

Homework Journaling in Undergraduate Mathematics

Alexis Larissa Johnston

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Jesse L. M. Wilkins, Chair  
Peter E. Doolittle  
Bettibel C. Kreye  
Anderson H. Norton III

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## ABSTRACT

Over the past twenty years, journal writing has become more common in mathematics classes at all age levels. However, there has been very little empirical research about journal writing in college mathematics (Speer, Smith, & Horvath, 2010), particularly concerning the relationship between journal writing in college mathematics and college students' motivation towards learning mathematics. The purpose of this dissertation study is to fill that gap by implementing homework journals, which are a journal writing assignment based on Powell and Ramnauth's (1992) "multiple-entry log," in a college mathematics course and studying the relationship between homework journals and students' motivation towards learning mathematics as grounded in self-determination theory (Ryan & Deci, 2000). Self-determination theory predicts intrinsic motivation by focusing on the fundamental needs of competence, autonomy, and relatedness (Ryan & Deci, 2000). In addition, the purpose of this dissertation study is to explore and describe the relationship between homework journals and students' attitudes towards writing in mathematics.

A pre-course and post-course survey was distributed to students enrolled in two sections of a college mathematics course and then analyzed using a  $2 \times 2$  repeated measures ANOVA with time (pre-course and post-course) and treatment (one section engaged with homework journals while the other did not) as the two factors, in order to test whether the change over time was different between the two sections. In addition, student and instructor interviews were conducted and then analyzed using a constant comparative method (Anfara, Brown, & Mangione, 2002) in order to add richness to the description of the relationship between homework journals and students' motivation towards learning mathematics as well as students' attitudes towards writing in mathematics.

Based on the quantitative analysis of survey data, no differences in rate of change of competence, autonomy, relatedness, or attitudes towards writing were found. However, based on the qualitative analysis of interview data, homework journals were found to influence students' sense of competence, autonomy, and relatedness under certain conditions. In addition, students' attitudes towards writing in mathematics were strongly influenced by their likes and dislikes of homework journals and the perceived benefits of homework journals.

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**Now all glory to God, who is able, through his mighty power at work within us, to accomplish infinitely more than we might *ask* or *imagine* or *dream*.**

**Ephesians 3:20**

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## **Chapter One: Introduction.**

### **Overview**

“The traditional view has been that students learn to write in English classes and to compute in mathematics classes and ‘never the twain shall meet’” (Davidson & Pearce, 1988, p. 42). However, over the past twenty years journal writing has become more common in mathematics classes at all age levels. With the increase of journal writing in mathematics classes has also come an increase in research to determine the effects of journal writing on students (Grossman, Smith, & Miller, 1993; Waywood, 1992), on teachers (Drake & Amspaugh, 1994; Greer & Reed, 2008), and on communication between students and teachers (Borasi & Rose, 1989; McIntosh & Draper, 2001) in those mathematics classes. Whereas most of this research is focused on K-12 mathematics teaching and learning (Pugalee, 2001, 2004; Miller & England, 1989; Waywood, 1992), there are only a few examples of research focused on college mathematics teaching and learning (Borasi & Rose, 1989; Grossman et al., 1993; Hamdan, 2005; Jurdak & Zein, 1998). Most of the current literature concerning journal writing and college mathematics teaching and learning seems to be examples of writing assignments and projects used in college mathematics courses (Beidleman, Jones, & Wells, 1995; Greer & Reed, 2008; Morrel, 2006; Shannon & Curtin, 1992) instead of empirical research attempting to determine the potential benefits of journal writing for students, for teachers, or for communication between students and teachers.

Mathematics education researchers do claim that the use of journal writing activities “can provide a valuable means to facilitate a personalized making-of-meaning approach to learning mathematics” (Borasi & Rose, 1989, p. 347). In addition, journal writing in college mathematics

courses has been shown to have a number of benefits for students, for teachers, and for communication between students and teachers. For example, journal writing has been shown to have a positive effect on students' procedural and conceptual mathematical knowledge; with the use of journal writing, students are better able to carry out mathematical operations and procedures and, more importantly, they are better able to understand the concepts behind the operations and procedures (Jurdak & Zein, 1998). Also, journal writing benefits teachers in the way it allows teachers to assess students' understanding (Miller & England, 1989). When a student writes a journal entry that is thoughtful and engaged, the teacher has a tangible and concrete snapshot of that students' thinking about that particular topic. Journal writing may inform teachers about what a student does and does not understand (Borasi & Rose, 1989; Drake & Amspaugh, 1994). Finally, journal writing provides a space for dialogue; specifically dialogue between the student and the teacher (Hamdan, 2005). Journal writing offers students the chance to talk individually with their teacher, to ask questions that they may not have asked in class, to get personal feedback directly from their teacher, and to communicate with their teacher (Borasi & Rose, 1989; Hamdan, 2005; Williams & Wynne, 2000).

According to a sweep of the current literature concerning journal writing and college mathematics teaching and learning, one potential benefit of journal writing that has not yet been studied is the relationship between journal writing in college mathematics courses and college students' motivation towards learning mathematics. Anecdotal evidence and informal analysis suggest that journal writing and college mathematics education have some relationship. For example, Sanders (2009) noticed that her students seemed to be more interested in learning mathematics when they were able to explore and investigate a topic on their own through the use of journal writing instead of just being told what to do, say, or write. Students seem to feel a

sense of excitement for mathematical inquiry (Borasi & Rose, 1989) and a sense of enthusiasm for mathematics (Powell & Ramnauth, 1992) when engaged with journal writing. These anecdotal reports from researchers and educators hint at the idea that journal writing in college mathematics and college students' motivation towards learning mathematics do have some relationship. However, based on a review of the current literature specific studies linking these two phenomena do not seem to exist. This dissertation seeks to fill that void in the current literature by examining the relationship between journal writing in college mathematics and college students' motivation towards learning mathematics.

### **Research Questions**

While there is an abundance of research about journal writing in K-12 mathematics, there is very little empirical research about journal writing in college mathematics (Speer, Smith, & Horvath, 2010). This study ties together journal writing and college mathematics in a unique way. The unique element of this study is that it seeks to examine whether or not journal writing affords any benefits for students' intrinsic motivation. A particular theory of motivation, called self-determination theory (Ryan & Deci, 2000), is especially suited for studying the relationship between journal writing in college mathematics and college students' motivation towards learning mathematics. Self-determination theory predicts intrinsic motivation by focusing on the fundamental needs of competence, autonomy, and relatedness (Ryan & Deci, 2000). Ryan and Deci (2000) describe competence, autonomy, and relatedness in the following ways: competence describes the degree to which an individual feels capable to complete a certain task or activity; autonomy describes the degree to which an individual feels that they are in control of their behavior; relatedness describes the degree to which an individual feels comfortable in their social environment (e.g., their classroom).

Powell and Ramnauth (1992) describe a type of journal writing called a “multiple-entry log” which is primarily a vehicle for students’ reflection. Students divide a sheet of paper into two vertical columns. In the column on the left, students write some mathematical text which can be any “combination of mathematical prose or notational expressions selected from a textbook, a lecture, a problem set, a calculator or computer screen, or from any other course material” include a mathematical discussion (Powell & Ramnauth, 1992, p. 14). In the column on the right, students reflect on the mathematical text they have just recorded. Similar to the “multiple-entry log,” homework journals are a particular type of writing in mathematics in which students select one problem from their assigned homework problems based on what interests them or what challenges them from their assigned homework problems. On the left-hand column of a piece of paper, students solve this one problem in detail showing all steps of their work and clearly communicating their problem-solving process. On the right-hand column of the same piece of paper, students then write a short reflection about this problem and their problem-solving process based on specific prompts. Teachers then read and respond to each student’s homework journal, creating an opportunity dialogue between students and teachers.

This type of writing potentially meets students’ needs for all three components associated with self-determination theory. Because students are successfully solving interesting or challenging mathematical problems, this may meet students’ need for competence. Because students get to choose which problem they want to solve, this may meet students’ need for autonomy. Because teachers respond individually to students’ writing and pay attention to what students are saying, this may meet students’ need for relatedness. These connections will be explored in further detail in later portions of this dissertation.

By targeting undergraduate mathematics students who are not majoring in mathematics or engineering and by implementing homework journals within a college mathematics course, this study seeks to determine the relationship between writing in mathematics and students' intrinsic motivation towards mathematics, students' attitudes towards writing in mathematics, and students' achievement in mathematics. More specifically: What is the relationship between homework journals and 1) students' psychological needs for competence, autonomy, and relatedness, 2) students' attitudes towards writing in mathematics, and 3) students' achievement in mathematics?

### **Outline**

In Chapter 2, I will present a review of the current literature and outline a conceptual framework for this dissertation study. First, I will review the current literature concerning journal writing in mathematics and teaching practices in college mathematics courses. Journal writing in mathematics and teaching practices in college mathematics courses are both relevant to this dissertation study because of the study's focus on journal writing in college mathematics courses. Secondly, I will outline a conceptual framework that explains the connections between journal writing in mathematics and self-determination theory. This framework is grounded in a perspective of learning and the reasons why journal writing in mathematics, particularly homework journals, fits into that perspective of learning and acts as a powerful learning tool for students will be discussed. I will then describe the connections between journal writing in mathematics and students' motivation for learning mathematics based on self-determination theory. Finally, I will discuss the connections between the research for this dissertation and the broader body of research about journal writing in college mathematics courses.

In Chapter 3, I will present the methods used to collect and analyze data for this study. I will describe the context of the study, the ways in which both quantitative and qualitative data were collected for the study, and the various methods used to analyze the data.

Chapter 4 will include the results from the study, particularly the analysis of both quantitative and qualitative data.

Chapter 5 will provide an overview of the entire study, connecting the results back to the literature and framework discussed in Chapter 2. The implications of the study will also be addressed in this chapter, particularly implications for future research connected to journal writing in mathematics and instructional implications for current instructors of college mathematics courses.

### **Summary**

The development and evaluation of homework journals, a particular type of journal writing in college mathematics courses, can provide college mathematics instructors with a valuable tool to foster students' mathematical learning and to increase students' intrinsic motivation. In particular, by connecting homework journals to the three main components of self-determination theory, competence, autonomy, and relatedness, it will be possible for researchers and college mathematics instructors to better understand practical ways to increase students' intrinsic motivation towards learning mathematics. Increasing students' intrinsic motivation towards learning mathematics could result in students' experiencing greater enjoyment of mathematics, more determination and perseverance in doing mathematics, and deeper understandings of mathematical content. And therefore the purpose of this study is to explore and describe the relationship between homework journals and students' psychological

needs for competence, autonomy, and relatedness, students' attitudes towards writing in mathematics, and students' achievement in mathematics.

## **Chapter Two: Literature Review and Conceptual Framework.**

### **Introduction**

This chapter contains two main sections. The first section is a review of the current literature concerning journal writing in mathematics and teaching practices in college mathematics courses. It is important to understand the current perceptions and uses of writing in mathematics courses – at any level – before embarking on a study concerning writing in mathematics. It is also important to understand the current climate and direction of literature concerning mathematics education in postsecondary classrooms. The second section of this chapter develops and outlines a conceptual framework that explains the connections between journal writing in mathematics and self-determination theory. This framework will begin with the perspective of learning that grounds this study and the reasons why journal writing in mathematics, particularly homework journals, fits into this perspective of learning and acts as a powerful learning tool for students. Next, the connections between journal writing in mathematics and students' motivation for learning mathematics through the lens of self-determination theory will be discussed. Finally, the connections between the research for this dissertation and the broader body of research about journal writing in college mathematics courses will be discussed.

### **Literature Review**

This literature review includes two components: literature related to writing in mathematics and literature related to teaching and learning in college mathematics courses. Although these two components are separated, they are closely related because the purpose of this study is to implement a journal writing assignment in a college mathematics course in order

to better understand the relationship between writing in mathematics and college students' intrinsic motivation towards mathematics, college students' attitudes towards writing in mathematics, and college students' achievement in mathematics.

### **Journal Writing in Mathematics**

The precise definition of journal writing is not widely agreed upon. Mathematics educators and teachers have many different definitions of journal writing. What one college mathematics instructor calls journal writing might be a very different practice from what a middle school teacher calls journal writing. However, the common theme throughout all varieties of journal writing is the simple fact that purposeful and intentional writing is involved (Emig, 1977). It may be a few sentences at the beginning or end of class or a thoughtful description of a problem solving process or a paragraph response to an affective prompt (Jurdak & Zein, 1998); no matter the form, writing is involved. The practices of journal writing are varied, journal writing can serve as a way to keep a record of students' thoughts or problems or questions throughout a course or even just throughout a unit of study (Hamdan, 2005). Journal writing can also be individual writing assignments (Sanders, 2009). I will provide a number of examples of journal writing from the literature in order to demonstrate the varied nature of journal writing. I will then discuss the commonalities of all types of journal writing.

#### **Journal writing examples.**

Journal writing includes many different writing practices, writing styles, and writing techniques. Multiple examples will be given in this section in order to illustrate the variety of journal writing assignments and describe how such journal writing assignments are put into practice.

One example of journal writing is set in a university linear algebra course (Hamdan, 2005). As a part of their coursework, students were asked to keep a journal focused on a particular topic from the course, specifically ways to show that a matrix is invertible. The structure of the journal writing in this example was based on Jerome Bruner's model of a spiral development (Bruner, 1996). Students would first be introduced to an idea, in this case the idea was the ways to show that a matrix is invertible, and then deepen their understanding of that idea by writing about the topic throughout the course. Bruner's spiral development model allows for a "gradual and dynamic construction of the...concept" (Hamdan, 2005, p. 610). The purpose of this journal writing was to raise students' awareness of the multiplicity of methods and definitions related to the subject of invertible matrices (Hamdan, 2005). By raising their awareness of multiple methods and definitions for this one mathematical concept, students would hopefully transfer that awareness and understanding to other mathematical concepts. This journal writing assignment progresses throughout the semester and the theme of describing multiple representations or methods of showing that a matrix is invertible weaves through each stage of the journal writing assignment.

Waywood (1992) provides another example of a journal writing assignment. At an all-girls school in Australia, journal writing assignments were given to students from grades seven to twelve. Journals were kept for the purpose of refining knowledge; the heart of the journal writing process was the use of prose to review, reflect on, and integrate concepts (Waywood, 1992). Students were instructed to use their journals in order to summarize, collect examples, ask questions, and discuss. Students wrote in their journals at the end of every mathematics lesson. Students in different grades are given similar tasks framed slightly differently. However, across all grades, the hope is that students would "be actively involved in constructing

knowledge for themselves” (Waywood, 1992, p. 43). Students are writing each day, collecting a history of the topics in the course as well as their understanding of those topics.

Powell (1997) describes a journal writing assignment in an urban high school. Writing was used in this case because it offered a way for both students and teachers to capture their mathematical thinking. The journal writing took the form of loose-leaf paper in a three-ring binder that students would keep throughout the school year. At the end of each lesson, students would take time to write about what they did and/or did not understand about the topic. For example, when studying methods for finding the greatest common factor of a group of positive integers, one student wrote:

I found that I could find the greatest common factor of two integers by first finding common factors of both integers and then by taking the largest common to both, e.g.,  $GCF(24,30) = 6$ ...because 6 is the largest factor common to both. (Powell, 1997, p. 21)

This student outlined her method of finding the greatest common factor of two integers, which is an illustrative example of what a student would write about their understanding of a certain topic.

To further describe the structure of the assignment, each loose-leaf piece of paper was divided lengthwise into three columns. In the left-most column, students would write the mathematical “text” relevant to the topic. The text could be any except from a textbook, class notes, example problem, class discussion, or technology (Powell, 1997). In the middle column, students would write their reflections, like the example above. This column was meant to be an elaboration of the student’s thoughts and to capture their mathematical thinking about the topic. The third column was originally left empty. Students would revisit the material again some time later and “reflect again on previous text-reflection entries and...revise, reconsider, refine, or otherwise comment on their previous reflections” (Powell, 1997, p. 25). With this type of

journaling, students keep a chronological journal of loose-leaf pieces of paper from each topic, and they return to previous topics. They are continuously revising their thoughts and reflections.

Another example of a journal writing assignment is a classroom blog (Greer & Reed, 2008). This example is representative of the many different types of online and electronic journal writing practices. Teachers and students can use blogs, class wikis, and online discussion forums to journal. In this particular example, the classroom blog was started in an upper level mathematics course at a small liberal arts college. The purpose of starting a classroom blog was to give students a space to pose questions, discuss class readings, write about problem solutions, and put course content into their own words. The blog was meant to stay very close to the idea of a paper and pencil log. However, one main difference is that everyone in the class can contribute, read, and respond to the electronic version of the log. The blog became a collective journal for the whole class, chronicling their discussions throughout the semester.

Powell & Ramnauth (1992) describe a writing assignment called a multiple-entry log. The name, multiple-entry log, comes from the fact that a student first writes an entry and then the teacher responds. This response pushes the student to further their thinking about the topic and to respond to the teacher's response. Because of the back-and-forth nature of the assignment, there can be multiple entries for each topic. This assignment was given in a college mathematics course as a way to promote reflection.

This writing assignment is implemented with a sheet of paper divided into two columns. The left-hand column contains a computation or a problem solution. The right-hand column contains reflections about that mathematical text. The reflection may include the reasons why a student chose to solve that particular problem, connections between that problem and other mathematical topics, questions, challenges, or issues that came up when solving that problem, or

other reflections. The teacher would then respond to the reflections. The teacher could affirm the student's thinking or challenge the student's thinking. Because teachers and students are writing back-and-forth, the reflections are generative; reflections generate more reflections.

Sanders (2009) describes a number of individual writing assignments that she regularly uses in her high school honors geometry class. Writing was a regular part of the students' course curriculum; there were a number of writing-intensive projects throughout the school year. These writing assignments were typically short essay questions or extended problems in which students were expected to explicitly show their work in a clear and comprehensible manner (Sanders, 2009). These assignments would take both very basic and very complicated topics and push students to dig deeper into the content.

One example was called the "Quads Project." Students were given the definitions of various types of quadrilaterals (parallelogram, rhombus, rectangle, square, kite, and trapezoid) and asked to construct an example of each. They were then given the following questions:

Use the parallelogram you constructed to make some conjectures about parallelograms in general. For example, what seems to be true about opposite sides? Construct the diagonals. What seems to be true about the diagonals? Are they congruent? Do they appear to bisect each other? When you have decided which properties are always true for a parallelogram, prove each property. (Sanders, 2009, p. 435)

The "Quads Project" and other writing-intensive projects in this class were included in the course curriculum in order to enrich students' understanding and experiences. These writing assignments could be turned into a portfolio, marking the progress of the student throughout the course.

High school students participated in a journaling study to test the effect of journal writing on achievement in and attitudes towards mathematics (Jurdak & Zein, 1998). The purpose of the study was to explore the potential benefits of journal writing: “engaging all students actively in the deliberate structure of meaning, allowing learners to learn at their own pace, providing immediate feedback by enabling learners to read the product of their own thinking on paper” (Jurdak & Zein, 1998, p. 412).

Students in this study were given about 10 minutes at the end of their mathematics class three times a week to write in their journals. Students were given specific writing assignments or prompts that were usually a question, statement, or set of instructions. There were two different types of prompts: cognitive and affective. Jurdak and Zein (1998) define cognitive prompts as mathematically oriented and focused specifically on the learning of mathematics. For example:

Imagine you have a friend who was absent today. You call him, and you do a summary of today’s math lesson, assuming that your friend will rely only on you and your explanation. So be as clear as possible, and do not miss any important point that was mentioned. (Jurdak & Zein, 1998, p. 415)

Jurdak and Zein define affective prompts as prompts directing students to express their goals, strategies, reactions, and feelings towards the learning of mathematics. For example:

How do you feel you did on your last math test? What concepts do you still find difficulty in? What did you do to overcome those difficulties? (Jurdak & Zein, 1998, p. 415)

The cognitive and affective prompts were alternated with each journal writing response time.

The teacher collected all the journals at the next class session. Even though the students were

given about 10 minutes at the end of the previous class session to begin working on their journal assignments, they did not turn them in until the next class session.

In summary, a journal writing assignment can provide a running commentary of mathematical learning (McIntosh & Draper, 2001). These examples all show how students regularly write in journals, notebooks, and even online to capture their mathematical thinking, to reflect on their learning, and to actively engage with new course material. Although these examples vary slightly, there are three major commonalities.

First, with journal writing assignments, students do more than simply solve problems. Journal writing assignments push students to think about mathematical problems or topics in ways that they would not necessarily think about them by simply solving problems and following steps of a procedure (Miller & England, 1989; Powell & Ramnauth, 1992; Sanders, 2009). Students ponder the problem solving process, the characteristics of mathematical quantities, even their own learning (Pugalee, 2001, 2004). When students write in mathematics they are building their mathematical understanding each time they write (Grossman, Smith & Miller, 1993). This mathematical understanding is more than applying simple procedures and algorithms; instead, students see that there are multiple methods to solve problems (Hamdan, 2005), that struggling with mathematical content is not bad (Greer & Reed, 2008), and that they can build their mathematical confidence (Borasi & Rose, 1989).

Second, students build their mathematical communication with journal writing assignments (Pugalee, 2004). In many journal writing assignments, students are prompted to describe their problem solving process. This helps foster students' mathematical communication; not only do their teachers need to be able to read and understand their problem

solving process, but students need to be able to read and understand their problem solving process when they read it again at a later time.

Third, journal writing assignments offer a snapshot into students' thinking about a particular mathematical topic (Powell, 1997). With writing assignments, teachers have a tangible glance into what and how students are thinking about a particular mathematical topic. In addition, journal writing assignments keep a running record of students' progress, which enables both students and instructors to monitor and record students' progress. Mathematics journals provide a history of students' mathematical work and reflections (Waywood, 1992).

### **Important characteristics of journal writing in mathematics.**

Mathematics educators and teachers clearly have many different perceptions and uses of journal writing. Journal writing assignments range from short responses (Jurdak & Zein, 1998; Miller & England, 1989) to regularly assigned tasks (Borasi & Rose, 1989; Greer & Reed, 2008; Lim & Pugalee, 2004; Powell, 1997; Waywood, 1992) to large projects (Hamdan, 2005; Sanders, 2009). However, the prevalent theme throughout all types of journal writing is the simple fact that purposeful and intentional writing is involved (Emig, 1977). It may be a few sentences at the beginning or end of class or a thoughtful description of a problem solving process or a paragraph response to an affective prompt; no matter the form, meaningful writing is involved. Based on the current literature related to journal writing in mathematics, there are three important characteristics of journal writing assignments, regardless of their format: 1) prompts (Borasi & Rose, 1989; Lim & Pugalee, 2004; Waywood, 1992), 2) clear expectations (Hamdan, 2005; Powell & Ramnauth, 1992), and 3) responses of the teacher (Borasi & Rose, 1989; Miller & England, 1989; Powell, 1997).

The first important characteristic of journal writing, which is common among almost all of the examples in the current literature, is the prompt. Most journal writing assignments have a prompt. The prompt can be relatively generic or extremely specific. Generic prompts are when a student is given a list of possible questions to answer (Borasi & Rose, 1989), or when a student is told to keep track of what they do and do not understand (Powell, 1997), or when a student is to summarize main ideas (Waywood, 1992). Specific prompts are when a student is given a very specific question to answer, task to complete, or statement to respond to (Miller & England, 1989; McIntosh, 1991; Jurdak & Zein, 1998; Williams & Wynne, 2000; Lim & Pugalee, 2004).

Prompts can be affective or cognitive, and can even range into broader types of writing like expository writing or creative writing. With creative writing, students can be encouraged to write poems or stories about mathematical concepts, letters to famous mathematicians, and even plays involving mathematical concepts (McIntosh, 1991). An excellent example of a mathematical story is *Flatland* (Abbott, 1884), a story about dimensions in which the characters are geometric figures. Students can be encouraged to write mathematical stories or plays similar to *Flatland* (Abbott, 1884). Prompts are a critical element of journal writing.

Another important characteristic of journal writing is communicating expectations. It is important for students to know what is expected of them. Many teachers may communicate expectations verbally or even informally on an assignment description (Hamdan, 2005; Powell & Ramnauth, 1992; Pugalee 2001; Pugalee, 2004; Sanders, 2009). Teachers can also communicate expectations through a detailed rubric (Clark, Waywood, & Stephens, 1993; Lim & Pugalee, 2004; Waywood, 1992; Williams & Wynne, 2000). However expectations are communicated, it is valuable for both teachers and students to know what the teacher is anticipating from the journal responses in order for the journal writing to be meaningful to both teachers and students.

A third important characteristic of journal writing is the response of the teacher; receiving feedback from their teacher is extremely important for students (Borasi & Rose, 1989; Greer & Reed, 2008; Jurdak & Zein, 1998; Miller & England, 1989; Powell, 1997; Powell & Ramnauth, 1992; Williams & Wynne, 2000). Students are not the only individuals in the classroom who should be writing in journals. Teachers should regularly collect and comment on students' journals. This opens up communication between teachers and students (Borasi & Rose, 1989), gives teachers insight into students' understanding (Powell, 1997), and gives students regular and constructive feedback (Sanders, 2009; Williams & Wynne, 2000). Teachers' responses, comments, affirmation, and challenges based on students' journals are all important aspect of journal writing, no matter the form.

#### **Benefits of journal writing in mathematics.**

There are many different forms of journal writing in mathematics classes. However, it remains to be seen what the impact of journal writing is for mathematics teaching and learning. Mathematics education researchers claim that the use of journal writing activities “can provide a valuable means to facilitate a personalized making-of-meaning approach to learning mathematics” (Borasi & Rose, 1989, p. 347). The value of journal writing extends from students to teacher and also to dialogue or communication between students and teachers. Journal writing “can contribute uniquely to the improvement of mathematics instruction” because of its impact on students, on teachers, and on dialogue between students and teachers (Borasi & Rose, 1989, p. 349). A discussion of the benefits of journal writing for students, for teachers, and for dialogue between students and teachers follow.

***Benefits for students.***

*Learning.* One of the primary benefits of journal writing for students is the strong connection to learning. “Writing serves learning uniquely because [it] possess a cluster of attributes that correspond uniquely to certain powerful learning strategies” (Emig, 1977, p. 122). Through journal writing, students are able to coordinate between old and new concepts. This type of learning strategy is not always natural for students, but writing places the responsibility of coordinating between new information and already existing information in the hands of the students. Through journal writing, students take an active role in constructing knowledge and making meaning of experiences, classroom activities, and mathematical content:

By writing, students can make meanings by coordinating between old and new concepts; they can write freely and reflect on the material learned in class and react to it, taking the responsibility of coordinating between the new received information and their existing cognitive structures. (Hamdan, 2005, p. 609)

With mathematical journal writing, students learn by constructing meaning (Pugalee, 2001; Sanders, 2009) because of the emphasis on summarizing, generalizing, reviewing, and relating information (Grossman et al., 1993; Powell, 1997). In one college linear algebra course, students participated in a journal writing activity that lasted the entire semester (Hamdan, 2005). The journal writing consisted of four stages. The first stage of the journal writing process was an introduction. At the beginning of the course, students were told that they should write in their journals as frequently as they thought seemed appropriate. The second stage of the journal writing process was a “first look.” Students submitted a preliminary report, which was mainly a list of properties or theorems related to proving that a matrix is invertible. While this was mostly an unsorted list without connections between statements, many students were beginning to see

connections between the ideas in their lists. The third stage of the journal writing process was a final product. Students sorted through their collections of definitions, methods, and theorems to clarify the different representations of the concept of invertible matrices and provide examples of each representation. In most cases, by the time students reached the third stage of the journal writing process, they had compiled a sophisticated set of notes on the topic written completely in their own words.

Through observation and analysis of the journal entries, through evidence on exams, and through informal interviews with students, the researchers came to a few conclusions. One of the main conclusions was that through the journal writing project, students were better able to synthesize new information (Hamdan, 2005). Through the use of writing, students synthesized their ideas and moved from concrete to abstract ideas relating to the mathematical content.

At an all-girls high school in Australia, students participated in journal writing from grades seven to twelve (Waywood, 1992). Students kept journals in order to summarize key content, collect relevant examples, ask meaningful questions, and discuss stimulating thoughts. The journals were analyzed using a rubric with three categories: recount mode, summary mode, and dialogue mode. Recount mode means that the student is basically just reporting what happened in a given class or a particular homework assignment. Summary mode means that the student is beginning to classify and synthesize content; “knowledge is to be used and as such it has to be integrated with what is already known” (Waywood, 1992, p. 35). Dialogue mode means that the student is moving back and forth between multiple different ideas; knowledge is created in dialogue mode. From the analysis, the students whose journal entries were regularly in “dialogue” mode were actively creating knowledge by connecting what they were studying in

class to their previous knowledge. Whenever students were writing in “dialogue” mode, then there were actively involved in constructing their own knowledge (Waywood, 1992).

Additionally, students claim that journal writing positively influences their learning. In a ninth grade algebra class, students engaged in regular journal writing activities where they were given a mathematical problem to solve and told to write every single thought that came to mind while solving the problems (Lim & Pugalee, 2004; Pugalee, 2004; Pugalee, 2001). From an anonymous survey given at the end of the course, one student commented on their perspective of journal writing: “[It] helps me understand the math. Writing journal entries is a good thing I think because it helps me to remember the math” (Lim & Pugalee, 2004, p. 12). This student had a positive view of journaling in mathematics; he believed that journaling in mathematics helped him understand and remember what he was learning. While there were some students who did not enjoy journal writing in mathematics, the majority of the students did.

Journal writing assignments seem to develop and support mathematical learning, both from a teacher’s point of view and from a student’s point of view. From a teacher’s point of view, students are more actively engaged in constructing knowledge and making connections with their learning. From a student’s point of view, they are able to dig deeper into concepts, which helps them understand and remember what they are learning.

*Procedural and conceptual knowledge.* Journal writing can have a positive effect on both procedural and conceptual mathematical knowledge (Sanders, 2009); “the act of writing helps clarify both the process and content of problems” (Borasi & Rose, 1989, p. 356). With the use of journal writing, students are better able to carry out mathematical operations and procedures and, more importantly, they are better able to understand the concepts behind the operations and procedures.

Borasi and Rose (1989) found that journal writing increased students' learning of mathematical content. In their study, 23 students enrolled in a college algebra course kept journals throughout the course and filled out evaluations at the end of the course. The students wrote in their journals three times per week; journals were collected at the end of the each week and returned to students at the beginning of the next week, along with teacher comments. Thirty-six suggested journal topics were given to the students. For example, "Describe your favorite math class." and "How do you go about solving word problems?" (Borasi & Rose, 1989, p. 351). Content analysis of the journals and evaluations was conducted in order to seek out recurring patterns related to journal writing. The analysis found that journal writing did increase mathematical learning. Specifically, "restating concepts and rules in one's own words can in fact facilitate their internalization" and "writing about a mathematical topic can also help identify learning difficulties and problems, and recognize connections previously unrealized" (Borasi & Rose, 1989, p. 355).

Jurdak and Zein (1998) also found that journal writing increased both procedural and conceptual understanding. Four different high school mathematics classes participated in the study, two of the classes were assigned to the journal-writing group (JW) and two of the classes were assigned to the no-journal-writing group (NJW). About ten minutes was given to all four classes at the end of each class meeting. The JW classes used that time for journaling. Students in the JW classes were given specific writing assignments or prompts that were usually a question, statement, or set of instructions. The prompts were both cognitive and affective prompts. The NJW classes used that time for completing problems from the textbook as practice problems. Other than the ten minutes at the end of class, all four classes covered the same material and were all taught by experienced teachers. There were five cognitive variables and

one affective variable used as dependent variables in the study. The cognitive variables were conceptual understanding, procedural understanding, problem solving, mathematical communication, and school achievement. The affective variable was attitudes towards mathematics. Conceptual understanding, procedural understanding, problem solving, and mathematical communication were measured with the Mathematics Evaluation Test (MET), which was administered as both a pretest and a posttest. Attitudes towards mathematics were measured with a questionnaire, which was also administered as both a pretest and a posttest. Those in the JW classes also wrote an open-ended evaluation of the journal writing experience.

A  $2 \times 2$  multivariate analysis of covariance (MANCOVA) was used to analyze the data. The mean score of the JW group was significantly higher, tested at an alpha level of 0.10, than that of the NJW group for conceptual understanding ( $p < 0.07$ ), procedural understanding ( $p < 0.07$ ), and mathematical communication ( $p < 0.01$ ). However, the mean score of the JW group was not statistically different than that of the NJW group for problem solving, school achievement, and attitudes towards mathematics (Jurdak & Zein, 1998). This study provides evidence that journal writing in mathematics classes can have a positive influence on both procedural and conceptual knowledge.

By engaging in journal writing activities in mathematics, students are able to develop both procedural knowledge and conceptual knowledge (Jurdak & Zein, 1998; Borasi & Rose, 1989). Students learn not only how to solve problems, but why those solutions make sense, what the deeper mathematical concepts involved in those problems are, and how those problem solving strategies can be applied to other similar problems.

*Problem solving.* Related to both procedural and conceptual knowledge is students' ability to problem solve. Conflicting results have been found concerning the effect of journal writing on building students' problem solving ability.

For example, in the Borasi and Rose (1989) study, students who were regularly writing in journals found that it was the first time they became aware of their own problem-solving processes. Students were then able to reflect on what seemed to work and what did not seem to work and change their problem-solving processes accordingly. On the end of the course evaluations, one student remarked: "I have been able to realize what I am doing wrong in my thinking process. Once I know what I am doing wrong, I have been able to change and thus do better" (Borasi & Rose, 1989, p. 357).

Grossman et al. (1993) also explored the relationship between writing about mathematics and problem solving in mathematics. Students in three undergraduate algebra courses engaged in writing activities throughout their course. Writing activities included problems on homework, quizzes, and tests. Based on a correlation of 0.67 ( $p < 0.001$ ), a strong relationship was found between a students' ability to write about mathematical problems and their ability to solve related problems. "Students who can write about concepts can also use those concepts correctly in problem solving" (Grossman et al., 1993, p. 4).

Contrary to the above-mentioned studies, the Jurdak and Zein (1998) study found that journal writing had no effect on problem solving. There was no significant difference between the mean score of the JW group and the NJW group for problem solving. Jurdak and Zein attribute this to "the complex nature of problem solving, the nature of journal writing, or both" (Jurdak & Zein, 1998, p. 417).

*Beliefs about mathematics.* Another benefit that students gain from journal writing in mathematics classes is the impact on students' beliefs about mathematics (Borasi & Rose, 1989). What students believe about the nature of mathematics and what they believe about mathematics as a discipline shapes the way that they engage with mathematical studies (Schoenfeld, 1989). A typical belief held by students about learning mathematics is that mathematics must be learned in a rote manner, which involves memorization and repetition (Ball, 1990). This is one belief that can change with the use of journal writing because students realize through journal writing that they can and do have ideas about mathematics and many of these ideas are valid (Hamdan, 2005). Another typical student belief about learning mathematics is that there is only one correct way to solve every mathematics problems (Ball, 1990). Again, the use of journal writing in mathematics can challenge this belief. Students are able to see that "one has at hand more than one available method or process" to solve a problem (Hamdan, 2005, p. 614).

One high school honors geometry class engaged in a number of writing activities throughout the year. One of these was writing test corrections (Sanders, 2009). The teacher chose to assign test corrections so that her students could learn where their mistakes were and how to correct them. The test corrections were presented to the students in the following way:

In doing test corrections, you are expected to follow these steps: 1) State the problem. 2) Write a complete and correct solution. If it is an algebraic problem, label your variables and write all the steps. If it is a proof, write the complete proof. Write at least one sentence beginning with the words 'I got this problem wrong because...' 3) Then write exactly why you got the problem wrong. You may not write 'because I didn't know how to do it' or any other 'generic' remark. (Sanders, 2009, p. 435-436)

Students' responses to the test corrections were that they were very time consuming, but also very useful. Students began to comment that the test corrections helped them value the process of solving a problem as much as the answer (Sanders, 2009). This belief about the nature of mathematics – valuing the process as much as the answer – is very powerful for students.

Explicitly discussing students' beliefs about the nature of mathematics and about learning mathematics is rare in many mathematics classes. In the Borasi and Rose (1989) study, students rarely wrote about their views on mathematics in their journals. In fact, only two students openly mentioned that journal writing had somehow affected their views of mathematics in their final evaluation (Borasi & Rose, 1989). Students and teachers discuss beliefs about mathematics infrequently, so it is difficult for journal writing to significantly change students' beliefs about mathematics. However, journal writing can at least make students aware of their beliefs about mathematics and how these beliefs impact their approach to learning mathematics.

*Mathematical confidence.* Through the use of journal writing in mathematics, students can build confidence in their mathematical ability. “A remarkable outcome from students' writing is their reclaim of their...enjoyment of the sense of empowerment and confidence” (Hamdan, 2005, p. 610). In addition, journal writing helps develop and support mathematical thinking; this helps students attain “mathematical power” (Lim & Pugalee, 2004, p. 12). Regular journal writing helps students gain a sense of boldness and courage when it comes to mathematics. Through the use of journal writing, students develop “faith in themselves as learners who are capable of doing and understanding mathematics” (Powell, 1997, p. 5).

For example, in one upper level undergraduate mathematics course, the teacher used a blog in order to give students a space to pose questions, discuss class readings, write about problem solutions, and put course content into their own words (Greer & Reed, 2008). The blog

helped students develop confidence by realizing that it was okay, even encouraged, to have questions about course content or to initially be confused about course content. One student commented: “When I ran into something I didn’t understand, I knew that it wasn’t that I was a moron; it was just that I was reading it for the first time, and I *should* have questions on it” (Greer & Reed, 2008, p. 146). Journaling on the class blog helped this student gain confidence in his ability to do and understand mathematics, even when he struggled to fully understand the content. When students have revelations like these – when they realize that it is a good thing to struggle with the content and to ask questions – they build their own self-beliefs about their ability to do mathematics.

The current literature about journal writing in mathematics does mention that one benefit of journal writing for students is that it increases their mathematical confidence. Through the use of regular and meaningful journal writing, students’ mathematical confidence may increase (Greer & Reed, 2008; Hamdan, 2005). However, the articles mentioned here are examples of what is commonly found in the current literature concerning students’ mathematical confidence: informal observations and analyses of teachers claiming that students’ mathematical confidence does increase because of journal writing in mathematics. What seems to be missing are empirical studies, studies that explicitly explore the relationship between journal writing in mathematics and mathematical confidence.

*Motivation.* Interest, enthusiasm, and motivation towards learning mathematics are all benefits of journal writing in mathematics (Borasi & Rose, 1989; Sanders, 2009). Students seem to be more interested in learning mathematics when they get to explore and investigate a topic on their own through the use of journal writing instead of just being told what to do, say, or write (Sanders, 2009). In the Borasi and Rose (1989) study, students reported that journal writing

served as a stimulus for new inquiry. Students seem to feel a sense of excitement for mathematical inquiry (Borasi & Rose, 1989) and a sense of enthusiasm about mathematics (Powell & Ramnauth, 1992) because of journal writing. In addition, students seem to “like” mathematics classes more when they are engaged in journal writing activities (Borasi & Rose, 1989).

Similar to the section on mathematical confidence, most of the current articles about journal writing that mention motivation are not empirical studies. Instead, what is mentioned is based on teacher’s perceptions of their students’ interest, enthusiasm, and motivation towards learning mathematics and on students’ informal comments. The one exception to this is the Borasi and Rose (1989) study; they applied content analysis to students’ open-ended evaluations and determined that students do feel an excitement for mathematical inquiry and that students seem to like mathematics more because of journal writing in mathematics. Therefore, there is a need to thoroughly explore the relationship between journal writing in mathematics and students’ motivation towards learning mathematics, which will be discussed further.

### ***Benefits for teachers.***

*Assessing students’ understanding.* One of the ways that journal writing benefits teachers is the way it allows teachers to assess students’ understanding. When a student writes a journal entry that is thoughtful and engaged, the teacher has a tangible and concrete snapshot of that students’ thinking about that particular topic. Journal writing may inform teachers about what a student does understand and also what a student does not understand (Borasi & Rose, 1989; Drake & Amspaugh, 1994; Miller & England, 1989).

“The diagnosis of errors in student *thinking* (as opposed to *procedures*) is not always easy to detect from analysis of computation exercises. Students’ writing can help teachers understand

the reasoning behind faulty algorithms” (Drake & Amspaugh, 1994, p. 44). Sometimes when just performing computations, a student may use an incorrect procedure, but arrive at a correct answer. Similarly, a student may use a correct procedure, but arrive at an incorrect answer due to a small miscalculation. These computations alone may not provide sufficient insight into the student’s thinking and lead the teacher to an incorrect judgment of what the student does and does not understand.

Similarly, students may be able to quote rules, properties, procedures, or algorithms, and they may even be able to apply rules properties, procedure, or algorithms, but they may not have a deeper understanding of the mathematical content behind them (Miller & England, 1989).

Writing helps teachers gain a more complete understanding of students’ thinking.

In addition to assessing students’ mathematical understanding, the use of journal writing can also help teachers assess students’ beliefs and attitudes about mathematics and about learning mathematics. For example, one possible belief about mathematics is that it is nothing more than a collection of facts. Most mathematics teachers recognize that there is much more to mathematics than rote memorization of facts, formulas, and computations. However, these teachers may not realize that their students do hold beliefs like this one. Writing is a very practical and tangible way for teachers to address students’ beliefs like this one (Drake & Amspaugh, 1994). Journal writing not only helps students become more aware of the beliefs and attitudes they hold about mathematics and about learning mathematics, but also helps teachers identify these beliefs and attitudes in students.

*Changing instruction.* Another way that journal writing benefits teachers is the way it gives teachers the opportunity to change or adjust their instruction based on the journal responses (Borasi & Rose, 1989; Drake & Amspaugh, 1994). Suppose a teacher assigns a journal prompt

focused on some particular mathematical content. The teacher reads all the journal entries at the end of the class and notices that more than half of the class has reported a concept incorrectly or made a major mistake in computation. The teacher is then able to address that issue during the next class by going over more examples or by describing the topic in a new and different way. One teacher who used a blog as a form of an electronic journal in her class reported that “based on the questions everyone asked [on the blog], I was able to frame the entire class discussion just about every day. I knew whether I needed to spend extra time on a specific theorem, or new notation, or whatever other part of the section the most questions came from” (Greer & Reed, 2008, p. 143). Teachers are able to use journal writing as a way to gain immediate feedback about instruction from students.

Changes to instruction may be immediate and potentially even long term (Borasi & Rose, 1989). Immediate changes to the course based on students’ journal entries might even be as simple as assigning more journal writing projects. Teacher may also make long-term improvements to their teaching methodologies based on feedback from students in journal entries (Borasi & Rose, 1989). One example of a long-term improvement to teaching methodologies is an awareness of classroom communication. Teachers become more aware of how to help students summarize information, collect important examples, ask questions, and discuss mathematical topics in depth (Waywood, 1992).

Journal writing can be a very powerful way for teachers to gain insight into students’ understanding and to improve their own teaching practices. Journal writing provides teachers with an avenue to understand what students do and do not understand and this can profoundly influence instruction. Teachers can challenge students when they know students understand a particular concept well and teachers can support students when they know students struggle with

another concept. In addition, the feedback about journals can even inspire teachers to make long-term changes to their instruction.

***Benefits for dialogue.***

Journal writing provides a space for dialogue, dialogue between the student and the teacher and dialogue between the student and the material (Hamdan, 2005). Journal writing offers students the chance to talk individually with their teacher, to ask questions that they may not have asked in class, to get personal feedback directly from their teacher, and to communicate with their teacher. Students receive individualized support and feedback from their teacher through the use of journal writing (Borasi & Rose, 1989; Williams & Wynne, 2000). Journal writing also offers students the chance to develop their mathematical communication, clearly describing and discussing mathematical ideas in detail. Journal writing is a way for students to learn to explain new material by building both an explanation of the material and a representation of the material (Miller & England, 1989; Williams & Wynne, 2000). In the Jurdak and Zein (1998) study, there was a significant increase in mathematical communication between the journal-writing group (JW) and the no-journal-writing group (NJW). Journal writing enabled those students to become more powerful and effective communicators of mathematics (Jurdak & Zein, 1998).

Journal writing promotes classroom discourse (Pugalee, 1998). Specifically, journal writing promotes discourse between the students and the teacher, between the student and the material and between students. Journal writing offers students the chance to discuss mathematical ideas and to have mathematical conversations with other students (Lim & Pugalee, 2004; Pugalee, 2004).

In addition, journal writing helps create a caring classroom atmosphere (Borasi & Rose, 1989). When teachers respond to students writing, this lets students know that what they write is actually being read (McIntosh & Draper, 2001). The two-way communication between students and teachers shows students that their teachers are listening to them and care about them (Watson, 1980).

### **Summary of journal writing in mathematics.**

Journal writing comes in many different forms; it may be a log or a personal notebook, kept by students throughout the entirety of a course or a school year. It may be an individual, stand-alone writing assignment. Regardless of the type of journal writing, journal writing in mathematics can be an incredibly rewarding practice for both students and teachers. However, there are also some challenges to implementing journal writing in mathematics classes. The biggest challenge is how students initially respond to journal writing, as this is typically a brand new assignment for students in a mathematics class. Students tend to perceive journal writing about mathematics as something new and different (Williams & Wynne, 2000). One of the biggest reasons for this challenge is that it can be difficult for students to express mathematical ideas in writing (Powell, 1997). However, the more students are given these types of writing assignments, especially if the journal writing assignment is continuous throughout the course, the more comfortable they feel about journal writing.

Another challenge is that it can be time consuming for teachers to respond to all the journals. This is a key element of journal writing; it is imperative that teachers respond to their students with personalized feedback, whether that feedback is an affirmation or a challenge. This can take up a great deal of time. Therefore, it is important for teachers to know how often

to assign journal writing projects and what type of journal writing projects to assign (Williams & Wynne, 2000).

Despite these challenges, journal writing does present many benefits for students, for teachers, and for communication between students and teachers. Benefits for students include increased learning through summarizing, generalizing, reviewing, and relating information (Grossman et al., 1993; Hamdan, 2005; Powell 1997), building conceptual knowledge as well as procedural knowledge (Borasi & Rose, 1989; Jurdak & Zein, 1998), fostering positive beliefs about mathematics (Borasi & Rose, 1989; Sanders, 2009), mathematical confidence (Greer & Reed, 2008; Hamdan, 2005), and motivation towards learning mathematics (Powell & Ramnauth, 1992; Sanders, 2009). Benefits for teachers include the ability to efficiently assess students' understanding (Borasi & Rose, 1989; Drake & Amspough, 1994; Miller & England, 1989) and the ability to tailor instruction to students' needs (Greer & Reed, 2008; Waywood, 1992). Benefits for communication between students and teachers include increased dialogue (Hamdan, 2005; Pugalee, 1998, 2004), individualized feedback and support for students (Borasi & Rose, 1989; Williams & Wynne, 2000), and a caring and creative classroom atmosphere (Borasi & Rose, 1989; McIntosh & Draper, 2001).

### **Mathematics Education in Postsecondary Classrooms**

There is an extensive and substantial body of literature providing insight into K-12 mathematics education. This body of literature encompasses mathematics teaching and learning (Brophy, 1986; Simon, Tzur, Heinz, & Kinzel, 2004). It encompasses classroom environments and social expectations (Goos, 2004; Huffered-Ackles, Fuson, & Sherin, 2004). It encompasses technology in the classroom and how it is used to impact student learning (Hollar & Norwood, 1999; Kaput & Thompson, 1994). This research has contributed to our understanding of

mathematics education and has “informed the design of teacher preparation and professional development programs for K-12 teachers” (Speer, Smith, & Horvath, 2010, p. 99). However, mathematics education research at the collegiate level is still in its beginnings. There are an abundance of articles that describe teaching practices (Gallegos & Flores, 2010; Oehrtman, 2009; Ward et al., 2010), classroom activities (Mumakata, 2005; Savitz & Savitz, 2010), physical and electronic (web-based) manipulative materials (Greer & Reed, 2008; Roh, 2010), and student assignments (Morrel, 2006; Shannon & Curtin, 1992). However, based on a review of the current literature, there seems to be very little empirical research dealing with teaching and learning mathematics at a college level.

The overwhelming trend of the current literature based on research studies, analytic reflections of instructors about particular courses, and articles describing teaching practices and instructional activities is for college mathematics instructors to move away from the traditional and dominant teaching method in college mathematics courses: lecture (Ahmadi, 2002; Bookman & Friedman, 1994; Castro, Gold, Marchisotto, Schilling, & Zeitlin, 1991; Hagerty, Smith, & Goodwin, 2010; Halmos, 1985; Ward et al., 2010). Moving away from lecture means that college mathematics instructors might integrate group projects into regular coursework, incorporate more writing assignments, provide historical context for materials, anchor new material in what students already know and be flexible with their course schedule (Castro et al., 1991). Relinquishing the lecture as the primary form of instruction also means that college mathematics instructors might challenge students to engage in “out-of-the-box” thinking (Morrel, 2006), encourage students to listen and respond to their classmates during class interaction and discussion (Rasmussen & Marrongelle, 2006; Stephen & Rasmussen, 2002), and provide opportunities for students to explain what they know about particular mathematical

topics (Beidleman, Jones, & Wells, 1995). I will provide a few examples from the literature that summarize and highlight the theme found in the literature: surrender the lecture and venture various other teaching practices that aim to engage students in actively doing and discussing mathematics.

### **Examples that highlight themes in the literature.**

Ward et al. (2010) provides an example of a course that incorporates inquiry as a key feature of the course instead of lecture. The mathematical inquiry course described in this article is designed as a general education course for students who are calculus ready. The purpose of the course is not to repeat topics that students most likely covered in high school. Instead, the purpose of the course is to engage students in relevant mathematics, to explore historical achievements, and to bring in creativity, critical thinking, and reasoning to mathematics all while using a variety of activities and assessments (Ward et al, 2010). Lecture is hardly ever used as an instructional tool in the mathematical inquiry course.

Ward et al. (2010) conducted a study to evaluate students' performance in and appreciation for mathematics after taking the course. Students' performance was evaluated using a pre-course and post-course test. Students' appreciation for mathematics was evaluated using a pre-course and post-course survey and four free-response questions. The survey consisted of three scales: 1) attitudes, to measure students' feelings about mathematics, 2) usefulness, to measure how much or how little students saw mathematics being used in everyday life, and 3) creativeness, to measure whether or not students saw creativity as a part of mathematics (Ward et al., 2010).

Results indicate that students' performance scores significantly increased ( $p < 0.001$ ) on the post-course test (Ward et al., 2010). This is not surprising because students' scores should

increase after taking a mathematics course. As far as students' appreciation for mathematics, attitudes showed a significant decrease, usefulness showed no significant difference, and creativeness showed a significant increase ( $p < 0.01$ ) (Ward et al., 2010). These results are surprising, but viewing these results in conjunction with the free-response questions, it seems as though students did gain an appreciation for the use of creativity in mathematics and the significance of mathematics. However, because the course topics were designed to be conceptual and abstract ideas, application-oriented topics were not included in the course curriculum. In relation to the course's purpose, it seems as though the instructional tools used in the mathematical inquiry course are successful. The description and analysis of this mathematics inquiry course provides an example of a college mathematics course that has moved away from using lecture as the primary instructional tool.

Hagerty et al. (2010) documented and analyzed instructional strategies such as using computer software, hedging topics in a historical perspective, engaging students in theoretical discussions, and providing plenty of application problems. The purpose of using these instructional strategies was to increase students' awareness of their attitudes about mathematics and to increase their performance in mathematics; the purpose was to improve students' ability and desire for mathematics (Hagerty et al., 2010).

By comparing students in courses that implement instructional strategies such as these, non-traditional courses, to students in more traditional courses that are largely lecture based, the results show that these instructional strategies do improve performance in and desire for mathematics (Hagerty et al., 2010). There is a 54% passing rate for students in traditional courses versus a 75% passing rate for students in non-traditional courses. Also, students in non-traditional courses performed better on a standardized end of the semester test than students in

traditional courses ( $p < 0.001$ ). In addition, attendance rates increased by 20% for the non-traditional courses and more students enrolled in upper level mathematics courses after taking a non-traditional course. Based on these results, this study indicates that teaching strategies like using computer software, starting whole-class discussions, and basing mathematics in its historical perspective are valuable strategies to improve students' ability and desire for mathematics. Again, this description and analysis provides an example of a college mathematics course that does not utilize the traditional teaching method of lecture, and finds positive results when comparing such a course to a lecture-based course.

Mumakata (2005) highlights a semester-long activity that engages students in group work, mathematical discussions, and mathematical writing and also provides students an opportunity to see and experience mathematical ideas in context. Mumakata (2005) describes an activity called the "math book club." The activity divides students into groups of four to read and discuss a book related to mathematics or the history of mathematics. At each book club meeting, students discuss themes, ideas, and questions they have from whatever section of the book they were reading for that meeting.

The activity first begins as a whole class activity before students are partitioned into groups of four. The entire class reads *Flatland* by Edwin A. Abbott (1884) and the class teacher models a typical book club meeting for the students. After this initial whole-class book club, students, in groups of four, choose a book to read from a list of recommended books. Students meet for book club meetings 30 minutes a week for four weeks. Students are given a pacing schedule, which tells them which portion of their book to read each week. At each book club meeting there are two roles: a host and a scribe. The host is charged with generating discussion questions and facilitating discussion. The scribe is charged with taking notes during the meeting

and writing a two-to-three page summary of the meeting. Since there are four members in each group, and four group meetings, each member of the group acts as the host and as the scribe exactly once (Mumakata, 2005).

At the end of the math book club students complete a writing assignment. The writing assignments ask students to describe their book to a friend, to elaborate on what they found most interesting about the book, and to describe their overall experience with the math book club. The responses are overwhelmingly positive. Students liked having a break from their “typical” mathematics class, they liked discussions, they liked the connections they found between their books and class topics, and they liked getting to know other students in their class (Mumakata, 2005). Because of how positively students perceive their overall experience with the math book club, this activity is an appropriate idea for liberal arts mathematics courses (Mumakata, 2005) and is an excellent example of a class activity – very distinct from lecture – that engages students in discussions and writing about mathematics, and also helps students make connections between mathematical topics.

One last example, set within a college calculus course, uses writing as a class activity and assignment to increase students’ conceptual understandings of mathematics. Beidleman, Jones, and Wells (1995) claim that students in calculus classes often have a good grasp of procedural knowledge, but not conceptual knowledge. The author’s noticed that students lacked deep understanding of calculus topics although they were capable of manipulating symbols and memorizing formulas. In addition, students seemed to be unable to communicate calculus topics verbally or in writing. Because of this lack of conceptual knowledge, the reform assignment introduced to a first-semester calculus course designed for non-mathematics majors was a writing assignment. The authors suggest that writing assignments help students gain conceptual

knowledge and take the focus of the classroom away from the teacher and towards the students. The authors' assumption, based on their past experiences, is that if a student cannot effectively explain a concept or an idea in writing, then they do not really understand that concept or idea (Beidleman et al., 1995).

Students were assigned three types of writing assignments throughout the semester. The first type was in-class writing assignments. Students were given 15 minutes of class time to respond to a prompt; for example, after being shown the definition of a limit, students were asked to interpret the definition by explaining what the notation in the definition stands for and what it conceptually means to find the limit of a function at a particular point (Beidleman et al., 1995). The second type was out-of-class writing assignments. These assignments were given once a week and required more thought and creativity than the in-class writing assignments. An example is using the Mean Value Theorem to discuss the velocity of two runners during a race (Beidleman et al., 1995). The third type was questions on exams that required writing; these were typically explanation problems where students were expected to explain what they know about particular mathematical topics, like continuity or differentiability (Beidleman et al., 1995). All three types of writing assignments demonstrate that students are actively engaging in critical thinking about conceptual knowledge and not just procedural knowledge. Beidleman et al. (1995) describe what these three writing assignments are and why they chose to implement them into their calculus courses. However, they do not provide more than anecdotal and informal evidence as to whether or not these writing assignments actually achieve their purpose.

### **Summary of mathematics education in postsecondary classrooms.**

There were many suggestions in this review of the literature on collegiate level mathematics education about how teachers can engage students in actively doing and discussing

mathematics. Some of the simple suggestions include asking meaningful questions and expecting well-reasoned responses (Shannon & Curtin, 1992), integrating group projects into coursework (Castro et al., 1991), and posing problems that require out-of-the-box thinking (Morrel, 2006; Ward et al., 2010) and multiple problem solving strategies (Rasmussen & Marrongelle, 2006). Other suggestions are incorporating games and technology into the classroom (Cascaval, Fogler, Abrams, & Durham, 2008; Gallegos & Flores, 2010; Hagerty et al., 2010) as well as a wide variety of writing and reading assignments (Beidleman et al., 1995; Kasman, 2006; Meel, 1999; Mumakata, 2005; Schurle, 1991; Weber, 2005).

Future research on collegiate level mathematics education is important because of the need to understand just how well these suggestions really do encourage students to be actively involved in their own learning, to deepen students' mathematical understanding, and to increase students' intrinsic motivation. Just as research on mathematics education in K-12 classrooms has been valuable for understanding the ways in which mathematics is taught and learned in these grades and also how to better prepare and develop K-12 mathematics teachers, it is equally as valuable to understand the ways in which mathematics is taught and learned at the collegiate level and how to better prepare and develop collegiate level mathematics teachers. Mathematics is, first and foremost, a human activity (Freudenthal, 1991), but it is also a student activity. "An important part of mathematics teaching [at any level] is responding to student activity, listening to student activity, notating student activity, learning from student activity" and most especially encouraging student activity (Rasmussen & Marrongelle, 2006, p. 414).

### **Development of Homework Journals**

Based on the literature related to journal writing in mathematics and mathematics education at a postsecondary level, I have developed a journal writing assignment, called

homework journals. Homework journals are an intentional journal writing assignment (Emig, 1977) in which students solve a mathematical problem and reflect on their problem solving process (Powell & Ramnauth, 1992). Homework journals give students the opportunity to be actively engaged in doing mathematics (Lave & Wenger, 1991) and reflect on mathematics and/or mathematics learning (Dewey, 2008b). Homework journals also have the potential to increase students' intrinsic motivation towards learning mathematics because of their strong connections to self-determination theory (Ryan & Deci, 2000).

Homework journals are a particular type of writing in mathematics in which students select one problem from their assigned homework problems to complete for their homework journals. Students choose this problem based on what interested them or what challenged them from their assigned homework problems. On the left-hand column of a piece of paper, students solve this one problem in detail showing all steps of their work and clearly communicating their problem-solving process. On the right-hand column of the same piece of paper, students then write a short reflection about this problem and their problem-solving process. This reflection is prompted by a number of suggested topics and questions that students choose from. Teachers read and respond to each student's homework journal. The structure of a homework journal is based on Powell and Ramnauth's (1992) description of a "multiple-entry log" as described in the previous chapter.

A homework journal example is provided in *Figure 2.1*. The mathematical content from this example is from an integral calculus course. The problem in the left-hand column deals with a method of integration called  $u$ -substitution. The reflection in the right-hand column addressed the mathematical content of the problem and the connection between this particular problem and previously learned mathematics. The homework journal in *Figure 2.1* provides a concrete

example of what a homework journal looks like and the types of problems and reflections that students will typically engage with when completing a homework journal.

<p>Solve <math>\int \frac{3x+6}{x^2+4x-3} dx</math>.</p> <p><math>u = x^2 + 4x - 3</math>  <math>du = (2x + 4)dx = 2(x + 2)dx \Rightarrow</math>  <math>(x + 2)dx = \frac{du}{2}</math></p> <p><math>\int \frac{3(x+2)}{x^2+4x-3} dx =</math></p> <p><math>\int \frac{1}{x^2+4x-3} \cdot 3(x + 2)dx =</math></p> <p><math>\int \frac{1}{u} \cdot 3 \cdot \frac{du}{2} =</math></p> <p><math>\frac{3}{2} \int \frac{1}{u} du =</math></p> <p><math>\frac{3}{2} \ln u  + c =</math></p> <p><math>\frac{3}{2} \ln x^2 + 4x - 3  + c</math></p>	<p>The content of this problem involves <math>u</math>-substitution. The purpose of <math>u</math>-substitution is to take an integral that is difficult (or impossible) to compute as it is and change it into a form that is easy to compute. The change happens by performing the method of <math>u</math>-substitution.</p> <p>I think that this method gets its name from the fact that you are taking all of the <math>x</math>'s out of the integral and replacing them (making a substitution) with <math>u</math>'s. You are changing the variable.</p> <p>I see a connection between the method of <math>u</math>-substitution for integration and the chain rule for differentiation. The method of <math>u</math>-substitution seems to be the "opposite" of the chain rule.</p>
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Figure 2.1. A typical homework journal.

There are very specific prompts given to students to aid in their homework journal reflections. The prompts are both cognitive and affective. Cognitive prompts are mathematically oriented and focused specifically on the learning of mathematics (Jurdak & Zein, 1998). Affective prompts direct students to express their goals, strategies, reactions, and feelings towards the learning of mathematics (Jurdak & Zein, 1998). The specific prompts given to students will be discussed in the next chapter.

Homework journals are based on a culmination of the ideas presented in the literature concerning writing in mathematics and the literature related to teaching and learning in college mathematics courses. Homework journals can be described as intentional and purposeful writing assignments in mathematics based on Emig's (1977) classifications. Similar to other studies related to journal writing in mathematics, homework journals have the potential to increase

students' mathematical learning (cf. Grossman et al., 1993), build students' procedural and conceptual knowledge (cf. Jurdak & Zein, 1998), foster positive beliefs about mathematics (cf. Borasi & Rose, 1989), increase students' motivation towards learning mathematics (cf. Powell & Ramnauth, 1992; Sanders, 2009), and develop meaningful relationships between teachers and students (cf. Borasi & Rose, 1989; McIntosh & Draper, 2001). In addition, similar to other studies concerning teaching and learning in college mathematics courses, homework journals are a way for teachers to engage students in actively doing and discussing mathematics (cf. Beidleman et al., 1995; Castro et al., 1991), a key theme found in the literature related to teaching and learning in college mathematics courses.

### **Conceptual Framework**

In this section, I will develop and outline a conceptual framework to elaborate on the connections between journal writing in mathematics and self-determination theory. This framework will begin with a general understanding of learning and the reasons why journal writing in mathematics, particularly the homework journal, fits into that understanding of learning and acts as a powerful learning tool for students. I will then describe the connections between journal writing in mathematics and students' motivation for learning mathematics through the lens of self-determination theory.

### **Homework Journals as a Powerful Learning Tool**

The process of coming to know, the process of learning, is many-faceted and complex. Knowledge is not to be simply transmitted from teacher to student (von Glasersfeld, 1983), or downloaded onto a student's "hard-drive" from the teacher the way files are downloaded onto computers' hard-drives from some other source. Instead, knowledge must be "actively built up by a cognizing subject"(von Glasersfeld, 1989, p. 162). Although to many constructivists, this

philosophical tenet is considered to be trivial, it is still of upmost importance and is the foundation of any branch of constructivism. At its most basic level, “constructivism involves the active creation and modification of thoughts, ideas, and understandings as the result of experiences that occur within socio-cultural contexts” (Doolittle & Hicks, 2003, p. 76). In addition, Lave and Wenger (1991) build an argument that learning is not simply listening to another person tell you something about a topic, regardless of whether that topic is apprenticing a craftsman or studying literature. Instead learning is participation; learning is interaction (Lave & Wenger, 1991). Their arguments and supporting case studies demonstrate that learning is best accomplished by doing.

We come to know through experience – whether that experience is a shared or an individual experience – but not *just* through experience. “Learning, contrary to the famous adage, does not occur from experiences alone. For it possibly to occur, learners must reflect on their experiences (Powell & Ramnauth, 1992, p. 12). Reflection simply means the careful inspection of a situation; reflection implies moving outside of a situation to gain a better understanding of or reorganizing that experience (Hickman, 1990).

The importance of reflection is primarily supported by the work of John Dewey (2008b). While he would not have labeled himself as a constructivist, his theories of learning might place him into a constructivist camp. For Dewey, the importance of reflection is found within the fluid cycle of learning that he describes. The cycle starts with equilibrium. This means that an individual feels cognitively and emotionally “comfortable.” There is no confusion or dissonance within that individual. The next stage of the cycle, usually brought on by some kind of stimulus, is an emotional response in which the individual feels fear, anger, confusion, or even frustration. After the emotional response comes an exploration or an intellectual response. Both inquiry and

reflection play a crucial role in this stage of the cycle. Following the ebb and flow of inquiry and reflection comes the solution or resolution. The individual is again restored to equilibrium, completing the cycle of learning.

An important part of Dewey's cycle of learning is the resolution of cognitive tension. This resolution is certainly a part of the rhythm but an individual must start with some kind of doubt, skepticism, or even wonder (brought about by a stimulus or emotional response) if that individual really wants to learn. Dewey (2008b) writes,

Take any incident of experience you choose, seeing a color, reading a book, listening to conversation, manipulating an apparatus, studying a lesson, and it has or has not intellectual, cognitive, quality according as there is deliberate endeavor to deal with the indeterminate so as to dispose of it, to settle it. Anything that may be called knowledge, or a known object, marks a question answered, a difficulty exposed of, a confusion cleared up, an inconsistency reduced to coherence, a perplexity mastered. (p. 181)

This emphasizes the need for reflection – no matter how big or how small the experience – in order to clear up any cognitive tension through an intentional engagement with the experience and its implications.

In summary, “knowledge is not passively received either through the senses or by way of communication, but it is actively built up by the cognizing subject” (von Glasersfeld, 1996, p. 24) and we can explore, through reflection, the operations and processes used to build up knowledge (von Glasersfeld, 1984). These philosophical tenets are summarized in this statement: learning is best accomplished by doing and by reflecting. Through the use of journal writing in mathematics, students have the opportunity to be engaged in doing mathematics and reflecting on mathematics and/or mathematics learning.

Within the context of homework journals, students have the opportunity to do and to reflect. Students are actively engaged in doing mathematics by solving problems in detail, showing all of their work and clearly communicating their problem solving process. Students are also actively engaged in reflection. They are reflecting on the aspects of the problem that brought about cognitive tension, reflecting on their problem solving process, and reflecting on their own feelings while doing mathematics. *Figure 2.2* takes a typical homework journal and explicitly shows how homework journals relate to these two philosophical tenets of constructivism. In the column on the left, where the  $u$ -substitution problem is solved in detail, the student is doing mathematics. In the column on the right, the student is reflecting on their learning experience. By purposefully engaging students with these two philosophical tenets of constructivism, homework journals seem to act as a powerful learning tool for students.

<p>Solve <math>\int \frac{3x + 6}{x^2 + 4x - 3} dx.</math></p> <p><math>u = x^2 + 4x - 3</math>  <math>du = (2x + 4)dx = 2(x + 2)dx \Rightarrow</math>  <math>(x + 2)dx = \frac{du}{2}</math></p> <p><math>\int \frac{3(x+2)}{x^2+4x-3} dx =</math></p> <p><math>\int \frac{1}{x^2+4x-3} \cdot 3(x + 2)dx =</math></p> <p><math>\int \frac{1}{u} \cdot 3 \cdot \frac{du}{2} =</math></p> <p><math>\frac{3}{2} \int \frac{1}{u} du =</math></p> <p><math>\frac{3}{2} \ln u  + c =</math></p> <p><math>\frac{3}{2} \ln x^2 + 4x - 3  + c</math></p>	<p>The content of this problem involves <math>u</math>-substitution. The purpose of <math>u</math>-substitution is to take an integral that is difficult (or impossible) to compute as it is and change it into a form that is easy to compute. The change happens by performing the method of <math>u</math>-substitution.</p> <p>I think that this method gets its name from the fact that you are taking all of the <math>x</math>'s out of the integral and replacing them (making a substitution) with <math>u</math>'s. You are changing the variable.</p> <p>I see a connection between the method of <math>u</math>-substitution for integration and the chain rule for differentiation. The method of <math>u</math>-substitution seems to be the "opposite" of the chain rule.</p>
 Solving (doing) mathematics.	 Reflection.

*Figure 2.2.* A typical homework journal with explicit connections to the tenets of constructivism.

## **Homework Journals Relationship to Motivation**

In order to describe the connections between journal writing in mathematics and students' motivation towards learning mathematics, it is necessary to begin by briefly describing motivation and various theories of motivation. Most people have a working definition or a natural understanding of what motivation is; words like "motivated" and "unmotivated" are used regularly. However, it is much more difficult to specifically define motivation and its effects. One way to define motivation is to say that it involves the processes that give behavior its energy and direction (Reeve, 2005). Internal processes that energize and direct behavior are internal motives, such as a deep personal enjoyment. External processes that energize and direct behavior are external environmental incentives, such as money or enforced deadlines. Energy of behavior refers to the strength of a particular behavior (Reeve, 2005). Energy of behavior can be strong, intense, and persistent; on the other hand, energy of behavior can be weak, apathetic, and intermittent (Reeve, 2005). Direction of behavior means that behavior is aimed toward a goal (Reeve, 2005). By describing motivation in this way, as a construct involving the processes that give behavior its energy and direction, motivation is like a mathematical vector. A mathematical vector is a geometric object that has both magnitude and direction. This analogy provides a vehicle for understanding motivation in a more concise way.

Even when thinking about motivation with the use of this metaphor, it is still a broad and extensive construct. Throughout the history of the study of motivation, there were several grand theories that attempted to wholly understand motivation. A grand theory is "an all-encompassing theory, one general model that seeks to explain the full range of motivated action – why we eat, drink, work, play, compete, fear certain things, read, fall in love, and so on" (Reeve, 2005, p. 24). There were three grand theories posed throughout the history of the study of motivation: 1) the

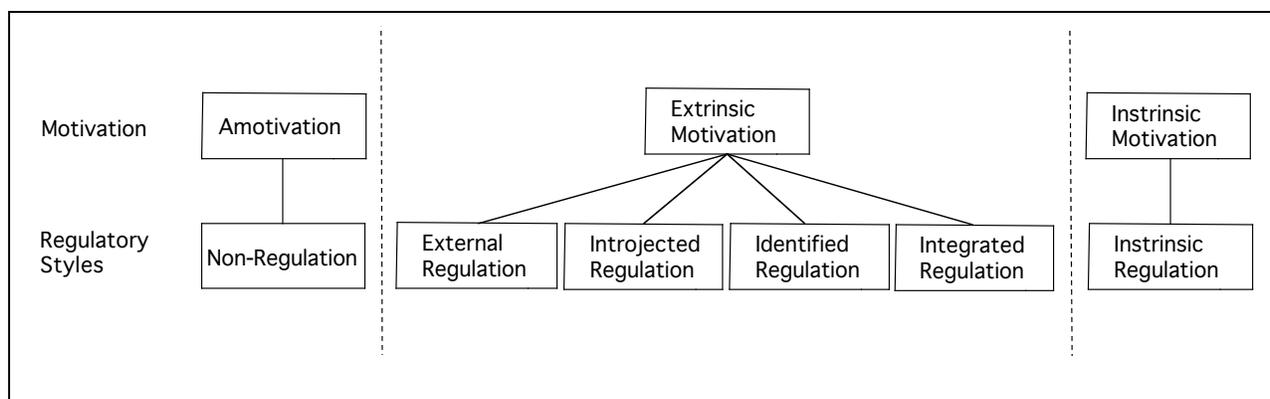
will, posed by Descartes, 2) instinct, posed by Darwin, and 3) drive, posed by Freud (Reeve, 2005). These all-encompassing theories of motivation, the grand theories, did not succeed in explaining “the full range of motivated action” (Reeve, 2005, p. 24). Therefore, researchers turned their sights to mini-theories of motivation. Mini-theories explain some, but not all of motivated behavior (Reeve, 2005). Many overlapping mini-theories of motivation have emerged; attribution theory (Weiner, 2000), expectancy-value theory (Eccles, 2005), flow theory (Csikszentmihalyi, 1990), goal theory (Anderman, Austin, & Johnson, 2002), self-determination theory (Ryan & Deci, 2000), and self-efficacy (Bandura, 1977) are a few examples.

Self-determination theory (Ryan & Deci, 2000), one mini-theory of motivation, has a natural relationship with writing in mathematics, especially homework journals. Before describing this relationship, it is first necessary to briefly explain self-determination theory. Self-determination theory is concerned with the degree to which motivation is regulated by external or internal factors (Ryan & Deci, 2000). Typically, motivation that is regulated by external factors is referred to as extrinsic motivation; this means that an individual is performing an activity in order to attain some separable outcome (Ryan & Deci, 2000). For example, a young child does chores around his house in order to earn his allowance. On the other hand, motivation that is regulated by internal factors is referred to as intrinsic motivation; this means that an individual is performing an activity for the inherent satisfaction of the activity itself (Ryan & Deci, 2000). For example, a high-school student walks her neighbors dogs each afternoon simply because she enjoys doing the activity. Teachers should strive to foster intrinsic motivation in their students (NRC, 2004). Offering students choices, acknowledging students’ feelings, and providing opportunities for self-direction are all ways that teachers can support intrinsic motivation. Tangible rewards, threats, deadlines, and directives are all ways that

teachers can diminish intrinsic motivation. A critical distinction is whether or not students feel as though their behavior is self-determined or determined by another. Self-determined action means that the regulatory style of action is based on choice. Controlled action, action determined by another, means that the regulatory style of action is based on compliance (Deci, Vallerand, Pelletier, & Ryan, 1991).

### **Self-determination continuum.**

Although it seems natural to assume that there are only two types of motivation, either extrinsic or intrinsic motivation, Ryan and Deci (2000) actually describe five different types of motivation. Each type of motivation is accompanied by its own regulatory style. The different types of motivation are illustrated in *Figure 2.3* (modified from Ryan & Deci, 2000) arranged from left to right in terms of the degree to which the motivation is self-determined or self-regulated.



*Figure 2.3.* Self-determination continuum showing various types of motivation along with their regulatory styles (Ryan & Deci, 2000).

At the far left side of the self-determination continuum is amotivation. Amotivation is the “state of lacking the intention to act” (Ryan & Deci, 2000, p. 72). When an individual experiences amotivation, they act without intent. Amotivation results from an individual who does not value an activity, does not feel competent to do the activity, or does not expect the

activity to yield a desired outcome (Ryan & Deci, 2000). An example would be a student in college who does not know why he is in college. He already owns his own business and attaining a college degree will not help his business. He experiences amotivation towards his coursework and is simply going through the motions.

In the middle of the self-determination continuum is extrinsic motivation. Extrinsic motivation varies in the extent to which behavior is self-regulated. External regulation, the regulatory style on the left-most side, means that the locus or cause of behavior is external to the individual (Ryan & Deci, 2000). External regulation implies the motivation is to satisfy an external demand. The external demand can be to gain something positive, such as a monetary reward, or to avoid something negative, such as a punishment. An example would be a student who chooses to attend medical school because he wants to make a great deal of money in the future.

Introjected regulation, the next step of regulation within extrinsic motivation, means that an individual accepts a regulation, but does not embrace it personally (Ryan & Deci, 2000). An individual may accept a regulation to avoid guilt or to attain ego enhancement. An example would be a student who arrives to class on time only so that she does not feel guilty about being late. The student's behavior, getting to class on time, is regulated by her desire to avoid feelings of guilt.

Identified regulation, the third type of regulation within extrinsic motivation, means that an individual accepts and embraces a regulation because they see some value in that particular behavior (Ryan & Deci, 2000). The individual values a goal or outcome and therefore accepts regulation as one's own; the behavior is personally important in some way. An example would be a high-school student who willingly does extra homework in mathematics class because she

knows it will help her in future mathematics classes. The student acknowledges value in extra homework and therefore her behavior is more self-regulated.

Integrated regulation is the most self-regulated style within extrinsic motivation. Integrated regulation means that behavior is fully assimilated to one's self, it is congruent with one's values and needs, but it is not performed for the inherent enjoyment of the behavior (Ryan & Deci, 2000). An example of integrated regulation is a college student who takes an economics course because he wants to better understand economics. Regulations are integrated when the behavior is an expression of who an individual is.

Finally, at the far right side of the self-determination continuum is intrinsic motivation along with intrinsic regulation. Intrinsic regulation means that an individual chooses to perform an activity simply because the activity is enjoyable in itself (Ryan & Deci, 2000). An example would be a college student who takes a literature course because the readings and class discussions are engaging and pleasant for that student. Intrinsic regulation does not necessarily mean internal. Some forms of extrinsic motivation are internally regulated. The main difference between intrinsic regulation and extrinsic regulation is that intrinsic regulation leads an individual to choose an activity or behavior just because that activity or behavior is inherently enjoyable.

These five types of motivation (excluding amotivation) form a continuum. While the development from externally regulated behavior to intrinsically regulated behavior is not necessarily a linear progression, the process of moving along the continuum is called internalization (Deci et al., 1991). There are many benefits of internalization such as increased motivation, performance, and development (Deci et al., 1991) and the process of internalization

is aided whenever particular psychological needs are met. The three psychological needs associated with self-determination theory are competence, autonomy, and relatedness.

**Competence, autonomy, and relatedness.**

When the three psychological needs associated with self-determination theory, competence, autonomy, and relatedness, are met, they form the perfect conditions for positive social development, personal well-being, and internalization (Ryan & Deci, 2000). Competence measures the degree to which an individual feels they are able to complete a task and succeed. Autonomy measures the degree to which an individual feels they have the ability to make meaningful choices concerning an activity. Relatedness measures the degree to which an individual feels related to or close to others participating in that activity.

There have been numerous studies that support these three psychological needs as being the three needs that are most necessary to foster intrinsic motivation (Hahn & Oishi, 2006; Ryan & Deci, 2000; Reeve & Sickenius, 1994; Deci et al., 1991). For example, Hahn and Oishi (2006) tested the relevance of 10 psychological needs for emotional well-being for both younger and older Korean and American adults. The 10 psychological needs identified from previous studies were: 1) autonomy, 2) competence, 3) relatedness, 4) physical thriving, 5) security, 6) self-esteem, 7) self-actualization, 8) pleasure-stimulation, 9) money-luxury, and 10) popularity-influence (Hahn & Oishi, 2006). Participants in the study were asked to recall the “most satisfying event in the past month” that they had personally experienced (Hahn & Oishi, 2006, p. 692). Participants were then asked a series of questions about that event, rating the relevance of 30 descriptive statements about the event. For each of the 10 psychological needs, there were three descriptive statements resulting in the 30 descriptive statements. When ignoring age and cultural factors, relatedness, autonomy, and self-esteem were the most important needs for all

participants. Competence and self-actualization were next most important needs for all participants. These findings provide more evidence that autonomy, competence, and relatedness are three of the most important psychological needs for emotional well-being across ages and cultures (Hahn & Oishi, 2006).

“Simply stated, motivation, performance, and development will be maximized within social contexts that provide people the opportunity to satisfy their basic psychological needs for competence, relatedness, and autonomy” (Deci et al., 1991, p. 327-328). It is incredibly important for teachers to facilitate learning by nurturing and encouraging their students’ competence, by supporting autonomy in their students, and by building positive relationships with their students.

#### **Connections to homework journals.**

Motivation in education is studied extensively and it is widely accepted that motivating students is important and valuable (NRC, 2004). Motivated students are more likely to think deeply about concepts, to be more focused in class, to regulate their learning by checking their understanding, and to ask questions (Reeve, 2005). To help foster students’ motivation, homework journals were created with the specific purpose of meeting the three psychological needs associated with self-determination theory, competence, autonomy, and relatedness, so that students can experience constructive social development and internalization.

#### ***Competence.***

Competence measures the degree to which an individual feels they are able to complete a task and succeed at completing that task. This is closely associated with self-beliefs, particularly self-efficacy (Bandura, 1977). Self-efficacy is a judgment that an individual makes about their

perceived abilities. Self-efficacy beliefs are powerful predictors of choice, effort, persistence, attitude, and motivation (Schunk & Pajares, 2005).

Similar to Bandura's (1977) description of performance accomplishments being a strong source of self-efficacy beliefs, personal mastery experiences also act as a way to meet an individual's need to feel competent (Ryan & Deci, 2000). When considering a particular task, success raises feelings of competence while failure lowers them. However, failures that are later overcome by success can raise feelings of competence significantly. By regularly solving challenging problems for homework journals, students are building performance accomplishments; successfully solving problems, having personal mastery experiences, helps meet students' needs for competence. In addition, when students reflect on their problem solving process in homework journals, they are taking a step back to examine their success. The combination of doing and reflecting in homework journals gives students the opportunity for personal mastery experiences.

Bandura (1977) describes verbal persuasion as another source of self-efficacy beliefs. Similarly, verbal persuasion or encouragement also acts as a way to meet an individual's need to feel competent (Ryan & Deci, 2000). Verbal persuasion is based on the encouragement of others. Although encouragement can help individuals believe that they will be able to perform a task or complete an activity that they struggled with in the past, it is a slightly weaker source of information for feelings of competence because it does not provide any type of authentic experience. However, individuals who have strong competence beliefs built by verbal persuasion are likely to persevere in the face of difficulty, even if they are not able to actually perform the task or complete the activity. When teachers respond to students' homework journals, both their problem solutions and their reflections, they ask students questions to clarify

what they wrote, challenge students to extend their thinking about a particular concept, affirm the work that students did or the reflection they wrote, or encourage students to try other problems.

This verbal persuasion can serve as the encouragement students' need to build their competence.

Homework journals have the potential to meet students' needs for feelings of competence by providing an opportunity for mastery experiences and through specific encouragement from teachers. By doing mathematics and reflecting on mathematics and/or mathematics learning, students are able to experience and examine their success. Students are also able to receive specific feedback in the form of encouragement and affirmation from teachers.

### *Autonomy.*

As important as competence and relatedness are, they must both be supported by autonomy (Deci et al., 1991). An individual can have great confidence in their ability to complete a task, but will still have little motivation if they do not feel autonomous. Similarly, an individual can experience a significant amount of relatedness with a teacher, but still have very little motivation towards an educational task if they do not feel autonomous. Competence or relatedness alone will not increase motivation; it must be coupled with autonomy (Ryan & Deci, 2000). Autonomy measures the degree to which an individual feels they have the ability to make meaningful choices concerning an activity.

Reeve and Jang (2006) identified autonomy-supportive teacher behaviors; behaviors that were found to predict whether or not a student would experience a sense of autonomy. Reeve and Jang (2006) found eight autonomy-supportive teacher behaviors to be statistically significant predictors of students' perceived sense of autonomy: time allowing students to work in their own way, time listening, time student talking, praise as information feedback, offering encouragement, offering hints, being responsive to student generated questions, and making

perspective-acknowledging statements (Reeve & Jang, 2006, p. 214). All eight autonomy-supportive teacher behaviors have strong connections to homework journals.

Homework journals do allow students to work in their own way. Although homework journals have a specific format and the content of homework journals is standard, students are able to work in their own way because of the choices they have. Students choose which problem to solve in the left-hand column of their homework journal and which prompt to respond to in the right-hand column of their homework journal.

Homework journals also encourage time for the teacher to listen and time for the students to talk. One of the most important aspects of homework journals is that teachers actually read them. By writing homework journals, students are “talking” and by reading homework journals, teachers are “listening”. “Talking” simply means that students are given space to demonstrate their mathematical understanding by solving a problem and to express their ideas about mathematical content through reflection. “Listening” simply means that teachers are taking the time to read and respond to each and every homework journal.

Praise as informational feedback, offering encouragement, offering hints, being responsive to student generated questions, and making perspective-acknowledging statements, the remainder of these autonomy-supportive teacher behaviors, are all aspects of the teacher responses. Praise as informational feedback means that teachers are offering statements such as “good job” or “great work” on students’ homework journals. This direct feedback about the student’s improvement or mastery of a topic is a quick and easy response for teachers to include on students’ homework journals. Offering encouragement is another natural response of teachers when responding to students’ homework journals. Statements like “you are really close” or “I know you can figure this out” are encouraging statements and ways for teachers to boost or

sustain the students' engagement. Offering hints means that teachers are giving suggestions about how to make progress when a student seems to be stuck. For example, a teacher might write, "have you tried using the method of integration by parts?" when a student is struggling with an integral in their homework journal. If a student poses a question in their homework journal, and if the teacher responds to that question – either immediately as a written response in that students' homework journal or by addressing the question during the next class session – then that is an autonomy-supportive teacher behavior. Communicating and affirming perspective-taking statements means that teachers are making empathetic statements that acknowledge a students' perspective or experience. An easy example of this autonomy-supportive teacher behavior would be if a student writes that they were extremely frustrated with a particular problem. A perspective-taking statement would be if the teacher responds with "yes, that problem was very difficult" or "a lot of students struggled with this problem." All of these teacher responses are ways to support students' sense of autonomy and to spark further reflection.

"Teachers cannot give students a sense of autonomy. Instead, teachers can provide students with high-quality interpersonal relationship – relationship rich in attunement and supportiveness – and out of that relationship context, students can experience and begin to exercise their own sense of autonomy" (Reeve & Jang, 2006, p. 217). Based on the structure of the homework journals and the teachers' responses to each and every students' homework journals, this particular writing assignment offers plenty of opportunities for teachers to demonstrate autonomy-supportive behaviors that can help students experience a sense of autonomy.

In addition to autonomy-supportive teacher behaviors, homework journals also allow students the opportunity to make choices. Choice is an important part of fostering students' sense of autonomy. Katz and Assor (2007) provide a few guidelines for offering choices to students. First of all, teachers should offer choices that have value to students or that are interesting to students (Katz & Assor, 2007). When students choose a problem to solve for their homework journal, they are explicitly told to choose a problem that is interesting to them or challenging to them. Secondly, teachers should offer a number of choices that are neither too few nor too many (Katz & Assor, 2007). Both the total number of problems that students choose from and the number of prompts – the two choices students make when completing their homework journals – are neither too few nor too many. The total number of problems that students choose from to solve in their homework journal is typically between 10 and 15 and there are only five prompts to choose from to reflect on in their homework journal. Similarly, teachers should offer a number of choices that are neither too simple nor too complex (Katz & Assor, 2007). Instead, choices should be in accordance with students' cognitive abilities and age level; choices should motivate students to demonstrate their competence (Katz & Assor, 2007). Again, students are able to choose problems that are interesting and challenging to them. The problem that one student chooses to solve for their homework journal might be different from the problem that another student chooses to solve. Students may reflect on why they chose one problem over another in the reflection portion of their homework journal. By following these guidelines from Katz and Assor (2007), the choices students' make regarding their homework journals are more meaningful and homework journals are likely to increase students' sense of autonomy.

***Relatedness.***

Relatedness measures the degree to which an individual feels related to or close to others participating in that activity. In order to foster relationships with students, teachers first need to know their students well. Teachers should be attuned to what and how their students are thinking and feeling (Reeve, 2006). This nurtures feelings of relatedness and helps students feel like they are special and important. By reading and responding to students' homework journals, teachers are able to get a first-hand perspective of what their students are thinking and feeling. Throughout the entirety of a course in which students are participating in homework journals, teachers will also be able to get to know their students, specifically, their style of communication and an idea of their interests.

Teachers also need to be supportive of their students; this means that they affirm what their students do well in order to help students feel more competent. It also means that they encourage their students when they struggle. Offering praise and encouragement are two ways that teachers' responses on homework journals help meet students' need for relatedness (Reeve, & Jang, 2006). Offering praise can be as simple as writing comments such as "great job" or "really nice work" on students' homework journals. Offering encouragement can also be simple; comments like "keep up the good work" and "I know you will eventually understand this difficult concept" are easy ways to encourage students.

Students' need for relatedness is also met when teachers directly respond to students' written questions in their homework journals. Teachers can respond to students' questions individually by writing a written response in students' homework journals or teachers can respond to students' questions corporately by addressing questions with the whole class during

the next class session. This demonstrates to students that teachers are actually reading their homework journal; teachers are “listening” to what they say (Reeve & Jang, 2006).

In addition, affirming students’ perspectives is another way to meet students’ need for relatedness. Affirming students’ perspectives means that the teacher supports and upholds what the students are expressing. This communicates to students that their teacher is supportive of them and that they are not alone in their learning and experiences (Reeve, 2006). For example, suppose a student expresses great joy and excitement over a particular method of solution in their homework journal. Affirming that students’ perspective means that the teacher would make a comment agreeing with the student or joining in excitement with the student. By offering praise and encouragement, by responding to students’ questions, and by affirming students’ perspectives, teachers will be exhibiting behaviors that are supportive of their students.

#### ***Importance of meaningful feedback.***

Receiving feedback from the teacher is something that is mentioned in each of the previous sections; receiving feedback from the teacher is critical to competence, autonomy, and relatedness. Receiving encouragement from their teacher is one way to help students believe that they will be able to perform a task or complete an activity that they struggled with in the past, such as solving a difficult problem. This helps meet students’ need for competence (Ryan & Deci, 2000). Receiving feedback from their teacher such as praise, encouragement, hints, and perspective taking statements helps meet students’ need for autonomy (Reeve & Jang, 2006). Receiving feedback from their teacher that is both supportive and encouraging is one way to help students feel reassured of their teacher’s care and concern for them and for their learning of mathematics. This helps meet students’ need for relatedness (Reeve, 2006; Reeve & Jang, 2006). In addition, positive feedback increases intrinsically motivated behavior (Butler, 1987;

Deci, 1972) and there is a positive association between positive feedback and both interest and persistence (Deci, Koestner, & Ryan, 1999). Because receiving feedback is such an important element of the homework journals, especially in relation to the three psychological needs associated with self-determination theory, it is important to briefly discuss feedback and elaborate on some considerations about feedback found in the literature.

First of all, what is considered “good” feedback and what is not? Students’ needs for competence, autonomy, and relatedness can be met when they receive feedback from their teacher, therefore the focus of “good” feedback should be on students and not on teachers. Because of this, it is wise to consider this question from the students’ perspective instead of from the teachers’ perspective. Students typically perceive poor feedback as being vague, overly critical, unrelated to communicated expectations, and containing no helpful guidance or suggestions about how to improve (Carless, 2006; Weaver, 2006). On the other hand, students typically perceive good feedback as being focused on how to improve, including specific suggestions and guidance (Brinko, 1993; Hounsell et al., 2008; Nicol & Macfarlane-Dick, 2006). Feedback should be specific, should include some type of praise or supportive expression, should directly connect to the communicated expectations of the assignment, problem, or task, and should provide specific suggestions for improvement (Hendry, Bromberger, & Armstrong, 2011). This is the type of feedback that students classify as good and meaningful feedback. How is such feedback accomplished? There are two essential elements: 1) feedback should be detailed and 2) feedback should include a combination of what students have done well and how students can improve (Hendry et al., 2011). The first essential element of good and meaningful feedback is fairly straightforward. The second essential of good and meaningful feedback requires some explanation.

Three specific types of feedback are praise, criticism, and suggestions. Hyland and Hyland (2001) define these three types of feedback in the following ways: Praise is when a teacher attributes credit to a student for some characteristic or accomplishment. Criticism is when a teacher expresses dissatisfaction or some other negative comment to a student about their performance. Suggestions are when a teacher gives a student an explicit recommendation for improvement. Individually, these three types of feedback are typically not helpful to students (Hyland & Hyland, 2001). On its own, students often dismiss praise as being placating. Criticism is seen as too harsh and even harmful on its own. And, on their own, suggestions are not helpful because students are not aware of why they need to improve. However, “by combining these acts into patterns of praise-criticism, criticism-suggestion, and praise-criticism-suggestion...[teachers] enhance relationships [with students], minimize the threat of judgment, and mitigate the full force of their criticisms and suggestions” (Hyland & Hyland, 2001, p. 207). By combining praise, criticism, and suggestions students are informed about what they did well and how they can improve.

Secondly, how is “good” feedback accomplished? Researchers have proposed numerous frameworks for providing quality feedback to students (Cardelle-Elawar, 1992; Hyland & Hyland, 2001; Mouratidis, Lens, & Vansteenkiste, 2010). One such framework defines specific guidelines that enable teachers to provide quality feedback to students (Parr & Timperly, 2010). Teachers should first identify where a student is positioned relative to the desired performance or outcome; this should be the first piece of feedback for the student. Teachers should then summarize key features of the desired performance or outcome, making clear to the student what is expected. Finally, teachers should describe a specific suggestion of what the student needs to

do in order to achieve the desired performance or outcome. This type of feedback (Parr & Timperly, 2010) is detailed, personal, and combines praise, criticism, and suggestions.

For example, consider a mathematics assignment where the desired performance or outcome is for students to successfully solve a problem and to clearly justify their solution. Suppose a student successfully solved the problem, but his justification lacked rigor and detail. With their feedback, the teacher would first explain that the student successfully solved the problem, incorporating praise into the feedback, but then point out that his justification was lacking, incorporating criticism into the feedback. The teacher would next explain why the justification is lacking; this might be in reference to a rubric or some other explanation of the expectations. Finally, the teacher would suggest one or two specific ways that the student could improve his justification, incorporating suggestions into the feedback. Parr and Timperly (2010) discuss how such feedback requires sufficient pedagogical content knowledge (Shulman, 1986) on the part of the teacher, but by following this framework students receive quality feedback.

In conclusion, good feedback should be detailed and should include a combination of what students have done well and how students can improve (Hendry et al., 2011). Receiving quality feedback from teachers is a critical element of homework journals because of the strong connections between feedback and competence, autonomy, and relatedness.

***Summary of the connection between HJ and SDT.***

Competence, autonomy, and relatedness are the three essential psychological needs that help motivation flourish (or diminish if they are absent). Homework journals provide opportunities for students' competence to increase. This is because students' are having personal experiences succeeding at solving problems as well as reflecting on their success, and because they are receiving feedback from their teacher. Homework journals are also a way for students

to experience a strong sense of autonomy. This is because of the many autonomy-supportive teacher behaviors associated with homework journals, such as providing constructive feedback, and because of the meaningful choices offered to students when completing homework journals. Finally, homework journals are a way for students to develop a sense of relatedness with their teacher. This is because teachers get to know their students through regularly reading their homework journals and because of the support that teachers give students through their responses and feedback to students' homework journals.

### **Overview and Conclusion**

This chapter contained on two main sections. The first section was a review of the current literature concerning journal writing in mathematics and teaching practices for postsecondary mathematics instructors. The second section was a conceptual framework that explained the relationship between homework journals, learning, and self-determination theory.

The story told by the current literature concerning journal writing in mathematics is that journal writing affords many benefits for students, for teachers, and for communication between students and teachers. One benefit for students is that journal writing helps build both procedural and conceptual knowledge (Borasi & Rose, 1989; Jurdak & Zein, 1998). One benefit for teachers is that journal writing gives teachers the ability to efficiently assess students' understanding (Borasi & Rose, 1989; Drake & Amspaugh, 1994; Miller & England, 1989). One benefit for communication between students and teachers is that journal writing provides individualized feedback and support for students from teachers (Borasi & Rose, 1989; Williams & Wynne, 2000).

The story told by the current literature concerning teaching practices for college mathematics instructors is to move away from lecture, the traditional and dominant teaching

method is college mathematics courses (Ahmadi, 2002; Bookman & Friedman, 1994; Castro et al., 1991; Hagerty et al., 2010; Halmos, 1985; Ward et al., 2010), and towards a model of teaching that integrates group projects into regular coursework, incorporates more writing assignments, provides historical context for materials, and anchors new material in what students already know (Castro et al., 1991).

Homework journals are a powerful learning tool that can help students actively engage in doing mathematics (Lave & Wenger, 1991), and can help promote active reflection on mathematics and on mathematics learning (Dewey, 2008b). Learning is a combination of doing and reflecting. “Knowledge, i.e., what is ‘known,’ cannot be the result of a passive receiving but originates as the product of an active subject’s activity” (von Glasersfeld, 1984, p. 28). Doing means intentional activity. In the case of homework journals, the intentional activity is solving mathematics problems in detail, showing the complete problem solving process. Reflection is the careful inspection of a situation (Hickman, 1990). Reflecting means taking a step back to examine and make sense of an experience. In the case of homework journals, examining and making sense of an experience is stepping back to examine the problem solving process or to make sense of the content of the problem.

When students complete a homework journal, they are doing mathematics; they are solving a mathematical problem. When students complete a homework journal, they are also reflecting on mathematics, on their mathematical understanding and confidence, and on their mathematical success. When teachers respond to students’ homework journals, they are encouraging and supporting students doing mathematics and reflecting on mathematics learning. Students are doing and reflecting and teachers are responding and encouraging. These are the key components of homework journals; they also provide the connection to self-determination

theory because of their potential to help meet students' needs of competence, autonomy, and relatedness.

Homework journals have the potential to increase students' intrinsic motivation by providing opportunities to meet the three psychological needs of competence, autonomy, and relatedness. By completing homework journals, students have mastery experiences and reflect on those experiences, demonstrating that they are capable of solving mathematics problems and receive positive feedback from teachers. These factors may lead to strengthened feelings of competence. When teachers exhibit autonomy-supportive teacher behaviors, such as positive feedback, and offer students meaningful choices, students' perceived sense of autonomy increases. By responding to students' homework journals with encouragement, supportive feedback and suggestions, empathetic comments, and informational praise, teachers are demonstrating autonomy-supportive behaviors. Meaningful choice is built into the structure of homework journals. In addition, the back and forth nature of homework journals also allow students and teachers to get to know each other, increasing students' sense of relatedness. The positive feedback offered from teachers – encouragements, praise, empathetic statements, and support – are also ways to increase students' sense of relatedness.

Although implementing and studying the effects of homework journals on college mathematics students' motivation through self-determination theory is a very specific task, it fits within the broader context of literature concerning writing in mathematics and its potential effect on students' motivation. The literature related to writing in mathematics in general, which implies through informal observations and analysis that writing increases motivation (Borasi & Rose, 1989; Sanders, 2009), influenced this study. This study takes these broad ideas about writing in mathematics and motivation and narrows the focus by examining a specific type of

writing in mathematics (homework journals) and its relationship to a specific mini-theory of motivation (self-determination theory). Based on the literature related to writing in mathematics in general, writing in mathematics should increase students' motivation towards learning mathematics. This study seeks to make that relationship explicit. And although this study concerns a specific type of writing assignment and a specific theory of motivation, it will contribute to the broader literature.

## **Chapter 3: Methods.**

### **Introduction**

One potential benefit of journal writing that has not yet been explicitly studied is the relationship between journal writing in college mathematics courses and college students' motivation towards learning mathematics. While there is some evidence that suggests there is a relationship between journal writing in college mathematics courses and college students' motivation towards learning mathematics, it is mostly anecdotal. For example, students seem to be more interested in learning mathematics (Sanders, 2009), feel a sense of excitement for learning mathematics (Borasi & Rose, 1989), and gain a sense of enthusiasm towards learning mathematics (Powell & Ramnauth, 1992) when engaged with journal writing. This study seeks to explicitly explore the relationship between journal writing in college mathematics courses and college students' motivation towards learning mathematics by answering the following research questions:

- What is the relationship between homework journals and students' psychological needs for competence, autonomy, and relatedness?
- What is the relationship between homework journals and students' attitudes towards writing in mathematics?
- What is the relationship between homework journals and students' achievement in mathematics?

### **Context**

The study took place during the Fall 2011 semester. Two sections of Math 2015, *Elementary Calculus with Trigonometry II*, were involved in the study. Most of the students

enrolled in Math 2015 come from various majors that do not have a concentrated focus in mathematics; students enrolled in Math 2015 are typically not mathematics or engineering majors. The purpose of the course is to provide opportunities and examples of mathematical problems to these students that may come up in their other courses and eventually in their careers. The content of the course includes antiderivatives, definite integrals, the Fundamental Theorem of Calculus, and elementary differential equations. A Senior Instructor in the Department of Mathematics taught both sections of Math 2015.

### Participants

One of the two sections engaged with homework journaling while the other section did not. The section that engaged with homework journaling will be referred to as the HJ section. There were 60 students enrolled in the HJ section at the beginning of the semester (40% male, 60% female). The section that did not engage with homework journaling will be referred to as the Non-HJ section. There were 59 students enrolled in the Non-HJ section at the beginning of the semester (44% male, 56% female). Descriptive information about the students in both the HJ section as well as the Non-HJ section can be found in Tables 3.1 and 3.2.

Table 3.1

#### *Gender of Students in Math 2015*

	HJ section	Non-HJ section
Male	40%	44%
Female	60%	56%

Table 3.2

*Majors of Students in Math 2015*

	HJ section	Non-HJ section
Science	30%	23%
Business	30%	37%
Natural Resources and Environmental Science	17%	9%
Mathematics or Engineering	0%	0%
Other	23%	31%

Interviews were conducted with nine students who were enrolled in the HJ section. In an attempt to choose student interview participants who would be representative of the entire HJ section, I used data from the pre-course survey to identify students who demonstrated low self-determination, high self-determination, low attitudes towards writing, and high attitudes towards writing. A classification of “low” meant that students fell within the lowest third of their class. A classification of “high” meant that students fell within the highest third of their class. There were four ways to pair up these different differentiations resulting in four categories: 1) low self-determination and low attitudes towards writing, 2) low self-determination and high attitudes towards writing, 3) high self-determination and low attitudes towards writing, and 4) high self-determination and high attitudes towards writing. I specifically recruited between three and five students from each of these four categories. From the category with low self-determination and low attitudes towards writing, two students responded: Ann and Melissa. From the category with low self-determination and high attitudes towards writing, one student responded, Mia. From the category with high self-determination and low attitudes towards writing, three students

responded: Adele, Kelly, and Lydia. From the category with high self-determination and high attitudes towards writing, two students responded: Joe and Sophie. When the search was widened to increase the number of student interview participants, only one other student responded: Chris. Chris was from the category with high self-determination and high attitudes towards writing. These nine students volunteered to participate in the interviews. In addition, one interview was conducted with a student who was enrolled in the Non-HJ section, who also volunteered to participate in the interview. Nate was from the category with high self-determination and low attitudes towards writing. The purpose for choosing at least one student who was enrolled in the Non-HJ section was to provide a comparison to the students who were enrolled in the HJ section. The ten student interview participants, their genders, and their majors are reported in Table 3.3. There were seven female and three male student interview participants. The students interview participants were biology, business, or psychology majors.

Table 3.3

*Student Interview Participants*

Student	Gender	Major
Adele	Female	Biology
Ann	Female	Business and Psychology
Chris	Male	Business
Joe	Male	Business
Kelly	Female	Biology
Lydia	Female	Psychology
Melissa	Female	Biology
Mia	Female	Environmental Science
Nate*	Male	Business
Sophie	Female	Biochemistry

*Note.* \*Non-HJ section. All other interview participants were from the HJ section.

### Variables

#### Independent Variable

The independent variable in the study was homework journaling. The independent variable had two levels: homework journaling (HJ) and non-homework journaling (Non-HJ). Students in both the HJ and the Non-HJ sections were assigned weekly homework problems. However, only students in the HJ section engaged with a homework journaling assignment based on these problems once a week. Other than the homework journaling assignment, the two sections (HJ and Non-HJ) were identical. Both of the sections were taught by the same

instructor, focused on the same mathematical content, were assigned the same labs and homework, and took the same tests. There was a common time for each test in Math 2015, which means that all Math 2015 students took the same test at the same time. Having an HJ section and a Non-HJ section included in the study allowed there to be a treatment group (HJ) and a control group (Non-HJ), which means that comparisons could be made between the two groups.

### **Dependent Variables**

There were five dependent variables in the study. Three of the dependent variables corresponded to students' degree of self-determination: competence, autonomy, and relatedness. The other two dependent variables were students' attitudes towards writing in mathematics and students' course achievement.

## **Measures and Instruments**

### **Homework Journals**

#### **Assignment description.**

Throughout the Fall 2011 semester, students in both the HJ and the Non-HJ sections of Math 2015 completed weekly homework assignments. The homework assignments were identical except that students in the HJ section also completed a homework journal. At the beginning of the course, the homework journaling assignment description was given to students in the HJ section (see Appendix B). The homework journaling assignment description explained both the purpose and the format of the homework journals. It also provided students with the specific prompts that they could choose from for each homework journal as well as an example homework journal. Finally, the homework journal assignment description outlined the guidelines and expectations for the homework journal assignment.

### **Format and content of homework journals.**

The format and content of homework journals is based on Powell and Ramnauth's (1992) idea of "multiple-entry logs" as described in the previous chapter. The homework journaling assignment required students to choose one problem from the assigned homework problems; they were asked to choose a problem that was personally challenging or interesting. For each homework journal, students divided a sheet of paper into two columns. In the column on the left, students solved the homework problem by showing and explaining all the steps of the problem solving process. In the column on the right, students wrote a short reflection of their understanding of the problem, based on one of the suggested prompts. After the students handed in their weekly homework journal assignment, the course instructor read each journal entry and responded with questions, challenges, affirmation, and encouragement. All of the weekly homework journals, including the instructor's responses, were kept together in a bluebook (provided by the researcher). An example homework journal is provided in Figure 2.1 in the previous chapter.

### **Homework journal prompts.**

The specific prompts given to students aided in their homework journal reflections; the purpose of the prompts was to allow students to engage in reflection. The prompts were both cognitive and affective. Cognitive prompts are mathematically oriented and focused specifically on the learning of mathematics (Jurdak & Zein, 1998). Affective prompts direct students to express their goals, strategies, reactions, and feelings towards the learning of mathematics (Jurdak & Zein, 1998). One of the main purposes of homework journals was to aid students to develop their understanding of mathematical concepts; therefore, the majority of the homework

journal prompts were cognitive. The homework journal prompts consisted of three cognitive prompts and two affective prompts. The three cognitive prompts were:

- 1) Narrate the process you used to solve this problem. What steps did you take to solve this problem and why did you choose those steps?
- 2) Describe the content of this problem. What are the conceptual ideas of the content of this problem?
- 3) In what ways does this problem relate to other mathematics that you know or to other problems that may come up in your other courses or eventually in your field of specialization?

The two affective prompts were:

- 4) How successful do you feel you were in solving this problem? Why? Do you feel you would be successful in solving a similar problem? Why or why not?
- 5) How did solving this problem make you feel about doing mathematics?

Students were told that they do not have to respond to every prompt in each homework journal they turn in. However, they were expected to choose and respond to at least one of the prompts. In addition to the prompts, students were told that they were free to discuss any other thoughts or ideas that they had about the problem they solved, their learning, or their feelings about mathematics. In the example homework journal (see *Figure 2.1*), the problem in the left-hand column deals with a method of integration called  $u$ -substitution. The reflection in the right-hand column addressed the mathematical content of the problem (Prompt 2) and the connection between this particular problem and previously learned mathematics (Prompt 3).

## **Quantitative Data**

### **Self-determination.**

A survey modified from Ryan and Deci (2000) was used to measure degree of self-determination (Kosko, 2010, 2011). This survey contained 30 items. These 30 items can be categorized into three different sub-scales: competence (9 items), autonomy (13 items), and relatedness (8 items). An example of an item from the competence sub-scale was, “I usually do well in mathematics.” An example of an item from the autonomy sub-scale was, “I feel like I can make a lot of decisions in how I do my math.” An example of an item from the relatedness sub-scale was, “I get along with the students in my math class.” Some of the items were reverse coded in order to make the sub-scales consistent. For example, an item from the competence sub-scale that was reverse coded was, “Mathematics is not one of my strengths.” To reverse code an item, it was necessary to assign a new value to that item by subtracting the old value from seven. Therefore an old value of five would be subtracted from seven, resulting in two for the new value. Once the necessary items were reverse coded, the items corresponding to each of the three sub-scales were averaged for each student using all available data, resulting in a comprehensive score for competence, autonomy, and relatedness for each student. All of the survey items can be found in Appendix A.

Reliability of the sub-scales in terms of internal-consistency was measured using Cronbach’s alpha. They are presented in Table 3.4 for both the pre-course and the post-course surveys. The Cronbach’s alphas for competence were all found to be above 0.90, which is considered excellent (George & Mallery, 2003). The Cronbach’s alphas for autonomy were all found to be between 0.80 and 0.90, which is considered good (George & Mallery, 2003). All of the Cronbach’s alphas for relatedness, except for the Non-HJ section on the post-course survey,

were found to be between 0.80 and 0.90, which is considered good (George & Mallery, 2003). The Cronbach's alpha for relatedness for the Non-HJ section on the post-course survey was found to be between 0.70 and 0.80, which is considered acceptable (George & Mallery, 2003).

### **Attitudes towards writing.**

In order to measure students' attitudes towards writing in mathematics, I created eight additional survey items related to writing in mathematics. These items all specifically mentioned or addressed writing in mathematics and created a fourth sub-scale on both the pre-course and post-course surveys. An example of an item from the attitudes toward writing sub-scale was, "It is important to write about problem solutions in math class." Similar to the sub-scales for competence, autonomy, and relatedness, some of the items related to writing in mathematics were reverse coded in order to make the sub-scale consistent. For example, an item that was reverse coded was, "Writing in math class is a waste of time." Reverse coding was accomplished in the manner described above. Once the necessary items were reverse coded, the eight items related to writing in mathematics were averaged for each student using all available data, resulting in a comprehensive score for attitudes towards writing for each student. All of the survey items can be found in Appendix A.

Reliability of the attitudes towards writing sub-scale in terms of internal-consistency was measured using Cronbach's alpha. They are presented in Table 3.4 for both the pre-course and the post-course survey. All of the Cronbach's alphas for attitudes towards writing, except for the HJ section on the pre-course survey, were found to be between 0.80 and 0.90, which is considered good (George & Mallery, 2003). The Cronbach's alpha for attitudes towards writing for the HJ section on the pre-course survey was found to be between 0.70 and 0.80, which is considered acceptable (George & Mallery, 2003).

Table 3.4

*Cronbach's  $\alpha$  for the Variables for Pre-Course and Post-Course*

	HJ Section		Non-HJ Section	
	Pre	Post	Pre	Post
Competence	0.92	0.93	0.95	0.95
Autonomy	0.84	0.85	0.90	0.84
Relatedness	0.83	0.83	0.81	0.75
Attitudes towards Writing	0.78	0.89	0.87	0.84

### **Course achievement.**

Course achievement was measured through students' final exam scores. The final exam score was chosen as the measure for course achievement because it was a comprehensive final exam and because all students in Math 2015 took the same tests throughout the semester, including the final exam. The course instructor reported students' final exam scores for students in both the HJ section and the Non-HJ section once the semester was over.

### **Qualitative Data**

#### **Student Interviews.**

During the month of November 2011, 10 students (9 HJ students, 1 Non-HJ student) were interviewed using a general interview guide (Patton, 1990). Using a general interview guide allowed the interview to be systematic and comprehensive, but also afforded flexibility with important issues that came up during the interview. The purpose of the student interviews was to discuss: 1) their background in mathematics, 2) their motivation towards learning mathematics, and 3) their attitudes towards writing in mathematics. An interview question related to students'

backgrounds in mathematics was, “Have you ever participated in a writing assignment in a mathematics class before?” The interview questions related to students’ backgrounds in mathematics allowed students to express their previous experiences in mathematics, particularly any previous experiences with writing in mathematics. Interview questions related to students’ motivation towards learning mathematics focused on their general feelings of motivation as well as their perceived sense of competence, autonomy, and relatedness. For example, an interview question related to students’ general feelings of motivation was, “In this mathematics class, do you feel motivated to come to class, do your homework, and study for tests?” and an interview question related to students’ perceived sense of autonomy was, “Do you feel a sense of freedom in this mathematics class? Why or why not?” The interview question related to students’ motivation towards learning mathematics allowed students to express their feelings of motivation as well as their perceived sense of competence, autonomy, and relatedness. Interview questions related to students’ attitudes towards writing in mathematics were general so that students in both the HJ section and the Non-HJ section could provide answers. An interview question related to students’ attitudes towards writing in mathematics was, “What benefits do you think there might be to writing in mathematics?” These interview questions provided insight into students’ attitudes towards writing in mathematics. The students in the HJ section were asked an additional set of questions that pertained specifically to the homework journals. For example, “What did you like and dislike about the homework journals?” The purpose of these interview questions was to allow the students in the HJ section to express their perceptions of the homework journals. A complete list of all the student interview questions can be found in Appendix C.

### **Instructor Interview.**

During the last week in November 2011, the course instructor teaching both the HJ section and the Non-HJ section was interviewed using a general interview guide (Patton, 1990). Using a general interview guide allowed the interview to be systematic and comprehensive, but also afforded flexibility with important issues that came up during the interview. The purpose of the instructor interview was to discuss 1) her perceptions of writing in mathematics and 2) her suggestions about how to better implement homework journals in the future. Related to her perceptions of writing in mathematics, an example interview question was, “Do you think writing in mathematics helped your students gain a better understanding of mathematics? Why or why not?” Related to her suggestions about how to better implement homework journals in the future, an example interview question was, “What changes would you make to the homework journals and why?” A complete list of all the instructor interview questions can be found in Appendix D. These interview questions were developed from the informal interview conducted with the course instructor from a pilot study in July 2011 (see Appendix F).

### **Procedures**

Data collection took place over the course of the Fall 2011 semester. Each step in the procedure, in chronological order, is discussed in detail in the following section.

#### **Preliminary Meeting with Course Instructor**

Before the Fall 2011 semester began, I met with the Senior Instructor who would be teaching the two sections of Math 2015. During that meeting, I explained the details of the research project, to orient the instructor to the homework journaling project and communicate all of the instructor’s responsibilities. There were five primary responsibilities conveyed to the instructor:

- 1) *Meet with the researcher 3 times during the semester.* Once during the first month of classes and once during the second month of classes, the instructor will meet with the researcher to discuss the progress of the students and to discuss any issues or suggestions for the homework journaling project. The third and final meeting will be a “formal” interview with the researcher at the end of the semester to discuss the instructor’s perceptions of the homework journaling project in particular and writing in mathematics in general.
- 2) *Describe the homework journaling project in detail to their students.* During the first week of classes, the instructor should take about 10 minutes to describe the homework journaling project to their students. This includes describing the purpose of the homework journaling project as well as a description of how each homework journal is to be completed. Specifically, examine an example homework journal (provided by the researcher) to give students an idea of what is expected of them. Additionally, pass out bluebooks to each student (provided by the researcher) in which the students are to keep all of their homework journals.
- 3) *Collect homework journals.* The course instructor is expected to collect a minimum of 10 homework journals throughout the semester.<sup>1</sup> If the course instructor collects one homework journal per week, excluding exam weeks, then at least 10 homework journals will be collected throughout the semester.

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<sup>1</sup> This is based on feedback from a course instructor who was involved in a pilot study (see Appendix F). Because the pilot study took place over the summer, there were only six weeks of the course and therefore only six homework journals were collected. The course instructor believed that collecting more than six homework journals would be beneficial to the students because it would give them a chance to develop their writing.

- 4) *Respond to homework journals.* When students turn in homework journals, the course instructor is expected to respond to every student in some way. The course instructor may choose to ask students questions to clarify what they wrote, challenge students to extend their thinking about a particular concept, affirm the work that students did or the reflection they wrote, or encourage students to try another problem.
- 5) *Grade homework journals.* Each homework journal will be worth a set number of points. Although grades will be somewhat subjective, the course instructor should not give full credit to homework journals that do not clearly show all mathematical work or that do not contain thoughtful and insightful reflections.<sup>2</sup>

### **Distributing the Pre-Course Survey**

During the first week of classes of the Fall 2011 semester, I visited both sections of Math 2015 to distribute the pre-course survey. The students were told that participation in the survey was voluntary and would in no way influence their course grade. It took students approximately 15 minutes to complete the pre-course survey.

### **Meetings with Course Instructor**

Twice during the semester, I met with the course instructor for a coaching and reviewing session. The first coaching session took place during the first week in September 2011 and the second coaching session took place during the first week in October 2011. These coaching sessions were motivated by suggestions from the interview that took place with the course instructor from a pilot study (see Appendix F). The pilot study course instructor thought it

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<sup>2</sup> Based on feedback from the pilot study course instructor, the course instructor needs to be strict when grading homework journals, especially during the first few weeks of the semester because this sets precedence for the entire semester and builds expectations.

would be valuable to have instruction and encouragement from the researcher throughout the semester. One purpose of these coaching sessions was to assess the implementation of the homework journals. Another purpose was to reiterate the expectations that the course instructor should have of students concerning their homework journals. Clearly communicating expectations with students is one of the most important aspects of writing in mathematics (Pugalee, 2001; Pugalee, 2004). Coaching sessions also offered the course instructor the opportunity to ask questions of the researcher and to address any problems that came up. Overall, the purpose of these meetings was to ensure that the course instructor felt supported.

### **Distributing the Post-Course Survey**

During the last week of classes of the Fall 2011 semester, I visited both sections of Math 2015 to distribute the post-course survey. The students were told that participation in the survey was completely voluntary and would in no way influence their course grade. The post-course survey took the students approximately 15 minutes to complete.

## **Design and Data Analysis**

### **Quantitative Data Analysis**

A 2×2 repeated measures analysis of variance (ANOVA) was conducted to determine the effects of homework journaling on students' psychological needs for competence, autonomy, and relatedness as well as the effect of homework journaling on students' attitudes towards writing in mathematics and students' achievement in mathematics. The two factors were time (pre-course survey and post-course survey) and treatment (HJ section and Non-HJ section). A 2×2 repeated measures ANOVA design suits this study because of these two factors. However, there was no time factor associated with achievement because the students took one final exam at the end of the semester. Therefore, an independent samples t-test was conducted to determine if

there were any differences between sections in students' achievement in mathematics at the end of the semester.

### **Qualitative Data Analysis**

A phenomenological design (Rossman & Rallis, 2003) was implemented to explore and describe the relationship between homework journals and students' motivation towards learning mathematics. Using a general interview guide (Patton, 1990), student and instructor interviews were conducted. The interviews were videotaped and transcribed. The transcriptions were then coded in order to organize, analyze, and interpret the interview data. A constant comparative method was used because it is an iterative process that sorts and organizes the data while remaining focused on the relevant research questions (Anfara, Brown, & Mangione, 2002). The "process of bringing order, structure, and interpretation to the mass of collected data" from the interview was accomplished with three iterations of code mapping analysis (Marshall & Rossman, 1999). The first iteration of code mapping analysis was the development of initial codes. The initial codes were based on the research questions and previous studies about writing in mathematics. The second iteration of code mapping analysis was the development of pattern variables. This involved looking for patterns in the codes. The third iteration of code mapping analysis was the development of themes. This involved generating holistic themes from the patterns that relate to the research questions. The three iterations of code mapping analysis kept the research questions the central focus of the analysis (Anfara et al., 2002). By analyzing the interview data through code mapping, the qualitative data added richness to the descriptive analysis of both the students' and the course instructor's experiences with writing in mathematics, their attitudes towards writing in mathematics, and the relationship between writing in mathematics and motivation towards learning mathematics.

## **Chapter Four: Results.**

### **Introduction**

This chapter contains two sections. The first section is the quantitative results based on the 2×2 repeated measures ANOVA, conducted to statistically determine the effects of homework journaling on students' psychological needs for competence, autonomy, and relatedness as well as the effect of homework journaling on students' attitudes towards writing in mathematics and students' achievement in mathematics. The second section is the qualitative results based on the constant comparative analysis of the student interviews, focused on the relationship between the homework journals and students' psychological needs for competence, autonomy, and relatedness as well as students' attitudes towards writing in mathematics. In addition, general descriptions of the students interview participants' levels of competence, autonomy, relatedness, and attitudes towards writing can be found in Appendix E. While these general descriptions do not directly influence the purpose of the study, they do provide a vivid context to better understand the students' perspectives.

### **Quantitative Results**

The means and standard deviations related to the competence, autonomy, relatedness, and attitudes towards writing scale responses for the pre-course and post-course surveys are illustrated in Table 4.1. The scores on the pre-course and post-course surveys range from 1 to 6. The means and standard deviations for achievement are also presented in Table 4.1.

Table 4.1

*Descriptive Statistics*

	Pre-Course Survey	Post-Course Survey	Differences
	$\bar{X}$ (SD)	$\bar{X}$ (SD)	
<b>Competence</b>			
HJ Section	3.69 (1.15)	3.65 (1.32)	-0.04
Non-HJ Section	3.92 (1.28)	4.08 (1.23)	0.16
<b>Autonomy</b>			
HJ Section	3.67 (0.74)	3.98 (0.70)	0.31
Non-HJ Section	3.77 (0.87)	4.08 (0.77)	0.31
<b>Relatedness</b>			
HJ Section	4.32 (0.74)	4.50 (0.84)	0.18
Non-HJ Section	4.14 (0.70)	4.40 (0.79)	0.26
<b>Attitudes Toward Writing</b>			
HJ Section	3.29 (0.95)	2.98 (1.05)	-0.31
Non-HJ Section	3.13 (0.90)	3.00 (0.94)	-0.13
<b>Achievement</b>			
HJ Section	74.75 (16.80)		n/a
Non-HJ Section	76.25 (13.53)		n/a

*Note:* For the pre-course survey, the sample size for the HJ section was  $N = 54$  and the sample size for the Non-HJ section was  $N = 54$ . However, not all students filled out the back page of the survey meaning that for attitudes towards writing, the sample size for the HJ section was  $N = 42$  and the sample size for the Non-HJ section was  $N = 44$ . For the post-course survey, the sample size for the HJ section was  $N = 49$  and the sample size for the Non-HJ section was  $N = 47$ .

In order to test whether the change over time was different between the HJ section and the Non-HJ section, a two-way analysis of variance design with time (pre-course and post-course) as the repeated measure was conducted. Because two different sections of Math 2015 were involved, it is important to include both section and time in the model to test whether the rate of change in competence, autonomy, relatedness and attitudes towards writing between the pre-course and post-course surveys was substantially different for the different sections. Given that no statistically significant interaction effects are found, then it is possible to analyze and interpret the main effects in a straightforward manner. If significant interaction effects are found, it is important to first analyze these interactions before analyzing the main effects.

### **Competence**

On average, students in the HJ section were found to have a mid-level sense of competence (Pre-Course:  $\bar{X} = 3.69, SD = 1.15$ ; Post-Course:  $\bar{X} = 3.65, SD = 1.32$ ), while the students in the Non-HJ section had a slightly higher sense of competence (Pre-Course:  $\bar{X} = 3.92, SD = 1.28$ ; Post-Course:  $\bar{X} = 4.08, SD = 1.23$ ). Considering responses related to the competence scale, no significant interaction effects between section and time were found,  $F(1,106) = 0.1317, p = 0.7175$ , suggesting that the rate of change was similar across the two different sections. The effect of time was also not statistically significant for the HJ section,  $F(1,101) = 0.0378, p = 0.8463$ , or the Non-HJ section,  $F(1,99) = 0.3799, p = 0.5390$ , indicating that students' sense of competence did not change from the pre-course survey to the post-course survey for either section.

### **Autonomy**

On average, students in the HJ section were found to have a mid-level sense of autonomy on the pre-course survey ( $\bar{X} = 3.67, SD = 0.74$ ), which increased on the post-course survey

( $\bar{X} = 3.98, SD = 0.70$ ). Similarly, the students in the Non-HJ section also had a mid-level sense of autonomy on the pre-course survey ( $\bar{X} = 3.77, SD = 0.87$ ), which increased on the post-course survey ( $\bar{X} = 4.08, SD = 0.77$ ). Considering responses related to the autonomy scale, no significant interaction effects between section and time were found,  $F(1,106) = 0.1644, p = 0.6861$ , suggesting that the rate of change was similar across the two different sections. The effect of time was found to be statistically significant at a 0.05 level for the HJ section,  $F(1,101) = 4.9891, p = 0.0277$ . While there was also a change over time for the Non-HJ section, the difference was not found to be statistically significant at a 0.05 level,  $F(1,99) = 3.7096, p = 0.0570$ . Both results indicate that students' sense of autonomy did increase from the pre-course survey to the post-course survey for both sections, but there was no difference in the rate of change by section.

### **Relatedness**

On average, students in the HJ section were found to have a high level of relatedness (Pre-Course:  $\bar{X} = 4.32, SD = 0.74$ ; Post-Course:  $\bar{X} = 4.50, SD = 0.84$ ). The same is true for students in the Non-HJ section (Pre-Course:  $\bar{X} = 4.14, SD = 0.70$ ; Post-Course:  $\bar{X} = 4.40, SD = 0.79$ ). Considering responses related to the relatedness scale, no significant interaction effects between section and time were found,  $F(1,106) = 0.0356, p = 0.8508$ , suggesting that the rate of change was similar across the two different sections. The effect of time was also not statistically significant for the HJ section,  $F(1,101) = 1.2868, p = 0.2593$ , or the Non-HJ section,  $F(1,99) = 3.1645, p = 0.0783$ , indicating that students' sense of relatedness did not change from the pre-course survey to the post-course survey for either section.

### Attitudes Towards writing

On average, given that we consider 3.50 to be neutral on a scale of one to six, students in the HJ section were found to have a relatively neutral attitude towards writing on the pre-course survey ( $\bar{X} = 3.29, SD = 0.95$ ), which decreased on the post-course survey ( $\bar{X} = 2.98, SD = 1.05$ ). Similarly, the students in the Non-HJ section also had a relatively neutral attitude towards writing on the pre-course survey ( $\bar{X} = 3.13, SD = 0.90$ ), which decreased on the post-course survey ( $\bar{X} = 3.00, SD = 0.94$ ). Considering responses related to the attitudes towards writing scale, no significant interaction effects between section and time were found,  $F(1,94) = 0.5693, p = 0.4527$ , suggesting that the rate of change was similar across the two different sections. The effect of time was also not statistically significant for the HJ section,  $F(1,89) = 2.1894, p = 0.1425$ , or the Non-HJ section,  $F(1,89) = 0.5068, p = 0.4784$ , indicating that students' attitudes towards writing did not change from the pre-course survey to the post-course survey for either section.

### Achievement

Students in both sections took the same comprehensive final exam; the exam consisted of mathematical content from the entire semester and was graded on scale from 0 to 100. On average, students in the HJ section and the Non-HJ section achieved middle-of-the-road grades on the final exam (HJ Section:  $\bar{X} = 74.75, SD = 16.80$ ; Non-HJ Section:  $\bar{X} = 76.25, SD = 13.53$ ). Because there was no time factor associated with achievement (the students took one final exam at the end of the semester) an independent samples t-test was conducted. Results of the t-test show that the final exam scores for the HJ section and the Non-HJ section were essentially the same ( $t(102) = 0.50, p > 0.05$ ). However, we cannot attribute any significance

to these results because there was no pre-course and post-course data; achievement data was only collected at the end of the semester.

### **Qualitative Results**

The qualitative results are based on the constant comparative analysis of the student interviews. The data and analysis help describe the relationship between the homework journals and students' psychological needs for competence, autonomy, and relatedness as well as students' attitudes towards writing in mathematics. Therefore, this section will have four major components, the relationship between homework journals and: 1) students' competence, 2) students' autonomy, 3) students' relatedness, and 4) students' attitudes towards writing in mathematics.

#### **Students' Competence**

The homework journal assignment description (Appendix B) instructs students to choose a problem from their assigned set of homework problems for their homework journal that interested them or challenged them. However, the course instructor allowed students to choose any problem they wanted. Based on the interview data, students chose a problem in one of two ways. Students either chose a problem that was personally easy, simple, and quick or they chose a problem that was personally challenging, interesting, or difficult. It is important to qualify the difficulty level as being personally easy or personally challenging. What may be challenging to one student may not be challenging to another student. Descriptive examples of students who chose easy problems for their homework journals and students who chose difficult problems for their homework journals will be provided in the following sections.

**Personally easy problems.**

Ann had very little confidence in her ability to learn the content in her current mathematics course. About her confidence in her own ability to learn the mathematics in her course, she said, “I’m not confident, not even a little bit!” She also did not believe that the homework journals helped her gain any confidence. She commented: “I know that [our instructor] said you should always do problems that you had a little trouble with, but I normally do the ones that I’m comfortable with and explain what I did. I’m not sure it helps me realize how to do problems I don’t know.” Ann typically chose problems from her assigned set of homework problems that were very easy to her; she did not choose problems that were personally difficult or challenging. In fact, she specifically said that she did not like to choose the difficult mathematical problems because she was “not sure [she] could put into words what [she] did and half the time [she] would probably do them wrong.” Ann was not confident in her ability to succeed in her current mathematics course and so she focused on problems that were easy to solve because they were the only problems she was confident she could solve. Because Ann was only solving problems that she was “comfortable with” and not problems that were personally challenging, she was not having what Bandura (1977) refers to as personal mastery experiences with the homework journals.

Melissa, on the other hand, was very confident in her ability to succeed in her current mathematics course. Even though she claimed that learning in mathematics does not come easily to her, she knows that in previous mathematics courses she has worked hard and done well. She described her confidence in this way, “I have a pretty high confidence level. Just because of the math classes I’ve taken in the past, I’ve done pretty well.” She also compared herself to her classmates and explained that on the last test, her score was well above the class average. She

said, “I guess I’m doing well, comparatively!” Melissa’s confidence is based on her success in previous mathematics courses and in her current mathematics course. Because of her confidence, Melissa typically breezed through her homework journal assignment each week. She candidly admitted that her mathematics course was on the “back burner” for her; Math 2015 was not her top priority. When she completed her homework journals each week, she typically chose a simple problem to solve from her assigned set of homework problems and she typically answered the prompt related to her success. This prompt asked, “How successful do you feel you were in solving this problem? Why? Do you feel you would be successful in solving a similar problem? Why or why not?” The purpose of this prompt was to help make students aware of the personal mastery experiences they were having as a result of the homework journals. Melissa commented, “It’s funny, when I do the homework journals the question that I end up answering in the homework journals is, ‘Do you feel successful doing this problem?’ And I guess I feel like just answering that question in the homework journals hasn’t really helped me [feel more confident].” Melissa chose easy problems to solve and she chose this prompt because it was the easiest one for her to answer. Melissa was confident in her ability to succeed in her current mathematics course, but the course was a very low priority to her and so she focused on problems that were easy to solve because they would take the least amount of time. Even though she was writing about how successful she did or did not feel, based on her response, this did not help meet her need for competence.

**Personally difficult problems.**

Sophie was very motivated and very positive. She had a high level of confidence in her ability to succeed in her current mathematics course because much of the mathematical content of the course was review for her and because of the pace of the course: “At the beginning, a lot

of [the course content] was kind of review and now it's not so much, but it's still, the pace that we go, because we do one lesson a day, and it's just kind of makes it easier because you can master that lesson." Sophie also solved personally challenging problems in her homework journals and these experiences did seem to help meet her need for competence. Sophie said the homework journals do help her feel more confidence in her mathematical learning: "I feel like it helps if you don't understand a problem or something you can write it out and that sort of helps. Because that's what questions I choose usually for the blue books are the ones that I didn't really know how to do as much as the other ones. That kind of helps to write it out, write the steps out." Sophie was confident in her ability to succeed in her current mathematics course and typically chose problems from her assigned set of homework problems that were personally challenging; the problems she chose were ones that she did not originally feel comfortable with. Based on her responses, the experiences of writing out the steps of challenging problems and thinking deeper about challenging problems, authentic personal mastery experiences (Bandura, 1977), did help meet Sophie's need for competence.

Chris was also very confident in his ability to learn the material in his mathematics course. He claimed, "Throughout all my math classes, not knowing how to do stuff and then learning and practicing and stuff, I've done that. And I've continued to do that throughout this course. I'm not worried about it. Like, I know I can learn everything and I can excel at it." His confidence is based on his previous experiences; he has worked hard in mathematics classes in the past and he has done well. He feels the same way about his current mathematics course. The prompt related to success was one of the prompts that Chris answered most often in his homework journals: "How successful do you feel you were in solving this problem? Why? Do you feel you would be successful in solving a similar problem? Why or why not?" Chris

explained that “throughout the course, [he has] gained an appreciation of the homework journals” because they can help him review difficult material and concepts because the problems that Chris typically chose to solve in his homework journals were problems that were personally challenging. He also explained that the homework journals do help improve his confidence level “to a certain extent.” He elaborated by saying, “In my answers, a lot of my answers, I’ll be like, ‘Yes, I was successful in completing this.’ So there’s that sense of accomplishment that will translate into being confident and know how to do that particular type of problem.” Chris was confident in his ability to learn the material in this course and he chose personally challenging problems to solve in his homework journals. Chris is describing the effect of personal mastery experiences (Bandura, 1977); successfully solving difficult problems, and being aware of that success, does seem to help meet his need for competence.

### **Students’ Autonomy**

The homework journals were specifically designed to offer students meaningful choices, because meaningful choices are an important part of fostering students’ sense of autonomy (Katz & Assor, 2007). However, based on their interview responses, only some students were aware of the choices they were free to make concerning the homework journals, while others were not. And what was a meaningful choice to one student was not necessarily a meaningful choice to another student. Three different classifications of students emerged from the interview data: 1) students who did not recognize the choices they were free to make, 2) students who did recognize the choices they were free to make, but did not think those choices were meaningful, or 3) students who did recognize the choices they were free to make and those choices were meaningful. Table 4.2 concisely outlines and organizes these three different categories and

shows which students fall into each category. Descriptive examples of at least one student from each category will be provided in order to serve as an illustration.

Table 4.2

*Three Categories Concerning Students' Sense of Autonomy*

Was the student aware of the choices they could make?	Were the choices meaningful to the student?	Students:
No	--	Mia Ann Kelly
Yes	No	Melissa Chris Adele
Yes	Yes	Lydia Sophie Joe Nate

**Unaware of choices.**

Kelly provides an example of the students who do not even recognize that they are free to make choices concerning the homework journals. When asked how she would describe a homework journal to a friend, Kelly said, "Um, it's basically where you solve one problem and explain how you solved it." Kelly's response does not include any words associated with choice, such as "choose," "pick," "decide," or "determine." She does not experience choice with the homework journals – neither the choice of which problem to solve nor the choice of which prompt to answer. For Kelly, you solve one problem, with no indication of which problem you solve, and you explain how you solved it, this is the only option for reflection. When asked whether or not she thought the homework journals helped her gain a sense of freedom, she responded, "I don't know. It kind of feels like just another homework assignment, I guess. So

I'm not sure that I feel freedom." Kelly's idea of the homework journals feeling like a "mandatory" assignment was common among these students who did not think that the homework journals helped them feel a sense of freedom in the course. When completing the homework journal each week, Kelly did not experience choice in any way.

**Choices were not meaningful.**

Chris represents the students who did recognize the choices they could make concerning the homework journals, but those choices were not meaningful to them. Chris was definitely aware of the choices he could make and even took advantage of those choices, but that made little difference; the choices were not meaningful to him. When asked how he would describe a homework journal to a friend, Chris answered, "From the homework assignment, from the list of problems, you pick one that you want to either work on more or one that you feel confident about doing" and solve this problem. He went on to say, "There's a number of questions you can answer. It's more like, how successful you were and how this relates to other mathematics and how this makes you feel about mathematics. Those are my three go-to's." Chris sees that there are choices built into the homework journals. There are choices regarding which problem to solve and which prompt to answer. In his response, he lists three different reflection prompts (out of the six total reflection prompts) and indicates that he usually chooses one of those three reflection prompts. This means that Chris is aware of the choices he can make and he takes advantage of those choices. However, Chris does not think that the homework journals help him feel any more of a sense of freedom. He said, "With the journals, I feel like we kind of gotta do a step-by-step process with that. Because we gotta write the problem out and then solve it and then explain it." Overall, Chris spoke very positively about the homework journals. He said, "I'm glad I've had the chance to do the journals just to try something new." And although Chris

believes that homework journals are a great assignment, there is no freedom with them. He says that he “feels forced” to do the homework journals. Unfortunately, the choices Chris makes are not meaningful.

### **Choices were meaningful.**

Sophie is one of the four students who did recognize the choices they could make and who did think those choices were meaningful. When asked to describe a homework journal to a friend, Sophie explained, “We pick one problem from our set of problems; we can pick any one that we want. And then there’s, like, different questions that we can choose from to write about, like, whether we want to write down the steps that we used or why we like doing the problem or what we found difficult. It’s kind of open-ended.” It is clear that Sophie experiences choice with the homework journals. She is clear about the fact that she has a choice for both which problem to solve and which prompt to answer. When asked about whether or not the homework journals help her feel more of a sense of freedom, she quickly affirmed that the homework journals do help her feel more of a sense of freedom. She said, “Yeah, because we can choose which [problem] we want to write about and, like, why we want to write about it, and what we want to say about it. And, like, each week we can change that.” Sophie recognizes the choices she has available to her, she takes advantage of those choices, and they are meaningful to her. Because of that, the homework journals do provide an opportunity to help meet her need for autonomy.

I specifically examined just one student from each of the three categories: 1) students who did not recognize the choices they were free to make, 2) students who did recognize the choices they were free to make, but did not think those choices were meaningful, and 3) students who did recognize the choices they were free to make and those choices were meaningful (see

Table 4.2). These three students provide a clarifying example of each category and are representative of all of the students within each category.

### **Students' Relatedness**

The homework journals required students to write reflections on mathematics and/or mathematics learning. In response, the instructor was to give written feedback. This provides a form of communication between students and the instructor. Based on the interview data, students either wrote thoughtful and meaningful reflections or wrote rushed and sloppy reflections. Some students wrote thoughtful and meaningful responses for their homework journals while others rushed through their homework journals, completing the assignment hastily and sloppily. In her interview, the course instructor described how much she liked reading homework journals that were thoughtful, where she could tell that the students took their time to intentionally solve and reflect, but she did not like reading homework journals that had sloppy and rushed responses. She said, "I really didn't do a lot [of feedback]. But if they gave me a lot, I would give them a lot. I do think that I should have taken more time with it." In the instructor's own words, if the students gave her a lot, then she would give them a lot. Based on her previous comments, "giving a lot" does not refer to the quantity of the homework journal, particularly the reflection, but the quality. This was also consistent throughout the interviews; the feedback given by the instructor mirrored the response given by the students. Adele, Mia, Lydia, and Chris serve as illustrations of this claim. Adele and Mia typically wrote rushed and sloppy responses while Lydia and Chris typically wrote thoughtful and meaningful responses.

#### **Rushed and sloppy responses.**

Throughout the interview, Adele assertively described how little she thought of the homework journals. The homework journals were not important to her and she had little

motivation to complete the assignment each week. She described completing her homework journals in this way: “Usually the homework journals were the last thing I would do. It was usually one of the last things, like, less than thirty minutes before class I’d do my homework journal.” Her solutions and her reflections were sloppy and rushed. The feedback she received was also rushed, with very little detail. Adele commented on this feedback: “She did give feedback, but usually it was just a grade or if I did make a mistake she would tell me what my mistake was. But if I got it, she was just like ‘Two out of two’ or ‘Good job’. It was just short and to the point.” Adele’s effort to complete her homework journal was minimal; her homework journal was the last thing on her mind, simply an assignment to complete before class. The feedback she received on her homework journals was also minimal; it was “short and to the point” to use her own words. Rushed and sloppy responses from Adele got short and to the point feedback from her instructor. Adele did not believe the homework journals helped her feel more related to her instructor. When asked about this directly, she said, “I don’t think so.”

Mia is another student who did not believe the homework journals helped her feel more related to her instructor. Mia expressed dissatisfaction with the homework journals in general. About the homework journals, she exclaimed, “I don’t like them. And that’s just my first gut reaction.” She also said that she had very little motivation to complete the homework journal assignment each week: “It’s always the last thing to do and always easy to forget.” Based on her comments, Mia’s responses on the homework journals were typically half-hearted and lacking detail. She described the feedback she received in the same way. She explained, “[My instructor] does leave comments back on the homework journals. They are comments on what I wrote, not just the problem. But I tend to find the test feedback a little bit more tangible.”

Although Mia does receive feedback on the homework journals, that feedback is not meaningful to her because it is short and intangible.

**Thoughtful and meaningful responses.**

Lydia typically wrote long, detailed responses for her homework journals. Lydia was highly motivated, highly confident, and talked very positively about the homework journals. The feedback she received mirrored her effort and enthusiasm. She described the type of feedback she received in detail: “[My instructor] will be like, ‘Well, this is really good, what about this and this and this?’ Or if there’s something I said I didn’t understand, she’ll try and write a note to try and help me or explain it.” The feedback she received was both supportive and encouraging; she was challenged to think about problems in different ways and she was praised for the responses she gave. Lydia believed the homework journals helped her feel more related to her instructor. She explained, “Yeah, I definitely think it helps that she writes back instead of just ‘Check, you did it.’ She actually reads what you say. I know she does. So I definitely think that’s helpful because it means that she really cares and she wants you to understand. She’s putting in this effort to write back and give you feedback.” The feedback that Lydia received further convinced her of her instructor’s care and concern. Based on her interview responses, this did seem to strengthen her sense of relatedness.

Similarly, Chris also believed that the homework journals helped him feel more related to his instructor. Chris described that at first he did not like the homework journal assignment, but “throughout the course, [he] gained an appreciation because they help [him] remember the steps and the processes of how to do the problems.” Because Chris sees value in the homework journal assignment, he typically puts a great deal of effort into his responses. The feedback he receives mirrors that effort. Chris said, “I really like the feedback from our instructor [in the

homework journals].” He went on to explain that with the homework journals, “[our instructor] actually takes the time to read them and look at what you wrote down and she’ll comment on it and give you feedback, which I think is really helpful and it just, like, really shows how much she cares about how you’re doing in this course.” Chris wrote thoughtful and meaningful responses in his homework journals and he received thoughtful and meaningful feedback from his instructor. For Chris, the feedback he received on his homework journals was another instance that demonstrated the level of care he felt from his instructor.

### **Students’ Attitudes Towards Writing in Mathematics**

The relationship between homework journals and students’ attitudes towards writing in mathematics is based on students’ descriptions of their likes and dislikes of the homework journals as well as what they perceive to be the benefits of the homework journals. Both the likes and dislikes of the homework journals and the perceived benefits of the homework journals indicate and describe students’ attitudes towards writing in mathematics.

#### **What the students liked.**

The students described a number of different aspects of the homework journals that they liked. Students liked 1) the consistency and the format of the homework journals, 2) the extra graded provided by the homework journals, 3) the feedback they received on the homework journals, 4) the way the homework journals made them slow down and think about the problems they were solving, 5) the opportunity to communicate with the instructor, 6) the way the homework journals help them make connections to other mathematics, and 7) the ability to make choices concerning the homework journals. I’ll elaborate on each of these “likes” and describe what the students meant.

***Consistency and format.***

Three students, Mia, Sophie, and Kelly, liked the consistency of the homework journals. Mia said, “I think my favorite thing about [the homework journals] was the consistency that they were in this one little book. That all the problems were in the same book as opposed to being spread out.” Each student received a bluebook at the beginning of the semester and was instructed to keep all of their homework journals from each week in this one bluebook. Mia liked the consistency of the homework journals in the sense that they were all kept together in one notebook throughout the course of the semester. Sophie also liked the consistency of the homework journals. She liked that it was something that she did every week and that all of the homework journals were kept together in a single location. She said, “Since it’s a consistent thing every week, that kind of helps, because it’s just a part of your routine and it just kind of helps.” Kelly liked the consistency of the homework journals, but in a different sense of the word. Kelly described a previous experience with writing in her high school calculus course. She disliked this experience because of the way her calculus teacher graded the writing assignment: “When he graded them, he would tell us one thing and then when he graded it, he would be extremely harsh and grade for things he hadn’t told us. So it kind of frustrated me more than anything.” When Kelly was asked what she liked about the homework journals, she mentioned the consistency between the expectations her instructor communicated and the way her instructor graded the homework journals. Kelly believed that the communicated expectations and the grading practices matched. She liked that her instructor clearly communicated expectations and followed through with those expectations.

*Extra grade contributing to overall course grade.*

Three students, Kelly, Melissa, and Adele, liked having an opportunity to earn an extra grade in the course; the grades they received on their homework journals helped them pull up their homework grade and influence their overall course grade. Kelly, for example, said, “It helps with your homework grade a little, so that’s always a plus.” Melissa, who was motivated by her desire to attend graduate school, valued doing well in her courses. She was trying to earn high grades in her courses in order to prepare for graduate school. She liked the homework journals because she didn’t have to work very hard in order to get full credit on the homework journals. She talked about how she thought she should have worked harder and if she had the homework journals would have been more meaningful to her, but she really just wanted to do whatever she had to in order to earn the maximum number of points on the assignment. Melissa described her work on the homework journals in this way, “I usually just sort of scribble stuff in there. Mostly because that’s all you have to do to get the points.” She went on to say, “I like that they were additional points, that if you put just a little bit in, you would get those points, so I liked that it contributed to my homework grade and wasn’t too strenuous.” Adele also liked earning “easy” points in the course that could influence her overall grade. She cited the homework journals as an example of “easy” points and specifically said, “I liked the [homework journal] grade.” These three students, Kelly, Melissa, and Adele were all vocal about the fact that they appreciated extra grades in the course that contributed to their overall grade; the homework journals gave them an opportunity to boost their homework grade and also their overall course grade.

***Feedback from the instructor.***

Two students, Lydia and Chris, liked the feedback they received on the homework journals. They both felt that this feedback was an important and meaningful component of the homework journals. Lydia said, “I did like that [my instructor] took it into account. Like, she’d look at it and be like, ‘Okay, they actually did it.’ and it wasn’t necessarily for correctness, it was ‘Did you actually try?’ I really like that she always wrote feedback.” Lydia went on to describe the feedback she received on her homework journals and how helpful that feedback was, both for gaining a better understanding of the mathematical content and for preparing for tests. Chris also really appreciated the feedback he received on his homework journals. He said, “I like the personal feedback from our instructor, I really like that.” The feedback they received from their instructor was very important to both Lydia and Chris and was one aspect of the homework journals that they liked the most.

***Slowing down and thinking deeply.***

Both Ann and Chris liked that the homework journals helped them slow down and really think about the mathematics, which helped them gain a better understanding of the mathematics. Throughout her responses and especially her response to what she liked best about the homework journals, Ann commented on how they helped her slow down and think about the mathematics. She said, “I guess I like that it makes you slow down because I definitely just try to get it all done. It makes you stop and think. I don’t know.” Ann values deeper thinking and the homework journals help her reach that goal when it comes to her homework; instead of racing through her homework assignment in order to complete the assignment and move on to the next task on her agenda, the homework journals help her slow down and think through the process she is using. This helps Ann better understand the mathematical content. Similarly, Chris also liked

that the homework journals helped him better understand the mathematical content. He said he liked “how writing stuff down will make [him] be able to retain that information and know the process.” Later in the interview, Chris described how writing about the problems he was solving helped him understand the process when he was originally going through that process, and it helped him understand the process when he was looking back on it to study for tests. For both Ann and Chris, writing about the mathematics that they are doing helps them slow down, think about what they are doing, understanding what they are doing, and remember what they are doing.

***Communicating with the instructor.***

Melissa reported that she liked being able to communicate with her instructor; therefore, she liked the homework journals because she was able to use the homework journals to communicate with her instructor. Melissa liked that if there was something she felt like she needed to communicate with her instructor, she could do that through the homework journal. She said, “I like the fact that I knew she was reading them and if there were something that I wanted to bring up to her, I could do it through the homework journal.” A few examples she gave were asking a question or telling the instructor about something she didn’t understand from class that week. Melissa liked that she could use the homework journals as a way to communicate with her instructor.

***Making connections to other mathematics.***

Joe liked that he could use the homework journals to make connections to other mathematics. He says, “I like how we can relate it to other math problems and stuff like that. I feel like it just sort of brings everything together instead of everything just being step by step.” Joe is referencing one of the specific reflection prompts that the students can choose to answer in

their homework journals. The prompt is: “In what ways does this problem relate to other mathematics that you know or to other problems that may come up in your other courses or eventually in your field of specialization?” Joe likes this prompt because of the way it helps him think about the big picture of mathematics, not just the step-by-step process he used to solve a particular problem, but the way that problem relates to other mathematics and to other applications.

### ***Making choices.***

Adele liked that she was able to choose which prompt she answered each week. She said, “I liked the choice to write about whatever you wanted.” Having the freedom to choose which prompt to answer was something that Adele liked about the homework journals.

The students from the HJ section shared what they liked about the homework journals. The students liked the consistency and the format of the homework journals, they liked the way the homework journals helped them slow down and better understand the mathematical content, they liked the feedback they receive on the homework journals, they liked having an extra grade in the course, they liked being able to communicate with their instructor through the homework journals, they liked using the homework journals to make connections with other mathematical ideas, and they liked the choices they were able to make with the homework journals.

### **What the students disliked.**

The students described a number of different aspects of the homework journals that they disliked. Students disliked 1) the way that the expectations about the homework journals were communicated to them and how their homework journals were graded, 2) the open-ended nature of the homework journals, 3) the extra time it took them to complete the homework journals in addition to the time it took them to complete their “regular” homework assignment, 4) the fact

that the homework journals were unnecessary, and 5) the restrictions of the format of the homework journals. I'll elaborate on each of these "dislikes" and describe what the students meant.

***Communicated expectations from the instructor.***

Mia and Chris disliked the way expectations were communicated and the way their instructor graded their homework journals. Mia felt very strongly about the fact that her instructor had not communicated expectations because, more than halfway through the semester, she still did not understand what was expected of her on the homework journals. She lamented, "I didn't like all the points I missed on them! I feel like that was where I lost most of my homework points because I wasn't able to fulfill expectations in the writing section." Mia is very clear that she is confused about what she is supposed to be writing about. She even says, "I still to this day, don't even know what I'm supposed to be writing about! I mean, we do have those questions and I try to answer them, but it's never good enough it seems like." Mia struggles because even though she knows what the different prompts are, she does not know what her responses should look like. She does not know what the expectations are and therefore is frustrated because she feels like she cannot fulfill the expectations. Chris also expresses frustrations about his grades on the homework journals, but while Mia's frustrations are connected to the reflection prompts, Chris' frustrations are connected to the mathematics problems. He said, "I guess a negative would be the grading, thinking that I did this perfectly and then I didn't change a limit or something and then just getting point-five taken off the homework assignment. It's like 'Dang it!' I guess that's kind of a negative thing." Chris does not indicate that he loses points from his reflection prompts; instead he is losing points from the

mathematics problems, from doing mathematics. Both Mia and Chris disliked the way the homework journals were graded, but for different reasons.

***Not enough structure or guidance.***

Adele and Melissa disliked the open-ended nature of the homework journals, they did not think there was enough structure or guidance to the homework journals. Adele did not like choosing which problem she wanted to solve. She did not like this choice because she felt like she was given too much freedom. With the freedom she was given, she would choose the easiest problem from the homework assignment. Because of this, the homework journals were typically meaningless to her. She explained, “I didn’t like picking any problem. Because with me, I always chose the easiest problem that I knew I could finish. Maybe if I had to write about a difficult problem, it would have made a lot more sense.” Adele could choose whatever problem she wanted to solve for her homework journal; she could choose any problem from her homework assignment to engage in doing mathematics and she usually choose the most basic problem. While Adele certainly could have chosen a more difficult problem, which would have made the homework journals much more meaningful to her, she did not. Therefore she disliked the freedom to choose any problem she wanted to solve for her homework journal. Melissa also did not like the freedom she was given, but instead of disliking the freedom to choose which problem to solve, she disliked the freedom to choose which reflection prompt to answer. She thought there were too many options for the reflection prompts and disliked having so many options. She explained, “I didn’t like that it is kind of open-ended. I’m someone who likes to have very specific directions. So I’ll be like, ‘What do you want me to write? I’ll write what you want me to write if you just tell me what you want me to write!’” Melissa wanted to be given specific instructions about what she was expected to write in her homework journal; she

did not like multiple choices for the reflection prompt. Both Adele and Melissa disliked the freedom of choices they were given with the homework journal, but for different reasons.

***Extra time required to complete homework journals.***

Both Kelly and Joe disliked the additional work required of them associated with the homework journals. For both these students, they knew that if they were just expected to complete the assigned homework problems, instead of the assigned homework problems as well as their homework journal, they would spend less time doing homework. Kelly disliked that the homework journals added another item to her to-do list: "It's just kind of one more thing that you have to remember to do." Joe disliked having to do extra work: "I guess it's extra work that you have to do and I don't like extra work. I guess that's the only thing I dislike." In general, both Kelly and Joe liked the homework journals. But as much as they liked the assignment in general, they disliked the extra work it required of them.

***Unnecessary nature of the homework journals.***

What Ann disliked about the homework journals was when she struggled to find something to write about the problem she chose to solve. She said that sometimes it was easy to write about the problem she chose to solve; this was especially true if the problem was complicated or involved multiple parts. However, she struggled to write about the problem she chose to solve if the problem was simple and straightforward. She said that some weeks this was unavoidable. She elaborated by saying, "Some of our homework assignments are just straightforward math and not using specific rules or anything, it's kind of plug-and-chug kind of deal. And it just feels like when you write about something like that it's like, 'I'm sorry, I didn't do anything other than this, that's all I can write!' It just seems like it's not worth it." Ann only thought there was value in writing about problems that were difficult or challenging, problems

that required multiple steps or that allowed her to choose between different methods in order to solve the problem. Most weeks, Ann enjoyed the homework journal assignment. It was only during the weeks that the assignment contained only “straightforward math” that she disliked having to write about the problem she chose to solve.

***Format restrictions.***

The aspect of homework journals that Lydia disliked was the format. She disliked being confined to solving the problem on the left-hand side of the paper and reflecting on the problem on the right-hand side of the paper. Lydia felt as though the homework journals limited her space: “I always felt like I was limited in space because you had to keep it to two sides and sometimes I had a lot more to write about the problem. I’d rather make my own space, like do the problem and then write about it.” Lydia suggests simply solving the problem and then reflecting on the problem, presenting her work sequentially instead of side-by-side. The format of the homework journals was the only aspect of the assignment that Lydia disliked.

***No dislikes.***

Only one student, Sophie, could not think of a single thing that she disliked about the homework journals. She explained, “There’s really not anything I dislike doing, there’s not any problem in doing it.” Sophie is the only student who had nothing to complain about or express dislike of concerning the homework journals.

The students from the HJ section shared what they disliked about the homework journals. The students disliked the way the homework journals were graded, they disliked extra work the homework journal required of them, they disliked the format of the homework journals, they disliked the choices they were able to make with the homework journals, and they disliked when the homework journals felt unnecessary.

**Connections between what the students liked and disliked.**

Interestingly enough some of the likes and the dislikes shared by the students are the same. For example, Mia liked the format of the homework journals. She liked being able to keep all her homework journals in a single location and she liked that each homework journal fit neatly onto one page. On the other hand, Lydia disliked the format of the homework journals. She disliked being confined to one page and keeping her work in a two-column format. Also, Kelly liked the consistency between the expectations her instructor communicated and the way her instructor graded the homework journals. She felt like the expectations were clearly communicated and she knew what was expected of her. However, Mia felt very strongly that her instructor had not communicated expectations and that she did not even know what was expected of her on the homework journals. As one final example, Kelly, Melissa, and Adele all liked the extra grade that the homework journals gave them. This extra grade helped boost their homework grade, which, in turn, helped influence their overall course grade. At the same time, both Kelly and Joe did not like having to do extra work in the course. Without extra work, there can be no extra grade. While there are many aspects of the homework journals that these students liked and disliked, it seems to come down to a matter of preference. One student may like a particular aspect of the homework journals and another student may dislike that same aspect.

**Perceived benefits of homework journals.**

Another aspect of students' attitudes towards writing that was influenced by the homework journals was the benefits students saw to the homework journals. The students in the HJ section mentioned four perceived benefits of homework journals: 1) more powerful learning, 2) gaining both procedural and conceptual knowledge, 3) building relationships with the course

instructor, and 4) boosting their overall course grade. I'll discuss each of the benefits that were mentioned by the students.

***More powerful learning.***

Ann, Joe, Melissa and Sophie referred to more powerful learning as one of the most important benefits of the homework journals. Each of these students phrased the idea of more powerful learning in different ways, but the basic idea was developing a stronger understanding of the mathematical concepts.

Ann, who previously described herself as “more towards writing than towards math,” believed that writing would help students like her learn the mathematical concepts better. Ann believed writing would be helpful for her and for other students who were also “more towards writing” in their mathematical learning. Ann’s thoughts about the benefits of the homework journals were very vague; for example, what did she mean by “help her learn?” However, as vague as her comments were, she did speak with conviction about the benefits of the homework journals. Melissa was very similar. She was another student who claimed to be “more of an English-based person instead of a math-based person” and for students like her, writing in the homework journals helps them understand the mathematics better. She said, “I think there’s a lot of benefits to writing! It makes you understand it better.” Again, it is not clear from what Melissa said or from the context of the interview what she meant by “understand it better” but she did believe that this was an important benefit of the homework journals.

While Ann and Melissa were vague about the benefits of the homework journals, Sophie gave clear and precise reasons that she thought the homework journals were helpful. She explained why the homework journals helped her understand the mathematical concepts in a deeper way: “Sometimes I feel like it helps if you don’t understand a problem or something you

can write it out and that sort of helps. I guess it just makes you think about it more. By writing each step out, it's not just numbers that you're looking at." Sophie believes that by writing out the steps and by describing those steps, students are able to better understand a particular problem or concept. Later in the interview, she elaborated, "[Homework journals] do help you understand because you have to think about it more, instead of just doing the examples. Because when we're learning the lesson, that's what we do is examples and writing make you kind of think about it more and break it down." Writing in the homework journals forces Sophie to break down the mathematical concepts and make sure that she understands the conceptual ideas behind the example or problem.

Joe's thoughts about the benefits of the homework journals are slightly different from the last three students. The last three students described the benefits of the homework journals as understanding the mathematical concepts *deeper*. Joe, on the other hand, described the benefits of the homework journals as understanding the mathematical concepts in a *fuller* sense. He said, "I can see the benefits of it. I think once you do write [in the homework journal] you can just sort of, get what's in your head out on the paper, you know, I just think it opens up more." Joe believes the one benefit of the homework journals is thinking more concretely about the mathematical ideas, getting the ideas from his head onto his paper. Joe is the student who routinely made connections between mathematical ideas in his homework journals and he said that one of the most important benefits is "opening up more" mathematical ideas and making connections between mathematical ideas. Joe believes that the homework journals allow him and his classmates to think about the problems they are solving and the concepts they are learning in a fuller and richer way.

Ann, Melissa, Sophie, and Joe all thought that one of the most important benefits of writing in mathematics, especially the homework journals that they completed throughout this semester, was that it helped students gain a stronger understanding of mathematical concepts. Whether that understanding was deeper or fuller, these students valued being able to think deeply and engage in the mathematics in a more meaningful way.

***Gaining procedural and conceptual knowledge.***

Three students, Chris, Joe, and Kelly, believed that an important benefit of homework journals was gaining a better understanding of the steps necessary to solve mathematical problems. Two students, Lydia and Mia, believed that an important benefit of homework journals was gaining a better understanding of the mathematical concepts, the reasons why the steps necessary to solve mathematical problems work the way they do. Gaining both procedural and conceptual mathematical knowledge are benefits of the homework journals.

Chris says that at first he did not particularly like the homework journals. But as the course went on, he said, “I’ve gained an appreciation for them and looking back and stuff, like re-reading it after I’ve written it down, really helps me to remember the steps and the process of how to do the problems.” By looking back on experiences and examples of explicitly doing mathematics by solving problems and by reflecting on those problems, Chris is able to remember how to solve similar problems; the homework journals build Chris’ procedural knowledge.

Joe described a second benefit of the homework journals. He explained, “It’s good to look back on it, like before an exam. You can look back and, yeah, see that this is how you did it and I felt confident then, so I can get it again. That’s probably the best benefit.” Not just solving problems and reflecting on those problems, but also looking back on those problems in his homework journals, helps Joe remember the steps and the processes he used to solve problems

throughout the semester. This helps him review for tests and gives him confidence that he will know how to do those same processes on the test. Again, the homework journals help build Joe's procedural knowledge.

Similar to what Chris and Joe described, Kelly also believed that homework journals helped build her procedural knowledge and this was especially evident as she was studying for tests. She described how she studied for the most recent test in the course. She said, "It really helps that you write the steps out. I guess going back and looking at other problems that are similar could help." Writing out the steps, the procedures, of problems in her homework journal gave her a collection of problems to study that would be similar to problems on the test. Kelly describes one the benefits of homework journals as being able to look back on previous work to help her understand the steps when studying for tests. Looking back on the homework journals help build her procedural knowledge.

Throughout her interview, Lydia continually returned to the idea of figuring out why mathematical procedures actually work. To her, this was the main benefit of the homework journals. She explained, "I really think it's the why. If you don't understand why you're doing something, you're not going to be able to apply it to other situations. I think it just helps you to talk about the process. I think that's the biggest thing that it's helped." Lydia emphasizes the importance of why the process she uses to solve problems makes sense. It is incredibly important to her to not only understand the procedures, but to also understand the conceptual ideas behind those procedures. This is the biggest reason that the homework journals are beneficial.

Mia similarly describes that the homework journals, because they require her to write about specific problems, help her slow down and get a better understanding of those problems.

She said the biggest benefit is that “you have a better understanding, you aren’t just going through the motions.” By writing about the problems in her homework journals she is not just going through the motions of finding a solution, she is not just focusing on the procedural operations and algorithms that result in the correct answer. Instead, she is gaining a better understanding of what is behind those operations and algorithms in order to build her conceptual knowledge.

Students are able to build both procedural and conceptual knowledge when completing homework journals. These are benefits that students value and appreciate about the homework journals.

***Building relationships.***

Melissa believes another benefit of the homework journals is the ability to communicate with her instructor. Building open lines of communication is important to Melissa and so she values the homework journals, and sees them as beneficial, because she can do just that through the reflection component of the homework journals. She said, “I like the fact that I knew she was reading [the homework journals] and if there were something that I wanted to bring up to her, I could probably do it through the homework journals.” She believed an important benefit of the homework journals is the way she could communicate with her instructor.

***Influencing course grade.***

Both Adele and Kelly believed one important benefit of the homework journal was how the grade they received for their homework journal influenced their overall course grade. Earning a high grade on each homework journal boosted their average homework grade, which, in turn, improved their overall course grade. Kelly explained, “I like the idea of having an extra grade in there! As bad as it sounds, if you are having trouble on the test, it’s always nice to have

a little extra thing to be graded!” Adele echoed this idea, “I think the only benefit is just grades. Just the grading part. I’m glad I got it graded ‘cause it did help my grade.” For both of these students, earning the maximum amount of points on their homework journals improved their overall course grade. This was another important benefit of the homework journals mentioned in the interviews.

When they were asked about the benefits of the homework journals, the nine students in the HJ section responded with a number of different benefits. These students believed that strengthening their mathematical understanding, gaining both procedural and conceptual knowledge, communicating with their instructor, and improving their overall course grade were four benefits of the homework journals.

## Chapter Five: Conclusions.

### Introduction

The goal of this study was to explore and describe the relationship between writing in mathematics and students' intrinsic motivation towards learning mathematics, specifically the relationship between homework journals and students' psychological needs for competence, autonomy, and relatedness, as well as the relationship between homework journals and students' attitudes towards writing in mathematics. Both quantitative and qualitative data analysis were employed to achieve this goal. Based on an analysis of survey data collected from two sections of *Elementary Calculus with Trigonometry II*, one that engaged in homework journals throughout the semester and one that did not, no differences in rate of change of competence, autonomy, relatedness or attitudes towards writing were found. However, an analysis of qualitative data found that 1) when students solve and reflect on personally challenging mathematical problems, homework journals can help meet students' need for competence, 2) when students recognize the choices they are free to make and when those choices are meaningful, homework journals can help meet students' need for autonomy, and 3) when students write thoughtful and meaningful responses and when the instructor writes thoughtful and meaningful feedback, homework journals can help meet students' need for relatedness. In addition, students' attitudes towards writing mathematics are strongly influenced by their likes and dislikes of homework journals and the perceived benefits of homework journals. This current chapter will discuss and elaborate on these conclusions in three main sections. The first main section relates to students' motivation towards learning mathematics, both students' general levels of motivation and the specific relationship between homework journals and self-

determination theory. The second main section relates to students' attitudes towards writing in mathematics, particularly students' likes and dislikes of the homework journals and the perceived benefits of the homework journals. The third main section relates to the implications for future research and instructional implications, all of which emerged from the current data collection and analysis.

## **Students' Motivation Towards Learning Mathematics**

### **General Descriptions of Students**

#### **General levels of motivation.**

Motivation involves the processes that give behavior its energy and direction (Reeve, 2005). Any level of motivation towards learning mathematics implies a degree of energy aimed in the direction of learning mathematics. In this study, students' levels of motivation were widely varied; some students were less self-determined and some were more self-determined. A broad range of self-determined behavior is typical because students' level of self-determination, and also their level of motivation, is based on their psychological needs for competence, autonomy, and relatedness (Ryan & Deci, 2000). These three needs can vary from student to student meaning that students experience a variety of levels of motivation.

#### **General levels of competence.**

In this study, students' confidence in their ability to learn the course content was incredibly varied. Based on the qualitative analysis, students' confidence levels are primarily based upon previous experiences. Those previous experiences can date all the way back to elementary school or can even be as recent as the last test in the current course. This is consistent with the literature. Ryan and Deci (2000) describe personal mastery experiences, successfully accomplishing a difficult or challenging task, as a way to meet students' need for

competence. In addition, both Ryan and Deci (2000) and Bandura (1977) describe encouragement from others, especially the teacher, as another way to meet students' need for competence. However, none of the students in this study mentioned their instructor's feedback or encouragement as an influence on their level of confidence. While this may have had an influence on students' levels of confidence, it is unknown from the interviews.

### **General levels of autonomy.**

In this study, students' sense of autonomy or freedom ranged from no freedom at all to very high levels of freedom. Students' levels of autonomy are based on feelings of being "listened to," on receiving praise, encouragement, and hints from teachers, and on teachers communicating perspective taking statements (Reeve & Jang, 2006). Students' levels of autonomy are also based on meaningful choice (Katz & Assor, 2007). Although the students in the interviews did comment on the opportunities for meaningful choice that they either did or did not experience, they did not comment on the influence of the other sources of autonomy. These other sources of autonomy are likely aspects of the feedback given by their instructor and not as easily seen as freedom-enhancing experiences. The interview questions (see Appendix C) regarding autonomy all used the word "freedom" in the question in order to make the question more accessible to the students. Because the interview questions were framed in this way, students most likely associated freedom with choice and not with aspects of the feedback given by their instructor. Therefore the only source of autonomy students cited was opportunities for meaningful choices. Students did not cite any sources of autonomy such as feeling listened to, receiving praise, encouragement, or hints, or having teachers communicate perspective taking statements that were outlined by Reeve and Jang (2006).

**General levels of relatedness.**

In this study, students in both sections experienced a high sense of relatedness with their classmates and with their instructor. This result was unanimous in the interviews. All ten students interviewed felt comfortable with and supported by their classmates and their instructor.

Students' feelings of relatedness are based on feeling known, receiving support and encouragement, having their questions answered, and feeling like teachers are on their side (Reeve, 2006; Reeve & Jang, 2006). In this study, because the students' level of relatedness was already so high, it was difficult to parse out the sense of relatedness strengthened by the instructor's behavior and attitude in general and what she specifically did with the homework journals. It seemed as though the feedback in the homework journals did give students support and encouragement, and also answered a lot of their questions. However, it was not clear from the interviews whether or not the homework journals helped students feel known by their instructor or feel like their instructor was on their side. It was also not clear whether students' need for relatedness would be met regardless of the homework journals. Results from the analysis of the quantitative data provided some insight into this, but do not fully answer this question.

**Relationship Between Homework Journals and Self-Determination Theory**

Based on the quantitative data and analysis, for all three psychological needs associated with self-determination theory, the rate of change from the pre-course survey to the post-course survey was similar for the HJ section and the Non-HJ section. This indicates that in general, the homework journals did not make a significant difference on students' need for competence, autonomy, or relatedness. One possible reason for the results related to competence and autonomy is the varied levels of these psychological needs among the students. For students

with lower levels of competence or autonomy, the homework journals may have made a significant difference from the pre-course survey to the post-course survey. However, it is difficult to tell since all students were included in the analysis, including the students with higher levels of competence or autonomy. In a future study, it might be beneficial to look specifically at students with lower levels of competence or autonomy to determine the influence of the homework journals on such students. One possible reason for the results related to relatedness is the high level of relatedness among the students in both the HJ section and the Non-HJ section. The relatedness responses were high based on the pre-course and post-course surveys and also based on the interview data. It is possible that because the students already had high levels of relatedness, there was very little room for the homework journals to change students' sense of relatedness. Another possible reason for the results related to relatedness is the lack of feedback the instructor gave. The instructor reported only giving minimal feedback, especially when students did not write thoughtful or meaningful responses, and this may have impacted students' levels of relatedness. It is possible that if the instructor gave feedback to all students, and not just the students who wrote thoughtful and meaningful responses, then there would be changes in students' levels of relatedness.

Although there were no overall differences in change between the HJ section and Non-HJ section for any of the three psychological needs connected to self-determination theory, results from the qualitative data analysis suggest that there may be dependent circumstances that influence students' psychological needs for competence, autonomy, and relatedness. Associated with competence: When students solve and reflect on personally challenging mathematical problems, homework journals can help meet students' need for competence. Associated with autonomy: When students recognize the choices they are free to make and when those choices

are meaningful, the homework journals can help meet students' need for autonomy. Associated with relatedness: When students write thoughtful and meaningful responses and when the instructor writes thoughtful and meaningful feedback, the homework journals can help meet students' need for relatedness. Each of these statements will be addressed and supported in the following sections.

### **Competence.**

When students solve and reflect on personally challenging mathematical problems, homework journals can help meet students' need for competence. As part of the homework journal assignment (see Appendix B), students were told to choose one problem from their assigned set of homework problems to solve on the left-hand column of their homework journal that interested them or challenged them; students were able to choose whichever problem they wanted. Students either chose personally easy mathematical problems or personally difficult mathematical problems. When students chose personally easy mathematical problems for their homework journals, they did not have the opportunity to solve and reflect on personally challenging mathematical problems and therefore they were not having personal mastery experiences (cf. Bandura, 1977; Ryan & Deci, 2000). This was true for students like Ann and Melissa. On the other hand, when students chose personally difficult mathematical problems for their homework journals, they did have the opportunity to solve and reflect on personally difficult mathematical problems and therefore they were having personal mastery experiences (cf. Bandura, 1977; Ryan & Deci, 2000). This was true for students like Sophie and Chris. From the qualitative data, which provides further insight into students' experiences, it seems as though when students solve and reflect on personally challenging mathematical problems, homework journals can help meet students' need for competence.

**Autonomy.**

When students recognize the choices they are free to make and when those choices are meaningful, the homework journals can help meet students' need for autonomy. Students were given a number of opportunities to make meaningful choices related to the homework journals; these choices were built into the homework journal assignment (see Appendix B). Students were able to choose whichever problem they wanted from their assigned set of homework problems to solve as a part of their homework journal. Students were also able to choose whichever prompt they wanted to respond to from the list of given prompts. These choices were not too simple nor too complex, offered too few or too many choices, and were interesting to students since they could choose whatever interested or challenged them. These are the three characteristics of meaningful choices (Katz & Assor, 2007). Related to the choices they were free to make concerning the homework journals, students in the current study could be categorized in one of three different ways: 1) unaware of the choices, 2) aware of the choices, but the choices were not meaningful, or 3) aware of the choices and the choices were meaningful. Some students were completely unaware of the choices available to them with the homework journals. For example, Kelly did not experience choice in any way when completing her homework journals. Other students were aware of the choices available to them, but those choices were not meaningful. For example, Chris was aware of the various prompts he was able to respond to and chose between these prompts, but those choices were not meaningful to him. Yet other students, like Sophie, were both aware of the choices available to them and those choices were meaningful. Of the three categories of students, only students in the third category, students who were aware of the choices they were free to make and who thought those choices were meaningful, believed that the homework journals helped them feel a greater sense of freedom, thus meeting their need

for autonomy. From the qualitative data, which provides further insight into students' experiences, it seems as though when students recognize the choices they are free to make and when those choices are meaningful, the homework journals can help meet students' need for autonomy.

### **Relatedness.**

When students write thoughtful and meaningful responses and when the instructor writes thoughtful and meaningful feedback, the homework journals can help meet students' need for relatedness. In this study, there seemed to be two different types of students: students who wrote thoughtful and meaningful responses for their homework journals and students who rushed through their homework journals, completing the assignment hastily and sloppily. The feedback given by the instructor seemed to mirror the responses given by the students. Some students, like Adele, rushed through the homework journals, completing the assignment as quickly as possible and with little thought or effort. Students like Adele typically received similar feedback from their instructor, feedback that was rushed and with little detail. These students typically did not express a sense that the homework journals helped them feel closer to their instructor; the homework journals did not help meet their need for relatedness. Other students, like Lydia, took their time with the homework journals, completing the assignment with care and thought. Students like Lydia also received similar feedback from their instructor, feedback that was thoughtful, meaningful, helpful, and specific. These students typically did express a sense that the homework journals helped them feel closer to their instructor; the homework journals did help meet their need for relatedness. From the qualitative data, which provides further insight into students' experiences, it seems as though when students write thoughtful and meaningful

responses and when the instructor writes thoughtful and meaningful feedback, the homework journals can help meet students' need for relatedness.

### **Students' Attitudes Towards Writing in Mathematics**

Based on the quantitative data and analysis, the rate of change from the pre-course survey to the post-course survey was similar for the HJ section and the Non-HJ section. This indicates that the homework journals did not make a significant difference on students' attitudes towards writing in mathematics. This is consistent with previous literature, which reports that attitudes towards writing in mathematics are not improved as a result of journal writing in mathematics (Jurdak & Zein, 1998). Both the HJ section and the Non-HJ section had more negative attitudes towards writing in mathematics at the end of the semester than they did at the beginning of the semester (see Table 4.1). Although it was not statistically significant, the difference in attitudes towards writing in mathematics for the HJ section (-0.31) was more negative than the difference in attitudes towards writing for the Non-HJ section (-0.13). One possible reason for this is that at the time of the post-course survey, the students in the HJ section had come to have a better idea about what it means to write in mathematics because of their experiences with the homework journals and had stronger opinions about writing in mathematics.

Although there were no overall differences between the HJ section and Non-HJ section for attitudes towards writing in mathematics, by looking at the qualitative data and analysis there are other indications about students' attitudes towards writing in mathematics. What the students liked and disliked about the homework journals, and what benefits students believed there were to the homework journals influenced students' attitudes towards writing in mathematics. In addition, by examining students' likes and dislikes of homework journals as well as students'

beliefs about the benefits of homework journals, it is possible to better understand how homework journals can help meet students' needs for competence, autonomy, and relatedness.

### **Likes and Dislikes of the Homework Journals**

The nine students in the HJ section expressed various likes related to the homework journals. Some students liked the consistency in the format of the homework journals; they liked that the homework journals were a weekly assignment and that all of the weekly homework journals were kept in the same place. Some students appreciated the extra grade provided by the homework journals; these students believed that this extra grade helped boost their overall grade. Some students liked the feedback that they received from their instructor. Some students valued the way the homework journals caused them to think deeply, to reflect, on the mathematical topics and to make connections to other mathematical topics. At least one student liked being able to communicate with their instructor through the use of the homework journals. Some students also liked being able to make choices when completing their homework journals, particularly the choices related to the prompts.

The aspects of homework journals that students liked may be connected to students' psychological needs for competence, autonomy, and relatedness. For example, boosting their overall course grade, thinking deeply, making connections, and receiving feedback are all connected to students' need for competence (Bandura, 1977; Ryan & Deci, 2000). Also, making choices and receiving feedback are related to students' need for autonomy (Katz & Assor, 2007; Reeve & Jang, 2006). Finally, communicating with their instructor and receiving feedback are all related to students' need for relatedness (Reeve, 2006; Reeve & Jang, 2006). Whereas these likes are a part of students' attitudes towards writing, they also help meet students' needs for competence, autonomy, and relatedness. Exploring and describing students' attitudes towards

writing in mathematics, specifically what they liked about homework journals, adds to the understanding of how homework journals can help meet students' psychological needs for competence, autonomy, and relatedness.

The nine students in the HJ section also expressed various dislikes related to the homework journals. Some students disliked the inconsistency between the expectations communicated to them and the way the homework journals were actually graded. These students thought that the expectations were too open-ended and this made it difficult to know what exactly was expected of them. Some students disliked having to spend extra time on their homework assignment each week in order to complete the homework journals. At least one student also disliked the format restrictions, being required to solve the mathematics problem in the left-hand column of the paper and reflect on the problem in the right-hand column of the paper.

The aspects of homework journals that students disliked may also be connected to students' psychological needs for competence, autonomy, and relatedness. For example, feeling as though the homework journal assignment was too open-ended and feeling overwhelmed with choices hinders students' sense of autonomy (Katz & Assor, 2007). Similarly, being restricted by the homework journal format also interferes with students' sense of autonomy (Reeve & Jang, 2006). Whereas these dislikes are a part of students' attitudes towards writing, they also detract from students' self-determination. By understanding students' attitudes towards writing, specifically what they disliked about homework journals, it is possible to provide insight into how homework journals detract from students' psychological needs for competence, autonomy, and relatedness.

The students' likes and dislikes related to the homework journals are also connected to three important characteristics of journal writing found in the literature. Those three characteristics are: the inclusion of a prompt (Borasi & Rose, 1989; Lim & Pugalee, 2004; Waywood, 1992), clear expectations (Hamdan, 2005; Powell & Ramnauth, 1992), and responses from the teacher (Borasi & Rose, 1989; Miller & England, 1989; Powell, 1997).

The first important characteristic of journal writing is the inclusion of a prompt. Most journal writing assignments have a prompt and the homework journals have six different prompts. The prompt can be relatively generic (Powell, 1997) or extremely specific (Lim & Pugalee, 2004). The prompt can also be cognitive or affective (Jurdak & Zein, 1998). The homework journal prompts were generic, specific, cognitive, and affective. The students in the HJ section were also given the opportunity to choose which prompt they wanted to respond to and reflect on each week. Some of the students expressed how much they liked being able to choose which prompt they wanted to respond to and reflect on each week as well as the variety of the prompts.

The second important characteristic of journal writing is communicating clear expectations. It is important for students to know what is expected of them (Sanders, 2009). This is one area of frustration for some of the students in the HJ section. Based on what they described as their dislikes about the homework journals, some students did not think that their instructor clearly communicated her expectations for the homework journals. These students were not sure about what they should or should not be writing about. One student, Mia, claimed to have no idea what she was expected to write about, even at the end of the semester. Another student, Melissa, said that she wanted to be told exactly what to write, she thought there was too much ambiguity with the prompt choices and with what the students were expected to respond to

and reflect on. Other students thought that the expectations were clearly communicated. Kelly, for example, thought there was consistency between what her instructor communicated about the homework journals and the way she graded the homework journals. The expectations of the homework journals were clear to some students, but not to others.

The third important characteristic of journal writing is the response of the teacher. When the course instructor regularly collects and comments on students' homework journals, this opens up communication between her and her students (Borasi & Rose, 1989), gives her insight into her students' understanding (Powell, 1997), and gives her students regular and constructive feedback (Williams & Wynne, 2000). The students in the study were thankful for the ability to communicate with their instructor through the homework journals, the students thought the homework journals helped their instructor get to know them, and their mathematical thinking, better, and the students valued the feedback they received from their instructor. Although the instructor self-described her feedback as minimal, it was still a vital part of the homework journals.

### **Perceived Benefits of the Homework Journals**

The benefits of the homework journals described and explained by students are also representative of their attitudes towards the homework journals. The use or purpose of the homework journals, basically what a student can get out of the homework journals, influences students' attitudes towards the homework journals in particular and writing in mathematics in general. That is why it is important to consider the students' beliefs about the benefits of the homework journals. The body of literature concerning writing in mathematics provides numerous examples of journal writing experiences that are beneficial to students, to teachers, and to communication between students and teachers. Some cognitive benefits are more powerful

learning and understanding (Emig, 1977), increasing procedural and conceptual mathematical knowledge (Jurdak & Zein, 1998; Sanders, 2009), and strengthening problem solving skills (Grossman, Smith, & Miller, 1993). Some affective benefits are encouraging positive beliefs about mathematics (Borasi & Rose, 1989), building mathematical confidence (Lim & Pugalee, 2004), increasing enjoyment of learning mathematics (Sanders, 2009), and creating avenues of communications between students and teachers (Borasi & Rose, 1989). Based on an analysis of the interviews, the students in the HJ section described and explained four benefits of the homework journals: 1) more powerful learning, 2) building both procedural and conceptual knowledge, 3) building relationships with their instructor, and 4) boosting their overall course grade. The first three benefits described and explained by the students are consistent with the previous literature, while the fourth benefit is not explicitly mentioned in the literature. Each of the benefits described and explained by the students in the HJ section will be discussed in the following sections.

### **More powerful learning.**

Students believed that the homework journals allowed them to gain a stronger understanding of the ideas they were learning in their mathematics course. Several students described homework journals as a way for them to better understand the course content. This is consistent with the literature, which suggests that students take an active role in constructing knowledge and making meaning of experiences, problem solving, and mathematical content when journal writing in mathematics (Hadman, 2005). Students gave two brief explanations about why homework journals helped them better understand the course content. One reason was that homework journals were different from the other aspects of the course, writing in their homework journals allowed students to think about the mathematics differently. The other

reason was that they had to take time to think about the mathematics, they had to actually slow down and ponder or wonder about the mathematics. By thinking differently and by slowing down, students learn by constructing meaning (Pugalee, 2001; Sanders, 2009) because this emphasizes summarizing, generalizing, reviewing, and relating information (Grossman et al., 1993; Powell, 1997).

This benefit also has a connection to students' need for competence. Competence, which measures the degree to which a student feels they are able to complete a task and succeed at completing that task, is strongly based on personal mastery experiences (Bandura, 1977; Ryan & Deci, 2000). When students understand the course content in both a fuller and a deeper sense, they are better able to experience success and accomplishment, thus helping meet their need for competence. This benefit of homework journals, reported by the students in the interviews, is related to students' attitudes towards writing in mathematics, and it is closely connected to students' need for competence.

### **Building procedural and conceptual knowledge.**

Students also believed that the homework journals helped them build both procedural and conceptual mathematical knowledge. Students who believed the homework journals helped them build procedural knowledge described how the homework journals were a way for them to describe and remember the steps they used to solve particular mathematics problems. Students who believed the homework journals helped them build conceptual knowledge described how the homework journals were a way for them to understand why the steps they used to solve particular mathematics problems actually worked and to see the bigger picture of the mathematics. This is also consistent with the literature, which suggests that journal writing in mathematics helps students be better able to carry out mathematical operations and procedures

and also be better able to understand the concepts behind those operations and procedures (Borasi & Rose, 1989; Jurdak & Zein, 1998; Sanders, 2009). Students valued knowing and remembering how to solve problems as well as knowing why those procedures are used and believed that homework journals helped them achieve these goals.

This benefit is also closely related to students' need for competence. Ryan and Deci (2000) describe competence as a measure of the degree to which students feel they are able to complete a task, such as solving problems. This need is strongly based on personal mastery experiences (Bandura, 1977; Ryan & Deci, 2000) and when students build both procedural and conceptual knowledge they are better able to solve problems and understand why those solutions make sense. Students perceive gains in both procedural and conceptual knowledge as a benefit of homework journals, which contributes to their attitudes towards writing in mathematics, and is also associated with students' need for competence.

#### **Building relationships with their course instructor.**

The other benefit mentioned by the students in the HJ section was that homework journals helped them build relationships with their course instructor. Students were thankful for the opportunity to communicate with their course instructor through the use of the homework journals. Students communicated questions about the course or about the mathematical content through their homework journals. Students also appreciated the feedback they received from their course instructor. These benefits of homework journals, both communication and feedback, are consistent with the literature. Individualized support and feedback from the teacher are two benefits of journal writing suggested by the literature (Borasi & Rose, 1989; Williams & Wynne, 2000). In addition, the two-way communication between students and their course instructor through the use of homework journals lets students know that what they write is actually being

read (McIntosh & Draper, 2001) and that their course instructor cares about them (Borasi & Rose, 1989; Watson, 1980).

This benefit also has a connection to students' need for relatedness. Relatedness, which measures the degree to which a student feels related to or close to their classmates and their instructor, is based on a number of different influences such as feeling known by their instructor (Reeve, 2006), receiving praise, encouragement, and feedback (Reeve & Jang, 2006), and communicating with their instructor (Reeve & Jang, 2006). Students recognize these influences as benefits of homework journals; these benefits are related to students' attitudes towards writing and students' need for relatedness.

### **Boosting their grade.**

Although this was not explicitly mentioned in the previous literature, another benefit reported by the students in the HJ section was the way the homework journals would boost their overall course grade as a benefit of the homework journals. Students were aware that homework journals contributed to their overall homework grades, which contributed to their overall course grades. Therefore, students thought that getting high grades on their homework journals was an easy way to boost their overall course grades.

This benefit, although indirectly, can be linked to students' need for competence. Many students associated success with high test scores and above average grades; personal mastery experiences (Bandura, 1977; Ryan & Deci, 2000) were based on test scores and grades instead of on understanding and personal accomplishment. Therefore, when students received a boost in their overall course grade because of the homework journals, they are experiencing success. While this was a commonly cited benefit of homework journals, thus contributing to students' attitudes towards writing in mathematics, it is also indirectly connected to students' need for

competence. Interestingly, this “benefit” might also be reported about any extra assignment that contributes to students’ overall course grades.

### **Implications for Future Research and Instruction**

This current study described the relationship between homework journals and self-determination theory as well as the relationship between homework journals and students’ attitudes towards writing in mathematics. From the data collection and analysis undertaken as a part of this study, implications for future research questions and instructional practices emerged. Two implications for future research are collecting and analyzing students’ homework journals and employing a triangulation mixed methods design for data collection and analysis. Two implications for instruction are connecting homework journals to the three important characteristics of writing in mathematics and providing specific feedback that targets students’ needs for competence, autonomy, and relatedness. These implications will be addressed in the following sections.

#### **Implications for Future Research**

##### **Collect and analyze students’ homework journals.**

In this current study, any information about feedback came from the students’ self-descriptions. It would be helpful to examine the feedback given by the instructor in order to see what kind of comments were made, especially any comments that should, in theory, have influenced students’ needs for competence, autonomy, or relatedness. In the future, in addition to gathering data in the form of the pre-course and post-course surveys and in form of student and instructor interviews, it would also be beneficial to collect student’s homework journals for further analysis. Collecting and examining students’ actual homework journals to determine the nature of the feedback that they received and to analyze the content of the homework journals

would be beneficial because it would provide a fuller picture of the students' experiences with the homework journals.

### **Triangulation mixed methods design.**

This current study employed a multiple methods design in which both quantitative and qualitative data were collected and analyzed, but the different types of data were not compared or used collectively in order to draw conclusions. In the future, a mixed methods design in which both quantitative and qualitative data and analysis are used to inform and complement each other (Creswell & Plano Clark, 2007) would be beneficial. Specifically, a triangulation mixed methods design would create a more robust description of the phenomenon of students' experiences with homework journals and students' levels of competence, autonomy, and relatedness because both the quantitative and qualitative data would be collected simultaneously and merged during analysis (Creswell & Plano Clark, 2007).

### **Implications for Instruction**

#### **Three important characteristics of writing in mathematics.**

Students' likes and dislikes of the homework journals are closely related to the three important characteristics of writing in mathematics: prompts (Borasi & Rose, 1989; Lim & Pugalee, 2004; Waywood, 1992), clear expectations (Hamdan, 2005; Powell & Ramnauth, 1992), and responses of the teacher (Borasi & Rose, 1989; Miller & England, 1989; Powell, 1997). These three important characteristics of journal writing lead to valuable instructional implications. First of all, precisely described prompts are essential when assigning homework journals. Students need to fully understand each prompt and the purpose of each prompt in order to meaningfully respond to and reflect on the prompt. This may require instructors to take time to explain each prompt to students instead of just giving students the list of prompts. Secondly,

instructors need to carefully and clearly communicate their expectations to students. One way to communicate expectations is to show students examples of homework journals. The students in the HJ section only saw one example homework journal at the beginning of the semester, but it is beneficial for students to see a number of examples. Instructors should model actual homework journals from the class. After the first and second homework journals have been collected, instructors could select two or three homework journal examples and show these examples to the whole class (being sure to remove any identifying marks or names from the homework journals). Instructors should then initiate a short class discussion about these homework journals with the whole class; this gives students an opportunity to see homework journals (other than their own) in order to gain a better idea of what is expected of them. Finally, instructors need to provide thoughtful and meaningful feedback to each student. Although this can be incredibly time consuming, even one small detailed or personal comment can be meaningful to a student (Hendry, Bromberger, & Armstrong, 2011). Thoughtful and meaningful feedback is a way for instructors to show students that they care about them and about their learning, which encourages feelings of relatedness (Reeve, 2006; Reeve & Jang, 2006).

### **Providing specific feedback.**

Receiving specific feedback is an important element of students' experiences with homework journals, especially because such feedback has strong connections to the three psychological needs associated with self-determination theory, competence, autonomy, and relatedness. For example, a comment such as, "Well done, you solved a really difficult problem." should influence students' need for competence because it serves as verbal persuasion or encouragement (Bandura, 1977; Ryan & Deci, 2000). Also, offering a hint such as, "Have you tried this method of integration?" should influence students' need for autonomy because it

offers students a suggestion on how to make progress on their own (Reeve & Jang, 2006). In addition, affirming a student's lament about a particular problem with, "Yes, this was a very difficult problem." should influence students' need for relatedness because it communicates that their teacher is "on their side" or "in their corner" (Reeve & Jang, 2006). Providing specific feedback that targets students' psychological needs for competence, autonomy, and relatedness is an important consideration for instructors who are planning to implement homework journals into their mathematics courses (Hendry et al., 2011; Hyland & Hyland, 2001).

### **Overview and Conclusion**

This study uniquely connected three areas of current research – mathematics education in colleges and universities, motivation, and journal writing in mathematics – by implementing and then studying the effects of homework journals on college mathematics students' motivation as grounded in self-determination theory. The relationship between homework journals and students' motivation towards learning mathematics, specifically students' level of self-determination, is made explicit through this study. Homework journals do have the potential to increase students' sense of self-determination, which is valuable in education.

Comparisons between people whose motivation is authentic (literally, self-authored or endorsed) and those who are merely externally controlled for an action typically reveal that the former, relative to the latter, have more interest, excitement, and confidence, which in turn is manifest as enhanced performance, persistence, and creativity. (Ryan & Deci, 2000, p. 69)

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## Appendix A

## List of Survey Items for Pre-Course and Post-Course Surveys

*The following prompt is given at the top of the survey:*

The following questions concern your beliefs about and your actions in **this mathematics course**. Please indicate your level of agreement with each of the following statements. Please fill in the number that best represents your answer.

1	2	3	4	5	6
Strongly Disagree	Somewhat Disagree	Slightly Disagree	Slightly Agree	Somewhat Agree	Strongly Agree

1. I feel like I can make a lot of decisions in how I do my math.
2. I really like the people in my math class.
3. People in my math class tell me I am good at math.
4. I usually do well in mathematics.
5. I get along with the students in my math class.
6. I typically keep to myself with I am in math class.
7. While in math class, I feel a relaxed sense of personal freedom.
8. When I am doing math, I often do not feel very capable.
9. I consider the people in my math class to be my friends.
10. People in my math class care about me.
11. There are few students in my math class that I am close to.
12. I am free to express my ideas and opinions about math in my math class.
13. The students in my math class do not seem to like me much.
14. When we discuss problems/topics in math class, my ideas are taken into consideration.
15. Mathematics is not one of my strengths.
16. I feel I can do math the way I want to do it.

17. People in my math class are typically friendly towards me.
18. I do not feel that I am very good at math.
19. I feel like I can be myself in math class.
20. Mathematics is more difficult for me than many of my classmates.
21. I am discouraged to solve math problems in my own way.
22. I feel I can do math only in the way the teacher wants me to do it.
23. I am confident I can do an excellent job on my math assignments.
24. During math class, I feel pressured.
25. I am encouraged to try my own strategies in solving math problems.
26. Writing solutions to problems in math class helps me think critically.
27. I am good at math.
28. I like to write about mathematics.
29. I usually have to do math my teacher's way.
30. Writing about problems in math class helps me connect what I'm learning to other topics in mathematics.
31. It is important to write about problem solutions in math class.
32. Sometimes, when I do not initially understand a topic in mathematics, I know that I will never understand it.
33. Explaining my answers in writing in math class has no effect on my learning of mathematics.
34. During math class, I feel free.
35. I feel more confident in math class when I am able to express my ideas through writing.
36. Writing in math class is a waste of time.
37. The teacher talks too much during class.

38. If I write about concepts from math class, I understand them better.

*The following two items were included only on the pre-course survey:*

39. What is your gender?                      1 = Male              2 = Female

40. Please write your major in the space below.

*The following two items were included only on the post-course survey:*

*The following prompt is given before the next two items:*

How often did you do the following things as a part of this mathematics course? Please fill in the number that best represents your answer.

1	2	3	4
Every or almost every day	1-2 times a week	1-2 times a month	Never or hardly ever

39. I write about my strategies for solving math problems.

40. I write about my math solutions.

## Appendix B

### Homework Journal Description

What is a homework journal? A homework journal is a weekly assignment in which you will turn in one homework problem and write a reflection about that homework problem.

The point of the homework journal is to increase your understanding of the topics that we will be covering in this course. The goal of the course is not for you to repeat steps and procedures without understanding what they mean and why they are important. These journals will help you make connections to previous mathematics classes, give you insights about what you are learning in this class, and allow you to develop your mathematical understanding and confidence.

The homework journal will have a specific format: two columns. In the column on the left, you will solve one homework problem – showing ALL your work. In the column on the right, you will write a reflection of your understanding of that problem.

This is unfamiliar to most of you, so let's break it up into pieces. What goes into a reflection? There are a number of things that you can “reflect” about in your homework journal. The following are suggestions for you to choose from:

- Narrate the process you used to solve this problem. What steps did you take to solve this problem and why did you choose those steps?
- Describe the content of this problem. What are the conceptual ideas of the content of this problem?
- In what ways does this problem relate to other mathematics that you know or to other problems that may come up in your other courses or eventually in your field of specialization?
- How successful do you feel you were in solving this problem? Why? Do you feel you would be successful in solving a similar problem? Why or why not?
- How did solving this problem make you feel about doing mathematics?

You do not have to answer every question listed above in your homework journal; however, you must answer at least one of the questions listed above in your homework journal. In addition, feel free to discuss any other thoughts or ideas you have about this problem, your learning, or your feelings about mathematics.

Here's an example of a journal reflection about the composition of functions. This is similar to what you will be expected to turn in.

Example homework journal:

<p>Composition of functions:</p> <p>Find <math>(g \circ f)(x) = g(f(x))</math> given that</p> $f(x) = \frac{x-3}{2} \text{ and } g(x) = x+2$ $g(f(x)) = g\left(\frac{x-3}{2}\right)$ $g(f(x)) = \frac{x-3}{2} + 2$ $g(f(x)) = \frac{x-3}{2} + \frac{4}{2}$ $g(f(x)) = \frac{x-3+4}{2}$ $g(f(x)) = \frac{x+1}{2}$	<p>The concept of “composition of function” is a new operation to me. If we have something like <math>(g \circ f)(x)</math>, I will ask myself what this means. The book says this means <math>g(f(x))</math>. We would compose the function like this. We would take <math>f(x)</math> and put it <i>into</i> <math>g(x)</math> like I did on the left.</p> <p>When I was in grade school my teachers introduced me to addition, subtraction, division and multiplication. Here I am in college taking pre-calculus and I am being introduced to a new member of the order of operation family.</p>
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Other guidelines:

- You are allowed to pick the problem that you want to solve. This problem should come from your current homework assignment.
- The problem that you choose must be a problem that challenges you, that interests you, or that you think you have learned something from. It’s okay if this is a different problem for different students.
- Your work should be neat and legible. If it is not neat and legible, it will not be accepted.
- When working your problem (in the column on the left), you **MUST SHOW ALL WORK**. Part of learning mathematics is learning to communicate mathematically and one way to increase your ability to communicate mathematically is by showing all your work. **YOU WILL NOT RECEIVE FULL CREDIT IF YOU DO NOT SHOW ALL OF YOUR WORK.**

## Appendix C

### Student Interview Questions

Follow up questions (indented) will be used to gather additional information from participants.

#### *Personal Background*

- P1: Please describe your previous experiences in mathematics classes.  
 P2: What is your reason for taking this particular mathematics class?  
 P3: Have you ever participated in a writing assignment in a mathematics class before?  
 P3a: If so, what was the writing assignment and how did you feel about it?

#### *Motivation*

- M1: In this mathematics class, do you feel motivated to...  
 • come to class?  
 • do your homework? (do your homework journals?)  
 • study for tests?  
 • participate in class?  
 M1a: Why or why not? (for all of the above questions)  
 M2: Do you feel personally motivated? Motivated by your classmates? Motivated by your instructor? Motivated by some other outside source?  
 M2a: Why or why not? (for all of the above questions)  
 M3: Do you feel freedom in this mathematics class? Why or why not? (autonomy)  
 M4: Do you feel confident about your ability to learn the mathematical content covered in this mathematics class? Why or why not? (competence)  
 M5: Do you feel comfortable with your instructor in this mathematics class? Why or why not? (relatedness)

#### *Writing in Mathematics*

- W1: How do you feel about writing in mathematics?  
 W2: Do you think writing in mathematics would help you feel more confident in your learning of mathematics? Why or why not? (competence)  
 W3: Do you think writing in mathematics would help you gain a better understanding of mathematics? Why or why not? (competence)  
 W4: Do you think writing in mathematics would help you to express your own ideas related to your understanding of mathematics? Why or why not? (autonomy)  
 W5: Do you think writing in mathematics would help you feel more freedom in your mathematics class? Why or why not? (autonomy)  
 W6: Do you think writing in mathematics would help you feel more related to your classroom instructor? Why or why not? (relatedness)  
 W7: What other benefits do you think there might be to writing in mathematics?  
 W8: Would you want to write in a future mathematics class? Why or why not?

*Homework Journaling (HJ sections only)*

- HJ1: What did you think of the homework journals?  
HJ1a: What did you like about the homework journals?  
HJ1b: What didn't you like about the homework journals?  
HJ2: Did you enjoy the homework journals? Why or why not?  
HJ3: If you were asked to describe the purpose of the homework journals to a friend from another mathematics class, how would you describe it?

## Appendix D

### Instructor Interview Questions

Follow up questions (indented) will be used to gather additional information from participants.

#### *Teaching Experience*

- T1: What previous teaching experience do you have?  
 T1a: Have you taught in elementary school, middle school, high school, or college?
- T2: Have you ever taught this course before?  
 T2a: What is your reason for teaching this class now?

#### *Journal Writing*

- J1: Tell me about this past semester...
- J2: What did you think of the homework journal?  
 J2a: What did you like about the homework journal?  
 J2b: What didn't you like about the homework journal?
- J3: What changes would you make to the homework journals? Why?
- J3: If you were asked to describe the purpose of the homework journal to another mathematics instructor, how would you describe it?
- J4: Do you think you will assign homework journals (or a similar writing assignment) in a future mathematics course in which you are the instructor? Why or why not?
- J5: Do you think writing in mathematics helps your students feel more confident in their learning of mathematics? Why or why not? (students' competence)
- J6: Do you think writing in mathematics helps your students gain a better understanding of mathematics? Why or why not? (students' competence)
- J7: Do you think writing in mathematics helps your students to express their own ideas related to their understanding of mathematics? Why or why not? (students' autonomy)
- J8: Do you think writing in mathematics helps your students feel more freedom in their mathematics class? Why or why not? (students' autonomy)
- J9: Do you think writing in mathematics helps you feel more related to your students? Why or why not? (relatedness)
- J10: What other benefits do you think there might be to writing in mathematics?  
 J10a: Benefits for students?  
 J10b: Benefits for instructors?

## Appendix E

### General Descriptions of Student Interview Participants

#### **General Description of Students**

While it is not directly related to the focus of this dissertation, it is helpful to describe students' general motivation, especially concerning their psychological needs for competence, autonomy, and relatedness, as well as their general attitudes towards writing. This provides a context to better understand the students' perspectives.

#### **Students' General Motivation**

All ten of the students interviewed were asked about their general motivation in their current mathematics course, this includes the nine students in the HJ section and the one student in the Non-HJ section. The responses to the interview questions concerning general motivation are not differentiated between the HJ section and the Non-HJ section students because the questions concerning general motivation were not specific to writing in mathematics or to homework journals. All ten students were asked to self-describe their level of motivation toward particular tasks or behaviors in their current mathematics course, such as coming to class, finishing homework assignments, studying for tests, and participating in class. All ten students were then asked to self-describe the source of their motivation. Based on these results, all ten students can be classified as exhibiting one of the five different types of motivation, each accompanied by its own regulatory style (Ryan and Deci, 2000). The different types of motivation are illustrated in *Figure 2.3* arranged from left to right in terms of the degree to which motivation is self-determined or self-regulated. Moving from left to right, I'll classify each student and provide at least one illustrative example of each of the five types of motivation/regulatory styles.

**Non-regulation.**

At the far left side of the continuum is amotivation. No students exhibited non-regulation. No students indicated that they were simply going through the motions of attending and passing this mathematics course.

**External regulation.**

In the middle of the self-determination continuum is extrinsic motivation. Extrinsic motivation varies in the extent to which behavior is self-regulated. External regulation, the regulatory style on the left-most side, implies that an individual's motivation toward a particular task or activity is to satisfy an external demand (Ryan & Deci, 2000). Only one student, Nate, exhibited external regulation. Nate did display a high level of motivation; his self-description indicated that he was very motivated to come to class, finish homework assignments, and study for tests. However, in each of the specific, behavior-based interview questions, Nate's responses indicated that his main concern was getting good grades on all the class tests. Nate claimed that he had only missed two classes throughout the entire semester; "I always want to come to class because if I miss a class, then I'm not in a good position to succeed." For Nate, coming to class is important because it helped him prepare for tests. Similarly for homework assignments, Nate regularly did his homework assignments because that would enable him to do well on tests: "Most of the homework questions pertain to the exam, she puts a lot of similar questions on it." Finally, when asked about the source of his motivation, Nate cited the need to have good grades in order to get a good internship. He said, "I'm getting to the point of applying for state internships and jobs, and everyone asks for you to have about a 3.0 and stuff like that. It's just like, alright, that's your benchmark, you want a 3.0 or above. So that's a pretty good motivation right there." Nate chooses to come to class, complete his homework assignments, and study for

tests because he wants to have a GPA above a 3.0, so that he can attain a good internship. His regulatory style is external.

### **Introjected regulation.**

Introjected regulation, the next step of regulation within extrinsic motivation, is when an individual accepts a regulation, but does not embrace it personally (Ryan & Deci, 2000).

Behavior is regulated by a desire to avoid guilt or to attain ego enhancement. Ann, Kelly, and Adele all demonstrated extrinsic motivation with introjected regulation. I'll elaborate with both Ann and Kelly in order to describe more fully.

Ann exhibits introjected regulation because she was motivated to get good grades in her current mathematics course, as well as all her other classes, to avoid feelings of guilt associated with "wasting" all the money her parents are spending for her to be in college. While Ann is motivated to come to class, do her homework, and study for tests, she also experiences introjected regulation. When asked about her motivation to complete her homework assignments, Ann responded, "Yeah, I need the grades. I need all the points I can get. I'm not great on the exams, so I need hundreds on all the homeworks and in-class questions." Ann's main purpose for completing homework assignments was to boost her overall course grade in case she did poorly on a test. When asked about her motivation to study for tests, Ann was adamant that she was very motivated to study for tests: "There's no point in paying how much I'm paying to be here every year if you're not going to try, you know?" However, when asked specifically about the source of her motivation, Ann claimed that it was personal. "I want to do my best." This is curious and seems contradictory because her previous answers to specific questions about motivation indicate that she is motivated by grades and even by finances. Despite her self-reported answer, it appears that Ann is achieving good grades so that she can

avoid guilt; she doesn't want to waste the money her parents are spending for her to be in college. Ann exhibits introjected regulation.

Kelly also exhibits introjected regulation because she is motivated to get good grades, but the source of her motivation is to avoid feelings of guilt associated with not doing well. She does proclaim to have motivation toward tasks like coming to class, completing her homework assignments, and studying for tests, but only so that she can avoid feeling guilty for not doing her best. Kelly comes to class regularly for two main reasons. One is that she feels as though she needs the material explained to her, she couldn't just read the book and understand the material on her own. The other reason is that there are in-class assignments and she wants to get as much credit as possible for those; this is very important to Kelly. When asked whether or not she felt motivated to complete her homework, she responded: "Yes, because of my grade." Similarly, when asked whether or not she felt motivated to study for tests, she responded: "Yes, I definitely want better grades!" Kelly is definitely motivated by her grades, however she also claims to be motivated by personal reasons. When asked directly what she would say is the main source of her motivation, she quickly said, "I'm definitely motivated by grades." But then after some thought added, "And then myself, obviously, because I want to be able to say that I did well here." For Kelly, her source of motivation is external, her grades, but her regulatory style is introjected because of her desire to attain good grades in order to avoid the guilt of feeling like she did not do her best.

### **Identified regulation.**

Identified regulation, the third type of regulation within extrinsic motivation, is when an individual values a goal or outcome and therefore accepts regulation as one's own; the behavior is personally important in some way (Ryan & Deci, 2000). An individual acknowledges value in

a particular behavior and therefore behavior is more self-regulated. Melissa, Joe, and Chris demonstrated extrinsic motivation with identified regulation. I'll elaborate with Melissa and Joe.

Melissa exhibits identified regulation because she is incredibly motivated to engage in behaviors like coming to class, finishing her homework, and studying for tests in order to achieve the highest possible grade. Melissa values a high grade in her current mathematics course because she desires to attend graduate school and believes that high grades in her undergraduate coursework will help her get into a respected graduate school. Melissa is motivated to come to each class meeting "mostly because [the instructor] takes attendance during every class and we have an in-class assignment every class." Melissa is also motivated to do her homework assignments "because I feel like tests can be kind of like a crap shoot, you never know what you're gonna get, so if you bomb a test somewhere along the line, at least you can be like, 'Well, thank God my homework grade is okay because it will boost it up!' It's like something to fall back on." For Melissa, her motivation to do well on her homework comes from the fact that it will help her overall course grade, especially if she does not do well on a test. When asked what she believes is her source of motivation, Melissa explains that getting good grades is her source of motivation. "In general, what motivates me is trying to go to grad school. I want to keep my grades high because I want to get into grad school." Melissa exhibits identified regulation because she values the behavior of coming to class, doing homework, and studying for tests because these behaviors will help her achieve a high grade in Math 2015 (and in all her other classes) so that she can get into graduate school. She knows that doing well in this class will help her in the future; therefore, she exhibits identified regulation.

Joe also exhibits identified regulation. Joe is motivated to do well and to study for Math 2015, along with the rest of his classes, because he is a student-athlete. As a student-athlete, he

is expected to maintain a certain GPA in order to remain eligible to compete. Joe values doing well in his classes so that he will be able to compete alongside with his teammates. Joe claims motivation to come to class regularly because he feels like he's learning and he will fall behind if he misses a class. Class is "not a drag" for him. Joe also claims motivation to do his homework; this is because doing homework "is a requirement to get good grades." Lastly, Joe claims motivation to study for tests because otherwise he will forget the material. He studies because he needs to refresh his mind before the tests. When asked to self-report the source of his motivation, Joe gives the reason of being a student-athlete. He said, "they keep on us pretty tight about getting good grades. So that sort of pushes you as well." Maintaining a certain GPA in order to remain on the team and eligible to compete is a strong motivation. Joe knows he needs to do well in order to compete alongside his teammates. Therefore, he values doing well in his classes, especially Math 2015. Joe experiences extrinsic motivation with identified regulation.

### **Integrated regulation.**

Integrated regulation is the most self-regulated style within extrinsic motivation (Ryan & Deci, 2000). Regulations are integrated when the behavior is an expression of who an individual is. Lydia and Sophie demonstrates extrinsic motivation with integrated regulation. I'll elaborate with Lydia in order to describe this type of regulation more fully.

Lydia exhibits integrated regulation; her behavior in her current mathematics course is an expression of who she is. She is someone who wants to feel accomplished and proud of what she does, in all areas of her life. For this reason she experiences motivation in this particular class. Lydia claims that she is definitely motivated to come to class, mostly because class serves as a review of the material for her, which reinforces the mathematical concepts. She is motivated to come to class and to actively participate in class in order to understand why certain concepts

and procedures are true; she claims that she will not attain this deeper understanding without being in class. Lydia also says she is motivated to do her homework assignments because “it’s good practice.” She is also motivated to study for tests: “I think I studied for at least three hours [for the last test], which is a lot for me since I’m fairly good and understand it. So there is some motivation, because I still want to do well. I mean, it’s an easy A for me, but it’s something that you still have to put in some effort for.” Lydia describes a desire to do well and to feel accomplished in herself and the effort she put forth in this class. When asked directly about it, Lydia says that her main source of motivation is a combination of getting good grades and the fact that she wants to feel proud of what she has done. She said she wants to know that when she earns a grade, “I put in the effort and I got the grade, I worked for it. I know I deserve this grade.” Working hard and earning excellent grades is an expression of who Lydia is and how dedicated she is to her schoolwork; therefore, she demonstrates integrated regulation.

### **Intrinsic regulation.**

Finally, at the far right side of the self-determination continuum is intrinsic motivation along with intrinsic regulation. Intrinsic regulation means that an individual chooses to perform an activity simply because the activity is enjoyable in itself (Ryan & Deci, 2000). There was only one student who demonstrated this type of motivation and regulation, Mia. Mia comes to class, does her homework, and studies for tests because she loves mathematics. Mia loves solving puzzles and riddles and in her eyes, mathematics is like a puzzle. She loves mathematics because “you have little tools and you sort of use them to figure out puzzles. And I love that! I think it’s so much fun! So just having the experience of figuring things out, I love that!” Mia is definitely motivated to come to class because of “the people [in her class] and because the subject matter is fun and interesting. [The instructor] goes at a pace that’s understandable. She

listens to students, and when we don't understand something, she's willing to go back over it, which is really nice. And I have friends in the class...my motivation is really the people." She is also motivated to complete her homework assignments because of the classmates she works with on a regular basis. About her classmates, she said, "we work on [homework] together and keep each other in check, and that's a big part of it." Lastly, she is motivated to study for tests. The main reason she is motivated to study for tests is because she enjoys the material and is comfortable with what is expected of her on the tests. Mia is highly motivated by the people in her class. The relationships that she has with her classmates and the comfort she feels with her instructor are critical to her motivation. To Mia, mathematics is like a giant puzzle and she simply enjoys the experience of solving the puzzle with other people. Her motivation is not driven by any kind of outside or external impetus; she exhibits intrinsic motivation and regulation.

### **Students' General Competence**

Competence measures the degree to which students feel they are able to complete a task or succeed at a particular endeavor, it is basically a measure of students' level of confidence for that task or endeavor (Ryan & Deci, 2000). Based on the interview data from all ten students, students' confidence in their ability to learn the material in this mathematics course ranged from very low to very high; students' confidence levels were incredibly varied. I'll provide descriptions and examples of each confidence level.

#### **Low level of competence.**

Ann and Adele both expressed very low confidence. Neither of these students felt that they would be able to learn and understand all of the material in their mathematics course; they both felt that they would struggle and not do well in the course.

Ann claimed that throughout elementary school and middle school, she excelled in mathematics. However, when she reached high school and took an introductory geometry course, she started falling behind. Now, as a sophomore in college, she said “No, I’m not confident, not even a little bit!” about her confidence in her own ability to learn the material in her current mathematics course. She elaborated, “I’m just not good at math. In class it seems to me like it makes sense, but then when I’m doing my homework or something, I’ll get to a question and I’m like ‘I have no idea what I’m doing!’ I don’t know, I’m just not a math person.” Ann’s confidence is based on her ability to solve problems on her own. Based on what she said, she is not able to solve problems on her own and therefore she has very little confidence.

Adele claimed that she has struggled with mathematics throughout her entire life. She said, “Math has always been difficult for me. It’s always been a struggle, no matter when, probably starting from elementary school. It’s always been a tug-of-war, a lot of work.” Starting with elementary school, Adele has struggled to comprehend the content in her mathematics courses. Adele mentioned that if she were to have an extra review session, in addition to class meetings, and if the pace of class was much slower, then she might have more confidence in her ability. However, in her current situation, she had very little confidence in her ability.

#### **Mid-level of competence.**

Joe, Mia, Kelly, and Nate all expressed a middle-of-the-road level of confidence; their confidence in their ability to learn the material in Math 2015 was neither high nor low. Mia and Joe’s example will provide a summary of these four students.

Mia described her own confidence level as mid-range. When Mia was in high school, the only course she failed was a mathematics course. “I was so afraid [of math classes in college] because the only course I failed in high school was actually math and so I was always the opposite of having confidence.” She started college with almost no confidence in her ability to learn the material in her mathematics courses in college (as an environmental science major, she is required to take at least three mathematics courses). However, since coming to college, mathematics has become her favorite subject and when asked to describe her confidence level for her current mathematics course, she said, “I certainly feel more comfortable and confident with this material than any that I have had in the past, but I don’t feel too confident.” Although she enjoys mathematics (Mia is the only student in the study who exhibited intrinsic motivation), her confidence level is still mid-range. The reason for that is most likely because she had to build her confidence up from so low when she started college; her confidence has gone from almost non-existent to “probably a five or six on a confidence scale of one-to-ten.”

Joe claimed that he has never excelled in his mathematics courses, nor has he done extremely poorly. He said, “I’ve always been sort of, I haven’t been great at math, but I’ve always been alright at math.” His confidence level is mid-range.

### **High level of competence.**

Sophie, Melissa, Chris, and Lydia all expressed a very high confidence level; they were all confident that they were able to learn the material in Math 2015. Lydia will provide an illustrative example of a student with a very high confidence level.

Lydia was extremely confident in her ability to learn the material in her current mathematics course. She was so confident because, “Up until now I’ve had no problem. Revolutions are iffy. But overall I feel fairly confident. I know my math fairly well, I grew up

with math, and I have a background in this information, so I'm not too worried about my grade.” Lydia’s father is a mathematics teacher and she grew up with his influence on her mathematics education. She also took a calculus course in high school, which meant that most of the material in Math 2015 was review for Lydia; she had learned the material in a previous course. However, even with material that was new to her – such as finding the volume of a solid generated by revolutions about an axis – she is still confident in her ability to learn that new material. Lydia’s confidence in her ability to learn that mathematical content was very high.

### **Students’ General Autonomy**

Autonomy measures the degree to which an individual feels they have freedom within an activity, especially the freedom to make meaningful choices concerning that activity (Ryan & Deci, 2000). Based on the interview data, the students expressed feeling varying levels of freedom. Of the ten students interviewed, nine in the HJ section and one in the Non-HJ section, there were only two students who did not feel any sense of freedom. The other eight students felt at least some degree of freedom.

#### **No sense of autonomy.**

Two out of the ten students interviewed, Melissa and Joe, did not feel any level of freedom in the course. Melissa said, “I would have to say no, [I don’t feel freedom]. I don’t think it’s our professor’s fault or anything, I don’t think she’s trying to, like, trap us.” Melissa then gave a number of examples of course activities that she felt confined her; she felt forced to engage in those activities in a certain way. The first example she gave was completing homework assignments. Melissa explained that the homework assignments were due at the beginning of the week, every week. “Considering [her] other classes, [she] really doesn’t feel like [she] has a huge amount of freedom in terms of when [she] does it.” Melissa explained that

there are certain days when she has to do her mathematics homework each week and even though she has an entire week to complete each assignment, she has limited options and availability. Melissa also explained that when studying for exams, the instructor gives students past exams as examples to study from. She said that when studying for exams, “I don’t feel like I have any other options than to do exactly that.” She believes that studying past exams provided by her instructor is the best study technique and does not believe there are any other options for studying. Melissa summarized her feelings by saying, “I don’t have any complaints, but I don’t feel like I have a lot of options to do things differently either.” Joe agrees. He also did not experience freedom in his current mathematics course. He said, “There’s definitely set ways in which we’re supposed to solve problems and do homework. I guess it’s pretty strict.” Both Melissa and Joe felt that there were not options for choice in the course and therefore did not experience a sense of autonomy.

**Some sense of autonomy.**

The other eight students interviewed experienced a sense of freedom to some degree. Kelly and Sophie provide illustrative examples. Kelly said, “I know [our instructor] gives us lots of different opportunities.” She then gave specific examples of opportunities for choice. One example she gave was that students “can come to class the day of the test or not come to class the day of test for extra review.” The instructor of the course offered students a choice about attending class for an extra review session the class meeting directly before the test. Another example Kelly gave was that the instructor allowed students to solve problems in different ways. She felt like there was choice in determining how she wanted to solve problems. A third example of an opportunity for choice that Kelly gave was when she wanted to do her homework. Kelly said, “[Our instructor] gives us a week to do our homework, so obviously we have freedom

in that.” Contrary to Melissa and her busy schedule, Kelly appreciated that homework was only due once a week and thought that this afforded her freedom. Based on these examples, Kelly did experience a sense of autonomy. Sophie gave similar examples of opportunities for meaningful choice. She also commented that the class meeting directly before each test was optional. She also commented on the homework journals, saying that they gave students an opportunity for meaningful choice: “We have freedom with our homework and stuff because we can choose what we want to write about in our homework journals and everything.” A third example Sophie gave was visiting her instructor’s open office hours. She said she felt like she had the freedom to visit open office hours and to ask whatever she wanted during open office hours. Her overall summary of the course was this: “I would say that I feel like I have freedom.” Both Kelly and Sophie exemplify the notion that there are some meaningful choices that students can make in this mathematics course. They were aware of the choices they could make and gave specific examples of such choices.

### **Students’ General Relatedness**

Relatedness measures the degree to which an individual feels they are related to or close to others participating in a particular activity (Ryan & Deci, 2000). All ten students interviewed, the nine students in the HJ section and the one student in the Non-HJ section, described a high level of general relatedness in their sections of Math 20215. Evidence from the interviews will demonstrate how the students experienced a high level of relatedness with both their classmates and their instructor.

#### **Relatedness with classmates.**

Kelly and Melissa describe how the instructor asked students to meet those sitting around them on the first day of class. The instructor’s purpose was to help students meet others in the

class so that they could work with them on class assignments, study with them for tests and quizzes, and call them for notes and class announcements in case they ever missed a class.

Melissa explained, “I actually feel really comfortable in the class. I think it’s because on the first day she had us meet the people around us, the people next to us, and that’s really helpful when professors do that because you can go through, like, a whole semester and not even have anybody to ask if you missed class for something.” Having students intentionally meet those around them helped them make friends in the class and have others in class that they were comfortable with. Kelly elaborated, “I met people the first day and they are actually in my major. So we always sit together and we’ll do the labs together, because we can work in groups. And we help each other with homework if we don’t understand. I like that.” Kelly is very comfortable in the class because she met other students who shared a major with her on the first day of class. These students help provide support for Kelly, both with in-class assignments, out-of-class assignments, homework assignments, and studying for tests.

While some students made friends on the first day of class, others came into the class already knowing a few of their classmates. Nate, the student from the Non-HJ section, explained, “In the class itself, I know a couple of girls in there I already knew from last year, so they’re kind of like in my group...everybody in the class is really nice.” He already knew some of the other students in the class and this helped him feel comfortable. Two of the students who participated in the interview are members of the university’s soccer team. Chris and Joe knew each other before the start of class because they were teammates on the soccer team. Chris exclaimed, “I have two other people on the soccer team and another guy that played on the soccer team before, so I’m pretty comfortable with the people I’m surrounded by in class.” Similarly, Joe was also comfortable with and supported by his teammates: “Two other guys in

the class are on the same soccer team, so we can sort of push each other along.” Nate, Chris, and Joe are all examples of students who already knew some of their classmates before the course started. This helped them feel comfortable with and related to their classmates.

**Relatedness with instructor.**

The students in both the HJ section and the Non-HJ section also felt a high level of general relatedness with their instructor. All ten students thought their instructor was kind, helpful, supportive, approachable, and encouraging. The students especially commented on two specific behaviors that made them feel such a high level of relatedness: their instructor was willing to help by answering their questions and their instructor took time to get to know them.

The course instructor was described to be extremely approachable and helpful, especially when students had questions. Mia said, “She listens to students, when we don’t understand something she’s willing to go back over it, which is really nice.” Ann made a similar comment: “She’s so nice! She’s ridiculously helpful!” She then told a story about how she usually had to come in at special times to take all of her tests because of unavoidable conflicts. Although this was not convenient for her instructor, she was very accommodating and willing to help. Ann later added that she could also go to her instructor for help at any time. “We always go to her and ask her for help. Like, she’s really nice.” Whenever Ann does ask her instructor for help, she is very patient and encouraging. Melissa was also very comfortable with her course instructor and thought she was very approachable. She said, “As far as the teacher goes, I think she’s a really sweet lady. I don’t think I’ve gone in [to office hours] to ask her anything, but I don’t think I would hesitate if I needed to.” The students have a high level of relatedness because they feel supported by their instructor by her willingness to answer questions and help in any way that she can.

The students also have a high level of relatedness because their instructor is very personable and takes time to get to know her students. Lydia described a time when she had a fifteen-minute conversation with her instructor after class; she said this was a great experience and really appreciated the time her instructor took to simply talk with her. Chris, one of the members of the soccer team, said, “She actually follows how we’re doing. She knows us by our names and I feel like I have a personal relationship with our teacher. That really helps out motivation-wise, wanting to do my work and going to class, too, because it just kind of shows respect and everything.” Chris was thankful that his instructor followed the progress and schedule of the soccer team and regularly asked them how they were doing. Both Nate and Joe also commented on how nice it was that their instructor knew them by name. Nate, a sophomore, claimed that only two other faculty members knew him by first name. “[My instructor] knows my name, she knows I’m a nice kid! It’s just kind of nice to know that my professor knows who I am. I’ve actually only had two other professors here at Tech who actually know me, so I like that a lot.” The fact that his instructor knew him by name and knew him personally helped Nate feel a sense of relatedness. Joe said something similar, “I think she knows all of us by first name basis and she takes an interest in us, which helps as well.” Being known is one of the ways to meet the need for relatedness and even something as simple as knowing her students’ names helps meet that need.

### **Students’ General Attitudes Towards Writing**

#### **HJ section.**

#### ***Previous experiences with writing in mathematics.***

Writing in mathematics was a new and slightly strange experience for most of the students in the HJ section. Of the nine students from the HJ section who participated in the

interview, only three of them had ever had an intentional previous experience with writing in mathematics, Ann, Kelly, and Sophie. The writing experiences of these three students varied in terms of both the purpose and the format of the writing assignment.

When she was in middle school, Ann was involved in an after-school program called “Basics to Beyond” and she was confident that some of the projects she completed as a part of that program required her to write about mathematics. Unfortunately, Ann could not remember specifically what those projects involved. She only remembered that she completed writing assignments as a part of that program; she did not remember any of the details of those writing assignments. Sophie also had a writing experience in middle school. Sophie’s writing experience was a regular part of her mathematics class; there were weekly writing assignments. She described the experience in this way: “My favorite math teacher that I had, throughout school, made us do a weekly writing. There was always a review question on the board on Monday and we’d have to turn it in on Friday. One that I had, for PEMDAS, we had to, like, create our own, our own words to go with it and make a book for it. I remember that one.” Sophie enjoyed this experience with writing in mathematics, mostly because it helped her relate to the mathematical content of her class that week.

As a student in a high school calculus course, Kelly was asked to write a paper on the quadratic formula. She described the experience: “My professor in my calculus class made us write a paper on the quadratic formula. Which I thought was kind of ridiculous because it was a calculus class and we didn’t use that as much in calculus. And when he graded them, he would tell us one thing and then when we graded it, he would be extremely harsh and grade for things he hadn’t told us. So it kind of frustrated me more than anything.” From this quote, it is clear that Kelly did not enjoy this previous experience; writing in mathematics was frustrating for

Kelly. However, it is important to make the following distinction: Kelly was not opposed to the idea of writing in mathematics in general, she was opposed to certain aspects of writing in mathematics. One aspect that Kelly was opposed to was the relevance of the writing assignment. Kelly didn't think that the quadratic formula was relevant to her studies in calculus. When I asked her if she would have a different perspective on writing if the topic of the paper had been more relevant to calculus, such as limits or derivatives, she said, "Yeah, I probably would have liked that." The other aspect that Kelly was opposed to was the imbalance between the expectations communicated to students and the way her teacher graded. She felt like her teacher communicated his expectations in one way, but then graded differently. This example emphasizes how important it is for teachers to communicate clear expectations about writing to their students and to follow through with those expectations.

The other six students (Mia, Melissa, Lydia, Joe, Chris, and Adele) had never had an intentional previous experience with writing in mathematics. Two students, Adele and Chris, described being asked to explain how they got a particular answer on a homework assignment, quiz, or test in previous mathematics courses. Two other students, Melissa and Lydia, described writing solutions to word problems in previous mathematics courses. However, none of these six students had ever completed a specific writing assignment in a previous mathematics course. The experience of writing in mathematics was entirely new to them. Even Kelly, one of the three students who did have a previous experience with writing in mathematics, thought that writing in mathematics was "a new concept". She said, "Before my calculus class [in high school], I'd never heard of it before and I thought it was very odd. So I never saw the two as going hand in hand, I don't really know what my opinion would be other than that. It's a new concept to me." Although Kelly did experience a writing assignment in high school, writing in

mathematics was still a new idea to her. It doesn't seem like she could quite wrap her head around the idea and many of the other students shared that opinion.

***Belief that writing is not applicable in mathematics.***

One of the reasons that writing in mathematics is such a new concept to the students in this mathematics course is because of the belief that writing typically “belongs” in other subjects, such as English or History. Adele specifically said, “I think of writing assignments belonging in English, not math courses.” Mia also described a dislike of having to write in a mathematics course. When asked about her attitude towards writing in mathematics, she said, “I don't like it. And that's my first gut reaction. I guess the fun for me, when it comes to math, is the problem solving. The actual having different parts so that when you put them together it makes sense to get the solution.” Mia, the one student who expressed intrinsic regulation towards this particular mathematics course, loves mathematics because it is like a puzzle. Writing about mathematics is not as enjoyable for Mia as actually doing mathematics. Students seem to believe that writing is not applicable to mathematics courses. Mathematics courses should include activities like solving problems instead of writing. Writing, on the other hand, is applicable to other classes, such as English or History.

***Belief that writing takes up too much time.***

In addition to whether or not writing is applicable in a mathematics course, students were also concerned about the amount of time required to complete a writing assignment. Students do not want to complete a writing assignment that takes a great deal of time. About writing in mathematics, Ann said, “As long as it doesn't take very much longer than a regular homework would, it wouldn't really be a bother or anything.” Ann clearly does not want to spend any additional time on her homework; only if a writing assignment does not take longer than a

typical homework assignment, would she want to engage in such an assignment. When asked whether or not she would want to participate in a writing assignment in a future mathematics course, she responded, “If it were something that doesn’t take up much more time, you might as well.” While she would be willing to participate in a writing assignment in a future mathematics course, her answer is based on the time it takes to complete the assignment, not on the value of the assignment. Ann is very focused on the time it takes to complete assignments, whether that is a regular homework assignment or a writing assignment. Kelly also focused on the time it takes to complete assignments. When Kelly was asked the same question, whether or not she would want to participate in a writing assignment in a future mathematics course, she responded, “It depends on the amount [of writing]. If it were a lot, I don’t know how I would feel about that.” Similar to Ann, her answer is not based on any value of a writing assignment, her answer is based on how much time that assignment would take. Students expressed concern over the amount of time and energy that would be required to expend to complete a writing assignment in a mathematics course.

***Belief that writing is only helpful for certain types of people.***

While some students expressed a dislike of or a concern about writing in a mathematics course, others were thankful for an opportunity to do something they were “good at” in a mathematics course. Students seem to polarize themselves as either a “math person” or “not a math person.” Melissa even specifically called herself an “English-based person instead of a math-based person.” Melissa explained that her score on the last test in Math 2015 was well above the class’ average score, although, “[she] has to work really hard to do well in math.” Because she has to work so hard, spending a great deal of time completing homework assignments and studying for tests, she does not think of herself as naturally “good at”

mathematics. What students imply by comments like “being a math person” or “being a math-based person” is an inherent ability to understand mathematics easily and earn high grades in mathematics courses with little effort or studying. When students claim to be “not a math person” they typically enjoy the opportunity to incorporate writing into a mathematics course. Ann, another student who does not believe she is a “math person,” said, “I’m not very good at math!” She then went on to say, “At least I’m good at something! I can write!” She is excited about writing in a mathematics course because writing is something she is “good at” instead of doing mathematics. These students also claim that writing in mathematics helps them understand the mathematical content more. Melissa said, “For somebody like me who is more of an English-based person instead of a math-based person, it makes you understand better.” Ann also said that for people in the class, like her, “who are more towards writing than they are towards math, it might help them more.” For students who are not “math people,” but who are “writing people,” writing in mathematics seems to help these students enjoy mathematics courses more and understand the material better.

#### **Non-HJ section.**

##### ***Previous experiences with writing in mathematics.***

Nate, the only student interviewed from the Non-HJ section had never had an intentional previous experience with writing in mathematics that he can remember. He did comment on his high school experience: “My high school experimented with that a little bit, sometimes they would have an open response. So not really, but somewhat, I guess.” Nate seems to be unsure about what is and what is not a writing assignment in mathematics. He assumes that problems on exams that are open-ended, which he describes as any problem that is not a multiple-choice

problem, involves writing in mathematics. While problems like these do involve writing, it is not the type of purposeful and intentional writing being studied.

Although Nate has never engaged in a writing assignment, I asked him to think about what writing in mathematics would be like. I asked him how he would feel about writing in mathematics and he responded, “It’s kind of strange because when you think of math, you think of one, two, three, four, not a, b, c, d. So it’s like, a little bit different. I have always been the kind of person that’s good with just, say if they give you the actual problem. I hate word problems. So that’s something, like when I have to write my own word problems, pretty much I don’t really enjoy it.” Writing in mathematics is a new and strange concept for Nate. One of the reasons that this is new and strange is simply because of his lack of experience. Another reason that this is new and strange is because he associates mathematics with numbers and not letters. He then recalls a previous experience that he does associate with writing in mathematics: writing and solving word problems. Nate describes himself as excelling at simply solving mathematics problems, what he calls “the actual problems.” What he means by this statement are bare-boned mathematical problems that are removed from any type of context. For example, an “actual problem” might ask a student to solve an integral or to find the area between two curves. Nate enjoys problems like these, but does not enjoy word problems, where mathematical problems are set within a context or situation. Again, while word problems do involve some kind of writing, they do not involve the type of purposeful and intentional writing being studied.

***Potential benefits of writing in mathematics.***

I also asked Nate what he thought might be potential benefits of writing in mathematics. He thoughtfully responded, “I know it definitely helps me understand, when you actually write something out it means you understand it, if you put it into words it means you actually

understand it. So that's my problem in math, I don't actually understand it, I just understand the steps." One benefit that Nate mentions is that when you write about something, it means that you fully understand it. Nate also unintentionally describes the difference between a procedural understanding of mathematics and a conceptual understanding of mathematics. He confesses that most of the time, he knows how to solve a problem, he has a procedural understanding, but he does not know why that works or the concepts behind the solution of that problem, he lacks a conceptual understanding. Nate implies that engaging in writing in mathematics would help students gain the conceptual understanding and not just the procedural understanding related to a particular mathematical problem. Nate goes on to say that another potential benefit of writing in mathematics is "Comprehending the material better 'cause you're putting it out in a different format so it's definitely another way to learn, I don't even know what the word would be. It's just a different style. It's proactive learning." Again, he talks about understanding the material better, but he also describes how writing could help students use different representations and be actively engaged in learning. Even though Nate has never personally participated in a writing assignment in mathematics, he does see multiple benefits to writing in mathematics, such as gaining both procedural and conceptual understanding, using multiple representations, and being actively engaged in learning.

***Personal disposition towards writing in mathematics.***

However, even though Nate believes that writing could bring about these very important and powerful benefits for students, he personally would not like to write in a mathematics course. He said, "I think it's a good idea, I just dislike the idea of having to do it because it's just different from what I've always learned." Nate believes writing in mathematics is a wonderful idea that has the potential to be very beneficial, but not for him. The reason he would not want

to engage in writing in mathematics is because it is so different from what he is accustomed to and expects in a mathematics course. This connects to Nate's initial remarks about writing in mathematics being a new and strange concept to him.

## Appendix F

### Pilot Study

#### **Introduction**

A pilot study was conducted during the summer of 2011. The pilot study was incredibly informative. The limitations of the pilot study as well as the results of the pilot study informed the dissertation study. I will briefly review the methods and results of the pilot study.

#### **Context**

The pilot study was conducted during a summer session of Math 1224, Vector Geometry. The primary purpose of Vector Geometry is to supplement content from differential calculus (Calculus I) and integral calculus (Calculus II) and to prepare students for multivariable calculus (Calculus III). Students engage in mathematical content including vectors, polar coordinates, parametric equations, and projectile motion. The secondary purpose of Vector Geometry is to encourage students to work in small groups of typically three or four students. Students gain an appreciation of the benefits of working in small groups and develop skills necessary for working cooperatively with peers. Students in Vector Geometry are typically mathematics or engineering majors; Vector Geometry is a required course for all mathematics and engineering majors.

#### **Participants**

There were 27 students enrolled in Vector Geometry at the time of the pilot study. The majority of the students in the class were mathematics or engineering majors ( $n = 17$ ). There were more male students ( $n = 24$ ) enrolled in Vector Geometry than female students ( $n = 3$ ). However, only 23 students took both the pre-course survey and the post-course survey, so only data from those 23 students were taken into consideration for the pilot study.

The participating instructor was a graduate student in mathematics education. He had two years of experience teaching middle school mathematics and four semesters of experience teaching Vector Geometry at the time of the pilot study. He was willing to participate in the pilot study by including homework journals in his course assignments because he values conceptual learning and teacher/student relationships.

### Procedures

I visited Math 1224 twice during the course, once on the first day of class and once on the last day of class. On the first day of class I administered a pre-course survey and on the last day of class I administered an identical post-course survey. All students in Math 1224 completed one homework journal each week throughout the course.

### Results

Based on data from a pilot study, Cronbach's alpha was computed for both the pre-course survey and the post-course survey (see Table G1).

Table G1

*A Comparison of Cronbach's  $\alpha$  from the Pilot Study*

	Pre-Course Survey	Post-Course Survey
Autonomy	0.77	0.71
Competence	0.88	0.86
Relatedness	0.65	0.75
Attitude toward writing	0.85	0.88

Quantitative data from the pilot study was used to compute means and standard deviations for each of the four sub-scales (autonomy, competence, relatedness, and attitudes

towards writing) for both the pre-course and post-course surveys (see Table G2). In addition to these descriptive statistics, differences in means for each sub-scale were also tested for statistical significance (see Table G2). For example, was there a significant difference between the means from the pre-course survey and the post-course survey for autonomy? Significant differences were tested at both the 0.05 level and the 0.10 level.

Table G2

*Descriptive Statistics from the Pilot Study*

	Pre-Course Survey		Post-Course Survey		Difference in Means
	Mean	SD	Mean	SD	
Autonomy	3.36	1.11	3.14	1.22	-0.22*
Competence	2.86	1.14	3.00	1.22	0.14
Relatedness	3.35	1.30	3.16	1.17	-0.19**
Attitude toward writing	3.81	1.30	3.71	1.38	-0.10

*Note:* \*  $p < 0.05$ , \*\*  $p < 0.10$

For each item on the surveys, students would respond with an answer between 1 and 6 (1 = strongly agree to 6 = strongly disagree). Therefore, if the difference in means between the pre-course survey and the post-course survey was a negative value for a particular sub-scale, then that subscale actually increased. The mean scores for autonomy significantly increased between the pre-course and the post-course survey ( $p < 0.05$ ). The mean scores for relatedness also increased between the pre-course and the post-course survey ( $p < 0.10$ ).

Qualitative data from the pilot study consisted only of an interview with the pilot study course instructor. The pilot study course instructor was asked about what benefits he thought

there were to homework journaling in Math 1224. He reported that there were benefits for him, as the course instructor, and benefits for the students.

One benefit for him was that the homework journals provided insight into what the students struggled with; they also provided feedback about which problems the students did and did not like. Another benefit for him was that even though the homework journals regularly took longer to grade than a standard homework assignment, “it was much better than reading ‘typical’ homework. Homework journals are narrow and deep instead of wide and shallow...which makes them seem like more than just an assessment; they are really informative.”

One benefit for the students was that they had the opportunity to ask questions about course content, but not in front of their peers. Another benefit for students was that the homework journal provided an opportunity for meaningful reflection, which the course instructor thought seemed to happen more towards the end of the semester. Even though not every student did meaningfully reflect for every homework journal, “even just the idea of it – for students to follow some train of thought is good.” A third benefit for students was that it was so easy for students to make connections between the content of Math 1224 and other mathematical topics and ideas. It was “especially easy for this class [Math 1224] because it is so closely connected with other calculus classes. Seeing connections is really good for students and it was great when they pointed out connections on their own.” Overall, the interview with the pilot study course instructor was informative in designing the methods for the actual study.

## Appendix G

## Annotated List of Figures

- Figure 2.1     A Typical Homework Journal (page 42)
- This figure is a concrete example of what a homework journal looks like and the types of problems and reflections that students will typically engage with when completing a homework journal.
- Figure 2.2     A Typical Homework Journal with Explicit Connections to the Tenets of Constructivism (page 46)
- This figure explicitly shows how a typical homework journal is related to two philosophical tenets of constructivism: being actively engaged in doing mathematics and being actively engaged in reflection.
- Figure 2.3     Self-Determination Continuum Showing Various Types of Motivation Along with Their Regulatory Styles (Ryan & Deci, 2000) (page 49)
- This figure illustrates the five different types of motivation, each accompanied by their own regulatory style as described by Ryan and Deci (2000). The five different types of motivation are arranged from left to right in terms of the degree to which the motivation is self-determined.

## Appendix H

## Institutional Review Board Approval Letter



VirginiaTech

Office of Research Compliance  
 Institutional Review Board  
 2000 Kraft Drive, Suite 2000 (0497)  
 Blacksburg, Virginia 24060  
 540/231-4606 Fax 540/231-0959  
 e-mail irb@vt.edu  
 Website: www.irb.vt.edu

**MEMORANDUM****DATE:** August 26, 2011**TO:** Jesse L. Wilkins, Alexis Johnston**FROM:** Virginia Tech Institutional Review Board (FWA00000572, expires May 31, 2014)**PROTOCOL TITLE:** Homework Journaling in Undergraduate Mathematics**IRB NUMBER:** 11-661

Effective August 26, 2011, the Virginia Tech IRB Chair, Dr. David M. Moore, approved the amendment request for the above-mentioned research protocol.

This approval provides permission to begin the human subject activities outlined in the IRB-approved protocol and supporting documents.

Plans to deviate from the approved protocol and/or supporting documents must be submitted to the IRB as an amendment request and approved by the IRB prior to the implementation of any changes, regardless of how minor, except where necessary to eliminate apparent immediate hazards to the subjects. Report promptly to the IRB any injuries or other unanticipated or adverse events involving risks or harms to human research subjects or others.

All investigators (listed above) are required to comply with the researcher requirements outlined at <http://www.irb.vt.edu/pages/responsibilities.htm> (please review before the commencement of your research).

**PROTOCOL INFORMATION:**

Approved as: **Expedited, under 45 CFR 46.110 category(ies) 5, 6, 7**

Protocol Approval Date: **8/22/2011**

Protocol Expiration Date: **8/21/2012**

Continuing Review Due Date\*: **8/7/2012**

\*Date a Continuing Review application is due to the IRB office if human subject activities covered under this protocol, including data analysis, are to continue beyond the Protocol Expiration Date.

**FEDERALLY FUNDED RESEARCH REQUIREMENTS:**

Per federal regulations, 45 CFR 46.103(f), the IRB is required to compare all federally funded grant proposals / work statements to the IRB protocol(s) which cover the human research activities included in the proposal / work statement before funds are released. Note that this requirement does not apply to Exempt and Interim IRB protocols, or grants for which VT is not the primary awardee.

The table on the following page indicates whether grant proposals are related to this IRB protocol, and which of the listed proposals, if any, have been compared to this IRB protocol, if required.

*Invent the Future*

Date*	OSP Number	Sponsor	Grant Comparison Conducted?

\*Date this proposal number was compared, assessed as not requiring comparison, or comparison information was revised.

If this IRB protocol is to cover any other grant proposals, please contact the IRB office ([irbadmin@vt.edu](mailto:irbadmin@vt.edu)) immediately.

cc: File