α-states in de Sitter space

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Field theory in de Sitter space admits a one-parameter family of vacua determined by a superselection parameter α. Of these vacua, the Euclidean vacuum uniquely extrapolates to the vacuum of flat Minkowski space. States which resemble the α-vacua can be constructed as excitations above the Euclidean vacuum. Such states have modes $\alpha(k)$ which decay faster that $k^{(1-d)/2}$. Fields in such states exhibit nonlocal correlations when examined from the perspective of fields in the Euclidean vacuum. The dynamics of such entangled states are fully consistent. If an α-state with properties that interpolate between an α-vacuum and the Euclidean vacuum were the initial condition for inflation, a signature for this may be found in a momentum dependent correction to the inflationary power spectrum. The functional formalism, which provides the tool for examining physics in an α-state, extends to fields of other spin. In particular, the extension to spin-2 may proffer a new class of infrared modifications to gravitational interactions. The implications of superselection sectors for the landscape of string vacua are briefly discussed.

I. INTRODUCTION

Cosmology provides a window into the Universe at early times, high energies, and large scales. That in the beginning the Universe was very nearly homogeneous and isotropic is a fact adduced from observations of identical structures today in causally disconnected regions of spacetime. The temperature of the cosmic microwave background radiation (CMBR), which maps out the surface of last photon scattering, has an angular distribution that varies at the level of one part in ten thousand. Small perturbations in the temperature are directly related to density perturbations at the time of recombination, when the Universe first became transparent to radiation. Large scale inhomogeneities in the Universe today (stars, galaxies, clusters, etc.) arise from cosmological perturbations at early times. Inflation provides a possible explanation for homogeneities and inhomogeneities in the density of matter when photons decoupled [1]. Recent astrophysical data [2] provide corroboration for the inflationary paradigm.

Understanding the large scale structure of the Universe today amounts to understanding in detail the initial conditions that preceded the inflationary epoch. During inflation, the large scale geometry is well approximated by a de Sitter spacetime, which is the maximally symmetric solution to the vacuum Einstein equations with a positive cosmological constant [3]. A pure de Sitter phase for the early Universe is only an approximation, however. The de Sitter geometry is empty and static. Moreover, quantum field theory in de Sitter space, even neglecting issues of back-reaction, is problematic from the point of view of

local observers. The usual S-matrix intuition of local quantum field theory seems to fail, or rather is unsuitable [4].

Quantum field theory in de Sitter space is further complicated by the existence of a one-parameter infinite family of vacuum states consistent with *CPT* invariance [5,6]. These vacua are labeled by a real number α. One of these vacua $|\alpha = 0\rangle$ is the unique vacuum that extrapolates to the standard Minkowski vacuum in the limit where the cosmological constant vanishes (i.e., the radius of curvature of the de Sitter geometry becomes infinite) and has the same short distance singularities along the light cone as the usual flat space two-point function [7–11]. The other vacua $|\alpha\rangle$ are *formally* realized as squeezed states over the Euclidean vacuum [12]. This is only a formal correspondence because each of the vacua $|\alpha\rangle$ is the ground state of a different Hilbert space; α is a superselection parameter. Understanding whether interacting field theory in any of the $|\alpha \neq 0\rangle$-vacua makes sense as a consistent theory of physics is still a matter of debate [13]. In this paper, we discuss a somewhat different scenario in which the notion of an α-vacuum leads to meaningful new physics.

Although there may be compelling arguments for preferring the Euclidean vacuum over the other α-vacua, there is no *a priori* justification for the belief that the Universe started out as a fluctuation about the adiabatic ground state. Rather, it is conceivable that the initial conditions were some fluctuation about some excited state. If this excited state is in the same Hilbert space as the Euclidean vacuum (i.e., it is a normalizable excitation over the Euclidean vacuum), the physics inherits the field theoretic structure of the Euclidean ground state and there is no inconsistency with observation or need to employ nonstandard technology. There are no unusual singularities in the Green function, for example, and the detector response function is perfectly thermal. Novel physics at late times can arise in

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such a scenario despite inflation if the excited state were special. We can consider physics in an α-state, which is like the α-vacuum in the infrared, but like the Euclidean vacuum in the ultraviolet. Such states are excited states in the same Hilbert space as the Euclidean vacuum and are perhaps natural because, from the point of view of effective field theory, it makes no sense to demand ''alphaness'' to arbitrary precision; we expect new degrees of freedom (such as strings) to emerge at least near the Planck scale. Preparing an α-state is fine-tuning, but this state is no more finely tuned than any other of the manifold of conceivable initial conditions.

An α-state is a squeezed state. Given an α-state as an initial condition, the field theory at late times will exhibit long-range (Hubble scale) correlations. Physics is nonlocal, but the long-range correlations are simply a manifestation of entanglement in the initial state. As no experiment has verified the locality of physics at Hubble distances, there is no inconsistency with observation.

Modifications in the inflaton power spectrum, i.e., the Fourier transform of the autocorrelation function of the inflaton field, would be one potential signature for α-like physics at early times. If the initial condition was the Euclidean vacuum, the modes of a scalar field are thermally populated. Inflation stretches the modes. The thermally populated modes in the ultraviolet descend below the cutoff scale ω that defines the effective field theory. Certain initial conditions in the Euclidean vacuum lead to $\mathcal{O}[(H/\omega)^2]$ deviations from the standard scale invariant spectrum [14]. Other authors consider initial conditions in an α-vacuum and obtain $\mathcal{O}(H/\omega)$ deviations, which are potentially observable by the WMAP and Planck experiments as a signature of trans-Planckian physics, although these results have engendered some controversy in the literature [15,16]. (See also Refs. [17,18].) The α-states are one more set of initial conditions to probe primeval anisotropies in the CMBR.

Another potential signature for new physics that is inspired by α-states exists in the gravitational sector. Einstein's theory of gravity is a classical field theory valid at least in the regime of distances larger than a few microns. Modifications to the gravitational interaction at astrophysical scales are constrained by the systematics of galactic rotation curves, the Tully-Fisher luminosity relation, and the dynamics of clusters [19]. There is, however, no observational check to the General Theory of Relativity in the very deep infrared. A number of scenarios have suggested various (possibly consistent) infrared modifications to the classical theory of gravitation [20]. If we treat fluctuations in the metric as being a spin-2 field in de Sitter space, then the technology of α-states for scalars will translate to a nonlocal redefinition of the spacetime metric. Exploring this hypothesis is an interesting problem.

The organization of the paper is as follows. In Sec. II, we briefly review scalar field theory in de Sitter space. In Sec. III, we discuss scalar fields in the operator formalism. We then present the construction of a normalizable squeezed state with α-vacuumlike properties in the same Hilbert space as the Euclidean vacuum. In Sec. IV, we revisit the definition of an α-vacuum, this time in the functional formalism. The wave functional for the Euclidean vacuum and the α-vacua are constructed using the functional Schrödinger equation, which respects the de Sitter isometries, and the antipodal map, $\hat{S}\colon\phi(x)\mapsto\phi(x_A)$, which sends a field at x to the field at x_A, the point antipodal to x, in the de Sitter spacetime. The definition of the α-vacuum wave functional is then modified to construct the wave functional for a scalar field that is an excitation of an α-state.

In Sec. V, we consider two cases. First, we suppose that $\alpha(k)$, the momentum space expression for the α-state, is some fixed profile. In particular, we consider the dynamical evolution of a field $\tilde{\phi}(x)$ in this state and compute its two-point function in the Euclidean vacuum. If $\tilde{\phi}(x)$ is the inflaton, the two-point function will imply a deviation from the predictions of standard inflationary cosmology. Our results are consistent with momentum dependent corrections to the power spectrum that scale as $\mathcal{O}[H^2\alpha(k)]$, where $\alpha(k)$ specifies the initial condition. Secondly, we make a first attempt at the more ambitious goal of treating α as a fully dynamical parameter that interpolates between α-vacuumlike behavior at early times and Euclidean behavior at late times. In a heuristic sense, the ''vacuum'' will roll during inflation. We argue that the intuition we inherit from entangled states in Minkowski space makes such dynamics reasonable in de Sitter space. To make our discussion more precise, we construct a toy model of harmonic oscillator squeezed states and consider the implications this has for the dynamics of α-states in de Sitter space.

In Sec. VI, we comment on how the formalism we have developed applies to other dynamical degrees of freedom besides scalars. In particular, we explore whether we can endow fluctuations in the background de Sitter metric with α-like character. Section VII offers some concluding remarks. We emphasize that superselection parameters must be part of any definition of the geography of the landscape of string vacua and comment upon some more general implications of superselection parameters in quantum gravity.

II. SCALAR FIELD THEORY IN DE SITTER SPACE

The maximally symmetric solution to the d-dimensional vacuum Einstein equations with constant, positive curvature is de Sitter space, dS_d. The de Sitter geometry is realized as the hyperboloid

$$\eta_{ab}X^aX^b = H^{-2} \equiv \ell^2 \tag{1}$$

embedded in $(d+1)$-dimensional Minkowski space, a construction which makes manifest the $SO(1,d)$ isometry

group of dS_d. The Hubble scale H sets the cosmological constant $\Lambda = \frac{1}{2}(d-1)(d-2)H^2$ and the curvature $R = d(d-1)H^2$. Spatial sections of the global manifold are spheres, S^{d-1}, but inflationary cosmology involves planar sections. A review of de Sitter geometry in various coordinate systems is contained in Refs. [21,22].

Let us consider the free scalar field $\phi(x)$ in de Sitter space.[1] The symmetric Green function

$$G_\alpha(x, y) = \langle\alpha|[\phi(x)\phi(y) + \phi(y)\phi(x)]|\alpha\rangle \quad (2)$$

in a de Sitter invariant state $|\alpha\rangle$ can depend only on the geodesic distance $d(x, y)$ [6,23,24]:

$$d(x, y) = H^{-1}\cos^{-1}Z(x, y),$$
$$Z(x, y) \equiv H^2 \eta_{ab} X^a(x) X^b(y). \quad (3)$$

$Z(x, y)$ is defined in terms of the embedding space $\mathbb{R}^{1,d}$. Notice that $Z(x, x) = 1$ so that $d(x, x) = 0$ and that $Z(x, x_A) = -1$, where x_A is the point antipodal to x; $X^a(x_A) = -X^a(x)$ in terms of the embedding geometry. (In global de Sitter space with spherical sections, the antipodal point to the north pole at time t is the south pole at time $-t$.) Given the maximal symmetry of the de Sitter geometry, $G_\alpha(x, y)$ must be a function only of $Z(x, y)$. The Klein-Gordon equation for $G_\alpha(x, y)$ implies that [6,24]

$$\left((Z^2 - 1)\frac{d^2}{dZ^2} + dZ\frac{d}{dZ} + m^2H^{-2}\right)F(Z) = 0. \quad (4)$$

The solution to this differential equation is a hypergeometric function. Given the invariance of the equation of motion under $Z \rightarrow -Z$, if $f(Z)$ is one solution, the other must be $f(-Z)$,[2] so that the general solution is of the form

$$F(Z) = af(Z) + bf(-Z). \quad (5)$$

The hypergeometric function $f(Z)$ has a singularity at $Z = 1$, which translates to $d(x, y) = 0$. In other words, the first term produces the well-known singularity along the light cone. The second term $f(-Z)$ is singular when $Z = -1$; that is, there is a singularity when points are antipodal to each other. The latter singularity is rather unconventional since antipodal points in de Sitter space are separated by a cosmological horizon. The vacuum labeled by $\alpha = 0$ corresponds to setting $b = 0$, thus removing the antipodal singularity and achieving a smooth Minkowski limit [8,9]. This vacuum is called the Euclidean vacuum, since it is selected by requiring a smooth Euclidean continuation, and the adiabatic (thermal) vacuum, since it ensures that the behavior of the detector response function of an Unruh detector is consistent with the principle of detailed balance [10,25].

III. THE OPERATOR FORMALISM: α-VACUA AND α-STATES

Consider the mode expansion for the free field in the Euclidean vacuum:

$$\phi(x) = \sum_n [a_n^E \phi_n^E(x) + a_n^{E\dagger} \phi_n^{E*}(x)]. \quad (6)$$

The vacuum $|\alpha = 0\rangle$ is annihilated by the operators a_n^E associated with the Euclidean mode basis $\phi_n^E(x)$. The modes can be chosen such that $\phi_n^E(x_A) = \phi_n^{E*}(x)$ [6].

We can then define a new mode basis and associated annihilation operators

$$\phi_n^\alpha(x) = \cosh\alpha_n \phi_n^E(x) + e^{-i\beta_n}\sinh\alpha_n \phi_n^{E*}(x), \quad (7)$$

$$a_n^\alpha = \cosh\alpha_n a_n^E - e^{i\beta_n}\sinh\alpha_n a_n^{E\dagger}, \quad (8)$$

with real $\alpha_n \in [0, \infty)$, $\beta_n \in [0, 2\pi)$. The vacuum $|\alpha\rangle$ arises as a Bogoliubov transformation of the Euclidean vacuum with mode and frequency independent coefficients α, β. It is annihilated by a_n^α. As time-reversal invariance requires that $\beta = 0$ [6], we will henceforth drop this parameter.

Formally, we can write the α-vacua as squeezed states of the Euclidean vacuum [12]:

$$a_n^\alpha = \mathcal{U} a_n^E \mathcal{U}^\dagger, \qquad |\alpha\rangle = \mathcal{U}|0\rangle,$$
$$\mathcal{U} = A\exp\left[\sum_n \frac{1}{2}[C_n(a_n^{E\dagger})^2 - C_n^*(a_n^E)^2]\right], \quad (9)$$

where $C_n = \alpha_n = \alpha = \text{const} \in \mathbb{R}$. Clearly, $a_n^E|0\rangle = 0$ implies that $a_n^\alpha|\alpha\rangle = 0$. Although in Eq. (9), $\mathcal{U}$ appears to implement a unitary rotation of $|0\rangle$ to give the vacuum state $|\alpha\rangle$, this is merely a formal statement: $|0\rangle$ and $|\alpha\rangle$ are not in the same Fock space, and α is a superselection parameter.[3] The overlap between the Euclidean vacuum and the α-vacuum is

$$\langle 0|\alpha\rangle = A\prod_n \cosh\alpha_n, \quad (10)$$

and the normalization condition for $|\alpha\rangle$ gives

$$1 = \langle\alpha|\alpha\rangle = |A|^2\left(\prod_n \cosh 2\alpha_n\right). \quad (11)$$

As there are an infinite number of modes, $|A|^2$ must vanish. Because the state $|\alpha\rangle$ cannot be normalized, it is not an excitation in the Fock space constructed over $|0\rangle$. Each of

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[1]Our discussion extends to free fields of other spins, as it depends only on properties of the de Sitter isometry group $SO(1, d)$.

[2]Note that $f(Z)$ and $f(-Z)$ are not linearly independent solutions when $m^2 = 0$. In this case, one must solve Eq. (4) for a second real solution.

[3]From the perspective of the conjectured duality between a quantum theory of gravity in de Sitter space and a Euclidean conformal field theory (CFT) on the boundary $\mathcal{I}^\pm$ [26–29], different values of α arise as marginal deformations of the CFT [30].

the α-vacua defines the de Sitter invariant ground state for a different Hilbert space.

Choosing α_n mode by mode, we can, however, construct a normalizable state in the same Hilbert space as $|0\rangle$. For small α, expanding the previous expressions requires that

$$0 \leq |A|^2 = \left[1 - 2\sum_n \alpha(n)^2 + \cdots \right] \leq 1. \quad (12)$$

More generally, if we are in d dimensions, we expect the momentum density to scale as k^{d-2}. Normalizability demands

$$\int dk k^{d-2} \log\cosh\alpha(k) < \infty, \quad (13)$$

which requires $\alpha(k)$ to decay faster than $k^{(1-d)/2}$. By choosing an $\alpha(k)$ that exhibits a sufficiently fast falloff, we can construct states with behavior characteristic of an α-vacuum at low mode numbers but exponentially close to being like the Euclidean vacuum at high mode numbers. (Of course, such states will not be de Sitter invariant.) A state with fixed α up to some cutoff would naturally fall within this setting. From the point of view of effective field theory, where we expect new physics to emerge beyond some cutoff, this is perhaps a compelling realization of the "α-vacuum" as an Euclidean excitation [13]. It is unreasonable to insist upon alphaness of the state at arbitrarily high mode number.

As well, it is tempting to think about generic initial conditions for inflation in this way. Inflation can begin and end in an excited state, which will be some unspecified excitation over the Euclidean ground state.[4,5] Such an excitation will have certain modes populated. That it is in the same Hilbert space as $|0\rangle$ ensures that the profile $\alpha(k)$ falls off sufficiently fast at large momenta. Low momentum modes, whose character differs significantly from that of the Euclidean vacuum, are greatly redshifted by inflation, so that the state today looks indistinguishable from the thermal vacuum. Thus, it is conceivable that the Universe was α-like at early times and is Euclidean at late times. Indeed, states which interpolate between Euclidean and α-vacua have previously been examined as probes of trans-Planckian physics [33–35]. We will argue that the α-states will be imprinted into fluctuations of the CMBR in a novel way. Deviations from the standard power spectrum will have explicit momentum dependence fixed by the shape of α.

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[4]The Hartle-Hawking no boundary prescription for the wave function of the Universe [31] selects the Euclidean vacuum [32].

[5]From the point of view of the Wheeler-de Witt equation, which is used in the minisuperspace approximation to justify the Hartle-Hawking vacuum, the issue of superselection sectors in the macroscopic theory is a real concern. The Hamilton constraint can be solved in the case of de Sitter space for general α.

IV. THE FUNCTIONAL FORMALISM

A. The wave functional of the α-vacuum

To further explore the physics of an α-state, we find it useful to work in the functional formalism. Following Refs. [10,32,36] consider a scalar field in dS$_d$ with stress-energy $T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L}$. The de Sitter symmetries (boosts and rotations) are generated by

$$M_{0i} = \int d^d x \sqrt{-g}(x^i T^{00} - t T^{0i})$$

$$= \int d^d x \sqrt{-g} \frac{1}{2} x^i [\pi^2 + (\nabla\phi)^2], \quad (14)$$

$$M_{ij} = \int d^d x \sqrt{-g}(x^i T^{0j} - x^j T^{0i})$$

$$= \int d^d x \sqrt{-g} \phi(x^i \partial_j - x^j \partial_i)\pi. \quad (15)$$

The scalar field $\phi(x)$ and its conjugate momentum $\pi(x) \equiv -i \frac{1}{\sqrt{-g(x)}} \frac{\delta}{\delta\phi(x)}$ satisfy canonical equal-time commutation relations $[\phi(x), \phi(x')] = [\pi(x), \pi(x')] = 0$ and $[\phi(x), \pi(x')] = \frac{i\delta^{(d-1)}(\vec{x}-\vec{x}')}{\sqrt{-g(x')}}$. To be de Sitter invariant, the wave functional of the vacuum must be annihilated by the symmetry generators.

Following [10], consider a Gaussian ansatz for the vacuum wave functional:

$$\langle\phi|0\rangle = \exp\left[-\frac{1}{2} \int d^d x \sqrt{-g(x)} \right.$$

$$\left. \times \int d^d x' \sqrt{-g(x')} \phi(x)F(x, x')\phi(x') \right]. \quad (16)$$

By imposing $M\langle\phi|0\rangle = 0$, one finds from rotational symmetry that $F(x, x') = F(|x - x'|)$. This means that $F(x, x')$ can be expanded in terms of $(d - 1)$-dimensional spherical harmonics:

$$F(|x - x'|) = \sum_{\ell,\vec{m}} f_\ell Y^*_{\ell\vec{m}} Y_{\ell\vec{m}}. \quad (17)$$

The boost condition for a fixed x^i determines the form of f_ℓ up to a one complex parameter ambiguity [10,36] corresponding to the choice of α. (The explicit formula for f_ℓ is not particularly illuminating and can be found in Ref. [10] for $d = 2, 3, 4$.) It can be shown that the Hartle-Hawking Euclidean prescription for the evaluation of the de Sitter wave functional restricts us to the adiabatic vacuum [31,32].

The general wave functional (for arbitrary α parameter) is given by [10]

$$\Phi_\alpha(\phi) = e^{i\alpha\hat{S}(\phi)}\Psi_{\alpha=0}(\phi), \quad (18)$$

where $\hat{S}$ provides the action of the antipodal map

$$[i\hat{S}, \phi(x)] = \tfrac{1}{2}\phi(x_A). \tag{19}$$

The antipodal map $\hat{S}$ is written as a spatial integral:

$$\hat{S} = \frac{1}{2}\int_{\Sigma_{t=0}} d^{d-1}x\sqrt{\gamma(x)}\phi(x_A)\pi(x). \tag{20}$$

The integration is in global coordinates over the $\Sigma_{t=0}$ surface. This is the unique spatial section which contains both a point and its antipode. $\gamma(x)$ is the pull-back of the de Sitter metric onto the $t = 0$ surface. To simplify our expressions, we shall suppress from now on factors of the determinant of the (pull-back) metric in our discussion of the scalar field. (We shall always treat the momentum as a scalar, rather than as a density, however.) In principle, a similar map $\hat{S}$ will exist for each field in the theory.

Setting $\alpha = 0$ in Eq. (18) corresponds to selecting the Euclidean vacuum wave functional Ψ, which satisfies the functional Schrödinger equation

$$\frac{1}{2}\int d^{d-1}x[\pi(x)^2 + (\nabla\phi)^2 + m_{\text{eff}}^2\phi(x)^2]\Psi(\phi)$$
$$= \omega\Psi(\phi). \tag{21}$$

We write the effective mass $m_{\text{eff}}^2 := m^2 + \xi R$ to include any quadratic coupling of $\phi(x)$ to the curvature. For nonzero, constant α, the Schrödinger equation is

$$e^{i\alpha\hat{S}(\phi)} \cdot \frac{1}{2}\int d^{d-1}x[\pi(x)^2 + (\nabla\phi)^2 + m_{\text{eff}}^2\phi(x)^2]$$
$$\cdot e^{-i\alpha\hat{S}(\phi)}\Phi(\phi)$$
$$= \omega\Phi(\phi). \tag{22}$$

We will consider only the Hamiltonian of a free scalar field in de Sitter space.[6]

1. Comparison to QCD θ-vacua

The functional description of the α-vacua is *formally* similar to that of θ-vacua in QCD [38]. Since we shall later consider promoting the parameter α to a dynamical field, it is useful to make a brief comparison with QCD.

The general gauge invariant wave functional for a θ-vacuum is

$$\Phi_\theta(\vec{A}) = e^{i\theta W(\vec{A})}\Psi_{\theta=0}(\vec{A}), \tag{23}$$

where by *vacuum* we mean a state that solves the appropriate Gauss constraint. $W(\vec{A})$ is the Chern-Simons form, which under large gauge transformations changes by an integer. The variation of $W(\vec{A})$ with respect to the gauge connection $\vec{A}$ gives the Yang-Mills magnetic field $\vec{B}_a = \nabla \times \vec{A}_a - \frac{1}{2}gf_{abc}\vec{A}_b \times \vec{A}_c$, where g is the Yang-Mills gauge coupling and f_{abc} are the structure constants of the

gauge group; $\frac{\delta W}{\delta A} = \frac{g^2}{8\pi^2}\vec{B}$. $\Psi(\theta = 0)$ satisfies a functional Schrödinger equation:

$$\int d^3x \frac{1}{2}\left(-\frac{\delta^2}{\delta\vec{A}_a^2} + \vec{B}_a^2\right)\Psi(\vec{A}) = \omega\Psi(\vec{A}). \tag{24}$$

The Hamiltonian density is the usual $\frac{1}{2}(\vec{E}^2 + \vec{B}^2)$, and $\vec{E}$ is the canonical momentum $-\dot{\vec{A}}$. The Schrödinger equation for Φ for nonzero θ reads

$$\int d^3x \frac{1}{2}\left[\left(-i\frac{\delta}{\delta\vec{A}_a} - \frac{\theta g^2}{8\pi^2}\vec{B}_a\right)^2 + \vec{B}_a^2\right]\Phi(\vec{A}) = \omega\Phi(\vec{A}). \tag{25}$$

By doing the phase space path integral one finds, upon integrating over $\vec{E}_a$, the expected term $\theta\dot{\vec{A}} \cdot \vec{B} = \theta\vec{E} \cdot \vec{B}$ in the effective Lagrangian [38]. In covariant notation, this is $\theta F \wedge \tilde{F}$. The parameter θ can be promoted into a field (the axion), by adding a canonical kinetic term to the Lagrangian [38]. The new operator $\theta F \wedge \tilde{F}$ is local, dimension five, and *CP* violating. This interaction is suppressed by some high-energy scale accommodating experimental bounds on *CP* violation. (Various phenomenological implications have been reviewed in Ref. [39].)

Although we may draw inspiration from this example, the two equations

$$\Phi_\theta(\vec{A}) = e^{i\theta W(\vec{A})}\Psi_{\theta=0}(\vec{A}), \tag{26}$$

$$\Phi_\alpha(\phi) = e^{i\alpha\hat{S}(\phi)}\Psi_{\alpha=0}(\phi) \tag{27}$$

exist on separate and unequal footings. The logic of $\Phi_\theta(\vec{A})$ is that it is the most general wave functional consistent with gauge invariance given that we are working with projective representations of the Hilbert space of Yang-Mills theory. The logic of $\Phi_\alpha(\phi)$ is that it is the most general wave functional consistent with de Sitter invariance. Gauge invariance is a *local* symmetry, whereas de Sitter invariance is a *global* statement about the geometry. (However, the large gauge transformations, under which the Yang-Mills vacuum wave functional is invariant, are an analogue of the de Sitter isometry, which is, in a sense, the set of large diffeomorphisms.) As well, while $W(\vec{A})$ is a *functional* of the gauge connection that rotates $\Psi_{\theta=0}(\vec{A})$ by a phase, $\hat{S}(\phi)$ is an *operator* that encodes a nonlocal interaction between $\pi(x)$ and $\phi(x_A)$. Indeed, Eq. (18) is yet another description of the formal unitary transformation of vacua that we have encountered previously. We also note that the short distance structure of the singularities is different in each of the α-vacua, whereas no analogous statement exists for the θ-vacua. While significant similarities do exist between the wave functional approaches to the θ-vacua of QCD and the α-vacua of de Sitter space, we emphasize that there are crucial differences.

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[6]For an analysis of interacting scalar fields in de Sitter space and the implications for cosmology, see Ref. [37].

2. Field theory in an α-vacuum

To consider fields in an α-vacuum, we compute the left-hand side of the functional Schrödinger Eq. (22). Taylor expanding the exponentials and iterating commutators with $i\alpha\hat{S}$, we find

$$\tilde{\pi}(x) := e^{i\alpha\hat{S}(\phi)} \cdot \pi(x) \cdot e^{-i\alpha\hat{S}(\phi)}$$

$$= \pi(x) - \frac{\alpha}{2}\pi(x_A) + \frac{\alpha^2}{8}\pi(x) + \cdots, \quad (28)$$

which can be resummed to give

$$\tilde{\pi}(x) = \cosh\frac{\alpha}{2}\pi(x) - \sinh\frac{\alpha}{2}\pi(x_A); \quad (29)$$

and similarly,

$$\tilde{\phi}(x) := e^{i\alpha\hat{S}(\phi)} \cdot \phi(x) \cdot e^{-i\alpha\hat{S}(\phi)}$$

$$= \cosh\frac{\alpha}{2}\phi(x) + \sinh\frac{\alpha}{2}\phi(x_A). \quad (30)$$

The gradient $\nabla\phi$ does not affect the analysis in a crucial way. In particular, we find

$$e^{i\alpha\hat{S}(\phi)} \cdot \nabla_x\phi(x) \cdot e^{-i\alpha\hat{S}(\phi)} = \nabla_x\tilde{\phi}(x). \quad (31)$$

The field redefinitions act like a canonical transformation. They preserve the commutation relations

$$[\phi(x), \pi(x')] = [\tilde{\phi}(x), \tilde{\pi}(x')] = i\delta^{(d-1)}(x - x'). \quad (32)$$

Conjugation by the antipodal map $\hat{S}$ implements the Bogoliubov transformation on the fields. Note that it is only for real α that the new Hamiltonian we obtain in this way is Hermitian.

Equations (29) and (30) imply that, if α is a nonzero, real constant, then $\tilde{\phi}$ is in the α-vacuum if the original field ϕ is in the Euclidean vacuum. The two-point Wightman function for $\tilde{\phi}$ is precisely as expected:

$$G_\alpha(x, y) := \langle\alpha|\tilde{\phi}(x)\tilde{\phi}(y)|\alpha\rangle$$

$$= \cosh^2\alpha\, G_E(x, y) + \sinh^2\alpha\, G_E(x_A, y_A)$$

$$+ \tfrac{1}{2}\sinh2\alpha[G_E(x_A, y) + G_E(x, y_A)]. \quad (33)$$

Equations of motion for $\tilde{\phi}(x)$ obtain from the Lagrangian

$$L = \int d^{d-1}x\, \tilde{\pi}(x)\dot{\tilde{\phi}}(x) - \hat{H} \quad (34)$$

with $\hat{H}$ the Hamiltonian

$$\hat{H} = \frac{1}{2}\int d^{d-1}x[\tilde{\pi}(x)^2 + (\nabla\tilde{\phi})^2 + m_{\text{eff}}^2\tilde{\phi}(x)^2] \quad (35)$$

defined using the relations (29) and (30). The action written with respect to the original variables $\phi(x)$ and $\pi(x)$ is clearly nonlocal, but in terms of the new variables $\tilde{\phi}(x)$ and $\tilde{\pi}(x)$ it is perfectly local. Conjugation by the antipodal map converts a *nonlocal* description involving fields in the Fock space over the Euclidean vacuum to a *local* description involving fields in the Fock space over the α-vacuum. This is as we expect. As $|0\rangle$ and $|\alpha\rangle$ are the ground states of different Hilbert spaces, $\phi(x)$ and $\tilde{\phi}(x)$ are local excitations of different vacua. The structure of the vacuum determines the most natural field variable for writing the Lagrangian. Regarding *either* the Hartle-Hawking vacuum of Eq. (35), with $\tilde{\phi}(x)$ as the fundamental field in the α-vacuum, or equivalently the Hartle-Hawking vacuum with $\phi(x)$ as the fundamental field, lead to the same α-states.

B. The wave functional of the α-state

We shall now adapt the functional formalism to the study of α-states. We want to write the analogue of the Schrödinger equation (22) for the wave functional $\langle\phi|\alpha\rangle$, where now $|\alpha\rangle$ is a normalizable excitation over the Euclidean vacuum $|0\rangle$.

It is instructive to recall that an α-state in the operator formalism corresponds to the profile $\alpha(k)$. This is an expression in *momentum* space. The function $\alpha(k) = $ const defines the α-vacuum. The antipodal map $\hat{S}$ in Eq. (20), by contrast, is defined as an integral over the $\Sigma_{t=0}$ spatial surface. In the expression

$$\Phi_\alpha(\phi) = e^{i\alpha\hat{S}(\phi)}\Psi_{\alpha=0}(\phi) \quad (36)$$

for the de Sitter vacuum wave functional, α simply appears as a coefficient in the exponential. We wish to promote this coefficient to a function of position consistent with the intuition that *constant* α in *momentum* space corresponds to a δ-*function* distribution of weight α in *position* space.

We make the ansatz

$$\hat{S}_\alpha = \lambda \int d^{d-1}x d^{d-1}y\, \alpha(x - y)\phi(x_A)\pi(y). \quad (37)$$

Note that λ may be dimensionful: $\dim\lambda + \dim\alpha = d - 1$. Clearly, putting $\alpha(x - y) = \alpha\delta^{(d-1)}(x - y)$ recovers the antipodal map $\hat{S}$ from before.

We note that α appears in the antipodal map (37) as a function of the *difference* in position of two points on the $\Sigma_{t=0}$ spatial surface integrated over the surface. Taking $\alpha(x - y) = \alpha(|x - y|)$ is consistent with physics in the α-state being homogeneous and isotropic. If we regard the scalar as being the inflaton field, this is a desirable feature of the cosmology. In global coordinates in $d = 4$, homogeneity and isotropy imply that $\alpha(x - y)$ is really $\alpha[\text{tr}(g^{-1}h)]$, where g and h are two points on S^3 via the identification of S^3 with $SU(2)$, so that g and h are $SU(2)$ matrices. This follows, for example, from the fact that $d(g, h)$, the distance on S^3, obeys $2\cos d(g, h) = \text{tr}(g^{-1}h)$. In inflationary coordinates, $\alpha(x - y)$ denotes a

function of the difference between x and y. The map between these two definitions of α is complicated as they live on different equal-time surfaces.

The Fourier transform[7] of Eq. (37) is

$$\hat{S}_\alpha = \lambda \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \alpha(k)\phi(k)\pi(k). \qquad (38)$$

(In writing this expression, we have employed the fact that de Sitter space is a maximally symmetric space.[8]) We note that the terms in the integrand are in accord with the expectation of the operator formalism:

$$\phi(k)\pi(k) \Rightarrow (a_k^\dagger + a_k)(a_k^\dagger - a_k) \sim [(a_k^\dagger)^2 - (a_k)^2]. \quad (39)$$

The right-hand side of Eq. (39), we recall, appears in the definition of $\mathcal{U}$ in the formal expression for the squeezed state $|\alpha\rangle = \mathcal{U}|0\rangle$ [cf. Eq. (9)].

To write the Schrödinger equation corresponding to a wave functional in an α-state, the analysis proceeds exactly as above. We simply conjugate the free field Hamiltonian by $e^{i\hat{S}_\alpha}$. We find that the transformed Hamiltonian is of the same form as the original Hamiltonian, but is written in terms of the new field variables

$$\tilde{\phi}(x) := e^{i\hat{S}_\alpha} \cdot \phi(x) \cdot e^{-i\hat{S}_\alpha}$$

$$= \phi(x) + \lambda \int d^{d-1}y\, \alpha(y-x)\phi(y_A)$$

$$+ \frac{1}{2}\lambda^2 \int d^{d-1}y\, d^{d-1}z\, \alpha(y-x)\alpha(z-y_A)\phi(z_A)$$

$$+ \cdots, \qquad (40)$$

[7]Strictly speaking, in global coordinates we should perform a discrete Fourier transform and write the result as a sum over momentum modes. For ease of notation, we prefer to write integral expressions instead.
[8]To be explicit, the calculation proceeds as below:

$$\hat{S}_\alpha = \lambda \int d^{d-1}x\, d^{d-1}y\, \frac{d^{d-1}k}{(2\pi)^{d-1}} \alpha(k)e^{-ik\cdot(x-y)}\phi(x_A)\pi(y)$$

$$= \lambda \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \alpha(k) \cdot \int d^{d-1}x\, e^{-ik\cdot x}\phi(x_A)$$

$$\cdot \int d^{d-1}y\, e^{+ik\cdot y}\pi(y)$$

$$= \lambda \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \alpha(k) \cdot \int d^{d-1}x\, e^{-ik\cdot x_A}\phi(x) \cdot \pi(k)$$

$$= \lambda \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \alpha(k)\phi(k)\pi(k).$$

Integrating the pair $[x, x_A(x)]$ over the spatial slice is the same as integrating the pair $[x_A(x), x]$ over the slice. In the last step we have used the Hermiticity properties of the spherical harmonics, $Y_{l,\tilde{m}}(x_A) = Y_{l,\tilde{m}}(x)^*$.

$$\tilde{\pi}(x) := e^{i\hat{S}_\alpha} \cdot \pi(x) \cdot e^{-i\hat{S}_\alpha}$$

$$= \pi(x) - \lambda \int d^{d-1}y\, \alpha(x_A - y)\pi(y)$$

$$+ \frac{1}{2}\lambda^2 \int d^{d-1}y\, d^{d-1}z\, \alpha(x_A - y)\alpha(y_A - z)\pi(z)$$

$$+ \cdots. \qquad (41)$$

The canonical equal-time commutation relation

$$[\phi(x), \pi(x')] = [\tilde{\phi}(x), \tilde{\pi}(x')] = i\delta^{(d-1)}(x - x') \quad (42)$$

is preserved by this transformation.[9]

In the case where $\alpha(x - y)$ is nearly a δ-function, the field redefinition of Eq. (40) presents a smearing of Eq. (30). The even powers of λ in the expansion of $\tilde{\phi}(x)$ coarsen out ϕ near x, while the odd powers coarsen out ϕ near x_A. This is, however, not a case of there being an extra antipodal source in an interaction Lagrangian, as has been proposed for an α-vacuum [40]. As we are working in the Euclidean vacuum, the singularity structure of the Green function is unchanged. We need not adopt a nonstandard Feynman prescription in this theory.

We can equally write momentum space expressions $\tilde{\phi}(k)$ and $\tilde{\pi}(k)$. The advantage of this is that we can sum the series to get closed form expressions:

$$\tilde{\phi}(k) = e^{i\hat{S}_\alpha} \cdot \phi(k) \cdot e^{-i\hat{S}_\alpha}$$

$$= \phi(k) + \lambda\alpha(-k)\phi(-k) + \frac{1}{2}\lambda^2\alpha(-k)\alpha(k)\phi(k)$$

$$+ \cdots$$

$$= \cosh s(k)\phi(k) + \lambda\alpha(-k)\frac{\sinh s(k)}{s(k)}\phi(-k), \qquad (43)$$

$$\tilde{\pi}(k) = e^{i\hat{S}_\alpha} \cdot \pi(k) \cdot e^{-i\hat{S}_\alpha}$$

$$= \pi(k) - \lambda\alpha(-k)\pi(-k) + \frac{1}{2}\lambda^2\alpha(-k)\alpha(k)\pi(k)$$

$$+ \cdots$$

$$= \cosh s(k)\pi(k) - \lambda\alpha(-k)\frac{\sinh s(k)}{s(k)}\pi(-k), \qquad (44)$$

where $s(k) := \lambda\sqrt{\alpha(k)\alpha(-k)}$. Fourier transformation recovers the position space expressions we have seen previously.

The Hamiltonian density, upon nonlocal field redefinition, is simply that of a free field:

$$\hat{\mathcal{H}} = \tfrac{1}{2}[\tilde{\pi}(x)^2 + (\nabla\tilde{\phi})^2 + m_{\text{eff}}^2\tilde{\phi}(x)^2]. \qquad (45)$$

Whereas the field $\tilde{\phi}(x)$ is the appropriate local variable to

[9]The cancellation of terms at $\mathcal{O}(\lambda^2)$ demands that we make use of the invariance of the integrand under $y \leftrightarrow y_A$, when y is the variable of integration. This once again invokes the maximal symmetry of de Sitter space.

work with when in an α-vacuum, it is not clear here *a priori* whether we should work with $\phi(x)$, which is an excitation of the Euclidean vacuum $|0\rangle$, or $\tilde{\phi}(x)$, which is an excitation of the state $|\alpha\rangle$, itself a squeezed excitation over $|0\rangle$. The Hamiltonian density expressed in terms of $\phi(x)$ and $\pi(x)$ is clearly nonlocal in that it involves integrations over the entire $\Sigma_{t=0}$ surface. We interpret the observation that the Hamiltonian density, when written in terms of the new field variables $\tilde{\phi}(x)$ and $\tilde{\pi}(x)$, looks local while secretly being nonlocal in terms of the old field variables $\phi(x)$ and $\pi(x)$ as a signal that the physics of an α-state is nothing but the highly entangled physics of scalar excitations over the Euclidean ground state. We are simply working with a state in which there are long-range (Hubble scale) correlations. There is no experiment that demonstrates that field theory exhibits the usual decoupling at the scale of the cosmological horizon. Local measurements by a de Sitter observer are completely consistent with this philosophy. Indeed, a signal sent from the antipode at global time $t = -\infty$ (on I^-) only reaches us at time $t = +\infty$ (on I^+). Antipodal correlations do not ring the death knell for causality. As we have performed a canonical transformation via a unitary rotation, there can be no inconsistency in the resulting physics. If we go beyond free field theory to include interactions, the perturbative consistency of the theory in the Euclidean vacuum ensures the perturbative consistency of the theory in the α-state.[10]

V. PHYSICS OF α-STATES

A. α as a fixed profile

Let us suppose that $\alpha(k)$ is a given function that defines an α-state. The Hamilton equations of motion for $\tilde{\phi}$ in this α-state on the $\Sigma_{t=0}$ surface simply give the Klein-Gordon equation:

$$0 = (\Box_x - m_{\text{eff}}^2)\tilde{\phi}(x), \tag{46}$$

where $\Box_x$ is the usual Laplace-Beltrami operator in curvilinear coordinates. We therefore expect that the functional form of $\tilde{\phi}(x)$ is the same as the functional form of $\phi(x)$ in the Euclidean vacuum. The only difference is that the mode expansion

$$\tilde{\phi}(x) = \sum_n [\tilde{a}_n \tilde{\phi}_n(x) + \tilde{a}_n^\dagger \tilde{\phi}_n^*(x)] \tag{47}$$

is written in terms of operators that act on the α-state: $\tilde{a}_n |\alpha\rangle = 0$. These operators satisfy the Heisenberg algebra

$[\tilde{a}_m, \tilde{a}_n^\dagger] = \delta_{mn}$. The $\tilde{\phi}_n(x)$ are linear combinations of the $\phi_n^E(x)$ mode solutions (6) as they solve the same wave equation.

Written in terms of the field $\phi(x)$, the Klein-Gordon equation is highly nontrivial:

$$0 = (\Box_x - m_{\text{eff}}^2)\bigg[\phi(x) + \lambda \int d^{d-1}y \alpha(y - x)\phi(y_A)$$
$$+ \frac{1}{2}\lambda^2 \int d^{d-1}y d^{d-1}z \alpha(y - x)\alpha(z - y_A)\phi(z_A)$$
$$+ \mathcal{O}(\lambda^3) \bigg]. \tag{48}$$

Note that $\Box_x$ acts on $\alpha(y - x)$ under the integrals. Equation (48) is a nonlocal integrodifferential equation of motion. It is nonlocal in the sense that it involves an integration over the entire $\Sigma_{t=0}$ surface.[11] Equation (46) dumps the nonlocal structure of the Klein-Gordon equations into the action of the $\tilde{a}_n$ on the α-state.

So far, we have worked exclusively on the $\Sigma_{t=0}$ surface. The usual Hamiltonian evolution

$$\tilde{\phi}(T, x) = e^{i\hat{H}T}\tilde{\phi}(x)e^{-i\hat{H}T} \tag{49}$$

propagates the solution from the $\Sigma_{t=0}$ spatial surface forward in time to the $\Sigma_{t=T}$ spatial surface. For any $t \neq 0$, the correlations in the expansion for $\tilde{\phi}(x)$ lie along a single spatial section. One would naively conclude that the antipodal correlation induced by the $\alpha(x - y)$ is *not* an antipodal interaction in the de Sitter spacetime. Rather the correlations are antipodal on the S^{d-1} sphere at time T. This is inconsistent with the expectation for physics in an α-vacuum. When $\alpha(x - y) = \alpha\delta^{(d-1)}(x - y)$, we expect to recover the singular behavior of the Green function both along the light cone *and* at the antipode in the de Sitter spacetime.

Recall that we can write the mode expansion for $\phi(x)$ in the Euclidean vacuum as

$$\phi(x) = \sum_n [a_n^E \phi_n^E(x) + a_n^{E\dagger} \phi_n^{E*}(x)]$$
$$= \sum_n [a_n^\alpha \phi_n^\alpha(x) + a_n^{\alpha\dagger} \phi_n^{\alpha*}(x)], \tag{50}$$

where the operators and mode functions are as in Eqs. (7) and (8). The scalar field in an α-vacuum is

$$\phi^\alpha(x) = \cosh\alpha \phi(x) + \sinh\alpha \phi(x_A)$$
$$= \sum_n [a_n^E \phi_n^\alpha(x) + a_n^{E\dagger} \phi_n^{\alpha*}(x)]. \tag{51}$$

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[10]Coupling matter to the metric beyond the free field level will generically stretch the Penrose diagram giving a tall de Sitter space [41]. In such a geometry, a point and its antipode are in causal contact before the end of time. One might naively conclude that antipodal nonlocalities will lead in such a setting to acausal or nonunitary physics. This is not the case. The initial conditions are responsible for an EPR-like entanglement.

[11]Such equations, we note, have appeared in the literature in other contexts, e.g., stress-strain relations [42], radiative transfer [43], and neurophysiology [44]. Wiener-Hopf techniques sometimes permit these equations to be solved for certain kernels when the integral is one-dimensional ($d = 2$) [45].

The Euclidean mode basis can be chosen [6] such that $\phi_n^*(x) = \phi_n(x_A)$. Thus, if one considers a field prepared in the Euclidean vacuum on the $\Sigma_{t=0}$ surface and implements the usual Hamiltonian evolution, $\phi(x, T) = e^{i\hat{H}T}\phi(x, 0)e^{-i\hat{H}T}$, the field knows about the backward time evolution as well because the mode functions are chosen to be antipodally related. The field $\phi^\alpha(x)$ in the α-vacuum and the field $\tilde{\phi}(x)$ in the α-state inherit this property.[12] The latter result, that $\tilde{\phi}^*(T, x) = \tilde{\phi}(-T, x_A)$, where x_A is the antipode on the spherical section, relies on the homogeneity and isotropy of the kernel $\alpha(x - y)$ in $\hat{S}_\alpha$.

1. A second look at time evolution

One can also apply the intuition of the eternal anti-de Sitter black hole to the problem of time evolution [46]. This alternate perspective is completely consistent with the prior reasoning in terms of an explicit mode analysis. The state $|\tilde{\phi}\rangle$ is doubled, as in thermofield dynamics. To every operator $\mathcal{O}$, we associate its conjugate $\overline{\mathcal{O}}$. (The algebraic rules for conjugation are given in Ref. [47], for example.) In particular, the free Hamiltonian, which is the generator of time translation, is

$$\hat{H}_{TFD} = H - \overline{H} \qquad (52)$$

and commutes with the squeezing operator

$$\mathcal{U}_{TFD} = A \exp\left[\sum_n \frac{1}{2}(C_n a_n^\dagger \bar{a}_n^\dagger - C_n^* a_n \bar{a}_n)\right]. \qquad (53)$$

The canonical equations of motion are

$$i\hbar \frac{\partial}{\partial t}\mathcal{O} = [\mathcal{O}, H], \qquad i\hbar \frac{\partial}{\partial t}\overline{\mathcal{O}} = -[\overline{\mathcal{O}}, \overline{H}]. \qquad (54)$$

Hamiltonian evolution carries us forward and backward in time from the $\Sigma_{t=0}$ surface:

$$|\tilde{\phi}(T, x)\rangle_{TFD} = e^{iHT}|\tilde{\phi}(0, x)\rangle \otimes e^{-i\overline{H}T}|\overline{\tilde{\phi}(0, x)}\rangle. \qquad (55)$$

The first term in the tensor product propagates the field $\tilde{\phi}(0, x)$ forward in time to the $\Sigma_{t=T}$ hypersurface. The second term propagates the conjugate field $\overline{\tilde{\phi}(0, x)}$ backward in time to the $\Sigma_{t=-T}$ hypersurface. In this way, antipodal correlations are maintained in the full de Sitter spacetime. As well, the boundary data on $\mathcal{I}^-$ are correlated to the boundary data on $\mathcal{I}^+$. Hence, if we accept the proposed de Sitter/CFT correspondence [26], the Euclidean conformal field theories on the spheres at $t = \pm\infty$ are naturally entangled. This is in the same spirit as the de Sitter holography discussed in Refs. [27,28].

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[12]We are working in the Heisenberg picture, where operators evolve. In the Schrödinger picture, the state $|\alpha\rangle$ evolves nontrivially, even for an α-vacuum. Translation in global time is not an isometry of de Sitter space.

The thermofield dynamics provides a compelling picture for the evolution of the state in static coordinates where there exits a timelike Killing isometry. Consider the two static patches N and S. The arrow of time will point in the opposite direction at the antipode, and these patches will cover the $\Sigma_{t=0}$ spatial surface. A squeezed state with energy ω is [28]

$$|\tilde{\phi}\rangle = C \exp\left[\frac{\cosh\alpha e^{-\pi\omega} + \sinh\alpha}{\cosh\alpha + \sinh\alpha e^{-\pi\omega}}a_N^\dagger a_S^\dagger\right]|0\rangle. \qquad (56)$$

(The expression as written is for an α-vacuum, but generalizes to an α-state.) The vacuum $|0\rangle = |0\rangle_N \otimes |0\rangle_S$. The operators $a_N^\dagger$ and $a_S^\dagger$ act on different sectors of the product Hilbert space. Preparing this state is in perfect accord with thermofield dynamics where a squeezed state is defined through the action of $a^\dagger \bar{a}^\dagger$. Time evolution on the static patches is just the usual Hamiltonian evolution. The state evolves such that the field at $t = +T$ is correlated to the field at the antipode at $t = -T$, as is required.

2. Phenomenology of α-states

How do we distinguish physics in an α-state? If we take the scalar field to be the inflaton, we can compute the power spectrum for $\tilde{\phi}(x)$, an excitation over $|\alpha\rangle$, and compare this to the power spectrum for $\phi(x)$, an excitation over $|0\rangle$. We shall now switch to conformal coordinates where dS_d is written in terms of the embedding

$$X^d - X^0 = H^{-2}\tau^{-1}, \qquad X^i = H^{-1}\tau^{-1}x^i,$$
$$i = 1, 2, \ldots, d - 1, \quad X^d + X^0 = \tau - \tau^{-1}\sum_i(x^i)^2, \qquad (57)$$

so that the de Sitter line element is

$$ds^2 = H^{-2}\tau^{-2}\eta_{\mu\nu}dx^\mu dx^\nu. \qquad (58)$$

The union of two coordinate patches, $\{\tau \in (-\infty, 0), \vec{x} \in \mathbb{R}^{d-1}\} \bigcup \{\tau \in (0, \infty), \vec{x} \in \mathbb{R}^{d-1}\}$, covers dS_d and the antipodal point of $x = (\tau, \vec{x})$ is $x_A = (-\tau, \vec{x})$. In these coordinates $\tau = 0$ corresponds to the boundaries $\mathcal{I}^\pm$. In defining the antipodal map, we have worked on the $t = 0$ hypersurface in global coordinates. In conformal coordinates, the S^{d-1} at $t = 0$ is given by

$$\tau = H^{-1}1/\omega^d = H^{-1}(\sin\theta_1 \ldots \sin\theta_{d-1})^{-1}, \qquad (59)$$

$$x^i = H^{-1}\omega^i/\omega^d, \qquad i = 1, 2, \ldots, d - 1, \qquad (60)$$

where the θ_i are angle variables on the sphere that define the coordinates ω^i. The antipodal transformation involves all (τ, x^i) consistent with this change of variables. In particular, it includes an integral over points in *both* flat patches. [Usually, one considers only a single coordinate patch and integrates (traces over) the other patch, or assigns vacuum expectation values to the degrees of freedom across the horizon and uses mean field theory.]

The calculation of the power spectrum relies on working in inflationary coordinates. We must choose an appropriate parametrization of the d-dimensional de Sitter hyperboloid in $(d + 1)$-dimensional Minkowski space. In defining the antipodal transformation, we have made repeated use of the maximal symmetry of the de Sitter geometry and the homogeneity and isotropy of the argument of α, which is a distance. Although in global coordinates we work with α on the $\Sigma_{t=0}$ surface, we could also define the kernel on other Cauchy surfaces that are preserved by the antipodal map. The most natural extension to these surfaces is to define α in terms of the embedding: $\alpha(x - y) = \alpha[|X(x) - X(y)|]$. [The distance in the embedding space $|X(x) - X(y)|$ on an equal τ surface is proportional to $|\vec{x} - \vec{y}|$.]

We shall compute $\tilde{P}(k) = \langle 0|[|\tilde{\phi}(k)|^2]|0\rangle$ and compare this to $P(k) = \langle 0|[|\phi(k)|^2]|0\rangle$. For definiteness, we choose to look at a massless, conformally coupled scalar field in $d = 4$, so that $m_{\text{eff}}^2 = 2H^2$. The conformal factor in the metric is absorbed via a field redefinition [11]. We follow Ref. [6] in our analysis and work to leading order in the expansion of $\tilde{\phi}(x)$. Anticipating our discussion of a dynamical α, we shall treat $\alpha(x - y)$ as an object with the mass dimension of a canonical scalar. This implies that the parameter λ in the definition of $\hat{S}_\alpha$ has dimension (mass)^2.[13] As the Hubble parameter is the only natural scale in the theory, we take $\lambda \simeq H^2$. The modes are

$$\phi_{\vec{k}}(\tau, \vec{x}) = \tfrac{1}{2} H \tau^{3/2} H_\nu^{(2)}(k\tau) e^{i\vec{k}\cdot\vec{x}}, \tag{61}$$

where $k = (\vec{k} \cdot \vec{k})^{1/2}$ and $\nu = (\tfrac{9}{4} - m_{\text{eff}}^2 H^{-2})^{1/2} = \tfrac{1}{2}$. Working with the mode expansion of $\tilde{\phi}(x)$ to leading order in the expansion parameter $\lambda\alpha(k)$, one finds

$$\tilde{\phi}(x) = \phi(x) + \lambda \int d^3 y\, \alpha(\vec{y}_A - \vec{x}) \phi(y) + \cdots \tag{62}$$

$$\simeq \int \frac{d^3 k}{(2\pi)^3} H \tau^{3/2} \{1 + \lambda\alpha(k) + \cdots\}[H_\nu^{(2)}(k\tau) e^{i\vec{k}\cdot\vec{x}} a_{\vec{k}} + H_\nu^{(1)}(k\tau) e^{-i\vec{k}\cdot\vec{x}} a_{\vec{k}}^\dagger]. \tag{63}$$

The integral (62) is strictly defined on the $t = 0$ surface in global coordinates. In practice, we substitute in the mode expansion (61) and Fourier transform. The expression in curly brackets determines the corrections to the power spectrum in this admittedly naive scheme:

$$\tilde{P}(k) = [1 + 2\lambda\alpha(k) + \cdots]P(k). \tag{64}$$

We find that there is a momentum dependent scaling in the power spectrum that arises from looking at the autocorrelation function of $\tilde{\phi}$ in the Euclidean vacuum. In four dimensions, a normalizable α-state obtains from $\alpha(k) \sim$

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[13]In general, λ has dimension (mass)$^{d/2}$.

k^{-2} scaling. For such a profile, we expect $\mathcal{O}[(H/k)^2]$ deviations from correlated initial conditions.

The correction to the power spectrum that we compute depends explicitly on the momentum. It is determined by the profile of the squeezed state $\alpha(k)$. The α-vacuum and adiabatic vacuum calculations of Refs. [14,15] predict either $\mathcal{O}(H/\omega)$ or $\mathcal{O}[(H/\omega)^2]$ deviations as signals of trans-Planckian physics. The scale ω is an ultraviolet cutoff, which is fixed. There is no explicit momentum dependence in the correction to the power spectrum. Reference [15] computes the power spectrum to be

$$\tilde{P}(k) = \left(\frac{H}{2\pi}\right)^2\left[1 - \frac{H}{\omega}\sin\left(\frac{2\omega}{H}\right)\right]. \tag{65}$$

This should correspond to $\alpha(k) \simeq 1/k$, which is not normalizable as an excitation over the Euclidean vacuum. The claim is that this details the physics of an α-vacuum. Using effective field theory in the Euclidean vacuum, Ref. [14] corrects the power spectrum as

$$\tilde{P}(k) = \left(\frac{H}{2\pi}\right)^2\left[1 + \chi\left(\frac{H}{\omega}\right)^2\right], \tag{66}$$

where χ is a model dependent numerical factor.

Kaloper and Kaplinghat [48] consider a sudden change in the background during the late stages of inflation. The inflaton, which defines the ground state of the quantum system prior to the sudden transition, is trapped in a squeezed state above the adiabatic vacuum after the transition. Because the inflaton is in an excited state, the power spectrum will deviate from the standard thermal result. Reference [48] computes the corrections to go as

$$\tilde{P}(k) = [1 + \mathcal{D}(p, H, \epsilon_H, \eta_H)]P(k), \tag{67}$$

$$\mathcal{D}(p, H, \epsilon_H, \eta_H) = \Phi(\epsilon_H, \eta_H) + \left(\frac{H}{p}\right)^2\cos\left(\frac{2p}{H}\right)$$
$$+ \Delta(\epsilon_H, \eta_H)\frac{H}{p}\sin\left(\frac{2p}{H}\right), \tag{68}$$

where Φ and Δ are functions of the slow-roll parameters and $p = k/a_0$. If we expand out the trigonometric functions in Eq. (68), we see that the leading order correction in the power spectrum $\Delta P(k) \simeq (H/k)^2 P(k)$, in agreement with our computations.

Initial conditions corresponding to squeezed excitations are imprinted in the CMBR. It is conceivable that deviations from the standard inflationary predictions can be detected in future observations. In principle, the α-state will affect higher order correlation functions as well as the two-point functions [37].

Our analysis applies as well when the asymptotics of the profile are fixed and the change in the profile near the asymptotic behavior is not too rapid (given, for example, that in the ultraviolet, we have nonzero α, and in the infrared $\alpha = 0$). This is consistent with adiabatic conditions used in quantum field theory in curved spacetime. In

principle, the analysis works for smoothly interpolating profiles given the adiabatic asymptotics. The most likely observed profiles are those which most closely saturate the normalization condition in Eq. (13).

B. α as a dynamical field

We know that in QCD the θ-angle, which is a super-selection parameter, can be promoted to a dynamical field, the axion, consistent with phenomenological constraints on *CP* violation in the strong sector [38]. The axion is the pseudoscalar Goldstone boson associated to a spontaneously broken global chiral $U(1)$ symmetry (the Peccei-Quinn symmetry). In an interacting scalar field theory in de Sitter space, quantum effects in the infrared *restore* spontaneously broken symmetries [11]. Giving α dynamics in de Sitter space must proceed in a different way.

We can approach the problem from a number of perspectives: Firstly, the dynamics of α may be induced by integrating out other fields to which $\phi(x)$ couples. As the ϕ-α system evolves in an inflationary cosmology, dynamics can then dump the energy from one field into the other so that at late times we end up in the Euclidean vacuum. Secondly, the dynamics of α may be induced by self-interactions of $\phi(x)$. The self-interactions can then be replaced by coupling to an auxiliary field α, just as one integrates in a new degree of freedom. Thirdly, as the antipodal map invokes $\alpha(x - y)$, the translational invariance may suggest that α is a condensate rather than a fundamental field: $\alpha(x - y) = \langle \overline{\psi}(x)\psi(y)\rangle$. The initial conditions in each case must be fine-tuned to ensure the proper entanglement between $\phi(x)$ and $\phi(x_A)$. As *any* particular initial condition *including the Euclidean one* is finely tuned, this is not a criticism unique to this model. It is, however, a challenge to obtain the particular excited states that we are interested in exploring as an effective field theory.

As a stepping stone to promoting the α superselection parameter to a fully dynamical field, the *alphon*, let us instead say a few words about the more modest effort of endowing the α-state with dynamics. To the Hamiltonian density $\hat{\mathcal{H}}$ from Eq. (45), we append

$$\hat{\mathcal{H}}_\alpha = \tfrac{1}{2}\pi_\alpha^2 + \tfrac{1}{2}(\nabla\alpha)^2 + V(\alpha). \qquad (69)$$

We treat α as an additional scalar field in the theory, with a canonical $[\alpha, \pi_\alpha]$ commutator and canonical mass dimension. This field couples to ϕ through the definition of $\tilde{\phi}$. The field $\tilde{\phi}$ is an excitation over the dynamically evolving state $|\alpha\rangle$.

We assume that α and π_α commute with ϕ and π. This immediately implies that α commutes with $\tilde{\phi}$ and $\tilde{\pi}$. However, π_α *will not commute* with either $\tilde{\phi}$ or $\tilde{\pi}$ because factors of α appear explicitly in the field redefinitions (40) and (41). We have

$$[\tilde{\phi}(x), \pi_\alpha(x')] = i\lambda\tilde{\phi}(x_A + x'_A) + \mathcal{O}(\lambda^2), \qquad (70)$$

$$[\tilde{\pi}(x), \pi_\alpha(x')] = -i\lambda\tilde{\pi}(x_A - x') + \mathcal{O}(\lambda^2). \qquad (71)$$

This will lead to a coupled set of nonlocal integrodifferential equations for the combined $\tilde{\phi}$-α system.

Although it is difficult to solve these equations even perturbatively in the parameter λ in low dimensions, we nevertheless expect that a solution exists and is stable. Our intuition for this arises from Minkowski space, where we can construct α-like squeezed states and consider excitations over them. The only difference in the formulas (37), (40), and (41) is that we remove the antipodal labels. Promoting α to a dynamical variable in Minkowski space results in a similar set of coupled nonlocal integrodifferential equations. As we expect the physics of such highly correlated entangled states to make perfect sense in flat Minkowski space, we expect nothing different in de Sitter space. The only new wrinkle in de Sitter space is the presence of antipodal labels on certain coordinates, but the structure of the equations of motion are the same. Indeed, from the point of view of the inflationary patch in de Sitter space where there is no clear notion of what the antipode is, Minkowski space reasoning is perhaps the most appropriate framework in which to address physics.

When α is dynamical and becomes an alphon field, given the nonlocal nature of the dynamics, we cannot fully analyze the effect on the power spectrum as we have with a fixed, static profile. When the strength of the alphon field is small (in natural units), we are, however, essentially dealing with the situation described by slowly varying α profiles and should reproduce the momentum dependent deviations from the standard power spectrum that we have discussed previously.

1. The rolling vacuum—a toy model

To investigate to what extent a slow-rolling alphon keeps the field $\tilde{\phi}(x)$ in the α-vacuum, we consider a similar problem for squeezed states in a harmonic oscillator. Thus, as in Sec. III, we take the operator

$$\mathcal{U}_\alpha = \exp\{\tfrac{1}{2}\alpha[(a^\dagger)^2 - a^2]\}. \qquad (72)$$

One easily verifies that $\mathcal{U}_\alpha$ creates a squeezed state when acting on the ground state of the harmonic oscillator,

$$\mathcal{U}_\alpha|0\rangle = \frac{1}{\sqrt{\cosh\alpha}}\exp\left[\frac{1}{2}\tanh\alpha(a^\dagger)^2\right]|0\rangle. \qquad (73)$$

We now construct a Hamiltonian $\hat{H}_\alpha$, whose ground state is $\mathcal{U}_\alpha|0\rangle$. This Hamiltonian is the analogue of the alphon Hamiltonian constructed before, and it is simply given by

$$\hat{H}_\alpha = \mathcal{U}_\alpha\hat{H}\mathcal{U}_\alpha^\dagger, \qquad (74)$$

with $\hat{H}$ the usual harmonic oscillator Hamiltonian. The exact eigenstates of $\hat{H}_\alpha$ are clearly the states $\mathcal{U}_\alpha|n\rangle$,

with $|n\rangle$ the usual harmonic oscillator states $|n\rangle = (n!)^{-1/2}(a^\dagger)^n|0\rangle$. Next, we are going to make α time dependent, and study the corresponding Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \mathcal{U}_{\alpha(t)}\hat{H}\mathcal{U}_{\alpha(t)}^\dagger \psi(t). \qquad (75)$$

Assuming that there is no back-reaction on α, a general solution will be of the form

$$\psi(t) = \sum_{k=0}^\infty c_k(t)e^{-iE_k t/\hbar}\mathcal{U}_{\alpha(t)}|k\rangle. \qquad (76)$$

To get an idea of the time dependence, we take $\alpha(t) = \alpha_0 \exp(-\epsilon t)$, and start with the state $\mathcal{U}_{\alpha_0}|0\rangle$ at $t = 0$. To leading order in ϵ, we find that the leakage out of the ground state is

$$\Delta \equiv \sum_{\ell>0} |c_\ell(t)|^2 = \frac{1}{8}\left|\frac{\alpha_0 \epsilon}{\omega}\right|^2. \qquad (77)$$

For a massive field in flat space, one can directly apply a similar calculation, with ω replaced by $\sqrt{k^2 + m^2}$, with k the spatial momentum and m the mass of the field. The parameter $\alpha(t)$ can now also be a nontrivial function of the momentum k, $\alpha(t) \to \alpha(k, t)$. In d-dimensional de Sitter space this situation is somewhat different. For a massive field with sufficiently large mass, $m \geq (d-1)/2$, the early and late time behavior of the solutions to the equations of motion is of the form $t^{\pm(d-1)/2\pm i\mu}$ with $\mu^2 = m^2 - (d-1)^2/4$. In particular, this does not depend on the momentum k of the mode under consideration. In contrast to what happened in flat space, we should replace $|\omega|$ by m, not by $\sqrt{k^2 + m^2}$. At early times, in the inflationary patch, the field shows the usual oscillatory behavior, and the transition between the two behaviors takes place at time $t^* \sim \log k$.

To summarize, we reinstate the scale M, which is of the order of the Hubble parameter H. The metric for de Sitter space is

$$ds^2 = -dt^2 + e^{2Ht}dx^2. \qquad (78)$$

Then as long as we consider a mode with momentum mode k at times t which satisfy $tM \gg \log(k/M)$, the deviation from the vacuum Δ defined in Eq. (77) generated by alphon decay is of the order of

$$\Delta \sim \left(\frac{\dot{\alpha}(k)}{m/M}\right)^2. \qquad (79)$$

2. Analogy with decoherence

Perhaps one route to further understanding the behavior of α is to proceed in analogy with decoherence and regard the combined ϕ-α system as a harmonic oscillator α coupled to an environment ϕ via a frictional term. The role of dissipation in such quantum systems has been addressed by Feynman and Vernon [49] and Caldeira and Leggett [50]. In the standard analysis, one consider the following toy Hamiltonian:

$$\hat{H} = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}m\omega^2 x^2 + \tfrac{1}{2}M\dot{q}^2 + V(q) - \gamma xq. \qquad (80)$$

The oscillator q is linearly coupled to the environment x by a damping term. To deduce the dynamics of q, one integrates out x. Taking $q(t)$ to be periodic with period T gives the effective action

$$S_{\text{eff}}[q] = \int_0^T dt\left[\frac{1}{2}M\dot{q}^2 + V(q)\right]$$
$$- \int_{-\infty}^\infty dt \int_0^T dt' A(t - t')q(t)q(t') + \text{const}, \qquad (81)$$

$$A(t - t') = \frac{\gamma^2}{4m\omega}e^{-\omega|t-t'|} =: \frac{1}{2\pi}\int d\omega J(\omega)e^{-\omega|t-t'|}. \qquad (82)$$

In the Feynman-Vernon formalism, the coupling to the environment is always linear, whereas at leading order the interaction between α and ϕ will include terms like

$$\hat{H}_{\text{int}} \supset \frac{1}{2}\lambda m_{\text{eff}}^2 \int d^{d-1}x d^{d-1}y \alpha(y-x)\phi(x)\phi(y_A) + \cdots. \qquad (83)$$

This differs in many respects from the standard dissipation analysis, which is purely Gaussian. Nevertheless, pursuing this reasoning further may offer insight into the dynamics of α.

VI. INFRARED CORRECTIONS TO GRAVITY?

As we have noted earlier, it is possible to extend our definition of the antipodal map to fields of other spin. The extension to vector fields is straightforward: We replace $\phi(x_A)$ with $A_\mu(x_A)$ and $\pi(y)$ with $\pi_A^\mu(y) = \frac{-i}{\sqrt{-g}}\frac{\delta}{\delta A_\mu(y)}$ in Eq. (37). The extension to fermions is complicated only in that we must preserve an anticommutator instead of a commutator in the canonical transformation. The Hamiltonian transforms by conjugation as

$$e^{i\hat{S}_\alpha} \cdot i\psi^\dagger(x)\partial_t\psi(x) \cdot e^{-i\hat{S}_\alpha} = ie^{i\hat{S}_\alpha} \cdot \psi^\dagger(x) \cdot e^{-i\hat{S}_\alpha}$$

$$\cdot e^{i\hat{S}_\alpha}\partial_t\psi(x) \cdot e^{-i\hat{S}_\alpha}$$

$$=: i\tilde{\phi}^\dagger(x)\partial_t\tilde{\phi}(x). \tag{84}$$

We can check that under the field redefinition of Eq. (84), $\{\tilde{\psi}(x), \tilde{\pi}_\psi(x')\} = \{\psi(x), \pi_\psi(x')\}$. [Recall that $\pi_\psi(x) = i\psi^\dagger(x)$.]

Considering an antipodal map for spin-2 fields, the metric, in particular, is more interesting.

It has been suggested in a number of contexts [20] that infrared modifications to gravity may play an important role in elucidating the physics of the vacuum, namely, in addressing the cosmological constant problems. Can we utilize the formalism we have developed to address matters of gravity?

We define the conjugate momentum to the metric as $\pi_g^{\mu\nu}(t, x) = \frac{-i}{\sqrt{-g(t,x)}}\frac{\delta}{\delta g_{\mu\nu}(t,x)}$. Note that the canonical equal-time commutator is

$$[g_{\mu\nu}(t, x), \pi_g^{\rho\sigma}(t, x')] = i\delta_\mu^\rho \delta_\nu^\sigma \frac{\delta^{(d-1)}(x - x')}{\sqrt{-g(t, x')}}. \tag{85}$$

We take

$$\hat{S}_\alpha = \lambda \int d^{d-1}x\sqrt{\gamma(x)} \int d^{d-1}y\sqrt{\gamma(y)}\alpha(x - y)$$

$$\times g_{\mu\nu}(t = 0, x_A)\pi_g^{\mu\nu}(t = 0, y). \tag{86}$$

The integrals are defined on the $\Sigma_{t=0}$ surface as before. As the de Sitter line element in global coordinates is

$$ds^2 = g_{\mu\nu}(t, x)dx^\mu dx^\nu = -dt^2 + H^{-2}(\cosh^2 Ht)d\Omega_{d-1}^2, \tag{87}$$

$\sqrt{-g(t, x)}|_{t=0} = \sqrt{\gamma(x)}$. (We have chosen a gauge with lapse set to unity and shift set to zero.) However, the formulas we obtain for

$$\tilde{g}_{\mu\nu}(x) = e^{i\hat{S}_\alpha}g_{\mu\nu}(x)e^{-i\hat{S}_\alpha}, \tag{88}$$

$$\tilde{\pi}_g^{\mu\nu}(x) = e^{i\hat{S}_\alpha}\pi_g^{\mu\nu}(x)e^{-i\hat{S}_\alpha} \tag{89}$$

are complicated by the action of $\delta/\delta g_{\mu\nu}$ on the various determinant factors.

Conjugation by the antipodal map gives

$$\tilde{g}_{\mu\nu}(0, x) = g_{\mu\nu}(0, x) + [i\hat{S}_\alpha, g_{\mu\nu}(0, x)]$$

$$+ \frac{1}{2}\{i\hat{S}_\alpha, [i\hat{S}_\alpha, g_{\mu\nu}(0, x)]\} + \cdots \tag{90}$$

$$= g_{\mu\nu}(0, x) + \lambda \int d^{d-1}y\sqrt{\gamma(y)}\alpha(y - x)g_{\mu\nu}(0, y_A)$$

$$+ \frac{1}{2}\lambda^2 \int d^{d-1}y\sqrt{\gamma(y)} \int d^{d-1}z\sqrt{\gamma(z)}\alpha(y - x)$$

$$\times \alpha(z - y_A)g_{\mu\nu}(0, z_A) + \frac{1}{4}\lambda^2 \int d^{d-1}y\sqrt{\gamma(y)}$$

$$\times \int d^{d-1}z\sqrt{\gamma(z)}\alpha(y - x)\alpha(z - y)g_{\mu\nu}(0, y_A)$$

$$\times g^{\rho\sigma}(0, y)g_{\rho\sigma}(0, z_A) + \mathcal{O}(\lambda^3). \tag{91}$$

Starting at $\mathcal{O}(\lambda^2)$, we find terms in $\tilde{g}_{\mu\nu}$ from the action of derivatives on the measure.

Let us work in the weak field approximation by treating $\lambda\alpha(k)$ as a small parameter. We take

$$\tilde{g}_{\mu\nu}(0, x) = g_{\mu\nu}(0, x) + h_{\mu\nu}(0, x),$$

$$h_{\mu\nu}(0, x) = [i\hat{S}_\alpha, g_{\mu\nu}(0, x)]. \tag{92}$$

Indices are lowered and raised using the background (inverse) metric $g_{\mu\nu}$ and $g^{\mu\nu}$. We define $h := g^{\mu\nu}h_{\mu\nu}$. In this linearized gravity approximation

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - h^{\mu\nu} + h^{\mu\rho}h_\rho^\nu + \cdots, \tag{93}$$

$$\sqrt{-\tilde{g}} = \sqrt{-g}(1 + \tfrac{1}{2}h + \tfrac{1}{8}h^2 - \tfrac{1}{4}h^{\mu\nu}h_{\mu\nu} + \cdots). \tag{94}$$

As $h_{\mu\nu}$ is itself proportional to λ, we shall drop quadratic and higher order terms in h.

At $\mathcal{O}(\lambda)$, we find that

$$\tilde{\pi}_g^{\mu\nu}(0, x) = \pi_g^{\mu\nu}(0, x) - \lambda\frac{\sqrt{\gamma(x_A)}}{\sqrt{\gamma(x)}} \int d^{d-1}y\sqrt{\gamma(y)}\alpha(x_A - y)\pi_g^{\mu\nu}(0, y) - \frac{1}{2}\lambda g^{\rho\sigma}(0, x) \int d^{d-1}y\sqrt{\gamma(y)}\alpha(y - x)$$

$$\times g_{\rho\sigma}(0, y_A)\pi_g^{\mu\nu}(0, x) - \frac{1}{2}\lambda g^{\mu\nu}(0, x)g_{\rho\sigma}(0, x_A) \int d^{d-1}y\sqrt{\gamma(y)}\alpha(x - y)\pi_g^{\rho\sigma}(0, y) + \mathcal{O}(\lambda^2). \tag{95}$$

A careful computation then gives

$$[\tilde{g}_{\mu\nu}(0, x), \tilde{\pi}_g^{\rho\sigma}(0, x')] = [g_{\mu\nu}(0, x), \pi_g^{\rho\sigma}(0, x')]$$

$$- \frac{i}{2}\delta_\mu^\rho \delta_\nu^\sigma \frac{\delta^{(d-1)}(x - x')}{\sqrt{-g(0, x')}}g^{\lambda\tau}(0, x')$$

$$\times [i\hat{S}_\alpha, g_{\lambda\tau}(0, x')] + \cdots \tag{96}$$

$$= i\delta_\mu^\rho \delta_\nu^\sigma \frac{\delta^{(d-1)}(x - x')}{\sqrt{-g(0, x')}}\left(1 - \frac{1}{2}h(0, x') + \cdots\right) \tag{97}$$

$$= i\delta_\mu^\rho \delta_\nu^\sigma \frac{\delta^{(d-1)}(x - x')}{\sqrt{-\tilde{g}(0, x')}}. \tag{98}$$

This is of course the expected result if we act by conjuga-

tion on the right-hand side of Eq. (85), but it is not a canonical transformation in that commutation relation for the metric and its conjugate momentum is not preserved.[14]

Working with the new metric $\tilde{g}_{\mu\nu}$, we can compute the change in the curvature:

$$\tilde{R}_{\mu\nu}[g, h] = R_{\mu\nu}[g] + \delta R_{\mu\nu}[g, h], \quad (99)$$

$$\delta R_{\mu\nu}[g, h] = \tfrac{1}{2}[\nabla_\lambda \nabla_\mu h_\nu^\lambda + \nabla_\lambda \nabla_\nu h_\mu^\lambda - \nabla_\lambda \nabla^\lambda h_{\mu\nu} - \nabla_\mu \nabla_\nu h], \quad (100)$$

where ∇_μ is the connection with respect to the fixed background metric $g_{\mu\nu}$. Since ∇_μ is a covariant derivative with respect to x, it acts on the factors of α in the integral expression for $h_{\mu\nu}$. The Einstein equation gives

$$\delta G_{\mu\nu} := \delta R_{\mu\nu} - \tfrac{1}{2}R h_{\mu\nu} - \tfrac{1}{2}\delta R g_{\mu\nu} + \Lambda h_{\mu\nu}$$
$$= 8\pi G_N T_{\mu\nu}. \quad (101)$$

Writing this explicitly in terms of α is not especially illuminating. We observe though that the effect of conjugation by the antipodal map can be absorbed into a modification of the stress-energy. Once we choose an α-state, the gravitational interaction—i.e., the way geometry couples to matter—involves the Hubble scale. This is an infrared effect in gravitational physics that can be treated perturbatively in the α-expansion of the antipodal map. We do not know, even in perturbation theory, how α-states in gravity generically decay. The only intuition we have for thermodynamics arises from local field theory, near to equilibrium. In an inherently nonlocal setting, it is not known how to confront the dynamical mixing of infrared and ultraviolet scales. Nevertheless, perhaps a detailed analysis of the perturbation theory will reveal interesting phenomenology. A second phenomenological manifesta-

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[14]To make a canonical transformation so that

$$[\tilde{g}_{\mu\nu}(0, x), \tilde{\pi}_g^{\rho\sigma}(0, x')] = [g_{\mu\nu}(0, x), \pi_g^{\rho\sigma}(0, x')]$$
$$= i\delta_\mu^\rho \delta_\nu^\sigma \delta^{(d-1)}(x - x'),$$

one can instead work with the momentum density, $\pi_g^{\mu\nu}(t, x) = -i\frac{\delta}{\delta g_{\mu\nu}(t,x)}$. We have

$$\tilde{g}_{\mu\nu}(0, x) = g_{\mu\nu}(0, x) + h_{\mu\nu}(0, x) + \cdots,$$

$$h_{\mu\nu}(0, x) = [i\hat{S}_\alpha, g_{\mu\nu}(0, x)]$$
$$= \lambda \int d^{d-1}y \sqrt{\gamma(y)} \alpha(y - x) g_{\mu\nu}(0, y_A),$$

$$\tilde{\pi}_g^{\mu\nu}(0, x) = \pi_g^{\mu\nu}(0, x) - \lambda\sqrt{\gamma(x_A)} \int d^{d-1}y \sqrt{\gamma(y)}$$
$$\times \alpha(x_A - y) \pi_g^{\mu\nu}(0, y) - \tfrac{1}{2}\lambda\sqrt{\gamma(x)} g^{\mu\nu}(0, x)$$
$$\times \int d^{d-1}y \sqrt{\gamma(y)} [\alpha(y - x) g_{\rho\sigma}(0, y_A) \pi_g^{\rho\sigma}(0, x)$$
$$+ \alpha(x - y) g_{\rho\sigma}(0, x_A) \pi_g^{\rho\sigma}(0, y)] + \cdots.$$

tion of α in the gravity sector may appear in deviations from the standard power spectrum for tensor fluctuations.

VII. THE ELEGANT DE SITTER UNIVERSE

The low energy effective dynamics of gravitation as a classical field theory and the physics of the Standard Model as a quantum field theory that captures a specific set of gauge principles are expected to descend from a consistent theory of quantum gravity. String theory is a very promising candidate theory of quantum gravity and matter. Among the many outstanding problems of string theory is the absence of a vacuum selection principle that marries with empirical data about the Universe. Our intuition for string theory/quantum gravity, and indeed our calculational technology, is essentially rooted in field theory. Perturbative string theory tells us how to compute S-matrix observables (scattering amplitudes) in ten-dimensional flat spacetime. The notion of well-defined particle excitations at asymptopia is, however, an artifact of the flat space limit. Though the classic techniques of field theory are the tools we rely upon to make sense of the initial singularity and the emergence of space, time, and matter, it is not clear that the tools we have are appropriate to answer the questions we ask. In de Sitter space there is no S-matrix. Different local observers have different horizons and make different measurements.

As well, the requirements of spacetime supersymmetry are incompatible with the symmetries of de Sitter space. There is no positive conserved energy because there is no global timelike Killing isometry. To date, there has been no construction of de Sitter space as a consistent background geometry for superstrings [51]. To realize de Sitter space in string theory, one may consider, for instance, Type IIB compactifications with a suitable choice of background flux [52]. Such de Sitter vacua are at best metastable and finely tuned. The true vacuum is ten-dimensional flat space.

Quantum field theory in de Sitter space, on the other hand, does make sense at least as an algebraic theory [53] and provides a framework to address a number of the issues that persist in quantum gravity. As de Sitter space admits an infinite family of vacua, we need a vacuum selection principle. One particular vacuum of this family is uniquely consistent with the Minkowski space limit of the de Sitter geometry. We have seen that squeezed states in the Fock space constructed from this unique vacuum share many of the features of the other de Sitter α-vacua. The functional formalism provides a crisp technology for addressing the implications of initial conditions with the long-range correlations native to these α-states. The effects of backreaction on α may modify some of this physics.

Initial conditions for inflation are generic in the sense that many sets of inputs lead to the same large scale structures today in causally disconnected regions of spacetime. Nevertheless some details about the initial conditions

can in principle be imprinted in the spectrum of the CMBR. If the Universe were to have begun in an α-state, the power spectrum for the inflaton may exhibit a novel dependence on momentum.

More radically, we can imagine that the α parameter that determines the vacuum and the Hilbert space is a fully dynamical superselection parameter, just as the θ parameter of QCD becomes the fully dynamical axion. In such a setting, what we mean by vacuum dynamically rolls toward the Euclidean value as the Universe inflates and drags the vacua of the other fields along with it. There are obvious phenomenological questions to address in this scenario. For example, is it consistent with the standard pictures of inflation, reheating, and big bang nucleosynthesis?

Recently it has been suggested that dynamics selects a corner of the moduli space of supersymmetric vacua of string theory (the so-called *landscape of vacua*) consistent with measurements of (some set of) physical parameters in our Universe [54]. The cosmological constant, for example, is its measured value in the corner of moduli space where our Universe resides. Superselection parameters, like θ of QCD or α for fields in de Sitter space (it is worth repeating here that each of the Standard Model degrees of freedom will have an associated α), will vary throughout the landscape just as the cosmological constant Λ does. The requirements of large scale structure in the Universe, inflation, and compatibility with big bang nucleosynthesis do not place restrictions on the numerical values of θ or α. Rather, like the existence of a third generation of Standard Model matter fields, the details of the inflationary power spectrum that is shaped by the α-state is an additional free parameter in the description of the landscape. It seems that this condition is generic for superselection parameters in quantum gravity. Perhaps this places additional limits on the predictivity of a theory of physics that includes anthropics as a defining principle.

It has been argued in Ref. [30] that the one-parameter family of de Sitter invariant α-vacua can be understood as a one-parameter family of marginal deformations of a possible CFT dual to the bulk de Sitter space. This interpretation is quite natural in view of the fact that the α parameter appears as a superselection parameter in the general de Sitter invariant wave functional. Alternatively,

it was noticed that entangled states preserving $SO(1, d)$ invariance in the product of two CFTs of the type discussed in Ref. [28] closely resemble boundary states for a free scalar field, for which a one-parameter ambiguity has been known in the literature. It was thus suggested that this translates into a one-parameter ambiguity of the de Sitter invariant vacua. It would be interesting to understand α-states in de Sitter space from the point of view of the conjectured de Sitter/CFT correspondence. The thermofield picture of evolution off the $\Sigma_{t=0}$ hypersurface on which the antipodal transformation is made may improve our knowledge about how boundary data on $I^{\pm}$ become entangled.

Finally, we have seen that the formalism for scalar fields extends to metric fluctuations. Hubble scale correlations in the geometry lead to infrared modifications of the gravitational interaction. This is a rich and promising direction with phenomenology to explore in greater depth.

We hope that continued analysis of physics in de Sitter space will shed new light on outstanding issues in cosmology and quantum field theory. Perhaps we are fortunate to live in an elegant de Sitter Universe.

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