Three-Point Functions of Aging Dynamics and the AdS-CFT Correspondence

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Aging can be realized as a subalgebra of Schrödinger algebra by discarding the time-translation generator. While the two-point functions of the age algebra have been known for some time, little else was known about the higher n-point correlators. In this Letter, we present novel three-point correlators of scalar primary operators. We find that the aging correlators are distinct from the Schrödinger correlators by more than certain dressings with time-dependent factors, as was the case with two-point functions. In the existing literature, the holographic geometry of aging is obtained by performing certain general coordinate transformations on the holographic dual of the Schrödinger theory. Consequently, the aging two-point functions derived from holography look as the Schrödinger two-point functions dressed by time-dependent factors. However, since the three-point functions obtained in this Letter are not merely dressed Schrödinger correlators and instead, depend on an additional time-translation breaking variable, we conclude that the most general holographic realization of aging is yet to be found. We also comment on various extensions of the Schrödinger and aging algebras.

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A remarkable example of *nonequilibrium criticality* is represented by the phenomenon of aging [1]. Such *nonequilibrium criticality* can be observed in a ferromagnetic spin system (an Ising model) prepared in a high-temperature state, which after being quenched to a temperature at or below its critical temperature, is left to evolve freely. It is observed that the size of the clusters of ordered spins (which form and grow) is time-dependent and scales as time to some power, the inverse of which defines the dynamical exponent. In addition, the two-point correlation functions in such systems depend on *both* time values (and not *only* on the their difference, as it is the case in other critical phenomena which do not break time-translation) [1,2]. The essential physics of aging (which is crucially a nonstationary process) [1] has been recently discussed in the context of the AdS-CFT duality [3,4]. In this Letter, we clearly distinguish between aging realized as *dressed* Schrödinger dynamics from pure aging. In particular, we explicitly demonstrate this difference at the level of the three-point function. Our *novel results*, regarding the three-point function should be of practical importance in both real and numerical experiments involving aging dynamics [1].

Let us begin by reviewing the current understanding of holographic aging [1,3,4]. The Schrödinger group is the group of symmetries of the free Schrödinger equation $(2i\mathcal{M}\partial_t + \nabla^2)\varphi_S(t, \vec{x}) = 0$. The age (or aging) group is the same as the Schrödinger group, minus the time translation. To break time-translation invariance, but maintain scale invariance, a simple modification can be made to the previous equation by adding a time-dependent potential $v(t)$ [1], so that $\{2\mathcal{M}[i\partial_t - v(t)] + \nabla^2\}\varphi_A(t, \vec{x}) = 0$. (For a general discussion regarding the breaking of time-translation invariance, consult Ref. [1].) However, $v(t)$ needs to transform the same way as ∂_t and ∇^2, so $v(t) = k/t$, where k is an arbitrary constant. The field transformation, which maps these two equations into each other, is given by $\varphi_A(t, \vec{x}) = \exp[-i\int_{t_0}^t d\tau v(\tau)]\varphi_S(t, \vec{x}) = (t/t_0)^{-ik}\varphi_S(t, \vec{x})$. If one considers operators with a certain mass $\mathcal{M}$, then $O_A(t, \vec{x}) = (t/t_0)^{-ik\mathcal{M}}O_S(t, \vec{x})$. This leads, straightforwardly, to a relationship between the age and Schrödinger n-point correlators: $G_A^{(n)}(t_i, \vec{x}_i; \mathcal{M}_i) = \prod_{i=1}^n t_i^{-ik\mathcal{M}_i} G_S^{(n)}(t, \vec{x}_i; \mathcal{M}_i)$, where the t_0 dependence cancels due to the Bargmann selection rule $\sum_i \mathcal{M}_i = 0$.

The holographic dual of a system which is invariant under the realization of age algebra using the above trick (and which takes into account the singularity at $t = 0$ in $v(t) = k/t$) was constructed in Ref. [4]. The relevant age metric ds_A^2 [4] reads as:

$$ds_A^2 = \frac{R^2}{z^2}\bigg[dz^2 + \frac{2\alpha\beta}{z}dzdt - \frac{\beta^2}{z^2}\bigg(1 + \frac{\alpha z^2}{\beta t}\bigg)dt^2 - 2dtd\xi + d\vec{x}^2\bigg]. \tag{1}$$

(This geometry is locally Schrödinger, but its global structure is not.) Then, it is easy to check that $\Phi_A(t, \xi, \vec{x}, z) = \Phi_S(t, \xi + \frac{\alpha\beta}{2}\ln\frac{\beta t}{z^2}, \vec{x}, z)$ obeys the equation of motion $\Box\Phi_A = 0$ in the age metric, if Φ_S obeys the equation of motion $\Box\Phi_S = 0$ in the Schrödinger background

$$ds_S^2 = \frac{R^2}{z^2}\left(dz^2 - \frac{\beta^2}{z^2}dt^2 - 2dtd\xi + d\vec{x}^2\right). \quad (2)$$

(This can be extended for fields of arbitrary spin.) In terms of the boundary field values $\bar{\Phi}_S$, the bulk Schrödinger field can be written as

$$\Phi_S(t,\xi,\vec{x},z) = \int_{t',\xi',\vec{x}'} \mathcal{G}_S(t-t',\xi-\xi',\vec{x}-\vec{x}',z)\bar{\Phi}_S(t',\xi',\vec{x}'), \quad (3)$$

which, after applying the above map becomes

$$\Phi_A\left(t, \xi - \frac{\alpha\beta}{2}\ln\frac{\beta t}{z^2}, \vec{x}, z\right) = \int_{t',\xi',\vec{x}'} \mathcal{G}_S(t-t', \xi-\xi', \vec{x}-\vec{x}', z) \times \bar{\Phi}_A\left(t', \xi' - \frac{\alpha\beta}{2}\ln\frac{\beta t'}{z_b^2}, \vec{x}'\right), \quad (4)$$

where z_b is the value of z at the regularized boundary ($z_b \ll 1$). Here, $\mathcal{G}_S(t-t', \xi-\xi', \vec{x}-\vec{x}', z)$ denotes the boundary-to-bulk propagator of a field in the Schrödinger background. Fourier transforming along the ξ direction, we find

$$\Phi_A(t,\vec{x},z;\mathcal{M}) = \int_{t',\vec{x}'} \exp\left[i\mathcal{M}\frac{\alpha\beta}{2}\left(\ln\frac{\beta t}{z^2} - \ln\frac{\beta t'}{z_b^2}\right)\right] \times \mathcal{G}_S(t-t',\vec{x}-\vec{x}',z;M)\bar{\Phi}_A(t',\vec{x}';\mathcal{M}). \quad (5)$$

This enables us to reconstruct, with relative ease, the holographic answer for the correlators of primary operators, with respect to the age algebra from those derived using the Schrödinger background holography [5,6]. For example, the three-point function of a scalar operator is

$$\delta(\mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_3)\Lambda\prod_{i=1,2,3}(t_i)^{-i\mathcal{M}(\alpha\beta/2)} \times \int_{t,\vec{x},z}\mathcal{G}_S(t-t_i,\vec{x}-\vec{x}_i;\mathcal{M}_i), \quad (6)$$

assuming that the scalar source, $\bar{\phi}(\mathcal{M})$, has a cubic coupling in the bulk: $S_{\text{grav}} = \int[\cdots + \frac{1}{3!}\Lambda\phi^3(t,\vec{x},z;\mathcal{M}) + \ldots]$. By using the Bargmann superselection rule $\sum_i \mathcal{M}_i = 0$ (which in this holographic context is simply the momentum conservation along the S^1 direction parametrized by ξ), the three-point function is reduced to the previous form. To get the age correlator, one performs a discrete light cone quantization projection along ξ followed by a functional differentiation with respect to the boundary fields. This guarantees that the age correlators differ from the Schrodinger correlators only by time-dependent phase factors, if the tensor indices are in the $\vec{x}$ directions, as in the case of scalar operators.

One of the main points of this Letter is that this realization of aging does not capture the most general aging dynamics and that what has been described above is just a particular realization of the Schrödinger dynamics. In what follows, we clearly distinguish between this special case and the most general aging dynamics.

For simplicity, let us consider a $(1+1)$-dimensional theory with coordinates t, r. We use ξ to denote the Fourier variable conjugate to the mass $\mathcal{M}$ of a certain primary operator. In the notation of Refs. [1,7], the Schrödinger and age algebras are, respectively, spanned by the generators $\{X_{-1}, X_0, X_1, M_0, Y_{1/2}, Y_{-(1/2)}\}$ and $\{X_0, X_1, M_0, Y_{1/2}, Y_{-(1/2)}\}$, which obey the following commutation relations

$$[X_n, X_{n'}] = (n-n')X_{n+n'}, \qquad [X_n, Y_m] = \left(\frac{n}{2} - m\right)Y_{n+m},$$
$$[X_n, M_{n'}] = -n'M_{n+n'}, \qquad [Y_m, Y_{m'}] = (m-m')M_{m+m'}. \quad (7)$$

The most general realization of these generators (which is apparently *new*) is

$$X_{-1} = -\partial_t + \frac{g(t)-\gamma}{t} + \frac{h(t)-\delta}{t}i\partial_\xi,$$
$$X_0 = -t\partial_t - \frac{1}{2}r\partial_r - \frac{\Delta}{2} + g(t) + h(t)i\partial_\xi,$$
$$X_1 = -t^2\partial_t - tr\partial_r - \Delta t + \frac{i}{2}r^2\partial_\xi + t(g(t)+\gamma) + t(h(t)+\delta)i\partial_\xi,$$
$$Y_{-(1/2)} = -\partial_r, \qquad Y_{1/2} = -t\partial_r + ir\partial_\xi,$$
$$M_0 = i\partial_\xi \equiv -\mathcal{M}, \quad (8)$$

where $g(t)$, $h(t)$ are arbitrary time-dependent functions and γ, δ are arbitrary constants. In arriving at Eq. (8), we have kept the form of the generators M_0 and of the spatial translation $Y_{-(1/2)}$ and generalized Galilean-invariance $Y_{1/2}$ unchanged. This general realization is central for the new results presented in what follows.

Next, we solve the partial differential constraints imposed on the three-point functions (namely, that they are left invariant by the age generators). The conclusion is that the most general scalar three-point function is

$$G_A(\{t_i, r_i, \xi_i\}) = \left(\prod_{i=1}^{3} t_i^{\gamma_i}\right)\exp\left(\sum_{i=1}^{3}\int^{t_i} d\tau\frac{g(\tau)}{\tau}\right) \times (t_3-t_1)^{-(1/2)\Delta_{31,2}+\gamma_{31,2}}(t_3-t_2)^{-(1/2)\Delta_{32,1}+\gamma_{32,1}} \times (t_2-t_1)^{-(1/2)\Delta_{21,3}+\gamma_{21,3}}\Theta_A\left[u_1, u_2, u_3, \frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}\right], \quad (9)$$

where $\gamma_{31,2} = \gamma_3 + \gamma_1 - \gamma_2$ etc. and $\Delta_{31,2} = \Delta_3 + \Delta_1 - \Delta_2$ etc. Here, $\Theta_A\Big[u_1, u_2, u_3, \frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}\Big]$ is some unconstrained function of:

$$\begin{aligned} u_1 &= -2i(\xi_2 - \xi_1) + \frac{(r_2 - r_1)^2}{t_2 - t_1} + 2\int_{t_1}^{t_2} d\tau \frac{h(\tau)}{\tau} + 2(\delta_2 - \delta_1)\ln(t_2 - t_1) + 2(\delta_1 + \delta_2)\ln\frac{t_3 - t_2}{t_3 - t_1} \\ &\quad - 2\delta_2 \ln t_2 + 2\delta_1 \ln t_1, \\ u_2 &= -2i(\xi_3 - \xi_1) + \frac{(r_3 - r_1)^2}{t_3 - t_1} + 2\int_{t_1}^{t_3} d\tau \frac{h(\tau)}{\tau} + 2(\delta_3 - \delta_1)\ln(t_3 - t_1) + 2(\delta_1 + \delta_3)\ln\frac{t_3 - t_2}{t_2 - t_1} \\ &\quad - 2\delta_3 \ln t_3 + 2\delta_1 \ln t_1, \\ u_3 &= -2i(\xi_3 - \xi_2) + \frac{(r_3 - r_2)^2}{t_3 - t_2} + 2\int_{t_2}^{t_3} d\tau \frac{h(\tau)}{\tau} + 2(\delta_3 - \delta_2)\ln(t_3 - t_2) + 2(\delta_2 + \delta_3)\ln\frac{t_3 - t_1}{t_2 - t_1} \\ &\quad - 2\delta_3 \ln t_3 + 2\delta_2 \ln t_2. \end{aligned} \tag{10}$$

For the Schrödinger three-point correlator, we find a similar expression, but without the dependence on the additional variable $\frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}$:

$$\begin{aligned} G_S(\{t_i, r_i, \xi_i\}) &= \left(\prod_{i=1}^{3} t_i^{\gamma_i}\right)\exp\left(\sum_{i=1}^{3}\int^{t_i} d\tau \frac{g(\tau)}{\tau}\right) \times (t_3 - t_1)^{-(1/2)\Delta_{31,2}+\gamma_{31,2}}(t_3 - t_2)^{-(1/2)\Delta_{32,1}+\gamma_{32,1}} \\ &\quad \times (t_2 - t_1)^{-(1/2)\Delta_{21,3}+\gamma_{21,3}}\Theta_S(u_1, u_2, u_3). \end{aligned} \tag{11}$$

Note that despite the presence of the time-dependent prefactors, this correlator is time-translation invariant. In fact, it is easy to check that a redefinition of the primary fields of the Schrödinger algebra, effected by factoring out appropriate time-dependent functions, gives the correlators of the type (11). However, this redefinition does not change the fact that $X_{-1}G_S(\{t_i, r_i, \xi_i\}) = 0$. The fundamental difference between age and Schrödinger three-point functions lies in the dependence of the former on $\frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}$.

At this stage, we pause to note that the analysis performed at the beginning of this Letter, regarding the form of the age correlators, was too restrictive. Since the time-dependent potential is introduced by a simple redefinition of the fields, $\varphi_S(t, \vec{x}) = \exp[i\int^t d\tau v(\tau)]\varphi_A(t, \vec{x})$, the relevant symmetry group is still the full Schrödinger and *not* the age group. One of the consequences of this observation is that the holographic realization of aging [Eq. (1)] is equally restrictive, and thus, the most general holographic age background is yet to be found. This is further evidenced by the fact that the three-point correlators implied by the holographic age metric [Eq. (1)] are dressed Schrödinger correlators (i.e., they are *fake* age correlators), whereas the ones in Eq. (9) are not.

For completeness, we also present the three-point correlators of the scalar fields in terms of their masses:

$$\begin{aligned} G_A(\{t_i, r_i, \mathcal{M}_i\}) &= 2\pi\delta\left(\sum_{i=1}^{3}\mathcal{M}_i\right)\left(\prod_{i=1}^{3} t_i^{-\gamma_i+\mathcal{M}_i\delta_i}\right) \times \exp\left(\int^{t_i} d\tau \frac{g(\tau) - \mathcal{M}_i h(\tau)}{\tau}\right) \times (t_3 - t_1)^{-(1/2)\Delta_{31,2}+\gamma_{31,2}-(\mathcal{M}\delta)_{31,2}} \\ &\quad \times (t_3 - t_2)^{-(1/2)\Delta_{32,1}+\gamma_{32,1}-(\mathcal{M}\delta)_{32,1}} \times (t_2 - t_1)^{-(1/2)\Delta_{21,3}+\gamma_{21,3}-(\mathcal{M}\delta)_{21,3}} \\ &\quad \times \exp\left[-\frac{\mathcal{M}_2(r_2 - r_1)^2}{2(t_2 - t_1)} - \frac{\mathcal{M}_3(r_3 - r_1)^2}{2(t_3 - t_1)}\right] \times \tilde{\Theta}_A\left[\mathcal{M}_2, \mathcal{M}_3, w, \frac{t_3(t_2 - t_1)}{t_2(t_3 - t_1)}\right]. \end{aligned} \tag{12}$$

Here, $(\mathcal{M}\delta)_{31,2} = \mathcal{M}_3\delta_3 + \mathcal{M}_1\delta_1 - \mathcal{M}_2\delta_2$, and $w = \frac{[(t_3-t_1)(r_2-r_1)-(t_2-t_1)(r_3-r_1)]^2}{(t_3-t_2)(t_2-t_1)(t_3-t_1)}$. For the Schrödinger three-point correlators, one obtains an expression similar to Eq. (12), but with an unconstrained function $\tilde{\Theta}_S = \tilde{\Theta}_S(\mathcal{M}_2, \mathcal{M}_3, w)$.

We mention, in passing, that there are extensions of both age and Schrödinger algebras which involve the addition of new generators that ensure the closure of the respective algebras. For example, provided that $\delta = 0$ in Eq. (8), we may add to both algebras $N = -t\partial_t + \xi\partial_\xi + g(t) - \gamma' + [h(t) + \int^t d\tau \frac{h(\tau)}{\tau} + \delta']i\partial_\xi$. The nonzero commutators that involve N are $[N, X_{\pm 1}] = \mp X_{\pm 1}$, $[N, M_0] = -M_0$, $[N, Y_{1/2}] = -Y_{1/2}$. In this case, the scalar three-point function of this extension of age algebra is given by

$$\hat{G}_A(\{t_i, r_i, \mathcal{M}_i\}) = 2\pi\delta\Big(\sum_{i=1}^{3}\mathcal{M}_i\Big)\Big(\prod_{i=1}^{3} t_i^{-\gamma_i}\Big)\times \exp\Big(\sum_{i=1}^{3}\int^{t_i} d\tau \frac{g(\tau)-\mathcal{M}_i h(\tau)}{\tau}\Big)(t_3-t_1)^{-(1/2)\Delta_{31,2}+\gamma_{31,2}}$$
$$\times (t_3-t_2)^{-(1/2)\Delta_{32,1}+\gamma_{32,1}}(t_2-t_1)^{-(1/2)\Delta_{21,3}+\gamma_{21,3}}\times \exp\Big[-\frac{\mathcal{M}_2(r_2-r_1)^2}{2(t_2-t_1)}-\frac{\mathcal{M}_3(r_3-r_1)^2}{2(t_3-t_1)}\Big]$$
$$\times w^{-(1/2)(\Delta_1+\Delta_2+\Delta_3)+\gamma_1+\gamma_2+\gamma_3+2}\times \tilde{\tilde{\Theta}}_A\Big[w\mathcal{M}_2, w\mathcal{M}_3, \frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}\Big]. \tag{13}$$

For the Schrodinger algebra extended by N, we encounter again an expression similar to Eq. (13) but with an unconstrained function $\tilde{\tilde{\Theta}}_S = \tilde{\tilde{\Theta}}_S(w\mathcal{M}_2, w\mathcal{M}_3)$. Yet another closed subalgebra extension of age algebra is obtained by adding both N and V_+, where $V_+ = -2tr\partial_t - 2\xi r\partial_\xi - (r^2 + 2i\xi t)\partial_r - 2\Delta r + 2r(g(t) + \gamma)$, and where $h(t) = 0$, $\delta = 0$ in Eq. (8), and $\delta' = 0$, $\gamma' = \gamma$ in the expression for N. On the other hand, adding both N and V_+ to the Schrödinger algebra and requiring its closure [8], leads to the full $(1 + 2)$-dimensional conformal algebra in a space parametrized by t, r, ξ coordinates. The generators take the form given by Eq. (8), the expressions for N and V_+, with $\delta = 0$, $h(t) = 0$, $\gamma' = \gamma$, $\delta' = 0$, supplemented by $V_- = -\xi\partial_r + ir\partial_t - \frac{ir}{t}(g(t) - \gamma)$ and

$$W = -\xi^2\partial_\xi - \xi r\partial_r + \frac{i}{2}r^2\partial_t - \Delta\xi + 2\gamma\xi - \frac{ir^2(g(t)-\gamma)}{2t}. \tag{14}$$

The familiar scalar three-point function is dressed by time-dependent factors which originate in a particular realization of the generators allowing for nonzero γ and $g(t)$:

$$G_c(\{t_i, r_i, \xi_i\}) = \Big(\prod_{i=1}^{3} t_i^{\gamma_i}\Big)\exp\Big(\sum_{i=1}^{3}\int^{t_i} d\tau\frac{g(\tau)}{\tau}\Big) \times X_{21}^{-(1/2)\Delta_{21,3}+\gamma_{21,3}}X_{31}^{-(1/2)\Delta_{31,2}+\gamma_{31,2}}X_{32}^{-(1/2)\Delta_{32,1}+\gamma_{32,1}}. \tag{15}$$

Here, X_{12}, etc. are the Lorentz invariant intervals $X_{12} = -2i(t_2 - t_1)(\xi_2 - \xi_1) + (r_2 - r_1)^2$, etc. However, we want to stress that the time-dependent factors in Eq. (15) once again do not signal any breaking of time-translation invariance, and can be obtained by redefining the primary operators of the type we have encountered earlier in this Letter.

Finally, we would like to comment on the holographic realization of the age algebra in terms of metric isometries of a $(1 + 3)$-dimensional space. The holographic dual space is parametrized by x^μ coordinates: t, r, ξ, and the holographic coordinate z. The main observation is that once one identifies the Killing vectors $K = K^\mu\frac{\partial}{\partial x^\mu}$ obeying the age algebra, one can reverse engineer the metric by solving the Killing vector equations for the components of the metric, i.e., $(g_{\rho\nu}\partial_\mu + g_{\rho\mu}\partial_\nu)K^\rho + K^\rho\partial_\rho g_{\mu\nu} = 0$. It is natural to assume that $Y_{-(1/2)}$ and M_0 are bulk Killing vectors. If one makes the additional assumption that $Y_{1/2}$, given by Eq. (8), is a bulk Killing vector then the problem becomes quite tractable. The bulk forms of the Killing vectors X_0 and X_1 are $X_0 = -t\partial_t + X_0^\xi\partial_\xi - \frac{1}{2}r\partial r + X_0^z\partial_z$ and $X_1 = -t^2\partial_t + X_1^\xi\partial_\xi - tr\partial_r + X_1^z\partial_z$, where $X_0^z = \frac{\partial_t(tg_{rr})}{\partial_z g_{rr}}$, $X_1^z = \frac{\partial_t(t^2 g_{rr})}{\partial_z g_{rr}}$, and

$$X_0^\xi = \frac{i}{2t\partial_z g_{rr}}\Big[-\partial_z(g_{rr}S) - t\partial_t g_{rr}\partial_z S + t\partial_t S\partial_z g_{rr} + 2tT\partial_z g_{rr} + 2tC_1\partial_z g_{rr}\Big],$$
$$X_1^\xi = \frac{i}{2\partial_z g_{rr}}\Big(-z^2\partial_z g_{rr} - 2g_{rr}\partial_z S - S\partial_z g_{rr} - t\partial_t g_{rr}\partial_z S + t\partial_t S\partial_z g_{rr} + 2tT\partial_z g_{rr}\Big). \tag{16}$$

Here, $g_{rr} = g_{rr}(t, z)$, $S = S(t, z)$, $T = T(t)$, and C_1 is an arbitrary constant. Solving the Killing vector equations corresponding to $Y_{-(1/2)}$ and M_0 leads to a metric which is ξ, r-independent. Furthermore, solving the $Y_{1/2}$ Killing equations brings the metric to a form which coincides with the initial ansatz of Ref. [4]: $ds^2 = g_{tt}(t, z)dt^2 + g_{rr}(t, z)dr^2 + g_{zz}(t, z)dz^2 - 2ig_{rr}(t, z)dtd\xi + 2g_{tz}(t, z)dtdz$. The other components of the metric are determined by the remaining Killing equations: $g_{zz} = \frac{C_2(\partial_z g_{rr})^2}{g_{rr}^2}$ and

$$g_{tz} = \frac{g_{rr}\partial_z S}{2t} + \frac{C_2\partial_z g_{rr}\partial_t(t^2 g_{rr})}{t^2 g_{rr}^2} + C_1\partial_z g_{rr},$$
$$g_{tt} = C_3 g_{rr}^2 + \frac{C_2[\partial_t(t^2 g_{rr})]^2}{t^4 g_{rr}^2} + \frac{2C_1\partial_t(t^2 g_{rr})}{t^2} + \frac{g_{rr}(2tT - S + t\partial_t S)}{t^2}, \tag{17}$$

where C_2, C_3 are additional integration constants. We stress that this metric is the *most general* solution of the reverse-engineering procedure, given the confines of the initial assumption that $Y_{1/2}$ becomes a bulk Killing vector while remaining unchanged. However, we cannot claim that we have identified the holographic dual of a general theory possessing the full symmetry of the age algebra. The reason for this is that we are able to identify one more Killing vector of the metric compatible with the following bulk extension of X_{-1}

$$-X_{-1} = \partial_t - \frac{\partial_t g_{rr}}{\partial_z g_{rr}}\partial_z + i\bigg(-\frac{2C_2}{t^2 g_{rr}} + \frac{S}{2t^2} - \frac{\partial_t S + 2T + 4C_1}{2t} + \frac{\partial_t g_{rr}\partial_z S}{2t\partial_z g_{rr}}\bigg)\partial_\xi. \quad (18)$$

Thus, the isometries of the above metric generate the full Schrödinger algebra, as in Ref. [4]. Naturally, the correlators computed from this metric using holography exhibit the kind of "fake" aging discussed earlier, and are constrained by the full Schrödinger algebra. For the time being, we can only trace this feature to the assumption made regarding the bulk realization of $Y_{1/2}$. (This assumption was also made in Ref. [4].) This condition was imposed at a purely technical level, such that the Killing vector equations became tractable. A logical course of action is to remove this restriction. However, in this case, we have not been able to solve the Killing vector equations and identify a metric. Perhaps a reasonable line of attack would be to try to solve them in a perturbative expansion in the bulk coordinate (a sort of Fefferman—Graham expansion) starting from the boundary. We leave this question for our future work.

In conclusion, in this Letter, we have clearly pointed out the difference between the aging dynamics realized as *dressed* Schrödinger dynamics from pure aging. In particular, we have obtained the three-point functions for aging which *cannot* be obtained by *dressing* the three-point Schrödinger correlators. Regarding the unconstrained function appearing in Eq. (9), which is similar to the unconstrained function present in scalar four-point correlators of the conformal algebra and which depend on the specific conformal field theory under consideration, it would be useful to determine this function in a particular case of aging dynamics. Perhaps it could be used to reverse engineer the particular holographic metric as in Ref. [5]. The physical implications of our new results are yet to be understood. Nevertheless, it is reasonable to expect that these results will have practical importance in the real and numerical experiments of aging dynamics [1] and that they should be generalizable to the relativistic context, with possible applications to the physics of the quark-gluon plasma. We plan to address these issues in our future work.

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