Dynamics of an Electrodynastic Tether System in a Varying Space-Plasma Environment

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(ABSTRACT)
Electrodynamic tethers have a wide range of proposed applications in the fields of satellite propulsion and space plasma research. The fundamental purpose of this dissertation is to improve the understanding of the behavior of an electrodynamic tether (EDT) system in Earth’s ionosphere. An electrodynamic tether system consists of two satellites connected by a long tether that generates current to produce either power or thrust via the system’s electromagnetic interaction with the space environment. Previous electrodynamic tether investigations decouple the interaction between the tether and the constantly changing plasma environment. The limiting factor inhibiting the development of a full system model that has an accurate characterization of the tether/plasma interaction is that the understanding of that interaction is not well developed over a wide range of system parameters. The EDT system model developed in this study uses a high fidelity dynamics model that includes a tether current described by an analytical current collection model whose plasma parameters are determined by the International Reference Ionosphere. It is first shown that new instabilities are induced in the system dynamics under a basic analytical current model versus a constant current model.

A 2-D3v Particle-in-Cell (PIC) code has been developed to study the plasma dynamics near a positively charged EDT system end-body and their impact on the current collected. Simulations are run over a range of system parameters that occur throughout a LEO orbit. The azimuthal current structures observed during the TSS-1R mission are found to enhance the current collected by the satellite when the magnetic field is slightly off of perpendicular to the orbital velocity. When the in-plane component of the magnetic field becomes large, the electrons are not able to easily cross the field lines causing plasma lobes form above and below the satellite. The lobes limit the current arriving to the satellite and also cause an enhanced wake to develop. A high satellite bias causes a stable bow-shock structure to form in the ram region of the satellite, which limits the number of electrons entering the sheath region and thus limiting the current collected. Electron-neutral collisions are found to destabilize the bow-shock structure and remove its current limiting effects. Additionally, as the magnetization of the plasma is increased, the current becomes limited by the charged particle’s inability to cross magnetic field lines. Analytical curve fits based on the simulation results are presented that characterize the dependence of the average current collected on the local magnetic field orientations, space plasma magnetization and satellite potential.

The results from the PIC simulations characterizing the magnetic field’s influence on the tether’s current are incorporated into the system dynamics model to study the behavior of the EDT system over a range of inclinations. The magnetic field is found to limit the diurnal variations in the current collected by the system throughout its orbit. As the inclination of the system’s orbit is increased, the impact of the magnetic field becomes more pronounced as its orientation sweeps through a larger range of angles. The impact of the magnetic field on the collected current is, therefore, found to limit the ability of an EDT system to boost the system’s orbit as the orbit’s inclination is increased. In summary, new system dynamics have been observed due to the previously unobserved behavior of the current over a range of end-body configurations.
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Chapter 1

Introduction

Electrodynamic tether systems typically consist of long, conducting tethers (10-100 km in length) connecting two satellites (end-bodies). In order for the EDT system to function, the central body in the system’s orbit must have a strong magnetic field and be surrounded by a dense space-plasma. The system’s end-bodies must be in electrical contact with the surrounding plasma in order to collect the electrons and ions that comprise the plasma. EDT systems generate a potential difference across the tether either by using a power source to force the potential difference or allowing an electromotive force to develop due to the tether’s motion across the central body’s magnetic field. The potential difference drives the electrons that are collected by the end-bodies through the tether to form a current. EDTs use the electromagnetic interaction between the current and the ambient space environment to create either power (generator mode) or thrust (motor mode). In either case, the current’s interaction with the local magnetic field develops a distributed Lorentz force on the tether. In generator mode, the electromotive force is generated due to the tether’s motion and not a power source. Therefore, the collected current can be used to power systems onboard the spacecraft. However, the electrodynamic force generated by the current opposes the tether’s motion and will de-orbit the system. Conversely, in motor mode, the force acts in the direction of the tether’s motion and gradually boosts the system’s orbit. The general configuration for an EDT system in motor mode is shown in Figure 1.1 oriented vertically relative to its central body. Electrons gyrate down the magnetic field lines and are collected by the upper end-body. The collected current is then driven through the tether and released at the lower end-body. The complex physics governing the processes inherent in current collection are not well understood. The physics that allows the released electrons to “close” the current loop is also not well understood. Since the current drives a forcing term in the system dynamics, an accurate characterization of the current that can be collected under a variety of plasma conditions is needed in order to completely characterize an EDT’s dynamics. EDT studies typically separate the system dynamics from the system’s interaction with the ambient space-plasma environment and assume that the electromagnetic interaction can be held constant. The fundamental purpose of the proposed research is to develop a better
understanding of the physics present in the plasma near a positively charged satellite in order to determine its impact on the current collected by an EDT end-body and then ultimately couple the new physics to a dynamics model of an EDT system in motor mode in order to study its impact on the tether system’s dynamics. The remainder of Chapter 1 provides an introduction to relevant fundamental theory and concepts that are necessary to frame the contributions of the work encompassed in this dissertation.

1.1 History and Motivation for the Study of Electrodynamic Tethers

The first space tether applications began with Tsiolkovskii who proposed several tether related concepts in his famous manuscript Day-Dreams of Earth and Heaven. In 1895, Tsiolkovskii introduced the ideas of a spinning tether system to create an artificial gravity environment on a spacecraft and a equatorial tower soaring above geostationary altitude to access an environment “free” of the effects of Earth’s gravity. Tsander later proposed a similar concept of stretching a tapered cable from the Moon to the vicinity of Earth in order
Table 1.1: A summary of the tethered satellite missions flown, reproduced from Table 1 in Reference 70.

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
<th>Orbit</th>
<th>Length</th>
<th>Agency^b</th>
<th>Comments</th>
</tr>
</thead>
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<tr>
<td>Gemini 11</td>
<td>1967</td>
<td>LEO</td>
<td>30 m</td>
<td>NASA</td>
<td>Spin stable 0.15 rpm</td>
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<tr>
<td>Gemini 11</td>
<td>1967</td>
<td>LEO</td>
<td>30 m</td>
<td>NASA</td>
<td>Local vertical, stable swing</td>
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<td>H-9M-69a</td>
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<td>Suborbital</td>
<td>500 m</td>
<td>NASA/ISAS</td>
<td>Partial deployment</td>
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<td>S-520-2a</td>
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<td>Suborbital</td>
<td>500 m</td>
<td>NASA/ISAS</td>
<td>Partial deployment</td>
</tr>
<tr>
<td>Charge-1a</td>
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<td>Suborbital</td>
<td>500 m</td>
<td>NASA/ISAS</td>
<td>Full deployment</td>
</tr>
<tr>
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<td>Suborbital</td>
<td>500 m</td>
<td>NASA/ISAS</td>
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<td>Canadian NRC/NASA</td>
<td>Spin stable 0.7 rpm, magnetic field aligned</td>
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<td>Suborbital</td>
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<td>Full deployment</td>
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<td>1992</td>
<td>LEO</td>
<td>260 m</td>
<td>NASA/Italian Space Agency</td>
<td>Partial deployment, retrieved</td>
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<td>NASA</td>
<td>Downward full deployment, swing, and cut</td>
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<tr>
<td>PMGa</td>
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<td>LEO</td>
<td>500 m</td>
<td>NASA</td>
<td>Upward deployment</td>
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<td>SEDS-2</td>
<td>1994</td>
<td>LEO</td>
<td>20 km</td>
<td>NASA</td>
<td>Full deployment, local vertical stable</td>
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<td>Oedipus-C</td>
<td>1995</td>
<td>Suborbital</td>
<td>1 km</td>
<td>Canadian NRC/NASA</td>
<td>Spin stable 0.7 rpm, magnetic field aligned</td>
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<tr>
<td>TSS-1Ra</td>
<td>1996</td>
<td>LEO</td>
<td>19.6 km</td>
<td>NASA/Italian Space Agency</td>
<td>Close to full deployment, severed by arcing</td>
</tr>
<tr>
<td>TiPS</td>
<td>1996</td>
<td>LEO</td>
<td>4 km</td>
<td>NRO/NRL</td>
<td>Long-life tether on-orbit (survived 12 years)</td>
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<tr>
<td>ATEx</td>
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<td>LEO</td>
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<td>ProSEDSa</td>
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<td>MAST</td>
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<td>LEO</td>
<td>1 km</td>
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<td>Did not deploy</td>
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<td>YES2</td>
<td>2007</td>
<td>LEO</td>
<td>30+ km</td>
<td>ESA</td>
<td>Full deployment</td>
</tr>
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^a Electrodynamic tether mission.
^b ISAS is the Institute of Space and Astronautical Science, NRC is the National Research Council, NRO is the National Reconnaissance Office, and NRL is the Naval Research Lab.
transfer material from the Moon’s surface to Earth orbit and completed the first numerical estimates of the cable’s dimensions using steel as the tether’s material. One of the first concepts of an electrodynamic tether was proposed by Alfvén’s solar wind engine. The engine consisted of two opposing, conducting tethers oriented parallel to the solar wind’s electric field. The system would generate a useable current, while also attaining relatively high velocities due to the solar wind. Dobrowolny et al. provide the first systematic analysis of a bare electrodynamic tether system in low-Earth orbit.

The work encompassed in this dissertation will focus on the classic ”dumbbell” configuration shown in Figure 1.1. The dumbbell system consists of two spherical end-bodies in electrical contact with the surrounding space-plasma (without use of active plasma contactors) connected by a fully insulated, conducting tether oriented vertically. An excellent collection of introductory papers can be found in Ref. 5. Section 1.1.1 gives an overview of the electrodynamic tether missions completed to date. Several EDT system configurations have been proposed for various applications, discussed in Section 1.1.2.

### 1.1.1 Missions

A total of eight EDT missions have been flown by a variety of agencies, excluding a mission whose hardware was built but not flown (ProSEDS). Table 1.1 summarizes all of the tether missions and their results to date. The Small Expendable Deployer System missions (SEDS-1 and SEDS-2) were meant to demonstrate the feasibility of the deployer systems that would be used on the up-coming Tethered Satellite System missions. SEDS-1 demonstrated that the system could successfully deploy a 2 km long tether. SEDS-2 was meant to show that a closed-loop control law could be used to stabilize the end-mass payload displace a small angle away from the local vertical (unfortunately the tether was severed before the mission was completed). The Cooperative High Altitude Rocket Gun Experiments (CHARGE-1 and CHARGE-2) sounding rocket missions showed that the current collected by an electrodynamic tether system can be significantly increased by the release of neutral gas clouds via neutral ionization by incoming, high-energy electrons. TSS-1R and the Plasma Motor Generator (PMG) mission are the most widely studied because they were flown in LEO (the projected ideal altitude range for EDT operations) and their on-board experiments were designed to investigate several key EDT concepts. Data from both missions will be used as validation for the models developed throughout the dissertation. The following two subsections give a brief summary of the contributions of each experiment to the understanding of EDT systems.

**Tethered Satellite System (TSS-1 and TSS-1R)**

The tethered satellite system (TSS) missions were meant to study to performance of an EDT system. The TSS-1R mission consisted of a 521 kg end-body connected by a 2.5 mm
copper tether 22 km in length to the Space Shuttle, making the TSS system the largest man-made space structure ever flown. The tether deployed to a length of 19.7 km before it was severed by arcing just above the deployment boom. The science instruments on the sub-satellite continued to collect scientific data for 3 days after severing from the deployer until its batteries ran out. Even though the tether was severed by arcing before it was completely deployed, a number of very useful observations were made. The results from the TSS-1R mission showed that the current collected by the TSS satellite was 2-6 times higher than the current predicted by the Parker-Murphy model, a model that was previously thought to predict the upper limit to the current collected by a positively charged satellite in the presence of a magnetic field.

A recent study by Mariani et al. brings to light the existence of azimuthal currents in the ram hemisphere of the TSS satellite, as observed in data from the TEMAG experiment, that are not characterized by existing theories of spacecraft charging. TEMAG consisted of two triaxial magnetometers mounted on a fixed boom 118 cm and 173 cm from the spin axis of the TSS sub-satellite. The sub-satellite was spin-stabilized, allowing the magnetometers to record a full 360° measurement of the magnetic field in the sub-satellite’s equatorial plane. Mariani et al. study two events where the current in the tether was held constant for longer than the full spin period of the satellite (which allows TEMAG to complete a full revolution around the satellite). TEMAG data showed significant deviations from the ambient local magnetic field (as predicted by the IGRF95 model) in both radial and azimuthal magnetic field components in the ram hemisphere. As a result of these observations, Mariani et al. proposed the existence of clockwise azimuthal currents ahead of the TSS satellite as the only explanation for the magnetic field deviations. Thus, the data from the TSS-1R experiment showed both the current collected by an EDT system is higher than the current predicted by the Parker-Murphy model and the existence of small scale azimuthal currents developing ahead of the TSS satellite.

**Plasma Motor Generator (PMG)**

The Plasma Motor Generator system is the most successful EDT mission to date. The PMG system consisted of a 0.5 km insulated copper tether connecting a Delta II second stage to a far end package (FEP), and each end-body was equipped with hallow cathode plasma contactors. The PMG tether mission produced several interesting results related to the system’s current closure, namely that the active plasma contactors allowed near-field current loop closure. The mission also demonstrated that the current in an EDT system is fully reversible, allowing the tether to operate in both motor and generator configurations. The PMG mission data exhibited significant differences in the current collected by the system in day and night ionospheric conditions while holding the collecting body at a constant potential and without any control on the tether’s current. The data set from the PMG mission is extremely valuable when trying to develop accurate models for an EDT system because it is the only existing data that shows the behavior of the collected current under
a variety of plasma conditions while the system parameters are held constant. The order of magnitude drop in collected current between the day and night sides that is seen in the PMG results shown Figure 1.2 is attributed to the drop in charge particle density and temperature, and the resulting decrease of plasma turbulence that aids in the transport of particles across field lines. In addition, the system’s orbit was chosen such that it flew over the Hilo, Hawaii, VHF radar site in order to detect plasma waves generated by the system’s motion through the ionosphere. The first ever observations of ion-acoustic solitons propagating along magnetic field lines were completed by the ground based radar as the PMG mission flew over.
1.1.2 Applications

Electrodynamic tethers have a wide variety of both science and engineering applications. As already mentioned, the anticipated primary application for EDT systems is to alter the system’s orbit via the electrodynamic force generated by the collection of current from the surrounding space-plasma acting on the tether. The EDT system’s ability to provide thrust without the use of propellent provides a tantalizing concept for extending satellite missions in Earth’s ionosphere without the need to replenish the system’s source of propellent. Another application that has been suggested for the generator mode configuration is space debris removal. In generator mode, the current driven through the tether by the emf interacts with Earth’s magnetic field to induce a drag force on the system that can be used to de-orbit space debris when a EDT is attached to the object. Additionally, EDT systems in generator mode offer an unlimited power source for LEO spacecraft that have an ready source of propellant to counteract the drag generated by the tether (EDT systems have been proposed as power systems for the International Space Station (ISS) and interplanetary exploration spacecraft). The current that is collected by system in generator mode can then be used to power systems aboard the spacecraft.

A recent study has shown that the number of space debris objects in LEO will continue to increase even if all satellite launches were suspended today. Many of the LEO debris objects are a result of the collision of dead satellites, such as the break up of the Fengyun-1C (FY-1C). Since LEO satellite operations have become an integral part of our society, the reduction of space debris is critical in order to maintain our ability to operate satellites in LEO. EDT systems in generator mode have been proposed as a means of removing large pieces of debris (e.g. the Electrodynamic Debris Eliminator). The basic concepts behind electrodynamic tether debris removal are momentum exchange (discussed later in this section) and electrodynamic drag. Large pieces of debris can be gradually de-orbited by attaching a long tether to them and allowing the electrodynamic force to act as an additional source of drag. Unfortunately, Levin et al. showed that the risk of collision between the tether attachment system and the debris mitigated the benefits of using EDT drag systems. However, the analysis of Levin et al. showed that momentum exchange electrodynamic tether systems are the most viable option for debris removal.

The concept of using a tether for momentum exchange to boost a payload into a higher orbit with out use of a second rocket stage has existed since Andreev and Konstantinov proposed the idea of a space escalator. The Momentum-Exchange/Electrodynamic-Reboost (MXER) tether system concept combines the concepts of momentum exchange and electrodynamic tethers. The concept of the MXER tether system is based on a rotating tether ~ 100 km in length, with various components (solar arrays, power systems, etc.) distributed along its length. A payload launched from Earth’s surface will rendezvous with an end of the tether as it reaches the lowest point in the tether’s rotation. Half a rotation later, the tether will release the payload, launching it into a higher orbit and transferring momentum to the payload. MXER will use its electrodynamic tether to regain the momentum.
that was lost to boosting the payload. The projected $\Delta v$ for the payload is $\sim 2.4 \text{ km/s}$.\textsuperscript{43} Momentum-exchange electrodynamic tether system, therefore, provides a re-useable means to boost a payload’s orbit without the need for additional (and expensive) rocket stages. A useful cross-section of literature pertaining to both electrodynamic tethers and momentum exchange tethers is provided in Ref. 14. A reversal of this concept (capturing the payload at the high point in the system’s rotation and releasing the payload at the system’s low point) could be used for debris removal and de-orbiting satellites.

Ionospheric scientists can also take advantage of the strong coupling between the ionospheric space-plasma and an electrodynamic tether system to design experiments to study the physics behind several types of plasma disturbances in the ionosphere. The most basic measurements possible involve high-impedance EDT systems and are based on deviations from the expected emf generated across the tether.\textsuperscript{6} The deviations from the expected emf will be driven by the presence of perturbing electric fields. High impedance EDT systems have the potential to provide new insight into the variations in weak electric fields that drive some ionospheric electrodynamics. Additionally, EDT systems at high magnetic latitudes can sample the difficult to measure component of the electric fields parallel to the magnetic field and provide new information about the polar ionospheric environment.

Low-impedance EDT systems can be used to generate large scale plasma disturbances in the ionosphere. A system that consists of two contactors connected by a fully insulated tether couples two physically separate regions of the ionosphere. Wave pulses are generated along magnetic field lines as current is absorbed and released. The frequency of the pulse is governed by the time the tether end-body is in contact with a given magnetic field line.\textsuperscript{6} In this manner, slow extraordinary (SE), fast magnetosonic (FM) and Alfvén waves can be radiated by the tether system into the cold ionospheric plasma.\textsuperscript{22,35,68} The PMG experiment was designed to generate ion-acoustic waves because the system was orbiting at a velocity greater than the ion-acoustic wave velocity in the ambient plasma. The VHF radar system placed in Hilo, HI detected ion-acoustic solitons modes radiating off of the system.\textsuperscript{87} Arrays of electrodynamic tethers time modulating their currents with a 90° phase shift can also be used to excite Whistler modes.\textsuperscript{68} Each of the waves modes discussed here occur naturally in the ionosphere, however they are governed by unpredictable processes in the space plasma. EDT systems would allow scientists to generate a known wave species at a known time, thus allowing them to design experiments around specific phenomena associated with each wave mode.

The ability of electrodynamic tethers to modify the surrounding space plasma environment enable them to modify particle populations present near their end-bodies. Incoming high-energy charged particles are trapped in Earth’s magnetosphere by magnetic mirroring in a region knowns as the Van Allen radiation belts.\textsuperscript{45} Spacecraft traveling through the radiation belts need to be protected from these high-energy particles via expensive shielding.\textsuperscript{88} Using ground stations to generate wave fields that will scatter the particles out of the Van Allen belts is relatively inefficient.\textsuperscript{68} Electrodynamic tethers can be used to either generate Whistler modes that mimic the natural diffusive processes that decay trapped particles or
scatter particles into the loss cone of the magnetic field line to which they are fixed off of the large potential structure that forms around a bare tether.\textsuperscript{69} Thus, electrodynamic tethers can also be used to remediate the Van Allen radiation belts.

The Jovian space environment offers the optimal operating conditions for electrodynamic tether systems to be used to explore Jupiter’s moons. The primary drawback to near-Earth operations is that EDT systems require a power source to maintain the potential difference across the tether that drives its current in motor mode. However, in the Jovian space environment offers two advantages when compared to Earth’s ionosphere: Jupiter’s magnetic field at its surface is an order of magnitude greater and the stationary orbit for Jupiter is about one-third the relative distance.\textsuperscript{68} Therefore, electrodynamic tether’s can operate above Jupiter’s stationary orbit in thrust mode without the use of a power supply because the Jovian magnetic field sweeps past the satellite system, inducing an emf in the tether in a direction that drives a current that will produce a thrust. The strong magnetic field and low stationary orbit altitude combine to create a co-rotating plasma environment that allows for “free” Lorentz force thrusting for EDT systems above the Jovian stationary orbit.\textsuperscript{13,68} The “free” thrusting allows the tether system to adjust its orbit so that it can maneuver between the Jovian moons.

In summary, the operation of electrodynamic tethers depends on the interaction between the space environment surrounding the EDT system. Several tether missions have successfully proved that EDT systems can provide both thrusting and power generation for satellite systems in LEO. The strong coupling between the tether and its space environment allow for EDT systems to be used as scientific instruments to study ionospheric phenomena that generate relatively weak electric fields. The next three sections provide the fundamental theory needed to understand the basic tether system dynamics, the system’s space environment and the physics governing the current collected by the tether system.

\section*{1.2 Tether Dynamics}

Tethered satellite systems provide a stable means to connect two structures in space due to gradients in the forces experienced by large tethered systems.\textsuperscript{23} This section will cover some of the basic operating principles and essential dynamics needed to understand the motion of a tether system. The failure of the ATEx open loop control system during deployment\textsuperscript{30} showed that additional dynamical issues exist in the system’s dynamics during deployment and retrieval, but the topics of deployment and retrieval are outside the scope of this dissertation. For the purposes of introducing the fundamental dynamics associated with tethered satellite systems, the dynamics of a simple, fully deployed dumbbell configuration oriented along the local vertical will be considered in order to facilitate a discussion of the basic motions of the system. Consider an electrodynamic tether consisting of two masses, $m_A$ and $m_B$, (where $m_A > m_B$ and $m_B$ is the upper mass), connected by a rigid tether of length $L$ and linear mass density, $\rho$. The tether’s dynamics can be broken down into two separate motions: the motion
of the system’s mass center, $\mathbf{R}_c$ and the pendular motion of the tether. The tether’s shape is defined by the general coordinate, $s$, where $s(\mathbf{R}_A) = s_A$ and $s(\mathbf{R}_B) = s_B$, shown relative to the geocentric reference frame in Figure 1.3. For a rigid tether, $s$ simply describes the line connecting $m_A$ and $m_B$. The equation of motion of the system’s mass center is governed primarily by the Newtonian gravitational field, with additional accelerations included in a perturbation function:

$$\ddot{\mathbf{R}}_C + \frac{\mu \mathbf{R}_C}{R^3} = \mathbf{w}_c$$

(1.1)

$$\mathbf{w}_c = (\nabla U)_C + \frac{G_2}{M} + \frac{I_T}{M} (s_A - s_B) \gamma_{AB} \times \mathbf{B} + \frac{\Phi_{oth}}{M}$$

where $\mathbf{R}_c$ is the vector stretching from Earth’s center to the center of mass of the EDT system, $\mu$ is the Earth’s gravitational constant, $\mathbf{w}_C$ is the perturbing acceleration function, and the total mass of the system, $M$, is given by

$$M = m_A + m_B + \int_{s_A}^{s_B} \rho ds = m_A + m_B + \rho L$$

(1.2)

The first term in the perturbation function is the gradient of the gravitational force function, $U$, taken at the system’s center of mass, which can include terms for the Sun, Moon and other planets, but does not include the main Newtonian component since it is already included on the left hand side of Equation 1.1. The second term in $\mathbf{w}_C$ defines the second order gravitational perturbations and is maximized in the vertical orientation. The maximum perturbation due to $G_2$ is given by

$$\frac{|G_2|}{M} = 3 \frac{\mu}{R_C^2} \left( \frac{r_i}{R_C} \right)^2$$

(1.3)
where \( r_i \) is the tethered system’s inertia radius. For the system in this example, the inertia radius is

\[
r_i \approx (s_A - s_B) \sqrt{\frac{1}{m_A} \left( m_B + \frac{m_T}{3} \right)}
\]  

(1.4)

where \( m_T \) is the total mass of the tether. The third term in the \( \mathbf{w}_C \) is the acceleration due to the Lorentz force acting on the tether due to the tether’s current, \( I_T \), and local magnetic field, \( \mathbf{B} \). Here, \( \gamma_{AB} \) is defined as

\[
\gamma_{AB} = \frac{R_A - R_B}{s_A - s_B}
\]  

(1.5)

The current, \( I_T \), is assumed to be positive in the direction of \( s \), that is from body \( B \) to \( A \). The last term in the equation for \( \mathbf{w}_C \) contains accelerations from aerodynamic drag, solar radiation pressure, micrometeorite impacts, and any thrusters attached to the tether system.\(^8\)

The primary motion describing the movement of the tether itself is the pendular motion that is driven by both the gradient of the gravitational force along the tether and the electrodynamic force. For the moment, consider a non-electrodynamic tether system with a rigid, massless tether of length \( L \) and whose end-bodies can be approximated by point masses orbiting in a circular orbit. The gravitational force acting on a point mass, \( m \), orbiting at a distance \( r \) from Earth, along the radial vector pointing from Earth’s center of mass to the body is\(^4\)

\[
F_g = -\frac{\mu m}{r^2}
\]  

(1.6)

The centrifugal force acting on the body is

\[
F_c = m r \omega^2_{sys}
\]  

(1.7)

where \( \Omega \) is the orbital angular velocity of the body. The net force acting on the point mass in a circular orbit must be zero. Therefore, the orbital velocity is

\[
\omega^2_{sys} = \frac{mu}{r^3}
\]  

(1.8)

Returning to the tether system in question, in order for the system to be oriented vertically, each element must have the same angular velocity, \( \omega_{sys} \). In order for the system to follow a circular orbit, there must exist a point, \( r_0 \), between the two masses where the net centrifugal force and the gravitational force are equal. If body \( A \) is the lower mass, then the net force acting on body \( A \) is dominated by the gravitational force (towards Earth). Conversely, the force acting on \( B \) is dominated by the centrifugal component (acting away from Earth). Therefore, a tension in the tether is developed. Now perturb the system slightly away from vertical by an angle \( \alpha \) in the orbital plane and an angle \( \beta \) out of the orbital plane. Since both the centrifugal and gravitational forces act in the system’s orbital plane, the vertical force acting on the end-body at distance \( l \) down the tether from \( r_0 \) (end-body \( A \)) can be
approximated by the sum of the net force acting at point \( r_0 \) and the gradient of the force in the vertical (radial) direction

\[
F_v = F_{r_0} + l \cos(\alpha) \nabla \left[ m_A r_0^2 \omega^{2}_{sys} - \frac{\mu m_A}{r^2} \right]
\]

(1.9)

where, by definition \( F_{r_0} = 0 \). Thus, using a small angle approximation, Equation 1.9 can be reduced to:

\[
F_v = m_A l \left[ \omega^{2}_{sys} + 2 \frac{\mu}{r^3} \right] = 3 m_A l \omega^{2}_{sys}
\]

(1.10)

The out-of-plane component of the net force is simply the out-of-plane component of the gravitational force:

\[
F_{o-p} = - \frac{m \mu l \sin(\beta)}{r^2} = -m \omega^{2}_{sys} l \beta
\]

(1.11)

Note that there is not a component of the net force acting on the body in the direction of the system’s motion. The in-plane torque is therefore

\[
\tau_{\alpha} = -l \sin(\alpha) F_v \approx -3 m_A l^2 \omega^{2}_{sys} \alpha
\]

(1.12)

The out-of-plane component of the torque is the sum of the torque due to the out-of-plane component of the force and the vertical component:

\[
\tau_{\beta} = -F_v l \cos(\alpha) + F_{o-p} l \sin(\beta) \approx -4 m_A l^2 \omega^{2}_{sys} \beta
\]

(1.13)

Since \( \tau_{\alpha} = m_A l^2 \ddot{\alpha} \) and \( \tau_{\beta} = m_A l^2 \ddot{\beta} \), the equations of motion for the tether’s libration for small angles are

\[
\ddot{\alpha} = -3 \omega^{2}_{sys} \alpha
\]

\[
\ddot{\beta} = -4 \omega^{2}_{sys} \beta
\]

(1.14)

The forces acting on the tether’s libration angles are therefore restoring forces on both the in-plane and out-of-plane angles. The system is therefore stabilized about the local vertical by the restoring forces acting on it. Each restoring force is primarily composed of the gravity gradient force. Thus, when oriented vertically, tether systems are said to be gravity gradient stabilized. Both the TSS-IR and the PMG tether system were gravity gradient stabilized.

Even though the electrodynamic force was not included in the discussion of gravity gradient stabilization, the development of the electrodynamic force is the primary focus of this dissertation and how it impacts both the orbital and pendular motions of the tether. The motion of the tether system throughout its orbit induces an electromotive force (emf) across the tether as it sweeps through Earth’s magnetic field. The emf generated by the tether’s motion is given by

\[
\mathcal{E}_i = - \int_L \langle \mathbf{B}, \mathbf{v} \times \mathbf{R}' \rangle ds
\]

(1.15)
where $s$ is again the parameter along the tether, the prime denotes differentiation with respect to the tether parameter $s$, $(\cdot)' = \partial/\partial s$, $\mathbf{R}(s, t)$ is a vector valued function that describes the tether’s shape in the Earth Centered Inertial (ECI) reference frame, $\mathbf{B}$ is the magnetic field vector in ECI and $\mathbf{v}$ is the velocity of tether element $ds$ relative to the moving space plasma. On board power systems can apply a control voltage, $\mathcal{E}_c$, to the tether, whose impedance is given by $G_t$. The remaining elements of the system (e.g. the plasma contactors, the ionospheric current path, on-board components, etc.) have a total impedance, $G_e$. The voltage balance for the entire system is thus,

$$ I_T (G_t + G_e) = \mathcal{E}_i + \mathcal{E}_t $$  \hspace{1cm} (1.16)

where $I_T$ is the current in the tether collected from the space-plasma. The quantities $\mathcal{E}$ and $I_T$ are again defined as positive in the direction of the tether parameter $s$. The local magnetic field, therefore, exerts an electrodynamic force acting on the tether per unit length, given by:

$$ \mathbf{F} = I_T \mathbf{R}' \times \mathbf{B} $$  \hspace{1cm} (1.17)

The electric power consumed (or produced) over the entire circuit defines the rate of the mechanical work performed on the tether by the distributed force:

$$ -\mathcal{E} I_T = \int_L (I_T \mathbf{R}' \times \mathbf{B}, \mathbf{v}) \, ds = I_T \int_L (\mathbf{B}, \mathbf{v} \times \mathbf{R}') \, ds $$  \hspace{1cm} (1.18)

In generator mode, $\mathcal{E} = \mathcal{E}_i$ and the current driven through the tether acts to brake its orbital motion. The electric power $\mathcal{E}_i I_T$ is generated at the cost of an equivalent amount of orbital energy.\textsuperscript{8} However, in thruster mode, a control voltage is implemented across the tether such that $\mathcal{E}_c = -k \mathcal{E}_i$ where $k > 1$. The control bias switches the direction of the current flow, aligns a component of the electrodynamic force in the direction of the tether’s motion and pumps energy into the system’s orbit.

An electrodynamic tether’s motion is impacted by the electrodynamic force that acts along the tether’s length. Both the system’s orbit and its pendular motion are affected by the magnitude of the electrodynamic force. This force is governed by the interaction of the tether system and the local space environment, which is in turn determined by the dynamics of the charged particles in the space-plasma. The next section focuses on introducing the basic physics governing charged particle motion in Earth’s ionosphere and the key parameters that characterize a space-plasma.

### 1.3 Particle Motion in an Ionospheric Plasma

The ionosphere is comprised of a space-plasma, a quasi-neutral gas of charged and neutral particles that display a collective behavior. A gas is said to be quasi-neutral if it can be
assumed that \( n_k \approx n_e \approx n \) (where \( n_e \) and \( n_k \) are the number densities of the species present in the plasma, and \( n \) defines the plasma density that is equal to the number density of each species), yet still enables the local electromagnetic forces driving the plasma’s dynamics to exist. The formation of an ionosphere is driven by the presence of a neutral atmosphere and an ionizing agent (e.g. high energy photons). The ionosphere is defined by three regions between approximately 60 \( km \) and 1000 \( km \): the D-region (60 \( km \) to 85 \( km \)), the E-region (85 \( km \) to 140 \( km \)) and the F-region (140 \( km \) to 1000 \( km \)). The ionization of the neutral atmosphere creates the ionosphere via photoionization (where most of the photons originate from the Sun) and energetic particle precipitation. The characteristics of a general electron density profile, shown in Figure 1.4a, are governed by several competing processes. As the neutral atmosphere absorbs the solar spectrum of EUV and UV radiation, it ionizes via photoionization to form a space-plasma and its temperature increases (the concept of temperature will be discussed later in this section). The balance of the recombination rates of ions and electrons (the rate at which ions and electrons combine to form a neutral atom) with the rate of photoionization provides an upper bound on the plasma density. The variation of the plasma’s density with altitude is dependent on the increasing recombination rate and neutral density, and decreasing photon flux with decreasing altitude, where a maximum density occurs around 300 \( km \) in the F-region. The ion populations in each region (see Figure 1.4b) are stratified by Earth’s gravitational field. The average electron density also varies by an order of magnitude between the midlatitude dayside ionosphere and the nightside ionosphere because of the decrease in the photon flux that drives the photoionization process. The density peak around 300 \( km \) in the profile in Figure 1.4a is known as the F-peak and its altitude ranges between 200 \( km \) and 400 \( km \) depending on the state of the ionosphere.

Electrodynamic tether applications assume satellite operation in Low Earth Orbit (LEO) around the F-region of the ionosphere due to the high electron density. The space environment in this region consists of Earth’s dipole magnetic field immersed in a plasma of neutral particles, oxygen ions and electrons (along with small densities of the other ion species). There are several fundamental parameters associated with charged particle dynamics that are used to characterize the state of a space-plasma. As charged particles move through Earth’s magnetic field, \( \mathbf{B} \), with velocity \( \mathbf{v} \), they experience a Lorentz force

\[
 m_k \frac{d\mathbf{v}}{dt} = q_k (\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

where \( m_k \) and \( q_k \) are the mass and charge of the \( k \)th species (e.g. electron, hydrogen ion, oxygen ion, etc.). The magnetic field in LEO is typically modeled as a dipole tilted at 11°. Additional perturbations can be added to the magnetic field dipole model to account for variations in Earth’s core and crust. The Lorentz force induces gyro-motion of charged particles about a guiding center corresponding to a magnetic field line. These particles gyrate with a cyclotron frequency

\[
 \Omega_{ck} = \frac{|q_k|B}{m_k}
\]
Figure 1.4: Average profiles for the charged particle densities in Earth’s ionosphere.

(a) The diurnal variation of the electron density with solar activity (Figure from Ref. 17).

(b) The electron and ion composition of the ionosphere as a function of height (Figure from Ref. 39).
and a radius from the gyro-motion’s guiding center

\[ \rho_{Lk} = \frac{m_k v_{\perp 0}}{|q_k| B} \]  

(1.21)

where the terms \( \rho_{Lk} \) and \( \Omega_{ck} \) denotes the Larmor radius and cyclotron frequency of particle species \( k \). The Larmor radius for ionospheric particles is approximately 3 cm for electrons and 6 m for oxygen ions. The gyro-motion of particles about field lines and their low thermal energy in the ionosphere result in particle motion that is fixed to magnetic field lines.

Another important concept that defines a space-plasma is the plasma’s temperature. The most probable distribution of the velocities of the particles in a gas in thermal equilibrium is known as a Maxwellian distribution.\(^\text{15}\) The Maxwellian distribution for the \( k \)th species of plasma particles is given by:

\[ f(\mathbf{v}) = n \left( \frac{m_k}{2\pi KT_k} \right)^\frac{3}{2} \exp \left( -\frac{1}{2} \frac{m_k ||\mathbf{v}||^2}{KT_k} \right) \]  

(1.22)

where \( K \) is Boltzmann’s constant. The width of the Maxwellian distribution is determined by the constant \( T_k \), which is used as the definition of the species’ temperature. Since the species average kinetic energy per degree of freedom is \( \frac{1}{2} KT_k \), temperature is usually defined in units of energy.\(^\text{15}\) Since the temperature of a species is defined in terms of its velocity distribution in a plasma, disturbances in a plasma that can affect a particle’s velocity will also affect its temperature. Thus, species in the same plasma can have unequal temperatures due to differing collision rates in situations where the plasma does not last long enough to attain thermal equilibrium. Additionally, due to the magnetic field’s significant influence on particle motion, plasmas tend to have varying temperatures parallel and perpendicular to the magnetic field. However, the temperature in a plasma decreases with decreasing altitude (a typical temperature profile is shown in Figure 1.5) due to the attenuation of the solar flux as the Sun’s photons are absorbed by photoionization processes. A plasma’s temperature should not be confused with the concept of heat because the low density of a space-plasma translates to a small heat capacity.\(^\text{15}\) Note that satellites in LEO near the F-peak are orbiting at \( v_0 \sim 0.045v_{\text{the}} \), where \( v_{\text{the}} \) is the electron thermal velocity.

Particle collisions are a means of transferring energy (and temperature) between space-plasma processes, such as waves and other plasma disturbances, and species. There are two different types of collisions in a plasma: coulomb collisions between charged particle species and neutral collisions between the plasma and neutral particles. Collisions between charged particles (electron-electron, ion-electron and ion-ion) can result in the diffusion for like particle collisions.\(^\text{15}\) The effect of charged particle collisions with neutrals is determined by the time scale of the plasma process being studied, \( \tau \), and the collision frequency. The collision frequency between the \( k \)th particle species and a neutral atom is given by

\[ \nu_{kn} = n_n v_{\text{thk}} \sigma_n \]  

(1.23)

where \( n_n \) is the neutral number density, \( v_{\text{thk}} \) is the thermal velocity for the \( k \)th species and \( \sigma_n \) is the collision cross-section of the neutral atoms.\(^\text{34}\) If \( \nu_{kn} \tau >> 1 \), then plasma is said to be
Figure 1.5: A typical temperature profile for the ionosphere plotted along with the electron thermal velocity, $v_{th}$, and relative orbital velocity, $v_0$, of a satellite as a function of altitude (Figure from Ref. 56).
collisional and its motion is dominated by the motion of the neutral gas. If \( \nu_{kn} \tau \ll 1 \), then the plasma is collision-less and collisional effects can be neglected. However, if the plasma is in the weakly collisional regime, then the slow rate of energy transferred to the neutrals by collisions will damp out any disturbances or waves present.

In addition to the single particle motion, plasmas display collective motions in the form of plasma oscillations. Suppose an electron is displaced from a background of uniform ions. The electric field induced by the electron’s displacement generate a restoring force that pull the electrons back to their original positions. The inertia of the electrons cause them to overshoot their original positions, causing another restoring force to develop. The oscillation of the electron is on such a small time scale that the much more massive ions do not have time to react. The electrons continue to oscillate around their original positions due to the restoring force of the ions. The frequency of electron’s oscillation defines the characteristic frequency of the plasma’s oscillation, termed the plasma frequency:

\[
\omega_p = \left( \frac{n_e e^2}{\epsilon_0 m_e} \right)^{1/2}
\]

where \( \epsilon_0 \) is the permittivity of free space. Since each of the parameters in Equation 1.24 is constant except the plasma density, the plasma frequency (and hence the frequency of oscillations in the plasma) depend primarily on the plasma’s density.

The plasma’s behavior is also defined by the shielding of applied electric field potentials. Suppose a satellite biased to a potential, \( \phi_0 \), is placed in an ionospheric plasma. The satellite will be surrounded by a non-neutralized region that will shield the electric field created by the satellite from penetrating into the rest of the plasma. The thickness of this region can then be calculated by determining the potential distribution in the plasma using Poisson’s Equation

\[
\epsilon_0 \nabla^2 \phi = -e(n_i - n_e)
\]

where \( n_i \) and \( n_e \) are the ion and electron number densities, respectively. For a single ion, the thickness of the non-neutralized region, the Debye length \( \lambda_D \), is defined as

\[
\lambda_D = \sqrt{\frac{\epsilon_0 K T_e}{n_e^2}}
\]

and is a measure of the distance over which the potentials that develop within the plasma are screened. For an ionospheric plasma, the Debye length is approximately 1 cm. Once a charged particle moves a Debye length away from another charged object, the electrostatic interaction between the two bodies can be assumed to be negligible. Therefore, the definition of quasineutrality can be updated to state that a quasineutral plasma is one where \( \lambda_D << L \), where \( L \) is the characteristic size of the plasma. The non-neutral region around a satellite defines its sheath and is typically several Debye lengths in thickness.

Each of the parameters mentioned (Debye length, species temperature and density, plasma frequency, cyclotron frequency and collision frequency) are required to define space plasmas in simulations. The fundamental parameters (temperature and density) on which the
characteristic time and length scales of the plasma depend can be accurately calculated by the International Reference Ionosphere (IRI).\textsuperscript{12} IRI is an internationally accepted model for the non-auroral ionosphere that is comprised of monthly data from a world wide network of facilities. IRI is used to generate the temperature and density parameters for several of the models in this dissertation in order to incorporate accurate variations in the plasma state into the tether/plasma interaction.

## 1.4 Fundamentals of Tether Current Collection

There is a wide range of techniques that have been proposed for collecting current in an EDT system. These techniques can generally be split into two categories: active and passive plasma contactors. Passive contactor electrodes offer the significant advantages of simplified end-body design, minimum mass and increased system robustness.\textsuperscript{79} The common configurations for passive contactors include conductive spherical end-bodies, partially bare tethers, and gridspheres. Active current collection enhance the end-body’s electrical connection with the surrounding plasma using a plasma generation source, such as the Hallow Cathode Assembly on the PMG mission.\textsuperscript{87} The tether system model developed in this dissertation will make use of a passive plasma contactor and thus, this section will focus on processes related to passive techniques for current collection.

The physics of the current collected by an EDT end-body is very similar to that of spacecraft charging, with the primary differences of higher bias potentials and a means to transport the collected charge away from the body (the conducting tether). Therefore, most of the theory pertaining to tether current collection has it roots in spacecraft charging. The text by Lai\textsuperscript{49} provides a thorough development of the spacecraft charging problem. First, some basic terminology needs to be introduced for specific charging situations as defined by Lai:\textsuperscript{49}

- **surface charging** occurs on conducting spacecraft where charges remain on the surface.
- **absolute charging** refers to a uniformly charged spacecraft, which as a result has only one potential $\Phi_0$.
- **differential charging** can occur on spacecraft composed of electrically separated surfaces. In this case, the surface potentials will depend on both the material and the surrounding space environment, which may be anisotropic.
- **deep dielectric charging** occurs due to the penetration of very energetic ions or electrons (MeV or higher) deep into dielectrics, which are nonconductors.

The electrodynamic tether end-bodies considered here collect current in the absolute charging state, where the end-body has a uniform potential.
There are several sources of charging or current flux to a positively biased satellite system. The primary current sources are the electrons and ions that are incident on the spacecraft’s surface and these sources will be discussed in detail throughout the rest of this section. Typically, the ambient electron flux is greater than the flux of ions simply because the electrons move faster due to the smaller mass (when $kT_e = kT_i$, $v_e \approx 46v_i$ for hydrogen ions). In the sunlight, energetic photons scattered electrons off of the satellite’s surface, creating a photoemission current away from the spacecraft that can exceed the ambient current sources arriving at an unbiased spacecraft. Secondary and backscattered provide a source of electron flux away from the satellite’s surface. Secondary and backscattered emissions are formed when a primary electron impacts a surface and either backscatters or energizes an electron near its impact site enough to cause the electron to escape. The magnitude of the backscattered and secondary electron flux therefore depends on the surface properties of the spacecraft. The final source of particle emission away from a satellite that needs to be discussed as active plasma contactors, such as the Hallow Cathode Array on the PMG mission, that release a beam of electrons or ions in order to modify the impedance of the surrounding space-plasma.

Orbit Motion Limited (OML) theory is one of the fundamental models used to describe charge particle collection by both Langmuir probes and spacecraft. A Langmuir probe is a charged body (typically a spherical, cylindrical or planar probe) that when immersed in a plasma collects a current based on a predetermined voltage. OML theory is used to relate the potential of the probe and the current collected to the charged particle density in the plasma, typically displayed via current-voltage curves, such as the one shown in Figure 1.6. OML theory can also be applied to charged spacecraft moving through a space-plasma. Consider

![Figure 1.6: Orbit-motion-limited current voltage characteristic for one, two and three dimensions (from Ref. 46), where $J$ and $J_0$ are the current and thermal current.](image)
a charged particle traveling at a velocity, $v$, infinitely far away from a biased spacecraft that attracts the particle. As the particle moves towards the spacecraft, conservation of energy states that:

$$\frac{mv(r)^2}{2} + q\phi(r) = \frac{mv^2}{2}$$

(1.27) where $q$ and $m$ are the charge of the attracted particle. The maximum impact distance is calculated using the conservation of angular momentum of the particle:

$$mvh = mva$$

(1.28) where $a$ is the radius of the probe, $v_a$ is the velocity of the particle $v(a)$ when it touches the probe tangentially at $r = a$, and $h$ is the distance from the center of the probe at which a particle can originate in order for it to intersect the surface of the probe tangentially. The kinetic energy of the particle at infinity is approximately $mv^2/2 = kT_e$. Substituting the particle’s kinetic energy and Equation 1.28 into Equation 1.27 results in impact distance

$$h = a \left(1 - \frac{q\phi}{kT}\right)^{1/2}$$

(1.29) and impact velocity:

$$v_a = v \left(1 - \frac{q\phi}{kT}\right)^{1/2}$$

(1.30) Assuming perfect collection by the surface of the spacecraft, every particle inside the radius $r = h$ will be collected. Therefore, the current collected will be the flux through the surface area through the sphere of radius $h$. Thus, the current collected by the satellite, $I(\phi)$, is given by:

$$I(\phi) = 4\pi h^2 nqv$$

(1.31) Substituting Equations 1.29 into 1.31 gives the current collected by a spherical satellite:

$$I(\phi) = 4\pi nqva^2 \left(1 - \frac{q\phi}{kT}\right)$$

(1.32) Note that if the spacecraft is unbiased ($\phi = 0$), then satellite collects the current $I(0) = 4\pi nqva^2$. Equation 1.32 is known as the OML-current collection formula for spherical spacecraft and was originally derived in Ref. 59. OML current curves for different Langmuir probe geometries are shown in Figure 1.6. Orbit motion limited theory, however, does not apply to a space-plasma that is partially magnetized. Parker and Murphy\textsuperscript{61} developed a similar model to account for the presence of a magnetic field. The Parker-Murphy model will be discussed in detail in Chapter 3.

Current collection models that based conserving aspects of single particle motion (such as OML theory and the Parker-Murphy model) do not take into account the collective behavior
of the plasma near the satellite. Sheath-based theories attempt to include the motion of the plasma motion as a whole by examining the behavior of the electron’s distribution function in the presence of a biased satellite. Consider a satellite positively biased relative to the ambient plasma potential. The potential in the plasma will vary from the satellite’s bias potential to the potential of the ambient plasma. Per the earlier discussion on Debye shielding, the plasma tends to shift around charged bodies to confine the area of potential variation to within a few Debye length’s of the satellite. The sheath is defined by the layer of potential variation near the boundary of the satellite. Sheath current collection models stem from the assumption that the edge of the sheath is defined by the boundary where the electric field abruptly changes sign and the electrons shift from one to two dimensional acceleration. The primary drawback of sheath based models for use in complex computational models is that they require the solution Poisson’s equation to determine the sheath’s thickness. Another disadvantage of sheath models is that they do not account for the sheath’s asymmetry in the presence of a magnetic field. Several sheath based models will be briefly discussed in Section 2.1.2.

In addition to a variety of sources of current collection, several different geometries for EDT plasma contactors have been proposed: passive metallic balloons, partially bare tethers, gridspheres and active plasma contactors. Current collection by passive metallic end-body (or satellite) is the subject of the remainder of this dissertation and will not be discussed here. Partially bare tethers have twenty to eighty percent of the tether’s length exposed to the plasma environment. The Momentum Exchange and Recovery (MXER) mission was going to be the first to use partially bare tethers. Since partially bare tethers can stretch over many kilometers, they can collect a constant current when passing through regions of variable charged particle densities, such as plasma bubbles. Hence, bare tethers have been considered due to their ability to maintain a constant current under a variety of plasma conditions. Gilchrist et al. compared the collection efficiency of a bare cylinder, tape tether and mesh tether in a dense, high speed plasma and concluded that tape tethers with widths above a Debye length are most efficient. However, Gilchrist et al. noted that mesh tethers may be a better design due to their survivability with respect to micro-meteoroid impacts. Stone et al. proposed the use of gridspheres as a lighter, more efficient means of current collection. Gridspheres are essentially a conducting mesh structure in the shape of a sphere. The mesh structures give incoming electrons two chances to be collected via the electron’s ability to cross the interior of the sphere. Therefore, gridspheres can collect current in both the ram and wake directions. Khazanov et al. compared the efficiency of gridspheres and solid spheres as passive plasma contactors and concluded that gridspheres would require about half of the mass of a solid sphere to collect the same amount of current. Active plasma contactors are systems that release particles into the vicinity of collecting body to modify the plasma around the contactor in order to reduce the impedance between the contactor and the plasma. Active plasma contactors have been demonstrated to improve the current collected by an EDT tether system during the PMG mission. The work in this dissertation, however will focus on current collection by passive spherical satellites that are positively biased and whose only source of current is from electron and ion fluxes from the
ambient space plasma.

1.5 Contributions of the Present Study

The survey of the introductory material discussed in the previous sections motivates the primary contributions of this dissertation. Even though there exists a large body of research investigating the dynamics of an electrodynamic tether system, the literature is relatively sparse on the topic of the coupling between the system’s dynamics and the state of the local space-plasma environment. A full review of the literature will be given in Chapter 2. For the purposes of understanding the contributions of this dissertation, it is sufficient to simply state that previous investigations of EDT systems decouple the tether’s dynamics from the space-plasma state’s effect on the tether’s current and vice versa. The primary factor limiting the ability to incorporate the physics of the current collection into the system’s dynamics is that the complex processes that govern the behavior of the current collection are not well understood. The goal of each section is to incrementally develop a better understanding of the interplay between the tether system’s dynamics and the state of the surrounding space-plasma.

1.5.1 Effects of the Local Plasma Environment on the Dynamics of Electrodynamic Tether Systems

A simple analytical model for the current collected by the EDT system governed by IRI 2012 is combined with a high fidelity tether dynamics model to create the first EDT system model that couples the affects of the diurnal variations in the tether’s current observed in the PMG mission into an EDT system’s dynamics. The dynamics of the new system model are compared to a model where the current is held at a constant value. The current oscillations from the simulation matches the characteristics of the data collected during the PMG mission. Additionally, new instabilities are observed in the tether’s dynamics that are driven by the oscillations in the state of the plasma throughout the system’s orbit. The oscillating current was found to pump energy into the librations of the tether out of the system’s orbital plane and drive the out of plane oscillations unstable. The diurnal variations in the current also introduced new modes into the tether’s oscillations in the orbital plane.

1.5.2 Investigation of the Current Collected by a Spherical Satellite with Application to Electrodynamic Tethers

As discussed in Section 1.4, the current collected by an electrodynamic tether end-body is typically modeled by a spherical satellite traveling through a space-plasma. A parametric
study of the variation of the current collected by a spherical satellite with both system and plasma parameters is completed. The research focused on improving the understanding of the plasma dynamics near a positively biased satellite moving through the ionosphere and to characterize the effects that the dynamics have on both the structures and the current accrued by the satellite. The simulations run for Chapter 4 comprise the first set of Particle-in-Cell simulations to investigate the effects of sweeping through a range of magnetic field orientations and collisions on the current collected by a positively biased satellite. The variation in the magnetic field’s orientation was found to induce the formation of two different types of structures as the orientation of the magnetic field varies from parallel to anti-parallel relative to the EDT end body’s orbital velocity. When the magnetic field is near perpendicular to the satellite’s orbital velocity, the magnetic field induces azimuthal current structures that enhance the current arriving at the satellite. A magnetic field that is parallel (or anti-parallel) to the satellite’s velocity forms lobed structures that limit the current. The satellite’s bias relative to the plasma’s potential was also found to induce the formation of a bow-shock structure when raised above a certain level. The bow-shock significantly limited the current arriving at the satellite. The introduction of collisions into the space-plasma inhibited the formation of the bow-shock structure and its current limiting affects were negated. Additionally, increasing the ratio between of the cyclotron frequency to plasma frequencies was found to limit to current collected. Analytical curve fits were generated to characterized impact that the variation each parameter has on the current collected by the satellite.

1.5.3 Electrodynmic Tether Instabilities Induced by the Dependence of the Collected Current on the Magnetic Field Orientation

Thompson et al.\cite{80} develop a curve fit function to the current-voltage characteristics of the TSS-1R sub-satellite based on the Parker-Murphy model. The variation in the current collected by an electrodynamic tether end-body with the local magnetic field orientation observed in the Particle-in-Cell simulations is incorporated into the curve fit form developed by Thompson et al. to build a plasma parameter dependent, numerical current collection model. The plasma parameters are again determined by the International Reference Ionosphere 2012 model. The dynamics of an electrodynamic tether system are studied under the influence of both a constant current and the plasma-parameter dependent model over a range of orientations. The results from this study show that as the inclination of the system’s orbit increases, the average current collected decreases. The decrease in average current cancels out an increase in the component of the electrodynamic force out of the system’s orbital plane known to drive an instability in the tether’s librations and reduces the ability of the electrodynamic force to increase the system’s altitude. The results indicate that electrodynamic tether systems will have smaller pendular motions on orbits with large inclinations, but will not be as effective at boosting the system’s orbit.
1.6 Organization of Dissertation

The remaining chapters of this dissertation are organized in a manner that will lead the reader through the development of the first electrodynamic tether system model that couples models for the dynamics and plasma interactions. Chapter 2 will review the literature pertaining to the study of both the dynamics of electrodynamic tether systems and plasma dynamics in the vicinity of charged bodies. Since the tether models vary from chapter to chapter, each chapter will contain an overview of the models and methods used to obtain the results presented in that chapter. Chapter 3 will focus on the implementation of a simple model for the tether’s current that is dependent on the local plasma state and the dynamical instabilities that result from the diurnal variation in the plasma parameters. A Particle-in-Cell model is then used to study the impact of the plasma structures that form around a charged body on the current it collects, presented in Chapter 4. The results from Chapter 4 are used to develop an improved current model for the system in Chapter 3. The improved current model is integrated into the system dynamics model and the system dynamics are studied over a range of inclinations in Chapter 5. Finally, Chapter 6 gives a summary of the conclusions of the dissertation and recommends directions for future research.
Chapter 2

Literature Review

Literature pertaining to electrodynamic tethers is generally sectioned into two categories: system dynamics and system interactions with the space environment. The following sections will reflect this separation. Section 2.1 will review the existing analytical current collection models. The next section will review the results from Particle-in-Cell simulations and how they have improved the understanding of the current collected by a circular spacecraft. Finally, the last section reviews the existing tether dynamics models.

2.1 Analytical Current Models

The study of current collection by a cylindrically or spherically symmetric charged body in a stationary space plasma began with the development of Langmuir probe theory and subsequently OML theory. Several groups have since developed analytic models for the upper bound on current collected. As mentioned above, Parker and Murphy rigorously conserved particle energy in the presence of an ambient magnetic field which resulted in the relationship \( I \propto \phi^{1/2} \). Linson considered the effects of turbulence on electron transport across magnetic field lines to create the constant density cylindrical shielding model: \( I \propto \phi / \ln \phi \). Rubenstein and Laframboise and Sanmartin developed complimentary models for an upper bound on current collection using kinetic theory.

Dobrowlony et al. and Khazanov et al. review several common current collection models and compare them to the results from the TSS-1R mission, shown in Figure 2.1. The curves labeled PM-1 and PM-2 represent the Parker-Murphy model and the Parker-Murphy model that has been adjusted by Singh and Chagenti for the motion of the tether end-body across the local magnetic field. The two Parker-Murphy models are discussed in Section 2.1.1. Lai’s text presents a fundamental description of existing spacecraft charging models and develops a sheath based model that determines the current-voltage characteristics of an EDT. The model results from equating the current entering the sheath with the current.
in the tether. However, the model assumes a spherical sheath and does not account for its deformation along field lines in the presence of a magnetic field. The red dots in Figure 2.1 represent a similar model that was developed by Khazanov et al.\textsuperscript{44} has the best correlation with the current voltage characteristics from the TSS-1R mission. All of the sheath based models require solving Poisson’s equation numerically for to find the sheath radius based on a specified density structure. Therefore, an analytical model that is both closed form and incorporates the physics of the relative motion between the satellite and the local magnetic field does not exist.

### 2.1.1 Parker-Murphy Model and Adjusted Parker-Murphy Model

The current collected by a spherical body in a space plasma is a well studied problem with applications including plasma diagnostics, spacecraft charging and electrodynamic tethers. For an ionospheric plasma, only two satellite environments have been considered: a stationary plasma with a uniform ambient magnetic field and a plasma that is drifting normal to the magnetic field simulating the satellite’s orbital motion, as shown in Figure 2.2.

The Parker-Murphy Model\textsuperscript{61} is an analytical solution used to predict an upper-bound on

![Figure 2.1: A comparison of the existing theoretical models and experimental results from TSS-1R (reproduced from Ref. 44).](image)
current collection by a spherical satellite in a non-flowing plasma with an ambient magnetic field. By conserving canonical angular momentum, Parker and Murphy found that the satellite will collect all of the electrons originating infinitely far away from the satellite within a collection area of radius $r_0$, as shown in Figure 2.2a. Since the thermal velocity of electrons in the ionosphere is approximately 300 km/s, the effects of the plasma drift (typically around $v_0 = 8$ km/s for satellites orbiting at 300 km) were initially thought to be insignificant. However, the results of the first electrodynamic tether experiment\textsuperscript{22} onboard the space shuttle, the Tethered Satellite System (TSS-1R), showed that currents 4-6 times greater than those predicted by Parker and Murphy are collected. There have been several analytical\textsuperscript{46} and numerical\textsuperscript{75} studies that have corroborated these observations by investigating the geometry shown in Figure 2.2b. Since the addition of a drift velocity caused a significant increase in current collection, it is important to investigate whether the current collected significantly varies as the drift velocity changes direction. The previously studied cases do not characterize the effects on charge collection by a satellite whose velocity vector is at an oblique angle to the ambient magnetic field. The theory needed to examine the oblique angle case has not been developed such that an analytical solution exists. Therefore, this study will be an important first step towards understanding the dependence of current collection by a spherical satellite on the direction of orbital motion.

In light of the results from the TSS-1R mission predicting currents 4-5 times higher than predicted by the Parker-Murphy model, Singh and Chagenti\textsuperscript{72} added an estimate for the sweeping effects of the magnetic field to the Parker and Murphy model. It should be stressed that this is a qualitative estimate of the additional current collected due to the orbital motion of the satellite and that it is only meant to provide a qualitative feel for the current enhancement. The additional current is calculated by estimating the charges in the sheath flux tube intercepted by the velocity flow. The actual current enhancement is governed by very complex plasma processes. As pointed out by Dobrowolny et al.,\textsuperscript{21} this estimation does not depend on the satellite’s velocity. Both the Parker-Murphy and Adjusted Parker-Murphy models will be covered in the methods section of Chapter 3.
2.1.2 Other Models

In addition to the aforementioned Parker-Murphy model, several numerical models for the current collected by a spherical end-body in an ionospheric plasma exist. The numerical models split the region near the satellite into two regions, where the inner region is known as the magnetic pre-sheath. The boundary of the pre-sheath is defined as the point where all of the ions are reflected due to the positive potential of the spacecraft. Due to the reflection of the ions, their density at the boundary is twice the density of the ambient plasma. Inside the pre-sheath, the electrons are one-dimensionally accelerated towards the satellite. Al’pert et al.\textsuperscript{2} assume an isotropic particle flux to the biased satellite, but do not take into account the effects of the orbital motion of the satellite nor the magnetic field. Laframboise\textsuperscript{46} estimates the extension of the electron collection region up magnetic field lines and calculate the electron flux into the magnetic pre-sheath region (whose boundary is defined by the number density contour where both the electron and ion number densities double their ambient levels, inside of which the gyromotion of electrons breaks down due to the presence of large electric fields). The resulting enhancement in current is 6.41 times the Parker-Murphy current.

Khazanov et al.\textsuperscript{44} extend the work of Laframboise\textsuperscript{46} by redefining the boundary of the magnetic presheath region as the location where the potential is near zero and the electric field changes sign. The current arriving at the boundary of the pre-sheath for a spherical satellite of radius $R$ is given by:

$$\frac{I}{I_0} = \frac{1}{\sqrt{\pi}} \frac{r_b^2}{R^2} \left( \sqrt{\chi_b} + \frac{1}{2\sqrt{\chi_b}} \right)$$

where $I_0$ is the random thermal current arriving to the surface of the pre-sheath, $r_b$ is the radius of the pre-sheath and $\chi_b = e\Phi_b/kT$ is the normalized potential at the pre-sheath boundary. It is assumed that only the ram hemisphere of the pre-sheath collects current. The pre-sheath radius is determined by solving the one-dimensional Poisson equation:

$$\frac{1}{x^2} \frac{d}{dx} x^2 \frac{d\chi}{dx} = \frac{R^2}{\lambda_D^2} \left( \frac{2}{\sqrt{\pi}\chi} - e^{-\chi} \right), \quad 1 \leq x = \frac{r}{R} \leq \frac{r_b}{R}$$

where the electron density is chosen to equal the ion density set at twice the ambient level. The red dots in Figure 2.1 show that the model developed by Khazanov et al. captures the I-V characteristics of the TSS-1R data.

Finally, Thompson et al.\textsuperscript{80} develop a Parker-Murphy based curve fit to the TSS-1R data of the form

$$\frac{I}{I_0} = \alpha \left[ 1 + \left( \frac{\Phi}{\Phi_0} \right)^\beta \right]$$

where the Parker-Murphy model is obtained if $\alpha = 1$ and $\beta = 0.5$. Throughout the TSS-1R experiment $\alpha$ ranged between 2.2 to 2.9 and $\beta$ varied between 0.47 and 0.56. Each of these
models contains a relatively significant assumption (e.g. limited parameter regime, orbital velocity perpendicular to magnetic field) due to the limited understanding of the physics near a positively charged body.

\section*{2.2 Investigation of Current Collection with PIC Models}

In the absence of a closed form, analytical model that captures the physics of current collection by an orbiting satellite, Particle-in-Cell (PIC) codes are the state of the art tool used to further the understanding of the behavior of the plasma around a positively charged satellite and thus its current-voltage properties. A summary of the PIC method is presented in Chapter 4. Several PIC codes\cite{29,38,60,71} have been developed to study the spacecraft charging problem. However, the work completed by N. Singh\cite{72,75–77,85} is directly applied to electrodynamic tether systems.

Singh and Vashi\cite{76} created a 2-D3$v$, two specie, PIC code that examined the effects of an ambient magnetic field on a long conducting cylinder with drift velocity $v_0$. Simulations were run for $B = 0$, $B = B_0\hat{z}$, $B = B_0\hat{y}$ and $B = B_0\hat{x}$ where the cylinder’s velocity is $v = v_0\hat{x}$. They found that in the perpendicular, out-of-plane case ($B = B_0\hat{z}$) the current collected by the satellite is significantly higher than predicted by the Parker-Murphy model. In the parallel case, the current is limit as predicted by the Parker-Murphy model and

Figure 2.3: A schematic of the field aligned currents to the azimuthal currents near an orbiting satellite at a positive potential (reproduced from Ref. 75). Note that the current has an increasing radial component as the current radial distance from the satellite decreases.
potential structures form along the magnetic field lines with their dimensions transverse to the magnetic field determined by the current limiting radius given in the Parker-Murphy model (equation 3.7). The case where $B = B_0\hat{y}$ resulted in an average current that lies between the other two cases. Vashi and Singh\(^8\) show that fan shaped potential structures form periodically and cause the current to vary with the same frequency over a range of satellite velocities. The observed potential structures produce a large collection area for electrons and are responsible for the enhancement of the collected current. Also, the period of the plasma oscillations is found to vary inversely with the satellite’s orbital velocity. Vashi and Singh also observe that the potential structures drive an azimuthal current towards the satellite. Singh et al.\(^7\) later determine via fully 3D PIC simulations that a.) field aligned currents contribute to the formation of the azimuthal current as shown in Figure 2.3, b.) the azimuthal current observed in the PIC simulations is likely the same current observed in the measurements from TEMAG on the TSS-1R satellite, and c.) the current varies with satellite radius as predicted by the Parker-Murphy model. Singh et al.\(^7\) also acknowledge that the key issue needing to be resolved is the variation of the current enhancement with satellite voltage, magnetic field and plasma density.

Shiah et. al.\(^7\) developed a fully three dimensional super-particle simulation (SUPS) such that each simulation particle represents a particle cloud in order to reduce the computational costs associated with modeling plasmas in LEO. The results from the SUPS code applied to the TSS-1R show that a toroidal electron structure forms in the equatorial plane perpendicular to the magnetic field. This structure was also observed in several laboratory experiments. However, as time passes, the toroidal structure elongates along the magnetic field lines to form the magnetic bottle formed in the Parker-Murphy model. These transient effects are thought to cause quick spacecraft charging, resulting in anomalies in plasma measurements and on board electronic failures. Ma and Schunk\(^5\) developed a fully three dimensional hydrodynamic model that has a simulation boundary far away from the satellite (in order to reduce boundary effects), has a realistic $O^+/\text{electron}$ mass ratio and takes into account the length of time the satellite stays on specific field lines. The satellite potentials used by this model were low (4 and 10 V) compared to the voltages seen by TSS-1R ($\sim 5\, \text{kV}$). Elongated cylindrical potential structures were observed along the field lines with radius slightly larger than that of the satellite. For the higher voltage satellite, $O^+$ ions were reflected in the ram direction, which led to a small density increase in front of the satellite. A study describing the parametric relationship between current collected and magnetic field orientation relative to drift velocity has yet to be published.

### 2.3 System Dynamics Models

Beletsky and Levin’s\(^8\) monograph presents a thorough discussion of the fundamental theory governing the dynamics of tethered satellite systems, including a chapter dedicated to EDT systems. Beletsky and Levin show that the operational equilibria of a massless tether system
orbiting a central body in the plane of the magnetic equator are unstable under a constant current. However, four tether stationary modes can be stabilized by a current modulation scheme based on the attitude and position of the secondary end-body as long as the current is below the critical value

$$I_{**} = \frac{B_0 E (s_A - s_B)}{8G (m_B + \frac{1}{3} m_t) \omega}$$

(2.4)

where $s$ is the parameter along the tether, $B_0$ is the local magnetic field, $E$ is the longitudinal tether stiffness, $G$ is the universal gravitational constant, $m_B$ and $m_t$ are the masses of the secondary end-body and the tether, and $\omega$ is the orbital frequency of the tether.

Estes and Lorenzini\textsuperscript{28} study a weak instability that occurs in a bare tether system on an inclined orbit due to the out-of-plane component of the electrodynamic force on the tether. Peláez et al.\textsuperscript{63} expand on the work by Estes and Lorenzini\textsuperscript{28} by analyzing the effect of two forcing terms on a rigid tethered system’s dynamics: eccentricity and the electrodynamic force. Under the assumptions of a non-tilted dipole model for Earth’s magnetic field, a constant current and no damping, the tether system developed a strong instability on orbits with an eccentricity larger than $e \approx 0.35$. The authors concludes with the recommendation that the next step be to investigate the effect of a current that varies throughout the tether’s orbit. Lanoix et al.\textsuperscript{51} incorporated the International Geomagnetic Reference Field in a tether dynamics model in order to investigate the effects of a realistic magnetic field on the Lorentz force on the tether. Finally, Cartmell and McKenzie\textsuperscript{14} present recent sources of fundamental theory, technical innovations and mission progress for momentum exchange and electrodynamic tethers with specific emphasis on the breadth of literature pertaining to tether dynamics modeling. Dobrowolny\textsuperscript{23} develops several electrodynamic tether models in order to characterize a variety tether modes of oscillation and derives a maximum, constant current under which the systems are stable. Dobrowolny also presents models for the maximum possible tether current based on the system resistance and potential distribution along the tether (defined as the system model) and an ionospheric current model based on the TSS-1R data\textsuperscript{82} where the variation in plasma density throughout an orbit is modeled via a simple sinusoidal function (defined as the ionospheric model). The actual current in the tether is the minimum of the system model and the ionospheric model. While Dobrowolny develops the theory for tether librations, longitudinal and transverse modes with the electrodynamic terms including the two previously mentioned current models, all of the numerical analysis is completed under the assumption that hollow cathodes are present to maintain a constant current that is chosen to be 20% below the minimum of the maximum tether current predicted by the system and ionospheric models.

Finally, the dissertation written by Ellis\textsuperscript{25} and the related papers Refs. 26 and 24 contain the development of the dynamics model used in the present dissertation. The dynamics model contains a tilted dipole magnetic field that rotates with the central body. Additionally, the model accounts for arbitrary tether end-bodies, torques due to the tether attachment on the end-body’s surface, a tether with mass that can deform and the elastic vibration of the tether. The resulting model is one of the most robust tether dynamics models developed to
date. The model developed by Ellis will be summarized in Chapter 3. Note that in all of the aforementioned models, the current is assumed to be governed by the system and not system-plasma interactions.

2.4 Summary

A review of the literature has shown that the dynamics of an EDT system is typically studied independently of the variations in the ionospheric space-plasma that governs the current collected by the tether system’s end-bodies. Investigations of the collected current, in turn, restrict the system’s configuration to the cases where the collecting body is stationary, or traveling perpendicular or parallel to the local magnetic field. Typical EDT system dynamics models hold the current in the tether constant throughout the simulations even though the aforementioned studies of the collected current show differences in the amount of current collected is dependent on whether or not the magnetic field is perpendicular or parallel to the system’s velocity. The following chapters will present the first models and results that investigate the end-body/space-plasma’s interaction over a variety of system states and incorporates the interaction into the tether system’s dynamics.
Chapter 3

Effects of the Local Plasma Environment on the Dynamics of Electrodynamic Tether Systems

3.1 Introduction

Electrodynamic tether (EDT) systems have the potential to expand satellite mission capabilities in Low Earth Orbit (LEO), extend missions near any body surrounded by a space plasma and a strong magnetic field, and provide a means to study plasma phenomena in the near-Earth space environment.\textsuperscript{6,14,16,23,68,70} The strong coupling between an EDT system and its surrounding space environment is what makes the EDT system an ideal solution for each of these applications. The TSS-1R mission demonstrated that EDT systems can collect currents 2-6 times greater than the limit predicted by Parker and Murphy.\textsuperscript{61} The Plasma Motor Generator (PMG) mission showed significant differences in the current collected by the system in day and night ionospheric conditions while holding the collecting body at a constant potential and without any control on the tether’s current.\textsuperscript{87} The order of magnitude drop in collected current between the day and night sides that is seen in the PMG current profiles is attributed to the drop in charge particle density and the resulting decrease of plasma turbulence that aids in the transport of particles across field lines. The data set from the PMG mission is invaluable for developing accurate models for an EDT system because it is the only existing data set that shows the behavior of the collected current under a variety of plasma conditions while other system parameters are held constant.

The purpose of this chapter is to fill a gap in the literature by connecting a model for the current collected by an EDT based on the characteristics of the local space plasma with a high fidelity system dynamics model in order to study the effects of an unconstrained tether current on the system’s dynamics. The remaining sections are organized as follows. The
methods section contains a description of the current collection model used to govern the current in the tether system and a brief discussion of the mathematical model governing the EDT system’s dynamics. Results from this model are discussed and a comparison is drawn between the dynamics of our model and the dynamics of the same model where the tether current is held constant.

3.2 Methods

The mathematical model developed for this study characterizes a classical EDT system: two satellites connected by a long, insulated conducting tether orbiting a central body with a strong, dipole magnetic field and a dense space plasma, as seen in Fig. 3.2a. The tether’s end-bodies are assumed to be in electrical contact with the plasma environment surrounding the satellites. A potential is set up across the tether either by the motion of the tether across the central body’s magnetic field or by a power source in the system. The potential difference drives a current that is collected from the surrounding plasma through the tether. The tether system can be in either motor or generator mode, depending on the direction of the current. For the purposes of our study, the tether will be in motor mode. The tether’s current and the local magnetic field that generate the electrodynamic force are governed by the system electrodynamics model. The current and the local magnetic field vector are then fed into the system dynamics model.

3.2.1 System Electrodynamics Model

The system electrodynamics model governs the interactions between the tether system and its local space environment. These interactions primarily occur through the electrodynamic force on the tether, which depends on the current collected by the positively biased end-body and the local magnetic field. Current is collected by the non-primary end-body, labeled $\mathcal{B}$ in Fig. 3.2a. The current balance equation for the collecting end-body immersed in a space plasma is

$$I_{tether} = I_e(\Phi_s) - I_i(\Phi_s) - I_b(\Phi_s) - I_{ph}(\Phi_s) - I_S(\Phi_s) - I_{pe}(\Phi_s)$$

(3.1)

where $\Phi_s$ is the potential of the collecting end-body, $I_e(\Phi_s)$ is the electron current, $I_i(\Phi_s)$ is the ion current, $I_b(\Phi_s)$ is the emitted backscattered electron current, $I_{ph}(\Phi_s)$ is the emitted photoelectron current, $I_S(\Phi_s)$ is the emitted secondary electron current and $I_{pe}(\Phi_s)$ is the current emitted by an active plasma contactor. In the presence of a strongly biased satellite, $I_i$, $I_b$, $I_S$ and $I_{ph}$ are negligible due to the presence of a sheath region where the emitted electrons are predominantly attracted back to the satellite and the ions are repelled. In the absence of an active plasma contactor, Equation 3.1 reduces to the current in the tether equating to the collected electron current.
The Parker-Murphy Model\(^{61}\) is an analytical model for the upper-bound on electron current collected by a spherical satellite in a non-flowing plasma with an ambient magnetic field. In a quasi-neutral plasma with a negligible electric field (i.e. the satellite potential is \(\Phi_s = 0\) relative to the plasma potential), the collected electrons will move along the ‘tube’ of magnetic field lines intersecting the satellite. The electrons gyrate around the magnetic field lines, where the radius of gyration is termed the Larmor radius. Due to the electron’s gyro-motion, the tube is slightly larger than the satellite’s cross-sectional area. However, the Larmor radius is small relative to a satellite’s radius in the presence of a strong magnetic field. Therefore, the additional area from to the gyro-motion is assumed to be negligible compared to the satellite’s collection area. The current collected by a satellite of radius \(a\) is the thermal current passing through the area of the sphere

\[
I_0 = 2\pi a^2 j_0
\]  

(3.2)

where \(j_0\) is the component of the ambient mean thermal current density parallel to the magnetic field. Assuming Maxwellian electrons, the thermal current density is given by

\[
 j_0 = n_e q_e \sqrt{\frac{kT_e}{2\pi m_e}}
\]  

(3.3)

where \(n_e\), \(T_e\) and \(m_e\) are the electron number density, temperature and mass, respectively. The saturation current of the sphere, \(I_0\), gives an upper limit on current collection for an unbiased satellite, assuming that electrons in the magnetic tube are not depleted.

When an electron beam is not present, there are at least two mechanisms causing the cross-field transport of electrons present near a satellite carrying potential \(\Phi_s > 0\): stochastic diffusion and diffusion due to the drift caused by a transverse electric field. The additional current due to stochastic diffusion is small relative to \(I_0\) and is neglected. However, in the presence of a strongly biased satellite there is a significant transverse electric field that causes electrons to drift across field lines. The satellite will then collect all of the electrons.
originating infinitely far away from the satellite within a collection area of radius $r_0$, as shown in Fig. 3.1. By conserving the canonical angular momentum of the gyrating electrons, Parker and Murphy found an upper limit to the collection radius and thus, the current collected by a satellite, given by

$$\frac{I_{PM}}{I_0} = \frac{r_0^2}{a^2} < 1 + \left( \frac{8\Phi_s}{m_e\Omega_e^2a^2} \right)^{1/2} \quad (3.4)$$

where $\Phi_s$ is the satellite’s potential and $\Omega_e$ is the electron cyclotron frequency. The Parker-Murphy model holds true for satellites that are stationary and at high potentials relative to the surrounding space plasma.

In light of the results from the TSS-1R mission, Singh and Chagenti added a term to the Parker-Murphy model to account for the sweeping effects of the magnetic field caused by the relative motion between the satellite and the field. It should be stressed that the adjustment to the Parker-Murphy model suggested by Singh and Chagenti is an estimate of the additional current collected due to the orbital motion of the satellite and that it is only meant to provide a qualitative current enhancement. The additional current is determined by calculating the electrons in the sheath flux tube intercepted by the velocity flow. The total current is thus the sum of the Parker-Murphy model and the current due to the sweeping effects, $\delta I$,

$$I_{APM} = I_{PM} + \delta I \quad (3.5)$$

where

$$\delta I = n_eq_eAV_0\sin(\theta_B) \quad (3.6)$$

$V_0$ is the satellite’s velocity relative to the surrounding plasma, $\theta_B$ is the angle between $V_0$ and $B$, $n_e$ is the electron number density and $A$ is the area of the sheath flux tube. The area of the sheath flux tube can be calculated from the perpendicular extension of the potential along the magnetic field lines. Unfortunately, there is not a consistent theory for predicting the length of the potential field’s extension along the magnetic field lines. However, physical considerations can lead to the following estimate. The radius of the sheath flux tube far away from the satellite will be the radius calculated by Parker and Murphy

$$r_0 = a \left[ 1 + \left( \frac{8\Phi_s}{m_e\Omega_e^2a^2} \right)^{1/2} \right]^{1/2} \quad (3.7)$$

Thus, an upper limit to the sheath size is the distance an electron will travel in the time it takes the flux tube to sweep across the magnetic field is

$$z_B = 2r_0 \frac{v_{the}}{V_0\sin(\theta_B)} \quad (3.8)$$

where $v_{the}$ is the electron thermal velocity. Assuming an elliptical shape for the sheath, the intercepted area is $A = \pi r_0 z_B$. Combining Equations 3.7 and 3.8, the current due to the
flux tube sweeping across magnetic field lines is

$$\delta I = 2\pi n_e q_e v_{the} a^2 \left[ 1 + \left( \frac{8\Phi_s}{m_e \Omega^2_e a^2} \right)^{1/2} \right]$$

(3.9)

The above current added to the original Parker-Murphy current produces qualitative estimate of the impact that orbital motion has on the current collected by the tether end-body and will be termed the adjusted Parker-Murphy model for the remainder of the paper. As pointed out by Dobrowolny et al., Singh and Chaganti’s estimate does not depend on the satellite’s velocity or $\theta_B$. The actual current enhancement is governed by complex plasma processes. Particle-in-cell simulations have shown that the electron current collected by a positively charged spherical body is dependent on the satellite’s orbital velocity and its orientation with the ambient magnetic field. Analytical models that accurately characterize the effects of orbital motion, magnetic field orientation or variations in plasma density have yet to be developed.

The plasma parameters used in the adjusted Parker-Murphy model (electron temperature, $T_e$ and number density, $n_e$) are calculated by the 2012 International Reference Ionosphere (IRI) based on the position of the collecting end-body. IRI is an internationally accepted model for generating worldwide, non-auroral, ionospheric parameters between 50 km and 2000 km from 1958 to present. The model is an analytical representation of charged particle species’ temperature and density based on data collected from a world-wide network of ionosondes, incoherent scatter radar, topside sounder satellites, in situ satellite measurements and lower ionosphere rocket observations. IRI allows the current collection model to take into account both the locational dependencies of the plasma parameters and the parameter’s dependence on solar activity. Thus, we can study the behavior of an EDT system over a wide range of plasma conditions.

The local magnetic field is modeled as a tilted dipole centered on and fixed to the central body as it rotates. The electrodynamic force acting at a given point on the tether is dependent on the local magnetic field vector. The magnetic field vector at some unstretched tether element $d\vec{s}$ is given by

$$\mathbf{B} = \frac{\mu_m}{||\mathbf{r}_T||^3} \left[ \hat{\mathbf{u}} - \frac{3(\hat{\mathbf{u}} \cdot \mathbf{r}_T)\mathbf{r}_T}{||\mathbf{r}_T||^2} \right]$$

(3.10)

where $\mathbf{r}_T = \mathbf{r}_A + \mathbf{p}_A + \mathbf{r}$ is the position of $d\vec{s}$ relative to $O$ (see Fig. 3.2a) and $\hat{\mathbf{u}}$ is the unit dipole axis. Similarly, the magnitude of the magnetic field that determines $\Omega_e$ in the adjusted Parker-Murphy model is given by the norm of the magnetic field vector at $\mathbf{r}_B$

$$B = \frac{\mu_m}{||\mathbf{r}_B||^2} \left| \hat{\mathbf{u}} - \frac{3(\hat{\mathbf{u}} \cdot \mathbf{r}_B)\mathbf{r}_B}{||\mathbf{r}_B||^2} \right|$$

(3.11)

The complete electrodynamics model for the system consists of both the magnetic field model and the IRI-Parker-Murphy current model. Equations 3.4, 3.5 and 3.9 combined with the
International Reference Ionosphere (where the local magnetic field is determined by Equation 3.11) define the IRI-Parker-Murphy model (hereafter referred to as the I-P-M model) and govern the current in the tether. Equation 3.10 defines the magnetic field vector along the tether. The results from Equations 3.5 and 3.10 will be used to determine the Lorentz force along the tether throughout the system’s orbit.

3.2.2 EDT System Dynamics Model

The following section gives a brief description of the system dynamics model. A complete derivation and description of the model for the system dynamics can be found in Ref. 25 and the related dissertation, Ref. 25. The EDT system model and coordinate frames of reference are shown in Fig. 3.2a and Fig. 3.2b. We assume that the central body is spherical, with a homogeneous mass distribution. Hence, the Newtonian component of the gravitational field is dominant. Excluding the electrodynamic and gravitational forces, all other forces acting on the system (e.g. atmospheric drag and solar radiation pressure) are assumed to be negligible. Let $\mathcal{F}_N$ be the Earth-centered inertial frame with coordinate axes, $\hat{n}_1$, such that the $\hat{n}_1 - \hat{n}_2$ plane is Earth’s equatorial plane and $\hat{n}_3$ is aligned with Earth’s spin axis. Define $\mathcal{F}_O$ as the orbital frame of the osculating orbit of the primary body’s mass center (end-body labeled $A$ in Fig. 3.2a), where $\hat{o}_3$ is in the direction of $r_A$, $\hat{o}_2$ is in the direction of the instantaneous angular momentum vector and $\hat{o}_1$ completes the right-handed triad.
Equations of Motion for Bodies $A$ and $B$.

The EDT end-bodies are assumed to be finite, rigid, and attached to the tether at points $P_A$ and $P_B$. Let $\mathcal{F}_A$ and $\mathcal{F}_B$ be the body-fixed principal coordinate frames for bodies $A$ and $B$. The translational equations of motion for end-bodies $A$ and $B$ relative to $\mathcal{F}_N$ are

$$m_A \ddot{\mathbf{r}}_A = -\frac{m_A \mu}{\| \mathbf{r}_A \|^3} \mathbf{r}_A + \mathbf{T}(0, t) \tag{3.12}$$

$$m_B \ddot{\mathbf{r}}_B = -\frac{m_B \mu}{\| \mathbf{r}_B \|^3} \mathbf{r}_B + \mathbf{T}(L, t) \tag{3.13}$$

where $\mathbf{T}(0, t)$ is the tension in the tether acting on $P_A$ and $\mathbf{T}(L, t)$ is the tension acting on $P_B$. A set of osculating orbit elements parameterizes the state of body $A$’s mass center, $\mathbf{e} = \{ a \; e \; \Omega \; I \; \omega \; \nu \}^T$. The Gauss variational equations describe the evolution of $\mathbf{e}$

$$\dot{a} = \frac{2a^2}{h} \left[ (e \sin \nu) a_{D3} + \left( \frac{l}{r_A} \right) a_{D1} \right] \tag{3.14}$$

$$\dot{e} = \frac{(l \sin \nu)a_{D3} + [(l + r_A) \cos \nu + r_A e] a_{D1}}{h} \tag{3.15}$$

$$\dot{i} = \frac{r_A \cos(\omega + \nu)a_{D2}}{h} \tag{3.16}$$

$$\dot{\Omega} = \frac{r_A \sin(\omega + \nu)a_{D2}}{h \sin I} \tag{3.17}$$

$$\dot{\omega} = \frac{-(l \cos \nu)a_{D3} + [(l + r_A) \sin \nu] a_{D1}}{h e} - \frac{[r_A \sin(\omega + \nu) \cos I] a_{D2}}{h \sin I} \tag{3.18}$$

$$\dot{\nu} = \frac{h}{r_A^2} + \frac{(l \cos \nu)a_{D3} - [(l + r_A) \sin \nu] a_{D1}}{h e} \tag{3.19}$$

where $a_{D_i}$ are the disturbance acceleration components of body $A$ expressed in $\mathcal{F}_o$, $h = \sqrt{\mu l}$, $l = a(1 - e^2)$ and $r_A = l/(1 + e \cos \nu)$. A 3-1-3 rotation sequence through angles $\Omega$, $I$ and $\theta = \omega + \nu$ (with an additional permutation to align the appropriate axes) relates the inertial frame to the orbital frame of body $A$. The angular velocity and acceleration of $\mathcal{F}_O$ with respect to $\mathcal{F}_N$ are

$$\omega_{O/N} = \frac{h}{r_A} \hat{\mathbf{0}_2} \tag{3.20}$$

$$\dot{\omega}_{O/N} = -\frac{2\mu e \sin \mu}{r_A^3} \hat{\mathbf{0}_2} \tag{3.21}$$

Assuming a first-order approximation of the gravity-gradient torque acting on each end-body, the rotational equation of motion for each end-body is

$$\mathbf{I}_A \cdot \dot{\omega}_{A/N} + \omega_{A/N} \times \mathbf{I}_A \cdot \dot{\mathbf{r}}_{A/N} = \frac{3\mu}{\| \mathbf{r}_A \|^3} \mathbf{r}_A \times \mathbf{I}_A \cdot \mathbf{r}_A + \mathbf{p}_A \times \mathbf{T}(0, t) \tag{3.22}$$
where $\omega_{A/N}$ and $\omega_{B/N}$ are the angular velocities of $F_A$ and $F_B$ relative to $F_N$.

**Equations of Motion for the Tether.**

The tether is modeled as a long, thin wire that can resist axial stretching, but cannot resist bending, compression or torsion. The tether is described by the unstretched tether length, $L$. Let $F_E$ be a tether fixed frame relative to $F_O$ such that $\hat{e}_3$ points from $P_A$ to $P_B$. The tether is allowed to deform in the $\hat{e}_1$ and $\hat{e}_2$ directions. A 2-1 rotation sequence through angles $\alpha$ and $-\beta$ describes the transformation between the orbital frame and $F_E$, where $\alpha$ and $\beta$ are the in- and out-of-plane libration angles of the tether relative to $F_O$ (see Fig. 3.2b). Note that the behavior of $\alpha$ and $\beta$ define the pendular motion of the tether. The angular velocity of $F_E$ relative to $F_O$ is

$$\omega_{E/O} = -\dot{\beta}\hat{e}_1 + \dot{\alpha}\cos\beta\hat{e}_2 + \dot{\alpha}\sin\beta\hat{e}_3$$  (3.24)

The displacement of a differential tether element $d\bar{s}$, from its unstretched state, $\bar{s} \in [0, L]$, is expressed relative to $F_E$ by

$$\mathbf{r}(\bar{s}, t) = U(\bar{s}, t)\hat{e}_1 + V(\bar{s}, t)\hat{e}_2 + (\bar{s} + W(\bar{s}, t))\hat{e}_3$$  (3.25)

The equations of motion for tether attitude angles $\alpha$ and $\beta$ in terms of the tether displacement at $P_B$ are

$$ (\ddot{\alpha} + \dot{\theta}) \cos \beta - 2(\dot{\alpha} + \dot{\theta}) \dot{\beta} \sin \beta = \frac{\ddot{v}_1(L, t) - 2\dot{W}(L, t)(\dot{\alpha} + \dot{\theta}) \cos \beta}{L + W(L, t)}$$  (3.26)

$$ \ddot{\beta} + (\dot{\alpha} + \dot{\theta})^2 \sin \beta \cos \beta = \frac{\ddot{v}_2(L, t) - 2\dot{W}(L, t)\dot{\beta}}{L + W(L, t)}$$  (3.27)

The electrodynamic force acting on an unstretched differential tether element, $d\bar{s}$, is

$$\mathbf{F}_{ED} = I_{IPM} \frac{\partial \mathbf{r}}{\partial \bar{s}} d\bar{s} \times \mathbf{B}$$  (3.28)

where $I_{IPM}$ is the current in the tether determined by the I-P-M model. Noting that the position of the tether relative to $O$ can be written as $\mathbf{r}_T = \mathbf{r}_A + \mathbf{p}_A + \mathbf{r}$. The partial differential equation governing the evolution of the tether shape can be written in terms of the unstretched tether length as

$$\ddot{\mathbf{r}} = -\frac{\mu}{\rho^2} \mathbf{r}_T + \frac{1}{\rho} \frac{\partial \mathbf{T}}{\partial \bar{s}} + I_{IPM} \frac{\partial \mathbf{r}}{\partial \bar{s}} \times \mathbf{B} - \ddot{\mathbf{r}}_A - \ddot{\mathbf{p}}_A$$  (3.29)
The tension in the tether is modeled by the linear Kelvin-Voigt law of viscoelasticity:

\[ T = EA(\varepsilon + c\dot{\varepsilon})\hat{\tau} \]  

(3.30)

where \( E \) is the Young’s modulus of the tether material and \( c \) is the structural damping constant. The strain and tether unit tangent vector are given by

\[ \varepsilon = \left\| \frac{\partial \mathbf{r}}{\partial s} \right\| - 1 \]  

(3.31)

\[ \hat{\tau} = \frac{\partial \mathbf{r}/\partial s}{\|\partial \mathbf{r}/\partial s\|} \]  

(3.32)

Since the acceleration of body \( B \) is equal to the acceleration of the tether end connected to \( B \) and the tether displacement is zero at its ends, the boundary conditions on the tether equations of motion are given by

\[ \ddot{\mathbf{r}}(L,t) = \ddot{\mathbf{r}}_B - \ddot{\mathbf{p}}_B - \ddot{\mathbf{p}}_A \]  

(3.33)

\[ \mathbf{r}(0,t) = \hat{\mathbf{0}} \]  

(3.34)

\[ \mathbf{r}(L,t) = [L + W(L,t)]\hat{\mathbf{e}}_3 \]  

(3.35)

Equation 3.29 governs the elastic vibrations of the tether while Equations 3.26 and 3.27 determine the tether’s pendular motions. These equations are not independent; the vibrational and elastic modes of the tether are coupled. In order to obtain a finite set of ODEs, Equations 3.25 and 3.29 have been discretized using the Finite Element Method.

Table 3.1: EDT System orbital elements for the Test Case and PMG models.

<table>
<thead>
<tr>
<th>Orbital Elements</th>
<th>Test Case</th>
<th>PMG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoch, day and UT</td>
<td>year 1999; day 128 year 1993; day 177; 1438:48:083</td>
<td>epoch 1993; day 177; 1438:48:083</td>
</tr>
<tr>
<td>Inclination, deg</td>
<td>50</td>
<td>25.7114</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.001</td>
<td>0.0489274</td>
</tr>
<tr>
<td>Right Ascension, deg</td>
<td>300</td>
<td>258.1846</td>
</tr>
<tr>
<td>Argument of Perigee, deg</td>
<td>45</td>
<td>198.6592</td>
</tr>
<tr>
<td>Mean motion, rev/day</td>
<td>15.5855</td>
<td>15.1123194</td>
</tr>
<tr>
<td>Perigee height, km</td>
<td>385.085</td>
<td>192.552</td>
</tr>
<tr>
<td>Apogee height, km</td>
<td>398.625</td>
<td>868.569</td>
</tr>
<tr>
<td>Mean anomaly, deg</td>
<td>0</td>
<td>194.9297</td>
</tr>
<tr>
<td>Period, min</td>
<td>92.4</td>
<td>95.2865</td>
</tr>
</tbody>
</table>

3.3 Results and Discussion

The dynamics of an EDT system under the influence of the I-P-M model have been analyzed using two representative systems. The first simulation set compares the dynamics of a system
Table 3.2: EDT system parameters for the Test Case and PMG models.

<table>
<thead>
<tr>
<th>System Parameters</th>
<th>Test Case</th>
<th>PMG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unstretched Length $L$, km</td>
<td>20</td>
<td>0.500</td>
</tr>
<tr>
<td>Longitudinal Stiffness $EA$, N</td>
<td>55000</td>
<td>96291</td>
</tr>
<tr>
<td>Unstretched linear mass density $\bar{\rho}$, kg/m</td>
<td>0.025</td>
<td>0.0074</td>
</tr>
<tr>
<td>Structural Damping constant $c$, s</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Body A mass $m_A$, kg</td>
<td>5000</td>
<td>6305</td>
</tr>
<tr>
<td>Body B mass $m_B$, kg</td>
<td>500</td>
<td>80</td>
</tr>
<tr>
<td>Body B potential $\Phi_B$, V</td>
<td>25</td>
<td>65</td>
</tr>
<tr>
<td>$\mu$, m$^3$s$^{-2}$</td>
<td>3.986×10$^{14}$</td>
<td>3.986×10$^{14}$</td>
</tr>
<tr>
<td>$\mu_m$, T·m$^3$</td>
<td>8×10$^{15}$</td>
<td>8×10$^{15}$</td>
</tr>
</tbody>
</table>

where the tether's current is calculated from the I-P-M model to a tether system where the current is maintained at a constant level equal to the average of the I-P-M current from the previous simulation. The second set models the PMG mission and compares the current variations predicted by the I-P-M model to the current profiles measured throughout the mission. The PMG tether system was chosen because it produced the most complete data set showing the behavior of the current throughout a complete orbit where the collecting end-body is held at a constant potential. The parameters for the system models are shown in Tables 3.1 and 3.2. The PMG model is based on the orbital elements and system parameters given in 87 and 33.

### 3.3.1 Effects of Diurnal Current Variation on System Dynamics

The effects of the variation in tether current on the system dynamics were investigated by running two simulations using an EDT system in motor mode over 20 orbits (system parameters are given in the first column of Tables 3.1 and 3.2). The first simulation implemented the I-P-M model to determine the current in the tether based on the system’s orbital position. The electron number density, electron temperature and resulting current during the first 10 orbits are shown in Fig. 3.3. In the second simulation, the constant current in the tether is equal to the mean current from the previous simulation ($I_{avg} \sim 0.263$ A). Since the primary application of an electrodynamic tether system in motor mode is to boost the system’s orbit, the amplitudes of the orbital elements were compared to examine the impact of the I-P-M current on the orbital motion of the system. As shown in Fig. 3.4, the semi-major axis of the system under the influence of both a constant current and the I-P-M current increases the osculating orbit’s semi-major axis by less than 2.75 km over the course of 20 orbits. The I-P-M current results in a maximum periodic increase in the orbit’s semi-major axis of less than 36 m above the constant current system. The implementation of the I-P-M current also causes a small and gradual periodic divergence in the remaining orbital elements. The most pronounced difference between the two systems occurs in the true anomaly, shown in Fig. 3.5, where a maximum difference of approximately 1.7° develops. Even though the differences in the orbital parameters between the two systems grow throughout the simulation, the variation in the orbital elements is small (< 0.5%). Thus, the influence of the I-P-M
The I-P-M model does, however, introduce a new instability in the tether’s pendular motion. Under a constant current, the tether’s in-plane attitude angle oscillates between 0.24° and −0.16° with small oscillations driven by the elastic vibration of the tether occurring every ∼1.5 oscillation periods (see Fig. 3.6). Figure 3.8 shows that the out-of-plane tether angle behaves similarly, oscillating between 0.11° and −0.11° with small amplitude oscillations driven by the tether’s vibrations occurring during every period. As noted by Peláez et al., the out-of-plane librations are primarily driven by the out-of-plane component of the electromagnetic force due to the inclination of the system’s orbit relative to the magnetic field. The in-plane attitude angle’s behavior is significantly different under the influence of the I-P-M current (see Fig. 3.6). The magnitude of the tether’s in-plane oscillations are amplified by 55%. Additionally, the tether’s in-plane pendular motion develops librations whose period is half of the orbital period. The most significant difference in the EDT system’s dynamics is the behavior of the tether’s out-of-plane motion. The periodicity of both the I-P-M current and the out-of-plane component of the electrodynamic force due to the tether’s inclined orbit super-impose to drive the out-of-plane motion towards instability. While the amplitude of the out-of-plane motion is small, it steadily increases without bound throughout the simulation and results in end-body $B$ swinging through a maximum arc-length of ∼300
Figure 3.4: A comparison of the time evolution of the osculating orbit’s semi-major axis between the system under a constant current and the I-P-M current over the system’s last 10 orbits.

$m$ (compared to an arc-length of $\sim 80 \, m$ in the constant current simulation).

A frequency analysis of the system’s pendular motion reveals that the I-P-M current modifies the system’s pendular oscillation modes and drives the new dynamic instability discussed above. Figures 3.7 and 3.9 show the frequency components (normalized for purposes of comparison) present in the in-plane and out-of-plane pendular motion, respectively. The two primary frequencies evident in the in-plane and out-of-plane motions of the constant current system dynamics occur at the system’s orbital frequency, $\omega_{sys}$, and twice the orbital frequency, $2\omega_{sys}$. As mentioned previously, these frequencies are driven by periodicities in the electrodynamic forcing term due to the inclination of the system’s orbit. The I-P-M current has a frequency component near $2\omega_{sys}$ due to the diurnal variation of the space plasma environment. The variation in the current causes the electrodynamic force to oscillate accordingly and introduces the oscillations at $2\omega_{sys}$ into the in-plane pendular motion. The in-plane motion of the EDT system also develops a larger periodic motion at the frequency component between the $\omega_{sys}$ and $2\omega_{sys}$ periodic motions. The I-P-M current also significantly reduces the amplitude of the out-of-plane $\omega_{sys}$ oscillation while increasing the relative amplitude of the $2\omega_{sys}$ oscillation. Since the tether current generated by the I-P-M model has frequency components near $2\omega_{sys}$, these variations seem to couple into the tether’s dynamics to amplify the oscillation modes near this frequency for both the in-plane and out-of-plane modes. Additionally, the introduction of the I-P-M current reduces the relative magnitude of the high-frequency oscillations driven by the elastic vibrations of the
3.3.2 I-P-M Model Compared to Plasma Motor Generator Data

The most complete set of data relating to the tether/plasma interactions of an EDT system was collected during the Plasma Motor Generator Mission (PMG). The PMG mission collected current-voltage data over several orbits at different end-body biases and showed that there was a strong diurnal variation in the current collected by an EDT system. The PMG tether consisted of a 500 m length of insulated 18-gauge copper wire connecting a spent second stage of a Delta II rocket to the 28 kg Far End Package (FEP). The FEP was deployed upwards, corresponding to end-body $B$ in our simulation model. A Hallow Cathode Assembly (HCA) mounted on both the FEP and the Delta II rocket produced xenon gas clouds near each end-body that enhanced the electrical contact between the system and the ionospheric plasma by reducing the electrical impedance in the plasma by an order of mag-
Figure 3.6: The in-plane tether attitude angle under the influence of a constant current and the I-P-M current over the system’s first 10 orbits.

Figure 3.7: The frequency spectrum of the in-plane tether attitude angle with a constant current and the I-P-M current.
Figure 3.8: The out-of-plane tether attitude angle with a constant current and the I-P-M current.

Figure 3.9: The frequency spectrum of the out-of-plane tether attitude angle with a constant current and the I-P-M current.
Figure 3.10: (a) The current predicted by the I-P-M model for the Test Case system throughout an orbit. (b) PMG data for a FEP bias level of $\Phi_s = 65V$ from Ref. 87.
A simulation of the PMG system was run using PMG system parameters given in the second column of Tables 3.1 and 3.2. Unfortunately, after an initial upward deployment, the simulated system stabilized in a downwards deployment such that the FEP was below Delta II stage after the system’s first orbit. The resulting downwards deployment produced a larger current due to an increased plasma density, which in turn pumped more energy into the system’s pendular motion and drove the system unstable before it completed 3 orbits.

However, it is still useful to compare the current predicted for the Test Case simulation to the data from the PMG mission. Even though the I-P-M current collection model does not include an estimate for the impact of the HCA on current exchange between the tether system and the plasma, it still captures the orbital day/night variation in current collected due to plasma density and temperature fluctuations seen throughout the PMG current collection profiles. It can be seen from a comparison in the current profiles shown in Fig. 3.10a and Fig. 3.10b that the current collected by the Test Case system and data from the PMG mission have the same characteristics throughout the systems’ orbit. Both curves show almost an order of magnitude change when the system crosses from day to night or vice versa, as well as a slight decrease in current collection in the middle of the day-side portion of the orbit. The outlying points in the PMG data correspond to when the HCA system was turned off. The differences in the system parameters (e.g. the HCA assembly, orbit inclination, mission year, end-body radius, etc.) contribute to the order of magnitude difference in the current collected by the two systems.

### 3.4 Conclusions

The dynamics of electrodynamic tether systems are typically analyzed using a current that is held at a fixed value determined by the system's parameters. However, the Plasma Motor Generator mission demonstrated that there is a strong coupling between the system's orbit and the current it can collect from the ionosphere. The International Reference Ionosphere has been integrated into the adjusted Parker-Murphy model to develop a model for the current collected by the tether system that accounts for the variation of the local plasma environment. A numerical model for an electrodynamic tether system's dynamics has been presented that uses the IRI-Parker-Murphy model to investigate the impact of a realistically oscillating Lorentz force on the system's dynamics. For a representative electrodynamic tether system, the IRI-Parker-Murphy current couples with the periodic out-of-plane electrodynamic force to drive the out-of-plane motion towards instability. The in-plane pendular motion under the influence of the IRI-Parker-Murphy model also shows three distinct periodicities; two present in a tether system under constant current and a new one introduced by effects of the plasma parameters' diurnal variations on the current collected by the system. The oscillations in the collected current were compared to data from the Plasma Motor Generator experiment and the IRI-Parker-Murphy model was shown to capture the oscillations in the current seen during the mission. The research presented in this paper represents an
initial step towards understanding the impact of local plasma variations on electrodynamic tether system dynamics. The resulting model has broad applicability and can model the dynamics of any electrodynamic tether system under a wide range plasma conditions, which will improve our understanding of key aspects in the system’s behavior.
Chapter 4

Investigation of the Current Collected by a Spherical Satellite with Application to Electrodynamic Tethers

4.1 Introduction

The charge accrued by a positively biased spacecraft has a number of applications to active space experiments and satellite systems. SPEAR-1 studied the altitude dependence of the current-voltage characteristic for a conducting sphere positively biased to tens of kilovolts in the ionosphere.\textsuperscript{65}\footnote{CHARGE 2 investigated the impact of a neutral gas release on the current collected by a positively charged spacecraft.\textsuperscript{32} SAMPIE was designed to measure both arcing and current collection by positively biased solar cells in order to characterize the interaction between high voltage space power systems and the ionospheric space plasma.\textsuperscript{36}} The focus of this chapter is to improve the understanding of the physics governing the current collected by an electrodynamic tether (EDT) end-body. An EDT end-body is essentially a positively charged satellite collecting current from the surrounding space-plasma. Parker and Murphy\textsuperscript{61} used the conservation of the canonical angular momentum of electrons gyrating around magnetic field lines to determine an upper bound for the current collected by a stationary, spherical satellite. The first electrodynamic tether experiments, TSS-1 and TSS-1R,\textsuperscript{21,80,82} showed that EDT systems collect currents 2-6 times higher than predicted by the Parker-Murphy model.\textsuperscript{61} Magnetometer measurements from the TEMAG experiment aboard TSS-1R indicated the presence of azimuthal current structures in the proximity of the high-voltage tether end-body.\textsuperscript{58} Three dimensional Particle-in-Cell (PIC) simulations by Singh et al.\textsuperscript{75} showed that the axial currents along the local magnetic field lines feed the azimuthal current structures observed in the TSS experiments. The azimuthal currents near
the satellite lie in the spacecraft’s equatorial plane and are largely two dimensional, with a gradually increasing radial component near the spacecraft’s surface. The plasma motor generator mission (PMG) not only demonstrated that the current in an EDT system can be reversed, but also that the current collected by the system varies by an order of magnitude between the day and night ionosphere. The specific focus of Chapter 4 is developing a better understanding of the plasma dynamics near a positively biased end-body of an electrodynamic tethered satellite system and the impact the resulting plasma structures have on the current collected by the system. The electromagnetic interactions between an EDT system and its environment couple into the system’s dynamics through the current’s influence on the electromagnetic forcing term. The model developed in the previous chapter used the International Reference Ionosphere and the Parker-Murphy model adjusted for the sweeping effects of the magnetic field to study the dynamics of an EDT system under the influence of a tether current that varies with the local space plasma properties. The diurnal variations in current were found to introduce new modes of oscillation in the tether’s pendular motion and drove the out-of-plane librations unstable. Therefore, an accurate model for the current collected by the system throughout its orbit is important for characterizing a system’s dynamics that do not have current regulation systems.

The rest of the chapter is organized as follows. Section 4.2 presents the PIC model developed to study the plasma dynamics around the EDT system end-body. In Section 4.3, results of the simulations using the model are presented. Finally, Section 4.4 summarizes the findings and presents the conclusions of this work.

4.2 Simulation Model

A Particle-in-Cell (PIC) model is used to study the current collected by a spherical end-body on an EDT system in thrust mode as a function of plasma magnetization, magnetic field orientation and satellite bias. PIC models are commonly used to study the dynamics of a space plasma, especially in the realm of spacecraft charging where the existing fundamental theory does not adequately describe the complex physics near a charged spacecraft. Singh et al. showed that the azimuthal currents observed during the TSS-1R experiment formed close to and in the equatorial region of the satellite. The simulations of Singh and Vashi, Vashi, and Singh et al. show that the field aligned currents near the satellite form primarily along the magnetic field lines that intersect the satellite, but do not contribute as much current as the structures that form in the satellite’s equatorial plane. Therefore, a simulation model that captures the equatorial plane near the satellite is sufficient to study the physics behind the formation of these azimuthal structures. Our model will use a 2-D3v PIC code to study the evolution of these azimuthal current structures in the equatorial plane of the tether system’s collecting end-body over a range of system parameters. For simplicity, the equatorial plane is described by the satellite’s orbital velocity vector and the vector...
Figure 4.1: Boundary conditions and simulation geometry for the Particle in Cell model. The magnetic field angle, plasma temperature and density, orbital velocity and satellite potential can be varied in order to study the behavior of the system under a variety of conditions.

normal to both the velocity and the magnetic field, as shown in Figure 4.1. The use of a 2-D code allows a simulation run strategy to be designed to focus on the evolution of the plasma structures that form in the equatorial plane and their impact on the current collected by an EDT system over a large parameter range.

The basic PIC algorithm is presented in depth in both Birdsall and Langdon, and Hockney and Eastwood. The simulation algorithm developed for the model used in the present work is shown in Figure 4.2. First, a particle population is initialized by loading the requisite velocity distribution and particle positions (this will be discussed in more detail in Section 4.2). The electric and magnetic fields are then interpolated to the particle positions using a bilinear area weighting method. The particle positions and velocities are then advanced using a Boris Mover integrator. The density source term for Poisson’s equation is then calculated by weighting the particles to the simulation grid using a similar bilinear area weighting scheme as is used to weight the fields to the particle positions. Finally, Poisson’s equation is solved over the grid space and an updated electric field is determined by solving \( \mathbf{E} = -\nabla \Phi \). The process is repeated for every time step. In order to include the satellite (or end-body in the case of an EDT system) into the PIC framework, the algorithm that solves Poisson’s equation and the particle mover are modified to include the appropriate boundary conditions.

Additionally, the Monte Carlo collision algorithm (PIC-MCC) described by Birdsall is implemented for several simulations in order to investigate the effects of electron-neutral collisions on the stable plasma structures that form in some simulations. The probability of
Figure 4.2: The satellite PIC algorithm. In order to apply the algorithm to simulating a plasma’s behavior around a spacecraft, the spacecraft geometry has to be included in the potential solver and the particle mover.

collision of the \( m \)th electron in a time step \( \Delta t \) is given by:

\[
P_{\text{col}} = 1 - \exp(-\nu_{\text{en}} \Delta t)
\]

where \( \nu_{\text{en}} \) is the electron-neutral collision frequency, \( \tilde{\nu}_{\text{en}} = \nu_{\text{en}}/\omega_{\text{pe}} \) is dimensionless electron-neutral collision frequency with electron plasma frequency, \( \omega_{\text{pe}} \). If \( P_{\text{col}} > R \), where \( R \) is a uniform random number generated for each particle, then the particle is scattered during the time step.

4.2.1 Boundary Conditions: Particle Injection and Current Collection

The boundary conditions and simulation geometry are shown in Figure 4.1. The satellite is modeled as a circular body held at a constant potential, \( \Phi_{\text{sat}} \). Using Singh’s tank method\(^7\) to simulate the motion of the spacecraft through the plasma, a constant population of Maxwellian particles is initialized in the “tank” on the left hand side of the grid.
space such that the particles are swept into the grid space at each time step. Each particle population’s velocity distribution is centered around the satellite’s orbital velocity. In order to account for the orbital motion of the satellite fixed frame relative to the inertial frame, a motional background electric field is added: \( \mathbf{E} = \mathbf{v}_{\text{orb}} \times \mathbf{B} \).

During every time step, an equal number of particles are injected of each species to ensure that no net charge is injected into the system. Each simulation particle represents many real electrons or ions. The magnitude of the charge (per unit length) of each simulation particle is determined by balancing the plasma flux across the boundary \((L_y v_0 n_0)\) with the flux of particles injected at each time step \((n_{\text{inj}})\)

\[
|q_i| = \frac{L_y v_0 n_0 \Delta t}{n_{\text{inj}}} \text{C/m}
\]  

(4.2)

where \(L_y\) is the \(y\)-dimension of the simulation box, \(v_0\) is the orbital velocity of the satellite, \(n_0\) is the number density of the plasma and \(n_{\text{inj}}\) is the number of particles injected during each time step. Note that even though \(q_i\) depends on the numerical factors \(L_y\) and \(n_{\text{inj}}\), Singh and Vashi\(^76\) determined that the collected current is independent of these parameters.

The boundary conditions on the simulation box are such that the electric potential at the edge is \(\Phi = 0\) and every particle leaving the simulation is assumed to be lost. The boundary conditions on the satellite surface require the electric potential to be uniform, constant and equal to the potential of the satellite. All of the particles striking the satellite are assumed to be collected and are removed from the simulation. Therefore, the flux of charged particles to the satellite surface gives the total current,

\[
I_{\text{sat}} = \sum_i q_i \frac{\delta N_i}{\Delta t}
\]  

(4.3)

where \(q_i\) is the charge of the \(i^{\text{th}}\) species, \(\delta N_i\) is the number of each species collected per time step, and the summation is performed over all particle species. Global quasi-neutrality is maintained by redistributing the deficit number of particles for the appropriate species from a Maxwellian distribution randomly over the entire simulation system. Singh et al.\(^74\) suggest that a more physical method for maintaining global quasi-neutrality is to inject particles along magnetic field lines, allowing the particles to flow along magnetic field lines to areas of large space charge densities. However, this method cannot be implemented in a 2-D PIC code. Additionally, the large negative potentials observed in Singh et al.\(^74\) simulations when the deficit particles are randomly redistributed throughout the simulation space were not observed in the simulations presented here.
4.2.2 Poisson Equation Solver

In order to determine the electric field needed for the Boris Mover, Poisson’s equation is solved using the density at the grid points as a source term. Poisson’s equation has been discretized over a cartesian grid space using a standard second order finite difference method,

$$\frac{\phi(i - 1, j) - 2\phi(i, j) + \phi(i + 1, j)}{\Delta x^2} + \frac{\phi(i, j - 1) - 2\phi(i, j) + \phi(i, j + 1)}{\Delta y^2} = -\frac{\rho(i, j)}{\epsilon_0} \quad (4.4)$$

Since Poisson’s equation is being discretized over a cartesian grid with a circular interior boundary condition, care must be taken to reduce the discretization errors near the spacecraft’s edge. Consider a point near the edge of the spacecraft, as shown in Figure 4.3, where the spacecraft’s boundary passes between grid points. Poisson’s equation can rewritten such that the points that fall within the spacecraft are weighted to account for the difference,

$$\nabla^2\phi(i, j) = \frac{2}{\Delta x^2} \left[ \frac{\phi(i - 1, j)}{l(r + l)} - \frac{(lr)\phi(i, j)}{lrns} + \frac{\phi(i + 1, j)}{r(l + r)} \right]$$

$$+ \frac{2}{\Delta y^2} \left[ \frac{\phi(i, j - 1)}{s(s + n)} - \frac{ns\phi(i, j)}{lrns} + \frac{\phi(i, j + 1)}{n(n + s)} \right] \quad (4.5)$$

where $l$, $r$, $s$ and $n$ represent the ratio of the distance between the spacecraft boundary and grid point $(i, j)$ to the grid spacing ($\Delta x$ or $\Delta y$). Note that if $l = r = s = n = 1$, the left hand side of equation 4.4 is equivalent to equation 4.5. Boundary conditions are applied by replacing the boundary term with the corresponding boundary’s potential. For an $N \times N$ grid, the set of $N^2$ equations represented by equation 4.4 can be solved by re-writing them in matrix form:

$$Au = (H + V)u = B \quad (4.6)$$
where \( \mathbf{u} = [\phi_{i,j}], \mathbf{B} = [\rho_{i,j}/\epsilon_0] \) and

\[
H = \frac{1}{\Delta x^2} \text{tridiag}(-1, 2, -1), \quad V = \frac{1}{\Delta y^2} \text{block\_tridiag}(-I, 2I, -I)
\]  

(4.7)

Note that for an \( N \times N \) grid, \( \text{tridiag} \) is an \( N^2 \times N^2 \) tridiagonal matrix whose three main diagonals are comprised of -1, 2, and -1. Additionally, \( \text{block\_tridiag} \) is an \( N^2 \times N^2 \) block tridiagonal matrix whose three main diagonal blocks consist of \( -I, -2I, \) and \( -I \), where \( I \) is an \( N \times N \) identity matrix.

The alternating direction iterative (ADI) method is implemented to efficiently solve the resulting matrix equation by solving the following equations implicitly:

\[
(H + Ir_m)u^{m+1/2} = (r_mI - V)u^m + \mathbf{B}
\]  

(4.8)

\[
(V + Ir_m)u^{m+1} = (r_mI - H)u^{m+1/2} + \mathbf{B}
\]  

(4.9)

where \( m \) represents the iteration number. Equations 4.8 and 4.9 can be reduced to the Peaceman-Rachford form

\[
u^{m+1} = T_{r_{m+1}}^{m+1}u^m + g_{r_{m+1}}^{(B)}
\]  

(4.10)

where

\[
T_r = (V + Ir)^{-1}(rI - H)(H + Ir)^{-1}(rI - V)
\]  

(4.11)

\[
g_r(B) = (V + Ir)^{-1}[(rI - H)(H + Ir)^{-1} + I]B
\]  

(4.12)

and \( r_m \) are relaxation parameters that are iteratively calculated to minimize the maximum spectral radius of the Peaceman-Rachford matrix, \( T_r \), for \( m = 2^k \) iterations using the method developed by Wachspress.\(^{83,86}\) As a point of comparison between this method and a standard method such as Successive Over-Relaxation (SOR), the rate of convergence for the ADI-Wachspress method for a \( 100 \times 100 \) grid with \( k = 5 \) is 15.5 times greater than the rate of convergence for SOR. Thus, the combination of ADI and Wachspress’ method results in an efficient Poisson Solver.

The following definitions and normalizations are used throughout the model: distance \( \tau = r/\lambda_{De} \), time \( \bar{t} = t\omega_{pe} \), velocity \( \bar{v} = v/v_{the} \) and potential \( \bar{\Phi} = \phi/\Phi_0 \), where \( \Phi_0 = kT_e/e, T_e, \omega_{pe}, v_{the} \), and \( \lambda_{De} \) are the electron temperature, plasma frequency, thermal velocity and Debye length, respectively.

### 4.3 Tether Current Dependence on System Parameters

Previous analyses of the current collected by a satellite or an EDT system use models where the system is traveling perpendicular to the magnetic field to derive an expression for a
constant current.\textsuperscript{44,46,75} Additionally, electrodynamic tether applications generally assume the perpendicular geometry because it maximizes the electrodynamic force in the direction of the system’s orbital velocity. However, it is unlikely that a tether system will be operating in these conditions (e.g. the PMG tether system’s orbit inclination was 25.7°). Thus, it is important to understand the behavior of the current collection under a variety of magnetic field orientations, plasma states and end-body biases. In order to maximize the current collected, EDT systems will likely operate near the F-2 electron density peak. The parameters that were used throughout the simulations were generated to build a reasonable approximation of the ionosphere at an altitude of 300 km. Simulations were performed for the following ionospheric parameters: plasma density \( n_0 = 10^{11} \, \text{m}^{-3} \), electron temperature given by \( T_e = 0.2 \, \text{eV} \), and the ambient magnetic field strength \( B_0 = 0.3 \, \text{G} \). The end-body has an orbital velocity of \( v_0 = 0.3 v_{\text{the}} \), where \( v_{\text{the}} \) is the electron thermal velocity (approximately 192 km/s) and radius \( r_{\text{sat}} = 10 \lambda_{\text{De}} \), where \( \lambda_{\text{De}} \approx 1 \, \text{cm} \) in the ionosphere. The large orbital velocity was chosen for computational economy (a typical orbital velocity for a LEO satellite is \( v_0 \sim 8 \, \text{km/s} \)). The ion temperature was set to \( T_i = 0 \, \text{eV} \) because the ion thermal velocity is an order of magnitude less than the satellite’s orbital motion if \( T_i = 0.2 \, \text{eV} \). In order to reduce the computational time while maintaining the relevant physics of the simulation, a reduced mass ratio is used: \( m_i/m_e = 1837 \). The reduced mass ratio corresponds to H\(^+\) ions, however the dominant species at 300 km are O\(^+\) ions that have a real mass ratio of \( m_i/m_e = 16 \times 1837 \). The simulation box was \( 256 \lambda_{\text{De}} \times 256 \lambda_{\text{De}} \) and the integration time step was \( \Delta t = 0.2 \omega_{\text{pe}}^{-1} \), where \( \omega_{\text{pe}} \) is the electron plasma frequency (at 300 km, \( \omega_{\text{pe}} = 1.78 \times 10^7 \, \text{rad/s} \)). Simulations were run over \( t = 7,200 \omega_{\text{pe}}^{-1} \), corresponding to a real time period of 0.404 ms, in order to allow several periods of the transient plasma structures around the satellite to evolve. Average current values are obtained by time-averaging the current collected over several oscillation periods.

The following discussion begins with the results for the well studied case of the magnetic field perpendicular to the orbital velocity of the satellite. The behavior of the current collected by the end-body is then examined when the satellite size and orbital velocity are held constant, but the magnetic field orientation, satellite potential and ratio of \( \omega_{\text{pe}}/\Omega_{\text{ce}} \) are varied. The simulations cover a parameter space that encompass space plasma states that would be encountered by an EDT system in LEO at an altitude of 300 km.

### 4.3.1 Benchmark Case \((\theta_B = 90^\circ, \phi_{\text{sat}} = 20 \, \text{V})\)

The configuration where the magnetic field is perpendicular to the satellite’s orbital velocity serves as a point of comparison between the present work and the previous work of Parker and Murphy,\textsuperscript{61} Vashi and Singh,\textsuperscript{85} Singh and Chaganti,\textsuperscript{72} Laframboise,\textsuperscript{46} and Singh et al.\textsuperscript{75} Figure 4.4 shows that the average current collected between \( t = 1500 \omega_{\text{pe}}^{-1} \) and \( t = 7,200 \omega_{\text{pe}}^{-1} \) is \( I = 11.26 \, \text{mA/m} \), which is directly between the values predicted by the adjusted Parker-Murphy model \((I = 6.23 \, \text{mA/m})\) and Laframboise\textsuperscript{46}(\(I = 19.98 \, \text{mA/m}\)). The current collected in the simulation is also within 2-6 times the Parker-Murphy current, matching the
Figure 4.4: The average steady state current collected by a spherical satellite biased at $\Phi_{sat} = 20V$ and orbital velocity $v_{orb} = 0.3v_{the}$ as a function the angle between the local magnetic field vector and its orbital velocity, $\theta_B$.

enhancement seen in the TSS-1R experiment.

The time evolution of the electric potential around the satellite is shown in Figure 4.5. Figure 4.5(a) shows the potential distribution at the start of the simulation runs. Figure 4.5(c), (e) and (f) correspond the maximum current collection states, while Figure 4.5(b) and (d) correspond to current minima. The fan shaped potential structures observed by Vashi and Singh\textsuperscript{85} to form in the ram region of the satellite are also observed in Figure 4.5(c), (e) and (f) during current maxima. The fan shaped potential surfaces create large azimuthal electric fields that capture electrons and drive them towards the satellite. The particle structures that form with these potential structures will be described in detail in the next section. Since the behavior of the current and potential structures throughout the simulations drive current maxima and minima, changes in these structures brought about by different system configurations will have a significant impact on the current collected by the satellite.

4.3.2 Impact of the Magnetic Field’s Orientation

In order to investigate the effect of the magnetic field’s orientation on the current structures described in the previous section, simulations have been run for $\theta_B \in [0^\circ, 180^\circ]$, where $\theta_B$ is the angle between the satellite’s velocity and the local magnetic field. The variation of the average current collected with $\theta_B$ is shown in Figure 4.4. Surprisingly, the maximum
Figure 4.5: The time evolution of the potential in the simulation box for the benchmark case \((\theta_B = 90^\circ, \Phi_{sat} = 20 \text{ V})\), where blue contours represent areas of negative potential. Figures 4.5(b) and (d) correspond to current minima, while Figures 4.5(d), (e) and (f) show the potential at current maxima. The fan shaped structures observed by Vashi and Singh\(^8\) form in the ram region of the box form during current maxima. A steady-state negative potential region forms in the wake of the satellite. The potential contours are separated by \(\Delta \Phi = 15\).
average current is collected when \( \theta_B = 75^\circ \) and \( 105^\circ \). A significant drop in the average current collected occurs when the magnetic field is near perpendicular to the satellite’s orbital velocity. When \( \theta_B \) is varied below \( 75^\circ \) and above \( 105^\circ \), the average current collected decays exponentially. A nonlinear, least squares curve fit results in the following bi-modal model for the tether current’s dependence on magnetic field orientation

\[
I(\theta_B) = a_1 \exp(-b_1^2(\theta_B - c_1)^2) + \exp(-b_2^2(\theta_B - c_2)^2) + \exp(-b_3^2(\theta_B - c_3)^2) + d
\]

(4.13)

where the values for the model’s parameters are given in Table 4.1. Equation 4.13 is plotted with the simulation data in Figure 4.4.

The variations in the current can be further illuminated by examining both the evolution of the current throughout the simulations and the plasma structures that form around the satellite. The transient behavior of the current, shown in Figures 4.6 and 4.7, clearly changes from small amplitude, high frequency oscillations to large amplitude, low frequency oscillations between \( \theta_B = 60^\circ \) and \( \theta_B = 90^\circ \). The change in the transient behavior suggests a change in the dominant physics present near the satellite. The transition occurs at an angle around \( \theta_B = 75^\circ \), as seen in Figure 4.6 and Figure 4.7, where the current collected by the satellite does not develop oscillations over the course of the simulation. The shift in the characteristic frequencies in the current collection has not been previously observed.

The change in the characteristic frequency of the collected current is due to a transition in the primary structures present in the plasma as the strength of the in-plane component of the magnetic field increases in strength. The variation of plasma structures across magnetic field orientations can be seen in Figure 4.8. Figure 4.8(a) and 4.8(b) show the electron structures present at selected current minima and maxima for \( \theta_B = 90^\circ \) (shown in Figure 4.7). The current minima occur during the formation of the large particle structures curving around the satellite at the edge of the simulation space, shown in Figure 4.8(a). As time progresses, the particle structures gradually translate in an azimuthal and radial direction until they reach the satellite (corresponding to a current maxima), shown in Figure 4.8(b). The current structures just described are the azimuthal current structures observed during events in the
Figure 4.6: The time evolution of the current collected by a spherical satellite biased at $\Phi_{sat} = 20V$ and orbital velocity $v_{orb} = 0.3v_{the}$ for $\theta_B = 60^\circ$, $70^\circ$, and $75^\circ$.

As the magnetic field tilts from $90^\circ$ to $75^\circ$, the magnetic field develops a component in the satellite’s equatorial plane. The in-plane component draws current along the field lines and bends the azimuthal current structure along the magnetic field line. The resulting effects on the plasma structure can be seen in Figure 4.8(c) and 4.8(d) for $\theta_B = 80^\circ$. When the magnetic field is perpendicular to the orbital velocity, most of the plasma particles trapped in the azimuthal current curve around the satellite and are not collected. However, in Figure 4.8(c) and 4.8(d) the additional component along the field lines directs the plasma particles into the satellite and the particles are collected. The additional collected electrons result in the current increasing by 67% between $\theta_B = 90^\circ$ and $75^\circ$.

When the magnetic field’s orientation decreases below $75^\circ$ (or increases above $105^\circ$), the dominant structure in the plasma is completely altered by the in-plane magnetic field component. The strong in-plane magnetic field causes plasma lobes to form on either side of the satellite (see Figure 4.8(e) and (f)). The formation of the lobes below $\theta_B = 75^\circ$ and above $\theta_B = 105^\circ$ results in potential barriers forming that prevent electrons from arriving at the satellite from the top or the bottom. Additionally, the current is limited in this orientation range due to the in-plane component of the magnetic field becoming large enough to prevent electrons from crossing the field lines. The breakdown of cross-field transport inhibits the formation of the azimuthal current structures, which seem to be a dominant mechanism.
Figure 4.7: The time evolution of the current collected by a spherical satellite biased at $\Phi_{sat} = 20V$ and orbital velocity $v_{orb} = 0.3v_{the}$ for $\theta_B = 75^\circ$, $80^\circ$, and $90^\circ$. Note that the oscillations in the currents for $\theta_B = 60^\circ$ (Figure 4.6) and $\theta_B = 80^\circ$ have distinctly different frequency characteristics. This suggests a transition in dominant physics in the space plasma around the satellite between these magnetic field angle (the behavior between $\theta_B = 100^\circ$ and $120^\circ$ is similar).
Figure 4.8: The plasma structures present at the current minima (a, c, and e) and maxima (b, d, and f) in Figure 4.6 and Figure 4.7 for $\theta_B = 60^\circ$, $80^\circ$ and $90^\circ$. The azimuthal current structures seen flowing in front of the satellite for $\theta_B = 80^\circ$ and $90^\circ$ are not present for $\theta_B = 60^\circ$. 
Figure 4.9: Steady state lobe structures that form near a spherical satellite biased at $\Phi_{sat} = 20V$, and orbital velocity $v_{orb} = 0.3v_{the}$ for $\theta_B = 0^\circ$. Note the formation of an enhanced wake due to ion scattering off of the potential surfaces created by the satellite’s bias. The collection radius predicted by the Parker-Murphy model is represented by the outer circle.
Figure 4.10: Simulations for both collision-less and MCC-PIC cases are plotted against the two Parker-Murphy models. The current is roughly proportional to the Parker-Murphy currents below $\Phi_{sat} = 150$. Above $\Phi_{sat} = 150$, a steady bow-shock forms around the satellite that inhibits the electrons from entering the sheath region. Electron-neutral collisions prevent the formation of the bow-shock structure inhibiting current collection. The analytical curve fit is based on the data points below $\Phi_{sat} = 150$.

for delivering current to the satellite. Figure 4.9 shows the lobed plasma structures that form above and below the satellite when $\theta_B = 0^\circ$. These lobes form at twice the collection radius predicted by the Parker-Murphy model (outer circle in Figure 4.9). The lobed structures form an enhanced wake around the spacecraft as described by Engwall et al.\textsuperscript{27} for an unmagnetized space plasma.

### 4.3.3 Current Dependence on Satellite Potential

The analytical models of Parker and Murphy,\textsuperscript{61} Singh and Chaganti,\textsuperscript{72} and Khazanov\textsuperscript{44} do not accurately capture all of the physics present in the current-voltage characteristic of the TSS-1R experiment, as seen in Figure 3 in Khazanov.\textsuperscript{44} In order to better understand the physics that are not incorporated in the analytical models, the satellite bias was varied between $\Phi_{sat} = 30$ and $\Phi_{sat} = 250$, which corresponds to $\phi_{sat} \in [6 \text{ V}, 50 \text{ V}]$ where $\Phi_0 \approx 0.2 \text{ V}$. The range of possible satellite potentials was limited by the validity of the simulation boundary conditions because the sheath intersected the edges of the simulation box at high potentials, violating the boundary condition of $\Phi = 0$. The solid dots in Figure 4.10 show the
results of the simulations and that the current roughly follows the Parker-Murphy models below $\Phi_{\text{sat}} = 150$. For purposes of comparison, the Parker-Murphy model and adjusted Parker-Murphy model are shown in Figure 4.10. The upper bound on current collection predicted by the Parker-Murphy model is given by:

$$I_{PM} = 2\pi r_0^2 J_r, \quad r_0^2 = r_{\text{sat}}^2 \left[ 1 + \left( \frac{8e\Phi_0}{m_e\Omega_{ce}^2 r_{\text{sat}}^2} \right)^{1/2} \right]$$

(4.14)

where $J_r$ is the random thermal current, $\Omega_{ce}$ is the electron cyclotron frequency and $m_e$ is the electron mass. The adjusted Parker-Murphy model adds an estimate for the current enhancement due to the satellite’s movement through the space plasma given by:

$$I_{APM} = I_{PM} + \delta I, \quad \delta I = 2\pi N e v_{\text{the}} r_\perp^2$$

(4.15)

where $N$ is the electron number density, $e$ is the elementary charge, $v_{\text{the}}$ is the electron thermal velocity and $r_\perp$ is the distance the sheath extends perpendicular to the satellite’s orbital velocity. Note that this estimation does not depend on the satellite’s velocity and therefore is only qualitative. Since there is not an analytical expression for the distance the sheath extends along magnetic field lines perpendicular to the satellite’s direction of motion, the collection radius, $r_0$, calculated by Parker and Murphy is used as an estimate. For low potentials ($\Phi_{\text{sat}} \leq 40$), the average collected current is between the Parker-Murphy model and adjusted Parker-Murphy model. Between $\Phi_{\text{sat}} = 40$ and $\Phi_{\text{sat}} = 150$, the current is above the values predicted by the adjusted Parker-Murphy model. Note that for the parameters of the TSS-1R collecting end-body, the two Parker-Murphy models bound the data from the tether experiment.

The representative time histories of the collected current presented in Figure 4.12 show the collected current’s behavior changes drastically as the satellite’s bias increases. At low potentials, the current remains relatively constant throughout the simulations. As the potential rises above $\Phi_{\text{sat}} = 75$, the current develops large amplitude oscillations. Above $\Phi_{\text{sat}} = 150$, an order of magnitude drop in average current collected is observed. The significant reduction in current collection is due to the sheath transitioning from a turbulent state to a steady state. As seen in the particle distributions shown in Figure 4.11 (a) and (b), the sheath remains small ($r_{\text{sheath}} << r_{\text{sat}}$) for low satellite bias potentials. However, the time histories of the current collected by the satellite in Figure 4.12 show a marked change in behavior above $\Phi_{\text{sat}} = 75$ due to the expansion and contraction of the sheath. The volatility of the sheath can be seen via a comparison of Figures 4.11(c) and (d). The contraction (Figure 4.11(c)) and expansion (Figure 4.11(d)) of the sheath corresponds to, as expected, current minima and maxima, respectively. The sheath steadies at satellite potentials above $\Phi_{\text{sat}} = 150$ and forms the steady state ion bow-shock seen in Figures 4.11(e) and (f). The formation of the ion barrier or bow-shock structure attracts electrons away from the sheath region with the result that the collected current is reduced by an order of magnitude.
Figure 4.11: (a) and (b) show that the ion-depletion region drops to zero in the ram direction for low potentials. Note that (c) and (d) correspond to current minima and maxima, respectively for $\Phi_{sat} = 125$. The sheath can be seen expanding and contracting. The current is limited when $\Phi_{sat} \geq 150$ by the formation of the steady-state ion structure ahead of the satellite, (e) and (f).
A curve fit to the data below $\Phi_{sat} = 150$ results in $I \propto \Phi_{sat}^{0.7}$, compared to the fully 3D model of Singh et al. $I \propto \Phi_{sat}^{0.62}$. For sufficiently large values of $\Phi_{sat}$, $I_{PM} \propto \Phi_{sat}^{0.5}$. Thus, the enhancement seen in the simulations above the Parker-Murphy model can be written as:

$$I/I_{PM} \propto \Phi_{sat}^{0.2} \quad (4.16)$$

where $I_{PM}$ is the current predicted by the Parker-Murphy model. Since the simulation data below $\Phi_{sat} = 150$ is greater than either Parker-Murphy model, the curve fit will likely predict currents larger than actual values.

### 4.3.4 Collisional Effects

Laboratory experiments have shown that the current collected continues to increase as the probe, or satellite, bias is increased. Electron-neutral collisions can have an impact on the electron velocity distribution and thus, impact the particle structures that form in a space-plasma. Typical ionospheric electron-neutral collision frequencies are on the order of $\nu_{en}/\omega_{pe} = 10^{-4}$ (e.g. Kelley). The incorporation of electron-neutral collisions into the simulation framework destabilizes the bow-shock that formed in the simulation discussed in the previous section. The resulting average collected current follows the trends observed in the laboratory experiments.
Figure 4.13: The current collected as a function of the magnetization of the space-plasma is shown, as well as an analytical curve fit to the data.

Simulations were run for $\tilde{\nu}_{en} = [1 \times 10^{-4}, 0.5 \times 10^{-3}, 1 \times 10^{-3}]$ and $\Phi_{sat} = [100, 150, 200]$ in order to capture the effects for both the variation of collision frequencies present in the ionosphere and the range of satellite potentials where the formation of a stable bow-shock structure occurs. The average current collected at each satellite voltage stay within 10% of the average current collected when $\tilde{\nu}_{en} = 1 \times 10^{-4}$ for all values of $\tilde{\nu}_{en}$. The average current for $\tilde{\nu}_{en} = 1 \times 10^{-4}$ is plotted against the collision-less simulation results in Figure 4.10. Comparisons between the average current above $\Phi_{sat} = 150$ show that the simulation results containing electron-neutral collisions do not display the current limiting behavior observed in the collision-less simulations. The current collected for $\Phi_{sat} = 100$ increases from $I = 11.3$ to $I = 15.6$ when collisions are included. For $\Phi_{sat} = 150$ and 200, the current remains close to the curve fit of the collision-less simulation data. Thus, electron-neutral collisions disrupt the formation of the bow-shock sheath structure responsible for limiting the current above $\Phi_{sat} = 150$. Note that the average current does not continue to increase as the satellite bias increases as it does in the experimental results of Gilchrist et al. However, both the amplitude of the current’s oscillation and the maximum current increase with satellite voltage. Thus, larger instantaneous currents are collected at higher voltages, but the average current is less due to periods of limited current collection.
Figure 4.14: A highly magnetized plasma limits the cross-field transport and thus limits current collected. A weakly magnetized plasma does not have the large amplitude oscillations present in the current collected in a magnetized plasma.

4.3.5 Magnetization Effects: Variation of $\omega_{pe}/\Omega_{ce}$

Since the magnetic field strength and plasma density can vary by an order of magnitude in LEO, simulations were run over a range of plasma frequency to cyclotron frequency ratios that can exist throughout LEO ($\omega_{pe}/\Omega_{ce} = 0.64, 3.33, 8.00, 13.00$ and $16.04$). Varying the frequency ratio captures the variation in plasma density and magnetic field strength that would be seen by an ionospheric satellite, while still maintaining the physics of the PIC simulations. Figure 4.13 displays the average current collected as a function of $\omega_{pe}/\Omega_{ce}$. An analytical curve fit to the simulation data shows that the average collected current varies as $I \propto \left[ \omega_{pe}/\Omega_{ce} \right]^{0.33}$. Figure 4.14 shows that a highly magnetized plasma, such as a space plasma found in Earth’s polar region where $\omega_{pe}/\Omega_{ce} = 0.64$, would significantly limit current collection ($I \approx 0.334 \text{mA/m}$) by a satellite due to the plasma’s inability to cross the strong magnetic field lines. However, the flow of a weakly magnetized plasma, such as a low latitude, daytime plasma at an altitude of 200 km, is not inhibited by magnetic field lines. The resulting effect is a relatively large, constant current flow with an average of $I \sim 24 \text{mA/m}$ (see Figure 4.14). A typical value for $\omega_{pe}/\Omega_{ce}$ in LEO is 3.33. The time evolution of the current displays large oscillations with an amplitude of $\Delta I \approx 19 \text{mA/m}$. At $\omega_{pe}/\Omega_{ce} = 3.33$, the electron cyclotron period is $\tau_{ce} \approx 18\omega_{pe}^{-1}$ and the ion cyclotron period is $\tau_{ci} = (m_i/m_e)\tau_{ce}$. Since $m_i/m_e = 1837$ for the simulations and the oscillations in the current have a period $\tau_i \approx 1200\omega_{pe}^{-1}$, the oscillatory behavior in the collected current is likely driven
4.3.6 Discussion and Applications

As described in Section 4.3, EDT end-body current collection is typically studied for the case when the collecting body is traveling perpendicularly to the local magnetic field because the electromagnetic force on the tether will act entirely in the direction of the system’s velocity. A system on an inclined orbit will have both an out-of-plane and in-plane component of the electromagnetic force. The out-of-plane component drives a well studied instability in the system’s dynamics. Assuming that the tether is perpendicular to the magnetic field and that its velocity vector is in the system’s orbital plane, then the in-plane component of the electromagnetic force is given by $F_{emf} = IL \sin(\theta_B)$, where $I$ is the current collected by the system’s end-body, $L$ is the tether length and $\theta_B$ is the previously defined angle of the magnetic field’s orientation. The magnitude of the force will remain constant for small departures from $\theta_B = 90^\circ$ if the current is held at a fixed value. However, the above results have shown that the current will double (and thus the electromagnetic force as well) at small departures away from $\theta_B = 90^\circ$ due to the modification of the plasma structures by the in-plane component of the magnetic field. Therefore, the enhancement of the current when the magnetic field orientation is slightly changed has significant ramifications for studying EDT system dynamics. Increasing the inclination of the EDT system’s orbit will also cause the magnetic field’s orientation to gradually sweep through larger angles. As the orbit’s inclination increases the orbital locations of maximum current collection will gradually approach apogee and perigee (i.e. the period between the current maxima near apsides will be reduced). The interaction of the already known instabilities in tether systems on inclined orbits and the periodic increases in tether current just described could result in new instability modes.

The majority of the chapter has focused on the application of satellite charging to EDT systems. However, the results encompassed in this paper have application to space-object charging in Earth’s dayside ionosphere. The satellite simulated in the present work is about double the size of a 1U Cubesat. Since satellites can charge up to 5 V positive on the day side, the low potential simulations have direct applicability to the charging of small satellites in LEO as they clear Earth’s shadow. The interaction of a charged surface with the local magnetic field can also perturb the orbits of high area-to-mass ratio (HAMR) space debris. While atmospheric drag forces typically dominate in LEO, the asymmetric nature of the plasma structures observed in Figure 4.8(a)-(d) would result in non-uniform electrostatic charging of the HAMR object. The resulting interaction with the magnetic field in LEO would produce both a torque and a force on the object, causing an error in the orbit and attitude determination. HAMR objects pose a serious collision risk for active GEO and LEO spacecraft. Thus, an accurate determination of the object’s location will depend on its interaction with the local space environment and is essential for collision avoidance. The asymmetry of the azimuthal current structure’s arrival to the surface of the satellite...
also has application to Langmuir probe design. Langmuir probes are commonly included on spacecraft to monitor the satellite’s potential and local charged particle densities (recent missions include C/NOFS\(^{18}\) and DEMETER\(^{52}\)). If the Langmuir probe is directional or planar, such as the ones on DEMETER and C/NOFS, then the asymmetry in the current structures will affect the data that are collected by the instruments.

### 4.4 Conclusions

In addition to building a better understanding of the dependence of the current collected by an EDT system on end-body bias, the fundamental purpose of this work is to show that the variations in the space plasma environment seen throughout an orbit will impact the current collected. A 2-D3\(\nu\) Particle-in-Cell model has been used to study the plasma dynamics in the vicinity of a positively charged electrodynamic tether end-body in LEO, with a specific focus on its applications to electrodynamic tether systems. Previous investigations have been limited to cases when the magnetic field is either perpendicular or parallel to the satellite’s orbital velocity due to the complex nature of the plasma’s behavior. The present work expands on previous studies by completing a parametric study over magnetic field orientation, sub-satellite bias and plasma magnetization.

As expected, a highly magnetized plasma inhibits particle transport across magnetic field lines and results in a significant reduction in current collected. However, a plasma with a ratio of \(\omega_{pe}/\Omega_{ce} = 16\) is typically only found in the polar regions where the magnetic field is the strongest. The satellite bias was also varied between \(\Phi_{\text{sat}} \in [6\text{V}, 50\text{V}]\). The average current varied as \(I \propto \Phi_{\text{sat}}^{0.7}\) below \(\Phi_{\text{sat}} = 125\). Above \(\Phi_{\text{sat}} = 125\), a steady-state bow shock formed in the ram region. The bow shock formed a large potential structure that prevented electrons from entering the satellite’s sheath and thus dropping the current collected by the satellite by an order of magnitude. Electron-neutral collisions were then introduced into the simulation model. The collisions destabilized the bow-shock structure and the average collected current.

A somewhat unexpected result was the increase in average current collected when the magnetic field is orientated \(\pm 15^\circ\) off of perpendicular from the satellite’s orbital velocity. The primary mechanism driving the current enhancement is the in-plane magnetic field guiding the azimuthal current structures into the satellite. Once the magnetic field was further than \(15^\circ\) off of perpendicular, the in-plane component of the magnetic field became large enough to limit the ability of the electrons to cross magnetic field lines, thus destroying the azimuthal current structure that enhanced the current collected. Additionally, ion lobes form above and below the satellite, further inhibiting the ability of electrons to enter the satellite’s sheath and to be collected. When the magnetic field is perpendicular to the satellite’s velocity, an enhanced wake forms, similar to the structures predicted by Laframboise\(^{46}\) to form around a positively charged satellite in a flowing, tenuous plasma.
In summary, PIC simulations have been run over a range of parameters for an electrodynamic tether system orbiting on an inclined orbit in LEO. Analytical curve fits have been generated to characterize the variation of the average current collected with both plasma and system parameters. The variations in average current are driven by changes in the plasma-satellite system altering the plasma structures that deliver current to the collecting end-body. The completed research can be applied to building an improved tether/plasma interaction model in order to better characterize the transient dynamics in large electrodynamic tether systems that develop due to the anisotropic nature of the ionospheric space plasma. The newly observed dependence of the collected current on the local magnetic field’s orientation will likely drive previously unobserved instabilities in the tether system’s dynamics.
Chapter 5

Electrodynamic Tether Instabilities Induced by the Variation of the Local Space Plasma Environment

5.1 Introduction

An electrodynamic tether system on an inclined orbit will travel through a constantly changing space-plasma environment. The current collected by the collecting end-body of the tether system will not only be affected by the diurnal variations in the plasma’s number density and temperature, but will also experience variations in the magnetic field’s orientation. Chapter 3 showed that new instabilities arise in the tether’s dynamics due to the diurnal variations in the collected current using a simple, analytical model to govern the current in the tether. The purpose of the work presented here is twofold. First, a rigorous investigation of the current collected by a spherical satellite is combined with a high-fidelity dynamics model to build a full electrodynamic tether system model that accurately incorporates the interaction between the tether and the ionosphere into the dynamics. Second, a study of the variation in the instabilities that arise in the dynamics of EDT systems on inclined orbits with increasing orbital inclination is presented. Increasing the inclination has several effects: it increases the out-of-plane component of Earth’s magnetic field that drives a known instability, changes the space-plasma environment through which the system is traveling and alters the current collected due to changes in the orientation of the local magnetic field relative to the collecting body’s velocity. As shown in Chapter 4, altering the magnetic field’s orientation has a significant effect on the electrodynamic force generated by the system due to the orientation of the magnetic field’s influence on the collected current.
5.2 Methods

The mathematical model developed for this study again characterizes a classical EDT system: two satellites connected by a long, insulated conducting tether orbiting a central body with a strong, dipole magnetic field and a dense space plasma, as seen in Figure 3.2a. The tether’s end-bodies are assumed to be in electrical contact with the plasma environment surrounding the satellites. A potential is set up across the tether either by the motion of the tether across the central body’s magnetic field or by a power source in the system. The potential difference drives a current that is collected from the surrounding plasma through the tether. The tether system can be in either motor or generator mode, depending on the direction of the current. For the purposes of our study, the tether will be in thruster mode. The tether’s current and the local magnetic field that generate the electrodynamic force are governed by the system electrodynamics model. The current and the local magnetic field vector are then fed into the model of the system’s dynamics.

5.2.1 System Electrodynamics Model

The system electrodynamics model is now updated to include the results from the PIC simulations from Chapter 4. The adjusted Parker-Murphy model used to govern the tether current in Chapter 3 is

\[
I_{APM} = I_{PM} + \delta I
\]  

where

\[
\delta I = 2\pi n_e q_e v_{the} a^2 \left[ 1 + \left( \frac{8\Phi_s}{m_e \Omega_e^2 a^2} \right)^{1/2} \right]
\]

Equation 5.1 produces a qualitative estimate of the impact that orbital motion has on the current collected by the tether end-body. Simplifying like terms simply results in:

\[
I_{APM} = 2I_0 \left[ 1 + \left( \frac{8\Phi_s}{m_e \Omega_e^2 a^2} \right)^{1/2} \right] = 2I_{PM}
\]

The results from 2-D3v PIC simulations in Chapter 4 show that the current varies as a function of the angle between the local magnetic field and the collecting body’s velocity, \(\theta_B\) as:

\[
I_{PIC}(\theta_B) = a_1 \exp(-b_1^2(\theta_B - c_1)^2) + \exp(-b_2^2(\theta_B - c_2)^2) + \exp(-b_3^2(\theta_B - c_3)^2) + d
\]

where the values for the model’s parameters are given in Table 4.1. The current was also found to vary with satellite potential by \(I_{PIC} \propto \Phi_s^{0.7}\). Thompson et al. develop a curve fit to the TSS-1R data based on the Parker-Murphy model of the form

\[
\frac{I_{PT}}{I_0} = \eta \left[ 1 + \left( \frac{8\Phi_s}{m_e \Omega_e^2 a^2} \right)^\gamma \right]
\]
Table 5.1: Parameter Values for Equation 5.8 and 5.7.

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<tr>
<th>Parameters</th>
<th>Values (95% confidence bounds)</th>
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<tr>
<td>$a$</td>
<td>$9.034$ mA ($8.616$ mA, $9.453$ mA)</td>
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<tr>
<td>$b_1$</td>
<td>$0.172$ ($0.1514$, $0.1926$)</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$0.02839$ ($0.02529$, $0.0315$)</td>
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<td>$c_1$</td>
<td>$77.53^\circ$ ($76.98^\circ$, $78.07^\circ$)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$88.59^\circ$ ($86.76^\circ$, $90.42^\circ$)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>$102.7^\circ$ ($102.1^\circ$, $103.3^\circ$)</td>
</tr>
<tr>
<td>$d$</td>
<td>$2.444$ mA ($2.922$ mA, $1.967$ mA)</td>
</tr>
</tbody>
</table>

where the Parker-Murphy model is obtained if $\eta = 1$ and $\gamma = 0.5$. Throughout the TSS-1R experiment $\eta$ ranged between 2.2 to 2.9 and $\gamma$ varied between 0.47 and 0.56. In order to build a model that captures both the enhancement above the Parker-Murphy current observed in the TSS-1R experiment and the behavior of the collected current observed in the PIC simulations, the coefficients, $\eta$ and $\gamma$, in Equation 5.5 will be chosen as:

$$\gamma = 0.5$$

$$c_{PIC}(\theta_B) = \exp(-b_1^2(\theta_B - c_1)^2) + \exp(-b_2^2(\theta_B - c_2)^2) + \exp(-b_1^2(\theta_B - c_3)^2)$$

$$\eta = 2 \times \frac{c_{PIC}(\theta_B)}{c_{PIC}(90^\circ)}$$

where $\gamma$ is chosen from the original Parker-Murphy model and $\eta$ is a combination of both the $\theta_B$ dependent portion of Equation 5.4 normalized to $c_{PIC}(90^\circ) = 1.02$ and the enhancement of the adjusted Parker-Murphy model. The inclination of the TSS-1R orbit was $28.45^\circ$. Throughout this orbit, $\theta_B$ will vary within $60^\circ$ and $120^\circ$, which corresponds to $\eta \in [1.02, 3.36]$. The plasma parameters used in the PIC-Parker-Murphy model (electron temperature, $T_e$ and number density, $n_e$) are calculated by the 2012 International Reference Ionosphere (IRI) based on the position of the collecting end-body. The local magnetic field is modeled as the tilted dipole model described in Section 3.2.1. The complete electrodynamics model for the system consists of both the magnetic field model and the PIC-Parker-Murphy current model. Equations 5.5-5.8 combined with the International Reference Ionosphere (where the local magnetic field is determined by Equation 3.11) define the PIC-Parker-Murphy model (hereafter referred to as the P-P-M model) and govern the current in the tether. Equation 3.10 defines the magnetic field vector along the tether. The results from Equations 5.5 and 3.10 is used to determine the Lorentz force along the tether throughout the system’s orbit.

### 5.2.2 EDT System Dynamics Model

The numerical model used for the system dynamics is the same as the one described in Section 3.2.2. The updated current collection model (Equations 5.5-5.8) is introduced into
the electrodynamic forcing term

$$F_{ED} = I_{PPM} \frac{\partial r}{\partial s} d\sigma \times B$$

(5.9)

where $I_{PPM}$ is the current in the tether given by Equations 5.5-5.8. As mentioned in Section 3.2.2, the partial differential equation governing the evolution of the tether shape can be written in terms of the unstretched tether length as

$$\ddot{r} = -\frac{\mu}{r_T^3} \mathbf{r}_T + \frac{1}{\rho} \frac{\partial T}{\partial \sigma} + \frac{I_{PPM}}{\rho} \frac{\partial r}{\partial \sigma} \times \mathbf{B} - \ddot{\mathbf{r}}_A - \ddot{\mathbf{p}}_A$$

(5.10)

As in the model from Chapter 3, equation 5.10 governs the elastic vibrations of the tether while Equations 3.26 and 3.27 determine the tether’s pendular motions. These equations are not independent; the vibrational and elastic modes of the tether are coupled. In order to obtain a finite set of ODEs, Equations 3.29 and 3.25 are discretized using the Finite Element Method.

### 5.3 Results and Discussion

The simulation parameters for the system are the same as the parameters from the system in Chapter 3 and are restated in Tables 5.2 and 5.3. Again, the system is modeled in thrust mode. The results presented in Section 5.3.1 will first focus on the new dynamics that develop as a result of the P-P-M model governing the tether’s interaction with the ionosphere. The dynamics under the P-P-M model are also compared to the system modeled in Chapter 3 where the tether’s current is modeled using the IRI-Parker-Murphy model. Since the range of magnetic field orientations observed by the system will increase when the inclination of the system increases, the variations in the current governed by the P-P-M model will also change. Therefore, the variation of the system’s dynamics with orbit inclination is presented in Section 5.3.2.

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<th>Orbital Elements</th>
<th>EDT System orbital elements.</th>
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<td>Inclination, deg</td>
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<td>Eccentricity</td>
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<tr>
<td>Right Ascension, deg</td>
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<td>Argument of Perigee, deg</td>
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<td>Mean motion, rev/day</td>
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<td>Perigee height, km</td>
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<td>Period, min</td>
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Table 5.3: EDT system parameters.

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<td>Longitudinal Stiffness $EA, \text{N}$</td>
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<tr>
<td>Unstretched linear mass density $\rho, \text{kg/m}$</td>
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</tr>
<tr>
<td>Structural Damping constant $c, \text{s}$</td>
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</tr>
<tr>
<td>Body A mass $m_A, \text{kg}$</td>
<td>5000</td>
</tr>
<tr>
<td>Body $B$ mass $m_B, \text{kg}$</td>
<td>500</td>
</tr>
<tr>
<td>Body $B$ potential $\Phi_B, \text{V}$</td>
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</tr>
<tr>
<td>$\mu, \text{m}^3\text{s}^{-2}$</td>
<td>$3.986\times10^{14}$</td>
</tr>
<tr>
<td>$\mu_m, \text{T}\cdot\text{m}^3$</td>
<td>$8\times10^{15}$</td>
</tr>
</tbody>
</table>

5.3.1 Effects Magnetic Field Orientation on Dynamical Instabilities

Two simulations were run to investigate the effects of including the current variations driven by changes in the magnetic field’s orientation on the system’s dynamics when it is in thrust mode. The first simulation used the P-P-M model to govern the interaction between the EDT system’s collecting end-body and the surrounding space-plasma environment. The second simulation model held the tether’s current constant at a value equal to the average of the tether’s current in the previous simulation. Simulations were run over 10 orbits, allowing the large oscillation’s in body $B$’s attitude to damp out and the motion of the end-body to reach steady-state, as seen in Figure 5.1. As expected, the 25° inclination of the system’s orbit...
Figure 5.2: The upper plot shows the temperature and number density calculated by IRI 2012 throughout the system’s orbits for an initial orbit inclination of 25°. The tether’s current, governed by the P-P-M model, is shown in the lower plot.

causes the oscillation $\theta_B$ to stabilize over a range of 50°. The following discussion will begin with an analysis of the magnetic field effects incorporated into the P-P-M model’s impact on the tether’s current. The remaining discussion will shift to analyzing the system’s overall motions, which are separated into the tether’s librations and the evolution of the system’s orbit.

The temperature and number density variations in the plasma surrounding the EDT system throughout the simulation, along with tether’s current, are plotted in Figure 5.2. While the large amplitude oscillations in the tether’s current are still clearly driven by the temperature and density oscillations, a comparison between Figures 3.3 and 5.2 shows that both the magnitude of the current and the amplitude of its variation are halved due to the magnetic field’s influence. The average current in the tether is $I_{avg} \sim 0.126$ A (compared to an average current of $I_{avg} \sim 0.263$ A for the IRI-Parker-Murphy model). A direct comparison to the current’s variation observed throughout the PMG mission is not necessarily warranted at this point because the experimental results showed that the charged particles released by the PMG’s Hallow Cathode Assembly (HCA) modified the plasma structure around the tether system. The PMG results in Figure 3.10b show that when the HCA is turned off, the collected current is reduced by an order of magnitude (the outlying data points near $5 \times 10^3$ s PMG time). The plasma modifications due to the presence of an active plasma contactor are not captured in the PIC results used to generate the P-P-M model. Additionally, the azimuthal current structures that were observed in the PIC simulations to strongly influence
The primary purpose of an EDT system in thrust mode is to raise or maintain the system's orbital altitude. The system's orbit was increased by $\sim 1$ km under the influence of both the P-P-M current and a constant current. However, the lower average current for the P-P-M model resulted less of an altitude gain than the IRI-Parker-Murphy model (the I-P-M model increased the system's semi-major axis by $\sim 1.4$ km). As observed in the results in Chapter 3, the oscillating orbital elements are not significantly affected by the variations in the P-P-M current (less than 0.4% difference). The differences do, however, continue to grow throughout the simulation (as an example, the absolute difference in the argument of periapsis, $\omega$, is shown in Figure 5.4). Therefore, the variations in the current predicted by the P-P-M model have little effect on the system's orbital elements and the magnetic field effects included in the P-P-M model limit the ability of the EDT system to raise its orbit.

The in-plane librations are shown in Figure 5.5. A comparison between the in-plane tether oscillations shows that the amplitude of the in-plane librations is unchanged between the two simulations. Figure 5.6 shows the frequency spectrum of the in-plane librations of the tether. The P-P-M current introduces a frequency component at twice the system's orbital frequency, $2\omega_{sys}$, but the two primary frequency components at $\omega_{sys}$ and slightly below $2\omega_{sys}$ that are induced in the constant current system remain the two strongest frequency components. Unlike the IRI-Parker-Murphy model, the P-P-M model does not significantly enhance the mode that is in between $\omega_{sys}$ and $2\omega_{sys}$. The impact of the P-P-M model on the in-plane component of the tether's libration is therefore limited.
Figure 5.4: The absolute difference in the argument of periapsis between the simulation where the tether’s current is held constant and where it is varied by the P-P-M model over 10 orbits.

The decreased amplitude of the P-P-M current (when compared to the IRI-Parker-Murphy current) results in a reduction of the amplitude of the tether systems out-of-plane librations (shown in Figure 5.7). As in the simulations under the influence of the IRI-Parker-Murphy current, the out-of-plane librations are no longer bounded when driven by the P-P-M current. The frequency spectrum of the tether’s out-of-plane motions shows that the high frequency oscillations due to the elastic vibrations of the tether are also damped out (see Figure 5.8). Note that the elastic vibrations were also damped out in the model under the influence of the IRI-Parker-Murphy current. Thus, the impact of the P-P-M model on $\beta$ is similar to the IRI-Parker-Murphy model.

5.3.2 Variation of Electrodynamic Tether Motions with System Orbit Inclination

The inclination of an EDT system’s orbit strongly affects the range of magnetic field orientations that are experienced throughout the system’s orbit, which in turn influence the current that is collected by the system. Note, however, that the variations in the out-of-plane component of the electrodynamic force acting on the tether will also increase. In order to investigate the change in dynamics that result from increasing the inclination of the system’s orbit, simulations were run over a range of initial inclinations: $I \in [15^\circ, 25^\circ, 40^\circ, 50^\circ, 65^\circ, 75^\circ,]$.
Figure 5.5: A comparison of the in-plane librations of the tether system when the current is held constant (top) and when it is determined by the P-P-M model.

simulations were also run over 10 orbits. Just as in the previous section, the focus of this discussion will be on the propagation of the EDT system’s orbit and its librations in order to understand how the general motion of the tether will change as a function of the system’s inclination.

The comparisons drawn between a tether system under a constant current and the time varying current calculated by the P-P-M model remain true as the inclination changes. The differences in the system’s orbital elements between a system with a constant tether current and a tether current driven by the P-P-M model remain small. Additionally, the librations of the tether in the system’s orbital plane are not significantly by the P-P-M current over the range of inclinations included in the simulations. The oscillations in the out-of-plane tether attitude angle continued to be driven unstable by the P-P-M model for each inclination.

The motions of the tether system do however show trends over the range of inclinations included in the simulations. The amplitude of the pendular motions of the tether system tend to decrease as the inclination of the system’s orbit increases. The maximum libration angles for both in and out-of-plane librations are shown in Figure 5.9. A comparison between the maximum libration angles when the current is held constant and when it is left to vary with the plasma parameters yields the following trends. The maximum angular displacements are greater under the influence of the P-P-M model for both modes of libration (with the exception for the in-plane libration angle for an inclination of 15°). The maximum difference in the tether’s in-plane oscillations between the two current models occurs when the
Figure 5.6: The frequency spectrum of the tether system’s in-plane librations.

Figure 5.7: A comparison of the out-of-plane librations of the tether system when the current is held constant (top) and when it is determined by the P-P-M model.
orbit’s initial inclination is 50°. The maximum out-of-plane libration angles double when the orbit’s inclination is less than 30°. For an initial inclination of 25°, the out-of-plane angle is maximized at $\beta_{PPM} = 0.13°$ for the P-P-M current and $\beta = 0.051°$ for a constant current system. While the maximum libration angles are small, the tether’s end-body swings through a maximum arc-length of 47.4 m under the P-P-M current and 17.5 m when the current is held constant. The differences in arc length are therefore significant, especially if the location of the tether’s collecting end-body is relevant to the system’s mission.

Figure 5.10 shows the average current collected by the system and the maximum semi-major axis of the system’s orbit over the range of initial inclinations. As the inclination system’s orbit decreases, both the average current and the orbit’s semi-major axis increase as well. As is expected, the increase in the system’s semi-major axis has the same behavior as the increase in the average current in the tether’s when the orbit’s inclination is varied. With respect to boosting tether operations, the most important observation is that the electrodynamic force is able to increase the orbit’s semi-major axis by an additional $\sim 1 km$ on less inclined orbits. Electrodynamic tethers will therefore be more effective at maintaining and/or raising a system’s orbit in LEO when on orbits that are closer to equatorial. A well know instability in an EDT system’s pendular motion, first reported by Lanoix et al.,51 is driven by the out-of-plane component of the electrodynamic force acting on the tether that develops due to the inclination of the system’s orbit. One would expect that increasing the system’s inclination would increase the instability in the system’s out-of-plane motion. However, the
Figure 5.9: The maximum in-plane and out-of-plane libration angles under a constant current and the P-P-M model are shown as a function of initial orbit inclination.

reduction in the amount of current that can be collected at higher latitudes counteracts the increase in the relative magnitude of the out-of-plane component of the electrodynamic force. As noted previously, the maximum out-of-plane libration angle actually decreases with increasing inclination. Even under a constant current, the magnitude of the out-of-plane librations does not significantly increase as the absolute value of the orbit’s inclination increases (see Figure 5.9).

5.4 Conclusions

In summary, a new approach to modeling the interaction between an electrodynamic tether and the space-plasma surrounding the system has been developed using the results from Particle-in-Cell simulations and parameters from the International Reference Ionosphere. A current collection model that includes the impact of the variations in the magnetic field’s orientation and the space-plasma’s state on the current collected by an electrodynamic tether system has been integrated into an electrodynamic tether dynamics model. Simulations have been completed that show differences in the system’s dynamics under both the varying current and a constant current, and over a range of initial orbit inclinations. The primary difference between the results presented here and the results shown in Chapter 3 is that the oscillations in the current collected by a tether system are limited when the effect of the magnetic field’s orientation is included in the system’s electrodynamic model. Additionally,
the variation of the system’s orbital inclination has several unexpected affects on the systems motion. Generally, the amplitude of the tether’s librations, the average current collected and the maximum semi-major axis of the system’s orbit decreases as the orbit’s inclination increases. The decrease in the current collected by the system counteracts the increase in the out-of-plane component of the electrodynamic force that is know the drive an instability in the tether’s dynamics to actually decrease the amplitude in the tether’s librations as the orbit becomes more inclined. Fundamentally, the inclusion of a more realistic current collection model has shown that an electrodynamic tether system will have larger librations on a near-equatorial orbit, but be more effective at increasing the altitude of the system’s orbit.
Figure 5.11: The upper plot shows the temperature and number density throughout the system’s orbits for an initial orbit inclination of 75°. The tether’s current, governed by the P-P-M model, is shown in the lower plot.
Chapter 6

Summary and Recommendations for Future Work

Electrodynamic tethers have the potential to aide several space-based applications and fields of research. The primary application of EDT systems is to satellite orbital maneuvering. However, before electrodynamic tether systems can be widely used, a better understanding of their interaction with the surrounding space-plasma needs to be developed. The results of this dissertations have shown that the coupling between the tether’s current and the space-plasma environment has a strong influence on the system’s dynamics. The key results from the research encompassed in this dissertation are summarized below, as well as some suggestions for future research directions.

6.1 Summary of Contributions

A review of the current literature has shown that a study of electrodynamic tether system dynamics under the influence of electrodynamic forcing varying with the local plasma parameters has not been completed. As a first order approach, the IRI-Parker-Murphy current collection model has been integrated into the electrodynamic forcing term in a high-fidelity tether system dynamics model. A comparison between a representative system with a constant current and the IRI-Parker-Murphy current has shown that the system’s orbit is not significantly affected by a plasma parameter dependent current. However, the IRI-Parker-Murphy current dampens the high-frequency, small amplitude oscillations in the tether librations and introduces a low frequency oscillation in the out-of-plane tether motion. A careful study of the dynamical behavior of the current collected by the system has been undertaken in order to understand the link between experimentally observed local plasma phenomena and the current fluctuations in a variety of plasma conditions in order to build a more higher fidelity current collection model. A Particle-in-Cell (PIC) code has been developed to study
the transient behavior of the plasma near a charged satellite. PIC simulations over a range of magnetic field orientations have shown that the azimuthal currents observed during the TSS-1R experiment play a significant role in current enhancement, the optimum magnetic field angle for current collection is $\theta_B = 80^\circ$ (previous studies have assumed that $\theta_B = 90^\circ$ or $\theta_B = 0^\circ$) and the azimuthal current structures break down for $\theta_B < 70^\circ$ and $\theta_B > 110^\circ$. A complete parametric study of both the local transient plasma processes in the vicinity of a spacecraft and current collected by a spacecraft as a function of satellite voltage, ratio of plasma frequency to cyclotron frequency and magnetic field orientation have been completed. Finally, the PIC results have been included into the system’s dynamics to create a robust system model. The completed research has made the following novel contributions to the understanding of the dynamics of electrodynamic tethers and spacecraft charging:

- A characterization of the plasma phenomena present near an EDT system as a function of satellite voltage, space-plasma frequencies and magnetic field orientation
- An improved understanding of the dependence of the current collected by a satellite on the local plasma fluctuations, satellite voltage, plasma magnetization and magnetic field orientation
- A new method for using Particle-in-Cell results to model the interaction between the tether system and the surrounding space plasma is developed
- An electrodynamic tether system model that incorporates the variation of the tether current with the local plasma parameters into its dynamics
- The observation of new instability modes in an electrodynamic tether system’s dynamics driven by the diurnal variations in the collected current
- A reduction of the system’s librations and ability to boost the system’s orbit when the inclination of the orbit is increased.

The completed research includes the first study of an electrodynamic tether system’s dynamics under a time-varying, plasma parameter dependent current and the first study of the plasma dynamics near a satellite where the magnetic field is not perpendicular or parallel to the satellite’s orbital velocity. An EDT system dynamics model has been developed that allows the study of the behavior of any EDT system under any plasma conditions. The completed body of work also encompasses a thorough study of the coupling between the local plasma environment and an electrodynamic tether system. The most significant results of this work pertaining to electrodynamic tether operations is that the current collected by an electrodynamic tether end-body is strongly dependent on the local magnetic field’s orientation and the tether system is more effective at increasing the system’s altitude on less inclined orbits.
6.2 Recommendations for Future Work

The primary direction of future research surrounds the interaction between the collecting end-body and the space-plasma. High fidelity models exist for a tether system’s dynamics, however our understanding of the plasma’s behavior over a wide range of system states is relatively limited. In order to accomplish a more thorough investigation via PIC simulations, both a full 3-D PIC code and a larger simulation box will need to be used in order to investigate the coupling between the field aligned current structures and the azimuthal current structures. The most significant limitation of the present work is that the interaction between the field aligned currents observed by Singh et al. and azimuthal current structures was not able to be investigated using the 2-D3v PIC model. A 3-D PIC model will allow the investigation of mechanisms that drive the interaction between the two types of current structures, which will connect the current transport processes that occur far away from the satellite to the near satellite processes. Additionally, a larger simulation would allow for the investigation of the current collection properties of a partially bare tether over a range of plasma states. Partially bare tethers have been proposed as a means to efficiently maintain a constant tether current due to the bare tether’s distributed collection area being able to account for variations in the space plasma. 3-D PIC simulations of a partially bare tether would allow for the investigation of the variation of the potential over the tether’s length, as well as the resulting variations in the current collected. The PIC simulation framework used in the plasma simulations for this work requires the normalization of the frequencies in the simulation model to the plasma frequency. Since the magnetic field strength is incorporated into the simulation through the species’ cyclotron frequency, the plasma density and magnetic field strength were inherently coupled. Therefore, decoupling the magnetic field’s strength from variations in the plasma density would allow for the investigation of the impact of the change in each parameter.

Several plasma processes are not included in the simulation model that likely affect the plasma structures that have been described in this paper: satellite surface secondary electron emission and photo ionization, particle scattering due to toroidal trapping, and neutral ionization processes. Depending on the surface properties of the satellite, incoming electrons may have enough energy to create one or more secondary electrons that will escape the surface. Satellite surfaces that are in the sunlight will also emit a photoelectron current depending on the satellite’s surface reflectance. Both of secondary electron and photoelectron emission will affect the total collected current and the sheath around the satellite. The particle trapping in the magnetic bottles that occur in the toroidal glow region affects the sheath’s structure in the presence of a magnetic field. For large satellite potentials, the accelerated, trapped particles have enough energy to ionize neutrals during collisions. The new electrons can be scattered off of trapped orbits and onto orbits that are collected by the satellite, thus enhancing the collected current. Lai show that neutral ionization in the satellite’s sheath significantly alters the sheath’s structure once ionization reaches its saturation point. Singh and Jaggernauth also show that in moderately dense plasmas ionization
processes drive recurrent sheath expansions and oscillations in the current. The inclusion of each of the described effects will allow for a more realistic investigation of the dynamic between the plasma structures and the current arriving at the satellite.

The data from the PIC simulations should also be validated via plasma chamber experiments. The results from Ref. 31 showed that the current will continue to increase with satellite potential. However, the behavior of the azimuthal structures and the impact of collisions have not yet been validated using laboratory experiments. Obviously, the perfect validation would be to compare the results to data from another electrodynamic tether experiment. There are a few EDT missions currently being developed, namely the Tether Electrodynamic Propulsion CubeSat Experiment (TEPCE) being developed by the Naval Research Laboratory. The ultimate direct extension of the present work would be to include the PIC framework into the system dynamics model. The inclusion of the PIC simulations would allow for the real-time calculation of the current collected by the tether system's end-body based on the parameters fed to it by IRI. Unfortunately, the PIC simulations run for this research required $\sim 1$ week to run on a single processor. Therefore, without parallelization and faster processors, the suggested simulations prohibitively long run times.
Bibliography


Chapter 6. Summary and Recommendations for Future Work


