Novel method for converting digital Fresnel hologram to phase-only hologram based on bidirectional error diffusion

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Abstract: We report a novel and fast method for converting a digital, complex Fresnel hologram into a phase-only hologram. Briefly, the pixels in the complex hologram are scanned sequentially in a row by row manner. The odd and even rows are scanned from opposite directions, constituting to a bidirectional error diffusion process. The magnitude of each visited pixel is forced to be a constant value, while preserving the exact phase value. The resulting error is diffused to the neighboring pixels that have not been visited before. The resulting novel phase-only hologram is called the bidirectional error diffusion (BERD) hologram. The reconstructed image from the BERD hologram exhibits high fidelity as compared with those obtained with the original complex hologram.

OCIS codes: (090.0090) Holography; (090.1995) Digital holography; (090.1760) Computer holography.

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1. Introduction

Computer-generated holography (CGH) has undergone encouraging development in the past two decades. One of the major factors leading to the success in CGH can be attributed to the emergence of fast algorithms that speed up the computation of the digital hologram by a significant amount. On top of that, the rapid advancement of computing and semiconductor technologies, have also enabled medium size digital hologram to be generated numerically swiftly with low cost commodity personal computer (PC) [1,2] and hardware (such as field programmable gate array (FPGA)). Despite the favorable outcome, the display of hologram is a difficult problem that imposes, to a certain extent, a bottleneck to the practical realization of the holographic technology. One of the major problems is that high resolution devices that are capable of displaying holograms, such as Liquid Crystal on Silicon (LCoS), are only capable of displaying either the magnitude or the phase component of a complex hologram. A straightforward solution to this problem, is to employ two spatial light modulators (SLMs) to display the real and imaginary components, or the amplitude and phase information, of a complex hologram [3–5]. Likewise, a complex hologram can be simulated with a double phase-only hologram [6], and displayed with a pair of phase-only SLMs. In some implementation, the pair of SLMs is replaced by a single device, displaying a pair of holograms and subsequently merging the reconstructed wavefront through a grating [7–10]. Although such approach is effective, the optical setups are rather complicated. Besides, the area of the SLM allocated to each component of the hologram is reduced to half of its original size. Alternatively, a complex hologram can be converted into either an amplitude-only, or a phase-only hologram (POH) so that it can be displayed directly with a single device. However, if an amplitude-only display is used to display a Fresnel hologram, the reconstructed image will be contaminated with a de-focused twin image. While the real twin image can be diverted away from the reconstructed virtual image by converting an on-axis hologram into an off-axis hologram, the angular separation between the two images is limited by the resolution of the display. Besides, the optical efficiency of an amplitude hologram is rather low as the illumination beam is attenuated by the opacity of the fringe patterns. A POH, on the other hand, can be displayed with a phase-only device and results in higher optical efficiency as well as rejection of the twin image. On the downside, removing the magnitude component will lead to heavy distortion on the reconstructed image. To alleviate this problem, the Gerberg-Saxton algorithm [11], or iterative Fresnel transform [12] are often adopted to compute the phase hologram in an iterative manner, so that the reconstructed image will match with a target planar image. However, the phase hologram generated with such approach is computationally intensive. The fastest method of generating a POH developed to date, is probably the "One-Step Phase Retrieval (OSPR) process" [13,14]. Briefly, a random phase is first added to the object points prior to the generation of the digital hologram. The phase component of the hologram, which may be quantized with thresholding or error diffusion [15], is displayed with a phase-only device. The reconstructed images of the holograms generated with the OSPR process are generally noisy and multiple sub-frames, each representing the same object scene added with different random phase patterns, have to be presented rapidly to the observers to average out the speckle noise. However, the sub-frames involve more computation to generate, and also need to be displayed at high frame rate to avoid flickering. In this paper, we propose a novel and fast method for converting a complex Fresnel hologram into a single phase-only hologram called the bidirectional error diffusion (BERD) hologram. The BERD hologram is capable of representing an object scene and preserving favorable visual quality on the reconstructed image. Different from the OSPR scheme, or other iterative approaches that generate a phase-only hologram from an object scene, our proposed method can be applied directly on a given digital complex hologram.

2. Bidirectional error diffusion (BERD) hologram as a novel phase-only hologram

2.1 Basic approach based on unidirectional error diffusion

To begin with, consider a complex Fresnel hologram that is generated from the object waves emitted from each point on an object scene. Each pixel in the hologram is denoted by $P_{u;v}$ and having a value $H(u,v)$ (where u and v are the vertical and horizontal axes of the co-ordinate system, respectively, as shown in Fig. 1(a)) given by

$$H(u,v)\Big|_{\substack{0\le u<X\\0\le v<Y}} = \sum_{x=0}^{X-1}\sum_{y=0}^{Y-1}\frac{I(x,y)\exp\left(i2\pi r_{x;y;u;v}/\lambda\right)}{r_{x;y;u;v}}, \tag{1}$$

where $I(x,y)$ and $r_{x;y;u;v}=\left[(u-x)^2\delta+(v-y)^2\delta+w_{x;y}^2\right]^{0.5}$ are the intensity of the point located at (x,y) in the object scene and its distance to the hologram, respectively. The perpendicular distance of a point at position (x,y) to the hologram is denoted by $w_{x;y}$, λ is the wavelength of the optical beam, and δ is the pixel size of the hologram. X and Y are the number of rows and columns of the hologram, and which are assumed to be the same as the object scene. We have assumed that no random phase has been added in the hologram generation process. The complex hologram can be converted into a POH, $H_p(u,v)$, by setting the magnitude of each pixel to be transparent with a value of unity, as given by

$$\left|H_p(u,v)\right|=1, \text{and} \arg\left(H_p(u,v)\right)=\arg\left(H(u,v)\right). \tag{2}$$

However, as we shall show later, the quality of the reconstructed image of a POH generated with Eq. (2) is extremely poor. Apparently, the heavy distortion on the reconstructed image is caused by the large amount of error in each hologram pixel after removing the magnitude information. In view of the above problem, we propose to employ the Floyd-Steinberg error diffusion technique [16] to compensate the error of each hologram pixel. The process is outlined as follows. To start with, each pixel in the hologram is scanned sequentially in a row by row, left to right manner, and the value of the pixel under evaluation is converted to a phase-only quantity $H_p(u,v)$ according to Eq. (2). Suppose $P_{u_j;v_j}$ is the current pixel that is being processed. By setting the magnitude to unity will result in an error given by

$$E(u_j,v_j)=H(u_j,v_j)-H_p(u_j,v_j). \tag{3}$$

Next, the error $E(u_j,v_j)$ is diffused to the neighborhood pixels that have not been visited previously. Its neighborhood members are updated according to the following equations:

$$H(u_j,v_j+1)\leftarrow H(u_j,v_j+1)+w_1E(u_j,v_j), \tag{4}$$

$$H(u_j+1,v_j-1)\leftarrow H(u_j+1,v_j-1)+w_2E(u_j,v_j), \tag{5}$$

$$H(u_j+1,v_j)\leftarrow H(u_j+1,v_j)+w_3E(u_j,v_j), \tag{6}$$

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$$H\left(u_j+1,v_j+1\right) \leftarrow H\left(u_j+1,v_j+1\right)+w_4E\left(u_j,v_j\right), \tag{7}$$

where the operator "$\leftarrow$" denotes updating the variable on the left hand side with the one on the right hand side of the expression. The values of the constant coefficients w_1 to w_4 are set to the values adopted in [16], with $w_1 = 7/16$, $w_2 = 3/16$, $w_3 = 5/16$, and $w_4 = 1/16$. According to Eqs. (4) to (7), we show, in Fig. 1(b), the spatial relation between pixel $P_{u_j;v_j}$ and its neighborhood pixels for compensating the error via diffusion. The figure shows how $P_{u_j;v_j}$ diffuses errors to its four unvisited neighborhood pixels. Equivalently, Eqs. (4-7) can be rearranged into a compact recursive expression given by

$$H\left(u_j,v_j\right) \leftarrow H\left(u_j,v_j\right)+w_1E\left(u_j,v_j-1\right)+w_4E\left(u_j-1,v_j-1\right)+w_3E\left(u_j-1,v_j\right)+w_2E\left(u_j-1,v_j+1\right), \tag{8}$$

and according to Eq. (8), Fig. 1(c) shows the updated value of each pixel can be visualized as the weighed sum of its four neighboring pixels. After all the hologram pixels have been visited and processed, the original hologram will be converted into a POH. As we shall illustrate in the experiment section, the reconstructed image of $H_p(u,v)$ generated with the proposed method is similar to the one obtained with the original complex hologram.

2.2 Enhanced method based on bidirectional error diffusion process

The basic approach described in section 2.1, though effective, results in noise contamination on the reconstructed image. A possible explanation is that in the error diffusion process, as given in Eq. (8), each hologram pixel is summed up with the errors diffused from neighboring pixels at fixed relative positions. This constitutes to a process which is similar to a predictive coder in which the error generated, to a certain extent, is correlated to the signal. As a result, the reconstructed image of the hologram, $H_p(u,v)$, will be superimposed with the noise signal corresponding to the correlated error. To overcome this problem, we propose an enhanced method to reduce the correlation between the holographic signal and the error. To begin with, we note that the error diffusion process represented in Eqs. (4-7) is independent on the direction of scanning the pixels along each row of the hologram image. As such, we change the scanning direction of each row of pixels so that the recursive process in Eq. (8), which leads to the correlated error, will be interrupted after each row is visited. The errors on the odd rows are diffused to the neighboring pixels in the same way as given in Eqs. (4-7), while the even rows are compensated by applying the error diffusion process in the opposite direction (from right to left), as given in Eqs. (9-12) below. The spatial relation between pixel $P_{u_j;v_j}$ and its neighborhood pixels is illustrated in Fig. 1(d).

$$H\left(u_j,v_j-1\right) \leftarrow H\left(u_j,v_j-1\right)+w_1E\left(u_j,v_j\right), \tag{9}$$

$$H\left(u_j+1,v_j+1\right) \leftarrow H\left(u_j+1,v_j+1\right)+w_2E\left(u_j,v_j\right), \tag{10}$$

$$H\left(u_j+1,v_j\right) \leftarrow H\left(u_j+1,v_j\right)+w_3E\left(u_j,v_j\right), \tag{11}$$

$$H\left(u_j+1,v_j-1\right) \leftarrow H\left(u_j+1,v_j-1\right)+w_4E\left(u_j,v_j\right). \tag{12}$$

The resulting generated hologram based on Eqs. (4-7) to process the even rows and on Eqs. (9)-(12) to proceed the odd rows of the complex hologram is called the bidirectional error diffusion (BERD) hologram.

#194122 - $15.00 USD Received 18 Jul 2013; revised 15 Sep 2013; accepted 15 Sep 2013; published 27 Sep 2013

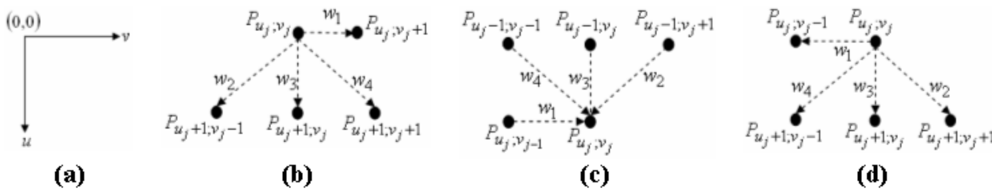


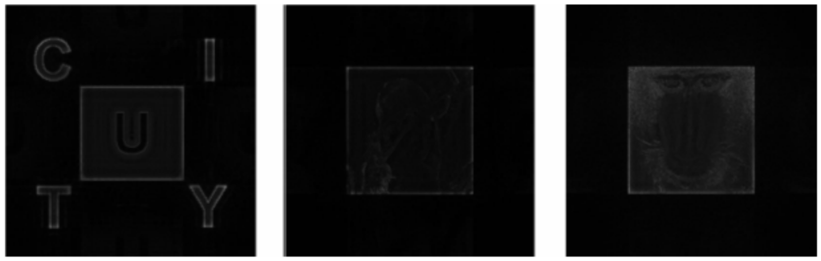
Fig. 1. (a) Horizontal and vertical axes of the co-ordinate system. (b) Diffusion of the error from the current pixel to its neighborhood, scanned from left to right. (c) An alternative representation of (a), showing the updating of a pixel from its neighborhood. (d) Diffusion of the error from the current pixel to its neighborhood, scanned from right to left.

3. Experimental results

A binary image "CITYU", a smooth image "Lena", and a highly textural image "Mandrill", as shown in Figs. 2(a)-2(c), respectively are employed to evaluate our proposed method. Equation (1) is applied to generate the complex Fresnel holograms of the 3 images. The hologram, as well as the test images, are comprising of 2048×2048 pixels, each having a square size of $7\mu m \times 7\mu m$. The wavelength of the optical beam is $650nm$. All the images are parallel to, and located at 0.3m from the hologram plane. The numerical reconstructed images of the 3 complex holograms are identical to the original images, and hence not shown in here. Next, we remove the magnitude component of the holograms with Eq. (2), and the numerical reconstructed images are shown in Figs. 3(a)-3(c). It can be seen that the shaded regions of the three images are removed extensively, leaving behind mostly the edges of the images. To overcome this problem, our proposed unidirectional error diffusion method described in section 2.1 is applied to convert the complex holograms into phase-only holograms (POHs). Reconstructed images of the POHs are shown in Figs. 4(a)-4(c). We observe that the reconstructed images are close to the original images in Figs. 2(a)-2(c). However, certain amount of noise contamination is noted, especially in the dark areas. Subsequently, we convert the complex holograms with our proposed bidirectional error diffusion method to obtain the BERD holograms as described in section 2.2. The numerical reconstructed images of the BERD holograms generated with such means, as shown in Figs. 5(a)-5(c), are very similar to the original images, and the noise signals are not noticeable. Quantitative evaluation on the quality of the reconstructed images of the POHs derived from the 3 aforementioned methods, as compared from the ones obtained with the complex holograms, are given in Table 1. Evidently, the proposed bidirectional error diffusion method generating BERD holograms results in significantly higher fidelity (in terms of PSNR) over its peers. For the sake of clarity, Figs. 2-5 are presented in their actual size in the media files MF2-MF5, respectively.

Fig. 2. (a)-(c) Test images "CITYU", "Lena", and "Mandrill".

#194122 - $15.00 USD Received 18 Jul 2013; revised 15 Sep 2013; accepted 15 Sep 2013; published 27 Sep 2013



(a) (b) (c)

Fig. 3. (a)-(c) Reconstructed images of the phase-only holograms derived from direct removal of the magnitude component of the complex holograms.

(a) (b) (c)

Fig. 4. (a)-(c) Reconstructed images of the phase-only holograms derived from our proposed unidirectional error diffusion method.

(a) (b) (c)

Fig. 5. (a)-(c) Reconstructed images of the BERD holograms (novel phase-only holograms derived from our proposed bidirectional error diffusion method).

Table 1. Quantitative comparison between the fidelity of the reconstructed images as shown in the Figs. 3(a)-3(c), 4(a)-4(c), and 5(a)-5(c) (with reference to the ones obtained from the original complex holograms in Figs. 2(a)-2(c)) from phase-only holograms derived from different means. The reported values are PSNR values.

	CITYU	Lena	Mandrill
Direct removal of the magnitude component	13.0 dB	16.1 dB	16.2 dB
Unidirectional error diffusion	24.3 dB	28.1 dB	27.0 dB
Bidirectional error diffusion	31.6 dB	38.6 dB	32.7 dB

4. Conclusion

This paper reports a novel and fast method for converting a digital, complex Fresnel hologram into a phase-only hologram called the BERD holograms. The process is accomplished with a BERD mechanism, with which each hologram pixel is scanned sequentially and forced to a constant magnitude, while the resulting error is diffused to the unvisited neighborhood. The odd and even rows are scanned from opposite directions, constituting to a bidirectional error diffusion process. Our proposed method has 3 major advantages. First, the reconstructed images of the novel BERD holograms generated are very similar to those obtained with the original complex holograms. Second, our proposed method only involves moderate amount of computation. Third, our method can be applied directly on an existing complex Fresnel hologram, without the presence of the object image. A potential research direction is to explore a better set of error diffusion coefficients (w_1 to w_4) that can enhance the quality of the reconstructed image of the BERD hologram.

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