CHAPTER 3
DOMINANCE BASED MEASUREMENT OF ENVIRONMENTAL PERFORMANCE AND PRODUCTIVE EFFICIENCY OF MANUFACTURING

3.1 Introduction

There were three criteria for the choice of method that formed the basis for the proposed method. These were:

1. Both productive efficiency as well as environmental efficiency need to be considered.
2. Data used in performance measures should be available from existing manufacturing information systems.
3. Performance measures should be based on data rather than a predictive model.

While the measurement of environmental performance is the focus of this research, an assumption of the research is that improvements in environmental performance should be made in the most cost effective way possible. Productive efficiency as evaluated in this research may also be called resource efficiency. Improvements in productive efficiency typically result in the cost of operations being reduced. This translates into improved profitability. Productive efficiency may therefore be used as a proxy for considering the overall business objective of increasing profitability. Of course, there are many other factors that affect profits, but improvements in productive efficiency are often the objective at the operational level in a manufacturing facility.

The many techniques for measuring productive efficiency are reviewed in Chapter 2. These methods only require information on inputs and outputs to be applied to a production system. This is information that is often available to a manufacturing facility and so is consistent with the second requirement that information be available from existing information systems. There are four basic approaches to the measurement of productive efficiency: Data Envelopment Analysis (based on linear programming); parametric (based on linear regression); Productivity Index (based on ratios or input and output indexes); FDH (based on the convexity assumption for the frontier being relaxed); and Benchmark Correspondence (based on dominance). Both the DEA and parametric approaches use observed production plans to define a reference technology against which particular observations may be compared to determine productive efficiency. Productivity Index methods are based on the ratio of the measurement of distance between production plans with one of the plans representing the reference technology. FDH defines a production frontier like DEA methods, but with the least restrictive assumptions. The Benchmark Correspondence method is based on the measurement of distance between two production plans, but with the selection of the reference production plan based on dominance.

Methods for defining the production frontier (also called the reference technology) depend on making different assumptions about the form of the production frontier. The less restrictive the assumptions, the more closely the frontier matches actual production plans. The advantage of more closely matching the production frontier to actual production plans is that measurements of efficiency are less subject to variation caused by the assumptions with respect to the shape of the production frontier and the choice of distance measure. The disadvantage is that an underlying model of the production technology, represented by the production frontier, is not being defined. As a result, methods that are less restrictive on the form of the production frontier (i.e., more closely match the data) tend to be more appropriate for performance measurements while methods that are more restrictive on the form of the production frontier (i.e., attempt to capture the underlying nature of the production technology) may be more appropriate for predictions.

The concept of dominance is an important aspect of frontier methods. The frontier represents the set of production plans that dominate other production plans not on the frontier. When metrics of efficiency are no longer based on a frontier, but on particular production plans, then dominance is the sole criterion that is applied to determine the set of production plans that are the reference against which measurements of efficiency are made. A dominant production plan simply uses less inputs than other production plans for given levels of outputs; or produces more outputs for given levels of inputs (Koopmans, 1951). This criterion then determines a reference set that measures how efficiently a production system converts inputs to outputs. The method called Benchmark Correspondence eliminates a frontier altogether and simply defines reference production plans based on dominance (Tulkens and Vanden Eeckaut, 1991, 1995, 1995a). Measures of performance are based on pairwise distance between a production plan and a benchmark (or reference) production plan. The Benchmark Correspondence meets the three criteria described above for a method of measuring environmental performance.

The Productivity Index methods in many respects meet the three criteria defined above. However, the Benchmark Correspondence method, as modified in this research, provides more detailed information at the operational level. Productivity Indexes simply provide a single metric of performance which is useful when comparing different production systems, but is not very informative when applied to measure the performance of a single manufacturing system.

In addition to being the least restrictive method in terms of the definition of the reference production plans (with the exception of Productivity Index methods), there are a number of other reasons that the Benchmark Correspondence method is chosen as the basis for this research. The nature of the data set being evaluated makes the Benchmark Correspondence method a more appropriate choice than many other methods. There is no limit on the number of inputs and outputs that can be potentially defined or specified when using the Benchmark Correspondence approach. This is because the method evaluates efficiency based on the direct comparison of production plans. DEA and parametric methods are in principle not limited in the number of inputs and outputs.

However, there is a practical limit based on ill-conditioning created by a large number of input/output variables.[1] For the measurement of productive efficiency using actual manufacturing data, large numbers of inputs and outputs are the norm. Aggregation of these inputs and outputs is a significant problem and can seriously bias results in DEA analysis (Thomas, 1994) (Primont, 1993).

Although the Benchmark Correspondence method does not define a frontier the basic elements of the measurement of productive efficiency are retained. These are the definition of reference production plans (actual data is used rather than a mathematically defined production frontier); and the measurement of productive efficiency based on the distance of a production plan to a reference production plan. The next section defines the variations on the Benchmark Correspondence method along with the notation. Then examples of sets are provided in Section 3.3 followed by a description of the metrics in Section 3.4.

3.2 Variations on the Benchmark Correspondence Method

The Benchmark Correspondence method is modified in three ways that are each described in the following sections. The actual application is based on the assignment of production plans to sets. Particular set assignments have particular meanings in terms of productive efficiency and environmental performance. Once production plans are assigned to sets, reference production plans may be selected and quantitative measures based on distance and set counts may be calculated. The application is described in more detail in Chapter 4. Figure 3-1 provides an overview of the different set partitions for the production plans. The standard sets defined as part of the Benchmark Correspondence method are shown on the left side of Figure 3-1.

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[1] Ill-conditioning occurs when there are so many inputs and outputs that all or almost all production plans define the production frontier. The result is that all or almost all production plans are judged to be efficient with little or no information being generated on relative performance.

The first modification to the Benchmark Correspondence Method changes the reference production plan being used as the basis for partitions to the production plan at time t. The set definitions of Tulkens and Vanden Eeckaut (1995) are the same, but different production plans now make-up the sets D_g (Dominating), D_d (Dominated), and D_i (Dominance Indifferent) for the proposed method. These sets are shown on the right side of Figure 3-1. The addition of undesirable outputs is another partition for all of the production plans and produces the sets TE (Technically Dominating - Preferred Environmentally), TI (Technically Dominating - Non-preferred Environmentally), and TU (Technically and Environmentally Dominance Indifferent). The set TU is further partitioned by ordinal ranking of inputs and outputs based on relative environmental preference into the sets SE (Substitution Environmental Performance - Dominating) and SI (Substitution Environmental Performance - Dominated). Note that production plans may remain in the set TU. The next section reviews the notation used. Then, each of the three modifications to the Benchmark Correspondence method are discussed in more detail.

3.2.1 *Notation and Dominance*

The notation used follows Färe, Grosskopf, Lovell (1994) and Tulkens, Vanden Eeckaut (1995). For a single manufacturing firm all possible inputs and outputs are part of the sets $\Re_+^I$ and $\Re_+^J$, respectively. Let $\mathbf{x} = (x_1,..,x_i, \ldots x_I) \in \Re_+^I$ denote the quantities of I inputs and let $\mathbf{u} = (u_1,\ldots,u_j, \ldots u_J) \in \Re_+^J$ denote the quantities of J outputs. A pair of these vectors $(\mathbf{x,u})$ is a production plan. There are t = 1, 2, … T production plans, where T is the number of observation periods.

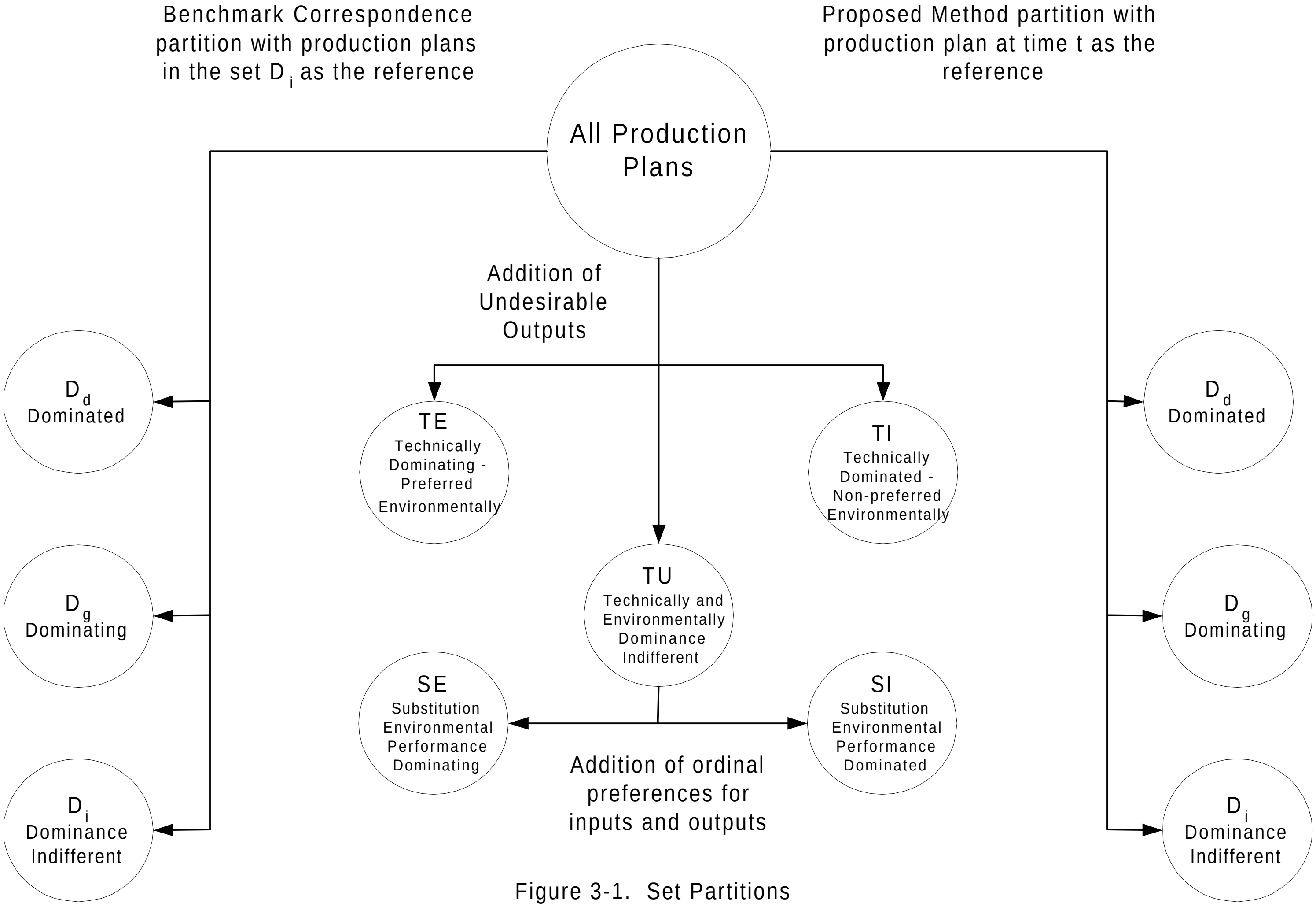

Figure 3-1. Set Partitions

A production technology is represented by the output correspondence set P(**x**) or the input correspondence set L(u).

$$P(\mathbf{x}) = \{\mathbf{u} : \mathbf{x} \in L(\mathbf{u})\} \qquad \text{(Eq. 3-1)}$$

$$L(\mathbf{u}) = \{\mathbf{x} : \mathbf{u} \in P(\mathbf{x})\} \qquad \text{(Eq. 3-2)}$$

The Input Correspondence Set, L(**u**), represents all input vectors $\mathbf{x} \in \Re_+^I$ that yield at least output vector $\mathbf{u} \in \Re_+^J$. The Output Correspondence Set, P(**x**), represents all output vectors $\mathbf{u} \in \Re_+^J$ that are obtainable from the input vector $\mathbf{x} \in \Re_+^I$. The sets P(**x**) and L(**u**) represent the feasible sets of outputs and inputs for a particular technology. The measurement of efficiency, however, depends on the definition of the subsets of P(**x**) and L(**u**) that represents the production frontier or reference technology. The sets P(**x**) and L(**u**) are partitioned according to the relative properties of the particular production plans.

3.2.2 *Change in the Benchmark Correspondence Algorithm*

The proposed method uses the same definitions of the sets D_g (dominating), D_d (dominated), D_i (dominance indifferent) as the Benchmark Correspondence method (refer to Section 2.2.3.1), but partitions the production plans using a different algorithm. In the proposed method at each time t in the time series of production plans all previous production plans are partitioned into the dominating, dominant, and dominance indifferent sets with the production plan at time t as the reference. This change in the algorithm is desirable for performance measurement since more information is provided on the production plan at time t

In contrast, the Benchmark Correspondence method evaluates each production plan in the time series compared to all of the production plans previously defined to be dominance indifferent. The first production plan in the time series is always dominance indifferent. The second production plan in the time series is then compared to this first plan. If it is dominant indifferent it is added to the set of dominance indifferent production plans. At each time t the production plan is compared to all previous production plans placed in the dominance indifferent set. Only if a production plan either dominates or is dominated by all of the production plans in the dominance indifferent set is a measure of progress or regress possible.

Improvement in productive efficiency may also represent improvement in environmental performance since less input is being used to produce the same or more product. However, if undesirable outputs are being increased then this is not an improvement in environmental performance. The addition of undesirable outputs allows a production plan to be judged to be an improvement or not in environmental performance.

3.2.3 *Addition of Undesirable Outputs*

The second variation on the Benchmark Correspondence method is to add undesirable outputs to the dominance criteria for inputs and product outputs. The result is that undesirable outputs must not increase for a production plan to be considered dominating and therefore an improvement in environmental performance. The sets defined by the inclusion of the undesirable outputs in the dominance criterion are: TE (Technically Dominating-Preferred Environmentally), TI (Technically Dominated-Non-preferred Environmentally) , and TU (Technically and Environmentally Dominance Indifferent). This variation requires that a production plan not only be an improvement in productive efficiency, but it must also at least not increase undesirable outputs to be considered dominating and part of the set TE.

In a variation on the dominance criteria, non-material inputs (like labor and capital) are excluded from the analysis of productive efficiency and environmental performance. This is because changes just in labor, capital, and other non-material inputs do not affect environmental performance. By removing these non-material inputs from consideration the measures of productive efficiency are more focused on changes that are also improvements in environmental performance.

The dominance criteria are based on increasing product outputs, reducing inputs, and reducing undesirable outputs. Let the first p outputs of the J outputs to be product outputs. The remaining outputs are undesirable. Two production plans are compared to determine if one is a member of the set, TE, according to the following definition.

Definition of Membership in the set, TE:

1. $(\mathbf{x}^r,\mathbf{u}^r)$ dominates $(\mathbf{x},\mathbf{u})$ in inputs if $(u_1^r,\ldots, u_p^r) \geqq (u_1,\ldots, u_p)$ and $(x_1^r,\ldots, x_I^r) \le (x_1,\ldots, x_I)$ and $(u_{p+1}^r,\ldots, u_J^r) \leqq (u_{p+1},\ldots, u_J)$.
2. $(\mathbf{x}^r,\mathbf{u}^r)$ dominates $(\mathbf{x},\mathbf{u})$ in outputs if $(u_1^r,\ldots, u_p^r) \ge (u_1,\ldots, u_p)$ and $(x_1^r,\ldots, x_I^r) \leqq (x_1,\ldots, x_I)$ and $(u_{p+1}^r,\ldots, u_J^r) \leqq (u_{p-1},\ldots, u_J)$.
3. $(\mathbf{x}^r,\mathbf{u}^r)$ dominates $(\mathbf{x},\mathbf{u})$ in undesirable outputs if $(u_1^r,\ldots, u_p^r) \geqq (u_1,\ldots, u_p)$ and $(x_1^r,\ldots, x_I^r) \leqq (x_1,\ldots, x_I)$ and $(u_{p+1}^r,\ldots, u_J^r) \le (u_{p-1},\ldots, u_J)$.
4. (x^r,u^r) dominates (x,u) if either (1) or (2) or (3) or all holds.

The comparison is between the same elements of the input and output vectors of two production plans with one being designated as $(\mathbf{x}^\mathbf{r},\mathbf{u}^\mathbf{r})$ and the other as $(\mathbf{x},\mathbf{u})$. The term $\geqq$ means "greater than or equal to" and the term $\leqq$ means "less than or equal to." The term $\le$ means "less than or equal to with at least one element of the vector $\mathbf{x}^\mathbf{r}$ or $\mathbf{u}^\mathbf{r}$ less than the corresponding element in the vector **x** or **u**, respectively ." The term $\ge$ means "greater than or equal to with at least one element of the vector $\mathbf{x}^\mathbf{r}$ or $\mathbf{u}^\mathbf{r}$ greater than the corresponding element in the vector **x** or **u**, respectively."

The membership in the sets TE (r), TI (r), and TU (r) are defined below in terms of set theory notation with (r) being the reference production plan. These definitions follow those provided by Tulkens and Vanden Eeckaut (1995), but with the inclusion of undesirable outputs. The notation differs from that for the sets defined Section 2.2.3.1 by the separation of product and undesirable outputs. Inputs are treated in the same way. The notation is as follows:

TI(r) is the set of production plans in the space that are weakly dominated by observation (**x**^{r}, **u**^{r})

TE(r) is the set of production plans in the space that are weakly dominating observation (**x**^{r}, **u**^{r})

TU(r) is the set of production plans in the space that are dominance indifferent and includes the observation (**x**^{r}, **u**^{r})

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$e_j^J, e_i^I, e_j^{J-P}, e_j^P$= J-dimensional, I dimensional, P dimensional, and (J-P) dimensional zero vectors where the subscript component (ith or jth) is equal to one with all other elements of the vector equal to zero.

O^I, O^J, O^P, $O^{I+(J-P)}$= I-dimensional, J-dimensional, I + (J - P)-dimensional, and P-dimensional zero vectors.

μ_j, v_i = scalars for all i and j.

I and J = Input and Output quantities, respectively.

P = Product outputs.

$\neg\begin{bmatrix} u^r \\ x^r \end{bmatrix}$ removes the reference production plan from the set.

TI (r) = (Eq. 3-3)

$$\left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in \Re_+^{I+J} \,\middle|\, \begin{bmatrix} u \\ x \end{bmatrix} = \begin{bmatrix} u^r \\ x^r \end{bmatrix} - \sum_{j=1}^{P} \mu_j \begin{bmatrix} e_j^P \\ O^{I+(J-P)} \end{bmatrix} + \sum_{j=P+1}^{J} \mu_j \begin{bmatrix} \begin{bmatrix} O^P \\ e_j^{J-P} \end{bmatrix} \\ O^I \end{bmatrix} + \sum_{i=1}^{I} \upsilon_i \begin{bmatrix} O^J \\ e_i^I \end{bmatrix}, \mu_j \geq O \forall j, \upsilon_i \geq O \forall i \right\} \neg \left\{ \begin{bmatrix} u^r \\ x^r \end{bmatrix} \right\}$$

TE (r) = (Eq. 3-4)

$$\left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in \Re_+^{I+J} \middle| \begin{bmatrix} u \\ x \end{bmatrix} = \begin{bmatrix} u^r \\ x^r \end{bmatrix} + \sum_{j=1}^{P} \mu_j \begin{bmatrix} e_j^P \\ O^{I+(J-P)} \end{bmatrix} - \sum_{j=P+1}^{J} \mu_j \begin{bmatrix} \begin{bmatrix} O^J \\ e_j^{J-P} \end{bmatrix} \\ O^I \end{bmatrix} - \sum_{i=1}^{I} \upsilon_i \begin{bmatrix} O^J \\ e_i^I \end{bmatrix}, \mu_j \geq O\forall_j, \upsilon_i \geq O\forall_i \right\} \neg \left\{ \begin{bmatrix} u^r \\ x^r \end{bmatrix} \right\}$$

TU (r) = $\left\{ \begin{bmatrix} \mathrm{u} \\ \mathrm{x} \end{bmatrix} \in \Re_+^{I+J} \neg \ (\mathrm{TI(r)} \cup \mathrm{TE(r)}) \right\}$ (Eq. 3-5)

Each of the sets, TE, TI, and TU are defined in the same way. The notation is discussed in terms of the definition of the set TE. The production plan, $\begin{bmatrix} u^r \\ x^r \end{bmatrix}$, is the basis of comparison for all other observed production plans in the space, $\Re_+^{I+J}$. The term, + $\sum_{j=1}^{P} \mu_j \begin{bmatrix} e_j^p \\ O^{I+(J-P)} \end{bmatrix}$, selects all of the production plans that have all product outputs (j,…,P) greater than or equal to the reference production plan, $\begin{bmatrix} u^r \\ x^r \end{bmatrix}$. The term, e_j^P, is a vector of dimension P with the, j, element equal to 1 and all other elements equal to 0. The term, $O^{I+(J-P)}$, is a vector of dimension, I+(J-P), with all elements equal to 0. The two vectors together form a single vector of dimension, I + J, equal to the total number of inputs (I) and outputs (J) for the production system. The sum is for j=1 to P and includes all product outputs. The sum of the vectors for products multiplied by the term, μ_j, produces: $\begin{bmatrix} \mu_1 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \mu_2 \\ 0 \\ . \\ . \\ 0 \end{bmatrix} + ... + \begin{bmatrix} 0 \\ . \\ \mu_P \\ . \\ . \\ 0 \end{bmatrix}$. If this sum is added to the reference production plan then all observed production plans with all outputs, j,…,P, greater than or equal to the reference may be included in the set. This is only one of three conditions for inclusion in the set TE. The remaining terms in the definition of the set, TE, require that both the undesirable outputs (j=P+1 to J) and the inputs (i=1 to I) be less than or equal to the reference production plan. The reference set itself, $\begin{bmatrix} u^r \\ x^r \end{bmatrix}$, is explicitly excluded from the sets TE and TI. The exclusion of the reference set means that there must be at least one input that is less than or one product output that is greater than or one undesirable output that is less than the corresponding product output, input, or undesirable output of the reference production plan to be placed in the set TE. This is how weak dominance is incorporated into the notation.

The set TI is defined in the same way as TE except that product outputs (j=1 to P) must be less than the reference or stay the same, the undesirable outputs must be greater than the reference or stay the same, and the inputs must be less than the reference or stay the same for a production plan to be part of the set TI. The set TU is simply defined as all production plans that are not members of the sets TE and TI.

3.2.4 *Changes in Input and Output Mix*

The third change in the Benchmark Correspondence Method is to include consideration of changes in input and output mix. Changes in input and output mix often have no affect on productive efficiency. However, such changes in input and output mix can be an improvement in environmental performance. Consideration of input and output mix is also an indirect way of evaluating how interconnected a production process is with other production processes. As discussed in Chapter 2 if a "waste" from one production process can be used in another then this is an improvement in terms of environmental performance. Inputs and outputs from Manufacture Inc. are categorized in terms of their environmental desirability. This categorization is then applied to determine which production plans have inferior or superior input and output mixes from an environmental performance perspective. The defined criteria are only applied to those production plans that are dominance indifferent with the consideration of undesirable outputs and, therefore, in the set TU. The two sets defined based on the relative desirability of input and output mixes are: SE (Substitution Environmental Performance - Dominating), and SI (Substitution Environmental Performance - Dominated).

Evaluating whether a substitution is desirable or not from an environmental perspective requires that the relative desirability of one input to another or one output to another be defined. This is analogous to the relative desirability of inputs and outputs that is the basis for the evaluation of economic efficiency. Practical methods for assigning weights (analogous to weights based on price in economic efficiency) are not available as discussed in Section 2.1. However, judgments can be made concerning the relative desirability of inputs and outputs. For example, non-material inputs, like labor and capital, are preferred to material inputs from an environmental performance perspective. This is because labor and capital do not have a direct environmental impact while material inputs must typically be mined or manufactured or processed in some way that results in pollution. Additionally, substitution of one material input that is environmentally preferable for another material input that is less environmentally preferable leads to an improvement in environmental performance.

The method that is described in this research is based on ordinal preferences. An increase in a preferred input (or output) that results in a decrease in a less preferred input (or output) results in an improvement in environmental performance. The advantage of this approach is that relatively little information about the relative desirability of inputs or outputs is required. The disadvantage is that there are situations where this ordinal approach can falsely show an increase in environmental performance. One potential difficulty in determining the relative desirability of input and output mixes is the rate at which change occurs. For example, determining the relative environmental desirability of 1 lb. of an input compared to 1 lb. of another input often requires extensive amounts of data or analysis. One lb. of an aqueous based solvent (say water and citric acid) is preferred to a petroleum based solvent (say kerosene). But, if 500 lbs. of the aqueous solvent are required in conjunction with heating and mechanical agitation (i.e., industrial washing machine) to clean a part as opposed to 1 lb. of the petroleum based solvent then the petroleum based solvent may be the better choice from an environmental perspective. Making this evaluation requires detailed life cycle cost analysis as described in Section 2.1.

Another limitation is that since only data typically available to a manufacturing facility is being used, environmental impacts associated with toxicity are not fully considered. A full consideration of toxicity, while certainly desirable, requires extensive analysis of environmental pathways, bioaccumulation, persistence, and health effects.[2] Much of this information is not readily available to manufacturers and in many instances it is not available at all. Distinctions are made for inputs and outputs that are regulated. Since regulation of particular pollutants is based on human toxicity (as well as other factors) an indirect, rough measure of toxicity is included.

The following classifications for inputs and outputs are applied to the data. This classification is illustrated in Figure 3-2. Other classifications are possible and will vary depending on the circumstances and what is to be measured. Outputs are classified in order of environmental preference based on the following distinctions:

- Primary Product: This is the output that the production process is primarily focused on producing.

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[2] These are the standard steps in determining the health effects of toxins in the environment. First, the path of the toxin from the source into the environment such as water bodies and the atmosphere must be determined. Then, the uptake and accumulation of these toxins into organisms or environmental sinks (e.g., sediments at the bottom of a river) are determined. The persistence of these toxins in the environment and in organisms must then be determined to find the level of exposure over time. Finally, health effects on humans are determined by a detailed analysis of direct exposure through the environment (e.g., drinking contaminated water) and indirect exposures (e.g., eating contaminated fish) along with any known or estimated health effects from the toxin.

- Recycled Output: This includes all outputs that are eventually used as inputs for other production processes. Categories of recycled outputs taken from Lave, Hendrickson, and McMichael (1994, p. 19A) are: waste to energy incineration, low value use (e.g., polystyrene cups made into park benches), high-value use (e.g., aluminum cans used to make new ones), re-manufacture (e.g., automobile water pumps), and re-used (e.g., refillable beverage bottles).

- Waste Discharge: These are discharges that must be disposed of in an environmentally acceptable manner (typically land disposal) to meet regulatory requirements.

- Permitted Discharge: These are waste discharges to the environment that are permitted by regulation. Treatment before release is often required. The treatment process itself can generate waste and can often result in the transfer of pollutants from one media (e.g., air) to another (e.g., scrubber sludge). A Permitted Discharge is less desirable than a Waste Discharge since waste is discharged directly to the environment. Environmental regulation often has the ultimate effect of moving a waste from a Permitted Discharge to a Waste Discharge.

Inputs may also be divided into classes depending on their environmental impact. In order of preference these classifications are:

- Non-Material Inputs: This category includes capital, labor, and overhead costs.

- Recycled Inputs: Input taken directly, or with some processing, from another production process that are not Primary Products.

- Renewable Inputs: Inputs from renewable resources. Renewable resources are those resources that can increase (or decrease) such as timber, fish, and grain. Whether or not a resource is renewable also depends upon how it is managed. Over harvesting of a fishery, for example, will eventually result in a collapse of the resource to a level from which it may not recover. Renewable inputs also include renewable energy resources such as solar, wind, and geothermal.

- Nonrenewable Inputs: Inputs such as oil or ores that are fixed in the quantity available on the planet.

Distinctions among inputs reflect their relative environmental impact. For example, an input that is recycled from another process (e.g., scrap steel) is preferable from an environmental impact perspective to the equivalent input from a raw material that is non-renewable (e.g., iron ore). Non-material inputs are the most preferred from an environmental impact perspective. Recycled inputs are expected to be the most preferred material inputs followed by renewable inputs. It is desirable to reduce non-renewable inputs where possible.

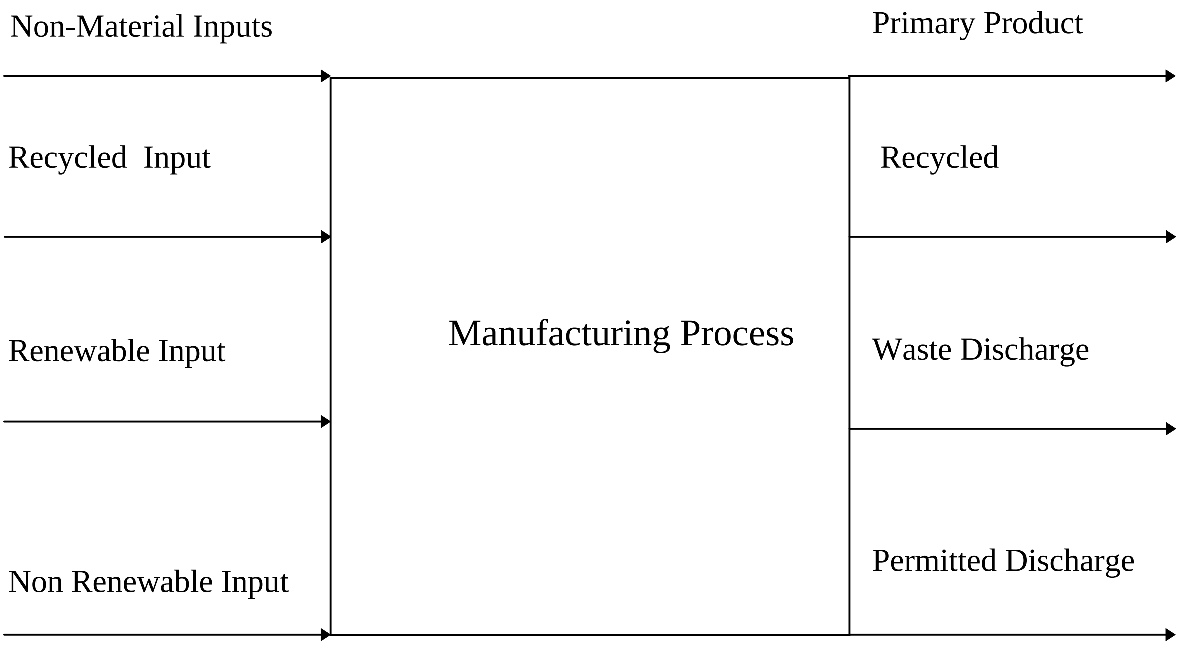

Figure 3-2. Input and Output Categories

The changes among the categories of inputs and outputs that constitute improvements in environmental performance are defined in the following two tables. Where an input in Table 3-1 (or an output in Table 3-2) is shown in a row that increases and an input (or an output in Table 3-2) in the column decreases this is an environmentally desirable change as indicated by a shaded cell. For example, in Table 3-1 if a recycled input in a row increases and a renewable input in a column decreases this is an improvement in terms of environmental performance and is so indicated with a shaded cell.

Table 3-1. Substitutions of Inputs (Rows) for Inputs (Columns) that are Improvements in Environmental Performance
(shown shaded)

	Non Material	**Recycle**	**Renewable**	**Nonrenew**
Non Material		▨	▨	▨
Recycle			▨	▨
Renewable				▨
NonRenew				

Table 3-2. Substitution of Outputs (Rows) for Outputs (Columns) that are Improvements in Environmental Performance
(shown shaded)

	Prim. Prod.	**Recycle.**	**Waste D.**	**Permitted D.**
Prim. Prod.		▨	▨	▨
Recycle			▨	▨
Waste D.				▨
Permitted D.				

Other schemes for ranking inputs and outputs are certainly possible. With more information the ranking scheme presented in this research can be further refined by the addition of more categories. However, the basic approach presented where some inputs are more preferred to others and some outputs are more preferred to others is generally applicable to many situations.

Dominance based techniques may be applied to determine whether or not a change in input or output mix moves a production system into a state that is more desirable from an environmental perspective. The most preferred production plans from an environmental perspective are part of the set TE. The least preferred production plans from an environmental perspective are part of the set TI. Given the dominance criterion, by definition, production plans with inputs both increasing and decreasing or with outputs both increasing or decreasing, or with undesirable outputs both increasing and decreasing will be dominance indifferent and part of the set TU. Given that inputs and undesirable outputs can be classified on an ordinal scale according to environmental preference at least some of the production plans in the set TU can be determined to be environmentally preferred or environmentally not preferred based on changes in input or output mixes.

The determination of whether or not a desirable or an undesirable change in input or output mix is occurring is divided into two parts. First, the inputs and outputs of the production system are grouped into subsets according to an ordinal classification based on environmental desirability. Given incomplete information on the relative desirability of inputs and outputs some inputs will be judged to be equivalent in terms of environmental desirability as will some outputs. Aggregation of inputs or outputs judged to be equivalent in terms of environmental desirability is not desirable since units are often not consistent (e.g., labor hours and pounds of material) and aggregation results in the loss of information. The definition of these subsets is necessary to avoid aggregating inputs and outputs. Therefore, there will be groups of inputs and outputs that are considered to be equivalent in terms of their environmental desirability.

For each production plan the inputs, $x_1,..,x_i, \ldots x_I$, are grouped according to environmental desirability. The subscript, I, represents the last of the inputs, x_i. The subscript, N, represents the last of the n sets, sx_n.

$sx_1 \sim sx_n \sim \ldots. \sim sx_{N-1}$

The symbol ~ means “preferred to” and is used to show the ordinal classification of inputs and outputs. Many inputs that are in small quantities may not be classified in terms of environmental desirability for convenience. Regardless of the reason for not classifying an input in terms of environmental desirability, it is included in the subset, sx_N.

A set, sx_n, is composed of those elements, x_i, that have the property defined by $Z(sx_n)$.

$sx_n = \{ x_i : Z(sx_n) \}$ where,

$Z(sx_n) =$ (Input Type 1, …Input Type n, …, Input Type N). (Eq. 3-6)

A definition of the set, sx_n, is provided where a set is defined to be all elements with the property $Z(sx_n)$ (Suppes, p. 33). Empty sets are allowed. Each input type corresponds to an input set sx_n. An example of one possible classification of inputs shown in Table 3-1 can be directly mapped onto the sets sx. Non-Material inputs are in the set sx_1, recycled inputs are in the set sx_2, renewable inputs are in the set sx_3, and non-renewable inputs are in the set sx_4. In addition, there is the set sx_5, that contains the inputs that are not classified.

Outputs are also grouped according to environmental desirability. It is assumed that outputs can always be categorized as product or as undesirable. However, there may be some undesirable outputs that cannot be rank ordered relative to other undesirable outputs. This classification is:

Products: su_1

Undesirable Outputs: $su_2 \sim su_m \sim \ldots \sim su_{M-1}$

Indifferent Undesirable Output: su_M

The subscript, M, represents the last of the m sets.

A set, su_m, is composed of those elements, u_j, that have the property defined by $\Psi(su_m)$.

$su_m = \{ u_j : \Psi(su_m) \}$ where,

$\Psi(su_m) =$ (Output Type 1, Output Type m, …,Output Type M). (Eq. 3-7)

A definition of the set, su_m, is provided where a set is defined to be all objects with the property $\Psi(sx_m)$ (Suppes, p. 33). Empty sets are allowed. Each output type corresponds to an input set, su_m. An example of one possible classification of outputs shown in Table 3-2 can be directly mapped onto the sets su. Products are in the set su_1, recycled outputs are in the set su_2, waste discharges are in the set su_3, and permitted discharges are in the set su_4. In addition, there is the set su_5, that contains the undesirable outputs that are not classified.

Where a production plan is either efficient (part of the set TE) or inefficient (Part of the set TI) relative to a reference set the additional distinction of whether or not desirable substitutions are occurring is not meaningful. Where a production plan is dominance indifferent (part of the set TU), the additional classification based on substitution allows a further refinement (or reduction of the dominance indifferent space). There are two tests that are applied to production plans that are in the dominance indifferent set TU to determine if they may be assigned to the sets SE and SI.

The first test is whether or not "external" change is occurring for a particular set, sx_n or su_m. The term external is used because change among inputs or outputs among different sets is being tested. Increasing one input in the set sx_n while decreasing another is neutral in terms of environmental desirability since all members of the set are of the same relative desirability. An external change is judged to occur when all the elements of a set are greater than or equal to a reference set with at least one element greater than; or external change is judged to occur where all the elements of a set are less than or equal to a reference set with at least one element less than. The definition of external change is:

> Definition: For the set, sx_n , containing elements, x_i, change external to the set is occurring compared to the reference set, sx_n^r , where $sx_n \geq sx_n^r$ (or $sx_n \leq sx_n^r$), for all elements x_i in sx_n with at least one element in sx_n greater than (or less than) the corresponding element in sx_n^r .

> Definition: For the set, su_m, containing elements, u_j, change external to the set is occurring compared to the reference set, su_m^r, where $su_m \geq su_m^r$ (or $sx_m \leq su_m^r$), for all u_j in su_m with at least one element in su_m greater than (or less than) the corresponding element in su_m^r.

If external change is occurring, it is assumed that there must be some outside influence. For example, if all of the inputs in the set containing non-material inputs sx_1 are greater compared to a reference production plan then there is possibly a decrease in some other set compared to the reference production plan. If all or some of the recycled outputs in the set, su_2, are less than a reference production plan then there is possibly some other set that is greater than the reference production plan.

If all inputs or outputs in a particular set, sx_n or su_m, are found to be greater than or less than a reference (external change is occurring) then the next test is to determine if there is change in input or output mix among sets that is also environmentally desirable. Environmentally desirable change in input mix is occurring if an input set is greater than a reference set and at least one less desirable input set is less than the reference set with all other sets no greater than the corresponding reference set. Environmentally desirable change in output mix is occurring if an output set is greater than a reference set and at least one less desirable output set is less than the reference set with all others not greater than the corresponding reference set. The last sets in the ordinal ranking, sx_N and su_M , are not included in this test since they are not distinguished by their relative environmental desirability. These tests for input sets and output sets are defined below.

> Definition: Given external change occurs for sx_n (i.e., $sx_n \geq sx_n^r$) environmentally desirable substitution occurs if $\exists$ a subset sx_k for which $sx_k \leq sx_k^r$, $k > n$ and where $sx_y \geqq sx_y^r$ for all subsets, sx_y, $y<n$ and where $sx_z \leqq sx_z^r$ for all subsets sx_z, $z > n$.

> Definition: Given external change occurs for su_m (i.e., $su_m \geq su_m^r$) environmentally desirable substitution occurs if $\exists$ a subset su_k for which $su_k \leq su_k^r$, $k > m$ and where $su_y \geqq su_y^r$ for all subsets, su_y , $y < m$ and where $su_z \leqq su_z^r$ for all subsets sx_z, $z > m$.

The definitions above are parallel for inputs and outputs since in both instances an increase in a more environmentally desirable input or output with a decrease in a less environmentally preferred input or output (with all else constant) is an improvement in the input or output mix.

It is possible for change to be occurring between input and output sets, sx_n and su_n, without this test detecting that change. This is because the relative desirability of, say, a recycled input compared to a permitted output is not known. However, the ordinal ranking of inputs and outputs does allow changes that either improve the input mix or the output mix from an environmental perspective to sometimes be detected.

The implementation of this approach is illustrated in Table 3-3 for inputs, but is the same for outputs. Here five inputs are shown divided into three sets of inputs. Inputs 1 and 2 are part of the most preferred set of inputs sx_1. Inputs 3 and 4 are part of the next preferred set. Input 5 is the only member of the least preferred set sx_3. An "UP" in a cell shown in Table 3-3 means that all of the elements x_i of the set sx_n are greater than or equal to a reference with at least one element in all of the "UP" cells in a column being greater than. The "UP" cells are further indicated by being shaded. A "DN" in a cell means that all element (x_i) of the set (sx_n) are less than or equal to a reference with at least one element in the "DN" cells in a column being less than.

Table 3-3. Substitution Among Input Categories

Input Category $sx_n(x_i)$	**N/A**	**SE**	**SE**	**N/A**	**SI**	**SI**	**N/A**	**N/A**
sx_1 (x_1,x_2)	UP	UP	UP	DN	DN	DN	DN	UP
sx_2 (x_3,x_4)	DN	UP	DN	DN	DN	UP	UP	DN
sx_3 (x_5)	UP	DN	DN	DN	UP	UP	DN	UP

The membership in the sets, SE and SI is defined below:

SE (r) = TU $\cap$

$$\left\{\left[\begin{bmatrix} su\\ sx\end{bmatrix} \in \Re_{+}^{I+J} \middle| \begin{bmatrix} su\\ sx\end{bmatrix} = \bigcup_{n=1}^{N-2}\left\{\left[\begin{bmatrix} su^{r}\\ sx^{r}\end{bmatrix} + \sum_{\forall i\in sx_{\le n}} \nu_{i}\begin{bmatrix} O^{J}\\ e_{i}^{I}\end{bmatrix} - \sum_{\forall_{i}\in su_{\substack{>n\\<N}}} \nu_{i}\begin{bmatrix} O^{J}\\ e_{i}^{I}\end{bmatrix}\right]\right\}, \nu_{i} \ge 0\forall_{i}\right]\right\}$$

$\cup$

$$\left\{\left[\begin{bmatrix} su\\ sx\end{bmatrix} \in \Re_{+}^{I+J} \middle| \begin{bmatrix} su\\ sx\end{bmatrix} = \bigcup_{m=1}^{M-2}\left\{\left[\begin{bmatrix} su^{r}\\ sx^{r}\end{bmatrix} + \sum_{\forall j\in sx_{\le m}} \mu_{j}\begin{bmatrix} e_{j}^{J}\\ O^{I}\end{bmatrix} - \sum_{\forall_{j}\in su_{\substack{>m\\<M}}} \mu_{j}\begin{bmatrix} e_{j}^{J}\\ O^{I}\end{bmatrix}\right]\right\}, \mu_{j} \ge 0\forall_{j}\right]\right\}$$

$$\neg\begin{bmatrix} su^{r}\\ sx^{r}\end{bmatrix} \qquad \text{(Eq. 3-8)}$$

SI(r)= TU $\cap$

$$\left\{\left[\begin{bmatrix} su\\ sx\end{bmatrix} \in \Re_{+}^{I+J} \middle| \begin{bmatrix} su\\ sx\end{bmatrix} = \bigcup_{n=1}^{N-2}\left\{\left[\begin{bmatrix} su^{r}\\ sx^{r}\end{bmatrix} - \sum_{\forall i\in sx_{\le n}} \nu_{i}\begin{bmatrix} O^{J}\\ e_{i}^{I}\end{bmatrix} + \sum_{\forall_{i}\in su_{\substack{>n\\<N}}} \nu_{i}\begin{bmatrix} O^{J}\\ e_{i}^{I}\end{bmatrix}\right]\right\}, \nu_{i} \ge 0\forall_{i}\right]\right\}$$

$\cap$

$$\left\{\left[\begin{bmatrix} su\\ sx\end{bmatrix} \in \Re_{+}^{I+J} \middle| \begin{bmatrix} su\\ sx\end{bmatrix} = \bigcup_{m=1}^{M-2}\left\{\left[\begin{bmatrix} su^{r}\\ sx^{r}\end{bmatrix} - \sum_{\forall j\in sx_{\le m}} \mu_{j}\begin{bmatrix} e_{j}^{J}\\ O^{I}\end{bmatrix} + \sum_{\forall_{j}\in su_{\substack{>m\\<M}}} \mu_{j}\begin{bmatrix} e_{j}^{J}\\ O^{I}\end{bmatrix}\right]\right\}, \mu_{j} \ge 0\forall_{j}\right]\right\}$$

$$\neg\begin{bmatrix} su^{r}\\ sx^{r}\end{bmatrix} \qquad \text{(Eq. 3-9)}$$

Where,

TU(r) is the set of production plans in the space that are dominance indifferent and includes the observation (x^r, u^r).

e_j^J, e_i^I ,= J-dimensional and I dimensional zero vectors where the subscript component (ith or jth) is equal to one with all other elements of the vector equal to zero.

O^I, O^J = I-dimensional and J-dimensional zero vectors.

μ_j, ν_i = scalars for all i and j.

i and j = Input and Output quantities, respectively.

The set, SE, is formed by the intersection of three terms. First, the production plans that are Dominance Indifferent, TU, are taken as the starting production plans for the definition of the set SE. By definition the production plans in TU have inputs or outputs that are both greater than and less than a reference production plan. The production plans that are both in the set TU and meet the defined criteria are partitioned into the set SE. The criteria are that an environmentally desirable substitution must occur for either inputs or outputs. For inputs, the definition requires that for each set, sx_n, all elements x_i of the sets sx_1 through sx_n are greater than or equal to a reference with at least one element in these sets greater than the corresponding element in the reference production plan; and that all elements x_i of the sets sx_{n+1} through sx_{N-1} are less than or equal to the corresponding elements in a reference production plan with at least one element in these sets less than the corresponding element in the reference production plan. The set sx_N is excluded since this includes inputs not distinguished by environmental preference. The set SE contains all production plans where a single instance of desirable input substitution was detected which is indicated by the union, $\bigcup_{n=1}^{N-2}$. The range is to N-2 since the set N-1 can only be compared to the set N which is not distinguished on environmental preference; so, this comparison is not useful. Desirable substitution for outputs is similarly defined. The definition of the set SI simply reverses the dominance requirements.

3.3 Examples of the Sets

The application of the set definitions presented above is illustrated by figures showing all the possible pairwise comparisons. For inputs, there are two groups of inputs - those that can be distinguished in terms of environmental preference and those that cannot. There is therefore the comparison between the two groups and within these two groups for a total of three pairwise comparisons. For outputs, there are three groups of output - products, undesirable outputs that can be distinguished based on relative environmental desirability, and undesirable outputs that cannot be distinguished based on relative environmental desirability. With these three groups there are three possible pairwise comparisons between groups with three more pairwise comparisons within each of the three groups for a total of six pairwise comparisons for outputs. These nine possibilities are illustrated in Figures 3-3 through 3-11.

Each of the following figures illustrates a reference production plan and four quadrants. A production plan compared to the reference production plan is classified into a quadrant that represents a set as previously defined. In terms of environmental performance the preferred set is TE where both manufacturing productivity and environmental performance are improved. Production plans that are members of the set SE have changes in inputs and outputs occurring that are an improvement in environmental performance.

First, there are the comparisons that occur within a subset, sx or su. Figure 3-3 illustrates the comparison of inputs where one is not preferred to another. Figure 3-4 illustrates the comparison of product outputs where one is not preferred to another. The sets TE, TI, and TU in these figures correspond exactly to the sets defined by Deprins, Simar, and Tulkens (1984): D_g, D_d, and D_i, respectively. Figure 3-5 illustrates the comparison of undesirable outputs where one is not preferred to another. This differs in that the set TE is in the lower left quadrant and the set TI is in the upper right quadrant. For undesirable outputs, a decrease in both is desirable and an increase in both is undesirable. An increase in one undesirable output and a decrease in another, where one is not preferred to another, is neutral in terms of environmental desirability. Since none of the inputs and outputs illustrated in these three figures is ranked by environmental preference there are no quadrants labeled SE or SI.

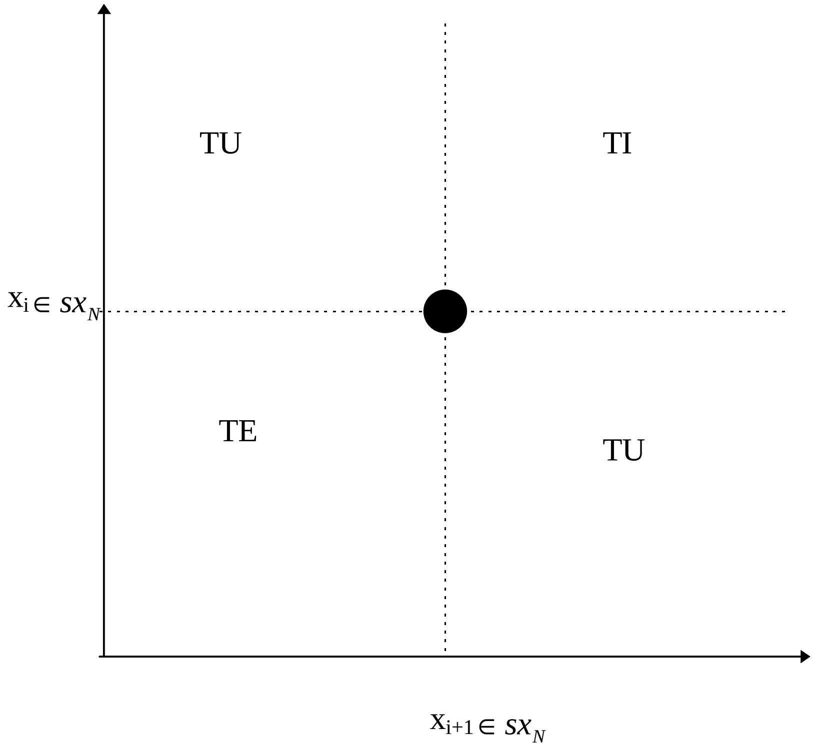

Figure 3-3. Two Inputs - with no environmental preference

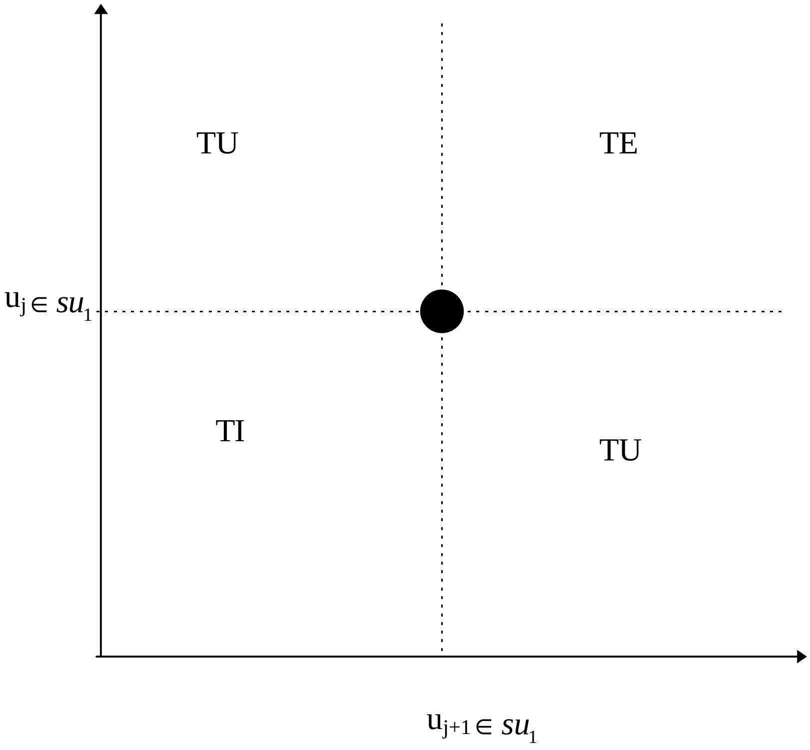

Figure 3-4. Two Product Outputs - with no environmental preference

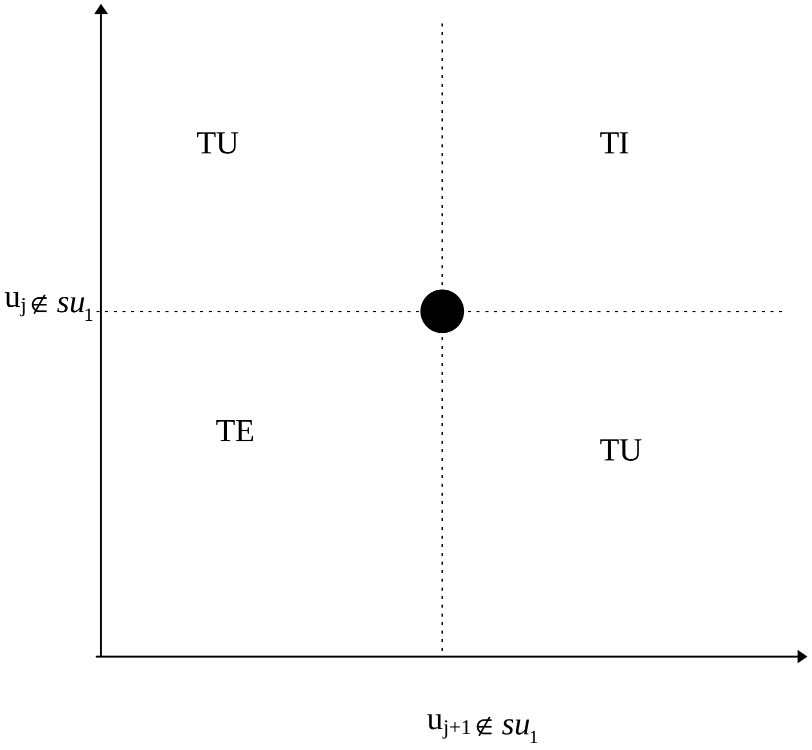

Figure 3-5. Two Undesirable Outputs - with no Environmental Preference

The next figures illustrate comparisons between inputs and outputs that are not part of the same subset. In other words, the comparisons are made where an environmental preference exists. The possible combinations given the preference structure previously defined are: two inputs - one preferred to another; two inputs - one not part of the ordinal ranking; two outputs - one product and one undesirable; two outputs - one undesirable and one more undesirable; and two inputs - one undesirable and one undesirable but not ranked. Each of these 5 possible two way comparisons are illustrated in the following figures.

Figure 3-6 illustrates the comparison of an input that is more preferred than another input. The quadrants that would be labeled TU without environmental preference are now labeled SE and SI. The upper left quadrant, labeled SE, is environmentally preferred since the more desirable input is greater than the reference and the less desirable input is less than the reference. The lower right quadrant, labeled SI, is less environmentally preferred since the less desirable input is greater than the reference and the more desirable input is less than the reference. Figure 3-7 illustrates the comparison of an input that is part of the ordinal ranking to an input that is not part of the ordinal ranking. This again corresponds the Deprins, Simar, Tulkens (1984) definitions since a preference between the inputs is not established.

Next, outputs that are from different sets su_m are compared. Figure 3-8 compares a product output to an undesirable output. A product output greater than the reference and an undesirable output less than the reference is TE. A product less than the reference and an undesirable output greater than the reference is TI. This comparison is not performed as part of the set definitions SE and SI. Therefore, the upper right and lower left quadrant are simply dominance indifferent TU.

Figure 3-9 compares an undesirable output to an even more undesirable output. Both undesirable outputs less than the reference (with inputs held constant) is TE, while both undesirable outputs greater than the reference is TI. Desirable output greater than the reference with the less desirable output less than the reference is SE. The more desirable output being less than the reference with the less desirable output being more than the reference is SI. Figure 3-10 compares an undesirable output that is part of the ordinal ranking with an undesirable output that is not part of the ordinal ranking. This corresponds to the Deprins, Simar, and Tulkens (1984) classification. Figure 3-11 again shows a comparison of product output to undesirable output not classified and therefore the upper right and lower left quadrants represent the dominance indifferent subset TU.

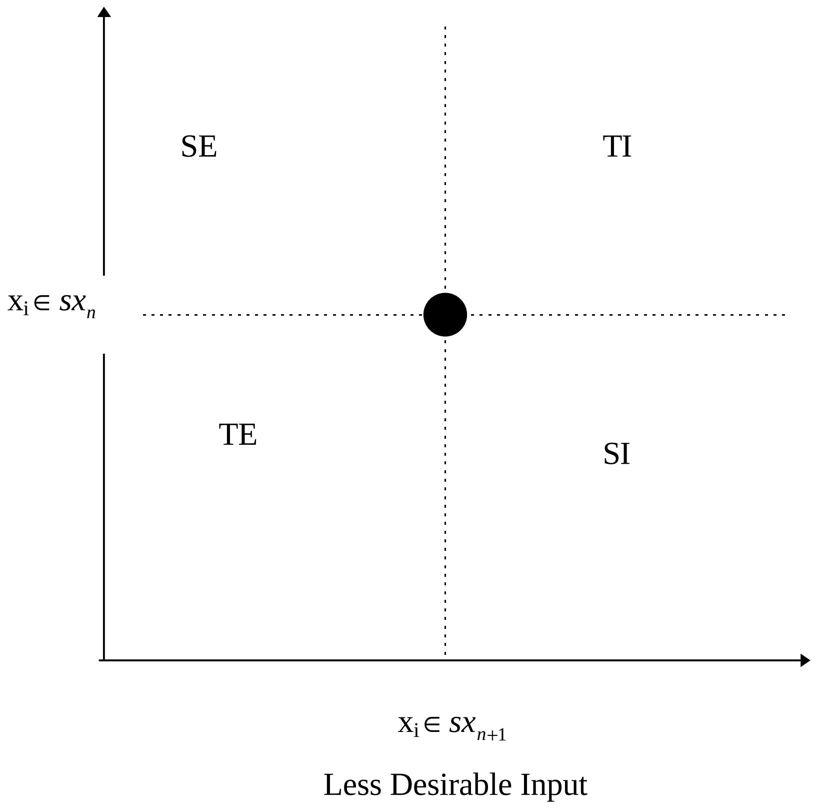

Figure 3-6. Two Inputs - with one Less Environmentally Desirable

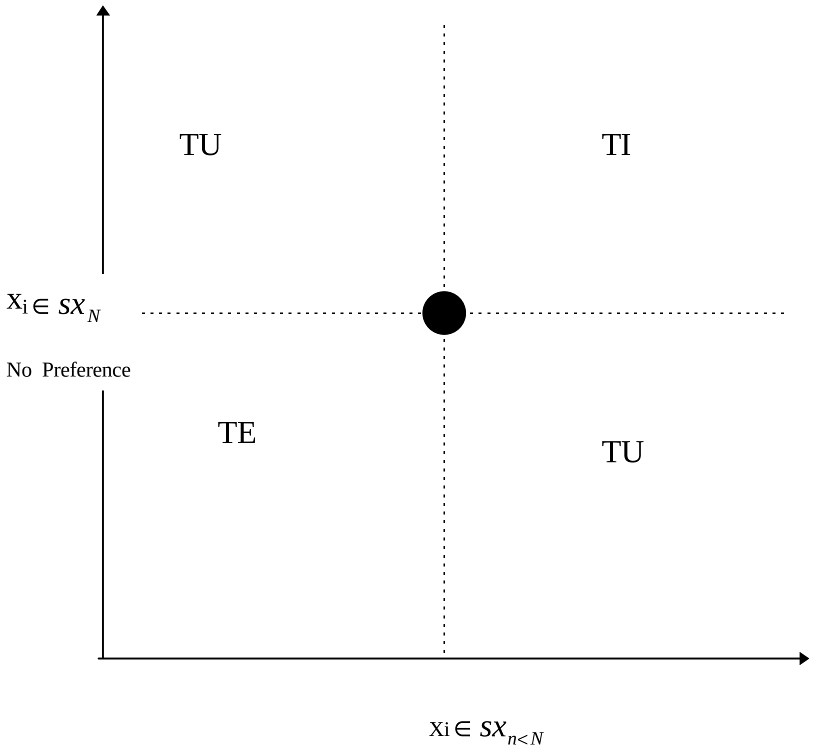

Figure 3-7. Two Inputs - with One not Part of the Ordinal Ranking

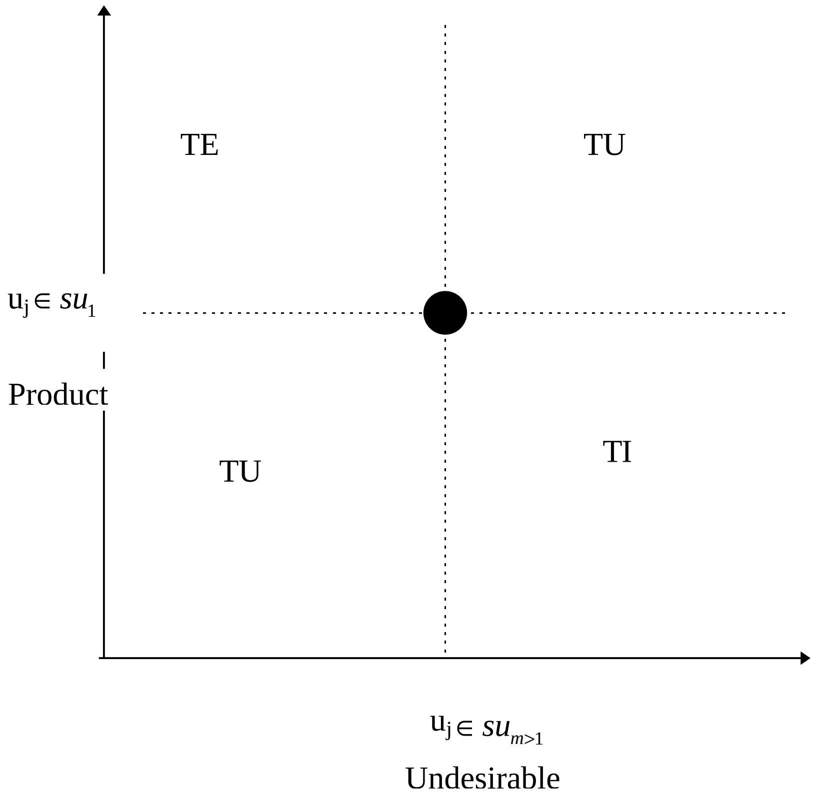

Figure 3-8. Two Outputs - with One a Product and One Undesirable.

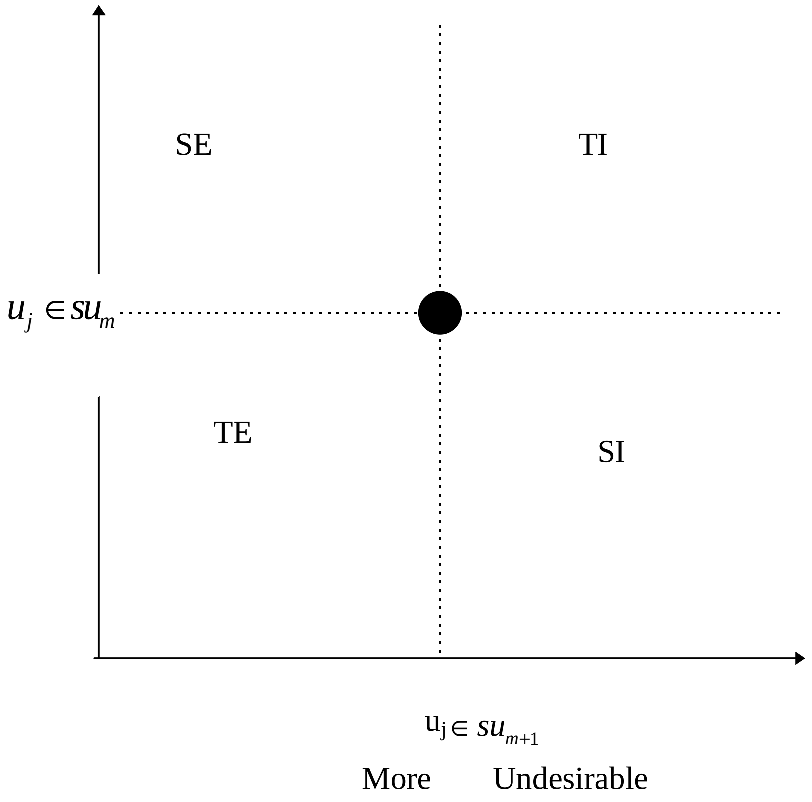

Figure 3-9. Two Outputs - One Undesirable and One More Undesirable.

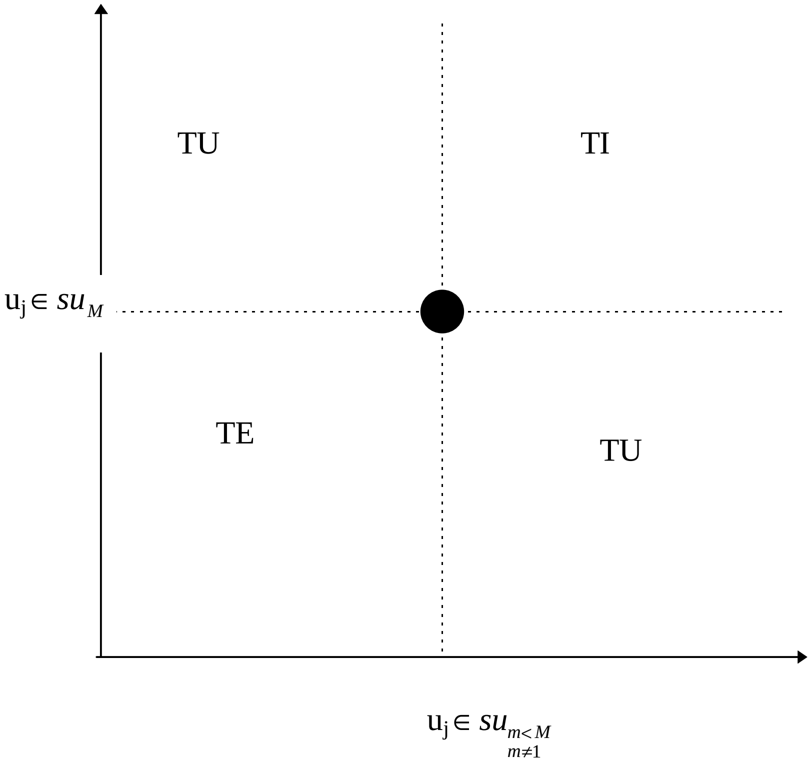

Figure 3-10. Two Outputs - with One not Part of the Ordinal Ranking.

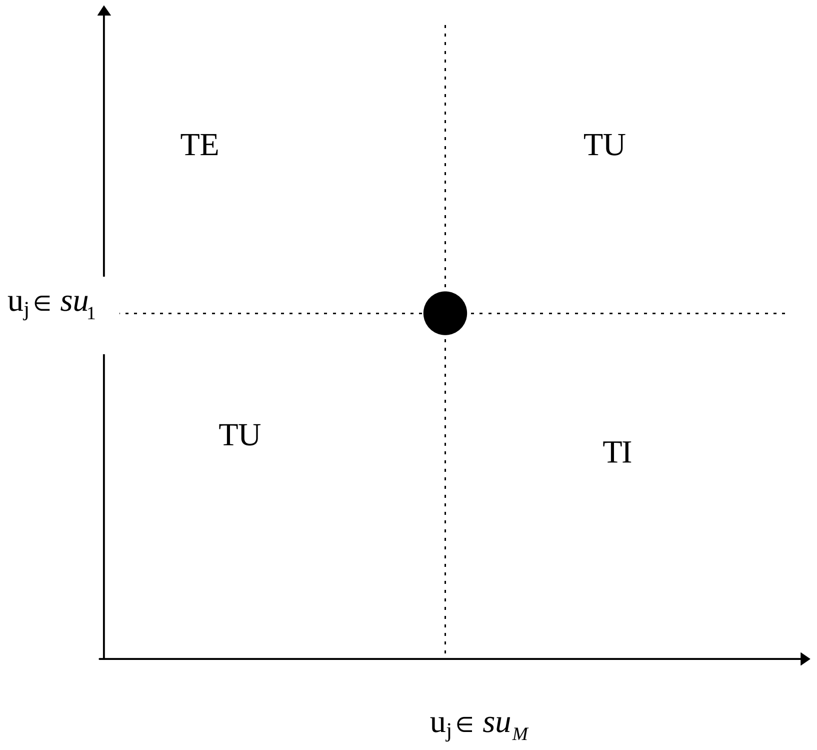

Figure 3-11. Two Outputs - One Product and One not Part of the Ordinal Ranking.

3.4 Environmental Performance Metrics

The previous sections are concerned with identifying the set to which a particular production plan belongs. This determination is the first step towards the measurement of performance based on dominance. A particular reference production plan must then be chosen from a particular set of production plans. For the proposed method the current production plan at time t is always the reference. Then, the actual metric of performance efficiency for a production plan may be calculated based on the distance to the reference production plan.

3.4.1 *Distance Measures*

There are a number of possibilities for distance measures between two production plans. The distance measure applied in this research is the Euclidean distance. This measure is the shortest distance between two production plans. Inputs, outputs, and undesirable output distances are each calculated and provide different measures of performance.

$$\text{D for inputs.} = \sqrt{\sum_{i=1}^{I} |x_i^r - x_i|^2} \qquad \text{(Eq. 3-10)}$$

$$\text{D for products}= \sqrt{\sum_{i=1}^{P} |u_i^r - u_i|^2} \qquad \text{(Eq. 3-11)}$$

$$\text{D for undesirable outputs}= \sqrt{\sum_{i=P+1}^{I} |u_i^r - u_i|^2} \qquad \text{(Eq. 3-12)}$$

In a modification of methods from cluster analysis the distance of points in a set to the production plan being evaluated are averaged. These metrics are expressed as:

$$\text{Average Euclidean Distance for Inputs} = \frac{\sum_{k=1}^{n} \sqrt{\sum_{i=1}^{I} |x_i^r - x_i^k|^2}}{n} \qquad \text{(Eq. 3-13)}$$

$$\text{Average Euclidean Distance for Products} = \frac{\sum_{k=1}^{n} \sqrt{\sum_{i=1}^{P} |u_j^r - u_j^k|^2}}{n} \qquad \text{(Eq. 3-14)}$$

$$\text{Average Euclidean Distance for Undesirable Outputs} = \frac{\sum_{k=1}^{n} \sqrt{\sum_{i=P+1}^{J} |u_j^r - u_j^k|^2}}{n} \qquad \text{(Eq. 3-15)}$$

The average Euclidean Distance provides a measure of relative environmental performance that is less sensitive to the effects of outlier observations. This measure will be less likely to detect changes in environmental performance.

The measures for inputs, products, and undesirable outputs provide different perspectives on environmental performance. The measure of distance for inputs is a measure of how well a production plan is using resources to produce product. The measure of distance for products is more a measure of productive efficiency than environmental performance. Increasing product is desirable for the manufacturing facility, but is neutral in terms of environmental performance (as defined in this research). The measure of distance for undesirable outputs provides a measure of how well a production plan is reducing pollution. This indicator will tend to be more important to a facility since regulatory agencies restrict many of these outputs.

3.4.2 *Property of a Set of Production Plans*

At each time the number of production possibilities for the sets D_g, D_d, D_i, TE, TU, TI, SE and SI are counted. The use of counts and normalized counts has been done with FDH analysis such as Vanden Eeckaut, Tulkens, Jamar (1993). These counts provide a way of distilling a large amount of data into summary form. This can indicate areas where additional analysis or assessment is needed to further define relative environmental performance.

3.5 Summary

Typically, productive efficiency techniques are used to compare the efficiency of similar systems (e.g., bank branches, municipal governments). This research will apply these techniques to evaluate the performance of a single system - a manufacturing facility. The basis for the proposed technique is the Benchmark Correspondence method and the three modifications described in Chapter 3. This approach allows metrics of efficiency to be based on data and not a mathematical model derived from data.

Chapter 4 applies the proposed approach to actual data. Although typically not addressed in the productive efficiency literature, model specification has a strong influence on results. Model specification issues are therefore addressed in Chapter 4 to include transformations of the data to prepare it for the proposed and standard analyses.