Chapter 6

Conclusions

The problem approached in this thesis was generated from a practical situation: the repeating failures of the diaphragm of an accumulator used to smooth the hydraulic pressure on the drifter HC150. For the explanation of the appearance of cracks on the lateral surface of the diaphragm and the diaphragm failure, we have proposed a structural point of view based on a physical model, which is sustained in this thesis by the results of numerical simulations, using the Finite Element Method (FEM).

We have identified that the diaphragm belongs to the class of pseudo-nonconservative systems. This characteristic is due to the fact that, in modeling the diaphragm deformation, we have neglected the nitrogen as a part of the system. If we consider the full model: oil + diaphragm + nitrogen, the system is conservative. The system is still conservative if we consider also only the diaphragm + nitrogen. If we consider only the diaphragm, then the system becomes pseudo-nonconservative. This suggests that such systems are generated from conservative systems in which a part is neglected in a model.

The mathematical problem of the diaphragm deformation was formulated from the view point of Continuum Mechanics. We have pointed out the transient and nonlinear character of the problem and we have presented the linearized form of the dynamic equations of motion.

For solving the mathematical problem, we have chosen the Finite Element Method (FEM). The general formulation of FEM was derived from the weak formulation of the problem. We have presented with details the weak form and the linearized form of the weak formulation and we have outlined the particularities of using a rubber-like material. The key steps in the derivation of the finite element equation were identified as well as the methodology of approaching a transient problem using FEM.

We have shown in detail the load/stiffness correction, which arises when a pressure load depends on the displacements and we have extended the previous work done by Schweizerhof and Ramm [46] in this direction with a new additional stiffness correction, required by the nature of the diaphragm problem. This correction is valid also for any other problems simmilar to the problem approached in this thesis. In the finite element approach, the character of pseudo-nonconservativeness of the diaphragm problem is translated into a correction of the tangent stiffness matrix with a symmetric matrix. We have determined the matrix and explained the nature of the correction. The load/stiffness correction was implemented in the code NIKE3D and we have used this program to study the diaphragm deformations and the influence of the stiffness correction term. The results obtained tend to confirm our physical model for the diaphragm and lead to possible explanation, from structural point of view, of the cracks formation. During this investigation we have shown that the study of the diaphragm deformation leads to a buckling, a post-buckling (snap-through), an inversion (eversion) and a load-response analysis from the inverted (everted) position.

Based on the interpretations of the diaphragm deformation we have arrived at the following conclusions

1. The cracks on the diaphragm are created as a result of a certain deformation pattern.

2. The cracks are likely to occur in the regions where the lobes are formed.

3. The nonsymmetry in the cracks development is due:

    - to a residual nonuniform stress created when the diaphragm is first inverted.

- to the imperfection of the geometry of the diaphragm in the shape of the first buckling mode (imperfect diaphragm).

4. The deformation pattern occurs on the inverted imperfect diaphragm and, based on our numerical results, it appears independent of the magnitude of a uniform applied pressure.

5. The stiffness correction plays an important roll in finding the correct inverted position of the diaphragm.

6. We expect the inertial effect in the reverse phase to increase the nonsymmetry in the deformation pattern.

Once the deformation pattern is installed it can either progress leading to a frictional contact between the lobe surfaces and consequently to damage of the material due to friction, or, without a contact between the surfaces of the lobes, a cycling load could lead in time to material fatigue and consequently to damage and cracks on the lobes. These conclusions are based on the physical interpretation of our numerical results since, as we mentioned before, we have used a perfect elastic material without a damage model. The important achievement is that we have identified the strategy for studying the deformation of such diaphragms. We expect that enhancing the material with a damage model will lead to even closer results to the real diaphragm.

In developing the physical model and the finite element analyses, we had to rely only on our physical interpretation mostly based on the visual observations of a damaged real diaphragm. Therefore the author is open to any suggestions and/or observations regarding the approach proposed in this thesis.

The explanations and the model proposed in this thesis are not restricted to the particular diaphragm analyzed here. It can be applied to any other diaphragms or shells subjected to similar loads and boundary conditions. Based on our results we can recommend the following steps for future design of new diaphragms of the same type

1. Buckling analysis. If the diaphragm is not flat the diaphragm can buckle. Since buckling is a nonlinear phenomenon, we recommend the determination of critical buckling using the load/frequency curve. The use of linear buckling analysis may lead to erroneous results in the buckling mode shape when there is an important deformation prior to the diaphragm losing its load carrying capacity.

2. Collapse analysis (post-buckling analysis) to find the inverted positions of the diaphragm. Collapse analysis can be performed by modifying the original geometry with an imperfection in shape of the first buckling mode. For each inverted position the response of the diaphragm to different types of loads can be performed. We recomend a dynamic analysis. If there is a deformation pattern it should appear at the beginning of the response even at lower aplied loads.

3. In using a FEM package we recomand the use of stiffness correction as described in this thesis.

6.1 Future work

The diaphragm problem mentioned in this thesis offers a starting point for further investigation of the rubber diaphragm deformation. We mention below some of additional directions:

Material models . We have used two rubber models: Blatz-Ko and Mooney-Rivlin due to lack of material data for our membrane. The problem can be restarted by changing either the rubber model or the material itself. This can affect the inverted (everted) shape of the diaphragm, an essential step in establishing a deformation pattern (if such deformation pattern exists).

Numerical investigation . The results using DYNA3D and the Blatz-Ko rubber model show a hourglass phenomenon. We have eliminated the hourglass modes by refining the mesh, but when a finite model is complex an hourglass control technique is recommended. The diaphragm problem can be used as an example and a model to test the efficiency of different hourglass control algorithms.

Different shapes . It would be interesting to check if a similar deformation occurs if the shape of the membrane is changed. This problem may be particularly interesting for industry.

Damage analysis A damage model for the rubber material which can take into consideration conditions where these diaphragms work will help in predicting the life-expectancy for a given diaphragm. This type of analysis can be very useful in industry.

At the end of this thesis, we would like to mention the importance of forensic and failure analyses especially for industrial components. While in general, most of the designs in mechanical engineering are based on linear theories, failure occurs almost always in a nonlinear range. By investigating the failures of either a component or a system, we can provide a better understanding of the problem and improve the design. Understanding a problem, we can predict that problem, and if we can predict then we can control. We would like to believe also that this thesis shows the importance of nonlinear analysis in industrial design and also the fact that real problems can lead and generate theoretical developments, in our case an inversion (eversion) problem and a new type of stiffness correction in the Finite Element Analysis.

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