

Finite Element Analysis of the Deformation of a Rubber Diaphragm

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(ABSTRACT)

Several rubber diaphragms, of the same type used inside an hydraulic accumulator, failed a short time after they were mounted. While there is nothing special with these failures the cost, in some cases can be high. A closer look, at the damaged diaphragms reveal an interesting nonsymmetric radial deformation accompanied in some cases by cracks. Most of the analyses regarding the failures of rubber diaphragms offer explanations only from a chemical or material science point of view. We propose in this thesis a new perspective from a mechanical-structural engineering view. Therefore the main goal of the thesis is to investigate the deformation of a diaphragm and based on this analysis to propose an explanation for formation of the cracks. It is shown that the analysis of the diaphragm problem leads to a pseudo-nonconservative system and involves a buckling, a post buckling (dynamic snap-through), an eversion, and a load response analysis. The problem is approached numerically using the finite element method. The character of pseudo-nonconservativeness of the system requires, in this case, an update of the tangent stiffness matrix with a certain stiffness correction. This new correction is proposed also. The result is valid not only for this particular problem but for the entire class of problems to which the diaphragm belongs. This correction is implemented in an existing finite element program (NIKE3D) and used to analyze the diaphragm deformation. The results indicate that under the typical load condition for a diaphragm a certain deformation pattern occurs, and this can lead to the formation of cracks. This deformation matches extremely well with the actual deformed shape of a typical failed diaphragm. It is shown that the deformation pattern depends on the structural properties of the diaphragm rather than on the magnitude of the applied load. The nonsymmetry in the diaphragm deformation and the difference in the crack development is explained also.

Dedication

To Bob and Emyl

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Chapter 1

Introduction

In engineering, designing a product or a component is a complex process involving several steps starting from the concept to the actual product. From the designer point of view a product needs to meet certain requirements defined or imposed at the beginning of the design process. From the user point of view, the product should meet expectations related to the role of the product in the system where it is used. In many cases a product may not be at the level of the user expectation and in this case observations made during the use can serve as a feedback for the designer to improve the product. Therefore the design activity is a cycling process: designer-user-designer. The improvement of the design of a product or component is in general required because of its low performance, new requirements from the user or, finally by the failure of that component. The first two situations are very normal in design but the last situation usually needs immediate special attention.

When a failure of a product or of a component occurs, the first step is to analyze the cause. This implies the need to verify whether the design has meet the requirements imposed and whether the technology for manufacturing that product or component has been followed accurately. However, there are situations when even if these standards are meet the product still fails. In this case it becomes necessary to the research part to provide explanations for failure and eventually to propose additional guidelines for the design. The failure of

a product cannot be always regarded as catastrophic, although there are such situations as a plane crash or a building collapse which obviously are. In most situations the low performance of a product or component are not life-threatening but rather economically deficient.

Failure is a wide concept and has different meanings in different engineering domains. In [17], for example, failure is defined as "unacceptable difference between expected and observed performance". The performance of a product or component has to be related to life expectancy and the degree of maintenance provided. When failure or low performance occur the product will be replaced. The replacement of a product is imposed in many cases not by failure or low expectations but also by new results in the research area.

In the present thesis we concentrate on a possible explanation of the failure of a rubber diaphragm. This diaphragm is a component of the accumulator used in the hydraulic system of the drilling drifter HC150-IR. This drifter is produced by the Ingersoll-Rand Co., Roanoke, VA., where the author worked as an intern in the period 1998-2000, in the Research and Development department.

Without entering into the details and classifications of such diaphragms, which goes beyond the topic of this thesis, we would like to mention that these diaphragms are designed basically to control the flow of a hydraulic system. A diaphragm separates two regions: one with hydraulic fluid and the other one with gas. They are flexible components, are made from rubber, and can have different shapes from flat to conical. The most common place where these diaphragms are used is inside pumps. By controlling the gas pressure the diaphragm will move back and forth adjusting the pressure in the hydraulic system. The situation is different in the case of an accumulator where the gas region is enclosed. Accidental peaks in the hydraulic pressure will be smoothed by the movement of the diaphragm inside the accumulator, allowing a variation of the flow debit and therefore an adjustment of the hydraulic pressure.

The diaphragms are consumable products and therefore periodically they have to be replaced.

However there are situations when failures occur prematurely as it is described in a technical article signed by the leader of the diaphragms team at W.L. Gore associates [54]. We mention below his observations, indicating also the domains where these diaphragms were used.

Chemical processing "The Grow Group in Harve de Grace Maryland produces professional and household products. One of the diaphragm pump applications was pumping a 32%HCL solution through a 250 ft line at ambient temperatures (60-70 F) and a 60 psi air supply. They were using a Wilden M-2 pump with a two piece diaphragm system consisting of a PTFE overlay and a Nordel backup. Their average life for the system was 5-6 weeks usually resulting in a catastrophic failure of the PTFE overlay (cracking from the center hole out). When this occurred the acid also destroyed the M-2's air valve, resulting in a costly pump rebuild."

Paints "At Sherwin-Williams Company in Baltimore (n.a.) they were pumping a chemical coatings product at 160 F with an air supply pressure at 60 psi, six hours/day. Up to this point (9/24/92 n.a.) the typical service life was from one to two weeks with failure resulting from dissolving of the PTFE overlay."

Resins "For a particular part of their process, a leading manufacturer of laminate boards for the PC industry uses Warren Rupp Sandpiper EB1A pumps which operates 24 hours/day. They are pumping resin for laminates which is a thick viscous slurry at 74 F with an air supply pressure of 40-60 psi. Typical life for the two-piece PTFE/Neoprene diaphragm system was 45 days."

Solvents "Montgomery Tank Lines in Saginaw, TX cleans its tank trucks, which mostly transport paint, with a MEK/Xylene solvent. Previously, the pump used PTFE/Neoprene overlays, with the service life of the diaphragms ranging between 3 days to 3 months (the mode of failure was holes developing in the diaphragms). In the previous 12 months (n.a Oct. 1993) Montgomery Tank Lines had spent \$16,000 on replacing the diaphragms and rebuilding the pumps."

The above examples refer to the failure of diaphragms in pumps. In our situation the diaphragm fails inside the accumulator. The type of diaphragm used in this thesis is produced at Montabert, France and is made from polyurethane. The life of such diaphragms is expected to be of several months before its replacement. However repetitive failures in a form of cracks on the lateral surface of this diaphragm have been reported (1999) in a relatively short time after the diaphragm was mounted. The failure of the diaphragm itself might not be so important (the cost to manufacture such a diaphragm being relatively small) but the auxiliary damages are important. When this diaphragm fails, it affects the hydraulic hoses, produces vibration of the machine (because the pressure pulsations are no longer smoothed), premature wear and, in the end, it diminishes the drill efficiency. According to the engineering team at Ingersoll-Rand, Roanoke, VA, the cost of failure of one diaphragm was up to \$ 5,000/day. At that moment (June 1999) an analysis of the cause of the failure of this type of membrane was requested at Research and Development (Roanoke, VA) group and the problem was assigned to the author of this thesis.

Analyzing the other reported failures of other types of diaphragms we can see that most of the explanations are given from a material-science/chemical engineer point of view and a possible mechanical-structural cause of failure has not been taken into consideration. On the other hand, several failed samples of the Montabert diaphragm present a similar deformation pattern as the sample that was given to the author (which is used in this thesis for comparison), These samples display a quite regular distribution of cracks. These suggest that it is possible that a certain deformation occurred prior to the crack formation, a deformation which ultimately, combined with material damage, could have lead to the appearance of cracks.

It is our purpose in this thesis to investigate if such a deformation could occur and eventually to give an explanation of the failure of the Montabert diaphragms. The point of view advanced in this thesis is a point of view of a mechanical-structural engineer. It may not be the only one or maybe not the complete one. However, from our references it seems that it is the first time a mechanical-structural explanation is proposed for explaining failure of

such diaphragms. For investigation we will use the Finite Element Method (FEM). It will be shown later in this thesis that the analysis of such diaphragms leads to a large displacements problem involving dynamic snap-through, eversion of diaphragm (we will use the engineering term: inversion) and a load response from the inverted (everted) position of diaphragm. Also the basic FEM formulation requires an update with a certain stiffness correction imposed by the nature of the problem. This correction is proposed also and derived in this thesis. The stiffness correction is implemented in an existing FEM code (NIKE3D) which is used to investigate the failure of the Montabert diaphragm.

The work on this subject was supported by Ingersoll-Rand Co., Roanoke, VA in the period of the internship and by the Engineering Science and Mechanics Department, Virginia Polytechnic Institute and State University, Blacksburg, VA.

1.1 Description of diaphragm failure

The accumulator shown in the Figures 1.1-1.2 is designed to absorb and smooth the hydraulic pulsation in the hydraulic system of the drifter HC150. The accumulator has small holes on one-side, which allow hydraulic fluid to enter/exit into the accumulator. Inside the accumulator there is a diaphragm made of rubber as shown in Figure 1.2 and Figure 1.3. The diaphragm separates the accumulator cavity into two regions: one region is connected to the hydraulic system filled with oil and the other region contains Nitrogen gas at high pressure. Initially the diaphragm is mounted as in the Figure 1.2 and precharged with Nitrogen at $p_{N_2} \approx 55 \text{ atm}$ ($= 5.5 \text{ MPa}$), ($1 \text{ atm} \approx 100 \text{ kPa}$). When the drifter is in a percussion mode (drilling process), the pressure in the hydraulic system can rise up to $p_h \approx 180 \text{ atm}$ ($= 18 \text{ MPa}$). This causes the diaphragm to move to a position such that $p_h = p_{N_2} = p_w$ as is shown in Figure 1.4. If pressure p_h increases above p_w , the oil flows into accumulator decreasing the debit in the hydraulic system and viceversa. As in reality the hydraulic pressure has oscillations around a mean value, the diaphragm will also exhibit vibrations

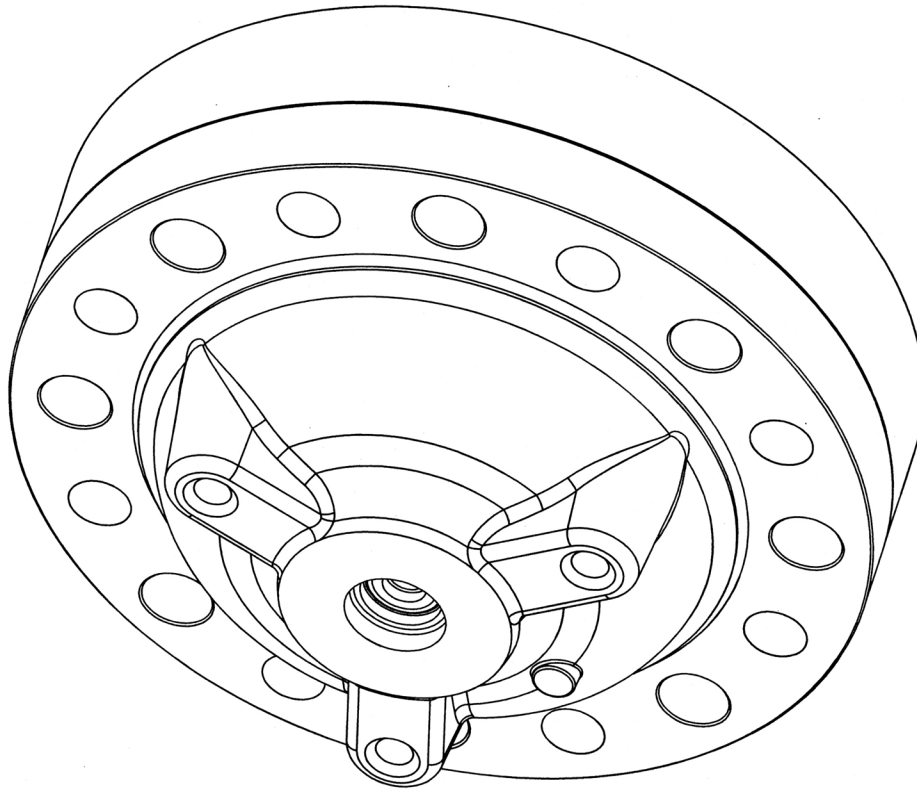


Figure 1.1: The accumulator-1.

around an equilibrium position. The failures of diaphragms occur in the form of cracks on the lateral surface having the edges with the appearance of being burned. Figures 1.5-1.7 show such a diaphragm, made from polyurethane, produced by Ingersoll-Rand/Montabert, France, after failure.

The aim of this thesis is to investigate the deformation of this diaphragm, to offer an explanation of the failure, and to propose a model for future studies and design. Failure is a complex process, which may involve mechanical, thermal, chemical, and fluid causes in our situation. Therefore we make the following hypothesis:

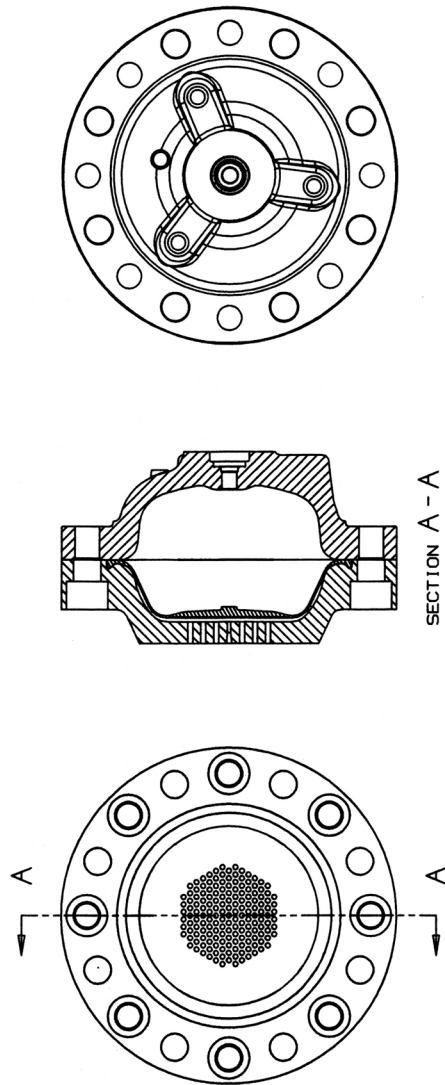


Figure 1.2: The accumulator-2.

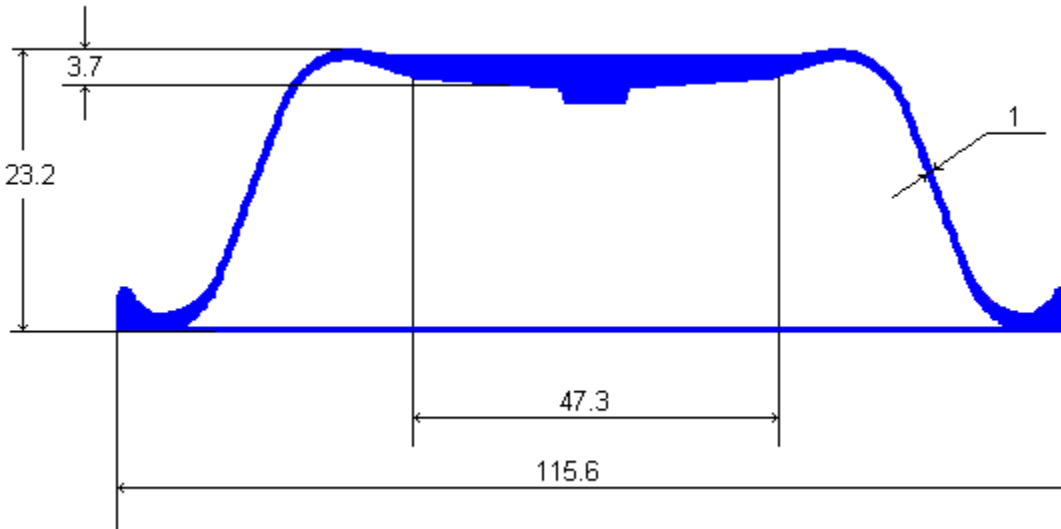


Figure 1.3: The diaphragm dimensions (mm).

The main cause of the creation of cracks, and ultimately the fail of the rubber diaphragm mentioned above is a mechanical cause, due to the deformation of the diaphragm.

Other effects such as thermal or chemical may accelerate the creation of the cracks and might be responsible for the crack initiation locally, but these will be considered as secondary effects in the proposed model and will be neglected.

1.2 Deformation of the diaphragm

Before the analysis of the diaphragm deformation, we make the observation that, due to relatively small dimensions of the accumulator, we can approximate the oil pressure with an inlet constant pressure. This means that the pressure on the side of the diaphragm in contact with the oil can be considered constant.

The thickness of the diaphragm is between 1–4 mm much smaller than the other dimensions

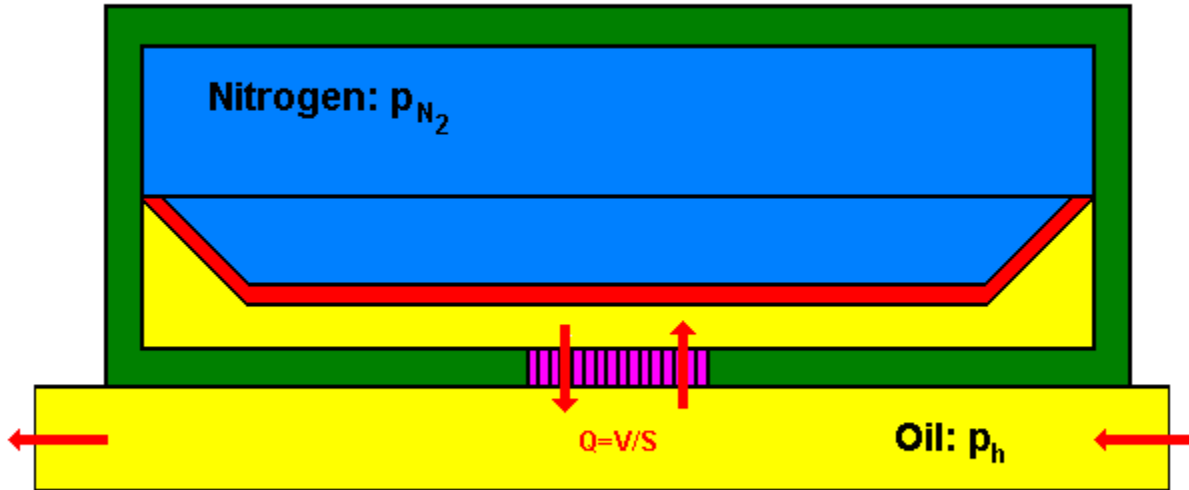


Figure 1.4: The accumulator model.

of the diaphragm. The diaphragm has on both sides very high pressures of 50-180 atm. High equal pressures on both sides compress the diaphragm to the limit of incompressibility and consequently we can consider the volumetric deformation very small. Physically, in such situations we do not expect a significant change in the shape of the diaphragm, but we do expect some shear deformations, but not very high. A slight difference between the oil pressure and the nitrogen pressure will cause the displacement of the diaphragm to one side or another reaching a new equilibrium position, corresponding to equalized pressures on both sides. In reality, because the oil pressure has relatively small perturbations around a mean value, it is likely that the diaphragm will vibrate around a mean position. However, in the new position, the diaphragm has its shape changed, and it is clear that important deformations occur during the movement of the diaphragm. To clarify these ideas, let us consider a diaphragm, as in the Figure 1.8, passing from an equilibrium position A to another equilibrium position B and further, to C . By equilibrium position we understand the position characterized by equal oil and nitrogen pressures. In the positions A , B , and C , we have

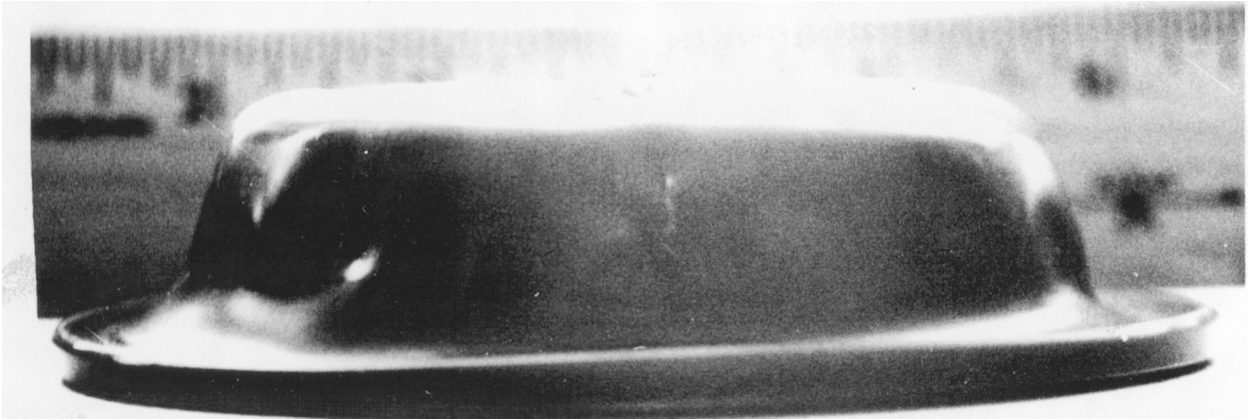


Figure 1.5: The diaphragm. Cracks are radial, on the lateral surface.

equal pressures. Assuming that the thickness of the diaphragm is small and the material is incompressible, the deformation energy of the diaphragm is stored in such positions as a shear deformation energy, i.e., we neglect the volumetric deformation energy. Let us assume that a difference in pressures occurs and the diaphragm reaches a new position characterized by new equal values of oil and nitrogen pressures. In the transition phase from $A \rightarrow B$, for example, the mechanical work done by Δp on the diaphragm is converted first in a kinetic energy, causing the movement of the diaphragm. As the diaphragm reaches the new position at B , the kinetic energy is transferred in a potential deformation energy. Based on our assumptions this energy is stored as shear deformation energy. The process of conversion of energy from mechanical work to kinetic energy and, at the end, to shear deformation energy is a continuum process and occurs during the transition phase: from A to B , from B to C etc. This can lead to important change in shape of the diaphragm. It follows also that for our problem the difference in pressures rather than the values on each side is more important. Consequently, instead of using high pressures on each side, we can use the pressure difference Δp on one of the diaphragm sides.

Therefore we can state our problem by saying that the deformation of the diaphragm occurs in the transition phase, and the objective is to find this deformation. We know the equilibrium position A which is the mounted diaphragm position. Physical conditions dictate that

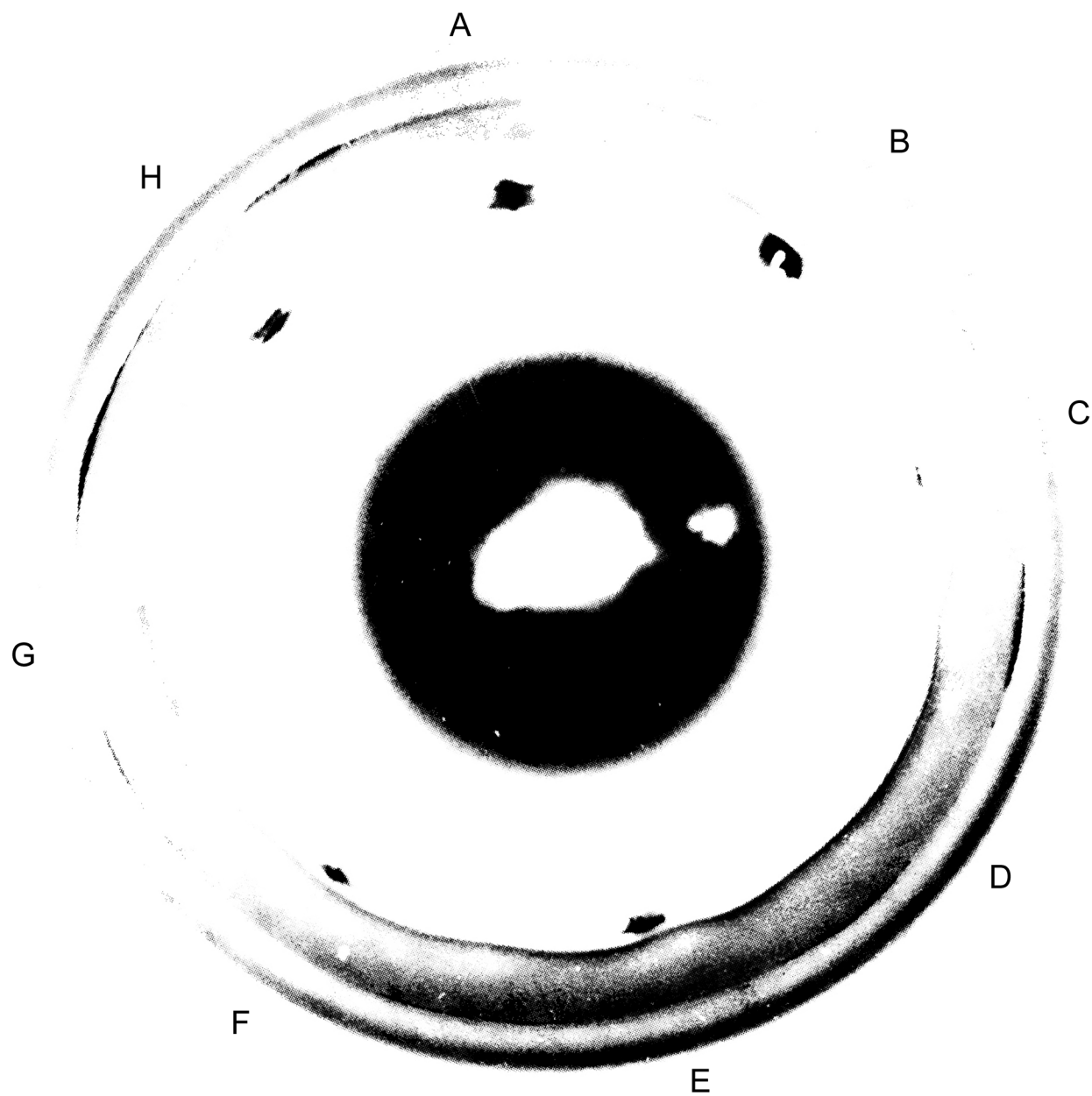


Figure 1.6: The diaphragm. Black points are the traces of the cracks

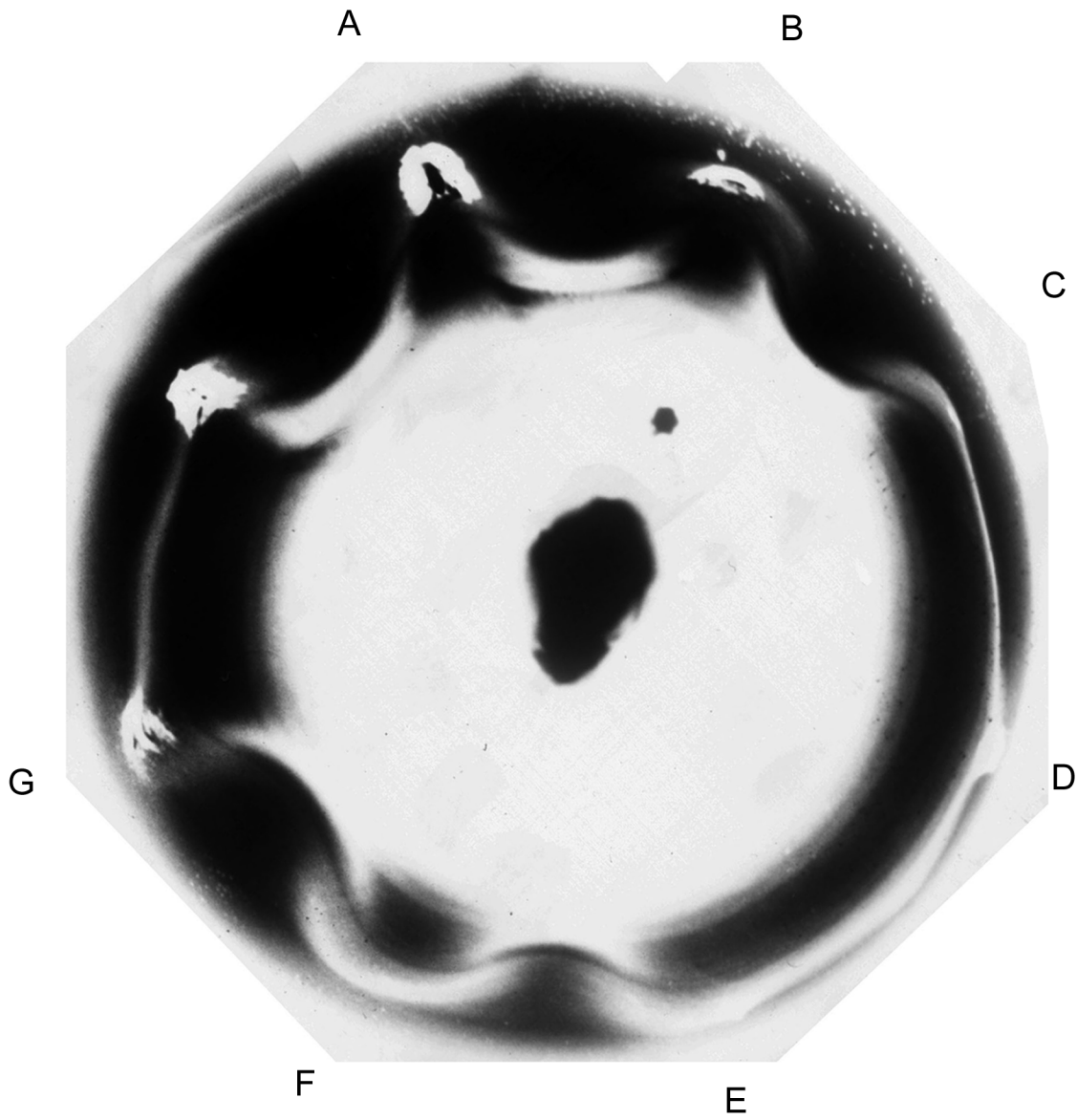


Figure 1.7: Diaphragm inverted.

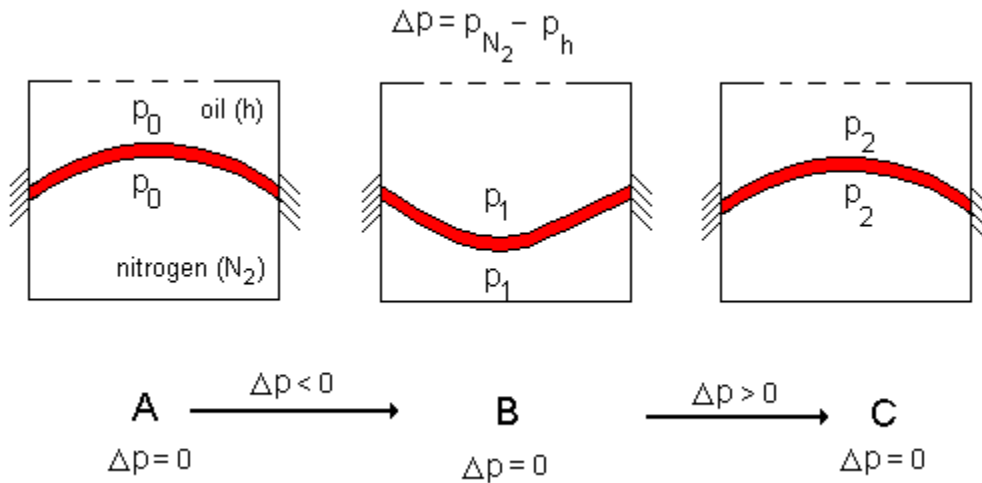


Figure 1.8: Diaphragm positions

there is a second equilibrium position B , at working pressure, when the drifter is in use. If cracks occur at certain locations, then, prior to the physical creation of cracks, there should be a *deformation pattern*, which will eventually favor the appearance of cracks. According to our observations, this deformation pattern is developed during the transition phase. The diaphragm can *arrive* at the position B with this deformation pattern, and equal pressures in B can *freeze* this pattern. Of course, the entire process is a dynamic one, but as we have mentioned above, we expect that there is a mean position around which the diaphragm will have some oscillations, while preserving a mean deformed shape.

1.3 The energy balance

We begin our investigation by analyzing the energy balance of the system. At this time, by system we understand the accumulator with oil (h) diaphragm (D) and nitrogen (N_2). The first principle of thermodynamics applied to the system states

$$Q = \Delta\mathcal{E} + \mathcal{L}, \quad (1.1)$$

where \mathcal{Q} is the heat, \mathcal{L} is the mechanical work, and $\Delta\mathcal{E}$ is the variation of the energy of the system. Consider now that the system is isolated, i.e. no heat is exchanged with the exterior and no mechanical work is done on the system, $\mathcal{L} = 0$, then the energy must be conserved.

$$\Delta\mathcal{E} = 0.$$

For the components of the system

$$\Delta\mathcal{E}_h + \Delta\mathcal{E}_D + \Delta\mathcal{E}_{N_2} = 0. \quad (1.2)$$

Applying the first principle of thermodynamics to the subsystems, we obtain

$$\Delta\mathcal{E}_h = - \int_{\Sigma_h} p_h u d\Sigma_h < 0, \quad (1.3)$$

$$\Delta\mathcal{E}_{N_2} = \int_{\Sigma_{N_2}} [p_{N_2} + \Delta p_{N_2}(u)] u d\Sigma_{N_2} > 0, \quad (1.4)$$

where Σ_h defines the surface of the diaphragm in contact with oil and Σ_{N_2} defines the surface of the diaphragm in contact with nitrogen. In (1.3) and (1.4), $p_h > p_{N_2}$ are the values of pressures before the diaphragm starts to move. u is the displacement, and $\Delta p_{N_2}(u)$ represents the increase of the nitrogen pressure as a result of the diaphragm movement. Consequently this variation depends on the displacement of the diaphragm. Introducing (1.3) and (1.4) into (1.2), we obtain

$$\int_{\Sigma_h} p_h u d\Sigma_h - \int_{\Sigma_{N_2}} p_{N_2} u d\Sigma_{N_2} = \Delta\mathcal{E}_D + \int_{\Sigma_{N_2}} \Delta p_{N_2}(u) u d\Sigma_{N_2}. \quad (1.5)$$

The equation (1.5) is a direct consequence of the conservation of energy of the system. Another balance which should be taken into consideration is the balance of mass of the entire system: oil+diaphragm+nitrogen. The movement of the diaphragm leads to an oil exchange between the accumulator and the hydraulic system and therefore we will have a change in the oil mass. During the process, the masses of diaphragm and nitrogen remain the same and therefore this equation applies only to oil.

The analysis of the full system implies a fluid model for oil and nitrogen flow and a solid model for the diaphragm. The analysis of such a system is very complicated, involving a solid-

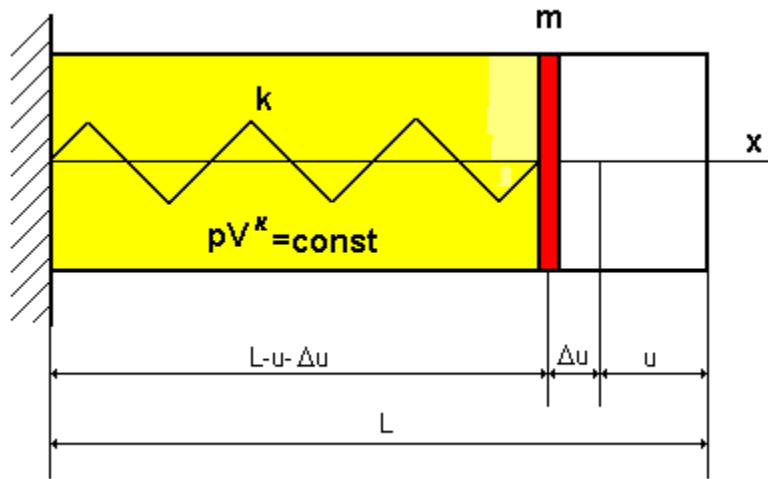


Figure 1.9: Spring model.

fluid interaction. Since we are interested only in the diaphragm failure, we can simplify the problem, without losing its generality, by considering the *system* being only the diaphragm.

In the equation (1.5) all terms are positive. This equation shows that an amount of energy is introduced in the system (left side term). A part of this amount is used to increase the energy of the diaphragm, and the remaining part is *consumed* as a mechanical work done on the diaphragm by the pressure forces generated by the motion of the diaphragm. If $p_h > p_{N_2}$, these forces extract energy from the system. Therefore we have a *nonconservative* system in the classical sense, with an applied pressure which depends on displacements. The discussion about the nature of this system will resume in Chapter 4.

1.4 Spring model

To illustrate an important aspect of the problem which will be addressed in the next chapters, let us consider a very simple model as is shown in Figure 1.9, where the diaphragm is represented as a nonlinear spring. Inside the cylinder we have a gas following an isentropic evolution $pV^\kappa = \text{const}$ where p is the gas pressure, V the gas volume, and κ the isentropic

exponent [36]. Let us assume that an external pressure p_a is applied to the diaphragm. This will cause a displacement u of the diaphragm. Let us assume further that, relative to this position, the external load is increased further causing a further displacement of the diaphragm with an increment Δu as shown in Figure 1.9. If we denote by S the diaphragm surface then the equilibrium equation at the position $u + \Delta u$ yields

$$p_a S - p(u + \Delta u) S - F(u + \Delta u) = 0, \quad (1.6)$$

where F is the restoring spring force, which can be a nonlinear function of displacement. Developing a Taylor series (1.6) at the position u , and keeping only the first order terms we obtain

$$p_a S - p(u) S - \frac{dp}{du} \Delta u - F(u) - \frac{dF}{du} \Delta u = 0.$$

Denoting by

$$F^{ext} = [p_a - p(u)] S, \quad F^{int} = F(u), \quad K_T = \frac{dF}{du}, \quad K_c = \frac{dp}{du},$$

equation (1.6) becomes

$$[K_T(u) + K_c(u)] \Delta u = F^{ext} - F^{int}(u). \quad (1.7)$$

Analyzing equation (1.7), it can be seen that the tangent spring stiffness K_T is corrected with an additional stiffness term K_c due to the variation of the gas pressure. Anticipating, if we transpose the entire problem into a three-dimensional model, equation (1.7) becomes

$$[\mathbf{K}_T(\mathbf{U}) + \mathbf{K}_c(\mathbf{U})] \mathbf{u} = \mathbf{F}^{ext} - \mathbf{F}^{int}(\mathbf{U}).$$

where \mathbf{K}_T is the tangent stiffness matrix of the diaphragm, \mathbf{K}_c is the correction stiffness matrix due to the presence of an external pressure which depends on the displacements of the diaphragm, \mathbf{U} is the displacements vector and \mathbf{u} the displacements increment vector.

Summarizing this chapter, we can say the diaphragm problem involves two major aspects

- a problem involving nonlinear analysis of a body made from a rubber material,
- a problem involving a pressure load depending on the displacements.

The main objective of this thesis is to find the circumstances in which the diaphragm deforms in such way that may lead to the creation of cracks. This analysis will also include the determination of the correction stiffness matrix and its effect on the diaphragm deformation.

In Chapter 5 it will be shown that, to find the deformation of diaphragm which could give the answer for our problem, we will have to pass through several steps: a buckling analysis, a post buckling analysis (a dynamic snap-through problem), an eversion (inversion) problem for the diaphragm, and, finally, a load response of the diaphragm from the everted (inverted) position. The analysis is complex, and further references to related works on each problem will be made on the corresponding section.