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MILLIMETER-WAVE SIGNATURE OF STRANGE MATTER STARS

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ABSTRACT

One of the most important questions in the study of compact objects is the nature of pulsars, including whether they consist of neutron star matter or strange quark matter (SQM). However, few mechanisms for distinguishing between these two possibilities have been proposed. The purpose of this Letter is to show that a strange star (one made of SQM) will have a vibratory mode with an oscillation frequency of approximately 250 GHz (millimeter wave). This mode corresponds to motion of the center of the expected crust of normal matter relative to the center of the strange quark core, without distortion of either. Radiation from currents generated in the crust at the mode frequency would be an SQM signature. We also consider effects of stellar rotation, estimate power emission and signal-to-noise ratio, and discuss briefly the particularly important, but unsolved, question of possible mechanisms for exciting the mode.

Subject headings: elementary particles — pulsars: general — stars: neutron — stars: oscillations

1. INTRODUCTION

Witten (1984) pointed out that strange quark matter (SQM) composed of roughly equal numbers of up, down, and strange quarks is more likely to be stable than nonstrange quark matter (which would have only up and down quarks but is known to be unstable). This is because conversion to strange quarks (for $m_s < m_N/3$) lowers the Fermi energy. This fact was known to others (Bodner 1971; Friedman & McLerran 1978). Witten, however, went on to suggest that nuggets of strange quark matter could be produced in phase transitions in the early universe or in supernova explosions and gave a possible scenario for the former, modified versions of which are still under debate (see, e.g., Cottingham, Kalafatis, & Vinh Mau 1994). Witten raised the possibility that such nuggets could solve the cosmological dark matter problem by evading the bound on the cosmological baryon density from the abundance of primordial deuterium. Farhi & Jaffe (1984) considered in some detail the properties of such nuggets as a function of nucleon number. De Rujula & Glashow (1984) considered terrestrial effects from incident strange quark nuggets.

Alcock, Fahri, & Olinto (1986) discussed the conversion of neutron stars to strange stars. The subject was reviewed thoroughly at a 1991 conference (Madsen & Haensel 1991). The consensus is that if SQM is stable, all "neutron stars" should in reality be strange quark stars. In a recent paper, Kettner et al. (1995, hereafter KWWG) have explored the nature of strange quark stars in further detail and computed important properties at nonzero temperatures; this Letter takes KWWG as its point of departure.

The object of this Letter is to speculate on a possible millimeter-wave radio signal that might be a signature of a strange quark star. Because of the fact that there are few observable differences between classical neutron stars and strange quark stars, such a signal could aid in identifying strange quark stars. As far as we are aware, to date only the existence of pulsars with shorter periods than permitted for classical neutron stars (Frieman & Olinto 1989; Glendenning 1989) and differences in cooling rates (Benvenuto & Vucetich 1991) have been discussed in detail in the literature. While our proposal is speculative, the impact of detecting strange quark matter would be so great that we believe it important to raise it for discussion and, hopefully, for observational efforts. Our proposal addresses the case generally contemplated by most workers that the strange quark core is surrounded by a crust of normal matter on the order of $10^{-5}\ M_\odot$. We note that, for the case in which the core is almost nude except for an atmosphere of normal matter of much smaller mass, Usov (1997) has recently proposed an X-ray signal. In § 2, we consider the frequency of the vibrational mode in which the crust of hadronic matter vibrates as a single entity, without distortion, with respect to the strange quark core. Radio waves generated from this vibration constitute our proposed signal. In § 3, the effects of the rotation of the strange star are included. In § 4, we estimate the power that might be radiated if such a mode were excited and the detectability of the resulting signal. In the discussion in § 5, we address the question of exciting the mode; it would be very hard to find a transient signal, but we show that there are directions in which one might look for mechanisms that would continuously feed energy into the mode.

The essential features of the core-crust system follow. (1) One is a strange quark core of roughly $A/3$ each of u, d, and s quarks (where A is the total baryon number) with a sharp boundary on the order of a fermi. The boundary is sharp because the core is bound by the strong force, not gravity. (2) Another feature is an electron gas extending a few hundred fermis beyond the core. The electron abundance, which is on the order of $10^{-4}A$, would be zero if the strange quark mass were essentially zero, as is true for up and down quarks, rather than the estimated 100–300 MeV c^{-2}. The electron gas beyond the core is held by, and accompanied by, a strong positive electric field resulting from the net positive charge on the sharp core; it is the gradient of a megavolt range potential. (3) The final essential feature is a hadron crust. Nonstrange matter attracted gravitationally by the core has its electrons repelled by the Pauli pressure of the electron gas, and its ions are repelled

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by the electric field. Neutrons suffer neither of these repulsions. A crust can therefore accumulate until it becomes energetically favorable for neutrons to leave nuclei at the base of the crust and to "drip" into the core.

The mass of the crust is bounded by $M_C \leq 10^{-5}\ M_\odot$ for SQM star mass, $M_Q \simeq M_\odot$. KWWG note that the electrostatic potential inside the strange quark core is $eV(r) = \mu_e(r)$, where μ_e is the chemical potential for which they solve numerically along with the quark chemical potential. They choose specific values for the mass of the strange quark (150 MeV) and the MIT bag constant (50 MeV fm^{-3}), which parameterizes quark confinement in QCD. They find, using local charge neutrality, $eV(r)$, near the surface ($r = R$) of the core, about 18.5 MeV for zero temperature, with quadratic corrections for finite temperature bringing $eV(r)$ near the surface down by about 0.5 MeV at $T = 50$ MeV. Using global charge neutrality just at the surface, $eV(r)$, for zero temperature, falls to $\frac{3}{4}$ of its (nearby) interior value; it falls to about half the interior value for $T =$ 50 MeV. KWWG solve Poisson's equation in the gap between the core and the hadron crust. They show that the width of the gap is on the order of a few hundred fermis. More precisely, they show

$$eV(r) = \frac{C}{r - R + r_0}, \qquad R < r < R_C, \tag{1}$$

with $r_0 \equiv C/\mathrm{eV}(R)$ and $C = (3\pi/2)^{1/2}/e = 5 \times 10^3$ MeV fm^{-1} $= 8.5 \times 10^{-16}$ ergs cm^{-1}.

2. VIBRATIONS

Figure 1 shows the centers of the core and crust displaced along the polar axis by $\xi < \Delta_G$, where Δ_G is the width of the gap. We need to compute the restoring force. First, we note that for $\xi = 0$, the electrostatic repulsion and gravitational attraction balance. The electrostatic repulsion pressure is given by $P_{\rm el} = Ze\eta_A C/(\Delta_G + r_0)^2$, where η_A is the number of ions per unit area at the base of the crust and Ze is their average charge. The gravitational attraction pressure is $P_G = GM_Q M_C/4\pi R^4$. For $\xi \ll (\Delta_G + r_0)$, we have $r^2 = R_C^2 \sin^2\phi + (R_C\cos\phi - \xi)^2$, $r \simeq R_C - \xi\cos\phi$, and so

$$\begin{aligned} F_z(\xi) &= \left| \xi Ze\eta_A C \int_0^\pi 2\pi \sin\phi d\phi \frac{d^2}{dz^2}(r - R_Q + r_0)^{-1} \right|_{z=0} \\ &= -\frac{8\pi}{3}\frac{\xi Ze\eta_A C}{(\Delta_G + r_0)^3} \\ &= -2/3\xi \frac{GM_Q M_C}{R^2}(\Delta_G + r_0)^{-1}. \end{aligned} \tag{2}$$

The result is

$$\omega^2 = \frac{2GM_Q}{3R^2(\Delta_G + r_0)}. \tag{3}$$

For a strange star at zero temperature with a maximal crust, we have $\Delta_G \sim 200$ fm and $r_0 \sim 300$ fm. From equation (3), we can see that $\nu_0 \simeq 2.5 \times 10^{11}$ Hz and $\lambda = 1.2$ mm. If the temperature rises to 50 MeV, $V(r)$ falls by 25% according to KWWG, so we then have $\nu_{50} = 2.6 \times 10^{11}$ Hz and $\lambda_{50} =$ 1.4 mm.

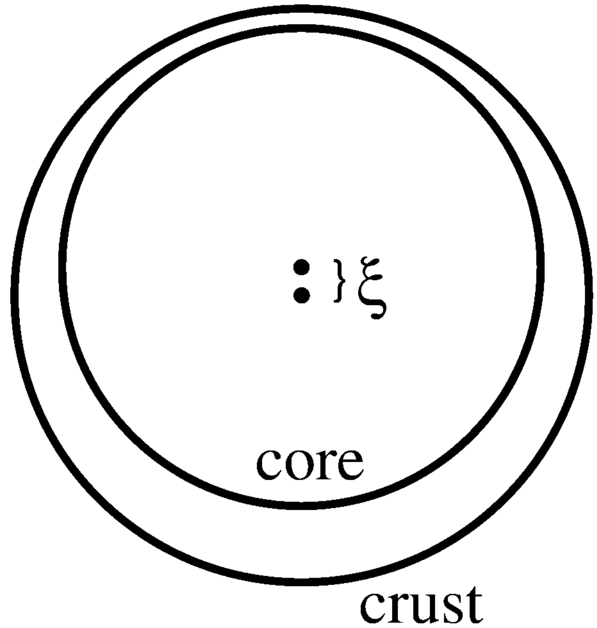

FIG. 1.—Centers of the core and crust being displaced. The resulting oscillation has a frequency of approximately 250 GHz, leading to the millimeter-wave signal discussed in the text.

We can also ask how ν varies if M_C is reduced from its maximum value when $\Delta_G \sim 200$ fm and $\rho \simeq 4.3 \times 10^{11}$ g cm^{-3}. The equality of $P_{\rm el}$ and P_G gives

$$(\Delta_G + r_0)^2 = R^4\beta^2, \tag{4}$$

where $\beta = (4\pi ZeC\eta_A/GM_Q M_C)^{1/2}$ and $\eta_A = (\rho/Am_p)^{2/3}$.

In the expression for η_A, the quantity A is the average atomic number and m_p is the mass of the proton. The dependence of ρ on M_C can be found by relating ρ to the pressure at the base of the crust in equation (4) by means of the equation of state. Using the results of Harrison et al. (1965) and of Baym, Pethick, & Sutherland (1971) as discussed in Shapiro & Teukolsky (1983), we have, for $10^8 < \rho < 4.3 \times 10^{11} = \rho_c$, by interpolating from the numerical results, $\rho \simeq \rho_c(P/P_c)^{5/6}$ and $P_c =$ $10^{29.5}$ dyn cm^{-2}. Inserting into equation (4), we see that $(\Delta_G + r_0) \propto M_C^{-2/9}$, and hence $\lambda \propto M_C^{-1/9}$. Thus, a decrease in M_C by a factor of 100 increases the wavelength by a factor of only 1.7.

We thus find that a low-temperature crust of maximal mass should exhibit a signal at about 1.2 mm; the wavelength increases with increasing temperature and decreasing crust mass. A 50 MeV crust with 1% of the maximal mass would have a wavelength of about 2.4 mm. We note that the lower bound on the wavelength (1.2 mm) is well above the region in which the atmosphere becomes opaque and that the effects on the above analysis of Pauli electron pressure can be shown to be small.

In summary, for strange quark star crusts of mass M_C in the range $10^{-7}M_Q < M_C < 10^{-5}M_Q$ with temperatures below about 50 MeV, the crust-core system has a normal mode corresponding to a wavelength, λ, roughly in the region 1.2–2.4 mm.

3. ROTATIONAL EFFECTS

Any real strange star will be rotating. One expects rotational periods ranging from milliseconds to a few seconds. This should have two effects on the millimeter-wave signature. The first is simply Doppler broadening—a 10 km diameter star rotating at 1 Hz would make the signal bandwidth $25 \sin\theta$ MHz, where θ is the angle between the rotation axis and the observer, at an observing frequency of 250 GHz. The bandwidth clearly scales proportional to the frequency. The second effect is caused by the fact that the rotation will cause the star to become oblate, leading to two normal modes of oscillation and thus splitting the signal. This is similar to the giant resonance mode in nuclei, in which the mode is split into two modes in nuclei with spin. In this section, we calculate the frequency splitting.

For a nonrotating strange star, the zero-temperature equation of state is $P = \frac{1}{3}(\rho - 4B)$, where B is the bag constant. This equation is inserted into the Oppenheimer-Volkov (OV) equation of hydrostatic equilibrium

$$\frac{d\rho}{dr} = -\frac{(\rho + P)(m + 4\pi r^3 P)}{r(r - 2m)}, \tag{5}$$

where we use units of $c = G = 1$ and m is the mass inside the radius r (and $2m$ is its Schwarzschild radius). The result of integrating the OV equation (Alcock, Fahri, & Olinto 1986) gives the structure of the star. If it is rotating, there will be an additional centrifugal pressure term added to P in the OV equation, $P \rightarrow P - \frac{1}{2}\rho\omega^2 r^2 \sin^2\theta$. Integrating the OV equation again with this term along the polar and equatorial directions gives the polar and equatorial radii, and the resulting eccentricity, as a function of the angular velocity of the star, f. We find that the eccentricity is given by $0.06(f/1000\ \mathrm{Hz})^2$. These results were obtained in the approximation of small eccentricity. At higher order, considerations of the mass distribution within the star, relativistic corrections, the difference between the shape of the inner edge of the crust and that of the core, and the like, must be included. These results are given for a strange star central density of $5.5B$, where B is the bag constant; the results will vary be roughly a factor of 2 for the expected range of this quantity.

The strange star is now an oblate spheroid. One will find two normal modes, corresponding to vibrations in the polar and equatorial directions. We find that (again, to leading order in the eccentricity)

$$\Delta\omega = \left(\frac{1}{R^2_{\mathrm{polar}}} - \frac{1}{R^2_{\mathrm{equat}}}\right)\frac{2GM}{3\beta^2}. \tag{6}$$

This frequency splitting, $\Delta\omega$, is then given by $15(f/1000\ \mathrm{Hz})^2$ GHz. The Doppler broadening is given by $25(f/1000\ \mathrm{Hz})\sin\theta$ GHz, where θ is the angle of the observer relative to the equatorial plane. We can see that for strange stars with periods of the order of seconds, the frequency splitting is negligible relative to the Doppler broadening, whereas for strange stars with periods of the order of milliseconds, the two are of the same order of magnitude. For such stars, this splitting would be a significant indicator of a strange star origin of a narrow line, millimeter-wave signal from a pulsar.

It should also be noted that if the pulsar is in a binary system, the tidal distortion would also cause a splitting of the mode, and the above equation would apply. (If the pulsar were rapidly rotating as well, the deformation would be quadrupolar.) The calculation of the tidal force (ignoring rotation) is straightforward, and it is easy to show that the results are identical with ω^2 replaced by $\omega_0^2/4$, where ω_0 is the frequency of revolution. Since the frequency of revolution, for all realistic pulsars, is much less than the frequency of rotation, the effects of tidal distortion will be negligible.

4. RADIATION AND DETECTABILITY

The energy stored in the vibrational mode of § 2 is given by $E \sim \frac{1}{2} M_C \xi^2 \omega^2$. Whereas the frequency of the mode is nearly independent of crust mass (varying roughly as $M_C^{1/9}$), the energy stored goes roughly as the 11/9 power. For $M_C \sim 10^{-5}\ M_\odot$ and $\xi \sim 200$ fm, E is on the order of 10^{31} ergs. We estimate the radiation rate in a simple model. When the crust center of mass is displaced downward, relative to the core center of mass by ξ, as shown in Figure 1, the electrostatic potential at the "top" rises by $-\xi\,(dV/dx) > 0$, and the potential at the "bottom" falls by the same amount. However, the crust is an equipotential, made from a material of very high conductivity, although likely not a superconductor, so charge must flow to cancel this change. Consider the "flat star approximation" in which the crust consists of two parallel planes, each of radius R and with separation R. To maintain the equipotential, a sheet of charge density σ must flow with current $I = \omega Q$, $\Delta V = 4\pi\sigma R = 2\xi\,(dV/dz)$, and $Q = \pi R^2 \sigma = R\,(\Delta V/4)$, giving radiation power on the order of

$$P = dE/dt = I^2/2c \simeq (\omega\xi R V')^2. \tag{7}$$

This expression will only be valid for temperature not too much smaller than the 5 MeV Fermi momentum of the electrons in the crust. For very small values of T, we would expect the radiation rate to fall like $(T/p_F)^3$, as Pauli blocking makes electrons (and holes) unable to radiate at frequency ν. For $\xi V' \sim 10$ MeV e^{-1}, equation (7) gives $P \sim (10\ eV\ e^{-1})^2 (R\omega)^2/c \sim 10^{34}$ ergs s^{-1}, which is a very large signal. This rate is reduced to the extent that radiation from electrons not at the surface either will not occur or will be absorbed and simply heat the crust. One rough estimate of this reduction would be to assume that the charge is spread evenly throughout the crust and, hence, reduce the intensity of the radiation by $\lambda/\Delta_C \sim 10^{-5}$, where $\Delta_C \sim 100$ m is the crust thickness. However the crust conductivity is so high, $\sigma \sim 10^{25}$ s^{-1} (Pethick & Sahrling 1995), that the skin depth is far less than the wavelength. For a conservative estimate of this effect, we take the geometric mean between λ and λ/Δ_C, giving $P_{\mathrm{rad}} \sim 10^{-3}P$. Thus, a rough power estimate might be 10^{31} ergs s^{-1} for maximal excitation of the mode and for a crust of maximum thickness. The power, but not the decay time, would scale with the energy in the mode. Roughly, the decay time would scale with the thickness of the crust, and the power radiated would scale inversely.

At a distance D in kiloparsecs from the pulsar, the flux density of radiation emitted at the rate of P Watts spread over a frequency f (Hz) is $S = 8.3 \times 10^{-15}(P/fD^2)$ Jy, where 1 Jy is 10^{26} W m^{-2} Hz^{-1}. A 10 m submillimeter telescope operating at a frequency of 250 GHz in good conditions near zenith has an rms noise of $\Delta S = 7000/(ft)^{1/2}$ Jy, where the integration time, t, is in seconds. The signal-to-noise ratio for a continuous signal is $S/\Delta S = 1.2 \times 10^{-18}\, P\,(t/f)^{1/2}\, 1/D^2$. For a pulsar of period 1 s and a consequential Doppler broadening of ∼25 MHz (at an observing frequency of 250 GHz) emitting a 10^{24} W signal, a 15 s integration time yields a signal-to-noise ratio of $\sim 10^3 D^{-2}$. A detectable signal (5 σ) could be achieved for pulsars as far away as 15 kpc (there are more than 100 with periods in excess of 1 s). For a pulsar as close as 1 kpc (there are about 10 with periods longer than 1 s), a signal of

10^{29} ergs s^{-1} could be detected. The strength of the signal is not the problem for detection; the problem is not knowing the frequency. Receivers in the 250 GHz range have instantaneous bandwidths of a few gigahertz that could be searched in a few minutes with a 1 GHz spectrometer. Since retuning the receiver to the next band would take perhaps a quarter hour, most of the time for the search would be spent in retuning the receivers, not in the observing.

5. DISCUSSION

One of the most important questions in the study of compact objects is the nature of pulsars, including whether pulsars consist of neutron matter or strange quark matter. In this Letter, we have identified an observable radio signal that would be characteristic of the latter possibility. We have, however, left important questions unanswered. How is the vibrational mode excited? A transient signal, such as that due to a starquake or cometary impact, would die off quickly and would thus be very difficult to detect. Does a mechanism for continuous excitation exist? The power of a detectable signal is only on the order of less than 10^{-9} of pulsar energy losses (less than 1% of the broadband radio signal). Thus, it would seem reasonable, in view of the many consequences, to make observations independent of theoretical considerations, in order to see if such a small fraction of the energy loss does, in fact, go into this mode. We may speculate, however, that one direction from which such a mechanism could come might be that of the interaction between superfluid vortices and (type 2) superconducting magnetic flux tubes. (We know that the magnetic field for a strange quark star must pass through the core because the crust is so thin.) It is the outward migration of the superfluid vortices that is responsible for pulsar spin-down, while M. Ruderman (1996, private communication) has conjectured that the latter could be effectively pinned by the interaction with the nonsuperconducting electron fluid. Some effective resonant coupling between those two systems, on the one hand, and the vibrational mode, on the other, could result in continuous excitation. For example, the coupling could be associated with discontinuous passage of a vortex through a pinned flux tube. Another question concerns the scale over which the coherence implicit in our calculations is maintained. If such coherence is only maintained over some fraction of the size of the star, then the power radiated would be reduced by that fraction.

The signal predicted in this Letter is a speculative one; however, the dearth of distinctive signatures for strange quark stars, in our view, makes the search for such a signal worthwhile.

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