CHAPTER 1

INTRODUCTION

1.1 Historical Survey

The pioneering work of Brittingham [1] suggested the possibility of solutions to Maxwell’s equations describing localized nondispersive transfer of electromagnetic energy in space. The originally proposed solutions, termed focus wave modes (FWMs), were continuous and nonsingular, had a three-dimensional pulse structure, were nondispersive for all time, and propagated at light velocity in straight lines. To render the energy content of those solutions finite, Brittingham introduced two surfaces of discontinuity infinitely extended along the direction of propagation, thus dividing space into three regions. The fields between the surfaces were chosen equal to the original FWMs, while the fields outside were set equal to zero. Wu and King [2] showed that the three-region extension proposed by Brittingham did not satisfy Maxwell's equations across the surfaces of discontinuity and, as a consequence, they established that the FWMs carried infinite energy. Wu and King's work was verified by Sezginer[6] and Wu and Lehmann [7], who proved that any finite energy solution led to dispersion and the spreading of energy.

The work of Belanger [4,10], Sezginer [6] and Ziolkowski [5] showed that the original FWMs could be related to exact solutions of the three-dimensional (3D) scalar wave equation. Ziolkowski [5] pointed out that plane waves shared with the FWMs their infinite energy content, and he showed that a superposition of those modes could result in finite energy solutions. Such pulses, characterized by high directionality and slow energy decay, have been called directed energy pulse trains (DEPTs), with a prefix A for acoustic and E for electromagnetic [27,42,54]. Beside the DEPT solutions, several papers have been published over the past several years on a variety of localized wave (LW) pulses, such as 'splash modes' [19], 'electromagnetic missiles' [8,22,23], 'monocrhomatic Bessel beams'

[16,17], 'transient Bessel beams' [132,133], 'transient beams' [11,14], 'acoustic and electromagnetic bullets' [9,94,121], 'X waves' or 'sling-shot pulses' [59] 'Bessel X waves' [59,70,116,117] and 'Bessel-Gauss pulses' [48].

In general, localized wave structures are carrier-free, ultra-wideband pulses that exhibit distinct advantages in their performance by comparison with conventional quasi-monochromatic signals. In particular, they have extended ranges of localization in the near-to-far field regions. This renders them useful in applications involving high resolution imaging, nondestructive testing, secure signaling and interference-free communications. For these reasons, the generation and propagation of these pulses has attracted considerable attention in recent years. [1-137]. This interest has been sustained by advancements in ultra-fast acoustic, optical and electrical devices capable of generating and shaping very short pulsed fields [41,118].

Wu [8] was first to address the issue of a source generating nondispersive wavepackets or electromagnetic missiles. He argued that the electromagnetic energy density transmitted by a finite aperture under transient excitation did not have to decrease as R^{-2} when $R \to \infty$. The energy reaching a receiver had to decay eventually to zero. He demonstrated that one could make the product of the missile's cross-sectional area and the average energy per unit area approach zero as slowly as one wished by choosing suitable frequency components of the exciting current. Wu deduced his results using the total received electromagnetic energy. Wu *et al.* [13] showed that electromagnetic missiles could be launched by a uniform dielectric sphere under transient excitation by a point source. A more recent study on the launchability of electromagnetic missiles deals with a line source within a cylindrical dielectric lens [23].

Yaghjian and Hansen [119] have shown that the generation of electromagnetic missiles requires infinite first-time derivatives of the current; as a consequence, the frequency spectrum of the current must decay as $\left(1/|\omega|^{3/2}\right)$, or slower, as $|\omega| \to \infty$. Fast switching currents are of great importance for future searching or fire-control radars. Success in this direction depends on the ability to generate short pulses with high peak power. The rapid progress in this direction is enhanced by the development of the bulk avalanche semiconductor [41] switch. This material, when illuminated by a low power laser beam, changes its state from nonconductor of electricity to an excellent conductor in picoseconds and can conduct gigawatts of energy.

On a completely different path, Ziolkowski [5,27] made the important observation that the scalar-valued FWMs describe fields that originate from moving complex sources. That observation linked the FWMs with earlier work by Deschamps [139] and Felsen [140] describing Gaussian beams as fields equivalent paraxially to spherical waves with centers at stationary complex locations.

Ziolkowski pointed out that a class of finite energy field solutions to the scalar wave equation could be obtained by using superpositions of FWMs [5,27]. He showed that one such solution, called the modified power spectrum (MPS) pulse, could be recovered to a high degree of accuracy from a finite planar array of point sources. In particular, a Huygens' reconstruction based on a finite planar array of point sources reproduced the MPS pulse at large distances away from the array [27]. In contrast to conventional CW-driven apertures, the array used by Ziolkowski was driven by a spatial distribution of wide bandwidth time waveforms. The array-generated MPS pulse appeared to be very robust and insensitive to perturbations in the initial aperture distributions. Ziolkowski *et al*. [28,33] performed an experiment investigating the feasibility of launching an acoustic directed energy pulse train (ADEPT). They established that an ADEPT launched from a linear synthetic array held itself and did not spread out until a distance equal to twice the Rayleigh length.

More recently, Ziolkowski considered the possibility of generating localized wave beams by means of ultra-wide bandwidth pulse-driven arrays [56]. He addressed the beam characteristics, such as the beam divergence, beam intensity and energy efficiency. He argued that new type of arrays were needed to generate LW pulses. Each array element, driven by broad bandwidth signals, must be independently addressable; that is, each element's time history is unique. Enhanced localization effects can be achieved by driving an array with a properly designed spatial distribution of broadband signals; that is, by controlling not only the amplitudes, but also the frequency spectra of the pulses driving the array. Along this framework, a simulation has been presented [54] that provides a method for calculating an optimum set of driving functions for an array of point sources given a scalar representation of the desired field. Comments on Ziolkowski's work were presented by Samaddar [69], who did not agree with Ziolkowski's work.

Ideas similar to those underlying the work on E(A)DEPTs and EM missiles were contemplated by Durnin [16] when he introduced the diffraction-free "Bessel beam' and he was able to demonstrate that such a beam had a larger depth of field compared to a conventional Gaussian beam, even if their central spots had the same radii. The increase in the depth of the Bessel beam was achieved, however, at the expense of the power utilized [17]. That behavior, which has been verified experimentally by Durnin *et al.* [21], can be attributed mainly to the differences in the energy distribution over the apertures. Durnin’s monochromatic beam is composed of different spatial spectral components. The depth of the monochromatic Bessel beam can be controlled by varying only its spatial spectral content or changing its energy distribution over the aperture. On the other hand, both temporal and spatial spectral components are required in synthesizing highly directional time-limited pulses, e.g., A(E)DEPTs and EM missiles.

Palmer and Donnelly [71] introduced a new approach for launching LW pulses using an infinite line source which radiates a field containing a FWM component and exhibiting a degree of localization perpendicular to the generation axis. The procedure is interesting, but the results are of limited practical value because of the infinite length of the line source. Along the same vein, Borisov and Utkin [91; cf., also, Ref. 134] have obtained solutions to the inhomogeneous scalar wave equation , the source being a pulse moving at the speed of light. The solution is a function describing both a transient and a steady-state wave process. The latter corresponds to Brittingham's FWM.

Lu and Greenleaf [59-61,76,101-104,112,113,124,130] have developed a new family of nondiffracting beams (called X waves because of the approach of their distribution in a plane through their axis). These wavefields are axially symmetric exact solutions of the free-space scalar wave equation. X waves travel to infinite distances without spreading, provided that they are produced by an infinite aperture. As in the case of a plane wave and Durnin's Bessel beam, the total energy of an X wave is infinite. But, the energy density of all these waves is finite. Approximate X waves with finite energy over a large depth of field can be realized with finite apertures. The real part of an X wave has a smooth phase change, over time t, across the transverse direction ρ. This makes it possible to realize X waves with physical devices. A broadband acoustic transducer can be used to produce this wave. Also, a band-limited transducer can generate a close approximation. This can be accomplished by controlling the electrodes of a PZT ceramic/polymer which can be cut into annular elements [36], each element driven with a proper waveform depending upon its radial position. Annular array transducers can be used both for transmitting and receiving X waves. In the transmission mode of the transducer, the material expands and contracts along the direction of the applied voltage signal launching a pressure wave; in the reciprocal (reception mode), an applied pressure wave deforms the piezoelectric material along this direction generating the measured voltage signal. Limited diffraction X wave beams have applications in medical imaging, tissue characterization and nondestructive evaluation, as well as other wave-related areas, such as electromagnetics and optics.

Heiki *et al*. [117,126] reported the results of an experiment involving a localized ultrashort pulse, the Bessel-X pulse, that maintained strong lateral and longitudinal localization in a bulk linear highly dispersive medium.

Besieris *et al.* [24] introduced a novel approach for synthesizing wave signals. This approach rests mainly on the decomposition of the exact solutions of the scalar wave equation into bidirectional, backward and forward plane waves traveling along a preferred direction z, viz., $e^{-i\alpha(z-ct)}e^{i\beta(z+ct)}$. These bilinear expressions can be elementary solutions to the Fourier-transformed (with respect to x and y) three-dimensional wave equation provided that a constraint relationship involving α, β and the Fourier variables dual to x and y is satisfied. Besieris *et al.* [24,43] used the bidirectional representation to group all FWM-like localized wave solutions in a single framework. The bidirectional representation method is applicable to other types of equations; specifically, it has been used to derive novel equations to the Klein-Gordon and Dirac equations [34].

Shaarawi *et al.* [25] applied the bidirectional method to the case of exciting a semi-infinite waveguide by an initial pulse with a shape related directly to parameters similar to those in the EDEPT solutions. They computed the radiated field using Kirchhoff's integral formula with a time-retarded Green's function. An approximate evaluation of the Kirchhoff integral gave rise to a LW train that was causal and had finite energy. Along the same vein, Vengsarkar *et al.* [53] applied the method of bidirectional decomposition to optical fiber waveguides and investigated the possibility of synthesizing pulsed LW solutions that could propagate along such guides with only local variations.

A modification to the bidirectional representation has been introduced by Chatzipetros [96]. As a result, one can select new elementary blocks resulting in different representations for a solution. The ordinary bidirectional method, as well as the aforementioned new superposition, have been applied to the forced scalar wave equation in order to compute fields generated by sources. This new approach provides an easy way to derive the sources of LW pulses.

Ziolkowski *et al.* [70] considered the realization of LW solutions from a finite aperture using the bidirectional representation. They argued that it was possible to reproduce approximately a causal Green's function in the near field of an aperture. It was demonstrated that only the forward propagating components of any solution to the homogeneous scalar wave equation were recovered in an open region in front of the aperture. By properly adjusting the parameters characterizing the source-free LW solutions, the radiated wavefields could be designed to minimize the contributions from backward propagating components. Hence, the causal fields generated by driving a finite aperture with LW solutions were demonstrated to be close approximations to source-free LW solutions everywhere in the near-to-far field region.

Shaarawi *et al*. [99] proposed the possibility of launching a good approximation to the source-free FWM solution from a Gaussian dynamic aperture. Such an aperture is characterized by an effective radius that shrinks from an infinite extension at $t \to \infty$ to a finite value at $t \to 0$, then expands once more to an infinite dimension as $t \to \infty$. The power density of the field illuminating the dynamic aperture decreases to zero at the same rate of expansion as that of its area extending towards an infinite value. These two effects balance each other and the power on the aperture remains constant for all time. However, the excitation of the FWM aperture utilizes infinite energy because it needs to be illuminated for an infinitely long period of time. The excitation of the FWM aperture does not need infinite power, in contradistinction to the excitation modes of infinite apertures, e.g., the Bessel beam and the plane wave illumination. Thus, there is no problem *per se* with the excitation of the FWM field, except for the need of an infinite time to illuminate the aperture. Shaarawi *et al*. [99,108] suggested that a finite energy FWM pulse could be excited by an aperture illuminated for a finite period of time. Thus, instead of using a superposition of FWM pulses to construct wide-band finite energy LW pulses, alternatively one may limit the initial excitation of the aperture over the time span $-4T \le t \le 4T$ by applying a Gaussian time window of the form $e^{-t^2/4T^2}$. The aperture shrinks from its maximum finite extension at $t = -4T$ to its minimum size at $t = 0$, then expands once more to its maximum extension at $t = 4T$. As a consequence, the aperture utilizes a finite excitation energy. The time limiting of the initial excitation of the aperture causes the radiated pulses to spread out at finite distances from the aperture [110]. The range of propagation depends mainly on the period of excitation, the smallest radius of the aperture and the 3-dB cutoff of the temporal frequency spectrum.

Along the same lines, the possibility of generating a time-limited X wave from a dynamic aperture antenna was addressed by Chatzipetros [96,137]. The decay rate of the radiated pulse was compared to that of a quasi-monochromatic signal. It was shown that the finite time X wave propagated to much further distances without significant distortion. A discussion of the diffraction limit of the time-limited X wave pulse was also provided.

It has been demonstrated [106,108] that the spectral depletion of the frequency components of LW pulses generated from dynamic apertures is completely different in nature from the spectral depletion of other transient pulsed wavefields having the same waist and longitudinal time duration. The spectral depletion of the Gaussian-waisted LW pulse generated by a FWM dynamic source has been compared to that of a quasi-monochromatic time-limited pulse having the same waist but launched from a static aperture. The switching time intervals of the excitations of those two pulsed fields have been taken to be equal.

Diffraction plays an important role in limiting the range of any propagating wavefield. Hafizi and Sprangle [51] have postulated that the diffraction length may be estimated by the distance at which the initial excitation of the beam is doubled, i.e., at $w = 2d$, where

w is the radius of the illuminated region and d is the radius of the aperture situated at $z = 0$. An alternative procedure for calculating the diffraction length of time-limited LW pulses has been proposed by Sedky [106; cf., also, Ref. 110]. The core of his procedure resides mainly on the determination of the maximum temporal frequency of the spectrum. The range of the pulse and its rate of decay depend on the spatial spectral bandwidth. These quantities, together with the maximum radius of the aperture, can be combined to give a good estimate of the diffraction length.

Recently, Shaarawi [129] presented a comparison of two localized wavefields, the X wave and the FWM, both generated from dynamic apertures. A study of their decay rates showed a difference in performance, even if they were excited from apertures of the same dimensions and with initial illumination wavefields of comparable frequency bandwidths. Based on the spectral depletion of those two pulses, a method was suggested for slowing down the rate of decay in the far field region.

A sequel to the bidirectional representation introduced recently by Besieris *et al.* [131] eliminates the difficulty in handling a seemingly disparate variety of LW wave solutions. The new method, called *boost variable representation*, is based on the Lorentz invariance and involves plane waves propagating along the subluminal and superluminal boost variables. This new superposition method is motivated by the need to derive new closed form X wave-type solutions.

1.2 Description of the Thesis

The organization of this thesis is as follows: In Chapter 2 we have carried out a detailed study of the numerical reconstruction of scalar-valued LW pulses generated from dynamic aperture antennas. This study is based on three well-known field reconstruction methods; specifically, Huygens' technique and the Rayleigh-Sommerfeld integral representations I and II, both in the time and frequency domain. A notable difference among these techniques depends on how they deal with acausal incoming components incorporated into the aperture excitation fields. This can occur if one uses source excitations that are superpositions of FWM-like solutions, e.g., the MPS pulse and the time-limited FWM. This is not the case for X wave pulses.

Chapter 3 deals with the propagation of LW in dispersive media modeled by the Klein-Gordon equation. The analysis focuses on the depletion of the spatio-temporal spectral components of the wavefields as they propagate away from their dynamic aperture sources. It is demonstrated that contrary to expectation, the depletion of the spectral components of the dispersive Klein-Gordon field may become slower than that associated with the free scalar field. It is shown that this behavior results from the proper tuning of the coupling of the spatio-temporal components of the excitation wavefield to the plasma frequency.

Chapter 4 deals with the scattering of an acoustic MPS pulse from compressible and rigid spheres immersed in a homogeneous and isotropic background. We have discussed the effect of the variation of sphere material parameters on the amplitude of the scattered spectrum in the acoustic frequency range $1\,\text{kHz} \rightarrow 150\,\text{kHz}$. The analysis allows the extraction of the radius of the sphere from the backscattered data for different metals.

In Chapter 5, we have carried out numerically the reconstruction of a novel class of electromagnetic LW solutions, called azimuthally polarized X waves, using Kirchhoff's surface integral representation with localized currents and tangential field components as exciting sources. Finally, Chapter 6 contains a summary of the work in this thesis, as well as conclusions and suggested future work.