Chapter 7 -- Concluding Remarks

7.1 Summary

It has been the objective of this work to determine how a designer might best utilize geometric and material properties to create a strong, lightweight dome for end closure of a pressure vessel. The problem is far too general to be completely exhausted in a single work, but the document presented here does provide a useful analysis which may be applied for a large number of cases. The problem has been addressed in this work by parametric studies of 8-ply, symmetric, composite laminate, ellipsoidal domes closing a right circular cylinder. In development of the analysis, a number of significant substeps have been required. These have been detailed in Chapters 2-6.

In Chapter 2, the basic equations of the theory of shells are presented. The derivation begins with a brief exposition on the elementary theory of surfaces, and proceeds to derive a shell theory by descent from the three-dimensional theory of elasticity. The equations are derived for the geometrically nonlinear (small strain, moderate rotations about the in-plane axes) theory. Along the way, specializations are made so that the linear equations are derived concurrently with the nonlinear. The equations for both linear and geometrically nonlinear theories include allowance for curvature in two dimensions and for general composite laminate construction, and incorporate first-order transverse shear deformation effects.

In Chapter 3 is derived the first-order state vector form of the governing ordinary differential equations (ODE's) for a shell of revolution, first for the linear and then for the geometrically nonlinear theory. The derivation of the linear equation closely follows the work of Steele and Kim (1992), but that work has been extended here to incorporate first-order transverse shear deformation effects. The derivation of Steele and Kim used the symmetric stress tensor of Sanders (1959), which is valid for classical theory only; furthermore, their derivation assumes that the two in-plane shear stress resultants are always equal to one another, regardless of shell geometry. The present work has removed that assumption. Derivation of the linear state vector equation is via the mixed variational principle of Reissner (1950).

Following the derivation of the linear state vector equation, the geometrically nonlinear form is developed by direct manipulation of the field equations for a shell of revolution. This derivation is, we believe, unique. In either case, linear or geometrically nonlinear, the assumption of axisymmetry of loading and response is used along with the asserted axial symmetry of the undeformed geometry to reduce the state vector equation from its partial differential form to the ordinary differential form. In addition, the process of derivation yields a set of boundary conditions which are appropriate for the shell problem, and which prescribe only variables which can be said to exist on meridional faces. Also, the derivation yields a set of linear algebraic equations which may be used to calculate the values of the stress resultants on circumferential faces of the shell, once the state vector equation has been solved. The derivation thus yields a two-point boundary value problem which may be integrated along the shell meridian, and whose solution is sufficient to completely describe the shell behavior under loading.

Following the derivations of the state vector equations, Chapter 3 concludes with a note on the application of elastic boundary conditions for the shell problem. In this way the work is made general, in the sense that the dome may be used to close any shape of pressure vessel, provided only that the pressure vessel demonstrates axisymmetry in the vicinity of the vessel/dome interface. Likewise, the dome may itself be capped by an arbitrary axisymmetric structural member.

Chapter 4 is devoted to the numerical solution techniques employed in this work. Specifically, it describes the method of multiple shooting, describes the application of Newton's method for nonlinear analysis, and describes the cubic spline interpolation. There is little unique in the presentation of Newton's method or in the cubic spline interpolation; those are standard techniques, presented here only for completeness. The method of multiple shooting as it is implemented here is, however, somewhat unique.

The shooting method is often employed in numerical integration of two-point boundary value problems, when numerical instability is not a problem. For the integration of numerically unstable problems, the method of *multiple* shooting is employed. The ODE's derived in Chapter 3 are numerically unstable, so we use a multiple shooting technique, specifically a modified version of the *Stabilized Marching Method* presented by Ascher, Mattheij and Russell (1988). The modifications made to the technique affect the choice of segment initial conditions, and allow for the incorporation of geometric and material discontinuities within the shell meridian, as well as discrete circumferential line loads and ring stiffeners along the meridian. The modifications made to the stabilized marching method are unique to this work.

The encoding of the equations developed in Chapter 3 with the solution techniques presented in Chapter 4 is next checked for accuracy in Chapter 5. Specifically, results obtained by multiple shooting are compared to results obtained by finite element analysis. The finite element codes used are the commercially available code ABAQUS, and the code STAGS, used at NASA/Langley Research Center. The verification cases presented were selected to include geometric and material discontinuities incorporated as elastic boundary conditions, non-spherical geometry and internal ring stiffeners. The verifications cover both linear and geometrically nonlinear analyses, and supplement previously published theoretical and numerical verifications (Steinbrink and Johnson, (1997)). It is shown that the multiple shooting technique presented here achieves good results for all test cases, agreeing quite well with the results of finite element analysis for most variables. It is even shown that the current analysis presents superior results for some cases, notably for the highest-order resultants. This is a virtue of the current method.

Finally, Chapter 6 presents the results of a set of parametric studies for dome closure of a quasi-isotropic, cylindrical pressure vessel. Appropriate material properties are assumed for eventual experimental testing. The cylinder is taken to be right circular in design, with a $[\pm 45,0,90]_s$ lamination, and 12-inch radius. There is a large stiffening ring at the dome/cylinder interface, and the dome is taken to be closed by a circular plate of titanium alloy. The dome is taken to have an elliptical meridian with the minor axis parallel to the axis of revolution, and also to have composite laminate wall construction. The class of laminates allowed for the dome wall is limited to 8-ply symmetric laminates, and the majority of attention is given to quasi-isotropic and balanced angle-ply laminates. There are no internal ring stiffeners on the dome. The entire body is under 55 psi internal hydrostatic pressure; geometrically nonlinear analysis is performed.

The parametric study begins with consideration of a quasi-isotropic $[\pm 45,0,90]_s$ dome of spherical shape with a cover plate of 3-inch radius. The results indicate that there is for that geometry, a small compressive stress resultant in the circumferential direction in the vicinity of the plate. This occurs as a result of the geometric discontinuity, and introduction of some ellipticity into the dome causes this effect to diminish, until at some point the entire dome meridian experiences a state of biaxial tension. Further increase in ellipticity has the effect of lowering the overall weight of the dome, but also drives the circumferential stress resultant to become compressive in a location near the cylinder/dome interface. The study was able to pinpoint the "best" geometry among the quasi-isotropic ellipsoids to achieve minimum dome weight without circumferential compression. Unlike the case of the spherical dome, the onset of circumferential compression cannot be relieved by further increases in ellipticity. In fact, the onset of compression is followed by a rapid deterioration in the overall response, such that the method soon fails to converge as the meridian is made more elliptical. A converged solution may again be obtained for a reduced pressure, possibly indicating the onset of limit point buckling.

The problem of compression in the dome is unidirectional, in the circumferential direction. In the meridional direction, the stress resultant remains tensile for all geometries. That being the case, it seems worthwhile to investigate the effects of laminate stacking sequence on the overall response. Specifically, it seems reasonable that a stiffening in the circumferential direction relative to the meridional direction might lead to a load transfer, relieving the compression. The analysis reveals, however, that such is not the case -- preferential stiffening, if it is to be done at all, is best done in the meridional direction. The effects of preferential stiffening are analyzed by simply changing the orientation of two of the innermost plies of the laminate: going from a quasi-isotropic laminate to either a $[\pm 45, 90_2]_s$ (circumferentially stiff) or a $[\pm 45, 0_2]_s$ (meridionally stiff) laminate. While the meridionally stiff laminate is superior in terms of response to the circumferentially stiff laminate, both are inferior to the quasi-isotropic laminate.

Continuing with the investigation of the effects of laminate stacking sequence, attention was next turned to balanced, symmetric angle-ply laminates, specifically to $[\pm\alpha]_{2s}$ laminates. Here again, as in the reported result of the previous paragraph, it was seen that the response of the shell is improved as a result of preferential stiffness in the meridional direction. That is, the overall response is "better" if the winding angle α is smaller. In fact, for winding angle α greater than 46 degrees, circumferential compression cannot be avoided for domes with semi-circular or more elliptical meridians. For a winding angle of 50 degrees, convergence cannot be obtained for the spherical dome. On the other hand, it was noted that reduction of the winding angle α soon results in a dome design which is as good, in terms of weight, as the "best" quasi-isotropic dome design. This occurs for $\alpha = 40$ degrees. Further reduction in the winding angle does not, however, result in significant further weight savings -- it is not until winding angle is reduced another 10 degrees that there is any change in the allowable ellipticity for avoidance of compression. Again, as for the quasi-isotropic laminate, the onset of compression is soon followed by a failure of the method to converge. It seems clear that the material properties cannot be randomly chosen, but that the most limiting factor in the onset of compression is the geometry of the dome. Finally, it is shown by direct comparison of the results obtained that the overall response of the "best" tested $[\pm\alpha]_{2S}$ laminate, that being $\alpha = 30$ degrees, exhibits a response which is generally inferior to the response of the quasi-isotropic laminate. In particular, the displacements of the symmetric angle-ply laminate are very large in comparison to the displacements of the quasi-isotropic laminate.

Overall, the results of Chapter 6 show that there seems to be no real utility in the use of any dome lamination scheme other than the quasi-isotropic $[\pm45,0,90]_S$ layup. Nevertheless, there may be an advantage in the use of the quasi-isotropic dome rather than an isotropic dome, for weight savings.

7.2 Achievements

The following items, detailed in this document, represent perhaps the most significant outcomes of the work which has been completed.

- The development of a mixed formulation of the governing ODE's for the geometrically nonlinear, static, axisymmetric response of shells of revolution. The mixed formulation is significant in that it provides a good representation of the gradients of the dependent variables. This formulation also contains only terms which may be prescribed on meridional boundaries, and hence represents a two-point boundary value problem in ordinary differential equations.
- Successful adaptation and implementation of the stabilized marching technique of the method of multiple shooting for numerical integration of the governing ODE's. These equations are both numerically stiff and numerically unstable, and their integration has

proven to be a challenging task. The application of the stabilized marching technique to their solution is unique to this work.

- The results of the shooting technique have been verified to have accuracy comparable to the results of finite element analysis; in some cases, the results of the shooting analysis are superior to those obtained by finite element analysis.
- Parametric studies have been performed to examine the effects of dome geometry and material properties on the overall response of a dome used to close a quasi-isotropic cylinder under pressure. The conclusion is reached that the dome geometry is the critical design factor in avoidance of circumferential compression under internal pressure load, while material properties govern the magnitude of the displacements. Generally speaking, isotropic or quasi-isotropic material properties provide the best response.

7.3 Recommendations for Future Work

The work presented here has provided a basis for the expansion of the technique to more advanced topics in structural mechanics of shells. Some such advanced topics which would seem to naturally follow are:

Buckling analysis. The geometrically nonlinear response has been derived, giving some knowledge as to the prebuckling equilibrium state. Inasmuch as knowledge of that prebuckling equilibrium state is crucial to a buckling analysis, it seems a reasonable task to make use of what we have. Stability of the calculated response could be assessed by the following additional steps, taken after completion of the calculation of static response.

Formulate the second variation of the total potential energy, assuming the varied displacements take the form of a Fourier trigonometric series in the circumferential coordinate. The second variation of the total potential energy may be obtained in a form which is quadratic in the varied displacements at the shooting points. This formulation requires the use of a finite difference scheme to approximate the derivatives of the varied displacements with respect to the meridional coordinate.

Assess the stability of the axisymmetric equilibrium states by an LDL^T factorization of the system matrix which appears in the quadratic form of the second variation of the total potential energy. The equilibrium state is judged to be stable if the system matrix is positive-definite. Factorization into an LDL^T form simplifies the numerical check of positive-definiteness. This approach has been utilized by Ley, et. al (1992).

<u>Investigation of interlaminar stresses</u>. It was shown in Foster and Johnson (1991) that the interlaminar stresses in a laminated plate may be estimated, if one has good knowledge of the values and gradients of the stress resultants acting upon the plate. The same process could be applied here, in that the state vector equation, which is solved by numerical integration, is actually an equation for those gradients. That is, the method is perfectly suited for calculation of the stress gradients in a way that the finite element method is not. The direct numerical integration technique may thus be the best tool for the job of estimation of interlaminar stresses.

<u>Structural Optimization</u>. The solution process as incorporated for this work made use of a cubic spline interpolation for the geometry. This was specifically chosen as a tool for an eventual optimization study. The cubic spline interpolation has been successfully used in structural optimization of shells by Hinton, Rao and Sienz (1992), by Hinton and Rao (1993) and by Weck and Steinke (1983-4).

In addition to the above broad topics for further study, a shooting code as presented here might be further refined to allow for the following additional features:

- <u>Automatic selection of shooting points</u>. It was argued by Ascher, et al., (1988) that the multiple shooting technique may be made more efficient by avoidance of the *a priori* selection of shooting points. Allowing the computer to select its own “targets” allows for the capture of local effects, but also allows for “rapid cruising through uninteresting terrain”.
- <u>Scaling of the dependent variables</u>. The dependent variables in the state vector equation are of two types: stress resultants/stress couples, and displacements. It is not unusual in stress analysis of shells to have those two types of variables differ from one

another by many orders of magnitude. For example, the largest stress resultant might be on the order of 10^3, while the smallest significant displacement might be on the order of 10^{-4}. Lumping the variables together in one equation without any allowance for the differences in magnitude may cause unnecessary calculation time as the computer struggles to provide the desired accuracy.

- Further incorporation of internal discontinuities. The multiple shooting technique as presented here is not limited to geometrically continuous structures, or to single material systems, but the program which has been created to do the shooting has not allowed for such discontinuities within the shell meridian. Discontinuities have been incorporated only by inclusion within the elastic boundary conditions. The necessary changes to incorporate internal geometric and material discontinuity would allow for "one shot" analysis of complex shell shapes.
- Expansion to two dimensional analysis. While the method as presented here is useful for analysis of axisymmetric response, the practical usefulness of the technique would be greatly increased if the method were expanded to two dimensions. Steele and Kim (1992) gave a nod to this idea in their paper by assumption of a harmonic response in the circumferential direction. In essence, we did the same thing, but choosing to allow only the zeroth harmonic. If a change were made in the derivation of the state vector equations, shooting could be done for any chosen circumferential wave number.