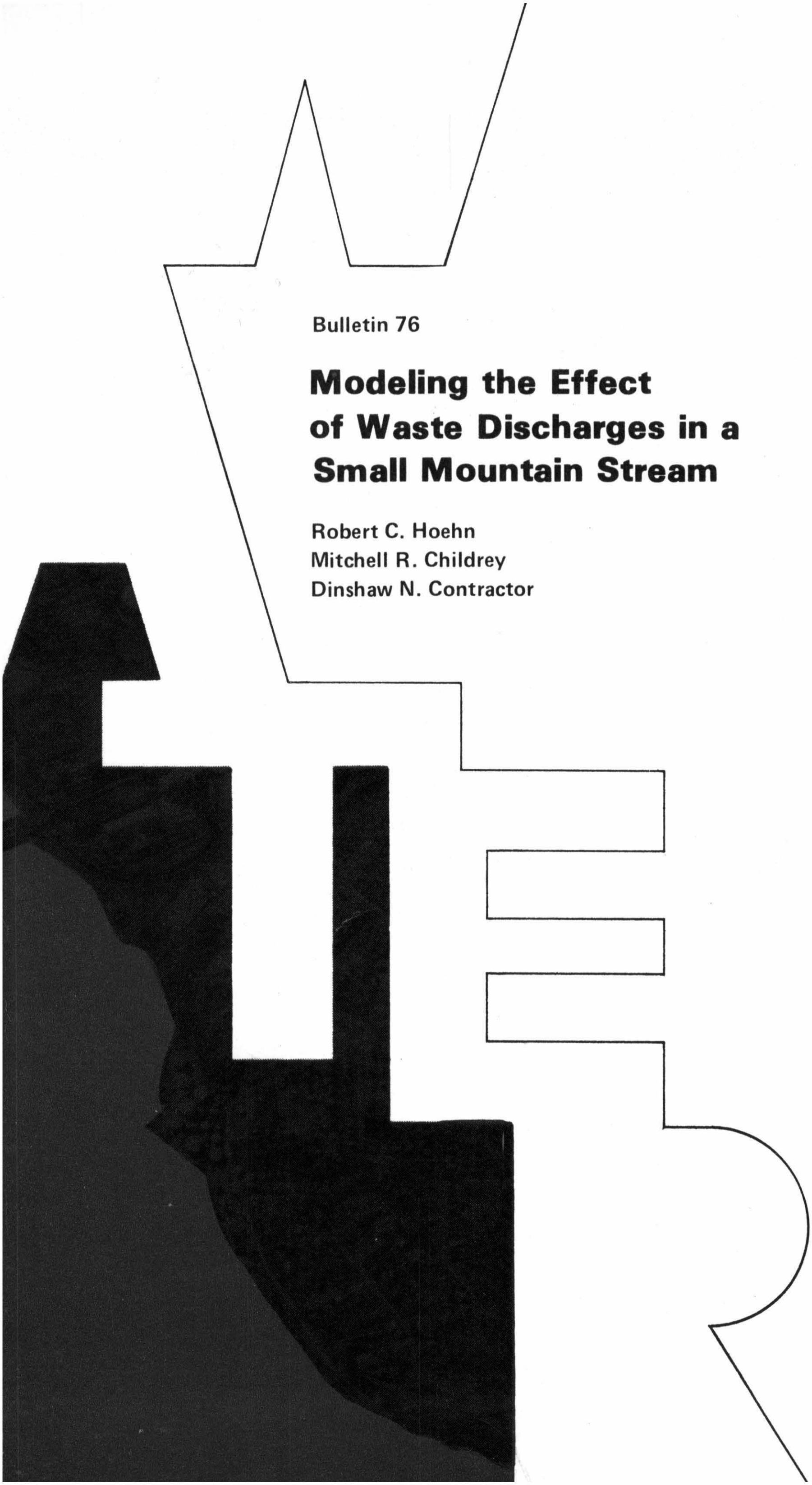
Bulletin 76

Modeling the Effect of Waste Discharges in a Small Mountain Stream

Robert C. Hoehn
Mitchell R. Childrey
Dinshaw N. Contractor



Bulletin 76
September 1975

Modeling the Effect of Waste Discharges in a Small Mountain Stream

Robert C. Hoehn,

Mitchell R. Childrey,

and

Dinshaw N. Contractor

Department of Civil Engineering
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

This research was supported in part by a Public Health Traineeship awarded to Virginia Polytechnic Institute and State University. Funds for this publication were provided by the Virginia Water Resources Research Center and the College of Engineering, Virginia Polytechnic Institute and State University.

VPI-VWRRC-BULL 76

A publication of
Virginia Water Resources Research Center
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

PREFACE

This research was conducted as part of the graduate study program of Mitchell R. Childrey, who completed his studies for the Master of Science in Sanitary Engineering at Virginia Polytechnic Institute and State University in June, 1974. The study was undertaken to provide practical information for those who may apply computer modeling of dissolved oxygen concentration to small streams. The study was particularly applicable to small streams in Virginia because so many of them receive treated sewage effluents.

Partial support for this study was provided by a Public Health Traineeship from the United States Department of Health, Education and Welfare. We gratefully acknowledge the assistance of Mr. Thomas Henry, of the Virginia State Water Control Board's Bureau of Water Control Management, who provided us with the State's DO—BOD computer model and technical assistance.

Special acknowledgement is accorded the following who generously gave their time to a critical review of the manuscript: Dr. Jonathan Young, Hydroscience, Westwood, New Jersey; and Mr. Tim Tischler, of the Texas Water Quality Board, Austin, Texas. Acknowledgement is also made to Katherine Miller Kurtz and Charlotte Riddle, who did the typesetting, and to Larry Wallace, who did the art work.

TABLE OF CONTENTS

Appendices

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

This research evaluated the applicability of a particular computer model, designed to predict dissolved oxygen (DO) and biochemical oxygen demand (BOD) concentrations in rivers, to similar studies of a small mountain stream in southwestern Virginia. The river model was developed by the Virginia State Water Control Board's Bureau of Water Control Management. This report presents a detailed explanation of the necessary computations involved, the methods used to obtain all necessary input data, and modifications that were made in the original model to adapt it for use with a small stream. Also included is a discussion of a modified sensitivity analysis of the model performed by the investigators to identify which input parameters, if varied slightly, would affect the model's DO and BOD predictions. Finally, data are presented showing that the modified version of the model is capable of predicting measured DO and BOD concentrations in the stream.

INTRODUCTION

A recent report (*Ref. 1*), submitted to the Environmental Studies Board of the National Academy of Sciences and the National Academy of Engineering, identifies one of the research needs in developing water quality criteria as being the establishment of "...methodologies and guidelines for determining the assimilative capacities of various kinds of receiving systems...." The research detailed in this Bulletin was directed toward fulfilling that need in regions where significant quantities of treated municipal wastes are discharged to small streams and where regional planners must make judgments regarding the extent of growth to be allowed in future years.

In large river basins, one assessment of the impact of further growth often is accomplished by the application of mathematical models that predict the effects of organic loads introduced at various points along a river receiving discharges from towns and industries. The more popular models are variations of the one developed by Streeter and Phelps (*Ref. 2*) for the Ohio River, which predicts the longitudinal dissolved oxygen (DO) profile resulting from the simultaneous exertion of biochemical oxygen demand (BOD) and the reaeration of the stream from the atmosphere. These models have been used successfully in large river basins where considerable hydrologic and water-quality data are available, but they are of little value in most small watersheds simply because of the lack of necessary data. However, even if the data were available, the models generally have a "black box" quality about them as far as most planners are concerned, and, as a result, few attempts have been made to use mathematical models in small watersheds as part of the analysis of the impact of predicted growth.

The specific objective of the research reported here was to determine whether a predictive model for DO and BOD, developed by the State of Virginia Water Control Board's Bureau of Water Control Management (BWCM), could be applied to a small mountain stream that received effluents of treated sewage. While the model has been applied to several large rivers in Virginia, this is the first reported intensive attempt of its verification on a small creek in which field data were taken to determine reaction rates and other significant variables. The results of this study identify the input data necessary for executing the model, and the methods of obtaining them. The model is discussed in detail in this Bulletin, and those input variables that have the greater influence on predicted DO concentrations have been identified by a sensitivity analysis and are also discussed. This report, therefore, is a practical guide for those who wish to consider using a more scientific approach than they have previously taken to studies of the assimilative capacity of small creeks.

THE STUDY AREA

Stroubles Creek is located in Montgomery County in the western mountain region of Virginia. It originates in the Town of Blacksburg and flows for 11.6 miles in a southwesterly direction to the New River. The drainage basin is 23.89 square miles. Two rather large tributaries to Stroubles Creek are Walls Branch and Slate Branch. Walls Branch has a drainage area of 1.69 square miles and enters Stroubles Creek 4.56 miles from its mouth. Slate Branch has a drainage area of 9.25 square miles and enters Stroubles Creek 2.41 miles from its mouth. The slope of the stream is rather steep, falling approximately 520 feet from its headwaters in Blacksburg to the New River.

The Blacksburg—VPI & SU sewage treatment plant is located 7.3 miles upstream from the mouth of Stroubles Creek. The segment of the creek on which the model was applied begins at a point just upstream of the sewage treatment plant and continues to a point 0.66 miles from the mouth.

The drainage area above the sewage treatment plant primarily is residential, commercial, and farm and pasture land. Below the sewage treatment plant the area is mostly forested, except at the mouth of Stroubles Creek, where an Army ammunition plant is located. The channel of the stream above the sewage treatment plant is narrow with a bed mostly of clay. As the stream passes the sewage treatment plant, the channel becomes considerably wider and the bottom is mostly rocky with many riffles.

THE MODEL

The model is a modified version of the Streeter and Phelps (*Ref. 2*) equation adapted for use with a digital computer and is a "steady-state model" in that it does not consider possible variations in stream flow, waste discharges, or other non-steady state conditions such as photosynthesis.

The data required for application of the model include stream geometry, stream flow data, waste discharges, biological and physical reaction rates, and a knowledge of the quality and quantity of tributary streams. The stream length must be divided into segments such that stream characteristics within a segment are as constant as possible; a new segment should begin at any point where a major waste discharge enters the stream, or a major withdrawal from the stream occurs, or where a major tributary enters the stream. The segments should be as short as is economically feasible in order to locate accurately the point where the lowest dissolved-oxygen concentration will occur (DO values are computed only at the beginning and end of each segment or at each station). The model equations are applied to each segment of the stream in a stepwise fashion. The results obtained from analysis of the first segment are applied to the second segment, and so on, along the reach of the stream.

Data Requirements

The data required for execution of the computer program are:

1. cross-sectional data, velocities, and flow rates at each station. These data then must be analyzed to obtain relations between average depth and flow rate and between velocity and flow rate for each station. These relations are assumed to follow the form $V = aQ^b$ and $h = cQ^d$, where Q is the stream flow rate and a, b, c, and d are coefficients obtained from log-log plots of V versus Q and h versus Q;

2. the distance between stations and the elevation of each station;

3. flow rates of waste discharges, BOD_5 of raw wastes, degree of treatment before discharge, and dissolved oxygen concentration in the waste discharges;

4. percent dissolved oxygen saturation of the stream at the point where modeling begins and of tributaries entering the stream;

5. flow rates at the initial station and of tributaries;

6. BOD_5 of stream at the initial station and of tributaries;

7. the flow rate of any withdrawals from the stream;

8. the long-term, biochemical reaction rate, the first-day biochemical reaction rate, the biological extraction rate, the rate of settling out of BOD, and the areal oxygen demand of bottom deposits at each station;

9. the thermal correction coefficient applied to reaction rates;

10. the coefficient for low profile dam effects for each station;

11. stream temperature at each station.

Model Equations

The flow chart presented in *Figure 1* outlines the major components of the DO–BOD model. A description of the steps in the flow chart is given, along with equations applicable to each step.

1. In steps 2 and 4, the required input data are read into the program.

2. Step 5 prints the input data.

3. In step 6, variables are initialized.

4. In step 8, the BOD_5 of the waste is computed by the equation:

$$\mathrm{CONC} = (\mathrm{TRT} \times \mathrm{BOD_5P})/(8.34 \times \mathrm{QE}) \qquad [1]$$

where CONC = effluent waste BOD_5 concentration, mg/l;
TRT = fraction of BOD_5 remaining after treatment;
$\mathrm{BOD_5P}$ = BOD_5 of raw waste, lb/day; and
QE = waste discharge, mgd.

5. In step 9, the rate coefficients are corrected for temperature by the equation:

$$k_T = k_{20^\circ C}\,\theta^{(T - 20^\circ C)} \qquad [2]$$

where k_T = rate coefficient at T°C,
$k_{20^\circ C}$ = rate coefficient at 20°C, and
θ = thermal coefficient

The thermal coefficient, θ is often accepted to be equal to 1.047 after Streeter and Phelps (*Ref. 2*), although it can vary.

6. In step 10, the total flow is computed after mix by summing the component flows.

7. In step 11, the stream DO saturation, C_s, is computed by the equation:

$$C_s = 14.4\,(10^{-.0102T})\,(1-.00003\ Ev) \qquad [3]$$

where C_s = DO saturation, mg/l;
T = temperature, °C, and
Ev = elevation at station, ft.

8. In step 12, DO is computed after mix by the mass balance equation:

$$DO = \sum_{i=1}^{n} Q_i\,DO_i / \sum_{i=1}^{n} Q_i \qquad [4]$$

where Q_i = flow rate from source i, cfs;
DO_i = DO of source i, mg/1; and
n = number of sources.

9. In steps 13 through 17, a maximum value is placed on the stream DO content. If the BOD_5 at the station is greater than 2 mg/l, the maximum value of DO is set at 85% of the DO saturation. If the BOD_5 at the station is less than or equal to 2 mg/l, then the maximum value of DO is set at 95% of the DO saturation.

10. In step 19, the value of BOD_5 after mix is computed from the mass balance equation:

$$BOD_5 = \sum_{i=1}^{n} Q_i\ BOD_{5i} / \sum_{i=1}^{n} Q_i \qquad [5]$$

where $BOD_{5i} = BOD_5$ from source i, mg/l.

11. In step 20, the velocity, V; depth, h; time of travel, t; and reaeration coefficient, k_2, are computed. The velocity is computed from the equation:

$$V = a\,Q^b \qquad [6]$$

The depth is computed from the equation:

$$h = cQ^d \qquad [7]$$

The time of travel from the previous station is computed as the distance between stations divided by the velocity. The reaeration coefficient is computed from the equation of Churchill *et al.* (*Ref. 3*):

$$k_{2(20^\circ C)} = 5V/h^{1.666} \qquad [8]$$

12. In step 21, the ultimate, first-stage BOD, L, is computed from the BOD_5 based on the first order rate equation. The equation for ultimate BOD is:

$$L = BOD_5/(1 - 10^{-5k_1}) \qquad [9]$$

where L = ultimate first stage BOD, mg/l; and

k_1 = biochemical reaction rate, days^{-1}

There is no statement in the model to account for nitrogenous oxygen demand.

13. In step 22, the reaeration coefficient is reduced due to the effects of the presence of organic matter measured in terms of ultimate BOD. If the ultimate BOD, L, is greater than 100 mg/l, the reaeration coefficient, k_2 is assumed to equal four-tenths of its original value. For L values less than 100 mg/l, k_2 is reduced according to the equation:

$$k_2 = k_2(1 - .006L) \qquad [10]$$

The reaeration coefficient is then adjusted for temperature by equation [2]. In this case θ is assumed equal to 1.016.

14. In steps 23 and 24, the station number, distance between stations, DO, L, total elapsed time from initial station, E_t; velocity, V; depth, h; time of travel from previous station, t; flow rate, Q; reaeration coefficient, k_2; stream reaction rate, k_r; and water temperature, T, are printed.

15. In step 25, the saturation deficit, D, is computed as the DO saturation minus the stream DO.

16. In steps 26, 27, and 28, the BOD from sludge deposits is computed. If the velocity is less than 0.6 ft/sec, settling out of BOD is assumed to occur. The equation used to compute the BOD addition from sludge deposits is:

$$p = 110\,p_1/h + 2.3\,k_3L \qquad [11]$$

where p = total addition of BOD from sludge deposits, mg/l;
p_1 = areal oxygen demand of sludge deposits in absence of settling, lbs/ft^2; and
k_3 = rate of settling out of BOD to stream bottom, days^{-}

If the stream velocity is greater than 0.6 ft/sec, the value of k_3 is set at 0 and p is reduced to:

$$p = 110\, p_1/h \qquad [12]$$

17. In steps 29 and 30, the reaeration coefficient is set to 0 if no saturation deficit exists.

18. In step 31, the saturation deficit at the end of the reach or at the next station before mix is computed by the equation:

$$D_b = \frac{k_1 + k_b}{k_2 - (k_1 + k_b + k_3)} \left[L_a - \frac{p}{2.3(k_1 + k_b + k_3)}\right] \left[10^{-(k_1 + k_b + k_3)t} - 10^{-k_2 t}\right] + \frac{(k_1 + k_b)p(1 - 10^{-k_2 t})}{2.3\, k_2\, (k_1 + k_b + k_3)} + D_a\, 10^{-k_2 t} \qquad [13]$$

where D_b = DO saturation deficit at end of reach, mg/l;
D_a = DO saturation deficit at beginning of reach, mg/l; and
k_b = biological extraction rate, days^{-1}.

Equation [13] is similar to that presented by Camp (*Ref. 4*) except Camp's equation did not include the biological extraction rate (k_b) but did include a correction for photosynthetic input of DO. Dobbin's (*Ref. 5*) equation also is similar to Equation [13].

19. In steps 32 and 33 a correction is made to the DO saturation deficit due to the effect of a low profile dam. This correction takes into account the initial DO saturation deficit, the water temperature, and the dam height. The corrected DO saturation deficit is computed from the equation:

$$D_b = D_{bi}/[1 + .11A_D\ (1 + .46R \times D_{AM}0)] \qquad [14]$$

where D_{bi} = initial DO saturation deficit, mg/l;
A_D = 1.25 for $D_{bi} \leqslant 2$,
A_D = 1.0 for $2 < D_{bi} \leqslant 4$,
A_D = 0.8 for $D_{bi} > 4$, and
D_{AM} = low profile dam effect coefficient, height in feet

20. In step 34, the ultimate, first-stage BOD applied to the end of the reach or the next station before mix, L_b, is computed by the equation:

$$L_b = [L_a - p/2.3\ (k_1 + k_b + k_3)]\ 10^{-(k_1 + k_b + k_3)t} + p/2.3\ (k_1 + k_b + k_3) \qquad [15]$$

21. In step 35, the percent DO saturation, PCT_b, at the next station (point b) is computed by the equation:

$$PCT_b = (C_s - D_b)/C_s \qquad [16]$$

22. In step 36, the BOD_5 in the stream before mix at the next station is computed from the ultimate, first-stage BOD at that point by the first order equation:

$$BOD_{5b} = L_b\ (1 - 10^{-5k_1}) \qquad [17]$$

23. In steps 37 and 39, the BOD_{5b} value computed in equation [17] is set such that its minimum value is 2 mg/l.

24. In step 40, the total distance and total time from the initial station is accumulated and stored.

25. In steps 41 and 42, if another station is to be considered, the station number is incremented and the program returns to step 7.

26. In step 43, the ending cumulative time and the ending ultimate BOD are printed.

27. In step 44, if another temperature set is to be considered, the program returns to step 3.

28. Step 45 ends the program.

A listing of the computer program is given in Appendix B. The program, listed as Version I, is the one received from the State Water Control Board, with the exception of several format changes that were required for use at the VPI & SU computer facility. A description of the code for variables used in the computer is given in Appendix A.

DATA COLLECTION AND ANALYSIS

In the following paragraphs, detailed descriptions are given of the procedures used to obtain and analyze the data necessary for execution of the model as it applies to Stroubles Creek. Past studies (*Ref. 6, 7*) of the area did not provide sufficient data; therefore, data provided by studies both *in situ* and in the laboratory were combined with existing data to obtain the necessary information.

In addition to determinations of the data actually needed to execute the model, photosynthetic effects on the DO concentration were evaluated by a diurnal study at Station 6. This study was performed to help determine the degree of importance that should be placed on photosynthesis even though no term for its consideration is included in the model used by the Virginia State Water Control Board.

Description of Station Layout

Sampling stations were established along a reach of Stroubles Creek from a point just upstream of the waste effluent outfall to a point near the New River. The factors governing the location of the sampling stations included the existence of previous data, location of waste effluent outfalls, physical characteristics of the stream, location of major tributaries, self-purification characteristics of the stream, distance between stations, accessibility, and time required for sampling. Considerable cross-section data and velocity data were available at several stations on Stroubles Creek from a study performed by Kelsey (*Ref. 6*). A total of eight sampling stations was established, and their locations are shown as bold dots in *Figure 2.* Station 8 was located just upstream of the Radford Arsenal waste effluent outfall.

Drainage Areas and Elevations

The total drainage area and the incremental drainage areas between stations were obtained by planimeter from the 7½-minute series of the U.S. Geological Survey topographic maps of the area. The distances between stations also were obtained from the 7½-minute series of the USGS topographic maps. The drainage divides are shown in *Figure 2.* The broad, solid line outlines the total drainage area of Stroubles Creek. The dotted lines indicate the drainage divides between stations and major tributaries. The elevations for each station were obtained from sewer plans for the proposed Stroubles Creek outfall sewer prepared by R. Stuart Royer and Associates (*Ref. 8*). A summary of physical data for Stroubles Creek is given in Table I.

Stream Hydrologic Parameters

Input required for execution of the model included data concerning flow rates and velocities in order to calculate, among others, the reaeration rates along the stream. Most of these data had to be generated by establishing cross-sections and measuring depth and velocity at each station on several days. Details are given in the following paragraphs concerning these measurements, their use, and their interpretation.

Cross sections. At each station, steel reinforcing rods were driven into the stream bed at several points across the stream. The rods were placed at points that would give an accurate profile of the stream bed. The cross-section at each station was obtained by measuring (a) the depth of the stream at each rod and at the banks along the cross-section and (b) the distance between rods and rod and bank. At some stations the stream bed was almost solid rock. In these cases, large stakes were placed on either side of the stream and a plastic-coated wire was stretched between the stakes. The points for depth measurements were marked off along the wire.

A typical cross-section is shown in ***Figure 3.*** While similar data were obtained at each station on any specified date, all station cross-sections are not presented in this Bulletin simply because they were valid only as long as the stream-bed profile remained unchanged. In a creek such as Stroubles, described by some as "flashy" (denoting a tendency to flood suddenly during a brief period of intense rainfall), the stream-bed profile changes quite often; therefore, new cross-sections should be prepared periodically if accurate rating curves are to be developed. The reader interested in past data of Stroubles Creek flow should refer to the theses by Childrey (*Ref. 9*) and Kelsey (*Ref. 6*).

Rating curves. On three different days, measurements of velocity and the water depth across the creek at each station were used to develop rating curves on log-log paper that related both depth (h) and velocity (V) to flow (Q). From these plots the coefficients in the equations $V = aQ^b$ and $h = cQ^d$ were evaluated and were used as input data in the computer program. The average velocity used for each cross-section was a weighted average according to the flow rate.

Velocities measured with a Price Pygmy current meter (Gurley Instruments, Troy, New York) were determined midway between points of depth measurement. The current meter, previously standardized in a flume, was placed at 0.6 of the depth for velocity measurement. The cross-section data provided incremental areas across the stream, and the velocity data provided incremental velocities for these areas. The flow rate of each segment was found by the continuity equation, $Q = AV$ where A = cross-sectional area, ft^2, and V = velocity, ft/sec. The flow rate of the stream at the station was found by summing all of the incremental flows along the cross-section. Velocity and cross-section data were taken at all stations during the same day, usually within a six-hour period, in order to include in the measurements the normal downstream increases in flow rate.

When samples were taken for DO and BOD analyses, there was not enough time to complete the measurements of velocity and cross-section. Instead, a reference depth was taken at a selected point in the cross-section, and a relation between the average stream depth and the reference depth could be determined. The flow rate for a particular station then could be found from the rating curve for that station. Knowing Q, the velocity for a particular station could be found from the V versus Q relationship for that particular station.

When some of the velocity and cross-section measurements were made, it was difficult to reach some of the stream stations. In these cases, the flow rate for the times in which DO and BOD samples were taken were proportioned according to drainage area. These proportions were determined from stations at which flow rate measurements had been made.

Sample rating curves are shown in *Figures 4 and 5* for two of the stations. *Figure 4* is representative of the type of curve one can expect at most stream stations—a well-defined linear relationship on log-log paper. *Figure 5* illustrates the scatter of points one occasionally will encounter when the flow is more turbulent and departs significantly from one-dimensional flow. The presence of eddy currents, streamlines, and other non-steady state phenomena increases the error in the measurements.

Although there is a fairly good fit of the data to the curves in the range of measurement, it is possible that extrapolation to extremely high or extremely low flow conditions would result in substantial errors of estimation, especially for the curves developed from only three data points. During low flow conditions, small channels may form in the stream bed, resulting in completely different flow conditions than those measured. During very high flow conditions, the stream may overflow its banks, resulting in increased resistance to flow as well as substantially changing other flow parameters. It was believed that the flow relationships developed for the stations were reasonably accurate for use in the computer model for the field comparison run. The same flow relationships also were used in the computer program for 7 day – 10 year low flow conditions. Although some degree of error due to extrapolation of the flow relations probably exists for this condition, these data were the best available.

Collection and Analysis of DO and BOD_5 Data

On several dates, water samples were collected at each station and analyzed in duplicate for DO and BOD_5. Composite samples for BOD analysis taken over a 24-hour period would have been better, but that type of sampling program requires the availability of either automatic sampling devices or many field personnel, neither of which were available during this study. On any particular day, the sampling time required was about eight hours, usually from 8:30 a.m. to 4:30 p.m. In addition, samples were collected from the effluent of the secondary clarifier of the sewage treatment plant, from the mouth of Walls Branch, and from the mouth of Slate Branch. Occasionally, other small tributaries to Stroubles Creek were sampled. Water temperatures were obtained whenever samples were collected. The uses made of these data were twofold: first, an analysis of the downstream changes could be used to pinpoint any significant, unsuspected alternations in the DO and BOD_5; second, the data would serve to verify the DO profiles predicted by the model.

The samples used for DO analysis were fixed in the field and analyzed later by the azide modification of the Winkler Method (*Ref. 10*). The samples taken for BOD_5 were kept on ice in plastic containers until they could be returned to the laboratory for analysis by standard procedures (*Ref. 10*), usually within 12 hours after collection.

On both dates when data were collected for comparison to model predictions (June 12 and June 25, 1974), significant growths of filamentous algae and of unidentified brown filamentous organisms resembling *Sphaerotilus* were seen attached to rocks at most stations. These growths proved to be important in two respects in altering the DO and BOD_5 profiles in the creek, namely input of DO during the daytime by the algae and extraction of DO, both by algae at night and by periphyton continuously. These are discussed in detail in subsequent sections concerning the effects of photosynthesis and biological extraction by periphyton.

The DO analyses along Stroubles Creek during the sampling periods showed the DO concentrations to be well above the limit of a 4 mg/l acceptable to the Virginia State Water Control Board for that stream at that time (present standards specify 5 mg/l daily minimum average. Values ranged from 7.1 mg/l near the sewage treatment plant to 9.85 at the station furthest from the plant. Greater DO deficits would have been more ideal for testing the predictive capacity of the model, but the existence of high concentrations of DO by no means precludes the model's application to any creek as long as the water is less-than-saturated for DO, a fact that will be shown by the analytical results.

All field data demonstrated that a drop in DO and an increase in BOD_5 occurred between Stations 3 and 4. Normally, such changes are indicative of a waste discharge between the stations, but none could be found upon reconnaissance of the area on several occasions. In certain circumstances, one might find that the conversion of ammonia to nitrate by nitrifying organisms is responsible for a sudden increase in BOD, but the concentrations of ammonia nitrogen in Stroubles Creek were too low to account for the observed oxygen demand. To account for these changes, a source of water discharge was assumed to originate between Stations 3 and 4, the DO and BOD values being determined by matching model results and field results at Station 4. Such assumptions, of course, were necessary because no actual waste discharge could be found from which to obtain direct measurements and if the changes were not considered, the predictive capability of the model would be weakened, just as it would be if the discharge of a waste to the stream were to be ignored.

Biochemical Reaction Rate Study

A laboratory study was conducted to determine the biochemical reaction rate (k_1) needed for execution of the model. Three samples from Station 4, collected on different dates, were dechlorinated, seeded with 1 ml of secondary effluent from the sewage treatment plant, and diluted with an equal volume of BOD dilution water (*Ref. 10*). The diluted samples were placed in the dark at 20°C for incubation for varying periods up to 20 days. At varying intervals, samples were analyzed for residual DO; the results were analyzed by the Thomas Slope Method (*Ref. 11*) shown in *Figure 6* and the Moore Moments Method (*Ref. 12*) shown in Table II.

Figure 6 indicates that a change in bacterial growth rate from 0.23 to 0.18 occurred after 4 to 5 days of incubation. This is probably a result of the nature of the organic matter present in the stream. If a portion of the organic matter is readily biodegradable and the other portion is more resistant to biochemical degradation, then a change in the bacterial growth rate will occur when the more easily biodegradable organic matter is exhausted. It is assumed that this phenomenon occurred in this biochemical reaction rate study.

Results of the analysis by the Moore Moments Method (*Ref. 12*) showed an average variation in k_1 of 0.025, with values ranging from 0.14 to 0.23 (Table II). The highest rate occurred in the early stages, as was true in the analysis by the Thomas Slope Method.

The k_1 of 0.23 days^{-1} from the first portion of the Thomas slope plot was selected for use in the DO–BOD model, primarily because the retention period of water in Stroubles Creek before entering the New River is within the time period where $k_1 = 0.23$ days^{-1} is operative (as shown in *Figure 6*). If the retention period were much longer, the use of some other rate (between 0.18 and 0.2) would be more appropriate.

Some investigators prefer to determine k_1 values by collecting composite samples at each station, determining the ultimate and 5-day BOD in the laboratory, and plotting the values as a function of the time of travel in the stream to each station. This plot then is analyzed by either the Thomas Slope Method or the Moore Moments Method. However, the value of k_1 obtained by this procedure is actually a "stream reaction rate" and does not allow differentiation between the biochemical reaction rate (k_1) and the biological extraction rate (k_b) used in this particular model. The latter rate will be discussed later. Furthermore, such a procedure is extremely time-consuming and requires not only several field personnel but also much laboratory equipment and space.

Wastewater Flow Determination

The rate of wastewater inflow, which was recorded at the sewage treatment plant, was used as one estimate of the effluent flow of the sewage treatment plant. The rate of inflow is considerably more variable than the effluent flow rate. Too, a lag period exists between inflow and outflow. The portion of record used to estimate the effluent flow rate at the time of DO and BOD sampling was an average of the previous 24-hour rate of inflow. Other estimates of the effluent flow rate of the sewage treatment plant were obtained by calculating the difference between the flow rates at Station 1 and Station 2 and by a mass balance of DO and BOD using the field data. All three methods used to estimate the effluent flow rate contained some degree of error, but it was believed that the mass-balance method would introduce the least error into the DO prediction. The difference between the flow obtained in this manner and that obtained from subtracting the flow at Station 2 from the flow at Station 1 was divided equally between Station 1 and Station 3 in order to retain the flow balance as before.

Variations in DO Caused by Photosynthesis

A study was made of the diurnal variation in DO at a representative station (Station 6) to determine the importance of photosynthesis as a contributor to the stream DO concentration. Samples were taken hourly for 24 hours, fixed, and later titrated in the laboratory. Temperature and incident light intensity also were measured hourly.

It should be pointed out that the model includes no term for photosynthetic effects. However, as is shown by *Figure 7*, there was considerable diurnal variation during the sampling period, concentrations of DO reaching supersaturation during the afternoon. The DO range during the period was 1.8 mg/l but probably would have been greater if the flow rate from the sewage treatment plant and the BOD_5 loading were steady.

Though the variation in DO was demonstrated only at one station, the data were sufficient to conclude that photosynthesis can indeed be a significant variable in a stream, which, if ignored, could introduce sizeable errors in the predicted DO concentrations. The importance of photosynthesis as a contributor of DO during daylight hours would be magnified in streams where the reaeration rate was not as high as it is in Stroubles Creek. However, one should remember that algae respire as well as photosynthesize, and the depletion of oxygen at night would be increased by their presence in abundance in any stream.

MODEL APPLICATION TO FIELD DATA AND TO EXTREME LOW-FLOW CONDITIONS

Several modifications of the computer program were deemed necessary before it could be applied to a small stream. These modifications, explained in subsequent paragraphs, were completed and the program was executed using stream conditions determined on June 12 and June 25, 1974. Predicted values for stream DO then were compared to measured stream DO. The flow rates for each station, the DO and BOD at Station 1, the DO and BOD of tributaries, and the temperature data for the two dates were used as input data for the program. The actual BOD values at the stations were used to evaluate the biological extraction rate.

Model Modifications

The following modifications were made to Version 1 of the DO—BOD model. Reference to the flow chart in *Figure 2* is useful in discussing these changes.

1. In step 2, the BOD_5 concentration of the sewage treatment plant effluent was used in the program instead of the sewage treatment plant loading in lb/day. This simplification was made because the effluent concentration was measured rather than the raw waste loading rate. This modification eliminated equation [1], which is step 8, the calculation of the effluent concentration of the sewage treatment plant.

2. In step 12, the DO after mix is computed. The value of the DO before mix as used in equation [4] is computed as the percent saturation times the saturation value. However, the saturation was computed based on the saturation value at the previous station (step 35 and equation [16]). This leads to some error if there is a difference in saturation values between the two stations. A modification was made in the program in which the percent saturation before mix is based on the saturation value at that station.

3. Steps 13 through 17, which placed a maximum allowable value on the stream DO, were eliminated. This original restraint was considered a built-in safety factor and would hinder model accuracy.

4. In step 31, the reaeration coefficient (k_2) is set to 0.0 if there is no DO deficit. When $k_2=0$ is used in equation [13], any supersaturation that may have existed is lost. Equation [13] was modified such that the value of k_2 in the term $D_a 10^{-k_2 t}$ was not set to 0 but was maintained at its original value. When supersaturation exists, the modified form of equation

[13] does not permit any more reaeration but does allow initial supersaturation conditions to approach saturation conditions at the same rate as it would in cases of undersaturation. Although there are no data to indicate that this rate for supersaturation is the same as that for undersaturation, it was felt that this modification was more realistic than to allow an automatic reduction of any supersaturated condition to one of saturation.

5. Steps 37 through 39 of *Figure 1*, which place a minimum value of 2 mg/l on the stream BOD_5 before mix, were eliminated. This restraint was considered to be another built-in safety factor in the original model, but it actually hindered the accurate prediction of actual DO data.

6. An additional write statement was installed in the program which would cause the value of DO and L before mixing occurred to be printed.

A listing of the modified version (Version II) of the DO–BOD program is given in Appendix D. A description of the variables used in the program is given in Appendix C.

Determination of Biological Extraction Rate

No direct method was available for determining the biological extraction rate, (k_b), so it was assumed, based on observations of stream conditions, that any observed BOD reduction over and above that exerted by planktonic microorganisms was a result of the "biological extraction" of organic matter by attached communities of microorganisms. The assumption that biological extraction was the only other operative mechanism of BOD removal was based on observations that there were no sludge or benthic deposits except in a reach extending approximately 100 yards downstream of the sewage treatment plant outfall. However, considerable quantities of filamentous, brown growths and filamentous algae were observed all along the reach of the stream. In order to obtain an estimate of the biological extraction rate, the computer program was executed for conditions of June 25 over a wide range of biological extraction rates. The results were analyzed to determine which value of biological extraction rate, k_b, resulted in the best fit of computed BOD to field measurements of BOD. The DO profile from this run was compared to the field DO measurements. Conditions of June 25 were selected because the stream flow rate was less than the stream flow rate of the other measurements. The k_b value determined by best fit of June 25 data was used as a condition in the analysis of the June 12 data.

Model Application to Low Flow Conditions

The DO—BOD program also was used to predict DO in Stroubles Creek for low flow conditions. The Virginia Water Quality Standards (*Ref. 13*) lists a minimum allowable DO concentration in streams like Stroubles Creek as 5 mg/l daily minimum average for stream flow conditions greater than or equal to 7 day—10 year low flow conditions. The 7 day—10 year low flow condition was used to determine if Stroubles Creek did meet the standards set forth by the State of Virginia.

The 7 day-10 year low flow for Stroubles Creek has been determined to be 0.105 cfs/sq mi (*Ref. 14*). The flow rate in cfs at each station, not including flow contributed by the sewage treatment plant, was determined by multiplying the drainage area (square miles) above the station by 0.105. The values of k_1 and k_b previously determined were used for the low flow condition. A sewage treatment plant flow rate of 2 mgd with an effluent BOD_5 concentration of 30 mg/l was used. The DO of the sewage treatment plant effluent was assumed to be 4 mg/l, based on field measurements. The design flow for the Blacksburg-VPI & SU sewage treatment plant was 2 mgd, and secondary treatment can be expected to produce an effluent quality of at least 30 mg/l, especially during summer months. A water temperature of 27°C was used for all stations. This value is in the range of maximum temperatures observed in Stroubles Creek (*Ref. 7*). A BOD_5 value at Station 1 was assumed to be 4 mg/l. The DO at Station 1 was assumed to be 85% of the saturation value. The BOD_5 contribution from the tributaries was assumed to be 2 mg/l. The DO of the tributaries was assumed to be 90% of the saturation value. These numbers are based on June 25 measurements, with a small margin of safety added.

PREDICTIONS OF STREAM CONDITIONS AND DISCUSSION

Comparisons with Observed Data

The value of the biological extraction rate, k_b—determined from successive iterations of the model until BOD reduction in the model agreed with June 25 field data—was 1.5 days^{-1}. It should be noted that biological extraction and biochemical reaction were assumed to account totally for the disappearance of oxygen-demanding materials from the stream. The justification for this assumption was that the stream velocity was too great to allow deposition of flocculated organics to the sediments, the only other major recognized mechanism for the removal of BOD. The fact that k_b proved to be large when compared to k_1 (1.5 versus 0.23) demonstrates vividly that the removal of dissolved organic matter by periphyton communities can be an extremely important consideration when determining the assimilative capacity of a stream. Recall that sizeable accumulations of these attached communities were evident in the stream bed at nearly every station.

Figures 8 and 9 show, respectively, the DO and BOD_5 profiles, both predicted and measured, for Stroubles Creek on June 12, 1973. *Figures 10 and 11* show, respectively, the same type of data for the creek on June 25, 1973. The predicted values shown in the latter figures were derived by execution of the model with the k_b value determined to give the best fit to data of June 25. In *Figures 8 and 10*, several measured DO values fall above the line that represents predicted values, reflecting, no doubt, the influence of photosynthesis on the oxygen content of the stream. Nevertheless, correlation between observed and predicted values seems quite good. Of course, one would expect good agreement for the BOD data of June 25 because of the method used to obtain the k_b value. The use of this k_b value in the model, along with other input data for June 12 conditions, resulted in a predicted DO profile that correlated well with observed DO data on that date.

The actual in-stream rate of BOD reduction on June 12 was slightly greater than was predicted by the model (*Figure 9*). Either a higher biochemical reaction rate or biological extraction rate could account for the differences. Little research has been conducted to determine factors that affect variations in biological extraction rates, while those factors that affect k_1 values are fairly well understood, and the authors believe the more likely errors in prediction occur from imprecise estimates of k_b, not of k_1. This point is discussed in more detail in the section concerning error analyses. The slight differences in BOD reduction apparently had little effect on the predicted DO profile for June 12 (*Figure 8*).

Low Flow Conditions

Figure 12 shows the predicted stream DO profile for the 7 day–10 year low flow conditions previously mentioned. The value of k_b used in the analysis was, as before, 1.5 $days^{-1}$. Under these conditions, the DO as predicted by the model would be depleted almost immediately downstream of the sewage treatment plant outfall. Although the stream may recover downstream, the model does not have the capability to predict recovery when anaerobic conditions occur. In order to determine what magnitude of stream flow would be required to maintain the DO minimum of 4 mg/l, the program was iterated several times, and each time the flow was increased incrementally to 14 times the 7 day–10 year low flow rate, Q_{7-10},would be required to maintain acceptable DO levels. This flow is approximately twice the flow rate of the sewage treatment plant effluent.

The curves depicted in *Figure 13*, especially the one for the condition of 7 day–10 year low flow, point out a problem common on many small creeks where large sewage effluents are discharged, namely that the initial oxygen deficit in the stream often is so great that standards are violated at the point of discharge. In streams where the reaeration rate is extremely high–usually those which have a steeply sloped bed and are shallow and winding–there is no critical point (i.e., no typical oxygen sag curve) and one needs to be concerned only about maintaining a high dissolved oxygen concentration in the effluent. Some plants actually aerate effluents before their discharge into a stream.

Error Analysis for Low Flow Conditions

Any particular DO profile that is generated when the model is executed is the result of interactions among many conditions that the modeler predicts will exist at some future time. Obviously, errors the modeler makes in quantifying these predicted conditions will affect the pattern and magnitude of DO distribution along the stream. In order to determine how important errors made in predicting the future conditions actually are, an analysis was conducted that involved successive iterations of the model with incremental variations over an arbitrarily defined range of each input parameter considered one at a time. The analysis was not a true error analysis because only one parameter at a time was varied. A true error analysis would require establishing and executing the model for a matrix of the parameters (11 x 11 in this case) that would account for DO changes caused by incremental variations of each parameter in combination with every other possible condition. Nevertheless, through the analysis that was conducted, the relative importance of errors made when selecting values for each of the input parameters was determined.

Conditions specified in the previous analysis of Stroubles Creek under low flow conditions were varied one at a time over the ranges shown in Table III. One change was made for most of the analyses—the value of k_b was reduced from 1.5 to 0.3 so as to maintain DO in the creek at Station 3. When k_b was the parameter being varied, however, the DO was depleted at Station 3 in every iteration for k_b values above 1.1. Station 3 was selected as the point at which to evaluate the effects of parameter variations because it was the point of minimum DO under Q_{7-10} conditions (*Figure 12*).

Table IV shows the results of the error analysis. The parameters are ranked in order of decreasing effect on DO at Station 3—that is, variations in the parameter listed first produced the greatest change in DO. Figures depicting the most significant changes in DO at Station 3 caused by variations in each parameter listed in Table IV are presented in subsequent paragraphs along with a discussion of each.

<u>Stream depth and velocity</u>. *Figures 14 and 15* show the effects of depth and velocity—which ranked first and fourth, respectively, in importance—and are considered together because both affect the value computed for the reaeration coefficient, k_2. The prediction of stream DO apparently is extremely sensitive to changes in k_2 and depth proved to be more important than velocity because it is raised to the 1.66 power in Churchill's (*Ref. 3*) equation.

Because k_2 is so important, an accurate method for measuring velocities should be insured in creeks like Stroubles that flow rapidly and have highly varied cross-sections. The use of a velocity meter does not necessarily insure a good *average* velocity at a particular cross-section because of these highly varied and turbulent conditions, and it is the average that is used in the computer model. Perhaps dye studies would provide a better evaluation of the average velocity throughout a reach in streams like Stroubles Creek. Dye studies would probably give better estimates of over-all travel time also.

<u>Effluent BOD_5</u>. *Figure 16* shows the relationship between DO and varying values for the BOD_5 of the effluent from the sewage treatment plant.

<u>Biological extraction rate</u>. This parameter ranked third in importance, and its relationship to DO at Station 3 is shown in *Figure 17* as an inverse one, which was expected. The major reason for the high ranking of k_b is that the range of variation permitted in the analysis was quite large because no information was

available from which to determine how much variation might be expected in streams. The obvious interpretation that could be stated is that if considerable periphyton biomass is observed in a stream to be analyzed, k_b will be high and, hence, will be extremely important in the analysis. A point made earlier should perhaps be reiterated—namely, that k_b should never be ignored when a shallow, turbulent stream is being analyzed.

Biochemical extraction rate. *Figure 18* depicts an unexpected relationship between predicted DO values and values of k_1. The effect of k_1 was unusual in that increases up to 0.20 day^{-1} caused an increased in DO, but greater values caused a decrease. For small k_1 values, the calculated, ultimate BOD was much larger than when large k_1 values were used because the ultimate was calculated by the function:

$$\mathrm{BOD}_5/(1 - 10^{-5k_1})$$

When k_1 is large the DO utilization rate is large.

Figure 18 shows that at some point ($k_1 = 0.20\ \text{days}^{-1}$ in this case) the effect of reducing the ultimate BOD (by the model equations) is overcome by the effect of increasing the oxygen utilization rate (reflected by higher k_1 values) and, beyond this point, the DO begins to decrease as k_1 increases.

Effluent DO concentration. The sewage treatment plant effluent DO ranked sixth in its effect on the DO at Station 3 for the range of estimated error. The DO at Station 3 increased linearly with increasing effluent DO up to 7 mg/l (*Figure 19*). The effect of effluent DO was substantial at Station 3 because the major portion of the stream flow was contributed by the sewage treatment plant.

A sharp decrease in DO at Station 3 occurred when the effluent DO was increased from 7 to 8 mg/l. This phenomenon was due to one of the limitations applied within the model and does not reflect an actual stream reaction. When oxygen is at saturation concentration at the beginning of a reach, reaeration is assumed in the model to be zero for that reach, and when, because of incrementing the effluent DO, the value of stream DO at Station 2 increased to the saturation value or greater, the model assumed no reaeration for the next reach. In streams where reaction rates are more in the normal range, the assumption of no reaeration probably would not drastically affect the DO within a short reach. However, in a stream such as Stroubles Creek, where the reaction rates (both reaeration and stabilization) are high, significant errors can occur when DO values are near saturation. This indicates that the magnitude of the reaction rates should be given due consideration along with other parameters when the maximum distance between stations is established.

Stream temperature. The stream temperature ranked seventh in its effect on the DO concentration (*Figure 20*). The effect was such that the DO at Station 3 was inversely proportional to the stream temperature. An increase in stream temperature increases the reaction rates and decreases the stream DO saturation value. The stream temperature would be expected to exhibit the same general relationship to stream DO regardless of the magnitude of other parameters.

Stream flow rates. The stream flow rate (tributary flow) ranked only eighth in its effect on the DO concentration at Station 3 (*Figure 21*). The direct relationship was approximately linear. The stream flow rate did not have as much effect on stream DO as might have been expected, primarily because its initial value was so small. An increase to twice the 7 day—10 year low flow rate did not significantly increase the over-all stream flow rate because the major contribution to stream flow was from the sewage treatment plant.

Other variables. Other parameters included in the error analysis which had little effect on the stream DO over the range of variation were the flow rate of the sewage treatment plant effluent, the tributary BOD_5, and the tributary percent DO saturation. It was expected that the tributary BOD_5 and the tributary percent DO saturation would have little effect on the stream DO within the range considered.

Greater effects than were demonstrated were expected when the flow rate of the sewage treatment plant effluent was varied. Increasing the effluent flow rate to approximately 0.7 times the 7 day—10 year low flow of the creek resulted in slight decreases in dissolved oxygen concentrations at Station 3, whereas increases beyond 0.7 times the low flow caused slight increases. When the effluent flow rate was in the lower range, an increase caused a decrease in stream DO because the stream BOD increased. When the effluent flow rate was in the higher range (above 0.7 times low flow of the stream), a small increase affected stream DO only slightly because little dilution was provided by the stream. But the increased effluent flow effectively increased the stream flow which, in turn, created conditions that increased the stream reaeration rate; hence, the dissolved oxygen concentration increased. Such would not occur, however, if the dissolved oxygen concentration of the effluent were extremely low.

An additional error analysis was conducted to determine a different type of information than was not provided by the previous one. The information desired was an assessment of the importance of errors associated with the measurement of the various input parameters. In the previous error analysis a reasonable range of variation for each parameter was assumed, but the position of any given variable in the ranking scheme was influenced by the selected width of the range.

Though the ranges of variation for each parameter were reasonable estimates of those expected under low flow conditions, their selection was somewhat arbitrary. Therefore, the question was asked, "what effect would an error of ± 10% in estimating or measuring a parameter have on the predicted DO at a particular station?" To answer that question, the value of each model input variable uses for the 7 day—10 year low flow analysis was varied one at a time, first by −10%, then by +10%, and the difference in DO at Station 3 predicted by these iterations was determined. These differences then were ranked to show the relative importance of the parameters.

Table V shows the results of this analysis and the ranking order is noticeably different than that shown in Table IV, which was derived from the former error analysis. Stream temperature ranked seventh in the former analysis but first in the latter, but no problem should be encountered in actual field operations because a variation of ± 10% represents an error in measurement or estimate of about 55°F. Usually the error is much less. However, temperature is demonstrated through this analysis to be an extremely important parameter, primarily because the model makes temperature adjustments to both the biochemical reaction rate and the reaeration rate.

As in the previous analysis, depth and velocity are shown to be important parameters, primarily because of their use in calculating k_2, the reaeration coefficient. It should be pointed out that an error of ± 10% in depth measurements could be on the order of 1 to 2 inches and in velocity measurements of 0.1 to 0.5 ft/sec. As was mentioned before, extreme care must be taken to insure accurate depth and velocity measurements.

Interestingly, the k_1 value ranked much lower in the latter analysis (Table V) than did k_b. The reason, no doubt, is because the value of k_b in Stroubles Creek is higher than k_1 and errors of ± 10% in measuring k_b results in changes of 0.1 to 0.2 $days^{-1}$,whereas for k^1 the error in absolute terms is an order of magnitude less.

It is important to notice that an error in measuring parameters ranked fourth or greater in Table V can cause predictive errors of between 0.6 and 1.2 mg/l

DO. However, errors in parameters ranked lower than four result in DO errors less than 0.5 mg/l, and the limits of accuracy of the DO test in practice are in the range of 0.1 to 0.3 mg/l. Therefore, a general conclusion of this particular error analysis that seems justified is similar to the conclusions derived from the first error analysis—namely, that those parameters used to calculate the reaeration coefficient should be measured with extreme care because slight deviations of the estimates or measurements of true values from the actual ones result in considerable predictive errors when the computer model is applied.

SUMMARY AND CONCLUSIONS

The objective of this Bulletin has been to detail how a computer model used for DO predictions in river systems could be adapted for use with small streams in which many problems develop because of sewage discharges to them. The model in the form given by the Virginia State Water Control Board had to be modified for use in Stroubles Creek, and those modifications have been explained. Based on the results of the study, the following conclusions can be made:

1. Version II (modified version) of the DO–BOD model exhibits sufficient capability for the prediction of dissolved oxygen in small, mountain streams when stream and effluent discharge characteristics are known.

2. Based on the assumption that $k_b = 1.5$ can be extrapolated to low flow conditions, the dissolved oxygen in Stroubles Creek will not meet stream standards when 7 day–10 year low flow conditions prevail. This situation probably is more common in Virginia than is recognized.

3. Relatively small errors in measurements of model input parameters—especially stream depth, effluent BOD_5 biological extraction rate, and stream velocity—may result in substantial errors in predicted dissolved oxygen concentrations. Thus, accurate determinations of the values of these parameters are extremely important.

4. Benthal respiration, or BOD_5 exerted by sludges, can be ignored if the velocity is great enough (greater than 0.6 fps) unless there are many pools where sludge can accumulate, but BOD_5 exertion by attached communities should not be ignored. The rate of O_2 uptake by attached communities in most small streams receiving sewage is probably quite high in comparison to the biochemical reaction rate, k_1.

5. Photosynthesis is likely to contribute significant dissolved oxygen to the stream and should be considered in analyzing the predictive results given by the model even though the model contains no term to account for the effects of algae activity. Too, large algae populations contribute significantly to the depletion of oxygen in darkness when they are not producing oxygen, and

this fact also should be considered when modeling streams where they abound.

RECOMMENDATIONS

1. Accurate procedures should be developed for assessing *in situ* the effects of photosynthesis and biological extraction. A term should be added to the model program that will allow consideration of photosynthesis.

2. A more thorough analysis is needed to more accurately determine the magnitude and variability of the biological extraction rate.

3. A more accurate method (e.g., dye-tracing techniques) of determining average stream velocities is needed for application of the DO—BOD model to streams which exhibit a high longitudinal variation in stream velocity (i.e., streams whose channel characteristics are highly variable).

4. When the stream is segmented for model application, consideration should be given to the magnitudes of the stream reaction rates.

5. A thorough sensitivity analysis of the variables for the DO—BOD model is needed in order to develop a more rational approach to the stream survey.

6. When conservative estimates of the stream dissolved oxygen are desired and uncertainties of input parameters exist, Version I (State version) of the DO—BOD model may be more desirable from a stream water quality standpoint than the modified version presented in this bulletin.

REFERENCES

1. "Research Needs in Water Quality Research—1972," A Report of the Committee on Water Quality Criteria to the National Academy of Science—National Academy of Engineering, Washington, D.C., 1973.

2. Streeter, H.W. and Earl B. Phelps, "A Study of the Pollution and Natural Purification of the Ohio River," Public Health Bull. No. 146, Washington, D.C., 1925.

3. Churchill, M.A., H.L. Elmore, and R.A. Buckingham, "The Prediction of Stream Reaeration Rates," J. San. Engineering Div., ASCE, 88:1-46, July 1972.

4. Camp, Thomas R., "Field Estimates of Oxygen Balance Parameters," J. San. Engineering Div., ASCE, 91:1-19, October 1965.

5. Dobbins, William E., "BOD and Oxygen Relationships in Streams," J. San. Engineering Div., ASCE, 90:53-78, June 1964.

6. Kelsey, Ronald G., "Some Effects of a Chlorinated, Municipal Sewage Effluent on a Small Creek," thesis, Virginia Polytechnic Institute and State University, 1973.

7. Hayles, Virginia M., "Biological and Chemical Monitoring of Three Streams in the Area of Blacksburg, Virginia," thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, l973.

8. R. Stuart Royer and Associates, Plans Sheets for "Stroubles Creek Outfall Sewer, Project No. 7302," March 30, 1973.

9. Childrey, Mitchell R., "The Application of a Mathematical Model for Dissolved Oxygen Concentrations in a Small Mountain Stream," thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 1973.

10. Standard Methods for the Examination of Water and Wastewater, 13th edition, American Public Health Association, Washington, D.C., 1971.

11. Thomas, Harold A., Jr., "Graphical Determination of BOD Curve Constants," Water and Sewage Works, 97:123-124, March 1950.

12. Ciaccio, Leonard L., Water and Water Pollution Handbook, Vol. 3, Marcel Dekker, Inc., New York, 1972.

13. Water Quality Standards, Virginia State Water Control Board, June 1970.

14. Sutherland, E.R., "Blacksburg–VPI Sanitation Authority: Plant Expansion," Virginia State Water Control Board Memorandum, February 26, 1969.

TABLES

TABLE I

Physical Data for Stroubles Creek

Location	Distance from New River (mi.)	Elevation (ft.)	Drainage Area (sq. mi.)
Station 1	7.39	1972	7.15
Sewage Treatment Plant Outfall	7.31	1971	7.16
Station 2	7.28	1971	7.16
Station 3	6.29	1949	8.54
Section 4	4.91	1911	9.05
Walls Branch	4.56	1900	1.69
Station 5	3.66	1861	11.51
Station 6	2.49	1789	12.11
Slate Branch	2.41	1784	9.25
Station 7	1.38	1734	21.84
Station 8	0.66	1706	22.99
New River	0	1681	23.89

TABLE II

Biochemical Reaction Rate Determined from the Moore Moments Method*

Time Series*	k_1** Values for Data Set 1	 2	 3
III	.185	.185	.23
IV	.165	.19	.195
V	.165	.185	.19
VII	.14	.16	.14
Average of the four series	.164	.18	.189
Over-all Average		.18	

* For details, see Reference 12.
** Base 10

TABLE III

Variables and Ranges of Variation Permitted During Error Analysis

Variable	Range of Variation	Increment of Variation
Tributary Flow Rate	–50% to +200%	25%
Effluent BOD_5	10 mg/l to 50 mg/l	5 mg/l
Effluent DO	0 mg/l to 8 mg/l	1 mg/l
Effluent Flow Rate	–50% to +200%	10%
Tributary BOD_5	0 mg/l to 8 mg/l	1 mg/l
Percent Tributary Saturation	70% to 100%	10%
Stream Velocity	–60% to +200%	20%
Stream Depth	–60% to +200%	20%
Biochemical Reaction Rate, k_1	.05 $days^{-1}$ to .4 $days^{-1}$	0.05 $days^{-1}$
Biological Extraction Rate, k_b	0 $days^{-1}$ to 3.0 $days^{-1}$	0.1 $days^{-1}$
Stream Temperature	20°C to 30°C	1°C

TABLE IV

Ranking of Effect of Input Parameters on DO at Station 3 for 7 Day—10 Year Low Flow Conditions for a Specified Range of Parameter Variation

1. Stream Depth
2. Sewage Treatment Plant Effluent BOD_5
3. Biological Extraction Rate
4. Stream Velocity
5. Biochemical Reaction Rate
6. Sewage Treatment Plant Effluent DO
7. Stream Temperature
8. Stream Flow Rate
9. Sewage Treatment Plant Flow Rate
10. Tributary BOD_5
11. Tributary DO Saturation

TABLE V

Variations in Dissolved Oxygen at Station 3 Associated with a Percentage Change of ±10% in Input Parameters for 7 Day–10 Year Low Flow Conditions

Rank	Parameter	ΔDO Caused by Variation from −10% to +10%	Rank in Previous Error Analysis, Table IV
1	Stream Temperature	1.12	7
2	Stream Depth	0.90	1
3	Sewage Treatment Plant Effluent	0.78	2
4	Stream Velocity	0.60	4
5	Biological Extraction Rate, k_b	0.38	3
6	Sewage Treatment Plant Effluent DO	0.30	6
7	Stream Flow Rate	0.30	8
8	Sewage Treatment Plant Flow Rate	0.20	9
9	Biochemical Reaction Rate, k_1	0.10	5
10	Tributary DO Saturation	0.09	11
11	Tributary BOD_5	0.05	10

FIGURES

Figure 1

Computer flow-chart for DO-BOD model

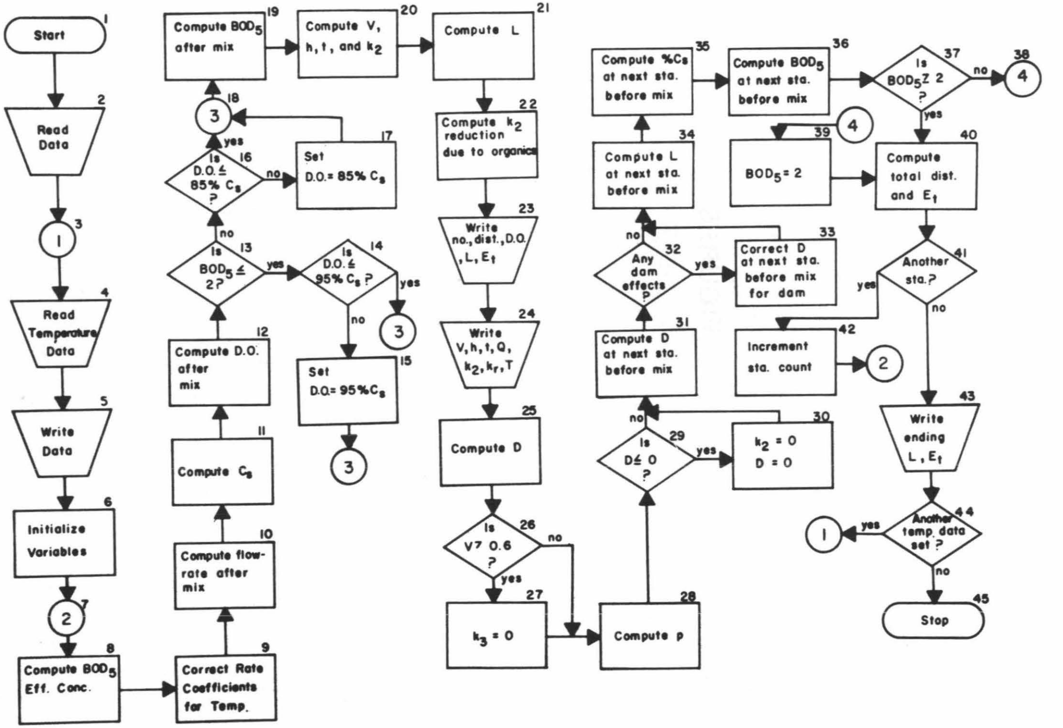


Figure 2

Stroubles Creek drainage area and sampling station locations

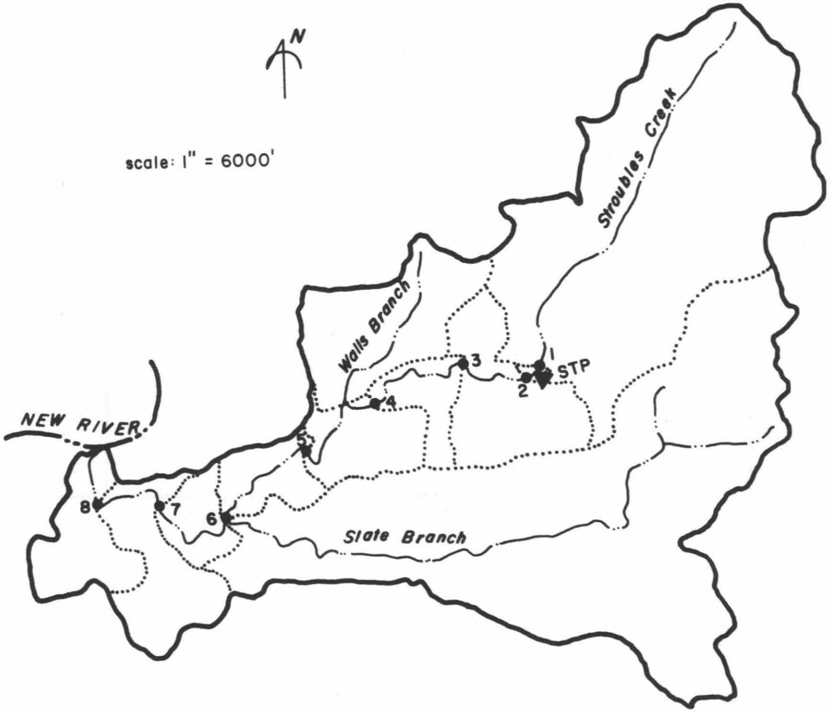


Figure 3

Typical cross section, Stroubles Creek, showing data needed to calculate flow

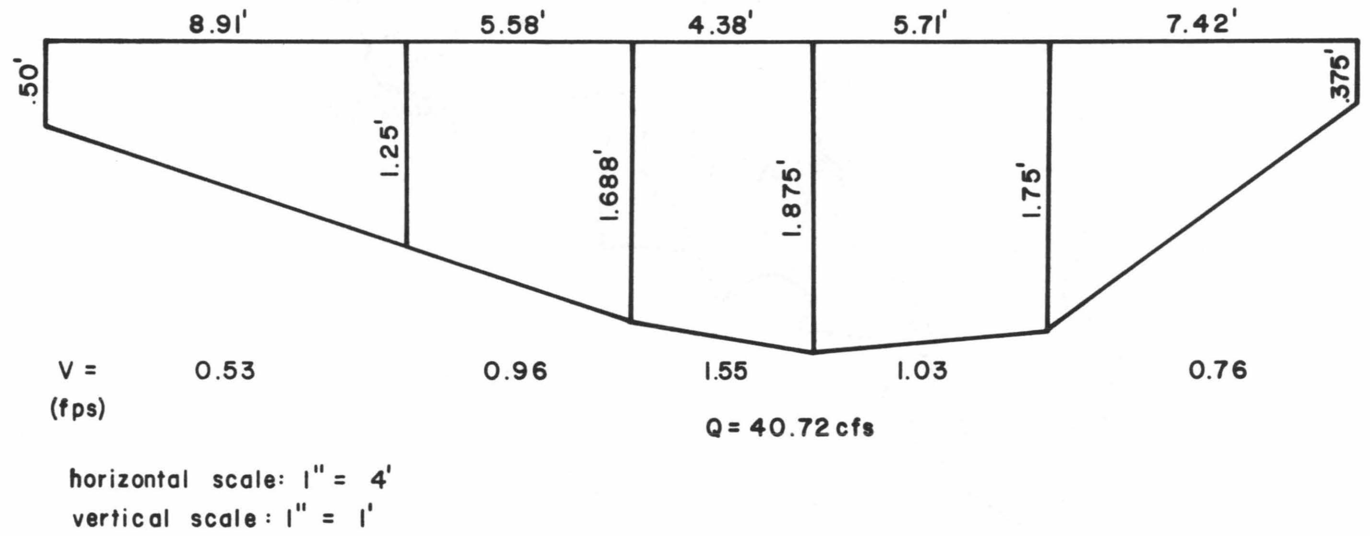


Figure 4

Sample rating curves for deriving coefficients used in computer model
(Good fit of data points)

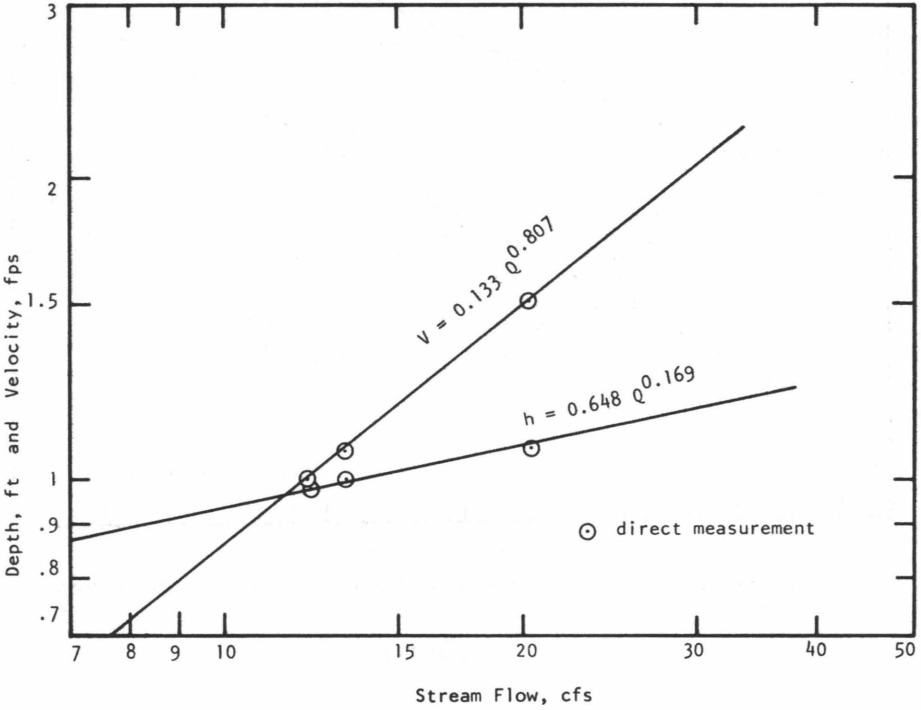


Figure 5

Sample rating curves for deriving coefficients used in computer model
(Fair fit of data)

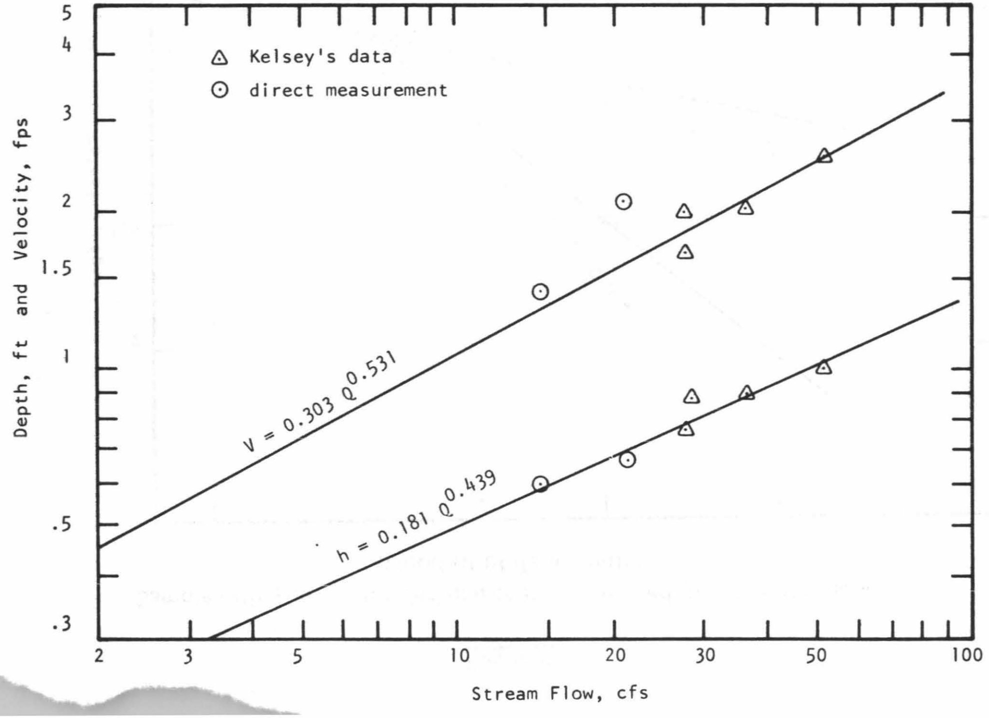


Figure 6

Determination of biochemical reaction rate
by Thomas Slope Method (Ref. 11)

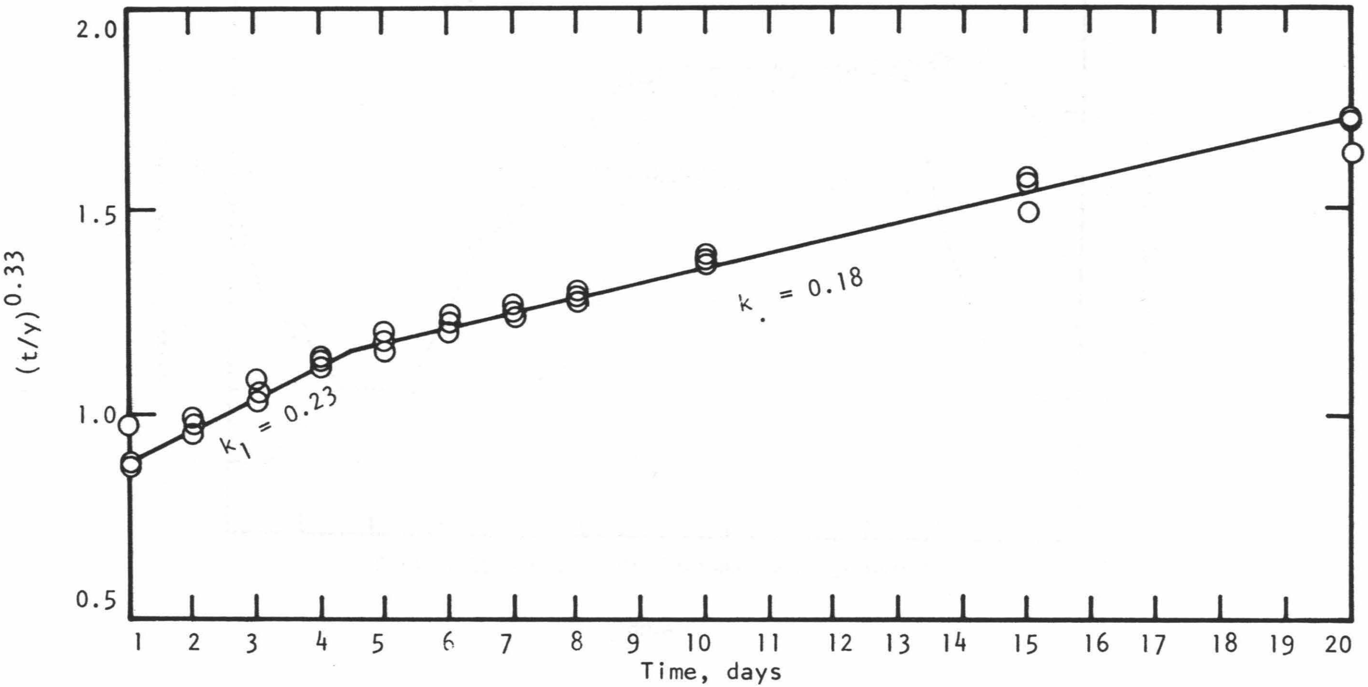


Figure 7

Diurnal DO variations in Stroubles Creek, Station 6

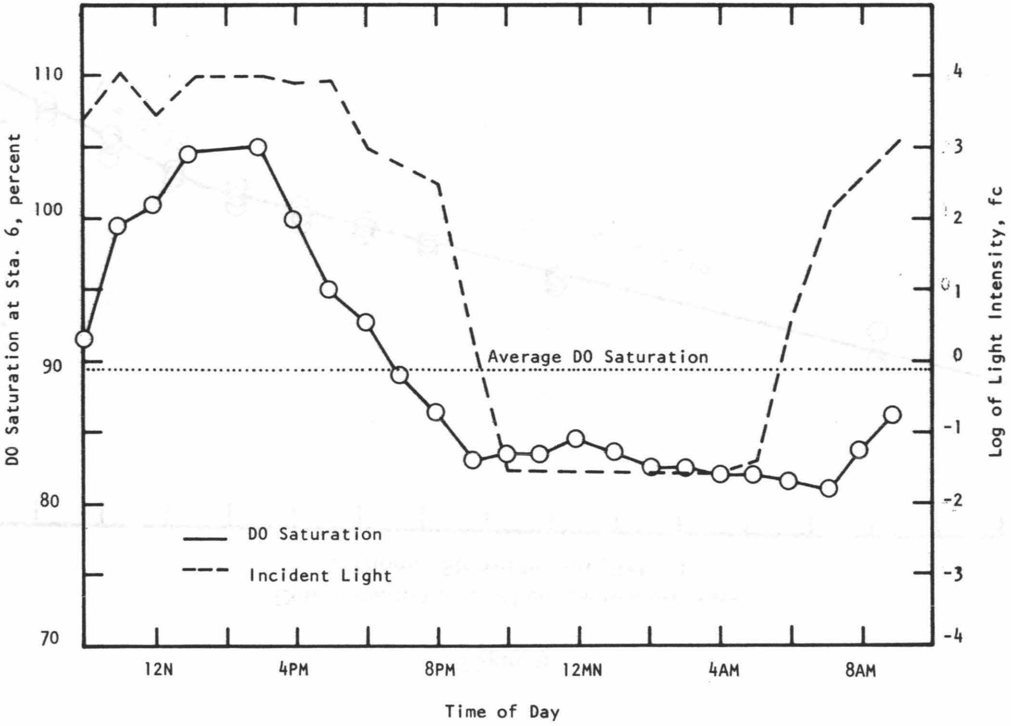


Figure 8

A comparison of computed and measured dissolved oxygen concentrations, June 12, 1973

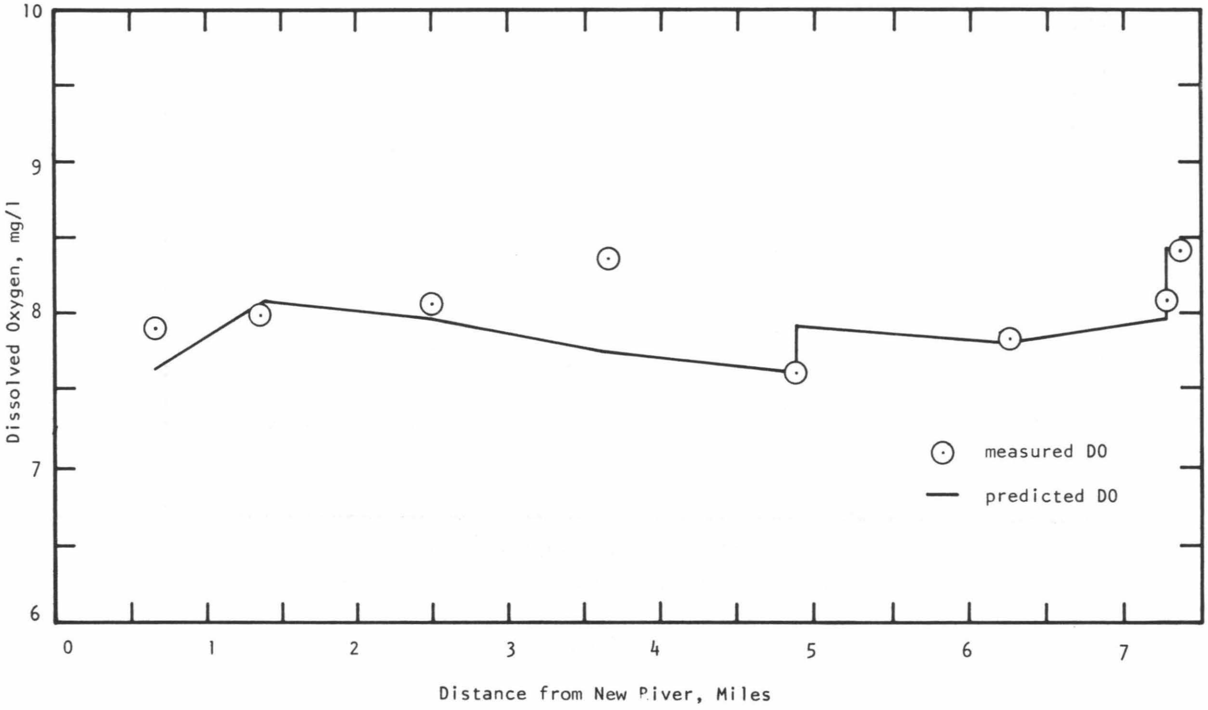


Figure 9

A comparison of computed and measured BOD_5 concentrations, June 12, 1973

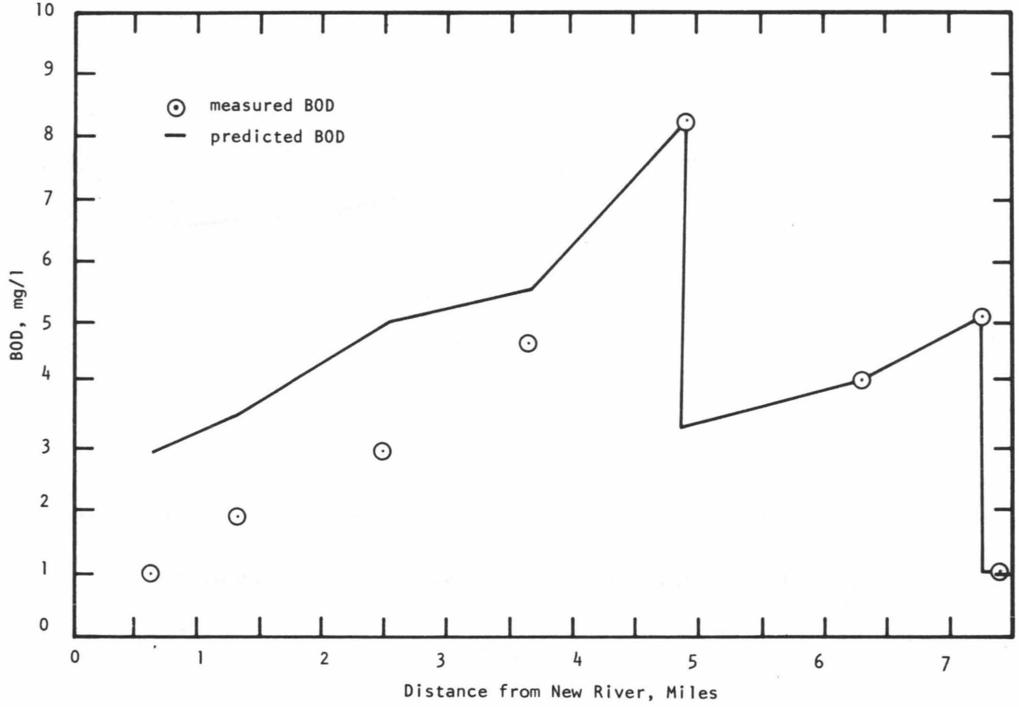


Figure 10

A comparison of computed and measured dissolved oxygen concentrations, June 25, 1973

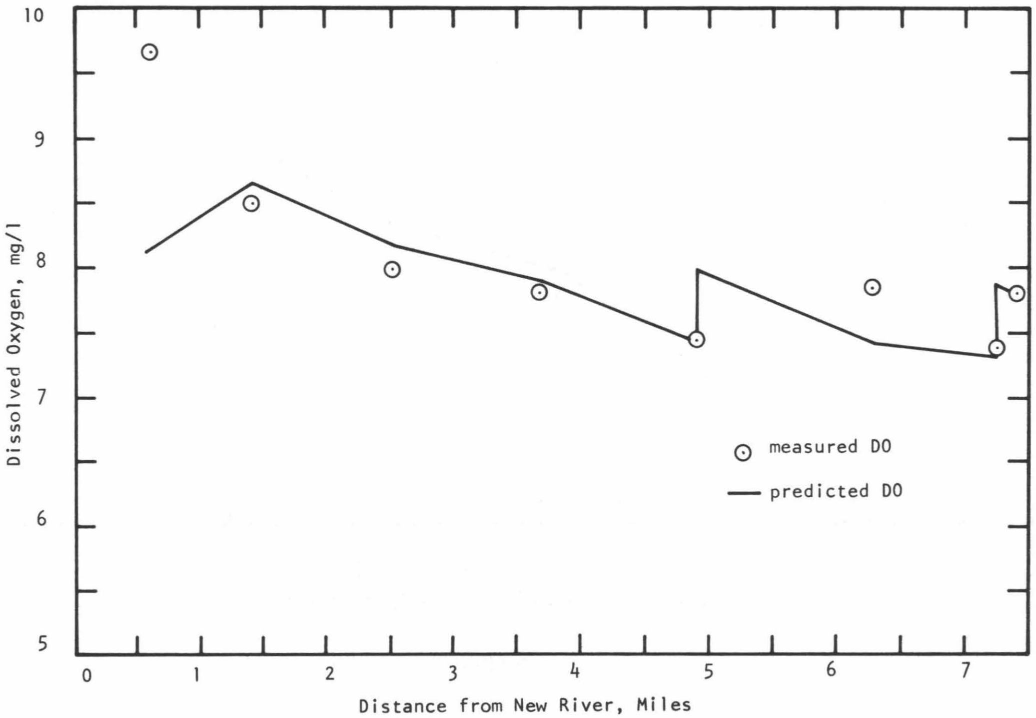


Figure 11

A comparison of computed and measured BOD_5 concentrations, June 25, 1973

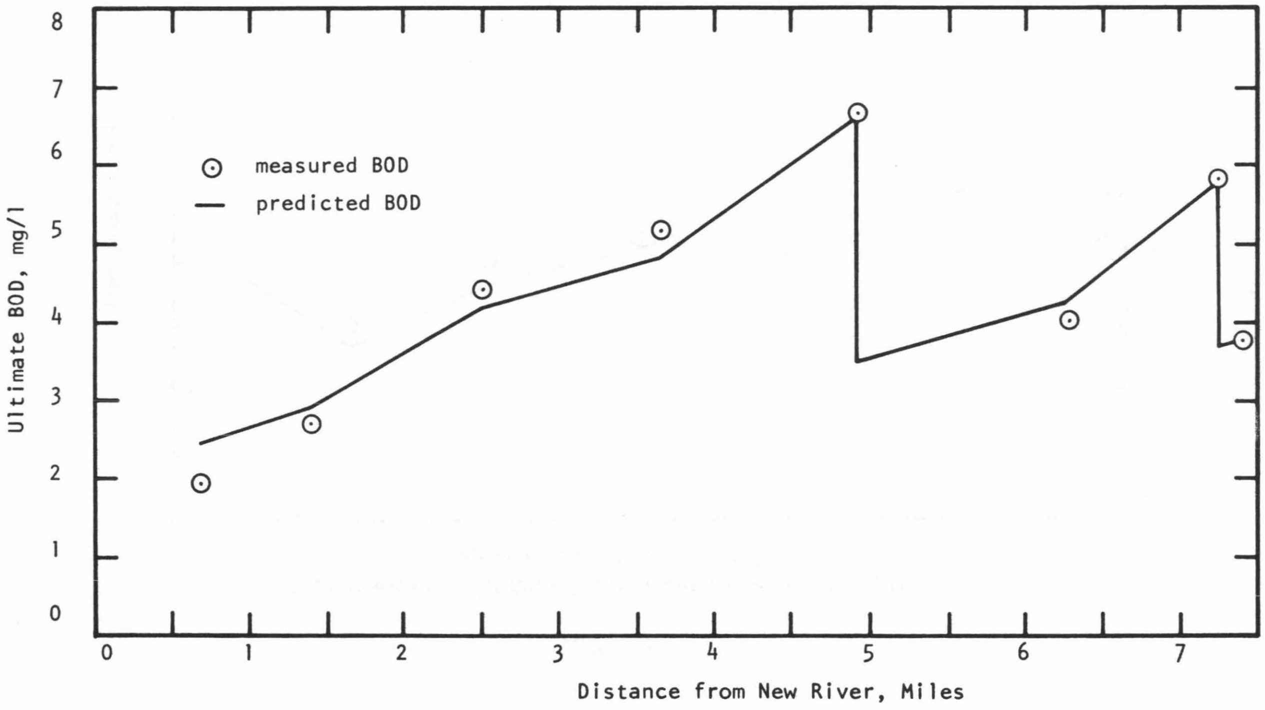


Figure 12

Computed DO profile for Stroubles Creek
under 7 day—10 year low-flow conditions

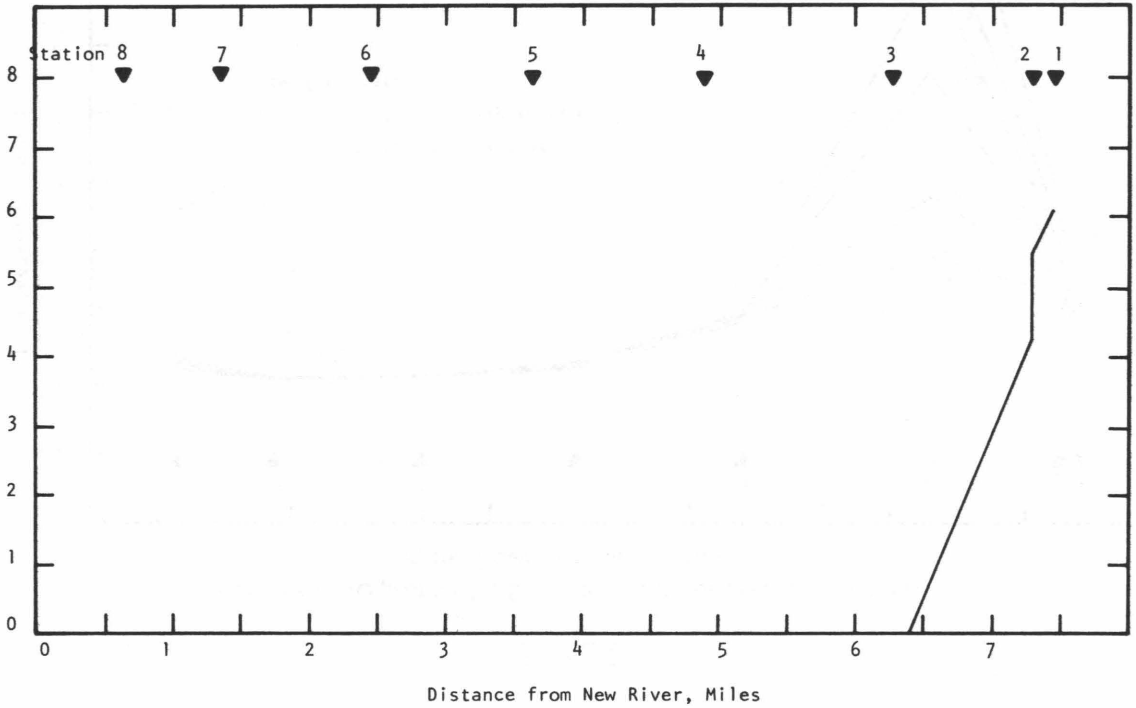


Figure 13

Computed DO profiles for Stroubles Creek for selected variations in the 7 day—10 year low flow

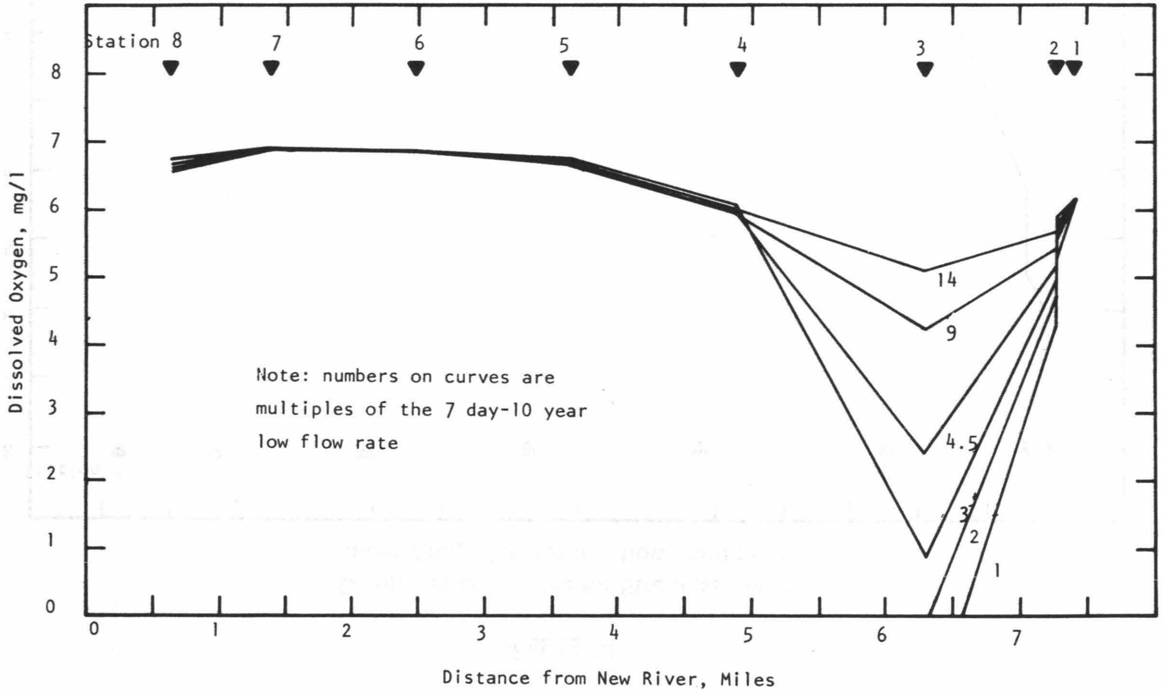


Figure 14

Effect of variations in stream depth on the dissolved oxygen concentration at Station 3 for 7 day—10 year low-flow conditions

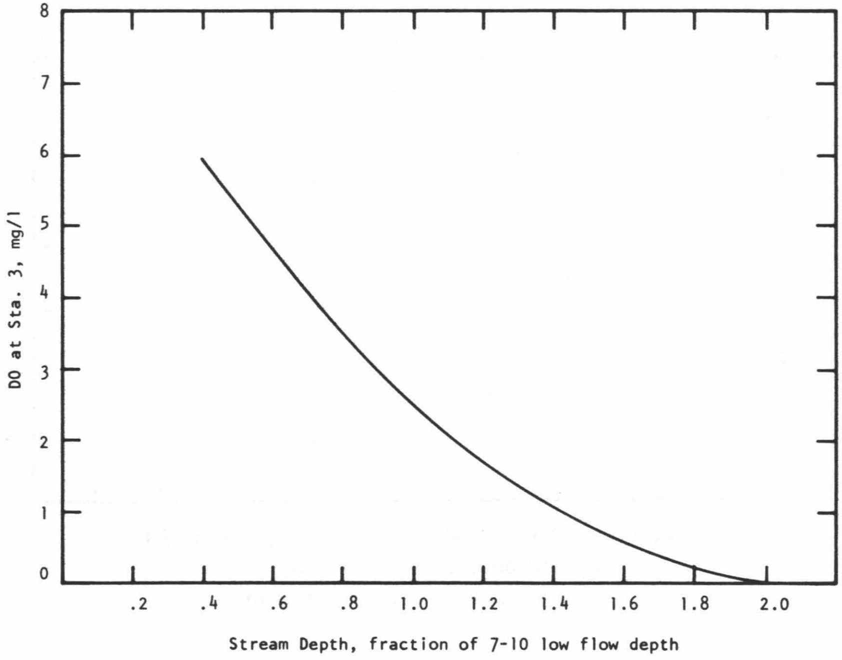


Figure 15

Effect of variations in stream velocity
on the dissolved oxygen concentration
at Station 3 for 7 day—10 year low-flow conditions

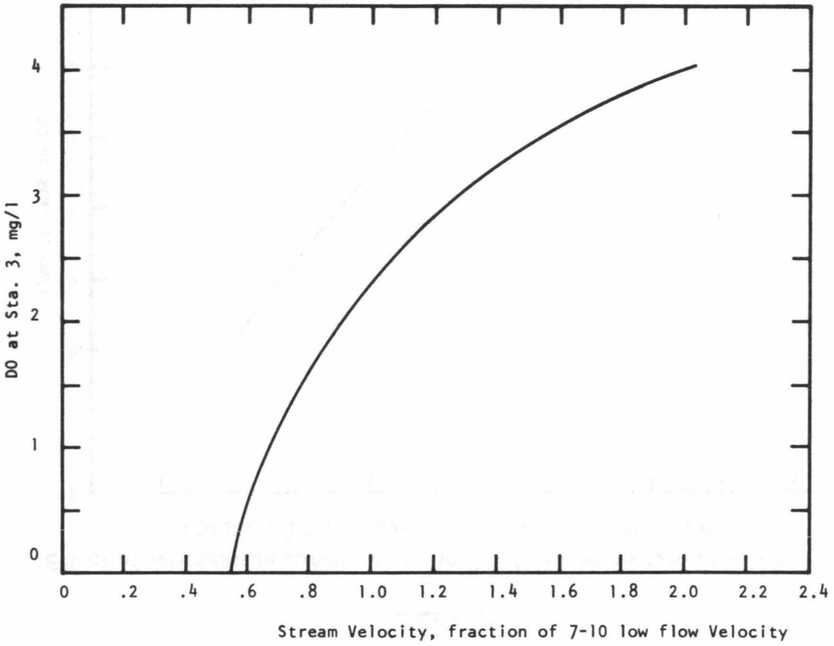


Figure 16

Effect of variations in sewage treatment plant effluent BOD_5
on the dissolved oxygen concentration
at Station 3 for 7 day—10 year low-flow conditions

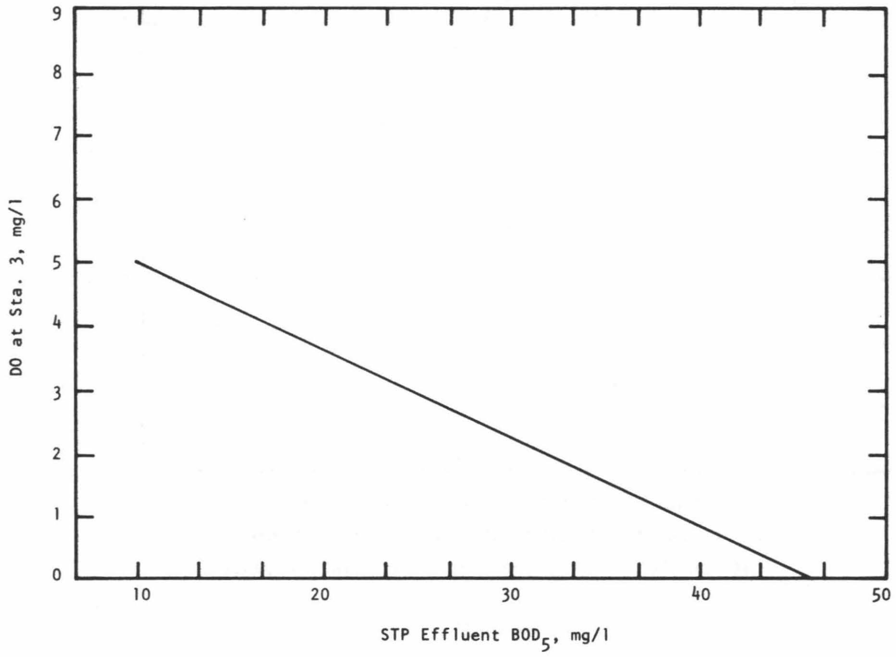


Figure 17

Effect of variations in the biological extraction rate
on the dissolved oxygen concentration at Station 3
for 7 day—10 year low-flow conditions

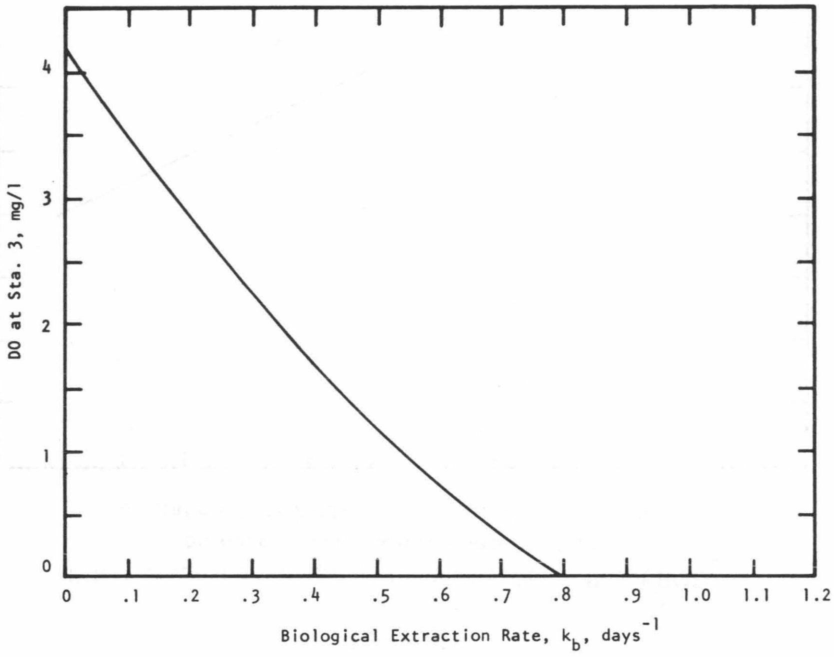


Figure 18

Effect of variations in the biochemical reaction rate
on the dissolved oxygen concentration
at Station 3 for 7 day—10 year low-flow conditions

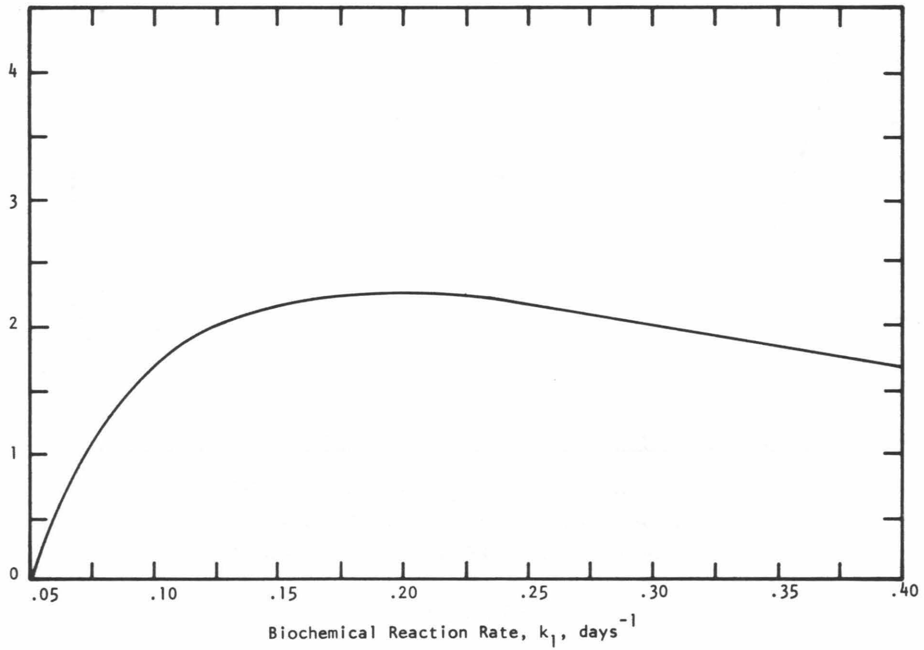


Figure 19

Effect of variations in sewage treatment plant effluent
on the dissolved oxygen concentration
at Station 3 for 7 day—10 year low-flow conditions

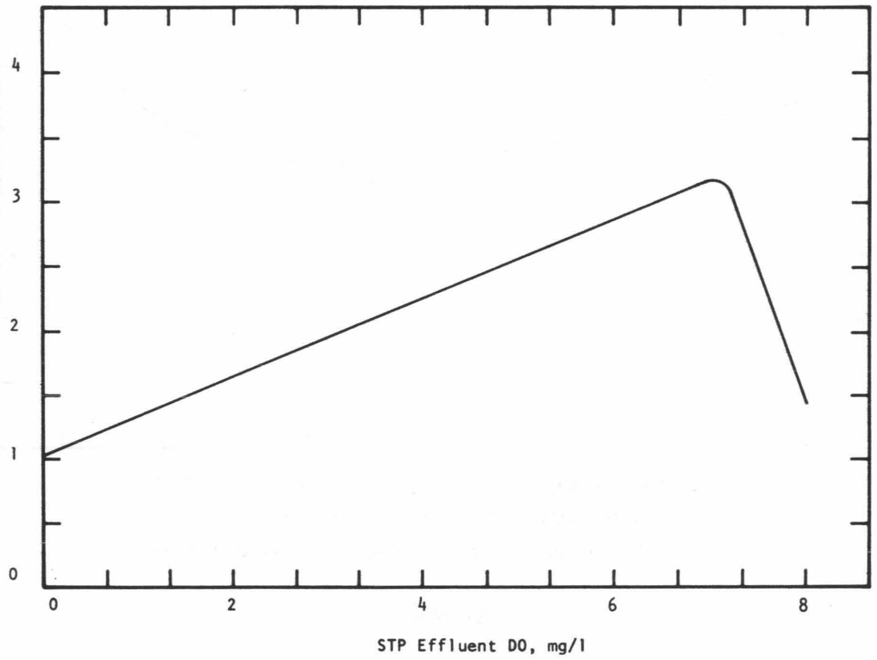


Figure 20

Effect of variations in stream temperature
on the dissolved oxygen concentration at Station 3
for 7 day—10 year low-flow conditions

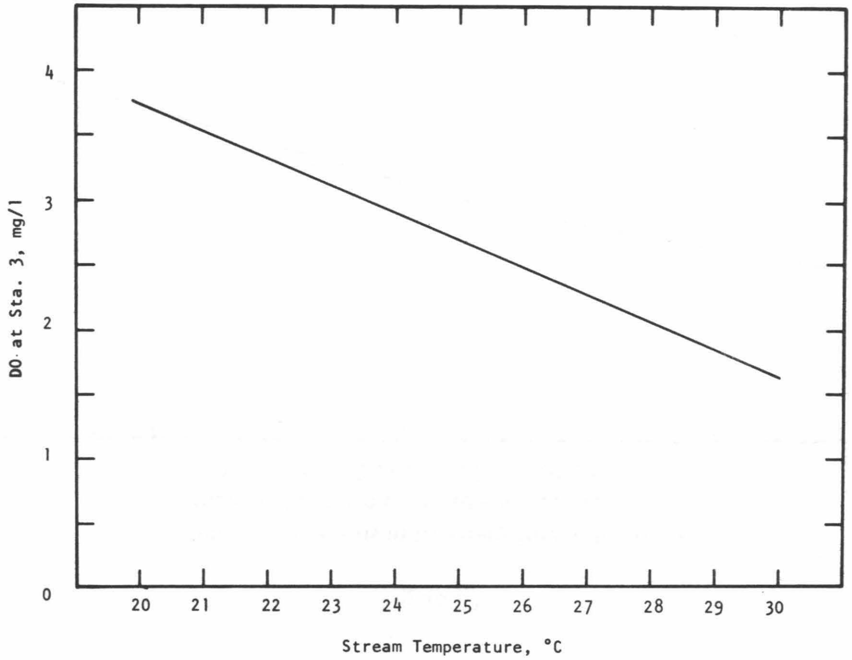
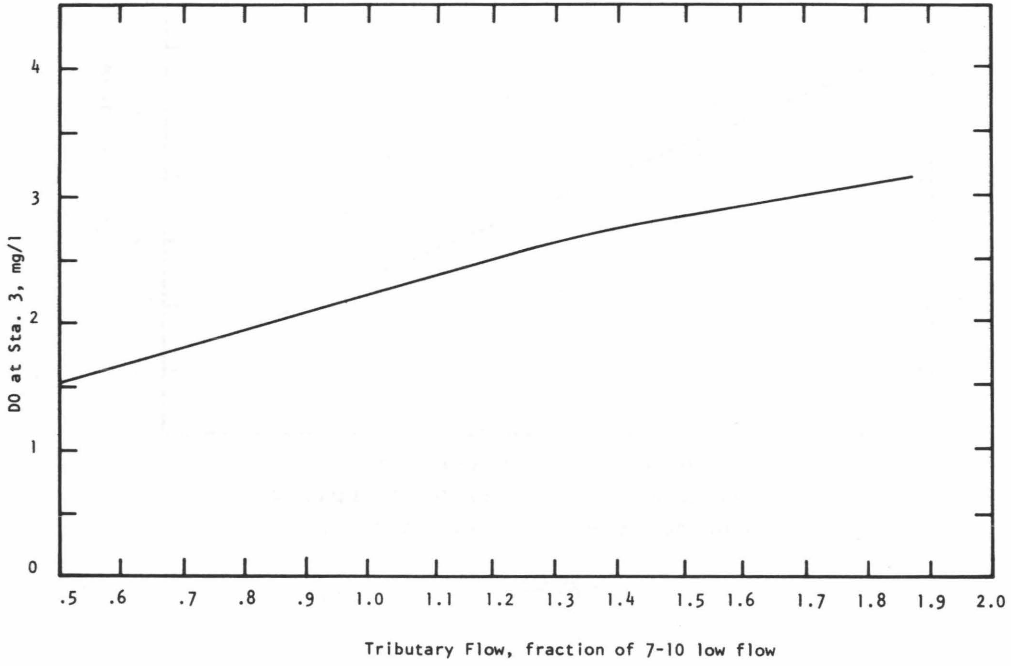


Figure 21

Effect of variations in tributary flow rate on the
dissolved oxygen concentration at Station 3
for 7 day—10 year low-flow conditions



APPENDIX A

TEXT NOTATION

The following notations are used in the text of this bulletin and are applicable to the equations stated:

BOD_5P = BOD_5 of raw waste, no./day;
CONC = effluent waste BOD_5 concentration, mg/l;
C_s = DO saturation, mg/l;
D = oxygen saturation deficit of the water, mg/l;
DAM = low profile dam effect coefficient, height in feet;
D_L = coefficient of molecular diffusion;
D_m = longitudinal mixing coefficient;
D_p = rate of sludge deposition;
E = energy dissipation per unit of mass per unit of time, sq. ft./cu. ft.;
Ev = elevation of station, ft.;
F = Froude number;
f = exchange coefficient;
g = gravitational constant;
h = mean water depth, ft.;
K_1 = biochemical reaction rate (deoxygenation coefficient) (base e), days^{-1};
K_2 = reaeration coefficient (base e), days^{-1};
K_3 = sludge deposit reaction rate (base e), days^{-1};
k_1 = biochemical reaction rate (deoxygenation coefficient) (base 10), days^{-1};
k_2 = reaeration coefficient (base 10), days^{-1};
k_3 = sludge deposit reaction rate (base 10), days^{-1};
k'_3 = sludge deposit reaction rate as defined by Thomas (base 10), days^{-1};
k_b = biological extraction rate (base 10), days^{-1};
k_n = nitrogenous reaction rate (base 10), days^{-1};
k_{4r} = river reaction rate (base 10), days^{-1};
L = initial ultimate oxygen demand of organic matter, mg/l;
PCT_b = percent DO saturation at point b;
p_m = maximum rate of photosynthetic oxygen production during the day, mg/l/day;
P-R = net photosynthetic oxygen production, mg/l/day;
p = rate of addition of BOD to the overlying water from the bottom deposits, mg/l/day;
p_1 = areal oxygen demand of sludge deposits in absence of settling, #/ft^2;
Q = flow rate, cfs;

QE = waste discharge, mgd;
R_n = net rate of removal of oxygen of benthal organisms, mg/l/day;
R_4 = algal respiration of dissolved oxygen;
S = oxidizability of sludge deposits;
Se = energy slope, ft/ft;
s = slope of channel, ft/ft;
s_c = slope of channel, ft/mi;
T = water temperature, °C;
TRT = fraction of BOD_5 remaining after treatment;
t = elapsed time, days;
t_D = time over which continuous sludge deposition can take place;
t_o = time of travel from x=0;
t_s = time of day at which solar radiation begins;
V = velocity of flow, ft/sec;
Y = mean relative increase in velocity per 5 ft increase in gauge height;
y = BOD utilized at time t, mg/l;
ϕ = fraction of the day over which solar radiation occurs;
σ = rate of production of DO by algae through photosynthesis, mg/l/day;
θ = thermal coefficient, and
μ = dynamic viscosity.

APPENDIX B

DO—BOD COMPUTER PROGRAM

VERSION I

```
C     DO AND BOD OF A STREAM     PROGRAM #1 - MOD 2  NO CONS
C     THIS PROGRAM DOES NOT USE CON1, CON2, OR CON3
      DIMENSION DO(150), BOD(150), FLOW(150), V(150), H(150)
      DIMENSION SAT(150), PCT(150), QE(150), BOD5P(150), DOE(150)
      DIMENSION P1(150), RR(150), TIME(150),R1A(150),R1B(150), R1C(150)
      DIMENSION QQE(150), TEMP(10,150), TITLE(13)
      DIMENSION PCTRB(150), BOD5T(150), QTRIB(150)
      DIMENSION DIST(150)
      DIMENSION TRT(150)
      DIMENSION A(150), B(150), C(150), D(150), ELEV(150)
      DIMENSION WTHDR(150), QWTHD(150)
      DIMENSION DAM(150)
      DIMENSION REDUC(150),CONC(150),BOD5E(150)
      NAMELIST /UPDATE/ A,B,C,D,DIST,ELEV,PCT,QE,BOD5P,DOE,P1,RR,R1A,
     1R1B,R1C,FLOW,PCTRB,BOD5T,QTRIB,WTHDR,THET,BOD5R,DAM,TEMP,TRT
C     NNN = NUMBER OF DATA POINTS READ IN OR NUMBER OF STATIONS
C     LMN = NUMBER OF TEMPERATURE SETS
      READ(5,910)   TITLE
  910 FORMAT(13A4)
      READ(5,1234)      NNN,LMN
 1234 FORMAT(2I5)
      READ(5,5) (A(I), I=1,NNN)
      READ(5,5) (B(I), I=1,NNN)
      READ(5,5) (C(I), I=1, NNN)
      READ(5,5) (D(I), I=1, NNN)
      READ(5,1) (DIST(I),   I = 1,NNN)
      READ(5,1) (ELEV(I), I = 1,NNN)
      READ(5,3) (PCT(I), I=1, NNN)
      READ(5,1) (QE(I)    , I = 1,NNN)
```

```
      READ(5,7) (BOD5P(I), I=1,NNN)                                     BOD  390
      READ(5,1) (DOE(I) , I=1,NNN)                                      BOD  400
      READ(5,6) (TRT(I), I=1,NNN)                                       BOD  410
      READ(5,1) (P1(I)    , I = 1,NNN)                                  BOD  420
      READ(5,1) (RR(I)    , I = 1,NNN)                                  BOD  430
      READ(5,1) (R1A(I) , I = 1,NNN)                                    BOD  440
      READ(5,3) (R1B(I), I=1, NNN)                                      BOD  450
      READ(5,3) (R1C(I), I=1,NNN)                                       BOD  460
      READ(5,1) (FLOW(I), I = 1, NNN)                                   BOD  470
      READ(5,3) (PCTRB(I), I=1,NNN)                                     BOD  480
      READ(5,1) (BOD5T(I), I = 1,NNN)                                   BOD  490
      READ(5,1) (QTRIB(I), I = 1,NNN)                                   BOD  500
      READ(5,1) (WTHDR(I), I = 1,NNN)                                   BOD  510
      READ(5,4) THET, BOD5R                                             BOD  520
      READ(5,1)  (DAM(I),I=1,NNN)                                       BOD  530
    1 FORMAT(16F5.1)                                                    BOD  540
    3 FORMAT(16F5.2)                                                    BOD  550
    4 FORMAT(16F5.3)                                                    BOD  560
    5 FORMAT(16F5.4)                                                    BOD  570
    6 FORMAT(20F4.2)                                                    BOD  580
    7 FORMAT(10F8.1)                                                    BOD  590
      DO 890 J=1,LMN                                                    BOD  600
      READ(5,2) (TEMP(J,I),I=1,NNN)                                     BOD  610
    2 FORMAT(16F5.1)                                                    BOD  620
  890 CONTINUE                                                          BOD  630
      WRITE(6,891)                                                      BOD  640
  891 FORMAT(1H1,10X,37HTHE ORIGINAL INPUT DATA IS AS FOLLOWS)          BOD  650
      WRITE(6,892)                                                      BOD  660
  892 FORMAT(//T3,'STA',T9,'A',T17,'B',T23,'C',T30,'D',T35,'DIST',T41,  BOD  670
```

```
     1'ELEV',T47,'PCT',T52,'QE',T59,' CONC',T70,'DOE',T76,'P1',T80,'RR')
      WRITE(6,893)
  893 FCRMAT(1X,T36,'MI',T42,'FT',T52,'MGD',T60,'LB/D',T70,'MG/L')
      DC 894 I=1,NNN
      WRITE(6,895) I,A(I),B(I),C(I),D(I),DIST(I),ELEV(I),PCT(I),QE(I),
     1 CONC(I),DOE(I),P1(I),RR(I)
  895 FCRMAT(1X,T4,I2,T7,F6.4,T14,F6.4,T21,F6.4,T28,F6.4,T35,F5.2,T41,
     1F5.0,T47,F4.2,T52,F5.2,T58,F9.1,T70,F3.1,T75,F4.2,T80,F4.2)
  894 CCNTINUE
      WRITE(6,896)
  896 FCRMAT(//T3,'STA',T8,'R1A',T14,'R1B',T19,'R1C',T24,'FLOW',T30,
     1'PCTRB',T37,'BOD5T',T44,'QTRIB',T51,'WTHDR',T60,'TRT',T70,'DAM')
      WRITE(6,897)
  897 FCRMAT(T25,'CFS',T37,'MG/L',T45,'CFS',T52,'MGD',T61,'%',T71,'FT')
      DC 898 I=1,NNN
      WRITE(6,899) I,R1A(I),R1B(I),R1C(I),FLOW(I),PCTRB(I),BOD5T(I),
     1QTRIB(I),WTHDR(I),TRT(I),DAM(I)
  899 FCRMAT(T4,I2,T7,F6.4,T14,F4.2,T19,F4.2,T24,F5.1,T31,F4.2,T37,F4.1,
     1T44,F5.1,T51,F6.2,T60,F4.2,T71,F4.1)
  898 CCNTINUE
  401 DC 340 J=1,LMN
      WRITE(6,1432)
 1432 FCRMAT(1H1,10X,22HDO AND BOD OF A STREAM)
      WRITE(6,911)  TITLE
  911 FCRMAT(10X,15A4)
C   ************************************************************
C   *                                                          *
C       CHANGE STATEMENT 2 TO HAVE NNN NUMBER FIELDS
C   *                                                          *
```

```
C  ****************************************************                BOD  970
      WRITE(6,8001)                                                    BOD  980
 8001 FORMAT(  //T3,'STA',T11,'VEL',T17,'HT',T22,'TIME',T30,' Q ',T37, BOD  990
     $'R2(K2)',T48,'DISTANCE',T62,'DO',T73,'BOD-ULT',T85,'R1-KR',T96,  BOD 1000
     #'TEMP',T106,'SUMTIM')                                            BOD 1010
      WRITE(6,8003)                                                    BOD 1020
 8003 FORMAT(1X,T11,'FPS',T22,'DAYS',T30,'CFS',T37,'@TEMP',T48,'TOTL-MI'BOD 1030
     $,T61,'MG/L',T74,'MG/L',T85,'@TEMP',T96,'CENT',T107,'HOURS')      BOD 1040
      XLED = 0.0                                                       BOD 1050
      DECD = 0.                                                        BOD 1060
      SUMTIM = 0.                                                      BOD 1070
      TODIST = 0.0                                                     BOD 1080
      BOD5R= 2.0                                                       BOD 1090
      TCP = PCT(1)                                                     BOD 1100
      DO 325 I = 1,NNN                                                 BOD 1110
      REDUC(I)=TRT(I)*BOD5P(I)                                         BOD 1120
      IF(QE(I) .LE. .0001) GO TO 19                                    BOD 1130
      CONC(I)=REDUC(I)/(8.34*QE(I))                                    BOD 1140
      BOD5E(I)=CONC(I)                                                 BOD 1150
      GO TO 30                                                         BOD 1160
   19 BOD5E(I)=1.0                                                     BOD 1170
   30 QQE(I)=1.547*QE(I)                                               BOD 1180
      QWTHD(I) = 1.547*WTHDR(I)                                        BOD 1190
      RR1AA = R1A(I)*(THET**(TEMP(J,I) - 20.0))                        BOD 1200
      RR1BB = R1B(I)*(THET**(TEMP(J,I) -20.0))                         BOD 1210
      RR1CC = R1C(I)*(THET**(TEMP(J,I) - 20.0))                        BOD 1220
      PCT(I) = TCP                                                     BOD 1230
      XLED= PCT(I)                                                     BOD 1240
      TCP = PCT(I)                                                     BOD 1250
```

```
      R3 = RR(I)                                                        BOD 1260
 8080 Q = FLOW(I) + QTRIB(I) + QQE(I) - QWTHD(I)                        BOD 1270
      SAT(I)=14.4*10.0**(-.0102*TEMP(J,I))*(1.00-.00003*ELEV(I))        BOD 1280
 3000 DO(I) = (FLOW(I)*SAT(I)*PCT(I) - QWTHD(I)*SAT(I)*PCT(I) + QQE(I)*DBOD 1290
     $OE(I) + QTRIB(I)*SAT(I)*PCTRB(I))/Q                               BOD 1300
 3004 IF(BOD5R-2.0)3006,3006,3008                                       BOD 1310
 3006 IF(DO(I)-0.95*SAT(I))3020,3020,3015                               BOD 1320
 3008 IF(DO(I) - 0.85*SAT(I)) 3020,3020,3010                            BOD 1330
 3010 DO(I) = 0.85*SAT(I)                                               BOD 1340
      GO TO 3020                                                        BOD 1350
 3015 DO(I)=0.95*SAT(I)                                                 BOD 1360
 3020 BOD5 =(BOD5R*FLOW(I) - BOD5R*QWTHD(I) + BOD5E(I)*QQE(I) + BOD5T(I)BOD 1370
     #*QTRIB(I))/Q                                                      BOD 1380
      V(I) = A(I)*Q**B(I)                                               BOD 1390
      H(I) = C(I)*Q**D(I)                                               BOD 1400
      TIME(I) = (.0613*DIST(I))/V(I)                                    BOD 1410
      RE = (5.0*V(I))/(H(I)**1.666)                                     BOD 1420
      BODA = BOD5/(1.0-(10.0**(-5.0*R1B(I))))                           BOD 1430
      IF(BODA-100.)3050,3050,3060
 3050 RE = RE*(1.0-0.006*BODA)                                          BOD 1450
      GO TO 3070                                                        BOD 1460
 3060 RE = 0.4*RE                                                       BOD 1470
 3070 R2 = RE*(1.016**(TEMP(J,I)-20.0))                                 BOD 1480
      R1 =RR1BB+RR1CC                                                   BOD 1490
  225 WRITE(6,8002) I,TODIST, DO(I), BODA, SUMTIM                       BOD 1500
 8002 FORMAT(1X,T2,I4,T46,F8.1,T56,F10.3,T70,F9.3,T104,F7.1)            BOD 1510
      WRITE(6,8004) V(I),H(I),TIME(I),Q,R2,R1,TEMP(J,I)                 BOD 1520
 8004 FORMAT(1X,T10,F5.1,T15,F5.1,T21,F6.3,T30,F5.0,T36,F8.4,T82,F8.3,  BOD 1530
     $T93,F7.1)                                                         BOD 1540
```

```
      FLOW(I+1) = Q                                                     BOD 1550
      DA = SAT(I) - DO(I)                                               BOD 1560
   61 IF(V(I) - 0.6) 75, 75, 70                                         BOD 1570
   70 R3 = 0.0                                                          BOD 1580
   75 P = (110.0*P1(I)/H(I)) + 2.303*R3*BODA                            BOD 1590
   90 R = R1 + R3                                                       BOD 1600
      TERM1 = 10.0**(-R*TIME(I))                                        BOD 1610
  145 IF(DA) 150,150,155                                                BOD 1620
  150 R2 = 0.0                                                          BOD 1630
      DA = 0.0                                                          BOD 1640
  155 TERM2 = 10.0**(-R2*TIME(I))                                       BOD 1650
      IF(R2) 161,161,162                                                BOD 1660
  161 FACT1 = -R1/R                                                     BOD 1670
      FACT2 = P/(2.303*R)                                               BOD 1680
      FACT4 = 0.0                                                       BOD 1690
      GO TO 170                                                         BOD 1700
  162 R4 = R2 - R                                                       BOD 1710
      IF(R4) 163,163,164                                                BOD 1720
  163 R4 = - R4                                                         BOD 1730
  164 IF(R4 - 0.001) 165,165,166                                        BOD 1740
  165 FACT1 = 0.0                                                       BOD 1750
      FACT2 = 0.0                                                       BOD 1760
      FACT4 = R1*P/(2.303*R2*R)                                         BOD 1770
      GO TO 170                                                         BOD 1780
  166 FACT1 = R1/(R2 -R)                                                BOD 1790
      FACT2 = P/(2.303*R)                                               BOD 1800
      FACT4 = R1*FACT2/R2                                               BOD 1810
  170 DB = FACT1*(BODA - FACT2)*(TERM1 - TERM2) + FACT4*(1.0 - TERM2) + BOD 1820
     1DA*TERM2                                                          BOD 1830
```

```
 171 IF( DAM(I) - 0.0 ) 178,178,172                                   BOD 1840
 172 IF ( DB .LE. 2.0 ) AD = 1.25                                     BOD 1850
     IF ( CB .GT. 2.0 .AND. DB .LE. 4.0 ) AD = 1.0                    BOD 1860
     IF ( DB .GT. 4.0 ) AD = 0.8                                      BOD 1870
 173 DR = 1.0 + 0.11 * AD * (1.0+0.46 * TEMP(J,I)) * DAM(I)           BOD 1880
 174 DS = DB/DR                                                       BOD 1890
 175 DB = DS                                                          BOD 1900
 178 IF ( SAT(I) - DB ) 4010,4011,4011                                BOD 1910
4010 DB = SAT(I)                                                      BOD 1920
4011 CONTINUE                                                         BOD 1930
        BOD(I) = (BODA - FACT2)*TERM1 + FACT2                         BOD 1940
     BODA = BOD(I)                                                    BOD 1950
     XLED = (SAT(I) - DB)/SAT(I)                                      BOD 1960
7081 TCP = XLED                                                       BOD 1970
 195 CONTINUE                                                         BOD 1980
     BOD5R = BODA*(1.0-10.0**(-(5.0*R1B(I))))                         BOD 1990
     IF(BOD5R-2.0)8005,8005,8006                                      BOD 2000
8005 BOD5R  = 2.0                                                     BOD 2010
8006 DECD = TIME(I)*24.                                               BOD 2020
     TODIST = TODIST + DIST(I)                                        BOD 2030
     SUMTIM = SUMTIM + DECD                                           BOD 2040
 325 CONTINUE                                                         BOD 2050
     WRITE(6,556) BODA;SUMTIM                                         BOD 2060
 556 FORMAT(1X,T45,'BODA(LAST) = ',F10.3,T85,'SUMTIM = ',F10.3)       BOD 2070
 340 CONTINUE                                                         BOD 2080
 400 READ(5,910,END=230)  TITLE                                       BOD 2090
     READ(5,UPDATE,ERR=402)                                           BOD 2100
     GO TO 401                                                        BOD 2110
 402 WRITE(6,403)  TITLE                                              BOD 2120
```

```
403 FORMAT('1',/////,10X,'**********  ERROR IN UPDATE **********',     BOD 2130
   1///,10X,'THE TITLE CARD WAS --   ',13A4)                            BOD 2140
    GO TO 400                                                           BOD 2150
230 CONTINUE                                                            BOD 2160
    STOP                                                                BOD 2170
```

APPENDIX C

DESCRIPTION OF MODEL VARIABLES

Description of Model Input Variables

Variable	Description
A	Coefficient in velocity-discharge relation $V = AQ^B$.
B	Coefficient in velocity-discharge relation $V = AQ^B$.
BOD5P	BOD_5 of raw waste entering treatment plant, lbs/day (input for Version I).
BOD5R	BOD_5 of receiving water, mg/l.
BOD5T	BOD_5 of the tributary, mg/l.
C	Coefficient in depth-discharge relation $H = CQ^D$.
CONC	BOD_5 of wastewater effluent, mg/l (input for Version II).
D	Coefficient of depth-discharge relation $H = CQ^D$.
DAM	Value used to account for the effect of a low profile dam, height in feet.
DIST	The distance between each station, miles.
DOE	Dissolved oxygen concentration of sewage treatment plant effluent, mg/l.
ELEV	Elevation at each station, feet.
FLOW	Flow rate of receiving stream above outfall, cfs.
LMN	Number of different temperature sets to be considered.
NNN	Number of stations to be read in.
P1	Areal oxygen demand of bottom deposits in absence of settling, lb/sq ft.
PCT	Fraction of Stream DO saturation at initial station.
PCTRB	Percent dissolved oxygen saturation of tributary.
QE	Mean daily effluent flow rate, mgd.
QTRIB	Flow rate contributed by tributary, cfs.
RR	Rate of settling of BOD from stream to bottom deposits, $days^{-1}$.
R1A	Laboratory first-day deoxygenation rate, $days^{-1}$.

R1B	Laboratory long-term Deoxygenation rate, days^{-1}.
R1C	Rate of biological extraction of oxygen in stream, days^{-1}.
TEMP	Stream temperature, °C.
TITLE	Title description for headings.
TRT	Fraction of the BOD$_5$ of the raw waste remaining after treatment.
WTHDR	The quantity of water withdrawn from the stream, mgd.

Description of Model Operating Variables

Variable	Description
AD	Correction for low profile dam depending on dissolved oxygen deficit.
BOD	Ultimate biochemical oxygen demand at a point in the stream below the outfall, mg/l.
BODA	Ultimate BOD in a particular reach of stream, mg/l.
BOD5	BOD_5 in stream, mg/l.
BOD_5E	Mean daily BOD_5 of effluent, mg/l.
DA	Dissolved oxygen deficit at upstream end of stream reach, mg/l.
DB	Dissolved oxygen deficit at downstream end of stream reach, mg/l.
DECD	Time of flow between two stations, hours.
DO	Dissolved oxygen at a point in the stream, mg/l.
DOB	Dissolved oxygen in stream before mix, mg/l.
DR	Correction applied to dissolved oxygen deficit for effects of a low profile dam, mg/l.
DS	Dissolved oxygen deficit corrected for effects of a low profile dam, mg/l.
FACT1	$(k_1 + k_b)/[k_2 - (k_1 + k_3 + k_b)]$
FACT2	$p/2.303\ (k_1 + k_3 + k_b)$.
FACT4	$(k_1 + k_b)\ p/2.303\ k_2\ (k_1 + k_3 + k_b)$.
H	Average stream depth, feet.
P	Total BOD of bottom deposits, sum of constant load + settling load, mg/l/day.
Q	Combined flow of effluent and stream, cfs.
QQE	The wastewater effluent flow rate, cfs.
QWTHD	Withdrawal rate from stream, cfs.
R	$k_1 + k_3 = k_b$ total rate of BOD satisfaction in stream, $days^{-1}$.
RE	Clean water reaeration rate, uncorrected for stream BOD load, $days^{-1}$.

REDUC	BOD_5 of wastewater effluent, lbs/day.
RR1AA	First day of 20°C laboratory deoxygenation rate adjusted for temperature, days^{-1}.
RR1BB	5-day 20°C laboratory deoxygenation rate adjusted for temperature, days^{-1}.
RR1CC	Rate of deoxygenation due to biological extraction adjusted for temperature, days^{-1}.
R1	Combined rate of deoxygenation by the biochemical reaction and by biological extraction, days^{-1}.
R2	Reaeration rate, corrected for temperature and stream BOD load, days^{-1}
R3	Rate of settling of BOD from stream to bottom deposits, days^{-1}.
R4	$k_2 - (k_1 + k_3 + k_b)$, test variable, days^{-1}.
SAT	Saturation concentration of oxygen in stream for given stream temperature, mg/l.
SUMTIM	Total time of flow between initial station and any other station, days.
TCP, XLED	
TERM1	$10^{-(k_1 + k_3 ++ k_b)t}$.
TERM2,	TERM2A
THET	Thermal coefficient for rate correction.
TIME	Time of flow, days.
TODIST	Distance between initial station and any other station, miles.
V	Stream velocity, fps.

APPENDIX D

DO–BOD COMPUTER PROGRAM

VERSION II

```
C     DO AND BOD OF A STREAM     PROGRAM #1 - MOD 2  NO CONS
C     THIS PROGRAM DOES NOT USE CON1, CON2, OR CON3
      DIMENSION DO(150), BOD(150), FLOW(150), V(150), H(150)
      DIMENSION SAT(150), PCT(150), QE(150), BOD5P(150), DOE(150)
      DIMENSION P1(150), RR(150), TIME(150),R1A(150),R1B(150), R1C(150)
      DIMENSION QQE(150), TEMP(10,150), TITLE(13)
      DIMENSION PCTRB(150), BOD5T(150), QTRIB(150)
      DIMENSION DIST(150)
      DIMENSION TRT(150)
      DIMENSION A(150), B(150), C(150), D(150), ELEV(150)
      DIMENSION WTHDR(150), QWTHD(150)
      DIMENSION DAM(150)
      DIMENSION REDUC(150),CONC(150),BOD5E(150)
      NAMELIST /UPDATE/ A,B,C,D,DIST,ELEV,PCT,QE,BOD5P,DOE,P1,RR,R1A,
     1R1B,R1C,FLOW,PCTRB,BOD5T,QTRIB,WTHDR,THET,BOD5R,DAM,TEMP,TRT
C     NNN = NUMBER OF DATA POINTS READ IN OR NUMBER OF STATIONS
C     LMN = NUMBER OF TEMPERATURE SETS
      READ(5,910)  TITLE
  910 FORMAT(13A4)
      READ(5,1234)      NNN,LMN
 1234 FORMAT(2I5)
      READ(5,5) (A(I), I=1,NNN)
      READ(5,5) (B(I), I=1,NNN)
      READ(5,5) (C(I), I=1, NNN)
      READ(5,5) (D(I), I=1, NNN)
      READ(5,1) (DIST(I),  I = 1,NNN)
      READ(5,1) (ELEV(I), I = 1,NNN)
      READ(5,3) (PCT(I), I=1, NNN)
      READ(5,1) (QE(I)    , I = 1,NNN)
```

```
      READ(5,7) ( CCNC(I), I=1,NNN)                                       BOD  390
      READ(5,1) (DOE(I) , I=1,NNN)                                        BOD  400
      READ(5,6) (TRT(I), I=1,NNN)                                         BOD  410
      READ(5,1) (P1(I)  , I = 1,NNN)                                      BOD  420
      READ(5,1) (RR(I)  , I = 1,NNN)                                      BOD  430
      READ(5,1) (R1A(I) , I = 1,NNN)                                      BOD  440
      READ(5,3) (R1B(I), I=1, NNN)                                        BOD  450
      READ(5,3) (R1C(I), I=1,NNN)                                         BOD  460
      READ(5,1) (FLOW(I), I = 1, NNN)                                     BOD  470
      READ(5,3) (PCTRB(I), I=1,NNN)                                       BOD  480
      READ(5,1) (BOD5T(I), I = 1,NNN)                                     BOD  490
      READ(5,1) (QTRIB(I), I = 1,NNN)                                     BOD  500
      READ(5,1) (WTHDR(I), I = 1,NNN)                                     BOD  510
      READ(5,4) THET, BOD5R                                               BOD  520
      READ(5,1)  (DAM(I),I=1,NNN)                                         BOD  530
    1 FCRMAT(16F5.1)                                                      BOD  540
    3 FCRMAT(16F5.2)                                                      BOD  550
    4 FCRMAT(16F5.3)                                                      BOD  560
    5 FCRMAT(16F5.4)                                                      BOD  570
    6 FCRMAT(20F4.2)                                                      BOD  580
    7 FCRMAT(10F8.1)                                                      BOD  590
      DC 890 J=1,LMN                                                      BOD  600
      READ(5,2) (TEMP(J,I),I=1,NNN)                                       BOD  610
    2 FCRMAT(16F5.1)                                                      BOD  620
  890 CCNTINUE                                                            BOD  630
      WRITE(6,891)                                                        BOD  640
  891 FCRMAT(1H1,10X,37HTHE ORIGINAL INPUT DATA IS AS FOLLOWS)            BOD  650
      WRITE(6,892)                                                        BOD  660
  892 FCRMAT(//T3,'STA',T9,'A',T17,'B',T23,'C',T30,'D',T35,'DIST',T41,    BOD  670
```

```
     1'ELEV',T47,'PCT',T52,'QE',T59,'BOD5P',T70,'DOE',T76,'P1',T80,'RR')BOD  680
      WRITE(6,893)                                                      BOD  690
  893 FORMAT(1X,T36,'MI',T42,'FT',T52,'MGD',T60,'LB/D',T70,'MG/L')      BOD  700
      DO 894 I=1,NNN                                                    BOD  710
      WRITE(6,895) I,A(I),B(I),C(I),D(I),DIST(I),ELEV(I),PCT(I),QE(I),  BOD  720
     1BOD5P(I),DOF(I),P1(I),RR(I)                                       BOD  730
  895 FORMAT(1X,T4,I2,T7,F6.4,T14,F6.4,T21,F6.4,T28,F6.4,T35,F5.2,T41,  BOD  740
     1F5.0,T47,F4.2,T52,F5.2,T58,F9.1,T70,F3.1,T75,F4.2,T80,F4.2)       BOD  750
  894 CONTINUE                                                          BOD  760
      WRITE(6,896)                                                      BOD  770
  896 FORMAT(//T3,'STA',T8,'R1A',T14,'R1B',T19,'R1C',T24,'FLOW',T30,    BOD  780
     1'PCTRB',T37,'BOD5T',T44,'QTRIB',T51,'WTHDR',T60,'TRT',T70,'DAM')  BOD  790
      WRITE(6,897)                                                      BOD  800
  897 FORMAT(T25,'CFS',T37,'MG/L',T45,'CFS',T52,'MGD',T61,'%',T71,'FT') BOD  810
      DO 898 I=1,NNN                                                    BOD  820
      WRITE(6,899) I,R1A(I),R1B(I),R1C(I),FLOW(I),PCTRB(I),BOD5T(I),    BOD  830
     1QTRIB(I),WTHDR(I),TRT(I),DAM(I)                                   BOD  840
  899 FORMAT(T4,I2,T7,F6.4,T14,F4.2,T19,F4.2,T24,F5.1,T31,F4.2,T37,F4.1,BOD  850
     1T44,F5.1,T51,F6.2,T60,F4.2,T71,F4.1)                              BOD  860
  898 CONTINUE                                                          BOD  870
  401 DO 340 J=1,LMN                                                    BOD  880
      WRITE(6,1432)                                                     BOD  890
 1432 FORMAT(1H1,10X,22HDO AND BOD OF A STREAM)                         BOD  900
      WRITE(6,911)  TITLE                                               BOD  910
  911 FORMAT(10X,15A4)                                                  BOD  920
C  ******************************************************               BOD  930
C  *                                                    *               BOD  940
C     CHANGE STATEMENT 2 TO HAVE NNN NUMBER FIELDS                      BOD  950
C  *                                                    *               BOD  960
```

```
C  ***************************************************************      BOD  970
      WRITE(6,8001)                                                     BOD  980
 8001 FCRMAT(  //T3,'STA',T11,'VEL',T17,'HT',T22,'TIME',T30,' Q  ',T37, BOD  990
     $'R2(K2)',T48,'DISTANCE',T62,'DO',T73,'BOD-ULT',T85,'R1-KR',T96,   BOD 1000
     #'TEMP',T106,'SUMTIM')                                             BOD 1010
      WRITE(6,8003)                                                     BOD 1020
 8003 FORMAT(1X,T11,'FPS',T22,'DAYS',T30,'CFS',T37,'@TEMP',T48,'TOTL-MI'BOD 1030
     $,T61,'MG/L',T74,'MG/L',T85,'@TEMP',T96,'CENT',T107,'HOURS')       BOD 1040
      XLED = 0.0                                                        BOD 1050
      DECD = 0.                                                         BOD 1060
      SUMTIM = 0.                                                       BOD 1070
      TCDIST = 0.0                                                      BOD 1080
      IF(BOD5R.LT.0.1) BOD5R=0.1                                        BOD 1090
      DC 325 I = 1,NNN                                                  BOD 1100
      IF(QE(I)  .LE. .0001) GO TO 19                                    BOD 1110
      BCD5E(I)=CONC(I)                                                  BOD 1120
      GC TO 30                                                          BOD 1130
   19 BCD5E(I)=1.0                                                      BOD 1140
   30 QQE(I)=1.547*QE(I)                                                BOD 1150
      QWTHD(I) = 1.547*WTHDR(I)                                         BOD 1160
      RR1AA = R1A(I)*(THET**(TEMP(J,I) - 20.0))                         BOD 1170
      RR1BB = R1B(I)*(THET**(TEMP(J,I) -20.0))                          BOD 1180
      RR1CC = R1C(I)*(THET**(TEMP(J,I) - 20.0))                         BOD 1190
      R3 = RR(I)                                                        BOD 1200
 8080 Q = FLOW(I) + QTRIB(I) + QQE(I) - QWTHD(I)                        BOD 1210
      SAT(I)=14.4*10.0**(-.0102*TEMP(J,I))*(1.00-.00003*ELEV(I))        BOD 1220
      IF(I.EQ.1) GO TO 3000                                             BOD 1230
      PCT(I)=DOB/SAT(I)                                                 BOD 1240
      WRITE(6,32) DOB,BODA                                              BOD 1250
```

```
   32 FORMAT(57X,4HBM= ,F6.3,4X,F9.3)                                   BOD 1260
 3000 DC(I) = (FLOW(I)*SAT(I)*PCT(I) - QWTHD(I)*SAT(I)*PCT(I) + QQE(I)*DBOD 1270
     $OE(I) + QTRIB(I)*SAT(I)*PCTRB(I))/Q                               BOD 1280
 3020 BOD5 =(BOD5R*FLOW(I) - BOD5R*QWTHD(I) + BOD5E(I)*QQE(I) + BOD5T(I)BOD 1290
     #*QTRIB(I))/Q                                                      BOD 1300
      V(I) = A(I)*Q**B(I)                                               BOD 1310
      H(I) = C(I)*Q**D(I)                                               BOD 1320
      TIME(I) = (.0613*DIST(I))/V(I)                                    BOD 1330
      RE = (5.0*V(I))/(H(I)**1.666)                                     BOD 1340
      BCDA = BOD5/(1.0-(10.0**(-5.0*R1B(I))))                           BOD 1350
      IF(BODA-100.)3050,3050,3060                                       BOD 1360
 3050 RE = RE*(1.0-0.006*BODA)                                          BOD 1370
      GC TO 3070                                                        BOD 1380
 3060 RE = 0.4*RE                                                       BOD 1390
 3070 R2 = RE*(1.016**(TEMP(J,I)-20.0))                                 BOD 1400
      R1 =RR1BB+RR1CC                                                   BOD 1410
  225 WRITE(6,8002) I,TODIST, DO(I), BODA, SUMTIM                       BOD 1420
 8002 FORMAT(1X,T2,I4,T46,F8.2,T56,F10.3,T70,F9.3,T104,F7.2)            BOD 1430
      WRITE(6,8004) V(I),H(I),TIME(I),Q,R2,R1,TEMP(J,I)                 BOD 1440
 8004 FORMAT(1X,T10,F5.2,T15,F5.2,T21,F6.4,T30,F5.2,T36,F8.4,T82,F8.3,  BOD 1450
     $T93,F7.1)                                                         BOD 1460
      FLOW(I+1) = Q                                                     BOD 1470
      DA = SAT(I) - DO(I)                                               BOD 1480
   61 IF(V(I) - 0.6) 75, 75, 70                                         BOD 1490
   70 R3 = 0.0                                                          BOD 1500
   75 P = (110.0*P1(I)/H(I)) + 2.303*R3*BODA                            BOD 1510
   90 R = R1 + R3                                                       BOD 1520
      TERM1 = 10.0**(-R*TIME(I))                                        BOD 1530
      TERM2A=10.0**(-R2*TIME(I))                                        BOD 1540
```

```
  145 IF(DA) 150,150,155                                                BOD 1550
  150 R2 = 0.0                                                          BOD 1560
  155 TERM2 = 10.0**(-R2*TIME(I))                                       BOD 1570
      IF(R2) 161,161,162                                                BOD 1580
  161 FACT1 = -R1/R                                                     BOD 1590
      FACT2 = P/(2.303*R)                                               BOD 1600
      FACT4 = 0.0                                                       BOD 1610
      GO TO 170                                                         BOD 1620
  162 R4 = R2 - R                                                       BOD 1630
      IF(R4) 163,163,164                                                BOD 1640
  163 R4 = - R4                                                         BOD 1650
  164 IF(R4 - 0.001) 165,165,166                                        BOD 1660
  165 FACT1 = 0.0                                                       BOD 1670
      FACT2 = 0.0                                                       BOD 1680
      FACT4 = R1*P/(2.303*R2*R)                                         BOD 1690
      GO TO 170                                                         BOD 1700
  166 FACT1 = R1/(R2 -R)                                                BOD 1710
      FACT2 = P/(2.303*R)                                               BOD 1720
      FACT4 = R1*FACT2/R2                                               BOD 1730
  170 DB = FACT1*(BODA - FACT2)*(TERM1 - TERM2) + FACT4*(1.0 - TERM2) + BOD 1740
     1DA*TERM2A                                                         BOD 1750
  171 IF( DAM(I) - 0.0 ) 178,178,172                                    BOD 1760
  172 IF ( DB .LE. 2.0 ) AD = 1.25                                      BOD 1770
      IF ( DB .GT. 2.0 .AND. DB .LE. 4.0 ) AD = 1.0                     BOD 1780
      IF ( DB .GT. 4.0 ) AD = 0.8                                       BOD 1790
  173 DR = 1.0 + 0.11 * AD * (1.0+0.46 * TEMP(J,I)) * DAM(I)            BOD 1800
  174 DS = DB/DR                                                        BOD 1810
  175 DB = DS                                                           BOD 1820
  178    BOD(I) = (BODA - FACT2)*TERM1 + FACT2                          BOD 1830
```

```
      BODA = BOD(I)                                                     BOD 1840
      DCB=SAT(I)-DB                                                     BOD 1850
  195 CCNTINUE                                                          BOD 1860
      BOD5R = BODA*(1.0-10.0**(-(5.0*R1B(I))))                          BOD 1870
      IF(BOD5R-0.1)8CC5,8005,8006                                       BOD 1880
 8005 BOD5R  = 0.1                                                      BOD 1890
 8006 DECD = TIME(I)*24.                                                BOD 1900
      TODIST = TODIST + DIST(I)                                         BOD 1910
      SUMTIM = SUMTIM + DECD                                            BOD 1920
  325 CCNTINUE                                                          BOD 1930
      WRITE(6,556) BODA,SUMTIM                                          BOD 1940
  556 FCRMAT(1X,T45,'BODA(LAST) = ',F10.3,T85,'SUMTIM = ',F10.3)        BOD 1950
  340 CCNTINUE                                                          BOD 1960
  400 READ(5,910,END=230)  TITLE                                        BOD 1970
      READ(5,UPDATE,ERR=402)                                            BOD 1980
      GC TO 401                                                         BOD 1990
  402 WRITE(6,403)  TITLE                                               BOD 2000
  403 FCRMAT('1',/////,10X,'**********  ERROR IN UPDATE  **********',   BOD 2010
     1///,10X,'THE TITLE CARD WAS  --   ',13A4)                         BOD 2020
      GC TO 400                                                         BOD 2030
  230 CCNTINUE                                                          BOD 2040
      STOP                                                              BOD 2050
      END                                                               BOD 2060
```

VIRGINIA WATER RESOURCES RESEARCH CENTER
BLACKSBURG, VIRGINIA 24061