CHAPTER 7: NEWMARK ANALYSES

The Newmark (1965) analyses described in this chapter were performed to compute dynamic displacements for the significant shaking events in centrifuge test CLM02. Comparison of the computed displacements with the displacements measured during the test provides a means of evaluating the cyclic shear resistance of the Rancho Solano #2 clay that was tested in the centrifuge model. This was done by repeated analyses, varying the clay strength until the calculated displacements matched the measured values.

Newmark's method is ideally suited for analyses of displacements for test CLM02, because test CLM02 exhibited sliding block behavior along a well defined shear interface. Newmark's inherent assumption of rigid-block shaking behavior is satisfied, because the clay layer was sandwiched between rigid steel plates, and the thickness of the clay layer was very small relative to its length. Consequently, later refinements to Newmark's method such as those proposed by Makdisi and Seed (1978) and Rathje and Bray (1999) are not needed for the analysis.

Applied Base Motions and Resulting Displacements for Test CLM02

Base acceleration time histories for each of the shaking events in test CLM02 were measured using accelerometers mounted parallel and perpendicular to the inclined steel base plates. The horizontal accelerations needed for the Newmark analyses were calculated by summing the horizontal components of the measured accelerations. The equation used to calculate the horizontal base acceleration time histories for each of the shaking events in test CLM02 is:

$$a_{\text{base horizontal}} = a_{\text{base parallel}} \cdot \cos(\beta) + a_{\text{base perpendicular}} \cdot \sin(\beta), \tag{7-1}$$

where: $a_{\text{base horizontal}}$ = calculated horizontal input acceleration at the base of the sliding block model,

$a_{\text{base parallel}}$ = base acceleration measured parallel to the direction of sliding,

$a_{\text{base perpendicular}}$ = base acceleration measured perpendicular to the direction of sliding, and

β = slope angle.

The values of $a_{base\ horizontal}$ calculated using data from the 10.5º slope and the 12º slope, which should be equal because both slopes were subjected to the same shaking, agreed within about 10 %.

In order to use the calculated horizontal base acceleration time histories to compute displacements using Newmark's method, it was necessary to invert the calculated acceleration records using D'Alembert's principle (which allows a dynamic system to be analyzed as an equivalent static system subjected to an inertial force and an inertial torque). The peak horizontal accelerations from the inverted base acceleration records are given in Table 7-1. Table 7-1 also lists the number of applied stress cycles and the shaking-induced displacement recorded for each of the slopes in test CLM02.

Table 7-1: Applied Base Motions and Resulting Displacements for Test CLM02

Event	Peak Horizontal Accel., Downslope, 10.5º Slope [1/(N·g)]	Peak Horizontal Accel., Upslope, 10.5º Slope [1/(N·g)]	Peak Horizontal Accel., Downslope, 12º Slope [1/(N·g)]	Peak Horizontal Accel., Upslope, 12º Slope [1/(N·g)]	No. of Applied Stress Cycles	Disp. for 10.5º Slope	Disp. for 12º Slope
Shake 1	0.05	-0.08	0.06	-0.08	5	0"	0"
Shake 2	0.32	-0.35	0.35	-0.33	5	0"	0.0015"
Shake 3	0.51	-0.60	0.51	-0.54	20	0.014"	0.037"

As shown in Table 7-1, no displacement was observed for either slope as a result of Shake 1, or for the 10.5º slope in Shake 2. Only a very small amount of displacement was observed for the 12º slope during Shake 2. Consequently, the Newmark analyses described in the following sections were only performed for the Shake 3 time histories.

Calculating Yield Acceleration from Undrained Shear Strength

Using Newmark's approach, the yield acceleration is defined as the smallest value of horizontal inertial acceleration that would induce slope failure. The yield acceleration is calculated by determining the horizontal acceleration that corresponds to a factor of safety of

1.0 against sliding, and is assumed to remain constant during shaking. The derivation of the equation for the downslope-directed horizontal yield acceleration using Newmark's approach is shown in Figure 7-1. The equation is:

$$a_{y\ ds} = \frac{S_u A - mgN \cdot \sin(\beta)}{mN \cdot \cos(\beta)}, \tag{7-2}$$

where: $a_{y\ ds}$ = downslope-directed horizontal yield acceleration,
S_u = undrained shear strength,
A = area of the slickensided plane,
m = mass of the sliding block,
g = acceleration of gravity,
N = centrifuge scale factor, and
β = slope angle.

For flat slopes with low friction angles, there is the possibility of upslope slip during shaking. At Newmark's recommendation, this effect is often neglected in practice, since it is conservative to assume that upslope slip will not occur. However, when comparing predicted displacements with measured displacements, this effect should be included for the sake of accuracy. The derivation of the equation for the upslope-directed horizontal yield acceleration using Newmark's approach is shown in Figure 7-2. The equation for the upslope yield acceleration is:

$$a_{y\ us} = \frac{S_u A + mgN \cdot \sin(\beta)}{mN \cdot \cos(\beta)}, \tag{7-3}$$

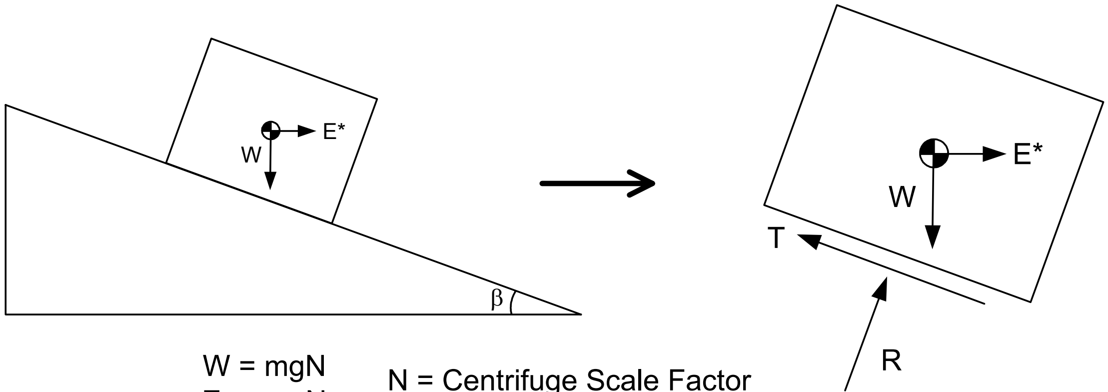
where: $a_{y\ us}$ = upslope-directed horizontal yield acceleration,
S_u = undrained shear strength,
A = area of the slickensided plane,
m = mass of the sliding block,
g = acceleration of gravity,
N = centrifuge scale factor, and
β = slope angle.

For test CLM02, m, g, N, and β are known for both slopes. This allows Equations 7-2 and 7-3 to be used to calculate the downslope and upslope yield accelerations as a function of the undrained shear strength. The resulting equations for yield acceleration can then be used to predict Newmark displacements as a function of undrained shear strength.

Newmark Displacement Analyses

Using Newmark's method, earthquake-induced slope displacements were calculated by double integration of the portion of the acceleration record that is larger than the yield acceleration (Newmark, 1965). Newmark displacement analyses were performed using a numerical integration routine written within the MathCAD computer platform. The numerical integration routine includes a provision for upslope sliding, in the event that the applied dynamic acceleration exceeds the upslope yield acceleration.

Figure 7-3 shows displacements predicted for the 10.5º slope for Shake 3, as a function of undrained shear strength. Figure 7-4 shows displacements predicted for the 12º slope for Shake 3, as a function of undrained shear strength. Figures 7-3 and 7-4 also show the relative displacements measured after Shake 3 for each of the slopes.



$W = mgN$
$E = ma_yN$

N = Centrifuge Scale Factor

a_y = Horizontal Yield Acceleration

*Note: For infinite slope mechanisms, it is more convenient to use components of acceleration that are parallel and perpendicular to the slope

Finding the value of yield acceleration that will cause downslope slip of the block:

$$F = 1 = \frac{S_u A}{T}$$

S_u = Undrained Shear Strength

A = Area of the Slickensided Plane

Apply equilibrium parallel to the sliding plane:

$$\Sigma F_{para} = 0 = mgN \sin(\beta) + ma_yN \cos(\beta) - T$$

$$T = mgN \sin(\beta) + ma_yN \cos(\beta)$$

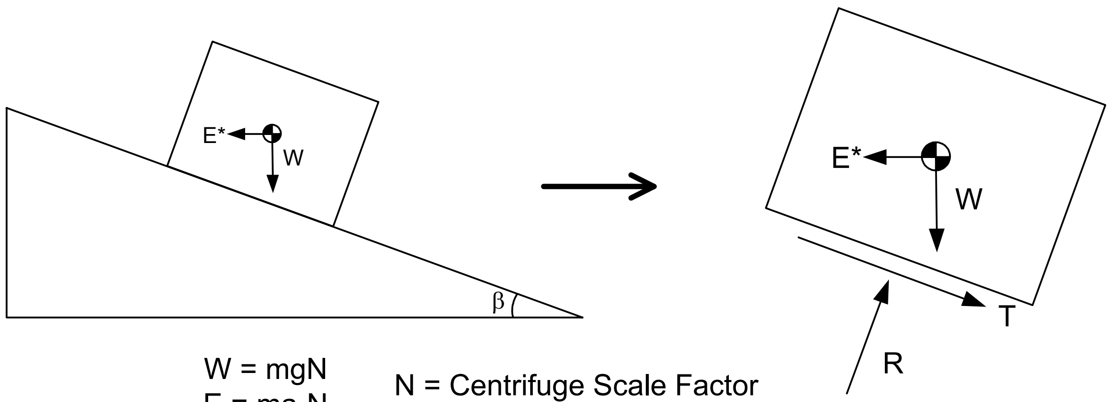
Solving for a_y:

$$S_u A = T$$

$$S_u A = mgN \sin(\beta) + ma_yN \cos(\beta)$$

$$a_y = \frac{S_u A - mgN \sin(\beta)}{mN \cos(\beta)}$$

Figure 7-1. Calculating the yield acceleration that causes downslope slip of the block.



$W = mgN$
$E = ma_yN$

N = Centrifuge Scale Factor

a_y = Horizontal Yield Acceleration

*Note: For infinite slope mechanisms, it is more convenient to use components of acceleration that are parallel and perpendicular to the slope

Finding the value of yield acceleration that will cause upslope slip of the block:

$$F = 1 = \frac{S_u A}{T}$$

S_u = Undrained Shear Strength

A = Area of the Slickensided Plane

Apply equilibrium parallel to the sliding plane:

$$\Sigma F_{para} = 0 = T + mgN\sin(\beta) - ma_yN\cos(\beta)$$

$$T = ma_yN\cos(\beta) - mgN\sin(\beta)$$

Solving for a_y:

$$S_u A = T$$

$$S_u A = ma_yN\cos(\beta) - mgN\sin(\beta)$$

$$a_y = \frac{S_u A + mgN\sin(\beta)}{mN\cos(\beta)}$$

Figure 7-2. Calculating the yield acceleration that causes upslope slip of the block.

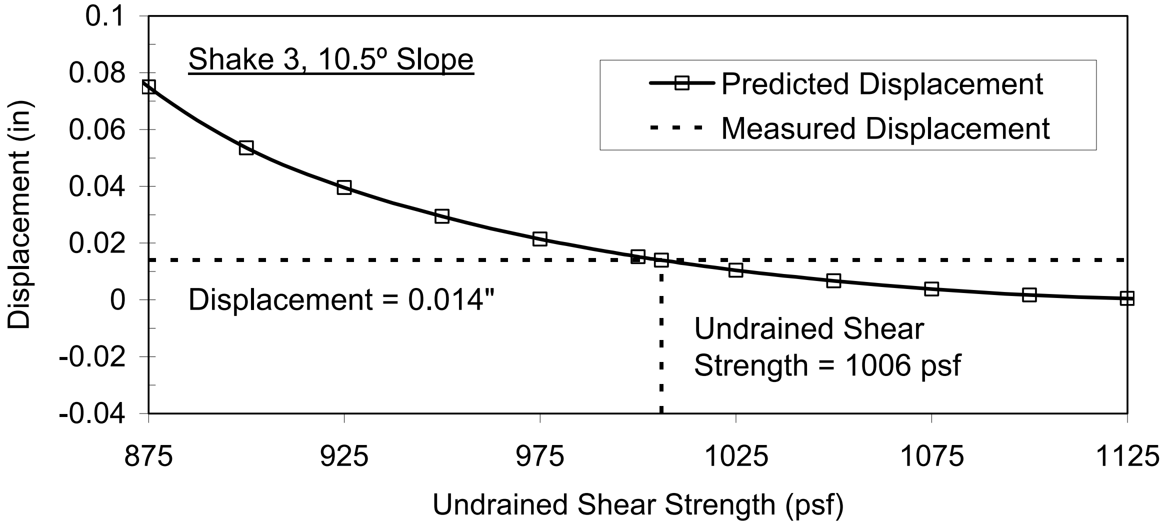


Figure 7-3. Newmark displacements calculated for the 10.5º slope as a function of undrained shear strength.

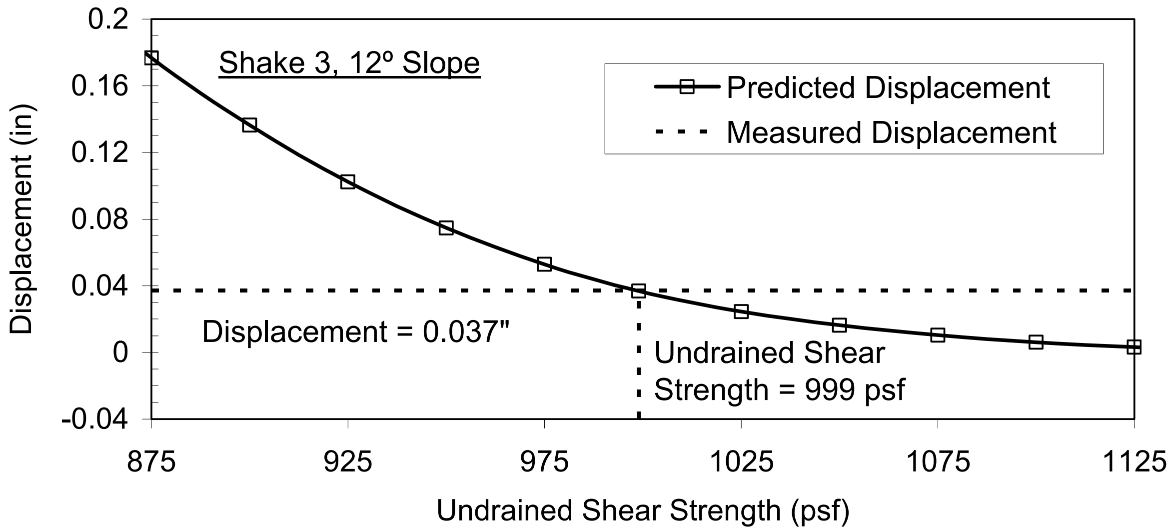


Figure 7-4. Newmark displacements calculated for the 12º slope as a function of undrained shear strength.

As shown in Figures 7-3 and 7-4, an undrained shear strength of approximately 1,000 psf produces good agreement between predicted and measured displacements for both slopes. Figure 7-5 shows the base acceleration, relative velocity and relative displacement predicted by Newmark's method for the 10.5º slope during Shake 3, assuming an undrained shear strength of 1,006 psf. Figure 7-6 shows the base acceleration, relative velocity and relative displacement predicted by Newmark's method for the 12º slope during Shake 3, assuming an undrained shear strength of 999 psf.

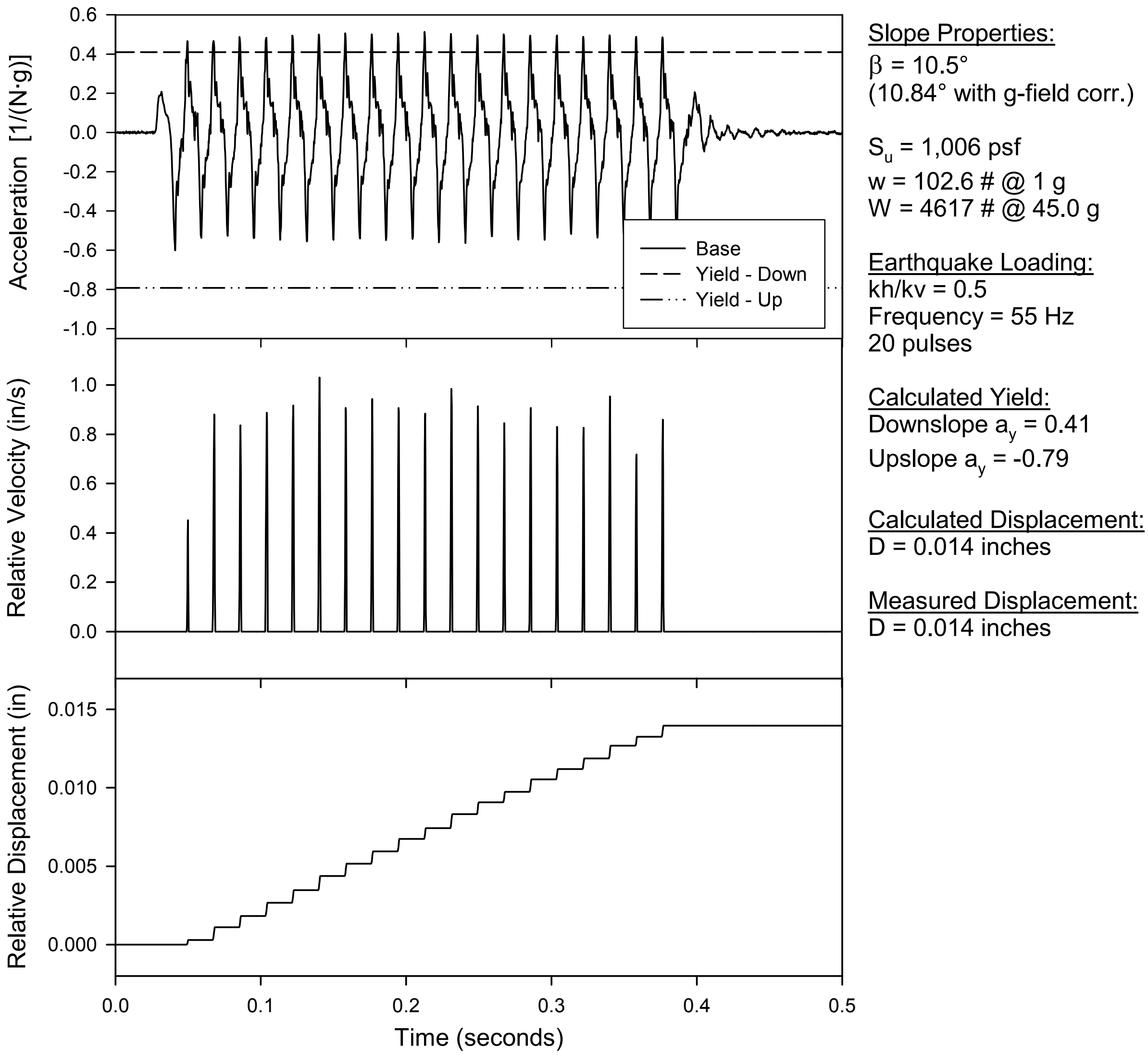


Figure 7-5. Newmark analysis of 10.5º slope for Shake 3.

Note that the details of the displacement responses predicted by Newmark's method (shown in Figures 7-5 and 7-6) are significantly different than the displacement responses that were observed during Shake 3 (shown in Figures 6-23 and 6-24). This difference is due to the assumption of rigid-plastic sliding behavior that is inherent to Newmark's method. Because Newmark's method assumes rigid-plastic sliding behavior, it cannot be used to capture the elastic, hysteretic load-displacement response that was observed during Shake 3 (shown in Figures 6-32 and 6-33), although it does match the post-shaking irrecoverable displacements (Pradel et al., 2005).

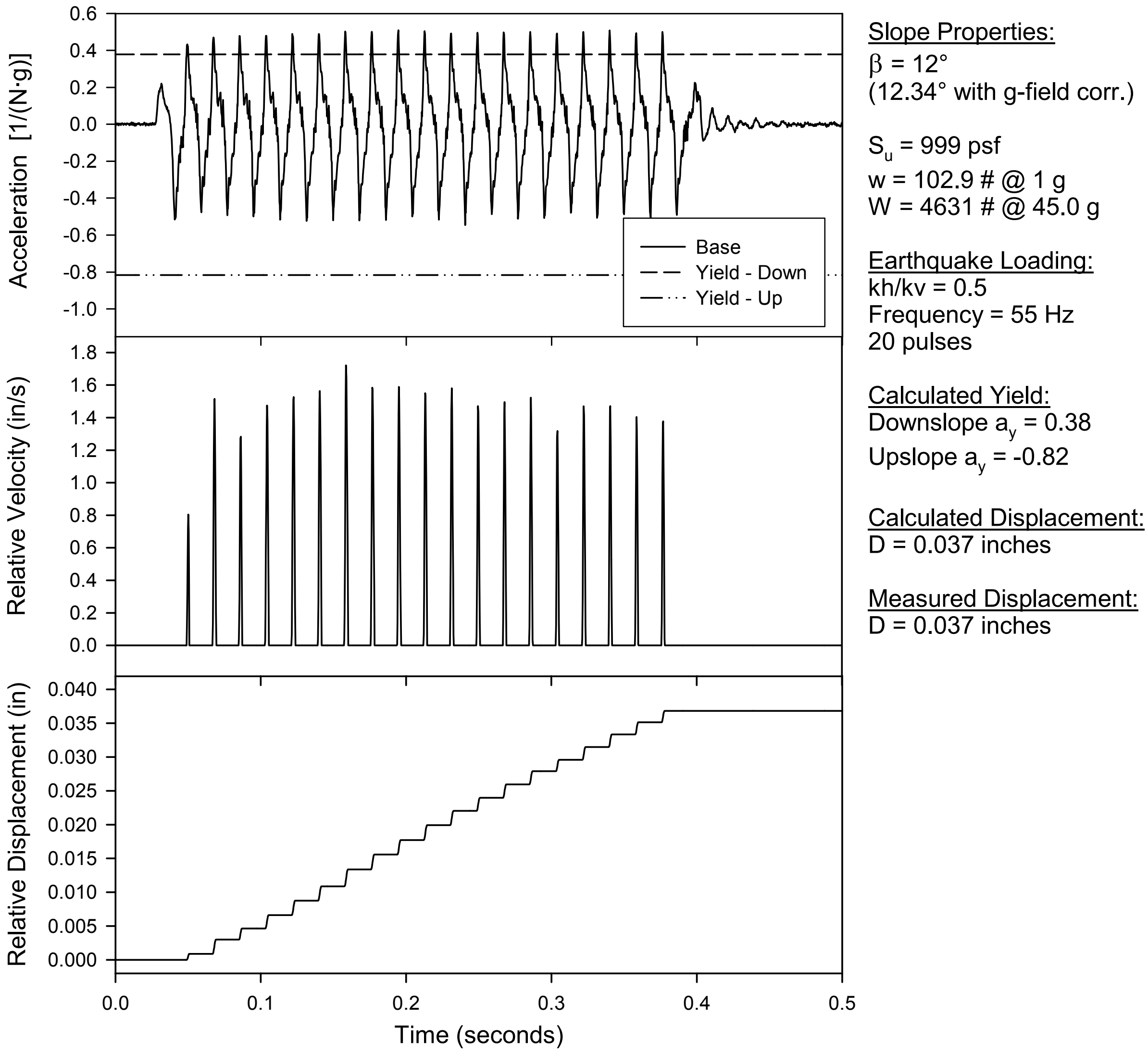


Figure 7-6. Newmark analysis of 12º slope for Shake 3.

Based on test CLM02, an undrained shear strength of 1,000 psf is appropriate for Rancho Solano Clay #2, in order to match slope displacements calculated using Newmark's method with the measured displacements. The calculated undrained strengths can also be expressed using cyclic shear strength ratios, as follows:

$$S_c = \frac{S_u}{\sigma'} = \frac{S_u}{\dfrac{mgN \cdot \cos(\beta)}{A} - U_o}, \tag{7-4}$$

where: S_c = cyclic shear strength ratio,
S_u = undrained shear strength,
σ' = initial effective normal stress on the slip plane,
m = mass of the sliding block,
g = acceleration of gravity,
N = centrifuge scale factor,
β = slope angle,
A = area of the slickensided plane, and
U_o = intial pore pressure acting on the slickensided plane.

Table 7-2 lists the values that were used to calculate the cyclic shear strength ratios for each slope, and the resulting cyclic strength ratios. A cyclic shear strength ratio between 0.63 and 0.69 is appropriate for Rancho Solano Clay #2.

Table 7-2: Calculated Cyclic Shear Strength Ratios

Slope Being Analyzed	S_u (psf)	mg (lb)	N (unitless)	β (degrees)	A (ft^2)	U_o (psf)	S_c (unitless)
10.5º Slope	1,006	102.6	45	10.84	2.71	79.2	0.63
12º Slope	999	102.9	45	12.34	2.71	216	0.69

Using Simplified Displacement-Based Approaches to Back-Calculate Strength

In engineering practice, simplified displacement-based screening approaches are often used to estimate upper bound earthquake-induced slope displacements. Two commonly used displacement-based screening approaches have been proposed by Newmark (1965) and Hynes-Griffin and Franklin (1984). Both of these screening approaches are semi-empirical, and were developed by performing Newmark analyses for various slope conditions, using a range of earthquake input motions.

Newmark (1965)

Newmark (1965) developed an upper bound estimate for earthquake-induced slope displacements using an empirical curve that bounded the results from his analyses of four earthquake time histories. The proposed upper-bound curve is given by the formula:

$$u = \frac{V^2}{2gN}\left[1 - \frac{N}{A}\right]\left[\frac{A}{N}\right], \qquad (7\text{-}5)$$

where:
u = maximum displacement (inches),
V = peak velocity (inches per second),
g = acceleration of gravity (inches per second squared),
N = horizontal acceleration that reduces factor of safety to 1.0 (fraction of g), and
A = peak horizontal acceleration (same units as N).

Cyclic shear strength ratios for Shake 3 were back-calculated for both centrifuge model slopes using Equation 7-5. The results from these simplified Newmark (1965) analyses are given in Table 7-3. Prototype displacements were used to back-calculate the cyclic strength ratios. Peak velocities were determined by integrating the horizontal input acceleration time histories.

Table 7-3: Cyclic Shear Strength Ratios Back-Calculated Using Equation 7-5

Slope Being Analyzed	Prototype Displacement (in)	Peak Horizontal Accel., Downslope, (1/g)	Peak Velocity (in/s)	Calculated N (1/g)	Strength Ratio Based on Equation 7-5 S_c	Strength Ratio From Detailed Numerical Analyses S_c	Diff-erence (%)
10.5º Slope	0.014 x 45 = 0.63	0.51	20.7	0.362	0.63	0.63	0
12º Slope	0.037 x 45 = 1.67	0.51	21.1	0.281	0.62	0.69	11

As shown in Table 7-3, the cyclic strength ratios back-calculated using Newmark's simplified method (Eq. 7-5) agree quite clearly with the strength ratios that were calculated using the detailed Newmark numerical integration approach. The cyclic strength ratios back-calculated using the simplified approach are equal to or smaller than those calculated from the numerical integration, which is contrary to what would be expected, given the upper-bound nature of Newmark's simplified approach, but this difference is not great.

Hynes-Griffin and Franklin (1984)

Hynes-Griffin and Franklin (1984) developed an upper bound estimate for earthquake-induced slope displacements based on the results of Newmark analyses of 348 earthquake motions and six synthetic acceleration time histories. The upper bound curve that envelops all 354 results is shown in Figure 7-7.

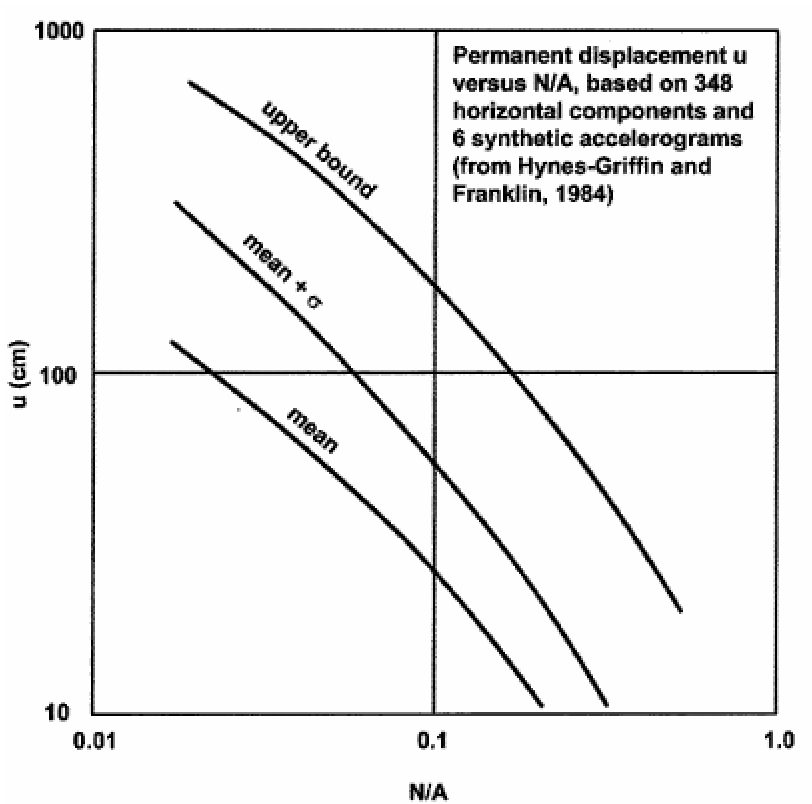


Figure 7-7. Earthquake-induced displacement vs. N/A (Hynes-Grifin and Franklin, 1984).

The prototype displacements that were observed for the 10.5° and the 12° slopes were 1.6 cm and 4.2 cm, respectively. These displacements are outside the range of the chart provided by Hynes-Griffin and Franklin. An empirical equation proposed by Duncan and

Brandon (2005) was used to extrapolate Hynes-Griffin and Franklin's upper bound curve and calculate displacements as a function of N. Their proposed equation is:

$$u = 7\left[\frac{N}{A}\right]^{-1.5}, \tag{7-6}$$

where: u = maximum displacement (cm),
N = horizontal acceleration that reduces factor of safety to 1.0, and
A = peak horizontal acceleration (same units as N).

Cyclic shear strength ratios for Shake 3 were back-calculated for both centrifuge model slopes using Equation 7-6. The results from these simplified Hynes-Griffin and Franklin (1984) analyses are given in Table 7-4. Prototype displacements were used to back-calculate the cyclic strength ratios.

Table 7-4: Cyclic Shear Strength Ratios Back-Calculated Using Equation 7-6

Slope Being Analyzed	Prototype Displacement (cm)	Peak Horizontal Accel., Downslope, (1/g)	Calculated N (1/g)	Strength Ratio Based on Equation 7-5 S_c	Strength Ratio From Detailed Numerical Analyses S_c	Difference (%)
10.5º Slope	0.0356 x 45 = 1.6	0.51	1.36	2.24	0.63	256
12º Slope	0.0940 x 45 = 4.2	0.51	0.72	1.32	0.69	91

As can be seen in Table 7-4, the cyclic strength ratios back-calculated using Hynes-Griffin and Franklin's simplified method result in much higher strength ratios than were calculated using Newmark's numerical integration approach. The difference between these two approaches is due to the fact that Hynes-Griffin and Franklin's method does not involve the actual peak velocity for the particular acceleration time history, but instead implicitly uses high velocities corresponding to the extremes of the 354 cases they considered.