EFFICIENT SAMPLING PLANS
FOR CONTROL CHARTS
WHEN MONITORING AN AUTOCORRELATED PROCESS

Xin Zhong

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M. R. Reynolds, Jr., Chair
J. B. Birch
G. G. Vining
W. H. Woodall

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(Abstract)

This dissertation contains the results of an investigation of the effects of autocorrelation on the performances of various sampling plans for control charts in detecting special causes that may produce sustained or transient shifts in the process mean and/or variance. Observations from the process are modeled as a first-order autoregressive process plus a random error. Combinations of two Shewhart control charts and combinations of two exponentially weighted moving average (EWMA) control charts based on both the original observations and on the process residuals are considered. Three types of sampling plans are investigated: samples of $n = 1$, samples of $n > 1$ observations taken together at one sampling point, or samples of $n > 1$ observations taken at different times. In comparing these sampling plans it is assumed that the sampling rate in terms of the number of observations per unit time is fixed, so taking samples of $n = 1$ allows more frequent plotting. The best overall performance of sampling plans for control charts in detecting both sustained and transient shifts in the process is obtained by taking samples of $n = 1$ and using an EWMA chart combination with a observations chart for mean and a residuals chart for variance. The Shewhart chart combination with the best overall performance, though inferior to the EWMA chart combination, is based on samples of $n > 1$ taken at different times and with a observations chart for mean and a residuals chart for variance.

Dedication

To my fiancé, for his love and support

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Chapter 1

Introduction

1.1 Brief Description of Control Charts

Statistical process control (SPC) is a set of statistical methods that are widely used to monitor, control and improve processes. One of the primary techniques of SPC is the control chart, which is employed to study the process variation that may affect the quality of the process outcome. The idea of the control chart was pioneered by Walter A. Shewhart in the 1920s, and the original applications were primarily in manufacturing processes. After decades of development and improvement, the practical applications of control charts nowadays extend into many areas such as engineering design, environmental science, pharmaceutical process, finance, accounting, and marketing (see, e.g., Ryan (2000); Montgomery (2005) and Stoumbos, Reynolds, and Woodall (2003)).

In any process, no matter how well it is designed, there exists a certain amount of variability in the outcome measurements of a quality characteristic. The origins of these process variations can be framed in terms of common (non-assignable) causes and special (assignable) causes. Variation due to common causes is the natural phenomenon constantly active within the system and is to some extent unavoidable. On the contrary, special causes may lead to excessive variation in process outcomes, resulting in malfunctioning of the product. Because special causes can be eliminated from the system if they are detected, quality improvement becomes possible by detecting and removing special causes of variation via the control chart.

In the basic form of a control chart, samples are taken from the process at regular time intervals, and measurements are obtained. Some sample statistic, say the sample mean, is plotted versus the sample time or versus the sample number, and then compared to a pair of so-called control limits. If the statistic falls within the bandwidth of two control limits, then it is assumed that the variability is only due to common causes and the process is in-control. If a statistic falls outside of control limits, it is called an out-of-control signal and indicates the presence of special causes of variation. As a result, investigation and corrective action is required to identify and eliminate the special causes responsible for this change in the process.

However, an out-of control signal may sometimes be a false alarm, i.e., an out-of-control signal is given when the process is actually in-control. Thus, in order to maintain the reliability of a control chart, a good choice of control limits is essential, which is a compromise between quick detection of a process change and avoidance of a high false alarm rate. In many applications, three-sigma control limits are customarily employed. When the statistics being plotted are normal and independent, using three-sigma control limits corresponds to a false alarm once in every 370.4 samples, on average, for a control chart in its basic form and with known parameters. Note that "sigma" refers to the standard deviation of the statistic plotted on the chart, not the standard deviation of the quality characteristic.

The most commonly used control charts are the Shewhart control charts, the cumulative sum (CUSUM) control charts, and the exponentially weighted moving average (EWMA) control charts. The simplicity and ease of interpretation makes

Shewhart control charts most popular in use, but they are relatively insensitive to small shifts in process parameters. When small process parameter shifts are of interest, CUSUM and EWMA control charts are two very effective alternatives to Shewhart control charts.

Control charts used for continuous process variables are usually called variables control charts, and control charts for discrete process variables are called attributes control charts. In this dissertation, only control charts for the continuous variables are considered, thus the discussion and description of control charts in this dissertation only refers to variables control charts.

1.2 The Presence of Autocorrelation

An important assumption usually made in constructing a control chart is that the process observations used for monitoring the process are independently distributed. However, in many practical situations, this assumption cannot be justified. Examples include chemical processes, high volume production processes, and processes using automated sampling technology, where consecutive observations are often close together and autocorrelated.

If the presence of autocorrelation is ignored, and a conventional control chart for independent observations is set up instead, many problems may arise. For example, the estimates of process parameters may be biased, the false-alarm rate may be much higher than expected, and the detection of out-of-control situations may be much slower than required.

When autocorrelation is detected in a process, it is necessary to first decide whether the autocorrelation is a special cause or part of the common cause variation. If the autocorrelation is due to a special cause, then an attempt should be made to identify the source and eliminate it. On the other hand, if the autocorrelation is an inherent part of the common cause variation that cannot feasibly be removed, then the autocorrelation must be taken into consideration when designing the control chart.

An easy solution to the problem of the autocorrelation is to sample less frequently to make the effects of the autocorrelation negligible. However, it is also obvious to see that this will reduce the amount of available information about the process, and thus make it much more difficult to detect process changes.

Several new approaches to dealing with the autocorrelated process have been developed in recent decades. One model-free approach is based on the unweighted batch means chart, which breaks successive groups of sequential observations into batches, with equal weights assigned to every point in the batch. This procedure retains the basic simplicity of the conventional control chart and dilutes the autocorrelation by choosing a sufficiently large batch size. However, it may not be as powerful as the model-based approaches, and may be inefficient when autocorrelated observations are not very closely spaced in time.

There are also two popular model-based approaches to dealing with the autocorrelation. In the first approach, a time series model is fitted to the process data, and then the residuals from the model are plotted using a conventional control chart. Control charts based on this approach will be called control charts of the residuals or residuals charts in the rest of this dissertation. The second approach is to plot the original observations on a control chart, and then adjust the control limits to account for the autocorrelation. Control charts based on this approach will be called control charts of the observations or observations charts. Note that this second approach may seem to avoid the necessity of fitting a time series model, but a time series model is still usually needed to properly adjust the control limits to account for the autocorrelation.

A large number of papers have investigated the effects of the autocorrelation on standard control charts and the properties of various approaches to monitoring autocorrelated processes. Additional details and references will be given in Chapter 2.

1.3 The Sampling Plans

A control chart is maintained by taking samples from the process. These samples are usually of a fixed size *n*, with a fixed sampling interval *d*, between successive sampling points. The choice of *n* and *d* affects the costs associated with sampling and the ability of the control chart to detect changes in process parameters. Thus an important issue in designing a control chart is how the sampling from the process should be conducted.

A critical decision in designing a sampling plan is the choice between taking process observations in groups of $n > 1$ and taking observations individually as $n = 1$. In this dissertation the statistical performance of the control charts and sampling plans will be investigated under the condition that n/d, the sampling rate per unit time, is fixed. It is assumed that the sampling cost depends only on this ratio and not on the individual values of *n* and *d*. In particular, the sampling rate per unit time will be taken to be $n/d = 1.0$, without loss of generality. Thus, for example, a sampling plan based on taking $n = 1$ observation every hour will be compared with a sampling plan based on taking samples of $n = 4$ observations every four hours. The issue being investigated is whether it is better to take small frequent samples, or larger but less frequent samples.

Another sampling issue is the question of whether, when taking samples of $n > 1$ observations, it is better to take all n observations as close together as possible or to spread them out over the sampling interval. This sampling issue was recently investigated in detail by Reynolds and Stoumbos (2004a) for the case of independent observations. Reynolds and Stoumbos (2004a) labeled sampling $n > 1$ observations as close together as possible as concentrated sampling, and sampling $n > 1$ observations spread over the sampling interval as dispersed sampling.

The conventional approach to sampling from a process is based on the concept of rational subgroups, and this corresponds to concentrated sampling when $n > 1$. The idea of rational subgroups is that sampling should maximize the possibility of a process change occurring between samples, and minimize the possibility of a change occurring within samples. However, the type of sampling that is best actually depends on what type of process parameter shift is expected. It is usually assumed that a special cause results in a sustained shift that lasts until it is detected by a control chart and the special cause is removed. Under this assumption, concentrated sampling based on the concept of rational subgroups may be the better choice. However, a special cause may result in a transient shift that lasts only for some short time period, say l time units. For example, if a transient shift lasts for only $l = 1$ hour, then a sampling plan based on a concentrated sample of $n = 4$ every 4 hours might completely miss the one-hour interval when the process is affected. In this case, it seems that it would be better to use dispersed sampling and spread the sample over the four-hour interval.

To illustrate the different sampling plans, suppose one item is produced by a product line every 10 seconds, but on average only one item is sampled every hour. Thus individual sampling is based on taking samples of $n = 1$ and then plotting a point on a control chart every hour. If concentrated sampling is based on taking a sample of $n = 4$ items every $d = 4$ hours, then a sample of four consecutive items would be taken every four hours, and a point would be plotted on a control chart every four hours. Here the production rate is high compared to the sampling rate, so it seems reasonable to assume that the 10 seconds between consecutive items in the sample can be neglected, i.e., it can be assumed that four items in one concentrated sample are produced at essentially the same time. On the contrary, although dispersed sampling is also based on a sample of $n = 4$ items with a point plotted on the control chart every $d = 4$ hours, each of the four items in one sample is actually taken as an individual item with one hour between each item.

1.4 The Objectives of This Dissertation

The main objective of this dissertation is to determine what combinations of sampling plans and control charts gives the best overall statistical performance when there is autocorrelation in the process being monitored. In general, the sampling plan that gives good performance depends on the type of control chart being used, so the investigations of the sampling plan and the control chart used need to be done together.

The control charts considered in this dissertation are Shewhart charts and EWMA charts. Studies have shown that EWMA and CUSUM control charts generally have similar detection efficiencies for both independent and autocorrelated data, so CUSUM control charts are not explicitly considered here. Control charts of the observations and control charts of the residuals are both considered. Because these two types of control charts are both model-based, it is assumed that the true model is known and the in-control parameter values have been estimated accurately enough that estimation error can be neglected. For each type of control chart, a combination of two charts is investigated, one chart designed to monitor the process mean and the other designed to monitor the process standard deviation.

Three basic types of sampling plans are investigated in this dissertation. One sampling plan is based on taking samples of $n = 1$ and plotting a point on a control chart after each observation. The second sampling plan is based on taking a concentrated sample of $n > 1$ and plotting a point on a control chart for each sample of n. With concentrated sampling it is assumed that the time between each observation in a sample is negligible. The third sampling plan is based on dispersed samples of $n > 1$ spread out over the sampling interval. With dispersed sampling the n observations may actually be taken as individual observations during the sampling interval, but a point is plotted on a control chart only at the end of the sampling interval after n observations have been accumulated.

The performance of the aforementioned sampling plans and control charts will be investigated when the autocorrelated observations can be expressed as a first-order autoregressive [AR(1)] process plus an additional random error. The AR(1) process has proved to be one of most useful time series models in practice, and the additional random error provides additional flexibility to model a process, for example, by representing measurement error. The range of the autocorrelation considered is from low to moderately high. This guarantees that the process may wander away from the target value, but will eventually return to the target value. The details of the AR(1) process will be given in Chapter 3.

To summarize, this dissertation considers the Shewhart and EWMA observations charts and residuals charts for simultaneously monitoring mean and standard deviation of an AR(1) process with additional random error. The objective is to investigate the effects of the autocorrelation on the statistical performance of various sampling plans for observations charts and residuals charts, and also to find efficient sampling plans for each type of control chart combination, so that recommendations can be made for practical applications.

Chapter 2

Background

2.1 The Effects of Autocorrelation on Control Charts

The effects of autocorrelation on the statistical performance of control charts designed under the independence assumption have been discussed by numerous authors. Goldsmith and Whitfield (1961) showed that positive autocorrelation can increase false alarm rates for CUSUM control charts, while negative autocorrelation decreases the rates. Johnson and Bagshaw (1974) and Bagshaw and Johnson (1975) investigated the effect of autocorrelation on the run length distribution when the process follows either an AR(1) model or a moving-average [MA(1)] model, where the run length is the number of observations plotted before an out-of-control signal. They also approximated the average run length (ARL) of CUSUM control charts for the autocorrelated data. Harris and Ross (1991) discussed the impact of autocorrelation on CUSUM and EWMA control charts and pointed out that the in-control ARL of these charts is sensitive to the presence of autocorrelation.

Alwan (1992) studied the effect of autocorrelation upon the Shewhart control chart, and found that there is an increased probability of both false and genuine alarms. Maragah and Woodall (1992) provided results on the effect of autocorrelation upon the retrospective Shewhart individuals control chart. Woodall and Faltin (1993) then gave a brief summary on the effect of autocorrelation on the performance of control charts and on how to deal with autocorrelation. VanBrackle and Reynolds (1997) evaluated the performance of EWMA and CUSUM observations charts for the process mean when the observations are from an AR(1) process with additional random error. They concluded that positive correlation may decrease the in-control ARL, shorten the time required to detect small to moderate shifts, and lengthen the detection time for large shifts. They also gave tables to aid in the design of the control charts.

2.2 Model-Based Approaches

2.2.1 Introduction to Two Model-Based Approaches

Two model-based approaches have been suggested in the literature to construct control charts when there is significant autocorrelation in the process observations. The first approach is based on modifications of standard control charts of the observations, where the control limits are adjusted to account for the effect of autocorrelation. Vasilopoulos and Stamboulis (1978) first proposed using a Shewhart observations chart with adjusted control limits. The run length of this chart was studied by Schmid (1995). Schmid and Schone (1997) applied an EWMA observations chart to monitor autocorrelated processes, and they studied the run length properties of this EWMA control chart.

The second approach is based on control charts of the residuals. This approach was advocated in an influential paper by Alwan and Roberts (1988). This method takes advantage of the fact that the process is autocorrelated by obtaining the process prediction errors (residuals) from a time series model fitted to the original observations. Then conventional control charts are used to monitor the residuals, which are statistically uncorrelated to each other if the time series model is correctly specified and there is no error in estimating process parameters. Control charts based on residuals are the most commonly used control charts for detecting process changes in autocorrelated processes. A large number of research papers have investigated this method. The advantage of this method is that it can be applied to any autocorrelated data, even if the data are from a non-stationary process. This method, however, also has its own drawbacks, which will be discussed below.

When Alwan and Roberts (1988) first recommended the use of residuals charts, they stated that if a shift occurs in the quality characteristic of interest, the identified model will no longer be correct, and this model misspecification will be transferred to the residuals. They assumed that the shift will be eventually detected by a control chart applied to the residuals. Woodall and Faltin (1993) argued that the shift in the level on the control chart of the residuals may not show up as clearly as the shift in the level on the process, because the effect of the shift is filtered by the time series model. Harris and Ross (1991) recognized that when the autocorrelation is positive, the residuals charts from a AR(1) process are extremely inefficient at detecting shifts in the process mean, comparing to the out-of-control ARL values of a corresponding conventional chart in the independent case. Yashchin (1993) showed that the ARL performance of CUSUM control chart is virtually identical by using original observations or their residuals when autocorrelation is mild. Also he suggested doing data transformations when the autocorrelation is strong. Wardell, Moskowitz, and Plante (1994) stated that the sensitivity of a Shewhart residuals chart is poor relative to a Shewhart conventional chart when the first lag autocorrelation is positive, but the probability of detecting shifts very early is substantially higher for the Shewhart residuals chart than for the Shewhart conventional chart. Zhang (1997) also showed the out-of-control ARL of the residuals chart is uniformly larger than that of the conventional chart to detect the mean shift, when the process has moderate positive autocorrelation.

Being aware of problems in the detection capability of the residuals charts, several papers suggested that these charts should be used in conjunction with time series plots of the original observations and their forecast values. Moreover, the validity of the model must be checked over time to reduce the mixed effect of modeling error and process change (see, e.g., MacGregor (1991); Woodall and Faltin (1993); and Zhang (1998)). Some other papers also discussed methods for modifying control charts of the residuals to get better performance, see, e.g., Runger, Willemain, and Prabhu (1995).

2.2.2 An Alternative to Control Chart of the Residuals

Montgomery and Mastrangelo (1991) proposed using one-step-ahead EWMA prediction errors as a simple alternative to using residuals based on a fitted time-series model. In this scheme, the parameter of the EWMA is determined by minimizing the sum of the squares of one-step-ahead EWMA prediction errors, and the standard deviation of one-step-ahead EWMA prediction errors can be estimated based on historical data. Alwan and Roberts (1988) briefly commented that this method provides an adequate approximation for many time series processes. In particular, Montgomery and Mastrangelo (1991) confirmed that this procedure is a satisfactory approximation of the exact time series model approach when the observations are positively autocorrelated at low lags and the process drifts moderately slowly.

Faltin and Woodall (1991) warned, however, that the chart based on EWMA residuals may not be sufficient to detect certain types of special causes, because residuals from an EWMA adapt to a level shift in the underlying process, which prevents such a change from being distinguished from the residuals. Superville and Adams (1994) compared Shewhart, CUSUM, and EWMA control charts of one-step-ahead EWMA forecast errors for AR(1) models. They found that the difference in the abilities of CUSUM and EWMA control charts and the Shewhart control chart to detect small shifts in this case is minor, and none of these control charts performed very well due to the quick recovery feature of the EWMA forecast. They recommended using the probability of a signal within a fixed number of periods beyond the shift point, instead of the ARL value, as the criterion of evaluating control charts that monitor autocorrelated observations. With this criterion, the Shewhart control chart provides a higher probability of signaling immediately following a shift than the other two control charts.

2.2.3 Comparisons of Two Methods of Constructing Control Charts

Many papers have compared the performance of control charts of the observations and control charts of the residuals in terms of the ability to detect changes in autocorrelated processes. The conclusions are mixed, and depend on the level and type of autocorrelation as well as on the type of control chart being considered.

Wardell, Moskowitz, and Plante (1992) compared the performance of Shewhart and EWMA observations charts to the performance of Shewhart residuals charts when the underlying process is a first-order autoregressive-first-order moving average [ARMA(1,1)]. The results showed that the ability of the EWMA observations chart to detect shifts in the process mean is quite robust, while the corresponding Shewhart observations chart rarely detects such shifts more quickly than the other charts. The Shewhart control chart of the residuals is shown to have best performance in most cases when a shift is large. However, Schmid (1995) and Kramer and Schmid (1997) showed that the Shewhart observations chart performs better for positively autocorrelated processes, while the Shewhart residuals chart is better when the autocorrelation is negative. They also found that the same conclusion can be drawn for Shewhart charts with estimated parameters.

Reynolds and Lu (1997) and Lu and Reynolds (1999a) stated that when the level of correlation is quite high, a control chart of the residuals is better than a control chart of the observations. In the second paper, they also investigated the effects of autocorrelation on control charts for monitoring the process variance and concluded that the combination of a Shewhart residuals chart and an EWMA observations chart works reasonably well in most cases for simultaneously monitoring the process mean and variance. MacCarthy and Wasusri (2001) compared the performance of a Shewhart residuals chart, an EWMA observations chart, and an EWMA residuals chart. They concluded that the EWMA residuals chart performs best in avoiding false alarms and in detecting process shifts. Koehler, Marks, and O'Connell (2001) also investigated the performance of the EWMA control chart of the residuals. Dyer, Adams, and Conerly (2003) argued that the combined EWMA-Shewhart and the reverse moving average (RMA) control chart are the only control charts that should be recommended because of their superior performance characteristics over a wide range of models and shift sizes.

Lu and Reynolds (1999b) did a systematic investigation to summarize the performance of various observations charts and residuals charts for the AR(1) plus error model. They found that the Shewhart observations chart is much better than the Shewhart residuals chart except for large shifts in which case there is not much difference between the two charts. They also showed that the EWMA residuals chart is better than the EWMA observations chart for large shifts, and the EWMA observations chart is a little better for small and moderate shifts. In the selection of control charts, they concluded that it is satisfactory to use control charts of the observations, which is much easier to interpret, rather than control charts of the residuals, unless the level of autocorrelation is relatively high. In addition, they stated that CUSUM and EWMA charts are much better than Shewhart charts unless interest is only in detecting large shifts.

2.3 Model-Free Approaches

Alwan and Radson (1992) proposed a subsample mean chart, which diminishes the autocorrelation by subsampling from the original data set. In this approach, a series of spaced subgroups of approximately four to six observations are selected to plot their average. Based on this procedure, Runger and Willemain (1996) introduced the unweighted batch means chart, a more general case, with much larger subgroups without spacing. They also provided a detailed analysis of batch size for AR(1) processes, and concluded that larger batches are more effective in detecting smaller shifts and vice versa. Runger and Willemain (1995) showed that the unweighted batch means chart has better ARL performance than the control chart of the residuals over a wide range of conditions for AR(1) processes.

Zhang (1998) proposed an EWMAST chart for weakly stationary process data, which uses the autocorrelation information to determine the control limits instead of fitting a time series model. When the process autocorrelation is not very strong and the mean changes are not large, this chart has larger in-control ARL's and smaller out-of-control ARL's than the model-based charts. However, this method is not particularly effective at giving a desired in-control ARL.

2.4 A Brief Discussion of Model Specification and Parameter Estimation

Most of the papers discussed above assumed perfect knowledge of the model describing process behavior, but this assumption is never met in practice. Consequently, the problems caused by parameter estimation error and model misspecification need to be taken into consideration when evaluating control charts for monitoring autocorrelated processes. In practice, a model must be fitted and in-control parameter values must be estimated using retrospective process data. This process of model fitting and estimation must be conducted during an initial phase, usually called Phase I, before the real-time process monitoring can begin.

To avoid problems with parameter estimation error, a relatively large amount Phase I process data is required to estimate in-control parameter values, even in the case of independent observations. In particular, a large Phase I data set is required to make the properties of the control chart with the estimated process model and parameters close to the properties in the case of known process model and parameters (see Ghosh, Reynolds, and Hui (1981) and Quesenberry (1993)). When autocorrelation is present, this requirement for a large Phase I data set is even more crucial. Adams and Tseng (1998) investigated the impact of parameter estimation error on the performance of control charts of the residuals. Lu and Reynolds (1999b) discussed how control charts of the observations behave when there are estimation errors of model parameters. Both Adams and Tseng (1998) and Lu and Reynolds (1999b) stressed that the size of the initial data set used to estimate process parameters is critical to autocorrelated processes. To account for model and parameter uncertainly in autocorrelated processes, Apley and Lee (2003) suggested using a modified EWMA control chart of the residuals with wider control limits.

2.5 The Sampling Plans

There are a few articles in the SPC literature dealing with the choice of *n* and *d* in specific control charts, but most of these articles are limited to the case in which the objective is to detect a sustained shift in only one process parameter (see, e.g., Hawkins (1992), Stoumbos and Reynolds (1996), Prabhu, Runger, and Montgomery (1997), and Hawkins and Olwell (1998)). The investigations of the choice of *n* and *d* have considered only the case of independent observations. In addition, there have been few comparisons between different types of control charts based on using the best sampling plan for each type of control chart.

Reynolds, Arnold, and Baik (1996) discussed the performance of different sampling intervals in the context of variable sampling interval (VSI) $\bar{X}$ control charts when the objective is to detect mean shifts in an autocorrelated process. They concluded that when autocorrelation is present the VSI $\bar{X}$ control chart is not necessarily better than the fixed sampling interval $\bar{X}$ control chart. They also questioned whether the use of Shewhart control chart of the observations is desirable when the autocorrelation is high. In this dissertation, only sampling plans with a fixed sampling size and sampling interval are investigated, and thus sampling plans based on a variable sampling size and/or interval are not considered.

Reynolds and Stoumbos (2004a) conducted systematic studies of sampling plans and control charts in the case of independent observations. They compared the statistical performance of control chart combinations with three sampling plans mentioned in Chapter 1 when simultaneously monitoring the process mean and variance. In particular, they assumed, without loss of generality, that $n/d = 1.0$, and considered the performance of control charts for various choices of *n* and *d*. They found that for a Shewhart control chart combination, when detecting sustained shift is of interest, the best performance is obtained using samples of a moderate size, such as $n = 4$ and $d = 4$, with concentrated sampling. When the objective is to detect transient shifts of short duration ($l < 4$), the best design for a Shewhart control chart combination is to use $n = 1$ and $d = 1.0$. However, if $n = 4$ is chosen in order to quickly detect sustained shifts, then dispersed sampling is best overall. For transient shifts of longer duration ($l = 4$), a sample size of $n = 4$ with concentrated sampling gives the best performance in a Shewhart control chart combination. For the CUSUM and EWMA control chart combination, the conclusion is much simpler: the best statistical performances in detecting small as well as large sustained or transient shifts in process mean or variance is achieved by using individual observations ($n = 1$). Furthermore, the overall conclusion is that a CUSUM or EWMA control chart combination with $n = 1$ is better than a Shewhart control chart combination with $n = 4$ at detecting both small and large sustained or transient shifts in the process mean or variance (see also Reynolds and Stoumbos (2004b)).

2.6 Summary

Although there have been many investigations of control charts for monitoring autocorrelated processes, and some investigations of sampling plans, there are very few investigations that have combined these two topics together. The main objective of this dissertation is to combine the problems considered by Lu and Reynolds (1999b) and Reynolds and Stoumbos (2004a), and investigate sampling plans and control chart combinations that give the best overall performance when there is autocorrelation in the process.

Chapter 3

Control Charts and an Autocorrelation Model

The first part of this chapter contains descriptions of different types of control charts for the case of independent observations. For convenience of comparison, the notation from Reynolds and Stoumbos (2004a) is used. Next the description of the AR(1) plus random error model is given. Then two forms of each type of control chart in the autocorrelated case are provided, one for control charts of the observations, the other for control charts of the residuals. Measures of performance and methods for evaluation are discussed in the rest of this chapter.

3.1 Notation and Assumptions

A process observation will be denoted by *X*, and it is assumed here that *X* has a normal distribution with mean μ_X and standard deviation σ_X. Thus, μ_X and σ_X will be called the process mean and standard deviation, respectively. The process is said to be in control when μ_X is at its target value μ_{X_0}, and σ_X is at its target value σ_{X_0}. A specific model for successive values of *X* when there is autocorrelation in the process will be discussed later.

When sampling from the process, suppose that a point will be plotted on the control chart at times $t_1, t_2, \ldots$, where the plotting time t_{k-1} and t_k are d time units apart, where $d = n > 0$. Let $\mathbf{X}_k = \left(X_{k1}, X_{k2}, \ldots, X_{kn}\right)$ represent the sample available at time t_k, where $X_{k1}, X_{k2}, \ldots, X_{kn}$ are the observations collected after time t_{k-1} and at or before time t_k.

When the individual sampling plan is employed, $d = n = 1$. In this case, $\mathbf{X}_k$ reduces to X_k, the individual observation at time t_k.

When $d = n > 1$ and concentrated sampling is used, the time between the individual observations $X_{k1}, X_{k2}, \ldots, X_{kn}$ is assumed to be negligible so this time is taken to be 0. In this case, it is assumed that $X_{k1}, X_{k2}, \ldots, X_{kn}$ are identically and independently distributed (i.i.d.) with the same conditioned mean.

When $d = n > 1$ and dispersed sampling is used, the observations $X_{k1}, X_{k2} \ldots, X_{kn}$ are assumed to be evenly distributed over the sample interval *d* with each observation one time unit apart. Note that no matter what sampling plan is chosen, the points for the control chart may only be plotted at plotting time t_k, based on the values of sample $\mathbf{X}_k$.

3.2 Description of Control Charts in the Independent Case

It is assumed that process monitoring will continue over a relative long period of time, and the objective is to detect any special cause that changes μ_X from μ_{X_0} or increase σ_X above σ_{X_0}. A decrease in σ_X may be of interest in some application, but control charts for detecting decreases in σ_X are not investigated in this dissertation. As stated in Chapter 1, it is assumed those target values have been estimated accurately enough through a preliminary phase that they can be regarded as known parameters in the process monitoring following in Phase II. In addition, in the numerical results of this dissertation, a number of combinations of two control charts are applied simultaneously to detect changes in μ_X and/or σ_X. When any one of the two charts signals, the control chart combination is said to signal.

3.2.1 Shewhart Control Charts

Shewhart control charts, originally proposed by Shewhart (1931), are the most widely used and simplest control charts. The Shewhart $\overline{X}$ chart for monitoring the mean μ_X is based on plotting the sample mean $\overline{X}_k$ of sample $\mathbf{X}_k$ versus time k, and a signal is generated at time k if $\overline{X}_k$ falls outside of control limits constructed at

$$\mu_{X_0} \pm h_X \sigma_{X_0} \big/ \sqrt{n} . \tag{3.2.1}$$

In practical applications, the chart parameter h_X is usually taken to be 3 in order to yield the three-sigma control limits.

The Shewhart S chart for monitoring σ_X is based on plotting the sample standard deviation S_k of sample $\mathbf{X}_k$ versus time k. The upper control limit for the S chart is represented as

$$h_S \sigma_{X_0} . \tag{3.2.2}$$

When $n=1$, the $\overline{X}$ chart reduces to the X chart (individuals chart), and the S chart cannot be used. In this case, Shewhart MR chart is usually employed instead for monitoring σ_X, which is based on plotting the control statistic

$$MR_k = \left| X_k - X_{k-1} \right|,\ k = 2,3,\ldots . \tag{3.2.3}$$

A signal is given that σ_X has increased if MR_k exceeds the upper control limit

$$h_{MR} \sigma_{X_0} . \tag{3.2.4}$$

However, research shows that there is essentially no advantage to using the *MR* chart with the *X* chart in the independent case (see, e.g., Reynolds and Stoumbos (2001)).

An alternative to the *S* chart is based on the squared deviations from target of sample $\mathbf{X}_k$. The control statistic for this Shewhart chart for σ_X is

$$X_k^2 = \sum_{i=1}^{n} (X_i - \mu_{X_0})^2 \big/ n, \; k = 1, 2, \ldots . \tag{3.2.5}$$

The upper control limit for the X_k^2 is

$$h_{X^2} \sigma_{X_0} . \tag{3.2.6}$$

The X^2 chart corresponds to the EWMA chart used here for monitoring σ_X, which will be discussed later. Note that X_k^2 is affected by changes in μ_X, which may be an disadvantage when diagnosing whether a shift is in the mean or variance is of interest. But two advantages of the X^2 chart make it more appealing than the *S* chart, which were presented by Reynolds and Stoumbos (2004b). One is that the squared deviation has *n* degrees of freedom rather than $n-1$, and the extra degree of freedom gives the X^2 chart better performance in numerical results. When *n* is small, the loss of information in the *S* chart can be significant. The other advantage is that the X^2 chart can be used when $n = 1$ (it reduces to the *X* chart in this case), while the *S* chart can not be used when $n = 1$.

It is well known that a Shewhart control chart is not effective in detecting small and moderate sustained process parameter shifts. To improve the performance of the Shewhart control chart, it is necessary that the control statistic in some way accumulate information across past samples. Champ and Woodall (1987) evaluated the runs rules method, which is based on patterns of points in a Shewhart control chart, and using runs rules helps to improve the ability to detect small shifts. However, a better approach is to use an EWMA or CUSUM control chart.

3.2.2 EWMA Control Charts

The EWMA control chart was introduced by Roberts (1959) for monitoring the process mean. Hunter (1986), Crowder (1989), and Lucas and Saccucci (1990) discussed the properties and performance of this chart in the case of independent observations. The EWMA chart is designed to detect small shifts in the mean more quickly than the Shewhart chart by giving exponentially decreasing weights to past observations.

The EWMA control chart for detecting changes in μ_X is based on the control statistic

$$E_k^X = (1-\lambda)E_{k-1}^X + \lambda \bar{X}_k,\ k = 1, 2, \ldots, \tag{3.2.7}$$

where λ is a smoothing parameter satisfying $0 < \lambda \le 1$, and the starting value is usually taken to be $E_0^X = \mu_{X_0}$. When λ is larger, relatively less weight is given to older observations. As λ becomes smaller, more weight is given to the older observations. A signal is given at time k if E_k^X falls outside of control limits constructed at

$$\mu_{X_0} \pm h_{E_X} \sqrt{\lambda/\left(n\left(2-\lambda\right)\right)}\sigma_{X_0}, \tag{3.2.8}$$

where $\sqrt{\lambda/\left(n\left(2-\lambda\right)\right)}\sigma_{X_0}$ is the asymptotic in-control standard deviation of E_k^X. The chart parameter h_{E_X} is usually chosen to yield a specified in-control average time to signal (ATS) for control chart comparisons. The EWMA chart based on the statistic E_k^X will be referred to as the EWMA_X chart.

For the effective detection of changes in σ_X, the EWMA chart of the squared deviations from target is considered. The reasons for using the squared deviation in EWMA chart, instead of the standard deviation, are similar to the reasons in the case of the Shewhart chart. One reason is that the squared deviation has 1 more degree of freedom, and the extra degree of freedom provides better performance in numerical results. A second reason is that this EWMA chart is still valid when n = 1. A third reason is that this EWMA chart is very effective in detecting large shifts in μ_X (see, for example, Reynolds and Stoumbos (2005)). Because the primary interest is in detecting an increase in σ_X, a one-sided version of this control chart is employed in this study. The control statistic for this one-sided EWMA chart is

$$E_k^{X^2} = (1-\lambda)\max\left\{E_{k-1}^{X^2}, \sigma_{X_0}^2\right\} + \lambda\sum_{i=1}^{n}(X_{ki}-\mu_{X_0})^2 \big/ n,\ k = 1,2,\ldots, \tag{3.2.9}$$

where λ is the smoothing parameter, and the starting value is usually taken to be $E_0^{X^2} = \sigma_{X_0}^2$ This control statistic is defined so that if $E_{k-1}^{X^2}$ is less than $\sigma_{X_0}^2$, then there is a reset back to $\sigma_{X_0}^2$ before computing $E_k^{X^2}$. A signal is generated if $E_k^{X^2}$ exceeds an upper control limit constructed at

$$\sigma_{X_0} \pm h_{E_{X^2}} \sqrt{2\lambda \big/ \left(n(2-\lambda)\right)}\,\sigma_{X_0}^2 . \tag{3.2.10}$$

Note that $\sqrt{2\lambda \big/ \left(n(2-\lambda)\right)}\,\sigma_{X_0}^2$ is not the asymptotic in-control standard deviation of $E_k^{X^2}$ due to the reset feature. This form is used to correspond to the form of the control limits of E_k^X. The chart parameter $h_{E_{X2}}$ is also chosen to yield specified in-control ATS for control chart comparisons. The EWMA chart that is based on the control statistic $E_k^{X^2}$ will be referred to as the EWMA$_{X^2}$ chart. Note that a special case is $\lambda = 1$, then Equation (3.2.9) reduces to (3.2.5).

To construct a two-sided EWMA$_{X^2}$ control chart for monitoring both increases and decreases in σ_X, one can simply eliminate the reset at $\sigma_{X_0}^2$ in the definition of the EWMA$_{X^2}$ statistic, then add a lower control limit. A better approach, suggested by Yashchin (1993), is to use two one-sided charts. For the details of adding another one-sided EWMA$_{X^2}$ control chart for monitoring a decrease in σ_X, see Reynolds and Stoumbos (2001).

The CUSUM control chart was originally proposed by Page (1954) and a relatively large number of papers dealing with this chart have been published over the years (see, for example, Lucas (1976), Woodall (1986), Hawkins and Olwell (1998)). Studies have shown that the EWMA and CUSUM charts generally have similar detection efficiencies for the autocorrelated data case (say, Lin and Adams (1996), Lu and Reynolds (2001)). Hence, the CUSUM chart will not be compared to the other charts in this study. For more studies about CUSUM control chart for autocorrelated processes, see Runger, Willemain, and Prabhu (1995), Vander Weil (1996), Timmer, Pignatiello, and Longnecker (1998), and Lu and Reynolds (2001).

3.3 Description of An Autocorrelated Process

3.3.1 When Samples are Taken using Individual Sampling ($n=1$)

In this subsection we consider the case in which $d=n=1$. Based on the notation and assumptions in Section 3.1, the sample (observation) available at time t_k is X_k. It is assumed here that X_k can be written as

$$X_k = \mu_k + \varepsilon_k \tag{3.3.1}$$

for $k=1,2,\dots$, where $\mu_1,\mu_2,\dots$ constitute an AR(1) process, and $\varepsilon_1,\varepsilon_2,\dots$ are i.i.d normal with mean 0 and variance σ_ε^2. In some cases it may be useful to think of μ_k as the process mean at time t_k. For example, in a batch process, X_k might be the observation taken from batch *k*, μ_k might represent the mean of batch *k*, and ε_k might represent variability within this batch.

When samples $k-1$ and k are one time unit interval apart and $\mu_1,\mu_2,\dots$ is an AR(1) process, then μ_k will be expressed as

$$\mu_k = (1-\phi)\xi + \phi\mu_{k-1} + \alpha_k \tag{3.3.2}$$

for $k = 1, 2, \ldots$, where ϕ is the autoregressive parameter and also the correlation between μ_k and μ_{k-1}. The symbol ξ represents the AR(1) process mean $E(\mu_k)$. The variables $\alpha_1, \alpha_2, \ldots$ are independent normal random variables with mean 0 and variance σ_α^2, also known as a white noise process. In order for the AR(1) process to be stationary, ϕ must be less than 1 in absolute value. Moreover, for most processes of interest in control chart applications, ϕ is nonnegative.

To model the special case in which the process observations are independent, take $\phi = 0$ and $\sigma_\alpha^2 = 0$ so that all μ_k's are equal to a constant variable ξ. It is assumed that ξ may only change to some other value when a special cause occurs, otherwise, it will remain constant at the target value because the process is in-control.

When there is autocorrelation in the process, the variance of X_k is not only due to σ_ε^2, but also due to the variance of μ_k, which is denoted by σ_μ^2. Note that in applications, σ_μ^2 can be regarded as long term variability, while σ_ε^2 can be thought of as short term variability or measurement error.

Equation (3.3.2) implies that the distribution of μ_k for $k = 1, 2, \ldots$ depends on a starting value μ_0 at the time $t_0 = 0$. It is easy to show that if it is assumed that μ_0 follows the same distribution as μ_k for all *k*, then $\mu_0, \mu_1, \mu_2, \ldots$ will be identically distributed with mean ξ and variance σ_μ^2, where

$$\sigma_\mu^2 = \frac{\sigma_\alpha^2}{1-\phi^2}. \tag{3.3.3}$$

Then, for this model, the process mean is

$$\mu_X = E(\mu_k + \varepsilon_k) = \xi, \tag{3.3.4}$$

and the process variance is

$$\sigma_X^2 = \sigma_\mu^2 + \sigma_\varepsilon^2 = \frac{\sigma_\alpha^2}{1-\phi^2} + \sigma_\varepsilon^2 . \tag{3.3.5}$$

Let ψ present the proportion of the variance σ_X^2 due to the AR(1) process, so that

$$\psi = \frac{\sigma_\mu^2}{\sigma_X^2} = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2} . \tag{3.3.6}$$

Then the correlation between two successive samples, X_k and X_{k+1}, denoted by ρ, is

$$\rho = \phi\psi . \tag{3.3.7}$$

For example, if the correlation between μ_k and μ_{k+1} is 0.8, and 50% of the variance σ_X^2 is due to σ_μ^2, then the correlation between X_k and X_{k+1} will be $\rho = (0.8)(0.5) = 0.4$. A special case is $\psi = 1$, which means $\sigma_\varepsilon^2 = 0$, thus the sequence $X_1, X_2, \ldots$ is itself an AR(1) process. This model is also very popular and easy to handle in applications, as many papers discussed in Chapter 2 are based on it.

Box, Jenkins, and Reinsel (1994) showed that the AR(1) process with random error can be viewed as an ARMA(1,1) process, which means that X_k can be written as

$$X_k = (1-\phi)\xi + \phi X_{k-1} - \theta\gamma_{k-1} + \gamma_k \tag{3.3.8}$$

for $k = 1,2,\ldots$, where $\gamma_1, \gamma_2, \ldots$ are normal independent random variable with mean 0 and variance σ_γ^2, known as the random shock components of the ARMA(1,1) process. In this model ξ is the overall mean, θ is the MA parameter, and ϕ is the AR parameter which is essentially same as in Equation (3.3.2). Reynolds, Arnold, and Baik (1996) provided equations to express the parameters ϕ, θ, and σ_γ^2 in the ARMA(1,1) model in terms of the parameters ϕ, σ_α^2, and σ_ε^2 in the AR(1) plus random error model, and vice versa. In particular, if $\phi > 0$ and $\sigma_\varepsilon^2 > 0$, then the ARMA(1,1) model parameters θ and σ_γ^2 can be obtained from the AR(1) plus random error model parameters using

$$\theta = \frac{\sigma_\alpha^2 + \left(1+\phi^2\right)\sigma_\varepsilon^2}{2\phi\sigma_\varepsilon^2} - \frac{1}{2}\sqrt{\left(\frac{\sigma_\alpha^2 + \left(1+\phi^2\right)\sigma_\varepsilon^2}{\phi\sigma_\varepsilon^2}\right)^2 - 4} \tag{3.3.9}$$

and

$$\sigma_\gamma^2 = \sigma_\varepsilon^2 \, \phi/\theta \, . \tag{3.3.10}$$

On the other hand, if $\phi > 0$, then the AR(1) plus random error model parameters can be obtained from the ARMA(1,1) model parameters using

$$\sigma_\alpha^2 = \sigma_\gamma^2 \left(\phi - \theta\right)\left(1 - \phi\theta\right)/\phi \tag{3.3.11}$$

and

$$\sigma_\varepsilon^2 = \sigma_\gamma^2 \, \theta/\phi \, . \tag{3.3.12}$$

For some applications, it will be more convenient to explain the process model in terms of the AR(1) plus random error model, but for other applications, the direct ARMA(1,1) model may be more appropriate. With Equations (3.3.9) and (3.3.10), and Equations (3.3.11) and (3.3.12), one can easily compute the estimates of parameters of one model from those of the other model. For example, some software can only provide the estimates of parameters of the ARMA(1,1) model. If the AR(1) plus random error model is of interest, then Equations (3.3.11) and (3.3.12) can be used to determine estimates of parameters of the AR(1) plus random error model from the estimates obtained from the software.

3.3.2 When Samples of $n > 1$ are Taken using Concentrated Sampling

In this subsection, $d = n > 1$ and it is assumed that there is 0 time between the individual observations in sample $\mathbf{X}_k = \left(X_{k1}, X_{k2}, \ldots, X_{kn}\right)$ at plotting time t_k. It will be assumed that X_{ki} can be written as

$$X_{ki} = \mu_k + \varepsilon'_{ki} \tag{3.3.13}$$

for $k = 1,2,\ldots$, and $i = 1,2,\ldots,n$, where $\mu_1, \mu_2, \ldots$ is an AR(1) process, and ε'_{ki}'s are normal independent random variables with mean 0 and variance $\sigma^2_{\varepsilon'}$. Equation (3.3.13) indicates that all the observations at sampling time t_k have the same mean μ_k, and this is actually a general case of Equation (3.3.1). The sample mean at the sampling time t_k, $\bar{X}_k$, can be expressed as

$$\bar{X}_k = \mu_k + \bar{\varepsilon}'_k \tag{3.3.14}$$

where $\bar{\varepsilon}'_k = \sum_{i=1}^{n} \varepsilon'_{ki} / n$, and $\bar{\varepsilon}'_1, \bar{\varepsilon}'_2, \ldots$ are i.i.d normal with mean 0 and variance $\sigma^2_{\varepsilon'} / n$.

Samples $k-1$ and k are now $d > 1$ time units apart, so the AR(1) model specified by Equation (3.3.2) for the case of $d = 1$ must be modified to account for the different spacing of the observations. In particular, assume that $\mu_1, \mu_2, \ldots$ is an AR(1) process with autoregressive parameter ϕ^d, so μ_k can be expressed as

$$\mu_k = \left(1 - \phi^d\right)\xi + \phi^d \mu_{k-1} + \alpha'_k \tag{3.3.15}$$

for $k = 1, 2, \ldots$. The variables $\alpha'_1, \alpha'_2, \ldots$'s are i.i.d. normal random variables with mean 0 and variance $\sigma^2_{\alpha'}$, where

$$\sigma^2_{\alpha'} = \sigma^2_{\alpha}\left(1 - \phi^{2d}\right) / \left(1 - \phi^2\right). \tag{3.3.16}$$

Note that when $d = 1$, Equation (3.3.15) reduces to Equation (3.3.2) , and Equation (3.3.16) reduces to σ^2_{α}. However, the distribution of μ_k when $d > 1$ is the same as that of μ_k when $d = 1$, which is normal with mean ξ and variance σ^2_{μ}, where σ^2_{μ} follows Equation (3.3.3), for any $k \ge 0$. Moreover, the process mean and variance are still of the same forms as Equations (3.3.4) and (3.3.5). Also the expressions of ψ and ρ are the same as Equations (3.3.6) and (3.3.7).

If the AR(1) process plus random error with $d > 1$ and concentrated sampling is viewed as an ARMA(1,1) process, the sample mean $\bar{X}_k$ can be written as

$$\bar{X}_k = \left(1 - \phi^d\right)\xi + \phi^d \bar{X}_{k-1} - \theta' \gamma'_{k-1} + \gamma'_k \tag{3.3.17}$$

for $k = 1,2,\ldots$, where θ' is the MA parameter, and γ'_k are normal independent random variable with mean 0 and variance $\sigma^2_{\gamma'}$. Note that the form of Equation (3.3.17) is similar to the form of Equation (3.3.8), but Equation (3.3.17) accounts for sample mean at plotting time t_k. If $\phi > 0$ and $\sigma^2_{\varepsilon'} > 0$, the ARMA(1,1) model parameters θ' and $\sigma^2_{\gamma'}$, when $d = n > 1$, can be obtained from the AR(1) plus random error model parameters using

$$\theta' = \frac{n\sigma^2_{\alpha'} + \left(1+\phi^{2d}\right)\sigma^2_{\varepsilon'}}{2\phi^d \sigma^2_{\varepsilon'}} - \frac{1}{2}\sqrt{\left(\frac{n\sigma^2_{\alpha'} + \left(1+\phi^{2d}\right)\sigma^2_{\varepsilon'}}{\phi^d \sigma^2_{\varepsilon'}}\right)^2 - 4} \tag{3.3.18}$$

and

$$\sigma^2_{\gamma'} = \sigma^2_{\varepsilon'}\phi^d \big/ n\theta' . \tag{3.3.19}$$

On the other hand, if $\phi > 0$, then the AR(1) plus random error model parameters can be obtained from the ARMA(1,1) model parameters using

$$\sigma^2_{\alpha'} = \sigma^2_{\gamma'}\left(\phi^d - \theta'\right)\left(1-\phi^d\theta'\right)\big/\phi^d \tag{3.3.20}$$

and

$$\sigma^2_{\varepsilon'} = n\sigma^2_{\gamma'}\theta'\big/\phi^d . \tag{3.3.21}$$

Note that Equations (3.3.18), (3.3.19), (3.3.20) and (3.3.21) are extensions of Equations (3.3.9), (3.3.10), (3.3.11) and (3.3.12), respectively.

3.3.3 When Samples of $n > 1$ are Taken using Dispersed Sampling

In this subsection, $d = n > 1$ and there is one time unit between each observation in a sample of n > 1. To obtain the model for $n > 1$ with dispersed sampling, use the model for the case of n = 1 with the modification that successive groups of n observations are called a sample. Recall that $\mathbf{X}_k = \left(X_{k1}, X_{k2}, \ldots, X_{kn}\right)$ is the sample at plotting time t_k.

The *i*th observation, X_{ki}, in this sample is actually taken at $d-i$ time units before time t_k (i.e., between plotting times t_{k-1} and t_k). Then X_{ki} can be expressed as

$$X_{ki} = \mu_{ki} + \varepsilon''_{ki}, \tag{3.3.22}$$

where the ε''_{ki}'s are i.i.d normal with mean 0 and variance σ_ε^2. Here Equation (3.3.22) is of the same form as Equation (3.3.13), but Equation (3.3.22) accounts for multiple observations at different time points between sampling times t_{k-1} and t_k, each of which has different process mean μ_{ki}. Note that μ_{ki} can be written as

$$\mu_{ki} = \begin{cases} (1-\phi)\xi + \phi\mu_{k-1,n} + \alpha''_{ki}, & i=1 \\ (1-\phi)\xi + \phi\mu_{k,i-1} + \alpha''_{ki}, & i=2,3,\ldots,n, \end{cases} \tag{3.3.23}$$

for $k=1,2,\ldots$, where α''_{ki}'s are normal independent random variables with mean 0 and variance σ_α^2. Sequentially, the sample mean at the plotting time t_k, $\bar{X}_k$, can be expressed as

$$\bar{X}_k = \bar{\mu}_k + \bar{\varepsilon}''_k \tag{3.3.24}$$

where $\bar{\mu}_k = \sum_{i=1}^{n} \mu_{ki}/n$, and $\bar{\varepsilon}''_k = \sum_{i=1}^{n} \varepsilon''_{ki}/n$. When $n=1$, Equation (3.3.24) reduces to Equation (3.3.1).

With $d=n>1$ and dispersed sampling, sample observations can be also viewed as an ARMA(1,1) process, i.e., X_{ki} can be written as

$$X_{ki} = \begin{cases} (1-\phi)\xi + \phi X_{k-1,n} - \theta\gamma''_{k-1,n} + \gamma''_{ki}, & i=1 \\ (1-\phi)\xi + \phi X_{k,i-1} - \theta\gamma''_{k,i-1} + +\gamma''_{ki}, & i=2,3,\ldots,n \end{cases} \tag{3.3.25}$$

for $k=1,2,\ldots$, where θ is the MA parameter, and γ''_{ki}'s are normal independent random variables with mean 0 and variance σ_γ^2. Note that θ and σ_γ^2 are of the same notation as for individual sampling. Moreover, if $d=n=1$, Equation (3.3.25) reduces to the form of Equation (3.3.8).

3.3.4 Changes in Process Parameters

It is assumed in this dissertation that a special cause will change one of the parameters μ_X, σ_α, and σ_ε, and the other two will remain at their in-control values. It is also assumed that ϕ does not change as a result of a special cause, and thus remains constant during the operation of the process. A shift in μ_X will be only due to a shift in AR(1) process mean ξ (see Equation (3.3.4)). An increase in σ_X will be only due to an increase in σ_α, contributed by the AR(1) process, or/and an increase in σ_ε, contributed by measurement error (see Equation (3.3.5)).

To explain the effect of a shift in a parameter, consider individual sampling as an example. Suppose a shift in ξ from ξ_0 to ξ_1 occurs between sample k^* and k^*+1, the AR(1) process before the shift can be expressed as Equation (3.3.2), where $\xi = \xi_0$. However, the AR(1) process immediately after the shift will be no longer in the same form as before the shift. Instead, the form changes to

$$\mu_{k^*+1} = (1-\phi)\xi_0 + \phi\mu_{k^*} + \alpha_{k^*+1} + (\xi_1 - \xi_0). \tag{3.3.26}$$

The right hand side of Equation (3.3.26) can be viewed as the mean μ'_{k^*+1} plus the mean shift $(\xi_1 - \xi_0)$, where μ'_{k^*+1} is process mean at time t_{k^*+1} if no shift occurs. It is easy to get similar formulas for the other two sampling plans based on Equation (3.3.26). For samples after time t_{k^*+1}, the form of AR(1) process can be still expressed as Equation (3.3.2), except now $\xi = \xi_1$. Thus the mean of AR(1) process can be written as

$$E(\mu_k) = \begin{cases} \xi_0 & \text{for } k = 1, 2, \ldots, k^* \\ \xi_1 & \text{for } k = k^*+1, k^*+2, \ldots \end{cases}.$$

Suppose that a shift in σ_X from σ_{X_0} to σ_{X_1} occurs between sample k^* and k^*+1, and this shift is caused by a shift in σ_μ, the standard deviation of AR(1) process. If the values of μ_X and σ_ε remain the same, then σ_α must shift from σ_{α_0} to

$$\sigma_{\alpha_1} = \sqrt{\sigma_{\mu_1}^2\left(1-\phi^2\right)} = \sqrt{\sigma_{\alpha_0}^2 + \left(\sigma_X^2 - \sigma_{X_0}^2\right)\left(1-\phi^2\right)} \qquad (3.3.27)$$

to produce the given shift in σ_X. Thus an increase in σ_α will correspond to a specified increase in σ_X via an increase in σ_μ.

Suppose that a shift in σ_X from σ_{X_0} to σ_{X_1} is only due to a shift in σ_ε, the standard deviation of measurement error. If the other parameters remain the same, then σ_ε will shift from σ_{ε_0} to

$$\sigma_{\varepsilon_1} = \sqrt{\sigma_{\varepsilon_0}^2 + \left(\sigma_X^2 - \sigma_{X_0}^2\right)}. \qquad (3.3.28)$$

to produce the given shift in σ_X.

Thus, when the process is in control, three process parameters in the autocorrelation model are at their in-control values, i.e., $\xi = \xi_0$, $\sigma_\alpha = \sigma_{\alpha_0}$, and $\sigma_\varepsilon = \sigma_{\varepsilon_0}$. When a shift occurs, either μ_{k^*+1}, σ_{α_1}, or σ_{ε_1} in Equations (3.3.26), (3.3.27) and (3.3.28) is employed instead, depending on whether the shift occurs in the mean or the standard deviation.

If the shift is a sustained shift, the process parameter will remain at the new value until the control chart signals. Note that if the shift is in μ_X, the AR(1) process mean after sample k^*+1 will follows Equation (3.3.2) with $\xi = \xi_1$. If the shift is in σ_X, either σ_α or σ_ε will remain at the new value σ_{α_1} or σ_{ε_1}.

If the shift is a transient shift, the process parameter will return to the in-control value after an out-of control period of duration *l*. Note that the AR(1) process at the sample plotted immediately after the duration ends will be similar to Equation (3.3.26) except for ξ_0 and ξ_1 switching positions.

3.4 Description of Control Charts in the Autocorrelated Case

In this dissertation, autocorrelation is regarded as an inherent characteristic of the process that cannot be eliminated. In this case, the process mean μ_X is continually wandering, even though the process is in control. Consequently, the definition of an in-control process is extended. For example, when the process observations follow the model in Equations (3.3.1) and (3.3.2) with all parameters at their target values, the process is said to be in control.

As mentioned in Section 1.2, two approaches to dealing with autocorrelated data are investigated in this dissertation. One is based on control charts of the observations, in which one plots the observations in conventional control charts but adjusts the control limits to account for the autocorrelations; the other is based on control charts of the residuals, in which one fits a time series model to the process data and then plots the residuals in conventional control charts. In this section, these two types of control charts are discussed.

3.4.1 Control Charts of the Observations

The Shewhart and EWMA control charts of the observations have the same formulas for the control statistics and control limits as the conventional control charts in the independent case (Equations (3.2.1) - (3.2.6) for the Shewhart control charts, and Equations (3.2.7) - (3.2.10) for the EWMA control charts), except that now the process observations are no longer independent from each other, and the factor *h* is adjusted in order to account for the process autocorrelation.

Two types of the observations charts are considered in this dissertation: control charts of the sample means, and control charts of the sample squared deviations of the observations from the target. When the Shewhart control charts are employed, these two types of the observations charts will be denoted as Shewhart X charts and Shewhart X^2 charts. When the EWMA control charts are used, the notation will be EWMA_X charts and EWMA_{X^2} charts.

3.4.2 Properties of the Residuals

When $d = n = 1$ and the process is in control, the minimum mean square error forecast for time t_k based on time t_{k-1} is

$$\hat{X}_k = \xi_0 + \phi\left(X_{k-1} - \xi_0\right) - \theta e_{k-1}, \tag{3.4.1}$$

where

$$e_k = X_k - \hat{X}_k \tag{3.4.2}$$

is the residual at time t_k, and θ and ϕ are parameters in the ARMA(1,1) model in Equation (3.3.8). The advantage to using e_k's is that when the process is in control, the e_k's are independent and normally distributed with mean 0 and variance σ_γ^2, where σ_γ^2 is given in Equation (3.3.12).

With $d = n > 1$ and concentrated sampling, n observations $X_{k1}, X_{k2}, \ldots, X_{kn}$ are taken at each plotting point, so the forecast for time t_k based on time t_{k-1} is based on the sample means. This forecast is

$$\hat{\bar{X}}_k = \xi_0 + \phi^d\left(\bar{X}_{k-1} - \xi_0\right) - \theta' \bar{e}'_{k-1}, \tag{3.4.3}$$

where

$$\bar{X}_k = \sum_{i=1}^{n} X_{ki} / n, \tag{3.4.4}$$

$$e'_{ki} = X_{ki} - \hat{X}_k , \tag{3.4.5}$$

and

$$\bar{e}'_k = \sum_{i=1}^{n} e'_{ki} / n . \tag{3.4.6}$$

Note that if $n = 1$, Equations (3.4.3), (3.4.5) and (3.4.6) reduce to Equations (3.4.1) and (3.4.2). The $\bar{e}'_k$ is the sample average residual at time t_k, and θ' and ϕ^d are parameters in the ARMA(1,1) model in Equation (3.3.17). When the process is in control, the $\bar{e}'_k$'s are independent and normally distributed with mean 0 and variance $\sigma^2_{\gamma'}$, where $\sigma^2_{\gamma'}$ is given in Equation (3.3.19).

When $d = n > 1$ and dispersed sampling is employed, the *i*th observation, X_{ki}, in sample $\mathbf{X}_k = (X_{k1}, X_{k2}, \ldots, X_{kn})$ at plotting time t_k is taken at $d - i$ time units before t_k. Thus the forecast made for time t_k is actually

$$\hat{X}_k = \sum_{i=1}^{n} \hat{X}_{ki} / n , \tag{3.4.7}$$

where

$$\hat{X}_{ki} = \begin{cases} \xi_0 + \phi\left(X_{k-1,n} - \xi_0\right) - \theta'' e''_{k-1,n}, & i = 1 \\ \xi_0 + \phi\left(X_{k,i-1} - \xi_0\right) - \theta'' e''_{k,i-1}, & i = 2,3,\ldots,n \end{cases} , \tag{3.4.8}$$

and

$$e''_{ki} = X_{ki} - \hat{X}_{ki} . \tag{3.4.9}$$

If $n = 1$, Equations (3.4.7), (3.4.8) and (3.4.9) reduce to Equations (3.4.1) and (3.4.2). The sample average residual at time t_k is $\bar{e}''_k$, which can be written as

$$\bar{e}''_k = \sum_{i=1}^{n} e''_{ki} / n .$$

Now consider the effect on the residuals of changes in the process parameters. Suppose the shift in μ_X from ξ_0 to ξ_1 occurs between sample k^* and $k^* + 1$, Lu and

Reynolds (1999b) provided the mean of e_k at various times after the shift for individual sampling, which can be written as

$$E\left(e_k\right)=\begin{cases} \xi_1-\xi_0 & \text{for } k=k^*+1 \\ \dfrac{\left[\theta^l\left(\phi-\theta\right)-\phi+1\right]}{1-\theta}\left(\xi_1-\xi_0\right) & \text{for } k=k^*+1+l,\ l=1,2\ldots \end{cases}. \qquad (3.4.10)$$

Thus the asymptotic mean of e_k after the shift in the process mean is $\left(\xi_1-\xi_0\right)\left(1-\phi\right)/\left(1-\theta\right)$. The variance of e_k remains as σ_γ^2. The residuals are still independent and normally distributed in this case. For the more details, refer to Lu and Reynolds (1999b) and the appendix therein.

For concentrated sampling with $d=n>1$, the mean of $\overline{e}'_k$ at various times after the shift can be written as

$$E\left(\overline{e}'_k\right)=\begin{cases} \xi_1-\xi_0 & \text{for } k=k^*+1 \\ \dfrac{\left[\theta'^l\left(\phi^d-\theta'\right)-\phi^d+1\right]}{1-\theta'}\left(\xi_1-\xi_0\right) & \text{for } k=k^*+1+l,\ l=1,2\ldots \end{cases}. \qquad (3.4.11)$$

Thus the asymptotic mean of e_k after the shift in the process mean is $\left(\xi_1-\xi_0\right)\left(1-\phi^d\right)/\left(1-\theta'\right)$. The variance of e_k remains as $\sigma_{\gamma'}^2$. When $d=n=1$, Equation (3.4.11) reduces to Equation (3.4.10).

The case of dispersed sampling with $d=n>1$ is more complicated. Suppose the shift in μ_X from ξ_0 to ξ_1 occurs between sample k^* and k^*+1. Because the sample observations $X_{k1},X_{k2},\ldots,X_{kn}$ is assumed to be evenly distributed over the sample interval d with each observation one time unit apart, it is likely that the shifts occurs between observations X_{k^*+1,i^*} and X_{k^*+1,i^*+1}. If the mean of e''_{ki} can be expressed as

$$E\left(e_{ki}\right)=\begin{cases} \xi_1-\xi_0 & \text{for } k=k^*+1\ ,i=i^*+1 \\ \dfrac{\left[\theta''^{l}\left(\phi-\theta''\right)-\phi+1\right]}{1-\theta''}\left(\xi_1-\xi_0\right) & \text{for } k=k^*+1, i=i^*+1+l, \\ & l=1,2,\ldots,n-i^*-1; \\ & \text{and for } k=k^*+1+l_1, i=l_2, \\ & l=n-i^*, n-i^*+1,\ldots, \\ & \text{where } l_1=1+\left(l-n+i^*+1-l_2\right)/n, \\ & l_2=\operatorname{mod}\left(l-n+i^*+1,n\right) \end{cases}.$$

Then the mean of $\overline{e}''_k$ can be expressed as

$$E\left(\overline{e}''_k\right)=\begin{cases} \sum\limits_{j=0}^{l}\dfrac{\left[\theta''^{j}\left(\phi-\theta''\right)-\phi+1\right]}{1-\theta''}\left(\xi_1-\xi_0\right) & \text{for } k=k^*+1,\ \ l=n-i^*-1; \\ \sum\limits_{j=0}^{n-1}\dfrac{\left[\theta''^{l+j}\left(\phi-\theta''\right)-\phi+1\right]}{1-\theta''}\left(\xi_1-\xi_0\right) & \text{for } k=k^*+2,\ldots, \\ & l=n-i^*, 2n-i^*, 3n-i^*,\ldots \end{cases}. \quad (3.4.12)$$

The variance of $\overline{e}''_k$ is denoted as $\sigma^2_{\gamma''}$, where $\sigma^2_{\gamma''}=\sigma^2_{\gamma}/n$. When $d=n=1$, Equation (3.4.12) reduces to Equation (3.4.10).

Suppose the shift that occurs between sample k^* and k^*+1 increases σ_{X_0} to σ_X, and is only due to an increase in either σ_α or σ_ε, then the values of σ_{α_1} or σ_{ε_1} after the shift is given in Equation (3.3.27) or (3.3.28). Lu and Reynolds (1999a) showed that the residuals after the shift for individual sampling are correlated normal random variable with mean 0 and variance which can be written as

$$\operatorname{var}\left(e_k\right)=\begin{cases} \sigma^2_{\gamma_0}+\left(\sigma^2_{\varepsilon_1}-\sigma^2_{\varepsilon_0}\right)+\left(\sigma^2_{\alpha_1}-\sigma^2_{\alpha_0}\right) & k=k^*+1 \\ \sigma^2_{\gamma_0}+\left[1+\left(\phi-\theta_0\right)^2\sum\limits_{i=0}^{l-2}\theta_0^{2(l-i-1)}\right]\left(\sigma^2_{\varepsilon_1}-\sigma^2_{\varepsilon_0}\right) & \\ \quad +\sum\limits_{i=0}^{l}\theta_0^{2(l-i)}\left(\sigma^2_{\alpha_1}-\sigma^2_{\alpha_0}\right) & k=k^*+1+l,\ l=1,2,\ldots \end{cases}, \quad (3.4.13)$$

where θ_0 is the in-control value of θ. For the more details, refer to Lu and Reynolds (1999a) and the appendix therein.

For concentrated sampling with $d = n > 1$, the variance of $\vec{e}'_k$ at various times after the shift is of the similar form as Equation (3.4.13), except $e_k = \vec{e}'_k$, $\sigma^2_{\gamma_0} = \sigma^2_{\gamma'_0}$, $\sigma^2_{\varepsilon_0} = \sigma^2_{\varepsilon'_0} / n$, $\sigma^2_{\varepsilon_1} = \sigma^2_{\varepsilon'_1} / n$, $\phi = \phi^d$ and $\theta_0 = \theta'_0$. The case of dispersed sampling with $d = n > 1$ when the shift is in σ_X is too complicated to work out explicitly, because some observations in the sample are from the in-control distribution, and the others are from the out-of-control distribution.

3.4.3 Control Charts of the Residuals

To construct a residuals chart, two main steps are: calculate process residuals, and then plot process statistics based on these residuals on a chart. The Shewhart and EWMA residuals charts have the same formulas of control statistics and control limits as their conventional control charts in the independent case (Equations (3.2.1) - (3.2.6) for the Shewhart charts, and Equations (3.2.7) - (3.2.10) for the EWMA charts), despite that now the control statistics are based on process residuals instead of process observations. For individual sampling, X_k is replaced with e_k, μ_{X_0} is replaced with 0, and σ_{X_0} is replaced with σ_γ. For concentrated sampling, X_k is replaced with $\vec{e}'_k$, μ_{X_0} is replaced with 0, and σ_{X_0} is replaced with $\sigma_{\gamma'}$. Finally for dispersed sampling, X_k is replaced with $\vec{e}''_k$, μ_{X_0} is replaced with 0, and σ_{X_0} is replaced with $\sigma_{\gamma''}$.

Compared to the observations charts, the residuals charts need an additional step of calculating the process residuals. However, because the residuals are obtained from the process observations, the process parameters (μ_X, σ_α, and σ_ε), their in-control values (μ_{X_0}, σ_{α_0}, and σ_{ε_0}), and the new parameter values after the shift occurs (Equations (3.3.26), (3.3.27) and (3.3.28)), are essentially the same for both charts. For more details, see Section 3.3.4.

Two types of the residuals charts are considered in this dissertation: control charts of the sample mean of the residuals, and control charts of the squared residuals. When the Shewhart control charts are employed, these two types of residuals charts will be referred as Shewhart $\overline{e}$ charts (or Shewhart e chart when n =1) and Shewhart e^2 charts. When the EWMA control charts are used, the notation will be EWMA_e charts and EWMA_{e^2} charts.

3.5 Measures of Performance

3.5.1 The Average Time to Signal

As emphasized in Section 1.2, the objective of this dissertation is to investigate the statistical performance of various sampling plans and control chart combinations in detecting process changes when autocorrelation is present. The ability of control charts to detect process changes can be measured in terms of the average time to signal (ATS), which is defined as the expected value of the time from the start of process monitoring until a signal is given. When the process is in control, it is desirable to have a large ATS so that the rate of false alarm produced by the control chart is low. When there is a shift in the process, a low ATS is needed in order to detect the change as quickly as possible.

In order to compare the performance of control charts more meaningfully for different models, all control chart combinations considered here are set up with the same false alarm rate when the process is in control. The standard false alarm rate is set here as 0.000675, corresponding to an in-control ATS of 1481.6 time units, which can be obtained when using a Shewhart $\overline{X}$ control chart with standard three-sigma control limits and with samples of $n = 4$ observations taken every $d = 4$ time units. This specified false alarm rate are obtained for each control chart combination by manipulating the control limits so that the in-control ATS is 1481.6. The control limits of each individual control chart in a combination are also chosen so that each individual chart has the same in-control ATS (while the in-control ATS of the combination is 1481.6).

In comparing different control charts and sampling plans, it is assumed that the sampling rate is fixed for all control chart combinations. In this dissertation, the sampling rate is set to be $n/d = 1$, and, for convenience, the time unit is referred to as an hour.

When different control chart combinations have the same false alarm rate and the same sampling rate, then the control chart combination with the lowest ATS when a shift in the process occurs is considered superior. Wardell, Moskowitz, and Plante (1992) stated that this is analogous to matching the Type I error (probability of an out-of-control signal given no shift has occurred), so that the Type II errors (probability of an in-control observation given a shift of a specific size has occurred) can be compared in a meaningful way.

Moreover, shifts in μ_X and σ_X are expressed in terms of $\delta_\mu = \left|\mu_X - \mu_{X_0}\right| / \sigma_{X_0}$ and $\delta_\sigma = \sigma_X / \sigma_{X_0}$, respectively, as the size of the shift in units of the process standard deviation.

3.5.2 The Steady-State ATS

The ATS is an appropriate measure only when the process change is present at the start of monitoring. However, in practical situations, a change may occur at some time after process monitoring has started. Therefore, the concept of steady-state ATS (SSATS) has been introduced to represent the expected length of time from the random time point that the change occurs to the time that a signal is produced. The SSATS is computed assuming that the control statistic has reached its steady-state by the random time point that the shift occurs. This feature is important to the numerical results, because for EWMA control charts, the ATS depends on the value of the control statistic at the time when the change occurs.

Furthermore, the process shift may occur in the time interval *d* between the plotting points. This means that, with dispersed sampling, a sample may include some observations from the in-control distribution and some from the out-of-control distribution. It is assumed that the time that the change occurs is uniformly distributed over the time interval *d*. Thus the SSATS can allow for fair comparisons among control charts with different simple sizes, sampling intervals, and types of sampling. For more details about the SSATS, see Reynolds and Stoumbos (2004b) and references therein.

3.5.3 Performance Measure for Transient Shifts

For sustained shifts, the main question is how long it will take the control charts to signal. However, when a transient shift occurs, if a control chart cannot detect it within the time period when it is present, it is unlikely to detect it afterwards either. In this case, the SSATS will essentially be the same as the in-control ATS. Thus, the SSATS is not an effective measure of performance for sustained shifts. Reynolds and Stoumbos (2004a) suggested using a simpler and more appropriate performance measure, which is the probability that the control chart signals within a period after the transient shift occurs. In this dissertation, this period is assumed to consist of the duration of the transient shift and four hours afterwards, i.e., $l+4$ hours.

3.6 Methods for Evaluating the Measures of Performance

In this dissertation, combinations of two control charts are applied simultaneously to detect changes in μ_X or σ_X. Thus the control limits of the chart combinations need to be adjusted so that the two charts have a joint in-control ATS of 1481.6 time units and also the same individual in-control ATS values.

It is very difficult to find the joint ATS value for the autocorrelated process with the integral equation or Markov chain methods. Thus, the simulation method with 1,000,000 runs is employed. Reynolds and Stoumbos (2004b) stated that when the process is in control, the standard deviation of the in-control time to signal is approximately equal to ATS, so the standard deviation of the simulated results is approximately equal to $\text{ATS}\big/\sqrt{1{,}000{,}000}$. They also stated that the same result also holds for SSATS unless the parameter change is large while the SSATS is very small. Thus the variations in in-control ATS and SSATS values due to simulation are very low in this case.

Note that the use of the SSATS is based on the assumption that the distribution of process observations immediately before changes occur is a stationary distribution conditional on no false alarm. Thus 600 initial in-control observations are generated before the process changes. If a false alarm occurred in the initial observations, the current sequence of observations is discarded, and a new sequence is generated. After a sequence of 600 in-control observations is obtained, a process change is chosen randomly from the *n* time points within the next sampling interval.

Moreover, in order to assure the accuracy of simulation, the integral equation method and the simulation method are respectively applied to some selected cases of the Shewhart control charts for monitoring μ_X, and the results are compared.

3.7 Structure of the Rest of the Work

In the following chapters, the performance of the Shewhart and EWMA observations and residuals chart combinations in detecting both sustained and transient shifts in μ_X and σ_X will be investigated. The comparisons are based on three sampling plans, two types of shifts and various combinations of different levels of ϕ and ψ.

The evaluations of the control charts of the observations and residuals, and of the Shewhart and EWMA control charts will be conducted separately. This is because efficient sampling plans for control charts of the observations may not necessarily work well for control charts of the residuals, efficient sampling plans for the Shewhart control charts may not work well for the EWMA control charts, etc. Then overall conclusions will be made based on the most efficient sampling plans for each type of control chart combination. A few illustrative examples of chart combination plots and discussion of some extensions will be also presented.

Chapter 4

Performance of Shewhart Observations Chart Combination

In this chapter, the performance of various sampling plans for the Shewhart observations chart combinations is investigated. All discussion of the Shewhart control charts in this chapter will refer to the Shewhart control charts of the observations (the Shewhart $\overline{X}$ chart or Shewhart X chart) and the Shewhart charts of the squared deviations of the observations from target (the Shewhart X^2 charts). The evaluations are done for both sustained shifts and transient shifts in the autocorrelated process.

There are three main issues taken into consideration. The first one is, when $n = 1$, whether adding the Shewhart *MR* chart improves the performance of the Shewhart X chart in detecting changes in μ_X and σ_X in the autocorrelated process. This issue has been previously investigated for the case of independent observations, but not for the case of autocorrelation. The second issue concerns an appropriate value for n, when $n > 1$, to obtain good overall performance of the Shewhart $\overline{X}$ and X^2 chart combination. The third issue is to find an efficient sampling plan for the Shewhart observations charts to detect both sustained and transient shifts in μ_X and σ_X.

An additional question of interest is the type of control chart combination with the best overall performance. Comparisons are between the Shewhart observations chart combination and the EWMA observations chart combination, and between the Shewhart observations chart combination and the Shewhart residuals chart combination. The investigation of this issue will be addressed later in Chapter 5 after the discussion of the EWMA observations chart combination, and also in Chapter 6 after the discussion of the Shewhart residuals chart combination.

Before carrying out any further simulations, the integral equation method is applied to the Shewhart control charts for monitoring sustained shifts in μ_X with both the individual and concentrated sampling plans in order to compare the performance of the integral equation method and the simulation method, and if possible, to assess the accuracy of the simulation.

4.1 The Integral Equation Method

4.1.1 Shewhart Charts of the Observations with Individual Sampling

Reynolds, Arnold, and Baik (1996) explained how to use the integral equation method to find the ATS and SSATS for the VSI Shewhart control charts when the autocorrelation is present in the process. In this section, their method is adjusted for application to the FSI Shewhart control charts with the individual sampling plan.

As discussed in Section 3.3.1, in the individual sampling case, the process observation at time t_k, X_k, is expressed as Equation (3.3.1) and the process mean at time t_k, μ_k, is expressed as Equation (3.3.2). Assuming the process is stationary, the transition distribution of the sample from time k to $k+1$ depends on μ_k, given that there have been no false alarm which in turns depends on X_k. The probability of no false alarm at time t_k, given that $\mu_k = u$, can be written as

$$P\left(X_k \in C \mid u\right) = \int_{\xi - h\sigma_X}^{\xi + h\sigma_X} f_Z\left(\frac{X_k - u}{\sigma_\varepsilon}\right)\frac{1}{\sigma_\varepsilon} dX_k \, , \tag{4.1.1}$$

where C is the in-control region, f_Z is the standard normal density, and σ_X^2 and σ_ε^2 are given in Equation (3.3.5). The transition density at $\mu_{k+1} = v$, given that $\mu_k = u$ and there is no false alarm at time t_k, is

$$f\left(v \mid u, X_k \in C\right) = f_Z\left(\frac{v - \xi}{\sigma_\alpha} - \phi\frac{u - \xi}{\sigma_\alpha}\right)\frac{1}{\sigma_\alpha} \, . \tag{4.1.2}$$

Therefore, the joint transition density at $\mu_{k+1} = v$ and no false alarm at time t_{k+1}, given that $\mu_k = u$ and no false alarm at time t_k, is $P\left(X_{k+1} \in C \mid v\right) f\left(v \mid u, X_k \in C\right)$.

Let $A(u)$ be the expected time from time t_k to the signal. Then $A\left(\mu_X\right)$ can be written as

$$\begin{aligned} A(u) &= \text{time from } t_k \text{ to } t_{k+1} + \text{expected time from } t_{k+1} \text{ to signal} \\ &= 1 + \int_{-\infty}^{+\infty} A(v) P\left(X_{k+1} \in C \mid v\right) f\left(v \mid u, X_k \in C\right) dv \\ &= 1 + \int_{-\infty}^{+\infty} \left[A(v) \cdot \left(\int_{\xi_0 - h\sigma_X}^{\xi_0 + h\sigma_X} f_Z\left(\frac{X_{k+1} - v}{\sigma_\varepsilon}\right)\frac{1}{\sigma_\varepsilon} dX_{k+1} \right) \cdot \right. \\ &\qquad\qquad \left. f_Z\left(\frac{v - (1-\phi)\xi_1 - \phi u}{\sigma_\alpha}\right)\frac{1}{\sigma_\alpha} \right] dv \end{aligned} \tag{4.1.3}$$

Thus the ATS, counted from time $t_0 = 0$, can be expressed as

$$\begin{aligned} ATS = {} & (\text{time from } t_0 = 0 \text{ to } t_1) + (\text{expected time from } t_1 \text{ to signal}) \\ &= 1 + \int_{-\infty}^{+\infty} A\left(u_1\right) P\left(X_1 \in C \mid u_1\right) f\left(u_1\right) du_1 \, , \end{aligned} \tag{4.1.4}$$

where $f\left(u_1\right) = f_Z\left(\sqrt{1-\phi^2}\left(u_1 - \xi\right) \big/ \sigma_\alpha\right)\sqrt{1-\phi^2} \big/ \sigma_\alpha$ is the marginal density of u_1, the process mean at time t_1.

The integral equation (4.1.3) is a Fredholm equation of the second kind, and the general equation of which can be expressed as

$$f(t) = g(t) + \int_a^b f(s) \lambda K(t,s) ds \, . \tag{4.1.5}$$

Here, $f(t) = A(u)$, $g(t) = 1$, $f(s) = A(v)$ and

$$\lambda K(t,s) = \left(\int_{\xi_0 - h\sigma_X}^{\xi_0 + h\sigma_X} f_Z\left(\frac{X_{k+1} - v}{\sigma_\varepsilon} \right) \frac{1}{\sigma_\varepsilon} dX_{k+1} \right) \cdot f_Z\left(\frac{v - (1-\phi)\xi_1 - \phi u}{\sigma_\alpha} \right) \frac{1}{\sigma_\alpha}.$$

The approximation to the integrals can be obtained by using numerical quadrature. For example, if a function $A(u)$ is to be integrated over the region $-\infty < u < \infty$ with r quadrature points, let $u_1, u_2, \ldots, u_r$ be the quadrature points, and $a_1, a_2, \ldots, a_r$ be the corresponding quadrature weights. Then the approximation is

$$\int_{-\infty}^{+\infty} A(u)\,du \approx \sum_{j=1}^{r} a_j A(u_j). \tag{4.1.6}$$

Thus the integral equation (4.1.3) can be written as a system of linear equations at $u = u_i, i = 1,2,...,r$, which are

$$\tilde{A}(u_i) = 1 + \sum_{j}^{r} a_j \tilde{A}(u_j) P(X_{k+1} \in C \mid u_j) f(u_j \mid u_i, X_k \in C). \tag{4.1.7}$$

Also, applying the approximation to the integrals in Equation (4.1.4) and substituting $\tilde{A}(u_j)$ for $A(u_j)$, the approximate in-control ATS can be expressed as

$$ATS \approx 1 + \sum_{j}^{r} a_j \tilde{A}(u_j) P(X_{k+1} \in C \mid u_j) f(u_j). \tag{4.1.8}$$

To compute the SSATS in the case of a shift in ξ, there are some changes required in the formula. Suppose the shift from ξ_0 to ξ_1 occurs between sample k^* and $k^* + 1$ so that sample k^* is the last sample before the shift. Let $\pi(u)$ be the conditional stationary distribution of μ_{k^*} and $\alpha(u)$ be the density of μ_{k^*}, given that there have been no false alarms. Then the relation between $\alpha(u)$ and $\pi(u)$ can be written as

$$\alpha(u) = \frac{\pi(u)}{\int_{-\infty}^{\infty} \pi(u)\,du}. \tag{4.1.9}$$

The relationship between μ_{k^*} and μ_{k^*+1} is given in Equation (3.3.26). The transition density between sample k^* and $k^* + 1$ is

$$f^*(v \mid u) = f_Z\left(\frac{v-\xi_0}{\sigma_X} - \phi\frac{u-\xi_0}{\sigma_X} - \frac{\xi_1-\xi_0}{\sigma_X}\right)\cdot\frac{1}{\sigma_X}. \tag{4.1.10}$$

Given $\mu_{k^*} = u$ and there is no false alarm at time t_{k^*}, the expected time from the shift to the signal is

$$\begin{aligned} A(shift) &= \frac{1}{2} + \text{expected time from } t_{k^*+1} \text{ to signal} \\ &= \frac{1}{2} + \int_{-\infty}^{+\infty} A(v) P\left(X_{k^*+1} \in C \mid v\right) f^*\left(v \mid u, X_{k^*} \in C\right) dv. \end{aligned} \tag{4.1.11}$$

Thus from the equation

$$SSATS = \int_{-\infty}^{+\infty} \alpha(u)\left[\frac{1}{2}t + \int_{-\infty}^{+\infty} A(v) P\left(X_{k^*+1} \in C \mid v\right) f^*\left(v \mid u, X_{k^*} \in C\right) dv\right] du \text{ ,} \tag{4.1.12}$$

the value of SSATS can be calculated from the following approximation equation:

$$SSATS \approx \sum_{i=1}^{r} a_i \tilde{\alpha}(u_i)\left[\frac{1}{2}t + \sum_{j=1}^{r} a_j \tilde{A}(u_j) P\left(X_{k^*} \in C \mid u_j\right) f^*\left(u_j \mid u_i\right)\right], \tag{4.1.13}$$

where $\tilde{A}(u_j)$ is given in Equation (4.1.7), and

$$\tilde{\alpha}(u_i) = \frac{\tilde{\pi}(u_i)}{\sum_{j=1}^{r} a_j \tilde{\pi}(u_j)}, \tag{4.1.14}$$

where $\tilde{\pi}(u_i)$ denotes "the normalized left eigen-function of the transition distribution corresponding to the largest eigenvalue". See Reynolds, Arnold, and Baik (1996), and for the details about $\tilde{\pi}(u_i)$ refer to Reynolds (1995).

4.1.2 Shewhart Charts of the Observations with Concentrated Sampling

As discussed in Section 3.3.2, the model of the autocorrelated process for multiple observations at a sampling point can be expressed as Equation (3.3.13). If samples $k-1$

and k are d time interval apart (note that here $d = n > 1$), μ_k can be written as Equation (3.3.15).

In this case, the probability of no false alarm at time t_k, given that $\mu_k = u$, is

$$P\left(\overline{X}_k \in C \mid u\right) = \int_{\xi - h\sigma_X}^{\xi + h\sigma_X} f_Z\left(\frac{\overline{X}_k - u}{\sigma_\varepsilon / \sqrt{2}}\right)\frac{1}{\sigma_\varepsilon / \sqrt{2}}\, d\overline{X}_k, \tag{4.1.15}$$

where f_Z, σ_X^2, and σ_ε^2 are of the same form with as in Equation (4.1.1). The transition density at $\mu_{k+1} = v$, given that $\mu_k = u$ and no false alarm at time t_k, is

$$f\left(v \mid u, \overline{X}_k \in C\right) = f_Z\left(\frac{v - \left(1-\phi^d\right)\xi_0 - \phi^d u}{\sqrt{\left(1-\phi^{2d}\right)/\left(1-\phi^2\right)}\sigma_\alpha}\right)\cdot\frac{1}{\sqrt{\left(1-\phi^{2d}\right)/\left(1-\phi^2\right)}\sigma_\alpha}. \tag{4.1.16}$$

The expected time from time t_k to the signal in this case can be written as

$A(u)$ = time from t_k to t_{k+1} + expected time from t_{k+1} to signal.

$$\begin{aligned}
&= 1 + \int_{-\infty}^{+\infty} A(v) P\left(\overline{X}_{k+1} \in C \mid v\right) f\left(v \mid u, \overline{X}_{k+1} \in C\right) dv \\
&= 1 + \int_{-\infty}^{+\infty}\left[A(v)\cdot\left(\int_{\xi - h\sigma_X}^{\xi + h\sigma_X} f_Z\left(\frac{\overline{X}_{k+1} - v}{\sigma_\varepsilon / \sqrt{2}}\right)\frac{1}{\sigma_\varepsilon / \sqrt{2}}\, d\overline{X}_{k+1}\right)\right. \\
&\qquad \left.\cdot f_Z\left(\frac{v - \left(1-\phi^d\right)\xi_0 - \phi^d u}{\sqrt{\left(1-\phi^{2d}\right)/\left(1-\phi^2\right)}\sigma_\alpha}\right)\frac{1}{\sqrt{\left(1-\phi^{2d}\right)/\left(1-\phi^2\right)}\sigma_\alpha}\right] dv.
\end{aligned} \tag{4.1.17}$$

Despite some differences in the parameter expressions, the calculation of the approximation of the in-control ATS for concentrated sampling is the same with that for individual sampling, which can be obtained via Equation (4.1.8) by plugging in Equations (4.1.15) and (4.1.16), and the approximation to Equation (4.1.17).

To compute the SSATS for concentrated sampling, the relationship between μ_{k^*} and μ_{k^*+1} is now

$$\mu_{k^*+1} = \left(1-\phi^d\right)\xi_0 + \phi^d \mu_{k^*} + \alpha_{k^*+1} + \left(\xi_1 - \xi_0\right). \tag{4.1.18}$$

The transition density between sample k^* and k^*+1 is

$$f\left(v \mid u\right) = f_Z\left(\frac{v-\left(1-\phi^d\right)\xi_0 - \phi^d u - \left(\xi_1 - \xi_0\right)}{\sqrt{\left(1-\phi^{2d}\right)/\left(1-\phi^2\right)}\sigma_\alpha}\right) \cdot \frac{1}{\sqrt{\left(1-\phi^{2d}\right)/\left(1-\phi^2\right)}\sigma_\alpha}. \tag{4.1.19}$$

The SSATS also can be easily calculated from Equation (4.1.13) by using Equations (4.1.11), (4.1.14) and (4.1.19), and the approximation to Equation (4.1.17).

4.1.3 Comparison of Numerical Values from the Integral Equation and Simulation Method

Table 1 shows the ATS and SSATS values from both the integral equation method and the simulation for individual sampling (n = 1) and concentrated sampling (n = 4). The cases of independent observations and four combinations of different values of the autocorrelated process parameters ϕ and ψ, such as ϕ = 0.2, 0.8 with ψ = 0.1, 0.9, are considered. The SSATS values are given for sustained shifts in μ_X from size δ_μ = 0.0 to 5.0, where $\delta_\mu = \left|\mu_X - \mu_{X_0}\right| / \sigma_{X_0}$. It appears that the results from the integral equation method and the simulation are very close, which implies that the simulation results are highly accurate. In the rest of this dissertation, only simulations will be conducted for control chart combinations.

4.2 Shewhart Observations Charts for Individual Sampling and Sustained Shifts

A number of papers have shown that there is essentially no advantage to using the Shewhart *MR* chart with the *X* chart in the case of independent observations. These papers concluded that the *X* chart alone is much more efficient in detecting changes in μ_X and only slightly worse in detecting changes in σ_X (see, e.g., Reynolds and Stoumbos (2001) and reference therein). However, studies should be done to check whether this statement is also true for autocorrelated processes. Tables 2 and 3 give the SSATS values for sustained shifts in μ_X and σ_X for the Shewhart *X* chart, *MR* chart and the combination of *X* and *MR* charts, and include the independent case and various combinations of ϕ and ψ, such as ϕ = 0.2, 0.6, 0.8 with ψ = 0.1, 0.9. The SSATS values are given for sustained shifts in μ_X, where the range of shift size δ_μ is from 0.0 to 15.0, where $\delta_\mu = \left|\mu_X - \mu_{X_0}\right| / \sigma_{X_0}$, and also in σ_α and σ_ε, respectively, where the range of δ_σ is from 1.0 to 15.0, where $\delta_\sigma = \sigma_X / \sigma_{X_0}$.

Note that when the autocorrelation is high, some charts may not perform well. However, such cases are still considered here, because the objective of this work is to investigate the effect of autocorrelation on the performance of these charts.

First consider the performance of the *X* chart and *MR* chart when each chart is used alone. Table 3 shows that when μ_X shifts the SSATS of the *MR* chart is almost as high as the in-control ATS unless the shift is extremely large. Thus, as expected, the *MR* chart alone is completely ineffective in detecting sustained shifts in μ_X regardless of the level of autocorrelation. This is because the distribution of the control statistic MR_k does not depend on μ_X, except at the shift point the expectation of MR_k is equal to the shift. Moreover, the *MR* chart is not even as good as the *X* chart in detecting increases in σ_X

for most of the cases. However, the performance of the *MR* chart improves considerately when the level of autocorrelation increases and the shift in σ_X is due to σ_ε, while the performance of the *X* chart given in Table 2 does not change much in this case. For example, an average of 65.3 hours is required for the *X* chart and 69.7 hours for the *MR* chart to detect a shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_ε when ϕ = 0.2 and ψ = 0.1; when ϕ = 0.6 and ψ = 0.9, a similar average of 65.8 hours is required for the *X* chart but only 21.2 hours is needed for the *MR* chart to detect the same type of shift. This is because the control limit h_{MR} is much smaller in the highly autocorrelated case in order to achieve the specified in-control ATS, and the same units of shift in σ_X may be more likely to cause a signal. Note that when the correlation gets higher, the *MR* chart is much more efficient when increases in σ_X are due to σ_ε than when they are due to σ_α. This is indicated by Equation (3.3.5), where a larger shift is needed in σ_ε than in σ_α in order to produce the same units of shift in σ_X. A conclusion can be drawn based on the above analysis, which is that the *MR* chart alone is much less efficient than the *X* chart alone in overall performance.

The SSATS values for the *X* and *MR* chart combination are also available in Table 3. A comparison between the chart combination and the individual *X* chart shows that the *X* chart alone is much more efficient in detecting shifts in μ_X in both the independent and the autocorrelated cases. For example, in the independent case, an average of 521.5 hours is required for the *X* chart and 709.4 hours for the *X* and *MR* chart combination to detect a shift in μ_X of size $0.5\sigma_{X_0}$; when ϕ = 0.6 and ψ = 0.9, an average of 555.7 hours is needed for the *X* chart and 750.8 hours for the *X* and *MR* chart combination to detect the same type of shift. When the shift in σ_X due to σ_α is of interest, the *X* chart alone is only slightly worse than the *X* and *MR* chart combination. However, when the shift in σ_X is due to σ_ε, the *X* chart is not as good as the *X* and *MR* chart combination, especially when the correlation is high. Although there is an increase in the time required to detect shifts in σ_X due to σ_ε, the big advantage of the *X* chart in detecting sustained shifts in μ_X seems to outweighs the disadvantage. This indicates that when the process is autocorrelated, the *X* charts alone also has better overall performance than the *X* and *MR* chart combination, with much quicker detection of shifts in μ_X and fair performance in detecting shifts in σ_X. Thus, if individual sampling is chosen, it is best to use the *X* chart alone for monitoring the process, whether or not there is autocorrelation among the observations.

4.3 Shewhart Observations Chart Combination with $n > 1$ and Sustained Shifts

Due to the advantages of the Shewhart X^2 chart over the traditional *S* chart mentioned in Section 3.1.2, the Shewhart $\overline{X}$ and X^2 chart combination is considered in this dissertation. Tables 4 and 5 give the SSATS values of the Shewhart $\overline{X}$ and X^2 chart combination with sample size n = 2, 4, and 8 for sustained shifts in μ_X, σ_α and σ_ε, where Table 4 is for the independent case and Table 5 is for the autocorrelated case, which includes various combinations of ϕ and ψ. This section is only for the study of sampling plans with $n > 1$. The comparison between sampling plans with $n > 1$ and individual sampling will be addressed later in Section 4.4.

4.3.1 The Independent Case

Reynolds and Stoumbos (2004a) showed in their work that the choice of sample size n = 4 or 5 for the Shewhart $\overline{X}$ and *S* chart combination would be reasonable for good overall performance in the independent case. The corresponding results of the Shewhart $\overline{X}$ and X^2 chart combination in the independent case are given in Table 4, which indicate that if detection of small sustained shifts in μ_X and σ_X is of interest, increasing *n* significantly improves the performance of the $\overline{X}$ and X^2 chart combination. For example, this combination requires 381.7 hours on average to detect a shift in μ_X of size $0.5\sigma_{X_0}$ for concentrated sampling when $n = 2$, while the average time for this shift is 228.6 hours when $n = 4$ and only 120.9 hours when $n = 8$. But this control chart combination with a large value of *n* performs poorly in detecting large sustained shifts in μ_X and σ_X, because the points are plotted $d = n$ hours apart and an average of at least $d/2$ hours is required to detect any shift, even when the shift is very large. As a result, the choice of *n* for the Shewhart $\overline{X}$ and X^2 chart combination in order to get a satisfactory overall performance is a compromise between quick detection of both small shifts and large shifts, i.e., $n = 4$ is a reasonable choice for both concentrated sampling and dispersed sampling.

4.3.2 The Autocorrelated Case

In the autocorrelated case, however, the conclusions about the value of *n* are mixed for different levels of ψ and the sampling plans.

For concentrated sampling, when ψ is small, increasing *n* results in improved performance for small sustained shifts in μ_X and σ_X and deteriorating performance for large shifts, which is similar to the independent case. For example, when $\phi = 0.6$ and $\psi = 0.1$, the Shewhart $\overline{X}$ and X^2 chart combination requires 390.4 hours on average to detect a shift in μ_X of size $0.5\sigma_{X_0}$ for concentrated sampling when $n = 2$, while the average time for this case is 274.4 hours when $n = 4$, and only 202.3 hours when $n = 8$. It is also shown that a change in ϕ does not significantly affect the SSATS when ψ is small. For example, when $\phi = 0.2$ and $\psi = 0.1$, 273.2 hours are required on average to detect a shift in μ_X of size $0.5\sigma_{X_0}$ for concentrated sampling when n = 4, while in the previous example, 274.4 hours are required when other conditions are the same except that ϕ = 0.6 and ψ = 0.1.

However, when ψ is large, increasing n always results in deteriorating performance. For example, when ϕ = 0.2 and ψ = 0.9, the Shewhart $\overline{X}$ and X^2 chart combination requires 547.9 hours on average to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$ for concentrated sampling when n = 2, however, the average time for this case when n = 8 increases to 641.6 hours. If detecting a shift in σ_X of size $1.4\sigma_{X0}$ due to σ_α when ϕ = 0.2 and ψ = 0.9 is of interest, an average of 83.2 hours is required for n = 2, and 149.7 hours for n = 8. Similarly, if the shift in σ_X of size $1.4\sigma_{X_0}$ is due to σ_ε when ϕ = 0.2 and ψ = 0.9, then an average of 172.6 hours is required when n = 2, and 547.4 hours when n = 8.

Based on above discussion, n = 2 seems to be a reasonable sample size for concentrated sampling due to its overall best performance.

For dispersed sampling, the performance of the Shewhart $\overline{X}$ and X^2 chart combination is the same as it in the independent case, i.e., increasing n improves the performance for small sustained shifts in μ_X and σ_X, and hurts the performance for large shifts, which means n = 4 is a reasonable sample size.

For example, when ϕ = 0.2 and ψ = 0.9, the Shewhart $\overline{X}$ and X^2 chart combination requires 399.0 hours on average to detect a shift in μ_X of size $0.5\sigma_{X_0}$ for dispersed sampling when n = 2, while the average time for this case when n = 8 reduces to 178.0 hours. If the interest is in detecting a shift in μ_X of size $5\sigma_{X_0}$ when ϕ = 0.2 and ψ = 0.9, it takes an average 1.1 hours when n = 2, and 4.2 hours when n = 8.

4.3.3 Choice of the Sample Size When $n > 1$

The comparison of $n = 2$, 4, and 8 shows that $n = 4$ is a reasonable sample size value for dispersed sampling when $n > 1$, which also corresponds to the choice of the sample size in the independent case. However, for concentrated sampling, $n = 2$ gives the overall best performance when $n > 1$. Thus in the comparisons in Section 4.4, $n = 2$ is chosen for the Shewhart $\overline{X}$ and X^2 chart combination with concentrated sampling in the autocorrelated case, and $n = 4$ is used for the $\overline{X}$ and X^2 chart combination with dispersed sampling in the autocorrelated case, along with both independent cases.

4.4 Efficient Sampling Plan for Shewhart Charts of the Observations for Sustained Shifts

Based on the conclusions of Sections 4.2 and 4.3, the final choice of an efficient sampling plan for the Shewhart observations chart combination in the autocorrelated case should be one of the following three chart combinations: the Shewhart X observations chart ($n = 1$), the Shewhart $\overline{X}$ and X^2 chart combination with $n = 2$ and concentrated sampling, or the Shewhart $\overline{X}$ and X^2 chart combination with $n = 4$ and dispersed sampling.

First, comparing the performance of two sampling plans of $n > 1$, Table 5 demonstrates that the conclusion that concentrated sampling gives slightly better overall performance in the independent case will no longer hold for the autocorrelated case. When there is autocorrelation in the process, dispersed sampling actually has better overall performance than concentrated sampling, except when a large shift is of interest or when the autocorrelation is very high (e.g. when $\phi = 0.8$ and $\psi = 0.9$). Because the observations charts may not be the best charts to use in highly correlated process, dispersed sampling with $n = 4$ is considered as the better choice.

Now consider the comparison of the X chart with $n = 1$ and $\overline{X}$ and X^2 chart combination with $n = 4$ and dispersed sampling. When a sustained shift in μ_X is of interest, dispersed sampling is more sensitive than individual sampling except when the shift is very large. When a sustained shifts in σ_X is of interest, dispersed sampling is better when the autocorrelation is low, while individual sampling is better when the autocorrelation is relatively high.

From the above discussion, the final choice of an efficient sampling plan for Shewhart control charts of the observations to detect sustained shift is dispersed sampling with $n = 4$ in the autocorrelated case. This conclusion is different from the conclusion solely drawn in the independent case, in which the concentrated sampling plan with $n = 4$ is considered superior. However, dispersed sampling with $n = 4$ in the dependent case is only slightly worse than concentrated sampling. Thus dispersed sampling with $n = 4$ is the best choice overall for the Shewhart chart combination of the observations to detect sustained shifts, unless there is some reason to believe the process observations are independent.

4.5 Efficient Sampling Plan for Shewhart Charts of the Observations for Transient Shifts

Sections 4.3 and 4.4 discuss efficient sampling plans to detect sustained shifts. In this section, the ability of Shewhart observations charts to detect transient shifts is under consideration. Tables 6 – 8 give the signal probabilities for various sizes of transient shifts in μ_X, σ_α and σ_ε for the Shewhart X control chart at various combinations of ϕ and ψ, where the duration of transient shifts is l = 1, 2, or 4 hours, respectively. Also, Tables 9 – 11 give the signal probabilities for various sizes of transient shifts in μ_X, σ_α and σ_ε for the Shewhart $\overline{X}$ and X^2 chart combination with n = 2, 4, and 8 at various combinations of ϕ and ψ, where the duration of transient shifts is l = 1, 2, or 4 hours, respectively. The signal probabilities are measured as the probabilities that a chart combination signals from the time the transient shift is present up to four hours after the shift has ended.

When a transient shift is of duration l = 1 hour, the probability of quickly detecting a small transient shift in μ_X and σ_X is low for all control chart combinations. When the transient is moderate or large, individual sampling (n = 1) has the overall highest detection ability, but it performs poorly in detecting small sustained shift as discovered in Section 4.4. So Shewhart $\overline{X}$ and X^2 chart combinations with concentrated and dispersed sampling are considered. Dispersed sampling is much better than concentrated sampling except for small shifts. Smaller n also produces better performance. The Shewhart $\overline{X}$ and X^2 chart combination with dispersed sampling and n = 4 only has slightly inferior performance in detecting moderate transient shifts but better performance in detecting sustained shifts than the chart combination with dispersed sampling and n = 2. Thus n = 4 will be considered as a better choice for dispersed sampling when l = 1 hours. The conclusions are the same in both the autocorrelated case and the independent case.

Now consider the case when l = 2 hours. Concentrated sampling with n = 2 is better when there is no autocorrelation or when ψ is small, while dispersed sampling with n = 4 is better when ψ is large. When a transient shift is of duration l = 4 hours, concentrated sampling with n = 4 is better when there is no autocorrelation or when ψ is small, but dispersed sampling with $n = 4$ is better when ψ is large. Thus these results show that when the transient shift duration increases, concentrated sampling becomes efficient, but dispersed sampling is almost as good, or even better when ψ is large.

As a result, it appears that dispersed sampling with $n = 4$ is the best choice overall for the Shewhart chart combination of the observations in detecting both sustained shifts and transient shifts when there is significant autocorrelation in the process. Note that there are multiple tradeoffs in sampling plan selection process, concerning sample size, shift type, shift size, the level of autocorrelation and so on. The dispersed sampling plan with $n = 4$ is not most efficient in every aspect, but it provides the most satisfactory performance overall. Also note that dispersed sampling violates the rational subgroup concept because in this case a shift is likely to occur within a sample.

Table 1: Comparisons of Numerical Results from Integral Equation and Simulation Method for Shewhart Chart X and $\overline{X}$ Chart

Performance Measure		δ_μ	ϕ = 0		ϕ = 0.2, ψ = 0.1		ϕ = 0.2, ψ = 0.9		ϕ = 0.8, ψ = 0.1		ϕ = 0.8, ψ = 0.9	
			Integral Eq.	Simulation	Integral Eq.	Simulation	Integral Eq.	Simulation	Integral Eq.	Simulation	Integral Eq.	Simulation
Individual Sampling (n = 1)	In-Control ATS	0.00	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	Out-of-Control SSATS	0.50	155.0	155.0	154.9	154.9	156.7	157.1	157.6	157.8	188.5	188.4
		1.00	43.4	43.4	43.5	43.5	44.9	45.0	45.5	45.6	66.9	66.8
		1.50	14.5	14.5	14.6	14.5	15.4	15.4	15.7	15.7	26.9	26.8
		2.00	5.8	5.8	5.9	5.9	6.3	6.3	6.4	6.4	12.0	12.0
		3.00	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	2.7	2.7
		4.00	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.8	0.8
		5.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	h_X		3.000	3.000	3.000	3.000	3.000	3.000	2.999	3.000	2.903	2.903
Concentrated Sampling (n = 4)	In-Control ATS	0.00	1481.6	1481.4	1481.6	1481.6	1481.6	1481.6	1481.6	1481.7	1481.6	1481.6
	Out-of-Control SSATS	0.50	173.7	174.0	173.6	174.0	173.6	174.0	175.2	175.4	192.6	192.8
		1.00	23.2	23.3	23.2	23.2	23.2	23.2	23.8	23.8	29.1	29.1
		1.50	6.0	6.0	6.0	6.0	6.0	6.0	6.1	6.1	7.4	7.4
		2.00	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.9	2.9
		3.00	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
		4.00	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
		5.00	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
	$h_{\overline{X}}$		3.000	3.001	3.000	3.001	3.000	3.000	3.000	3.000	2.991	2.992

Table 2: SSATS Values of Shewhart X Chart for Sustained Shifts

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0$	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
In-Control ATS		0.00	1.00	1481.6	1481.5	1481.4	1481.3	1481.5	1481.6	1481.4
Out-of-Control SSATS	μ_X	0.25	1.00	1052.3	1054.8	1056.8	1052.6	1078.5	1056.3	1112.1
		0.50	1.00	521.5	521.9	525.2	522.8	555.7	525.9	604.8
		0.75	1.00	246.8	247.5	249.5	248.5	274.3	251.2	314.7
		1.00	1.00	121.4	121.7	123.9	122.8	141.7	124.7	171.1
		1.50	1.00	34.3	34.4	35.8	35.1	44.7	36.3	59.0
		2.00	1.00	11.9	12.0	12.8	12.4	17.2	12.9	24.1
		3.00	1.00	2.4	2.4	2.6	2.5	3.9	2.6	5.1
		4.00	1.00	0.9	0.89	0.9	0.9	1.1	0.9	1.3
		5.00	1.00	0.6	0.6	0.6	0.6	0.6	0.6	0.6
		7.00	1.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5
		10.0	1.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5
		15.0	1.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	σ_α	0.00	1.20	216.5	216.2	218.7	219.5	236.6	228.6	271.1
		0.00	1.40	65.5	66.0	66.2	69.7	77.0	78.5	97.4
		0.00	1.60	29.3	29.7	29.9	33.0	36.4	40.2	49.4
		0.00	1.80	16.5	16.8	16.9	19.7	21.4	25.3	30.3
		0.00	2.00	10.7	11.0	11.0	13.3	14.4	17.9	20.9
		0.00	2.40	5.9	6.0	6.1	7.7	8.2	10.9	12.2
		0.00	3.00	3.4	3.5	3.5	4.5	4.7	6.6	7.1
		0.00	5.00	1.5	1.5	1.5	1.9	1.9	2.7	2.7
		0.00	7.00	1.1	1.1	1.1	1.3	1.3	1.8	1.7
		0.00	10.0	0.9	0.9	0.9	1.0	1.0	1.2	1.2
		0.00	15.0	0.7	0.7	0.7	0.9	0.8	0.9	0.9
	σ_ε	0.00	1.20	216.5	216.9	216.6	216.7	217.7	217.0	213.8
		0.00	1.40	65.5	65.3	65.6	65.2	65.8	65.6	65.1
		0.00	1.60	29.3	29.2	29.3	29.1	29.3	29.3	29.0
		0.00	1.80	16.5	16.5	16.5	16.6	16.5	16.5	16.1
		0.00	2.00	10.7	10.8	10.7	10.7	10.7	10.7	10.4
		0.00	2.40	5.9	5.9	5.9	5.9	5.8	5.9	5.7
		0.00	3.00	3.4	3.4	3.4	3.4	3.4	3.4	3.3
		0.00	5.00	1.5	1.5	1.5	1.5	1.5	1.5	1.5
		0.00	7.00	1.1	1.1	1.1	1.1	1.1	1.1	1.1
		0.00	10.0	0.9	0.9	0.9	0.9	0.9	0.9	0.9
		0.00	15.0	0.7	0.7	0.7	0.7	0.7	0.7	0.7
		h_X		3.400	3.400	3.399	3.400	3.384	3.400	3.338

Table 3: SSATS Values of Shewhart MR Chart and X & MR Chart Combination for Sustained Shifts

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0$		$\phi = 0.2$, $\psi = 0.1$		$\phi = 0.2$, $\psi = 0.9$		$\phi = 0.6$, $\psi = 0.1$		$\phi = 0.6$, $\psi = 0.9$	
				MR	*X & MR*	*MR*	*X & MR*	*MR*	*X & MR*	*MR*	*X & MR*	*MR*	*X & MR*
In-Control ATS		0.00	1.00	1481.6	1482.2	1481.8	1481.2	1484.0	1481.5	1481.6	1480.4	1481.6	1482.4
Out-of-Control SSATS	μ_X	0.25	1.00	1481.0	1197.2	1479.3	1192.6	1483.8	1208.8	1481.4	1200.7	1481.1	1220.8
		0.50	1.00	1480.4	709.4	1478.6	707.1	1483.2	720.4	1480.8	714.5	1479.5	750.8
		0.75	1.00	1479.1	364.6	1477.2	363.2	1481.6	374.0	1479.8	366.8	1476.2	406.3
		1.00	1.00	1476.9	181.9	1475.2	180.7	1478.9	189.0	1477.2	183.4	1469.3	213.0
		1.50	1.00	1467.8	49.3	1466.1	49.2	1467.0	51.9	1466.1	50.3	1429.5	64.0
		2.00	1.00	1446.7	16.1	1445.3	16.1	1437.0	17.4	1442.8	16.7	1340.8	22.6
		3.00	1.00	1331.4	2.9	1324.6	2.9	1271.8	3.2	1310.7	3.1	895.1	3.8
		4.00	1.00	1056.8	1.0	1043.1	1.0	908.0	1.0	1012.6	1.0	321.3	0.9
		5.00	1.00	654.9	0.6	633.6	0.6	454.9	0.6	590.5	0.6	52.2	0.5
		7.00	1.00	88.2	0.5	78.6	0.5	27.3	0.5	61.4	0.5	0.6	0.5
		10.0	1.00	0.7	0.5	0.6	0.5	0.5	0.5	0.6	0.5	0.5	0.5
		15.0	1.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	σ_α	0.00	1.20	226.9	199.6	289.8	219.4	231.7	198.0	555.7	267.8	262.0	212.3
		0.00	1.40	72.0	59.1	96.9	65.5	72.5	57.7	242.3	83.2	84.9	63.81
		0.00	1.60	33.7	26.4	44.9	29.0	33.8	25.8	121.2	37.9	38.5	28.8
		0.00	1.80	19.7	15.0	25.5	16.2	19.5	14.7	68.9	21.7	21.8	16.4
		0.00	2.00	13.2	9.9	16.7	10.5	13.0	9.6	42.9	14.3	14.2	10.8
		0.00	2.40	7.6	5.6	9.1	5.8	7.4	5.4	20.7	7.9	7.7	6.0
		0.00	3.00	4.6	3.3	5.2	3.3	4.4	3.2	9.9	4.6	4.4	3.6
		0.00	5.00	2.1	1.5	2.2	1.5	2.0	1.5	3.1	1.9	1.8	1.6
		0.00	7.00	1.4	1.1	1.5	1.1	1.4	1.1	1.9	1.3	1.3	1.2
		0.00	10.0	1.1	0.9	1.1	0.9	1.0	0.9	1.3	1.0	1.0	0.9
		0.00	15.0	0.8	0.7	0.8	0.7	0.8	0.7	0.9	0.8	0.8	0.7
	σ_ε	0.00	1.20	226.9	199.6	220.7	196.7	171.4	169.3	208.7	189.8	70.2	84.0
		0.00	1.40	72.0	59.1	69.7	57.9	52.2	49.0	65.0	55.8	21.2	23.7
		0.00	1.60	33.7	26.4	32.6	26.0	24.8	22.2	30.5	25.2	11.0	11.7
		0.00	1.80	19.7	15.0	19.1	14.8	14.9	12.9	18.1	14.4	7.3	7.5
		0.00	2.00	13.2	9.9	12.9	9.7	10.3	8.6	12.2	9.5	5.5	5.5
		0.00	2.40	7.6	5.6	7.5	5.5	6.2	5.0	7.2	5.4	3.7	3.5
		0.00	3.00	4.6	3.3	4.5	3.2	3.9	3.1	4.3	3.2	2.5	2.4
		0.00	5.00	2.1	1.5	2.1	1.5	1.9	1.5	2.0	1.5	1.4	1.3
		0.00	7.00	1.4	1.1	1.4	1.1	1.3	1.1	1.4	1.1	1.0	1.0
		0.00	10.0	1.1	0.9	1.1	0.9	1.0	0.9	1.0	0.9	0.8	0.8
		0.00	15.0	0.8	0.7	0.8	0.7	0.8	0.7	0.8	0.7	0.7	0.7
		h_X		---	3.555	---	3.555	---	3.566	---	3.559	---	3.565
		h_{MR}		4.793	5.015	4.746	4.967	4.347	4.560	4.648	4.869	3.259	3.430

Table 4: SSATS Values of Shewhart $\overline{X}$ & X^2 Chart Combination for Sustained Shifts in Independent Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.5		1482.1		1481.5	
Out-of-Control SSATS	μ_X	0.25	1.00	938.4	931.7	742.2	741.9	533.1	535.8
		0.50	1.00	381.7	384.7	228.6	231.4	120.9	126.0
		0.75	1.00	153.5	154.6	76.3	78.4	35.5	40.7
		1.00	1.00	66.3	67.1	29.7	32.0	13.9	19.0
		1.50	1.00	16.0	16.9	7.1	9.4	4.9	9.3
		2.00	1.00	5.4	6.2	3.0	5.0	4.0	7.1
		3.00	1.00	1.4	2.1	2.0	3.1	4.0	5.4
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0
	σ_X	0.00	1.20	203.6	204.1	189.1	191.4	165.2	169.5
		0.00	1.40	60.3	60.7	53.6	55.3	44.0	48.1
		0.00	1.60	26.7	27.2	23.4	25.2	19.3	23.4
		0.00	1.80	15.0	15.6	13.2	15.0	11.2	15.2
		0.00	2.00	9.8	10.4	8.7	10.5	7.8	11.5
		0.00	2.40	5.5	6.0	5.1	6.6	5.3	8.4
		0.00	3.00	3.2	3.7	3.3	4.6	4.3	6.7
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3
		$h_{\overline{X}}$		3.343		3.177		2.983	
		h_{X^2}		6.099		4.812		3.865	

Table 5: SSATS Values of Shewhart $\bar{X}$ & X^2 Chart Combination for Sustained Shifts in Autocorrelated Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$, $\psi = 0.1$						$\phi = 0.2$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.1	1481.5	1481.8	1482.4	1481.7	1481.4	1481.6	1482.0	1481.9	1481.3	1481.7	1480.9
Out-of-Control SSATS	μ_X	0.25	1.00	935.5	937.6	795.2	752.9	675.4	551.7	1071.1	942.7	1101.0	801.7	1134.7	637.5
		0.50	1.00	389.9	386.0	273.2	237.5	202.1	132.4	547.9	399.0	587.5	280.2	641.6	178.0
		0.75	1.00	160.7	156.5	99.9	81.8	70.2	43.0	268.1	169.2	299.6	105.0	343.1	61.3
		1.00	1.00	71.3	68.5	41.6	33.6	29.3	20.0	136.0	77.4	158.2	45.4	189.8	27.8
		1.50	1.00	18.0	17.4	10.5	9.8	8.5	9.5	41.2	21.0	51.5	13.2	67.0	11.3
		2.00	1.00	6.2	6.4	4.1	5.2	4.6	7.2	15.3	7.9	20.4	6.3	28.5	7.8
		3.00	1.00	1.6	2.1	2.1	3.1	4.0	5.4	3.5	2.4	5.3	3.3	8.5	5.6
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.5	1.4	2.6	2.5	4.7	4.7
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.1	1.1	2.1	2.1	4.1	4.2
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	132.0	192.3	87.5	173.9	74.5	156.7	247.7	217.2	294.0	206.9	355.2	184.3
		0.00	1.40	40.4	57.7	31.3	51.9	32.2	46.3	83.2	66.0	110.5	61.4	149.7	53.4
		0.00	1.60	20.2	26.5	18.1	24.4	21.1	23.1	40.2	29.8	57.6	28.0	84.1	25.6
		0.00	1.80	12.8	15.5	12.8	14.8	16.2	15.2	23.9	17.0	36.2	16.5	55.5	16.4
		0.00	2.00	9.2	10.4	10.0	10.5	13.4	11.6	16.3	11.3	25.6	11.4	40.6	12.3
		0.00	2.40	5.9	6.1	7.2	6.7	10.5	8.5	9.5	6.5	15.7	7.1	26.3	8.8
		0.00	3.00	3.9	3.8	5.4	4.7	8.5	6.7	5.8	4.0	10.0	4.9	17.4	6.9
		0.00	5.00	2.2	2.0	3.6	3.0	6.2	5.1	2.8	2.1	5.1	3.1	9.5	5.2
		0.00	7.00	1.8	1.6	3.0	2.6	5.4	4.7	2.1	1.6	3.9	2.7	7.4	4.7
		0.00	10.0	1.5	1.4	2.7	2.4	4.9	4.4	1.6	1.4	3.2	2.4	6.1	4.4
		0.00	15.0	1.3	1.2	2.4	2.2	4.6	4.3	1.4	1.2	2.7	2.3	5.3	4.3
	σ_ε	0.00	1.20	217.9	207.0	219.8	193.6	210.2	171.4	459.2	235.7	729.9	231.3	925.8	203.2
		0.00	1.40	63.9	61.3	60.8	55.8	52.8	48.5	172.6	69.6	356.3	65.6	547.4	55.5
		0.00	1.60	28.0	27.4	25.8	25.4	21.9	23.5	75.9	30.5	175.5	28.7	309.2	25.8
		0.00	1.80	15.7	15.7	14.3	15.0	12.3	15.2	38.2	17.1	87.4	16.5	165.2	16.2
		0.00	2.00	10.1	10.4	9.3	10.5	8.4	11.5	22.1	11.2	45.9	11.3	84.8	12.1
		0.00	2.40	5.6	6.0	5.3	6.7	5.5	8.4	10.1	6.3	17.1	7.0	26.2	8.7
		0.00	3.00	3.3	3.8	3.4	4.6	4.4	6.7	5.0	3.9	7.1	4.8	9.4	6.8
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	2.0	2.0	2.7	3.1	4.2	5.1
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.4	1.6	2.2	2.6	4.0	4.7
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.2	1.4	2.1	2.4	4.0	4.4
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	4.0	1.2	2.0	2.2	4.0	4.3
		$h_{\bar{X}}$, h_{X^2}		3.473 7.073	3.371 7.082	3.576 4.448	3.229 4.393	3.818 3.022	3.045 2.931	4.424 9.839	3.571 7.158	5.779 8.427	3.590 4.519	7.528 7.172	3.478 3.019

Table 5: SSATS Values of Shewhart $\bar{X}$ & X^2 Chart Combination for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.1						ϕ = 0.6, ψ = 0.9					
				n = 2		n = 4		n = 8		n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.7	1481.1	1481.3	1480.7	1482.0	1481.2	1481.6	1480.2	1481.9	1481.5	1481.5	1481.7
Out-of-Control SSATS	μ_X	0.25	1.00	936.1	935.3	796.1	773.2	675.1	596.2	1022.0	1023.0	1102.3	953.6	1133.5	861.8
		0.50	1.00	390.4	388.3	274.4	255.2	202.3	155.7	492.0	495.2	591.5	420.8	641.7	338.7
		0.75	1.00	161.9	160.0	100.9	91.4	70.4	52.3	232.6	233.7	303.5	188.7	343.5	143.0
		1.00	1.00	72.3	71.3	42.3	38.5	29.4	23.9	117.1	117.2	161.6	92.6	190.2	69.1
		1.50	1.00	18.6	18.7	10.7	11.2	8.5	10.3	36.6	36.5	53.7	29.2	67.3	23.7
		2.00	1.00	6.4	6.9	4.1	5.6	4.6	7.4	14.5	14.5	21.6	12.5	28.7	12.2
		3.00	1.00	1.6	2.2	2.1	3.2	4.0	5.4	3.8	3.8	5.6	4.6	8.5	6.7
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.7	1.7	2.6	2.9	4.7	5.2
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.2	1.2	2.1	2.3	4.1	4.5
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	134.1	165.1	87.9	132.5	74.6	119.6	254.4	254.7	294.8	273.3	354.8	273.6
		0.00	1.40	42.3	52.8	31.6	44.7	32.2	42.2	87.5	87.4	111.3	97.0	149.6	95.4
		0.00	1.60	21.6	26.6	18.3	24.0	21.1	23.9	43.0	43.0	58.1	48.3	84.1	47.5
		0.00	1.80	13.8	16.7	12.9	15.9	16.2	16.7	25.9	25.9	36.6	29.3	55.5	29.3
		0.00	2.00	10.0	11.9	10.1	11.9	13.4	13.1	17.6	17.6	25.9	20.1	40.6	20.7
		0.00	2.40	6.4	7.4	7.3	8.0	10.5	9.7	10.2	10.2	15.9	11.9	26.3	13.3
		0.00	3.00	4.3	4.7	5.5	5.6	8.5	7.5	6.0	6.0	10.1	7.4	17.4	9.2
		0.00	5.00	2.3	2.4	3.6	3.4	6.2	5.5	2.7	2.7	5.1	3.9	9.5	6.0
		0.00	7.00	1.8	1.8	3.0	2.8	5.4	4.9	1.9	2.0	3.9	3.1	7.4	5.2
		0.00	10.0	1.5	1.5	2.7	2.5	4.9	4.6	1.6	1.6	3.2	2.6	6.1	4.7
		0.00	15.0	1.3	1.3	2.4	2.3	4.6	4.3	1.3	1.3	2.7	2.4	5.3	4.4
	σ_ε	0.00	1.20	217.7	211.9	219.7	202.5	210.0	179.6	357.0	357.3	727.9	467.2	924.9	472.1
		0.00	1.40	63.8	62.7	60.7	57.9	52.8	50.0	115.5	115.0	355.8	154.7	546.6	141.8
		0.00	1.60	28.0	27.9	25.8	26.1	21.9	23.9	48.2	48.4	175.5	61.0	308.8	53.9
		0.00	1.80	15.6	15.9	14.3	15.3	12.3	15.4	25.1	25.1	87.2	30.3	164.9	27.7
		0.00	2.00	10.1	10.6	9.3	10.6	8.4	11.6	15.4	15.4	45.9	18.2	84.7	17.9
		0.00	2.40	5.6	6.1	5.3	6.7	5.5	8.5	8.0	8.0	17.1	9.6	26.2	11.0
		0.00	3.00	3.3	3.8	3.4	4.7	4.4	6.7	4.5	4.5	7.1	5.8	9.4	7.8
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	2.2	2.2	2.7	3.3	4.2	5.5
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.7	1.7	2.2	2.8	4.0	4.9
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.4	1.4	2.1	2.5	4.0	4.5
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.3	1.3	2.0	2.3	4.0	4.3
		$h_{\bar{X}}$, h_{X^2}		3.473 7.073	3.472 7.068	3.576 4.448	3.370 4.395	3.818 3.021	3.266 2.937	4.418 9.813	3.996 8.262	5.777 8.419	4.549 5.753	7.527 7.170	4.871 3.883

Table 5: SSATS Values of Shewhart $\bar{X}$ & X^2 Chart Combinations for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8,\ \psi = 0.1$						$\phi = 0.8,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.9	1481.1	1482.0	1481.6	1481.3	1481.9	1481.7	1481.6	1481.6	1481.7	1481.9	1480.4
Out-of-Control SSATS	μ_X	0.25	1.00	939.1	938.6	798.8	787.4	680.1	636.7	1110.2	1088.3	1118.1	1059.0	1143.8	1010.7
		0.50	1.00	393.8	393.0	278.4	269.0	205.7	180.3	603.2	576.9	616.5	540.4	652.7	486.7
		0.75	1.00	164.6	164.1	104.0	100.0	72.4	63.4	314.9	295.2	325.4	270.8	353.2	236.4
		1.00	1.00	74.6	74.4	44.3	43.3	30.5	28.9	171.3	158.9	178.2	144.3	197.7	124.6
		1.50	1.00	19.6	20.0	11.4	12.6	8.7	11.5	59.2	54.7	62.6	50.1	71.9	44.5
		2.00	1.00	6.8	7.4	4.3	6.0	4.7	7.7	24.3	22.7	26.2	21.5	31.2	21.0
		3.00	1.00	1.6	2.2	2.1	3.2	4.0	5.5	5.4	5.5	6.5	6.5	9.0	8.6
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.7	2.0	2.7	3.4	4.7	5.9
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.1	1.2	2.1	2.6	4.1	5.0
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.1
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	142.2	158.6	92.6	118.4	75.9	104.0	279.7	287.7	307.6	311.9	358.8	338.5
		0.00	1.40	48.6	55.6	34.6	45.0	32.8	43.0	103.6	108.1	119.9	122.8	151.8	137.9
		0.00	1.60	26.2	30.3	20.3	26.5	21.5	26.9	53.5	56.5	64.1	66.2	85.8	75.3
		0.00	1.80	17.4	20.2	14.4	18.8	16.4	19.8	33.3	35.6	40.9	42.6	56.7	48.8
		0.00	2.00	12.8	15.0	11.2	14.6	13.6	15.9	23.3	25.0	29.1	30.5	41.5	35.1
		0.00	2.40	8.3	9.8	8.0	10.2	10.6	11.9	13.8	15.1	17.9	18.8	26.8	22.0
		0.00	3.00	5.4	6.5	5.9	7.2	8.5	9.1	8.3	9.2	11.3	11.7	17.7	14.3
		0.00	5.00	2.8	3.1	3.7	4.1	6.2	6.2	3.6	3.9	5.5	5.4	9.6	7.8
		0.00	7.00	2.1	2.2	3.1	3.3	5.5	5.3	2.5	2.6	4.1	3.9	7.4	6.2
		0.00	10.0	1.7	1.7	2.7	2.8	5.0	4.8	1.9	1.9	3.3	3.1	6.2	5.3
		0.00	15.0	1.4	1.4	2.5	2.4	4.6	4.5	1.5	1.5	2.8	2.6	5.3	4.7
	σ_ε	0.00	1.20	217.6	214.5	219.6	209.8	210.3	188.7	459.1	405.9	731.5	609.7	927.5	723.0
		0.00	1.40	63.8	63.5	60.6	59.7	52.8	51.8	173.3	142.3	359.1	250.1	549.2	305.9
		0.00	1.60	28.0	28.3	25.8	26.6	21.9	24.5	75.7	60.2	177.1	105.4	309.9	123.7
		0.00	1.80	15.6	16.0	14.2	15.6	12.3	15.6	37.8	30.5	87.6	49.7	166.1	55.6
		0.00	2.00	10.1	10.6	9.3	10.8	8.4	11.8	21.7	18.1	45.9	27.5	84.8	30.5
		0.00	2.40	5.6	6.1	5.3	6.8	5.5	8.5	9.9	8.9	17.0	12.6	26.3	15.0
		0.00	3.00	3.3	3.8	3.4	4.7	4.4	6.7	4.9	4.9	7.1	6.9	9.4	9.3
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	1.9	2.3	2.7	3.5	4.2	5.8
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.4	1.7	2.2	2.9	4.0	5.1
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.2	1.4	2.1	2.5	4.0	4.7
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.1	1.3	2.0	2.3	4.0	4.4
		$h_{\bar{X}}$, h_{X^2}		3.472 7.065	3.446 7.059	3.574 4.445	3.4623 4.409	3.818 3.021	3.478 2.952	4.379 9.642	4.158 8.797	5.759 8.366	5.091 6.795	7.524 7.166	5.938 4.945

Table 6: Signal Probabilities of Shewhart X Chart for Transient Shifts when l = 1 hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0$	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		2.00	1.00	0.08	0.08	0.08	0.08	0.09	0.08	0.09
		3.00	1.00	0.35	0.35	0.35	0.34	0.35	0.35	0.37
		4.00	1.00	0.73	0.73	0.73	0.72	0.73	0.73	0.75
		5.00	1.00	0.95	0.95	0.95	0.94	0.95	0.95	0.95
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.02	0.02	0.01	0.01	0.01	0.01
		0.00	1.60	0.04	0.03	0.03	0.02	0.02	0.01	0.01
		0.00	1.80	0.06	0.06	0.06	0.04	0.04	0.02	0.02
		0.00	2.00	0.09	0.09	0.09	0.06	0.05	0.03	0.03
		0.00	2.40	0.16	0.15	0.15	0.10	0.10	0.06	0.05
		0.00	3.00	0.26	0.25	0.25	0.18	0.18	0.12	0.11
		0.00	5.00	0.50	0.49	0.49	0.42	0.41	0.33	0.30
		0.00	7.00	0.63	0.62	0.62	0.56	0.55	0.48	0.45
		0.00	10.0	0.73	0.73	0.73	0.68	0.68	0.62	0.60
		0.00	15.0	0.82	0.82	0.82	0.79	0.78	0.74	0.72
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.02	0.02	0.02	0.02	0.02	0.02
		0.00	1.60	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		0.00	1.80	0.06	0.06	0.06	0.06	0.06	0.06	0.07
		0.00	2.00	0.09	0.09	0.09	0.09	0.09	0.09	0.10
		0.00	2.40	0.16	0.16	0.16	0.16	0.16	0.16	0.17
		0.00	3.00	0.26	0.26	0.26	0.26	0.26	0.26	0.27
		0.00	5.00	0.50	0.50	0.50	0.50	0.50	0.50	0.51
		0.00	7.00	0.63	0.63	0.63	0.63	0.63	0.63	0.63
		0.00	10.0	0.73	0.73	0.73	0.73	0.74	0.73	0.74
		0.00	15.0	0.82	0.82	0.82	0.82	0.82	0.82	0.82
		h_X		3.400	3.400	3.399	3.400	3.384	3.400	3.338

Table 7: Signal Probabilities of Shewhart X Chart for Transient Shifts when l = 2 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0$	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.02	0.02	0.02	0.02	0.02	0.02	0.02
		1.50	1.00	0.06	0.06	0.06	0.06	0.06	0.06	0.06
		2.00	1.00	0.16	0.16	0.15	0.15	0.14	0.16	0.14
		3.00	1.00	0.57	0.57	0.55	0.56	0.50	0.56	0.48
		4.00	1.00	0.92	0.92	0.90	0.92	0.86	0.92	0.84
		5.00	1.00	1.00	1.00	0.99	1.00	0.99	1.00	0.98
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.03	0.03	0.03	0.02	0.02	0.02	0.02
		0.00	1.60	0.07	0.07	0.07	0.05	0.05	0.03	0.03
		0.00	1.80	0.12	0.11	0.11	0.08	0.08	0.05	0.05
		0.00	2.00	0.17	0.17	0.17	0.12	0.12	0.08	0.07
		0.00	2.40	0.29	0.28	0.28	0.21	0.20	0.15	0.13
		0.00	3.00	0.45	0.44	0.44	0.35	0.34	0.25	0.23
		0.00	5.00	0.75	0.74	0.74	0.66	0.66	0.55	0.52
		0.00	7.00	0.86	0.86	0.86	0.80	0.80	0.72	0.70
		0.00	10.0	0.93	0.93	0.93	0.90	0.90	0.84	0.83
		0.00	15.0	0.97	0.97	0.97	0.95	0.95	0.93	0.92
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		0.00	1.60	0.07	0.07	0.07	0.07	0.07	0.07	0.07
		0.00	1.80	0.12	0.12	0.12	0.12	0.12	0.12	0.12
		0.00	2.00	0.17	0.17	0.17	0.17	0.17	0.17	0.18
		0.00	2.40	0.29	0.29	0.29	0.29	0.29	0.29	0.30
		0.00	3.00	0.45	0.45	0.45	0.45	0.45	0.45	0.46
		0.00	5.00	0.75	0.75	0.75	0.75	0.75	0.75	0.76
		0.00	7.00	0.86	0.86	0.86	0.86	0.86	0.86	0.87
		0.00	10.0	0.93	0.93	0.93	0.93	0.93	0.93	0.93
		0.00	15.0	0.97	0.97	0.97	0.97	0.97	0.97	0.97
		h_X		3.400	3.400	3.399	3.400	3.384	3.400	3.338

Table 8: Signal Probabilities of Shewhart X Chart for Transient Shifts when l = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0$	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.02	0.02	0.02	0.02	0.02	0.02	0.02
		1.00	1.00	0.04	0.03	0.03	0.03	0.03	0.03	0.03
		1.50	1.00	0.11	0.11	0.11	0.11	0.10	0.11	0.09
		2.00	1.00	0.29	0.29	0.28	0.28	0.24	0.28	0.21
		3.00	1.00	0.82	0.81	0.78	0.80	0.68	0.79	0.62
		4.00	1.00	0.99	0.99	0.99	0.99	0.96	0.99	0.93
		5.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.02
		0.00	1.40	0.06	0.06	0.06	0.05	0.05	0.04	0.03
		0.00	1.60	0.13	0.13	0.13	0.10	0.10	0.08	0.07
		0.00	1.80	0.22	0.21	0.21	0.17	0.16	0.12	0.11
		0.00	2.00	0.31	0.31	0.30	0.25	0.23	0.18	0.15
		0.00	2.40	0.49	0.49	0.48	0.40	0.38	0.30	0.26
		0.00	3.00	0.70	0.69	0.68	0.59	0.58	0.46	0.43
		0.00	5.00	0.94	0.93	0.93	0.89	0.88	0.80	0.78
		0.00	7.00	0.98	0.98	0.98	0.96	0.96	0.92	0.91
		0.00	10.0	0.99	0.99	0.99	0.99	0.99	0.97	0.97
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	0.99	0.99
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.02
		0.00	1.40	0.06	0.06	0.06	0.06	0.06	0.06	0.06
		0.00	1.60	0.13	0.13	0.13	0.13	0.13	0.13	0.14
		0.00	1.80	0.22	0.22	0.22	0.22	0.22	0.22	0.23
		0.00	2.00	0.31	0.31	0.31	0.31	0.31	0.31	0.32
		0.00	2.40	0.49	0.50	0.50	0.49	0.50	0.50	0.51
		0.00	3.00	0.70	0.70	0.70	0.69	0.70	0.70	0.71
		0.00	5.00	0.94	0.94	0.94	0.94	0.94	0.94	0.94
		0.00	7.00	0.98	0.98	0.98	0.98	0.98	0.98	0.98
		0.00	10.0	0.99	0.99	1.00	0.99	1.00	0.99	1.00
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		h_X		3.400	3.400	3.399	3.400	3.384	3.400	3.338

Table 9: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$						$\phi = 0.2,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.02	0.02	0.02	0.02	0.01	0.01
		2.00	1.00	0.14	0.07	0.16	0.05	0.12	0.03	0.06	0.06	0.05	0.05	0.03	0.02
		3.00	1.00	0.39	0.28	0.25	0.23	0.12	0.12	0.23	0.27	0.14	0.21	0.08	0.10
		4.00	1.00	0.49	0.65	0.25	0.58	0.13	0.31	0.41	0.64	0.22	0.55	0.12	0.28
		5.00	1.00	0.50	0.91	0.25	0.87	0.13	0.51	0.49	0.91	0.25	0.86	0.12	0.49
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	1.00	0.13	0.62
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.03	0.02	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.60	0.05	0.03	0.05	0.03	0.04	0.02	0.03	0.03	0.02	0.02	0.01	0.01
		0.00	1.80	0.07	0.05	0.07	0.04	0.05	0.02	0.04	0.05	0.03	0.04	0.02	0.02
		0.00	2.00	0.10	0.08	0.08	0.07	0.06	0.04	0.06	0.07	0.04	0.06	0.02	0.03
		0.00	2.40	0.15	0.14	0.11	0.12	0.07	0.07	0.10	0.13	0.06	0.11	0.03	0.06
		0.00	3.00	0.20	0.23	0.14	0.21	0.08	0.12	0.15	0.23	0.08	0.20	0.05	0.11
		0.00	5.00	0.31	0.47	0.18	0.45	0.10	0.27	0.27	0.46	0.14	0.44	0.07	0.26
		0.00	7.00	0.36	0.61	0.20	0.59	0.11	0.36	0.33	0.60	0.17	0.58	0.09	0.35
		0.00	10.0	0.40	0.72	0.21	0.71	0.11	0.43	0.38	0.71	0.19	0.70	0.10	0.43
		0.00	15.0	0.44	0.81	0.23	0.80	0.12	0.49	0.42	0.81	0.21	0.80	0.11	0.49
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.60	0.04	0.03	0.04	0.03	0.04	0.01	0.01	0.03	0.01	0.02	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.03	0.05	0.01	0.04	0.01	0.02
		0.00	2.00	0.09	0.08	0.09	0.07	0.08	0.04	0.04	0.07	0.02	0.06	0.01	0.03
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.07	0.09	0.13	0.05	0.11	0.03	0.06
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.17	0.23	0.11	0.20	0.07	0.11
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.27	0.34	0.47	0.21	0.44	0.12	0.26
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.36	0.41	0.60	0.24	0.58	0.12	0.35
		0.00	10.0	0.47	0.72	0.25	0.71	0.13	0.43	0.45	0.72	0.25	0.70	0.12	0.43
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.48	0.81	0.25	0.80	0.13	0.49
		$h_{\bar{X}}$, h_{X^2}		3.473 7.073	3.371 7.082	3.576 4.448	3.229 4.393	3.818 3.022	3.045 2.931	4.424 9.839	3.571 7.158	5.779 8.427	3.590 4.519	7.528 7.172	3.478 3.019

Table 9: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.1$						$\phi = 0.6,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.03	0.02	0.02	0.01	0.01	0.01
		2.00	1.00	0.14	0.06	0.16	0.05	0.12	0.03	0.06	0.04	0.05	0.02	0.03	0.01
		3.00	1.00	0.39	0.28	0.25	0.23	0.12	0.11	0.23	0.17	0.14	0.09	0.08	0.04
		4.00	1.00	0.49	0.65	0.25	0.57	0.13	0.30	0.41	0.51	0.22	0.31	0.12	0.12
		5.00	1.00	0.50	0.91	0.25	0.87	0.13	0.51	0.49	0.86	0.25	0.69	0.12	0.32
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	1.00	0.13	0.61
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.01	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.60	0.04	0.03	0.05	0.03	0.04	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.80	0.06	0.04	0.07	0.04	0.05	0.02	0.04	0.03	0.03	0.02	0.02	0.01
		0.00	2.00	0.09	0.06	0.08	0.07	0.06	0.03	0.05	0.04	0.04	0.03	0.02	0.02
		0.00	2.40	0.13	0.12	0.11	0.12	0.07	0.06	0.09	0.08	0.06	0.06	0.03	0.03
		0.00	3.00	0.19	0.20	0.13	0.20	0.08	0.11	0.14	0.15	0.08	0.13	0.05	0.06
		0.00	5.00	0.30	0.44	0.18	0.44	0.10	0.25	0.25	0.37	0.14	0.34	0.07	0.19
		0.00	7.00	0.36	0.58	0.20	0.58	0.11	0.34	0.32	0.52	0.17	0.49	0.09	0.28
		0.00	10.0	0.40	0.70	0.21	0.70	0.11	0.42	0.37	0.65	0.19	0.63	0.10	0.37
		0.00	15.0	0.43	0.79	0.23	0.79	0.12	0.49	0.41	0.76	0.21	0.75	0.11	0.45
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.60	0.04	0.03	0.04	0.03	0.04	0.01	0.01	0.02	0.01	0.01	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.03	0.03	0.01	0.02	0.01	0.01
		0.00	2.00	0.09	0.08	0.09	0.07	0.08	0.04	0.04	0.05	0.02	0.03	0.01	0.02
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.07	0.09	0.11	0.05	0.07	0.03	0.03
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.17	0.19	0.11	0.14	0.07	0.07
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.27	0.34	0.43	0.21	0.37	0.12	0.21
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.36	0.41	0.57	0.24	0.53	0.12	0.30
		0.00	10.0	0.47	0.72	0.25	0.71	0.13	0.43	0.45	0.69	0.25	0.66	0.12	0.39
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.48	0.79	0.25	0.77	0.13	0.47
		$h_{\bar{X}}$, h_{X^2}		3.473	3.472	3.576	3.370	3.818	3.266	4.418	3.996	5.777	4.549	7.527	4.871
				7.073	7.068	4.448	4.395	3.021	2.937	9.813	8.262	8.419	5.753	7.170	3.883

Table 9: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8,\ \psi = 0.1$						$\phi = 0.8,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.03	0.01	0.02	0.01	0.01	0.01
		2.00	1.00	0.14	0.06	0.16	0.05	0.12	0.03	0.07	0.03	0.05	0.01	0.03	0.01
		3.00	1.00	0.39	0.28	0.25	0.22	0.12	0.11	0.23	0.14	0.14	0.05	0.08	0.02
		4.00	1.00	0.49	0.65	0.25	0.57	0.13	0.30	0.41	0.45	0.22	0.18	0.12	0.05
		5.00	1.00	0.50	0.91	0.25	0.87	0.13	0.50	0.49	0.82	0.25	0.49	0.12	0.14
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	0.99	0.13	0.57
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.01	0.01	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		0.00	1.60	0.03	0.02	0.04	0.02	0.04	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.80	0.05	0.03	0.06	0.04	0.05	0.02	0.02	0.02	0.02	0.01	0.02	0.01
		0.00	2.00	0.06	0.05	0.08	0.05	0.06	0.03	0.03	0.03	0.03	0.02	0.02	0.01
		0.00	2.40	0.10	0.08	0.10	0.09	0.07	0.04	0.06	0.05	0.05	0.04	0.03	0.01
		0.00	3.00	0.16	0.16	0.13	0.17	0.08	0.08	0.10	0.09	0.08	0.07	0.05	0.03
		0.00	5.00	0.27	0.38	0.17	0.39	0.10	0.21	0.22	0.27	0.13	0.24	0.07	0.11
		0.00	7.00	0.33	0.52	0.20	0.54	0.11	0.30	0.29	0.42	0.16	0.39	0.09	0.20
		0.00	10.0	0.38	0.65	0.21	0.66	0.11	0.39	0.35	0.57	0.19	0.54	0.10	0.30
		0.00	15.0	0.42	0.76	0.22	0.77	0.12	0.46	0.40	0.70	0.21	0.68	0.11	0.39
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.60	0.04	0.03	0.04	0.02	0.04	0.01	0.01	0.02	0.01	0.01	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.03	0.03	0.01	0.01	0.01	0.01
		0.00	2.00	0.09	0.08	0.09	0.06	0.08	0.03	0.05	0.05	0.02	0.02	0.01	0.01
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.06	0.09	0.09	0.05	0.05	0.03	0.02
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.17	0.18	0.11	0.11	0.07	0.04
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.26	0.34	0.42	0.21	0.33	0.12	0.16
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.35	0.41	0.56	0.24	0.48	0.12	0.26
		0.00	10.0	0.47	0.72	0.25	0.70	0.13	0.43	0.45	0.68	0.25	0.62	0.12	0.36
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.48	0.79	0.25	0.74	0.13	0.44
		$h_{\bar{X}}$, h_{X^2}		3.472 7.065	3.446 7.059	3.574 4.445	3.4623 4.409	3.818 3.021	3.478 2.952	4.379 9.642	4.158 8.797	5.759 8.366	5.091 6.795	7.524 7.166	5.938 4.945

Table 10: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 2 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$						$\phi = 0.2,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.75	1.00	0.01	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.02	0.02	0.02	0.01	0.02	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.05	0.07	0.04	0.06	0.03	0.04
		2.00	1.00	0.28	0.22	0.33	0.19	0.23	0.11	0.13	0.19	0.09	0.16	0.06	0.10
		3.00	1.00	0.78	0.65	0.49	0.62	0.25	0.39	0.45	0.59	0.27	0.56	0.16	0.36
		4.00	1.00	0.98	0.93	0.50	0.93	0.25	0.64	0.82	0.91	0.44	0.90	0.23	0.62
		5.00	1.00	1.00	1.00	0.50	1.00	0.25	0.72	0.97	0.99	0.49	0.99	0.25	0.72
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.05	0.03	0.06	0.03	0.06	0.02	0.03	0.03	0.02	0.03	0.01	0.02
		0.00	1.60	0.10	0.07	0.10	0.06	0.08	0.04	0.05	0.06	0.03	0.05	0.02	0.03
		0.00	1.80	0.15	0.11	0.14	0.10	0.10	0.06	0.08	0.10	0.05	0.09	0.03	0.05
		0.00	2.00	0.20	0.17	0.17	0.15	0.11	0.09	0.12	0.16	0.07	0.14	0.04	0.08
		0.00	2.40	0.29	0.28	0.22	0.26	0.14	0.16	0.19	0.27	0.11	0.24	0.07	0.15
		0.00	3.00	0.41	0.44	0.27	0.42	0.16	0.27	0.30	0.43	0.17	0.40	0.09	0.25
		0.00	5.00	0.62	0.74	0.36	0.72	0.20	0.49	0.53	0.73	0.28	0.71	0.15	0.48
		0.00	7.00	0.73	0.86	0.40	0.85	0.21	0.59	0.66	0.85	0.34	0.84	0.18	0.59
		0.00	10.0	0.81	0.93	0.43	0.92	0.22	0.66	0.76	0.92	0.39	0.92	0.20	0.65
		0.00	15.0	0.87	0.97	0.45	0.96	0.23	0.70	0.83	0.97	0.42	0.96	0.21	0.70
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.01	0.03	0.01	0.02	0.00	0.02
		0.00	1.60	0.07	0.07	0.07	0.06	0.08	0.03	0.03	0.06	0.01	0.05	0.01	0.03
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.05	0.10	0.02	0.09	0.01	0.05
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.09	0.16	0.04	0.14	0.02	0.08
		0.00	2.40	0.30	0.28	0.27	0.26	0.21	0.16	0.18	0.27	0.11	0.25	0.07	0.15
		0.00	3.00	0.47	0.44	0.37	0.42	0.24	0.27	0.34	0.43	0.22	0.40	0.15	0.26
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.50	0.68	0.74	0.43	0.72	0.24	0.49
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.60	0.82	0.86	0.48	0.84	0.25	0.59
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.91	0.93	0.49	0.92	0.25	0.66
		0.00	15.0	0.97	0.97	0.50	0.97	0.25	0.70	0.96	0.97	0.50	0.96	0.25	0.70
		$h_{\bar{X}}$, h_{X^2}		3.473 7.073	3.371 7.082	3.576 4.448	3.229 4.393	3.818 3.022	3.045 2.931	4.424 9.839	3.571 7.158	5.779 8.427	3.590 4.519	7.528 7.172	3.478 3.019

Table 10: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when *l* = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6$, $\psi = 0.1$						$\phi = 0.6$, $\psi = 0.9$					
				n = 2		*n* = 4		*n* = 8		*n* = 2		*n* = 4		*n* = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.75	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.02	0.02	0.02	0.01	0.01	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.05	0.05	0.04	0.03	0.03	0.02
		2.00	1.00	0.28	0.21	0.33	0.18	0.23	0.11	0.13	0.12	0.09	0.08	0.06	0.04
		3.00	1.00	0.78	0.63	0.49	0.60	0.25	0.38	0.45	0.43	0.27	0.34	0.16	0.18
		4.00	1.00	0.98	0.93	0.50	0.92	0.25	0.64	0.82	0.79	0.44	0.73	0.23	0.46
		5.00	1.00	1.00	1.00	0.50	0.99	0.25	0.72	0.97	0.97	0.49	0.95	0.25	0.66
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.04	0.03	0.06	0.03	0.06	0.02	0.02	0.02	0.02	0.02	0.01	0.01
		0.00	1.60	0.08	0.06	0.10	0.06	0.08	0.04	0.04	0.04	0.03	0.03	0.02	0.02
		0.00	1.80	0.13	0.10	0.13	0.10	0.10	0.06	0.07	0.06	0.05	0.05	0.03	0.03
		0.00	2.00	0.18	0.14	0.17	0.15	0.11	0.09	0.10	0.10	0.07	0.08	0.04	0.04
		0.00	2.40	0.27	0.24	0.22	0.24	0.14	0.14	0.17	0.17	0.11	0.14	0.07	0.08
		0.00	3.00	0.38	0.38	0.27	0.38	0.16	0.24	0.27	0.30	0.17	0.26	0.09	0.15
		0.00	5.00	0.60	0.68	0.36	0.68	0.20	0.45	0.51	0.62	0.28	0.57	0.15	0.37
		0.00	7.00	0.71	0.82	0.40	0.82	0.21	0.56	0.64	0.77	0.34	0.74	0.18	0.50
		0.00	10.0	0.80	0.90	0.43	0.90	0.22	0.64	0.74	0.88	0.39	0.86	0.20	0.60
		0.00	15.0	0.86	0.96	0.45	0.96	0.23	0.69	0.82	0.94	0.42	0.94	0.21	0.67
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.01	0.02	0.01	0.01	0.00	0.01
		0.00	1.60	0.07	0.06	0.07	0.06	0.08	0.03	0.03	0.04	0.01	0.02	0.01	0.01
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.05	0.07	0.02	0.04	0.01	0.02
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.09	0.12	0.04	0.08	0.02	0.04
		0.00	2.40	0.30	0.28	0.27	0.26	0.21	0.16	0.18	0.22	0.11	0.16	0.07	0.09
		0.00	3.00	0.47	0.44	0.37	0.42	0.24	0.27	0.34	0.37	0.22	0.30	0.15	0.17
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.50	0.68	0.70	0.43	0.65	0.24	0.42
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.60	0.82	0.83	0.48	0.80	0.25	0.55
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.91	0.91	0.49	0.90	0.25	0.63
		0.00	15.0	0.97	0.97	0.50	0.96	0.25	0.70	0.96	0.96	0.50	0.95	0.25	0.68
		$h_{\bar{X}}$, h_{X^2}		3.473 7.073	3.472 7.068	3.576 4.448	3.370 4.395	3.818 3.021	3.266 2.937	4.418 9.813	3.996 8.262	5.777 8.419	4.549 5.753	7.527 7.170	4.871 3.883

Table 10: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8,\ \psi = 0.1$						$\phi = 0.8,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.75	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.01	0.02	0.02	0.01	0.01	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.05	0.04	0.04	0.02	0.03	0.01
		2.00	1.00	0.28	0.20	0.33	0.17	0.23	0.10	0.13	0.10	0.09	0.05	0.06	0.02
		3.00	1.00	0.78	0.63	0.49	0.59	0.25	0.37	0.46	0.37	0.28	0.22	0.16	0.09
		4.00	1.00	0.98	0.93	0.50	0.92	0.25	0.64	0.83	0.72	0.44	0.57	0.23	0.28
		5.00	1.00	1.00	1.00	0.50	0.99	0.25	0.72	0.98	0.95	0.49	0.88	0.25	0.55
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.74
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.03	0.02	0.05	0.03	0.05	0.02	0.02	0.01	0.02	0.01	0.01	0.01
		0.00	1.60	0.06	0.05	0.09	0.05	0.08	0.03	0.03	0.03	0.03	0.02	0.02	0.01
		0.00	1.80	0.09	0.07	0.12	0.09	0.10	0.05	0.05	0.04	0.04	0.03	0.03	0.02
		0.00	2.00	0.13	0.11	0.15	0.12	0.11	0.07	0.07	0.06	0.06	0.05	0.04	0.02
		0.00	2.40	0.21	0.19	0.20	0.20	0.14	0.11	0.12	0.11	0.10	0.09	0.06	0.04
		0.00	3.00	0.31	0.30	0.25	0.32	0.16	0.18	0.20	0.20	0.15	0.16	0.09	0.08
		0.00	5.00	0.55	0.59	0.35	0.60	0.20	0.38	0.44	0.47	0.27	0.42	0.15	0.23
		0.00	7.00	0.67	0.75	0.39	0.75	0.21	0.50	0.57	0.66	0.33	0.61	0.18	0.36
		0.00	10.0	0.76	0.86	0.42	0.86	0.22	0.59	0.69	0.81	0.38	0.77	0.20	0.49
		0.00	15.0	0.84	0.93	0.45	0.93	0.23	0.66	0.79	0.91	0.42	0.89	0.21	0.60
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.01	0.02	0.01	0.01	0.00	0.01
		0.00	1.60	0.07	0.06	0.07	0.05	0.08	0.03	0.03	0.03	0.01	0.02	0.01	0.01
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.06	0.06	0.02	0.03	0.01	0.01
		0.00	2.00	0.18	0.16	0.18	0.15	0.16	0.09	0.09	0.10	0.04	0.05	0.02	0.02
		0.00	2.40	0.30	0.28	0.27	0.26	0.21	0.16	0.19	0.20	0.11	0.11	0.07	0.05
		0.00	3.00	0.47	0.44	0.37	0.41	0.24	0.26	0.34	0.35	0.22	0.24	0.15	0.11
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.49	0.68	0.68	0.43	0.59	0.24	0.35
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.59	0.82	0.82	0.48	0.76	0.25	0.50
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.91	0.91	0.49	0.88	0.25	0.60
		0.00	15.0	0.97	0.97	0.50	0.96	0.25	0.70	0.96	0.96	0.50	0.94	0.25	0.67
		$h_{\bar{X}}$, h_{X^2}		3.472 7.065	3.446 7.059	3.574 4.445	3.4623 4.409	3.818 3.021	3.478 2.952	4.379 9.642	4.158 8.797	5.759 8.366	5.091 6.795	7.524 7.166	5.938 4.945

Table 11: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$						$\phi = 0.2,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.03	0.04	0.03	0.05	0.03	0.02	0.02	0.02	0.03	0.01	0.03
		1.00	1.00	0.06	0.05	0.09	0.07	0.12	0.06	0.03	0.05	0.03	0.06	0.02	0.05
		1.50	1.00	0.20	0.19	0.32	0.26	0.32	0.20	0.09	0.16	0.08	0.20	0.06	0.16
		2.00	1.00	0.48	0.45	0.66	0.55	0.46	0.44	0.23	0.39	0.18	0.46	0.12	0.37
		3.00	1.00	0.95	0.93	0.98	0.95	0.50	0.82	0.69	0.88	0.55	0.91	0.32	0.78
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.93	0.96	1.00	0.88	1.00	0.46	0.92
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98	1.00	1.00	0.99	1.00	0.50	0.97
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.03	0.02	0.05	0.02	0.05	0.02	0.02	0.02	0.02	0.02	0.01	0.02
		0.00	1.40	0.10	0.07	0.12	0.07	0.11	0.05	0.05	0.06	0.04	0.06	0.03	0.04
		0.00	1.60	0.18	0.14	0.20	0.14	0.16	0.10	0.10	0.12	0.07	0.12	0.05	0.09
		0.00	1.80	0.27	0.23	0.27	0.23	0.20	0.17	0.16	0.21	0.11	0.20	0.07	0.16
		0.00	2.00	0.36	0.32	0.33	0.32	0.23	0.25	0.22	0.30	0.15	0.30	0.09	0.23
		0.00	2.40	0.50	0.50	0.43	0.50	0.28	0.41	0.35	0.48	0.23	0.48	0.13	0.38
		0.00	3.00	0.65	0.70	0.54	0.70	0.32	0.58	0.51	0.68	0.34	0.68	0.19	0.56
		0.00	5.00	0.86	0.94	0.72	0.94	0.39	0.83	0.78	0.93	0.56	0.93	0.30	0.83
		0.00	7.00	0.93	0.98	0.80	0.98	0.42	0.91	0.88	0.98	0.68	0.98	0.35	0.90
		0.00	10.0	0.96	1.00	0.86	1.00	0.45	0.94	0.94	0.99	0.77	0.99	0.40	0.94
		0.00	15.0	0.98	1.00	0.91	1.00	0.46	0.97	0.97	1.00	0.85	1.00	0.43	0.97
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.01	0.02	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.05	0.03	0.06	0.01	0.05	0.01	0.04
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.10	0.05	0.12	0.03	0.12	0.01	0.09
		0.00	1.80	0.23	0.22	0.25	0.22	0.24	0.17	0.10	0.21	0.05	0.20	0.02	0.16
		0.00	2.00	0.33	0.32	0.36	0.32	0.32	0.25	0.17	0.30	0.09	0.30	0.04	0.23
		0.00	2.40	0.51	0.50	0.55	0.51	0.42	0.41	0.33	0.49	0.21	0.48	0.13	0.39
		0.00	3.00	0.71	0.70	0.74	0.71	0.48	0.59	0.56	0.69	0.44	0.69	0.30	0.57
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.89	0.94	0.85	0.93	0.49	0.83
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.97	0.98	0.95	0.98	0.50	0.91
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	0.99	0.99	0.99	0.99	0.50	0.94
		0.00	15.0	0.97	0.97	0.50	0.97	0.25	0.70	0.96	0.97	0.50	0.96	0.25	0.70
		$h_{\bar{X}}$, h_{X^2}		3.473 7.073	3.371 7.082	3.576 4.448	3.229 4.393	3.818 3.022	3.045 2.931	4.424 9.839	3.571 7.158	5.779 8.427	3.590 4.519	7.528 7.172	3.478 3.019

Table 11: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 4 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6$, $\psi = 0.1$						$\phi = 0.6$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.03	0.04	0.03	0.05	0.03	0.02	0.02	0.02	0.02	0.01	0.02
		1.00	1.00	0.06	0.05	0.09	0.07	0.12	0.05	0.03	0.03	0.03	0.03	0.02	0.03
		1.50	1.00	0.20	0.18	0.32	0.23	0.32	0.18	0.09	0.10	0.08	0.10	0.06	0.08
		2.00	1.00	0.48	0.43	0.66	0.51	0.46	0.41	0.22	0.25	0.18	0.25	0.12	0.19
		3.00	1.00	0.94	0.92	0.98	0.94	0.50	0.80	0.65	0.70	0.55	0.69	0.32	0.57
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.93	0.94	0.96	0.88	0.96	0.46	0.84
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98	1.00	1.00	0.99	1.00	0.50	0.93
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.03	0.02	0.05	0.03	0.05	0.02	0.02	0.02	0.02	0.02	0.01	0.01
		0.00	1.40	0.09	0.07	0.12	0.08	0.11	0.06	0.04	0.04	0.04	0.04	0.03	0.03
		0.00	1.60	0.16	0.13	0.20	0.14	0.16	0.10	0.09	0.08	0.07	0.07	0.05	0.05
		0.00	1.80	0.25	0.21	0.27	0.22	0.20	0.16	0.14	0.14	0.11	0.12	0.07	0.08
		0.00	2.00	0.33	0.29	0.33	0.31	0.23	0.23	0.20	0.20	0.15	0.17	0.09	0.12
		0.00	2.40	0.47	0.44	0.43	0.46	0.28	0.35	0.32	0.33	0.23	0.30	0.13	0.21
		0.00	3.00	0.62	0.63	0.54	0.64	0.32	0.51	0.47	0.52	0.33	0.48	0.19	0.36
		0.00	5.00	0.84	0.90	0.72	0.90	0.39	0.78	0.76	0.86	0.56	0.84	0.30	0.70
		0.00	7.00	0.92	0.97	0.80	0.97	0.42	0.87	0.87	0.95	0.68	0.94	0.35	0.83
		0.00	10.0	0.96	0.99	0.86	0.99	0.45	0.93	0.93	0.99	0.77	0.98	0.40	0.90
		0.00	15.0	0.98	1.00	0.91	1.00	0.46	0.96	0.97	1.00	0.85	1.00	0.43	0.95
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.05	0.03	0.04	0.01	0.02	0.01	0.02
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.10	0.05	0.08	0.02	0.06	0.01	0.04
		0.00	1.80	0.23	0.22	0.25	0.22	0.24	0.17	0.10	0.15	0.05	0.11	0.02	0.07
		0.00	2.00	0.33	0.32	0.36	0.32	0.32	0.25	0.17	0.23	0.09	0.18	0.05	0.12
		0.00	2.40	0.52	0.50	0.55	0.50	0.42	0.40	0.33	0.40	0.21	0.34	0.13	0.25
		0.00	3.00	0.72	0.70	0.74	0.70	0.48	0.59	0.56	0.62	0.44	0.57	0.30	0.45
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.89	0.92	0.85	0.90	0.49	0.78
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.97	0.97	0.95	0.97	0.50	0.88
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	0.99	0.99	0.99	0.99	0.50	0.93
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.96
		$h_{\bar{X}}$, h_{X^2}		3.473 7.073	3.472 7.068	3.576 4.448	3.370 4.395	3.818 3.021	3.266 2.937	4.418 9.813	3.996 8.262	5.777 8.419	4.549 5.753	7.527 7.170	4.871 3.883

Table 11: Signal Probabilities of Shewhart $\bar{X}$ & X^2 Chart Combination for Transient Shifts in Autocorrelated Case when $l = 4$ hours (con’td)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8$, $\psi = 0.1$						$\phi = 0.8$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.02	0.04	0.03	0.05	0.03	0.02	0.02	0.02	0.02	0.01	0.01
		1.00	1.00	0.06	0.05	0.09	0.07	0.12	0.05	0.03	0.03	0.03	0.03	0.02	0.02
		1.50	1.00	0.20	0.18	0.32	0.22	0.32	0.16	0.09	0.09	0.08	0.07	0.06	0.04
		2.00	1.00	0.47	0.42	0.66	0.49	0.46	0.38	0.21	0.20	0.18	0.17	0.12	0.10
		3.00	1.00	0.94	0.91	0.98	0.93	0.50	0.79	0.61	0.61	0.55	0.54	0.32	0.38
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.92	0.92	0.92	0.88	0.89	0.46	0.72
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98	0.99	0.99	0.99	0.99	0.50	0.88
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	0.99
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.02	0.02	0.04	0.03	0.05	0.02	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.07	0.06	0.10	0.07	0.11	0.05	0.03	0.03	0.03	0.03	0.03	0.02
		0.00	1.60	0.13	0.11	0.17	0.13	0.16	0.09	0.06	0.06	0.06	0.05	0.04	0.03
		0.00	1.80	0.19	0.17	0.24	0.19	0.20	0.13	0.10	0.09	0.09	0.08	0.07	0.04
		0.00	2.00	0.26	0.23	0.30	0.26	0.23	0.18	0.14	0.13	0.12	0.11	0.09	0.06
		0.00	2.40	0.38	0.36	0.40	0.39	0.27	0.27	0.24	0.23	0.20	0.19	0.13	0.11
		0.00	3.00	0.53	0.52	0.51	0.54	0.32	0.40	0.38	0.37	0.30	0.32	0.18	0.20
		0.00	5.00	0.79	0.82	0.70	0.83	0.39	0.68	0.68	0.73	0.53	0.67	0.29	0.49
		0.00	7.00	0.89	0.93	0.78	0.93	0.42	0.81	0.82	0.88	0.66	0.85	0.35	0.68
		0.00	10.0	0.94	0.98	0.85	0.98	0.45	0.89	0.90	0.96	0.75	0.95	0.39	0.82
		0.00	15.0	0.97	0.99	0.90	1.00	0.46	0.94	0.96	0.99	0.83	0.99	0.43	0.90
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.04	0.03	0.03	0.01	0.02	0.01	0.01
		0.00	1.60	0.14	0.13	0.15	0.12	0.15	0.09	0.06	0.07	0.03	0.03	0.01	0.02
		0.00	1.80	0.23	0.22	0.25	0.21	0.24	0.16	0.11	0.12	0.05	0.07	0.02	0.03
		0.00	2.00	0.33	0.32	0.36	0.31	0.32	0.24	0.17	0.20	0.09	0.12	0.04	0.06
		0.00	2.40	0.52	0.50	0.55	0.50	0.42	0.40	0.34	0.37	0.21	0.26	0.13	0.15
		0.00	3.00	0.72	0.70	0.74	0.70	0.48	0.58	0.57	0.59	0.44	0.48	0.30	0.32
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.90	0.91	0.85	0.86	0.49	0.71
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.97	0.97	0.95	0.96	0.50	0.84
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	0.99	0.99	0.99	0.99	0.50	0.91
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.95
		$h_{\bar{X}}$, h_{X^2}		3.472 7.065	3.446 7.059	3.574 4.445	3.4623 4.409	3.818 3.021	3.478 2.952	4.379 9.642	4.158 8.797	5.759 8.366	5.091 6.795	7.524 7.166	5.938 4.945

Chapter 5

Performance of EWMA Observations Chart Combination

In this chapter, the performance of various sampling plans for the EWMA observations chart combination is investigated. All discussion of the EWMA control charts in this chapter refers to the EWMA control chart of the sample means (the EWMA_X chart) and the EWMA control chart of the squared deviations of the observations from target (the EWMA_{X^2} chart). Both sustained shifts and transient shifts in the AR(1) plus error process are evaluated.

The main issue for this chapter is to find an efficient sampling plan for the EWMA observations chart combination in detecting both sustained shifts and transient shifts in μ_X and σ_X in the presence of autocorrelation. Then the comparison with the Shewhart observations chart combination with the most efficient sampling plan obtained in Chapter 4 will be made in order to find an observations chart combination with the efficient sampling plan which has the best overall performance. Because the EWMA chart with individual sampling ($n=1$) has the best overall performance in the independent case, three sampling plans are taken into consideration in this chapter: (1) individual sampling ($n=1$); (2) $n=4$ with concentrated sampling; (3) $n=4$ with dispersed sampling. If either of the sampling plans with $n=4$ in the dependent case performs better than the one with $n=1$, more investigation with more values of n will be added for finding the best choice when $n>1$.

5.1 The Smoothing Parameter

As mentioned in Section 3.1.2, λ is a smoothing parameter satisfying $0 \le \lambda < 1$. When λ is larger, relatively less weight is given to older observations and the chart is more sensitive to large parameter shifts; when λ is smaller, more weight is given to the older observations and the chart is more sensitive to small parameter shifts. When an EWMA control chart with n = 1 (individual sampling) is compared to an EWMA chart with $n > 1$ (using either concentrated sampling or dispersed sampling), the choice of λ is an important issue. Reynolds and Stoumbos (2004b) introduced a reasonable way to ensure fair comparison, which chooses different values of λ when the sample size n is different so that the sum of the weights for n samples of size n = 1 equals to the weight of one sample of n > 1. If let λ_n denote the value of λ when the sample size is n, then the relationship between λ_1 and λ_n can be expressed as

$$\lambda_n = 1 - \left(1 - \lambda_1\right)^n \text{ or } \lambda_1 = 1 - \left(1 - \lambda_n\right)^{1/n}. \tag{5.1.1}$$

For example, if λ_4 = 0.1 is used for n = 4, then the corresponding value of λ_1 for n =1 is $\lambda_1 = 1 - \left(1 - \lambda_4\right)^{1/4}$ = 0.026. In this way, the individual observations in the control charts with different sample sizes have approximately the same weight, and the effects of historic observations on the control charts are the same. This also means that the control charts with different sample sizes have approximately the same sensitivity to any given shift (except for very large shifts), which leads to fair comparisons of the performance of EWMA control charts with different sample sizes. Note that for the EWMA control chart combination, it is not necessary that both of the EWMA charts use the same value of λ, but the numerical results in this dissertation are based on using the same value of λ in both charts for simplicity.

Lu and Reynolds (1999b) showed that in the autocorrelated case, a relatively small value of λ works well across a range of shifts for both the EWMA chart of the observations and the EWMA chart of the residuals. Thus in this dissertation, $\lambda_1 = 0.026$ for $n = 1$, and the corresponding value $\lambda_4 = 0.1$ for $n = 4$, are employed.

5.2 Efficient Sampling Plan for EWMA Observations Chart Combination

5.2.1 Sustained Shifts

Table 12 gives SSATS values for the EWMA observations chart combination for three sampling plans for sustained shifts in μ_X or σ_X in the independent case. Table 13 gives SSATS values for the EWMA observations chart combination for three sampling plans for sustained shifts in μ_X, σ_α and σ_ε in the autocorrelated case. The sizes of the shifts and the values of the autocorrelated process parameters are of the same as in the tables in Chapter 4. It appears that the SSATS values for all three sampling plans increase when the level of autocorrelation increases. For example, to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, individual sampling requires an average of 41 hours when the process observations are independent. When the observations are correlated, an average of 53.5 hours is needed for individual sampling when $\phi = 0.2$ and $\phi = 0.9$, while an average of 114.5 hours is needed when $\phi = 0.6$ and $\phi = 0.9$. To detect a sustained shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_α when $n = 4$, it takes concentrated sampling 29.9 hours on average in the independent case, 79.8 hours on average when $\phi = 0.2$ and $\phi = 0.9$, and 83.4 hours on average when $\phi = 0.6$ and $\phi = 0.9$.

To compare the three sampling plans, the independent case is considered first. Table 12 shows that $n = 4$ and concentrated sampling is slightly better than the other two sampling plans when the interest is of small sustained shifts in μ_X and σ_X. But for large sustained shifts, individual sampling is much better than n = 4 concentrated or dispersed sampling.

When the autocorrelation is present as in Table 13, individual sampling has the best overall performance in detecting sustained shifts from small size to large size. When the shifts are small or moderate, dispersed sampling also has good performance, but increasing the shift size shows quickly deteriorating performance. When ψ is small and a very small sustained shift in σ_α is of interest, concentrated sampling has slightly better performance than individual sampling. For instance, when ϕ = 0.6 and ψ = 0.1, and the shift in σ_X is of size $1.2\sigma_{X_0}$ and due to σ_α, individual sampling requires an average of 98.7 hours to detect the shift, but concentrated sampling only requires an average of 77.4 hours. However, if the shift size increases to 1.6 σ_{X_0} and the other conditions remain the same, individual sampling only needs an average of 18.6 hours to detect the shift, while concentrated sampling needs an average of 19.6 hours. However, when ψ is large, concentrated sampling performs worst even in detecting small shifts in σ_α. For example, when ϕ = 0.6 and ϕ = 0.9, if a sustained shift in σ_X of size $1.2\sigma_{X_0}$ due to σ_α is of interest, it takes individual sampling, concentrated sampling and dispersed sampling about 179.9 hours, 228.5 hours and 84.6 hours on average to detect this shift, respectively.

Thus the efficient sampling plan for the EWMA observations chart combination is $n = 1$ which gives better overall performance in detecting sustained shifts in μ_X and σ_X.

5.2.2 Transient Shifts

Tables 14 and 15 contain the signal probabilities for various sizes of transient shifts in μ_X, σ_α and σ_ε for the EWMA observations chart combination in the independent case. Tables 16 – 18 give the signal probabilities for various sizes of transient shifts in μ_X and σ_X for the EWMA observations chart combination for various combinations of ϕ and ψ. The duration of transient shifts is l = 1, 2 or 4 hours, respectively. Recall that the signal probabilities are the probabilities that a chart combination signals during the time when the transient shift is present plus four hours after the transient shift has ended.

It appears that the probability of detecting a transient shift decreases when the process autocorrelation increases for all three sampling plans. When the duration is $l = 1$ hour, the signal probability for small transient shifts in μ_X, σ_α and σ_ε is small for all three sampling plans, especially when the autocorrelation is relatively high. The signal probability is higher for transient shifts of moderate size, and in this case concentrated sampling is better than the other two sampling plans which have similar performance. However, for large transient shifts, concentrated sampling is much worse than the other two plans, and individual sampling is slightly better than dispersed sampling. For example, if a transient shift of $l = 1$ hour in μ_X of size $7\sigma_{X_0}$ occurs, both individual and dispersed sampling are able to detect this shift with a probability very close to 1. However, the signal probability for concentrated sampling is only 0.25. This is because the transient shift only lasts 1 hour and there is only a 25 percent chance that this shift occurs within a hour before a concentrated sample is collected when $n = 4$.

The conclusions about the performance of the three sampling plans in detecting a transient shift of duration $l = 2$ hours are similar, i.e., individual sampling is best overall, dispersed sampling is slightly worse, and concentrated sampling is the worst one.

When a transient shift is of duration l = 4 hours, the performance of concentrated sampling improves a lot because a sample will always be taken while the transient shift is present. When the process observations are independent, concentrated sampling has the same performance as the other two plans when the transient shift is small or large, and is slightly better when the shift is moderate. However, when autocorrelation is present, concentrated sampling is still not as good as individual sampling, especially when there is a large shift that is due to σ_α. Thus, in the case of l = 4 hours, individual sampling still has the best overall performance.

To summarize, when a transient shift is present, the efficient sampling plan for the EWMA observations chart combination is $n = 1$, which gives best overall performance in detecting transient shifts of various sizes and durations in μ_X, σ_α and σ_ε. If individual sampling is not applicable in some process, dispersed sampling should be conducted. Note that this conclusion for the transient shifts is the same as the conclusion for the sustained shifts, i.e., individual sampling is the best sampling plan in detecting both sustained and transient shifts in μ_X, σ_α or σ_ε for the EWMA observations chart combination.

5.3 Comparison between Shewhart and EWMA Chart Combinations of the Observations

Chapter 4 shows the most efficient sampling plan overall for the Shewhart chart combination of the observations is $n = 4$ with dispersed sampling. Section 5.2 in this chapter draws the conclusion that individual sampling ($n = 1$) is most efficient sampling plan for the EWMA observations chart combination. The comparison between Tables 5 and 13 shows that the EWMA observations chart combination with individual sampling is always better than the Shewhart observations chart combination with dispersed sampling in detecting sustained shifts, but not as good as the Shewhart chart combination with dispersed sampling in detecting large transient shifts, especially when the transient shift duration is small (say, l = 1 hour). For example, when ϕ = 0.6 and ϕ = 0.1, to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, the EWMA observations chart combination with individual sampling requires an average of 49.1 hours, while the Shewhart observations chart combination with dispersed sampling requires an average of 255.2 hours. To detect a sustained shift in μ_X of size $3\sigma_{X_0}$, the EWMA chart combination requires an average of 2.4 hours, while the Shewhart chart combination requires an average of 3.2 hours. To detect a transient shift in μ_X of size $3\sigma_{X_0}$ with 1 hour duration, the signal probability within 4 hours after the shift ends for the EWMA chart combination with individual sampling is 0.13, while the probability for the Shewhart chart combination with dispersed sampling is 0.23. If the size of the transient shift is the same but the duration increases to 4 hours, the signal probability within 4 hours after the shift ends for the EWMA chart combination is 0.91, while the probability for the Shewhart chart combination with dispersed sampling is 0.94.

Thus the overall performance of the EWMA observations chart combination with individual sampling is better than the overall performance of the Shewhart observations chart combination with dispersed sampling and $n = 4$. The Shewhart chart combination is recommended only when it is known that special causes will produce a transient shift in μ_X, σ_α or σ_ε with very short duration.

Table 12: SSATS Values of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Sustained Shifts in Independent Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	Individual Sampling (n = 1)	Concentrated Sampling (n = 4)	Dispersed Sampling (n = 4)
In-Control ATS		0.00	1.00	1481.5	1481.8	1481.8
Out-of-Control SSATS	μ_X	0.25	1.00	131.3	130.7	132.1
		0.50	1.00	41.0	40.3	41.9
		0.75	1.00	23.1	22.5	24.0
		1.00	1.00	15.6	15.1	16.6
		1.50	1.00	8.6	8.2	9.8
		2.00	1.00	5.2	4.9	6.5
		3.00	1.00	2.3	2.3	3.8
		4.00	1.00	1.3	2.0	2.8
		5.00	1.00	0.8	2.0	2.3
		7.00	1.00	0.5	2.0	2.0
		10.0	1.00	0.5	2.0	2.0
		15.0	1.00	0.5	2.0	2.0
	σ_X	0.00	1.20	90.6	89.8	91.4
		0.00	1.40	30.2	29.9	31.5
		0.00	1.60	16.9	16.7	18.3
		0.00	1.80	11.4	11.3	12.9
		0.00	2.00	8.5	8.5	10.0
		0.00	2.40	5.6	5.6	7.0
		0.00	3.00	3.6	3.8	5.1
		0.00	5.00	1.8	2.3	3.3
		0.00	7.00	1.3	2.1	2.8
		0.00	10.0	1.0	2.0	2.5
		0.00	15.0	0.8	2.0	2.3
		h_{EWMA_X}		3.072	2.941	2.941
		$h_{\text{EWMA}_{X^2}}$		3.635	3.414	3.414

Table 13: SSATS Values of EWMA_X & EWMA_{X^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 =$ 0.1) for Sustained Shifts in Autocorrelated Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$, $\psi = 0.1$			$\phi = 0.2$, $\psi = 0.9$			$\phi = 0.6$, $\psi = 0.1$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
In-Control ATS		0.00	1.00	1481.8	1481.4	1482.8	1481.0	1481.4	1481.9	1481.9	1481.8	1479.8
Out-of-Control SSATS	μ_X	0.25	1.00	137.1	165.6	138.0	182.4	410.0	183.4	164.7	177.9	166.1
		0.50	1.00	42.4	49.0	43.3	53.5	120.0	54.5	49.1	52.0	50.0
		0.75	1.00	23.7	26.5	24.7	28.9	58.6	29.8	26.8	27.8	27.7
		1.00	1.00	16.0	17.4	17.0	18.9	36.8	19.9	17.7	18.1	18.7
		1.50	1.00	8.7	9.1	9.9	9.9	19.5	11.1	9.3	9.4	10.5
		2.00	1.00	5.3	5.3	6.5	5.8	11.9	7.1	5.5	5.4	6.8
		3.00	1.00	2.4	2.4	3.8	2.5	5.4	4.0	2.4	2.4	3.8
		4.00	1.00	1.3	2.0	2.8	1.4	3.0	2.9	1.3	2.0	2.8
		5.00	1.00	0.8	2.0	2.3	0.9	2.2	2.4	0.8	2.0	2.3
		7.00	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		10.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		15.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
	σ_α	0.00	1.20	89.0	77.8	89.8	98.7	219.1	99.2	83.4	77.4	84.6
		0.00	1.40	30.0	31.1	31.4	32.4	79.8	33.8	31.3	31.3	32.6
		0.00	1.60	17.0	19.4	18.3	18.1	44.3	19.4	18.6	19.6	20.0
		0.00	1.80	11.5	14.2	13.0	12.3	29.8	13.6	13.1	14.4	14.5
		0.00	2.00	8.7	11.3	10.1	9.1	22.4	10.5	10.1	11.5	11.5
		0.00	2.40	5.7	8.3	7.1	5.9	14.8	7.4	6.8	8.4	8.3
		0.00	3.00	3.7	6.2	5.2	3.9	10.1	5.3	4.6	6.3	6.1
		0.00	5.00	1.8	3.9	3.3	1.8	5.4	3.3	2.2	4.0	3.7
		0.00	7.00	1.3	3.3	2.8	1.3	4.1	2.8	1.5	3.3	3.0
		0.00	10.0	1.0	2.8	2.5	1.0	3.3	2.5	1.1	2.8	2.6
		0.00	15.0	0.8	2.5	2.2	0.8	2.8	2.3	0.9	2.5	2.4
	σ_ε	0.00	1.20	91.1	95.2	91.9	101.3	409.2	101.5	93.1	95.6	93.9
		0.00	1.40	30.2	31.1	31.5	32.4	111.4	33.6	30.7	31.1	32.0
		0.00	1.60	16.9	17.2	18.3	17.9	47.8	19.2	17.1	17.3	18.5
		0.00	1.80	11.4	11.6	12.9	12.1	28.0	13.4	11.6	11.7	13.0
		0.00	2.00	8.6	8.7	10.0	8.9	19.1	10.4	8.6	8.7	10.1
		0.00	2.40	5.6	5.7	7.0	5.8	11.3	7.3	5.6	5.7	7.1
		0.00	3.00	3.6	3.8	5.1	3.8	6.7	5.2	3.7	3.8	5.1
		0.00	5.00	1.8	2.3	3.3	1.8	3.0	3.3	1.8	2.3	3.3
		0.00	7.00	1.3	2.1	2.8	1.3	2.3	2.8	1.3	2.1	2.8
		0.00	10.0	1.0	2.0	2.5	1.0	2.1	2.5	1.0	2.0	2.5
		0.00	15.0	0.8	2.0	2.3	0.8	2.0	2.3	0.8	2.0	2.3
h_{EWMA_X}, $h_{\text{EWMA}_{X^2}}$				3.141 3.637	3.348 3.485	3.011 3.418	3.633 3.786	5.634 7.189	3.503 3.560	3.436 3.652	3.465 3.485	3.306 3.435

Table 13: SSATS Values of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.9			ϕ = 0.8, ψ = 0.1			ϕ = 0.8, ψ = 0.9		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
In-Control ATS		0.00	1.00	1481.9	1481.4	1481.6	1481.9	1481.8	1481.8	1481.4	1481.5	1481.5
Out-of-Control SSATS	μ_X	0.25	1.00	395.7	482.9	395.1	216.0	221.3	216.7	647.3	672.5	647.5
		0.50	1.00	114.5	147.2	115.6	61.8	62.7	62.6	224.5	237.4	225.1
		0.75	1.00	55.7	70.3	56.6	32.1	32.2	33.0	105.3	111.9	106.2
		1.00	1.00	34.3	43.0	35.2	20.5	20.4	21.5	61.6	65.4	62.7
		1.50	1.00	17.0	21.9	18.1	10.3	10.1	11.5	29.1	31.1	30.2
		2.00	1.00	9.7	13.0	10.9	5.8	5.6	7.1	16.3	17.7	17.6
		3.00	1.00	4.0	5.7	5.4	2.5	2.5	3.9	6.5	7.2	7.9
		4.00	1.00	2.1	3.1	3.6	1.3	2.0	2.8	3.3	3.6	4.7
		5.00	1.00	1.3	2.2	2.8	0.8	2.0	2.3	2.0	2.3	3.4
		7.00	1.00	0.6	2.0	2.1	0.5	2.0	2.0	0.9	2.0	2.4
		10.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		15.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
	σ_α	0.00	1.20	179.9	228.5	180.4	83.6	79.4	83.1	289.4	292.3	289.6
		0.00	1.40	58.8	83.4	59.9	35.4	34.1	35.2	107.9	112.0	108.0
		0.00	1.60	31.7	46.0	32.9	22.2	21.7	22.1	58.1	61.4	58.2
		0.00	1.80	21.0	30.9	22.3	16.3	16.0	16.1	38.1	40.6	38.2
		0.00	2.00	15.5	23.1	16.8	12.8	12.8	12.7	28.0	29.9	27.9
		0.00	2.40	10.0	15.3	11.4	9.0	9.3	8.9	17.9	19.4	17.8
		0.00	3.00	6.4	10.3	7.9	6.2	6.9	6.1	11.5	12.7	11.4
		0.00	5.00	2.9	5.5	4.4	3.1	4.2	3.2	5.1	6.4	5.1
		0.00	7.00	1.9	4.2	3.4	2.1	3.4	2.5	3.2	4.7	3.3
		0.00	10.0	1.3	3.4	2.8	1.4	2.9	2.1	2.1	3.7	2.5
		0.00	15.0	1.0	2.8	2.5	1.0	2.6	2.0	1.4	3.0	2.1
	σ_ε	0.00	1.20	230.8	429.7	230.2	97.0	97.7	96.0	507.3	588.2	512.2
		0.00	1.40	58.7	115.9	59.7	31.4	31.5	31.1	129.8	167.2	130.6
		0.00	1.60	28.6	49.0	30.0	17.4	17.4	17.2	51.8	64.7	52.1
		0.00	1.80	18.2	28.5	19.6	11.7	11.7	11.6	29.6	35.5	29.7
		0.00	2.00	13.1	19.5	14.4	8.7	8.7	8.7	20.2	23.5	20.1
		0.00	2.40	8.1	11.5	9.5	5.7	5.7	5.7	11.8	13.4	11.8
		0.00	3.00	5.0	6.8	6.5	3.7	3.8	3.8	7.0	7.8	7.0
		0.00	5.00	2.2	3.0	3.7	1.8	2.3	2.3	2.9	3.3	3.1
		0.00	7.00	1.5	2.3	3.0	1.3	2.1	2.1	1.8	2.4	2.3
		0.00	10.0	1.1	2.1	2.6	1.0	2.0	2.0	1.3	2.1	2.1
		0.00	15.0	0.9	2.0	2.4	0.8	2.0	2.0	1.0	2.0	2.0
h_{EWMA_X} , $h_{\text{EWMA}_{X^2}}$				5.472 5.335	6.216 7.274	5.351 5.085	3.888 3.695	3.828 3.501	3.758 3.473	7.536 7.636	7.784 8.364	7.439 7.398

Table 14: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Independent Case when l = 1 hour and 2 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	Individual Sampling (n = 1)	Concentrated Sampling (n = 4)	Dispersed Sampling (n = 4)
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01
		1.00	1.00	0.01	0.01	0.01
		1.50	1.00	0.01	0.04	0.01
		2.00	1.00	0.03	0.10	0.03
		3.00	1.00	0.13	0.23	0.12
		4.00	1.00	0.39	0.25	0.36
		5.00	1.00	0.73	0.25	0.70
		7.00	1.00	0.99	0.25	0.99
		10.0	1.00	1.00	0.25	1.00
		15.0	1.00	1.00	0.25	1.00
	σ_X	0.00	1.20	0.01	0.00	0.01
		0.00	1.40	0.01	0.01	0.01
		0.00	1.60	0.02	0.02	0.02
		0.00	1.80	0.03	0.04	0.03
		0.00	2.00	0.04	0.06	0.04
		0.00	2.40	0.08	0.11	0.08
		0.00	3.00	0.16	0.16	0.16
		0.00	5.00	0.39	0.23	0.39
		0.00	7.00	0.54	0.24	0.53
		0.00	10.0	0.67	0.25	0.66
		0.00	15.0	0.78	0.25	0.77
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01
		1.00	1.00	0.01	0.02	0.01
		1.50	1.00	0.04	0.08	0.04
		2.00	1.00	0.11	0.21	0.10
		3.00	1.00	0.47	0.47	0.44
		4.00	1.00	0.88	0.50	0.86
		5.00	1.00	0.99	0.50	0.99
		7.00	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	0.50	1.00
	σ_X	0.00	1.20	0.01	0.01	0.01
		0.00	1.40	0.02	0.02	0.02
		0.00	1.60	0.04	0.05	0.03
		0.00	1.80	0.06	0.08	0.06
		0.00	2.00	0.10	0.13	0.10
		0.00	2.40	0.19	0.22	0.19
		0.00	3.00	0.34	0.33	0.33
		0.00	5.00	0.67	0.46	0.67
		0.00	7.00	0.82	0.49	0.81
		0.00	10.0	0.91	0.50	0.90
		0.00	15.0	0.96	0.50	0.96
		h_{EWMA_X}		3.072	2.941	2.941
		$h_{\text{EWMA}_{X^2}}$		3.635	3.414	3.414

Table 15: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Independent Case when *l* = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	Individual Sampling ($n = 1$)	Concentrated Sampling ($n = 4$)	Dispersed Sampling ($n = 4$)
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01
		0.75	1.00	0.02	0.02	0.02
		1.00	1.00	0.04	0.05	0.04
		1.50	1.00	0.15	0.16	0.14
		2.00	1.00	0.39	0.42	0.37
		3.00	1.00	0.93	0.94	0.92
		4.00	1.00	1.00	1.00	1.00
		5.00	1.00	1.00	1.00	1.00
		7.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00
	σ_X	0.00	1.20	0.02	0.02	0.01
		0.00	1.40	0.04	0.04	0.04
		0.00	1.60	0.09	0.09	0.09
		0.00	1.80	0.17	0.17	0.16
		0.00	2.00	0.25	0.26	0.24
		0.00	2.40	0.43	0.44	0.42
		0.00	3.00	0.65	0.66	0.64
		0.00	5.00	0.92	0.93	0.92
		0.00	7.00	0.98	0.98	0.98
		0.00	10.0	0.99	0.99	0.99
		0.00	15.0	1.00	1.00	1.00
		h_{EWMA_X}		3.072	2.941	2.941
		$h_{\text{EWMA}_{X^2}}$		3.635	3.414	3.414

Table 16: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when l = 1 hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.2, ψ = 0.1			ϕ = 0.2, ψ = 0.9			ϕ = 0.6, ψ = 0.1		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.01
		1.00	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.01	0.04	0.01	0.01	0.01	0.01	0.01	0.04	0.01
		2.00	1.00	0.03	0.10	0.03	0.03	0.03	0.02	0.03	0.10	0.03
		3.00	1.00	0.13	0.23	0.12	0.11	0.10	0.10	0.13	0.22	0.12
		4.00	1.00	0.39	0.25	0.36	0.35	0.19	0.33	0.38	0.25	0.36
		5.00	1.00	0.72	0.25	0.70	0.69	0.24	0.67	0.72	0.25	0.70
		7.00	1.00	0.99	0.25	0.99	0.99	0.25	0.99	0.99	0.25	0.99
		10.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
		15.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.01
		0.00	1.40	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01
		0.00	1.60	0.02	0.03	0.02	0.02	0.01	0.01	0.02	0.03	0.02
		0.00	1.80	0.03	0.05	0.03	0.02	0.02	0.02	0.03	0.05	0.02
		0.00	2.00	0.04	0.06	0.04	0.04	0.03	0.04	0.04	0.06	0.04
		0.00	2.40	0.08	0.09	0.08	0.08	0.05	0.07	0.08	0.09	0.07
		0.00	3.00	0.16	0.11	0.16	0.15	0.07	0.15	0.15	0.11	0.14
		0.00	5.00	0.39	0.17	0.39	0.38	0.13	0.37	0.38	0.17	0.37
		0.00	7.00	0.54	0.19	0.53	0.53	0.16	0.52	0.53	0.19	0.52
		0.00	10.0	0.67	0.21	0.66	0.66	0.19	0.65	0.66	0.21	0.65
		0.00	15.0	0.77	0.22	0.77	0.77	0.21	0.77	0.77	0.22	0.76
	σ_ε	0.00	1.20	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
		0.00	1.40	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
		0.00	1.60	0.02	0.02	0.01	0.01	0.00	0.01	0.02	0.02	0.01
		0.00	1.80	0.03	0.04	0.03	0.02	0.01	0.02	0.03	0.04	0.02
		0.00	2.00	0.04	0.06	0.04	0.04	0.01	0.04	0.04	0.06	0.04
		0.00	2.40	0.08	0.11	0.08	0.08	0.03	0.07	0.08	0.11	0.08
		0.00	3.00	0.16	0.16	0.15	0.15	0.08	0.15	0.16	0.16	0.15
		0.00	5.00	0.39	0.23	0.39	0.38	0.19	0.38	0.39	0.23	0.38
		0.00	7.00	0.54	0.24	0.53	0.53	0.23	0.52	0.54	0.24	0.53
		0.00	10.0	0.67	0.25	0.66	0.66	0.24	0.66	0.67	0.25	0.66
		0.00	15.0	0.78	0.25	0.77	0.77	0.25	0.77	0.77	0.25	0.77
h_{EWMA_X}, $h_{\text{EWMA}_{X^2}}$				3.141 3.637	3.348 3.485	3.011 3.418	3.633 3.786	5.634 7.189	3.503 3.560	3.436 3.652	3.465 3.485	3.306 3.435

Table 16: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when *l* = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.9			ϕ = 0.8, ψ = 0.1			ϕ = 0.8, ψ = 0.9		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00
		1.00	1.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00
		1.50	1.00	0.01	0.01	0.01	0.01	0.04	0.01	0.01	0.01	0.01
		2.00	1.00	0.01	0.03	0.01	0.03	0.09	0.03	0.01	0.02	0.01
		3.00	1.00	0.04	0.10	0.03	0.12	0.22	0.11	0.02	0.07	0.01
		4.00	1.00	0.13	0.19	0.12	0.37	0.25	0.35	0.04	0.17	0.04
		5.00	1.00	0.36	0.24	0.33	0.71	0.25	0.69	0.11	0.23	0.10
		7.00	1.00	0.94	0.25	0.93	0.99	0.25	0.99	0.65	0.25	0.60
		10.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
		15.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
	σ_α	0.00	1.20	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.01	0.01	0.01	0.01	0.02	0.01	0.00	0.00	0.00
		0.00	1.60	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.01	0.01
		0.00	1.80	0.01	0.02	0.01	0.02	0.04	0.02	0.01	0.01	0.01
		0.00	2.00	0.02	0.03	0.02	0.03	0.06	0.03	0.01	0.02	0.01
		0.00	2.40	0.04	0.04	0.04	0.07	0.08	0.05	0.02	0.03	0.01
		0.00	3.00	0.09	0.07	0.08	0.13	0.11	0.11	0.04	0.06	0.03
		0.00	5.00	0.28	0.13	0.27	0.35	0.16	0.32	0.17	0.11	0.15
		0.00	7.00	0.43	0.16	0.42	0.50	0.19	0.47	0.31	0.15	0.28
		0.00	10.0	0.58	0.19	0.57	0.63	0.20	0.61	0.47	0.18	0.45
		0.00	15.0	0.71	0.21	0.71	0.75	0.22	0.73	0.63	0.20	0.61
	σ_ε	0.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	1.40	0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00
		0.00	1.60	0.01	0.00	0.01	0.02	0.02	0.01	0.01	0.00	0.00
		0.00	1.80	0.01	0.01	0.01	0.03	0.04	0.02	0.01	0.01	0.01
		0.00	2.00	0.02	0.01	0.02	0.04	0.06	0.04	0.01	0.01	0.01
		0.00	2.40	0.04	0.03	0.03	0.08	0.11	0.08	0.02	0.02	0.01
		0.00	3.00	0.09	0.08	0.08	0.16	0.16	0.15	0.04	0.06	0.04
		0.00	5.00	0.29	0.19	0.28	0.39	0.23	0.38	0.20	0.18	0.20
		0.00	7.00	0.45	0.23	0.44	0.54	0.24	0.53	0.36	0.23	0.35
		0.00	10.0	0.60	0.24	0.59	0.67	0.25	0.66	0.52	0.24	0.51
		0.00	15.0	0.72	0.25	0.72	0.77	0.25	0.77	0.67	0.25	0.66
h_{EWMA_X} , $h_{\text{EWMA}_{X^2}}$				5.472 5.335	6.216 7.274	5.351 5.085	3.888 3.695	3.828 3.501	3.758 3.473	7.536 7.636	7.784 8.364	7.439 7.398

Table 17: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when $l = 2$ hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$			$\phi = 0.2,\ \psi = 0.9$			$\phi = 0.6,\ \psi = 0.1$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01
		1.50	1.00	0.04	0.08	0.04	0.04	0.03	0.04	0.04	0.08	0.04
		2.00	1.00	0.11	0.19	0.10	0.10	0.06	0.09	0.11	0.19	0.10
		3.00	1.00	0.46	0.45	0.44	0.42	0.20	0.40	0.45	0.45	0.43
		4.00	1.00	0.88	0.50	0.86	0.84	0.38	0.82	0.87	0.50	0.85
		5.00	1.00	0.99	0.50	0.99	0.99	0.48	0.98	0.99	0.50	0.99
		7.00	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.04	0.02	0.02	0.01	0.02	0.02	0.04	0.02
		0.00	1.60	0.04	0.06	0.03	0.03	0.02	0.03	0.04	0.06	0.04
		0.00	1.80	0.06	0.09	0.06	0.06	0.04	0.06	0.07	0.09	0.06
		0.00	2.00	0.10	0.12	0.10	0.09	0.06	0.09	0.10	0.12	0.10
		0.00	2.40	0.19	0.17	0.19	0.18	0.09	0.17	0.18	0.17	0.17
		0.00	3.00	0.34	0.23	0.33	0.32	0.14	0.31	0.31	0.23	0.30
		0.00	5.00	0.67	0.33	0.66	0.66	0.26	0.65	0.62	0.33	0.61
		0.00	7.00	0.81	0.38	0.81	0.81	0.32	0.80	0.78	0.38	0.77
		0.00	10.0	0.90	0.41	0.90	0.90	0.37	0.90	0.88	0.41	0.88
		0.00	15.0	0.96	0.44	0.95	0.95	0.41	0.95	0.94	0.44	0.94
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.02	0.02	0.02	0.01	0.01	0.02	0.02	0.02
		0.00	1.60	0.04	0.04	0.03	0.03	0.01	0.03	0.03	0.04	0.03
		0.00	1.80	0.06	0.08	0.06	0.06	0.02	0.05	0.06	0.08	0.06
		0.00	2.00	0.10	0.12	0.10	0.09	0.03	0.09	0.10	0.12	0.09
		0.00	2.40	0.19	0.21	0.19	0.18	0.06	0.17	0.19	0.21	0.18
		0.00	3.00	0.34	0.32	0.33	0.33	0.16	0.32	0.34	0.32	0.33
		0.00	5.00	0.67	0.46	0.67	0.66	0.39	0.65	0.67	0.46	0.66
		0.00	7.00	0.82	0.49	0.81	0.81	0.46	0.80	0.82	0.49	0.81
		0.00	10.0	0.91	0.50	0.90	0.90	0.49	0.90	0.90	0.50	0.90
		0.00	15.0	0.96	0.50	0.96	0.95	0.50	0.95	0.96	0.50	0.96
h_{EWMA_X}, $h_{\text{EWMA}_{X^2}}$				3.141 3.637	3.348 3.485	3.011 3.418	3.633 3.786	5.634 7.189	3.503 3.560	3.436 3.652	3.465 3.485	3.306 3.435

Table 17: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when l = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.9			ϕ = 0.8, ψ = 0.1			ϕ = 0.8, ψ = 0.9		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		1.00	1.00	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
		1.50	1.00	0.02	0.02	0.02	0.04	0.07	0.03	0.01	0.02	0.01
		2.00	1.00	0.04	0.05	0.04	0.10	0.19	0.09	0.02	0.04	0.02
		3.00	1.00	0.19	0.19	0.18	0.44	0.45	0.42	0.07	0.14	0.06
		4.00	1.00	0.55	0.38	0.53	0.86	0.50	0.84	0.24	0.33	0.22
		5.00	1.00	0.89	0.48	0.88	0.99	0.50	0.99	0.61	0.46	0.58
		7.00	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.01	0.01	0.01	0.02	0.03	0.02	0.01	0.01	0.01
		0.00	1.60	0.02	0.02	0.02	0.04	0.06	0.03	0.01	0.02	0.01
		0.00	1.80	0.03	0.04	0.03	0.06	0.09	0.05	0.02	0.03	0.01
		0.00	2.00	0.05	0.05	0.05	0.09	0.11	0.08	0.03	0.04	0.02
		0.00	2.40	0.10	0.09	0.09	0.16	0.16	0.14	0.05	0.07	0.04
		0.00	3.00	0.19	0.14	0.18	0.27	0.22	0.24	0.10	0.11	0.09
		0.00	5.00	0.49	0.26	0.48	0.55	0.32	0.53	0.32	0.23	0.30
		0.00	7.00	0.68	0.32	0.68	0.71	0.37	0.70	0.51	0.30	0.49
		0.00	10.0	0.82	0.37	0.82	0.84	0.41	0.83	0.69	0.35	0.67
		0.00	15.0	0.92	0.41	0.91	0.92	0.44	0.92	0.84	0.40	0.83
	σ_ε	0.00	1.20	0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00
		0.00	1.40	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.00	0.01
		0.00	1.60	0.01	0.01	0.01	0.03	0.04	0.03	0.01	0.01	0.01
		0.00	1.80	0.02	0.01	0.02	0.06	0.08	0.06	0.01	0.01	0.01
		0.00	2.00	0.04	0.02	0.04	0.10	0.12	0.09	0.02	0.02	0.02
		0.00	2.40	0.09	0.06	0.09	0.19	0.21	0.18	0.04	0.04	0.03
		0.00	3.00	0.20	0.15	0.19	0.33	0.32	0.32	0.10	0.12	0.10
		0.00	5.00	0.55	0.39	0.54	0.67	0.46	0.66	0.42	0.36	0.41
		0.00	7.00	0.74	0.46	0.73	0.81	0.49	0.81	0.64	0.45	0.63
		0.00	10.0	0.86	0.49	0.86	0.90	0.50	0.90	0.80	0.49	0.80
		0.00	15.0	0.94	0.50	0.93	0.96	0.50	0.95	0.91	0.50	0.90
h_{EWMA_X} , $h_{\text{EWMA}_{X^2}}$				5.472 5.335	6.216 7.274	5.351 5.085	3.888 3.695	3.828 3.501	3.758 3.473	7.536 7.636	7.784 8.364	7.439 7.398

Table 18: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when *l* = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.2, ψ = 0.1			ϕ = 0.2, ψ = 0.9			ϕ = 0.6, ψ = 0.1		
				Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02
		1.00	1.00	0.04	0.05	0.04	0.04	0.02	0.04	0.04	0.05	0.04
		1.50	1.00	0.15	0.16	0.14	0.13	0.05	0.13	0.14	0.15	0.13
		2.00	1.00	0.39	0.39	0.37	0.35	0.12	0.33	0.37	0.38	0.36
		3.00	1.00	0.92	0.90	0.91	0.88	0.40	0.87	0.91	0.90	0.90
		4.00	1.00	1.00	1.00	1.00	1.00	0.76	1.00	1.00	1.00	1.00
		5.00	1.00	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.02	0.03	0.02	0.01	0.01	0.01	0.02	0.03	0.02
		0.00	1.40	0.04	0.07	0.04	0.04	0.03	0.04	0.05	0.07	0.05
		0.00	1.60	0.09	0.13	0.09	0.08	0.05	0.08	0.10	0.13	0.10
		0.00	1.80	0.17	0.19	0.16	0.15	0.08	0.14	0.17	0.19	0.16
		0.00	2.00	0.25	0.24	0.24	0.23	0.11	0.22	0.24	0.24	0.23
		0.00	2.40	0.43	0.34	0.42	0.40	0.18	0.39	0.39	0.34	0.38
		0.00	3.00	0.64	0.46	0.63	0.62	0.28	0.61	0.58	0.45	0.57
		0.00	5.00	0.92	0.66	0.92	0.91	0.52	0.91	0.88	0.66	0.88
		0.00	7.00	0.98	0.76	0.97	0.97	0.64	0.97	0.96	0.75	0.96
		0.00	10.0	0.99	0.83	0.99	0.99	0.75	0.99	0.99	0.83	0.99
		0.00	15.0	1.00	0.88	1.00	1.00	0.83	1.00	1.00	0.88	1.00
	σ_ε	0.00	1.20	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
		0.00	1.40	0.04	0.04	0.04	0.04	0.01	0.03	0.04	0.04	0.04
		0.00	1.60	0.09	0.09	0.09	0.08	0.02	0.08	0.09	0.09	0.08
		0.00	1.80	0.16	0.16	0.16	0.15	0.03	0.14	0.16	0.16	0.15
		0.00	2.00	0.25	0.25	0.24	0.23	0.05	0.22	0.25	0.25	0.24
		0.00	2.40	0.43	0.43	0.42	0.41	0.13	0.40	0.43	0.43	0.42
		0.00	3.00	0.65	0.65	0.64	0.63	0.31	0.62	0.65	0.65	0.64
		0.00	5.00	0.92	0.92	0.92	0.92	0.78	0.92	0.92	0.92	0.92
		0.00	7.00	0.98	0.98	0.98	0.98	0.92	0.97	0.98	0.98	0.98
		0.00	10.0	0.99	0.99	0.99	0.99	0.98	0.99	0.99	0.99	0.99
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
h_{EWMA_X} , $h_{\text{EWMA}_{X^2}}$				3.141 3.637	3.348 3.485	3.011 3.418	3.633 3.786	5.634 7.189	3.503 3.560	3.436 3.652	3.465 3.485	3.306 3.435

Table 18: Signal Probabilities of EWMA_X & EWMA_{X^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when *l* = 4 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.9			ϕ = 0.8, ψ = 0.1			ϕ = 0.8, ψ = 0.9		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		1.00	1.00	0.02	0.02	0.02	0.04	0.04	0.04	0.01	0.01	0.01
		1.50	1.00	0.07	0.05	0.06	0.13	0.14	0.13	0.03	0.03	0.03
		2.00	1.00	0.17	0.11	0.16	0.36	0.37	0.34	0.07	0.07	0.07
		3.00	1.00	0.60	0.38	0.58	0.89	0.90	0.88	0.32	0.28	0.30
		4.00	1.00	0.94	0.76	0.93	1.00	1.00	1.00	0.75	0.66	0.73
		5.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00	0.97	0.93	0.97
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.40	0.03	0.03	0.02	0.05	0.06	0.04	0.02	0.02	0.01
		0.00	1.60	0.05	0.05	0.05	0.09	0.12	0.08	0.03	0.03	0.02
		0.00	1.80	0.08	0.07	0.08	0.15	0.17	0.14	0.05	0.05	0.04
		0.00	2.00	0.12	0.11	0.12	0.21	0.23	0.19	0.07	0.08	0.06
		0.00	2.40	0.23	0.18	0.22	0.33	0.32	0.31	0.12	0.13	0.11
		0.00	3.00	0.40	0.28	0.39	0.49	0.44	0.47	0.23	0.22	0.21
		0.00	5.00	0.79	0.51	0.78	0.80	0.65	0.79	0.56	0.46	0.55
		0.00	7.00	0.92	0.64	0.92	0.92	0.74	0.91	0.77	0.60	0.76
		0.00	10.0	0.98	0.74	0.98	0.97	0.82	0.97	0.91	0.71	0.91
		0.00	15.0	0.99	0.83	0.99	0.99	0.88	0.99	0.98	0.80	0.97
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.01	0.02	0.04	0.04	0.04	0.01	0.01	0.01
		0.00	1.60	0.03	0.02	0.03	0.09	0.09	0.08	0.01	0.01	0.01
		0.00	1.80	0.06	0.03	0.06	0.16	0.16	0.15	0.02	0.02	0.02
		0.00	2.00	0.11	0.05	0.10	0.24	0.24	0.23	0.04	0.03	0.04
		0.00	2.40	0.24	0.13	0.23	0.42	0.42	0.41	0.11	0.09	0.10
		0.00	3.00	0.46	0.31	0.45	0.64	0.64	0.63	0.28	0.24	0.27
		0.00	5.00	0.86	0.78	0.85	0.92	0.92	0.92	0.76	0.72	0.75
		0.00	7.00	0.95	0.92	0.95	0.98	0.98	0.97	0.92	0.90	0.91
		0.00	10.0	0.99	0.98	0.99	0.99	0.99	0.99	0.98	0.97	0.97
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99
h_{EWMA_X} , $h_{\text{EWMA}_{X^2}}$				5.472 5.335	6.216 7.274	5.351 5.085	3.888 3.695	3.828 3.501	3.758 3.473	7.536 7.636	7.784 8.364	7.439 7.398

Chapter 6

Performance of Shewhart Residuals Chart Combination

In this chapter, the performance of various sampling plans for the Shewhart chart combination of residuals is investigated. All discussion of the Shewhart control charts in this chapter refers to the Shewhart control charts of sample mean of residuals (Shewhart $\overline{e}$ Charts, or Shewhart e Charts, when $n = 1$) and the Shewhart control charts of squared residuals (Shewhart e^2 Charts). The evaluations are based on both sustained shifts and transient shifts only in the autocorrelated processes.

Similar to the previous discussion in Chapter 4 on the Shewhart chart combination of the observations, there are three main issues taken into consideration. The first issue concerns an appropriate value for n, when $n > 1$, to obtain good overall performance of the Shewhart $\overline{e}$ and e^2 chart combination. The second issue is to find an efficient sampling plan for Shewhart residuals chart combination in detecting both sustained and transient shifts in μ_X and σ_X. The third one is on whether to use Shewhart observations chart combination or Shewhart residuals chart combination in order to get the best overall performance.

6.1 Performance of Shewhart Residuals Chart Combination for Sustained Shifts

6.1.1 Individual Sampling

The Shewhart chart of the residuals with individual sampling is actually only the Shewhart *e* chart. Table 19 gives the SSATS values for sustained shifts in μ_X and σ_X for the Shewhart *e* chart, and includes various combinations of ϕ and ψ to account for the autocorrelation level from low to high. The range of δ_μ is from 0.0 to 15.0, where $\delta_\mu = \left|\mu_X - \mu_{X_0}\right| / \sigma_{X_0}$, and the range of δ_σ is from 1.0 to 15.0, where $\delta_\sigma = \sigma_X / \sigma_{X_0}$. It appears that when the autocorrelation increases, the average time to detect a signal increases when a shift occurs in μ_X or σ_α, but it decreases when a shift occurs in σ_ε. For example, when ϕ = 0.2 and ψ = 0.9, a Shewhart *e* chart needs an average of 671.6 hours to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, an average of 66.3 hours to detect a sustained shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_α, and an average of 59.0 hours to detect a shift of the same size due to σ_ε. When ϕ = 0.8 and ψ = 0.9, the corresponding times for a Shewhart *e* chart are 1236.8 hours, 98.2 hours, and 12.7 hours, respectively. Note that the change in the values of SSATS as a function of the level of autocorrelation is more significant when the sustained shifts are small. When shifts are large, the values of the SSATS do not change much, especially when shifts occur in μ_X.

6.1.2 Concentrated Sampling and Dispersed Sampling

Similar to the previous discussion about the Shewhart chart combination of the observations, when the Shewhart $\overline{e}$ and e^2 chart combination is considered, the sample size *n* is an important issue. Table 20 shows the SSATS values for sustained shifts in μ_X

and σ_X for the Shewhart $\bar{e}$ and e^2 chart combination, and also includes various combinations of ϕ and ψ to account for the autocorrelation level from low to high. Also similar to the Shewhart chart combination of the observations, the choice of the sample size for the residuals chart combination is hard to make because there is not a simple trend for the performance of the control chart combinations with different levels of autocorrelation and sampling plans.

For concentrated sampling, when sustained shifts in μ_X are of interest, increasing n results in improved performance for small shifts and deteriorating performance for large shifts. When there are sustained shifts in σ_α or σ_ε, when ψ is small, the trend is the same as when shifts occur in μ_X; but when ψ is large, increasing n always results in deteriorating performance. Thus it appears that $n = 2$ is a reasonable sample size for concentrated sampling, especially as it gives good performance when the autocorrelation is high. Note that this conclusion is the same as the conclusion obtained when concentrated sampling is used in the Shewhart observations chart combination in Chapter 4.

For dispersed sampling, when the autocorrelation is from low to moderate, increasing n results in improved performance for small shifts and deteriorating performance for large shifts in either of μ_X, σ_α or σ_ε. When the autocorrelation is high, increasing n always results in deteriorating performance. Thus it also appears that $n = 2$ is a reasonable sample size for concentrated sampling, as it gives good performance when the autocorrelation is high. Note that this conclusion is different from the conclusion in Chapter 4 for the Shewhart observations chart combination.

To summarize, the comparison of n = 2, 4, and 8 shows that $n = 2$ is a reasonable sample size value to used in both concentrated sampling and dispersed sampling to detect sustained shifts.

6.1.3 Efficient Sampling Plan for Shewhart Chart Combination of the Residuals

When $n = 2$, it appears that concentrated sampling performs better overall than dispersed sampling when a sustained shift occurs in μ_X or σ_α, but worse when a shift occurs in σ_ε. For example, when ϕ = 0.8 and ψ = 0.9, it takes concentrated sampling 1069 hours on average to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, 91.4 hours on average to detect a sustained shift in σ_X of size $1.4\sigma_{X0}$ due to σ_α, and 58.5 hours on average to detect a shift of the same size due to σ_ε; while it takes dispersed sampling 1259.1 hours, 94.5 hours, and 36.0 hours on average, respectively.

The comparison between concentrated sampling with $n = 2$ and individual sampling shows that concentrated sampling has better performance in detecting small or moderate sustained shifts in μ_X at all levels of autocorrelation, and also in detecting small or moderate sustained shift in σ_α or σ_ε when autocorrelation is low.

As a compromise, the best overall choice for the Shewhart chart combination of the residuals in detecting sustained shifts is $n = 2$ with concentrated sampling.

6.2 Performance of Shewhart Residuals Chart Combination for Transient Shifts

Tables 21 – 23 give the signal probabilities for various sizes of transient shifts in μ_X and σ_X for the Shewhart *e* chart with various combinations of ϕ and ψ. Tables 24 – 26 provide the signal probabilities for the Shewhart $\overline{e}$ and e^2 chart combination. The duration of transient shifts is l = 1, 2 or 4 hours, respectively.

When the duration of the transient shift is 1 hours, individual sampling has the best performance. Dispersed sampling with $n = 2$ is almost as good, being only slight worse when moderate transient shifts are of interest. Concentrated sampling with $n = 2$ performs badly for large transient shifts. This is because the transient shift only lasts 1 hour and there is only a 50 percent chance that this shift occurs within the hour before a concentrated sample is collected when $n = 2$. But if the interest is in small or moderate transient shifts, concentrated sampling is as good as, or even better than the other two sampling plans.

When the duration is $l = 2$ hours, the three sampling plans have similar performance. Concentrated sampling with $n = 2$ is slightly better than the other two when the autocorrelation is low, while individual sampling is slightly better when the autocorrelation is high.

The three sampling plans also have similar performance in detecting a transient shift of duration l = 4 hours. Concentrated sampling with $n = 2$ is slightly better than the other two when the autocorrelation is low, while individual sampling is slightly better when the autocorrelation is high.

It appears that individual sampling has the best overall performance in detecting transient shifts, especially when the duration of transient shifts is short (say, l = 1 hour). However, concentrated sampling with $n = 2$ also performs well in most cases in detecting transient shifts and it is best overall in detecting sustained shifts. Thus, concentrated sampling with $n = 2$ is recommended as the most efficient sampling plan for the Shewhart chart combination of the residuals to detect both sustained and transient shifts in either μ_X, σ_α or σ_ε. Individual sampling is advisable only when it is known for sure that only transient shifts in short duration will occur in the process.

6.3 Comparison between Shewhart Observations Chart Combination and Shewhart Residuals Chart Combination

In Chapter 4, Sections 6.1 and 6.2, the performance of three sampling plans for the Shewhart chart combination of the observations and the Shewhart chart combination of the residuals have been discussed, and some conclusions about efficient sampling plan have been drawn, respectively. The next question that naturally arises is: which control chart combination is better, the Shewhart observations chart combination or the Shewhart residuals chart combination?

6.3.1 Sustained Shifts

Tables 2, 5, 19 and 20 are used to answer the above question when sustained shifts are of interest. It appears that when the autocorrelation is low, the Shewhart residuals chart combination has similar performance with the Shewhart observations chart combination for all three sampling plans in most cases, but it is worse in detecting small sustained shifts in μ_X. Take dispersed sampling with n = 4 as an example. When

ϕ = 0.2 and ψ = 0.9, it takes the Shewhart observations chart combination an average of 280.2 hours to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, 61.4 hours to detect a sustained shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_α, and 65.6 hours to detect a shift of the same size due to σ_ε; while it takes the Shewhart residuals chart combination 410.2 hours, 53.8 hours, and 65.8 hours on average for same shifts, respectively.

When autocorrelation is moderate or large, the Shewhart residuals chart combination has much worse performance than the observations chart combination in detecting small or moderate sustained shifts in μ_X, similar or slightly better performance in detecting shifts in σ_α, and much better performance in detecting small or moderate shifts in σ_ε. Also take dispersed sampling with n = 4 as an example. When ϕ = 0.8 and ψ = 0.9, it takes the Shewhart observations chart combination an average of 540.4 hours to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, 122.8 hours to detect a sustained shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_α, and 250.1 hours to detect a shift of the same size due to σ_ε; while it takes the Shewhart residuals chart combination 1298.8 hours, 112 hours, and 125.3 hours on average, respectively. As another example consider concentrated sampling with n = 2. When ϕ = 0.6 and ψ = 0.9, the Shewhart observations chart combination requires an average of 603.2 hours, 103.6 hours, or 173.3 hours to detect aforementioned three sustained shifts, respectively; while the Shewhart residuals chart combination requires 1069.4 hours, 91.4 hours, and 58..5 hours on average, respectively.

6.3.2 Transient Shifts

When transient shifts are of interest, Tables 6 – 11 and Tables 21 – 26 are used for the comparison of signal probabilities for the Shewhart observations chart combination and the Shewhart residuals chart combination. It appears that the Shewhart residuals chart combination has almost the same performance as the Shewhart observations chart combination for all three sampling plans in detecting transient shifts of duration l = 1 hour, 2 hours, or 4 hours when the autocorrelation is from low to moderate. When the autocorrelation is high, the Shewhart residuals chart combination is actually better than the observations chart in detecting transient shifts in either μ_X, σ_α or σ_ε for all three sampling plans. Again, take dispersed sampling with n = 4 as an example. Consider the case of ϕ = 0.8, ψ = 0.9, and the signal probability in 4-hour period after a transient shift of duration l = 1 hour. When the transient shift is in μ_X and of size $4\sigma_{X_0}$, the signal probability is 0.18 for the Shewhart observations chart combination, and 0.31 for the Shewhart residuals chart combination. When the transient shift is in σ_X and of size $7\sigma_{X0}$ due to σ_α, the signal probability is 0.39 for the Shewhart observations chart combination, and 0.43 for the Shewhart residuals chart combination. When the transient shift is in σ_X and of size $7\sigma_{X_0}$ due to σ_ε, the signal probability is 0.48 for the observations chart combination and 0.53 for the residuals chart combination.

6.4 Shewhart Observations and Residuals Chart Combination

Section 6.3.2 shows that the Shewhart residuals chart combination has a clear advantage over the Shewhart observations chart combination in detecting transient shifts. However, it is still hard to say that the residuals chart combination should be chosen over the observations chart combination in general cases, because Section 6.3.1 concludes that neither the Shewhart observations chart combination nor the residuals chart combination has overall satisfactory performance in detecting sustained shifts.

The previous analysis indicates that the control chart combination based on the observations tends to give better performance in detecting shifts in μ_X, while the control chart combination based on the residuals tends to perform better in detecting shifts in σ_α and σ_ε. Therefore, the combination of a mean chart based on the observations and a squared deviation chart based on the residuals (i.e., the Shewhart $\bar{X}$ and e^2 chart combination) should also be taken into consideration. Such a combination would presumably combine the advantages of the Shewhart $\bar{X}$ and X^2 chart combination and the Shewhart $\bar{e}$ and e^2 chart combination. Table 27 confirms that this type of control chart combination is indeed a better approach. It appears that the Shewhart $\bar{X}$ and e^2 chart combination is more effective than the other two Shewhart chart combinations that have been considered to detect sustained shifts in either μ_X, σ_α or σ_ε in all levels of autocorrelation.

Table 27 also shows that dispersed sampling with n = 4 has the overall best performance, because this sampling plan can detect sustained shifts in μ_X and σ_α very efficiently and also has a good performance in detecting shifts in σ_ε. Consider the case when ϕ = 0.8 and ψ = 0.9 as the example of high autocorrelation. It takes the Shewhart $\bar{X}$ and e^2 chart combination an average of 698.3 hours to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$ for dispersed sampling with n = 4, 103.2 hours to detect a shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_α, and 155.9 hours to detect a shift with same size in σ_X due to σ_ε. This is very good performance comparing to the corresponding SSATS values for the other two combinations, which are 540.4 hours, 122.8 hours and 250.1 hours on average for the observations chart combination from Table 5, and 1298.8 hours, 112 hours, and 125.3 hours on average for the residuals chart combination from Table 20. Also take the case when ϕ= 0.2 and ψ = 0.9 as the example of low correlation. Three corresponding values of SSATS for the Shewhart $\bar{X}$ and e^2 chart combination are 299.7 hours, 61.8 hours, and 66.0 hours, while the corresponding values of SSATS for the other two charts are 280.2 hours, 61.4 hours, and 65.6 hours for the observations chart combination, and 410.2 hours, 62.1 hours and 65.8 hours for the residuals chart combination. The

Shewhart $\overline{X}$ and e^2 chart combination also has a relatively good performance in detecting shifts in μ_X, σ_α and σ_ε.

Tables 28 – 30 provide the signal probabilities for the Shewhart $\overline{X}$ and e^2 chart combination in detecting transient shifts, where the duration of sustained shifts is *l* = 1, 2 or 4 hours, respectively. The signal probabilities in Tables 28 – 30 can be compared to the probabilities for the other two chart combinations in Tables 6 – 11 and Tables 21 – 26. Although the Shewhart $\overline{X}$ and e^2 chart combination is not as good as Shewhart residuals chart combination, it still performs well, especially comparing to the Shewhart observations chart combination.

The above analysis concludes that the Shewhart $\overline{X}$ and e^2 chart combination with dispersed sampling and n = 4 gives the best overall performance in detected both sustained and transient shifts in the process mean and standard deviation.

Table 19: SSATS Values of Shewhart e Chart for Sustained Shifts in Autocorrelated Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
In-Control ATS		0.00	1.00	1481.6	1481.3	1481.2	1481.6	1481.8	1481.9
Out-of-Control SSATS	μ_X	0.25	1.00	1068.7	1160.7	1130.7	1340.8	1209.7	1411.3
		0.50	1.00	540.4	671.6	628.0	1027.4	758.2	1236.8
		0.75	1.00	260.5	359.5	324.5	719.7	434.5	1013.8
		1.00	1.00	129.6	194.8	170.6	486.4	247.6	793.7
		1.50	1.00	37.2	62.6	52.3	215.4	84.1	434.6
		2.00	1.00	13.0	22.8	18.4	93.2	30.1	199.8
		3.00	1.00	2.5	4.0	3.1	13.4	4.0	17.3
		4.00	1.00	0.9	1.0	0.9	1.4	0.9	0.8
		5.00	1.00	0.6	0.6	0.6	0.5	0.6	0.5
		7.00	1.00	0.5	0.5	0.5	0.5	0.5	0.5
		10.0	1.00	0.5	0.5	0.5	0.5	0.5	0.5
		15.0	1.00	0.5	0.5	0.5	0.5	0.5	0.5
	σ_α	0.00	1.20	219.2	218.7	247.1	244.8	302.5	297.4
		0.00	1.40	66.4	66.3	79.0	76.2	105.2	98.2
		0.00	1.60	29.9	29.6	36.9	34.1	51.4	44.3
		0.00	1.80	16.9	16.7	21.4	19.1	30.8	24.6
		0.00	2.00	11.0	10.8	14.3	12.3	20.9	15.6
		0.00	2.40	6.1	5.9	8.0	6.6	12.1	8.2
		0.00	3.00	3.5	3.4	4.6	3.7	7.0	4.4
		0.00	5.00	1.5	1.5	1.9	1.6	2.8	1.8
		0.00	7.00	1.1	1.1	1.3	1.1	1.8	1.2
		0.00	10.0	0.9	0.9	1.0	0.9	1.2	0.9
		0.00	15.0	0.7	0.7	0.8	0.7	0.9	0.8
	σ_ε	0.00	1.20	216.5	198.6	214.1	91.4	209.9	40.6
		0.00	1.40	65.4	59.0	64.6	26.1	63.2	12.7
		0.00	1.60	29.3	26.5	28.9	12.8	28.3	7.0
		0.00	1.80	16.5	15.1	16.3	8.0	16.0	4.8
		0.00	2.00	10.7	9.9	10.6	5.7	10.5	3.6
		0.00	2.40	5.9	5.6	5.9	3.6	5.8	2.5
		0.00	3.00	3.4	3.3	3.4	2.4	3.4	1.8
		0.00	5.00	1.5	1.5	1.5	1.3	1.5	1.1
		0.00	7.00	1.1	1.1	1.1	1.0	1.1	0.9
		0.00	10.0	0.9	0.9	0.9	0.8	0.9	0.7
		0.00	15.0	0.7	0.7	0.7	0.7	0.7	0.6
		h_e		3.400	3.399	3.400	3.400	3.400	3.400

Table 20: SSATS Values of Shewhart $\bar{e}$ & e^2 Chart Combination for Sustained Shifts in Autocorrelated Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$						$\phi = 0.2,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.5	1481.7	1482.1	1482.1	1481.8	1481.2	1481.6	1481.3	1481.7	1482.0	1481.5	1481.6
Out-of-Control SSATS	μ_X	0.25	1.00	938.1	954.4	794.3	773.6	674.3	569.3	1091.4	1069.9	1098.8	957.6	1129.8	799.3
		0.50	1.00	392.7	403.3	273.0	250.9	201.7	139.6	574.9	545.1	587.4	410.2	638.4	274.6
		0.75	1.00	162.3	166.3	99.9	87.4	70.1	45.1	287.8	263.0	299.8	173.6	341.7	100.0
		1.00	1.00	72.2	73.5	41.6	35.8	29.3	20.5	148.2	130.9	158.4	78.5	189.0	41.9
		1.50	1.00	18.3	18.7	10.5	10.2	8.4	9.5	45.7	37.5	51.6	20.4	66.8	12.9
		2.00	1.00	6.2	6.7	4.1	5.2	4.6	7.2	16.9	12.9	20.5	7.7	28.5	7.9
		3.00	1.00	1.6	2.1	2.1	3.1	4.0	5.4	3.7	2.8	5.3	3.4	8.5	5.6
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.5	1.4	2.6	2.5	4.7	4.7
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.1	1.1	2.1	2.1	4.1	4.2
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	132.0	193.0	87.4	174.4	74.4	156.3	247.5	216.9	293.5	208.9	353.7	185.6
		0.00	1.40	40.4	57.9	31.3	52.0	32.1	46.3	83.1	65.9	110.3	62.1	149.2	53.8
		0.00	1.60	20.2	26.6	18.1	24.5	21.1	23.1	40.2	29.7	57.5	28.3	83.9	25.8
		0.00	1.80	12.8	15.5	12.8	14.8	16.1	15.2	23.9	17.0	36.2	16.7	55.4	16.5
		0.00	2.00	9.2	10.4	10.0	10.5	13.4	11.6	16.3	11.3	25.6	11.5	40.6	12.3
		0.00	2.40	5.9	6.1	7.2	6.7	10.5	8.5	9.5	6.4	15.7	7.2	26.2	8.8
		0.00	3.00	3.9	3.8	5.4	4.7	8.4	6.7	5.8	4.0	10.0	4.9	17.4	6.9
		0.00	5.00	2.2	2.0	3.6	3.1	6.2	5.1	2.8	2.1	5.1	3.1	9.5	5.2
		0.00	7.00	1.8	1.6	3.0	2.6	5.4	4.7	2.1	1.6	3.9	2.7	7.4	4.7
		0.00	10.0	1.5	1.4	2.7	2.4	4.9	4.4	1.6	1.4	3.2	2.4	6.1	4.4
		0.00	15.0	1.3	1.2	2.4	2.2	4.6	4.3	1.4	1.2	2.7	2.3	5.3	4.3
	σ_ε	0.00	1.20	217.9	206.8	219.6	193.6	209.8	171.1	457.6	227.1	728.2	231.9	920.8	204.2
		0.00	1.40	63.9	61.3	60.7	55.8	52.7	48.4	172.0	67.0	355.6	65.8	544.4	55.9
		0.00	1.60	28.0	27.4	25.8	25.4	21.9	23.5	75.5	29.5	175.1	28.8	307.5	25.9
		0.00	1.80	15.6	15.7	14.3	15.1	12.3	15.2	38.1	16.6	87.2	16.6	164.4	16.3
		0.00	2.00	10.2	10.4	9.3	10.5	8.4	11.5	22.0	11.0	45.9	11.3	84.3	12.1
		0.00	2.40	5.6	6.0	5.3	6.7	5.5	8.4	10.0	6.3	17.1	7.0	26.2	8.7
		0.00	3.00	3.3	3.8	3.4	4.6	4.4	6.7	5.0	3.8	7.1	4.8	9.4	6.8
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	2.0	2.0	2.7	3.1	4.2	5.1
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.4	1.6	2.2	2.6	4.0	4.7
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.2	1.4	2.1	2.4	4.0	4.4
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.1	1.2	2.0	2.2	4.0	4.3
		$h_{\bar{e}}$, h_{e^2}		3.473 7.072	3.372 7.081	3.575 4.448	3.230 4.395	3.816 3.021	3.045 2.931	4.423 9.829	3.618 7.240	5.777 8.423	3.671 4.624	7.524 7.164	3.571 3.083

Table 20: SSATS Values of Shewhart $\bar{e}$ & e^2 Chart Combination for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.1						ϕ = 0.6, ψ = 0.9					
				n = 2		n = 4		n = 8		n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.6	1481.4	1481.8	1481.7	1482.0	1481.5	1481.7	1482.2	1481.6	1481.8	1481.7	1481.4
Out-of-Control SSATS	μ_X	0.25	1.00	973.6	1033.1	806.5	881.5	675.0	704.7	1244.9	1339.0	1159.2	1336.2	1137.5	1306.2
		0.50	1.00	428.3	487.8	282.2	333.1	202.6	206.2	822.8	1028.4	679.8	1024.3	650.0	961.3
		0.75	1.00	183.4	218.7	104.5	126.7	70.6	67.9	496.7	719.7	372.0	716.6	351.1	642.5
		1.00	1.00	83.6	102.3	43.8	53.0	29.5	28.0	296.7	486.2	207.1	481.1	195.4	411.1
		1.50	1.00	21.5	26.5	11.0	13.2	8.5	10.6	110.2	214.3	71.5	207.9	69.3	155.3
		2.00	1.00	7.1	8.7	4.2	5.8	4.6	7.5	43.4	92.0	28.2	84.7	29.5	52.0
		3.00	1.00	1.6	2.3	2.1	3.2	4.0	5.4	7.2	13.7	6.3	11.2	8.6	8.1
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.6	2.1	2.6	3.2	4.7	5.4
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.0	1.1	2.1	2.4	4.1	4.7
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	138.3	177.1	88.0	136.8	74.5	121.1	252.0	251.6	293.6	276.1	353.7	292.4
		0.00	1.40	43.3	56.1	31.6	45.7	32.1	42.5	84.9	83.3	110.4	98.1	149.2	105.9
		0.00	1.60	22.0	27.8	18.3	24.3	21.1	24.0	41.0	39.5	57.5	48.9	83.9	53.4
		0.00	1.80	14.0	17.2	12.9	16.1	16.2	16.8	24.4	23.1	36.2	29.7	55.4	32.8
		0.00	2.00	10.1	12.2	10.1	11.9	13.4	13.1	16.6	15.5	25.6	20.4	40.6	23.0
		0.00	2.40	6.4	7.5	7.3	8.0	10.5	9.7	9.6	8.8	15.7	12.0	26.2	14.4
		0.00	3.00	4.3	4.8	5.4	5.6	8.5	7.5	5.8	5.2	10.0	7.4	17.4	9.8
		0.00	5.00	2.3	2.4	3.6	3.4	6.2	5.5	2.8	2.4	5.1	3.9	9.5	6.2
		0.00	7.00	1.8	1.8	3.0	2.8	5.4	4.9	2.1	1.8	3.9	3.1	7.4	5.3
		0.00	10.0	1.5	1.5	2.7	2.5	4.9	4.6	1.7	1.5	3.2	2.6	6.1	4.8
		0.00	15.0	1.3	1.3	2.4	2.3	4.6	4.3	1.4	1.3	2.7	2.4	5.3	4.4
	σ_ε	0.00	1.20	216.7	210.9	219.5	202.9	209.7	180.2	369.0	216.3	715.4	407.2	920.7	501.8
		0.00	1.40	63.5	62.3	60.7	57.9	52.7	50.2	126.7	62.7	344.8	133.1	544.2	163.9
		0.00	1.60	27.9	27.8	25.8	26.0	21.9	24.0	54.1	27.5	168.2	54.3	307.3	63.5
		0.00	1.80	15.6	15.8	14.2	15.3	12.3	15.4	27.7	15.6	83.3	27.9	164.3	32.0
		0.00	2.00	10.1	10.5	9.3	10.6	8.4	11.7	16.6	10.3	43.9	17.3	84.3	20.1
		0.00	2.40	5.6	6.1	5.3	6.7	5.5	8.5	8.1	6.0	16.5	9.4	26.1	11.8
		0.00	3.00	3.3	3.8	3.4	4.7	4.4	6.7	4.3	3.8	6.9	5.8	9.4	8.2
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	1.8	2.0	2.7	3.3	4.2	5.6
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.4	1.6	2.2	2.8	4.0	4.9
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.2	1.4	2.1	2.5	4.0	4.6
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.1	1.2	2.0	2.3	4.0	4.3
		$h_{\bar{e}}$, h_{e^2}		3.469 7.060	3.426 7.067	3.575 4.447	3.378 4.407	3.816 3.021	3.278 2.946	4.398 9.202	4.311 8.222	5.771 8.345	5.375 6.836	7.522 7.161	6.113 5.005

Table 20: SSATS Values of Shewhart $\overline{e}$ & e^2 Chart Combination for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8,\ \psi = 0.1$						$\phi = 0.8,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.3	1481.7	1481.8	1482.0	1481.8	1482.4	1481.8	1481.4	1481.3	1481.8	1481.6	1482.3
Out-of-Control SSATS	μ_X	0.25	1.00	1038.1	1154.7	845.5	1029.1	692.8	883.0	1356.1	1422.4	1279.1	1427.5	1202.7	1427.1
		0.50	1.00	499.9	627.2	312.1	462.2	212.7	317.1	1069.4	1259.1	895.8	1298.8	758.6	1309.5
		0.75	1.00	229.7	314.8	119.9	195.7	75.3	110.1	771.4	1054.7	571.7	1120.3	442.7	1142.9
		1.00	1.00	109.3	157.9	51.2	84.1	31.5	41.5	533.8	839.8	357.8	925.0	259.8	954.8
		1.50	1.00	28.6	42.0	12.5	17.9	8.7	12.1	241.7	482.1	142.5	569.1	97.2	594.5
		2.00	1.00	8.8	12.0	4.4	6.5	4.7	7.8	100.9	239.7	57.9	305.7	40.8	316.0
		3.00	1.00	1.7	2.4	2.1	3.2	4.0	5.5	11.5	33.9	9.5	51.9	9.9	49.6
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.4	2.6	2.6	5.4	4.7	7.5
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.0	1.0	2.0	2.3	4.1	5.0
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.1
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	159.6	191.8	94.9	129.3	75.7	107.8	267.3	275.1	296.8	297.2	353.8	340.0
		0.00	1.40	53.1	64.6	34.9	47.5	32.7	43.8	91.4	94.5	111.8	112.0	149.4	140.0
		0.00	1.60	27.9	33.8	20.4	27.4	21.4	27.2	44.1	45.8	58.3	58.6	84.0	77.3
		0.00	1.80	18.1	21.9	14.4	19.2	16.4	20.0	26.2	27.2	36.6	36.9	55.5	50.4
		0.00	2.00	13.1	15.9	11.3	14.8	13.6	16.0	17.7	18.4	25.9	26.1	40.6	36.5
		0.00	2.40	8.4	10.1	8.0	10.2	10.6	11.9	10.2	10.6	15.9	16.0	26.2	22.9
		0.00	3.00	5.4	6.6	5.9	7.2	8.5	9.1	6.1	6.3	10.1	10.0	17.4	14.8
		0.00	5.00	2.7	3.1	3.7	4.1	6.2	6.2	2.9	2.8	5.1	4.8	9.5	7.9
		0.00	7.00	2.1	2.2	3.1	3.3	5.5	5.3	2.1	2.0	3.9	3.6	7.4	6.3
		0.00	10.0	1.7	1.7	2.7	2.8	5.0	4.8	1.7	1.6	3.2	2.9	6.1	5.3
		0.00	15.0	1.4	1.4	2.5	2.4	4.6	4.5	1.4	1.4	2.7	2.5	5.3	4.8
	σ_ε	0.00	1.20	212.2	212.6	217.0	209.9	209.3	190.3	203.8	130.9	602.1	384.3	904.2	663.8
		0.00	1.40	62.1	62.4	59.9	59.4	52.6	52.0	58.5	36.0	257.2	125.3	525.2	278.6
		0.00	1.60	27.3	27.8	25.5	26.5	21.8	24.5	25.0	16.6	115.5	51.0	292.2	117.1
		0.00	1.80	15.3	15.8	14.1	15.5	12.3	15.7	13.8	10.1	55.2	26.1	153.7	54.8
		0.00	2.00	10.0	10.5	9.2	10.7	8.4	11.8	8.9	7.1	29.4	16.2	78.0	30.7
		0.00	2.40	5.5	6.1	5.3	6.8	5.5	8.5	5.0	4.5	12.0	8.9	24.4	15.3
		0.00	3.00	3.3	3.8	3.4	4.7	4.4	6.7	3.0	3.1	5.6	5.6	9.0	9.5
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	1.6	1.8	2.5	3.3	4.2	5.9
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.3	1.5	2.1	2.8	4.0	5.1
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.1	1.3	2.0	2.5	4.0	4.7
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.1	1.2	2.0	2.3	4.0	4.4
		$h_{\overline{e}}$, h_{e^2}		3.458 7.022	3.452 7.063	3.562 4.432	3.474 4.430	3.812 3.018	3.505 2.974	4.316 7.666	4.593 7.592	5.706 7.646	6.500 7.891	7.506 7.051	8.672 7.371

Table 21: Signal Probabilities of Shewhart e Chart for Transient Shifts in Autocorrelated Case when l = 1 hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.01	0.00	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.01	0.01	0.01	0.02	0.01	0.03
		1.50	1.00	0.03	0.03	0.03	0.06	0.03	0.12
		2.00	1.00	0.08	0.09	0.08	0.16	0.08	0.35
		3.00	1.00	0.35	0.37	0.35	0.59	0.35	0.88
		4.00	1.00	0.73	0.75	0.73	0.93	0.74	1.00
		5.00	1.00	0.95	0.95	0.95	1.00	0.95	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.01
		0.00	1.40	0.02	0.02	0.01	0.02	0.01	0.01
		0.00	1.60	0.03	0.04	0.02	0.03	0.01	0.02
		0.00	1.80	0.06	0.06	0.03	0.05	0.02	0.04
		0.00	2.00	0.09	0.09	0.05	0.08	0.03	0.06
		0.00	2.40	0.15	0.16	0.10	0.14	0.05	0.11
		0.00	3.00	0.25	0.26	0.18	0.24	0.11	0.20
		0.00	5.00	0.49	0.50	0.41	0.48	0.32	0.44
		0.00	7.00	0.62	0.63	0.56	0.61	0.47	0.58
		0.00	10.0	0.73	0.73	0.68	0.72	0.61	0.70
		0.00	15.0	0.82	0.82	0.78	0.81	0.73	0.80
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.02
		0.00	1.40	0.02	0.02	0.02	0.03	0.02	0.06
		0.00	1.60	0.04	0.04	0.04	0.06	0.04	0.11
		0.00	1.80	0.06	0.06	0.06	0.10	0.06	0.17
		0.00	2.00	0.09	0.10	0.09	0.15	0.09	0.22
		0.00	2.40	0.16	0.17	0.16	0.23	0.16	0.32
		0.00	3.00	0.26	0.27	0.26	0.34	0.26	0.44
		0.00	5.00	0.50	0.50	0.50	0.57	0.50	0.65
		0.00	7.00	0.63	0.63	0.63	0.69	0.63	0.75
		0.00	10.0	0.73	0.74	0.73	0.78	0.74	0.82
		0.00	15.0	0.82	0.82	0.82	0.85	0.82	0.88
		h_e		3.400	3.399	3.400	3.400	3.400	3.400

Table 22: Signal Probabilities of Shewhart e Chart for Transient Shifts in Autocorrelated Case when l = 2 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.02
		1.00	1.00	0.02	0.02	0.02	0.02	0.02	0.04
		1.50	1.00	0.06	0.05	0.05	0.07	0.05	0.14
		2.00	1.00	0.15	0.13	0.14	0.18	0.14	0.38
		3.00	1.00	0.56	0.48	0.54	0.62	0.53	0.91
		4.00	1.00	0.92	0.87	0.91	0.94	0.91	1.00
		5.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.03	0.03	0.02	0.03	0.01	0.02
		0.00	1.60	0.07	0.07	0.04	0.06	0.03	0.04
		0.00	1.80	0.11	0.12	0.08	0.10	0.04	0.08
		0.00	2.00	0.17	0.17	0.12	0.15	0.07	0.12
		0.00	2.40	0.28	0.29	0.21	0.26	0.13	0.22
		0.00	3.00	0.44	0.45	0.35	0.42	0.23	0.36
		0.00	5.00	0.74	0.75	0.66	0.73	0.54	0.69
		0.00	7.00	0.86	0.86	0.80	0.85	0.71	0.82
		0.00	10.0	0.93	0.93	0.90	0.92	0.84	0.91
		0.00	15.0	0.97	0.97	0.95	0.96	0.92	0.96
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.02	0.01	0.04
		0.00	1.40	0.03	0.03	0.03	0.07	0.03	0.13
		0.00	1.60	0.07	0.07	0.07	0.14	0.07	0.24
		0.00	1.80	0.12	0.12	0.12	0.21	0.12	0.34
		0.00	2.00	0.17	0.18	0.17	0.29	0.17	0.42
		0.00	2.40	0.29	0.30	0.29	0.42	0.29	0.56
		0.00	3.00	0.45	0.46	0.45	0.58	0.45	0.70
		0.00	5.00	0.75	0.76	0.75	0.82	0.75	0.88
		0.00	7.00	0.86	0.87	0.86	0.90	0.86	0.94
		0.00	10.0	0.93	0.93	0.93	0.95	0.93	0.97
		0.00	15.0	0.97	0.97	0.97	0.98	0.97	0.99
		h_e		3.400	3.399	3.400	3.400	3.400	3.400

Table 23: Signal Probabilities of Shewhart e Chart for Transient Shifts in Autocorrelated Case when l = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$ $\psi = 0.1$	$\phi = 0.2$ $\psi = 0.9$	$\phi = 0.6$ $\psi = 0.1$	$\phi = 0.6$ $\psi = 0.9$	$\phi = 0.8$ $\psi = 0.1$	$\phi = 0.8$ $\psi = 0.9$
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.02	0.02	0.02	0.01	0.02	0.02
		1.00	1.00	0.03	0.03	0.03	0.02	0.03	0.04
		1.50	1.00	0.11	0.08	0.09	0.07	0.08	0.15
		2.00	1.00	0.27	0.20	0.23	0.19	0.22	0.40
		3.00	1.00	0.79	0.65	0.74	0.65	0.71	0.92
		4.00	1.00	0.99	0.96	0.98	0.95	0.98	1.00
		5.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.02	0.02	0.02	0.02	0.01	0.02
		0.00	1.40	0.06	0.06	0.05	0.05	0.03	0.04
		0.00	1.60	0.13	0.13	0.10	0.11	0.06	0.09
		0.00	1.80	0.21	0.21	0.16	0.19	0.10	0.15
		0.00	2.00	0.31	0.31	0.23	0.28	0.15	0.23
		0.00	2.40	0.48	0.49	0.39	0.46	0.27	0.39
		0.00	3.00	0.69	0.69	0.58	0.66	0.44	0.60
		0.00	5.00	0.93	0.94	0.89	0.93	0.79	0.90
		0.00	7.00	0.98	0.98	0.96	0.98	0.92	0.97
		0.00	10.0	0.99	0.99	0.99	0.99	0.97	0.99
		0.00	15.0	1.00	1.00	1.00	1.00	0.99	1.00
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.04	0.02	0.09
		0.00	1.40	0.06	0.07	0.06	0.14	0.06	0.26
		0.00	1.60	0.13	0.14	0.13	0.26	0.13	0.43
		0.00	1.80	0.22	0.23	0.22	0.39	0.22	0.57
		0.00	2.00	0.31	0.33	0.31	0.50	0.32	0.68
		0.00	2.40	0.50	0.52	0.50	0.67	0.50	0.81
		0.00	3.00	0.70	0.71	0.70	0.82	0.70	0.91
		0.00	5.00	0.94	0.94	0.94	0.97	0.94	0.99
		0.00	7.00	0.98	0.98	0.98	0.99	0.98	1.00
		0.00	10.0	0.99	1.00	1.00	1.00	1.00	1.00
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00
		h_e		3.400	3.399	3.400	3.400	3.400	3.400

Table 24: Signal Probabilities of Shewhart $\bar{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$, $\psi = 0.1$						$\phi = 0.2$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.03	0.02	0.02	0.02	0.01	0.01
		2.00	1.00	0.14	0.07	0.16	0.05	0.12	0.03	0.06	0.06	0.05	0.04	0.03	0.02
		3.00	1.00	0.39	0.28	0.25	0.23	0.12	0.12	0.23	0.27	0.14	0.20	0.08	0.10
		4.00	1.00	0.49	0.65	0.25	0.58	0.13	0.31	0.41	0.64	0.22	0.54	0.12	0.28
		5.00	1.00	0.50	0.91	0.25	0.87	0.13	0.51	0.49	0.91	0.25	0.86	0.12	0.49
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	1.00	0.13	0.62
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.03	0.02	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.60	0.05	0.03	0.05	0.03	0.04	0.02	0.03	0.03	0.02	0.02	0.01	0.01
		0.00	1.80	0.07	0.05	0.07	0.05	0.05	0.02	0.04	0.05	0.03	0.04	0.02	0.02
		0.00	2.00	0.10	0.08	0.08	0.07	0.06	0.04	0.06	0.07	0.04	0.06	0.02	0.03
		0.00	2.40	0.15	0.14	0.11	0.12	0.07	0.07	0.10	0.13	0.06	0.11	0.03	0.06
		0.00	3.00	0.20	0.23	0.14	0.21	0.08	0.12	0.15	0.23	0.08	0.20	0.05	0.11
		0.00	5.00	0.31	0.47	0.18	0.45	0.10	0.27	0.27	0.47	0.14	0.44	0.07	0.26
		0.00	7.00	0.36	0.61	0.20	0.59	0.11	0.36	0.33	0.60	0.17	0.58	0.09	0.35
		0.00	10.0	0.40	0.72	0.21	0.71	0.11	0.43	0.38	0.71	0.19	0.70	0.10	0.43
		0.00	15.0	0.44	0.81	0.23	0.80	0.12	0.49	0.42	0.81	0.21	0.80	0.11	0.49
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.02	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.60	0.04	0.03	0.04	0.03	0.04	0.01	0.01	0.03	0.01	0.02	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.03	0.05	0.01	0.04	0.01	0.02
		0.00	2.00	0.09	0.08	0.09	0.07	0.08	0.04	0.04	0.07	0.02	0.06	0.01	0.03
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.07	0.09	0.14	0.05	0.11	0.03	0.06
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.17	0.23	0.11	0.20	0.07	0.11
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.27	0.34	0.47	0.21	0.44	0.12	0.26
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.36	0.41	0.60	0.24	0.58	0.12	0.35
		0.00	10.0	0.47	0.72	0.25	0.71	0.13	0.43	0.45	0.72	0.25	0.70	0.12	0.43
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.48	0.81	0.25	0.80	0.13	0.49
		$h_{\bar{e}}$, h_{e^2}		3.473	3.372	3.575	3.230	3.816	3.045	4.423	3.618	5.777	3.671	7.524	3.571
				7.072	7.081	4.448	4.395	3.021	2.931	9.829	7.240	8.423	4.624	7.164	3.083

Table 24: Signal Probabilities of Shewhart $\overline{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.1$						$\phi = 0.6,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.03	0.02	0.02	0.01	0.01	0.01
		2.00	1.00	0.14	0.06	0.16	0.05	0.12	0.03	0.08	0.06	0.05	0.02	0.03	0.01
		3.00	1.00	0.39	0.28	0.25	0.23	0.12	0.11	0.27	0.26	0.14	0.09	0.08	0.03
		4.00	1.00	0.49	0.65	0.25	0.57	0.13	0.30	0.44	0.67	0.22	0.31	0.12	0.10
		5.00	1.00	0.50	0.91	0.25	0.87	0.13	0.50	0.49	0.94	0.25	0.69	0.12	0.27
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	1.00	0.13	0.60
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.01	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.60	0.04	0.03	0.05	0.03	0.04	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.80	0.06	0.04	0.07	0.04	0.05	0.02	0.04	0.03	0.03	0.02	0.02	0.01
		0.00	2.00	0.09	0.06	0.08	0.07	0.06	0.03	0.06	0.05	0.04	0.03	0.02	0.02
		0.00	2.40	0.13	0.12	0.11	0.12	0.07	0.06	0.09	0.10	0.06	0.06	0.03	0.03
		0.00	3.00	0.19	0.20	0.13	0.20	0.08	0.11	0.15	0.18	0.08	0.12	0.05	0.06
		0.00	5.00	0.30	0.43	0.18	0.44	0.10	0.25	0.27	0.41	0.14	0.34	0.07	0.18
		0.00	7.00	0.36	0.57	0.20	0.58	0.11	0.34	0.33	0.55	0.17	0.49	0.09	0.27
		0.00	10.0	0.40	0.69	0.21	0.69	0.11	0.42	0.38	0.68	0.19	0.63	0.10	0.36
		0.00	15.0	0.43	0.79	0.23	0.79	0.12	0.49	0.42	0.78	0.21	0.74	0.11	0.45
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.60	0.04	0.03	0.04	0.02	0.04	0.01	0.02	0.03	0.01	0.01	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.03	0.05	0.01	0.02	0.01	0.01
		0.00	2.00	0.09	0.08	0.09	0.07	0.08	0.04	0.06	0.07	0.02	0.03	0.01	0.01
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.07	0.11	0.14	0.05	0.07	0.03	0.03
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.19	0.23	0.11	0.14	0.07	0.06
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.27	0.35	0.47	0.21	0.37	0.12	0.19
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.36	0.42	0.61	0.24	0.53	0.12	0.29
		0.00	10.0	0.47	0.72	0.25	0.71	0.13	0.43	0.46	0.72	0.25	0.66	0.12	0.38
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.48	0.81	0.25	0.77	0.13	0.46
		$h_{\overline{e}}$, h_{e^2}		3.469 7.060	3.426 7.067	3.575 4.447	3.378 4.407	3.816 3.021	3.278 2.946	4.398 9.202	4.311 8.222	5.771 8.345	5.375 6.836	7.522 7.161	6.113 5.005

Table 24: Signal Probabilities of Shewhart $\bar{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.8, ψ = 0.1						ϕ = 0.8, ψ = 0.9					
				n = 2		n = 4		n = 8		n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.02	0.01	0.01	0.01	0.01	0.00
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.06	0.03	0.03	0.01	0.01	0.01
		2.00	1.00	0.14	0.06	0.17	0.05	0.12	0.03	0.15	0.09	0.06	0.02	0.03	0.01
		3.00	1.00	0.39	0.28	0.25	0.22	0.12	0.11	0.39	0.44	0.17	0.08	0.08	0.02
		4.00	1.00	0.49	0.65	0.25	0.57	0.13	0.30	0.49	0.88	0.24	0.31	0.12	0.05
		5.00	1.00	0.50	0.92	0.25	0.87	0.13	0.50	0.50	1.00	0.25	0.74	0.12	0.13
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	1.00	0.13	0.55
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.01	0.01	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		0.00	1.60	0.03	0.02	0.04	0.02	0.04	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.80	0.04	0.03	0.06	0.03	0.05	0.02	0.04	0.03	0.03	0.02	0.02	0.01
		0.00	2.00	0.06	0.04	0.08	0.05	0.06	0.02	0.05	0.04	0.04	0.03	0.02	0.01
		0.00	2.40	0.10	0.08	0.10	0.09	0.07	0.04	0.09	0.08	0.06	0.05	0.03	0.01
		0.00	3.00	0.15	0.15	0.13	0.17	0.08	0.08	0.14	0.15	0.08	0.09	0.05	0.03
		0.00	5.00	0.27	0.37	0.17	0.39	0.10	0.21	0.26	0.37	0.14	0.28	0.07	0.11
		0.00	7.00	0.33	0.52	0.20	0.54	0.11	0.30	0.32	0.52	0.17	0.43	0.09	0.19
		0.00	10.0	0.38	0.65	0.21	0.66	0.11	0.39	0.37	0.65	0.19	0.58	0.10	0.29
		0.00	15.0	0.42	0.76	0.22	0.77	0.12	0.46	0.41	0.76	0.21	0.71	0.11	0.39
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.02	0.00	0.01	0.00	0.00
		0.00	1.60	0.04	0.03	0.04	0.02	0.04	0.01	0.03	0.04	0.01	0.01	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.06	0.07	0.02	0.02	0.01	0.01
		0.00	2.00	0.09	0.08	0.09	0.06	0.08	0.03	0.09	0.11	0.03	0.03	0.01	0.01
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.06	0.16	0.19	0.07	0.07	0.04	0.02
		0.00	3.00	0.23	0.24	0.19	0.21	0.12	0.12	0.25	0.29	0.13	0.15	0.08	0.04
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.26	0.39	0.53	0.22	0.38	0.12	0.16
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.35	0.44	0.66	0.24	0.53	0.12	0.26
		0.00	10.0	0.47	0.72	0.25	0.70	0.13	0.43	0.47	0.76	0.25	0.66	0.12	0.35
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.49	0.84	0.25	0.77	0.13	0.44
		$h_{\bar{e}}$, h_{e^2}		3.458 7.022	3.452 7.063	3.562 4.432	3.474 4.430	3.812 3.018	3.505 2.974	4.316 7.666	4.593 7.592	5.706 7.646	6.500 7.891	7.506 7.051	8.672 7.371

Table 25: Signal Probabilities of Shewhart $\overline{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when $l = 2$ hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$, $\psi = 0.1$						$\phi = 0.2$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.75	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.02	0.02	0.02	0.01	0.02	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.05	0.07	0.04	0.06	0.03	0.04
		2.00	1.00	0.28	0.21	0.33	0.19	0.23	0.11	0.13	0.18	0.09	0.16	0.06	0.10
		3.00	1.00	0.78	0.64	0.49	0.62	0.25	0.39	0.45	0.57	0.27	0.55	0.16	0.35
		4.00	1.00	0.98	0.93	0.50	0.93	0.25	0.64	0.82	0.89	0.44	0.89	0.23	0.62
		5.00	1.00	1.00	1.00	0.50	1.00	0.25	0.72	0.97	0.99	0.49	0.99	0.25	0.72
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.05	0.03	0.06	0.03	0.06	0.02	0.03	0.03	0.02	0.02	0.01	0.02
		0.00	1.60	0.10	0.07	0.10	0.06	0.08	0.04	0.05	0.06	0.03	0.05	0.02	0.03
		0.00	1.80	0.15	0.11	0.14	0.10	0.10	0.06	0.08	0.10	0.05	0.09	0.03	0.05
		0.00	2.00	0.20	0.17	0.17	0.15	0.11	0.09	0.12	0.16	0.07	0.14	0.04	0.08
		0.00	2.40	0.29	0.28	0.22	0.26	0.14	0.16	0.19	0.27	0.11	0.24	0.07	0.15
		0.00	3.00	0.41	0.44	0.27	0.42	0.16	0.27	0.30	0.43	0.17	0.40	0.09	0.25
		0.00	5.00	0.62	0.74	0.36	0.73	0.20	0.49	0.53	0.73	0.28	0.71	0.15	0.48
		0.00	7.00	0.73	0.86	0.40	0.85	0.21	0.59	0.66	0.85	0.34	0.84	0.18	0.59
		0.00	10.0	0.81	0.93	0.43	0.92	0.22	0.66	0.76	0.92	0.39	0.92	0.20	0.65
		0.00	15.0	0.87	0.97	0.45	0.96	0.23	0.70	0.83	0.97	0.42	0.96	0.21	0.70
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.01	0.03	0.01	0.02	0.00	0.02
		0.00	1.60	0.07	0.07	0.07	0.06	0.08	0.03	0.03	0.06	0.01	0.05	0.01	0.03
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.05	0.11	0.02	0.09	0.01	0.05
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.09	0.16	0.04	0.14	0.02	0.08
		0.00	2.40	0.30	0.28	0.27	0.26	0.21	0.16	0.18	0.27	0.11	0.24	0.07	0.15
		0.00	3.00	0.47	0.44	0.37	0.42	0.24	0.27	0.34	0.43	0.22	0.40	0.15	0.26
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.50	0.68	0.74	0.43	0.72	0.24	0.49
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.60	0.82	0.86	0.48	0.84	0.25	0.59
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.91	0.93	0.49	0.92	0.25	0.66
		0.00	15.0	0.97	0.97	0.50	0.97	0.25	0.70	0.96	0.97	0.50	0.96	0.25	0.70
		$h_{\overline{e}}$, h_{e^2}		3.473 7.072	3.372 7.081	3.575 4.448	3.230 4.395	3.816 3.021	3.045 2.931	4.423 9.829	3.618 7.240	5.777 8.423	3.671 4.624	7.524 7.164	3.571 3.083

Table 25: Signal Probabilities of Shewhart $\overline{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.1						ϕ = 0.6, ψ = 0.9					
				n = 2		n = 4		n = 8		n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.75	1.00	0.01	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.02	0.02	0.02	0.01	0.01	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.06	0.07	0.04	0.03	0.03	0.02
		2.00	1.00	0.28	0.21	0.33	0.18	0.23	0.11	0.16	0.17	0.09	0.08	0.06	0.04
		3.00	1.00	0.78	0.63	0.49	0.60	0.25	0.38	0.53	0.53	0.28	0.33	0.16	0.15
		4.00	1.00	0.98	0.92	0.50	0.92	0.25	0.64	0.88	0.85	0.44	0.72	0.23	0.41
		5.00	1.00	1.00	0.99	0.50	0.99	0.25	0.72	0.99	0.98	0.49	0.94	0.25	0.63
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.74
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.04	0.03	0.06	0.03	0.06	0.02	0.03	0.02	0.02	0.02	0.01	0.01
		0.00	1.60	0.08	0.06	0.10	0.06	0.08	0.04	0.05	0.04	0.03	0.03	0.02	0.02
		0.00	1.80	0.13	0.10	0.13	0.10	0.10	0.06	0.08	0.08	0.05	0.05	0.03	0.03
		0.00	2.00	0.18	0.14	0.17	0.15	0.11	0.09	0.12	0.11	0.07	0.08	0.04	0.04
		0.00	2.40	0.27	0.24	0.22	0.24	0.14	0.14	0.19	0.20	0.11	0.14	0.07	0.07
		0.00	3.00	0.38	0.38	0.27	0.38	0.16	0.23	0.29	0.34	0.17	0.25	0.09	0.13
		0.00	5.00	0.60	0.68	0.36	0.68	0.20	0.45	0.53	0.66	0.28	0.57	0.15	0.34
		0.00	7.00	0.71	0.82	0.40	0.81	0.21	0.56	0.65	0.81	0.34	0.74	0.18	0.48
		0.00	10.0	0.80	0.90	0.43	0.90	0.22	0.64	0.75	0.90	0.39	0.86	0.20	0.59
		0.00	15.0	0.86	0.96	0.45	0.96	0.23	0.69	0.83	0.95	0.42	0.94	0.21	0.66
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.02	0.03	0.01	0.01	0.00	0.01
		0.00	1.60	0.07	0.06	0.07	0.06	0.08	0.03	0.04	0.06	0.01	0.02	0.01	0.01
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.07	0.11	0.02	0.05	0.01	0.02
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.11	0.16	0.04	0.08	0.02	0.03
		0.00	2.40	0.31	0.28	0.27	0.26	0.21	0.16	0.22	0.28	0.11	0.16	0.07	0.07
		0.00	3.00	0.47	0.44	0.37	0.42	0.24	0.27	0.38	0.44	0.22	0.30	0.15	0.16
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.50	0.71	0.74	0.43	0.65	0.24	0.40
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.59	0.84	0.86	0.48	0.80	0.25	0.53
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.92	0.93	0.49	0.90	0.25	0.62
		0.00	15.0	0.97	0.97	0.50	0.96	0.25	0.70	0.96	0.97	0.50	0.95	0.25	0.68
		$h_{\overline{e}}$, h_{e^2}		3.469 7.060	3.426 7.067	3.575 4.447	3.378 4.407	3.816 3.021	3.278 2.946	4.398 9.202	4.311 8.222	5.771 8.345	5.375 6.836	7.522 7.161	6.113 5.005

Table 25: Signal Probabilities of Shewhart $\overline{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8,\ \psi = 0.1$						$\phi = 0.8,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00
		0.75	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.02	0.02	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.01	0.03	0.03	0.02	0.01	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.12	0.11	0.05	0.03	0.03	0.01
		2.00	1.00	0.29	0.20	0.33	0.17	0.23	0.10	0.29	0.27	0.12	0.08	0.06	0.02
		3.00	1.00	0.78	0.62	0.49	0.59	0.25	0.37	0.78	0.70	0.33	0.34	0.16	0.08
		4.00	1.00	0.98	0.92	0.50	0.92	0.25	0.64	0.98	0.95	0.47	0.74	0.23	0.26
		5.00	1.00	1.00	0.99	0.50	0.99	0.25	0.72	1.00	1.00	0.50	0.95	0.25	0.53
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.73
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.03	0.02	0.05	0.03	0.05	0.02	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.60	0.05	0.04	0.09	0.05	0.08	0.03	0.05	0.04	0.03	0.03	0.02	0.01
		0.00	1.80	0.09	0.07	0.12	0.09	0.10	0.05	0.07	0.06	0.05	0.04	0.03	0.02
		0.00	2.00	0.13	0.10	0.15	0.12	0.11	0.07	0.11	0.09	0.07	0.06	0.04	0.02
		0.00	2.40	0.20	0.18	0.20	0.20	0.14	0.11	0.18	0.17	0.11	0.11	0.07	0.04
		0.00	3.00	0.31	0.29	0.26	0.32	0.16	0.18	0.28	0.29	0.17	0.20	0.09	0.07
		0.00	5.00	0.54	0.58	0.35	0.60	0.20	0.38	0.52	0.60	0.28	0.47	0.15	0.22
		0.00	7.00	0.66	0.74	0.39	0.75	0.21	0.49	0.64	0.76	0.34	0.66	0.18	0.36
		0.00	10.0	0.76	0.86	0.42	0.86	0.22	0.59	0.75	0.87	0.39	0.81	0.20	0.49
		0.00	15.0	0.84	0.93	0.45	0.93	0.23	0.66	0.83	0.94	0.42	0.91	0.21	0.60
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.03	0.04	0.01	0.01	0.00	0.01
		0.00	1.60	0.07	0.06	0.07	0.05	0.08	0.03	0.07	0.09	0.02	0.02	0.01	0.01
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.12	0.16	0.03	0.05	0.01	0.01
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.19	0.23	0.06	0.08	0.02	0.02
		0.00	2.40	0.31	0.28	0.28	0.26	0.21	0.16	0.32	0.37	0.14	0.16	0.07	0.04
		0.00	3.00	0.47	0.44	0.37	0.41	0.24	0.26	0.49	0.54	0.26	0.31	0.15	0.11
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.49	0.78	0.80	0.44	0.66	0.24	0.35
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.59	0.88	0.89	0.48	0.81	0.25	0.49
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.94	0.95	0.50	0.90	0.25	0.60
		0.00	15.0	0.97	0.97	0.50	0.96	0.25	0.70	0.97	0.98	0.50	0.95	0.25	0.67
		$h_{\overline{e}}$, h_{e^2}		3.458 7.022	3.452 7.063	3.562 4.432	3.474 4.430	3.812 3.018	3.505 2.974	4.316 7.666	4.593 7.592	5.706 7.646	6.500 7.891	7.506 7.051	8.672 7.371

Table 26: Signal Probabilities of Shewhart $\overline{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$						$\phi = 0.2,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.02	0.04	0.03	0.05	0.03	0.02	0.02	0.02	0.03	0.01	0.03
		1.00	1.00	0.06	0.05	0.09	0.07	0.12	0.06	0.03	0.04	0.03	0.06	0.02	0.05
		1.50	1.00	0.20	0.18	0.32	0.25	0.32	0.20	0.09	0.13	0.08	0.19	0.06	0.16
		2.00	1.00	0.48	0.44	0.66	0.54	0.46	0.44	0.22	0.31	0.18	0.43	0.12	0.36
		3.00	1.00	0.95	0.93	0.98	0.95	0.50	0.82	0.67	0.82	0.55	0.89	0.32	0.78
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.93	0.96	0.99	0.88	1.00	0.46	0.92
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98	1.00	1.00	0.99	1.00	0.50	0.97
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.03	0.02	0.05	0.02	0.05	0.02	0.02	0.02	0.02	0.02	0.01	0.02
		0.00	1.40	0.10	0.07	0.12	0.07	0.11	0.05	0.05	0.06	0.04	0.06	0.03	0.04
		0.00	1.60	0.18	0.14	0.20	0.14	0.16	0.10	0.10	0.12	0.07	0.12	0.05	0.09
		0.00	1.80	0.27	0.23	0.27	0.23	0.20	0.17	0.16	0.21	0.11	0.20	0.07	0.15
		0.00	2.00	0.36	0.32	0.33	0.32	0.23	0.25	0.22	0.30	0.15	0.29	0.09	0.23
		0.00	2.40	0.50	0.50	0.43	0.50	0.28	0.41	0.35	0.48	0.23	0.47	0.13	0.38
		0.00	3.00	0.65	0.70	0.54	0.70	0.32	0.58	0.51	0.68	0.34	0.68	0.19	0.56
		0.00	5.00	0.86	0.94	0.72	0.94	0.39	0.84	0.78	0.93	0.56	0.93	0.30	0.83
		0.00	7.00	0.93	0.98	0.80	0.98	0.42	0.91	0.88	0.98	0.68	0.98	0.35	0.90
		0.00	10.0	0.96	1.00	0.86	1.00	0.45	0.94	0.94	0.99	0.77	0.99	0.40	0.94
		0.00	15.0	0.98	1.00	0.91	1.00	0.46	0.97	0.97	1.00	0.85	1.00	0.43	0.97
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.01	0.02	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.05	0.03	0.06	0.01	0.05	0.01	0.04
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.10	0.05	0.12	0.03	0.11	0.01	0.09
		0.00	1.80	0.23	0.22	0.25	0.22	0.24	0.17	0.10	0.21	0.05	0.20	0.02	0.15
		0.00	2.00	0.33	0.32	0.36	0.32	0.32	0.25	0.17	0.31	0.09	0.29	0.05	0.23
		0.00	2.40	0.51	0.50	0.55	0.51	0.42	0.41	0.33	0.49	0.21	0.48	0.13	0.39
		0.00	3.00	0.71	0.70	0.74	0.71	0.48	0.59	0.56	0.69	0.44	0.69	0.30	0.57
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.89	0.94	0.85	0.93	0.49	0.83
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.97	0.98	0.95	0.98	0.50	0.90
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	0.99	1.00	0.99	0.99	0.50	0.94
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.97
		$h_{\overline{e}}$, h_{e^2}		3.473 7.072	3.372 7.081	3.575 4.448	3.230 4.395	3.816 3.021	3.045 2.931	4.423 9.829	3.618 7.240	5.777 8.423	3.671 4.624	7.524 7.164	3.571 3.083

Table 26: Signal Probabilities of Shewhart $\bar{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when $l = 4$ hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.1$						$\phi = 0.6,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.02	0.04	0.03	0.05	0.03	0.02	0.02	0.02	0.02	0.01	0.01
		1.00	1.00	0.05	0.05	0.09	0.07	0.12	0.05	0.03	0.03	0.03	0.03	0.02	0.02
		1.50	1.00	0.19	0.17	0.32	0.23	0.32	0.18	0.08	0.09	0.08	0.09	0.06	0.07
		2.00	1.00	0.46	0.40	0.66	0.50	0.46	0.40	0.19	0.22	0.18	0.22	0.12	0.16
		3.00	1.00	0.94	0.90	0.98	0.93	0.50	0.80	0.61	0.65	0.56	0.63	0.32	0.52
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.93	0.93	0.94	0.88	0.93	0.46	0.81
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98	1.00	1.00	0.99	1.00	0.50	0.92
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	0.99
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.03	0.02	0.05	0.03	0.05	0.02	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.40	0.09	0.06	0.12	0.07	0.11	0.06	0.05	0.05	0.04	0.03	0.03	0.02
		0.00	1.60	0.16	0.13	0.20	0.14	0.16	0.10	0.09	0.09	0.07	0.07	0.05	0.04
		0.00	1.80	0.25	0.20	0.27	0.22	0.20	0.16	0.15	0.15	0.11	0.11	0.07	0.07
		0.00	2.00	0.33	0.28	0.33	0.30	0.23	0.23	0.22	0.23	0.15	0.17	0.09	0.10
		0.00	2.40	0.46	0.44	0.43	0.46	0.28	0.35	0.34	0.38	0.23	0.29	0.13	0.19
		0.00	3.00	0.62	0.62	0.54	0.64	0.32	0.51	0.50	0.58	0.33	0.48	0.19	0.33
		0.00	5.00	0.84	0.90	0.72	0.90	0.39	0.78	0.78	0.89	0.56	0.84	0.30	0.67
		0.00	7.00	0.92	0.97	0.80	0.97	0.42	0.87	0.88	0.96	0.68	0.94	0.35	0.81
		0.00	10.0	0.96	0.99	0.86	0.99	0.45	0.93	0.94	0.99	0.77	0.98	0.40	0.90
		0.00	15.0	0.98	1.00	0.91	1.00	0.46	0.96	0.97	1.00	0.85	1.00	0.43	0.94
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.01	0.01	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.05	0.03	0.06	0.01	0.03	0.01	0.02
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.10	0.07	0.13	0.03	0.06	0.01	0.03
		0.00	1.80	0.23	0.22	0.25	0.22	0.24	0.17	0.13	0.22	0.05	0.11	0.02	0.06
		0.00	2.00	0.33	0.32	0.36	0.32	0.32	0.25	0.21	0.32	0.09	0.18	0.05	0.10
		0.00	2.40	0.52	0.50	0.55	0.50	0.42	0.40	0.39	0.51	0.22	0.35	0.13	0.22
		0.00	3.00	0.72	0.70	0.74	0.70	0.48	0.59	0.62	0.71	0.45	0.58	0.30	0.41
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.91	0.94	0.86	0.90	0.49	0.76
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.97	0.98	0.95	0.97	0.50	0.87
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	0.99	1.00	0.99	0.99	0.50	0.92
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.96
		$h_{\bar{e}}$, h_{e^2}		3.469 7.060	3.426 7.067	3.575 4.447	3.378 4.407	3.816 3.021	3.278 2.946	4.398 9.202	4.311 8.222	5.771 8.345	5.375 6.836	7.522 7.161	6.113 5.005

Table 26: Signal Probabilities of Shewhart $\bar{e}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 4 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.8, ψ = 0.1						ϕ = 0.8, ψ = 0.9					
				n = 2		n = 4		n = 8		n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.02	0.04	0.03	0.05	0.02	0.02	0.02	0.02	0.02	0.01	0.01
		1.00	1.00	0.05	0.05	0.09	0.06	0.12	0.05	0.04	0.04	0.04	0.03	0.02	0.02
		1.50	1.00	0.18	0.16	0.33	0.22	0.32	0.16	0.13	0.14	0.10	0.10	0.06	0.04
		2.00	1.00	0.45	0.39	0.66	0.48	0.46	0.38	0.31	0.34	0.24	0.23	0.13	0.10
		3.00	1.00	0.93	0.89	0.99	0.92	0.50	0.79	0.81	0.80	0.67	0.64	0.33	0.36
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.92	0.99	0.98	0.94	0.93	0.46	0.70
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.87
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	0.98
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.02	0.02	0.04	0.03	0.05	0.02	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.40	0.06	0.05	0.10	0.07	0.11	0.05	0.04	0.04	0.04	0.03	0.03	0.02
		0.00	1.60	0.12	0.10	0.17	0.13	0.16	0.09	0.09	0.08	0.07	0.06	0.05	0.03
		0.00	1.80	0.18	0.16	0.24	0.19	0.20	0.13	0.14	0.13	0.11	0.09	0.07	0.04
		0.00	2.00	0.25	0.22	0.30	0.26	0.23	0.18	0.20	0.19	0.15	0.13	0.09	0.06
		0.00	2.40	0.38	0.34	0.40	0.38	0.27	0.27	0.33	0.32	0.23	0.23	0.13	0.11
		0.00	3.00	0.53	0.51	0.51	0.54	0.32	0.40	0.48	0.51	0.33	0.37	0.19	0.19
		0.00	5.00	0.79	0.82	0.70	0.83	0.39	0.68	0.77	0.84	0.56	0.73	0.30	0.48
		0.00	7.00	0.88	0.93	0.78	0.93	0.42	0.81	0.87	0.94	0.68	0.89	0.35	0.67
		0.00	10.0	0.94	0.98	0.85	0.98	0.45	0.89	0.94	0.98	0.77	0.96	0.39	0.81
		0.00	15.0	0.97	0.99	0.90	1.00	0.46	0.94	0.97	1.00	0.85	0.99	0.43	0.90
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.01	0.01	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.04	0.06	0.10	0.02	0.03	0.01	0.01
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.09	0.14	0.20	0.03	0.06	0.01	0.02
		0.00	1.80	0.23	0.22	0.25	0.21	0.25	0.16	0.24	0.32	0.07	0.11	0.03	0.03
		0.00	2.00	0.33	0.32	0.36	0.31	0.32	0.24	0.36	0.44	0.12	0.19	0.05	0.06
		0.00	2.40	0.52	0.50	0.55	0.50	0.42	0.40	0.55	0.63	0.28	0.36	0.14	0.14
		0.00	3.00	0.72	0.70	0.75	0.70	0.48	0.58	0.75	0.80	0.52	0.59	0.31	0.31
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.95	0.96	0.89	0.91	0.49	0.70
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.99	0.99	0.97	0.97	0.50	0.84
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.94	1.00	1.00	0.99	0.99	0.50	0.91
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.95
		$h_{\bar{e}}$, h_{e^2}		3.458 7.022	3.452 7.063	3.562 4.432	3.474 4.430	3.812 3.018	3.505 2.974	4.316 7.666	4.593 7.592	5.706 7.646	6.500 7.891	7.506 7.051	8.672 7.371

Table 27: SSATS Values of Shewhart $\bar{X}$ & e^2 Chart Combination for Sustained Shifts in Autocorrelated Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$						$\phi = 0.2,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.5	1481.6	1481.9	1482.2	1481.5	1481.5	1481.6	1481.1	1481.4	1481.8	1481.5	1480.9
Out-of-Control SSATS	μ_X	0.25	1.00	938.1	941.6	795.4	756.5	675.5	552.9	1080.5	980.7	1101.2	834.1	1134.4	659.0
		0.50	1.00	392.7	389.2	273.3	239.3	202.1	132.9	558.6	432.3	587.8	299.7	641.5	185.3
		0.75	1.00	162.3	158.1	99.9	82.4	70.2	43.1	275.2	185.8	299.8	112.0	343.1	63.2
		1.00	1.00	72.2	69.2	41.7	33.8	29.3	20.0	139.8	85.1	158.4	47.9	189.8	28.2
		1.50	1.00	18.3	17.5	10.5	9.8	8.5	9.5	42.4	22.6	51.6	13.5	67.0	11.3
		2.00	1.00	6.2	6.4	4.1	5.2	4.6	7.2	15.6	8.3	20.4	6.3	28.5	7.8
		3.00	1.00	1.6	2.1	2.1	3.1	4.0	5.4	3.5	2.4	5.3	3.3	8.5	5.6
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.5	1.4	2.6	2.5	4.7	4.7
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.1	1.1	2.1	2.1	4.1	4.2
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	132.0	192.5	87.5	174.1	74.5	156.6	246.1	214.5	293.9	207.4	355.1	185.1
		0.00	1.40	40.4	57.8	31.3	51.9	32.2	46.3	82.5	65.0	110.4	61.8	149.7	53.6
		0.00	1.60	20.2	26.5	18.1	24.4	21.1	23.1	39.9	29.4	57.6	28.2	84.1	25.8
		0.00	1.80	12.8	15.5	12.8	14.8	16.2	15.2	23.7	16.8	36.2	16.6	55.5	16.5
		0.00	2.00	9.2	10.4	10.0	10.5	13.4	11.6	16.1	11.2	25.6	11.5	40.6	12.3
		0.00	2.40	5.9	6.1	7.2	6.7	10.5	8.5	9.4	6.4	15.7	7.2	26.3	8.8
		0.00	3.00	3.9	3.8	5.4	4.7	8.5	6.7	5.7	4.0	10.0	4.9	17.4	6.9
		0.00	5.00	2.2	2.0	3.6	3.1	6.2	5.1	2.8	2.1	5.1	3.1	9.5	5.2
		0.00	7.00	1.8	1.6	3.0	2.6	5.4	4.7	2.1	1.6	3.9	2.7	7.4	4.7
		0.00	10.0	1.5	1.4	2.7	2.4	4.9	4.4	1.6	1.4	3.2	2.4	6.1	4.4
		0.00	15.0	1.3	1.2	2.4	2.2	4.6	4.3	1.4	1.2	2.7	2.3	5.3	4.3
	σ_ε	0.00	1.20	217.9	207.1	219.8	193.7	210.1	171.3	462.9	230.5	728.9	232.9	923.0	204.3
		0.00	1.40	63.9	61.3	60.8	55.8	52.8	48.5	174.4	67.7	355.8	66.0	545.8	55.9
		0.00	1.60	28.0	27.4	25.8	25.4	21.9	23.5	76.6	29.7	175.2	28.9	308.2	25.9
		0.00	1.80	15.6	15.7	14.3	15.0	12.3	15.2	38.6	16.7	87.3	16.6	164.8	16.3
		0.00	2.00	10.2	10.4	9.3	10.5	8.4	11.5	22.3	11.0	45.9	11.3	84.5	12.2
		0.00	2.40	5.6	6.0	5.3	6.7	5.5	8.4	10.1	6.3	17.1	7.0	26.2	8.7
		0.00	3.00	3.3	3.8	3.4	4.6	4.4	6.7	5.0	3.9	7.1	4.8	9.4	6.8
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	2.0	2.0	2.7	3.1	4.2	5.1
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.4	1.6	2.2	2.6	4.0	4.7
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.2	1.4	2.1	2.4	4.0	4.4
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.1	1.2	2.0	2.2	4.0	4.3
		$h_{\bar{X}}$, h_{e^2}		3.473 7.072	3.372 7.083	3.576 4.448	3.229 4.395	3.818 3.021	3.045 2.931	4.439 9.895	3.589 7.303	5.779 8.424	3.594 4.636	7.530 7.169	3.478 3.087

Table 27: SSATS Values of Shewhart $\bar{X}$ & e^2 Chart Combination for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.1$						$\phi = 0.6,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.6	1481.6	1481.7	1481.7	1481.5	1481.6	1481.5	1481.8	1481.6	1482.0	1481.6	1480.9
Out-of-Control SSATS	μ_X	0.25	1.00	946.4	958.6	799.3	795.8	675.2	611.3	1147.2	1131.1	1127.5	1076.2	1137.9	966.5
		0.50	1.00	399.0	406.8	276.4	265.7	202.5	159.8	646.1	620.3	624.9	536.0	646.6	413.2
		0.75	1.00	166.0	168.2	101.7	94.9	70.5	53.1	337.0	309.8	326.3	247.9	347.3	173.2
		1.00	1.00	74.2	74.7	42.5	39.6	29.4	24.0	177.6	156.7	174.7	119.5	192.4	80.7
		1.50	1.00	18.9	19.3	10.7	11.3	8.5	10.3	55.9	46.4	57.6	35.0	68.0	25.8
		2.00	1.00	6.5	7.0	4.2	5.6	4.6	7.4	20.8	17.0	22.8	14.0	29.0	12.8
		3.00	1.00	1.6	2.2	2.1	3.2	4.0	5.4	4.2	3.7	5.7	4.7	8.6	7.0
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.4	1.6	2.6	3.0	4.7	5.5
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.0	1.1	2.1	2.4	4.1	4.7
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.1
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	134.9	167.7	87.9	133.5	74.6	120.0	239.4	232.8	288.9	261.6	354.1	283.4
		0.00	1.40	42.5	53.4	31.6	45.0	32.2	42.3	79.9	76.5	108.1	92.8	149.2	103.9
		0.00	1.60	21.7	26.8	18.3	24.1	21.1	23.9	38.7	36.8	56.3	47.0	83.9	53.2
		0.00	1.80	13.9	16.8	12.9	16.0	16.2	16.7	23.2	22.0	35.5	29.0	55.4	33.2
		0.00	2.00	10.0	12.0	10.1	11.9	13.4	13.1	15.9	15.0	25.1	20.2	40.6	23.5
		0.00	2.40	6.4	7.5	7.3	8.0	10.5	9.7	9.4	8.8	15.5	12.2	26.2	14.7
		0.00	3.00	4.3	4.8	5.4	5.6	8.5	7.5	5.7	5.3	9.9	7.6	17.4	10.0
		0.00	5.00	2.3	2.4	3.6	3.4	6.2	5.5	2.8	2.5	5.1	3.9	9.5	6.2
		0.00	7.00	1.8	1.8	3.0	2.8	5.4	4.9	2.1	1.9	3.9	3.1	7.4	5.3
		0.00	10.0	1.5	1.5	2.7	2.5	4.9	4.6	1.7	1.5	3.2	2.7	6.1	4.8
		0.00	15.0	1.3	1.3	2.4	2.3	4.6	4.3	1.4	1.3	2.7	2.4	5.3	4.5
	σ_ε	0.00	1.20	217.3	211.8	219.7	202.9	210.0	180.0	404.7	251.0	753.9	446.6	931.9	531.5
		0.00	1.40	63.6	62.5	60.7	58.0	52.8	50.1	142.7	72.7	372.7	152.1	550.7	183.0
		0.00	1.60	27.9	27.8	25.8	26.1	21.9	24.0	61.2	31.1	183.1	62.1	310.9	71.1
		0.00	1.80	15.6	15.9	14.2	15.3	12.3	15.4	31.1	17.2	90.7	31.4	166.1	35.2
		0.00	2.00	10.1	10.5	9.3	10.6	8.4	11.7	18.3	11.2	47.4	19.0	85.2	21.6
		0.00	2.40	5.6	6.1	5.3	6.7	5.5	8.5	8.7	6.4	17.5	10.0	26.4	12.4
		0.00	3.00	3.3	3.8	3.4	4.7	4.4	6.7	4.5	3.9	7.2	6.0	9.4	8.4
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	1.9	2.1	2.7	3.4	4.2	5.6
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.4	1.6	2.2	2.8	4.0	5.0
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.2	1.4	2.1	2.5	4.0	4.6
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.1	1.2	2.0	2.3	4.0	4.3
		$h_{\bar{X}}$, h_{e^2}		3.474 7.062	3.425 7.072	3.576 4.447	3.371 4.409	3.818 3.021	3.267 2.947	4.556 9.776	4.152 8.774	5.853 8.564	4.703 7.240	7.538 7.183	4.972 5.212

Table 27: SSATS Values of Shewhart $\bar{X}$ & e^2 Chart Combination for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8$, $\psi = 0.1$						$\phi = 0.8$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
In-Control ATS		0.00	1.00	1481.7	1481.2	1481.4	1481.5	1481.5	1481.4	1481.4	1481.9	1481.6	1481.6	1481.2	1482.0
Out-of-Control SSATS	μ_X	0.25	1.00	963.5	992.1	811.8	838.0	683.8	674.8	1201.6	1209.6	1182.1	1184.3	1167.7	1139.1
		0.50	1.00	414.3	429.4	286.0	292.9	207.4	191.9	733.7	737.5	707.1	698.3	691.7	626.3
		0.75	1.00	174.5	180.1	106.9	107.7	72.9	66.0	406.4	404.3	389.4	370.4	381.9	314.8
		1.00	1.00	78.8	80.9	45.4	45.8	30.7	29.5	224.5	219.7	216.1	197.3	215.0	162.7
		1.50	1.00	20.4	21.2	11.5	12.9	8.7	11.5	74.3	70.8	74.1	63.7	77.4	53.6
		2.00	1.00	6.9	7.6	4.3	6.0	4.7	7.7	27.2	25.7	29.2	24.8	32.8	23.4
		3.00	1.00	1.6	2.3	2.1	3.2	4.0	5.5	4.1	4.2	6.2	6.1	9.0	9.0
		4.00	1.00	1.0	1.4	2.0	2.5	4.0	4.6	1.2	1.3	2.4	3.1	4.7	6.2
		5.00	1.00	1.0	1.1	2.0	2.1	4.0	4.2	1.0	1.0	2.0	2.4	4.1	5.2
		7.00	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.2
		10.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
		15.0	1.00	1.0	1.0	2.0	2.0	4.0	4.0	1.0	1.0	2.0	2.0	4.0	4.0
	σ_α	0.00	1.20	144.9	164.1	93.1	120.4	75.8	104.7	247.6	250.3	284.8	276.2	348.9	320.3
		0.00	1.40	49.2	56.9	34.6	45.5	32.8	43.2	84.6	86.0	106.8	103.2	146.3	131.1
		0.00	1.60	26.4	30.8	20.4	26.7	21.5	26.9	41.5	42.5	56.0	54.7	82.2	73.4
		0.00	1.80	17.5	20.5	14.4	18.8	16.4	19.9	25.1	25.9	35.4	35.1	54.5	48.8
		0.00	2.00	12.8	15.2	11.3	14.6	13.6	16.0	17.2	17.9	25.2	25.2	40.0	35.9
		0.00	2.40	8.3	9.9	8.0	10.2	10.6	11.9	10.1	10.6	15.6	15.9	26.0	23.2
		0.00	3.00	5.4	6.5	5.9	7.2	8.5	9.1	6.1	6.5	10.0	10.2	17.3	15.3
		0.00	5.00	2.8	3.1	3.7	4.1	6.2	6.2	2.9	2.9	5.1	5.0	9.5	8.2
		0.00	7.00	2.1	2.2	3.1	3.3	5.5	5.3	2.1	2.1	3.9	3.7	7.4	6.4
		0.00	10.0	1.7	1.7	2.7	2.8	5.0	4.8	1.7	1.6	3.2	3.0	6.1	5.4
		0.00	15.0	1.4	1.4	2.5	2.4	4.6	4.5	1.4	1.4	2.7	2.6	5.3	4.8
	σ_ε	0.00	1.20	214.7	214.6	217.8	210.5	209.7	189.8	253.2	167.1	681.7	454.2	979.4	729.7
		0.00	1.40	62.8	63.0	60.0	59.5	52.7	52.0	72.7	44.1	310.6	155.9	592.2	337.4
		0.00	1.60	27.6	28.0	25.5	26.6	21.8	24.5	30.2	19.4	142.7	63.0	332.7	149.2
		0.00	1.80	15.4	15.9	14.1	15.5	12.3	15.7	16.2	11.4	68.1	31.4	175.7	69.1
		0.00	2.00	10.0	10.6	9.2	10.7	8.4	11.8	10.3	7.9	35.6	18.8	89.0	37.3
		0.00	2.40	5.5	6.1	5.3	6.8	5.5	8.5	5.6	4.9	13.9	9.9	27.0	17.2
		0.00	3.00	3.3	3.8	3.4	4.7	4.4	6.7	3.3	3.2	6.2	6.0	9.6	10.1
		0.00	5.00	1.6	2.0	2.2	3.0	4.0	5.1	1.6	1.9	2.6	3.4	4.3	6.0
		0.00	7.00	1.3	1.6	2.1	2.6	4.0	4.7	1.3	1.5	2.2	2.8	4.0	5.2
		0.00	10.0	1.1	1.4	2.0	2.4	4.0	4.4	1.1	1.3	2.0	2.5	4.0	4.7
		0.00	15.0	1.1	1.2	2.0	2.2	4.0	4.3	1.1	1.2	2.0	2.3	4.0	4.4
		$h_{\bar{X}}$, h_{e^2}		3.476 7.029	3.455 7.081	3.576 4.432	3.469 4.433	3.818 3.018	3.483 2.975	4.593 8.375	4.378 8.245	5.976 8.215	5.359 8.575	7.660 7.311	6.219 7.991

Table 28: Signal Probabilities of Shewhart $\bar{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.2, ψ = 0.1						ϕ = 0.2, ψ = 0.9					
				n = 2		n = 4		n = 8		n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.03	0.02	0.02	0.02	0.01	0.01
		2.00	1.00	0.14	0.07	0.16	0.05	0.12	0.03	0.06	0.06	0.05	0.04	0.03	0.02
		3.00	1.00	0.39	0.28	0.25	0.23	0.12	0.12	0.23	0.27	0.14	0.20	0.08	0.10
		4.00	1.00	0.49	0.65	0.25	0.58	0.13	0.31	0.41	0.64	0.22	0.54	0.12	0.28
		5.00	1.00	0.50	0.91	0.25	0.87	0.13	0.51	0.49	0.91	0.25	0.86	0.12	0.49
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	1.00	0.13	0.62
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.03	0.02	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.60	0.05	0.03	0.05	0.03	0.04	0.02	0.03	0.03	0.02	0.02	0.01	0.01
		0.00	1.80	0.07	0.05	0.07	0.04	0.05	0.02	0.04	0.05	0.03	0.04	0.02	0.02
		0.00	2.00	0.10	0.08	0.08	0.07	0.06	0.04	0.06	0.07	0.04	0.06	0.02	0.03
		0.00	2.40	0.15	0.14	0.11	0.12	0.07	0.07	0.10	0.13	0.06	0.11	0.03	0.06
		0.00	3.00	0.20	0.23	0.14	0.21	0.08	0.12	0.15	0.23	0.08	0.20	0.05	0.11
		0.00	5.00	0.31	0.47	0.18	0.45	0.10	0.27	0.27	0.46	0.14	0.44	0.07	0.26
		0.00	7.00	0.36	0.61	0.20	0.59	0.11	0.36	0.33	0.60	0.17	0.58	0.09	0.35
		0.00	10.0	0.40	0.72	0.21	0.71	0.11	0.43	0.38	0.71	0.19	0.70	0.10	0.43
		0.00	15.0	0.44	0.81	0.23	0.80	0.12	0.49	0.42	0.81	0.21	0.80	0.11	0.49
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.02	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.60	0.04	0.03	0.04	0.03	0.04	0.01	0.01	0.03	0.01	0.02	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.03	0.05	0.01	0.04	0.01	0.02
		0.00	2.00	0.09	0.08	0.09	0.07	0.08	0.04	0.04	0.07	0.02	0.06	0.01	0.03
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.07	0.09	0.13	0.05	0.11	0.03	0.06
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.17	0.23	0.11	0.20	0.07	0.11
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.27	0.34	0.47	0.21	0.44	0.12	0.26
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.36	0.41	0.60	0.24	0.58	0.12	0.35
		0.00	10.0	0.47	0.72	0.25	0.71	0.13	0.43	0.45	0.72	0.25	0.70	0.12	0.43
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.48	0.81	0.25	0.80	0.13	0.49
		$h_{\bar{X}}$, h_{e^2}		3.473 7.072	3.372 7.083	3.576 4.448	3.229 4.395	3.818 3.021	3.045 2.931	4.439 9.895	3.589 7.303	5.779 8.424	3.594 4.636	7.530 7.169	3.478 3.087

Table 28: Signal Probabilities of Shewhart $\bar{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.1$						$\phi = 0.6,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.03	0.02	0.02	0.01	0.01	0.01
		2.00	1.00	0.14	0.06	0.16	0.05	0.12	0.03	0.08	0.05	0.05	0.02	0.03	0.01
		3.00	1.00	0.39	0.28	0.25	0.23	0.12	0.11	0.26	0.23	0.14	0.07	0.08	0.03
		4.00	1.00	0.49	0.65	0.25	0.57	0.13	0.30	0.43	0.62	0.22	0.26	0.12	0.09
		5.00	1.00	0.50	0.91	0.25	0.87	0.13	0.50	0.49	0.92	0.25	0.63	0.12	0.24
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	0.99	0.13	0.60
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.01	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.60	0.04	0.03	0.05	0.03	0.04	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.80	0.06	0.04	0.07	0.04	0.05	0.02	0.04	0.03	0.03	0.02	0.02	0.01
		0.00	2.00	0.09	0.06	0.08	0.07	0.06	0.03	0.06	0.05	0.04	0.03	0.02	0.01
		0.00	2.40	0.13	0.12	0.11	0.12	0.07	0.06	0.09	0.09	0.06	0.06	0.03	0.03
		0.00	3.00	0.19	0.20	0.13	0.20	0.08	0.11	0.14	0.17	0.08	0.12	0.05	0.05
		0.00	5.00	0.30	0.44	0.18	0.44	0.10	0.25	0.26	0.40	0.14	0.33	0.07	0.17
		0.00	7.00	0.36	0.58	0.20	0.58	0.11	0.34	0.33	0.55	0.17	0.48	0.09	0.27
		0.00	10.0	0.40	0.70	0.21	0.70	0.11	0.42	0.38	0.67	0.19	0.62	0.10	0.36
		0.00	15.0	0.43	0.79	0.23	0.79	0.12	0.49	0.42	0.78	0.21	0.74	0.11	0.44
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.60	0.04	0.03	0.04	0.02	0.04	0.01	0.02	0.02	0.01	0.01	0.00	0.01
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.03	0.04	0.01	0.02	0.01	0.01
		0.00	2.00	0.09	0.08	0.09	0.07	0.08	0.04	0.05	0.07	0.02	0.03	0.01	0.01
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.07	0.10	0.12	0.05	0.06	0.03	0.03
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.18	0.22	0.11	0.13	0.07	0.06
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.27	0.35	0.46	0.21	0.36	0.12	0.19
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.36	0.41	0.60	0.24	0.51	0.12	0.29
		0.00	10.0	0.47	0.72	0.25	0.71	0.13	0.43	0.46	0.71	0.25	0.65	0.12	0.38
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.48	0.81	0.25	0.76	0.13	0.45
		$h_{\bar{X}}$, h_{e^2}		3.474 7.062	3.425 7.072	3.576 4.447	3.371 4.409	3.818 3.021	3.267 2.947	4.556 9.776	4.152 8.774	5.853 8.564	4.703 7.240	7.538 7.183	4.972 5.212

Table 28: Signal Probabilities of Shewhart $\bar{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8,\ \psi = 0.1$						$\phi = 0.8,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00
		1.00	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		1.50	1.00	0.05	0.03	0.08	0.02	0.08	0.01	0.05	0.03	0.02	0.01	0.01	0.01
		2.00	1.00	0.14	0.06	0.16	0.05	0.12	0.03	0.13	0.07	0.06	0.02	0.03	0.01
		3.00	1.00	0.39	0.28	0.25	0.22	0.12	0.11	0.37	0.37	0.16	0.06	0.08	0.01
		4.00	1.00	0.49	0.65	0.25	0.57	0.13	0.30	0.49	0.84	0.23	0.24	0.12	0.04
		5.00	1.00	0.50	0.92	0.25	0.87	0.13	0.50	0.50	0.99	0.25	0.65	0.12	0.10
		7.00	1.00	0.50	1.00	0.25	1.00	0.13	0.62	0.50	1.00	0.25	1.00	0.13	0.50
		10.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
		15.0	1.00	0.50	1.00	0.25	1.00	0.13	0.63	0.50	1.00	0.25	1.00	0.13	0.63
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.01	0.01	0.03	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00
		0.00	1.60	0.03	0.02	0.04	0.02	0.04	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.80	0.05	0.03	0.06	0.03	0.05	0.02	0.03	0.03	0.03	0.02	0.02	0.01
		0.00	2.00	0.06	0.05	0.08	0.05	0.06	0.03	0.05	0.04	0.04	0.02	0.02	0.01
		0.00	2.40	0.10	0.08	0.10	0.09	0.07	0.05	0.09	0.07	0.05	0.05	0.03	0.01
		0.00	3.00	0.16	0.15	0.13	0.17	0.08	0.08	0.14	0.14	0.08	0.09	0.05	0.03
		0.00	5.00	0.27	0.38	0.17	0.39	0.10	0.21	0.25	0.36	0.14	0.27	0.07	0.10
		0.00	7.00	0.33	0.52	0.20	0.54	0.11	0.30	0.32	0.50	0.17	0.42	0.09	0.19
		0.00	10.0	0.38	0.65	0.21	0.66	0.11	0.39	0.37	0.64	0.19	0.57	0.10	0.28
		0.00	15.0	0.42	0.76	0.22	0.77	0.12	0.46	0.41	0.75	0.21	0.70	0.11	0.38
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.02	0.00	0.01	0.00	0.00
		0.00	1.60	0.04	0.03	0.04	0.02	0.04	0.01	0.03	0.03	0.01	0.01	0.00	0.00
		0.00	1.80	0.06	0.05	0.06	0.04	0.06	0.02	0.05	0.06	0.01	0.02	0.01	0.01
		0.00	2.00	0.09	0.08	0.09	0.06	0.08	0.03	0.08	0.09	0.03	0.03	0.01	0.01
		0.00	2.40	0.15	0.14	0.14	0.12	0.10	0.06	0.15	0.17	0.06	0.06	0.03	0.01
		0.00	3.00	0.23	0.23	0.19	0.21	0.12	0.12	0.23	0.27	0.12	0.13	0.07	0.03
		0.00	5.00	0.38	0.47	0.24	0.45	0.12	0.26	0.38	0.52	0.22	0.36	0.12	0.14
		0.00	7.00	0.43	0.61	0.25	0.59	0.12	0.35	0.44	0.64	0.24	0.51	0.12	0.24
		0.00	10.0	0.47	0.72	0.25	0.71	0.13	0.43	0.47	0.75	0.25	0.65	0.12	0.34
		0.00	15.0	0.48	0.81	0.25	0.80	0.13	0.49	0.49	0.83	0.25	0.76	0.13	0.43
		$h_{\bar{X}}$, h_{e^2}		3.476 7.029	3.455 7.081	3.576 4.432	3.469 4.433	3.818 3.018	3.483 2.975	4.593 8.375	4.378 8.245	5.976 8.215	5.359 8.575	7.660 7.311	6.219 7.991

Table 29: Signal Probabilities of Shewhart $\overline{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 2 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$						$\phi = 0.2,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.75	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.02	0.02	0.02	0.01	0.02	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.05	0.07	0.04	0.06	0.03	0.04
		2.00	1.00	0.28	0.22	0.33	0.19	0.23	0.11	0.13	0.18	0.09	0.16	0.06	0.10
		3.00	1.00	0.78	0.64	0.49	0.62	0.25	0.39	0.45	0.57	0.27	0.55	0.16	0.35
		4.00	1.00	0.98	0.93	0.50	0.93	0.25	0.64	0.82	0.89	0.44	0.89	0.23	0.62
		5.00	1.00	1.00	1.00	0.50	1.00	0.25	0.72	0.97	0.99	0.49	0.99	0.25	0.72
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.05	0.03	0.06	0.03	0.06	0.02	0.03	0.03	0.02	0.02	0.01	0.02
		0.00	1.60	0.10	0.07	0.10	0.06	0.08	0.04	0.05	0.06	0.03	0.05	0.02	0.03
		0.00	1.80	0.15	0.11	0.14	0.10	0.10	0.06	0.08	0.10	0.05	0.09	0.03	0.05
		0.00	2.00	0.20	0.17	0.17	0.15	0.11	0.09	0.12	0.16	0.07	0.14	0.04	0.08
		0.00	2.40	0.29	0.28	0.22	0.26	0.14	0.16	0.19	0.27	0.11	0.24	0.07	0.15
		0.00	3.00	0.41	0.44	0.27	0.42	0.16	0.27	0.30	0.42	0.17	0.40	0.09	0.25
		0.00	5.00	0.62	0.74	0.36	0.72	0.20	0.49	0.53	0.73	0.28	0.71	0.15	0.48
		0.00	7.00	0.73	0.86	0.40	0.85	0.21	0.59	0.66	0.85	0.34	0.84	0.18	0.59
		0.00	10.0	0.81	0.93	0.43	0.92	0.22	0.66	0.76	0.92	0.39	0.92	0.20	0.65
		0.00	15.0	0.87	0.97	0.45	0.96	0.23	0.70	0.83	0.97	0.42	0.96	0.21	0.70
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.01	0.03	0.01	0.02	0.00	0.02
		0.00	1.60	0.07	0.07	0.07	0.06	0.08	0.03	0.03	0.06	0.01	0.05	0.01	0.03
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.05	0.10	0.02	0.09	0.01	0.05
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.09	0.16	0.04	0.13	0.02	0.08
		0.00	2.40	0.30	0.28	0.27	0.26	0.21	0.16	0.18	0.27	0.11	0.24	0.07	0.15
		0.00	3.00	0.47	0.44	0.37	0.42	0.24	0.27	0.33	0.43	0.22	0.40	0.15	0.26
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.50	0.67	0.74	0.43	0.72	0.24	0.49
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.60	0.82	0.86	0.48	0.84	0.25	0.59
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.91	0.93	0.49	0.92	0.25	0.66
		0.00	15.0	0.97	0.97	0.50	0.97	0.25	0.70	0.96	0.97	0.50	0.96	0.25	0.70
		$h_{\overline{X}}$, h_{e^2}		3.473 7.072	3.372 7.083	3.576 4.448	3.229 4.395	3.818 3.021	3.045 2.931	4.439 9.895	3.589 7.303	5.779 8.424	3.594 4.636	7.530 7.169	3.478 3.087

Table 29: Signal Probabilities of Shewhart $\bar{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.1$						$\phi = 0.6,\ \psi = 0.9$					
				n = 2		n = 4		n = 8		n = 2		n = 4		n = 8	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.75	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.02	0.02	0.02	0.01	0.01	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.06	0.06	0.04	0.03	0.03	0.02
		2.00	1.00	0.28	0.21	0.33	0.18	0.23	0.11	0.15	0.15	0.09	0.07	0.06	0.03
		3.00	1.00	0.78	0.63	0.49	0.60	0.25	0.38	0.52	0.50	0.28	0.30	0.16	0.14
		4.00	1.00	0.98	0.92	0.50	0.92	0.25	0.64	0.87	0.83	0.44	0.68	0.23	0.38
		5.00	1.00	1.00	0.99	0.50	0.99	0.25	0.72	0.99	0.97	0.49	0.92	0.25	0.62
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.74
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.04	0.03	0.06	0.03	0.06	0.02	0.03	0.02	0.02	0.02	0.01	0.01
		0.00	1.60	0.08	0.06	0.10	0.06	0.08	0.04	0.05	0.04	0.03	0.03	0.02	0.02
		0.00	1.80	0.13	0.10	0.13	0.10	0.10	0.06	0.08	0.07	0.05	0.05	0.03	0.02
		0.00	2.00	0.18	0.14	0.17	0.15	0.11	0.09	0.11	0.11	0.07	0.07	0.04	0.04
		0.00	2.40	0.27	0.24	0.22	0.24	0.14	0.14	0.19	0.20	0.11	0.14	0.07	0.07
		0.00	3.00	0.38	0.38	0.27	0.38	0.16	0.23	0.29	0.33	0.17	0.25	0.09	0.13
		0.00	5.00	0.60	0.68	0.36	0.68	0.20	0.45	0.53	0.65	0.28	0.56	0.15	0.34
		0.00	7.00	0.71	0.82	0.40	0.82	0.21	0.56	0.65	0.80	0.34	0.73	0.18	0.47
		0.00	10.0	0.80	0.90	0.43	0.90	0.22	0.64	0.75	0.90	0.39	0.86	0.20	0.58
		0.00	15.0	0.86	0.96	0.45	0.96	0.23	0.69	0.83	0.95	0.42	0.93	0.21	0.66
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.02	0.02	0.01	0.01	0.00	0.01
		0.00	1.60	0.07	0.06	0.07	0.06	0.08	0.03	0.03	0.05	0.01	0.02	0.01	0.01
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.06	0.09	0.02	0.04	0.01	0.02
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.10	0.15	0.04	0.07	0.02	0.03
		0.00	2.40	0.31	0.28	0.27	0.26	0.21	0.16	0.20	0.26	0.10	0.14	0.07	0.07
		0.00	3.00	0.47	0.44	0.37	0.42	0.24	0.27	0.36	0.42	0.22	0.28	0.15	0.15
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.50	0.69	0.73	0.43	0.63	0.24	0.39
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.59	0.83	0.85	0.48	0.79	0.25	0.52
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.91	0.92	0.49	0.89	0.25	0.62
		0.00	15.0	0.97	0.97	0.50	0.96	0.25	0.70	0.96	0.97	0.50	0.95	0.25	0.68
		$h_{\bar{X}}$, h_{e^2}		3.474 7.062	3.425 7.072	3.576 4.447	3.371 4.409	3.818 3.021	3.267 2.947	4.556 9.776	4.152 8.774	5.853 8.564	4.703 7.240	7.538 7.183	4.972 5.212

Table 29: Signal Probabilities of Shewhart $\overline{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8,\ \psi = 0.1$						$\phi = 0.8,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00
		0.75	1.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.03	0.02	0.05	0.02	0.06	0.01	0.03	0.03	0.02	0.01	0.01	0.01
		1.50	1.00	0.11	0.08	0.16	0.07	0.16	0.04	0.10	0.09	0.05	0.03	0.03	0.01
		2.00	1.00	0.28	0.20	0.33	0.17	0.23	0.10	0.25	0.23	0.11	0.07	0.06	0.02
		3.00	1.00	0.78	0.62	0.49	0.59	0.25	0.37	0.74	0.65	0.32	0.29	0.16	0.07
		4.00	1.00	0.98	0.92	0.50	0.92	0.25	0.64	0.98	0.93	0.47	0.69	0.23	0.22
		5.00	1.00	1.00	0.99	0.50	0.99	0.25	0.72	1.00	1.00	0.50	0.92	0.25	0.48
		7.00	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.72
		10.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
		15.0	1.00	1.00	1.00	0.50	1.00	0.25	0.75	1.00	1.00	0.50	1.00	0.25	0.75
	σ_α	0.00	1.20	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.03	0.02	0.05	0.03	0.05	0.02	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.60	0.06	0.05	0.09	0.05	0.08	0.03	0.04	0.04	0.03	0.03	0.02	0.01
		0.00	1.80	0.09	0.07	0.12	0.09	0.10	0.05	0.07	0.06	0.05	0.04	0.03	0.02
		0.00	2.00	0.13	0.11	0.15	0.12	0.11	0.07	0.10	0.09	0.07	0.06	0.04	0.02
		0.00	2.40	0.21	0.18	0.20	0.20	0.14	0.11	0.17	0.16	0.11	0.11	0.07	0.04
		0.00	3.00	0.31	0.30	0.25	0.32	0.16	0.18	0.27	0.28	0.16	0.19	0.09	0.07
		0.00	5.00	0.55	0.59	0.35	0.60	0.20	0.38	0.51	0.59	0.28	0.46	0.15	0.21
		0.00	7.00	0.67	0.75	0.39	0.75	0.21	0.49	0.64	0.75	0.34	0.65	0.18	0.34
		0.00	10.0	0.76	0.86	0.42	0.86	0.22	0.59	0.74	0.87	0.38	0.80	0.20	0.47
		0.00	15.0	0.84	0.93	0.45	0.93	0.23	0.66	0.82	0.94	0.42	0.90	0.21	0.59
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00
		0.00	1.40	0.03	0.03	0.03	0.03	0.04	0.02	0.02	0.04	0.01	0.01	0.00	0.01
		0.00	1.60	0.07	0.06	0.07	0.05	0.08	0.03	0.06	0.08	0.01	0.02	0.01	0.01
		0.00	1.80	0.12	0.11	0.12	0.10	0.12	0.06	0.10	0.14	0.03	0.04	0.01	0.01
		0.00	2.00	0.18	0.17	0.18	0.15	0.16	0.09	0.16	0.21	0.05	0.06	0.02	0.02
		0.00	2.40	0.31	0.28	0.28	0.26	0.21	0.16	0.29	0.34	0.12	0.14	0.06	0.04
		0.00	3.00	0.47	0.44	0.37	0.41	0.24	0.26	0.46	0.51	0.24	0.28	0.15	0.09
		0.00	5.00	0.76	0.74	0.48	0.73	0.25	0.49	0.76	0.79	0.44	0.63	0.24	0.32
		0.00	7.00	0.87	0.86	0.49	0.85	0.25	0.59	0.87	0.89	0.48	0.79	0.25	0.47
		0.00	10.0	0.93	0.93	0.50	0.92	0.25	0.66	0.94	0.94	0.49	0.89	0.25	0.59
		0.00	15.0	0.97	0.97	0.50	0.96	0.25	0.70	0.97	0.97	0.50	0.95	0.25	0.66
		$h_{\overline{X}}$, h_{e^2}		3.476 7.029	3.455 7.081	3.576 4.432	3.469 4.433	3.818 3.018	3.483 2.975	4.593 8.375	4.378 8.245	5.976 8.215	5.359 8.575	7.660 7.311	6.219 7.991

Table 30: Signal Probabilities of Shewhart $\overline{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$, $\psi = 0.1$						$\phi = 0.2$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.03	0.04	0.03	0.05	0.03	0.02	0.02	0.02	0.03	0.01	0.03
		1.00	1.00	0.06	0.05	0.09	0.07	0.12	0.06	0.03	0.05	0.03	0.06	0.02	0.05
		1.50	1.00	0.20	0.19	0.32	0.25	0.32	0.20	0.09	0.15	0.08	0.20	0.06	0.16
		2.00	1.00	0.48	0.45	0.66	0.54	0.46	0.44	0.23	0.37	0.18	0.44	0.12	0.37
		3.00	1.00	0.95	0.93	0.98	0.95	0.50	0.82	0.69	0.87	0.55	0.90	0.32	0.78
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.93	0.96	1.00	0.88	1.00	0.46	0.92
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98	1.00	1.00	0.99	1.00	0.50	0.97
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.03	0.02	0.05	0.02	0.05	0.02	0.02	0.02	0.02	0.02	0.01	0.02
		0.00	1.40	0.10	0.07	0.12	0.07	0.11	0.05	0.05	0.06	0.04	0.06	0.03	0.04
		0.00	1.60	0.18	0.14	0.20	0.14	0.16	0.10	0.10	0.12	0.07	0.12	0.05	0.09
		0.00	1.80	0.27	0.23	0.27	0.23	0.20	0.17	0.16	0.21	0.11	0.20	0.07	0.15
		0.00	2.00	0.36	0.32	0.33	0.32	0.23	0.25	0.22	0.30	0.15	0.29	0.09	0.23
		0.00	2.40	0.50	0.50	0.43	0.50	0.28	0.41	0.35	0.48	0.23	0.47	0.13	0.38
		0.00	3.00	0.65	0.70	0.54	0.70	0.32	0.58	0.51	0.69	0.34	0.68	0.19	0.56
		0.00	5.00	0.86	0.94	0.72	0.94	0.39	0.83	0.78	0.93	0.56	0.93	0.30	0.83
		0.00	7.00	0.93	0.98	0.80	0.98	0.42	0.91	0.88	0.98	0.68	0.98	0.35	0.90
		0.00	10.0	0.96	1.00	0.86	1.00	0.45	0.94	0.94	0.99	0.77	0.99	0.40	0.94
		0.00	15.0	0.98	1.00	0.91	1.00	0.46	0.97	0.97	1.00	0.85	1.00	0.43	0.97
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.01	0.02	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.05	0.03	0.06	0.01	0.05	0.01	0.04
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.10	0.05	0.12	0.03	0.11	0.01	0.09
		0.00	1.80	0.23	0.22	0.25	0.22	0.24	0.17	0.10	0.21	0.05	0.20	0.02	0.15
		0.00	2.00	0.33	0.32	0.36	0.32	0.32	0.25	0.17	0.31	0.09	0.29	0.05	0.23
		0.00	2.40	0.51	0.50	0.55	0.51	0.42	0.41	0.33	0.49	0.21	0.48	0.13	0.39
		0.00	3.00	0.71	0.70	0.74	0.71	0.48	0.59	0.56	0.69	0.44	0.69	0.30	0.57
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.89	0.94	0.85	0.93	0.49	0.83
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.97	0.98	0.95	0.98	0.50	0.90
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	0.99	1.00	0.99	0.99	0.50	0.94
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.97
		$h_{\overline{X}}$, h_{e^2}		3.473 7.072	3.372 7.083	3.576 4.448	3.229 4.395	3.818 3.021	3.045 2.931	4.439 9.895	3.589 7.303	5.779 8.424	3.594 4.636	7.530 7.169	3.478 3.087

Table 30: Signal Probabilities of Shewhart $\overline{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 4 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.1$						$\phi = 0.6,\ \psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.02	0.04	0.03	0.05	0.03	0.02	0.02	0.02	0.02	0.01	0.01
		1.00	1.00	0.06	0.05	0.09	0.07	0.12	0.05	0.03	0.03	0.03	0.03	0.02	0.02
		1.50	1.00	0.20	0.18	0.32	0.23	0.32	0.18	0.09	0.11	0.08	0.10	0.06	0.07
		2.00	1.00	0.47	0.43	0.66	0.51	0.46	0.40	0.22	0.26	0.18	0.23	0.12	0.16
		3.00	1.00	0.94	0.91	0.98	0.93	0.50	0.80	0.66	0.71	0.56	0.64	0.32	0.50
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.93	0.95	0.96	0.88	0.93	0.46	0.80
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98	1.00	1.00	0.99	1.00	0.50	0.91
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	0.99
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.03	0.02	0.05	0.03	0.05	0.02	0.02	0.02	0.02	0.02	0.01	0.01
		0.00	1.40	0.09	0.07	0.12	0.08	0.11	0.06	0.05	0.05	0.04	0.04	0.03	0.02
		0.00	1.60	0.16	0.13	0.20	0.14	0.16	0.10	0.10	0.09	0.07	0.07	0.05	0.04
		0.00	1.80	0.25	0.21	0.27	0.22	0.20	0.16	0.16	0.16	0.11	0.11	0.07	0.07
		0.00	2.00	0.33	0.29	0.33	0.31	0.23	0.23	0.22	0.23	0.15	0.17	0.09	0.10
		0.00	2.40	0.47	0.44	0.43	0.46	0.28	0.35	0.35	0.38	0.23	0.29	0.13	0.18
		0.00	3.00	0.62	0.63	0.54	0.64	0.32	0.51	0.50	0.57	0.33	0.47	0.19	0.32
		0.00	5.00	0.84	0.90	0.72	0.90	0.39	0.78	0.78	0.88	0.56	0.83	0.30	0.66
		0.00	7.00	0.92	0.97	0.80	0.97	0.42	0.87	0.88	0.96	0.68	0.94	0.35	0.81
		0.00	10.0	0.96	0.99	0.86	0.99	0.45	0.93	0.94	0.99	0.77	0.98	0.40	0.89
		0.00	15.0	0.98	1.00	0.91	1.00	0.46	0.96	0.97	1.00	0.85	1.00	0.43	0.94
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.01	0.01	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.05	0.03	0.05	0.01	0.02	0.01	0.01
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.10	0.06	0.11	0.02	0.05	0.01	0.03
		0.00	1.80	0.23	0.22	0.25	0.22	0.24	0.17	0.12	0.20	0.05	0.10	0.02	0.05
		0.00	2.00	0.33	0.32	0.36	0.32	0.32	0.25	0.19	0.29	0.08	0.16	0.04	0.09
		0.00	2.40	0.52	0.50	0.55	0.50	0.42	0.40	0.37	0.48	0.21	0.32	0.13	0.20
		0.00	3.00	0.72	0.70	0.74	0.70	0.48	0.59	0.59	0.69	0.43	0.55	0.30	0.39
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.91	0.93	0.85	0.89	0.49	0.75
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.97	0.98	0.95	0.97	0.50	0.86
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	0.99	0.99	0.99	0.99	0.50	0.92
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.96
		$h_{\overline{X}}$, h_{e^2}		3.474 7.062	3.425 7.072	3.576 4.447	3.371 4.409	3.818 3.021	3.267 2.947	4.556 9.776	4.152 8.774	5.853 8.564	4.703 7.240	7.538 7.183	4.972 5.212

Table 30: Signal Probabilities of Shewhart $\bar{X}$ & e^2 Chart Combination for Transient Shifts in Autocorrelated Case when l = 4 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.8$, $\psi = 0.1$						$\phi = 0.8$, $\psi = 0.9$					
				$n = 2$		$n = 4$		$n = 8$		$n = 2$		$n = 4$		$n = 8$	
				Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling	Conc. Sampling	Disp. Sampling
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.50	1.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.03	0.02	0.04	0.03	0.05	0.02	0.02	0.02	0.02	0.02	0.01	0.01
		1.00	1.00	0.06	0.05	0.09	0.06	0.12	0.05	0.04	0.04	0.03	0.03	0.02	0.02
		1.50	1.00	0.20	0.17	0.32	0.22	0.32	0.16	0.12	0.13	0.10	0.09	0.06	0.04
		2.00	1.00	0.47	0.41	0.66	0.48	0.46	0.38	0.30	0.32	0.23	0.22	0.13	0.09
		3.00	1.00	0.94	0.90	0.98	0.92	0.50	0.79	0.80	0.79	0.64	0.61	0.33	0.33
		4.00	1.00	1.00	1.00	1.00	1.00	0.50	0.92	0.99	0.98	0.93	0.92	0.46	0.67
		5.00	1.00	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.85
		7.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	0.97
		10.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.02	0.02	0.04	0.03	0.05	0.02	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.40	0.07	0.06	0.10	0.07	0.11	0.05	0.04	0.04	0.04	0.03	0.03	0.02
		0.00	1.60	0.12	0.11	0.17	0.13	0.16	0.09	0.09	0.08	0.07	0.06	0.04	0.03
		0.00	1.80	0.19	0.16	0.24	0.19	0.20	0.13	0.14	0.13	0.10	0.09	0.07	0.04
		0.00	2.00	0.26	0.23	0.30	0.26	0.23	0.18	0.20	0.19	0.14	0.13	0.09	0.06
		0.00	2.40	0.38	0.35	0.40	0.39	0.27	0.27	0.32	0.32	0.22	0.22	0.13	0.10
		0.00	3.00	0.53	0.52	0.51	0.54	0.32	0.40	0.48	0.50	0.33	0.37	0.19	0.18
		0.00	5.00	0.79	0.82	0.70	0.83	0.39	0.68	0.76	0.83	0.56	0.72	0.30	0.46
		0.00	7.00	0.89	0.93	0.78	0.93	0.42	0.81	0.87	0.94	0.67	0.88	0.35	0.65
		0.00	10.0	0.94	0.98	0.85	0.98	0.45	0.89	0.93	0.98	0.77	0.96	0.39	0.80
		0.00	15.0	0.97	0.99	0.90	1.00	0.46	0.94	0.97	1.00	0.84	0.99	0.43	0.89
	σ_ε	0.00	1.20	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.00	0.01
		0.00	1.40	0.06	0.06	0.07	0.06	0.07	0.04	0.05	0.08	0.01	0.02	0.01	0.01
		0.00	1.60	0.14	0.13	0.15	0.13	0.15	0.09	0.12	0.17	0.03	0.05	0.01	0.02
		0.00	1.80	0.23	0.22	0.25	0.21	0.24	0.16	0.21	0.29	0.06	0.09	0.02	0.03
		0.00	2.00	0.33	0.32	0.36	0.31	0.32	0.24	0.32	0.40	0.10	0.16	0.04	0.05
		0.00	2.40	0.52	0.50	0.55	0.50	0.42	0.40	0.51	0.60	0.25	0.32	0.13	0.12
		0.00	3.00	0.72	0.70	0.75	0.70	0.48	0.58	0.72	0.78	0.49	0.55	0.29	0.27
		0.00	5.00	0.94	0.94	0.95	0.94	0.50	0.84	0.94	0.96	0.87	0.89	0.48	0.68
		0.00	7.00	0.98	0.98	0.99	0.98	0.50	0.91	0.98	0.99	0.96	0.97	0.50	0.82
		0.00	10.0	1.00	1.00	1.00	1.00	0.50	0.95	1.00	1.00	0.99	0.99	0.50	0.90
		0.00	15.0	1.00	1.00	1.00	1.00	0.50	0.97	1.00	1.00	1.00	1.00	0.50	0.95
		$h_{\bar{X}}$, h_{e^2}		3.476 7.029	3.455 7.081	3.576 4.432	3.469 4.433	3.818 3.018	3.483 2.975	4.593 8.375	4.378 8.245	5.976 8.215	5.359 8.575	7.660 7.311	6.219 7.991

Chapter 7

Performance of EWMA Residuals Chart Combination

Similar to the previous discussion in Chapter 6 on the Shewhart residuals chart combinations, the performance of various sampling plans for the EWMA residuals chart combination is investigated in this chapter. The EWMA residuals chart combination includes the EWMA control chart of the residuals (the EWMA_e chart) for detecting shifts in process mean and the EWMA control chart of the squared residuals (the EWMA_{e^2} chart). The evaluations are based on both sustained shifts and transient shifts in the AR(1) plus error process.

7.1 Performance of EWMA Residuals Chart Combination

7.1.1 Sustained Shifts

Table 31 provides SSATS values for sustained shifts in μ_X, σ_α and σ_ε for the EWMA_e and EWMA_{e^2} chart combination. The three sampling plans considered are individual sampling (n = 1), concentrated sampling with n = 4, and dispersed sampling with n = 4. These are the same sampling plans considered in Chapter 5 when the EWMA observations chart combination is investigated. The smoothing parameters used in this chapter are also λ_1 = 0.026 and λ_4 = 0.1.

When autocorrelation is low, Table 31 shows that individual sampling is slightly better than the other two sampling plans with n = 4 in detecting small or moderate sustained shifts in both mean and variance, and it is much better in detecting large sustained shifts. For example, when ϕ = 0.2 and ϕ = 0.9, an average of 54.4 hours is needed for individual sampling to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, while concentrated sampling and dispersed sampling requires 119.8 hours and 55.6 hours, respectively. Similarly, to detect a sustained shift in σ_X of size $1.4\sigma_{X_0}$, it takes individual sampling an average of 30.4 hours if the shift is due to σ_α and an average of 28.1 hours if the shift is due to σ_ε, while concentrated sampling with n = 4 needs 79.7 hours and 11.5 hours on average, respectively, and dispersed sampling with n = 4 needs 34.1 hours and 34 hours on average, respectively.

When the autocorrelation in the process increases, the advantage of individual sampling over the other two sampling plans is much clearer, especially when sustained shifts in process variance are of interest. For example, when ϕ = 0.8 and ϕ = 0.9, it only takes individual sampling 41.4 hours on average to detect a sustained shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_α. But with n = 4 it takes concentrated sampling 81.2 hours and dispersed sampling 67.9 hours, on average, to detect the same shift. If the shift is due to σ_ε, individual sampling only requires an average of 9.7 hours to detect this shift, while the other two sampling plans need 77.6 hours and 47.1 hours, respectively.

Thus the most efficient sampling plan for the EWMA residuals chart combination is n = 1, which gives better performance in detecting sustained shifts in both process mean and variance. This is the same conclusion reached previously for detecting sustained shifts when the EWMA observations chart combination is used.

7.1.2 Transient Shifts

Tables 32 – 34 give probabilities of a signal for various sizes of transient shifts in the process mean or standard deviation, for the EWMA residuals chart combinations considered in previous section. The structure of Tables 32 – 34 is the same as the structure of Tables 16 – 18, which give transient-shift signal probabilities for the EWMA observations chart combination in Chapter 5.

The conclusions from Table 32 – 34 for the EWMA residuals chart combination are also of the same as the conclusions for the EWMA observations chart combination in Tables 16 – 18. When a transient shift is present, the most efficient sampling plan for the EWMA observations chart combination is $n = 1$, which gives best overall performance in detecting transient shifts of various sizes and durations in μ_X and σ_X. This conclusion for transient shifts is also the same as the conclusion for sustained shifts. Thus individual sampling is the best sampling plan for detecting both sustained and transient shifts in μ_X and σ_X for the EWMA residuals chart combination.

7.2 Most Efficient Residuals Chart Combination and Sampling Plan

The discussion in Section 6.2 shows that concentrated sampling with $n = 2$ is the most efficient sampling plan for Shewhart residuals chart combination. Section 7.1 shows that individual sampling is the most efficient sampling plan for EWMA residuals chart combination. Then if both of these two sampling plans are feasible in a process, one may want to know which residuals chart combination is more efficient. Comparisons between Tables 19 – 20 and Table 31 for sustained shifts and comparisons between Tables 21 – 26 and Tables 32 – 34 for transient shifts provide the answer.

For sustained shifts, Tables 19 – 20 and Table 31 show that the EWMA_e and EWMA_{e^2} chart combination with individual sampling performs much better than the Shewhart e and e^2 chart combination with concentrated sampling and $n = 2$ in detecting sustained shifts in both in μ_X and σ_X at all levels of autocorrelation.

The comparisons between two chart combinations for transient shifts are given in Tables 21 – 26 and Tables 32 – 34. When the duration of transient shifts is $l = 1$ hour, the EWMA_e and EWMA_{e^2} chart combination with individual sampling is much better than the Shewhart $\bar{R}$ and R^2 chart combination with concentrated sampling and $n = 2$ in detecting transient shifts in both in μ_X and σ_X at all levels of autocorrelation. When the duration of transients shifts gets longer, say 2 or 4 hours, the performance of the Shewhart e and e^2 chart combination with concentrated sampling and $n = 2$ is slightly better than the EWMA_e and EWMA_{e^2} chart combination with individual sampling when the transient shift is small.

However, because of the dominant advantage of the EWMA_e and EWMA_{e^2} chart combination with individual sampling in detecting sustained shifts, it is considered to be the residuals chart combination and sampling plan with the best overall efficiency in detecting sustained shifts and transient shifts.

7.3 Most Efficient EWMA Chart Combination and Sampling Plan

7.3.1 Comparisons between EWMA Observations Chart Combination and EWMA Residuals Chart Combination

Another interesting comparison is between the EWMA observations chart combination and the EWMA residuals chart combination. The most efficient sampling plan for both chart combinations is individual sampling. Tables 13 and 31 show that when autocorrelation is low, the EWMA_X and EWMA_{X^2} chart combination and the EWMA_e and EWMA_{e^2} chart combination have similar performance in detecting sustained shifts in both μ_X and σ_X. When the autocorrelation increases, the EWMA_e and EWMA_{e^2} chart combination performs a little bit worse than the EWMA_X and EWMA_{X^2} chart combination in detecting small sustained shifts in μ_X, but performs much better in detecting small sustained shifts in σ_X, especially when the shift is due to σ_α. For example, when ϕ = 0.2 and ϕ = 0.9, the EWMA_X and EWMA_{X^2} chart combination with individual sampling requires average 53.5 hours to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, while the EWMA_e and EWMA_{e^2} chart combination requires average 54.4 hours to detect the same shift. When ϕ = 0.8 and ϕ = 0.9, the SSATS values to detect the same shift for the two control chart combinations are 224.5 hours and 249.5 hours, respectively. However, if detecting a sustained shift due to σ_ε in σ_X of size $1.4\sigma_{X_0}$ is of interest, the EWMA_X and EWMA_{X^2} chart combination and the EWMA_e and EWMA_{e^2} chart combination require 32.4 hours and 28.1 hours on average when ϕ = 0.2 and ψ = 0.9, respectively, or 129.8 hours and 9.7 hours on average when ϕ = 0.8 and ψ = 0.9, respectively.

The signal probabilities for the two chart combinations for detecting transient shifts are given in Tables 16 – 18 and Tables 32 – 34. It appears that for either a short or a long duration of the transient shift, the performance of the EWMA_e and EWMA_{e^2} chart combination with individual sampling and the performance of the EWMA_X and EWMA_{X^2} chart combination with individual sampling is about the same when the autocorrelation is low, but the EWMA_e and EWMA_{e^2} chart combination performs much better when the autocorrelation is high.

Thus, the EWMA_e and EWMA_{e^2} chart combination with individual sampling gives better overall performance in detecting sustained shifts in both mean and variance.

7.3.2 EWMA Observations and Residuals Chart Combination

Section 6.4 shows that a Shewhart observations and residuals chart combination combines the advantages of both the Shewhart observations chart combination and the Shewhart residuals chart combination. It is also interesting to see if an EWMA observations and residuals chart combination is able to give improved performance. Thus the combination of a mean chart based on the observations and a squared deviation chart based on residuals (i.e., the EWMA_X and EWMA_{e^2} chart combination) is considered next.

Tables 35 – 38 confirm that this type of control chart combination is indeed a better approach. In the same format as previous tables with the SSATS values, Table 35 shows that the EWMA_X and EWMA_{e^2} chart combination with individual sampling can detect sustained shifts in standard deviation as quickly as the EWMA_e and EWMA_{e^2} chart combination, and it is also very effective in detecting sustained shift in the process mean, regardless of whether the autocorrelation in the process is low or high. Take ϕ = 0.8 and ψ = 0.9 as an example. It takes the EWMA_X and EWMA_{e^2} chart combination with individual sampling an average of 243.4 hours to detect a sustained shift in μ_X of size $0.5\sigma_{X_0}$, 41.7 hours to detect a sustained shift in σ_X of size $1.4\sigma_{X_0}$ due to σ_α, and 9.7 hours to detect a shift of the same size due to σ_ε. For the same shifts in the mean and variance, it takes the EWMA_X and EWMA_{X^2} chart combination with individual sampling 224.3 hours, 107.9 hours and 129.9 hours on average to signal, respectively, and it takes the EWMA_e and EWMA_{e^2} chart combination with individual sampling about 249.5 hours, 41.4 hours and 9.7 hours on average to signal, respectively. The signal probabilities at different durations of transient shifts given by Tables 36 – 38 also show that the transient-shift performance of the EWMA_X and EWMA_{R^2} chart combination with individual sampling is as good as the performance of the EWMA_e and EWMA_{e^2} chart combination.

Due to the overall best performance at various levels of the process autocorrelation, the EWMA_X and EWMA_{e^2} chart combination with individual sampling is strongly recommended to detect both sustained and transient shifts in any situation, especially when process autocorrelation is believed to be very high (say, $\phi = 0.8$ and $\psi = 0.9$). When process autocorrelation is low, the EWMA_X and EWMA_{X^2} chart combination is also a good choice. Its performance is only slightly worse than the performance of the EWMA_X and EWMA_{e^2}, and the interpretation of the observations charts is probably more straightforward for most practitioners.

7.4 Most Efficient Chart Combination and Sampling Plan

Section 6.4 concludes that among various Shewhart chart combinations, the Shewhart $\bar{X}$ and e^2 chart combination with dispersed sampling and $n = 4$ gives best overall performance in detected both sustained and transient shifts. On the other hand, Section 7.3 demonstrates that the EWMA_X and EWMA_{e^2} chart combination with individual sampling is the most efficient EWMA chart combination in detecting both sustained and transient shifts. Thus a comparison between these two chart combinations along with their corresponding sampling plans will be able to give the most efficient chart combination and sampling plan.

For sustained shifts, Tables 27 and 35 show that the EWMA_X and EWMA_{e^2} chart combination with individual sampling performs much better than the Shewhart $\overline{X}$ and e^2 chart combination with dispersed sampling and n = 4 in detecting shifts in both process mean and standard deviation from low correlation to high correlation.

The performance of the two chart combinations in detecting different durations of sustained shifts is given by Tables 28 – 30 and Tables 36 – 38. When the process autocorrelation is low, the Shewhart $\overline{X}$ and e^2 chart combination with dispersed sampling and n = 4 is slightly better than the EWMA_X and EWMA_{e^2} chart combination with individual sampling. When the autocorrelation is high, the EWMA_X and EWMA_{e^2} chart combination with individual sampling is slightly better.

Thus the EWMA_X and EWMA_{e^2} chart combination with individual sampling has the overall most efficient performance in detecting sustained and transient shifts at all levels of autocorrelation.

Table 31: SSATS Values of EWMA_e & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Sustained Shifts in Autocorrelated Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$, $\psi = 0.1$			$\phi = 0.2$, $\psi = 0.9$			$\phi = 0.6$, $\psi = 0.1$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
In-Control ATS		0.00	1.00	1482.4	1481.6	1482.7	1481.7	1481.4	1481.7	1482.1	1481.8	1482.3
Out-of-Control SSATS	μ_X	0.25	1.00	137.3	165.8	138.4	184.7	409.4	187.6	166.9	178.6	172.0
		0.50	1.00	42.5	49.0	43.4	54.4	119.8	55.6	49.8	52.1	51.0
		0.75	1.00	23.8	26.5	24.7	29.6	58.6	30.4	27.3	27.8	27.9
		1.00	1.00	16.1	17.4	17.0	19.8	36.7	20.6	18.3	18.2	18.8
		1.50	1.00	8.9	9.1	9.9	11.1	19.5	11.8	10.1	9.4	10.6
		2.00	1.00	5.4	5.3	6.6	6.8	11.9	7.6	6.1	5.4	6.8
		3.00	1.00	2.4	2.4	3.8	2.9	5.4	4.0	2.6	2.4	3.8
		4.00	1.00	1.3	2.0	2.8	1.4	3.0	2.9	1.3	2.0	2.8
		5.00	1.00	0.8	2.0	2.3	0.8	2.2	2.4	0.8	2.0	2.3
		7.00	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		10.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		15.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
	σ_α	0.00	1.20	89.8	77.9	89.7	91.6	218.8	98.7	91.2	77.2	85.0
		0.00	1.40	30.3	31.1	31.4	30.4	79.7	34.1	33.3	31.3	32.7
		0.00	1.60	17.0	19.4	18.3	17.0	44.2	19.7	19.5	19.6	20.0
		0.00	1.80	11.6	14.2	13.0	11.5	29.8	13.9	13.6	14.4	14.5
		0.00	2.00	8.7	11.3	10.1	8.6	22.3	10.7	10.4	11.5	11.5
		0.00	2.40	5.7	8.3	7.1	5.6	14.8	7.5	7.0	8.4	8.3
		0.00	3.00	3.7	6.2	5.2	3.7	10.1	5.4	4.6	6.3	6.1
		0.00	5.00	1.8	4.0	3.3	1.8	5.4	3.4	2.2	4.0	3.7
		0.00	7.00	1.3	3.3	2.8	1.3	4.1	2.8	1.5	3.3	3.0
		0.00	10.0	1.0	2.8	2.5	1.0	3.3	2.5	1.1	2.8	2.6
		0.00	15.0	0.8	2.5	2.3	0.8	2.8	2.3	0.9	2.5	2.4
	σ_ε	0.00	1.20	90.8	95.2	91.8	83.5	409.3	100.7	90.3	95.6	93.5
		0.00	1.40	30.1	31.1	31.5	28.1	111.5	34.0	29.9	31.1	31.9
		0.00	1.60	16.9	17.3	18.3	15.9	47.9	19.5	16.7	17.3	18.5
		0.00	1.80	11.4	11.6	12.9	10.8	28.0	13.7	11.4	11.7	13.0
		0.00	2.00	8.5	8.7	10.0	8.1	19.1	10.5	8.5	8.7	10.0
		0.00	2.40	5.6	5.7	7.0	5.4	11.3	7.4	5.6	5.7	7.1
		0.00	3.00	3.6	3.8	5.1	3.5	6.7	5.3	3.6	3.8	5.1
		0.00	5.00	1.8	2.3	3.3	1.7	3.0	3.3	1.8	2.3	3.3
		0.00	7.00	1.3	2.1	2.8	1.2	2.3	2.8	1.3	2.1	2.8
		0.00	10.0	1.0	2.0	2.5	1.0	2.1	2.5	1.0	2.0	2.5
		0.00	15.0	0.8	2.0	2.3	0.8	2.0	2.3	0.8	2.0	2.3
h_{EWMA_e}, $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.847 7.771	2.953 3.423	3.073 3.638	5.850 7.771	3.036 3.904	3.073 3.636	5.847 7.772	3.033 3.476

Table 31: SSATS Values of EWMA_e & EWMA_{e^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.9$			$\phi = 0.8,\ \psi = 0.1$			$\phi = 0.8,\ \psi = 0.9$		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
In-Control ATS		0.00	1.00	1481.7	1481.2	1481.2	1482.1	1481.9	1482.6	1481.7	1481.8	1482.0
Out-of-Control SSATS	μ_X	0.25	1.00	414.7	492.6	461.0	223.6	223.6	253.9	701.8	713.2	870.3
		0.50	1.00	120.9	150.1	134.8	63.7	63.3	67.8	249.5	256.9	348.1
		0.75	1.00	58.9	71.7	64.3	33.3	32.5	34.0	116.2	120.3	158.5
		1.00	1.00	37.0	44.1	39.7	21.7	20.7	21.6	67.8	70.1	87.0
		1.50	1.00	19.9	22.8	20.7	11.6	10.3	11.5	32.6	33.5	37.2
		2.00	1.00	12.3	13.8	12.7	6.8	5.7	7.2	18.3	19.1	19.6
		3.00	1.00	4.5	5.9	5.9	2.7	2.4	3.9	4.6	6.6	7.0
		4.00	1.00	1.4	3.0	3.7	1.3	2.0	2.8	0.8	2.8	3.7
		5.00	1.00	0.6	2.2	2.9	0.8	2.0	2.3	0.5	2.1	2.8
		7.00	1.00	0.5	2.0	2.1	0.5	2.0	2.0	0.5	2.0	2.1
		10.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		15.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
	σ_α	0.00	1.20	103.5	219.0	152.0	96.9	79.6	85.9	129.1	222.9	183.2
		0.00	1.40	33.8	79.9	54.5	39.5	34.0	36.8	41.4	81.2	67.9
		0.00	1.60	18.7	44.3	31.2	24.2	21.6	23.6	22.4	45.0	39.0
		0.00	1.80	12.6	29.9	21.6	17.3	15.9	17.6	14.9	30.2	27.0
		0.00	2.00	9.4	22.4	16.6	13.5	12.7	14.2	11.0	22.6	20.7
		0.00	2.40	6.1	14.9	11.4	9.3	9.2	10.4	7.0	15.0	14.2
		0.00	3.00	3.9	10.1	7.9	6.3	6.8	7.7	4.5	10.1	9.9
		0.00	5.00	1.9	5.4	4.4	3.0	4.2	4.5	2.1	5.4	5.4
		0.00	7.00	1.3	4.1	3.4	2.0	3.4	3.5	1.4	4.1	4.1
		0.00	10.0	1.0	3.3	2.8	1.4	2.9	2.9	1.1	3.3	3.2
		0.00	15.0	0.8	2.8	2.5	1.0	2.6	2.5	0.8	2.8	2.7
	σ_ε	0.00	1.20	40.1	402.2	164.3	89.0	95.9	96.5	21.6	308.7	155.0
		0.00	1.40	15.8	107.8	50.7	29.5	31.1	32.4	9.7	77.6	47.1
		0.00	1.60	9.7	46.4	27.6	16.5	17.2	18.7	6.3	35.4	25.8
		0.00	1.80	7.0	27.3	18.7	11.2	11.6	13.1	4.8	21.7	17.7
		0.00	2.00	5.5	18.7	14.1	8.4	8.6	10.1	3.9	15.3	13.4
		0.00	2.40	3.9	11.1	9.5	5.5	5.7	7.1	2.8	9.3	9.2
		0.00	3.00	2.7	6.6	6.6	3.6	3.8	5.2	2.1	5.7	6.4
		0.00	5.00	1.5	3.0	3.8	1.8	2.3	3.3	1.2	2.8	3.7
		0.00	7.00	1.1	2.3	3.0	1.3	2.1	2.8	1.0	2.2	3.0
		0.00	10.0	0.9	2.1	2.6	1.0	2.0	2.5	0.8	2.1	2.6
		0.00	15.0	0.7	2.0	2.4	0.8	2.0	2.3	0.7	2.0	2.4
h_{EWMA_e} , $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.850 7.771	3.544 9.028	3.072 3.636	5.847 7.773	3.233 3.558	3.073 3.638	5.850 7.770	4.338 13.899

Table 32: Signal Probabilities of EWMA_e & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when *l* = 1 hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$			$\phi = 0.2,\ \psi = 0.9$			$\phi = 0.6,\ \psi = 0.1$		
				Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00
		1.00	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.01	0.04	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		2.00	1.00	0.03	0.10	0.03	0.03	0.03	0.02	0.03	0.03	0.03
		3.00	1.00	0.13	0.23	0.12	0.14	0.10	0.10	0.13	0.10	0.12
		4.00	1.00	0.39	0.25	0.36	0.42	0.19	0.31	0.39	0.19	0.35
		5.00	1.00	0.73	0.25	0.70	0.77	0.24	0.64	0.73	0.24	0.69
		7.00	1.00	0.99	0.25	0.99	1.00	0.25	0.99	0.99	0.25	0.99
		10.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
		15.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.01
		0.00	1.40	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.60	0.02	0.03	0.02	0.02	0.01	0.01	0.01	0.01	0.02
		0.00	1.80	0.03	0.05	0.03	0.03	0.02	0.02	0.02	0.02	0.02
		0.00	2.00	0.04	0.06	0.04	0.04	0.03	0.04	0.04	0.03	0.04
		0.00	2.40	0.08	0.09	0.08	0.08	0.05	0.07	0.07	0.05	0.07
		0.00	3.00	0.16	0.11	0.16	0.16	0.07	0.14	0.14	0.07	0.14
		0.00	5.00	0.39	0.17	0.39	0.39	0.13	0.37	0.37	0.13	0.37
		0.00	7.00	0.54	0.19	0.53	0.54	0.16	0.52	0.52	0.16	0.52
		0.00	10.0	0.67	0.21	0.66	0.67	0.19	0.65	0.65	0.19	0.65
		0.00	15.0	0.77	0.22	0.77	0.77	0.21	0.76	0.76	0.21	0.76
	σ_ε	0.00	1.20	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00
		0.00	1.40	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.01
		0.00	1.60	0.02	0.02	0.01	0.02	0.00	0.01	0.02	0.00	0.01
		0.00	1.80	0.03	0.04	0.03	0.03	0.01	0.02	0.03	0.01	0.02
		0.00	2.00	0.04	0.06	0.04	0.05	0.01	0.03	0.04	0.01	0.04
		0.00	2.40	0.08	0.11	0.08	0.09	0.03	0.07	0.09	0.03	0.08
		0.00	3.00	0.16	0.16	0.15	0.17	0.08	0.14	0.16	0.08	0.15
		0.00	5.00	0.39	0.23	0.39	0.41	0.19	0.37	0.40	0.20	0.38
		0.00	7.00	0.54	0.24	0.53	0.55	0.23	0.52	0.54	0.23	0.53
		0.00	10.0	0.67	0.25	0.66	0.68	0.24	0.65	0.67	0.24	0.66
		0.00	15.0	0.78	0.25	0.77	0.78	0.25	0.76	0.78	0.25	0.77
h_{EWMA_e}, $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.847 7.771	2.953 3.423	3.073 3.638	5.850 7.771	3.036 3.904	3.073 3.636	5.847 7.772	3.033 3.476

Table 32: Signal Probabilities of EWMA_e & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when $l = 1$ hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6$, $\psi = 0.9$			$\phi = 0.8$, $\psi = 0.1$			$\phi = 0.8$, $\psi = 0.9$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
		0.75	1.00	0.01	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00
		1.00	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.00
		1.50	1.00	0.03	0.01	0.01	0.01	0.04	0.01	0.07	0.02	0.01
		2.00	1.00	0.08	0.03	0.01	0.03	0.10	0.03	0.21	0.04	0.01
		3.00	1.00	0.37	0.10	0.04	0.13	0.23	0.11	0.75	0.14	0.04
		4.00	1.00	0.80	0.19	0.12	0.40	0.25	0.34	0.99	0.22	0.12
		5.00	1.00	0.98	0.24	0.32	0.74	0.25	0.68	1.00	0.25	0.34
		7.00	1.00	1.00	0.25	0.90	0.99	0.25	0.99	1.00	0.25	0.94
		10.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
		15.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
	σ_α	0.00	1.20	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
		0.00	1.60	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.01	0.01
		0.00	1.80	0.02	0.02	0.01	0.02	0.04	0.02	0.02	0.02	0.01
		0.00	2.00	0.04	0.03	0.02	0.03	0.06	0.03	0.03	0.03	0.02
		0.00	2.40	0.08	0.05	0.04	0.06	0.08	0.05	0.06	0.04	0.03
		0.00	3.00	0.15	0.07	0.08	0.11	0.11	0.11	0.12	0.07	0.06
		0.00	5.00	0.37	0.13	0.26	0.32	0.16	0.32	0.34	0.13	0.22
		0.00	7.00	0.52	0.16	0.42	0.47	0.19	0.47	0.49	0.16	0.37
		0.00	10.0	0.65	0.19	0.57	0.61	0.20	0.61	0.63	0.19	0.52
		0.00	15.0	0.76	0.21	0.70	0.74	0.22	0.73	0.75	0.21	0.67
	σ_ε	0.00	1.20	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.00	0.01	0.01	0.01	0.01	0.04	0.00	0.01
		0.00	1.60	0.04	0.00	0.01	0.02	0.02	0.01	0.08	0.01	0.01
		0.00	1.80	0.06	0.01	0.01	0.03	0.04	0.02	0.12	0.01	0.01
		0.00	2.00	0.10	0.01	0.02	0.04	0.06	0.04	0.17	0.02	0.02
		0.00	2.40	0.17	0.03	0.04	0.09	0.11	0.08	0.27	0.05	0.04
		0.00	3.00	0.27	0.08	0.08	0.16	0.16	0.15	0.38	0.10	0.08
		0.00	5.00	0.51	0.20	0.28	0.40	0.23	0.38	0.61	0.21	0.29
		0.00	7.00	0.64	0.23	0.44	0.55	0.24	0.53	0.72	0.24	0.45
		0.00	10.0	0.75	0.24	0.59	0.67	0.25	0.66	0.80	0.25	0.59
		0.00	15.0	0.83	0.25	0.72	0.78	0.25	0.77	0.87	0.25	0.72
h_{EWMA_e}, $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.850 7.771	3.544 9.028	3.072 3.636	5.847 7.773	3.233 3.558	3.073 3.638	5.850 7.770	4.338 13.899

Table 33: Signal Probabilities of EWMA_e & EWMA_{e^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when l = 2 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.2, ψ = 0.1			ϕ = 0.2, ψ = 0.9			ϕ = 0.6, ψ = 0.1		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.04	0.08	0.04	0.03	0.03	0.03	0.04	0.03	0.04
		2.00	1.00	0.10	0.19	0.10	0.08	0.06	0.09	0.10	0.06	0.10
		3.00	1.00	0.45	0.45	0.44	0.37	0.20	0.38	0.43	0.21	0.43
		4.00	1.00	0.87	0.50	0.86	0.80	0.38	0.80	0.85	0.39	0.85
		5.00	1.00	0.99	0.50	0.99	0.98	0.48	0.98	0.99	0.48	0.99
		7.00	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.04	0.02	0.02	0.01	0.02	0.02	0.01	0.02
		0.00	1.60	0.04	0.06	0.03	0.04	0.02	0.03	0.04	0.02	0.04
		0.00	1.80	0.06	0.09	0.06	0.06	0.04	0.05	0.06	0.04	0.06
		0.00	2.00	0.10	0.12	0.10	0.10	0.06	0.08	0.09	0.05	0.10
		0.00	2.40	0.19	0.17	0.19	0.19	0.09	0.17	0.17	0.09	0.17
		0.00	3.00	0.34	0.23	0.33	0.34	0.14	0.30	0.30	0.14	0.30
		0.00	5.00	0.67	0.33	0.66	0.67	0.26	0.64	0.61	0.26	0.61
		0.00	7.00	0.81	0.38	0.81	0.82	0.32	0.80	0.77	0.32	0.77
		0.00	10.0	0.90	0.41	0.90	0.90	0.37	0.89	0.88	0.37	0.88
		0.00	15.0	0.96	0.44	0.95	0.96	0.41	0.95	0.94	0.41	0.94
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.01
		0.00	1.40	0.02	0.02	0.02	0.02	0.01	0.01	0.02	0.01	0.02
		0.00	1.60	0.04	0.04	0.03	0.04	0.01	0.03	0.04	0.01	0.03
		0.00	1.80	0.06	0.08	0.06	0.07	0.02	0.05	0.06	0.02	0.06
		0.00	2.00	0.10	0.12	0.10	0.11	0.03	0.08	0.10	0.03	0.09
		0.00	2.40	0.19	0.21	0.19	0.21	0.07	0.17	0.20	0.07	0.18
		0.00	3.00	0.34	0.32	0.33	0.36	0.16	0.31	0.34	0.16	0.33
		0.00	5.00	0.67	0.46	0.67	0.69	0.39	0.65	0.68	0.39	0.66
		0.00	7.00	0.82	0.49	0.81	0.82	0.46	0.80	0.82	0.46	0.81
		0.00	10.0	0.91	0.50	0.90	0.91	0.49	0.90	0.91	0.49	0.90
		0.00	15.0	0.96	0.50	0.96	0.96	0.50	0.95	0.96	0.50	0.96
h_{EWMA_e}, $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.847 7.771	2.953 3.423	3.073 3.638	5.850 7.771	3.036 3.904	3.073 3.636	5.847 7.772	3.033 3.476

Table 33: Signal Probabilities of EWMA_e & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when $l = 2$ hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.9$			$\phi = 0.8,\ \psi = 0.1$			$\phi = 0.8,\ \psi = 0.9$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
$l = 2$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.01	0.01
		1.50	1.00	0.04	0.03	0.02	0.04	0.07	0.03	0.10	0.03	0.02
		2.00	1.00	0.11	0.06	0.04	0.10	0.19	0.09	0.29	0.08	0.04
		3.00	1.00	0.51	0.21	0.18	0.43	0.45	0.42	0.87	0.28	0.18
		4.00	1.00	0.91	0.39	0.51	0.85	0.50	0.84	1.00	0.45	0.53
		5.00	1.00	1.00	0.48	0.85	0.99	0.50	0.99	1.00	0.50	0.89
		7.00	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.01	0.01	0.02	0.03	0.02	0.01	0.01	0.01
		0.00	1.60	0.03	0.02	0.02	0.03	0.06	0.03	0.03	0.02	0.02
		0.00	1.80	0.06	0.04	0.03	0.05	0.09	0.05	0.04	0.04	0.03
		0.00	2.00	0.09	0.05	0.05	0.07	0.11	0.08	0.07	0.05	0.04
		0.00	2.40	0.17	0.09	0.09	0.14	0.16	0.14	0.14	0.09	0.08
		0.00	3.00	0.31	0.14	0.18	0.24	0.22	0.24	0.27	0.14	0.15
		0.00	5.00	0.65	0.26	0.48	0.53	0.32	0.53	0.60	0.26	0.40
		0.00	7.00	0.80	0.32	0.67	0.70	0.37	0.70	0.77	0.32	0.59
		0.00	10.0	0.90	0.37	0.82	0.83	0.41	0.83	0.88	0.37	0.75
		0.00	15.0	0.95	0.41	0.91	0.92	0.44	0.92	0.94	0.41	0.88
	σ_ε	0.00	1.20	0.01	0.00	0.01	0.01	0.01	0.01	0.03	0.00	0.01
		0.00	1.40	0.04	0.01	0.01	0.02	0.02	0.01	0.09	0.01	0.01
		0.00	1.60	0.09	0.01	0.01	0.04	0.04	0.03	0.18	0.01	0.01
		0.00	1.80	0.15	0.02	0.02	0.07	0.08	0.06	0.27	0.02	0.02
		0.00	2.00	0.22	0.03	0.04	0.10	0.12	0.09	0.36	0.04	0.04
		0.00	2.40	0.35	0.07	0.09	0.20	0.21	0.18	0.50	0.09	0.09
		0.00	3.00	0.51	0.16	0.19	0.35	0.32	0.32	0.65	0.20	0.20
		0.00	5.00	0.78	0.39	0.54	0.68	0.46	0.66	0.86	0.42	0.55
		0.00	7.00	0.88	0.46	0.73	0.82	0.49	0.81	0.93	0.47	0.74
		0.00	10.0	0.94	0.49	0.86	0.91	0.50	0.90	0.96	0.49	0.86
		0.00	15.0	0.97	0.50	0.93	0.96	0.50	0.95	0.98	0.50	0.94
h_{EWMA_e}, $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.850 7.771	3.544 9.028	3.072 3.636	5.847 7.773	3.233 3.558	3.073 3.638	5.850 7.770	4.338 13.900

Table 34: Signal Probabilities of EWMA_e & EWMA_{e^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when l = 4 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.2, ψ = 0.1			ϕ = 0.2, ψ = 0.9			ϕ = 0.6, ψ = 0.1		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.01	0.02
		1.00	1.00	0.04	0.05	0.04	0.03	0.02	0.04	0.04	0.02	0.04
		1.50	1.00	0.14	0.16	0.14	0.10	0.05	0.12	0.12	0.05	0.13
		2.00	1.00	0.37	0.39	0.37	0.26	0.12	0.31	0.32	0.12	0.35
		3.00	1.00	0.91	0.90	0.91	0.79	0.40	0.84	0.87	0.41	0.89
		4.00	1.00	1.00	1.00	1.00	0.99	0.77	0.99	1.00	0.78	1.00
		5.00	1.00	1.00	1.00	1.00	1.00	0.96	1.00	1.00	0.96	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.02	0.03	0.02	0.02	0.01	0.01	0.02	0.01	0.02
		0.00	1.40	0.04	0.07	0.04	0.04	0.03	0.04	0.04	0.03	0.05
		0.00	1.60	0.09	0.13	0.09	0.09	0.05	0.08	0.09	0.05	0.09
		0.00	1.80	0.16	0.19	0.16	0.16	0.08	0.14	0.15	0.08	0.16
		0.00	2.00	0.25	0.24	0.24	0.25	0.11	0.21	0.22	0.11	0.23
		0.00	2.40	0.43	0.34	0.42	0.43	0.18	0.38	0.37	0.18	0.38
		0.00	3.00	0.64	0.46	0.63	0.65	0.28	0.60	0.57	0.28	0.57
		0.00	5.00	0.92	0.66	0.92	0.92	0.52	0.91	0.88	0.52	0.88
		0.00	7.00	0.98	0.76	0.97	0.98	0.64	0.97	0.96	0.64	0.96
		0.00	10.0	0.99	0.83	0.99	0.99	0.75	0.99	0.99	0.74	0.99
		0.00	15.0	1.00	0.88	1.00	1.00	0.83	1.00	1.00	0.83	1.00
	σ_ε	0.00	1.20	0.02	0.01	0.01	0.02	0.01	0.01	0.02	0.01	0.01
		0.00	1.40	0.04	0.04	0.04	0.05	0.01	0.03	0.04	0.01	0.04
		0.00	1.60	0.09	0.09	0.09	0.10	0.02	0.07	0.09	0.02	0.08
		0.00	1.80	0.16	0.16	0.16	0.18	0.03	0.14	0.17	0.03	0.15
		0.00	2.00	0.25	0.25	0.24	0.27	0.05	0.21	0.25	0.05	0.24
		0.00	2.40	0.43	0.43	0.42	0.46	0.13	0.39	0.44	0.13	0.42
		0.00	3.00	0.65	0.65	0.64	0.67	0.31	0.61	0.65	0.32	0.63
		0.00	5.00	0.92	0.92	0.92	0.93	0.78	0.91	0.93	0.78	0.92
		0.00	7.00	0.98	0.98	0.98	0.98	0.92	0.97	0.98	0.93	0.98
		0.00	10.0	0.99	0.99	0.99	0.99	0.98	0.99	0.99	0.98	0.99
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
h_{EWMA_e} , $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.847 7.771	2.953 3.423	3.073 3.638	5.850 7.771	3.036 3.904	3.073 3.636	5.847 7.772	3.033 3.476

Table 34: Signal Probabilities of EWMA_e & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when l = 4 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.9$			$\phi = 0.8,\ \psi = 0.1$			$\phi = 0.8,\ \psi = 0.9$		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.01	0.01
		1.00	1.00	0.02	0.02	0.02	0.03	0.04	0.04	0.03	0.02	0.02
		1.50	1.00	0.07	0.05	0.07	0.11	0.15	0.12	0.11	0.07	0.07
		2.00	1.00	0.18	0.12	0.16	0.30	0.38	0.34	0.33	0.17	0.17
		3.00	1.00	0.67	0.41	0.55	0.85	0.90	0.88	0.90	0.55	0.56
		4.00	1.00	0.97	0.78	0.90	1.00	1.00	1.00	1.00	0.90	0.91
		5.00	1.00	1.00	0.96	0.99	1.00	1.00	1.00	1.00	0.99	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.40	0.04	0.03	0.03	0.04	0.06	0.04	0.03	0.03	0.02
		0.00	1.60	0.08	0.05	0.05	0.08	0.12	0.08	0.06	0.05	0.04
		0.00	1.80	0.14	0.08	0.08	0.13	0.17	0.13	0.11	0.08	0.07
		0.00	2.00	0.22	0.11	0.12	0.18	0.23	0.19	0.18	0.11	0.10
		0.00	2.40	0.39	0.18	0.22	0.30	0.32	0.31	0.33	0.18	0.18
		0.00	3.00	0.61	0.28	0.39	0.46	0.44	0.47	0.55	0.28	0.31
		0.00	5.00	0.91	0.52	0.78	0.80	0.65	0.79	0.89	0.51	0.67
		0.00	7.00	0.97	0.64	0.91	0.92	0.74	0.91	0.96	0.64	0.85
		0.00	10.0	0.99	0.74	0.97	0.97	0.82	0.97	0.99	0.74	0.95
		0.00	15.0	1.00	0.83	0.99	0.99	0.88	0.99	1.00	0.83	0.99
	σ_ε	0.00	1.20	0.03	0.01	0.01	0.02	0.01	0.01	0.07	0.01	0.01
		0.00	1.40	0.11	0.01	0.02	0.04	0.04	0.04	0.23	0.01	0.02
		0.00	1.60	0.22	0.02	0.03	0.09	0.09	0.08	0.40	0.02	0.03
		0.00	1.80	0.35	0.03	0.06	0.17	0.16	0.15	0.55	0.04	0.07
		0.00	2.00	0.46	0.05	0.11	0.26	0.25	0.23	0.66	0.08	0.11
		0.00	2.40	0.64	0.13	0.24	0.44	0.43	0.41	0.80	0.19	0.25
		0.00	3.00	0.81	0.32	0.45	0.66	0.65	0.63	0.90	0.40	0.47
		0.00	5.00	0.96	0.78	0.85	0.93	0.92	0.92	0.98	0.83	0.86
		0.00	7.00	0.99	0.93	0.95	0.98	0.98	0.98	1.00	0.95	0.96
		0.00	10.0	1.00	0.98	0.99	0.99	0.99	0.99	1.00	0.99	0.99
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
h_{EWMA_e} , $h_{\text{EWMA}_{e^2}}$				3.073 3.637	5.850 7.771	3.544 9.028	3.072 3.636	5.847 7.773	3.233 3.558	3.073 3.638	5.850 7.770	4.338 13.899

Table 35: SSATS Values of EWMA_X & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Sustained Shifts in Autocorrelated Case

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2$, $\psi = 0.1$			$\phi = 0.2$, $\psi = 0.9$			$\phi = 0.6$, $\psi = 0.1$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
In-Control ATS		0.00	1.00	1481.8	1481.6	1482.1	1482.0	1481.4	1482.1	1481.7	1481.8	1482.5
Out-of-Control SSATS	μ_X	0.25	1.00	137.2	165.7	138.1	184.6	409.6	185.1	166.3	178.4	167.1
		0.50	1.00	42.5	49.0	43.3	54.4	119.8	55.1	49.7	52.1	50.4
		0.75	1.00	23.8	26.5	24.7	29.6	58.5	30.4	27.3	27.8	28.0
		1.00	1.00	16.1	17.4	17.0	19.8	36.8	20.7	18.3	18.2	19.0
		1.50	1.00	8.9	9.1	10.0	11.1	19.4	12.0	10.1	9.4	10.8
		2.00	1.00	5.4	5.3	6.6	6.9	11.9	7.7	6.1	5.4	6.9
		3.00	1.00	2.4	2.4	3.8	2.9	5.4	4.1	2.6	2.4	3.8
		4.00	1.00	1.3	2.0	2.8	1.4	3.0	2.9	1.3	2.0	2.8
		5.00	1.00	0.8	2.0	2.3	0.8	2.2	2.4	0.8	2.0	2.3
		7.00	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		10.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		15.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
	σ_α	0.00	1.20	89.9	77.8	89.7	91.6	218.9	99.0	91.4	77.3	85.0
		0.00	1.40	30.3	31.1	31.4	30.4	79.7	34.2	33.4	31.3	32.7
		0.00	1.60	17.0	19.3	18.3	17.0	44.2	19.8	19.5	19.6	20.0
		0.00	1.80	11.6	14.2	13.0	11.5	29.8	13.9	13.6	14.4	14.5
		0.00	2.00	8.7	11.3	10.1	8.6	22.3	10.7	10.4	11.5	11.5
		0.00	2.40	5.7	8.3	7.1	5.6	14.9	7.5	7.0	8.4	8.3
		0.00	3.00	3.7	6.2	5.2	3.7	10.1	5.4	4.7	6.3	6.1
		0.00	5.00	1.8	4.0	3.3	1.8	5.4	3.4	2.2	4.0	3.7
		0.00	7.00	1.3	3.3	2.8	1.3	4.1	2.8	1.5	3.3	3.0
		0.00	10.0	1.0	2.8	2.5	1.0	3.3	2.5	1.1	2.8	2.6
		0.00	15.0	0.8	2.5	2.3	0.8	2.8	2.3	0.9	2.5	2.4
	σ_ε	0.00	1.20	90.9	95.2	91.8	83.4	409.6	100.8	90.4	95.6	93.3
		0.00	1.40	30.1	31.1	31.5	28.1	111.6	34.0	29.9	31.1	31.9
		0.00	1.60	16.9	17.3	18.3	15.9	47.8	19.6	16.8	17.3	18.4
		0.00	1.80	11.4	11.6	12.9	10.8	28.0	13.7	11.4	11.6	13.0
		0.00	2.00	8.5	8.7	10.0	8.1	19.2	10.6	8.5	8.7	10.0
		0.00	2.40	5.6	5.7	7.0	5.4	11.3	7.4	5.6	5.7	7.1
		0.00	3.00	3.6	3.8	5.1	3.5	6.7	5.3	3.6	3.8	5.1
		0.00	5.00	1.8	2.3	3.3	1.7	3.0	3.3	1.8	2.3	3.3
		0.00	7.00	1.3	2.1	2.8	1.2	2.3	2.8	1.3	2.1	2.8
		0.00	10.0	1.0	2.0	2.5	1.0	2.1	2.5	1.0	2.0	2.5
		0.00	15.0	0.8	2.0	2.3	0.8	2.0	2.3	0.8	2.0	2.3
h_{EWMA_X}, $h_{\text{EWMA}_{e^2}}$				3.143 3.638	3.349 3.484	3.011 3.423	3.718 3.651	5.634 7.190	3.511 3.906	3.444 3.637	3.467 3.483	3.312 3.474

Table 35: SSATS Values of EWMA_X & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Sustained Shifts in Autocorrelated Case (cont'd)

Performance Measure	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.9$			$\phi = 0.8,\ \psi = 0.1$			$\phi = 0.8,\ \psi = 0.9$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
In-Control ATS		0.00	1.00	1481.6	1481.5	1481.8	1481.7	1482.1	1481.1	1481.7	1481.4	1481.7
Out-of-Control SSATS	μ_X	0.25	1.00	411.0	490.7	413.3	220.7	222.7	222.3	689.7	701.6	693.2
		0.50	1.00	120.2	149.7	121.4	63.2	63.2	63.8	243.4	251.1	245.7
		0.75	1.00	58.8	71.4	59.9	33.3	32.6	33.7	114.3	118.0	115.7
		1.00	1.00	37.2	44.0	38.3	21.9	20.7	22.2	67.4	69.2	68.7
		1.50	1.00	20.2	22.8	21.5	11.9	10.3	11.9	33.1	33.5	34.7
		2.00	1.00	12.6	13.8	14.2	7.0	5.7	7.3	19.0	19.3	21.4
		3.00	1.00	4.6	5.9	6.7	2.7	2.4	3.9	4.9	6.8	8.7
		4.00	1.00	1.4	3.1	3.8	1.4	2.0	2.8	0.8	2.8	4.0
		5.00	1.00	0.6	2.2	2.9	0.8	2.0	2.3	0.5	2.1	2.8
		7.00	1.00	0.5	2.0	2.1	0.5	2.0	2.0	0.5	2.0	2.1
		10.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
		15.0	1.00	0.5	2.0	2.0	0.5	2.0	2.0	0.5	2.0	2.0
	σ_α	0.00	1.20	103.9	220.4	154.0	98.2	79.8	85.9	130.6	224.9	187.6
		0.00	1.40	33.9	80.2	55.3	39.9	34.1	36.9	41.7	81.9	69.8
		0.00	1.60	18.8	44.5	31.7	24.4	21.6	23.7	22.6	45.3	40.2
		0.00	1.80	12.6	30.0	22.0	17.5	15.9	17.6	15.0	30.5	27.9
		0.00	2.00	9.4	22.4	16.8	13.6	12.7	14.2	11.1	22.8	21.4
		0.00	2.40	6.1	14.9	11.5	9.4	9.3	10.4	7.1	15.1	14.7
		0.00	3.00	3.9	10.1	8.0	6.4	6.8	7.7	4.5	10.2	10.2
		0.00	5.00	1.9	5.4	4.4	3.1	4.2	4.5	2.1	5.5	5.5
		0.00	7.00	1.3	4.1	3.4	2.0	3.4	3.5	1.4	4.1	4.1
		0.00	10.0	1.0	3.3	2.9	1.4	2.9	2.9	1.1	3.3	3.2
		0.00	15.0	0.8	2.8	2.5	1.0	2.6	2.5	0.8	2.8	2.7
	σ_ε	0.00	1.20	40.1	403.2	165.0	89.3	95.9	95.3	21.6	310.1	156.7
		0.00	1.40	15.8	108.0	50.9	29.5	31.1	32.3	9.7	77.9	47.5
		0.00	1.60	9.7	46.5	27.7	16.6	17.2	18.6	6.4	35.5	26.0
		0.00	1.80	7.0	27.3	18.8	11.3	11.6	13.1	4.8	21.8	17.8
		0.00	2.00	5.5	18.7	14.2	8.4	8.7	10.1	3.9	15.3	13.5
		0.00	2.40	3.9	11.1	9.6	5.5	5.7	7.1	2.8	9.3	9.2
		0.00	3.00	2.7	6.6	6.6	3.6	3.8	5.2	2.1	5.7	6.4
		0.00	5.00	1.5	3.0	3.8	1.8	2.3	3.3	1.2	2.8	3.7
		0.00	7.00	1.1	2.3	3.0	1.3	2.1	2.8	1.0	2.2	3.0
		0.00	10.0	0.9	2.1	2.6	1.0	2.0	2.5	0.8	2.1	2.6
		0.00	15.0	0.7	2.0	2.4	0.8	2.0	2.3	0.7	2.0	2.4
h_{EWMA_X}, $h_{\text{EWMA}_{e^2}}$				5.543 3.638	6.248 7.174	5.428 9.050	3.909 3.639	3.835 3.476	3.780 3.546	7.737 3.639	7.931 6.981	7.644 13.982

Table 36: Signal Probabilities of EWMA_X & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when $l = 1$ hour

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$			$\phi = 0.2,\ \psi = 0.9$			$\phi = 0.6,\ \psi = 0.1$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
$l = 1$	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.75	1.00	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.01
		1.00	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.50	1.00	0.01	0.04	0.01	0.01	0.01	0.01	0.01	0.04	0.01
		2.00	1.00	0.03	0.10	0.03	0.03	0.03	0.02	0.03	0.10	0.03
		3.00	1.00	0.13	0.23	0.12	0.14	0.10	0.10	0.13	0.23	0.12
		4.00	1.00	0.39	0.25	0.36	0.42	0.19	0.31	0.39	0.25	0.36
		5.00	1.00	0.73	0.25	0.70	0.77	0.24	0.64	0.73	0.25	0.69
		7.00	1.00	0.99	0.25	0.99	1.00	0.25	0.99	0.99	0.25	0.99
		10.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
		15.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.01
		0.00	1.40	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01
		0.00	1.60	0.02	0.03	0.02	0.02	0.01	0.01	0.01	0.03	0.01
		0.00	1.80	0.03	0.05	0.03	0.03	0.02	0.02	0.02	0.05	0.02
		0.00	2.00	0.04	0.06	0.04	0.04	0.03	0.03	0.04	0.06	0.04
		0.00	2.40	0.08	0.09	0.08	0.08	0.05	0.07	0.07	0.09	0.07
		0.00	3.00	0.16	0.11	0.15	0.16	0.07	0.14	0.14	0.11	0.14
		0.00	5.00	0.39	0.17	0.39	0.39	0.13	0.37	0.37	0.17	0.37
		0.00	7.00	0.54	0.19	0.53	0.54	0.16	0.52	0.52	0.19	0.52
		0.00	10.0	0.67	0.21	0.66	0.67	0.19	0.65	0.65	0.21	0.65
		0.00	15.0	0.77	0.22	0.77	0.77	0.21	0.76	0.76	0.22	0.76
	σ_ε	0.00	1.20	0.01	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.00
		0.00	1.40	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
		0.00	1.60	0.02	0.02	0.01	0.02	0.00	0.01	0.02	0.02	0.01
		0.00	1.80	0.03	0.04	0.03	0.03	0.01	0.02	0.03	0.04	0.02
		0.00	2.00	0.04	0.06	0.04	0.05	0.01	0.03	0.04	0.06	0.04
		0.00	2.40	0.08	0.11	0.08	0.09	0.03	0.07	0.09	0.11	0.08
		0.00	3.00	0.16	0.16	0.15	0.17	0.08	0.14	0.16	0.16	0.15
		0.00	5.00	0.39	0.23	0.39	0.41	0.19	0.37	0.40	0.23	0.38
		0.00	7.00	0.54	0.24	0.53	0.55	0.23	0.52	0.54	0.24	0.53
		0.00	10.0	0.67	0.25	0.66	0.68	0.24	0.65	0.67	0.25	0.66
		0.00	15.0	0.78	0.25	0.77	0.78	0.25	0.76	0.78	0.25	0.77
h_{EWMA_X}, $h_{\text{EWMA}_{e^2}}$				3.143 3.638	3.349 3.484	3.011 3.423	3.718 3.651	5.634 7.190	3.511 3.906	3.444 3.637	3.467 3.483	3.312 3.474

Table 36: Signal Probabilities of EWMA_X & EWMA_{e^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when *l* = 1 hour (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.9			ϕ = 0.8, ψ = 0.1			ϕ = 0.8, ψ = 0.9		
				Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)
l = 1	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
		0.75	1.00	0.01	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00
		1.00	1.00	0.01	0.01	0.00	0.01	0.01	0.01	0.02	0.01	0.00
		1.50	1.00	0.03	0.01	0.01	0.01	0.04	0.01	0.07	0.02	0.01
		2.00	1.00	0.08	0.03	0.01	0.03	0.09	0.03	0.21	0.04	0.01
		3.00	1.00	0.37	0.10	0.04	0.13	0.23	0.11	0.75	0.14	0.03
		4.00	1.00	0.80	0.19	0.12	0.40	0.25	0.35	0.99	0.22	0.12
		5.00	1.00	0.98	0.24	0.32	0.74	0.25	0.69	1.00	0.25	0.33
		7.00	1.00	1.00	0.25	0.90	0.99	0.25	0.99	1.00	0.25	0.94
		10.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
		15.0	1.00	1.00	0.25	1.00	1.00	0.25	1.00	1.00	0.25	1.00
	σ_α	0.00	1.20	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
		0.00	1.40	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
		0.00	1.60	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.01	0.01
		0.00	1.80	0.02	0.02	0.01	0.02	0.04	0.02	0.02	0.02	0.01
		0.00	2.00	0.04	0.03	0.02	0.03	0.06	0.03	0.03	0.03	0.01
		0.00	2.40	0.07	0.04	0.03	0.05	0.08	0.05	0.06	0.04	0.02
		0.00	3.00	0.15	0.07	0.07	0.11	0.11	0.11	0.12	0.07	0.05
		0.00	5.00	0.37	0.13	0.25	0.32	0.16	0.32	0.34	0.13	0.21
		0.00	7.00	0.52	0.16	0.41	0.47	0.19	0.47	0.49	0.16	0.35
		0.00	10.0	0.65	0.19	0.56	0.61	0.20	0.61	0.63	0.19	0.51
		0.00	15.0	0.76	0.21	0.70	0.73	0.22	0.73	0.74	0.21	0.66
	σ_ε	0.00	1.20	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00
		0.00	1.40	0.02	0.00	0.01	0.01	0.01	0.01	0.04	0.00	0.01
		0.00	1.60	0.04	0.00	0.01	0.02	0.02	0.01	0.08	0.01	0.01
		0.00	1.80	0.06	0.01	0.01	0.03	0.04	0.02	0.12	0.01	0.01
		0.00	2.00	0.09	0.01	0.02	0.04	0.06	0.04	0.17	0.02	0.02
		0.00	2.40	0.17	0.03	0.03	0.09	0.11	0.08	0.27	0.05	0.03
		0.00	3.00	0.27	0.08	0.08	0.16	0.16	0.15	0.38	0.10	0.08
		0.00	5.00	0.51	0.20	0.28	0.40	0.23	0.38	0.61	0.21	0.28
		0.00	7.00	0.64	0.23	0.44	0.55	0.24	0.53	0.72	0.24	0.44
		0.00	10.0	0.75	0.24	0.58	0.67	0.25	0.66	0.80	0.25	0.59
		0.00	15.0	0.83	0.25	0.72	0.78	0.25	0.77	0.87	0.25	0.72
h_{EWMA_X} , $h_{\text{EWMA}_{e^2}}$				5.543 3.638	6.248 7.174	5.428 9.050	3.909 3.639	3.835 3.476	3.780 3.546	7.737 3.639	7.931 6.981	7.644 13.982

Table 37: Signal Probabilities of EWMA_X & EWMA_{e^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when l = 2 hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.2, ψ = 0.1			ϕ = 0.2, ψ = 0.9			ϕ = 0.6, ψ = 0.1		
				Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)	Indiv. Sampling (n = 1)	Conc. Sampling (n = 4)	Disp. Sampling (n = 4)
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01
		1.50	1.00	0.04	0.08	0.04	0.03	0.03	0.03	0.04	0.08	0.04
		2.00	1.00	0.10	0.19	0.10	0.08	0.06	0.09	0.10	0.19	0.10
		3.00	1.00	0.45	0.45	0.44	0.37	0.20	0.38	0.43	0.45	0.43
		4.00	1.00	0.87	0.50	0.86	0.80	0.38	0.80	0.85	0.50	0.85
		5.00	1.00	0.99	0.50	0.99	0.98	0.48	0.98	0.99	0.50	0.99
		7.00	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.04	0.02	0.02	0.01	0.02	0.02	0.04	0.02
		0.00	1.60	0.04	0.06	0.03	0.04	0.02	0.03	0.04	0.06	0.04
		0.00	1.80	0.06	0.09	0.06	0.06	0.04	0.05	0.06	0.09	0.06
		0.00	2.00	0.10	0.12	0.10	0.10	0.05	0.08	0.09	0.12	0.09
		0.00	2.40	0.19	0.17	0.19	0.19	0.09	0.16	0.17	0.17	0.17
		0.00	3.00	0.34	0.23	0.33	0.34	0.14	0.30	0.30	0.23	0.30
		0.00	5.00	0.67	0.33	0.66	0.67	0.26	0.64	0.61	0.33	0.61
		0.00	7.00	0.81	0.38	0.81	0.82	0.32	0.79	0.77	0.38	0.77
		0.00	10.0	0.90	0.41	0.90	0.90	0.37	0.89	0.88	0.41	0.88
		0.00	15.0	0.96	0.44	0.95	0.96	0.41	0.95	0.94	0.44	0.94
	σ_ε	0.00	1.20	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.02	0.02	0.02	0.01	0.01	0.02	0.02	0.02
		0.00	1.60	0.04	0.04	0.03	0.04	0.01	0.03	0.04	0.04	0.03
		0.00	1.80	0.06	0.08	0.06	0.07	0.02	0.05	0.06	0.08	0.06
		0.00	2.00	0.10	0.12	0.10	0.11	0.03	0.08	0.10	0.12	0.09
		0.00	2.40	0.19	0.21	0.19	0.21	0.07	0.17	0.20	0.21	0.18
		0.00	3.00	0.34	0.32	0.33	0.36	0.16	0.31	0.34	0.32	0.33
		0.00	5.00	0.67	0.46	0.67	0.69	0.39	0.65	0.67	0.46	0.66
		0.00	7.00	0.82	0.49	0.81	0.82	0.46	0.80	0.82	0.49	0.81
		0.00	10.0	0.91	0.50	0.90	0.91	0.49	0.90	0.91	0.50	0.90
		0.00	15.0	0.96	0.50	0.96	0.96	0.50	0.95	0.96	0.50	0.95
h_{EWMA_X} , $h_{\text{EWMA}_{e^2}}$				3.143 3.638	3.349 3.484	3.011 3.423	3.718 3.651	5.634 7.190	3.511 3.906	3.444 3.637	3.467 3.483	3.312 3.474

Table 37: Signal Probabilities of EWMA_X & EWMA_{e^2} Chart Combination (λ_1 = 0.026 and λ_4 = 0.1) for Transient Shifts in Autocorrelated Case when *l* = 2 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	ϕ = 0.6, ψ = 0.9			ϕ = 0.8, ψ = 0.1			ϕ = 0.8, ψ = 0.9		
				Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)
l = 2	μ_X	0.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.50	1.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01
		0.75	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1.00	1.00	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.01	0.01
		1.50	1.00	0.04	0.03	0.02	0.04	0.07	0.03	0.09	0.03	0.02
		2.00	1.00	0.11	0.06	0.04	0.09	0.19	0.09	0.29	0.08	0.04
		3.00	1.00	0.50	0.20	0.17	0.42	0.45	0.42	0.86	0.27	0.17
		4.00	1.00	0.91	0.39	0.49	0.85	0.50	0.84	1.00	0.45	0.52
		5.00	1.00	1.00	0.48	0.85	0.99	0.50	0.99	1.00	0.50	0.88
		7.00	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		10.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
		15.0	1.00	1.00	0.50	1.00	1.00	0.50	1.00	1.00	0.50	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.00	1.40	0.02	0.01	0.01	0.01	0.03	0.02	0.01	0.01	0.01
		0.00	1.60	0.03	0.02	0.02	0.03	0.06	0.03	0.02	0.02	0.01
		0.00	1.80	0.05	0.04	0.03	0.05	0.09	0.05	0.04	0.04	0.02
		0.00	2.00	0.09	0.05	0.04	0.07	0.11	0.07	0.07	0.05	0.03
		0.00	2.40	0.17	0.09	0.08	0.13	0.16	0.14	0.14	0.09	0.06
		0.00	3.00	0.31	0.14	0.17	0.24	0.22	0.24	0.26	0.14	0.13
		0.00	5.00	0.65	0.26	0.47	0.53	0.32	0.53	0.60	0.26	0.38
		0.00	7.00	0.80	0.32	0.66	0.70	0.37	0.70	0.77	0.32	0.57
		0.00	10.0	0.90	0.37	0.81	0.83	0.41	0.83	0.88	0.37	0.74
		0.00	15.0	0.95	0.41	0.91	0.92	0.44	0.92	0.94	0.41	0.87
	σ_ε	0.00	1.20	0.01	0.00	0.01	0.01	0.01	0.01	0.03	0.00	0.01
		0.00	1.40	0.04	0.01	0.01	0.02	0.02	0.02	0.09	0.01	0.01
		0.00	1.60	0.09	0.01	0.01	0.04	0.04	0.03	0.18	0.01	0.01
		0.00	1.80	0.15	0.02	0.02	0.06	0.08	0.06	0.27	0.02	0.02
		0.00	2.00	0.22	0.03	0.04	0.10	0.12	0.09	0.36	0.04	0.04
		0.00	2.40	0.34	0.07	0.09	0.20	0.21	0.18	0.50	0.09	0.09
		0.00	3.00	0.51	0.16	0.19	0.35	0.32	0.32	0.65	0.20	0.20
		0.00	5.00	0.78	0.39	0.54	0.68	0.46	0.66	0.86	0.42	0.55
		0.00	7.00	0.88	0.46	0.73	0.82	0.49	0.81	0.93	0.47	0.73
		0.00	10.0	0.94	0.49	0.85	0.91	0.50	0.90	0.96	0.49	0.86
		0.00	15.0	0.97	0.50	0.93	0.96	0.50	0.95	0.98	0.50	0.93
h_{EWMA_X}, $h_{\text{EWMA}_{e^2}}$				5.543 3.638	6.248 7.174	5.428 9.050	3.909 3.639	3.835 3.476	3.780 3.546	7.737 3.639	7.931 6.981	7.644 13.982

Table 38: Signal Probabilities of EWMA_X & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when $l = 4$ hours

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.2,\ \psi = 0.1$			$\phi = 0.2,\ \psi = 0.9$			$\phi = 0.6,\ \psi = 0.1$		
				Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)	Indiv. Sampling ($n = 1$)	Conc. Sampling ($n = 4$)	Disp. Sampling ($n = 4$)
$l = 4$	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02
		1.00	1.00	0.04	0.05	0.04	0.03	0.02	0.03	0.04	0.05	0.04
		1.50	1.00	0.14	0.16	0.14	0.10	0.05	0.11	0.12	0.15	0.13
		2.00	1.00	0.37	0.39	0.37	0.26	0.12	0.30	0.32	0.39	0.35
		3.00	1.00	0.91	0.90	0.91	0.79	0.40	0.83	0.87	0.90	0.89
		4.00	1.00	1.00	1.00	1.00	0.99	0.76	0.99	1.00	1.00	1.00
		5.00	1.00	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.02	0.03	0.02	0.02	0.01	0.01	0.02	0.03	0.02
		0.00	1.40	0.04	0.07	0.04	0.04	0.03	0.04	0.04	0.07	0.05
		0.00	1.60	0.09	0.13	0.09	0.09	0.05	0.08	0.09	0.13	0.09
		0.00	1.80	0.16	0.19	0.16	0.16	0.08	0.14	0.15	0.19	0.16
		0.00	2.00	0.25	0.24	0.24	0.25	0.11	0.21	0.22	0.24	0.23
		0.00	2.40	0.43	0.34	0.42	0.43	0.18	0.38	0.37	0.34	0.37
		0.00	3.00	0.64	0.46	0.63	0.65	0.28	0.60	0.57	0.45	0.57
		0.00	5.00	0.92	0.66	0.92	0.92	0.52	0.91	0.88	0.66	0.88
		0.00	7.00	0.98	0.76	0.97	0.98	0.64	0.97	0.96	0.75	0.96
		0.00	10.0	0.99	0.83	0.99	0.99	0.75	0.99	0.99	0.83	0.99
		0.00	15.0	1.00	0.88	1.00	1.00	0.83	1.00	1.00	0.88	1.00
	σ_ε	0.00	1.20	0.02	0.01	0.01	0.02	0.01	0.01	0.02	0.01	0.01
		0.00	1.40	0.04	0.04	0.04	0.05	0.01	0.03	0.04	0.04	0.04
		0.00	1.60	0.09	0.09	0.09	0.10	0.02	0.07	0.09	0.09	0.08
		0.00	1.80	0.16	0.16	0.16	0.18	0.03	0.13	0.17	0.16	0.15
		0.00	2.00	0.25	0.25	0.24	0.27	0.05	0.21	0.25	0.25	0.24
		0.00	2.40	0.43	0.43	0.42	0.46	0.13	0.39	0.44	0.43	0.42
		0.00	3.00	0.65	0.65	0.64	0.67	0.31	0.61	0.65	0.65	0.64
		0.00	5.00	0.92	0.92	0.92	0.93	0.78	0.91	0.92	0.92	0.92
		0.00	7.00	0.98	0.98	0.98	0.98	0.92	0.97	0.98	0.98	0.98
		0.00	10.0	0.99	0.99	0.99	0.99	0.98	0.99	0.99	0.99	0.99
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
h_{EWMA_X} , $h_{\text{EWMA}_{e^2}}$				3.143 3.638	3.349 3.484	3.011 3.423	3.718 3.651	5.634 7.190	3.511 3.906	3.444 3.637	3.467 3.483	3.312 3.474

Table 38: Signal Probabilities of EWMA_X & EWMA_{e^2} Chart Combination ($\lambda_1 = 0.026$ and $\lambda_4 = 0.1$) for Transient Shifts in Autocorrelated Case when *l* = 4 hours (cont'd)

Duration of Transient Shift	Shifted Process Parameter	δ_μ	δ_σ	$\phi = 0.6,\ \psi = 0.9$			$\phi = 0.8,\ \psi = 0.1$			$\phi = 0.8,\ \psi = 0.9$		
				Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)	Indiv. Sampling (*n* = 1)	Conc. Sampling (*n* = 4)	Disp. Sampling (*n* = 4)
l = 4	μ_X	0.25	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.50	1.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		0.75	1.00	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.01	0.01
		1.00	1.00	0.02	0.02	0.02	0.03	0.04	0.04	0.03	0.02	0.02
		1.50	1.00	0.06	0.05	0.06	0.10	0.15	0.12	0.11	0.07	0.06
		2.00	1.00	0.18	0.12	0.14	0.29	0.38	0.33	0.32	0.16	0.15
		3.00	1.00	0.66	0.41	0.52	0.85	0.90	0.88	0.89	0.55	0.54
		4.00	1.00	0.97	0.78	0.89	1.00	1.00	1.00	1.00	0.90	0.90
		5.00	1.00	1.00	0.96	0.99	1.00	1.00	1.00	1.00	0.99	1.00
		7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	σ_α	0.00	1.20	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01
		0.00	1.40	0.04	0.03	0.02	0.04	0.06	0.04	0.03	0.03	0.02
		0.00	1.60	0.08	0.05	0.04	0.07	0.12	0.08	0.06	0.05	0.03
		0.00	1.80	0.14	0.08	0.07	0.12	0.17	0.13	0.11	0.08	0.05
		0.00	2.00	0.22	0.11	0.11	0.18	0.22	0.19	0.17	0.11	0.08
		0.00	2.40	0.39	0.18	0.21	0.29	0.32	0.31	0.33	0.18	0.16
		0.00	3.00	0.61	0.28	0.37	0.46	0.43	0.47	0.54	0.28	0.29
		0.00	5.00	0.91	0.52	0.77	0.79	0.65	0.79	0.88	0.51	0.66
		0.00	7.00	0.97	0.64	0.91	0.92	0.74	0.91	0.96	0.64	0.84
		0.00	10.0	0.99	0.74	0.97	0.97	0.82	0.97	0.99	0.74	0.95
		0.00	15.0	1.00	0.83	0.99	0.99	0.88	0.99	1.00	0.83	0.99
	σ_ε	0.00	1.20	0.03	0.01	0.01	0.02	0.01	0.01	0.07	0.01	0.01
		0.00	1.40	0.11	0.01	0.02	0.04	0.04	0.04	0.23	0.01	0.02
		0.00	1.60	0.22	0.02	0.03	0.09	0.09	0.08	0.40	0.02	0.03
		0.00	1.80	0.35	0.03	0.06	0.17	0.16	0.15	0.54	0.04	0.06
		0.00	2.00	0.46	0.05	0.11	0.26	0.25	0.23	0.66	0.08	0.11
		0.00	2.40	0.64	0.13	0.23	0.44	0.43	0.41	0.80	0.19	0.24
		0.00	3.00	0.81	0.32	0.45	0.66	0.65	0.63	0.90	0.40	0.47
		0.00	5.00	0.96	0.78	0.85	0.93	0.92	0.92	0.98	0.83	0.86
		0.00	7.00	0.99	0.93	0.95	0.98	0.98	0.98	1.00	0.94	0.95
		0.00	10.0	1.00	0.98	0.99	0.99	0.99	0.99	1.00	0.99	0.99
		0.00	15.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
h_{EWMA_X} , $h_{\text{EWMA}_{e^2}}$				5.543 3.638	6.248 7.174	5.428 9.050	3.909 3.639	3.835 3.476	3.780 3.546	7.737 3.639	7.931 6.981	7.644 13.982

Chapter 8

Illustrative Examples and Conclusions

Previous chapters show that the Shewhart $\bar{X}$ and e^2 chart combination with dispersed sampling and n = 4 is the most efficient Shewhart chart combination, and the EWMA_X and EWMA_{e^2} chart combination with individual sampling is the most efficient EWMA chart combination. In this chapter, various chart combination plots of process data corresponding to several out-of-control conditions will be used to illustrate the response of these two control chart combinations with their efficient sampling plans to these out-of-control conditions.

8.1 Illustrative Example of Chart Combination Plots

8.1.1 In-Control Process

Consider two autocorrelated processes: one has a relatively low autocorrelation with ϕ = 0.2 and ψ = 0.9, and the other has a relatively high autocorrelation with ϕ = 0.8 and ψ = 0.9. The variables of two processes are simulated from the same random number generator via Equations (3.3.1) and (3.3.2), and have been standardized so that their in-control mean are μ_{X_0} = 0 and in-control standard deviations are σ_{X_0} = 1. Suppose that one observation is sampled from each process every hour, and a five-day period with three eight-hour work shifts per day is monitored. Thus a total of 120 observations are obtained from each process.

Figures 1 shows five control chart plots corresponding to 120 observations that have been simulated from the autocorrelated process with $\phi = 0.2$ and $\psi = 0.9$ when it is in control. The first row is a Shewhart X chart. The control limits are set at ± 3.399 with an in-control ATS of 1481.6 hours and are provided in Table 2. Figure 1 also provides the plots of two control chart combinations using these 120 observations. The first chart combination (in the second row of Figure 1) is the Shewhart $\bar{X}$ and e^2 chart combination based on samples of $n = 4$ observations. Each sample is obtained by taking successive groups of four observations, thus this corresponds to dispersed sampling, which is the most efficient sampling plan for the Shewhart $\bar{X}$ and e^2 chart combination. The control limits can be derived from the control limits provided in Table 27, which generate a joint in-control ATS of 1481.6 hours. The second chart combination in Figure 1 (in the third row) is the EWMA_X and EWMA_{e^2} chart combination with $\lambda = 0.026$ and $n = 1$, because individual sampling is the most efficient sampling plan for this chart combination. The joint in-control ATS of the EWMA_X and EWMA_{e^2} chart combination is also set as 1481.6 hours with the control limits given in Table 35. Note that the lower limit of EWMA_{e^2} chart corresponds to the reset value $\sigma_{\gamma_0}^2$, which is discussed in Section 3.2.2 when introducing EWMA control charts. In the case of EWMA residuals chart with individual sampling, when the statistic $E_{k-1}^{e^2}$ is less than $\sigma_{\gamma_0}^2$, there is a reset back to $\sigma_{\gamma_0}^2$ before computing $E_k^{e^2}$. Also, there are some distinct patterns on the EWMA_{e^2} chart with sequences of decreasing observations. This is just due to this random number generator. Finally, note all of the plotted points fall within the bandwidth of the control limits of all the control charts.

Figure 2 also gives five control chart plots, which correspond to 120 observations that are simulated from the in-control autocorrelated process with ϕ = 0.8 and ψ = 0.9. Same as in Figure 1, the first row of Figure 2 is a Shewhart X chart with an in-control ATS of 1481.6 hours. The control limits are set at ± 3.338, which also can be found in Table 2. The second row is the Shewhart $\bar{X}$ and e^2 chart combination based on dispersed samples of n = 4 observations with the control limits given in Table 27. Also the third row is the EWMA_X and EWMA_{e^2} chart combination with λ = 0.026 and n = 1 with the control limits given in Table 35. The values of joint in-control ATS for both control chart combinations are also set as 1481.6 hours.

8.1.2 Sustained Shifts

First a special cause that produces a sustained shift in μ_X of size $1.5\sigma_{X_0}$, starting from observation 61, is taken into consideration. The effect of this special cause is employed by adding 1.5 to original observations 61 – 120 in the two control chart combinations in Figures 1 and 2. The plots of the two control chart combinations using the data with the shifted process mean for two autocorrelated processes are shown in Figures 3 and 4, respectively. The two plots in the first row of each figure are the Shewhart $\bar{X}$ and e^2 chart combination with dispersed sampling and n = 4, and the two plots in the second row are the EWMA_X and EWMA_{e^2} chart combination with individual sampling.

The Shewhart $\bar{X}$ chart and e^2 chart in Figure 3 with ϕ = 0.2 and ψ = 0.9 do not signal that μ_X has increased until observations 80 and 112, respectively, while the EWMA_X chart and EWMA_{e^2} chart signal at observations 78 and 79, respectively. With ϕ = 0.8 and ψ = 0.9, the Shewhart $\bar{X}$ chart in Figure 4 signals at observation 112 and the Shewhart e^2 chart has no signal up to 60 observations after μ_X increases, while the EWMA_X chart signals at observations 103 and the EWMA_{e^2} chart does not signal either within 60 observations. Both Figures 3 and 4 show that the detection capability of the EWMA chart combination in μ_X when the sustained shift is of size $1.5\sigma_{X_0}$ is better than that of the Shewhart chart combination. Furthermore, when the autocorrelation is relatively small, the performance difference between two charts is slight; while when the autocorrelation is large, the advantage of the EWMA chart combination is significant. The numerical results provided in Tables 27 and 35 also demonstrate the same conclusion. The SSATS values of the EWMA chart combination (an average of 11.1 hours) is slightly smaller than the SSATS value of the Shewhart chart combination (an average of 13.5 hours) when ϕ = 0.2 and ψ = 0.9; however, the SSATS values of the EWMA chart combination (an average of 33.1 hours) is significantly smaller than the SSATS value of the Shewhart chart combination (an average of 63.7 hours) when ϕ = 0.8 and ψ = 0.9.

Then consider the situation in which a special cause produces a sustained increase in σ_α resulting in an increase in σ_X of size $2.4\sigma_{X_0}$ starting at observation 61. The effect of this special cause is simulated by using σ_{α_1} via Equation (3.3.27) in place of σ_α, where $\sigma_X = 2.4\sigma_{X_0}$. The same sets of control chart combinations are plotted in Figures 5 and 6 using the data with the increased process standard deviation.

In Figure 5 with ϕ = 0.2 and ψ = 0.9 , the Shewhart $\bar{X}$ chart and Shewhart e^2 chart signals at observations 68 and 64, respectively, while the EWMA_X chart does not signal before observation 120 but the EWMA_{e^2} chart signal at as early as observation 63. In Figure 6 with ϕ = 0.8 and ψ = 0.9, the Shewhart $\bar{X}$ chart does not signal within 60 observations after σ_X shifts but the Shewhart e^2 chart signals at observation 100, while the EWMA_X chart failed to signal again and the EWMA_{e^2} chart signal at observations 65. Similar to the conclusion for sustained shifts in μ_X, both Figures 5 and 6 show that the detection capability of the EWMA chart combination in detecting sustained shifts in σ_X of size $2.4\,\sigma_{X_0}$ due to σ_α is better than that of the Shewhart chart combination, especially when the process autocorrelation is relatively high. The numerical results provided in Tables 27 and 35 also show that the SSATS values of the EWMA chart combination (an average of 5.6 hours) is only slightly smaller than the SSATS value of the Shewhart chart combination (an average of 7.2 hours) when ϕ = 0.2 and ψ = 0.9; while the SSATS values of the EWMA chart combination (an average of 7.1 hours) is much smaller than the SSATS value of the Shewhart chart combination (an average of 15.9 hours) when ϕ = 0.8 and ψ = 0.9.

Thirdly, consider a similar situation with the previous one except that the sustained increase in σ_X of size $2.4\,\sigma_{X_0}$ is due to σ_e. The effect of this special cause is simulated by using σ_{ε_1} via Equation (3.3.28) in place of σ_e, where $\sigma_X = 2.4\sigma_{X_0}$. The plots for this situation are presented in Figures 7 and 8. These plots show that when ϕ = 0.2 and ψ = 0.9, both the Shewhart $\bar{X}$ chart and Shewhart e^2 chart signal at observation 68, while the EWMA_X chart and the EWMA_{e^2} chart signal at observations 101 and 66, respectively. When ϕ = 0.8 and ψ = 0.9, both the Shewhart $\bar{X}$ chart and EWMA_X chart cannot detect the shift within 60 observations, but the Shewhart e^2 chart and EWMA_{e^2} chart show signals as early as Observations 80 and 66, respectively. Tables 27 and 35 also provide SSATS values for the two chart combinations – 7.0 hours on average for the Shewhart chart combination and 5.4 hours on average for the EWMA chart combination when ϕ = 0.2 and ψ = 0.9; 9.9 hours on average for the Shewhart chart combination and 2.8 hours on average for the EWMA chart combination when ϕ = 0.8 and ψ = 0.9. The conclusions in this case is the same as in the previous two cases, i.e., the EWMA chart combination provides better performance than the Shewhart chart combination does, especially when the process autocorrelation is relatively large.

8.1.3 Transient Shifts

First consider the situation in which a special cause produces transient shifts in μ_X of size $5.0\sigma_{X_0}$, and these shifts last for only one hour. Suppose that these transient shifts affect only observations 24, 64 and 96. The effect of this special cause is simulated by adding 5.0 to observations 24, 64 and 96 in the two control chart combinations in Figures 1 and 2. The plots of the two control chart combinations using the data with the shifted process mean for two autocorrelated processes are shown in Figures 9 and 10.

In Figure 9, it appears that there is no signal at observations 24, 64 or 96, or within four observations after observations 24, 64 or 96, respectively, on the plots of either the Shewhart $\overline{X}$ chart or the EWMA_X chart. On the other hand, both the Shewhart e^2 chart and the EWMA_{e^2} chart have significant jumps and signals at observations 24, 64 and 96. The control chart combinations in Figure 10 show similar performance, except the signals on the Shewhart e^2 chart plot are less significant but the signals on the EWMA_{e^2} chart plot are more significant. Note that with multiple transient shifts, if there is a signal by a control chart and the cause is found and eliminated, then the chart could be restarted from the starting value. In this example, the control chart is not restarted after a significant jump occurs, assuming no immediate action has been done. Without resetting, there are many out-of-control observations after sustained shifts, because a previous observation has effect on the future observations in the EWMA_{e^2} chart. But the jumps at 64 and 96 are both large enough to signal even if the EWMA statistic is reset after previous signal(s). Tables 28 and 36 show that the signal probabilities of the Shewhart chart combination and the EWMA chart combination for this transient shift are 0.86 and 0.77 on average within four hours after the shift ends when ϕ = 0.2 and ψ = 0.9, while the signal probabilities are 0.65 and 1.00 on average when ϕ = 0.8 and ψ = 0.9. Both plots and numerical results demonstrate that the detection capability of the Shewhart chart combination in μ_X decreases as the process autocorrelation increases when the sustained shift is of size $5.0\sigma_{X_0}$ and of duration 1 hour, while the detection capability of the EWMA chart combination in the same situation increases up to probability 1.

Next consider the situation in which a special cause produces transient shifts in σ_α resulting in an increase in σ_X of size $7.0\sigma_{X_0}$ at observations 24, 64 and 96, and these shifts last only one hour. The same sets of control chart combinations are plotted in Figures 11 and 12 using the data with the increased process standard deviation. It appears that in both figures either the Shewhart $\bar{X}$ chart nor the EWMA$_X$ chart can detect these transient shifts. When ϕ = 0.2 and ψ = 0.9, Figure 11 shows that both the Shewhart e^2 chart and the EWMA$_{e^2}$ chart signal only at observation 64. There is only one signal because the detection ability of both charts for transient shifts in the process standard deviation is not high. According to Tables 28 and 36, the signal probabilities of the Shewhart chart combination and the EWMA chart combination for this transient shift are only 0.58 and 0.54 on average, respectively, within four hours after the shift ends. When ϕ = 0.8 and ψ = 0.9, the Shewhart e^2 chart fails to signal as well. Tables 28 indicates that the signal probabilities of the Shewhart chart combination in this case decreases to 0.42 on average. The EWMA$_{e^2}$ chart shows large jump and signal only at observation 64, and the signal probabilities for the EWMA chart combination in this case is 0.49 on average.

Finally, consider transient shifts of the same size and location in σ_X except that these shifts are due to σ_ε in this case. The plots of two control chart combinations using the data with the shifted process standard deviation are given in Figures 13 and 14. When ϕ = 0.2 and ψ = 0.9, Figure 13 indicates that the Shewhart $\bar{X}$ chart and the Shewhart e^2 chart detects the shifts at observations 24 and 96. The EWMA$_X$ chart does not detect the transient shifts within 4 observations after sustained shifts ends, while the

EWMA_{e^2} chart has large jumps and signals at observations 24 and 96. Tables 28 and 36 show that the average signal probabilities of the Shewhart chart combination and the EWMA chart combination for this transient shift within four hours after the shift ends are 0.58 and 0.55, respectively. When ϕ = 0.8 and ψ = 0.9, the Shewhart $\bar{X}$ chart only signals at observation 24 and the Shewhart e^2 chart detects the shifts at observations 24 and 96. The EWMA_X chart does not signal at all, while the EWMA_{e^2} chart has large jumps and signals at observations 24 and 96. Their average signal probabilities, provided in Tables 28 and 36, are 0.51 and 0.72.

After taking three cases of different transient shifts into consideration, the conclusion is that the EWMA chart combination almost always performs better than the Shewhart chart combination, especially when the process autocorrelation is relatively large. This conclusion is the same as the conclusion drawn for the sustained shifts.

8.1.4 Summary

The plots of the control chart combination in Figures 3 – 14 reinforce the conclusions that have been obtained from evaluating the numerical results of the control chart combinations. In particular, the EWMA chart combination based on individual sampling usually detects sustained shifts faster than the Shewhart chart combination based on dispersed sampling, especially when the process autocorrelation is high. When transient shifts is of interest, both chart combinations have similar performance when the autocorrelation is low, and the EWMA chart combination has significantly better performance as the autocorrelation increases.

Moreover, when the Shewhart e^2 chart signals, it is hard to tell whether the shift is a sustained one or a transient one from the plots. On the other hand, the impacts of sustained shifts and transient shifts on the EWMA_{e^2} chart are apparently distinguishable. Thus, in addition to faster detection of the shifts, the EWMA chart combination is also better than the Shewhart chart combination at showing the type of these shifts.

To summarize the conclusions in Section 8.1, if feasible, the EWMA_X and EWMA_{e^2} chart combination with individual sampling is always preferred over the Shewhart $\overline{X}$ and e^2 chart combination with dispersed sampling and $n = 4$.

8.2 Conclusions

The performance of various control charts and sampling plans in detecting both sustained and transient shifts at different levels of autocorrelation has been investigated in Chapters 4 – 7.

The results of these investigations show that the Shewhart $\overline{X}$ and X^2 chart combination is most efficient when it is based on dispersed sampling of size $n = 4$, while the Shewhart $\overline{e}$ and e^2 chart combination has best overall performance based on concentrated sampling of size $n = 2$. Individual sampling is the most efficient sampling plan for both the EWMA_X and EWMA_{X^2} chart combination and the EWMA_e and EWMA_{e^2} chart combination.

The EWMA_X and EWMA_{X^2} chart combination based on individual sampling gives best overall performance if only the observations chart combinations are considered, and the EWMA_e and EWMA_{e^2} chart combination based on individual sampling gives best overall performance if only the residuals chart combinations are considered. If both types of the charts are feasible options in a combination, the Shewhart $\bar{X}$ and e^2 chart combination based on dispersed sampling of $n = 4$ is best among the Shewhart chart combinations; while the EWMA_X and EWMA_{e^2} chart combination based on individual sampling is best among the EWMA chart combinations. Moreover, the EWMA_X and EWMA_{e^2} chart combination based on individual sampling is best among all chart combinations and sampling plans discussed in this dissertation.

Note that there are multiple tradeoffs concerning sample size, anticipated shift type and size, the level of autocorrelation involved in the sampling plan selection process, especially for the Shewhart control chart combinations. The choice made to optimize performance for one type and size of shift may be far from optimal for another type and size of shift. In most of practical applications, the specific type and size of parameter shifts is unknown and unpredictable. Thus the recommended efficient sampling plan for each chart combination is not required to be most efficient in every aspect, but to provide the most overall satisfactory performance compared to other choices. Moreover, it is possible that the most efficient sampling plan for one control chart combination is not feasible for some processes. A compromise has to made is to use another control chart or sampling plan, which can provide a fairly good performance, though inferior to the performance of ideal situation. For example, suppose that items are selected from a product line every hour, but the actual measurement on items is not available every hour. Thus the inspection can be done only on a group of n items at the end of the n-hour period. In this case, dispersed sampling also has fairly good performance, though inferior to the performance of individual sampling.

8.3 Additional Discussion

Discussions in this dissertation are based on the assumptions that the true model is known and the in-control process parameter values are known, so that model specification error and parameter estimation error can be neglected. But in practice, the only information that can be obtained directly from a process is the observations. Thus the impacts on a control chart of specifying model and estimating parameters can be severe. Based on previous research discussed in Section 2.4, more work can be done in this aspect for different sampling plans.

All of the EWMA chart combinations considered here require that the value of λ be specified. In this dissertation, $\lambda_1 = 0.026$ and $\lambda_4 = 0.1$ are employed, because Lu and Reynolds (1999b) showed that in the autocorrelated case, a relatively small value of λ works well across a range of shifts for both the EWMA chart of the observations and the EWMA chart of the residuals. More investigation can be done regarding to the optimal choice of λ in the control chart combinations of both the residuals and the observations.

The results on chart combination performance that have been presented here are based on the assumption that the observations from the process are normally distributed. However, this assumption may be unrealistic in some processes, and this can seriously hinder the practical usefulness of the chart combination along with its efficient sampling plan. Thus the effect of non-normality can be also studied to test the robustness of the recommended chart combinations and sampling plans.

In addition to producing a sudden sustained or transient shift in a process parameter, which has been discussed in this dissertation, a special cause may result in a drift of a process parameter away from its in-control value. For example, drifts frequently occur in chemical processes because of the aging of a catalyst. They are also attributed to the gradual wear-out or deterioration of tools, fixtures or other critical components.

The final point concerns another issue regarding to shifts in a process parameter. In this dissertation, it is assumed that ϕ is constant during the process operation, and a special cause will change only one of the parameters μ_X, σ_α, and σ_ε, and the other two will remain at their in-control values. It is quite common that more than one parameter are affected by a special cause at the same time. Moreover, ϕ can also be shifted during the process, and very little literature exists on this topic except Bagshaw and Johnson (1977).

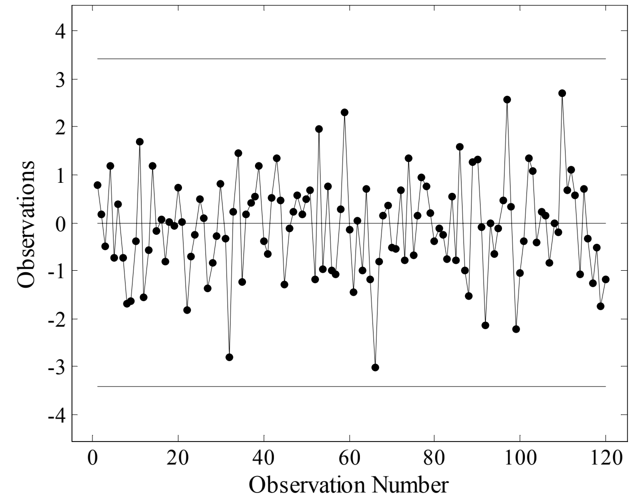
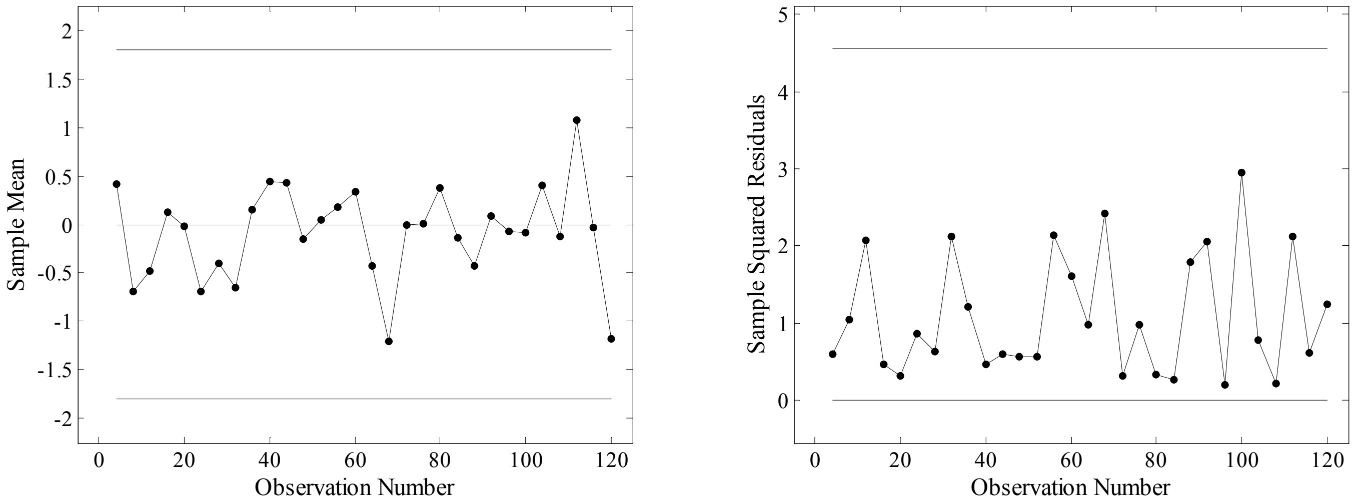
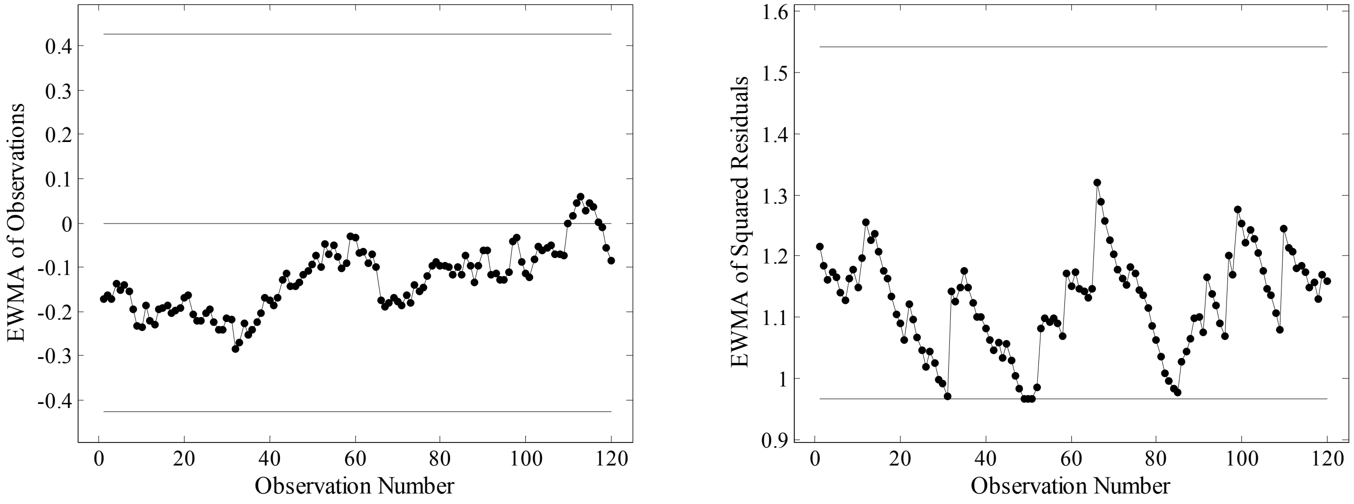


Figure 1: Control Chart Plots for the 120 Autocorrelated In-Control Observations with μ_{X_0} = 0 and σ_{X_0} = 1 when ϕ = 0.2 and ψ = 0.9.

First Row: Shewhart X Chart. Second Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Third Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).





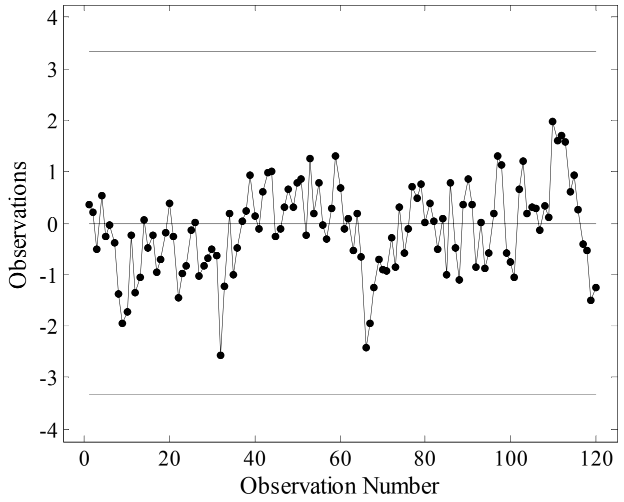
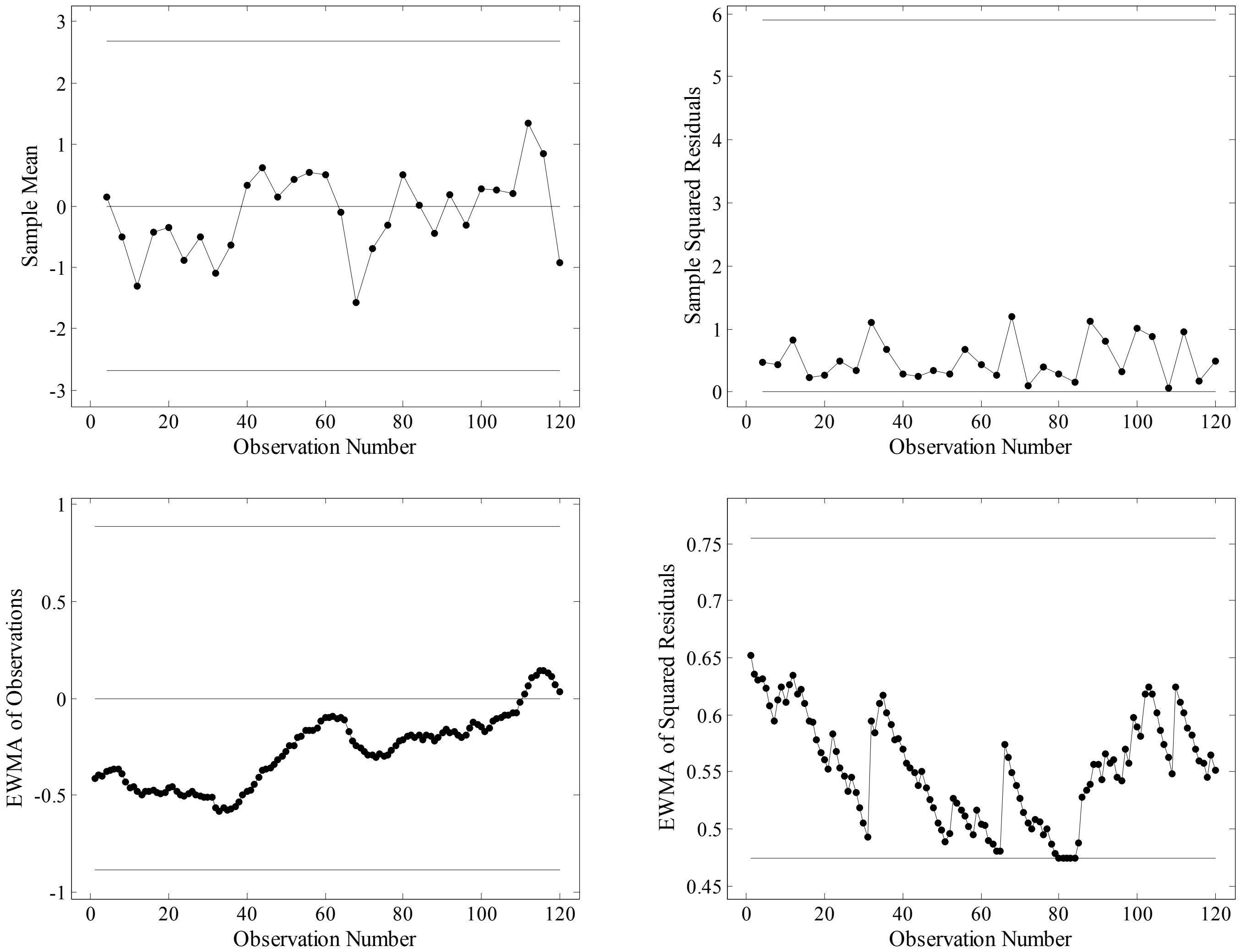


Figure 2: Control Chart Plots for the 120 Autocorrelated In-Control Observations with μ_{X_0} = 0 and σ_{X_0} = 1 when ϕ = 0.8 and ψ = 0.9.

First Row: Shewhart X Chart. Second Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Third Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).



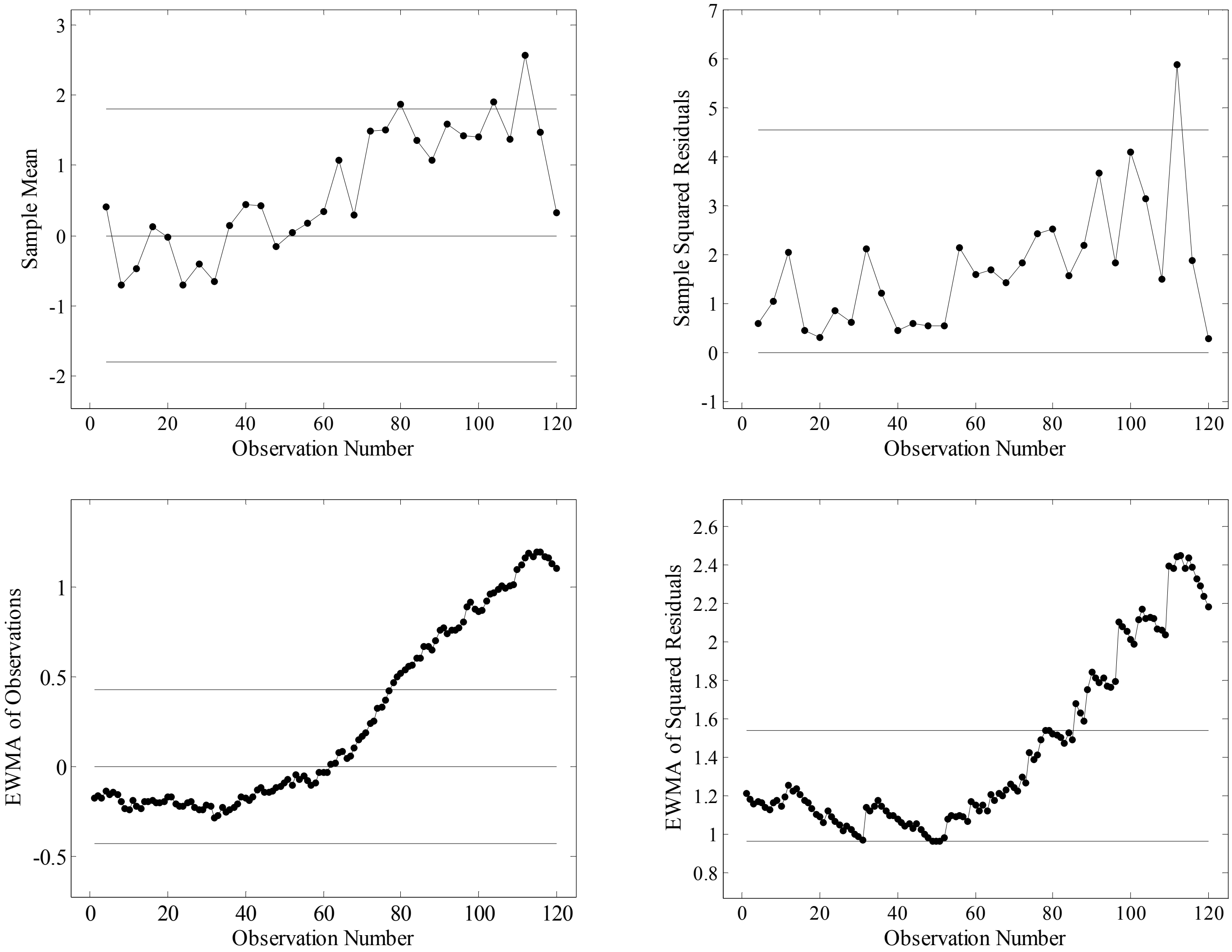


Figure 3: Control Chart Plots for a Sustained Shift in μ_X of Size $1.5\sigma_{X_0}$ Starting at Observation 61 when ϕ = 0.2 and ψ = 0.9.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).

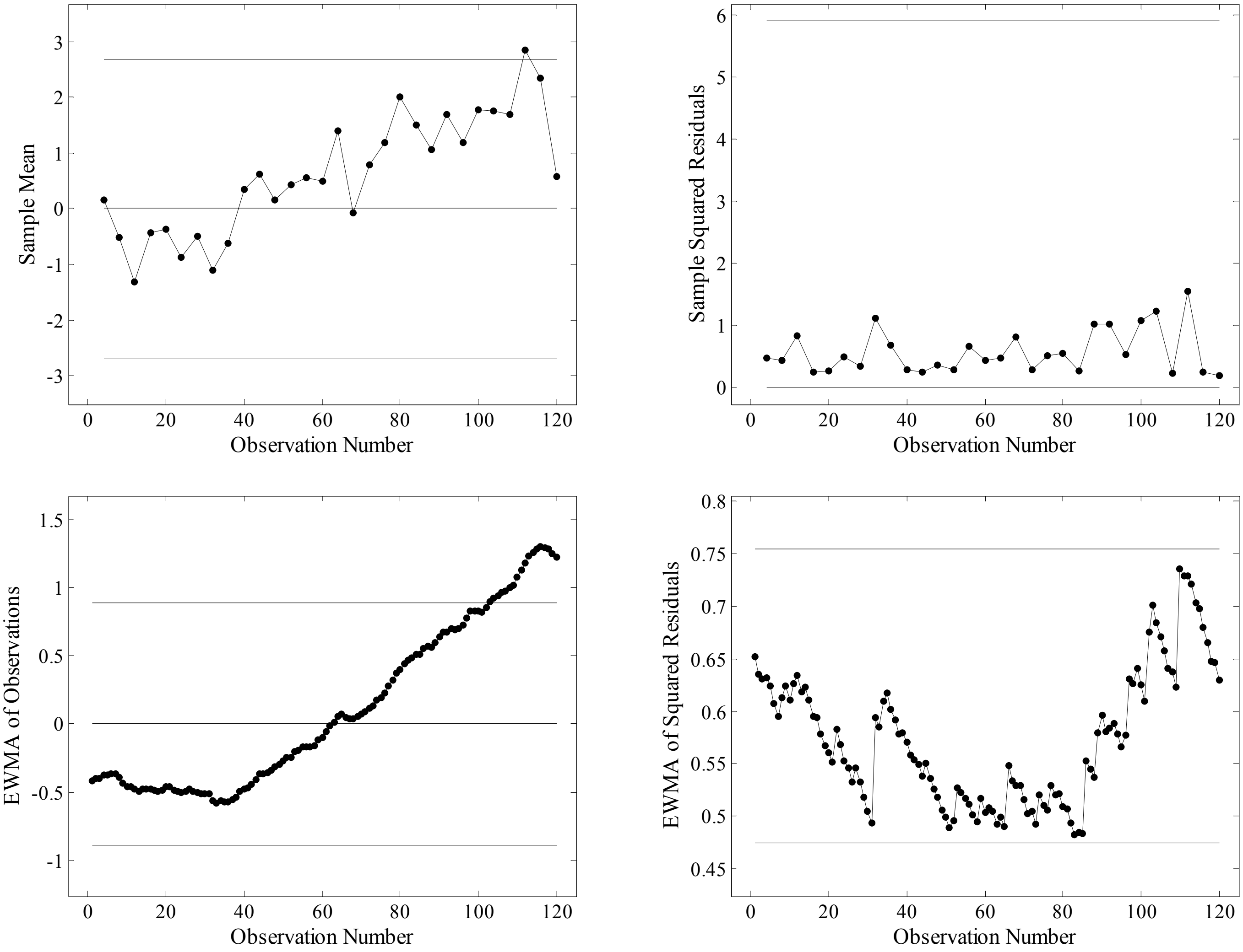


Figure 4: Control Chart Plots for a Sustained Shift in μ_X of Size $1.5\sigma_{X_0}$ Starting at Observation 61 when ϕ = 0.8 and ψ = 0.9.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).

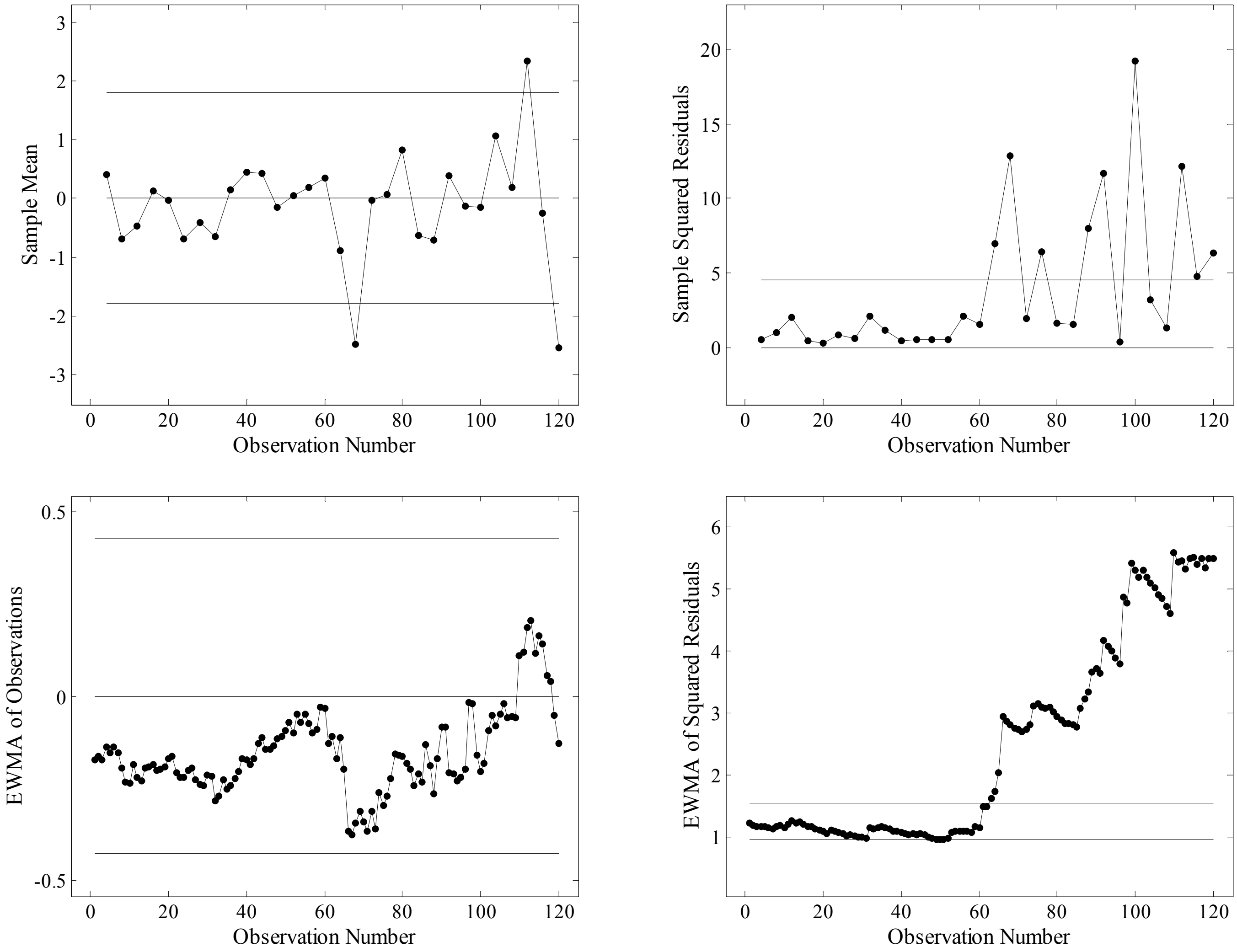


Figure 5: Control Chart Plots for a Sustained Shift in σ_X of size $2.4\sigma_{X_0}$ due to σ_α Starting at Observation 61 when $\phi = 0.2$ and $\psi = 0.9$.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and $n = 4$. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling ($n = 1$).

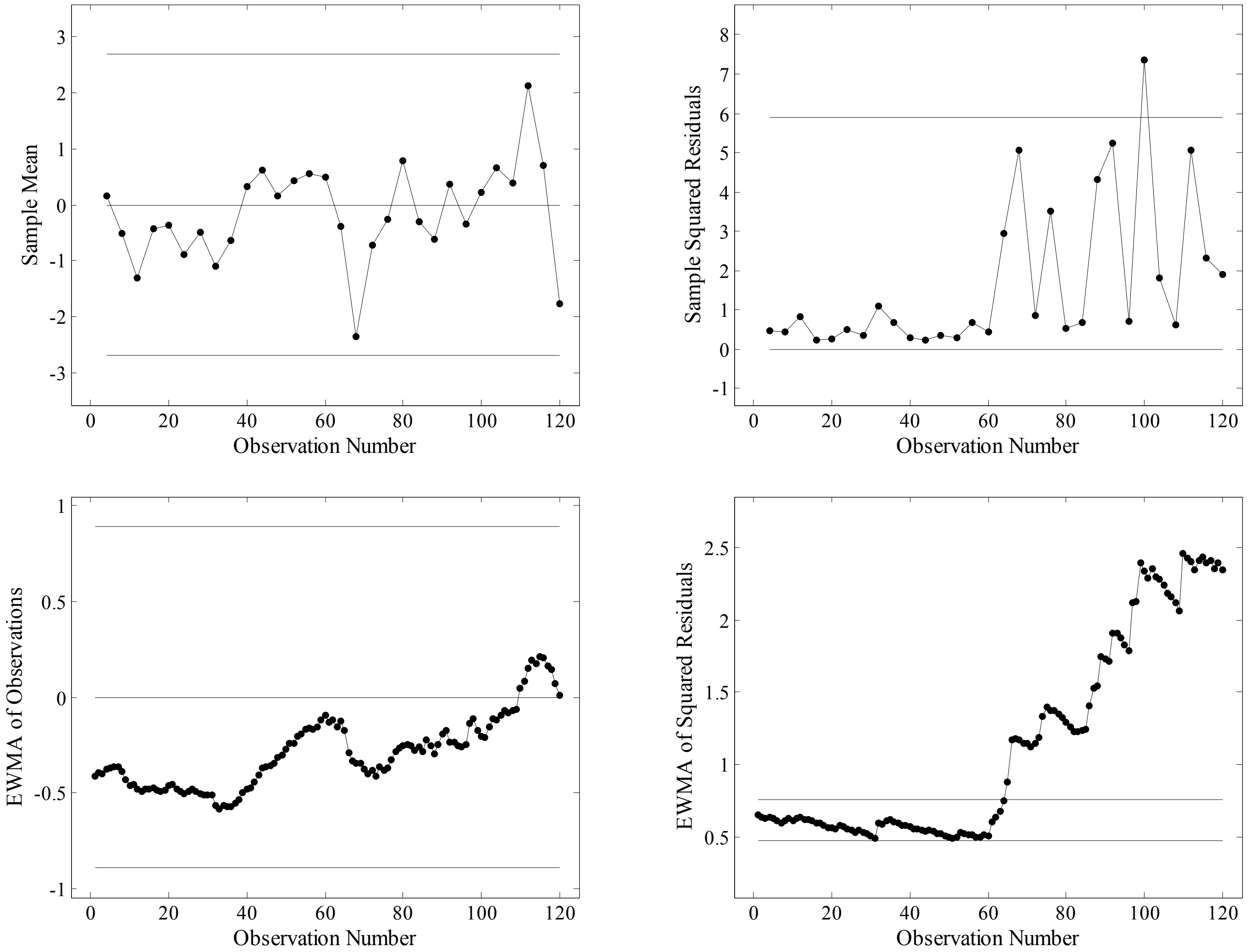


Figure 6: Control Chart Plots for a Sustained Shift in σ_X of size $2.4\sigma_{X_0}$ due to σ_α Starting at Observation 61 when $\phi = 0.8$ and $\psi = 0.9$.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and $n = 4$. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling ($n = 1$).

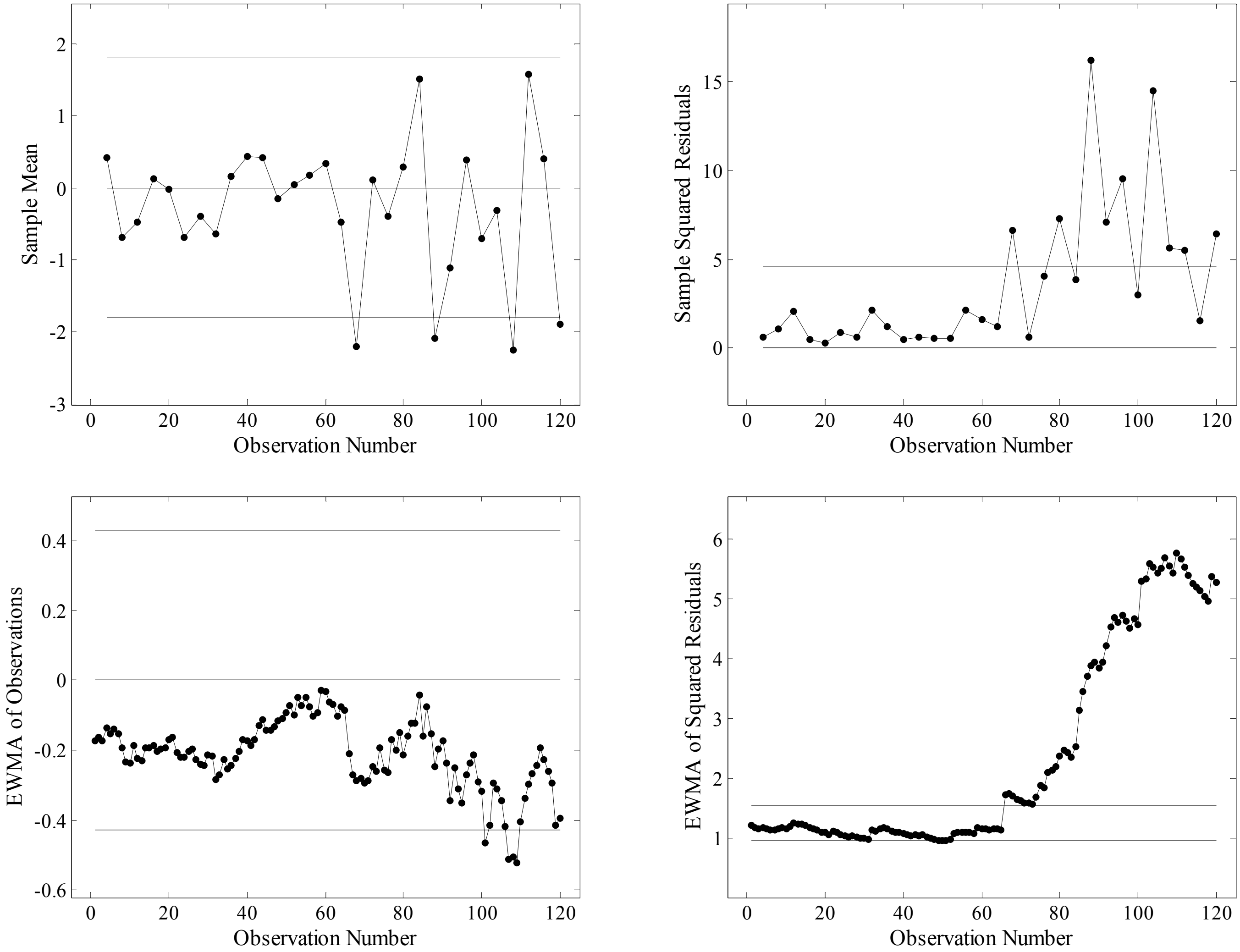


Figure 7: Control Chart Plots for a Sustained Shift in σ_X of size $2.4\sigma_{X_0}$ due to σ_e Starting at Observation 61 when $\phi = 0.2$ and $\psi = 0.9$.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and $n = 4$. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling ($n = 1$).

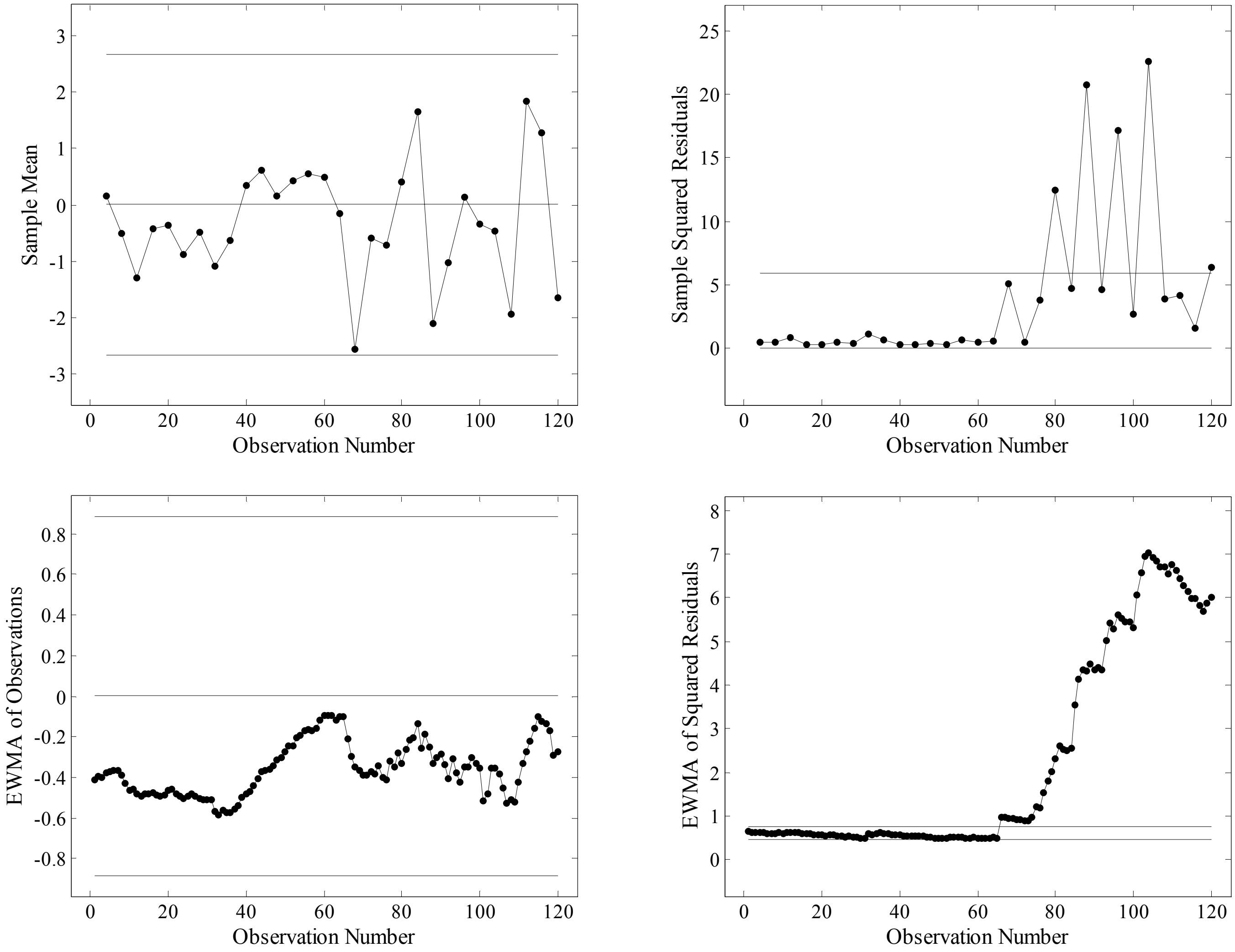


Figure 8: Control Chart Plots for a Sustained Shift in σ_X of size $2.4\sigma_{X_0}$ due to σ_e Starting at Observation 61 when ϕ = 0.8 and ψ = 0.9.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).

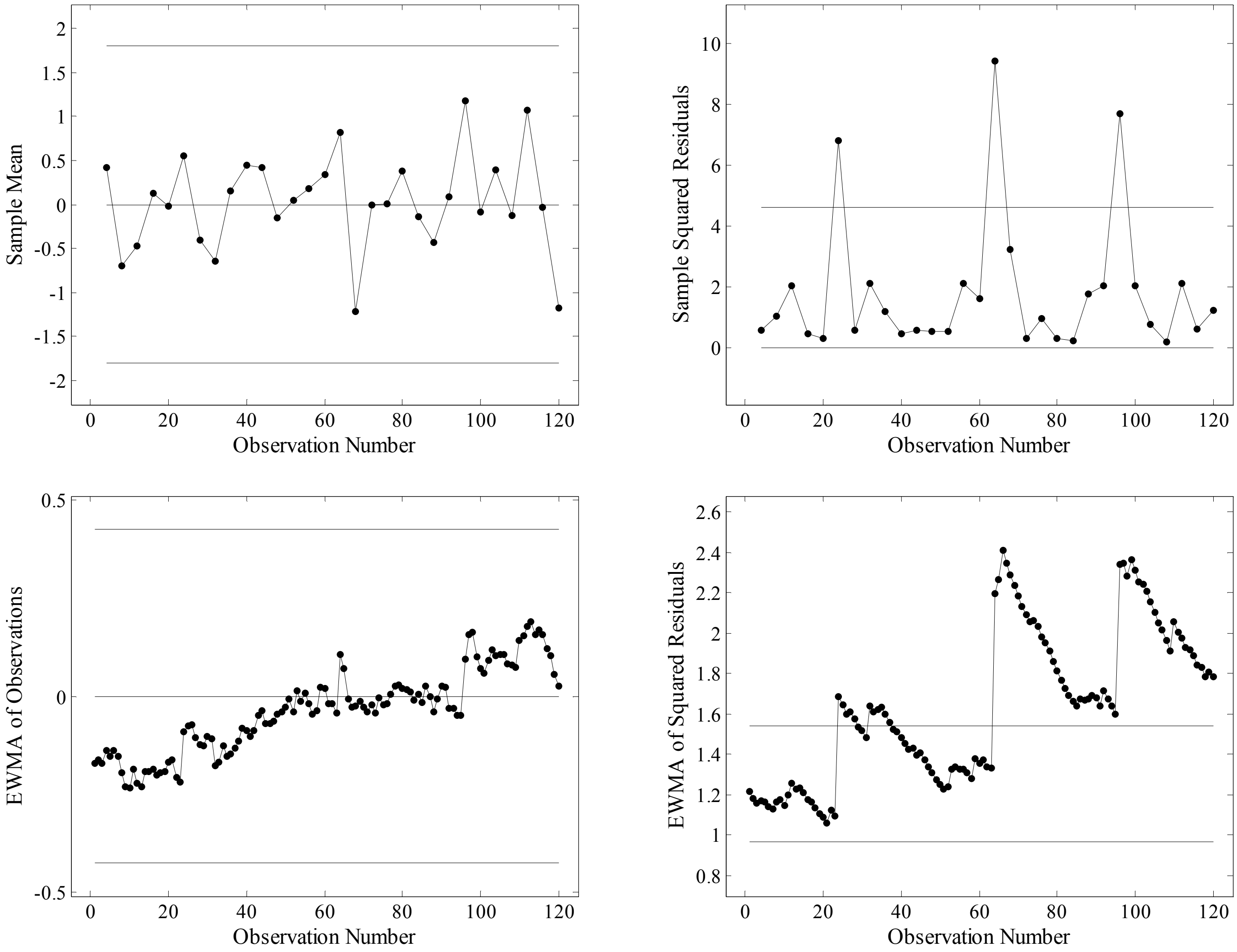


Figure 9: Control Chart Plots for One-Hour Transient Shifts in μ_X of size 5.0 σ_{X_0} that Affects Only Observations 24, 64 and 96 when ϕ = 0.2 and ψ = 0.9.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).

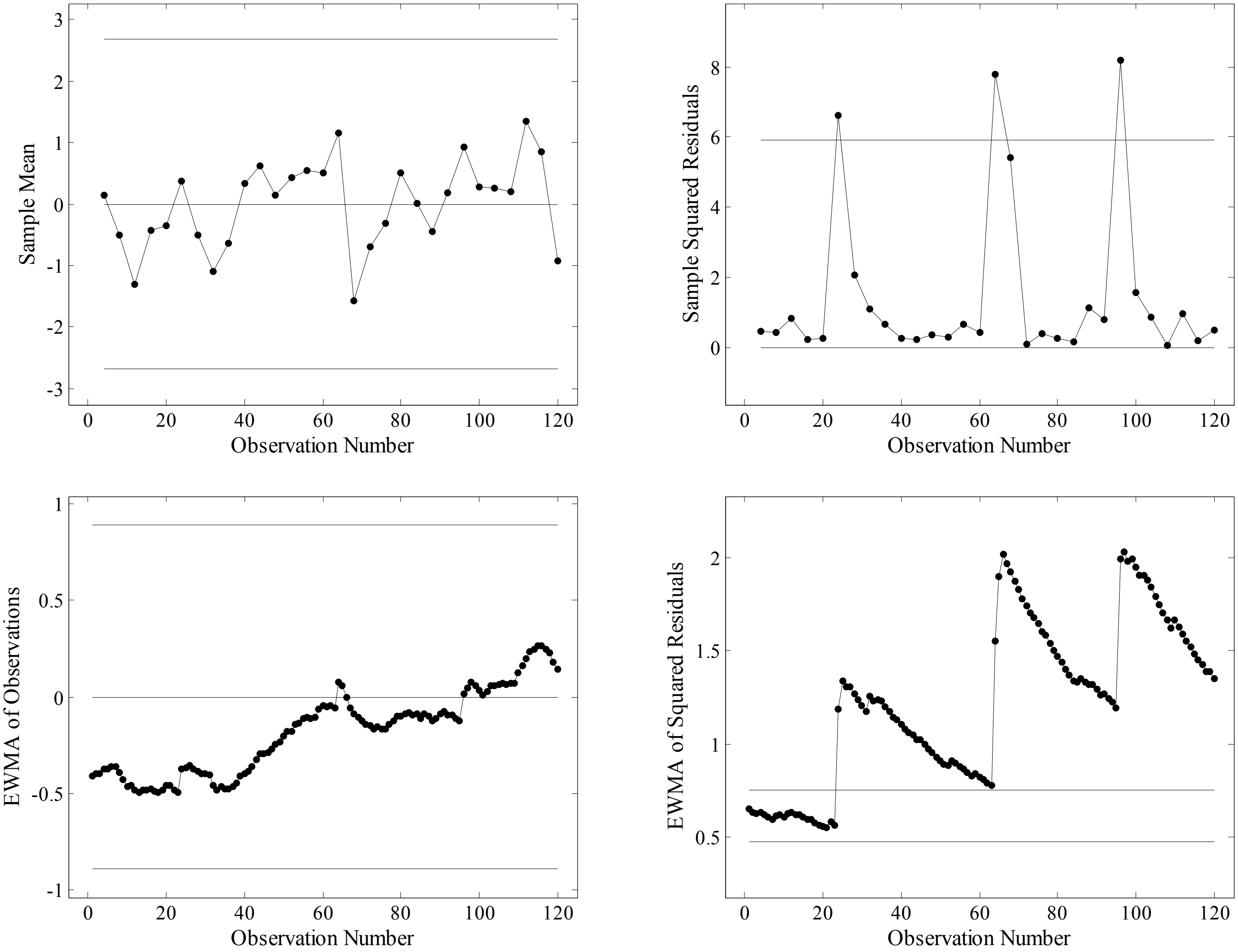


Figure 10: Control Chart Plots for One-Hour Transient Shifts in μ_X of size 5.0 σ_{X_0} that Affects Only Observations 24, 64 and 96 when ϕ = 0.8 and ψ = 0.9.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).

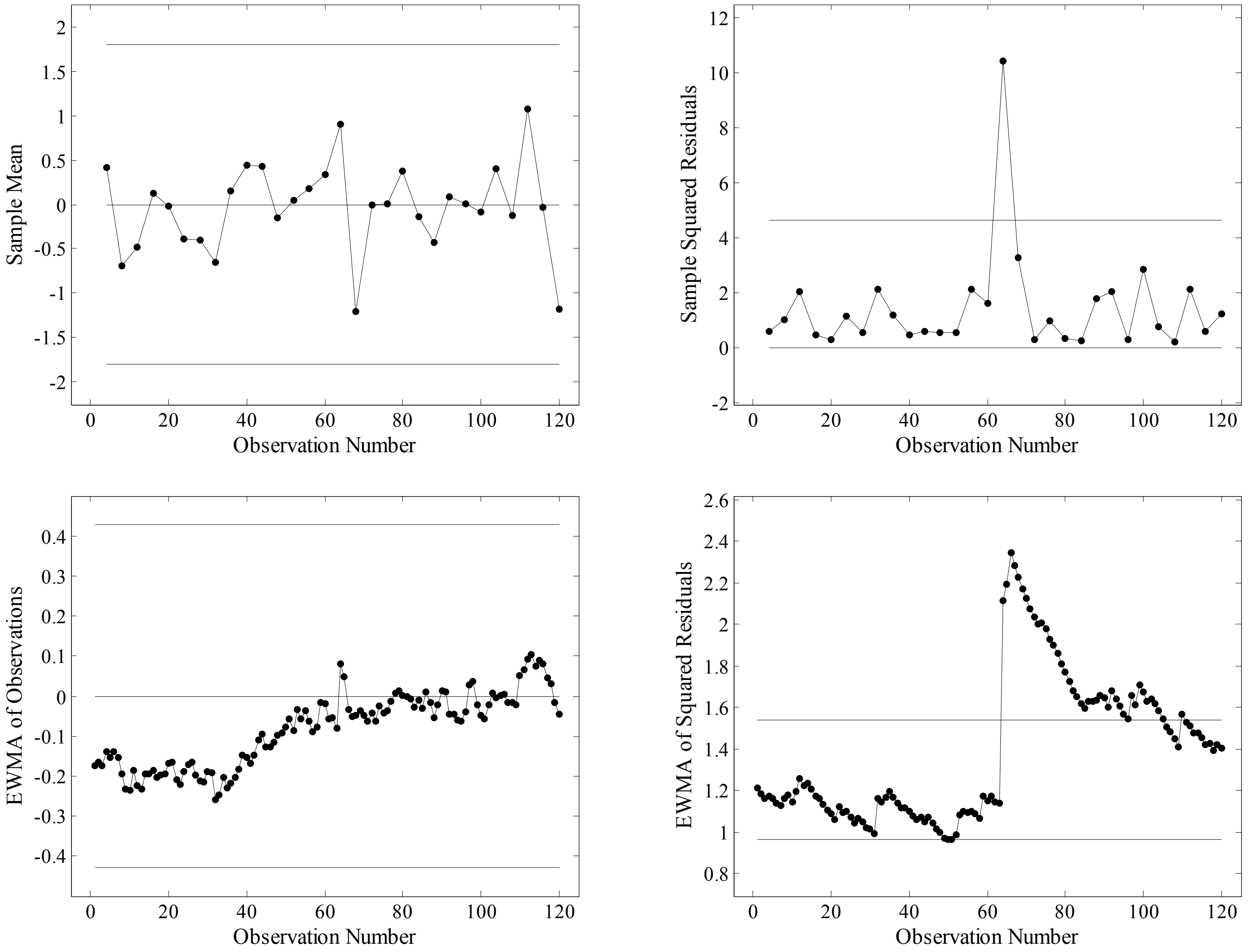


Figure 11: Control Chart Plots for One-Hour Transient Shifts in σ_X of size $7.0\sigma_{X_0}$ due to σ_α that Affects Only Observations 24, 64 and 96 when $\phi = 0.2$ and $\psi = 0.9$.

First Row: Shewhart $\bar{X}$ and e^2 Chart Combination with Dispersed Sampling and $n = 4$. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling ($n = 1$).

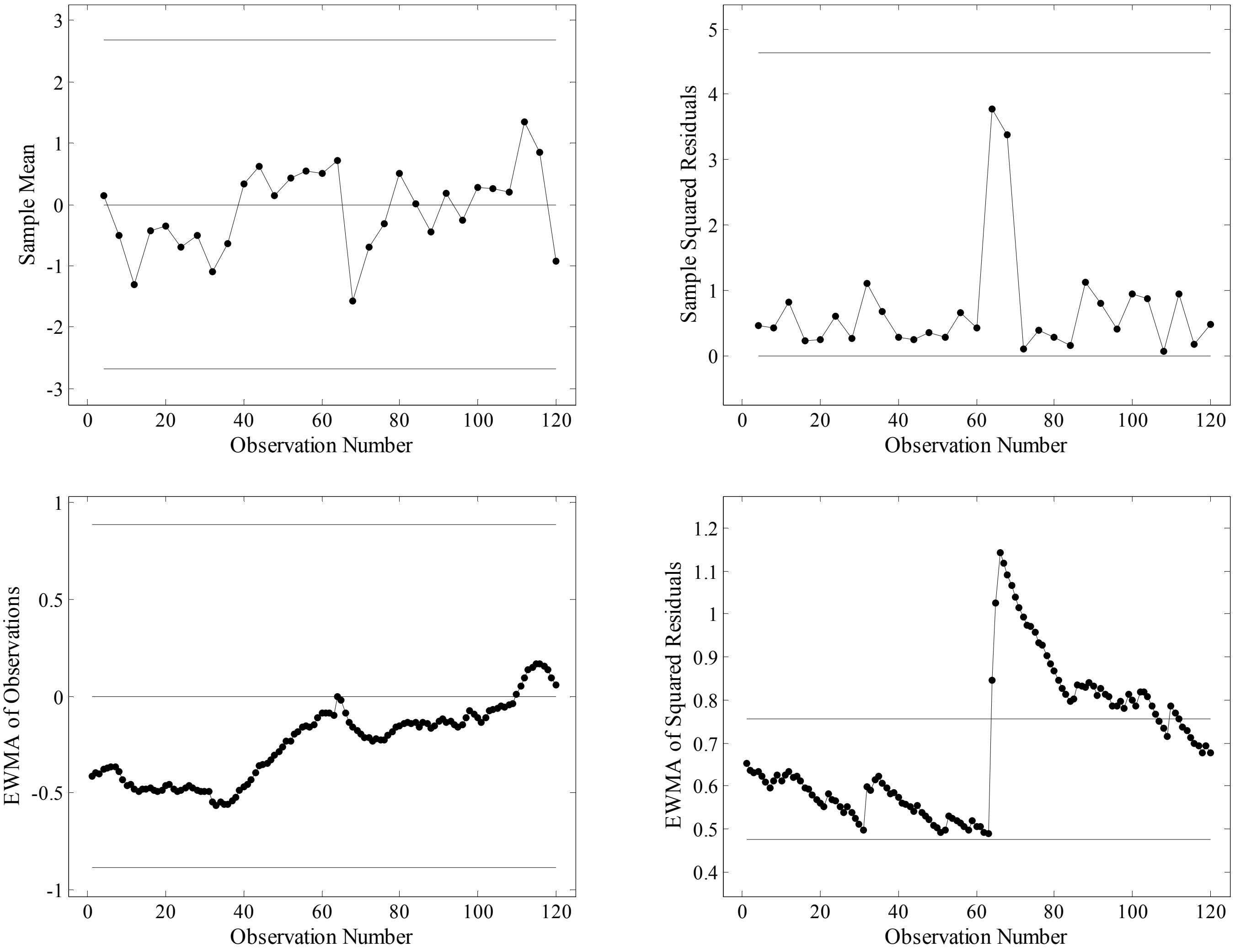


Figure 12: Control Chart Plots for One-Hour Transient Shifts in σ_X of size $7.0\sigma_{X_0}$ due to σ_α that Affects Only Observations 24, 64 and 96 when ϕ = 0.8 and ψ = 0.9.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).

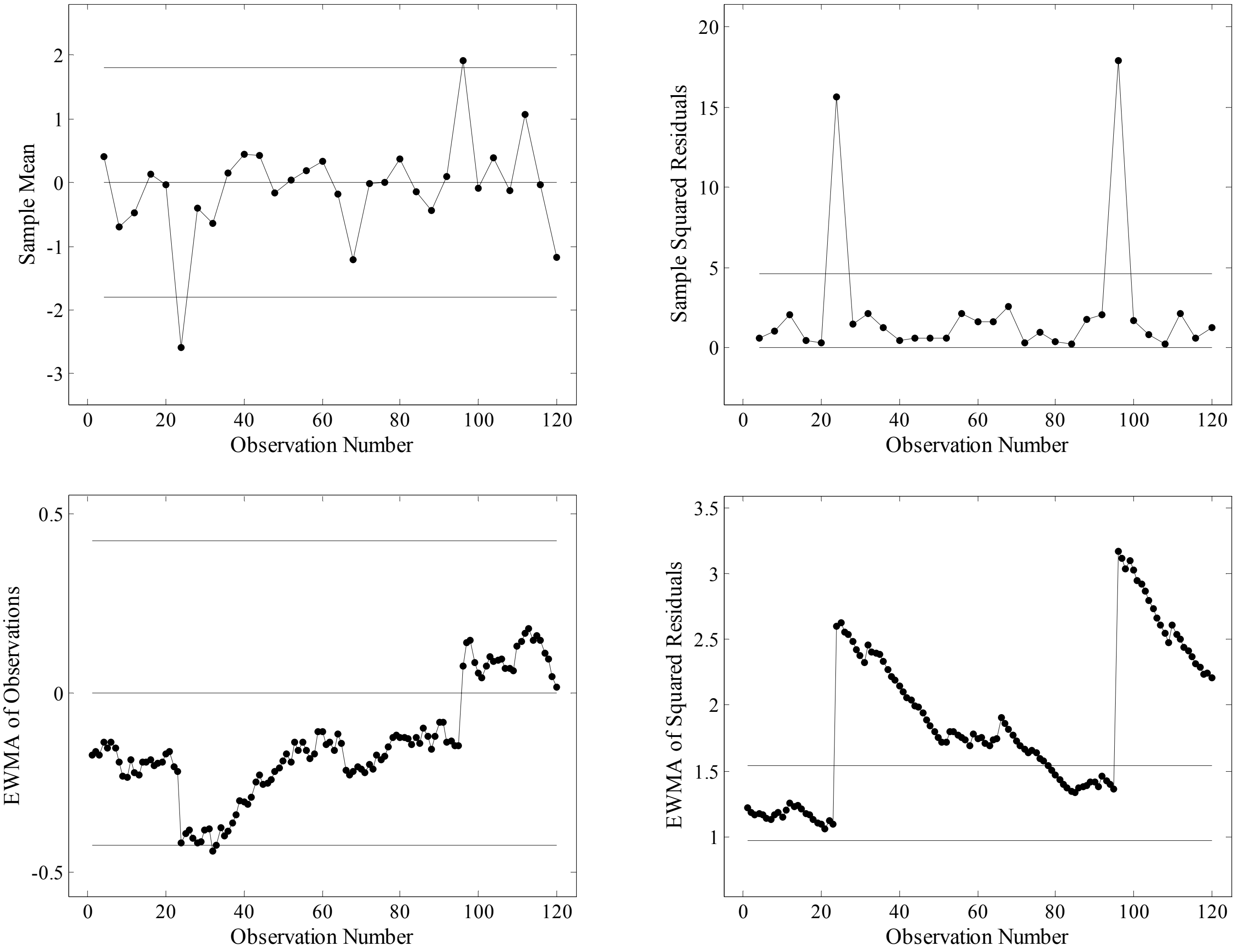


Figure 13: Control Chart Plots for One-Hour Transient Shifts in σ_X of size $7.0\sigma_{X_0}$ due to σ_ε that Affects Only Observations 24, 64 and 96 when ϕ = 0.2 and ψ = 0.9.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and n = 4. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling (n = 1).

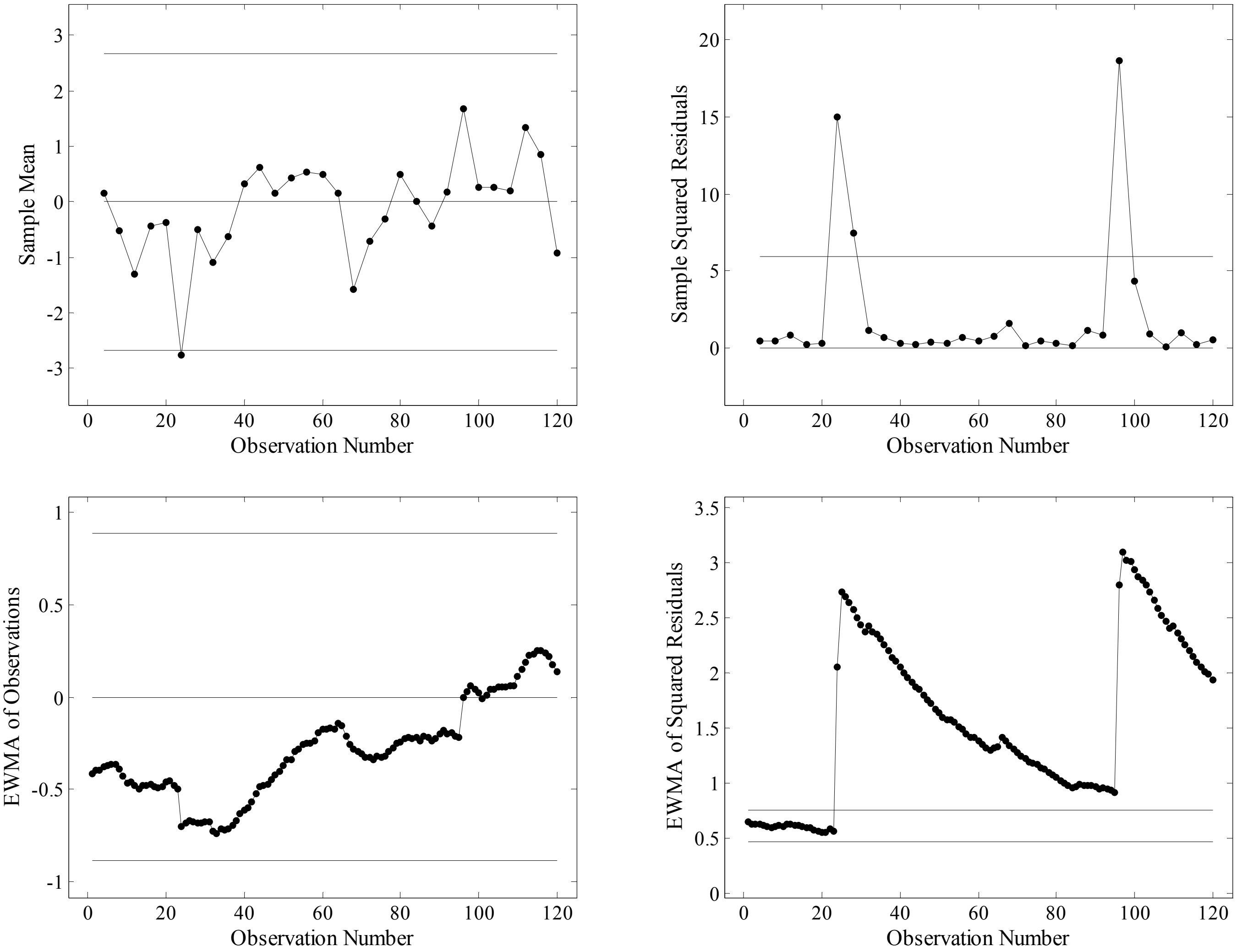


Figure 14: Control Chart Plots for One-Hour Transient Shifts in σ_X of size $7.0\sigma_{X_0}$ due to σ_ε that Affects Only Observations 24, 64 and 96 when $\phi = 0.8$ and $\psi = 0.9$.

First Row: Shewhart $\overline{X}$ and e^2 Chart Combination with Dispersed Sampling and $n = 4$. Second Row: EWMA_X and EWMA_{e^2} Chart Combination with Individual Sampling ($n = 1$).

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Vita

Xin Zhong was born on February 6, 1979 in Sichuan, P. R. China. She joined the Department of Statistics and Finance at University of Science and Technology of China in September 1996 and received her Bachelor of Science degree in Finance in June 2000. She started her graduate study in the Department of Statistics at Virginia Polytechnic Institute and State University in August 2000, and earned a Master of Science degree in Statistics in December 2001. In January 2002, she enrolled in the doctoral program in the same department and successfully defended her dissertation in February 2006, to earn a Doctor of Philosophy degree in Statistics. Currently she is working as Quantitative Analyst in Equity Trading Research at Fidelity Management and Research Company.