

Prospects for the Collision-Free Car:
The Effectiveness of Five Competing Forward Collision Avoidance Systems

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ABSTRACT

Rear-end collisions in which the leading vehicle was stationary prior to impact and at least one vehicle was towed from the crash site represent 18% of all yearly crashes in the United States. Forward Collision Avoidance Systems (FCASs) are becoming increasingly available in production vehicles and have a great potential for preventing or mitigating rear-end collisions. The objective of this study was to compare the effectiveness of five crash avoidance algorithms that are similar in design to systems found on production vehicles of model year 2011. To predict the effectiveness of each algorithm, this study simulated a representative sample of rear-end collisions as if the striking vehicle was equipped with each FCAS.

In 2011, the ADAC (Allgemeiner Deutscher Automobil-Club e.V) published a test report comparing advanced emergency braking systems. The ADAC tested production vehicles of model year 2011 made by Audi, BMW, Infiniti, Volvo, and VW. The ADAC test results were used in conjunction with video evidence and owner's manual information to develop mathematical models of five different FCASs. The systems had combinations of Forward Collision Warning (FCW), Assisted Braking (AB), and Autonomous Emergency Braking (AEB).

The effectiveness of each modeled system was measured by its ability to prevent collisions or reduce the collision severity of reconstructed crashes. In this study, 977 rear-end crashes that occurred from 1993 to 2008 were mathematically reconstructed. These crashes were investigated as part of NHTSA's National Automotive Sampling System, Crashworthiness Data System (NASS/CDS). These crashes represent almost 800,000 crashes during that time period in which the struck vehicle was stationary. Part of the NASS/CDS investigation was to reconstruct the vehicle change in velocity during impact, ΔV . Using energy and Newtonian based methods, the ΔV in each crash was calculated as if the vehicle was equipped with each modeled FCAS. Using the predicted reduction in crash ΔV , the expected reduction in the number of moderately-to-fatally injured (MAIS2+) drivers was predicted.

This study estimates that the most effective FCAS model was the Volvo algorithm which could potentially prevent between 79% and 92% of the crashes simulated in this study and between 76% and 94% of associated driver injuries. This study estimates that the BMW algorithm would prevent the fewest number of crashes (between 11% and 14%), but would provide admirable benefits to driver safety by preventing between 21% and 25% of driver injuries. The VW algorithm would be the least effective at preventing driver injuries if the system were to be implemented across the U.S. fleet. This algorithm offers a 19% reduction in crashes, but only prevents 15% of driver injuries.

This study introduces and demonstrates a unique method of comparing potential benefits of competing FCAS algorithms. This method could be particularly useful to system designers for comparing the expected effects of design decisions on safety performance. This method could also be useful to government officials who wish to evaluate the effectiveness of FCASs.

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1. INTRODUCTION

From 2007-2011, rear-end collisions represented 24.7% of all tow-away crashes in the United States. On an annual average, there were 2,053,570 rear-end tow-away crashes per year and 70,129 annual serious-to-fatal occupant injuries (MAIS3+) that were caused by these crashes [1]. Forward Collision Avoidance Systems (FCASs) are automobile active safety systems that are designed to either reduce the severity of or completely prevent rear-end crashes. These systems are becoming increasingly available in production vehicles and have a great potential for preventing or mitigating rear-end collisions.

Figure 1 provides an example of a crash that was mitigated with an FCAS, and illustrates three sub-systems that compose an FCAS: Forward Collision Warning (FCW), Assisted Braking (AB), and Autonomous Emergency Braking (AEB). It is important to note that these sub-systems are sometimes referred to by different names. Assisted Braking (AB) is sometimes called Pre-Crash Brake Assist (PBA), Dynamic Brake Assist (DBA), or Dynamic Brake Support (DBS). Autonomous Emergency Braking (AEB) is sometimes called autonomous Pre-crash Brake (PB) or Crash Imminent Braking (CIB).

FCW is an audible, visual, or tactile stimulus, intended to alert the driver of a potential collision. The purpose of an FCW is to alert a drowsy, inattentive, or distracted driver of an upcoming potential collision. The FCW is arguably the least intrusive of the three sub-systems. During or shortly after this warning, some systems will prepare for Assisted Braking (AB). If a driver applies insufficient brake pressure in a pre-crash event, AB will magnify the braking level in order to reduce the likelihood or severity of a collision. The AB sub-system will address the problem of drivers misjudging the brake force required to prevent a collision. Systems that have an AEB sub-system will autonomously apply a braking acceleration (a_{AEB}) when the system deems a

crash imminent. The AEB sub-system helps prevent or mitigate the severity of crashes for cases in which the driver reacts too slowly to an FCW.

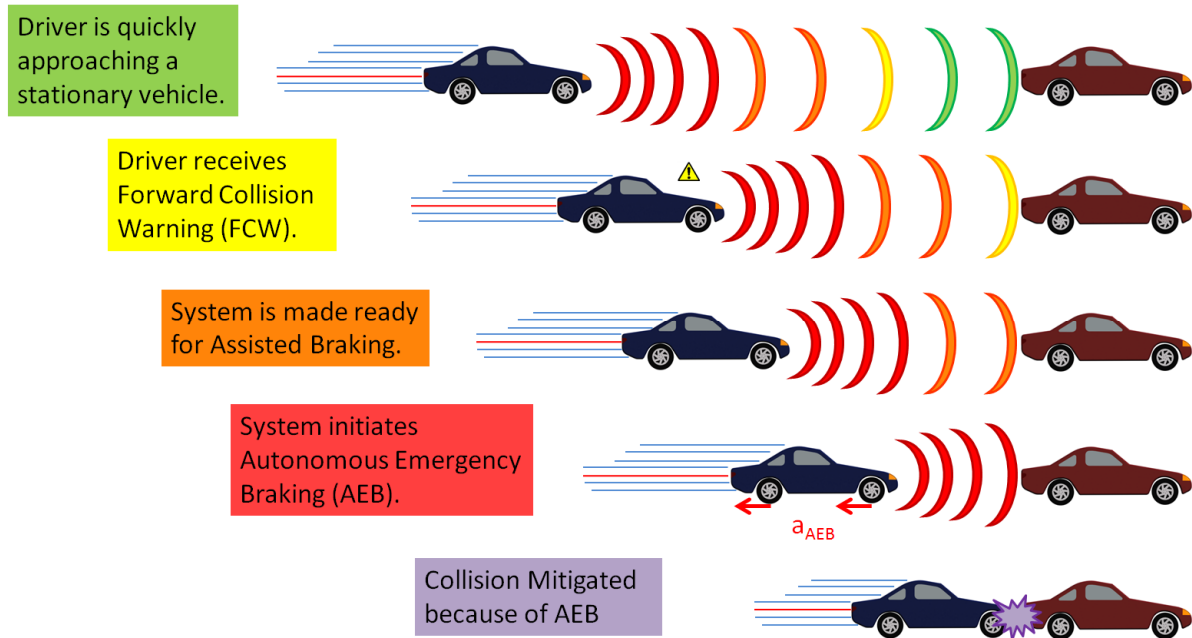


Figure 1. Components of Forward Collision Avoidance System

All components of FCASs rely on sensors to track vehicles and objects in front of the equipped vehicle. Automakers often use millimeter-wave radar scanning technology as a means for sensing vehicles and objects. Some systems use cameras instead of or in addition to radar technologies.

1.1 FCAS COMPONENT DESIGN PHILOSOPHIES

The different approaches taken by automakers in the design of FCASs reflects their different design philosophies. One philosophy is that the driver should always maintain complete control of the vehicle and no system should interfere with that control, even in emergency situations. A philosophy like this one would likely yield an FCAS that only has an FCW. Some automakers (or

perhaps more importantly, their customers) might fear that systems such as AB or AEB could result in a collision due to system malfunction or a hazardous driver reaction to the system's activation.

A second philosophy might permit the use of an AB system in addition to an FCW. Automakers that adhere to this design philosophy might be wary of an AEB system for the same reasons described above. The justification for the use of an AB system would be that the driver is still in complete control of the vehicle.

A third design philosophy would permit the use of all three systems, but AEB would only activate in emergency situations. An emergency situation would be a scenario in which the driver could not avoid the leading vehicle by means of braking, steering, or a combination of braking and steering.

A fourth design philosophy would be to give more control to the AEB system, allowing it to activate in non-emergency situations. This design philosophy will yield systems that are precursors to systems that will likely be found in automated vehicles.

Another contributing factor of design variation among different FCASs is cost. The effectiveness of a system is likely very dependent on development and implementation costs. For example, a system that uses a radar or LiDAR system in addition to a camera might be more effective than a system that only uses a camera, but the later system will most likely be less expensive to install into a vehicle.

1.2 CHOOSING ACTIVATION TIMES FOR FCAS COMPONENTS

Apart from choosing which systems to encapsulate into an FCAS, automakers also have to decide on activation times for each system. The decision for system activation times is a trade-off between customer safety and customer satisfaction. If the activation time is too early, drivers might deem the systems annoying and turn the systems off. As the activation time to collision decreases,

the benefits of the system are diminished because the driver does not have as much time to react to the emergency situation.

1.3 CHOOSING BRAKING MAGNITUDES FOR AEB SYSTEMS

Automakers have to decide on a braking magnitude for AEB systems. Choosing a high braking magnitude would likely prevent more crashes and could reduce the number of injuries of vehicle occupants. High braking magnitudes, however, could cause unwanted effects. If the braking magnitude is high enough, occupants could be injured or could shift out of a safe position, putting them at greater risk of injury in the event of a crash. Another factor that automakers have to consider in the selection of a braking magnitude for an AEB system is customer satisfaction. Some customers might not want a system that will brake very hard and cause the driver to think that he or she is not in control of the vehicle.

Our hypothesis is that the algorithm which controls the FCAS has a direct effect on the safety benefits which could be realized by each system. The specific design of FCASs is rapidly evolving and automakers are taking drastically different approaches to the design of these systems. Needed is an assessment of how current algorithm design affects safety performance.

1.4 OBJECTIVE

The objective of this study was to compare the estimated effectiveness of FCAS algorithms in five different model year 2011 production vehicles.

1.5 METRICS FOR EVALUATING EFFECTIVENESS OF FCASs

The five FCASs were analyzed and compared on a basis of their abilities in preventing crashes and trailing vehicle driver injuries of at least moderate severity. The classification of injury severity is based on the Maximum Abbreviated Injury Score (MAIS). Injuries that are considered to be moderate in severity are assigned a score of 2. This study estimated the number of moderate-

to-fatal injuries (MAIS2+) that could be prevented by each FCAS if it were installed in all vehicles in the United States fleet.

1.6 TARGET POPULATION

To estimate the effectiveness of the modeled FCASs, crashes that occurred between 1993 and 2008 were mathematically reconstructed and simulated as if each striking vehicle were equipped with each FCAS. The population of crashes reconstructed in this study comprises rear-end crashes in which the leading (struck) vehicle was stationary prior to the crash. Crashes of interest were limited to these lead vehicle stationary scenarios because there was insufficient data to model the functionality of FCASs when the leading vehicle was moving prior to the crash. Crashes in which the leading vehicle was stationary prior to the collision account for 70% of all rear-end crashes in which at least one vehicle was towed from the crash site.

As a result of the database chosen for this study, the population of crashes was limited to those crashes in which at least one of the vehicles was towed from the crash site. This limited the analysis of cases to those with impact speeds of approximately 20 km/h and above. Crashes in which the leading vehicle was stationary prior to the collision account for 18% of all U.S. crashes in which at least one vehicle was towed from the crash site.

Injury reduction benefits were only calculated for the driver of the trailing vehicle. Injury prevention benefits were restricted to those cases in which the driver was wearing a seatbelt. Injury prevention benefits were not calculated for unbelted drivers because the potential dynamics of the driver are unknown during FCAS operation. It is possible that an FCAS could cause an unbelted driver to shift into an unsafe position prior to a crash, thereby, influencing the probability of an injury.

1.7 PRIOR RESEARCH

It is unsurprising that studies with an objective of designing Collision Avoidance Systems often have an evaluation component. One of these studies that is noteworthy was a design and evaluation study for a Rear-End Collision-Warning System [2]. The objective of the proposed design (which would be installed on a lead vehicle) was to alert a trailing vehicle of a potential rear-end crash situation by flashing its rear lights. The researchers limited the scope of the evaluation of the system to straight, dry, paved, arterial roads. The researchers estimated that their system would be over 99% effective at preventing crashes and would have an expected nuisance probability of 5%.

The remaining papers that were read in preparation for this thesis focused primarily on the evaluation of Collision Avoidance Systems. Godbole et al. presented a hierarchical framework for evaluating crash avoidance systems [3]. Although the paper was demonstrative of the framework, no systems were evaluated to estimate potential benefits. The paper emphasizes the need for “real-world data” that would be used in the process of evaluating a Crash Avoidance System.

Several studies used driving simulators to estimate the effectiveness of Collision Avoidance Systems. Lee et al. conducted a simulator study with the objective of determining if collision warning systems mitigate distraction due to in-vehicle devices [4]. In the study, three different simulated scenarios were presented to various human subjects. The baseline scenario in which no warning was provided to the “driver” resulted in a simulated crash proportion of 45.5%. Another scenario was defined by a late crash warning. In this scenario, 22.5% tests yielded simulated crashes which translates to 50.5% effectiveness for the late crash warning. The final scenario was defined by an early crash warning. In this scenario, 8.8% tests yielded simulated crashes which translates to 80.6% effectiveness for the early crash warning. This study also discusses the potential hazards of an FCAS. For example, “If a driver’s trust exceeds [Rear-End Collision Avoidance

System] capabilities then the warnings may be relied upon excessively and drivers' attention to the roadway may decline.”

The studies of greatest interest to this thesis were those that used theoretical and/or computational models to estimate the potential benefits of FCASs. As part of an analysis of emergency situations Drahos et al. carried out a special analysis of 215 accidents in order to assess the collision avoidance potential of radar warning, radar-actuated brakes, and anti-lock brakes [5]. The researches found that a combination of these systems would have had a beneficial effect for 38% of the analyzed crashes. Farber et al. estimated the potential benefits of collision warning systems with a quasi-Monte Carlo routine [6]. They estimated that for a collision warning system with a range of approximately 250 feet, the potential benefits in terms of preventing crashes could exceed 50%. McLaughlin used naturalistic data and a computational method to predict a 50%-70% reduction in crashes for three different CAS algorithms [7]. Lindman and Tivesten presented a method for estimating benefits of AEB systems and estimated that 50% of the crashes of interest could be prevented [8]. Kusano et al. estimated the effectiveness of an FCAS that had an FCW system, AB functionality, and an AEB system [9]. The study estimated that 7.7% of rear-end crashes would be prevented and 50% of moderate to fatal injuries would be prevented for drivers wearing a seatbelt.

1.8 OUTLINE OF THESIS

Chapter 2 of this study discusses the methods that were used to model the five FCASs that were analyzed in this study and also provides defining features of each system. Three separate methods were used to model the FCASs. All three methods are described in great detail in the chapter.

Chapter 3 provides an in-depth description of the methods used to compute benefits of FCASs. The chapter discusses how crashes were mathematically reconstructed to estimate the

dynamics of a trailing vehicle prior to a crash. The chapter continues by discussing how each case was simulated several times to estimate the impact severity of a crash or near-crash event. Lastly, the chapter describes a methodology of computing the probability of a driver being injured and estimating the fleet-wide effectiveness of FCASs.

Chapter 4 provides the results of simulating reconstructed crashes as if they were equipped with each modeled FCAS. The results for each method of modeling the FCASs are described in great detail followed by a summary of all of the results. The chapter concludes with an assessment of the effectiveness of each system.

Chapter 5 presents conclusions drawn from the results, states the limitations of this study, and gives recommendations for future studies.

2. METHODS AND RESULTS OF MODELING FCASS

As a first step toward estimating the potential benefits of production FCASSs, this chapter describes a method for modeling the functionality of each FCAS as an algorithm. The modeled algorithms were based on systems found on the Audi A7, BMW 530d, Infiniti M, Volvo V60, and VW Passat. Although these algorithms are only models of these five systems, this thesis refers to the algorithms by the name of the manufacturer of the associated vehicle. The reader should keep in mind that the models presented in this chapter are only models, and not true representations of the actual systems found on the aforementioned vehicles. The algorithms were modeled using test track data, data found in owner's manuals, European New Car Assessment Programme (Euro NCAP) information, and videos of the FCASSs during operation.

2.1 OVERVIEW OF DATA SOURCES

In 2011, the ADAC (Allgemeiner Deutscher Automobil-Club e.V) published a test report comparing advanced emergency braking systems. The ADAC tested FCASSs available for the Volvo V60, the Mercedes CLS, the Audi A7, the VW Passat, the BMW 530d, and the Infiniti M37S. Each FCAS was tested in various scenarios that were representative of typical rear-end collision situations. [10]

Owner's manuals for the Volvo V60, Infiniti M, and BMW 5series were downloaded from automaker websites [11]–[13]. Equivalent information for the VW and Audi was obtained from the Euro NCAP program website. The European New Car Assessment Programme (Euro NCAP) is a voluntary vehicle safety system that was modeled after the U.S. New Car Assessment Program (NCAP). The U.S. NCAP was introduced in 1979 by the U.S. National Highway Traffic Safety Administration. The Euro NCAP publishes safety reports on new cars, and recently, new active safety systems. “Euro NCAP Advanced is a new reward system for advanced safety technologies [14].” “Many of these technologies focus on avoiding the crash by informing, advising and alerting

you about dangerous situations and by assisting you to avoid the accident. Some technologies optimally prepare the vehicle's safety systems, just milliseconds before a crash, in order to provide the best possible protection. Even other technology saves crucial minutes for emergency services to arrive at the accident scene, helping medical personnel to deliver the best support given the circumstances [15].” “By rewarding technologies, Euro NCAP provides an incentive to manufacturers to accelerate the standard fitment of important safety equipment across their model ranges and helps the car buyer to make the right purchase decision [15].” Active safety systems that were rewarded by Euro NCAP include the Volkswagen City Emergency Brake and the Audi Pre Sense Front. Information concerning these systems was obtained from the Euro NCAP website [16], [17].

Three videos were used in this study to help reconstruct the FCAS algorithms. One of the videos was taken of the ADAC tests and shows the VW system during operation [18]. The video clip used from this video spans from 1:31 to 1:35 minutes. Just prior to this video clip of the VW approaching the stationary balloon car, the narrator states “[The first test begins].” We interpreted this statement as meaning that this test was of the scenario in which the VW was approaching a stationary balloon car with a speed of 20 km/h. The driver did not brake and the AEB sub-system completely prevented the crash.

The second video analyzed was taken during a test/demonstration done by the Insurance Institute for Highway Safety (IIHS) [19]. The video clip used from this video was from 0:17 to 0:20 minutes. This video clip shows a 2012 Volvo XC60 approaching a stationary balloon car with a speed of 10 mph (16 km/h). The driver did not brake and the AEB sub-system completely prevented the crash. A still frame at time 2:42 minutes was analyzed to determine the final stopping distance between the Volvo's front bumper and the balloon car's rear “bumper.”

The third video was taken of a test/demonstration done at the Volvo Safety Center in Sweden. The video clip used in this study was of taken between 1:54 and 1:56 minutes. The Volvo was approaching the rear of a stationary tractor trailer with an initial speed of 45 km/h. There was no driver present in the vehicle. The crash was not prevented, but the severity was mitigated by the AEB sub-system.

2.2 FCAS ALGORITHM DESCRIPTION

Forward Collision Avoidance Systems are often combinations of Adaptive Cruise Control (ACC) systems and other systems that go by a variety of names. Table 1 shows the names and prices of the systems that were available for the vehicles that were tested by the ADAC. These names and numbers come from the ADAC test report.

Table 1. List of available systems and respective prices for five ADAC test vehicles

ADAC Test Vehicles	Available Systems	System Price (€)
Audi A7 3.0 TFSI quattro	ACC including pre-sense	1460
BMW 530d Automatic	ACC including Adaptive Brake Assistant	1550
Infiniti M37S Premium	ACC including Intelligent Brake Assist	standard
VW Passat Variant 2.0 TFSI DSG Highline	ACC including Front Assist	1195
Volvo V60 D5 AWD Geartronic SUMMUM	ACC including Full Auto Brake	1700
	City Safety	standard

Table 2 presents the variables necessary to completely define the functionality of an FCAS. The Lower Speed Threshold of Operation (*LSTO*) and the Upper Speed Threshold of Operation (*USTO*) were defined based on statements in owner's manuals and Euro NCAP reports and validated by data and statements in the ADAC test report. $FCW?$, $AB?$, $AEB?$, and TTC_{FCW} were defined based on the text and data in the ADAC test report. TTC_{AEB} , a_{AEB} , and r_{AEB} were defined by using a combination of the ADAC test report data and video analysis. The techniques used to define these three variables will be discussed in the following sections.

Table 2. List of variables that define the functionality of a Forward Collision Avoidance System

Variables that Define an FCAS	Description	Primary Data source(s)	Secondary Data Source
<i>LSTO</i>	Lower Speed Threshold of Operation. FCAS does not function below this speed.	Owner's Manuals Euro NCAP	ADAC Test Report
<i>USTO</i>	Upper Speed Threshold of Operation. FCAS does not function above this speed.	Owner's Manuals Euro NCAP	ADAC Test Report
<i>FCW?</i>	Does the FCAS provide an FCW when the leading vehicle is stationary?	ADAC Test Report	–
<i>AB?</i>	Does the FCAS offer Assisted Braking when the leading vehicle is stationary?	ADAC Test Report	–
<i>AEB?</i>	Does the FCAS provide AEB when the leading vehicle is stationary?	ADAC Test Report	–
<i>TTC_{FCW}</i>	Time at which driver is warned of a potential collision	ADAC Test Report	–
<i>TTC_{AEB}</i>	Time at which FCAS initiates Autonomous Emergency Braking	ADAC Test Report Video Analysis	–
<i>a_{AEB}</i>	Acceleration due to Autonomous Emergency Braking (AEB)	ADAC Test Report Video Analysis	–
<i>r_{AEB}</i>	Braking ramp due to Autonomous Emergency Braking (AEB)	ADAC Test Report Video Analysis	–

2.3 ADAC TEST DATA

A list of all tests and the data reported for each test is shown in Table 3. Most of the tests were designed to analyze the effectiveness of the FCW systems and the AEB systems. With the exception of the test in which a vehicle traveling at 70 km/h was approaching a stationary vehicle, the impact velocity was reported in addition to the time at which the driver was warned. In the test scenario in which the test vehicle was traveling at a speed of 70 km/h and the balloon car was stationary, the AEB systems were not tested due to safety concerns. The balloon car was only deemed safe for collisions up to relative impact speeds of 40 km/h.

In addition to these tests, each FCAS was tested for AB functionality. In the AB functionality test, the test vehicle was traveling at 100 km/h, approaching a vehicle traveling at a speed of 20 km/h. Approximately one second after the driver was warned by the FCW the driver stepped on the brake pedal and maintained a force that, without AB, would have resulted in a -3 m/s^2 acceleration. Without an AB system, this force would not have been sufficient to prevent a collision with the leading vehicle. For all vehicles with an AB system, the system applied the amount of force required to prevent the collision. This study used the ADAC AB functionality test results as indicators for the presence of AB in each system. Our assumption was that AB systems functionalities were independent of the leading vehicle's speed and the trailing vehicle's speed.

This study used the ADAC test results of model year 2011 vehicles made by Audi, BMW, Infiniti, Volvo, and VW to develop mathematical models of five different FCAS algorithms. When available, owner's manuals, Euro NCAP information, and video evidence were used to help define the algorithms. Because the ADAC test results were limited to a few scenarios and speeds, our models only approximate the systems' behavior for scenarios in which the leading vehicle was stationary prior to the crash. The items in Table 2 that are emphasized in bold signify the information that was used in this study to model the FCASs. Some data published by the ADAC was not used in this study because the focus of this study was on benefits estimations for scenarios in which the lead vehicle was stationary during the emergency event.

Table 3. Information reported by all ADAC tests

Test Scenario	TTC_{FCW}	AB?	TTC_{AEB}	a_{AEB}	Impact Speed	Used in this Study
20 km/h vehicle approaching stationary vehicle	Yes	No	No	No	Yes	Yes
30 km/h vehicle approaching stationary vehicle	Yes	No	No	No	Yes	Yes
40 km/h vehicle approaching stationary vehicle	Yes	No	No	No	Yes	Yes
70 km/h vehicle approaching stationary vehicle	Yes	No	No	No	No	Yes
50 km/h vehicle approaching 20 km/h vehicle	Yes	No	No	No	Yes	No
100 km/h vehicle approaching 60 km/h vehicle	Yes	No	Yes	Yes	Yes	Yes
60 km/h vehicle approaching 60 km/h vehicle, accelerating at a rate of -3 m/s/s	Yes	No	No	No	Yes	No
50 km/h vehicle approaching 40 km/h vehicle, accelerating at a rate of -3 m/s/s	Yes	No	No	No	Yes	No
Brake Assist Test	No	Yes	No	No	No	Yes

2.4 FUNCTIONALITY OF SYSTEMS

All algorithms in the test set had a Forward Collision Warning (FCW) component. Each FCW operated within a specified range of vehicle speeds. Table 4 specifies the speed ranges of operability in addition to the sub-system functionalities for all of the algorithms evaluated in this

study. The speed thresholds of operations for the BMW, Infiniti, and Volvo were defined by using information in owner's manuals.

The owner's manual for the 2011 BMW 5 series states, "The system issues a two-phase warning of a danger of collision at speeds above approx.. 10 mph/15 km/h." Because of this statement, a Lower Speed Threshold of Operation (LSTO) of 15 km/h was chosen to model the BMW algorithm. The owner's manual did not indicate any Upper Speed Threshold of Operation (USTO) for the BMW system. The owner's manual was downloaded from the BMW USA website [13].

The owner's manual for the 2011 Infiniti M states, "The Forward Collision Warning (FCW) system will warn the driver by the vehicle ahead detection indicator light and chime when your vehicle is getting close to the vehicle ahead in the traveling lane. The FCW system will function when your vehicle is driven at speeds of approximately 10 MPH (15 km/h) and above." The owner's manual also states, "The Intelligent Brake Assist (IBA) system warns the driver by a warning light and chime when there is a risk of a collision with the vehicle ahead in the traveling lane and the driver must take avoidance action immediately. The system helps reduce the rear-end collision speed by applying the brakes when the system judges that the collision cannot be prevented. The IBA system will function when your vehicle is driven at speeds of approximately 10 MPH (15 km/h) and above, and when your vehicle is driven at speeds approximately 10 MPH (15 km/h) faster than the vehicle ahead." Because of these statements, a Lower Speed Threshold of Operation (LSTO) of 15 km/h was chosen to model the Infiniti algorithm. The owner's manual did not indicate any Upper Speed Threshold of Operation (USTO) for the Infiniti system. [12]

The owner's manual for the Volvo V60 states, "City Safety™ is not active if your vehicle's speed is below approximately 2 mph (4km/h). This means that the City Safety™ will not react if your vehicle approaches another vehicle at very low speed, for example, when parking." Because

of this statement, a Lower Speed Threshold of Operation (LSTO) of 4 km/h was chosen to model the Volvo algorithm. The owner's manual did not indicate any Upper Speed Threshold of Operation (USTO) for the Volvo system. [11]

In the Euro NCAP Advanced section of the Euro NCAP website, which provides information on rewarded active safety systems, a report on the Audi system states, "between 30 km/h and 80 km/h there is no automatic braking but the system will warn the driver if it detects a stationary object." Because this study evaluates FCAS benefits only for scenarios in which the leading vehicle was stationary prior to the collision, these values were used to define the speed thresholds of operation for the Audi algorithm. [16]

In the "Euro NCAP Advanced" section of the Euro NCAP website, a report on the VW system states the following:

At vehicle speeds between 5km/h and 30km/h, City Emergency Brake monitors an area 10m ahead of the car for vehicles which might present a threat of collision. If a collision is likely, City Emergency Brake first pre-charges the brakes and makes the Emergency Brake Assist system more sensitive: if the driver should notice the risk, the car is ready to respond more quickly to his braking action. However, if the driver still takes no action and a collision becomes imminent, City Emergency Brake independently applies the brakes very hard. [17]

Based on this quote, this study chose a Lower Speed Threshold of Operation (LSTO) of 5 km/h and an Upper Speed Threshold of Operation (USTO) of 30 km/h for the VW FCAS algorithm.

Table 4. Algorithm component functionalities

FCAS Algorithm	Lower Speed Threshold of Operation (km/h)	Upper Speed Threshold of Operation (km/h)	FCW?	AB?	AEB?
Audi	30	80	YES	YES	NO
BMW	15	None	YES	NO	NO
Infiniti	15	None	YES	NO	YES
VW	5	30	YES	YES	YES
Volvo	4	None	YES	YES	YES

The Audi, VW, and Volvo possessed Assisted Braking (AB), which was modeled as an amplifier of driver braking input in emergency situations. The ADAC test report identified these algorithms as possessing a brake assist that “effectively optimizes braking pressure to prevent [collisions].” Test data were not available to fully define the functionality of each AB. This study assumed that they could be modeled as systems that amplify the brake input to the maximum braking level permitted by friction.

Only the Infiniti, VW, and Volvo systems possessed an Autonomous Emergency Braking (AEB) component that functions when the leading vehicle is stationary. AEB was modeled as a braking ramp (r_{AEB}) followed by a constant brake input (a_{AEB}) as shown in Figure 2. These parameters were modeled for each AEB subsystem with the assistance of ADAC test data and video analysis of test track experiments. Values for a_{AEB} vary from -2 m/s^2 to -10 m/s^2 . Reasons for specifying specific values for r_{AEB} and a_{AEB} will be discussed in the video analysis section in greater detail.

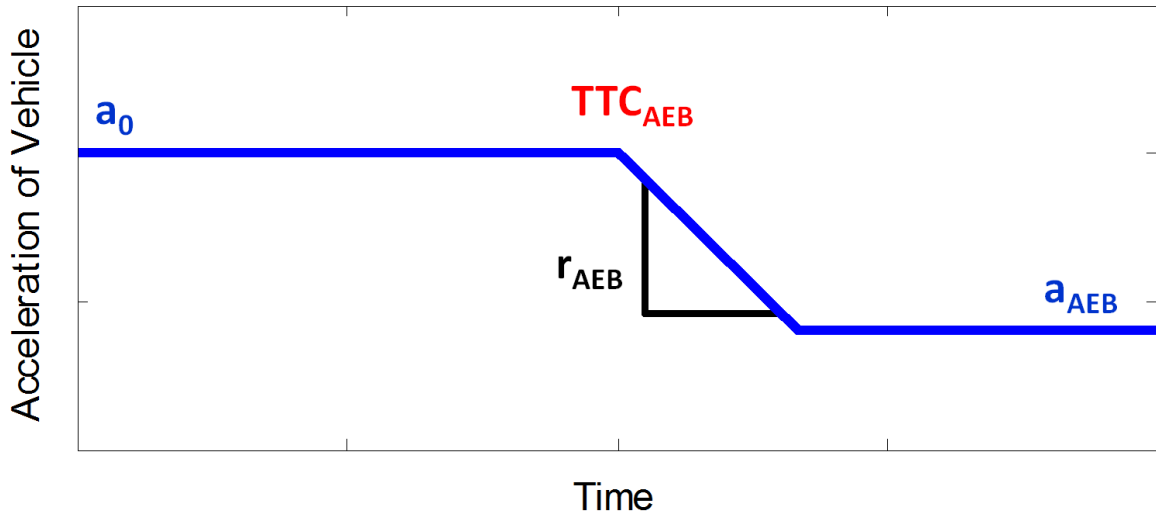


Figure 2. Autonomous Emergency Braking (AEB) was modeled as a braking ramp followed by a constant acceleration.

2.5 MODELING TIME TO COLLISION AT WHICH FCW WAS INITIATED (TTC_{FCW})

Included in the ADAC test report were measurements of when each system provided a warning to the driver. This measurement of when FCW activates is known as Time to Collision (TTC) and is defined as follows:

$$TTC_{FCW} = \frac{-x_0}{v_0} \quad (1)$$

x_0 is the position of the approaching vehicle relative to the leading vehicle and v_0 is the velocity of the approaching vehicle relative to the leading vehicle when the FCW activates. The ADAC measured TTC_{FCW} for speeds of 20, 30, 40, and 70 km/h for the scenario in which the leading vehicle was stationary prior to the collision. These data collected by the ADAC are shown in Figure 3. Note that there are missing values for the Audi at a speed of 20 km/h and for the VW at speeds of 40 km/h and 70 km/h. The ADAC reported that the systems failed to warn the driver at these speeds. These failures support the information acquired from the Euro NCAP website, reflected in Table 4.

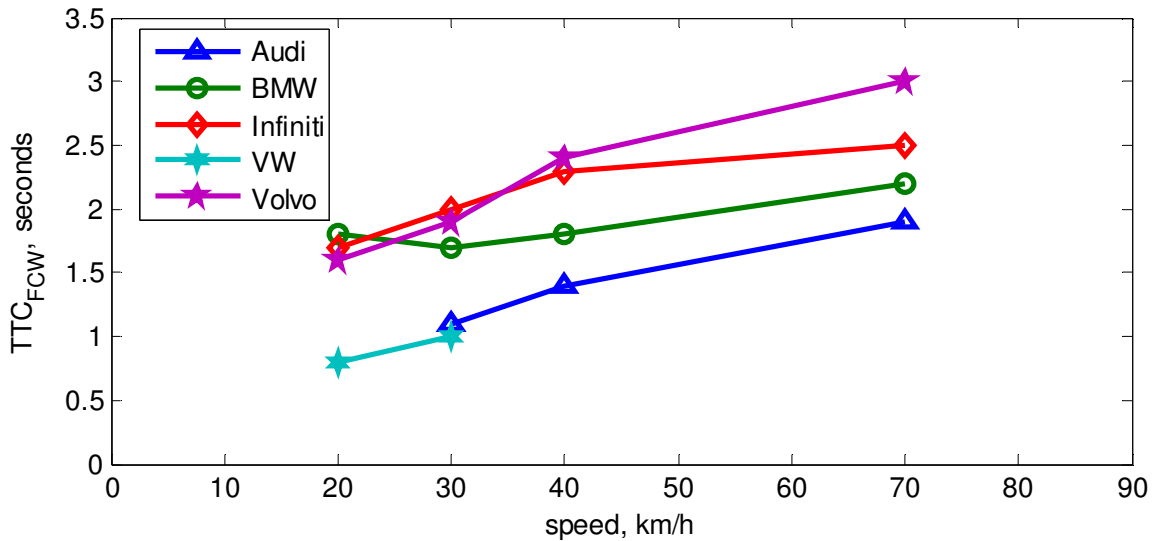


Figure 3. TTC_{FCW} data collected by ADAC for approaching speeds of 20, 30, 40, and 70 km/h

Based on evidence provided by owner’s manuals and the Euro NCAP website, we believe, but cannot confirm, that these data do not encompass the entire range of operation for the five FCAS. If we were to only use the test data to model the algorithm relationships of TTC_{FCW} against speed and refrain from extrapolating outside of the test conditions, the FCAS algorithms would not provide benefits for cases in which the vehicle was traveling at speeds below 20 km/h or above 70 km/h. Therefore, modeling the relationship only in the tested speed range will provide a conservative baseline.

Three methods were used to model the relationships between speed and TTC_{FCW} for all the algorithms analyzed in this study. The first method uses linear interpolation within the bounds of known test results to provide a relationship that will yield a conservative lower bound for expected benefits. The second method uses linear interpolation within the bounds of known test results and linear extrapolation to provide relationships that span the systems’ operation ranges defined by the

LSTO and USTO. The third method uses linear fits to provide relationships that extrapolate outside the known test results, spanning the systems' operation ranges defined by the LSTO and USTO.

2.5.1 METHOD 1 OF MODELING THE RELATIONSHIP BETWEEN SPEED AND TTC_{FCW}

The first method that was used to define a relationship between speed and TTC_{FCW} is conservative and was used to help estimate a lower bound for expected benefits by FCAS. This method uses interpolation to estimate the relationship between the test conditions. This method is conservative because the relationship was not extrapolated outside of the ADAC test conditions. 20 km/h was set as the minimum lower threshold of operation for all algorithms. 70 km/h was set as the maximum upper threshold of operation for all algorithms. Note that the lower speed threshold of operation for the Audi algorithm is 30 km/h because the FCW did not activate for the test condition of an initial speed of 20 km/h. The upper speed threshold of operation for the VW is 30 km/h because the FCW did not activate for the test conditions in which the vehicle was traveling 40 km/h and 70 km/h. Method 1 employs the assumption that measurement variation in the region of known test data accurately reflects the associated algorithm.

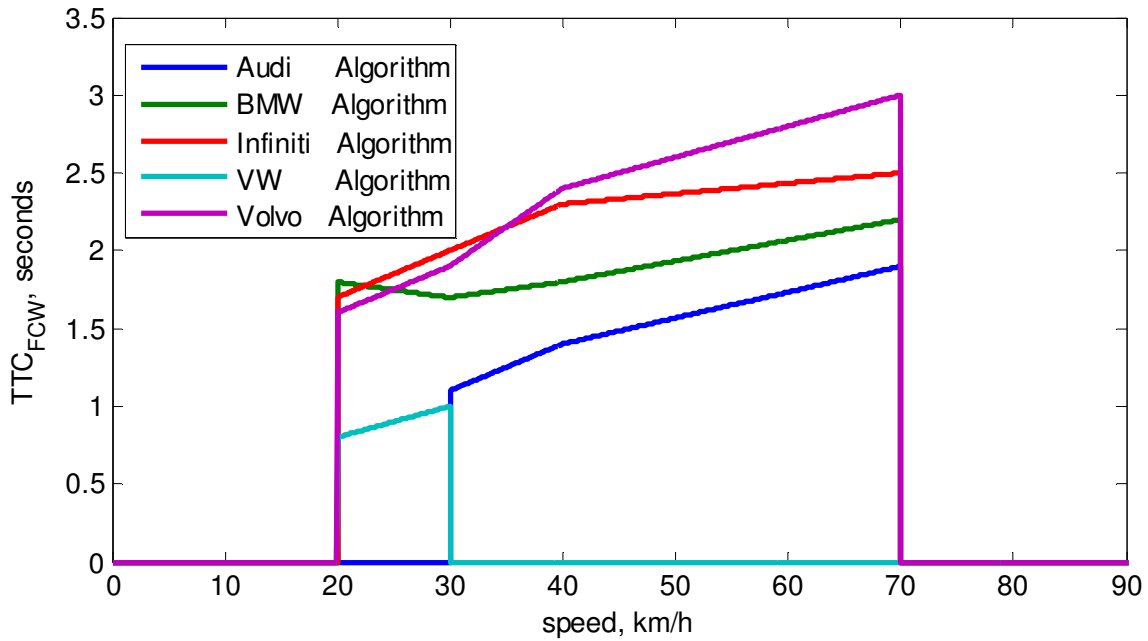


Figure 4. TTC_{FCW} against initial speeds for each algorithm modeled with Method 1. Vertical lines denote velocity thresholds of operation.

2.5.2 METHOD 2 OF MODELING THE RELATIONSHIP BETWEEN SPEED AND TTC_{FCW}

The second method that was used to model the relationship between speed and TTC_{FCW} also used linear interpolation of the data shown in Figure 3 to model the relationships within the range of test speeds. Unlike the first method, the second method used linear extrapolation to estimate the relationships outside of the test conditions. This method used the speed thresholds of operation listed in Table 4 as boundaries for the functionality of the FCAS. We do not know the TTC_{FCW} vs. speed relationships, at speeds below 20 km/h and above 70 km/h, but assume that a linear extrapolation of the ADAC data provides a reasonable estimate. Method 2 employs the assumption that measurement variation in the region of known test data accurately reflects the associated algorithm.

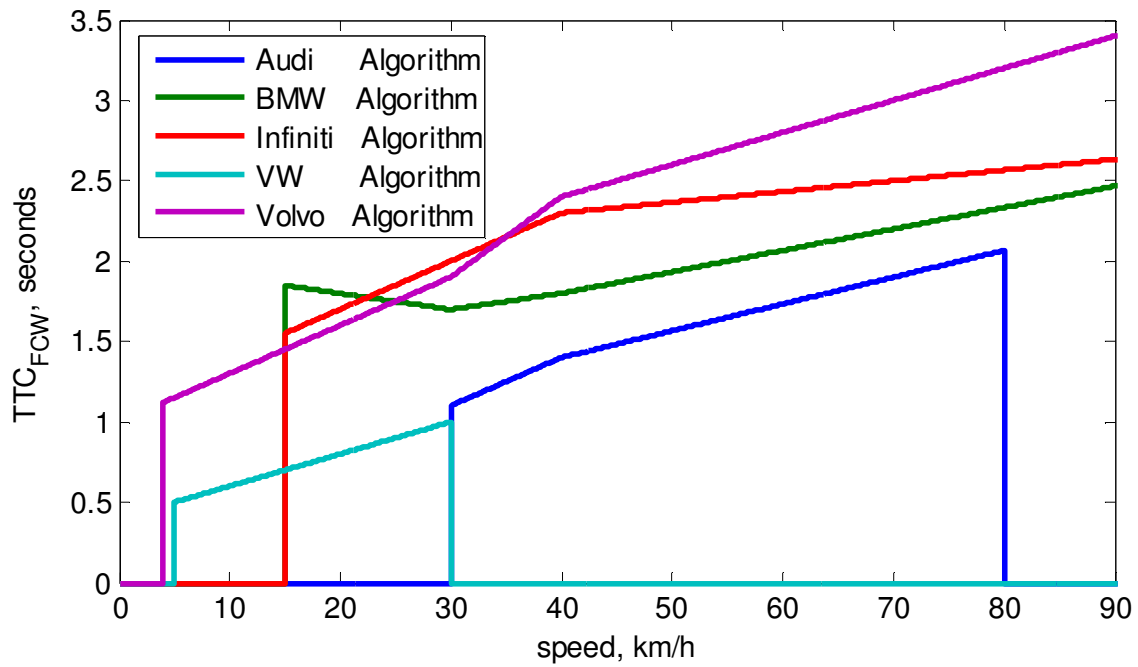


Figure 5. TTC_{FCW} against initial speeds for each algorithm modeled with Method 2. Vertical lines denote velocity thresholds of operation.

2.5.3 METHOD 3 OF MODELING THE RELATIONSHIP BETWEEN SPEED AND TTC_{FCW}

The third method that was used to model the relationship between speed and TTC_{FCW} used a linear fit of the data provided in Figure 3 to estimate the relationships between speed and TTC_{FCW} . The linear fit was generated from the data in the ADAC test range, and extended outside the range to the known limits of FCAS functionality shown in Table 4. Method 3 assumes that the relationship between TTC_{FCW} and speed is linear and that the variation of the results of each algorithm test is due to experimental fluctuations.

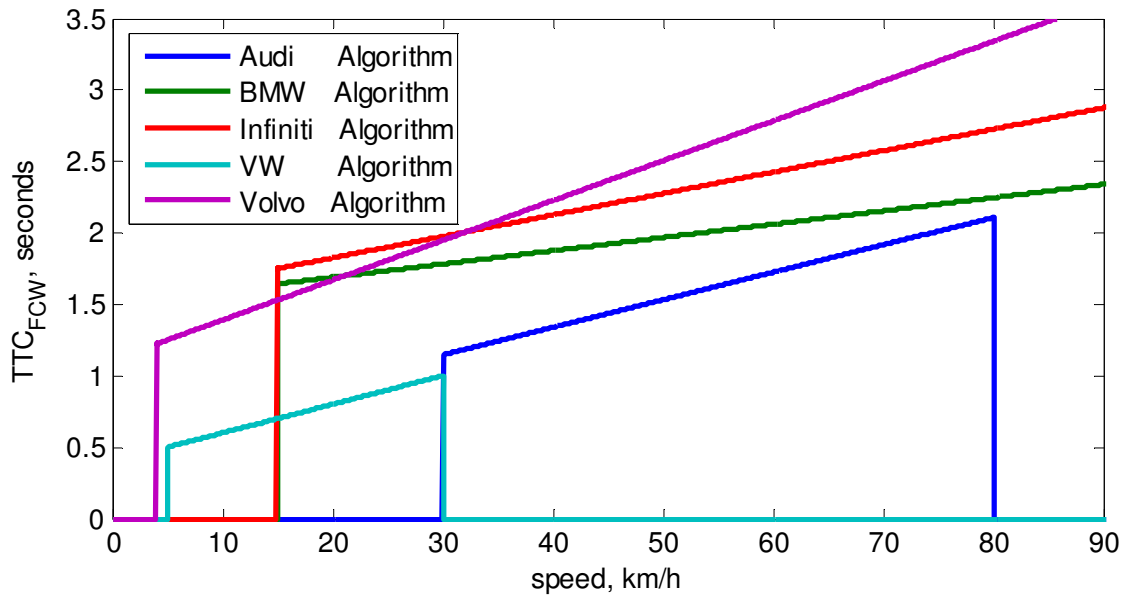


Figure 6. TTC_{FCW} against initial speeds for each algorithm modeled with Method 3. Vertical lines denote velocity thresholds of operation.

2.6 MODELING TIME TO COLLISION AT WHICH AEB WAS INITIATED (TTC_{AEB})

The Infiniti, VW and Volvo systems were equipped with AEB. In order to accurately model the behavior of these systems, it was necessary to model the relationship between speed and the time of AEB activation (TTC_{AEB}). Two methods were used to obtain two different sets of models that provide estimates for the relationships between v_0 and TTC_{AEB}: Methods A and B. Method A is unique for each AEB system, but yields a reasonable approximation for the actual relationships that were engineered by the automakers. Method B is more conservative and does not permit extrapolation outside of ADAC and video analysis test conditions to model the relationships.

2.6.1 METHOD A OF MODELING THE RELATIONSHIP BETWEEN v_0 AND TTC_{AEB}

Method A is unique for each AEB system, but yields what we believe, but cannot verify, are reasonable approximations for the actual relationships that were engineered by the automakers. Method A was used in conjunction with Methods 2 and 3 from section 2.5 to estimate FCAS benefits. The relationship between v_0 and TTC_{AEB} for the VW algorithm was modeled to mimic the relationship of v_0 and TTC_{FCW} . This modeling decision was based on the ADAC test report statement: “The VW Passat does not provide an early warning since it would make autonomous braking redundant. It alerts the driver only before initiating the autonomous braking.” [10] The relationship between the initial speed and TTC_{AEB} for the Volvo Algorithm was modeled by means of video analysis, which will be discussed in section 2.8.

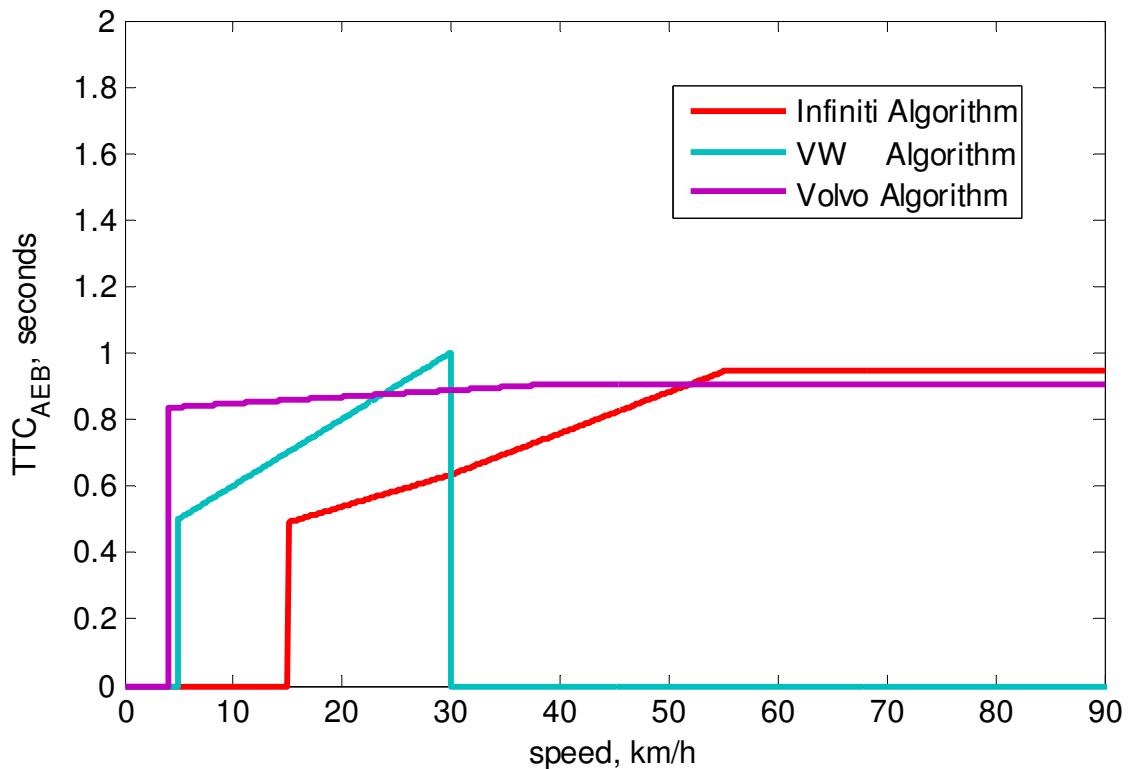


Figure 7. TTC_{AEB} against initial speeds for each algorithm (Method A). Vertical lines denote velocity thresholds of operation.

The relationship between the initial speed and TTC_{AEB} for the Infiniti algorithm was modeled by using the ADAC test report data. Figure 8 was adapted from a figure in the ADAC test report that shows a model of the braking behavior exhibited for the Infiniti AEB system when the trailing vehicle has an initial velocity of $v_1=40$ km/h relative to the leading vehicle. After a constant braking ramp that starts at a TTC_{AEB} of 0.7 seconds, a constant braking pressure yields a deceleration of 5.8 m/s². The trailing vehicle crashes into the leading vehicle with a relative velocity of $v_{impact}=29$ km/h. By solving Equations 2-7 simultaneously, the value of the ramp (r_{AEB}) was determined to equal -12 m/s³.

$$TTC_{AEB} = -\frac{x_1}{v_1} \quad (2)$$

$$a_{AEB} = r_{AEB}t_{1,2} \quad (3)$$

$$v_2 = v_1 + \frac{1}{2}r_{AEB}t_{1,2}^2 \quad (4)$$

$$x_2 = x_1 + v_1t_{1,2} + \frac{1}{6}r_{AEB}t_{1,2}^2 \quad (5)$$

$$v_{impact} = v_2 + a_{AEB}t_{2,3} \quad (6)$$

$$0 = x_2 + v_2t_{2,3} + \frac{1}{2}a_{AEB}t_{2,3}^2 \quad (7)$$

The unknowns in this instance of the system of equations are x_1 , r_{AEB} , $t_{1,2}$, v_2 , x_2 , and $t_{2,3}$. x_1 is the position of the trailing vehicle relative to the leading vehicle in dynamical state 1 which is defined by the start of AEB. $t_{1,2}$ is the time it takes to get from state 1 to state 2 which is defined by the moment r_{AEB} ends. v_2 and x_2 are the velocity and position of the trailing vehicle relative to the leading vehicle in dynamical state 2. $t_{2,3}$ is the time it takes to get from state 2 to state 3 which is defined by the crash.

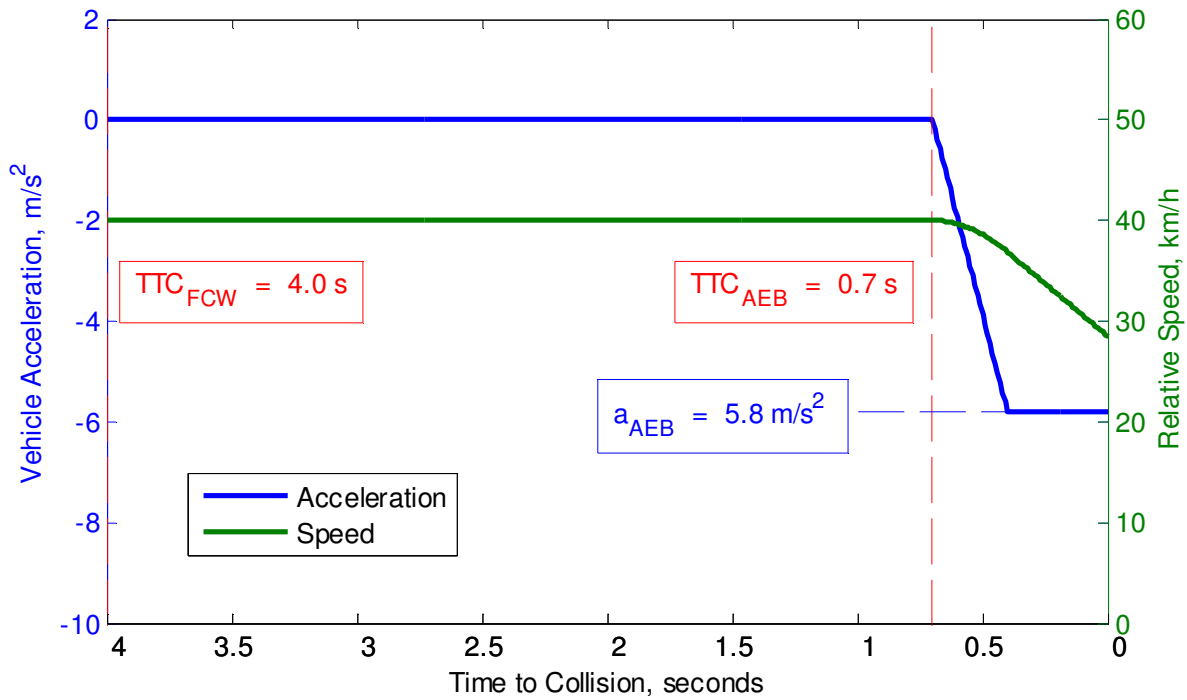


Figure 8. ADAC model of Infiniti AEB system with leading vehicle constant speed of 60 km/h and trailing vehicle initial speed of 100 km/h.

To model the behavior of the system when the vehicle approaches a stationary object, we assumed that the Infiniti system always has a ramp of -12 m/s^3 followed by a constant acceleration of -5.8 m/s^2 . Given this assumption and ADAC test data concerning the impact speed of the vehicle for initial speeds of 20, 30, and 40 km/h, 3 different TTC_{AEB} values were determined by simultaneously solving Equations 2-7 for x_1 , TTC_{AEB} , $t_{1,2}$, v_2 , x_2 , and $t_{2,3}$. Table 5 lists the impact speeds provided by the ADAC test report as well as the estimated TTC_{AEB} for all three speeds. These results were linearly interpolated and extrapolated to yield the Infiniti curve shown in Figure 7. We assumed that there exists an upper TTC_{AEB} threshold because automakers do not want to be responsible for causing an unwelcome instance of AEB. A maximum TTC_{AEB} was set to 0.955 seconds because it is the average of the maximum TTC_{AEB} for the VW and Volvo Algorithms.

Table 5. Impact speeds and estimated TTC_{AEB} for ADAC tests of the Infiniti

Initial Speed (km/h)	Impact Speed (km/h)	TTC_{AEB} (seconds)
20	12	0.538
30	20	0.634
40	27	0.758

2.6.2 METHOD B OF MODELING THE RELATIONSHIP BETWEEN V_0 AND TTC_{AEB}

Method B is more conservative than Method A and does not extrapolate outside of ADAC and video analysis test conditions to model the relationships. Method B was used in conjunction with Method 1 from section 2.5 to estimate a lower bound for FCAS benefits.

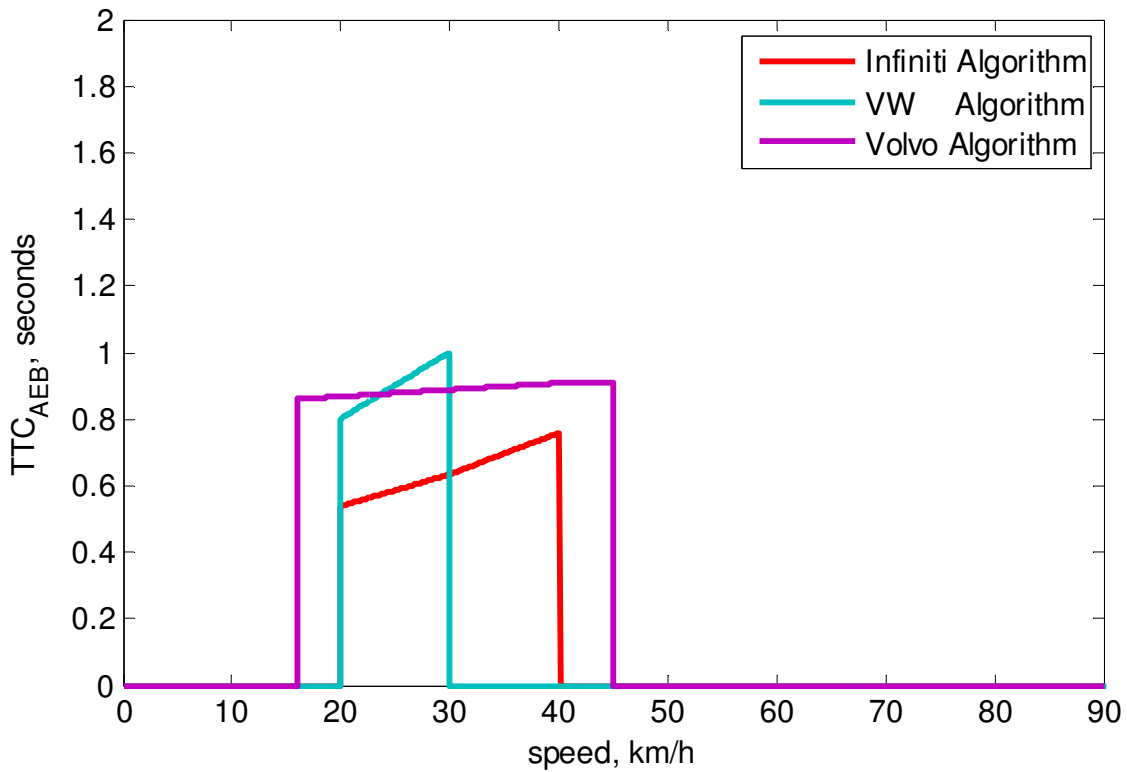


Figure 9. TTC_{AEB} against initial speeds for each algorithm (Method B). Vertical lines denote velocity thresholds of operation.

2.7 THE NECESSITY OF VIDEO ANALYSIS TO HELP MODEL AEB SYSTEMS

The ADAC test report, owner's manuals, and Euro NCAP website provided enough information to model all aspects of an FCAS for all five vehicles except for the VW FCAS and the Volvo FCAS. For all FCASs, the functionalities of the sub-systems have been modeled, including the lower and upper speed thresholds of operation. The relationships between speed and TTC_{FCW} have been modeled for all FCAS algorithms. Only for the Infiniti algorithm, have all AEB system parameters been modeled (a_{AEB} , r_{AEB} , and the relationship between speed and TTC_{AEB}). For the VW AEB system, only the relationship between speed and TTC_{AEB} has been modeled. All of the Volvo AEB parameters have yet to be modeled.

Up to this point in this thesis, the VW algorithm is completely defined except for a_{AEB} and r_{AEB} . This study assigned a value of -10 m/s^2 for the VW a_{AEB} . This means that the peak deceleration is always assumed to equal the maximum deceleration permitted by friction. Although the magnitude of this value is an overestimate of the actual deceleration, the results will be accurate because the system is designed to prevent all collisions between the lower and upper speed thresholds of operation (5 km/h - 30 km/h), permitted that sufficient friction exists between the vehicle's tires and the road. In some simulations, the friction between the tires and road will not be sufficient for the VW AEB system to completely prevent a crash, so the VW r_{AEB} will have an effect on the impact speed.

Recall from section 2.6.1 that for the Infiniti, r_{AEB} was defined by using the data provided in Figure 8. The corresponding plot for the ADAC test of the VW is shown in Figure 10. The plot represents an ADAC model of the VW's FCAS functionality as a function of Time to Collision (TTC) for the situation in which a leading vehicle is traveling at a constant speed of 60 km/h and the trailing vehicle is has an initial speed of 100 km/h. For this scenario the driver was warned by the VW's FCW 2.8 seconds prior to a potential collision. The VW FCAS provided a haptic

warning 2.2 seconds prior to the potential collision. For this scenario the VW AEB system provided two levels of braking. This complexity makes it impossible to use this ADAC model to estimate a value of t_{AEB} with good confidence. Instead of using the ADAC model of acceleration against TTC, video analysis was used to estimate a value for the VW t_{AEB} .

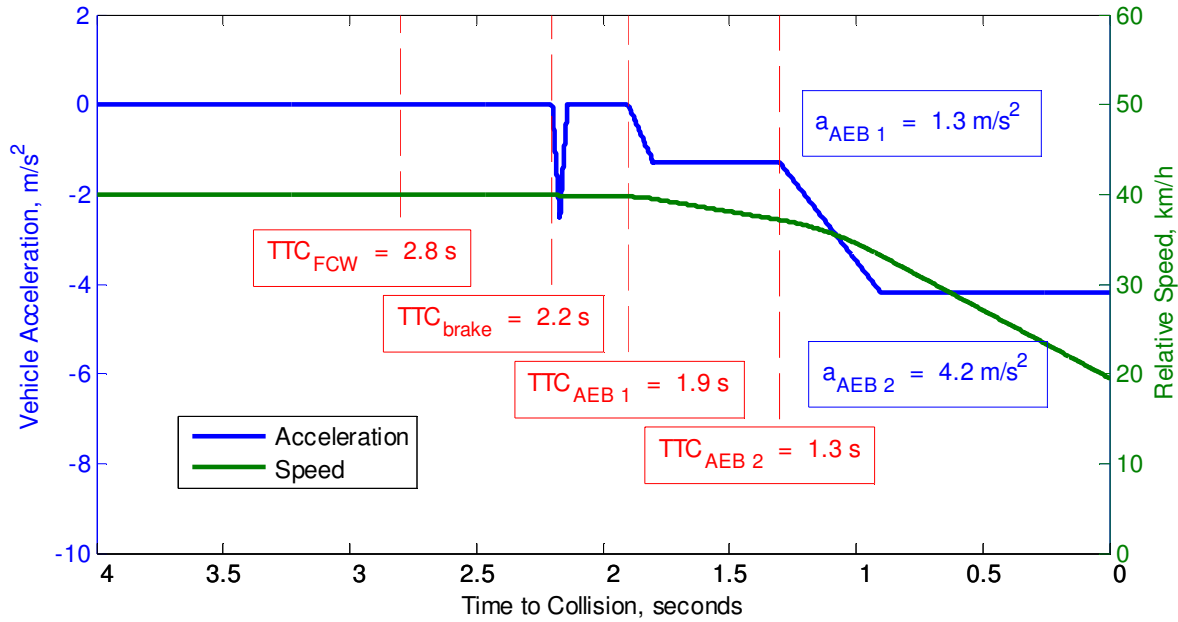


Figure 10. ADAC model of VW AEB system with leading vehicle constant speed of 60 km/h and trailing vehicle initial speed of 100 km/h.

Up to this point in this thesis, the Volvo algorithm is completely defined except for a_{AEB} , t_{AEB} , and the relationship between speed and TTC_{AEB} . Figure 11 shows the ADAC model of AEB acceleration as a function of time. The ADAC test report did not provide a value for the Volvo a_{AEB} , so it was not possible to use this plot to estimate values for a_{AEB} and t_{AEB} with good confidence. Without these values, a relationship between speed and TTC_{AEB} could not be estimated. Video analysis was used to define a_{AEB} , t_{AEB} , and the relationship between speed and TTC_{AEB} for the Volvo algorithm.

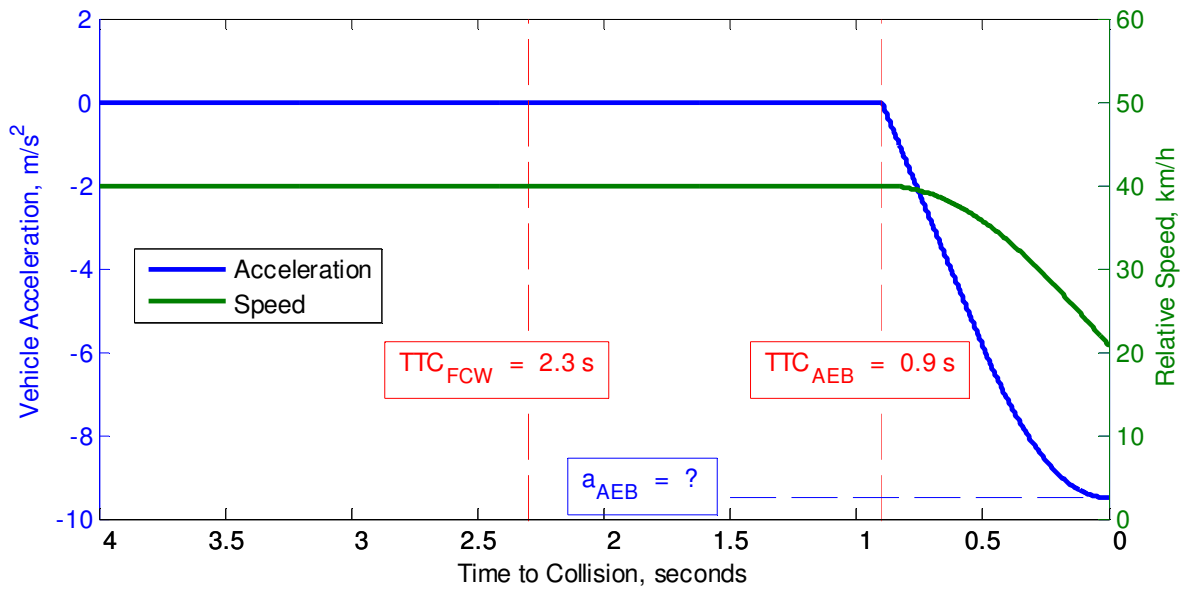


Figure 11. ADAC model of Volvo AEB system with leading vehicle constant speed of 60 km/h and trailing vehicle initial speed of 100 km/h.

2.8 METHODS AND RESULTS OF VIDEO ANALYSIS

Table 6 lists descriptions of the three videos that were analyzed in this study. The first video analyzed in this study was taken with a stationary (non-panning) video camera during a test conducted by the Insurance Institute for Highway Safety (IIHS) of the Volvo's AEB system. The vehicle that is used in this demonstration appears to be a Volvo V60. The video was uploaded to youtube.com on July 19, 2011, so the model year is likely a 2011 or 2012. Because the generation of the 2011 Volvo V60 spans from model years 2010-present, our study assumed that the Volvo systems tested by the ADAC and the IIHS are one and the same. The purpose of analyzing this video was to determine the Volvo AEB braking rate (r_{AEB}) and the time at which the Volvo's AEB initiates (TTC_{AEB}) for initial speeds of 16 km/h. [19]

The second video analyzed in this study was that of a VW with an initial speed of 20 km/h. This video was taken during a test performed by the ADAC with a stationary (non-panning) video

camera. The purpose of analyzing this video was to determine the braking ramp (r_{AEB}) of the VW's AEB system. [18]

The third video analyzed in this study was that of a Volvo approaching a stationary semi-trailer with an initial speed of 45 km/h. This video was posted to youtube.com on March 17, 2011, so the vehicle used in the demonstration likely has a model year of 2011 or 2012. The vehicle that is used in this demonstration appears to be a Volvo S60. Because the Volvo V60 is an estate version of the Volvo S60, this study assumed that the Volvo systems tested by the ADAC and the Volvo system tested at the Volvo Safety Center are one and the same. This video demonstrated that the Volvo's AEB system did not prevent the collision, but only mitigated the severity of the collision. The purpose of analyzing this video was to confirm the Volvo's AEB braking ramp (r_{AEB}), determine the time at which the Volvo's AEB starts (TTC_{AEB}) for initial speeds of 45 km/h, determine the impact speed for initial speeds of 45 km/h, and determine the maximum braking acceleration used by the Volvo's AEB (a_{AEB}). [20]

Table 6. List of videos analyzed in this study

Vehicle	Trailing Vehicle Speed (km/h)	Lead Vehicle Speed (km/h)	Type of Lead Vehicle	Video Camera Stationary/Panning	Source
Volvo	16	0	Balloon Car	Stationary	[19]
VW	20	0	Balloon Car	Stationary	[18]
Volvo	45	0	Semi-Trailer	Panning	[20]

2.8.1 ANALYSIS OF VIDEO TAKEN WITH A STATIONARY (NON-PANNING) CAMERA

Tracker, an open source video analysis tool, was used to obtain data from videos [21]. Figure 12 shows a drawing of the tracker environment during video analysis of a video that was taken with a stationary camera (i.e a video camera that is not panning, moving, or zooming). This video demonstrates the Volvo City Safety automatic braking system at 10 mph (16 km/h). In this demonstration, the AEB system completely prevented a collision.

The orange diamonds represent the pixel positions of the test vehicle's rear wheel for the current and previous five video frames. The blue diamonds represent the pixel positions of the test vehicle's front wheel for the current and previous five video frames. The purple arrows represent a user-defined 2-dimensional coordinate system. The coordinate system was defined such that the tracking data (diamonds) fell on the x-axis which defines the vehicle's heading. In other words, the Volvo is traveling purely in the x-axis.

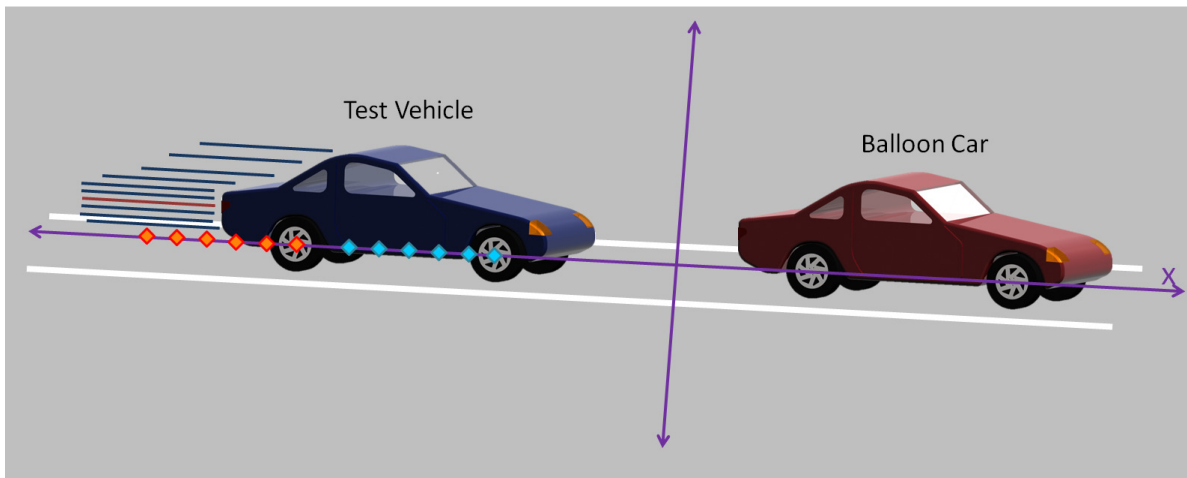


Figure 12. Analyzing video taken with a stationary camera in the Tracker environment

Figure 13 shows the position data collected from tracker plotted against time. Because the frame rate of the video was not specified in the video, time does not have units. Due to the effects of perspective, the wheelbase appears to increase as the vehicle approaches the camera.

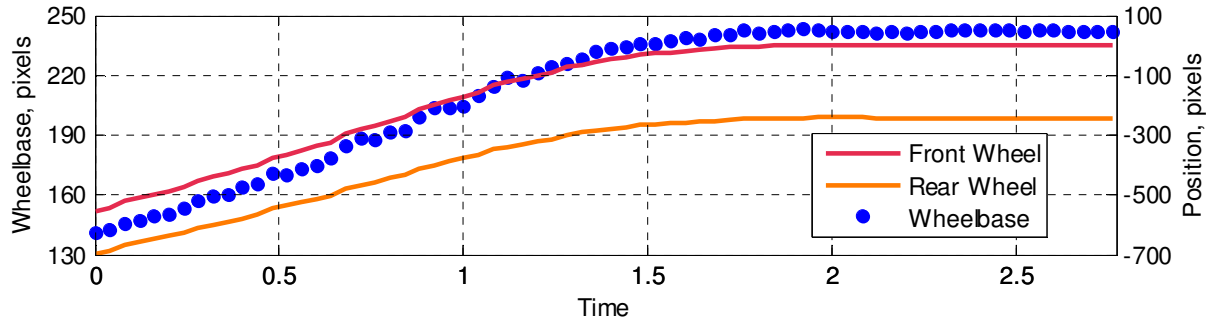


Figure 13. Data obtained from video taken with stationary camera

Figure 14 demonstrates the effect of perspective on the video analysis process. As the trailing vehicle approaches the stationary balloon car (and the video camera) the vehicle appears to get larger. This enlargement in appearance is due to perspective and results in an enlargement of the wheelbase measured in pixels. For this particular video, the first wheelbase that was measured was 130 pixels long. The last (40th) wheelbase that was measured was 240 pixels long.

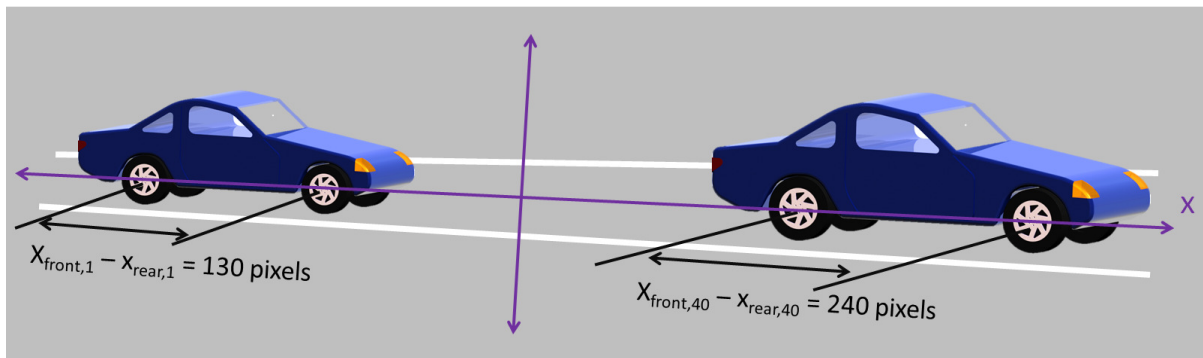


Figure 14. Demonstration of the effect of perspective on the video analysis process

The position in physical space of the vehicle's wheels relative to the leading vehicle can be determined by adjusting for perspective with Equation 8. The position of a wheel in physical space, x' , is a function of the pixel location of the wheel's center, x , and three constants: c_1 , c_2 , and c_3 . x has units of pixels and x' has units of meters.

$$x' = c_1 \frac{x}{c_2 x + 1} + c_3 \quad (8)$$

Figure 15 shows a physical interpretation of Equation 8. This diagram depicts the physical space of the camera sensor plane (pixel plane) in relation to the plane in which a vehicle is traveling (physical plane of travel). In the physical plane of travel there exists a 2-dimensional coordinate system. The abscissa is defined to align with the heading of the vehicle. The coordinate system is projected onto the pixel plane to create another coordinate system which is local to the pixel plane. The pixel position of the rear wheel of the vehicle relative to the origin of the 2-dimensional pixel plane coordinate system is denoted by x . The position of the rear wheel of the physical vehicle relative to the origin of the physical plane of travel coordinate system is denoted by x' . Given a value for x and a few spatial parameters, it would be possible to solve for x' .

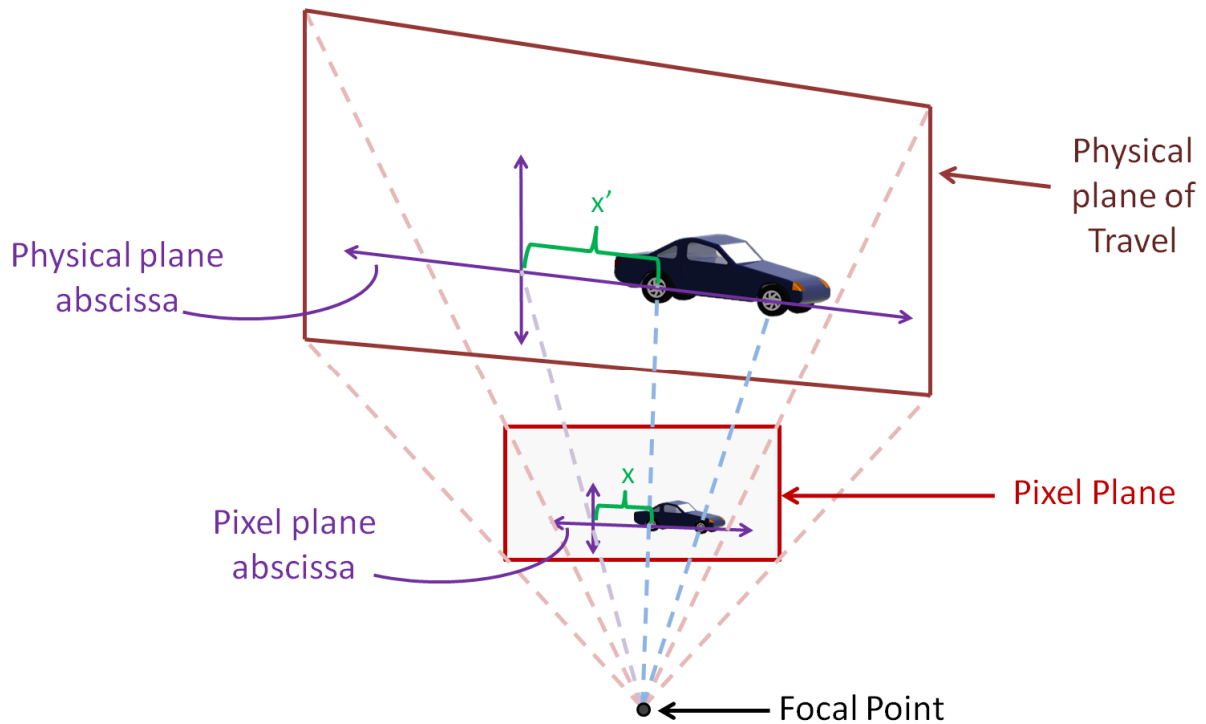


Figure 15. Schematic representation of transformation from pixel to spatial measurements

Figure 16 shows a Bird's-eye view of Figure 15. Points “a” and “b” represent the coordinate system origins for the pixel plane and the physical plane of travel, respectively. Point “m” represents either the front or rear wheel's center in the pixel. Point “p” represents either the front or rear wheel's center in the physical plane of travel.

Four spatial parameters that could be used to solve for $x'(\overline{bp})$ are the distance between the focal point and the pixel coordinate system origin (\overline{fa}), the distance between the focal point and the physical coordinate system origin (\overline{fb}), the angle between \overline{fa} and \overline{fh} (τ), and the angle between the abscissa in the physical plane of travel and the abscissa in the pixel plane (θ). Using trigonometry, these four spatial parameters and $x(\overline{am})$ can be used to derive an equation that simplifies to Equation 8. This derivation is shown in Appendix A.

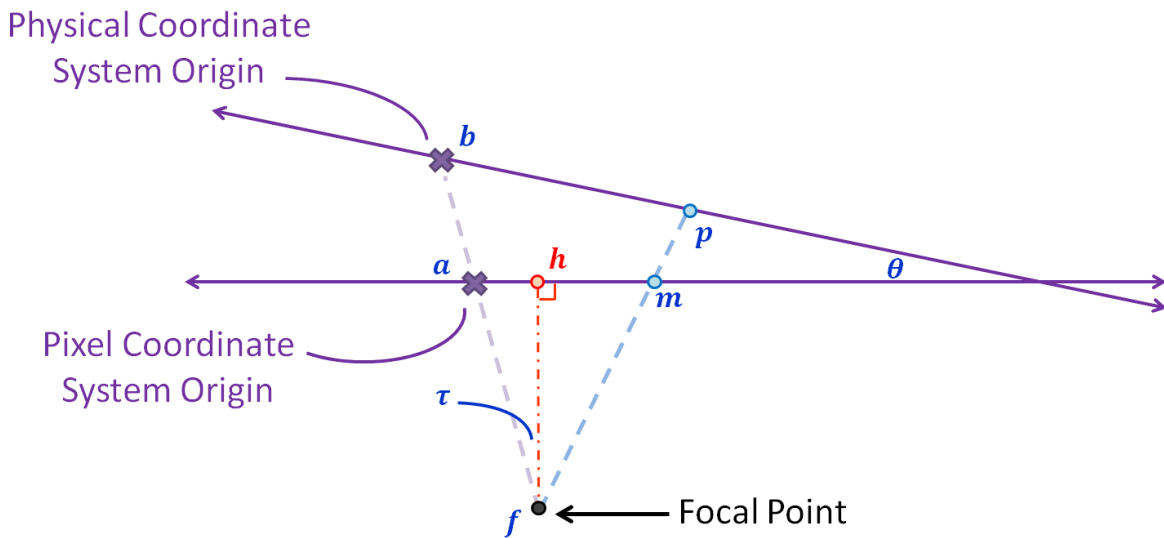


Figure 16. Bird's eye view of schematic representation of pixel-spatial transformation

In order to calculate the spatial position of the wheel positions, it was necessary to estimate c_1 , c_2 , and c_3 . This was accomplished with an optimization problem.

For each video frame, the wheelbase in real space, WB' , could be found with Equation 9:

$$WB'_i = x'_{front,i} - x'_{rear,i} \quad (9)$$

Combining Equations 8 and 9 yields:

$$WB'_i = c_1 \left(\frac{x_{front,i}}{c_2 x_{front,i} + 1} - \frac{x_{rear,i}}{c_2 x_{rear,i} + 1} \right) \quad (10)$$

where x_{front} is the pixel position of the front wheel's center, x_{rear} is the pixel position of the rear wheel's center, and i is the frame of interest. To determine the optimal values of c_1 and c_2 , the following unconstrained objective function was minimized.

$$Objective\ Function = \sigma_{WB'} + \sum_{i=1}^{N_{frames}} (WB'_i - wheelbase)^2 \quad (11)$$

where $\sigma_{WB'}$ is the standard deviation of the wheelbases calculated with Equation 10 and $\sum (WB' - wheelbase)^2$ is the sum of the squared differences between the actual wheelbase and the wheelbase calculated with Equation 10. For this video, the actual wheelbase was assumed to equal 2776 millimeters which is the nominal wheelbase of the Volvo V60 [22].

The optimal values of c_1 and c_2 were determined to be 0.01022 and 4.51×10^{-4} , respectively. A value for c_3 was selected so that the true stopping distance, which was measured in a different video clip, equaled the computed stopping distance. Using Equation 8, the positions in physical space of the front and rear wheels were calculated. They are shown in Figure 17 plotted against arbitrary time. Note that the calculated wheelbase has a mean of 2776 millimeters and a standard deviation of 14.3 mm. Also note that the positions of the wheels level off at a time of approximately 1.7 (unknown units). This indicates the moment at which the trailing vehicle came to a complete stop.

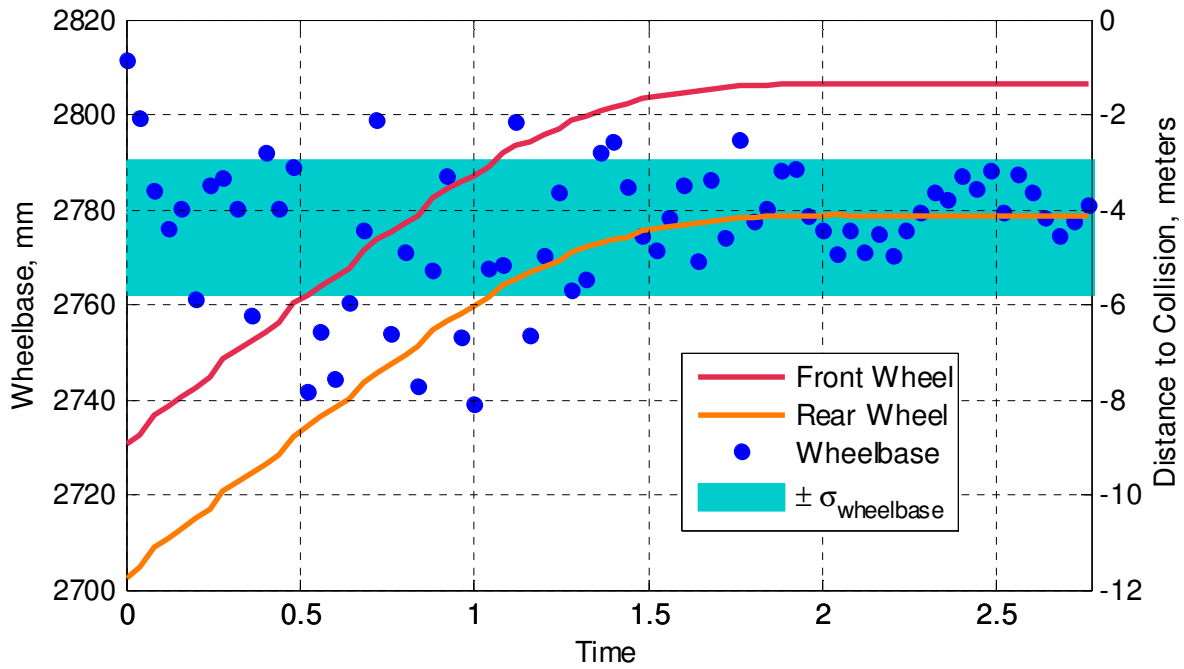


Figure 17. Data obtained from video taken with stationary camera, distance corrected

The final step in determining the position of the vehicle as a function of time is to adjust the time by multiplying it by a time adjustment factor, TAF, defined in Equation 12:

$$TAF = \frac{\text{apparent speed}}{\text{actual speed}} \quad (12)$$

where apparent speed is the slope of the data in the constant slope (constant speed) section of the data. In this video, the TAF is equal to 1.29. In other words, the assumed video frame rate was off by a factor of $TAF^{-1} = 0.775$. The apparent speed was 25.8 km/h while the assumed actual speed was 20 km/h.

The corrected position of the vehicle's front bumper relative to the rear bumper of the leading vehicle is plotted against corrected time in Figure 18. The hypothetical behavior of the Volvo had it not been equipped with an AEB system was modeled by applying a least squares method to the

straight portion of the blue curve. The point at which the straight pink line and the blue curve deviate indicates the time and location of the vehicle when the AEB system activated and the braking ramp began. The difference between the time at which AEB activated and the time at which a collision would have occurred, had the vehicle not been equipped with an AEB system, is the Time to Collision of AEB system activation (TTC_{AEB}). This video analysis indicates that for a Volvo approaching a stationary vehicle with an initial speed of 16 km/h, a proper model of the AEB system dictates that the AEB system will activate at a TTC_{AEB} of 0.86 seconds.

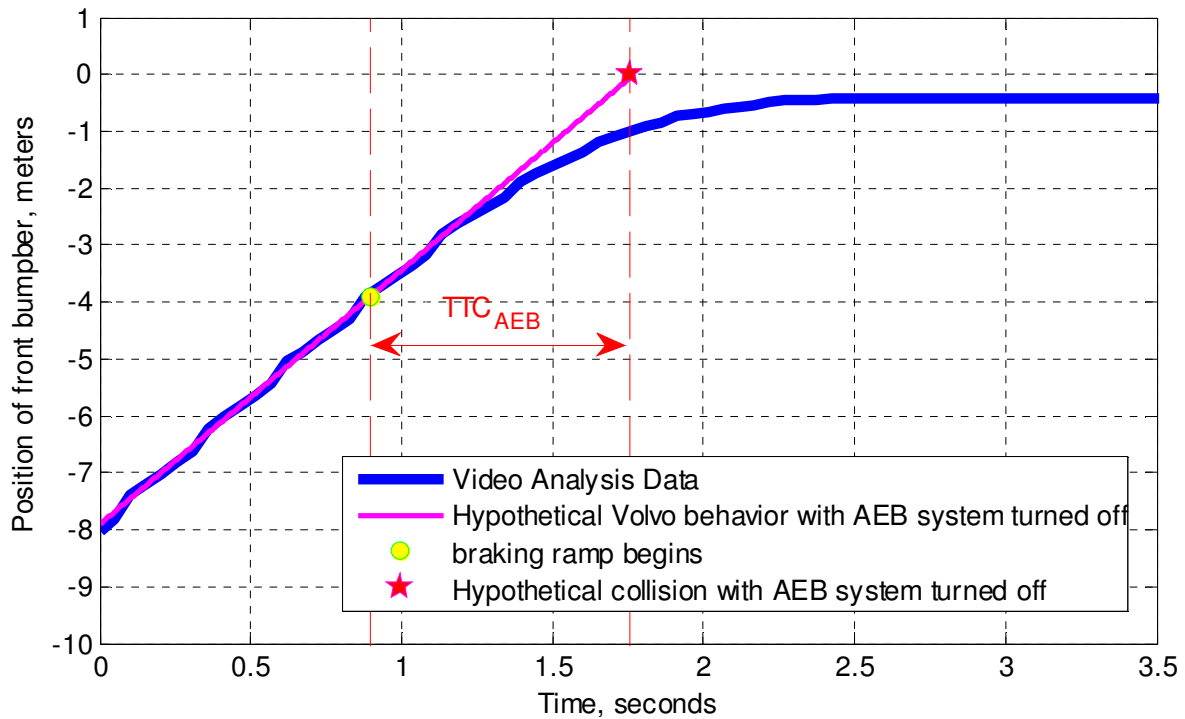


Figure 18. Position of vehicle's front bumper against corrected time.

In order to estimate a_{AEB} and r_{AEB} for this test scenario with a Volvo, a piecewise curve was fit to the video analysis position vs. time data. The objective of this curve fitting was to minimize the sum of the position residuals. The curve fitting was constrained by Equations 13-17. The unknowns in this system of equations were a_{AEB} , r_{AEB} , x_2 , v_2 , $t_{1,2}$, and $t_{2,3}$. For this particular fit, x_1

was equal to -3.91 meters and the final resting position of the front bumper, x_3 , was -0.42 meters as shown in Figure 18.

$$a_{AEB} = r_{AEB}t_{1,2} \quad (13)$$

$$v_2 = v_1 + \frac{1}{2}r_{AEB}t_{1,2}^2 \quad (14)$$

$$x_2 = x_1 + v_1t_{1,2} + \frac{1}{6}r_{AEB}t_{1,2}^2 \quad (15)$$

$$0 = v_2 + a_{AEB}t_{2,3} \quad (16)$$

$$x_{final} = x_2 + v_2t_{2,3} + \frac{1}{2}a_{AEB}t_{2,3}^2 \quad (17)$$

Figure 19 shows position of the vehicle in meters plotted against actual time in seconds. The modeled position of the vehicle's front bumper is shown in green, orange, and red. The green section of the modeled position corresponds to the vehicle traveling with a constant velocity. The orange section of the modeled position corresponds to vehicle braking at a rate of $r_{AEB} = -16 \text{ m/s}^3$. The red section of the modeled position corresponds to the vehicle braking at a constant $a_{AEB} = -3.3 \text{ m/s}^2$.

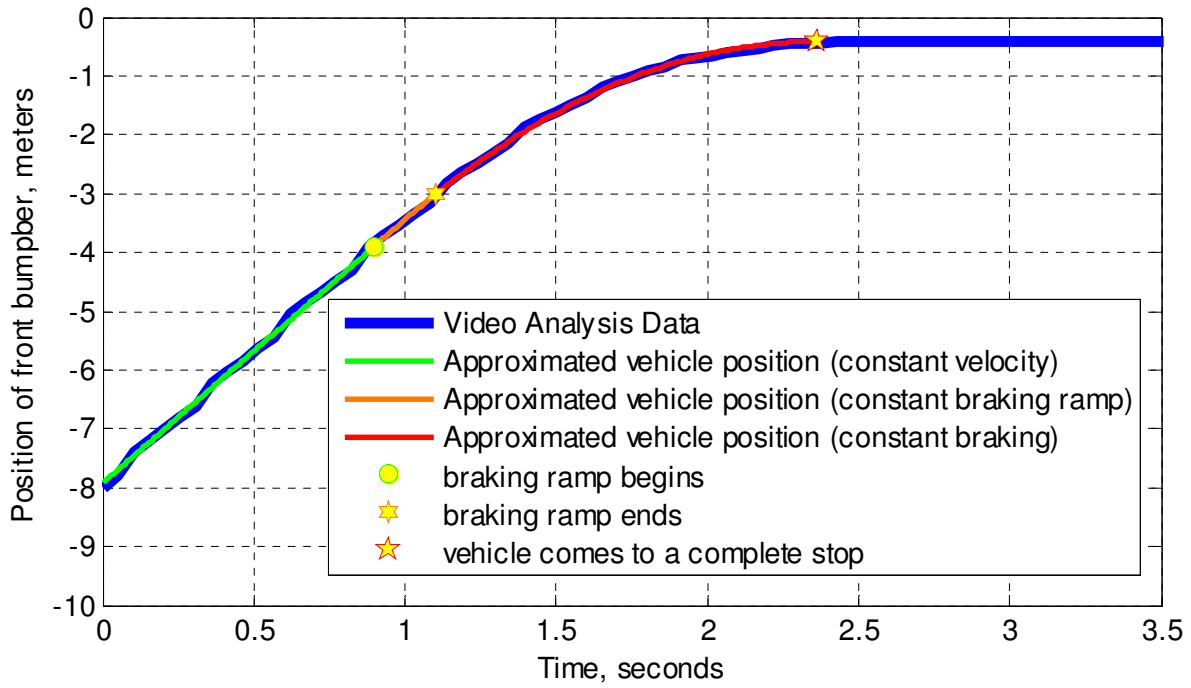


Figure 19. Data-driven and modeled position of Volvo. Data obtained from video taken with a stationary camera.

The same method was used to model the actual position of a vehicle equipped with the VW algorithm as a piecewise function of time, given an initial velocity of 20 km/h. The results of this method in application to the VW FCAS are shown in Figure 20. A braking ramp of -40 m/s^3 was chosen because it yields a good fit to the data (in blue) and also yields a TTC of 0.9 seconds which is close to the value that the ADAC report provided for this speed (0.8 seconds).

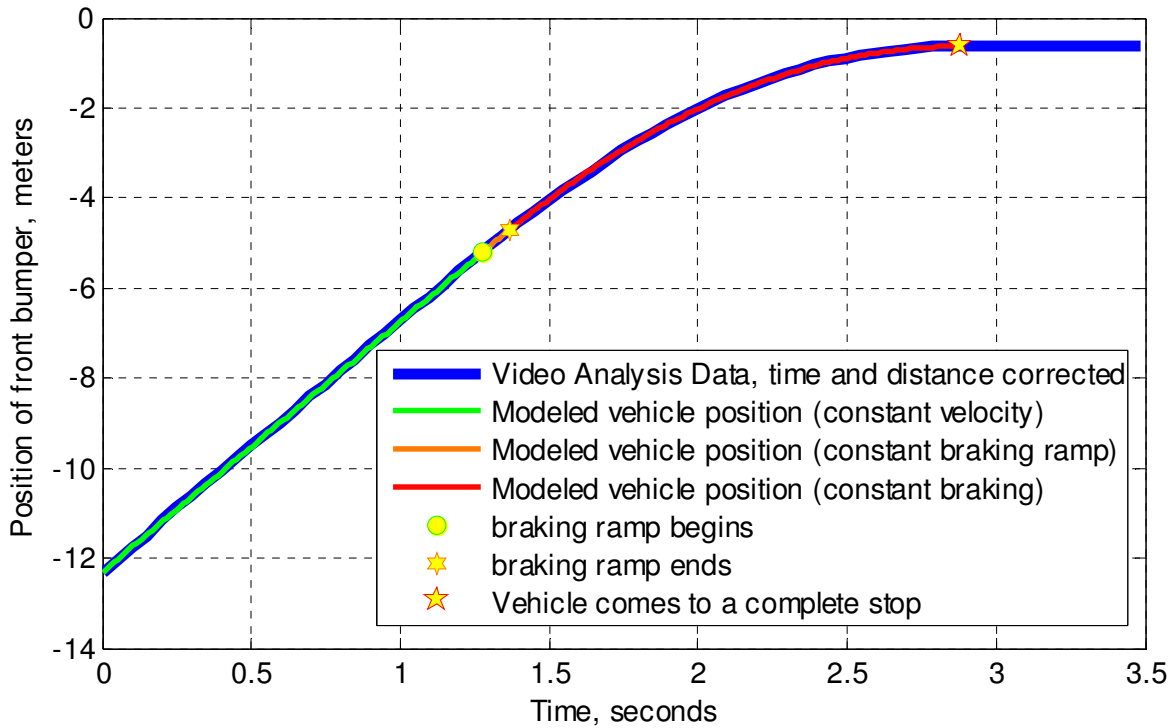


Figure 20. Actual and modeled position of Volvo. Data obtained from video taken with a non-panning, stationary camera.

2.8.2 ANALYSIS OF VIDEO TAKEN WITH A PANNING CAMERA

The video clip of the 45 km/h Volvo's AEB system demonstration was taken with a panning video camera. Because the perspective changes with each frame, the previously described video analysis method does not yield accurate results for this video. Instead, the road geometry was used to determine the required perspective corrections for each video frame.

Figure 21 shows a drawing of the video analysis in progress for the Volvo 45 km/h video clip. White lines were painted on the road, perpendicular to the vehicle's heading, spaced 5 meters apart. Due to the poor quality of the video, blue arrows were drawn on the frame to help define the lines, ensuring that they converged on a single vanishing point. Parallel to the vehicle's heading are rough edges on both sides of the track which blend in with darker pavement on both sides of the

track. Blue lines were also drawn to help define these boundaries. The diamonds indicate points of interested that were tracked with Tracker for the current video frame as well as the past three frames.

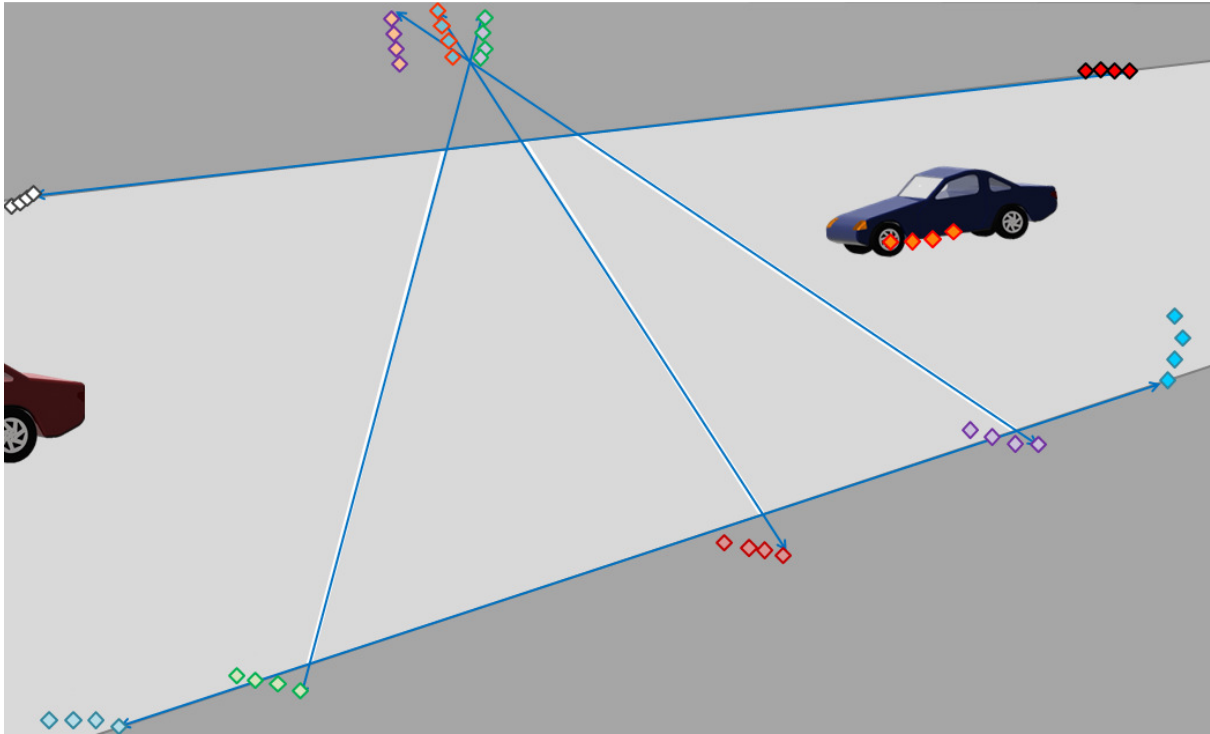


Figure 21. Analyzing video taken with a panning camera

Figure 22 shows a sketch of a video frame taken with a panning camera. Thick white lines represent the white lines perpendicular to the vehicle's heading. Thick yellow lines represent the edges of the track that are parallel to the vehicle's heading. The three dark blue lines and the two green lines represent the lines drawn on the actual frame as shown in Figure 21. Points outlined in black (points A-K) represent locations measured in the video frame in relation to a common 2-dimensional coordinate system. Point V represents the vanishing point of the edges of the track and was calculated by finding the intersection of the two green lines (line AB and line CD). Because the vehicle is assumed to be traveling parallel to the track, point V also serves as a vanishing point

for the vehicle. The light blue line, KV, represents the path of the vehicle. Points X, Y, and Z represent points in space, 5 meters apart, through which the vehicle's left wheels eventually pass. The location of point X was calculated by finding the intersection of EF and KV. The location of point Y was calculated by finding the intersection of GH and KV. The location of point Z was calculated by finding the intersection of IJ and KV.

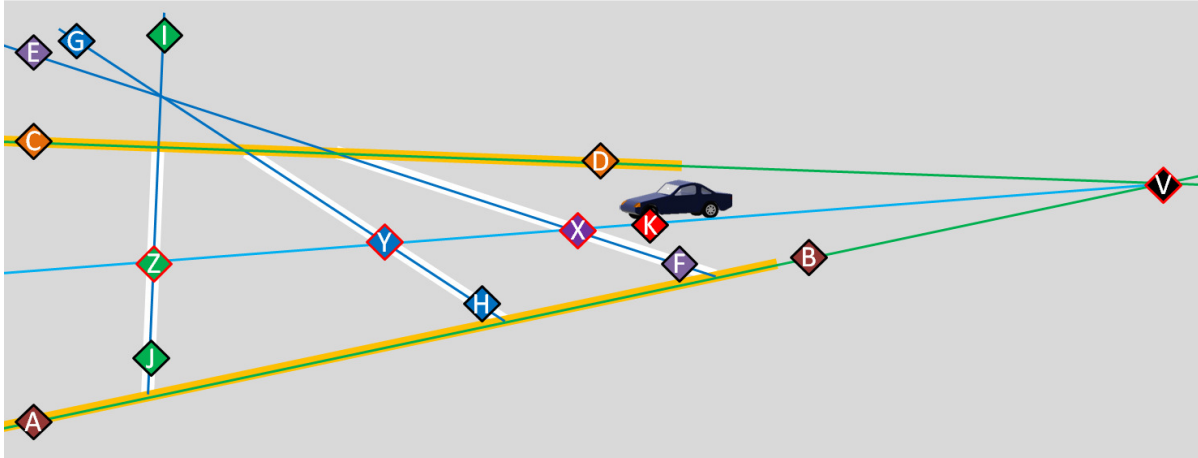


Figure 22. Method of determining position of vehicle in video taken with a panning camera

After the locations of X, Y, and Z are calculated, a coordinate system is defined such that the origin is Z and the x-axis is collinear with the light blue line, KV. The locations of Y, X, and K in this new coordinate system are calculated by using the Pythagorean Theorem. The actual distances between pairs of white lines are determined by

$$D(i) = c_1 \left(\frac{x_{i+1}}{c_2 x_{i+1} + 1} - \frac{x_i}{c_2 x_i + 1} \right) \quad (18)$$

where $D(1)$ is the distance between Z and Y, $D(2)$ is the distance between Y and X, x_1 is the location of Z, x_2 is the location of Y, and x_3 is the location of X. The distance between pairs of

white lines should always be 5 meters, so the following objective function was minimized to find optimal values of c_1 and c_2 :

$$\text{Objective Function} = \sigma_D + \sum_{i=1}^{N_{lines}-1} (D(i) - 5)^2 \quad (19)$$

where σ_D is the standard deviation of the distances computed with Equation 14 and N_{lines} is the number of white lines observed in the video frame. The position of the vehicle's front wheel was computed with Equation 3, with c_3 set to 5 meters. This process was completed for all 65 frames of the video clip.

After the physical position of the vehicle was determined for each video frame, the time was adjusted by multiplying the arbitrary time by a TAF, defined in Equation 12. For this test scenario in which the Volvo was traveling with an initial speed of 45 km/h, a_{AEB} and r_{AEB} were determined by fitting a piecewise curve to the corrected test data. The objective of this curve fitting was to minimize the sum of the residuals. Equations 3-7 were used as constraints.

Figure 23 shows the calculated position of the front bumper of the vehicle against time in blue. The modeled position of the vehicle is shown in green, orange, and red. The vehicle started off with a constant speed of 45 km/h. The AEB system initiated at a TTC_{AEB} of 0.91 seconds. The AEB system then provided a braking ramp of -16 m/s^3 followed by a constant acceleration of -10 m/s^2 . The Volvo struck the leading vehicle with an impact speed of 12.3 km/h (3.4 m/s).

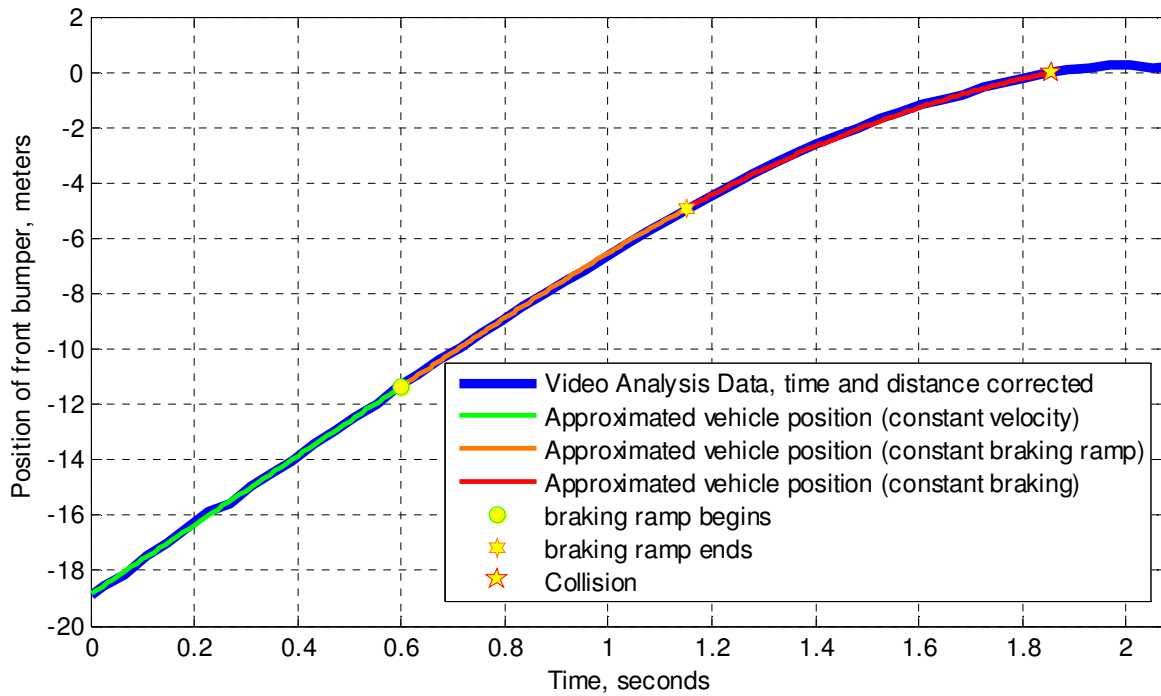


Figure 23. Corrected video analysis data with modeled vehicle position overlaid

Recall from Figure 7 the relationship between v_0 and TTC_{AEB} for the Volvo Algorithm. Note that the maximum TTC_{AEB} was chosen to equal 0.91 seconds which is the actual TTC_{AEB} value found by analyzing a 45 km/h system demonstration. TTC_{AEB} is assumed to level off at a velocity of 40 km/h because collisions can be fully prevented at speeds of 40 km/h, but not at speeds of 45 km/h. A linear fit was used to model the relationship of v_0 and TTC_{AEB} for speeds between 16 km/h and 40 km/h and the fit was extrapolated to provide a relationship for speeds between 4 km/h and 16 km/h. Note that with a coefficient of friction equal to 1.0, the modeled Volvo FCAS will prevent all crashes for initial speeds between 4 km/h and 40 km/h. This aspect of the Volvo's model is supported by the owner's manual and the ADAC test results.

2.9 SUMMARY OF FCAS ALGORITHMS

Table 7 summarizes the functionalities of all of the FCASs analyzed in this study and the AEB parameter values. Note that a_{AEB} is listed as -10 m/s^2 for the VW and Volvo Algorithms. This means that the deceleration is always assumed to equal the maximum deceleration permitted by friction. Although this is often an overestimate of the actual deceleration, the results will be accurate because the systems are designed to prevent all collisions above the lower speed threshold of operation and below 40 km/h as long as there exists sufficient friction between the vehicle's tires and the road.

Table 7. Summary of Forward Collision Avoidance System (FCAS) functionality and Autonomous Emergency Braking (AEB) parameter values

FCAS Algorithm	Lower Speed Threshold of Operation (km/h)	Upper Speed Threshold of Operation (km/h)	FCW?	AB?	AEB?	$a_{AEB} \left(\frac{m}{s^2}\right)$	$r_{AEB} \left(\frac{m}{s^3}\right)$
Audi	30	80	YES	YES	NO	–	–
BMW	15	None	YES	NO	NO	–	–
Infiniti	15	None	YES	NO	YES	-5.8	-12
VW	5	30	YES	YES	YES	-10.0	-40
Volvo	4	None	YES	YES	YES	-10.0	-16

2.10 LIMITATIONS OF METHODS USED TO MODEL FCASs

The FCAS models presented in this chapter are approximations rather than exact representations of the actual FCASs, but we are confident in the models enough to state that each FCAS modeled in this study is unique. The FCAS models should not be thought of as exact representations of their corresponding actual FCASs because several assumptions were used in the process of developing the models.

2.10.1 FCAS MODELING ASSUMPTIONS CONCERNING FUNCTIONALITY OF FCASs

The first assumptions made in the development of the methods concern the functionality of the systems, described in section 2.4. Assumptions were made about the lower and upper speed thresholds of operation. Assumptions were made concerning the functionality of the FCW, AB, and AEB systems. Assumptions were made concerning the overall functionality of the FCASs.

This study used owner's manuals and information on the Euro NCAP website to define the lower and upper speed thresholds of operation for all FCASs. This study assumed that the nominal speed thresholds were the same as those on the physical systems that were tested by the ADAC. Another limitation of this study was that when upper thresholds were not listed in owner's manuals, we assumed that there was no upper speed threshold of operation.

This study assumed that drivers were provided an FCW in all situations, given that the vehicle speed was within the operating range of speeds. In real world crashes the driver might brake lightly or steer slightly during an emergency situation. We had no test data to determine if early light braking or slight steering would prevent the initiation of an FCW.

The ADAC test report provided very limited data on Assisted Braking (AB) systems. This study had to make several assumptions concerning the functionality of (AB) systems. This study assumed that AB systems always functioned, even when the leading vehicle was stationary prior to the crash or when the driver was braking or accelerating. This study assumed that all three AB systems modeled in this study, functioned identically. This study assumed that the functionality of the AB systems was constrained by the lower and upper speed thresholds of operation. This study assumed that AB systems increased the braking force to the maximum force permitted by friction. This study assumed that AB systems activated at the same time that drivers were warned by an FCW system.

For the Infiniti, VW, and Volvo systems, this study assumed that the AEB systems functioned in all scenarios in which the vehicle was traveling at a speed within the range of system operation. This assumption might not hold true because some AEB systems might not activate if the driver is braking, accelerating, or steering. This study assumed that for the Audi and BMW FCASs, the AEB system never activated when the leading vehicle was stationary prior to the crash. This assumption might not hold true in a scenario in which the leading vehicle comes to a complete stop while in range of the FCAS sensing apparatus.

A few assumptions were made concerning the overall functionality of the FCASs that were analyzed in this study. All tests conducted by the ADAC were done on flat terrain on straight roads. This study assumed that system functionality was not influenced by terrain or the horizontal curvature of the road. This study assumed that systems always performed reliably with 100% correct target classification. In general the functionalities of the systems were simplified.

2.10.2 LIMITATIONS CONCERNING RELATIONSHIPS BETWEEN TTC_{FCW} AND SPEED

This study used three methods to model the relationships between TTC_{FCW} and speed. The purpose of using three different methods was to provide a range of results. Despite the ranges of potential effectiveness for each system, we cannot be sure that the ranges encompass the true effectiveness of each system. This study was limited by the data that was available to model the relationships between TTC_{FCW} and speed. Only between two and four data points were available to model these relationships for each FCAS. The confidence in these few data points is limited, because the ADAC test report did not provide confidence intervals or results from multiple tests for the same vehicle and scenario. Even if the data recorded by ADAC is truly representative of the systems at the speeds that were tested, the TTC_{FCW} recorded by the ADAC was only reported with 1/10 of a second accuracy.

2.10.3 LIMITATIONS CONCERNING a_{AEB} , r_{AEB} , AND THE RELATIONSHIPS BETWEEN TTC_{AEB} AND SPEED

Several assumptions were made when estimating values for a_{AEB} and r_{AEB} and when modeling the relationships between TTC_{AEB} and speed. These methods are described in sections 2.6 and 2.8. The first assumption concerning AEB was that all AEB systems can be modeled as a braking ramp followed by a constant acceleration, as shown in Figure 2. The second major assumption was that all AEB systems were modeled as having constant values of a_{AEB} and r_{AEB} for all emergency situations.

For the method used to model the VW AEB system, we assumed that the function $TTC_{AEB}(\text{speed})$ was equal to the function $TTC_{FCW}(\text{speed})$. Recall from section 2.10.2 that using this function is a limitation within itself. In addition, using this function was an assumption because this modeling decision was based on a vague statement in the ADAC test report. A value for r_{AEB} was defined for the VW system by means of video analysis. We assumed that our video analysis methods were valid and we also assumed that the video clip taken of the VW was during the test in which the VW was approaching the stationary balloon car at an initial speed of 20 km/h.

For the method used to model the Infiniti AEB system, we used ADAC test information from a test in which the test vehicle was traveling at an initial speed of 100 km/h and the leading vehicle was traveling at an initial speed of 60 km/h. We assumed that the braking ramp and braking acceleration would be constant across all scenarios, including those scenarios in which the leading vehicle was stationary prior to the crash. We also assumed that the Infiniti had a maximum TTC_{AEB} of 0.955 seconds. This is perhaps one of the largest assumptions of this thesis because it is based solely on a design philosophy for AEB systems.

We used video analysis to model the Volvo AEB system. The major assumption with this method was that the video analysis methods were valid. Future work should validate these methods

with test vehicles. Another assumption was that the first vehicle analyzed in this study was that of a Volvo approaching a vehicle with an initial speed of 16 km/h. This video showed that the vehicle was operated by a human who was supposedly trying to maintain a constant speed of 16 km/h prior to the crash. It is possible that the chosen value of 16 km/h is not accurate. The third video analyzed in this study was of a Volvo approaching a stationary semi-trailer with an initial speed of 45 km/h. We assumed that the reported initial speed of 45 km/h was accurate.

Although several assumptions were made when modeling the three AEB systems, the conclusions of this thesis were not affected. The great number of assumptions makes it impossible to precisely compare the effectiveness of each physical system, but the modeled systems can still provide insight into the design of AEB systems.

2.11 CONCLUSIONS CONCERNING THE METHODS USED TO MODEL FCASs

This chapter introduced the methods that were used to model five FCASs. This chapter also described the results of these methods. The specifications of the actual algorithms used in the five FCASs are proprietary, and not publicly available. One of the novel accomplishments of this thesis was the development of methods to “reverse-engineer” these algorithms from publicly available system descriptions, test reports, and videos. Due to the limited availability of information, we emphasize that the algorithms presented in this chapter are approximations of the physical systems – rather than exact representations.

3. METHOD OF COMPUTING BENEFITS BY FCAS

The effectiveness of each modeled system was measured by its ability to prevent collisions and reduce collision severity of reconstructed crashes. The study is based on 977 rear-end crashes, representing 799,960 crashes, extracted from the National Automotive Sampling System/Crashworthiness Data System (NASS/CDS) from year 1993 to 2008. NASS/CDS is a nationally representative sample of U.S. crashes in which at least one vehicle was towed from the scene. This limitation of only analyzing crashes in which at least one vehicle was towed will skew the results, but only slightly when analyzing the number of injuries prevented because the probability of injury is low in crashes that did not result in a towed vehicle. This study further limited the analysis to rear-end collisions in which the leading vehicle was stationary prior to the collision. This restriction was due to the limited availability of ADAC test results for scenarios in which the leading vehicle was not stationary prior to collision. The crashes that were mathematically reconstructed and simulated represent 18% of all crashes annually in the U.S., accounting for 70% of the 25% of annual rear-end crashes in the U.S. Part of the NASS/CDS investigation was to determine the vehicle change in velocity during impact (delta-V) which is a good indication of the likelihood of injury [23]–[25].

Benefits of each algorithm were estimated by computing the reduction of expected delta-V for each case. Figure 24 illustrates the process used to determine these benefits. This study used a modified method previously developed by Kusano et al. for predicting the delta-V based on algorithm functionality and driver reaction to an FCW [9]. Each case was mathematically reconstructed several times with a range of values for the parameters not provided by the NASS/CDS database. These parameters were the driver braking magnitudes and reaction times to FCWs. The expected delta-V was calculated as if each car was equipped with an FCAS. In

addition, this study estimated the expected reduction in the number of moderately-to-fatally injured drivers (MAIS2+) with an injury risk function dependent on the crash delta-V.

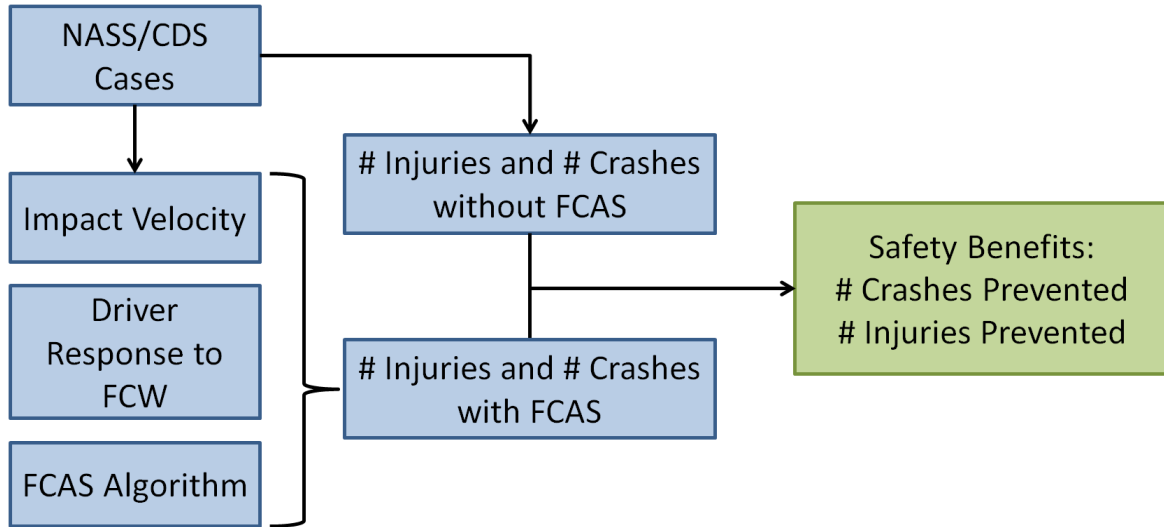


Figure 24. Process used to determine safety benefits of each FCAS algorithm

3.1 CALCULATING DELTA-V AS IF THE VEHICLE WERE EQUIPPED WITH AN FCAS

In a previous study, Kusano et al. developed a method that computes the delta-V of a trailing vehicle in a rear-end crash as if the vehicle were equipped with an FCAS. This method used a conservation of momentum approach to determine the reduction in delta-V due to braking caused by an FCAS. This method is effective for some cases, but is not accurate for others. Cases in which this method does not work include those in which the driver was assumed to brake when the vehicle was not equipped with an FCAS. In the current study, the momentum approach is replaced with a new method.

Instead of calculating the reduction in delta-V as a means for calculating the delta-V when the vehicle is equipped with an FCAS, the current study's method involves calculating the impact speed

as an intermediate step. Another difference is the equation used to compute the relative velocity at impact. Kusano et al. used

$$V_{12,impact} = \Delta V_1 + \Delta V_2 \quad (20)$$

where $V_{12,impact}$ is the relative speed at impact, ΔV_1 is the trailing vehicle's change in speed during the collision, and ΔV_2 is the leading vehicle's change in speed during the collision. The current study adopts Equation 21 from Rose et al. [26]

$$v_{impact} = \sqrt{2E_T \frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_1 m_1 \gamma_2 m_2} \sec \theta_1 + V_2 [\cos(\theta_1 - \theta_2) - \tan \theta_1 \sin(\theta_2 - \theta_1)]} \quad (21)$$

v_{impact} is the speed of the trailing vehicle prior to impact, E_T is the total energy absorbed in the crash as determined by NASS/CDS investigators, m_1 is the mass of the trailing vehicle, m_2 is the mass of the leading vehicle, γ_1 is the effective mass coefficient for the trailing vehicle, γ_2 is the effective mass coefficient for the leading vehicle, and V_2 is the speed of the leading vehicle prior to impact.

Figure 25 illustrates that θ_1 and θ_2 are the angles between the Principal Direction of Force (PDOF) line and the headings of vehicles 1 and 2, respectively. The figure also illustrates the moment arm of the resultant collision force, h . This parameter is necessary to calculate the effective mass coefficient, γ , for each vehicle as shown in Equation 22, which was also adopted from Rose [26].

$$\gamma = \frac{k^2}{k^2 + h^2} \quad (22)$$

In this study, the radius of gyration for the vehicle, k , was estimated to equal one third of the length of the vehicle. The effective mass coefficient is a fraction between 0 and 1 and represents the proportion of the mass that contributes to the change in velocity along the vehicle's heading.

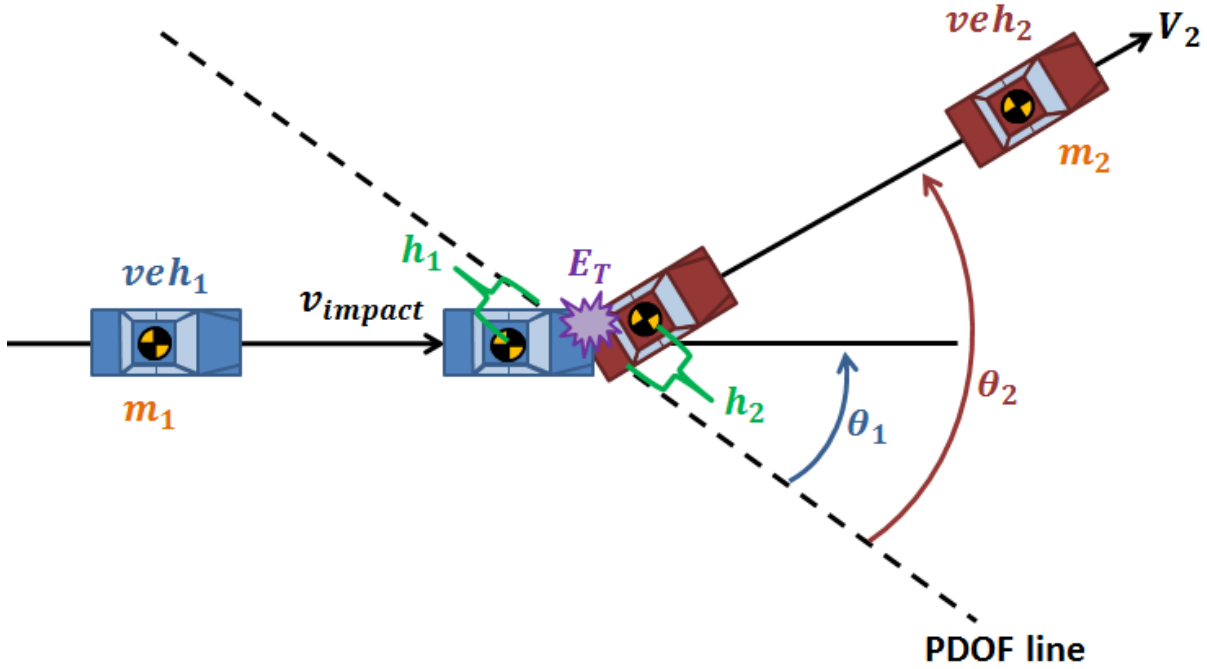


Figure 25. Illustration of Equation 21 parameters.

Because only cases in which the leading vehicle was stationary prior to impact were simulated in this study, Equation 21 simplifies to

$$v_{impact} = \sqrt{2E_T \frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_1 m_1 \gamma_2 m_2}} \sec \theta_1 \quad (23)$$

The distribution of impact speeds that were reconstructed for all simulations is shown in Figure 26. Note that this distribution was not weighted to represent the distribution of reconstructed impact speeds for all U.S. rear-end crashes in which the leading vehicle was stationary prior to impact and at least one vehicle was towed from the crash site. The lowest impact speed

reconstructed from the crashes had a value of 18 km/h. Because lower impact speed crashes were not reconstructed and simulated, this study offers little insight into the benefit differences of the five FCASs for low speed impact scenarios.

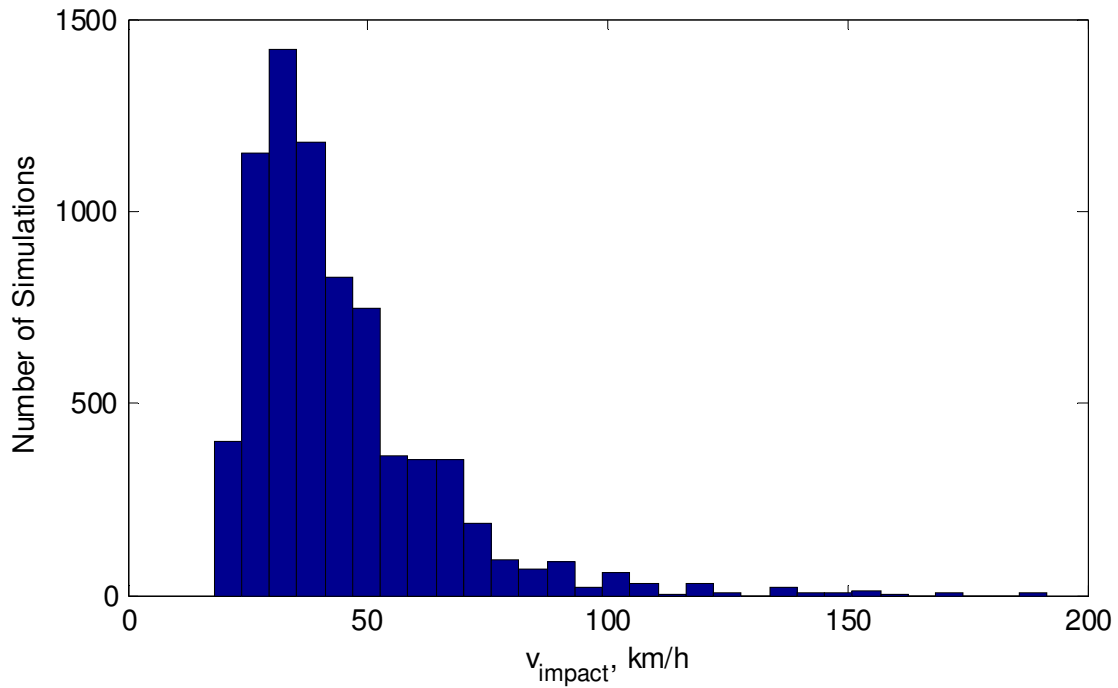


Figure 26. Distribution of reconstructed impact speeds for all simulations

After reconstructing the impact speed, the next step in the revised method was to compute the velocity of the trailing vehicle at the time at which the FCW would have been given, provided the vehicle was equipped with an FCAS. This step in the process required an assumption of how the driver of the trailing vehicle behaved prior to the crash (in the actual vehicle that was not equipped with an FCAS – not the hypothetical vehicle with an FCAS). This study adopts the same assumptions used by Kusano et al. Based on the pre-crash maneuver (MANEUVER), the driver was determined to be accelerating, braking, or maintaining constant speed. It was also assumed that the driver did not steer in the moments before the collision.

If the MANEUVER NASS/CDS variable indicated that the trailing driver was accelerating prior to the collision, the acceleration of the vehicle was assumed to equal 0.2g. If MANEUVER indicated that the driver was braking, the acceleration of the vehicle was assumed to either have a constant acceleration of -0.2g to simulate an early, weak braking force or -0.4g to simulate a late, strong braking force. Both of these scenarios were given an equal weight of occurrence. In the case of a late, strong braking force, the driver was assumed to apply the brakes at a TTC of 0.4 seconds. Table 8 shows the probability of each driver action, given the value of MANEUVER. When the driver maneuver was unknown, the driver was assumed to have a 29% percent chance of maintaining a constant speed and a 71% chance of braking because these proportions were consistent with the proportions of known maneuvers. In other words, excluding the cases with an unknown value for MANEUVER, in 71% of cases the MANEUVER variable indicated that the striking vehicle's driver braked prior to impact. The proportion of drivers that accelerated was ignored because it was less than 1%.

Table 8. Probabilities of possible driver actions, given MANEUVER.

MANEUVER	Late Braking (a = -0.4g)	Early Braking (a = -0.2g)	Constant Speed (a = 0)	Accelerating (a = 0.2g)
Braking	50%	50%	0%	0%
Accelerating	0%	0%	0%	100%
Neither	0%	0%	100%	0%
Unknown	35.5%	35.5%	29%	0%

In the cases of early braking, maintaining constant speed, and accelerating, the acceleration of the vehicle was assumed to be constant between the time of collision and the time at which the driver would have been given an FCW by an FCAS, had it been installed in the vehicle. For these scenarios, Equations 24 - 27 were solved simultaneously for the position and speed of the trailing vehicle relative to the leading vehicle at the time of the FCW.

$$v_{impact} = v_{FCW} + a_0(t_{FCW,impact}) \quad (24)$$

$$0 = x_{FCW} + v_{FCW}(t_{FCW,impact}) + \frac{1}{2}a_0(t_{FCW,impact})^2 \quad (25)$$

$$TTC_{FCW} = -\frac{x_{FCW}}{v_{FCW}} \quad (26)$$

$$TTC_{FCW} = f(v_{FCW}) \quad (27)$$

Figure 27 provides an example of a collision that would be reconstructed with this system of equations. v_{FCW} is the speed of the trailing vehicle when FCW would have been given, x_{FCW} is the position of the trailing vehicle relative to the leading vehicle when the FCW would have been given, a_0 is the assumed acceleration of the vehicle prior to impact given that the vehicle is not equipped with an FCAS, and $t_{FCW,impact}$ is the difference of the time of collision and the time at which an FCW would have been given. Equation 27 refers to the relationship between v_{FCW} and TTC_{FCW} for each FCAS as shown in Figures 4-6.

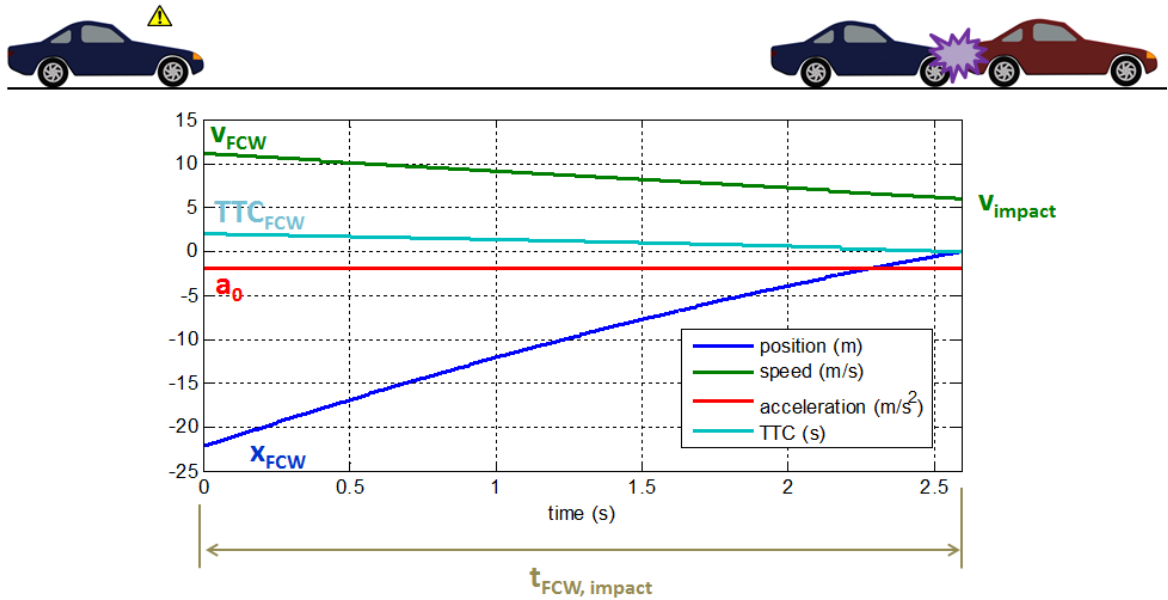


Figure 27. Example collision in which leading driver accelerated, braked early and weak, or maintained a constant speed prior to the collision.

For the scenario in which the driver was assumed to brake late with an acceleration of -0.4g, Equations 28 - 37 are solved simultaneously for v_{FCW} and x_{FCW} . Note that because the initial acceleration was assumed to equal zero, all values of a_1 were set equal to zero while solving this system of equations.

$$v_{SLB,begin} = v_{FCW} + a_1 t_{FCW,SLB} \quad (28)$$

$$x_{SLB,begin} = x_{FCW} + v_{FCW} t_{FCW,SLB} + \frac{1}{2} a_1 (t_{FCW,SLB})^2 \quad (29)$$

$$TTC_{FCW} = -\frac{x_{FCW}}{v_{FCW}} \quad (30)$$

$$TTC_{FCW} = f(v_{FCW}) \quad (31)$$

$$a_{SLB} = r_{SLB}(t_{SLB}) + a_1 \quad (32)$$

$$v_{SLB,end} = \frac{1}{2} r_{SLB}(t_{SLB})^2 + a_1(t_{SLB}) + v_{SLB,begin} \quad (33)$$

$$x_{SLB,end} = \frac{1}{6} r_{SLB}(t_{SLB})^3 + \frac{1}{2} a_1(t_{SLB})^2 + v_{SLB,begin}(t_{SLB}) + x_{SLB,begin} \quad (34)$$

$$TTC_{SLB} = -\frac{x_{SLB,begin}}{v_{SLB,begin}} \quad (35)$$

$$v_{impact} = v_{SLB,end} + a_{SLB}(t_{SLB,impact}) \quad (36)$$

$$0 = \frac{1}{2} a_{SLB}(t_{SLB,impact})^2 + v_{SLB,end}(t_{SLB,impact}) + x_{SLB,end} \quad (37)$$

Figure 28 provides an example of a collision that would be reconstructed with this system of equations. a_1 , which for all Strong, Late Braking (SLB) cases was set equal to zero, is the acceleration of the vehicle prior to late, strong braking. $v_{SLB,begin}$ is the speed of the trailing vehicle when the driver starts to brake strong, and late. $t_{FCW,SLB}$ is the time between the driver's onset of braking and the time at which an FCW would have been delivered, had the vehicle been equipped with an FCAS. $x_{SLB,begin}$ is the position of the trailing vehicle relative to the leading vehicle when the driver starts to brake. a_{SLB} , which is always set equal to -0.4g, is the steady-state

acceleration of the vehicle when the driver brakes strong and late. r_{SLB} is the braking ramp braking ramp due to strong, late braking. t_{SLB} is the time it takes for the acceleration due to strong, late braking to reach steady-state. $v_{SLB,end}$ is the speed of the vehicle when the braking ramp ends. $x_{SLB,end}$ is the position of the trailing vehicle relative to the leading vehicle when the braking ramp ends. TTC_{SLB} is the Time to Collision at which the driver starts to brake strong, and late. $t_{SLB,impact}$ is the difference between the time at which the collision occurs and the time at which the braking ramp ends and the acceleration of the vehicle reaches a steady-state condition of $-0.4g$.

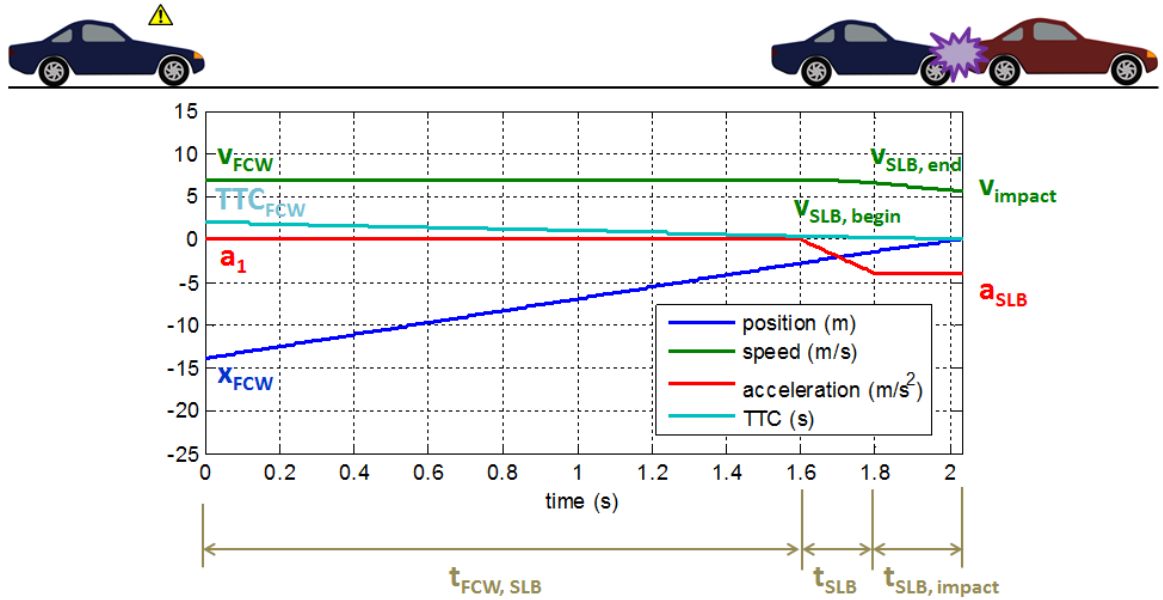


Figure 28. Example collision in which the driver braked late with an acceleration of $-0.4g$

Both systems of equations used to determine v_{FCW} and x_{FCW} have several constraints. All values of speed (v_{FCW} , $v_{SLB,begin}$, $v_{SLB,end}$, v_{impact}) must be positive. All values of position (x_{FCW} , $x_{SLB,begin}$, $x_{SLB,end}$) must be less than or equal to zero. All values of time ($t_{FCW,impact}$, $t_{FCW,SLB}$, t_{SLB} , $t_{SLB,impact}$) must be positive.

After the values of v_{FCW} and x_{FCW} have been determined, the crash is simulated as if the vehicle was equipped with an FCAS. In some scenarios, the FCAS offers no benefit because the driver fails to react to the FCW prior to the crash and the FCAS does not come with an AEB component or no systems are functional for the vehicle's speed. In many scenarios, however, the FCAS does offer a benefit because the driver reacts to an FCW in time to mitigate a crash or an AEB system activates. The reaction time of the driver, the presence of AB, and a_{AEB} are used to define the acceleration of the vehicle as a function of time. The reaction time of the driver to an FCW is discussed in section 3.1.1. Using basic kinematic equations and this acceleration function, the impact speed, v_{impact}^* , is determined as if the vehicle was equipped with an FCAS. Equation 23 is re-arranged to solve for the new impact energy, E_T^* .

$$E_T^* = \frac{\gamma_1 m_1 \gamma_2 m_2}{2(\gamma_1 m_1 + \gamma_2 m_2)} (v_{impact}^*)^2 \cos^2 \theta_1 \quad (38)$$

The new delta-V is calculated with Equation 39 from Rose:

$$\Delta V_1^* = \sqrt{\frac{2\gamma_1 E_T^*}{m_1 \left(1 + \frac{\gamma_1 m_1}{\gamma_2 m_2}\right)}} \quad (39)$$

Similarly, the new delta-V for the leading vehicle was also calculated:

$$\Delta V_2^* = \sqrt{\frac{2\gamma_2 E_T^*}{m_2 \left(1 + \frac{\gamma_2 m_2}{\gamma_1 m_1}\right)}} \quad (40)$$

The process for determining the new delta-V is shown in Figure 29. Note that there are five unique methods of modeling the acceleration as a function of time, $a(t)$. The method choice is dependent on the functionality of the FCAS and the reaction time of the driver.

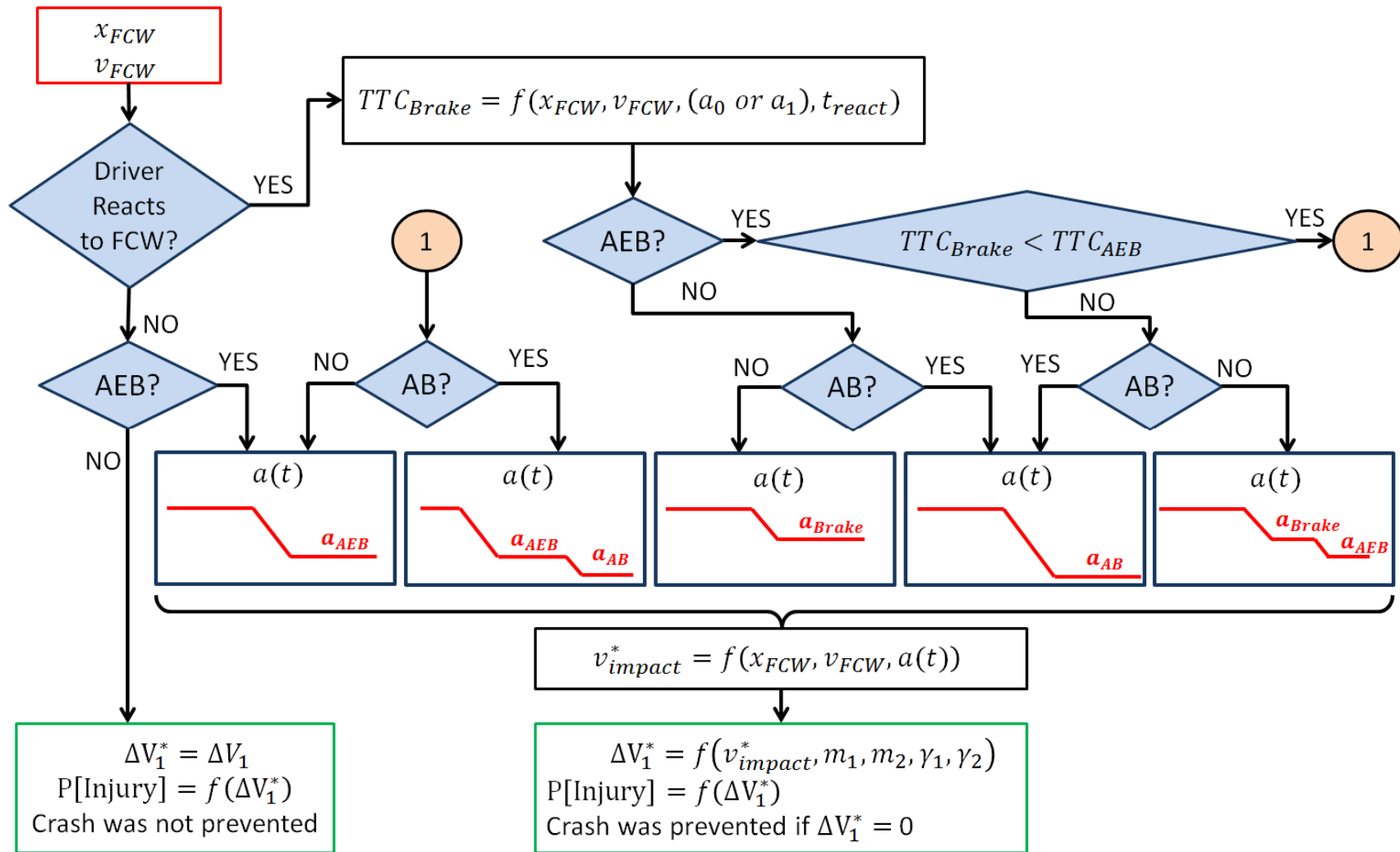


Figure 29. Method used in this study to determine the delta-V of the trailing and leading vehicles during the collision, had the trailing vehicle been equipped with an FCAS

3.1.1 DRIVER REACTION TIME TO FCW

A value for driver reaction time in response to an FCW is necessary to estimate the kinematics of the trailing vehicle prior to a simulated crash or near-crash event. Because the reaction times of the drivers in the NASS/CDS cases are unknown, the cases were simulated several times with different values of driver reaction times.

This study used a range of reaction times based on a study by Sivak et al. [27]. The objective of the study was to determine the effects of high-mounted brake lights on the behavior of following drivers. The study collected naturalistic driver response times to a leading vehicle's brake light initiation and reported a discrete distribution.

The current study fit a lognormal distribution to the data obtained from Sivak. This distribution was discretized as shown in Table 9. Four reaction times were chosen to represent the entire population of driver reaction times. Each reaction case represents 25% of the entire population of reaction times. The value for each reaction time was chosen based on cumulative distribution values centered for each range of reaction times represented by the single reaction time. This study assumed that all drivers reacted to an FCW if one was provided.

Table 9. Driver reaction times used in this study

	Case 1	Case 2	Case 3	Case 4
Reaction Time (seconds)	0.55	0.87	1.25	1.98
Reaction Time Weight (%)	25%	25%	25%	25%
CDF value (%)	12.5%	37.5%	62.5%	87.5%

Although the Sivak study provided a reasonable distribution of reaction times to be used in this study, it is not perfect. Naturalistic data pertaining to driver reaction times to FCWs would be very beneficial in future benefits estimation studies.

3.1.2 CALCULATING EXPECTED NUMBER OF CRASHES PREVENTED

In many simulations, the potential crash was completely prevented. The total expected number of crashes was calculated by

$$\mathbb{E}[Crashes] = \sum_{j=1}^{N_{cases}} \begin{pmatrix} W_j, & \text{if crash occurred} \\ 0, & \text{if crash did not occur} \end{pmatrix} \quad (41)$$

W_j is the weight of the simulation case and is calculated by

$$W_j = W_{NASS}W_{brake}W_{react} , \quad (42)$$

where W_{NASS} is the weight assigned to the NASS/CDS case that was simulated several times. W_{brake} is the weight assigned to how the driver behaved prior to the crash that actually happened. Recall that W_{brake} is defined in Table 8. W_{react} is the weight assigned to the simulation reaction time which is defined in Table 9.

3.2 ESTIMATING NUMBER OF INJURIES

Several studies have found that the risk of crash injury is correlated with vehicle crash ΔV [23]–[25]. In this section, the probability of injury was modeled with a logistic regression. The independent variable was delta-V and the dependent variable was a moderate to fatal injury indicator. The moderate to fatal injury indicator was assigned a value of 1 if the driver

experienced a MAIS2+ injury and was assigned a value of zero, otherwise. Using a weighted logistic regression, a probability of injury curve was modeled for the driver of the trailing vehicle. It is important to note that the reduction in injures for the driver of the trailing vehicle is only a portion of the expected safety benefits due to FCASs. These systems are also expected to reduce the probability of injury for the driver of the leading vehicle and the passengers of both vehicles.

The model that was used for calculating the probability of injury for the driver of the trailing vehicle, given ΔV_1^* is shown in Table 10. This injury risk model is identical to the model discussed by Kusano and Gabler [9], [23]. The only difference is that this model is only applicable to drivers that were belted. The c-statistic is a measure between zero and unity that approximates the area under the Receiver Operator Characteristic curve, a measure of model discrimination. A predictor that is a random guess has a c-statistic of 0.5. As a guideline, models with a c-statistic greater than 0.7 have adequate predictive power.

Table 10. Model and regression fitting information for probability of injury of trailing vehicle driver

Predictor Variables	Coefficient Values
β_1 : Intercept	-6.068
β_2 : Delta-V (km/h)	0.1
c-statistic	0.805
Number of cases	1960
Weighted Frequency	405,487

The probability of injury is calculated with

$$P[Injury_{Leading Driver}] = \left\{ \begin{array}{ll} \frac{1}{1 + e^{-(B_1+B_2\Delta V_1)}}, & \text{if crash occurred} \\ 0, & \text{if crash did not occur} \end{array} \right\} \quad (43)$$

The curve defined by this model is shown in Figure 30.

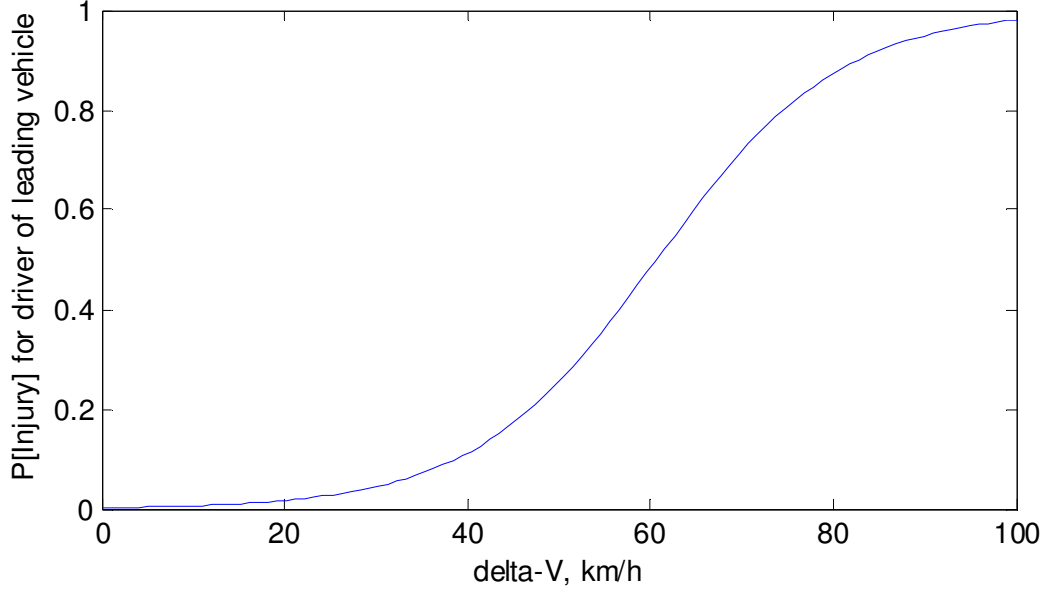


Figure 30. Moderate-to-Fatal Injury (MAIS2+) Model for Trailing Vehicle Drivers

The expected number of injuries was computed with

$$\mathbb{E}[Injuries] = \sum_{j=1}^{N_{cases}} (P[Injury_j] * w_j) \quad (44)$$

where $P[Injury_j]$ is the probability of a moderate to fatal injury (MAIS2+) for a simulated case and w_j is the weight of that simulated case.

4. RESULTS

This chapter presents the expected safety benefits of each of the FCAS if implemented across the entire U.S. fleet. The effectiveness of each system will be presented in terms of crashes avoided and injuries prevented. As described earlier in this thesis, each FCAS algorithm was modeled using three methods. The first method did not extrapolate outside of test conditions and interpolated between test conditions to fully define the algorithms. The second method used linear interpolation and extrapolation of the test data to fully define the algorithms. The third method used the same AEB algorithm found in method 2, but used a single linear fit to define the relationship between TTC_{FCW} and speed instead of using linear interpolation. Method 3 extended this fit outside of the test conditions.

For each method of modeling the FCAS, the NASS/CDS cases were reconstructed several times to account for varying driver reactions to the collisions that actually happened and the forward collision warnings that were introduced by the FCAS algorithms. There were 977 cases simulated several times that represented 799,960 crashes that occurred between 1993 and 2008. The total number of simulations for each FCAS was 7,460.

4.1 RESULTS FOR FCASs MODELED WITH METHOD 1

Figure 31 compares the cumulative distributions of impact speeds for vehicles without an FCAS versus the same vehicles equipped with all of the FCASs that were modeled with Method 1. The cumulative distributions for the Audi and BMW algorithms were very similar. Both modeled systems lacked an AEB component when the leading vehicle was stationary prior to the crash. The Audi's distribution was more concentrated in the lower range of impact speeds, likely

because it possessed an AB sub-system, whereas the BMW algorithm did not possess an AB system. The Infiniti’s distribution of impact speeds indicates that the system offered benefits for a wide range of approaching speeds. The VW’s distribution clearly shows that the system functioned very well, but only at low speeds. The Volvo’s distribution indicates that the system was very effective across all speeds.

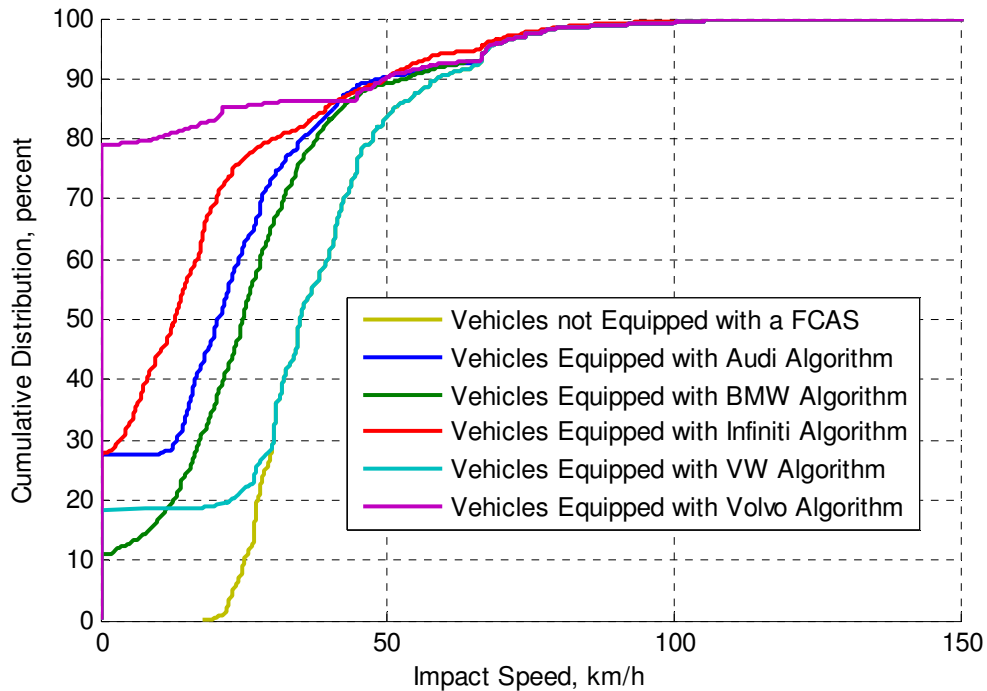


Figure 31. Cumulative Distributions of impact speeds for Method 1 of modeling FCASs

Figure 32 shows the cumulative number of crashes that were either prevented or at least mitigated by an FCAS modeled with Method 1. These weighted cumulative distributions were plotted against the impact speeds of the reconstructed cases. The Method 1 VW algorithm offered benefits to the fewest number of vehicles because it only functions at speeds between 20 and 30 km/h. The Method 1 Audi algorithm did not offer benefits to as many simulated drivers

as did the Method 1 BMW, Infiniti, and Volvo algorithms because it only operates at speeds between 30 km/h and 70 km/h instead of 20 km/h and 70 km/h.

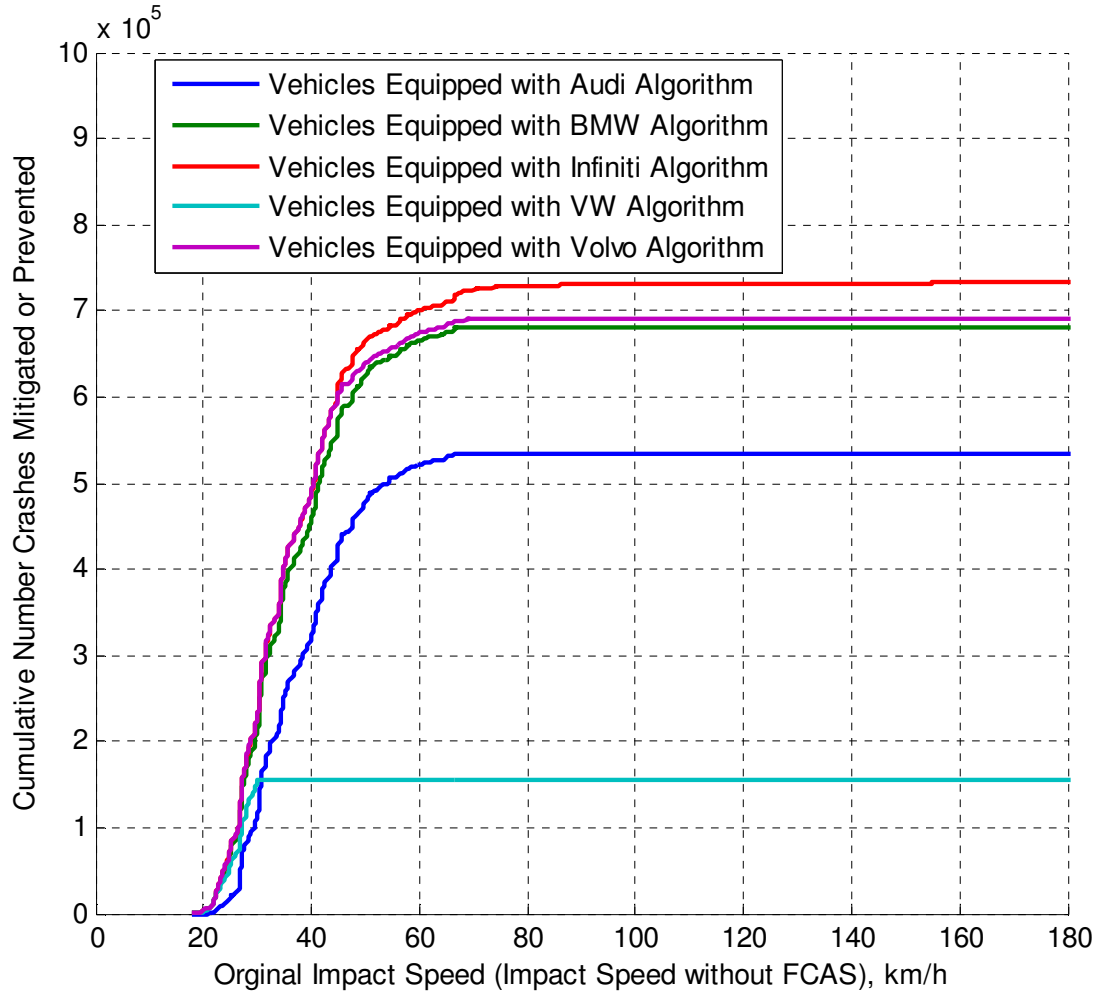


Figure 32. Cumulative Number of crashes that were either prevented or mitigated due to FCASs modeled with Method 1

Figure 33 shows the cumulative number of crashes that were prevented by an FCAS modeled with Method 1. These weighted cumulative distributions were plotted against the impact speeds of the reconstructed cases. The Method 1 Audi algorithm does fairly well in preventing collisions, offering a 27% reduction in crashes. The Audi system modeled with

Method 1 prevents collisions up to its upper speed threshold of 70 km/h. The BMW prevents the least number of crashes and only up to reconstructed case impact speeds of approximately 30 km/h. The Infiniti algorithm modeled with Method 1 is second best at preventing crashes when compared to the other systems that were modeled with Method 1, outperforming the Method 1 Audi algorithm by less than 1%. The VW curve is almost identical to the curve found in Figure 32. This means that if the modeled system activates, it will almost always completely prevent the potential crash. The Volvo system modeled with Method 1 is best at preventing crashes when compared to the other systems modeled with Method 1. This modeled system offers a crash reduction of 79%.

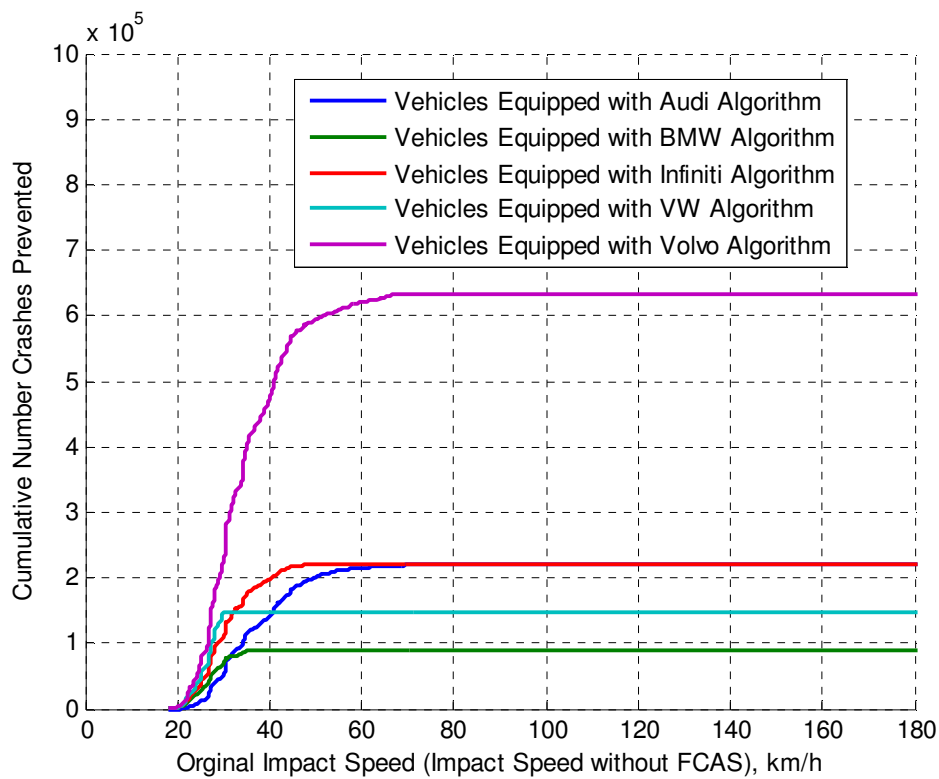


Figure 33. Cumulative Number of crashes that were prevented due to FCASs modeled with Method 1

Figure 34 shows the cumulative number of MAIS2+ injured trailing vehicle drivers. The modeled system for the VW prevented the fewest number of MAIS2+ injuries, followed closely by the modeled BMW system. The modeled Audi system prevented more injuries, likely because of its AB system. The modeled Infiniti and Volvo modeled systems prevented the most injuries, likely because they both have FCW, AB, and AEB systems.

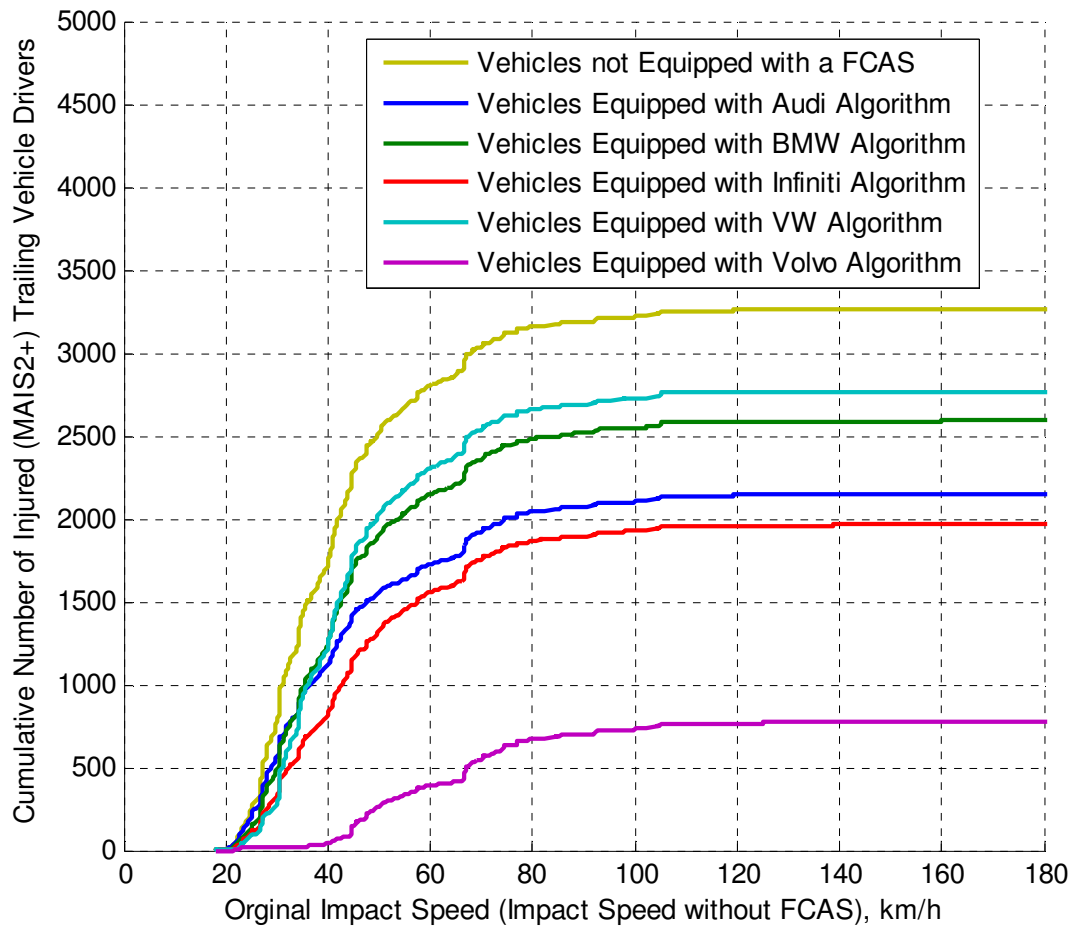


Figure 34. Cumulative Number of MAIS2+ injured drivers of trailing vehicles for FCASs modeled with Method 1

Table 11 shows the expected number of rear-end crashes in the United States in 2012 if the entire U.S. fleet were to be equipped with one of the systems analyzed in this study. It also shows the expected number of injuries caused by this mode of collision for the driver of the striking vehicle.

Table 11. Expected Number of Rear End crashes in which the leading vehicle was stationary during impact and related injuries from 1993-2011 if U.S. fleet had been equipped with one of the FCASs modeled with Method 1

Algorithm	Expected Number of Crashes	Expected Number of Injuries for Drivers in Striking Vehicle
None	799,960	3,274
Audi	580,618	2,156
BMW	711,593	2,598
Infiniti	578,980	1,974
VW	652,127	2,775
Volvo	167,058	779

4.2 RESULTS FOR FCASs MODELED WITH METHOD 2

Figure 35 shows the cumulative distributions of impact speeds for all of the FCASs that were modeled with Method 2. These curves are very similar to those curves obtained by modeling FCASs with Method 1, shown in Figure 31. The Audi curve obtained with Method 2 is almost identical to the curve obtained with Method 1. The only difference is that for Method 2, there is a lower concentration of crashes that occurred with impact speeds between 70 km/h and 80 km/h. The Method 2 BMW curve did not change much either compared to the Method 1 BMW curve. There was a slight shift of impact speeds from 70 km/h and above to impact speeds between 40 km/h and 70 km/h, which shows that the main difference between the Method 1 and Method 2 algorithms is the upper speed threshold of operation. The Method 2 Infiniti

algorithm offers benefits that substantially outweigh the benefits of the Method 1 Infiniti algorithm. The number of prevented crashes increased from 28% to 36% and for this second method, there were fewer simulated cases that resulted in high impact speeds. The Method 2 VW algorithm offers slightly better benefits than the Method 1 VW algorithm because the lower speed threshold was extended from 20 km/h to 5 km/h. If this study analyzed crashes in which no vehicles were towed from the scene, the increased effectiveness of the Method 2 algorithm would be more pronounced. The Volvo algorithm yielded the greatest improvement from Method 1 to Method 2.

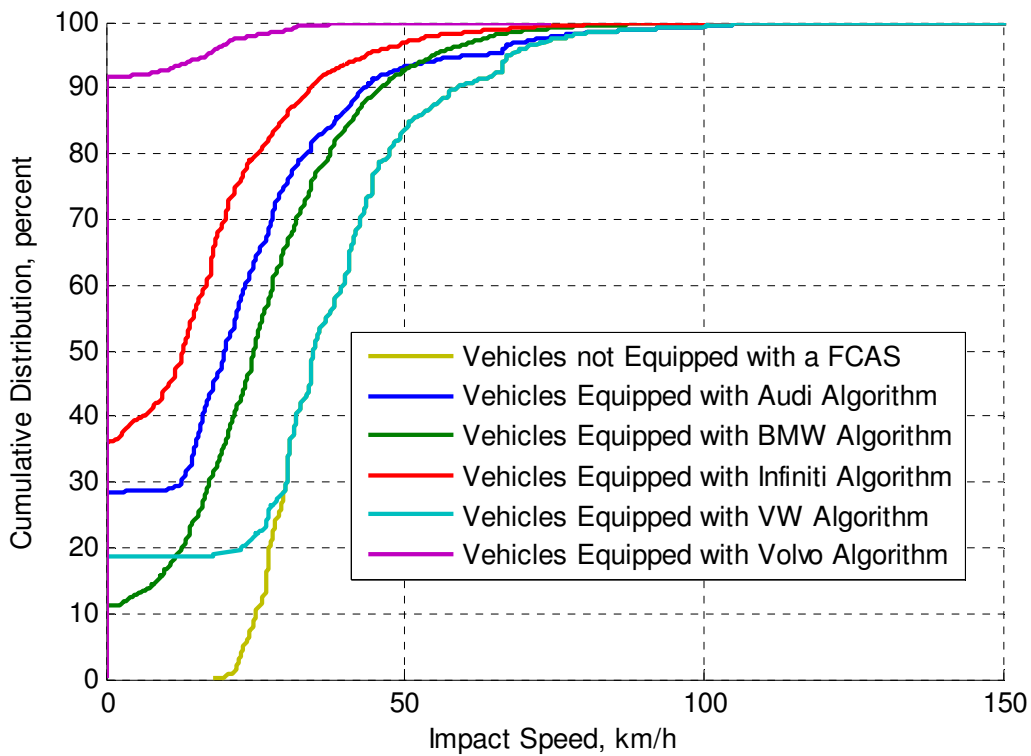


Figure 35. Cumulative Distributions of impact speeds for Method 2 of modeling FCASs

Figure 36 shows the cumulative number of crashes that were either prevented or at least mitigated by an FCAS modeled with Method 2. These weighted cumulative distributions were plotted against the impact speeds of the reconstructed cases. As expected, these curves are very similar to their Method 1 counterparts. The Audi algorithm modeled with Method 2 offers extra benefits between original impact speeds of 70 km/h and 80 km/h. The BMW, Infiniti, and Volvo algorithms modeled with Method 2 offer more benefits than their Method 1 counterparts, primarily above original impact speeds of 70 km/h. These three algorithms and the VW algorithm modeled with Method 2 also experienced limited benefits below reconstructed crash impact speeds (original impact speeds) below 20 km/h.

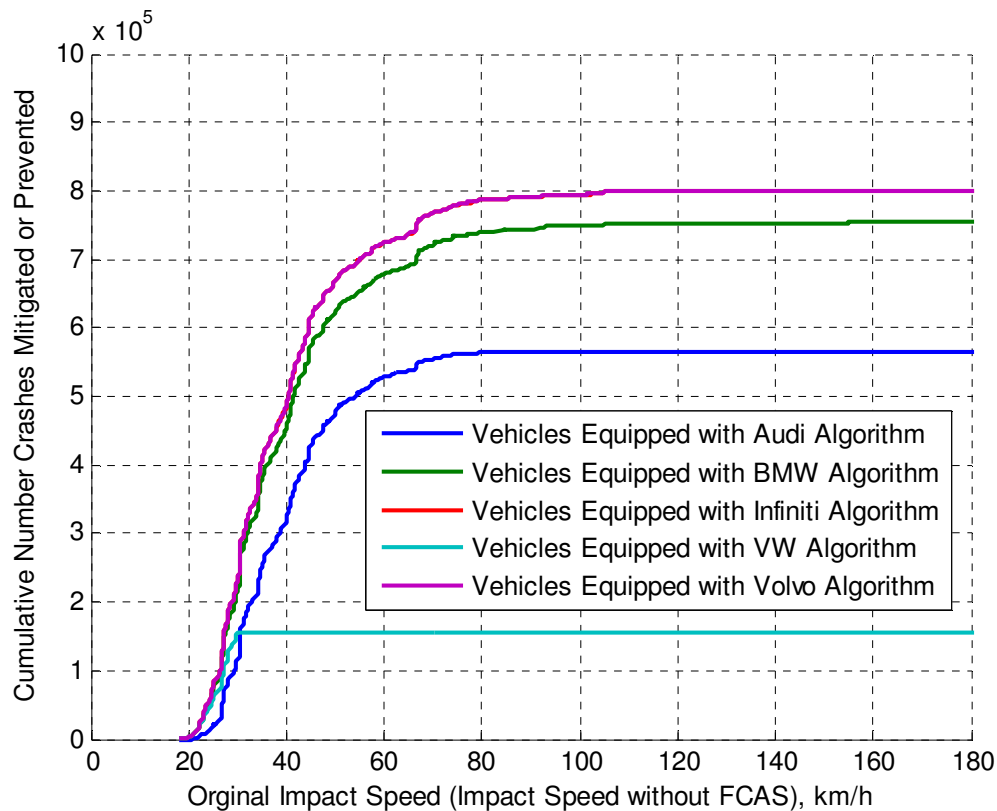


Figure 36. Cumulative Number of crashes that were either prevented or mitigated due to FCASs modeled with Method 2

Figure 37 shows the cumulative number of crashes that were prevented by an FCAS modeled with Method 2. These weighted cumulative distributions were plotted against the impact speeds of the reconstructed cases. The cumulative distribution for the Audi system modeled with Method 2 is very similar to its counterpart from Method 1. The total number of crashes is slightly higher for the Method 2 system because the FCW and AB system upper speed thresholds of operation were extended from 70 km/h to 80 km/h. The BMW and VW Method 2 algorithms offered almost no benefits over their Method 1 counterparts in terms of crashes prevented. Compared to their Method 1 counterparts, the Infiniti and Volvo algorithms modeled with Method 2 offered the most benefits in terms of crash prevention.

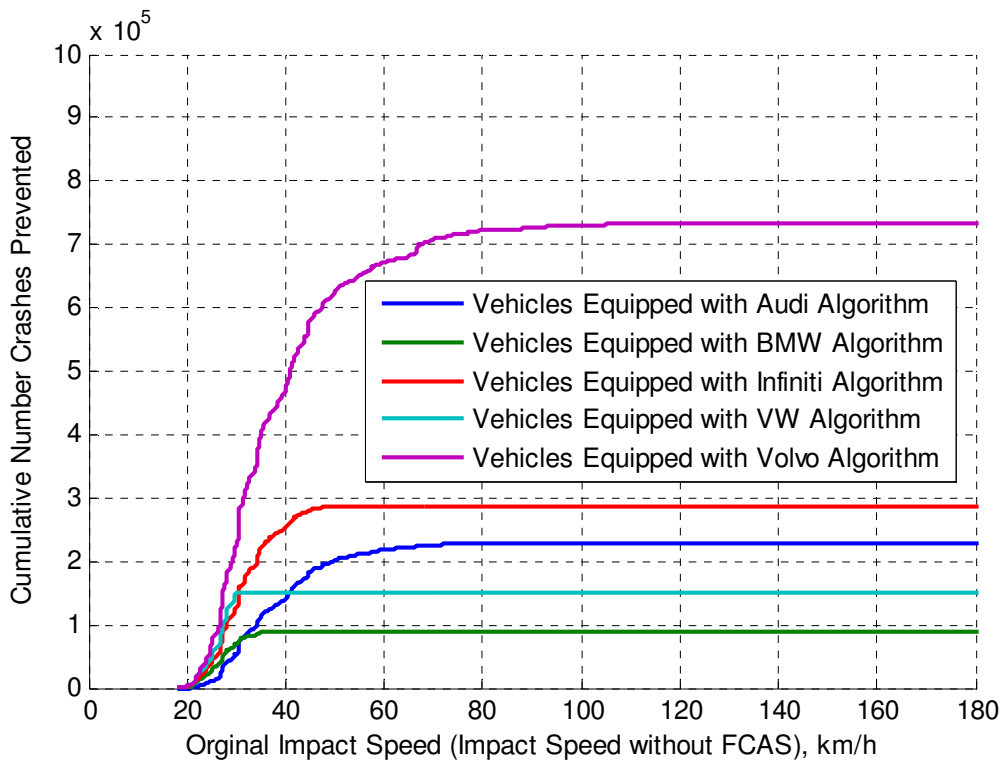


Figure 37. Cumulative Number of crashes that were prevented due to FCASs modeled with Method 2

Figure 38 shows the cumulative number of MAIS2+ injured trailing vehicle drivers. Compared to their Method 1 counterparts, the Audi and VW algorithms modeled with Method 2 offer very little extra benefit in terms number of driver injuries prevented. The BMW, Infiniti, and Volvo algorithms modeled with Method 2, however, do offer significant injury reduction benefits compared to their Method 1 counterparts.

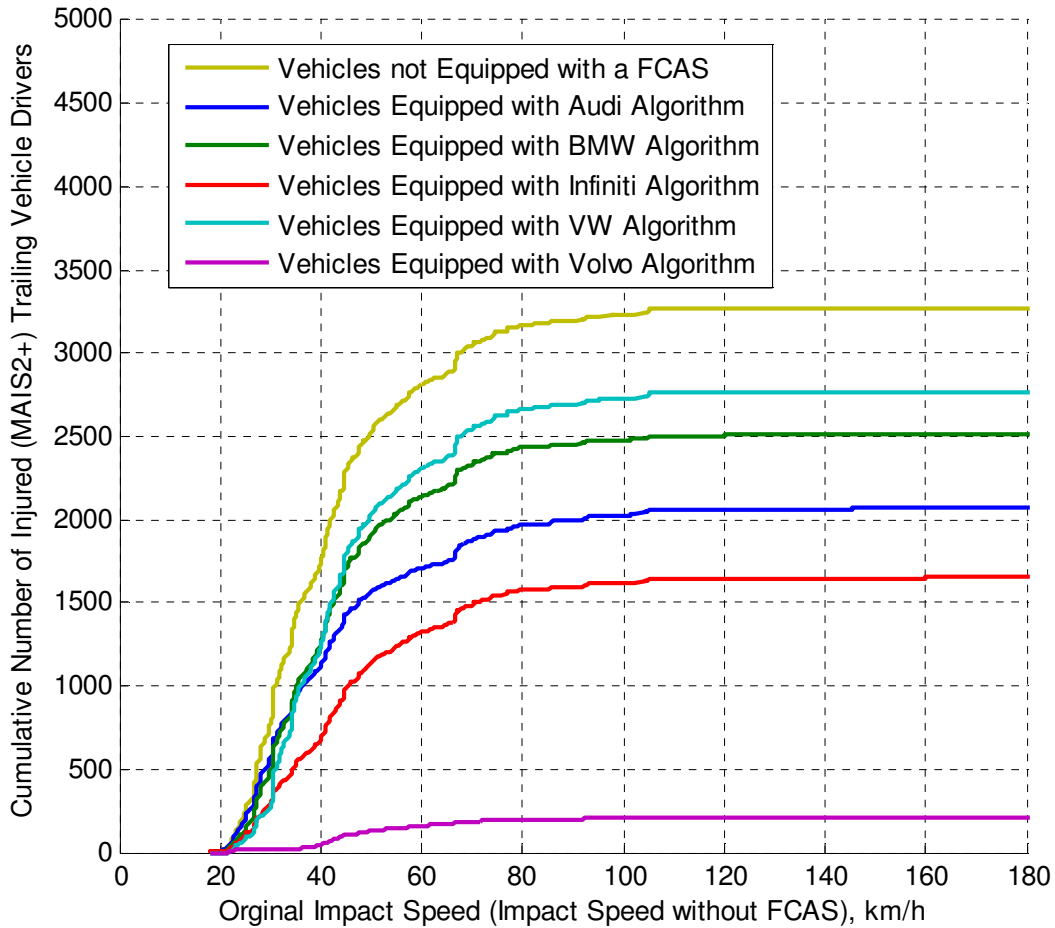


Figure 38. Cumulative Number of MAIS2+ injured drivers of trailing vehicles for FCASs modeled with Method 2

Table 12 shows the expected number of rear-end crashes in the United States in 2012 if the entire U.S. fleet were to be equipped with one of the Method 2 algorithms analyzed in this study. It also shows the expected number of injuries caused by this mode of collision for the driver of the trailing (striking) vehicle. Compared to its Method 1 counterpart, the Audi system modeled with Method 2 prevented 9,099 more crashes and 84 more trailing vehicle driver injuries. Compared to its Method 1 counterpart, the BMW system modeled with Method 2 prevented 4 more crashes and 85 more trailing vehicle driver injuries. Compared to its Method 1 counterpart, the Infiniti system modeled with Method 2 prevented 66,252 more crashes and 324 more trailing vehicle driver injuries. Compared to its Method 1 counterpart, the VW system modeled with Method 2 prevented 1,389 more crashes and 4 more trailing vehicle driver injuries. Compared to its Method 1 counterpart, the Method 2 Volvo algorithm prevented 100,098 more crashes and 573 more trailing vehicle driver injuries.

Table 12. Expected Number of Rear End crashes in which the leading vehicle was stationary during impact and related injuries from 1993-2011 if U.S. fleet had been equipped with one of the FCASs modeled with Method 2

Algorithm	Expected Number of Crashes	Expected Number of Injuries for Drivers in Striking Vehicle
None	799,960	3,274
Audi	571,519	2,072
BMW	711,589	2,513
Infiniti	512,728	1,650
VW	650,738	2,771
Volvo	66,960	206

4.3 RESULTS FOR FCASs MODELED WITH METHOD 3

Figure 39 shows the cumulative distributions of impact speeds for all of the FCASs that were modeled with Method 3. Compared to its Method 2 counterpart, the Audi algorithm

modeled with Method 3 produced a similar, but differently shaped cumulative distribution curve. This is because of the change in the relationship between TTC_{FCW} and v_{FCW} . The BMW, Infiniti, and Volvo algorithms modeled with Method 3 yielded cumulative distributions that did not differ much from their Method 2 counterparts. The Method 2 and Method 3 VW algorithm results do not differ at all because the modeled algorithms are identical.

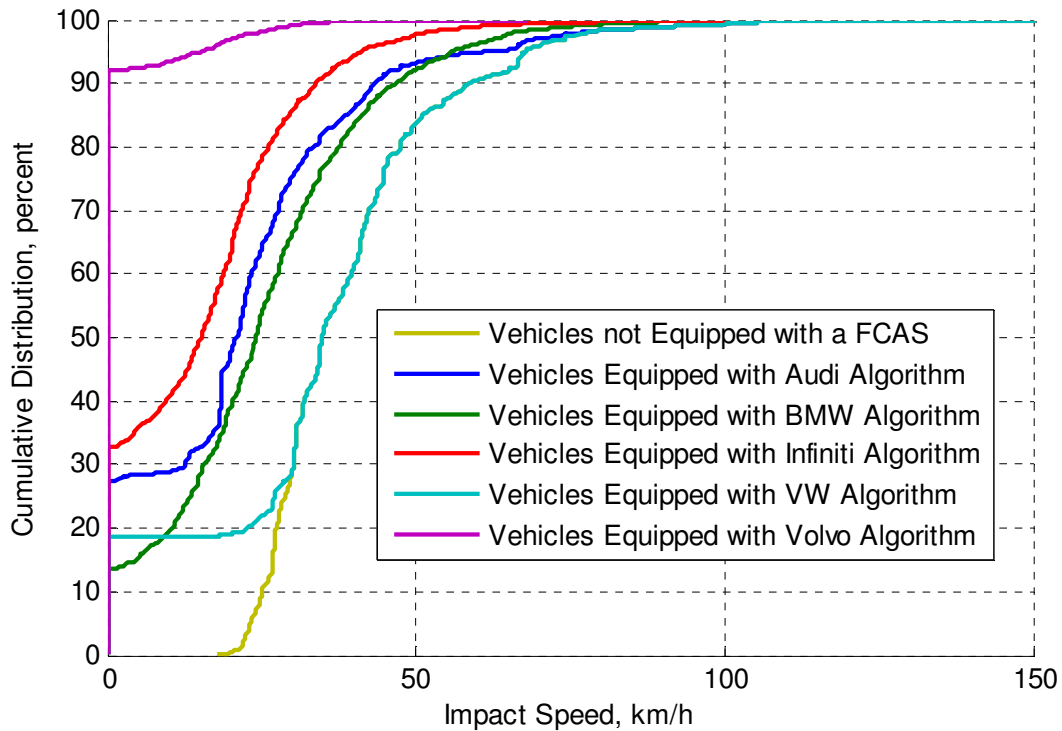


Figure 39. Cumulative Distributions of impact speed for Method 3 of modeling FCASs

Figure 40 shows the cumulative number of crashes that were either prevented or at least mitigated by an FCAS modeled with Method 3. These weighted cumulative distributions were plotted against the impact speeds of the reconstructed cases. As expected, these cumulative distributions are identical to those from Method 2 of modeling the algorithms, shown in Figure

36. This is because the lower and upper speed thresholds are the same for the two sets of algorithms.

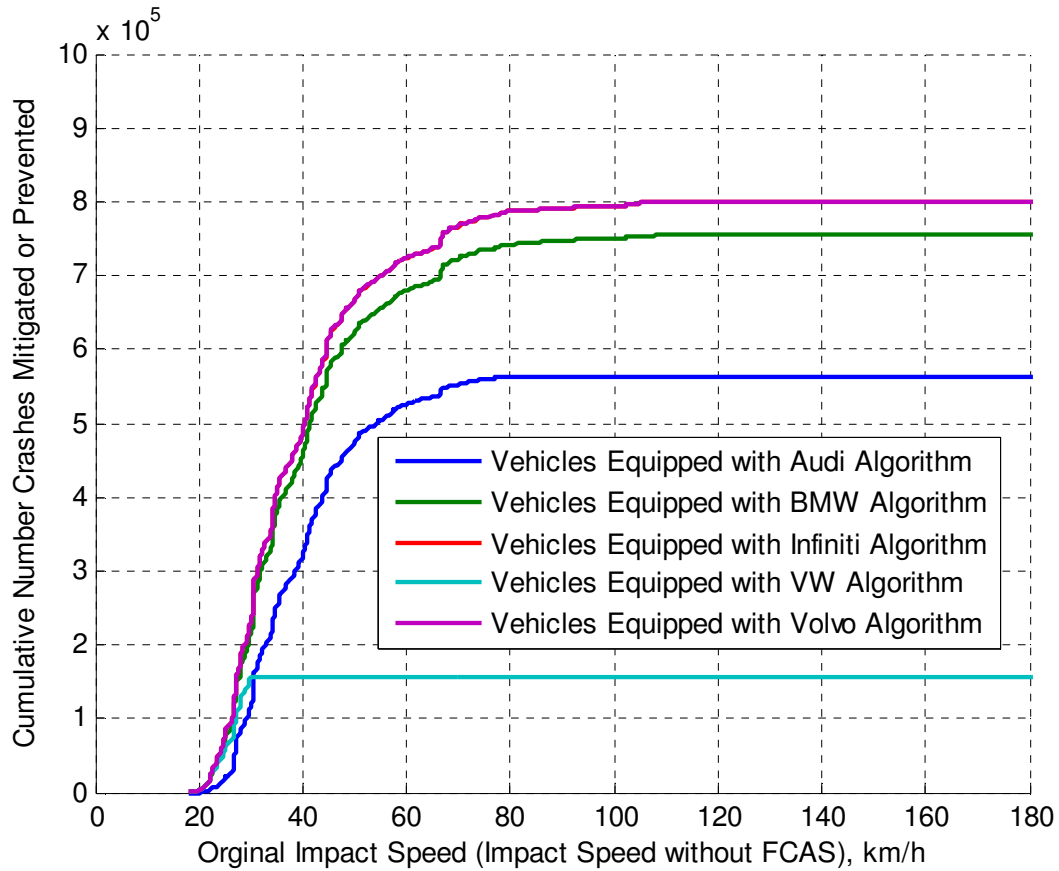


Figure 40. Cumulative Number of crashes that were either prevented or mitigated due to FCASs modeled with Method 3

Figure 41 shows the cumulative number of crashes that were prevented by an FCAS modeled with Method 2. These weighted cumulative distributions were plotted against the impact speeds of the reconstructed cases. The Audi and Infiniti algorithms modeled with Method 3 were slightly less beneficial than their Method 2 counterparts in terms of crash prevention. This is because of the change in relationships between TTC_{FCW} and v_{FCW} .

Compared to their Method 2 counterparts, these two Method 3 algorithms are less likely to prevent collisions with impact speeds close to 40 km/h. Compared to their Method 2 counterparts, the BMW and Volvo algorithms modeled with Method 3 are more likely to prevent crashes due to their increased effectiveness at speeds close to 30 km/h. The Method 2 and Method 3 VW algorithm results do not differ at all because the modeled algorithms are identical.

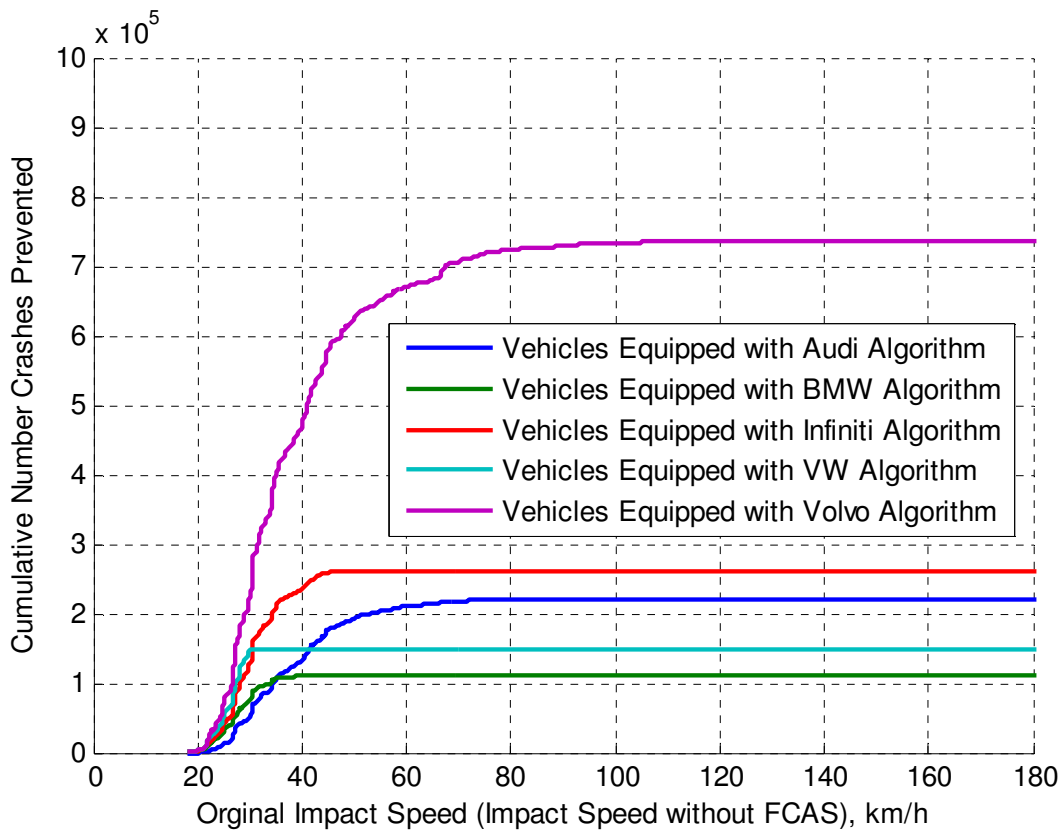


Figure 41. Cumulative Number of crashes that were prevented due to FCASs modeled with Method 3

Figure 42 shows the cumulative number of MAIS2+ injured trailing vehicle drivers. Compared to their Method 2 counterparts, all of the algorithms that were modeled with method 3 offered very similar injury prevention benefits. The Method 3 Audi and Infiniti algorithms were

slightly worse than their Method 2 counterparts. The Method 3 BMW and Volvo algorithms were slightly better than their Method 2 counterparts. The Method 2 and Method 3 VW algorithm results do not differ at all because the modeled algorithms are identical.

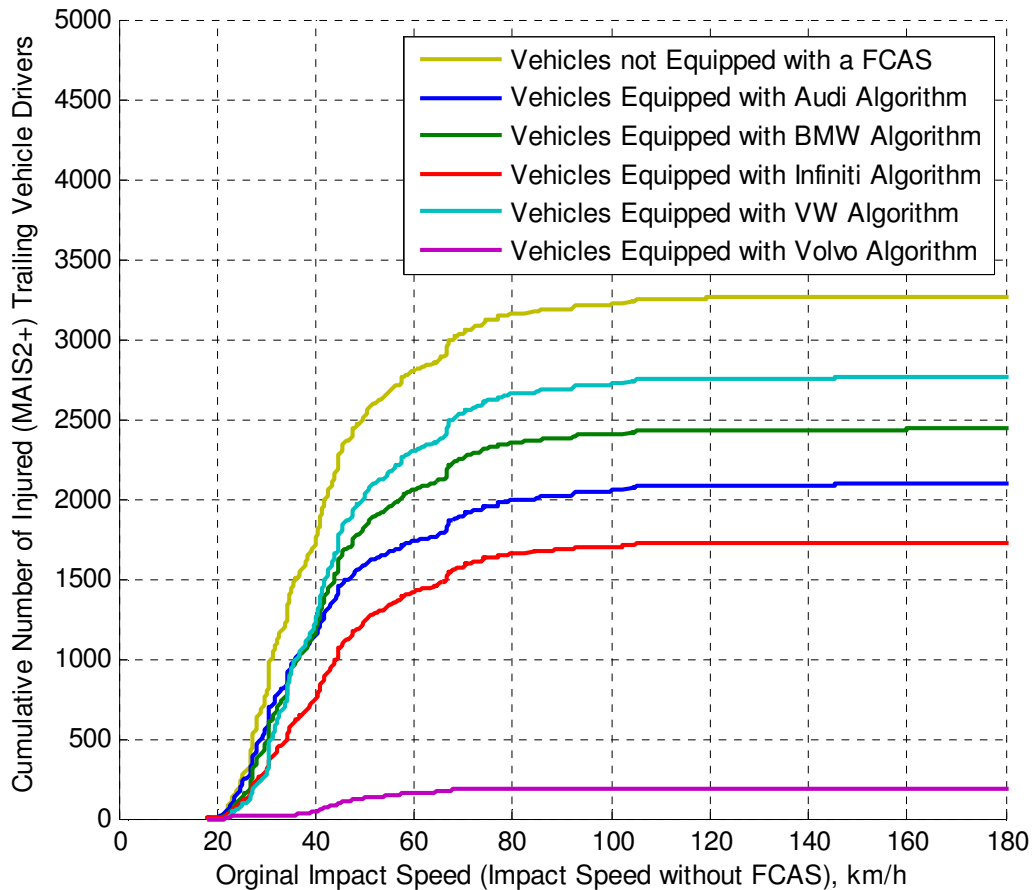


Figure 42. Cumulative Number of MAIS2+ injured drivers of trailing vehicles for FCASs modeled with Method 3

Table 13 shows the expected number of rear-end crashes in the United States in 2012 if the entire U.S. fleet were to be equipped with one of the Method 3 systems analyzed in this study. It also shows the expected number of injuries caused by this mode of collision for the driver of the

trailing (striking) vehicle. Compared to its Method 2 counterpart, the Audi algorithm modeled with Method 3 prevented 7,812 fewer crashes and 30 fewer trailing vehicle driver injuries. Compared to its Method 2 counterpart, the BMW algorithm modeled with Method 3 prevented 21,164 more crashes and 68 more trailing vehicle driver injuries. Compared to its Method 2 counterpart, the Infiniti algorithm modeled with Method 3 prevented 25,748 fewer crashes and 85 fewer trailing vehicle driver injuries. Compared to its Method 2 counterpart, the VW system modeled with Method 3 did not offer any extra benefits because the two modeled systems are identical. Compared to its Method 2 counterpart, the Volvo system modeled with Method 3 prevented 4,107 more crashes and 15 more trailing vehicle driver injuries.

Table 13. Expected Number of Rear End crashes in which the leading vehicle was stationary during impact and related injuries from 1993-2011 if U.S. fleet had been equipped with one of the FCASs modeled with Method 3

Algorithm	Expected Number of Crashes	Expected Number of Injuries for Drivers in Striking Vehicle
None	799,960	3,274
Audi	579,331	2,102
BMW	690,425	2,445
Infiniti	538,476	1,735
VW	650,738	2,771
Volvo	62,853	191

4.4 SUMMARY OF RESULTS

Figure 43 shows the percent of crashes prevented by the various systems that were modeled and analyzed in this study. This study estimates that if implemented across the U.S. fleet the Audi system will prevent between 27% and 29% of rear-end crashes in which the leading vehicle was stationary prior to the crash and at least one of the vehicles was towed from the crash site. The BMW system would prevent between 11% and 14% of these types of

crashes. The Infiniti system would prevent between 28% and 36% of these crashes. The VW system would prevent approximately 19% of these types of crashes. The Volvo system would prevent between 79% and 92% of these crashes.

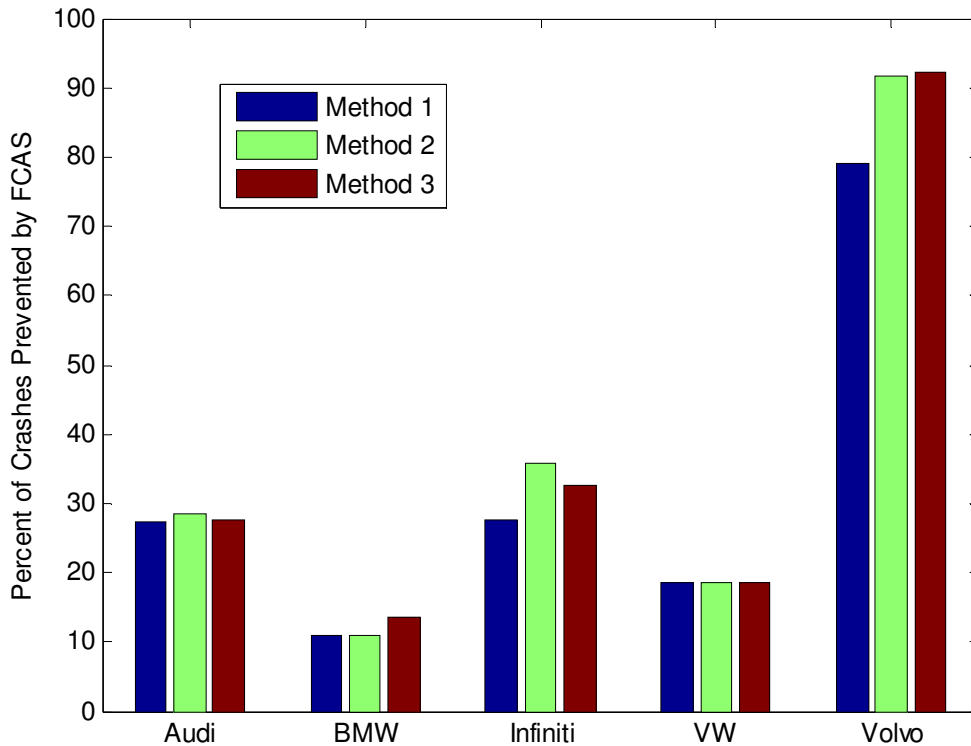


Figure 43. Percent of Crashes prevented due to FCASs

Figure 44 shows the percent of trailing vehicle driver injuries prevented by the various systems that were modeled and analyzed in this study. This study estimates that if implemented across the U.S. fleet the Audi system will prevent between 34% and 37% of trailing vehicle driver injuries caused by rear-end crashes in which the leading vehicle was stationary prior to the crash and at least one of the vehicles was towed from the crash site. The BMW system would prevent between 21% and 25% of these types of injuries. The Infiniti system would prevent

between 40% and 50% of these injuries. The VW system would prevent approximately 15% of these types of injuries. The Volvo system would prevent between 76% and 94% of these injuries.

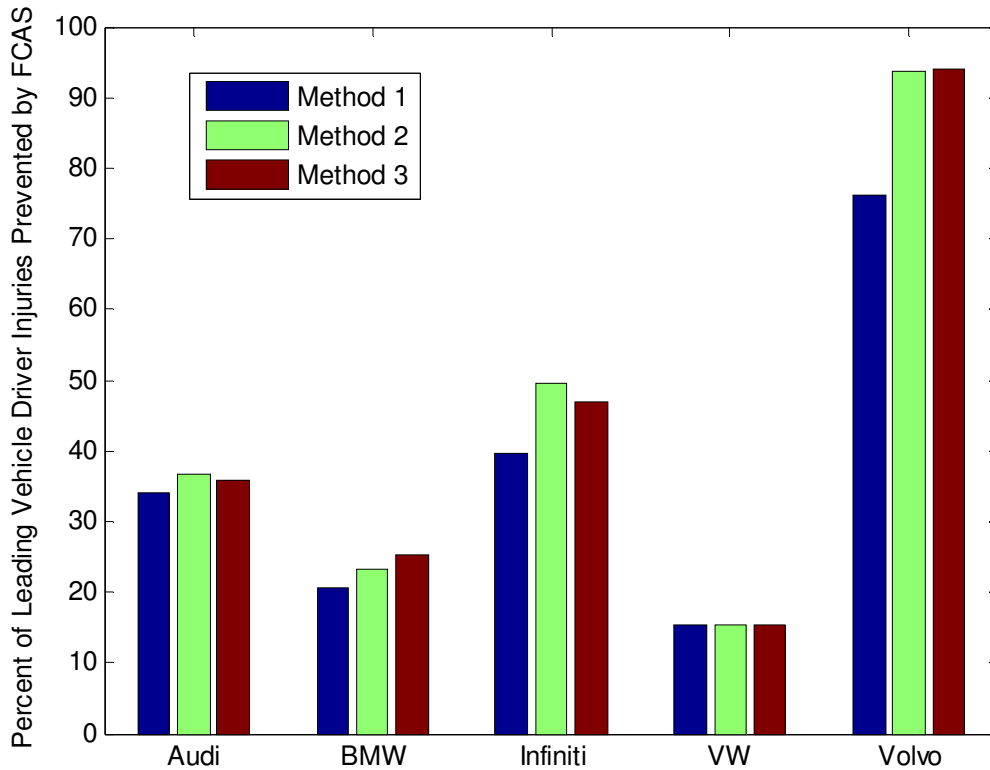


Figure 44. Percent of Striking Driver Injuries prevented due to FCASs

The results of this study indicate that the Volvo FCAS offers the most benefits in terms of crashes that could be prevented and driver injuries that could be prevented. Methods 2 and 3 of modeling the Volvo FCAS yielded very similar benefits estimations, providing significantly

more benefits than the Volvo algorithm that was modeled with Method 1. All models of the Volvo FCAS were very effective across the whole range of modeled system functionality.

This study estimates that of the five systems that were modeled and analyzed, the Infiniti system is second best at offering potential benefits to the U.S. driving community. The three methods used to model the Infiniti system all yield similar estimates for potential reduction in crashes and driver injuries.

All three methods used to model the Audi FCAS resulted in very similar predictions for potential crash prevention effectiveness and driver injury effectiveness. The results of this study indicate that of the five systems modeled and analyzed in this study, the Audi FCAS is third best at preventing crashes and driver injuries.

All three methods used to model the VW FCAS resulted in very similar predictions for potential crash prevention effectiveness and driver injury effectiveness. This study estimates that of the five FCASs that were modeled and analyzed in this study, the VW offers the fewest benefits in terms of potential injury reduction and is the fourth best in terms of preventing crashes.

Although the modeled BMW systems are fourth best at preventing injuries, these modeled systems are least effective at preventing crashes. All three methods of modeling the BMW system yielded similar results in terms of expected potential reductions in crashes and driver injuries.

5. CONCLUSIONS

The design of FCASs is currently unregulated and current production systems behave very differently and offer a wide range of benefits. This study modeled five production FCASs and estimated the potential benefits of these algorithms through simulations of reconstructed crashes that occurred from 1993 to 2008. The modeled algorithms were based on systems found on the Audi A7, BMW 530d, Infiniti M, Volvo V60, and VW Passat. Three major conclusions were drawn from the results of this study.

The first major conclusion that can be drawn from this thesis is that, if the assumptions used to model the AB systems are reasonable, Assisted Braking (AB) systems are effective components of Forward Collision Avoidance Systems. This conclusion was drawn from the results of Audi and BMW algorithms. Both systems have FCW systems and neither of the systems possesses an AEB system that functions when the leading vehicle is stationary. The BMW FCW system warns the driver sooner than does the Audi FCW system. Another strength the BMW algorithm has is that it operates over a wider range of speeds. Based on the earlier FCW alone, one would expect the BMW algorithm to outperform the Audi algorithm in terms of effectiveness in preventing crashes and injuries. However, the Audi algorithm outperforms the BMW algorithm in terms of effectiveness in preventing crashes and injuries. The difference is that the Audi system was modeled as possessing an AB system.

The second major conclusion that can be drawn from this thesis is that operation thresholds can greatly influence the effectiveness of Forward Collision Avoidance Systems. This conclusion was drawn by comparing the system effectiveness results of the VW algorithm to the system effectiveness results of the Infiniti algorithm. The operating range of speeds for the VW

system was modeled as [5,30] km/h whereas the modeled operating range of speeds for the Infiniti system was [15,∞) km/h. Up to an impact speed of 30 km/h, the VW algorithm is on par with the Infiniti algorithm in terms of effectiveness in preventing crashes. Up to the original impact speed of 30 km/h the VW algorithms decidedly outperforms the BMW and Audi systems in terms of effectiveness in preventing crashes. If the VW algorithm was not limited by its upper speed threshold of operation of 30 km/h, perhaps it would have been as effective as or even more effective than the Infiniti algorithm in terms of preventing crashes instead of performing worse than the Audi algorithm which the VW algorithm had outperformed at original impact speeds below 30 km/h. Up to original impact speeds of 30 km/h, the VW algorithm was on par with the Infiniti system in terms of preventing the number of MAIS 2+ injured drivers. Beyond original impact speeds of 30 km/h, the VW algorithm proved ineffective and the Audi and BMW algorithms surpassed the VW algorithm in terms of preventing driver injuries.

The final major conclusion that can be drawn from this study is that AEB improves effectiveness of Forward Collisions Avoidance Systems. This conclusion was drawn by comparing the system effectiveness results of the BMW algorithm to the system effectiveness results of the Infiniti algorithm. The Infiniti and BMW algorithms are very similar. They both have lower speed thresholds of 15 km/h. Neither of them was modeled as having upper speed thresholds of operation. Both of them have FCW systems and neither of them has an Assisted Braking system. The relationship between TTC_{FCW} and speed for the BMW algorithm is similar to that of the Infiniti algorithm. The TTC_{FCW} differs by a maximum of approximately 0.5 seconds for all speeds. The primary difference between the two systems is that the Infiniti algorithm was modeled as possessing an AEB system, whereas the BMW system was modeled as not having an AEB system for scenarios in which the leading vehicle was stationary prior to

the crash. The Infiniti algorithm was much more effective than the BMW algorithm at preventing crashes and injuries. Because the difference in TTC_{FCW} is minimal between the systems, the difference in system effectiveness can be attributed to the fact that, for the crash scenarios simulated in this study, the Infiniti was modeled as having an AEB system and the BMW was modeled as not having an AEB system.

5.1 CONTRIBUTIONS OF THIS STUDY

This study improves and expands upon a previous method for estimating potential benefits of FCASs. Using this method, this study estimated the potential effectiveness of five FCASs that were released for public use on vehicles with a 2011 model year.

The results of this study provide insight into the effectiveness of various designs of FCASs. The methods described in Chapter 3 of this report could be used in conjunction with SiL (Software-in-the-Loop) tests to evaluate systems, helping government officials regulate the design and implementation of FCASs. The results of this study and future studies using the same methods could prove useful to customers when making a vehicle purchasing decision.

In an effort to model some of the FCASs analyzed in this study, extensive video analysis methods were explored, some of which may be novel. This study details these methods which could be used or further explored in future studies.

5.2 LIMITATIONS OF THIS STUDY

The designs of Forward Collision Avoidance Systems are trade secrets tightly guarded by manufacturing companies. The governing algorithms of these systems are not available to the public and are likely updated often. Without the designs of the governing algorithms of the five FCASs studies in this thesis, we had to resort to publicly available system descriptions, limited

test data, and videos. The assumptions made to “reverse engineer” the systems comprise a substantial limitation of this study.

Many of the limitations of the study stem from the population of crashes that were reconstructed and simulated to estimate the potential benefits of FCASs. The population was limited to crashes in which at least one of the vehicles was towed from the crash site. This limits the analysis of cases to those with impact speeds of approximately 20 km/h and above.

The population was further limited to rear-end crashes in which the leading (struck) vehicle was stationary prior to the crash. The population was limited to these cases because the ADAC did not test enough scenarios to model the functionality and behavior of the FCASs. Another limitation concerning the target population was that safety benefits were only computed for drivers wearing seatbelts.

This study only analyzed five of the six systems tested by the ADAC. The Mercedes system was not analyzed in this study because ample test data were not available to model the complexity of this system. Apart from the six systems that were tested by the ADAC, there are many other systems that were not analyzed in this study.

In the NASS/CDS cases that were reconstructed in this study, information on driver behavior was limited. The MANEUVER variable indicated if the driver was braking, accelerating, or maintaining a constant speed prior to the crash, but no information was available concerning potential braking or acceleration levels. This study made assumptions on the braking and acceleration levels and also assumed that the driver did not steer prior to the crash. Additionally, no information was available on the timing of when a driver may have started to

brake prior to the crash. Information on driver reactions to FCASs is also limited. Few studies have researched driver reaction times to and braking levels due to FCW systems.

5.3 FUTURE WORK

The automotive industry is producing and updating FCASs every year and offering these systems an increasing number of vehicles. The methods described in this study could be applied to other FCAS tests. Future studies could model FCAS behavior and estimate potential benefits in terms of crashes prevented and driver injuries prevented. The methods described in Chapter 3 could be used in conjunction with SiL tests to more accurately estimate the benefits of individual FCASs. If future FCAS tests or SiL tests are performed on a wide range of scenarios, the method described in this study could potentially be applied to rear-end crashes in which the leading vehicle was not stationary prior to the crash.

In addition to estimating driver injuries, future studies could estimate the potential safety benefits for passengers. Future studies could also estimate the potential cost benefits to society. Future studies could also address limitations of the target population, simulating cases in which neither of the vehicles was towed from the crash site. To address the limited information concerning individual driver behaviors in reconstructed cases, future studies could introduce more complex models to estimate the behavior of drivers, both to crash situations without an FCAS and to the initiation of an FCAS.

Within the next 20 years, automakers will potentially start releasing vehicles with Vehicle-2-Vehicle (V2V) and/or Vehicle-to-Infrastructure (V2I) capabilities in addition to, or in

replacement of FCASs. Some methods introduced in this study could be applied to the estimation of potential benefits of these systems.

5.4 THE PROSPECTS OF A COLLISION FREE CAR

This thesis provides some insight into the prospects of a collision-free car. For cases in which the leading vehicle was stationary and the trailing vehicle had an original impact speed less than 40 km/h, the Volvo algorithm prevented almost every rear-end crash. If all cars were required to travel at speeds less than 40 km/h, the prospects for a collision-free car would seem very good. However, this thesis only provides insight into the effectiveness of FCASs in preventing rear-end collisions in which the leading vehicle was stationary prior to the crash. This crash mode only accounts for 18% of all annual crashes in the United States.

Hypothetically, if we were to assume that active safety systems completely prevented all other crash modes, the prospects for a collision free car would still not be great because the modeled systems are not completely effective at speeds above 40 km/h. With the current technology, a large portion of rear-end crashes can be prevented with FCASs, but we believe that several decades will pass before a car is made that is guaranteed to be completely collision-free.

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APPENDIX A: DERIVATION OF EQUATION 8, THE PIXEL-SPATIAL TRANSFORMATION

EQUATION

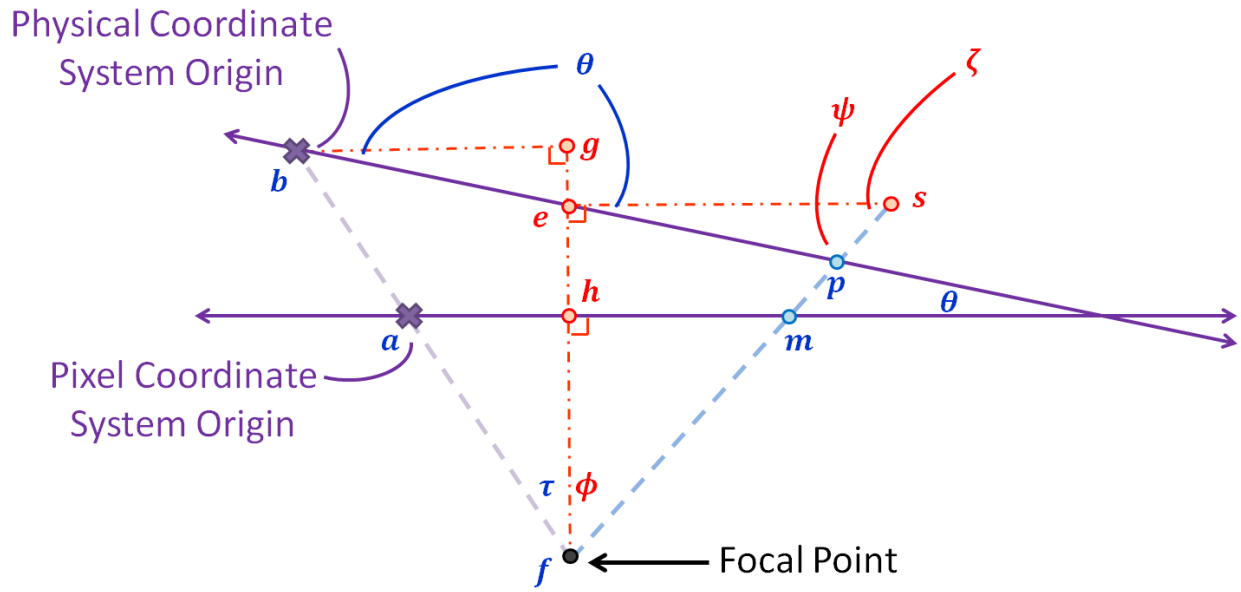


Figure 45. Bird's Eye view of Figure 15 with all geometrical points listed that are necessary for the derivation of the pixel-to-spatial transformation equation.

Given:

$$\overline{am} = x, \overline{fa}, \overline{fb}, \theta, \tau$$

Find:

$$x' = \overline{bp}$$

Solution:

$$\overline{bp} = \overline{be} + \overline{ep} \tag{A1}$$

Solving for \overline{be} :

$$\overline{bg} = \overline{fb} * \sin(\tau) \quad (\text{A2})$$

$$\overline{be} = \frac{\overline{bg}}{\cos(\theta)} \quad (\text{A3})$$

Combining (A2) and (A3):

$$\overline{be} = \frac{\overline{fb} * \sin(\tau)}{\cos(\theta)} \quad (\text{A4})$$

Solving for \overline{ep} :

By Law of Sines:

$$\overline{ep} = \overline{es} * \frac{\sin(\zeta)}{\sin(\psi)} \quad (\text{A5})$$

Solve for \overline{es} :

$$\overline{es} = \frac{\overline{hm} * \overline{fe}}{fh} \quad (\text{A6})$$

Solve for \overline{hm} :

$$\overline{hm} = \overline{am} - \overline{ah} \quad (\text{A7})$$

$$\overline{ah} = \overline{fa} * \sin(\tau) \quad (\text{A8})$$

Combine:

$$\overline{hm} = \overline{am} - \overline{fa} * \sin(\tau) \quad (\text{A9})$$

Solve for \overline{fe} :

$$\overline{fe} = \overline{fg} - \overline{eg} \quad (\text{A10})$$

$$\overline{fg} = \overline{fb} * \cos(\tau) \quad (\text{A11})$$

$$\overline{eg} = \overline{be} * \sin(\theta) \quad (\text{A12})$$

Combining (A4) and (A12):

$$\overline{eg} = \frac{\overline{fb} * \sin(\tau)}{\cos(\theta)} * \sin(\theta) \quad (\text{A13})$$

Simplify:

$$\overline{eg} = \overline{fb} * \sin(\tau) * \tan(\theta) \quad (\text{A14})$$

Combining (A10), (A11), and (A14):

$$\overline{fe} = \overline{fb} * \cos(\tau) - \overline{fb} * \sin(\tau) * \tan(\theta) \quad (\text{A15})$$

Solve for \overline{fh} :

$$\overline{fh} = \overline{fa} * \cos(\tau) \quad (\text{A16})$$

Combining (A6), (A9), (A15), and (A16):

$$\overline{es} = \frac{(\overline{am} - \overline{fa} * \sin(\tau))(\overline{fb} * \cos(\tau) - \overline{fb} * \sin(\tau) * \tan(\theta))}{(\overline{fa} * \cos(\tau))} \quad (\text{A17})$$

Solve for $\sin(\zeta)$:

$$\sin(\zeta) = \cos(\phi) = (\overline{fh})/(\overline{fm}) \quad (\text{A18})$$

Combining (A16) and (A18):

$$\sin(\zeta) = \frac{\overline{fa} * \cos(\tau)}{\overline{fm}} \quad (\text{A19})$$

Solve for $\sin(\psi)$:

$$\sin(\psi) = \sin(180^\circ - \theta - \zeta) = \sin(\theta + \zeta) \quad (\text{A20})$$

$$\sin(\psi) = \sin(\theta) \cos(\zeta) + \cos(\theta) \sin(\zeta) \quad (\text{A21})$$

Solve for $\cos(\zeta)$:

$$\cos(\zeta) = \sin(\phi) = (\overline{hm})/(\overline{fm}) \quad (\text{A22})$$

Combine:

$$\cos(\zeta) = \frac{\overline{am} - \overline{fa} * \sin(\tau)}{\overline{fm}} \quad (\text{A23})$$

Combining (A19), (A21), and (A23):

$$\sin(\psi) = \sin(\theta) \frac{\overline{am} - \overline{fa} * \sin(\tau)}{\overline{fm}} + \cos(\theta) \frac{\overline{fa} * \cos(\tau)}{\overline{fm}} \quad (\text{A24})$$

$$\sin(\psi) = \frac{\sin(\theta) (\overline{am} - \overline{fa} * \sin(\tau)) + \cos(\theta) (\overline{fa} * \cos(\tau))}{\overline{fm}} \quad (\text{A25})$$

Let:

$$k_1 = \overline{fa} * \sin(\tau)$$

$$k_2 = \overline{fb} * \cos(\tau) - \overline{fb} * \sin(\tau) * \tan(\theta)$$

$$k_3 = \overline{fa} * \cos(\tau)$$

$$k_4 = \sin(\theta)$$

$$k_5 = \cos(\theta)$$

Recall:

$$\overline{am} = x$$

Simplify:

$$\overline{es} = \frac{(x-k_1)(k_2)}{(k_3)} \quad (\text{A26})$$

$$\sin(\zeta) = \frac{k_3}{fm} \quad (\text{A27})$$

$$\sin(\psi) = \frac{k_4(x-k_1)+k_5k_3}{fm} \quad (\text{A28})$$

Combining (A5), (A26), (A27), and (A28):

$$\overline{ep} = \frac{(x-k_1)(k_2)}{(k_3)} * \frac{\frac{k_3}{fm}}{\frac{k_4(x-k_1)+k_5k_3}{fm}} \quad (\text{A29})$$

$$\overline{ep} = \frac{(x-k_1)(k_2)}{(k_3)} * \frac{k_3}{k_4(x-k_1)+k_5k_3} \quad (\text{A30})$$

$$\overline{ep} = \frac{(x-k_1)(k_2)}{k_4(x-k_1)+k_5k_3} \quad (\text{A31})$$

$$\overline{ep} = \frac{k_2x-k_1k_2}{k_4x+k_3k_5-k_1k_4} \quad (\text{A32})$$

Let:

$$k_6 = -k_1k_2$$

$$k_7 = \frac{k_3k_5-k_1k_4}{k_4}$$

$$k_8 = \frac{1}{k_4}$$

$$\overline{ep} = k_8 \left(\frac{k_2x+k_6}{x+k_7} \right) \quad (\text{A33})$$

Let:

$$k_9 = \frac{k_6}{k_7}$$

$$k_{10} = \frac{k_2}{k_9}$$

$$k_{11} = k_8 k_9$$

$$\overline{ep} = k_8 \left(\frac{k_9 k_{10} x + k_9 k_7}{x + k_7} \right) \quad (\text{A34})$$

$$\overline{ep} = k_{11} \left(\frac{k_{10} x + k_7}{x + k_7} \right) \quad (\text{A35})$$

Let:

$$k_{12} = k_{10} - 1$$

$$\overline{ep} = k_{11} \left(\frac{(x + k_7) + k_{12} x}{x + k_7} \right) \quad (\text{A36})$$

$$\overline{ep} = k_{11} k_{12} \left(\frac{x}{x + k_7} \right) + k_{11} \quad (\text{A37})$$

Let:

$$k_{13} = \frac{1}{k_7}$$

$$\overline{ep} = k_{11} k_{12} k_{13} \left(\frac{x}{k_{13} x + 1} \right) + k_{11} \quad (\text{A38})$$

Combining (A1), (A4), and (A38):

$$x' = \overline{bp} = \overline{be} + \overline{ep} = \frac{\overline{fb} \sin(\tau)}{\cos(\theta)} + k_{11} k_{12} k_{13} \left(\frac{x}{k_{13} x + 1} \right) + k_{11} \quad (\text{A39})$$

Let:

$$c_1 = k_{11} k_{12} k_{13}$$

$$c_2 = k_{13}$$

$$c_3 = \frac{\overline{fb} \sin(\tau)}{\cos(\theta)} + k_{11}$$

$$x' = c_1 \left(\frac{x}{c_2 x + 1} \right) + c_3 \quad (8)$$