to be constructed and loaded in a way that is common or wanted in a real world case. There are two goals to accomplish now, the first being to structure the array so that the elements are in a pattern that would reduce grating lobe effects, which is a common array design practice, and to load each element with a generator load that would be within a reasonable margin of the antenna loads seen as the beam scan-angle is changed. The first goal was meet by separating the elements by half a wave length, which is a common spacing used to reduce side lobe issues from appearing as the array is scanned. Each array is constructed in a rectangular pattern due to the ease of creation when using the periodic boundary tool in FEKO. The generator load for the elements was determined by using the periodic boundary function in FEKO to simulate an infinite number of elements in beam scan-angles from 0 to 90 degrees in theta and 0 to 90 degrees in phi. From those simulations the antenna impedance at beam scan-angle phi 45° and theta 45° was taken as the generator load because it was the impedance that was within the 2:1 ratio with all the other impedances. This array and load setup up is realistic and should provide results that could be applied outside of a simulation environment.

One of the important aspects of the research was to determine at what size array this Floquet technique would be applicable. Therefore, the simulation setup would have to have a way to test if the size of the array made a difference in the comparison between the active-element patterns obtain using the different arrays. This was solved by setting up three different sized arrays that increase in element number by about hundred at a time. The array sizes are 25, 121, and 225 elements, which stop at 225 elements because that number of elements was the last symmetric array size that was able to stay with in the FEKO computation requirements of 512 MB of processing power.

### 4.2 SIMULATION OUTPUT SETUP

The output of the comparison will have to make the differences between the patterns as easy as possible to notice. Therefore, all of the active element-pattern results will be overlaid so that
comparison can be done on the same plot. There will also have to be comparison of the radiation pattern from more than one plane. Due to the pattern being produced from a half wave dipole not being omni-directional there is difference in the pattern from angles that are parallel and perpendicular to the element. Specifically as the angle of interest starts aligning with the structure of the half wave dipole the pattern will change more extremely due to the location of a null in that direction. This obstacle will be overcome by taking two, two-dimensional cuts of the average active-element pattern that are 90 degrees apart. The first cut will be along the X-axis, which is the parallel with the structure of the dipole and the second will be along the Y-axis, which is perpendicular. These two cuts should give a sufficient view of the average active-element patterns from each array so that they can be compared.
5 RESULTS

The chapter provides the comparisons and differences between the outputs of the full-element arrays and the Floquet arrays. First the difference in computation time and power will be shown between using the Floquet array and the full-element array. This comparison will be specific to a certain computers processing power, but the computer is a good representation of an average PC. Second there will be a discussion of the differences and similarities of the two-dimensional plane cuts at phi angles of 0 and 90 of the two array configurations average active-element radiation patterns. There will also be the addition of the free-space element radiation pattern to use as a comparison for the reason of using an active-element pattern instead of a free-space element pattern when conducting array analysis. From the different array sizes, a specific size array will be chosen to do a more in-depth comparison to achieve a better understanding of the difference between the radiation patterns. This more in-depth comparison will not include a free-space element pattern as well.

5.1 DIFFERENCE IN COMPUTATION TIME

One of the most important aspects of this new technique is that it saves time and computation power while finding the average active-element pattern for a large array. The average active-element patterns are also very important, but if the method does not save computation time or power then there is no need to put any more research into the area. For this research the computer used for the computation had a 2 GHz processor and was always limited to 512 MB due to the restraints of the FEKO version used on computer. Table 5.1 below contains the computation power and time need to compute all the data need to find the average active-element pattern for all the arrays that are considered in this research.
As expected, when looking at the computation time and power results for the full-element arrays the power and time increase as the number of elements increases. The amount of computation power is not extreme by common computer standards, but then most of these array sizes are small when compared to real world arrays. Realistic large arrays in use today are usually on the order of thousands of elements and would easily require several GB of computation power to work through. The worst part about the full-element array computation is the time it takes to complete the simulations. At the smallest number of elements it takes about an hour to finish the simulations and at the largest it takes almost 16 hours. That 16 hours is then small when thinking about trying to analyze an array that has over a thousand elements. Not only would an array over a thousand elements take far more power than capable on most FEKO licenses but it would probably take days to complete.

The computation power and time results from the Floquet array are much more realistic for common PC computers and a first-order analysis of a large array. As mentioned in the methods section, due the way that the Floquet array is computed inside FEKO the results never change due the process taking the same time to compute an infinite array of elements. This lack changing computation process lead to the computation time always being the same. The Floquet method also allows for the computation time to be lower than the lowest full element array computation time. Attached to this extremely low computation time is a very reasonable computation power requirement that does not break half a MB of RAM. Computation power for the Floquet array is extremely low because there was only one element cell that needs to be

<table>
<thead>
<tr>
<th></th>
<th>25 Full Element</th>
<th>121 Full Element</th>
<th>225 Full Element</th>
<th>25 Floquet Array</th>
<th>121 Floquet Array</th>
<th>225 Floquet Array</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (Sec)</td>
<td>4094</td>
<td>21517</td>
<td>56382</td>
<td>3070</td>
<td>3070</td>
<td>3070</td>
</tr>
<tr>
<td>Time (Min)</td>
<td>68.2</td>
<td>358.62</td>
<td>939.7</td>
<td>51.17</td>
<td>51.17</td>
<td>51.17</td>
</tr>
<tr>
<td>Time (Hour)</td>
<td>1.14</td>
<td>5.98</td>
<td>15.66</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Table 5-1. Computation power and times needed to gather the far-field values for all the array types and sizes needed for the research using a 2 GHz processor.
actually meshed and virtually created. The remainder of the array is mathematically created through periodic boundaries. Therefore, the new technique has most definitely achieved the goal of reducing the computation time and power required for a large array to something that can be managed on an average PC.

5.2 ACTIVE-ELEMENT PATTERN OVERLAYS

This section presents the comparison plots of all the average active-element patterns from each array size. There will be a separate section for each of the two dimensional planar cuts of the average active-element patterns.

5.2.1 PHI CUT OF 0 DEGREES

Figure 5:1 is an overlay of the average active-element patterns produced from each array that is considered in this research when looking at a two dimensional cut at phi equals zero. The Floquet arrays will be the solid lines and the full element arrays will be the dashed lines. The free-space element pattern will be dash-dotted line.

One of the confusing aspects of this plot is the fact that there is only one visible Floquet average active-element pattern. There is only one average active-element pattern for the Floquet arrays because all of the patterns are the same. As mentioned in the methods section, once the power factor is reduced from the far-field results all the values for the Floquet arrays become the same due to the infinite array calculation. This outcome makes the need to designate a large number of elements unnecessary when finding that average active-element pattern. Therefore, if the large array in question was 3000 elements the average active-element pattern could be found using a Floquet array no larger than 9 elements, assuming that the large array is symmetric.

As expected the average active-element patterns for the Floquet and full-element arrays do not match up with the free-space element pattern. In the non-ideal array and active-element pattern vs. free-space element pattern background sections it was stated that a free pattern cannot be used due to the fact that the pattern ignores the effects of mutual coupling. The mutual coupling effects of the adjacent elements that are not shown in the free-space pattern. However, the coupling effects are shown in the average active-element patterns. One of the effects is that the null values for this specific two-dimensional plane cut are very different for the average
active-element patterns and the free-space element pattern. The difference between the full element average active-element pattern and the free-space element pattern null is about five to six dB. That difference is not a major problem since it is a null location, but could become an issue if not dealt with correctly when the array beam-scan angle is pointing in direction that is close to the null area of the average active-element patterns. If the proper precautions are not taken the result would be the summation of the five to six dB difference summed across all the elements in the array and projected into the far-field. Another difference in the patterns is the fall-off rate of the main-beam lobe. Due to the mutual coupling, the rate at which the main beam of the average active-element patterns is less step then the free-space element pattern. When looking at the plot below the average active-element patterns do not reach a directivity of zero until about 60 degrees in theta, whereas the free-space element pattern achieves zero dB at about 35 degrees in theta. This change in the main beam lobe decay rate could cause issues with steering the main lobe of the entire array in the correct direction. Once again it is much more beneficial to use an active-element pattern than a free-space element pattern when analyzing an array.

The comparison between the average active-element patterns for the full element array and the Floquet array are good until reaching the null section of the patterns. The main-beam portion of each average active-element pattern are very similar. That is to say, where the active-element patterns are above zero dB in directivity they are almost identical. Once the directivity drops below zero dB the Floquet array pattern does not have the same drop of rate as the full element arrays into the null section of the patterns. The comparison of the null sections of the patterns becomes worse as the number of elements in the full arrays gets lower. When looking at the 225 full element average active-element pattern its null section is about 6 dB away from the Floquet average active-element pattern, whereas the 25 element array average active-element pattern is about 9 dB. This observation makes sense because the Floquet array is based on an infinite array that is then truncated to the number of elements designated. Therefore, the larger the full element array gets the closer the comparison between the average active-elements patterns will become. The null section not comparing well is an issue, but not a major one, as mentioned before it would only make a difference when the array point in a direction that is lined up with the null section of the average active-element patterns. This issue may be a path for more research in the future to come up with a power factor correction for the null-section roll-off rate.
Figure 5:1. Average active-element patterns from each of the array types and sizes and half wave dipole free-space pattern overlayed for a phi angle cut of zero degrees.

5.2.2 PHI CUT OF 90 DEGREES

Figure 5:2 has the same setup as Figure 5:1, but the two-dimensional cut of the average active-element pattern is at the phi angle of 90 degrees. This cut gives a completely different view of the average active-element pattern and is in a direction where the free-space pattern of the half wave dipole has no null or deviation in pattern. This plot will show the most difference between a free-space pattern and an active-element pattern.

As in the last sections figure the Floquet average active-element patterns are all the same, as expected. The figure also shows the same relationship between the Floquet array and the full-element array patterns, the more elements that are simulated in the full element array the closer the Floquet array output comes in comparison. Another similarity in the figures is that the issue
of the null section of the average active-element pattern between the full-element array and the Floquet array is still obvious. In this two-dimensional cut the difference in the nulls is less extreme as the difference between the 225 null value and the Floquet null values, being only about 4 dB instead of the 6 dB from the last figure. One of the positive outputs from this figure is that the average active-element patterns continue to match up well until about -3 dB. This may be due to the fact that the null section of a single element pattern has less effect in this region.

The major observation to notice in this plot is the fact that there is a beam-shape pattern in the average active-element patterns were there is not in the free-space pattern. This difference is major when thinking about the usual half-wave dipole free-space pattern. In that pattern there is only one null and the rest of the pattern is omni-directional about the axis of the element. This difference in pattern can account for many difference seen in the far-field pattern of an array if one is not using the correct active-element patterns for the elements. Specifically the difference in pattern can cause the position of the main beam to not be at the specified beam scan-angle and the difference can cause the side lobes to be at the levels that are not expected.

An observation has to be made on how many elements are needed to make the Floquet method reasonable for being applied to large arrays. The only sizes that are able to be considered are the 121 and 225 elements arrays because the 25 is always closer to the free-space pattern and is quite different in pattern then the other full-element patterns. Between the full element patterns there is not much of difference for the 121 and the 225 element patterns. In both figures the results from those two full-element array sizes only differ by about .5 dB and that is at the exact null of each pattern. There is no difference between the two signals until the patterns bottom out at the null, which mean that the patterns only differ by about .5 dB for 15 degrees at the nulls. Therefore, the minimum number of elements that can have the Floquet method applied to it would be the 121 element array. This observation would match up with other research on the matter of determining how many elements are needed to use an infinite array calculation technique (Deshpande 1989, Wasylkiwskyj 1973). Any further research into this area or comparison between results will be done with the 121 full element array because of its reduced computation time and power.
Figure 5:2. Average active-element patterns from each of the array types and sizes and half wave dipole free-space pattern overlayed for a phi angle cut of 90 degrees.

This section discussed the pattern overlays for the average active-element patterns that were found using the Floquet array and a full-element array. The overlays showed the two arrays produce patterns that match up well until the pattern rolls off into a null section. This null section issue is not a major one but something that could be addressed in future research on this subject. The overlays also showed that the average active-element pattern is completely different than the free-space pattern. Another aspect of the Floquet method that the figures showed was that one does not have to designate a large number of elements when using the Floquet array approach since the number of elements does not make a difference after the processing of the data. Last this section has suggested that further research in the technique can be done with the 121 element arrays due to the similarity of the 121 and 225 element arrays patterns and the lessened computation power and time needed when computing the full-element array.
5.3 ACTIVE-ELEMENT DIFFERENCE PLOTS

This section covers the comparison of the full-element and Floquet arrays in more depth using a simulation size of 121 elements. As in the last section the results will be analyzed in two two-dimensional planar cuts, with one plane being a phi angle of zero degrees and the other 90 degrees. The comparison will be looking at the percentage difference between the plots to give a better understanding of where and how large the differences occur between the two average active-element patterns.

5.3.1 PHI CUT OF 0 DEGREES

Figure 5:3 is the percentage difference between the average active-element patterns for the full-element array and the Floquet array, using a two-dimensional cut at the phi angle of zero degrees. As mentioned in the last section the comparison between the two plots starts to degrade quickly at about 60 degrees in theta, which is at about 0 dB directivity on the overlay of Figure 5.1. This area of the average active-element patterns after 60 degrees in theta is where the two patterns have the null areas and they do not match up well there. The difference between the two is not extreme since the maximum difference between the patterns is under ten percent, but there does need to be some precaution taken when using the Floquet average active-element pattern over the pattern produced from a full-element array. The main beam of the patterns, designated as the pattern that is outside the region from theta of 60 to 120 degrees, has a good match and will be more important due to the fact that the majority of the power in the radiation will go into those areas. The null area is also not an issue due to that fact that most large arrays due not operate down to a beam scan-angle of 60 degrees or more very often. The bottom portion of the null area starts to reduce the difference between the patterns, but it is the change in the decay rate to the null section that is the biggest difference between the patterns. The difference decay rate can be seen in the sharp incline between 60 to 80 degrees in theta.
5.3.2 PHI CUT OF 90 DEGREES

Figure 5:3 shows the percentage difference between the average active-element patterns for the full-element array and the Floquet array for a two-dimensional cut at the phi angle of 90 degrees. As with the last percentage-difference plot, the comparison between the patterns holds until about 60 degrees in theta and falls out until theta of 120 degrees. This two-dimensional cut has a better comparison in the main beam of the pattern but the null section is even further apart. Unlike the other percentage-difference plot (Figure 5:3) there is no reduction in the null section when the patterns reach the minimum area of each pattern. This difference is most likely due to infinite array excitation and the fact that a half-wave dipole pattern does not naturally have any change in its pattern in that particular area and therefore will need many more antennas to be part of the pattern before the average active-element pattern will start to have the same effects as the
Floquet array results. Therefore, when the array is being steered in the angles close to phi of 90 degrees even more precautions would have to be taken if the null areas of the pattern become important.

![Graph showing percent difference between Floquet and 121 Element array active element patterns](image)

**Figure 5.4**. Plot of the difference between the average active-element patterns from the 121 full element array and the Floquet array when looking at the phi angle cut of 90 degrees.

### 5.4 LARGE ARRAYS WITH GROUND PLANES

This research was done with an array that does not include a ground plane. A ground plane was not added due to fact that it was not necessary to the process of coming up with a new active element pattern technique. However, all arrays have some sort of ground plane and it has to be included in to the electromagnetic environment for computation. A ground plane will focus most
of the array pattern energy in the direction away from the ground plane face but does cause diffraction effects towards the edges of the array. The issue with using a ground plane and the Floquet technique described in this paper is that the infinite array excitation will cause the diffraction to not be calculated because there are no array edges for the pattern energy to diffract around. Therefore, when the ground plane is included in the environment another technique is needed to include the diffraction around the edges of the array that the Floquet technique misses. This other technique is called uniform theory of diffraction and is an extension of geometric theory of diffraction. Uniform theory of diffraction can be applied through FEKO after the average active-element pattern is returned to FEKO. This addition of uniform theory of diffraction process will cause for more computation time but will give a much more accurate result overall for an array with a ground plane.
6 CONCLUSION

This document characterizes a method for efficient computation of active-element patterns for large arrays. Information includes the minimum number of elements needed for the Floquet technique to be applicable to the array for analysis. There is also a summary on how the Floquet technique reduced the power and computation time needed to find the required data mentioned in the methods chapter. Last there is a discussion on the use of UTD for finite ground planes and possible future research that could be done in this area.

6.1 LEAST NUMBER OF ELEMENTS NEEDED IN PHYSICAL ARRAY TO BE LARGE

One of the major parts of the research was to determine how many elements where needed in an array to make it “large” so that the Floquet technique could be accurately applied. Since the Floquet technique is based on an infinite array, there would have to be a minimum number of elements in an array so that the differences between the average active-element patterns would not be too great. The array with the minimum number of elements would have to have a pattern that would be able to produce a pattern that would compare to the Floquet average active-element pattern well for most if not all of the main beam or stronger areas of radiation of the patterns. If there are differences in the null areas of the pattern the issue could be ignored for now since the main beam and stronger radiation areas would carry most of the power in the pattern and affect the overall far-field of the array more when used for simulations, instead of a physical element.

The results show that the 121 and 225 element full element array patterns hold true to the criteria of following the main beam pattern up to the null section. The 121 and 225 element arrays are almost identical until the floor of the null section. At this floor level the difference between the 121 and the 225 full element array patterns is less than one dB. Therefore, 121 can be assumed to be the lowest number of elements that are needed to be described as a large array and can be analyzed using the new Floquet technique based on this small study. This result of 121 elements matches with research that was done on other large arrays were it was found that
when the array reaches an element number of 100 or more an infinite array calculation method was acceptable (Wasylkiwskyj, 1973).

The results in this research are also provide a worst case scenario when considering the mutual coupling effects of a large array. The array used was a half-wave dipole array with no backing or mitigation effects used between elements. Due to the semi omni-directional pattern of the half wave dipole pattern the mutual coupling effects between elements is high. This mutual coupling effect was advantageous for finding a new method but not when designing a working array. Therefore, when looking at elements that have a less omni-directional pattern or have less space coupling, such as a Vivaldi or slotted-wave guide antennas, this Floquet technique will have better results. The number of elements at 100 could possibly be dropped by using more directional array elements and having some type of mutual coupling mitigation.

6.2 DIFFERENCES IN COMPUTATION TIME

The biggest purpose of the research was to find a new computational method to find average active-element patterns that reduces the time and computation power needed. This was obviously accomplished using the new technique. The full element arrays took from one hour to almost 16 hours to finish their runs and the computation power was on the order of tens of MB. The computation power need is not extreme, but the array sizes that are used in this research are on the low side for the number of elements in a large array and a common large array is on the order of 1000 elements or more. The computation time need for the full elements is one of the biggest problems, having something run for that long is to long for a first-order design or analysis work of a phased array. The Floquet approach reduced both these problems to a much more reasonable constraint for an average PC and for first-order design work of an array. Due to the solution method for the Floquet approach, the computation time does not vary with array size and that time is less than an hour. This computation time is less than the 25 element array which is not considered a large array at all. In addition, the computation power need for the Floquet approach is in the kB instead of MB which is due to only needing a single-element meshed in the simulation. These computation times and power levels are also based on a very simple half wave dipole element. Using elements that are more complex, such as a Vivaldi or open-ended waveguide, that have more surface area, will make the computation power need grow greatly.
With a limit of 512 MB of processing RAM certain antenna elements could not be simulated in even a 25 full element array. The only way to simulate large arrays with complex antennas would be to use infinite array assumptions and this Floquet technique matches that need. The computation time was based on an array without a ground plane. The time for calculation for a large array with a ground plane will be slightly higher due to inclusion of the UTD method mentioned in Section 5.4. This UTD method is a required computation addition needed to accurately find the array pattern past the edges of the ground plane.

6.3 POSSIBLE AREAS FOR FURTHER RESEARCH

The one array of the research that has not returned a positive result is the null section results for the Floquet technique. For the 121 element full array, which is the smallest array that can be compared to the Floquet results, the difference at the null is a max around 16 percent. This difference will only become less as the number of elements being simulated increases but at the lower end there still need to be precautions taken to reduce the difference or at least mitigate it to something that can be used without having to ignore the results from that area of the pattern. Therefore, future research could be done on techniques to be added into the manipulation of the far-field results to reduce this problem or adding in a power factor in that area to bring the Floquet pattern to a more comparable level.
REFERENCES


