

Modeling General Response to Silvicultural Treatments in Loblolly Pine Stands

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(ABSTRACT)

Basal area and dominant height growth and survival models incorporating general response to silvicultural treatments for loblolly pine stands were developed using data from various silvicultural experiments across Southern United States. Growth models for treated stands were developed by multiplying base-line growth models with modifier response functions/multipliers accounting for effects of thinning, fertilization, and control of competing vegetation. Chapman-Richards functions were used to model the base-line growth. Separate response functions to mid-rotation thinning and fertilization effects were developed. The thinning response function was based on duration and rate parameters and is sensitive to stand age at the time of thinning, time since thinning, and intensity of thinning. The fertilization response functions were based on Weibull distribution and the magnitude of responses varies with time since application of fertilizers, type of fertilizer elements applied, and rate of application. Response functions were integrated as a multiplier to base-line models. Response to early control of competing vegetation was incorporated into base-line models through multiplier factors. Multiplier factors were calculated based on growth difference between treated and untreated stands. A difference function, derived from differential equation with age, initial stand density, and site index served as the base-line survival model. The survival model was adjusted for thinning treatment by including an additional independent variable that represents thinning intensity. No adjustment was required for survival model in response to fertilization and competing vegetation control. All growth models were unbiased and had adequate performance in predicting basal area and dominant height following treatments. Models were developed to represent general growth trends in response to treatments. The response functions developed here can be viewed as general response functions.

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Chapter 1

Introduction

Growth and yield models that quantify forest stand dynamics have been widely used in forest operations and management as an effective tool in decision making. The scale of growth and yield models ranges from simple whole-stand level models to more complex individual-tree level models that require information on individual trees (Munro, 1974, Clutter et al., 1983, Burkhart and Tomé, 2012). Whole-stand models are used to forecast quantities such as volume, basal area, and/or number of trees per unit area. The predictor variables for these models for even-aged stands are generally age, site index, and stand density (Burkhart and Tomé, 2012).

Typically growth and yield models are fitted with data from permanent sample plots established in stands of varying initial conditions that were measured periodically. Some permanent plot installations also include plots with different levels of thinning treatments. In addition to thinning, other silvicultural practices such as fertilization, vegetation control and the use of improved genetic stock are now commonly applied in order to control the establishment, growth, quality, and composition of forest stands which are eventually expected to increase the productivity of plantations. In order to make silvicultural decisions, forest managers should use growth models that are designed to incorporate these silvicultural practices and that can also accurately project growth. Most existing growth models, however, were developed from limited data from untreated and genetically unimproved stands and are therefore expected to under predict the growth of today's more intensively managed forest stands. There have been many works aimed at incorporating silvicultural treatments into existing growth models; however, most of the past studies have focused on a single silvicultural treatment (Short and Burkhart, 1992, Amateis, 2004, Bailey et al., 1989, Hynynen et al., 1998, Quicke et al., 1999). Only a few studies have reported work on incorporating multiple treatments effects. Pienaar and Rheney (1995) proposed the use of an additional treatment response term to a base-line growth model to account for expected response due to a variety of silvicultural treatments. The height growth model (1.1) they developed was

shown to provide an accurate description of average dominant height growth response of slash pine plantations to several silvicultural treatments that were applied at different times of stand development:

$$domH = a_0 \left(1 - e^{-a_1 A}\right)^{a_2} + b_1 y_{st} e^{-b_2 y_{st}} \quad (1.1)$$

where, $domH$ is average dominant stand height, A is stand age, y_{st} is time (yrs) since treatment was applied, a_0 , a_1 , a_2 , b_1 and b_2 are parameters that define a particular growth curve.

The expected treatment response is explicitly represented in the model through readily interpretable parameters rather than implicitly by attempting to relate the parameters of the basic height growth model (a_0 , a_1 , a_2) to different silvicultural treatments. The second term in equation (1.1) represents the cumulative effect of an additional treatment on average dominant height over time. Parameters b_1 and b_2 determine the magnitude and pattern of the response. When different levels of treatment are involved, one or both of these parameters must be considered to be a function of the treatment level. When no additional treatment is applied, the second term in the equation becomes zero and the model represents the base-line height growth for a given site.

Pienaar and Rheney (1995) mentioned that different silvicultural practices may have differential effects on height growth over the life of a stand. Some treatments may have brief positive effects compared to the standard treatment, while others may have large and long lasting effects. Snowdon (2002) described the concept of Type 1 and Type 2 responses to silvicultural treatments. Type 1 response occurs when there is a temporary increase in growth rate that advances the stage of stand development. Thereafter, the treated and non treated stands follow parallel growth trajectories. Weed control and application of N fertilizer are

examples of Type 1 responses. And such responses are believed to have little or no long term effects on soil or site properties. Type 2 response typically occurs when the treatment has a large and long term effect on site properties. Application of P fertilizer to P deficient sites would be a good example of Type 2 response that can enhance the site over the rotation period. Snowdon (2002) tested the Schumacher yield and projection models incorporating these concepts of Type 1 and Type 2 responses with data from field experiments and compared results with the models by Pienaar and Rheney (1995). Models by Snowdon (2002) performed better than Pienaar and Rheney's model in yield form but the Pienaar and Rheney model performed better in projection form. Pienaar and Rheney's sub-model did not perform well in the yield model because this model couldn't adequately capture Type 2 response. Although both of these modeling approaches were able to capture the effects of various silvicultural treatments to some extent, these models were not devised to separate the response to individual treatment and also did not consider the effect of different levels of treatments directly in the growth models.

Modern forest management practices are more intensive and often consider combinations of various silvicultural treatments. With an increasing trend of intensive management practices, there is also a demand for a growth and yield model structure that can accommodate a wide range of treatment options. But the challenge is experiments that encompass all treatments of interest do not exist. Most of the existing long-term data available are for a single treatment. Thus, the inclusion of silvicultural treatment effects is a modeling problem that goes far beyond fitting equations to data (Burkhart and Tomé, 2012). Therefore, we are forced to model response to given treatments separately and integrate these response functions into an overall model structure. While modeling responses to selected silvicultural treatments have been developed to some extent, there has been very little advancement in modeling common silvicultural practices into unified modeling framework. Hence, this study aims to develop an approach for modeling growth response to a wide range of silvicultural

treatments utilizing data from various silvicultural experiments. The primary objectives of this study were 1) to develop a base-line model structure at stand level, 2) to develop general response functions to silvicultural practices- primarily thinning, fertilization, and control of competing vegetation, and 3) to integrate response functions into base-line models.

Each chapter of this dissertation describes separate but integrated parts of the research. Chapter 2 describes the base-line models for basal area, dominant height and stand mortality. In Chapters 3, 4, and 5, base-line growth models were adjusted and refitted with modifier response functions or multipliers accounting for the effects of thinning, fertilization, and competing vegetation control, respectively. Chapter 6 focuses on general discussion and conclusions.

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Chapter 2

General Base-line Model Structure at Stand Level

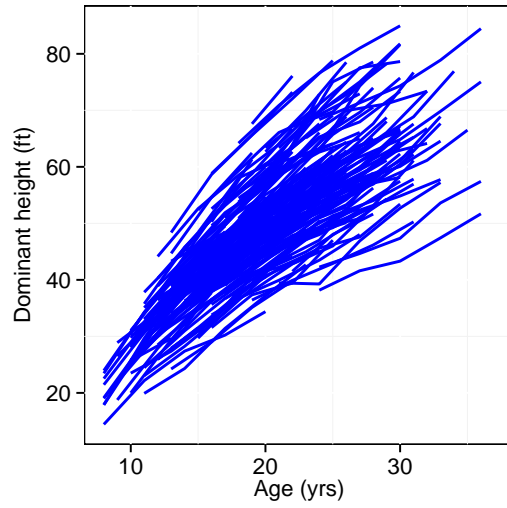
2.1 Introduction

Reliable growth models are essential in forest management planning and decision making. Models that describe and generalize observed stand behavior are developed in this chapter. Intensive cultural practices are common in today's forest management. Therefore, along with general stand growth trends, managers also need to evaluate alternative silvicultural practices before making decisions. Response models with various silvicultural treatments, that can be used in analysis of stand-level management decision problems, are presented in separate chapters. The models developed in this chapter are simple and robust base-line models for loblolly pine that can be easily modified or adjusted to include effects of silvicultural treatments. Base-line models refer to the models developed with data from untreated stands. Unthinned stands with no application of fertilizers or control of competing vegetation are considered to be untreated stands for this study.

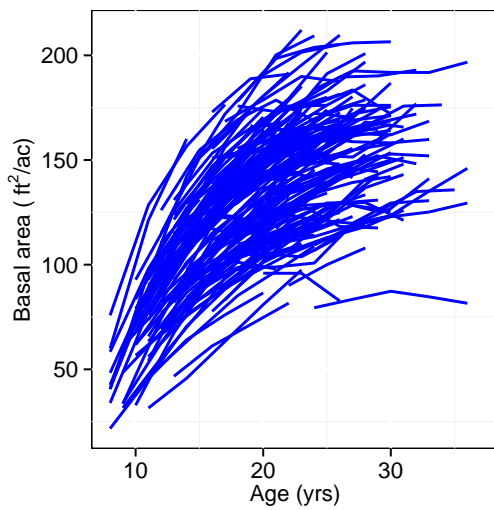
A trio of equations- dominant height, basal area, and number of trees per acre- was developed to describe the general growth dynamics in a stand. Information needed for many forest management decisions can be adequately estimated by a growth model that includes these three core equations (Garcia, 1994).

2.2 Data

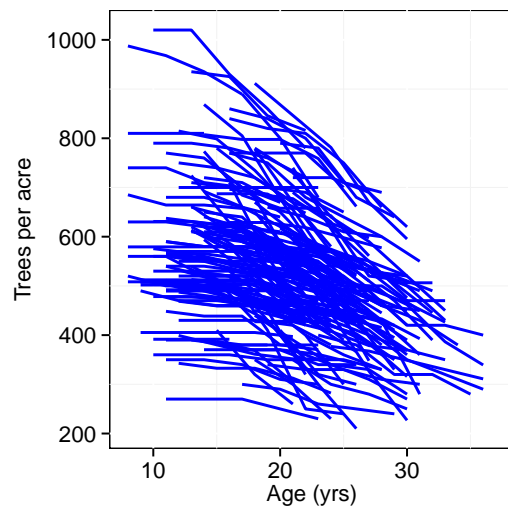
The Forest Modeling and Research Cooperatives (FMRC) installed a region wide thinning study in loblolly pine plantations, distributed throughout the Piedmont and Coastal plain regions of Southeastern United States. In this study, only control- plots data were used to develop the models. Permanent plots in 8-25 years old loblolly pine plantations were established during the dormant seasons of 1980-1982 (Burkhart et al., 1985). This study is characterized by cutover site-prepared areas, nongenetically improved planting stock, and no intermediate nutritional or competition control treatments (Amateis et al., 2006). Plots were established at 186 locations and at each location three comparable plots were established. Each plot was randomly assigned to a treatment category; 1) unthinned control 2) lightly thinned and 3) heavily thinned. All thinnings were from below. The light thinning removed approximately one-third of the basal area while the heavy thinning removed about one-half of the basal area (Burkhart et al., 1985). For each planted pine the following data were measured: dbh, total height, height to the base of live crown, and stem quality. Plots were remeasured at three years intervals for a 21-year period after establishment. Second thinnings were imposed at fourth remeasurement on certain plots. By the end of the study, a total of eight measurements (including initial measurement) had been collected. Due to loss of plots over time, this study utilized data through the fifth remeasurement only. Measurements with unusual growth or mortality were dropped from the study. The final data set for analysis consisted of over 600 consecutive measurement pairs from 186 unthinned control plots. The selected data are plotted in Figure 2.1.



(a) Dominant height



(b) Basal area



(c) Number of trees per acre

Figure 2.1: Measurements from 186 control plots each having atleast three remeasurements forming the data set.

2.3 Model Development and Selection

The models consist of a set of differential equations describing stand growth and survival. Stand growth (dominant height and basal area) models are based on Chapman-Richards function. The Chapman-Richards generalization of von Bertalanffy's growth model (Richards, 1959, Chapman, 1961, von Bertalanffy, 1949 1957) is derived from basic biological considerations and has proven to be very flexible in application. The model is formulated based on the biological concept that the growth rate of an organism or a population can be expressed as the difference between anabolic rate (constructive metabolism) and catabolic rate (destructive metabolism). The anabolic rate is assumed to be proportional to the size of the organism or population, raised to a power, while the catabolic rate is assumed to be directly proportional to size. The relationships can be expressed mathematically as

$$\frac{dY}{dA} = \eta Y^m - \gamma Y \quad (2.1)$$

where Y is size of the organism or population, A is time or, more specifically, age in years in the context of even-aged forest stands, and η, m, γ are parameters.

Integration of equation (2.1), accomplished by employing Bernoulli's equation for integration of differential equations, leads to the growth model

$$Y^{1-m} = \frac{\eta}{\gamma} + constant \times \exp[-\gamma(1-m)A]$$

Solving this equation given the initial condition that at time $A = A_0$, $Y = 0$ and taking the limit $A \rightarrow \infty$ results in

$$Y_{A \rightarrow \infty} \rightarrow \left(\frac{\eta}{\gamma} \right)^{\frac{1}{1-m}}$$

Let's denote this asymptotic value by a , then we have

$$\frac{\eta}{\gamma} = a^{1-m}$$

Substitute $constant = -\frac{\eta}{\gamma}$, when $A = 0$

$$Y^{1-m} = \frac{\eta}{\gamma} - \frac{\eta}{\gamma} \times \exp[-\gamma(1-m)A]$$

And, if $\gamma(1-m) = k$, then

$$Y = a \left(1 - e^{-kA}\right)^{\frac{1}{1-m}} \quad (2.2)$$

Equation (2.2) is sigmoid in shape with upper asymptote a , k is a growth rate related parameter, and m is a shape parameter. This is the Chapman-Richards function that has been frequently used for developing site index curves (Burkhart and Tennent, 1977, Goelz and Burk, 1992, Rennolls, 1995, Amaro et al., 1998, Fang and Bailey, 2001) and modeling other forest stand variables (Pienaar and Turnbull, 1973, Somers and Farrar, 1991).

The Chapman-Richards function is usually presented in integral or differential form; however, when using data from remeasurement plots to project growth over discrete time intervals, such as years, difference or projection forms are more appropriate (Amaro et al., 1998). The projection form of equation (2.2) can be derived by constraining one of the three parameters using initial stand measurements. For example, assuming the ratio of Y in equation (2.2) at two points in time A_1 , and A_2 , is independent of the asymptotic parameter a , gives

$$\frac{Y_1}{Y_2} = \left(\frac{1 - e^{-kA_1}}{1 - e^{-kA_2}} \right)^{\frac{1}{1-m}}$$

which simplifies to

$$Y_2 = Y_1 \left(\frac{1 - e^{-kA_2}}{1 - e^{-kA_1}} \right)^{\frac{1}{1-m}} \quad (2.3)$$

Equation (2.3) is a projection form of the Chapman-Richards growth model with constraining parameter a . Two other difference equations can be derived in a similar manner by constraining parameter k or m , respectively.

The survival model considered in this study is also a differential equation. In general, the best approach for developing a suitable differential equation for modeling survival involves an initial exploration of instantaneous mortality models (Clutter et al., 1983). If the proportional instantaneous mortality rate is assumed to be a constant, that is

$$\frac{1}{N} \frac{dN}{dA} = \alpha \quad (2.4)$$

where N is number of trees per unit area at age A , and $\frac{dN}{dA}$ is instantaneous mortality rate operating at age A , and α is a parameter.

Integration of equation (2.4) with initial condition that $N = N_1$, when $A = A_1$, gives the difference equation model for predicting $N = N_2$ at remeasurement age $A = A_2$

$$N_2 = N_1 e^{\alpha(A_2 - A_1)} \quad (2.5)$$

Equation (2.5) would be an appropriate survival model only for populations where proportional mortality rate is constant at all ages, site indices, and stand densities. However, in most cases involving forest stands, the proportional mortality rate is related to such stand variables. Pienaar and Shiver (1981), Pienaar et al. (1990), and Amateis et al. (1997) have

fitted differential survival functions in which the relative rate of instantaneous mortality is proportional to a power of age. Clutter and Jones (1980) presented a more flexible difference equation assuming that the relative rate of instantaneous mortality is proportional to stand age and density.

$$\frac{1}{N} \frac{dN}{dA} = \alpha A^\delta N^\beta \quad (2.6)$$

Solving equation (2.6) as a difference equation gives

$$N_2 = \left[N_1^{b_1} + b_2 \left(A_2^{b_3} - A_1^{b_3} \right) \right]^{\frac{1}{b_1}} \quad (2.7)$$

Model (2.7) exhibits several desirable properties in survival models mentioned by Clutter et al. (1983), such as, when $A_1 = A_2$, $N_2 = N_1$ and as A_2 approaches infinity N_2 approaches zero. Lemin and Burkhart (1983) found that the difference equation (2.7) performed best among four equations evaluated for predicting mortality after thinning in loblolly pine plantations. Although site index frequently does not contribute significantly after including age and number of trees when predicting mortality, it is occasionally also included (Burkhart and Tomé, 2012). This gives additional flexibility to the model. Including site index (S) as an independent variable in equation (2.6) the relative rate of instantaneous mortality can be expressed as

$$\frac{1}{N} \frac{dN}{dA} = \alpha A^\delta N^\beta S^\gamma \quad (2.8)$$

Integrating (2.8) results in difference equation

$$N_2 = \left[N_1^{b_1} + b_2 S^{b_3} \left(A_2^{b_4} - A_1^{b_4} \right) \right]^{\frac{1}{b_1}} \quad (2.9)$$

The relative rate of mortality can be assumed to be related to combinations of stand age, density, and site index in various functions giving differential equations and then integrating over initial conditions to yield corresponding difference equation models. Zhao et al. (2007) listed over 25 such differential equations, some of which have been used in several studies (Clutter and Jones, 1980, Pienaar and Shiver, 1981, Lemin and Burkhart, 1983, Bailey et al., 1985, Pienaar et al., 1990, Amateis et al., 1997, Dieguéz-Aranda et al., 2006).

Clutter et al. (1983) noted several desirable properties of functions used for growth and yield models: *(i)* representation of growth and yield should be compatible, *(ii)* the functions should be consistent, *(iii)* they should be path invariant, and *(iv)* should possess logical asymptotic limits to stand growth and mortality. The path invariance property is emphasized while selecting growth models to ensure that models do not suffer from compounded errors in the simulation process. Base-line models used here possess most of these features. All growth and mortality models were fitted by least squares estimation method using the PROC MODEL procedure in SAS software (SAS Institute Inc., 2011). The Gauss-Newton or Marquardt iterative method was applied. Each fitted equation was tested against observations, experience with other studies, and/or generally accepted theory. Graphical analyses along with the following fit statistics were used for model evaluation:

Mean residual, \bar{E} :

$$\bar{E} = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)}{n}$$

Root mean square error, $RMSE$:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n - p}}$$

Coefficient of determination for nonlinear model, R^2 :

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y}_i)^2}$$

where y_i , \hat{y}_i , \bar{y} are the observed, predicted, and average value of dependent variables, respectively, n is the total number of observations used to fit the model, and p is number of model parameters.

2.4 Model fitting and Results

2.4.1 Height growth

In the USA, the most commonly used stand height measure for site index determination is the mean height of trees classified as dominant and co-dominant, usually known as dominant height (Sharma et al., 2002). However, the information on classification of trees as dominant and co-dominant may not be always available in given data sets. An alternative definition of dominant height i.e., average height of tallest 80% of trees (if unthinned plots) and tallest 90% of trees (if thinned plots), was used in this study. Percentage of tree heights to include was determined ensuring the heights obtained from this method matches the dominant heights calculated as average height of undamaged trees in the dominant and co-dominant crown classes that remained in the dominant canopy over all plot remeasurements.

The difference equation (2.3) was found to be best among the models obtained from constraining one of the three parameters in the Chapman-Richards equation (2.2) in terms of explaining observed height growth trajectories. The selected model for dominant height growth was

$$domH_2 = domH_1 \left(\frac{1 - e^{-kA_2}}{1 - e^{-kA_1}} \right)^{\frac{1}{b}} \quad (2.10)$$

where $domH_2$ is dominant height of a stand at age A_2 and $domH_1$ is dominant height at age A_1 . All parameters were significant ($\alpha = 0.05$) when equation (2.10) was fitted using data from unthinned plots. Parameters estimates and fit statistics are shown in Table 2.1. Equation (2.10) explained about 97% of total variance, and provided a random pattern of residuals with no strong trends (Figure 2.2(a)). When the equation of Dieguéz-Aranda et al.

Table 2.1: Parameter estimates and fit statistics of base-line equations fitted with only data from unthinned control plots.

Equations	Parameters	Estimates	SE	$P > t $	\bar{E}	RMSE	R^2
Dominant height (2.10)	k	0.0365	0.00402	<0.0001			
	b	0.8997	0.0389	<0.0001	0.0584	1.7174	0.9746
Basal area (2.11)	k	0.1344	0.00494	<0.0001			
	b	0.3324	0.179	<0.0001	0.5160	5.8640	0.9617
Survival (2.9)	b_1	-1.0023	0.1675	<0.0001			
	b_2	2.147×10^{-8}	2.775×10^{-8}	0.4393			
	b_3	1.5661	0.2960	<0.0001			
	b_4	3.3207	0.1883	<0.0001	0.1634	21.1400	0.9722

(2006), which is based on the region-wide thinning study data, was used to predict heights, very similar results were obtained (Figure 2.2(b)) indicating that if there was any weak trend in prediction it must be due to available data not due to model structure. Equation (2.10) also described the data well when compared to observed height growth (Figure 2.3(a)).

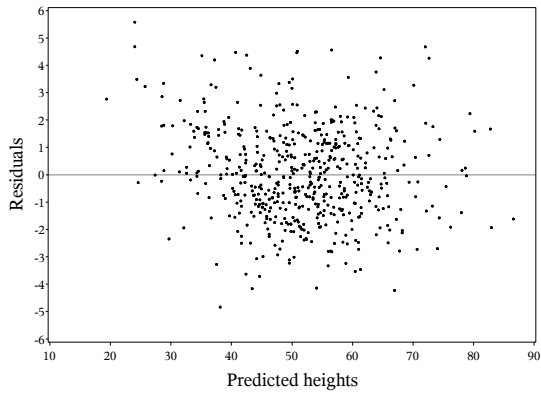
2.4.2 Basal area growth

Similar to dominant height growth, the difference equation (2.3) was found to be appropriate in explaining basal area growth over time. The fitted model is

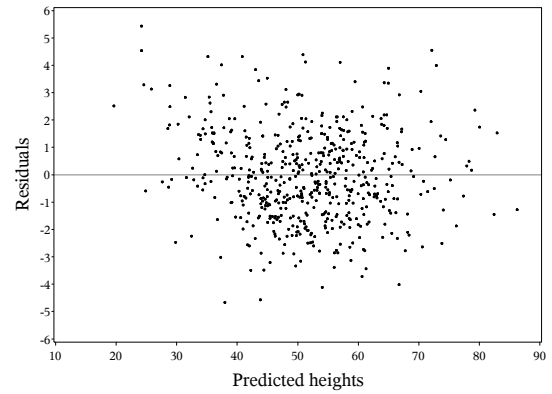
$$B_2 = B_1 \left(\frac{1 - e^{-kA_2}}{1 - e^{-kA_1}} \right)^{\frac{1}{b}} \quad (2.11)$$

where B_2 is basal area at stand age A_2 and B_1 is basal area at age A_1 . Parameter estimates and fit statistics are shown in Table 2.1. All parameters were found to be significant ($\alpha = 0.05$). Equation (2.11) explained about 96% of total variance, and with the exception of the few over predictions for lower basal areas, the equation provided random pattern of residuals with no strong trends (Figure 2.2(c)). Figure 2.3(b) shows fitted curves represent

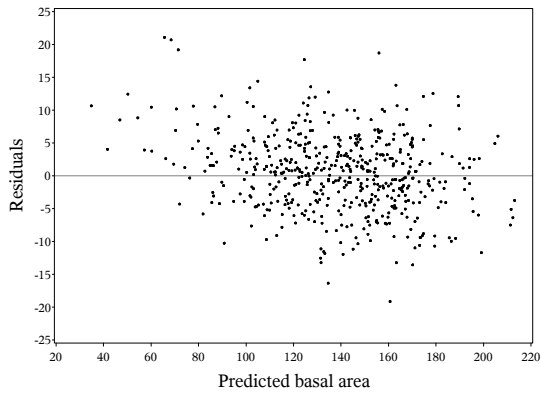
well the range of trajectories of observed basal area growth over time.



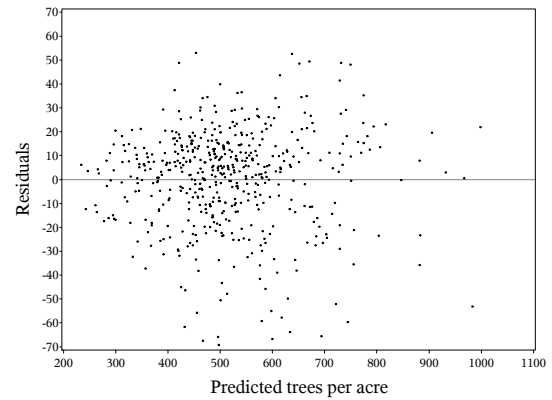
(a) Residuals versus predicted dominant heights (ft) for Equation (2.10)



(b) Residuals versus predicted dominant heights (ft) for Equation Dieguéz-Aranda et al. (2006)



(c) Residuals versus predicted basal area (ft^2/ac)



(d) Residuals versus predicted number of trees per acre

Figure 2.2: Residual versus predicted values for base-line equations for dominant heights (2.10), basal area (2.11), and survival (2.9).

2.4.3 Survival

The difference equation (2.9) was found to be the best among survival models mentioned in Section 2.3 when fitted to data using observations from control plots. The fitted equation

can be expressed as

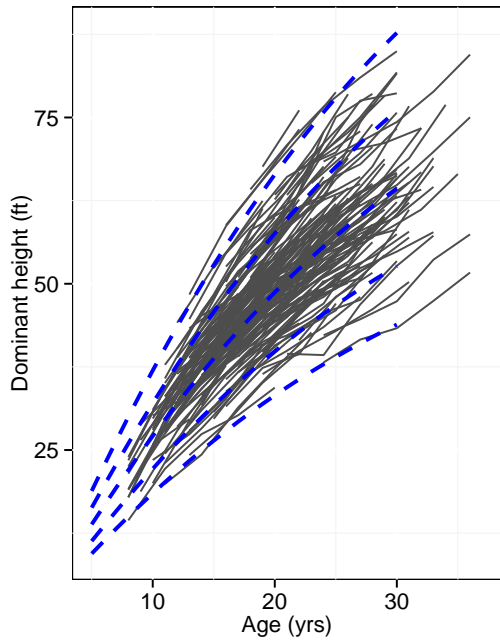
$$N_2 = \left[N_1^{b_1} + b_2 \left(\frac{S}{100} \right)^{b_3} (A_2^{b_4} - A_1^{b_4}) \right]^{\frac{1}{b_1}} \quad (2.12)$$

Parameter estimates along with fit statistics are shown in Table 2.1. About 97% of total variability in survival was explained by the fitted model. Residual plot indicated unbiased prediction with no trend in residuals (Figure 2.2(d)). The estimated value for parameter b_2 was not significant ($\alpha = 0.05$) and close to zero. However, parameter b_2 was kept in the model because it is often found to be more appropriate if age, number of trees, and site index are included when predicting mortality (Zhao et al., 2007, Burkhart and Tomé, 2012). Moreover, without a scalar in front of term $\left(\frac{S}{100} \right)^{b_3}$ in equation (2.12), the function collapses to $N_2 = N_1$, which clearly is not true based on data which shows $N_2 \leq N_1$, and decreasing with age.

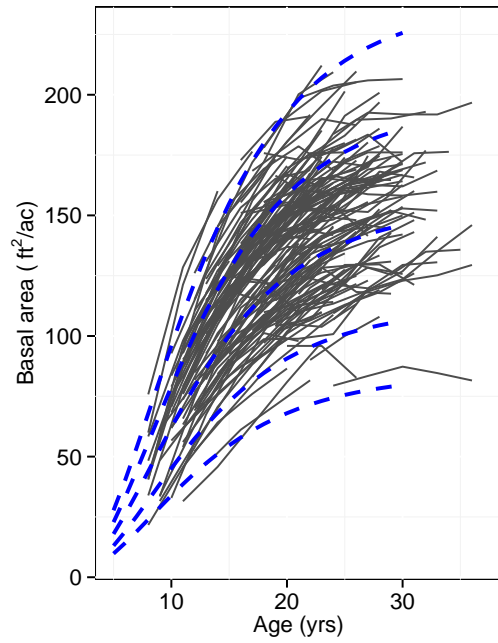
2.5 Discussion and Conclusion

Model selection was accomplished as a compromise between biological and statistical considerations. Selected growth and survival base-line models are expected to represent the general growth and survival pattern in loblolly pine stands. A difference form of the Chapman-Richards function with constraining asymptotic parameter a fitted available data well and gave satisfactory biological interpretation. There was no particular prior reason to constrain parameter a in the height and basal area growth functions. But it gives a simple and easily interpretable difference equation form of the Chapman-Richards function. One of the properties of this particular constraint is that height equals site index at base age. The function was also found to fit best when compared to difference equations obtained by constraining parameter k or m . Selected height and basal area models are flexible enough to cover a wide range of growth patterns in the available data set (Figure 2.3(a) and 2.3(b)). The survival function derived from a differential equation where relative rate of mortality is proportional to a function of age, initial stand density, and site index produced satisfactory results in predicting surviving number of tree in loblolly pine stands. The fitted survival model exhibits slow mortality for the first few years before canopy closure, afterwards the slope of the curve increases rapidly indicating large number of trees dying due to competition and eventually it levels off; this is the expected trend in stand mortality (Figure 2.3(c)).

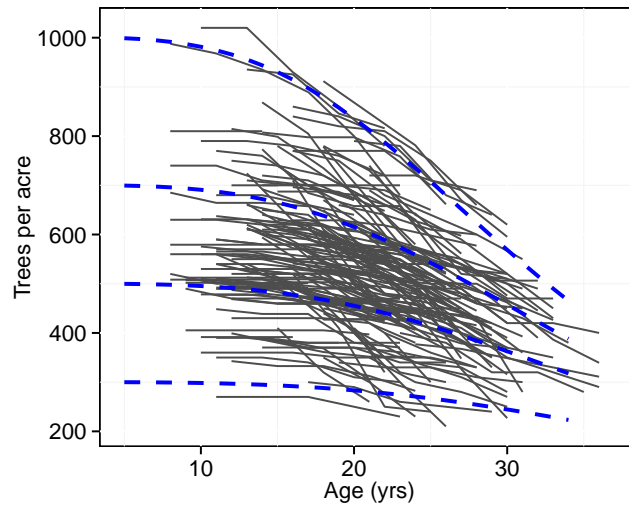
All models were developed based on basic biological concepts, and, therefore, the curves produced by the models mimic the biological processes of growth. These models are expected to provide realistic growth and survival patterns even in the case where the model is extrapolated beyond the range of the data used to fit the models. These base-line equations were adjusted and refitted with modifier response functions accounting for effects of silvicultural treatments as described in Chapters 3, 4, and 5.



(a) Dominant height projected curves



(b) Basal area projected curves



(c) Number of trees per acre projected curves

Figure 2.3: Fitted curves (dotted line) obtained using different initial values overlaid on observed growth and mortality over time. Site index assumed for survival curves was $58.7ft$, which is an average value for control plots in data set.

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Chapter 3

Modeling Response to Thinning

3.1 Introduction

Thinning is an important silvicultural practice generally applied in plantation forestry to improve stand structure. Several studies have shown that thinning produces significant change in growth response (Short and Burkhart, 1992, Zhang et al., 1997, Westfall and Burkhart, 2001, Sharma et al., 2006) Thinning influences the growth by alerting stand density, decreasing potential volume loss to mortality, and distributing volume to trees of greater vigor, size and quality (Bailey and Ware, 1983). In the past, efforts have been made to incorporate the response to thinning in growth and yield model in various forms. Bailey and Ware (1983) used a measure of thinning based on the ratio of the quadratic mean diameter of trees removed in thinning to the quadratic mean diameter of all trees before thinning, which significantly improved the predictive ability of their basal area growth projection model. This index is an indicator for thinning method representing various thinning intensities and remains constant over time. Short and Burkhart (1992) found that thinning, both its intensity and the elapsed time since its occurrence, had a significant effect on crown-height growth and developed a function using the proportion of remaining basal area after thinning to express thinning effect. The function can be expressed as

$$t_{res} = \left(\frac{G_a}{G_b} \right)^{\frac{A_t}{A}} \tag{3.1}$$

where, t_{res} is thinning response, G_a is basal area after thinning, G_b basal area before thinning, A_t is stand age at the time of thinning, and A is stand age.

This thinning response function monotonically decreases over time, producing the highest level of thinning effect right after thinning. Biologically, there should be no immediate response at the time of thinning for any growth function. Keeping in mind that thinning

response should begin at zero and increase to some maximum point before it starts diminishing when stands close a few years after thinning, Liu et al. (1995) developed a thinning response function based on biological considerations and sensitive to thinning intensity, age of stand at the time of thinning, and elapsed time since thinning:

$$t_{res} = \left(\frac{G_a}{G_b} \right) \frac{r[-(A_s - A_t)^2 + k(A_s - A_t)]}{A_s^2} \quad (3.2)$$

where, A_s is stand age, A_t is age of stand at time of thinning, G_a is basal area after thinning, G_b is basal area before thinning, k is a duration parameter for thinning effect. The unit of k is years and r is a rate parameter. The response function was found to be effective in predicting the response to thinning for an increment or an allometric crown size model in loblolly pine trees.

Moore et al. (1994) introduced a relative size-relative growth (RSG) function (3.3) for Douglas-fir that performed well for distributing stand basal area growth to individual trees in response to thinning and fertilization treatments. The statistical test and validation they did on the RSG function indicated that thinning and fertilization do not alter the characteristic relationships between tree size, stand density, stand structure, and the relative distribution of growth across size classes within a stand. Their RSG function can be expressed as

$$\frac{\Delta g}{\Delta G} = b_0 + b_1 \frac{g}{G} + b_2 \frac{g^2}{G^2} \quad (3.3)$$

where, g is tree basal area, G is stand basal area, Δg and ΔG are basal area growth at tree and stand level, respectively.

Westfall and Burkhart (2001) developed separate thinning response variables for core equa-

tions in an individual-tree model, PTAEDA2. Results showed significant improvement in prediction ability for the diameter model when a thinning response variable was included. However, the authors cautioned that improvements in predictive ability shown by given equations do not necessarily equate to a similar level of performance when incorporated into stand simulators. Some growth and yield models operating at tree-level introduce the effect of thinning in a model implicitly. The PTAEDA2 individual-tree growth and yield model is one example where response to thinning is implicitly expressed through changes in each tree's competitive status (Burkhart et al., 1987). Although growth response to thinning has been incorporated into growth and yield models in some studies, there is a lack of simple models that can describe general growth response due to thinning in loblolly pine plantations.

The objectives of this part of study were 1) to develop response functions to thinning treatment at various levels and 2) to integrate the base-line growth model (developed in Chapter 2) and response function for predicting basal area growth, dominant height growth, and surviving number of trees in loblolly pine plantations.

3.2 Data

The region-wide thinning study data were utilized in developing and analyzing a thinning response function. The Forest Modeling Research Cooperatives (FMRC) installed a region-wide thinning study in loblolly pine plantations, distributed throughout the Piedmont and Coastal plain physiographic regions of Southeastern United States. Permanent plots in 8-25 years old loblolly pine plantations were established during the dormant seasons of 1980-1982 (Burkhart et al., 1985). This study is characterized by cutover site-prepared areas, non-genetically improved planting stock, and no intermediate nutritional or competition control treatments (Amateis et al., 2006). Plot installations were established at 186 locations and, at each location, three comparable plots were established. Each plot was randomly assigned to a treatment category; 1) unthinned control 2) lightly thinned and 3) heavily thinned. All thinnings were from below. The light thinning removed approximately one-third of the basal area, while the heavy thinning removed about one-half of the basal area. For each planted pine tree the following data were measured: dbh, total height, height to the base of live crown, and stem quality. Plots were remeasured at three years intervals for a 21-year period after establishment. Second thinnings were imposed at the fourth remeasurement on certain plots. By the end of the study, a total of eight measurements (including initial measurement) had been collected. Due to loss of plots over time, this study utilized data through the fifth remeasurement including only those plots installations (thinned once and unthinned) that had at least four remeasurements. Measurements with very inconsistent data or with substantial environmental damage were excluded from analysis. The final data set for analysis consisted over 1900 consecutive measurement pairs from thinned and unthinned plots. Summary statistics of data are given in Table 3.1 and the extent of observed data are shown in Figure 3.1. The data cover a wide range of stand densities, tree ages, and site qualities.

Table 3.1: Summary statistics at time of establishment by treatments for the loblolly pine thinning study data.

Variables	Control			Light thin			Heavy thin		
	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max
Age (years)	15.2	8	25						
Site index ¹ (<i>ft</i> , base 25)	59.0	35.4	84.5						
Trees/ <i>ac</i>	570.2	270	1020	341.8	191.1	632.6	259.9	129.5	478.7
Basal area (<i>ft</i> ² / <i>ac</i>)	106.2	21.6	190.5	75.1	18.0	155.4	60.6	12.8	117.4
Dominant height ² (<i>ft</i>)	39.7	14.4	68.3	40.5	15.3	70.5	40.9	15.5	68.2

¹Site index was calculated using equation given by Dieguéz-Aranda et al. (2006).

²Dominant height is an average height of 80% tallest trees in a plot, if unthinned and 90% tallest trees, if thinned.

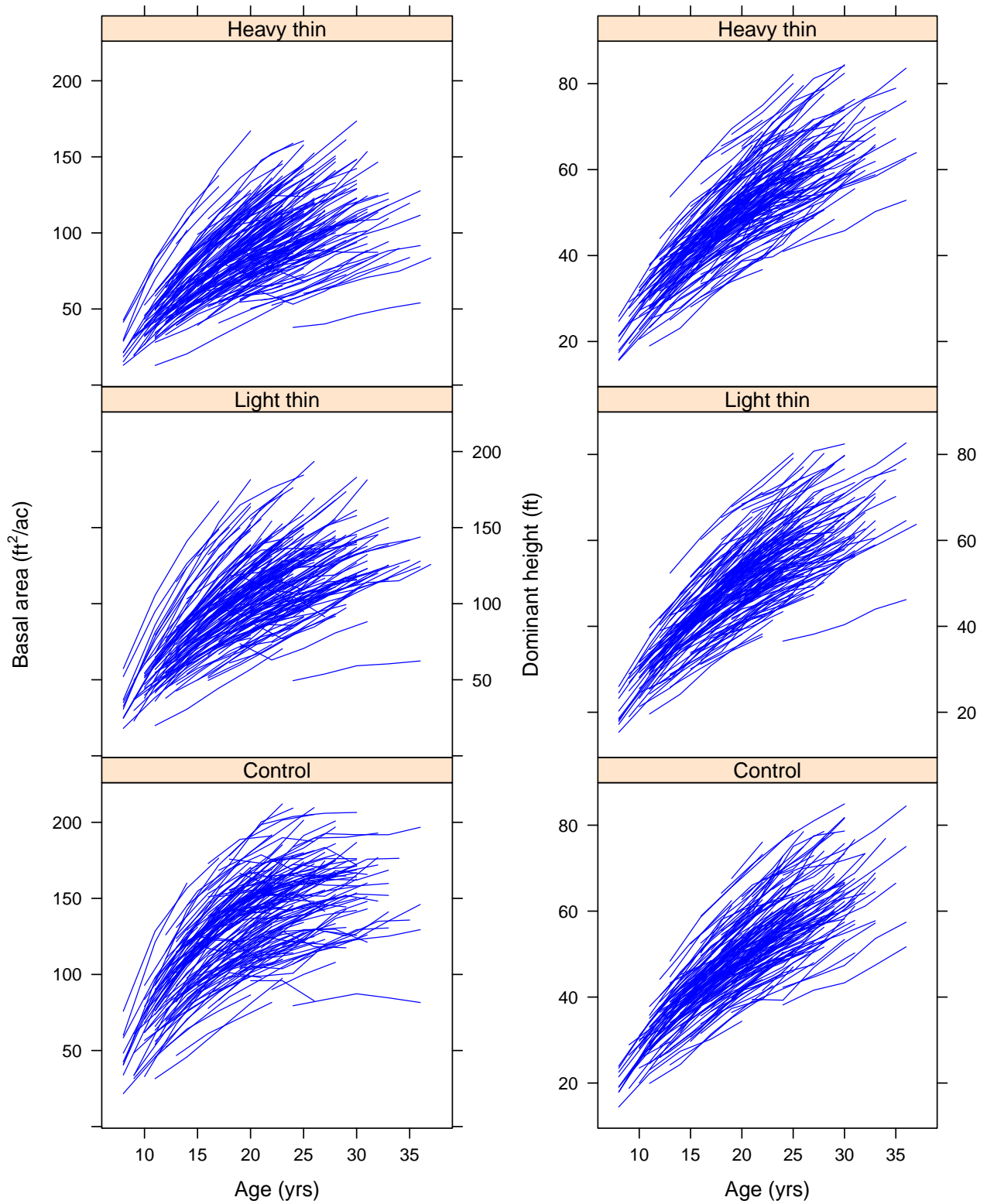


Figure 3.1: Observed basal area growth and dominant height growth over time in control, lightly thinned, and heavily thinned plots for the region-wide thinning study data.

3.3 Model Development and Methods

The approach adopted in modeling stand growth following silvicultural treatments was to incorporate effects of treatments as a response function in base-line models. Thus, the base-line models were first developed using data from untreated stands (Chapter 2) and response functions for each silvicultural treatment were developed utilizing representative data sets. Response functions were then coupled with base-line models. The following model structure with a response function multiplier was fitted and tested using data from both treated and untreated stands:

$$\hat{y} = f_{base} \times f_{res} \quad (3.4)$$

where \hat{y} is estimated growth, f_{base} is base-line growth function, and f_{res} is treatment response function.

In the model structure (3.4), the treatment response is represented explicitly by the second term denoted by f_{res} . Response functions were developed in such a way that when no silvicultural treatments are applied the f_{res} term equals one, and estimated growth is provided by the first term, f_{base} , which represents the base-line for a given site. It is assumed that the effect of each treatment response interacts multiplicatively with the base-line. Model structure (3.4) was analyzed for each treatment separately using selected data sets for corresponding treatment. This chapter particularly develops response functions to thinning treatment.

3.3.1 Thinning response function

The thinning response function developed here was based on the function developed by Liu et al. (1995). The function was modified and adjusted to correctly describe the change in loblolly pine stand growth (basal area and dominant height) due to thinning. The various factors such as age of stand at the time of thinning, the time elapsed since thinning, and intensity of thinning were considered as predictor variables while developing the response function. The developed response function for thinning treatment effect, f_{thin} , can be expressed as:

$$f_{thin} = \begin{cases} 1 & (A - A_t) > k \\ \left(\frac{1}{B_r}\right) \frac{r[-(A-A_t)^2+k(A-A_t)]}{A^2} & 0 < (A - A_t) \leq k \end{cases} \quad (3.5)$$

where, A is stand age, A_t is age of stand at time of thinning, $A - A_t$ thus, represents time elapsed since thinning, B_r is ratio of basal area after thinning to basal area before thinning, k is duration parameter for thinning effect. The unit of k is years and r is rate parameter.

The function (3.5) is biologically consistent and has the following properties: response to thinning starts at zero ($f_{thin} = 1$) at the time of thinning and gradually increases, reaches a maximum before it starts diminishing, and eventually thinning effect becomes zero again. The duration of thinning response (in yrs) is determined by the value of the duration parameter, k . It can be easily shown that the maximum response to thinning occurs at

$$A - A_t = \frac{kA_t}{k + 2A_t} \quad (3.6)$$

years after thinning. Thus, age of maximum response depends on age of the stand at the

time of thinning, A_t . In equation (3.5), thinning response also depends on thinning intensity represented by the ratio of basal area after to before thinning, B_r . According to this function, response increases with increasing thinning intensity.

3.3.2 Basal area growth model

The base-line growth model for basal area (2.11) developed in Chapter 2, when coupled with thinning response function (3.5), gives the basal area growth model. Therefore, the model structure (3.4) for basal area can be expressed as:

$$B_2 = B_1 \underbrace{\left(\frac{1 - e^{-aA_2}}{1 - e^{-aA_1}} \right)^{\frac{1}{b}}}_{f_{base}} \times \underbrace{\left(\frac{1}{B_r} \right)^{\frac{r[-(A_2 - A_t)^2 + k(A_2 - A_t)]}{A_2^2}}}_{f_{thin}} \quad (3.7)$$

for $A_2 - A_t \leq k$, where B_2 is basal area at stand age A_2 , B_1 is basal area at age A_1 , and all other variables and parameters are as defined previously.

3.3.3 Height growth model

Dominant height growth was also modeled using a model structure (3.4) similar to that of the basal area growth model. The standard growth or base-line model was based on Chapman-Richard growth function developed in Chapter 2. The thinning response function applied was similar to that used in the basal area growth model. The developed height growth model can be expressed as:

$$domH_2 = \underbrace{domH_1 \left(\frac{1 - e^{-aA_2}}{1 - e^{-aA_1}} \right)^{\frac{1}{b}}}_{f_{base}} \times \underbrace{\left(\frac{1}{B_r} \right)^{\frac{r[-(A_2 - A_t)^2 + k(A_2 - A_t)]}{A_2^2}}}_{f_{thin}} \quad (3.8)$$

for $A_2 - A_t \leq k$, where $domH_2$ is dominant height at stand age A_2 , $domH_1$ is dominant height at age A_1 , and all other variables and parameters are as defined previously.

3.3.4 Survival model

The base-line survival function developed in Chapter 2 was modified to adjust for thinning treatment. Thinning intensity was added to model (2.9) as an independent variable to represent levels of thinning. The survival function adjusted for thinning treatment can be expressed as:

$$N_2 = \left[N_1^{b_1} + b_2 B_r \left(\frac{S}{100} \right)^{b_3} (A_2^{b_4} - A_1^{b_4}) \right]^{\frac{1}{b_1}} \quad (3.9)$$

where N_2 is number of trees per acre at stand age A_2 , N_1 is number of trees per acre at age A_1 , B_r is ratio of basal area after thinning to before thinning representing intensity of thinning, S is site index, and b_1, b_2, b_3, b_4 are parameters.

3.3.5 Model fitting, evaluation and validation

Nonlinear least squares fits of all equations were accomplished by using the PROC MODEL procedure on SAS/ETS software (SAS Institute Inc., 2011). To ensure the solution is global, rather than local minimum, different initial values of parameters were provided for the fits.

Fitted equations were tested against observations. Graphical analysis along with the following statistics were used in evaluating and validating models:

Mean residual, \bar{E} :

$$\bar{E} = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)}{n}$$

Root mean square error, $RMSE$:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n - p}}$$

Coefficient of determination for nonlinear model, R^2 :

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

Mean PRESS prediction error, \bar{E}_p :

$$\bar{E}_p = \frac{\sum_{i=1}^n (y_i - \hat{y}_{i,-i_s})}{n}$$

where y_i , \hat{y}_i , \bar{y} are the observed, predicted, and average value of dependent variables, respectively, $\hat{y}_{i,-i_s}$ is predicted value for i^{th} observation, where second subscript $-i_s$ indicates that i_s observations (the set of remeasured observations for plot i) were omitted when the

regression function was fitted, n is the total number of observations, and p is number of model parameters.

Since independent data were not available, a leave-one-out cross-validation approach (PRESS measure) was used to evaluate the model's predictive ability. Because the data used in this study were from repeated measures, rather than deleting a single observation, an entire set of remeasured observations in a plot was left out one at a time during the estimation procedure. Estimated parameters were subsequently applied to predict the stand growth in left out plot. The mean PRESS prediction error was then calculated using the foregoing expression. The leave-one-out cross-validation technique can be considered as a generalization of data splitting and it has been used in forestry to evaluate model prediction performance when no independent data were available for validation or available data were not sufficient for data splitting (Huang et al., 2003, Timilsina and Staudhammer, 2013).

3.4 Results and Discussion

3.4.1 Basal area growth

Results from fitting basal area growth model (3.7) on region-wide thinning study data are presented in Table 3.2. Parameter estimates, standard errors (Std. Err.), P -values, as well as model's mean residual (\bar{E}), root mean squares error ($RMSE$), and R^2 are shown in Table 3.2. All parameters were significant ($\alpha = 0.05$). Model fitting was done on annualized data. All parameters became significant only when all measurements were converted into one year interval periods. Since the base-line models used here are time invariant, this should not be a problem even when the projection interval used is greater than one year.

Table 3.2: Parameter estimates and fit statistics of basal area growth model (3.7) fitted to the thinning study data.

Parameters	Estimates	Std. Err.	$P > t$
a	0.1488	0.00381	<0.0001
b	0.2978	0.0113	<0.0001
r	0.1190	0.0427	0.0053
k	30.8038	8.1529	0.0002
\bar{E}	0.2728		
$RMSE$	1.9652		
R^2	0.9966		

The positive sign of the thinning response rate parameter, r , in model (3.7) indicates that thinned stands had a greater basal area growth rate than unthinned stands. The response also increased with thinning intensity. The thinning response duration parameter, k , was positive in sign indicating that basal area growth response to thinning increased after thinning to some level and then gradually decreased. In addition to intensity of thinning, magnitude of response to thinning also varied with age of stand at the time of thinning. Response was

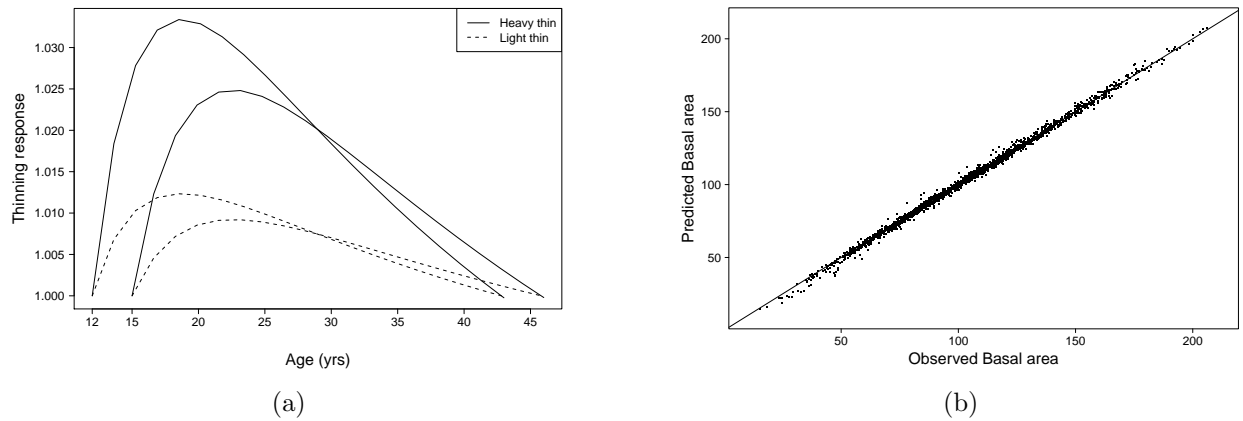


Figure 3.2: (a) Thinning response function plotted against stand age for light and heavily thinned plots at assumed stand age of 12 and 15 years at the time of thinning. (b) Predicted basal area plotted against observed basal area.

greater in younger stands. Figure 3.2(a) demonstrates the behavior of the thinning response function over time for four hypothetical stands. Two were thinned at age 12 years with basal area ratios of 0.5 (heavy thinning) and 0.66 (light thinning). The other two stands were thinned at age 15 years with the same thinning intensities. Response was greater in younger stands and heavier thinning produced greater responses. There was no thinning response at the time of thinning, the response function gradually increased over time and once maximum response was achieved, it started diminishing. This function is conditioned to equal one when time lapsed since thinning exceeds duration parameter, k , otherwise it is always greater than one for thinned stands.

Fit statistics in Table 3.2 indicate the model describes the data well. This is also evident in Figure 3.2(b), which shows that the model was unbiased and it described well the basal area growth following thinning treatment.

To further evaluate model (3.7), basal area growth curves were predicted with estimated coefficients (Table 3.2) for stands with initial basal area of $45 ft^2/ac$ (\approx average observed value) at age 8 years. Curves were generated for unthinned stands and for stands that were

thinned at age 15 years with light and heavy thinning intensities (Figure 3.3(a)). Cumulative basal areas in thinned stands were also computed (Figure 3.3(b)). As expected, different growth patterns were observed between thinned and unthinned stands. Thinned stands were growing faster than unthinned stands. Basal area in heavily thinned stands tended to grow at a greater rate than in lightly thinned stands. Cumulative basal area of total production to age 30 years exceeded its unthinned counterpart only when the stand was thinned heavily. Figures B.1 in Appendix A further compare the generated curves for all levels of thinning with observed basal area growth curves. Close agreement between projected growth and observed growth of basal area for all levels of thinning suggested that the models describe the data well.

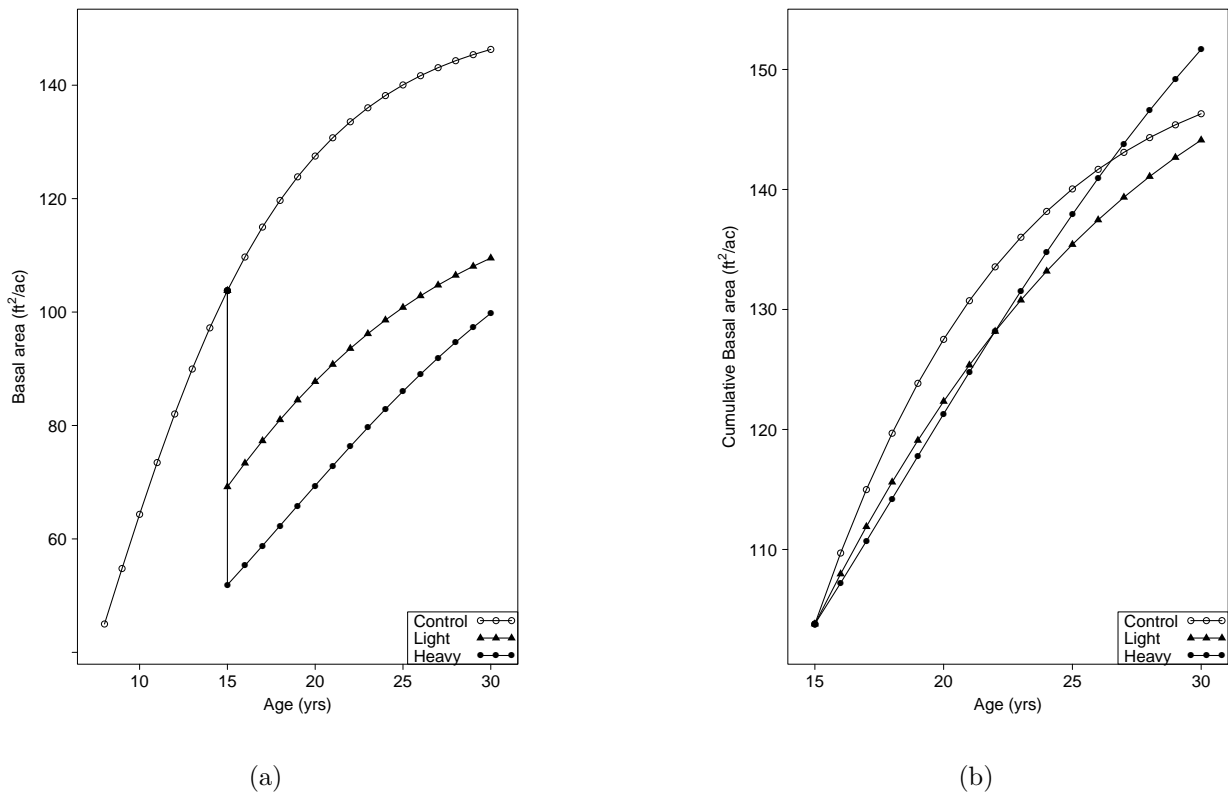


Figure 3.3: (a) Comparison of development of basal area generated by Model (3.7) for unthinned and thinned stands with an initial stocking of $45 \text{ ft}^2/\text{ac}$ at age 8 years. (b) Comparison of cumulative basal area growth for thinned stands with unthinned counterpart.

Several researchers in the past have also reported responses to thinning on basal area growth similar to the results described here. Pienaar and Shiver (1986) observed that magnitude of the difference in basal area growth between thinned and unthinned stands depends on the age when the thinning occurred and the intensity of thinning. Hasenauer et al. (1997) and Bailey and Ware (1983) also found similar responses to thinning treatments and developed basal area projection equation with an adjustment factor representing timing and intensity of thinning. Both studies found significant improvement in basal area projection when an adjustment factor was included in the equation.

Although, basal area projection model (3.7) indicated basal areas of thinned stands grew faster, it is unlikely that basal areas of thinned stands will exceed those of unthinned counterparts within normal rotation periods.

3.4.2 Dominant height growth

Parameter estimates for the dominant height growth model (3.8) are given in Table 3.3. Estimated value of duration parameter, k , in the response function was very short and the value of rate parameter, r , had a negative sign.

Table 3.3: Parameter estimates of dominant height growth model (3.8) fitted to the thinning study data.

Parameters	Estimates	Std. Err.	$P > t$
r	-0.1591	0.0240	<0.0001
k	5.5642	0.6000	<0.0001
a	0.0516	0.00261	<0.0001
b	0.8338	0.0226	<0.0001

Results indicated a brief negative impact on dominant height growth due to thinning; however, the impact was relatively small and can be safely ignored. Several other studies in the past have also reported negative thinning impacts on the stand dominant height growth

(Ginn et al., 1991, Peterson et al., 1997). Sharma et al. (2006) found reduced but negligible height growth of loblolly pine stands for the initial few years following thinning. They used the same thinning study data that was used for the present study to evaluate thinning impact on height growth of dominant and codominant loblolly pine trees.

3.4.3 Survival model

Parameter estimates and fit statistics for the survival model (3.9) are given in Table 3.4. All parameters were significant except b_2 ($\alpha = 0.05$). Although not significant parameter b_2 was kept in the model for the same reason described in Chapter 2 Section 2.4.3. Excluding few lower density observations, residual plot indicated the model was unbiased in overall prediction (Figure 3.4(a)). Residual plots and fitted survival curves were also plotted by treatment levels including control plots. The model was unbiased for all treatment levels (Figure C.1). Mortality rate was reduced with increasing intensity of thinning (Figure 3.4(b) and 3.4(c)). Higher site index resulted in increased mortality rate.

Table 3.4: Parameter estimates of survival model (3.9) fitted to the thinning study data.

Parameters	Estimates	Std. Err.	$P > t$
b_1	-1.2680	0.0907	<0.0001
b_2	5.144×10^{-9}	3.862×10^{-9}	0.1831
b_3	1.9926	0.1960	<0.0001
b_4	3.3910	0.1215	<0.0001
\bar{E}	0.140		
<i>RMSE</i>	15.04		
<i>R</i>²	0.988		

One of the possible reason for a decrease in mortality rate in thinned plot is initial reduction in stand density that leads to a decrease in competition among trees. Moreover, in this study, thinnings were mainly from below and because the suppressed trees were most likely to die were already taken out, the thinned plots resulted in reduced mortality compared

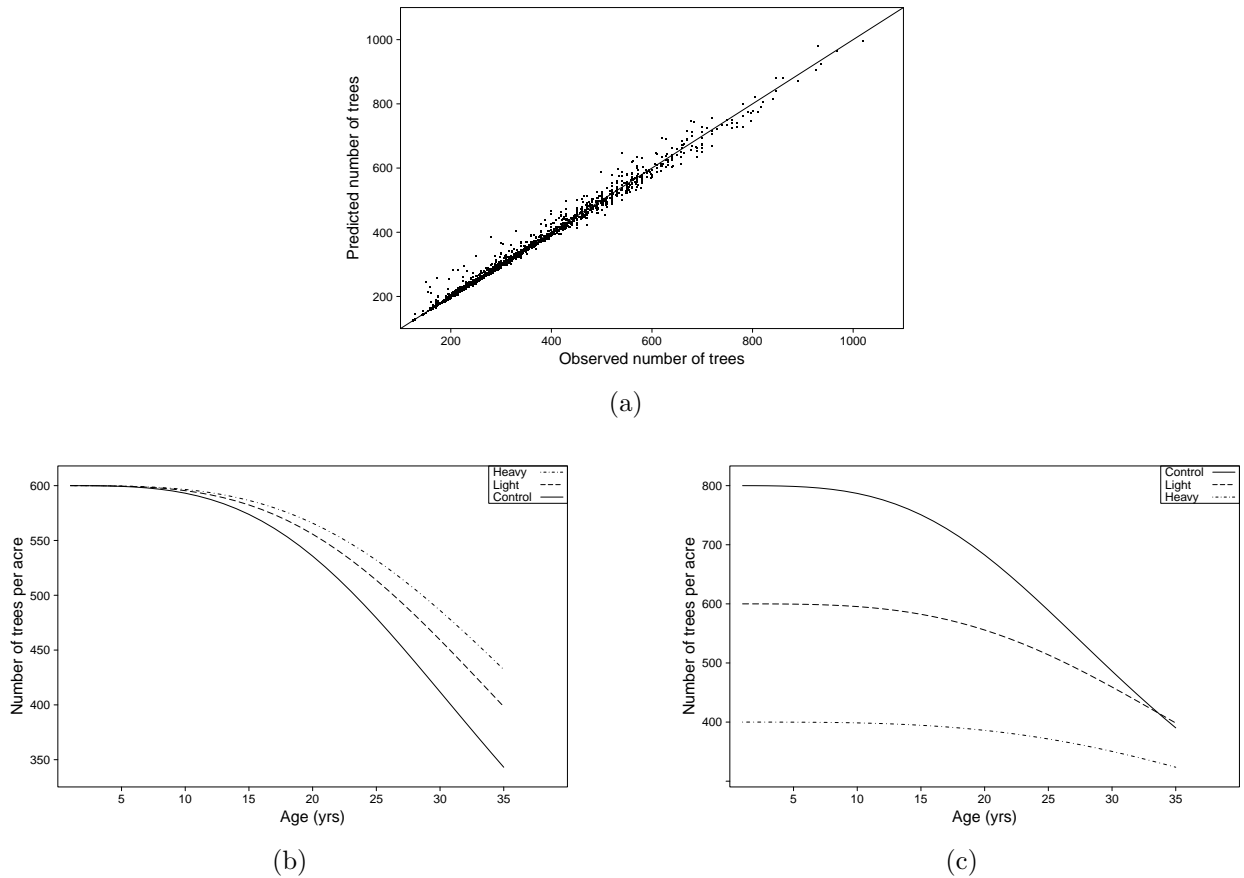


Figure 3.4: (a) Predicted number of trees plotted against observed number of trees per acre. (b) Fitted survival function plotted against stand age for control, light, and heavily thinned plots with assumed initial density 600 trees per acre. (c) Fitted survival function plotted against stand age for control, light, and heavily thinned plots with initial density 800, 600, and 400 trees per acre, respectively. Site index assumed for survival curves was $59 ft$, which is an average value in observed data set

to the unthinned plots. Amateis (2000) reported a similar effect of thinning on survival of loblolly pine trees. He found thinning reduced mortality rates allowing number of trees in the unthinned plots to converge toward number of trees in the thinned plots.

3.4.4 Model validation

Along with graphical evaluation, models were also cross-validated using the leave-one-out technique. Mean PRESS prediction errors (\bar{E}_p) were calculated for the basal area growth

model and survival model (Table 3.5). Both models yielded small mean predictive errors. The percentage errors relative to the observed mean were less than 0.3% for both models. The validation results suggested that the models (3.7) and (3.9) had adequate predictive ability.

Table 3.5: The mean PRESS prediction error (\bar{E}_P), standard deviation of prediction error (SE_P), and percentage error relative to observed mean of the basal area growth and survival models for thinning treatment.

Models	\bar{E}_P	SE_P	%Error
Model (3.7)	0.27	1.95	0.26
Model (3.9)	-0.99	16.22	0.27

3.5 Conclusion

A general response model for projecting stand basal area for various levels of thinning treatments was developed. The effects of thinning were incorporated into the model as a multiplier function. The thinning response function is sensitive to stand age at the time of thinning, time since thinning, and thinning intensity. The response function presented here was developed using data from a wide variety of growth sites for loblolly pine across the Southern United States, and therefore, is believed to represent general growth trends in response to thinning. In addition to observed data, the function was also based on biological considerations. Because of the model structure (3.4), this response function can be easily integrated with base-line growth models other than the one used in this study. The response function can be viewed as a general response function.

A Response function integrated with base-line model was fitted and tested to data from the region-wide thinning study. Analyses suggested that the model described well the effect of thinning on stand growth and was unbiased in projecting basal area for these loblolly pine stands. The base-line survival model was adjusted for thinning effects by including an additional independent variable representing thinning intensity. The adjusted survival model (Equation 3.9) fitted data well and it was unbiased in prediction at all levels of thinning treatments. No adjustment was required for dominant height growth model in response to thinning treatment.

Data used in these analyses came from repeated measurements of permanent plots and are likely to have correlated observations. Since the main interest of the present study was prediction rather than testing of hypotheses, the auto-correlation issue should not be a problem. Prediction capability of models developed in this study was not influenced by the applied estimation method. Ordinary least square estimation with correlated data might

give biased estimates of parameter variance but parameter estimates are unbiased.

Model (3.7), developed here considered age of a stand, time since application of treatments, and intensity of treatment as predictor variables. There are many other factors that could potentially affect the variability in response to thinning. Initial stand and site conditions and other climatic and geographic factors can affect response to these treatments (Amateis, 2000, Pretzsch, 2005). Type of thinning applied also alters the response in stand growth. All these factors were not considered directly in this study. Nevertheless, based on results from model fitting and evaluation, the basal area growth model presented here should provide satisfactory stand average response to a wide range of stand and site conditions with various levels of thinning.

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Chapter 4

Modeling Response to Fertilization

4.1 Introduction

Forest fertilization is an important silvicultural tool for increasing stand productivity. Fertilization, mainly Nitrogen (N) and Phosphorus (P), has been reported to produce significant growth responses in mid-rotation loblolly pine plantations (Hynynen et al., 1998, Amateis et al., 2000, Bataineh et al., 2006, Fox et al., 2007, Antony et al., 2009, Jokela et al., 2010). Researchers have incorporated fertilization response in growth and yield models in many ways such as modifying site index (Daniels and Burkhart, 1975), developing new models to include fertilizer response directly (Bailey et al., 1989, Amateis et al., 2000), and using multipliers or additional terms to scale existing models (Pienaar and Rheney, 1995, Hynynen et al., 1998).

The site index adjustment approach involves adjusting the height growth equation to reflect effect of fertilizer. Early work, such as Daniels and Burkhart (1975), used this approach to model the effect of fertilization on loblolly pine plantations. Bailey et al. (1989) developed a stand structure and yield prediction model for fertilized mid-rotation slash pine stands by including the fertilizer rates directly into their growth and yield models. Their equations included N and P fertilization rates and indicator variables accounted for different soil types. Amateis et al. (2000) used the data from a fertilizer response study in loblolly pine plantations to develop response models for dominant height and basal area following mid-rotation N and P fertilization. They constructed a nonlinear regression model that can predict total cumulative response as a function of interaction of N and P application rates, drainage class of the site, stand conditions and time since fertilization. Hynynen et al. (1998) constructed multiplicative diameter and height growth models for individual trees following fertilization using data from mid-rotation loblolly pine plantations in Southeastern United States. Their method predicts tree growth in fertilized stands from a growth function of trees without fertilization (a reference growth model) adjusted by a multiplier function predicting the

relative growth response following fertilization. In another study, Carlson et al. (2008) used age-shifts approach to model mid-rotation fertilizer response. Age-shifts are defined as the time reduction needed to reach a specific height, diameter or volume when observed growth exceeds that of control trees. Their age-shift prediction models for dominant height and basal area were functions of the rate and combination of fertilizers applied, as well as stand density and age at fertilization. These age-shift relationships were incorporated in growth and yield models.

The purpose of this part of the overall study was to develop growth models incorporating response to fertilization treatment. Unlike most of the past studies, this investigation emphasized modeling general response to treatment effects at stand-level. The objectives of this study were 1) to develop response functions to fertilization treatment and 2) to integrate the base-line growth model (developed in Chapter 2) and response functions for predicting basal area and dominant height growth in loblolly pine plantations following mid-rotation fertilization.

4.2 Data

The region-wide 13 (RW13) data were used to develop and analyze the effect of application of fertilizer in loblolly pine stands. The Forest Productivity Cooperative's (FNC) region-wide 13 study was established in loblolly pine plantations averaging 13 years of age during the early spring 1984-1987 across the Southern United States (Amateis et al., 2000). At each study location four levels of nitrogen (0, 100, 200, 300 *lb/ac*) and three levels of phosphorus (0, 25, 50 *lb/ac*) were applied. Two or four replicates were established at each installation. Tree dbh and total height of all trees in plots were measured prior to fertilization and again at 2, 4, 6, and 10 years following treatment. Measurements with very inconsistent data or with substantial environmental damage were excluded from analysis. The final data set used for analysis consisted over 4800 consecutive measurement pairs. Table 4.1 gives the summary statistics of all data and the extent of observed data for basal area and dominant height growth for selected treatments can be seen in Figure 4.1.

Table 4.1: Summary statistics at time of study installation and fertilization for loblolly pine RW13 data.

Variables	Mean	Minimum	Maximum
Age (years)	13.3	9	19
Site index	62.5	45.0	82.0
Trees/<i>ac</i>	464.2	191.0	1043.5
Basal area (<i>ft</i>²/<i>ac</i>)	90.0	42.0	138.6
Dominant height (<i>ft</i>)	38.4	22.4	59.2

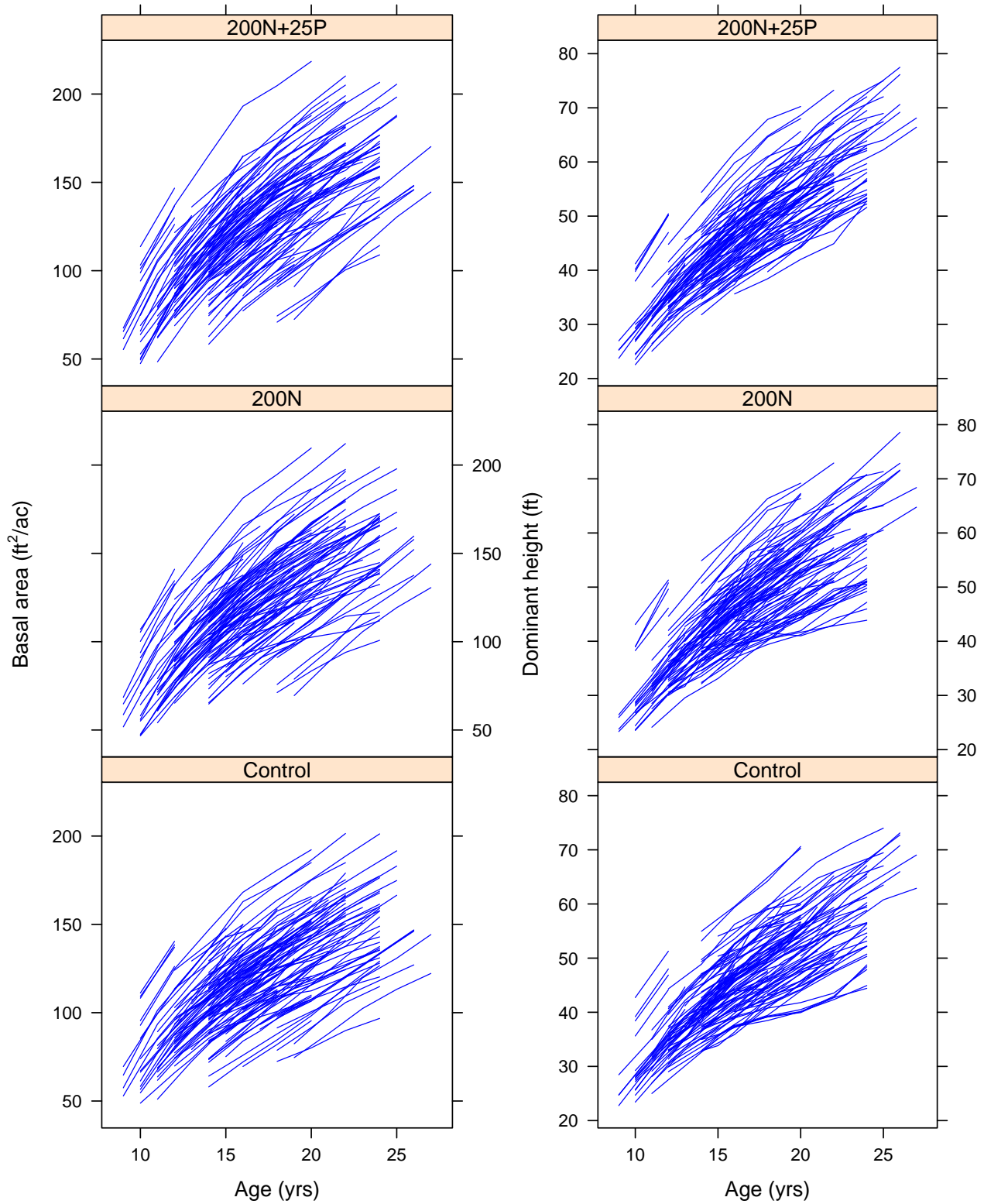


Figure 4.1: Observed basal area growth and dominant height growth over time on unfertilized (Control) plots and fertilized plots with varying rates of N and P for RW13 data.

4.3 Model Development and Methods

A similar approach to that described in Chapter 3 for thinning treatments was followed for modeling response to fertilization applications. The base-line models were first developed using data from untreated stands (Chapter 2) and response functions for fertilization treatment were developed for basal area and dominant height growth. Response functions were then coupled with base-line models. The following model structure with a response function multiplier was fitted and tested using data from both treated and untreated stands:

$$\hat{y} = f_{base} \times f_{res} \quad (4.1)$$

where \hat{y} is estimated growth, f_{base} is base-line growth function, and f_{res} is treatment response function.

In this model structure (4.1), the treatment response is represented explicitly by the second term denoted by f_{res} . Response functions were developed in such a way that when no additional silvicultural treatments are applied the f_{res} term equals one, and estimated growth is provided by the first term, f_{base} , which represents the base-line for a given site. It is assumed that the effect of each treatment response interacts multiplicatively with base-line. Model structure (4.1) was analyzed for basal area and dominant height growth using RW13 data.

4.3.1 Fertilization response function

Growth response following fertilization was modeled using a two-parameter Weibull distribution. The probability density function for two-parameter Weibull distribution for random

variable y_{st} (year since treatment) can be expressed as:

$$f(y_{st}) = \begin{cases} \frac{\gamma}{\beta} \left(\frac{y_{st}}{\beta}\right)^{\gamma-1} e^{-\left(\frac{y_{st}}{\beta}\right)^\gamma} & y_{st} \geq 0, \beta > 0, \gamma > 0 \\ 0 & otherwise \end{cases} \quad (4.2)$$

The Weibull function is very flexible. It can assume a variety of forms described by scale parameter β and the shape parameter γ (Figure 4.2). If $\gamma < 1$, the curve is reversed J shape. For $\gamma = 1$, the density function becomes exponential distribution (Equation 4.3). For $\gamma > 1$, the density function is mound shaped and positively skewed.

$$f(y_{st}) = \begin{cases} \frac{1}{\beta} e^{-\left(\frac{y_{st}}{\beta}\right)} & y_{st} \geq 0, \beta > 0 \\ 0 & otherwise \end{cases} \quad (4.3)$$

In past studies, the response curve due to fertilization has typically been found to follow a form of Weibull or exponential trend (NCSFNC, 1992, Hynynen et al., 1998). The magnitude of growth response also varies according to the amounts and the fertilizer elements applied (Hynynen et al., 1998). Therefore, the Weibull based fertilization response function with adjustments for fertilizer elements and their doses was developed in this study.

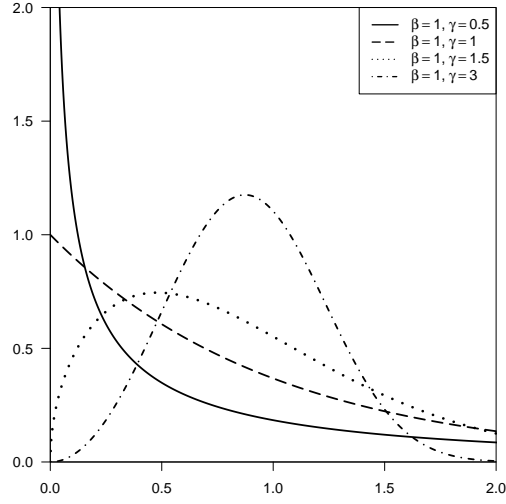


Figure 4.2: Two-parameter Weibull probability density function with different values of shape parameter, γ .

4.3.2 Basal area growth model

The basal area growth model is similar in structure (4.1) as that used in modeling response to thinning treatments. A base-line growth model with a fertilization response curve was fitted to predict basal area growth following fertilizer applications. Data analyses suggested the function given below gave satisfactory results:

$$B_2 = B_1 \underbrace{\left(\frac{1 - e^{-aA_2}}{1 - e^{-aA_1}} \right)^{\frac{1}{b}}}_{f_{base}} \times \underbrace{\left(1 + [(\delta_0 + \delta_1 P)N] \frac{1}{\beta} e^{-\left(\frac{y_{st}}{\beta}\right)} \right)}_{f_{fert}} \quad (4.4)$$

where B_2 is basal area at stand age A_2 , B_1 is basal area at age A_1 , N is nitrogen doses (lb/ac), P is a categorical variable, $P = 1$ if fertilized with phosphorus, otherwise $P = 0$,

and $a, b, \delta_0, \delta_1, \beta$ are parameters.

The f_{fert} term in equation (4.4) that represents response to fertilization is a modified exponential density function, where $\beta > 0$ is a scale parameter. The magnitude of response depends on years since treatment, y_{st} , and presence of fertilizers. The dose of N affects the response curve, however application of P has been included as a categorical variable. Therefore, the response to N also depends on whether or not P is applied. Studies have shown that increasing phosphorus dosage does not increase growth (Hynynen et al., 1998, Carlson et al., 2008). The developed response function follows a decreasing exponential trend.

4.3.3 Height growth model

Similar to the basal area growth model, height growth was also modeled using model structure (4.1). The developed model can be expressed as

$$domH_2 = \underbrace{domH_1 \left(\frac{1 - e^{-aA_2}}{1 - e^{-aA_1}} \right)^{\frac{1}{b}}}_{f_{base}} \times \underbrace{\left(1 + [(\delta_0 + \delta_1 P)N] \frac{\gamma}{\beta} \left(\frac{y_{st}}{\beta} \right)^{\gamma-1} e^{-\left(\frac{y_{st}}{\beta} \right)^\gamma} \right)}_{f_{fert}} \quad (4.5)$$

where $domH_2$ is dominant height at stand age A_2 , $domH_1$ is dominant height at age A_1 and all other variables and parameters are as defined previously.

The term f_{base} in equation (4.5) is the base-line model that represents standard (unfertilized) dominant height growth in a stand. The term f_{fert} represents response due to fertilization. This fertilization response curve is an adjusted Weibull function. The Weibull probability density function is described in section 4.3.1 in detail. f_{fert} is expressed as a function of time

since fertilization, fertilizer elements added, and dose levels. The magnitude and duration of response to fertilization mostly depends on the shape parameter, γ , and scale parameter, β ; the magnitude of response curves changes with types of fertilizer added and their doses.

4.3.4 Model fitting, evaluation and validation

Nonlinear least squares fits of all equations were accomplished by using the PROC MODEL procedure on SAS/ETS software (SAS Institute Inc., 2011). To ensure the solution is global, rather than local minimum, different initial values of parameters were provided for the fits. Fitted equations were tested against observations. Graphical analysis along with the following statistics were used in evaluating and validating models:

- Mean residual, \bar{E}
- Root mean square error, $RMSE$
- Coefficient of determination for nonlinear model, R^2
- Mean PRESS prediction error, \bar{E}_p

All terms were defined in Section 3.3.5, Chapter 3.

4.4 Results and Discussion

4.4.1 Basal area growth

Results from fitting Equation (4.4) on RW13 data are presented in Table 4.2. All parameters were significant ($\alpha = 0.05$). Small mean residuals (\bar{E}), small $RMSE$ and large value of R^2 indicated the model fitted well for given data. With exception of few lower basal area values, the model was unbiased in predicting basal area growth following treatments (Figure 4.3(a)).

Table 4.2: Parameter estimates and fit statistics of basal area growth model (4.4) fitted to the region-wide 13 data.

Parameters	Estimates	Std. Err.	$P > t $
a	0.1263	0.00292	<0.0001
b	0.2998	0.00798	<0.0001
δ_0	0.000738	0.000123	<0.0001
δ_1	0.000538	0.000094	<0.0001
β	6.9419	1.0245	<0.0001
\bar{E}	0.6275		
$RMSE$	5.2645		
R^2	0.9632		

Years since treatment (y_{st}), along with N and P fertilizer variables, were found to have a significant effect on basal area response. The positive sign on estimated parameters for N indicated that basal area growth response increased with amount of N applied. The amount of N applied also had a positive effect on duration of response. Figure 4.4(a) demonstrates the behavior of the fertilization response function (f_{fert}) in Model (4.4). Response curves were plotted for various rates of N with and without the presence of P. The response function was always greater than one when fertilizers were applied and followed the decreasing exponential trend. Response increased with increasing rate of N. Adding P together with N produced greater response than N alone. The magnitude of response varied with the amount of N applied and the presence of P.

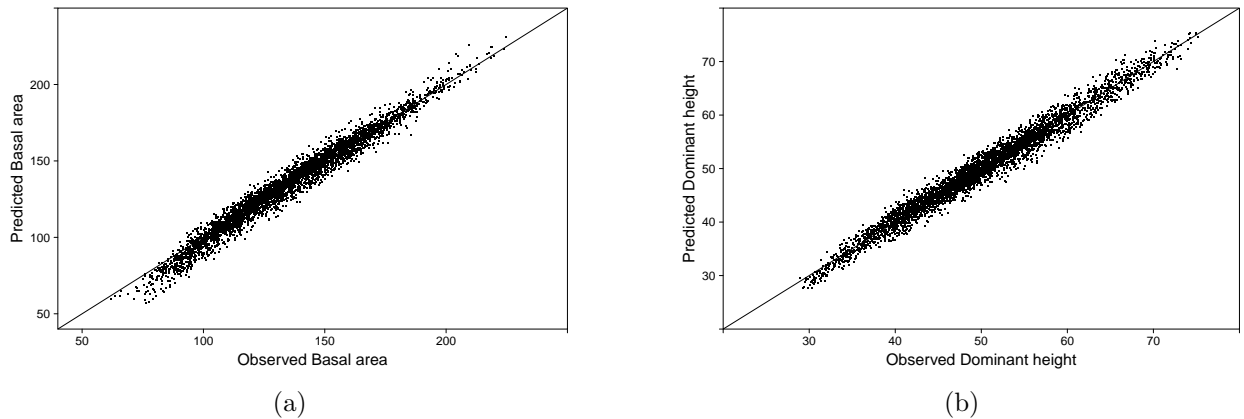


Figure 4.3: (a) Predicted basal area plotted against observed basal area. (b) Predicted dominant height plotted against observed dominant height.

The fertilization response function (f_{fert}) by itself may give the general idea of how growth responded to fertilization treatments, but actual effects can be examined only when it is predicted in conjunction with a base-line equation. Therefore, to further evaluate the model (4.4), basal area growth curves were predicted with estimated coefficients (Table 4.2) for stands with assumed initial basal area of $80 ft^2/ac$ (\approx average observed value) at age 12 years. Curves were generated for unfertilized stands (base-line model) and for stands that were fertilized with different rates of N with or without inclusion of P (Figure 4.5(a)). All fertilized stands had higher growth rates compared to unfertilized stands. Growth response was affected by both rate and element of fertilizer added. According to the model, basal area growth rate increased with increasing rate of N. Treatments with P and N together resulted in greater growth response than nitrogen alone. P only treatments were not included in this study. Generated growth curves were also compared to observed basal area growth (Appendix D, Figure D.1 and D.2). Projected curves agreed closely with the observed growth trend for all levels of fertilization treatment.

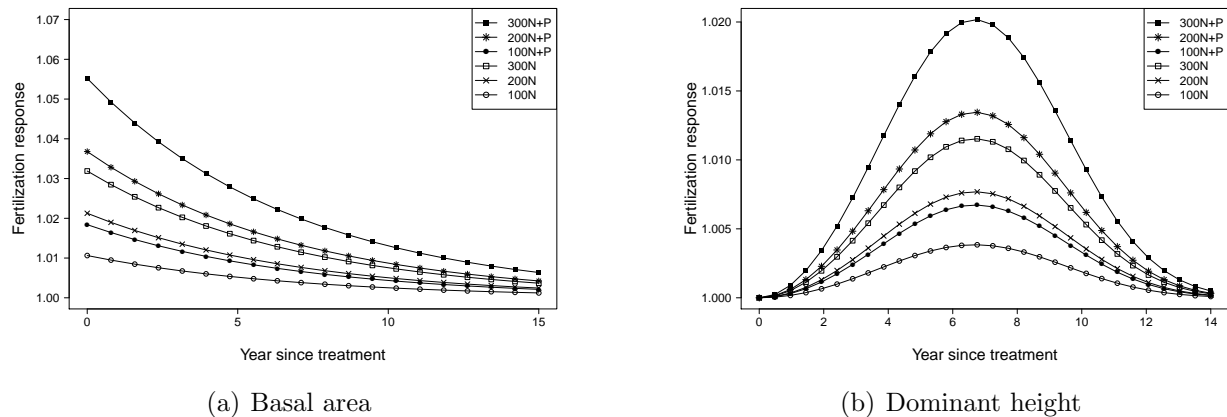


Figure 4.4: Fertilization response function for basal area and dominant height plotted against year since treatment for various levels of fertilization treatments. Stands were 12 years at the time of fertilization.

4.4.2 Dominant height growth

Model (4.5) was fitted to RW13 data. Parameter estimates and fit statistics are presented in Table 4.3. All parameters were significant ($\alpha = 0.05$). Small mean residual (close to zero), small *RMSE*, and large value of R^2 suggested the model fitted well to data. Residual analysis showed no bias in predicting dominant height growth (Figure 4.3(b)).

Table 4.3: Parameter estimates and fit statistics of dominant height growth model (4.5) fitted to the region-wide 13 data.

Parameters	Estimates	Std. Err.	$P > t $
a	0.0987	0.00283	<0.0001
b	0.4969	0.0125	<0.0001
δ_0	0.000256	0.000057	<0.0001
δ_1	0.000192	0.000047	<0.0001
β	7.7046	0.2949	<0.0001
γ	2.9453	0.3006	<0.0001
\bar{E}	0.05272		
<i>RMSE</i>	1.4634		
R^2	0.9743		

Years since treatment (*yst*) and fertilizer elements N and P had significant effects in dominant

height response in a similar manner as in basal area growth response function. Figure 4.4(b) demonstrates the behavior of fertilization response function (f_{fert}) in Model (4.5). The two-parameter Weibull function was found to describe the data well for dominant height response to fertilization. The response function followed a mound shape. Magnitude of the response function varied with rate of N and presence of P. Performance of model (4.5) in predicting dominant height following various levels of fertilization treatments can be seen in Figure 4.5(b). Height growth curves were generated using fitted model (4.5) for stands with assumed initial dominant height of $25ft$ (\approx average observed value) at age 10 years. Height growth also responded in a similar fashion as the basal area growth. Growth rate increased with increasing rate of N and always resulted in greater growth response when P was added. Projected growth curves were also plotted over observed dominant height growth for all levels of treatments (Appendix C, Figure E.1 and E.2). Plotted trends indicated that the model described well the observed growth trend at all levels of treatments.

Basal area and dominant height growth responses to fertilization treatments in this study were similar to responses reported by several previous studies. Amateis et al. (2000) and Carlson et al. (2008), when analyzing the same data used in this study, observed greater response on both basal area and height growth for higher rate of N application and when N and P were applied together. Carlson et al. (2008) also reported that basal area growth responses were typically greater than dominant height, which is consistent with the current study. Bataineh et al. (2006), Fox et al. (2007), Antony et al. (2009), Jokela et al. (2010), and Liechty and Fristoe (2013) all found similar results on responses to N and P fertilization that are comparable to this study.

Models (4.4) and (4.5) developed in this study included age of a stand, the N rate and an indicator variable of P as predictor variables. Carlson et al. (2008) and Amateis et al. (2000) tried including rate of P but did not find any improvement in their models. There might be many other factors that can affect response to fertilization, such site drainage condition and

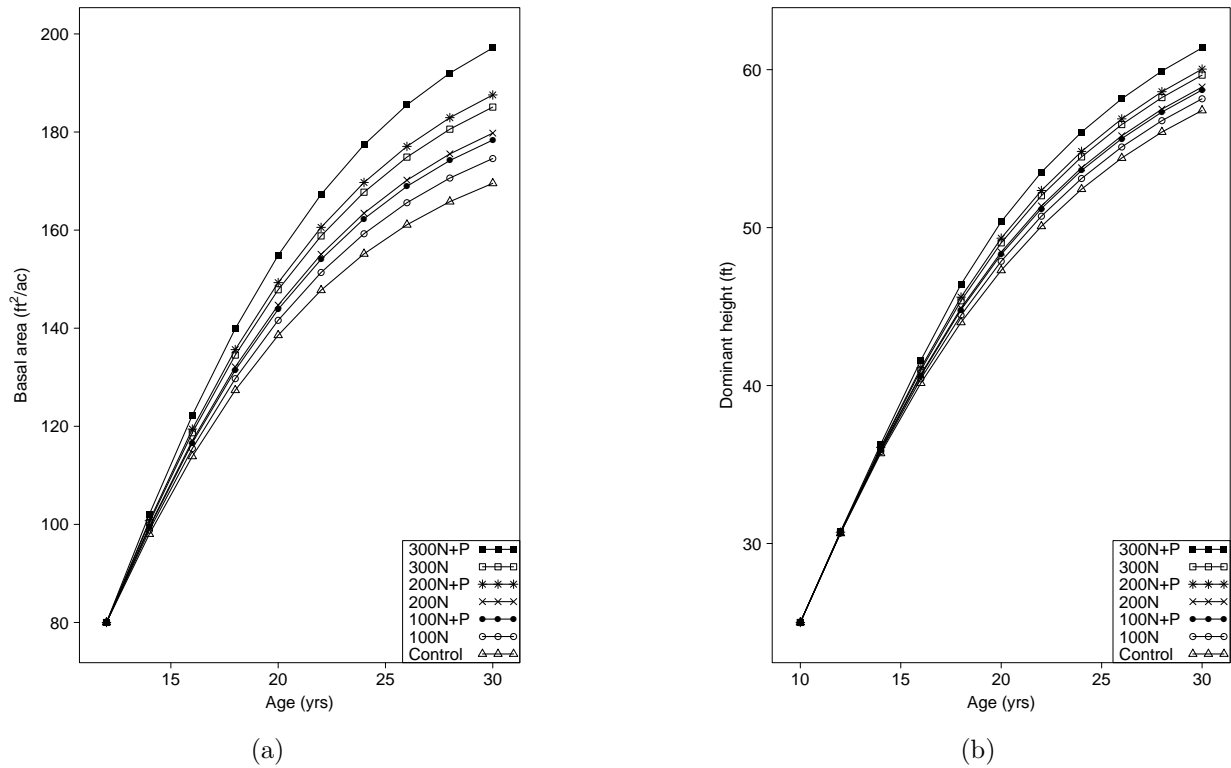


Figure 4.5: (a) Comparison of development of basal area generated by Equation (4.4) for various levels of fertilization treatments with an initial stocking of $80 \text{ ft}^2/\text{ac}$ at age 12 years. (b) Comparison of development of dominant height generated by Equation (4.5) for various levels of fertilization treatments with an initial height of 25 ft at age 10 years.

stand density (Amateis et al., 2000, Carlson et al., 2008). These factors were not directly considered in this study because the objective was to develop the general response function across a range of conditions.

4.4.3 Model validation

Along with graphical evaluation, models were also cross-validated using the leave-one-out technique. Mean PRESS prediction errors (\bar{E}_p) were calculated for basal area and dominant height growth models. The values of \bar{E}_p and its standard error (SE_p), and percentage error relative to observed mean ($\%Error$) are presented in Table 4.4.

Table 4.4: The mean PRESS prediction error (\bar{E}_p), standard deviation of prediction error (SE_p), and percentage error relative to observed mean of the basal area and dominant height growth models for fertilization treatment.

Models	\bar{E}_p	SE_p	%Error
Model (4.4)	0.6280	5.2383	3.89
Model (4.5)	0.0530	1.4639	0.10

Both models yielded very small mean predictive errors. The percentage error was 0.1% for Model (4.5) and comparatively somewhat higher but acceptable for Model (4.4). Overall, the validation results suggested that both models had adequate predictive ability.

4.5 Conclusion

General response models for projecting stand basal area and dominant height for various levels of fertilization treatment were developed. The effects of treatments were incorporated into the models as a multiplier function. The fertilization response function was based on Weibull distribution and the magnitude of response varies with time since application of fertilizers, type of fertilizer elements applied, and their doses. Response functions presented here were developed using data from a wide variety of growth sites for loblolly pine across the Southern United States, and therefore, believed to represent general growth trends in response to treatment. In addition to observed data, these functions were also based on biological considerations. Because of the model structure (4.1), these response functions can be easily integrated with base-line growth models other than those used in this study. These response functions can be viewed as general response functions.

Response functions integrated with base-line models were fitted and tested using data from a fertilization experiment (RW13). Analyses suggested that the fitted models described well the effect of fertilization on stand growth and were also unbiased in projecting basal area and dominant height within these loblolly pine stands.

Data used in these analyses came from repeated measurements of permanent plots and are likely to have correlated observations. Since the main interest of the present study was prediction rather than testing of hypotheses, the auto-correlation issue should not be a problem. Prediction capability of models developed in this study was not influenced by the applied estimation method. Ordinary least square estimation with correlated data might give biased estimates of parameter variance but parameter estimates are unbiased.

Models (4.4), and (4.5) developed here considered time since application of treatments, and intensity of treatment as predictor variables. There are many other factors that could poten-

tially affect the variability in response to fertilization. Initial stand and site condition and other climatic and geographic factors can affect response to treatments (Amateis et al., 2000, Carlson et al., 2008). These factors were not considered directly in this study. Nevertheless, based on results from model fitting and evaluation, the growth models presented here should provide satisfactory stand average response to a wide range of stand and site conditions with various levels of fertilization treatments.

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Chapter 5

Modeling Response to Competing Vegetation Control

5.1 Introduction

Competing vegetation control is one of several silvicultural practices used to enhance productivity in forest stands. Numerous studies have documented that early herbaceous and longer term woody vegetation control can enhance the growth of loblolly pine (Tiarks and Haywood, 1986, Creighton et al., 1987, Glover et al., 1989, Jokela et al., 2000, Miller et al., 2003) by decreasing competition for space, water, nutrients and light. There are many reports on early increased growth of loblolly pine (less than 10 years) from control of competing vegetation, altering mainly height and diameter growth (Glover et al., 1989, Haywood and Tiarks, 1990), and some studies have also reported longer term effects on growth of pine by early vegetation control (Clason, 1989, Glover and Zutter, 1993, Miller et al., 2003). Growth models that accurately reflect response to control of various levels of vegetation are required for making wise management decisions.

A stand-level model developed by Burkhart and Sprinz (1984) showed hardwood competitors increased pine mortality and shifted diameter distributions to the left reducing basal area per acre but had no influence on dominant height. Their models were based on the assumption that the differences in growth between weed-free former agricultural lands (old-fields) and cutover, site-prepared areas were due to hardwood competition. Models based on old-field plantations were adjusted by reduction factors computed from the level of hardwood competition measured in a region-wide set of plots in plantations established on cutover, site-prepared areas. Work of Knowe (1992) followed a similar pattern on basal area growth and diameter distribution due to increased hardwood competition. Lauer et al. (1993) studied loblolly pine response to herbaceous weed control on eight sites in the Southeastern US through age 9 years. Their findings supported the hypothesis that weed control does not change the site quality but rather advanced early height and dbh growth. They used the age-shift method to project future height for treated stands. Quicke et al. (1999) evaluated

growth of *Pinus taeda* L. with and without control of competing herbaceous vegetation over 15 years at various planting densities. The authors used separate adjustment terms in height and basal area models that successfully captured the short-term height growth and longer-term basal area responses to vegetation control. The adjustment model they used was that of the Pienaar and Rheney (1995) yield model modified to account for planting density effects. The Pienaar and Rheney (1995) model requires information on years since treatment, therefore this model may not be appropriate where treatments are applied continuously for multiple years.

Objectives of this study were to examine response to vegetation control treatments and to develop adjustments required to account for observed response in basal area and height growth. While most previous studies have focused on a particular level of vegetation control, this study included response to various levels. This study also utilized long-term observations from a wide variety of sites.

5.2 Data

USDA Forest Service’s Competition Monitoring Project (COMP) data were utilized in studying the weed control effects in growth modeling. The project established permanent plots at 13 plantation sites in 1984 to examine the influence of herbaceous and woody vegetation on the growth of loblolly pine across the Southern United States (Miller et al., 2003). Treatments included were: no control, woody control for 5 years, herbaceous control for 4 years, and total control after site preparation. Four complete blocks were established in 11 of 13 locations. One of the remaining locations had five complete blocks and in another location a completely randomized design was used. Pines were measured for total height and dbh from year 3 – 11 and 15. All hardwood and shrub rootstocks were also recorded by species and height class. Data consisted over 2300 observations. The overall average summary statistics including mean, minimum and maximum values for various stand attributes are given in Table 5.1. Observed basal area and dominant height growth over time for different treatments are shown in Figure 5.1. These figures give an idea of growth patterns for various treatments.

Table 5.1: Summary statistics at stand age 15 years for COMP plots established to examine influence of vegetation control on the growth of loblolly pine.

Variables	Mean	Minimum	Maximum
Site index	70.0	55.2	87.7
Trees/ <i>ac</i>	492.5	340.0	539.0
Basal area (<i>ft</i> ² / <i>ac</i>)	133.1	71.2	187.5
Dominant height (<i>ft</i>)	49.7	37.7	61.7

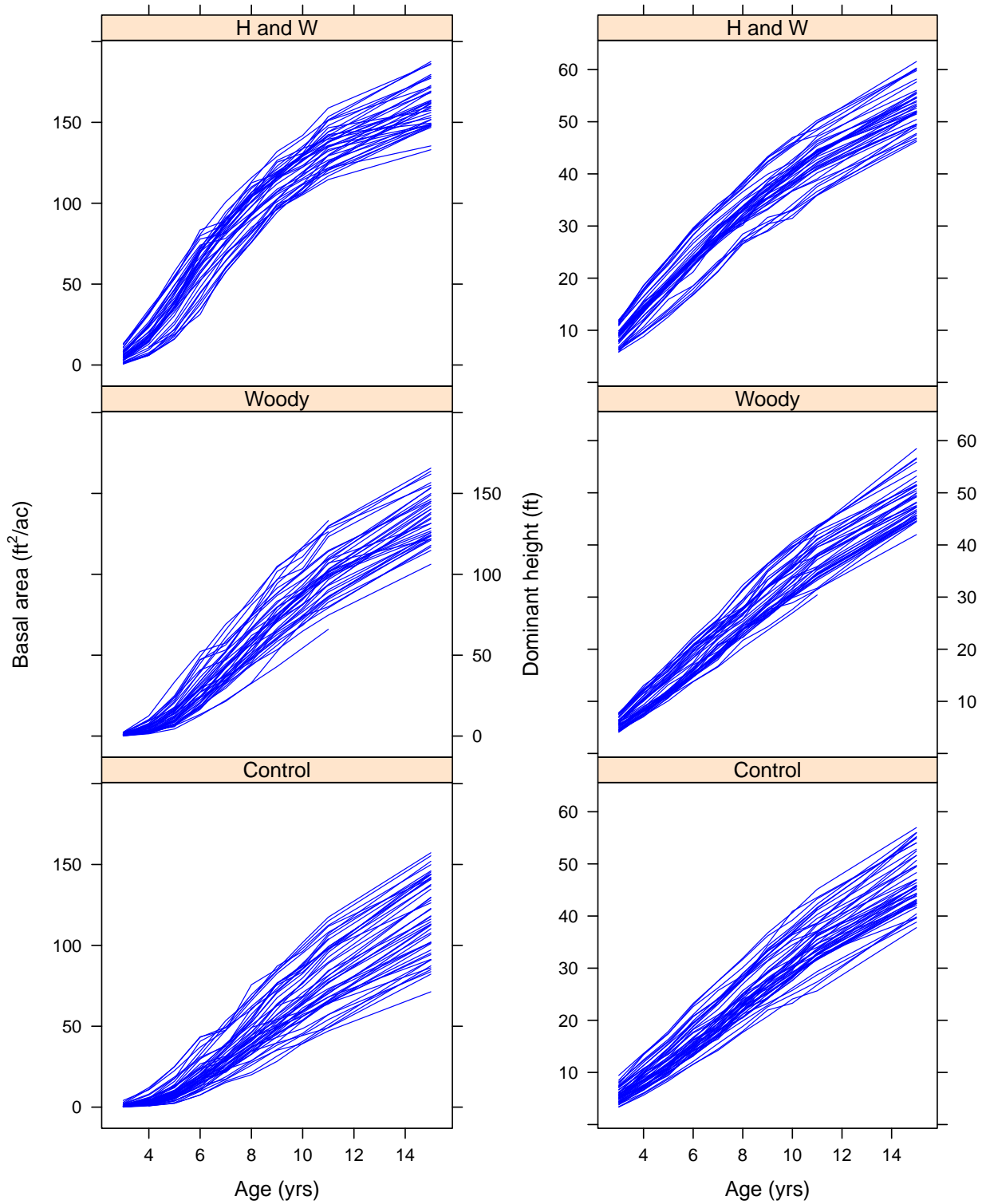


Figure 5.1: Observed basal area growth and dominant height growth over age in plots with no vegetation control (Control), woody vegetation control (Woody), and herbaceous and woody vegetation control (H and W) for the COMP data set.

5.3 Methods

Growth responses to different vegetation control treatments were first examined and growth models were then adjusted to account for observed trends. Stand growth gain due to control of competing vegetation was adjusted using multiplier factors. In this approach, stand growth over time was fitted for both treated and non treated stands and then multipliers were calculated based on growth differences. The Chapman-Richards model with indicator variables (Equation 5.1), representing each treatment type, was fitted to periodic remeasurement data ranging from age 3 to 15 years. Equation (5.1) is an integral form of the Chapman-Richards growth function with asymptote parameter b_1 , growth rate related parameter b_2 , and shape parameter b_3 . Details on Chapman-Richards model are given in Chapter 2. The model can be expressed as

$$y = \sum_{i=1}^4 D_i b_{1i} \left(1 - e^{-\sum_{i=1}^4 D_i b_{2i} A} \right)^{\sum_{i=1}^4 D_i b_{3i}} \quad (5.1)$$

where y is stand growth, D_i is indicator variable

$$D_i = \begin{cases} 1 & \text{if treatment } i, \quad i = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

A is stand age, and b_1 , b_2 , b_3 are parameters and 1 = no control, 2 = woody control, 3 = herbaceous control, and 4 = herbaceous and woody control.

Multiplier ratio factors for each treatment were then determined by comparing its estimated parameter values with those estimated for the growth curve representing no vegetation control plots. A fitted curve from Equation (5.1) for no vegetation control treatment serves as

a base-line growth model for both basal area and height growth. Growth response for other treatments can then be easily obtained by adjusting selected parameters on the base-line equation by the corresponding multiplier ratio factors for each treatment.

Nonlinear least squares fit of the selected equation was accomplished by using the PROC MODEL procedure on SAS/ETS software (SAS Institute Inc., 2011). To ensure the solution was global rather than local minimum, different initial values of parameters were provided for the fits. Fitted equations were tested against observations. Graphical analysis along with the following statistics were used in evaluating and validating models:

- Mean residual, \bar{E}
- Root mean square error, $RMSE$
- Coefficient of determination for nonlinear model, R^2
- Mean PRESS prediction error, \bar{E}_p

All terms were defined in Section 3.3.5, Chapter 3.

5.4 Results and Discussion

Model (5.1) was fitted to the COMP data for basal area growth and dominant height growth. Asymptotic parameters were constrained while fitting the basal area growth model to ensure that the upper asymptote for basal area growth in control plot is always less than or equal to that of plots with herbaceous vegetation control treatments. Parameter estimates and fit statistics are presented in Table 5.2.

Table 5.2: Parameter estimates and fit statistics of basal area growth and height growth model (5.1) fitted to COMP data.

Parameters	Basal area			Dominant height		
	Estimates	Std. Err.	95% C.I.	Estimates	Std. Err.	95% C.I.
b_{11}	147.3	4.6337	(138.2, 156.4)	72.099	4.9003	(62.489, 81.710)
b_{12}	157.4	6.1190	(145.4, 169.4)	70.543	4.1871	(62.331, 78.755)
b_{13}	147.3	4.6337	(138.2, 156.4)	66.304	3.0076	(60.406, 72.203)
b_{14}	170.8	3.9113	(163.2, 178.5)	66.757	2.5887	(61.680, 71.834)
b_{21}	0.2278	0.0149	(0.2003, 0.2554)	0.1100	0.0122	(0.0861, 0.1339)
b_{22}	0.2616	0.0198	(0.2228, 0.3004)	0.1231	0.0126	(0.0984, 0.1478)
b_{23}	0.2293	0.0154	(0.1992, 0.2595)	0.1263	0.0116	(0.1035, 0.1491)
b_{24}	0.2968	0.0169	(0.2637, 0.3299)	0.1438	0.0122	(0.1199, 0.1676)
b_{31}	6.9007	0.7327	(5.4636, 8.3377)	2.0368	0.1345	(1.7730, 2.3005)
b_{32}	7.5298	0.9952	(5.5778, 9.4817)	2.1321	0.1452	(1.8473, 2.4169)
b_{33}	4.7188	0.4590	(3.8187, 5.6190)	1.7775	0.1062	(1.5992, 1.9858)
b_{34}	5.9138	0.5822	(4.7718, 7.0558)	1.8922	0.1182	(1.6603, 2.1241)
\bar{E}	-0.1503			0.00118		
$RMSE$	14.676			3.5012		
R^2	0.8962			0.9299		

All parameters were significant ($\alpha = 0.05$) for the basal area growth model. Fit statistics in Table 5.2 indicated the model fitted the data well. Predicted basal areas were plotted against observed basal area values (Figure 5.2(a)). Residual plots indicated model (5.1) was unbiased in predicting basal area growth. To further analyze the basal area growth trend, fitted curves were plotted against age for different competing vegetation control treatments including no vegetation control (Figure 5.2(b)). Early competition control had a significant positive

influence on basal area growth. Basal area gain for herbaceous and woody vegetation control over no control was larger compared to other treatments. Basal area gains from herbaceous control treatments were greater than those from woody control treatments for growth at early ages but had no lasting effect. Dunnet test ($\alpha = 0.05$) also suggested herbaceous control did not differ significantly from control treatment at age 15 years. Release from herbaceous vegetation might have enhanced the woody competition that eventually led to no gain in basal area in later years of growth. Basal area gain for woody control treatment surpassed the herbaceous control treatment and became more prominent as stands got older. Gain in growth also depends on site conditions i.e., level of available competing vegetation before the application of treatments (Miller et al., 2003).

Several other studies have also reported comparable results on growth response due to control of competing vegetation. Miller et al. (2003) reported significant gains in basal area for herbaceous plus woody vegetation control when compared to no control for loblolly pine plantations. They also found that early increase in pine growth from herbaceous control were not sustained through 15 years. Short-term response to herbaceous control was also consistent with findings by Lauer et al. (1993). In another study for loblolly pine in Georgia, Borders and Bailey (2001) reported a significant gain in volume growth for complete control over no control treatment. Similar results have been reported in region-wide site preparation study with loblolly pine by Shiver and Martin (2002). They found that the control of both woody and herbaceous vegetation increased basal area by 130% relative to the untreated control in 12 years. Early hardwood development had considerable effects on 27-years loblolly pine basal area growth in another study by Glover and Zutter (1993).

Based on the fit results for basal area growth (Table 5.2), multiplier ratio factors were computed. Multipliers needed to adjust the base-line model for the performance of treated stands (control of competing vegetation) are reported in Table 5.3. Considering the fact that early growth gain in pine from herbaceous control was not sustained, the only treatments consid-

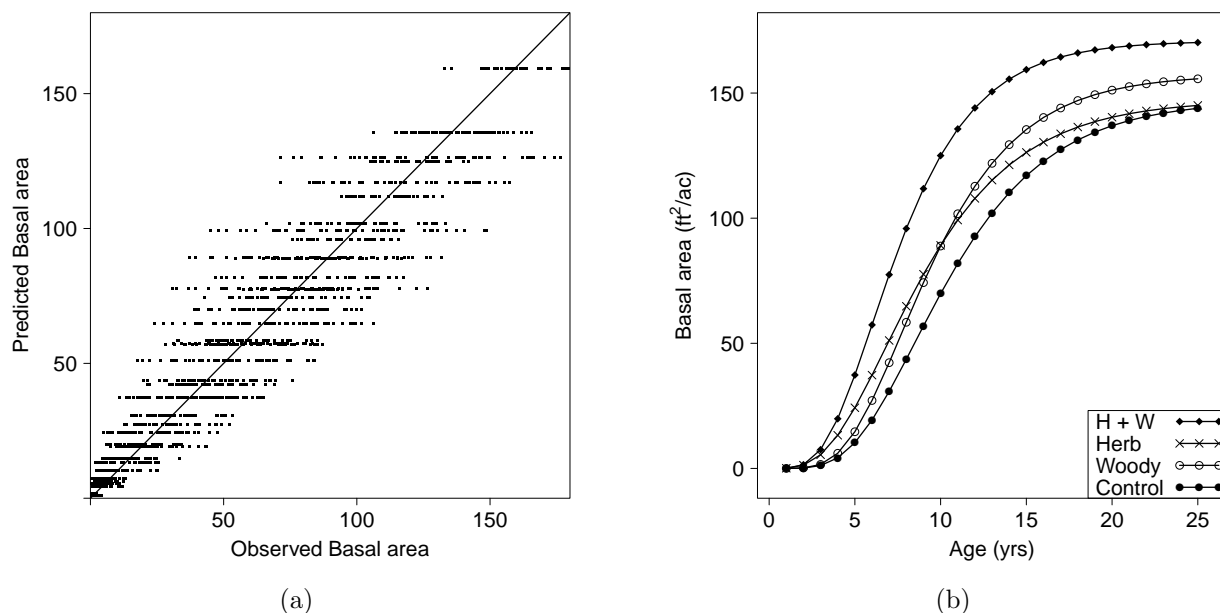


Figure 5.2: (a) Predicted basal area plotted against observed basal area. (b) Fitted basal area curve plotted against age for different competing vegetation control treatments.

ered for this part of the analyses were woody control (Woody) and herbaceous and woody control (H and W). Herbaceous and woody control treatments had larger multipliers for corresponding asymptotic and rate parameters. These multipliers were based on an assumption that observed data represent the general growth trend for loblolly pine plantations with early control of competing vegetation across the Southeastern United States.

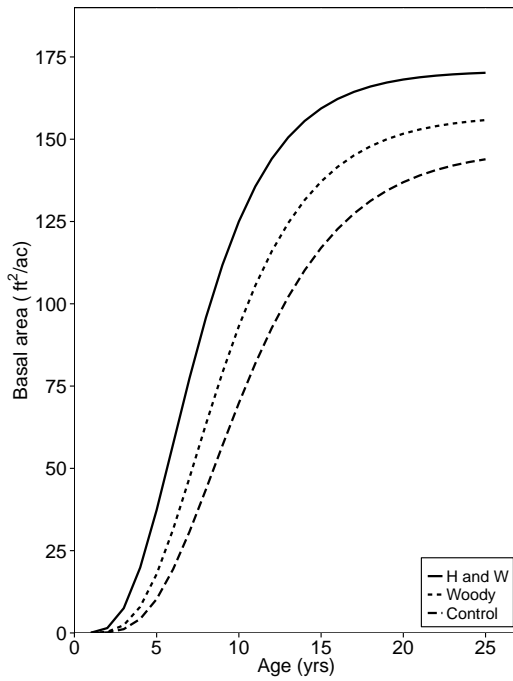
Table 5.3: Ratio factors as parameter multiplier describing basal area growth for stand with control of competing vegetation.

Parameters	Multiplier ratio factor, m_f	
	Woody control	Herbaceous and woody control
b_1	1.0685	1.1595
b_2	1.1483	1.3029
b_3	1.0911	0.8569

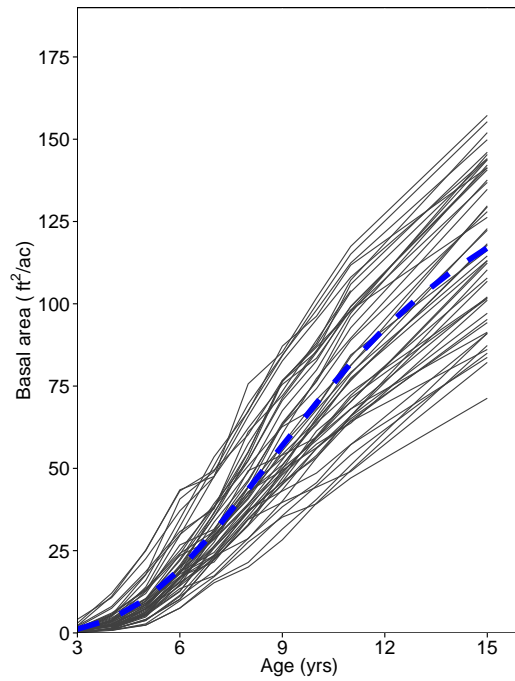
Analyses suggested that all three parameters were needed to adjust in the base-line model to capture the additional growth due to control of herbaceous and woody (H and W) vegetation

but adjusting only asymptotic and rate parameters were sufficient in case of woody vegetation control treatment. Estimated asymptotic and rate parameters in herbaceous and woody control treatments differed significantly from those in the no control as the 95% confidence intervals were not overlapping (Table 5.2). These parameters were larger but not significantly different from no control treatment in the case of woody control. Estimated shape parameters did not differ significantly among all treatments. To further evaluate, the basal area growth curve for woody control and herbaceous and woody control treatments were generated using multipliers in Table 5.3. Figure 5.3 compares the generated curves for different treatments with base-line growth (no control) and observed growth curves. Adjusted curves in Figure 5.3(a) illustrate the gain in basal area growth for woody control and H and W control treatments relative to base-line growth. Although treatment response through rotation age was not available, adjusted curves agreed closely with average observed growth trend through age 15 years (Figure 5.3(b), 5.3(c), and 5.3(d)).

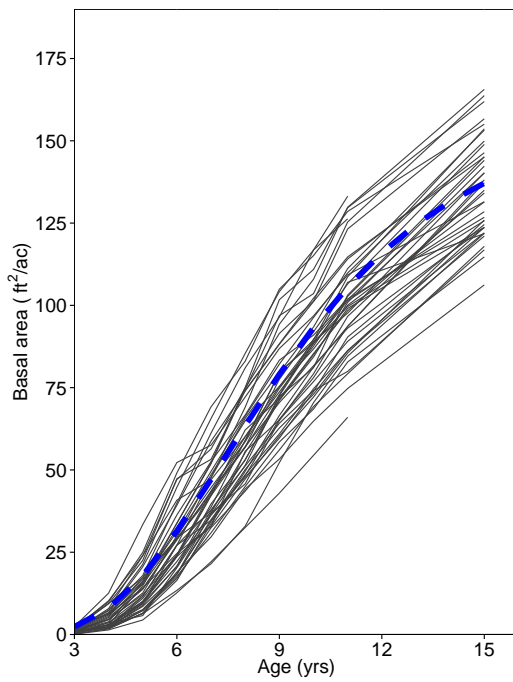
All parameter estimates for the dominant height growth model (5.1) were significant ($\alpha = 0.05$). Model (5.1) for dominant height growth was unbiased and fitted well to the data (Figure 5.4(a), Table 5.2). Fitted curves were plotted against age for different competing vegetation control treatments including no control (Figure 5.4(b)). Fitted curves indicated gain in early height growth for herbaceous only and herbaceous and woody control treatments; however, the gain in growth was not sustained throughout rotation. Woody control did not differ significantly from no control and other treatments had small differences at age 15 years (Dunnet test at $\alpha = 0.05$). Furthermore, all 3 parameters, b_1 , b_2 , and b_3 , did not differ significantly among treatments (Table 5.2). Results suggested that the change in growth was relatively small for dominant height compared to basal area growth due to competing vegetation control and can be ignored safely. Burkhart and Sprinz (1984) reported differences in height growth were not large enough from a practical point of view in their study of



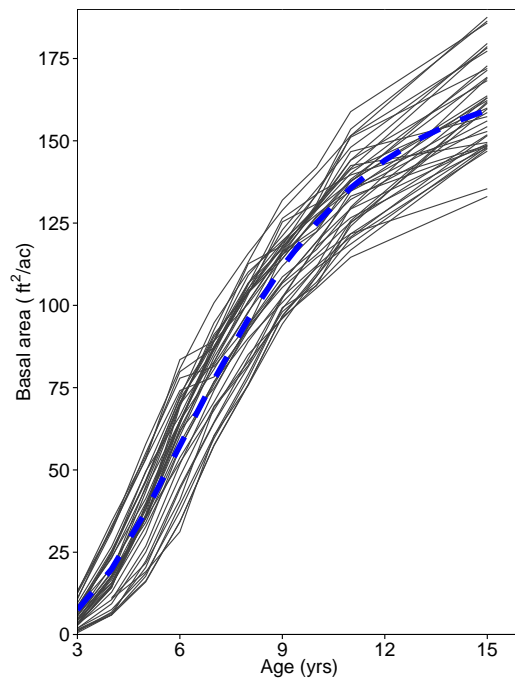
(a) Adjusted curves



(b) Control



(c) Woody



(d) H and W

Figure 5.3: (a) Comparison of adjusted basal area growth curves with base-line growth curves. (b), (c), and (d) Adjusted basal area growth curve (dotted line) generated using multipliers overlaid on observed growth over time for no competing vegetation control (Control), woody vegetation control (Woody), and herbaceous and woody vegetation control (H and W) treatment.

hardwood competition effects on yield for unthinned loblolly pine plantations. Glover and Zutter (1993) indicated loblolly pine height was density independent within a wide range of combined pine and hardwood densities and cautioned that differences in height at a young stand age due to competition control treatments may not be maintained throughout the life of stand.

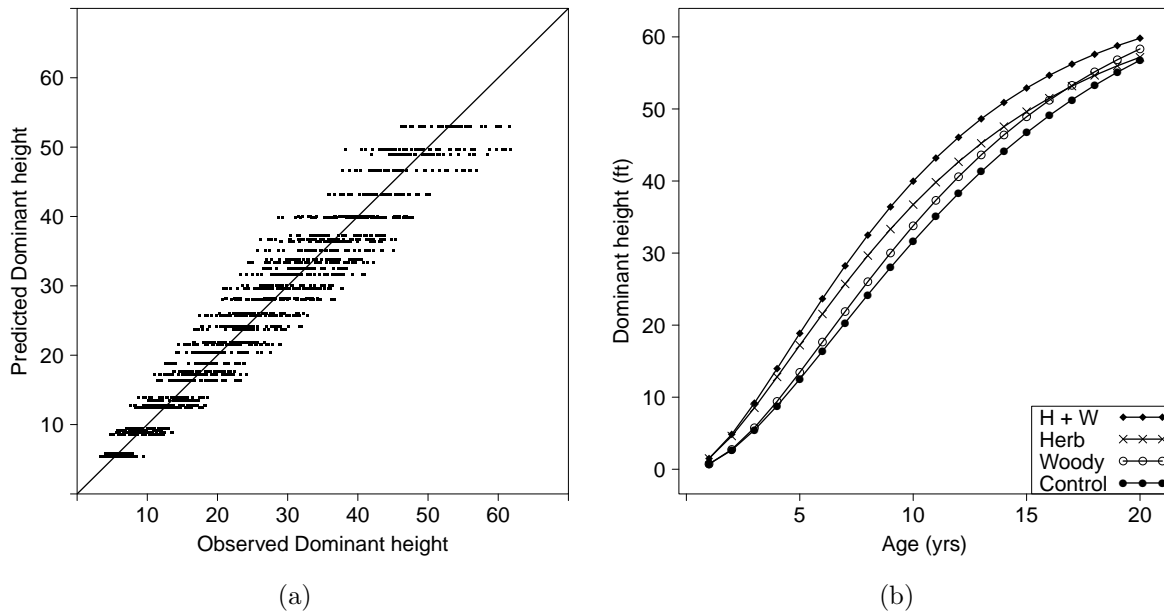


Figure 5.4: (a) Predicted dominant height plotted against observed dominant height. (b) Fitted height curve plotted against age for different competing vegetation control treatments.

5.4.1 Model validation

In addition to graphical evaluation, models were also cross-validated using the leave-one-out technique. Mean PRESS prediction errors (\bar{E}_p) was calculated for basal area growth model. The model yielded very small mean predictive errors (-0.148) with standard deviation (15.1). The percentage error relative to observed mean was less than 0.3% (0.248%). The validation results suggested that the model (5.1) for basal area growth had adequate predictive ability.

5.5 Conclusion

Response to early control of competing vegetation was prominent on basal area growth; however, controlling only herbaceous vegetation did not have a long term effect. Change in dominant height growth was relatively small and can be ignored safely. Ratio multiplier factors were used to adjust the base-line growth equation to predict basal area growth for stands treated with various levels of vegetation control. Analyses based on available data used for this study suggested this method captured well the positive effect of vegetation control in the basal area growth model. No adjustment was required for the dominant height growth model in response to vegetation control.

The focus of this study was to develop a modeling method that can capture the general stand response to vegetation control. Multipliers presented here were derived using data from a wide variety of growing sites for loblolly pine across the Southern United States, and, therefore, believed to represent general growth trends to various levels of vegetation control. Treatment effects were based on differences in growth trends for treated over untreated stands. Therefore, this simple method allows managers to evaluate the worth of alternative vegetation management treatments when a base-line growth model is available. However, the managers should always be aware that vegetation control might influence crop dimensions and quality in addition to total productivity. The effects of vegetation control on crop dimensions and quality were not directly examined in this study.

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Chapter 6

General Discussion and Conclusion

Stand level growth and survival models incorporating responses to silvicultural treatments were developed for loblolly pine stands. This dissertation emphasized 1) developing robust base-line growth and survival models 2) incorporating appropriate response functions for thinning, fertilization, and vegetation control into base-line growth models.

Difference forms of the Chapman-Richards growth functions were selected as base-line models. Selected basal area and dominant height models were flexible enough to cover a wide range of growth patterns in the data set. The difference survival function, derived from a differential equation with age, initial stand density, and site index produced satisfactory results in predicting surviving number of trees in loblolly pine stands. Selected growth and survival models are expected to represent the general growth and survival trends in untreated loblolly pine stands.

Base-line growth models were adjusted with modifier response functions or multipliers accounting for effects of silvicultural treatments - thinning, fertilization, and competing vegetation control. Separate response functions for thinning and fertilization effects were developed. The thinning response function was based on duration and rate parameters and is sensitive to stand age at the time of thinning, time since thinning, and intensity of thinning. The fertilization response functions were based on the Weibull distribution; the magnitude of responses varies with time since application of fertilizers, type of fertilizer elements applied, and their doses. Response functions were integrated as a multiplier to base-line models. The adjusted base-line models were then fitted and tested separately to their corresponding data sets for thinning and fertilization. Models were unbiased and exhibited adequate performance in predicting basal area and dominant height following treatments. No adjustment was required for dominant height growth model in response to thinning treatment. The base-line survival model was adjusted for thinned plots by including an additional independent variable that represents thinning intensity. No adjustment was required for the survival model in response to fertilization treatment, because site index is incorporated as a predictor

of survival. Thinning response functions developed here were based on data from plots where all thinning were done from below. Data employed from a fertilization experiment represent growth response to one-time mid-rotation fertilizers application. Therefore, these growth models are expected to perform best in the stands where the treatment methods are similar.

Response to early control of competing vegetation was incorporated into base-line models through multiplier factors. A Chapman-Richards model with indicator variables, representing each treatment level including the control treatment, were fitted to data from a vegetation control experiment. Multiplier factors were calculated based on growth difference between treated and untreated stands. Analysis suggested the adjusted model captured well the positive effect of vegetation control on basal area growth at all levels of treatments. No adjustment was required for dominant height growth and survival models in response to vegetation control.

The focus of this dissertation work was to develop growth models that can capture the general response to silvicultural treatments. All models presented here were based on data from a wide variety of growth sites for loblolly pine across the Southern United States, and therefore, believed to represent general growth trend. In addition to testing against observed data, response functions developed for thinning and fertilization were also based on biological considerations. Furthermore, because of the model structure, these response functions can be easily integrated with base-line growth models other than those used in this study. Therefore, these functions can be viewed as general response functions.

Models developed in this study will eventually be put together into a stand projection system and software for stand growth simulation will be developed. Response functions to treatments will be incorporated into a system as additive effects. Silvicultural treatment effects have been reported to be additive in past studies. Preliminary analysis on Forest Productivity Cooperative's the Regionwide 19 data indicated thinning and fertilization to be additive

in loblolly pine plantations. Effects of weed control and fertilization were also reported to be additive by Albaugh et al. (2012) and Liechty and Fristoe (2013). Predictive ability shown by individual equations may not be realized when all response models are incorporated into a stand simulator. Therefore, further screening and evaluation might be necessary before implementing the overall stand simulation system.

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Appendix A

Location map of permanent plots for silvicultural experiments

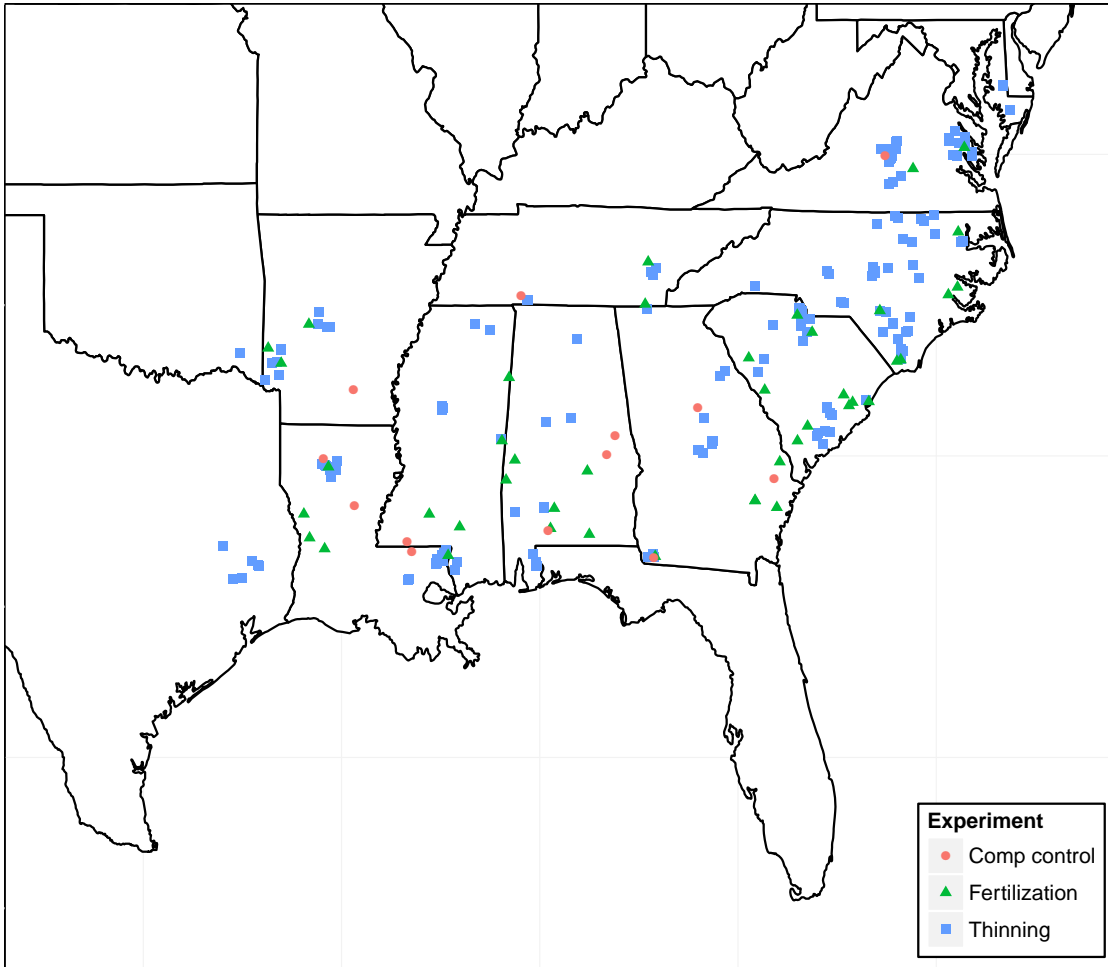
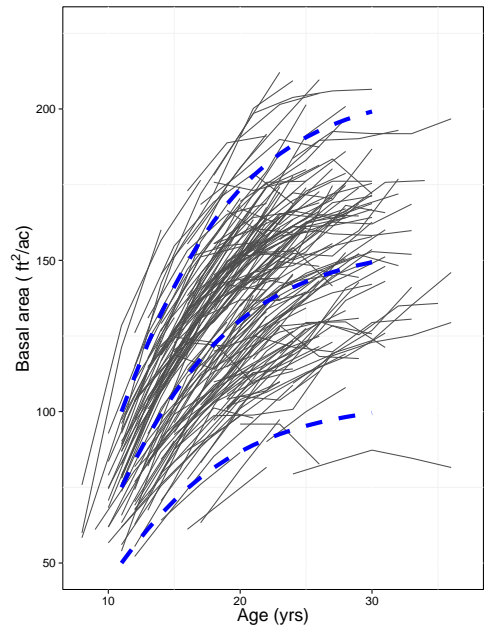


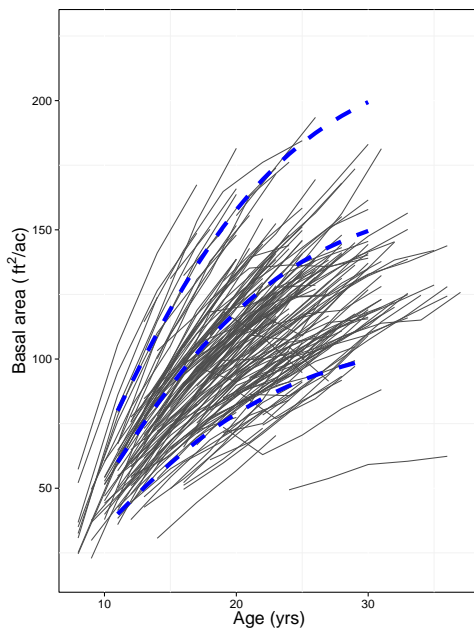
Figure A.1: Locations of permanent plots established across Southeastern United States for silvicultural experiments- thinning, fertilization, and competing vegetation control.

Appendix B

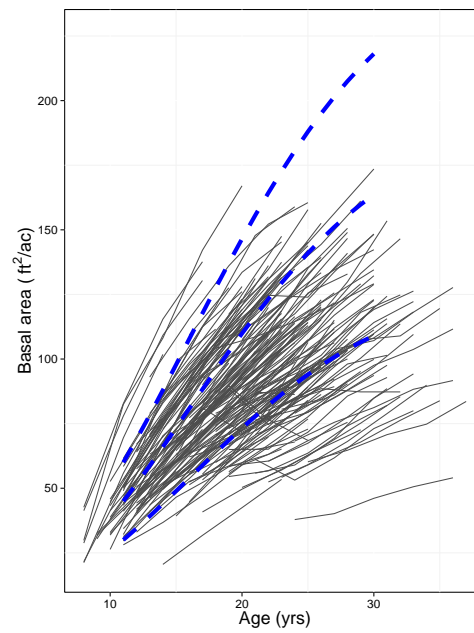
Basal area growth projection following thinning



(a) Control



(b) Light thin



(c) Heavy thin

Figure B.1: Basal area projection curves (dotted line), obtained using different initial values, overlaid on observed basal area growth for various levels of thinning intensities. Curves were generated for stands that were 11 yrs of age at the time of thinning. Observed data (light solid line) includes plots thinned at different ages.

Appendix C

Residual plots for Survival function by thinning treatments

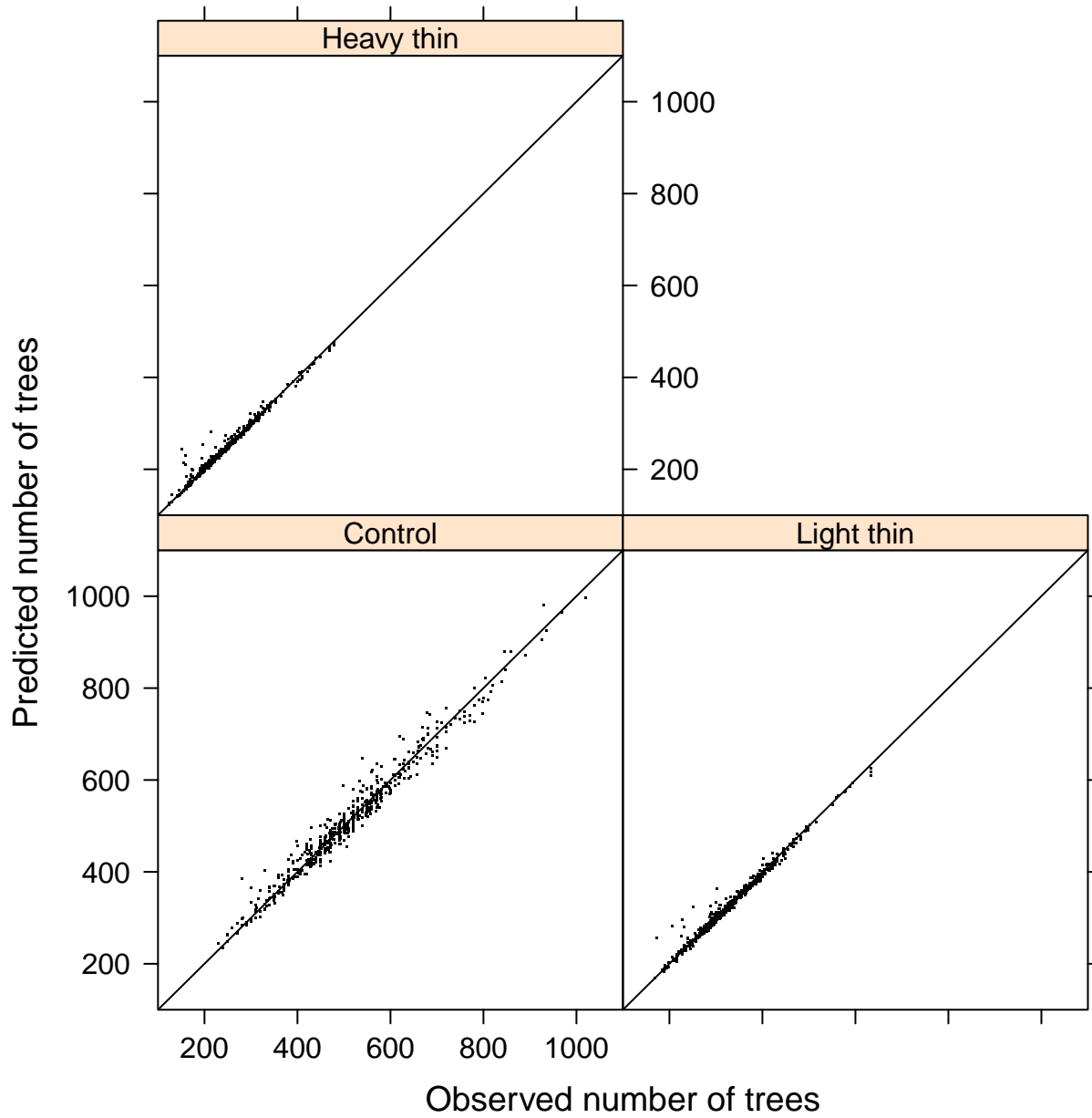


Figure C.1: Residual plots for Survival function by thinning treatments.

Appendix D

Basal area growth projection following fertilization

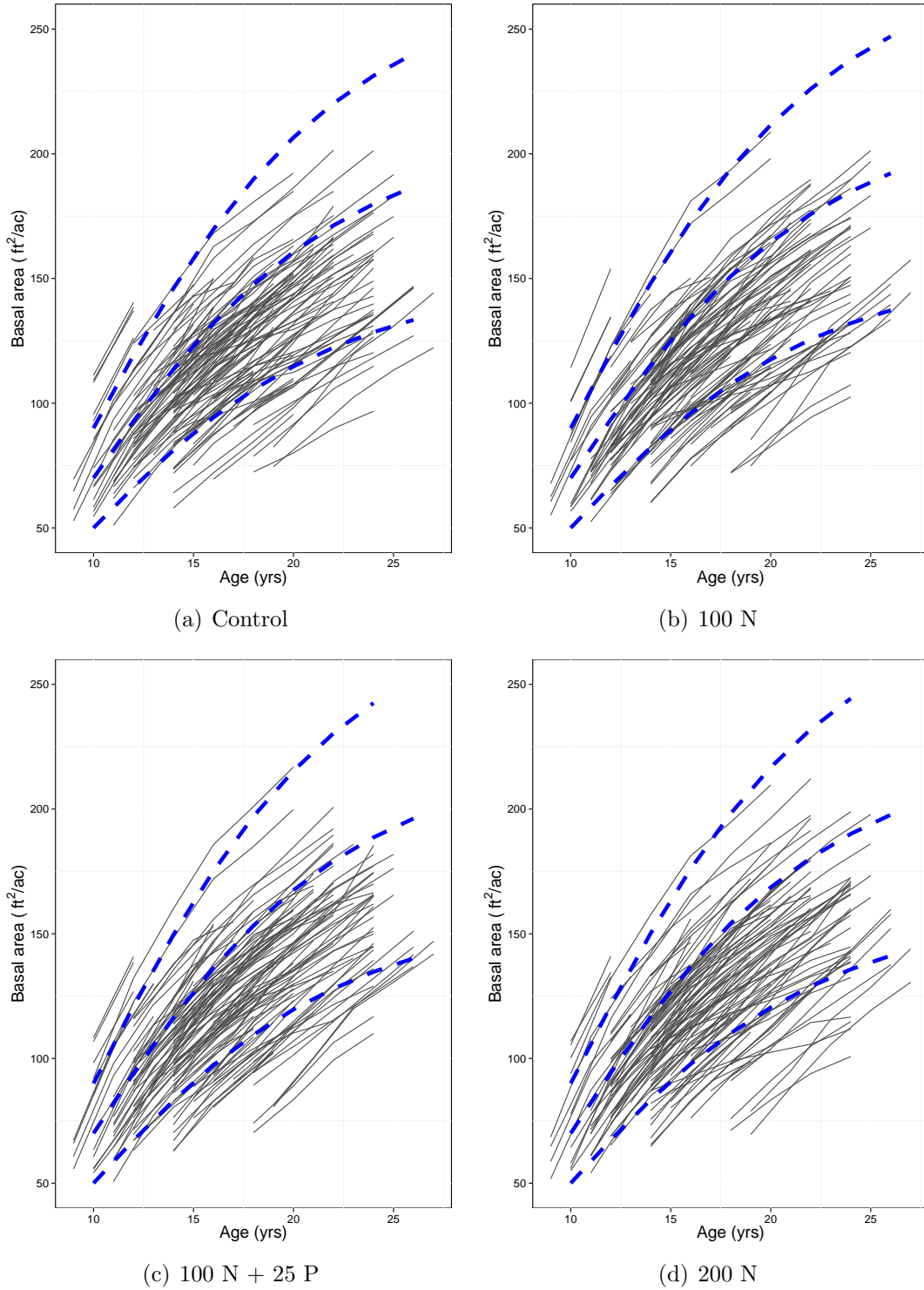
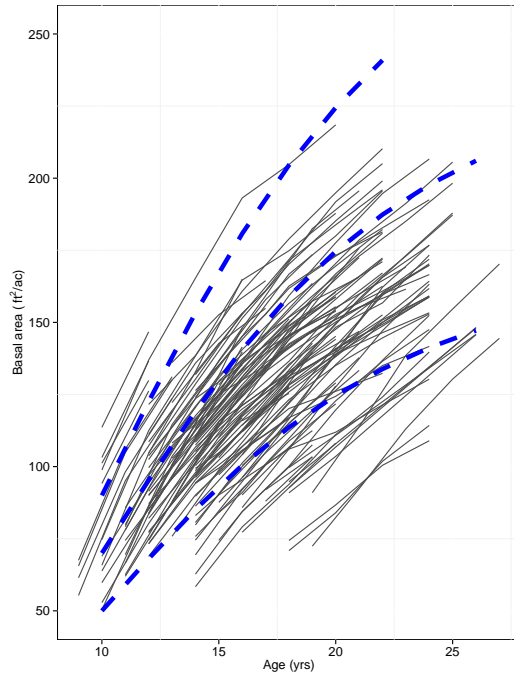
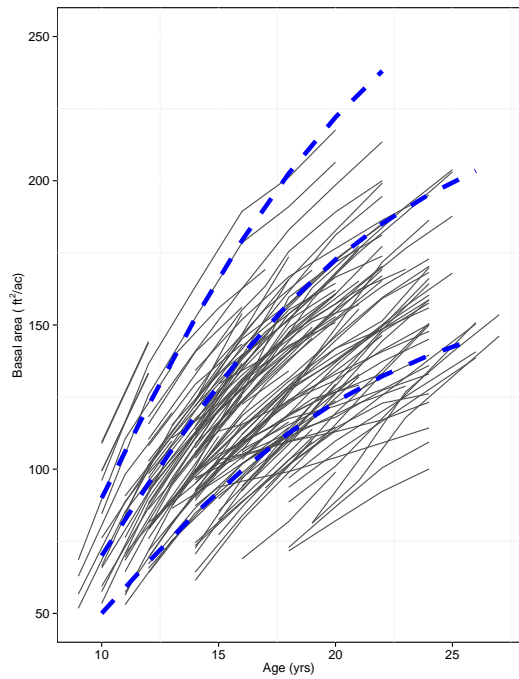


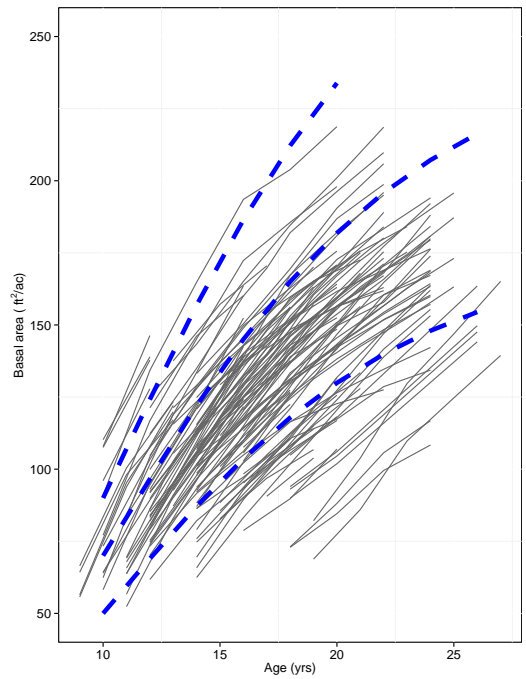
Figure D.1: Basal area projection curves (dotted line), obtained using different initial values, overlaid on observed basal area growth for various levels of fertilization treatments. Curves were generated for stands that were 10 yrs of age at the time of application of fertilization. Observed data (light solid line) include plots fertilized at age 9-19 years.



(a) 200 N + 25 P



(b) 300 N

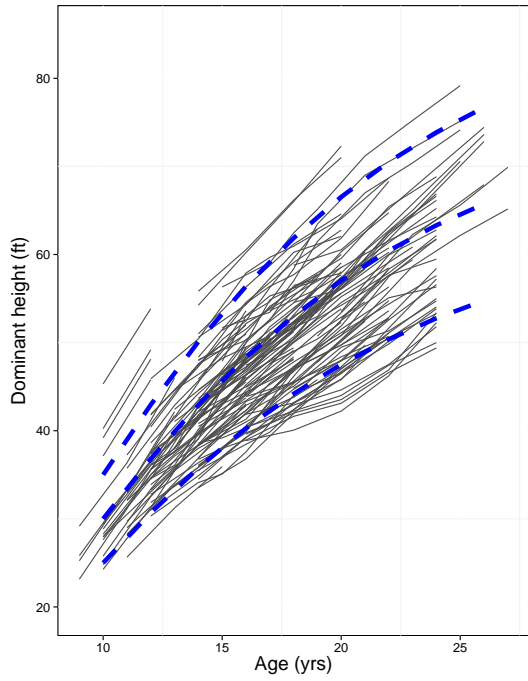


(c) 300 N + 25 P

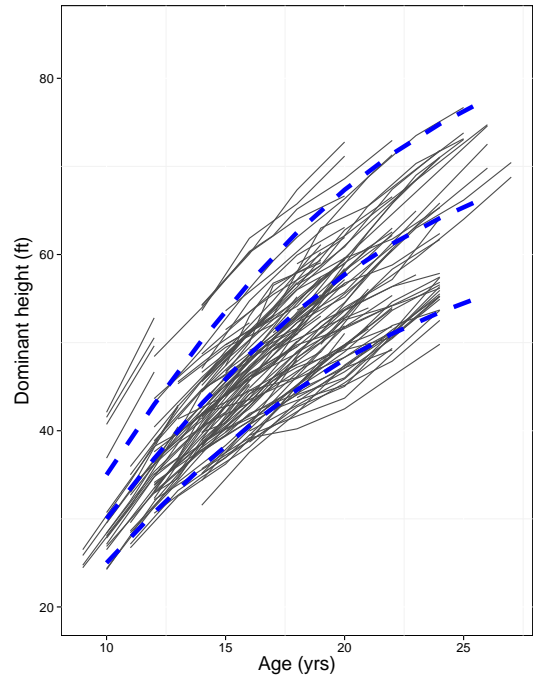
Figure D.2: Basal area projection curves (dotted line), obtained using different initial values, overlaid on observed basal area growth for various levels of fertilization treatments. Curves were generated for stands that were 10 yrs of age at the time of application of fertilization. Observed data (light solid line) include plots fertilized at ages 9-19 years.

Appendix E

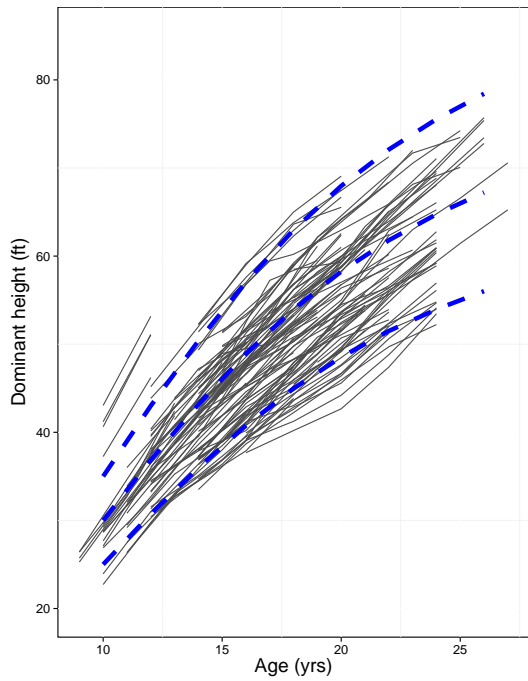
Dominant height projection following fertilization



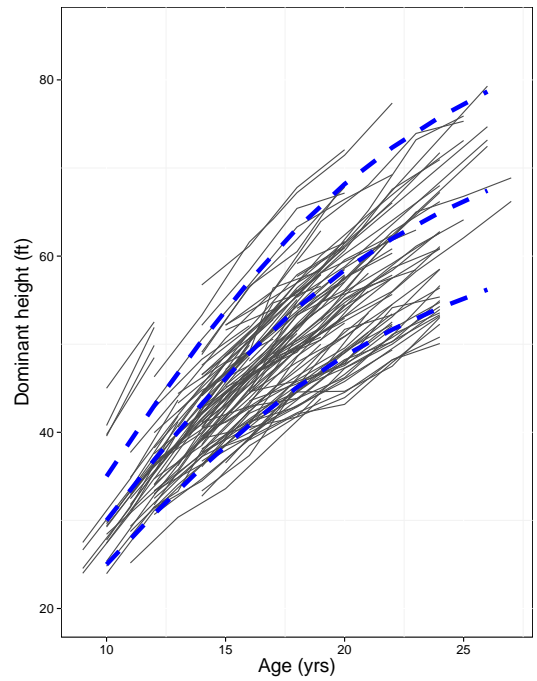
(a) Control



(b) 100 N

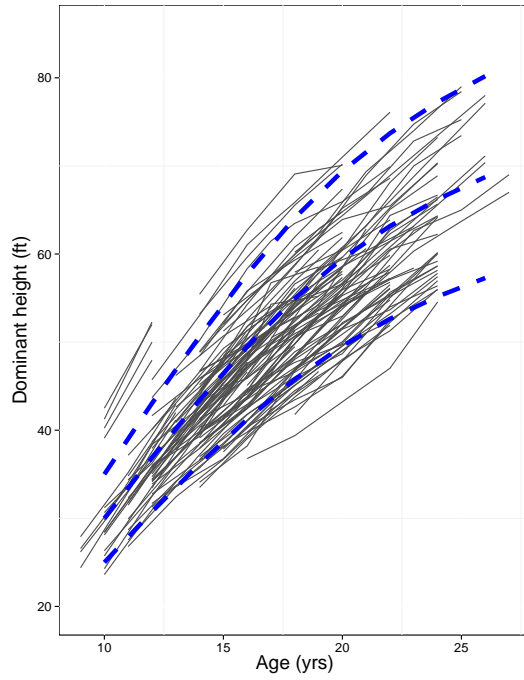


(c) 100 N + 25 P

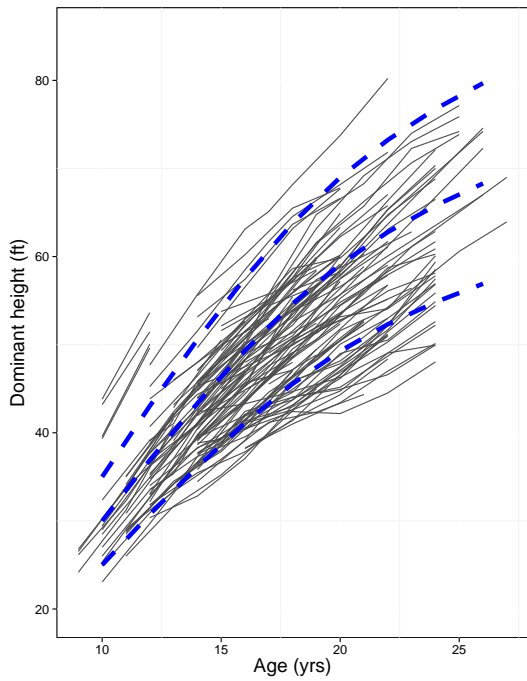


(d) 200 N

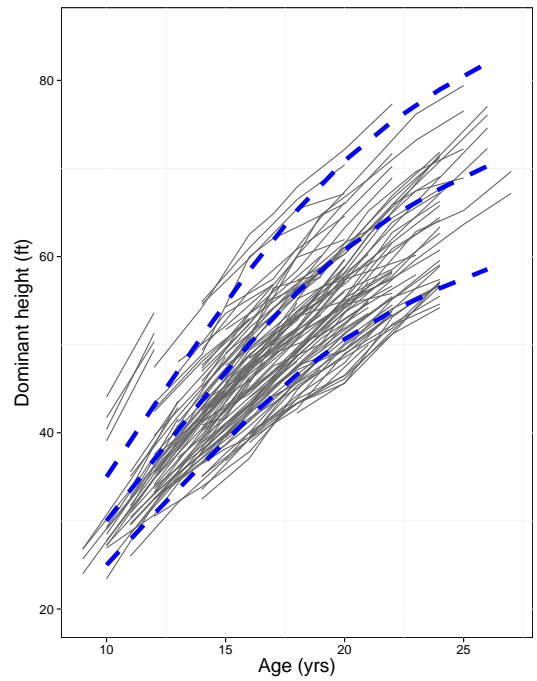
Figure E.1: Dominant height projection curves (dotted line), obtained using different initial values, overlaid on observed dominant height growth for various levels of fertilization treatments. Curves were generated for stands that were 10 yrs of age at the time of application of fertilization. Observed data (light solid line) include plots fertilized at ages 9-19 years.



(a) 200 N + 25 P



(b) 300 N



(c) 300 N + 25 P

Figure E.2: Dominant height projection curves (dotted line), obtained using different initial values, overlaid on observed dominant height growth for various levels of fertilization treatments. Curves were generated for stands that were 10 yrs of age at the time of application of fertilization. Observed data (light solid line) include plots fertilized at ages 9-19 years.