APPLICATION OF STOCHASTIC CHOICE MODELING TO POLICY ANALYSIS OF PUBLIC GOODS: A CASE STUDY OF AIR QUALITY IMPROVEMENTS

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1. Introduction

ONE of the outstanding issues of applied welfare economics is how to obtain appropriate welfare measures for policy analysis of changes in public goods when market demand information is not available. Surveys and elections provide alternative forums for revealing preference information. Recently, Randall and Stoll (1980) discussed the use of willingness to pay as a welfare measure and Brookshire, Randall, and Stoll (1980) demonstrated use of a survey to obtain willingness to pay information. Here, we also measure willingness to pay for public goods using survey data but we use an alternative questionnaire design and different estimation methods.

To analyze the survey data obtained, we use a stochastic model of choice. The basis for stochastic choice models is that there is randomness in observed choice behavior. Because of uncontrollable factors in survey execution and unobservable characteristics of survey respondents, the stochastic model is particularly appropriate for analysis of choices obtained from a survey. By estimating parameters in a distribution function representing probability of choice, the stochastic model can be used to explain and predict choice probabilities for individuals in a population.

Choice data obtained from our survey are “paired comparisons” of changes in a public good with changes in income. Here, we use the stochastic model to predict the probability of a person being willing to pay an amount of money rather than experience a decrease in a public good. A “representative” bid curve is then obtained by defining indifference (similar to Davis, DeGroot, and Hinich, 1972) to be a probability of one-half.

In comparison, the method used by Brookshire, Randall, and Stoll (1980) requires that a respondent identify an exact maximum willingness to pay bid; because of the bidding procedure, use of a trained interviewer is needed. Because of our simpler questionnaire design, our type of survey can be distributed by mail rather than administered by interviewers. A mail survey is cheaper to administer and eliminates variation in responses due to influence of interviewers on respondents. However, the method is still subject to other types of “biases” discussed by Brookshire et al. (1979), such as the free rider problem.

In addition to differences in surveying techniques, the comparison between the resulting willingness to pay function and that obtained by methods used by Brookshire, Randall, and Stoll (1980) has to do with estimation properties. In one case, an arbitrary functional form is hypothesized for a bid curve and individual bids are assumed to be normally distributed about a mean bid. In the other case, an arbitrary form is specified for a distribution function relating to choice behavior.

To demonstrate use of the stochastic model to estimate willingness to pay from survey data, we present an analysis of air pollution control for the Tampa Bay area of Florida. The succeeding sections describe the policy problem, survey design, choice model, and analysis.
II. Policy Problem and Survey Design

Two policy alternatives concerning the public good of producing air quality were evaluated. One potential policy evaluated was to control power plant emissions in the entire Tampa Bay area of Florida ("control all") at a cost of $98 million and a reduction of power plant emission of 83%. An alternative policy was to reduce emissions from plants in the urban areas only ("control urban") at a cost of $23 million and a reduction in total emissions of 39%.

To evaluate these policies, we assume that willingness to pay to avoid air pollution may be associated with willingness to pay to avoid health effects (e.g., asthma, bronchitis, emphysema, heart and lung problems) related to pollution. Since respondents might not have been familiar with these diseases, they were asked to consider more familiar characteristics of the diseases (e.g., coughing, sneezing, shortness of breath). The health effects on the questionnaire are described in terms of types of symptoms, duration of symptoms (1, 7, 90 days), and severity (mild or severe). "Severe" refers to restricted activity with possible bed confinement.

The survey was designed to provide observations of individual choices between decreased income and increased health effects related to pollution. Odor and haze were also included as examples of aesthetic and psychological effects.

The survey is similar to others of the willingness to pay type (e.g., Brookshire et al., 1980). However, to reduce the cost of obtaining variation in income and health status among respondents, the survey was distributed by mail. Therefore, rather than being able to determine the exact bid of willingness to pay for each respondent, a more standardized approach had to be used. Instead of giving an exact value, respondents chose the highest value they were willing to pay from a standardized list. For each health effect, a range of dollar amounts from $0 to $1000 is used to provide enough "spread" for low and high income respondents and diversity in seriousness of health effects. A typical choice question on the questionnaire is of the form:

To avoid one day per year minor head congestion, the most I would pay is:

$0 $0.50 $1 $2 $10 $15 $50 $120 $250 $1000 per year.

The respondent circled the highest value that he/she was willing to pay.

For each given dollar amount, the response to the question is similar to a "paired comparison" between less income and worse health. The paired comparison method has been utilized in marketing and psychological studies and is considered to be a reliable method of obtaining responses since the choices required are relatively simple.²

Since health status and household income are hypothesized to affect choice behavior, questions about these were included on the questionnaire. Questions about other factors which may affect choice, such as age and insurance, were also included.

The questionnaire was mailed in May 1977 to 1,800 people selected randomly in the Tampa Bay area; the return response was 404. The return responses were tested and found to be representative of the Tampa Bay population but included a slightly higher proportion of higher income persons.

III. Stochastic Choice Model and Willingness to Pay

On the survey, the respondent is asked to compare a situation with worse health to one with less income and choose the lesser of the two "evils." This choice may be modeled as follows. Define the indirect utility function as

\[ U(M - E, P_x, D) = \max \ U(x, D) \]

subject to \( P_xx = M \)

where \( x \) are private goods at a cost \( P_x \), \( M \) is income of the consumer, and \( D \) is the initial health status. Let \( m \) denote the decrease in income and \( d \) the increase in days of illness. The respondent makes a choice corresponding to the larger utility level. The maximum willingness to pay is the amount \( E \) such that

\[ U(M - E, P_x, D) = U(M, P_x, D + d). \] ²

\( E \) is the "equivalent variation" for a given

² There is good evidence from previous research that responses to questionnaires of this type are reasonably valid; see Stouffer et al. (1950) and Shaw and Wright (1967). The questionnaire was distributed to a sample of 47 persons drawn randomly from a college population and readministered to the same persons after three weeks. Test-retest correlations ranged from 0.82 to 0.95, averaging 0.86.
change in public good. The respondent is better off choosing the public good decrease for any payment larger than \( E \). As \( d \) is varied, a bid curve (Bradford, 1970) is obtained; the exact position of this bid curve depends on income, health status, and other socioeconomic factors.

We use a zero–one variable to denote choice between increased illness and decreased income and assign

\[
z = 1 \text{ if } E \geq m \\
z = 0 \text{ if } E < m.
\]

(3)

If a person with socioeconomic characteristics \( S \) is selected randomly from a population, the probability that the person will choose to pay \( m \) rather than have health effect \( d \) is denoted by \( P(m, d, S) \):

\[
P(m, d, S) = \text{prob}(z = 1|m, d, S).
\]

(4)

Following McFadden (1973, 1976) we hypothesize that choices observed on the survey can be explained using a “representative” utility function; for individuals sampled from a population, choices are represented by utility functions which are distributed about a representative utility function for the population. Here, we use the indirect utility function to represent choices between income and health; from the theory, choices will be affected by income and health status. Thus, if \( \hat{U} \) denotes the individual’s indirect utility function and \( V \) the “representative” indirect utility,

\[
\hat{U}(M, D) = V(M, D) + \epsilon(M, D)
\]

(5)

where \( \epsilon(M, D) \) denotes the stochastic effect. McFadden (1973) showed that under a set of axioms, the choice probabilities may be defined in a particular way in terms of values of the representative utility for each alternative. In particular, for two alternatives

\[
P(m, d, S) = \text{prob}(V(M - m, D) - V(M, D + d) > \epsilon(M, D + d) - \epsilon(M - m, D)) = \frac{\exp[V(M - m, D)]}{\exp[V(M - m, D)] + \exp[V(M, D + d)]}.
\]

(6)

The distribution function is in fact the logistic distribution function

\[
F(v) = \frac{1}{1 + \exp(-v)}
\]

(7)
evaluated at

\[
v = V(M - m, D) - (VM, D + d).
\]

(8)

The logistic model has been validated as an appropriate choice model for binary choices by Cox (1970). The “paired comparison” method used here is a case of binary choices.

Following Davis, DeGroot, and Hinich (1972), we define a preference relation between two alternatives \( A \) and \( B \) as

\[
A \sim B \Leftrightarrow \text{Prob}(U_A > U_B) > 1/2.
\]

(9)

Thus, a proposal will be preferred to an alternative if and only if the value of the representative utility is higher. Indifference (lack of preference) between two alternatives then corresponds to a choice probability equal to one-half. Similar to willingness to pay for the individual, “representative” willingness to pay is defined from an indifference relation using the representative utility function.4

IV. Estimation

In order to predict the probability of choice and representative bid curve, a functional form must be hypothesized for the utility difference. From the choice theory, relevant variables are the money payments \( m \), health effects \( D \), income \( M \), health status \( d \), and other socioeconomic characteristics. Rather than specifying a linear utility function as McFadden (1976) did, following Theil (1969) we specify a nonlinear functional form for the utility difference. The specification of the utility difference used here is

\[
v = \alpha_0 + \alpha_1 \ln m + \alpha_2 \ln d + \alpha_3 \ln M + \alpha_4 \ln D + \alpha_5 S
\]

(10)

where \( S \) denotes socioeconomic characteristics other than income and health. Taking the logit transformation and substituting (10) provides the model for estimation:

4 By specifying a functional form for the utility function, it is theoretically possible to specify the stochastic relationship between the representative bid and individual bids. For example, for a constant marginal utility of income, the distribution of the bids is also logistic and the representative bid is the mean of the individual bids.
\[
\ln \frac{p}{1-p} = (\alpha_0 + \alpha_1 \ln m + \alpha_2 \ln d + \alpha_3 \ln M + \alpha_4 \ln D + \alpha_5 S). \quad (11)
\]

Hypothesized signs for the parameters may be derived from theory assuming that the representative utility function has properties

\[V_d < 0, \quad V_m > 0, \quad V_{mm} \leq 0, \quad V_{dd} \leq 0, \quad V_{md} \leq 0\]
similar to those of an individual utility function (here \(D\) is measured in days of illness). Then, using the definition of the distribution function, we obtain that the probability of preferring a payment \((m_i)\) to avoid an increase in days of illness \((d)\) will increase with income, increase with days of illness, decrease with the amount of payment, and increase with worse initial health status.

The representative bid curve \(E\) is obtained from setting the choice probability equal to one-half:

\[E = kd^\delta_2 M^\delta_3 D^\delta_4 \quad (12)\]

where \(\delta_i = - (\alpha_i/\alpha_1) > 0\). The choice probabilities may be represented in terms of the representative bid as

\[P(m, d, S) = 1/(1 + (m/E)^{-\alpha_1}). \quad (13)\]

Thus if the representative bid exceeds the required payment the choice probability will be greater than one-half.

A separate choice probability estimation combining data on number of days and dollar bids was made for each symptom and severity combination. Individual data and maximum likelihood estimation could have been used but there would have been about twelve thousand observations for each estimation. To reduce the number of data inputs, respondents were grouped; income and health status were used to define groups since these factors were hypothesized to determine choice behavior. Table 2 shows income and health categories used to define groups. Proportions \(\pi\) of persons in each group willing to pay each amount listed on the questionnaire were used to estimate choice probabilities. Proportions were obtained by cumulating responses for each dollar amount listed on the questionnaire; that is, a person willing to pay at most ten dollars to avoid illness would also be willing to pay less than ten dollars to avoid the same illness.

Log \(\pi/(1 - \pi)\) is approximately normally distributed (Cox, 1970); thus the method of ordinary least squares (OLS) can be applied to estimate parameters. However, group size and standard deviations of bids vary by group. Therefore, we used generalized least squares (GLS) to estimate parameters; the OLS estimates give initial parameter estimates for the GLS procedure. The generalized least squares method used followed the method of Walker and Duncan (1967). GLS estimates obtained are very similar to the initial OLS estimates.

Table 1 gives the results of OLS estimation used here for hypothesis testing. In each equation the coefficients of \(m, d, M\) and \(D\) all have the hypothesized sign. The \(t\)-statistics for \(m, d, M\) and \(D\) are all significant beyond the 99% level. The coefficients for health status \(D\) are significant above the 95% level only for severe shortness of breath.

Other socioeconomic variables included in the analysis were “percentage employed,” “percentage retired” and “percentage insured.” These variables affect demand because of their effect on the real price of illness. The sign of the employment variable indicates that as “percentage employed” increases, there is a larger probability of willingness to pay a given amount; this variable is significant at the 99% level for all effects, indicating the larger opportunity cost of illness for the employed population. The sign of the “percentage retired” variable indicates that retired persons are willing to pay more than unemployed persons with the same income, indicating a wealth effect. The insurance variable was most significant in the case of severe illness; as the percentage with insurance increases, the willingness to pay decreases since insurance is a “substitute” for preventative health activities. The “percentage female” dummy variable was not significant except in the case of odor and haze. “Age” was not used as a socioeconomic variable because it is correlated

Use of OLS after correcting for differences in group variances and group sizes has been termed the minimum chi squared approach by Berkson (1955). Amemiya (1974) has shown the equivalence of these approaches and maximum likelihood.
TABLE I.—OLS REGRESSION RESULTS—DEPENDENT VARIABLE IS LOG OF THE "ODDS"

<table>
<thead>
<tr>
<th>Symptom Coefficient</th>
<th>ln m</th>
<th>ln d</th>
<th>ln M</th>
<th>ln D</th>
<th>% Retired</th>
<th>% Female</th>
<th>% Insured</th>
<th>% Employed</th>
<th>C</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSB</td>
<td>-0.7106</td>
<td>0.2809</td>
<td>0.4301</td>
<td>0.0464</td>
<td>2.9435</td>
<td>-0.1397</td>
<td>-1.9471</td>
<td>2.1251</td>
<td>-3.1291</td>
<td>.874</td>
</tr>
<tr>
<td>MHC</td>
<td>-0.8654</td>
<td>0.2805</td>
<td>0.3087</td>
<td>0.0444</td>
<td>1.1857</td>
<td>0.3031</td>
<td>-0.6987</td>
<td>0.9904</td>
<td>-2.3686</td>
<td>.878</td>
</tr>
<tr>
<td>SHC</td>
<td>-0.7976</td>
<td>0.2496</td>
<td>0.2032</td>
<td>0.0464</td>
<td>2.0208</td>
<td>-0.0278</td>
<td>-1.3534</td>
<td>1.8090</td>
<td>-1.0966</td>
<td>.879</td>
</tr>
<tr>
<td>MCS</td>
<td>-0.8761</td>
<td>0.3274</td>
<td>0.2345</td>
<td>0.0529</td>
<td>1.6784</td>
<td>0.0511</td>
<td>-0.4891</td>
<td>1.2993</td>
<td>-2.2366</td>
<td>.869</td>
</tr>
<tr>
<td>O</td>
<td>-0.7891</td>
<td>0.2742</td>
<td>0.3169</td>
<td>-0.0105</td>
<td>2.0662</td>
<td>1.0585</td>
<td>-0.4191</td>
<td>1.2993</td>
<td>-3.5108</td>
<td>.848</td>
</tr>
<tr>
<td>MSB</td>
<td>-0.8065</td>
<td>0.2599</td>
<td>0.3010</td>
<td>0.0118</td>
<td>2.9355</td>
<td>0.0789</td>
<td>-1.1225</td>
<td>2.0280</td>
<td>-2.6799</td>
<td>.877</td>
</tr>
<tr>
<td>H</td>
<td>-0.7637</td>
<td>0.2872</td>
<td>0.3287</td>
<td>0.0318</td>
<td>2.2899</td>
<td>0.9006</td>
<td>-0.3322</td>
<td>1.3728</td>
<td>-3.8998</td>
<td>.835</td>
</tr>
<tr>
<td>SCS</td>
<td>-0.7914</td>
<td>0.2757</td>
<td>0.3899</td>
<td>0.0346</td>
<td>2.6803</td>
<td>0.3048</td>
<td>-1.4973</td>
<td>1.7944</td>
<td>-3.0782</td>
<td>.872</td>
</tr>
</tbody>
</table>

Note: SSB = severe shortness of breath  
MSB = minor shortness of breath  
SHC = severe head congestion  
MHC = minor head congestion  
m = amount of money to be paid  
d = increase in days of illness  
M = income  
C = total willingness to pay

SSC = severe coughing and sneezing/eye irritation  
MCS = minor coughing and sneezing/eye irritation

"odds" = probability of a vote in favor of paying m to avoid d divided by one minus this probability

with variables already in the analysis (income, health, employment status).

V. Policy Analysis and Aggregation

The air pollution example here illustrates the use of a stochastic choice model for policy analysis and some of the problems encountered. Policy analysis requires prediction of preferences regarding a change in a public good for levels which could be different than those presented on a questionnaire. The methods described above can be used to predict choice probabilities and willingness to pay for any level of changes in a public good using estimated parameter values. The validity of the prediction, as with any economic analysis, depends on how far the change is outside the ranges on the questionnaire.

For the two air pollution control policies given above, expected health effects were computed using dose response relations. The bid for the expected health effect was used in the policy analysis; this corresponds to an assumption of risk neutrality. We did not obtain option values on the questionnaire (willingness to pay to avoid risk). If risk aversion holds, then the predicted bids are too low.⁶

To apply the bid curves obtained above for policy analysis, two aggregation problems are encountered. The first is a problem of aggregation over goods. Since there are several health effects related to air quality changes, an aggregate willingness to pay value is needed. In general, this will not be the same as the sum of willingness to pay for each effect separately. Also, as discussed by Chipman and Moore (1980) for price changes, different values may be obtained depending on the ordering of the effects considered. For the purpose of policy analysis, we ignore these problems and estimate the aggregate bid by adding the bids for each separate effect. These procedures correspond to an assumption of constant marginal utility of income and a util-

⁶ In order to evaluate pollution controls, both the relationship between emissions and ambient air quality levels and ambient levels and health effects are needed. Average ages for income and health groups, data by census tract on numbers of persons in each group and pollution levels by tract were also required. A more complete description of these details are given in Loehman et al. (1979).
ity function which is separable in the separate health effects.

Another aggregation issue has to do with ranking social policies by comparing costs of a policy with total benefits where total benefits are estimated as the sum (over all individuals) of willingness to pay. In order to estimate total willingness to pay for a reduction in pollution, here we multiply the representative group bids by the number of persons in each group and add over groups. It can be shown that using the sum of willingness to pay over individuals to rank social outcomes corresponds to use of a social welfare function with welfare weights inverse to the marginal utility of income (Loehman, 1978).

Table 2 shows the result of the policy analysis described above for the two control scenarios, "control all" and "control urban." For comparison to willingness to pay to avoid health effects, we have also given the estimated cost by income group (allocated in proportion to electricity use) required to implement the pollution control policies. Note that the "control all" option results in negative net benefits for all socioeconomic groups whereas the "control urban" policy results in positive net benefits for all but three groups.7

The model given here may also in theory be used to predict the probability of preference for a given policy as compared with another policy and the probabilities could then be used to rank policies. From (13), estimation of the preference probability requires the ratio of payment to willingness to pay raised to a power (α). We have not utilized this method here because willingness to pay for each separate health effect has a different coefficient associated with it. In addition to this aggregation problem, there would also be a problem of transitivity of probabilistic preferences when more than two choices are being compared, similar to that discussed in voting literature.

VI. Conclusion

This paper demonstrates the applicability of the stochastic choice method for the purpose of

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### Table 2.—Policy Analysis of Two Pollution Control Policies by Socioeconomic Groups

<table>
<thead>
<tr>
<th>Socioeconomic Group</th>
<th>Willingness to Pay Expected Health Effects</th>
<th>Cost of Control ($ per person)</th>
<th>Willingness to Pay Expected Health Effects</th>
<th>Cost of Control ($ per person)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Severe Cough</td>
<td>Severe Shortness of Breath</td>
<td>Minor Eye Irritation</td>
<td>Total</td>
</tr>
<tr>
<td></td>
<td>1 1</td>
<td>1.97</td>
<td>5.11</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>2 1</td>
<td>1.79</td>
<td>4.85</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>3 1</td>
<td>2.13</td>
<td>7.00</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>4 1</td>
<td>2.92</td>
<td>8.30</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>1 2</td>
<td>1.66</td>
<td>4.76</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>2 2</td>
<td>1.39</td>
<td>4.88</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>3 2</td>
<td>2.36</td>
<td>7.32</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>4 2</td>
<td>2.81</td>
<td>9.11</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>1 3</td>
<td>2.48</td>
<td>8.00</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>2 3</td>
<td>2.61</td>
<td>8.54</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>3 3</td>
<td>3.02</td>
<td>10.03</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>4 3</td>
<td>3.58</td>
<td>12.37</td>
<td>1.49</td>
</tr>
</tbody>
</table>
| Total               | $6,169,390   | $5,189,870                | Total $6,450,290         | $21,759,340

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7 There are health and aesthetic benefits associated with these policies which are not included in the analysis. Thus, the numbers given here do not provide a complete benefit-cost analysis.
performing policy analysis of changes in public goods using survey data. Although air quality control policies have been analyzed here, the methods used are also applicable to other public good valuation problems given the appropriate model of choice and choice data.

The policy problem studied here involved relatively simple choices between changes in a public good and disposable income; more complex tradeoff situations with tradeoffs among several public goods could also be analyzed with a properly designed survey.

A survey can provide a public forum in which preferences about public goods are revealed. Though not widely used by economists as a source of information, surveys may be considered as a potential source of choice and preference information when market data or election data are not available. To implement the method given here for analysis of public good issues, economists will need to work closely with psychometricians in order to develop appropriate survey instruments. On the other hand, as demonstrated by this paper, survey instruments need to be designed according to appropriate economic models of individual choice if measurement of economic concepts is a concern.

REFERENCES


