Cumulative Sum Control Charts for Censored Reliability Data

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(ABSTRACT)

Companies routinely perform life tests for their products. Typically, these tests involve running a set of products until the units fail. Most often, the data are censored according to different censoring schemes, depending on the particulars of the test. On occasion, tests are stopped at a predetermined time and the units that are yet to fail are suspended. In other instances, the data are collected through periodic inspection and only upper and lower bounds on the lifetimes are recorded. Reliability professionals use a number of non-normal distributions to model the resulting lifetime data with the Weibull distribution being the most frequently used. If one is interested in monitoring the quality and reliability characteristics of such processes, one needs to account for the challenges imposed by the nature of the data. We propose likelihood ratio based cumulative sum (CUSUM) control charts for censored lifetime data with non-normal distributions. We illustrate the development and implementation of the charts, and we evaluate their properties through simulation studies. We address the problem of interval censoring, and we construct a CUSUM chart for censored ordered categorical data, which we illustrate by a case study at Becton Dickinson (BD). We also address the problem of monitoring both of the parameters of the Weibull distribution for processes with right-censored data.

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Dedication

To my mother, Luci Olteanu, an example of strength, vision, dedication, and dependability. Thank you for everything. Cu dedicaţie pentru mama mea, Luci Olteanu, un exemplu de tărie, viziune şi caracter. Mulţumesc pentru tot.

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Chapter 1

Introduction

1.1 Background and Applications

All products are subject to failure. In its simplest form, failure is any change in a component that prevents the satisfactory performance of its intended function. Failure rates are a common method for summarizing the product's reliability. A failure mode is the appearance, manner or form in which a product's failure manifests itself, the effect produced by a certain failure cause. Service life or the lifetime of the product designates the time-span during which a product can be expected to operate safely and meet specified performance standards. It is assumed that the product is maintained in accordance with the standard and not subjected to environmental or operational stresses beyond specified limits. Some people call reliability "quality over time". Reliability is the ability of a system to perform a required function under stated conditions for a stated period of time. Early detection of faults with a monitoring program of the reliability characteristics would allow for repairs to be performed in situations at much less expense than totally rebuilding a unit. Furthermore, reliability of products is important to companies in order to reduce warranty costs. As a result, many companies routinely perform life tests, often under accelerated conditions. Typically, companies put n items on a test stand. In some cases, they run the test until all items fail, which is the uncensored case. More often, however, they run the test to a specified upper limit for the time, which leads to the censored case when not all of the items fail. Such censoring results in some complications to the statistical analysis of the test data.

Lifetime monitoring appears to be quite similar to the problem of waiting-time monitoring, where successive waiting times are usually modeled by the exponential distribution. However, lifetime monitoring and the waiting time problem are two clearly distinct problems that should not be confused. Lifetime monitoring usually involves testing groups of n ($n \geq 1$) products in successive order over time, while the waiting-time applications study the distribution of the time between individual events as they occur. One important characteristic of lifetime monitoring is that there is typically a physical mechanism in place that controls the failures. This underlying physical mechanism controls the specific failure mode. The primary purpose of lifetime monitoring is the stability of a product with regard to failures associated with specific, often known, failure modes. Censoring is not an issue for the waiting-time area; however, it is of vital importance in lifetime monitoring. Most waiting-time problems center on inter-arrival times, and as a result, they are of great interest in queuing theory. Our focus in this is research is exclusively in monitoring aspects of sone non-normal distributions with censored data.

In the area of reliability, practitioners are interested in different aspects of the manufacturing process. The practitioner's main interest from the quality point of view is to maintain or improve the process's performance. Even small shifts in the different aspects of the process can indicate quality problems that should be identified and tackled in due time. These processes are usually modeled by one of the described distributions and the different aspects of interest are characterized by the different distributional parameters. For example a negative shift in the scale parameter of the Weibull distribution can indicate an undesirable drop in the mean life of the product with causes that should be investigated. A positive shift is desirable and indicates that the product became more reliable, however the underlying causes should be known for future decisions involving similar processes. In this case the timely detection of the shift is desirable. Often the shape parameter of the Weibull distribution is considered known and fixed, indicating a certain failure mechanism specific to the process. Nevertheless, one can suspect on occasions that the failure mechanism has changed due to different factors, such as the environment. In that case the detection of this parameter's shift can warn the practitioner of the change. If shifts occur in both parameters, the combined effect should also be detected, studied, and the root causes identified.

1.2 Censoring of Lifetime Data

Two aspects specific to lifetime data make the development of a control chart more demanding:

- The data tend to be skewed and non-normally distributed.
- The data typically are censored.

Right censoring is most common. It occurs whenever at least one item does not fail over the course of the test. Type I right-censoring occurs when one stops the test at a certain predetermined time, and the testing of the remaining units that did not fail by that time is suspended. Type II right censoring occurs when one stops the test after a predetermined number of items fail. Random (or non-informative) censoring occurs when each unit has a censoring time that is statistically independent of their failure time. The observed lifetime of an item is the minimum of the censoring and failure times.

Left censoring occurs when a unit fails before a certain threshold time for beginning to observe the process, for example, before the first 100 hours of running time. In this case, one does not know the actual failure time; rather, one only knows that it was before the threshold value. Schneider et al. (1995) considered this case for control charts with skewed and censored data, where the left-censoring occurred due to measurement precision limitations in the chemical field.

Interval censoring occurs when, for example, a process is subjected to periodic inspection at predetermined time intervals. The operator cannot know the actual failure time for units failing between inspection times. Instead, the analyst only knows the specific time interval in which each unit failed. Meeker and Escobar (1998) considered interval censoring throughout their book on statistical methods for reliability data.

1.3 Distributions Typically Used to Model Lifetime Data

Most life data follow skewed, non-normal distributions. A number of distributions have been proposed for machinery failure probabilities. The exponential distribution is often used because of its universal applicability to systems that are repairable. The lifetimes of several kinds of electronic components can be represented closely by exponential distributions. Machinery parts behave in this mode when they succumb to brittle failure. For example, diesel engine control unit failures typically tend to follow an exponential distribution.

The normal distribution has limited applicability to life data. Some analysts use it to model failures due to wear-out processes. The hazard function for a normal distribution cannot be expressed in a simple form.

The log-normal distribution enjoys wide acceptance in reliability work. One of the typical applications is for failures due to crack propagation or corrosion. It is one of the most common distributions used to model lifetime data, together with the Weibull distribution, which includes the exponential distribution as a special case.

There are instances when lifetime data are interval censored, due to periodic inspection, and one uses a multinomial distribution to conduct statistical analysis.

1.3.1 The Weibull Distribution

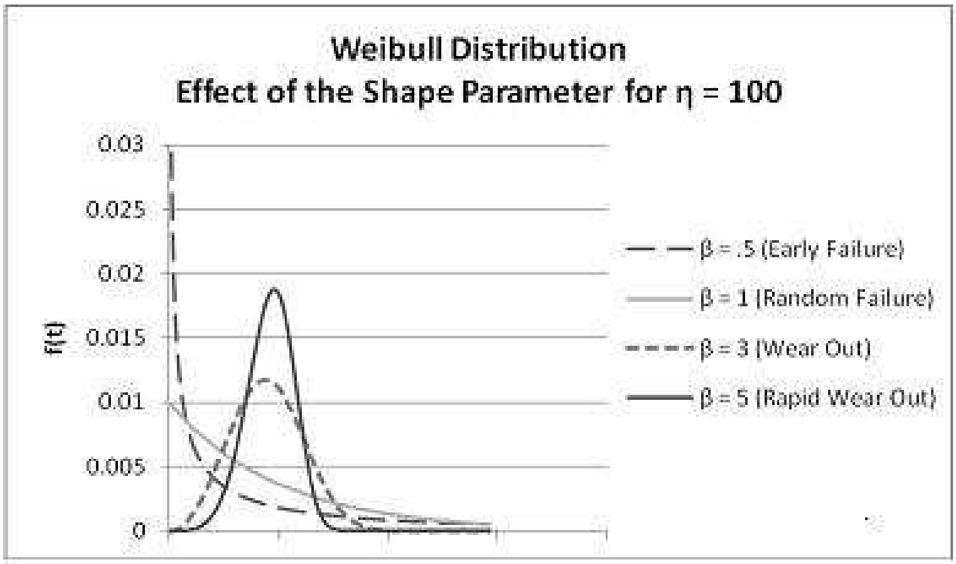
The Weibull distribution is a popular distribution for modeling failure times because it reflects the physics of failure and it models either increasing or decreasing failure rates simply. The Weibull is the preferred distribution for the time effects due to fatigue, erosion and wear. It is also used as the distribution for product properties such as strength (electrical or mechanical), elongation, and resistance. It is used to describe the life of roller bearings, electronic components, ceramics, capacitors and dielectrics in accelerated tests (Bloch and Geitner (1994)). The Weibull distribution is also used in biological and medical applications to model the time to occurrence of tumors (Lawless (2002, p. 18)).

Many analysts prefer to model lifetime data by the two-parameter Weibull distribution, with scale parameter η and shape parameter β, since it is a versatile distribution that can take many shapes depending on the value of the shape parameter. Suppose that T is a variable denoting failure times. A common two-parameter Weibull parametrization is

$$f(t, \beta, \eta) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}e^{-(\frac{t}{\eta})^{\beta}},$$

where $\beta > 0$ is the shape parameter, $\eta > 0$ is the scale parameter. The scale parameter, η, has the same units as T, for example, hours, months, or cycles. The scale parameter indicates the characteristic life of the product, the time by which roughly 63.2% of the units are expected to fail. The shape parameter β is a unitless number. For most products and materials, β is in the range 0.5 to 5. The parameter β determines the shape of the distribution, and η determines the spread. The Weibull distribution is a member of the log-location-scale family of distributions. When $\beta = 1$, the Weibull distribution becomes an exponential distribution. When $\beta = 2$, the distribution is called a Rayleigh Distribution. The Weibull distribution is also a special case of an extreme value distribution for the weakest link. In many applications with lifetime data the Weibull distribution fits better than the exponential, normal, and log-normal distributions. Often the Weibull and log-normal distributions may fit a set of data

Figure 1.1: Shape of the Weibull Distribution for Various β Values.



equally well, especially over the middle of the distribution. When both distributions are fitted to a data set, we tend to be more conservative when using the Weibull distribution (Nelson (1990)). That is, a low Weibull percentile is below the corresponding log-normal percentile, in other words we forecast that more products fail earlier when fitting a Weibull distribution than when fitting a log-normal distribution.

Different values of the shape parameter β model several different failure modes. For example, reliability practitioners associate values of β near 0.5 with infant mortality, and they associate values of β near 5 with rapid wear-out. Figure 1.1 illustrates the flexibility of the Weibull distribution, showing the range of shapes that can be modeled by it.

The cumulative distribution function (cdf) for the Weibull distribution is

$$F(t, \beta, \eta) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}.$$

The survival function is

$$S(t, \beta, \eta) = 1 - F(t, \beta, \eta).$$

The central moments of the Weibull distribution have the form $E(T^m) = \eta^m \Gamma\left(1 + \frac{m}{\beta}\right)$. Therefore the mean and variance respectively are

$$E(T) = \eta\Gamma\left(1 + \frac{1}{\beta}\right),$$

and

$$Var(T) = \eta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\eta\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2.$$

The hazard function is defined as the instantaneous rate of failure, given by

$$h(t) = \frac{f(t)}{S(t)}.$$

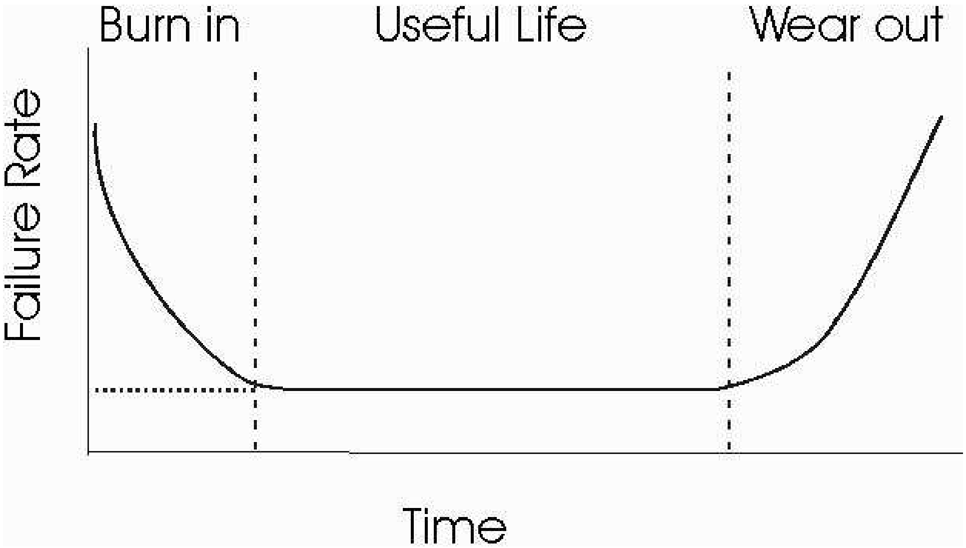
For the Weibull distribution, the hazard function is

$$h(t) = \frac{\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}}{1 - \left(1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}\right)} = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}.$$

- For $\beta < 1$ the hazard function is decreasing. Decreasing failure rate is indicated for components where initial self-accommodation takes place. Mechanical shaft seals would be a typical example.

- For $\beta > 1$ the hazard function is increasing. An increasing failure rate is symptomatic of wear-out failures. In cases where the shape factor is greater than 1, the distribution mean is nearly equal to the characteristic life η.

Figure 1.2: The Hazard Function of the Weibull Distribution for Various β Values.



- For $\beta = 1$ the hazard function is constant and $h(t) = \frac{1}{\eta}$. In this case the Weibull distribution reduces to the exponential distribution. Practitioners often associate the exponential distribution with random failures since the hazard function is constant.

For many products, such as computer processors, the hazard function graphical representation resembles a bathtub, with a decreasing hazard rate early during product life, accounting for infant mortality, a constant and low hazard rate accounting for the product lifetime, and then increasing hazard function, accounting for rapid wear-out, during the later age of the product. Figure 1.2 illustrates the bathtub hazard function.

The three-parameter Weibull has limited applicability in the reliability area due to the obvious relationship with the simpler two-parameter Weibull. The three-parameter Weibull has the following probability density function:

$$f(t, \beta, \eta, \gamma) = \frac{\beta}{\eta}(\frac{t-\gamma}{\eta})^{\beta-1} e^{-(\frac{t-\gamma}{\eta})^{\beta}},$$

where $\beta > 0$ is the shape parameter, $\eta > 0$ is the scale parameter, γ is the location parameter, and $t > \gamma$. The two-parameter Weibull can be considered a special case of the three-parameter Weibull, with $\gamma = 0$. In practical terms, this means that the product lifetime starts at $t = 0$ cycles.

1.3.2 The Smallest Extreme Value (SEV) Distribution and its Relationship to the Weibull Distribution

When dealing with the Weibull distribution, especially for the purpose of parameter estimation, researchers often use the monotonic transformation $Y = log(T)$. If $T \sim Weibull(\beta, \eta)$ and $Y = log(T)$ then $Y \sim SEV(u, b)$ where $b = \frac{1}{\beta}$ is the scale parameter and $u = log(\eta)$ is the location parameter. The relationship of the Weibull distribution to the SEV distribution illustrates the Weibull distribution's membership to the log-location-scale family. We can parallel the relationship between the Weibull distribution and the SEV distribution with the relationship between the log-normal distribution and the normal distribution.

The smallest extreme value (SEV) distribution is a member of the larger family of extreme value distributions. Kotz and Nadarajah (2000) studied the types and properties of these distributions and their relationship with other distributions. There are three types of extreme value distributions: type I, also called Gumbel-type, which is the most commonly used; type II, also called Fréchet-type; and type III, also called Weibull-type.

The largest and the smallest observations from a sample follow an extreme value distribution. If the parent distribution has a bounded tail, such as the Weibull distribution of lifetimes which are intrinsically bounded by zero, than the smallest observation in a sample of size n has a Type III smallest extreme value distribution, as n increases. This is a limiting distribution. The Type III SEV limiting distributions are useful in describing physical phenomena where the outcome is determined by the worst performing unit in the sample, for example if a system has n components in series, the system fails when the first component fails. The extreme value distributions are appropriate for modeling the smallest value in samples from a parent distribution with tails that drop exponentially fast, such as the normal or the Weibull distributions. Although the SEV distributions are introduced as the distributions of minima and maxima, they can be used empirically, without an extreme value model (Kotz and Nadarajah (2000)).

The relationship of the Type I SEV (also called the Gumbel distribution) to the Weibull distribution is often used. The log-data are easier to analyze with the simpler smallest extreme value (SEV) distribution, because its parameters are location and scale parameters, similar to the normal distribution. The parameters of the SEV distribution are easier to estimate than the original parameters of the Weibull distribution, and the monotonic transformation to the original parameters of the Weibull distribution makes the relationship popular in the practice of reliability. For a discussion of the estimation methods for the parameters of the two-parameter Weibull distribution see Olteanu and Freeman (2010) and Genschel and Meeker (2010). We can write the pdf and cdf of the Weibull distribution in log-location scale form as

$$f(t,u,b) = \frac{1}{bt}\phi_{SEV}\left[\frac{log(t)-u}{b}\right]$$

and

$$F(t,u,b) = \Phi_{SEV}\left[\frac{log(t)-u}{b}\right],$$

where,

$$\phi_{SEV}(z) = \exp[z-\exp(z)],$$

$$\Phi_{SEV}(z) = 1-\exp[-\exp(z)],$$

are the pdf and cdf of SEV, and $z = \frac{y-u}{b}$. We can write the pdf of the random variable $Y = log(T)$ as

$$f(y, u, b) = \frac{1}{b} \exp\left[\frac{y-u}{b}\right] \exp\left[-\exp\left(\frac{y-u}{b}\right)\right].$$

Another quantity of interest in reliability data analysis is the failure time for the p^{th} percentile. This quantity under the log-location scale parametrization of the Weibull distribution is

$$t_p = \exp\left[u + \Phi^{-1}_{SEV}(p)b\right].$$

For $p = .6321$, $\Phi^{-1}_{SEV}(p) = 0$ and therefore $t_p = \eta$. The time by which 63.21% of items are expected to fail is also known as the characteristic life.

1.3.3 The Log-Normal Distribution

Another distribution of choice for practitioners is the log-normal distribution. One of the advantages of this distribution is the relationship with the normal which facilitates statistical analysis. A random variable X follows a log-normal distribution with parameters μ and σ if its probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}x\sigma} \exp[-\frac{1}{2\sigma^2}[\log(x) - \mu]^2],$$

where $x > 0$ and the random variable $Y = \log(X)$ follows a normal distribution with mean μ and variance σ^2. The expected value and variance for the random variable X are respectively

$$E(X) = \exp\left[\mu + \sigma^2/2\right],$$

and

$$Var(X) = \exp[2\mu + \sigma^2/2](\exp \sigma^2 - 1).$$

1.3.4 The Multinomial Distribution

On occasions reliability data are collected through periodic inspection. In this instances, the inspection times delimit ordered intervals and only upper and lower bounds are known for a particular unit failure time. For this case the data are gauged into ordered categories and the counts of number of failures occurring in each category follow a multinomial distribution. This particular case is similar to other ordered categories problems in other fields. In Section 3 we develop a monitoring method for interval censored data, and we implement it in a case study from the medical field.

In this chapter we have presented our research goal to develop monitoring methods for data types specific to the reliability field, i.e., censored data following typically non-normal distributions. We have given an overview of the censoring schemes and distributions of interest. Next we will give an overview of the relevant literature in Chapter 2. In Chapter 3 we construct a likelihood-ratio cumulative sum control chart for interval censored data based on a multinomial distribution, with an application to the medical equipment testing field. The case study has been supported through a grant from Becton Dickinson and implemented in their operations. In Chapters 4 and 5 we address monitoring needs for right-censored lifetimes with a Weibull distribution. Beside focusing on the censoring problem, we also focus on monitoring different aspects of the Weibull distribution, challenging the perspectives encountered throughout the relevant literature.

Chapter 2

Literature Review

The existing literature predominantly presents monitoring methods for uncensored lifetimes with different underlying distributions. Some papers consider right-censoring while the interval censoring case is overlooked. One can extend the methods proposed for monitoring ordered categorical data to inspection lifetime data, based on the similarity of the problems.

2.1 Monitoring Methods for Ordered Categorical Data

Steiner et al. (1996) constructed CUSUM charts monitoring for shifts in the mean of the process based on sequential probability ratio tests for grouped data with any number of groups. They motivated economically the gauging of continuous data and classifying them into groups. The nature of the data is intrinsically continuous (measurements on a variable from an industrial process) but for economical reasons (it takes less skill, it is faster and less costly) the data are gauged and items classified into groups, of which binomial attribute data represent a special case. Steiner et al. (1996) compared the performance of their chart to the equivalent chart monitoring for shifts in the mean of the underlying continuous random variable. In this case the first chart was chosen due to economical reasons and not required by the intrinsic nature of data. The authors derived exact expressions for the operating characteristics and the average sampling number using the theory of sequential analysis. They illustrated the implementation and application of the methodology for the normal distribution with known standard deviation and gave the steps of the implementation algorithm in an example concerning the production of metal fasteners in a progressive die environment. Only the $n = 1$ case was covered in detail, indicating that the cases with $n > 1$, which are typical for industrial processes testing, should be approached in a similar way. The authors presented the theoretical development of the chart in detail but briefly outlined the practical details. They did not discuss the determination of the in-control characteristics of the process. The binomial CUSUM chart is a particular case of the CUSUM chart for grouped data and was developed by Reynolds and Stoumbos (2000) in their paper on monitoring proportions, demonstrating the superiority of the chart as compared to a traditional Shewhart p-chart for detecting both small and large shifts in the proportion.

Franceschini et al. (2004) presented two charts for the process control of quality characteristics measured on ordinal scale that do not assume any distributional properties and use a sample space ordering based on a dominance criteria. Their Shewhart-type charts are not designed to monitor the mean of the process.

Marcucci (1985) proposed two Shewhart-type charts for proportions to monitor for grouped quality characteristics: one based on the Pearson χ^2 statistic and one based on the multinomial distribution for the case when only increases in all but one category are of interest. Nelson (1987) later built on Marcucci's χ^2 approach for grouped data and plotted the statistic obtained from a goodness of fit test applied to each subgroup. He applied the chart to an example with four categories and provided control limits and median values. He argued that the subgroup size can vary but the larger the value was, the more sensitive the chart was.

Tucker et al.(2002) constructed a robust control chart for ordinal data and presented advantages over the χ^2 approach. The Shewhart type chart assumed a certain underlying distribution of interest for each category. The authors used the inverse of the cumulative distribution function to determine the assignment of units to an interval. The charted quantity was the maximum likelihood estimator of the location parameter normalized by the estimate of its asymptotic variance.

2.2 Monitoring Lifetimes

In the following paragraphs we present the popular approaches to lifetime monitoring found in the literature. Our focus on papers relevant to our paradigm, i.e., censored lifetimes, with samples of n items put on a test stand, which only very few papers address. The following table gives a brief overview of the papers discussed in this section.

Table 2.1: Approaches to Monitoring Lifetime Data.

Paper	n	Allow Censoring	Distribution	Chart Type	Approach
Steiner&MacKay(2000)	≥ 1	YES	Normal	Shewhart	CEV
Steiner&MacKay(2001a)	≥ 1	YES	Weibull	Shewhart	CEV
Vargas&Montaño(2005)	≥ 1	YES	Weibull	Shewhart	CEV
Zhang&Chen(2004)	≥ 1	YES	Weibull	EWMA	CEV
Padgett&Spurrier(1990)	≥ 1	NO	Weibull	Shewhart	Percentiles
Padgett&Spurrier(1990)	≥ 1	NO	Log-Normal	Shewhart	Percentiles
Nichols&Padgett(2006)	≥ 1	NO	Weibull	Shewhart	Percentiles
Erto, Pallotta&Park(2008)	≥ 1	NO	Weibull	Shewhart	Percentiles
Nelson(1979)	≥ 1	NO	Weibull	Shewhart	Median
Ramalhoto&Morais(1999)	≥ 1	NO	Weibull	Shewhart	Scale Parameter

2.2.1 Applications for Uncensored Data

There are a number of methods developed for uncensored reliability data, designed to detect shifts in different aspects of the process.

Methods for Monitoring Percentiles. One group of methods developed for uncensored data, when $n \geq 1$, in the reliability field can be used to monitor for changes in specific lower percentiles of the distribution. Interestingly, these methods are all developed for the uncensored case. The focus of these methods is to detect decreases in the percentiles of interest, for which a target value is usually given. Padgett and Spurrier (1990) considered monitoring the breaking strength of carbon fibers used in fibrous composite materials, as quantified by the tenth or lower percentiles of interest from the assumed distribution, providing several justifications for their choice. They argued that monitoring the tenth, or smaller, percentiles is as important, or even more important than monitoring for decreases in the mean, since there can be client specifications requiring a minimum strength, quantified by percentiles, or survival probabilities. They advocated the use of percentile monitoring in conjunction with the more traditional monitoring for the mean and variance. They also noted that monitoring percentiles covered the situation where a small decrease in mean, combined with a small increase in the variance, produced a larger decrease in the lower percentiles of interest. The impact on the percentile of interest can be significant and best captured by a chart designed for monitoring percentiles.

Padgett and Spurrier (1990) observed that the Weibull distribution modeled well the breaking strengths for brittle materials such as carbon and that the log normal distribution was more appropriate for ductile materials such as stainless steel. They thus proposed Shewhart-type charts for percentiles of both the Weibull and the log normal distributions, assuming all in-control parameters were unknown.

For the Weibull case, Padgett and Spurrier (1990) allowed both the shape and the scale parameters to vary and built a Shewhart-type chart to monitor for changes in a percentile of interest, which is a function of both shape and scale parameters. They considered that there is a target value for the percentile of interest and we can monitor for departures from the target. If there is not a given target value, it is estimated from past k samples as an average percentile estimate. The control limits for the chart are determined using the distribution of the percentile of interest and specifying a desired false alarm probability. Then they plotted the percentiles estimated from each sample of data from the production line against these control limits. The estimated percentiles are functions of the shape and scale parameters estimated using best linear invariant estimators, based on the data provided in each sample. The details of the chart development follow.

To develop their chart, Padgett and Spurrier (1990) used the relationship of the Weibull distribution to the extreme value distribution. If T follows a two-parameter Weibull distribution, then $Y = \log(T)$ follows the smallest extreme value distribution with pdf

$$f(y, b, u) = \frac{\beta}{\eta} e^{\frac{(y-u)}{b}} \exp[-e^{\frac{(y-u)}{b}}],$$

where $u = \log(\eta)$ and $b = \beta^{-1}$. Padgett and Spurrier (1990) assumed that one has random samples of n uncensored observations. They focused on the best linear invariant estimators (BLIE) of the smallest extreme value distribution parameters (b and u) proposed by Mann et al. (1974, pp. 222-239), who showed that the BLIE of the $p100^{th}$ percentile of the extreme value distribution is

$$\tilde{x_p} = \tilde{b} + \tilde{u} \log[-\log(1-p)]),$$

where $\tilde{b}$ and $\tilde{u}$ are the BLIEs of b and u respectively. The chart construction used a pivotal quantity, which is a function of the data whose distribution is independent on the parameter values. Starting from these tabled values, Padgett and Spurrier (1990) expressed the quantity of interest in terms of the pivotal quantity and determined its properties. For this situation, Lawless (2002, p. 214) and Mann et al. (1974, p. 221) showed that the pivotal quantity is

$$V_{1-p} = (\tilde{b} - x_p)/\tilde{u}.$$

Mann et al. (1974, p. 221) provided in their book a few tables outlining the pivotal quantity distribution. Padgett and Spurrier (1990) used a simulation study to determine values of this pivotal quantity for different values of the Type I error probability. They proposed using either a target value for the x_p percentile or the average of the estimates from the last k samples and then signaling whenever the calculated pivotal quantity for a particular sample is extreme in either direction. Padgett and Spurrier (1990) used a simulation study to determine the properties of their proposed chart and to compare it to traditional $\overline{X}$ and R charts. They concluded that their proposed chart performed better since the traditional $\bar{X}$ and R charts tended to signal inconsistently.

Padgett and Spurrier (1990) also constructed a similar chart for the log-normal distribution. They applied the $p100^{th}$ percentile chart based on the log-normal distribution in conjunction with the $\bar{X}$ and R charts to a carbon fiber breaking strength example and found that the signalling was mixed. They thus concluded that the three charts should be used in parallel.

Nichols and Padgett (2006) proposed the use of a bootstrap chart for detecting a shift in a lower percentile of the Weibull distribution. They showed through simulation that this chart would signal faster in an out-of-control situation than the chart proposed by Padgett and Spurrier (1990). Nichols and Padgett constructed their chart using the maximum likelihood estimators from past data for the shape and scale parameters of the Weibull distribution. They then used the bootstrap method to get the percentile estimate. Once again, the authors assumed there was no censoring.

Recently, Erto, Pallotta and Park (2008) built upon the basic idea from Padgett and Spurrier (1990) to develop a Shewhart-type chart for monitoring percentiles using Bayes estimators for the Weibull distribution parameters. They assumed that the target value for the percentile was known. They intended to develop a method for small sample sizes (less than three) that exploits the amount of prior information on the process. They asserted that their method really does not have a competitor for such small samples. Once again, their proposed procedure was not designed for censored data.

Methods with Order Statistics. Nelson (1979) addressed a few problems encountered when charting non-normally distributed processes, such as the Weibull, using a Shewhart-type chart monitoring for changes in the median or the range. His derivations apply to cases with $n \geq 1$ and he gave examples of charts implementation for a sample size $n = 5$. This paper did not address censoring. Traditionally, practitioners have assumed a normal distribution to design Shewhart charts to detect shifts in the process median or range. They often designed these charts such that there is a probability of 0.003 that a single point falls outside the control limits when the process is in-control. Nelson (1979) identified that if one wants to set up a Shewhart chart to detect shifts in the median and the range of the process, one needs to approach the calculation of the control limits differently when the data are non-normal, such as Weibull distributed. He proposed a methodology to compute the control limits for these charts such that the probability of a point falling outside the control limits is 0.003 when the process is stable and data follow a three-parameter Weibull distribution. He also constructed two additional charts, beside the more traditional median and range charts: the location and the scale charts. The location chart monitors for changes in the first order statistic calculated from each sample of data (the time of first failure recorded in each sample). The scale chart monitors for changes in the statistic $(1/n)\sum_{i=1}^{n}(X_i - \hat{\gamma_0})^{\hat{\beta_0}}$. For each of these charts Nelson (1979) determined the control limits corresponding to a probability of a point falling outside the control limits of 0.003, based on appropriate distributions of the chart statistics. He also gave numerical examples for different values of the Weibull parameters, providing guiding tables of control limits and chart center lines for the practitioner's use. To construct the charts, Nelson (1979) assumed that the practitioner estimates the in-control Weibull parameters values from past data. Shifts in any of the Weibull parameters can cause shifts in the median, range, location chart statistic or scale chart statistic. Noticing that the first order statistic follows a Weibull distribution, when sampling from a Weibull, Nelson (1979) derived the proper control limits for the location chart as

$$\eta_0[\frac{-\log(1-p)}{n}]^{1/\beta_0} + \gamma_0,$$

where $p = 0.5$ for the center line, $p = 0.0015$ for the lower limit and $p = 0.9985$ for the upper limit. He pointed out that the appropriate control limits are given by

$$[\eta_0^{\beta_0}/(2n)]\chi^2(p;2n),$$

where, η, β, and γ are the scale, shape and location parameters of the Weibull distribution, respectively, p is the probability of a point falling below the control limit of choice ($p = 0.5$ for the center line, $p = 0.0015$ for the lower limit and $p = 0.9985$ for the upper limit), and $\chi^2(p;2n)$ is the p^{th} percentile of the χ^2 distribution with $2n$ degrees of freedom.

The control limit for the median chart when the sample size is an odd number and the shape parameter is 1 is

$$limit = -\log(1-q),$$

where $q = [1 + F(p; n+1, n+1)]^{-1}$ and F is the cumulative distribution function of an F distribution with $n+1$ numerator and $n+1$ denominator degrees of freedom. Nelson (1979) provided tables with control limits for the even sample size and different values of the shape parameter. He also applied each of the four different charts to four different sets of data that included shifts in different parameters of the Weibull distribution and recommended that the optimum use of these charts is in pairs. The location and range chart combination appeared to be superior due to relative ease in calculating control limits and plotting points.

Nelson (1979) provided tables with computer-generated control limits for the four charts, corresponding to different values of the in-control parameters of the Weibull distribution, for different sample sizes. There is not an easy apparent way to extend these results to include censoring, to different values of the starting parameters, or to a different distribution.

Kanji and Arif (2001) proposed a median rankit control chart for the three-parameter Weibull distribution. The method plotted the individual data points on a Shewhart-type chart with control limits determined using the quantile distribution described as

$$Q(p) = \gamma + \eta(-\log(1-p))^{\beta},$$

where γ, η, β are the location, scale and shape parameters of the Weibull distribution, estimated from data using the rather non-standard method of percentiles, detailed in their paper. Here, $p = 0.5$ for the central line, described in this paper as the median rankit line, $p = 0.05$ and $p = 0.95$ for the lower and upper warning limits and $p = 0.01$ and $p = 0.99$ for the lower and upper action limits. The authors first check the fit of the Weibull distribution to the data using a probability plot and then they construct the chart. The authors did not address censoring.

Chart for the scale parameter with fixed and variable sampling intervals. Ramalhoto and Morais (1999) proposed a series of two-sided and one-sided Shewhart control charts for shifts in the scale parameter of the three-parameter Weibull distribution from a known target value, for cases with fixed and variable sampling intervals. They also studied the performance of the charts theoretically and by providing a numerical example that supported the theoretical results. They focused on shifts in the scale parameter, which results in a shift in the mean and variance of the distribution. They assumed that there was no censoring. The pivotal statistic used for the construction of the charts was the maximum-likelihood estimator of the scale parameter η derived from each sample. The data were grouped and sample sizes are either fixed (the fixed sampling interval approach- FSI) or alternating between two values (the variable sampling interval approach- VSI). The authors worked with the monotonic transformation of the Weibull to the smallest extreme value distribution to derive the control limits and properties of the charts. They used two different criteria to evaluate the charts in terms of the average run length and the average time to signal. The "comparability criterion" stated that two or more control charts are considered to be comparable under control conditions if and only if their average times to false alarm are equal. The "primordial criterion" stated the comparability of two charts in terms of their average run lengths. Ramalhoto and Morais (1999) proved theoretically that VSI are preferable to

FSI in terms of the average time to signal. Also, they conjectured that the VSI version of the one-sided control charts performs better in terms of the comparability criterion than the FSI version, especially with an increase of the shape parameter and for larger magnitudes of the shift. Overall, their conclusion was that one-sided charts performed better than the two-sided counterparts. Although Ramalhoto and Morais (1999) provided thorough mathematical derivations in their paper, they did not give clear practical guidelines for the chart selection and design.

Hawkins and Olwell (1998) developed cumulative sum (CUSUM) charts based on a sequential probability testing approach, considering different distributions, without censoring. The details of the chart development are given for the case with $n = 1$. Their book is intended to cover the development of CUSUM charts for quality improvement, in general. Hawkins and Olwell (1998) proposed that the optimal diagnostic for a step change in the parameter of any distribution is a CUSUM with the score statistic

$$z_i = \log \frac{f(X|\theta_1)}{f(X|\theta_0)},$$

where f represents the probability density function of the random variable X and z_i is the score statistic corresponding to the i^{th} observation from the process flow. Here θ is a parameter of interest, with the values θ_0 in an in-control situation and θ_1 in an out-of-control situation. The CUSUM chart signals if the statistic $D_i = max(0, D_{i-1} + z_i)$ crosses a threshold h, determined through simulations to achieve a desired in-control average run length. If T follows a two-parameter Weibull distribution with shape parameter β and scale parameter η, then Y^β follows an exponential distribution with mean η^β. Then Hawkins and Olwell (1998) provided a general result for distributions that are members of the exponential family. The authors proposed a CUSUM chart monitoring for changes in the scale parameter of the Weibull distribution, holding the shape parameter fixed. Applying known properties of the exponential family, the CUSUM statistics becomes

$$C_0 = 0$$
$$C_i = max(0, C_{i-1} + y^\beta - k)$$

where $k = \frac{\beta \log(\eta_1/\eta_0)}{\eta_1{}^{-\beta} - \eta_0{}^{-\beta}}$. The chart signals when $C_i > h$, where h is the threshold. This chart was designed to detect an increase in the scale parameter, and the authors proposed a similar chart for a decrease in the scale parameter.

2.2.2 Applications that Allow Censoring

Two aspects specific to lifetime data make the development of a control chart challenging:

- The data tend to be skewed and non-normally distributed.
- The data typically are censored.

In the first chapter we discussed some of the preferred non-normal distributions for the reliability area, and we presented the censoring problem. Although there are several types of censoring, the right-censoring seemed to be predominant in the relevant work, and frequently encountered in reliability practice. The methods presented in this paragraph can be applied to the uncensored case as well, by ignoring the treatment of the censored observations. The real value of these methods resides nevertheless in the inclusion of censoring.

Most of the monitoring methods proposed in the literature for processes with censored data suggest a special treatment of the censored observations, replacing them with a known quantity that should minimize the censoring impact on the test decisions. Also, the designed charts' performance depends on the amount of censoring. Steiner and MacKay (2000) determined that for very high censoring proportions (a large number of the units included in a sample of n observations are censored), the information contained in the sample is reduced.

Then a chart monitoring for a proportion is feasible, where we are interested only in the number of censored observations (usually cheaper). They also advocated that it is reasonable to apply control charts directly to data, ignoring censoring, for censoring proportions less than 25%. In this case the censored observations take the value given by their stopping time. These applications considered type I right-censored data. For censoring proportions between 25% and 95%, they proposed the replacement of censored observations with their conditional expected value, which they claimed was superior to the more traditional chart using the raw data. Steiner and MacKay (2000) first constructed Shewhart-type $\bar{X}$ and S charts for detecting decreases in the process mean and increases in the standard deviation, respectively, based on the CEV approach. The authors assumed a normal distribution for the data, which clearly has limited application for most lifetime data. Nevertheless, when data are log-normally distributed, a simple log transformation would make Steiner and MacKay's (2000) normal approach applicable. Their proposed charts were plots of the sample means and standard deviations calculated from the data with the CEV replacements. The authors provided graphs for choosing control limits for different sample sizes and censoring proportions, assuming a false alarm rate of 0.0027. In their 2001 paper, Steiner and MacKay (2001a) proposed Shewhart charts to monitor for shifts in mean of processes with censored lifetimes and Weibull distribution. They proposed several types of charts, some of them with the CEV approach. Zhang and Chen (2004) implemented the CEV approach to construct an EWMA chart for lifetimes with a Weibull distribution and right-censoring that monitors for shifts in the process mean.

These methods are based on the assumption that the in-control distribution parameters or target values are known or accurately estimated from past data. Another assumption is that, when the Weibull distribution is used, the shape parameter is fixed. Steiner and MacKay (2000) also noticed that it is more difficult to detect increases in the process mean rather than decreases because such changes increase the censoring proportion (considering a fixed stopping time for the testing process, more observations become censored when the process mean increases). Nevertheless, a manufacturer is usually more concerned with accurate detection of deterioration of the process quality (decreases in the process mean is usually associated with deterioration of some reliability characteristic of the product).

As other authors observed, such as Nelson (1979), the design of a control chart for lifetime data is also complicated by the skewness of the distribution. The control limits for a Shewhart chart cannot be set at plus or minus three standard errors away from the estimated process mean, as for traditional charts under normality. Such a naive application of the chart, as Steiner and MacKay (2000) noticed, results in a lower false alarm rate than desired and lower power. This problem is alleviated in the literature by using simulations to find the control limits for desired false alarm rates. Next, we discuss several charts proposed in the literature for censored data.

Conditional Expected Value (CEV) Methods. The CEV approach replaces each censored observation with its conditional expected value calculated based on the initial parameter estimates (in-control). Instead of working with samples of n observations, with several of them censored at a time C, Steiner and MacKay (2001a) proposed that we should work with a sample of n CEV weights, where the censored observations are replaced by some value larger than C. One motivation for this replacement is that in this case the expected sample average equals the process mean.

Let T be a variable representing the failure time for a product of interest and let C denote the censoring time. For each observation, the CEV weight w is

$$w = \begin{cases} t & \text{if } t \leq C \text{ (not censored)} \\ w_C & \text{if } t > C \text{ (censored)} \end{cases} , \tag{2.1}$$

where

$$w_C = E(T|T \geq C). \tag{2.2}$$

Lawless (2002, pp. 223-228) gave the appropriate formulae to find these weights for several common distributions. Since the determination of the CEV weights depends on the assumed distribution, different distributions result in different values for CEV. Therefore, different transformations of the random variables are of interest. Steiner and MacKay (2001a) studied the impact of different transformations on the Shewhart chart design.

Steiner and MacKay (2001a) derived Shewhart-type charts for detecting shifts in the process mean for the Weibull and exponential models based on the following competing methods:

- the Weibull CEV Method,
- the Weibull MLE approach,
- the Score Method,
- the Exponential CEV Method, and
- the Extreme Value CEV Method.

In each case they assumed that the shape parameter remained fixed at a known value because the specific failure mode, which does not change often, determines this parameter. Thus, they assumed that a decrease in the scale parameter caused the decrease in the process mean. Under these assumptions they also noted that a decrease in the mean lifetime resulted in a decrease in the process variability.

We first consider their Weibull CEV method. Let T be a random variable representing a right-censored lifetime following the Weibull distribution. Censored observations are replaced with their CEV values (or weights, as the authors called them), calculated using (2.2) for the Weibull distribution with

$$w_C = \frac{\eta_0 \Gamma^*(x_C, 1 + 1/\beta_0)}{\exp(-x_C)}, \tag{2.3}$$

where $x_C = (C/\eta_0)^{\beta_0}$ and Γ^* is the incomplete gamma function. Here, η_0 and β_0 are the assumed known in-control values for the scale and shape parameters, respectively. On the Shewhart-type chart one plots the sample means calculated using the CEV weights and monitors for a decrease in the mean. The assumption without loss of generality that the in-control value for the scale parameter is unity removes the dependency of the weights on η_0 in (2.3). We can determine the control limit through simulations for a desired false alarm rate, usually corresponding to a probability of a false alarm equal to .0027.

From the expression for w_C, we can see that it depends on the values of the in-control shape parameter β_0. This means that we should determine the control limit for the chart separately for different values of the in-control shape parameter β_0.

Two of the other methods proposed by Steiner and MacKay (2001a), the Score method and Exponential CEV method, are essentially equivalent in construction. Only their derivation and physical interpretation are different because scores do not have a physical interpretation. The score chart plots the sample averages of the $\eta - scores$, which are the first derivatives of the log-likelihood function with respect to the scale parameter η, evaluated at the parameter values of the stable (in-control) process. These $\eta - scores$ are

$$\left.\frac{\partial \log L}{\partial \eta}\right|_{\beta=\beta_0,\eta=\eta_0} = \begin{cases} -\frac{\beta_0}{\eta_0} + \frac{\beta_0}{\eta_0}(\frac{t}{\eta_0})^{\beta_0} & \text{if } t \leq C \text{ (not censored)} \\ \frac{\beta_0}{\eta_0}(\frac{C}{\eta_0}) & \text{if } t > C \text{ (censored)} \end{cases} .$$

The equivalent Exponential CEV Method plots sample averages of CEV weights calculated on the exponential scale. These sample averages are simple linear functions of the previous η-scores. The authors replaced the censored observations by their CEV weights calculated using the transformed data and (2.1) as

$$\begin{cases} x = (t/\eta_0)^{\beta_0} & \text{if } t \leq C \;\text{ (not censored)} \\ w_C & \text{if } t > C \;\text{ (censored)} \end{cases} ,$$

where

$$w_C = E(X|X \geq (C/\eta_0)^{\beta_0}) = (C/\eta_0)^{\beta_0} + 1.$$

The appropriate control limits are derived through simulations for starting in-control values of one. The form of the test statistics for both charts removes dependency on the starting in-control values. Steiner and MacKay (2001a) provided a graph with appropriate control limits corresponding to different sample sizes n and censoring proportions.

Steiner and MacKay (2001a) proposed an alternative Shewhart chart based on a maximum likelihood (MLE) approach. The test statistic is here $\frac{1}{\hat{\eta}}$, where $\hat{\eta}$ is the maximum likelihood estimator for the scale parameter, given a known shape parameter. This procedure replaced censored observations with C, their censoring time. The resulting chart was a plot of the test statistic from each sample of n observations, which is equal to $(r/\sum_{i=1}^{n}(t_i))^{1/\beta_0}$. The authors provided a graph for choosing the appropriate control limit for different sample sizes, a theoretical false alarm rate of .0027 and in-control parameter values equal to one. They use the standard exponential transformation to determine the control limit through simulations.

One last chart proposed by Steiner and MacKay (2001a) for censored lifetime data, the Extreme Value CEV chart, uses a Shewhart-chart to monitor for decreases in mean. This chart plots the sample averages on the extreme value scale, or equivalently, the geometric sample average on the original Weibull scale. Let T be a random variable following the Weibull distribution. If we apply the logarithm transformation to the data, then $Y = \log(T)$ is a random variable that follows the smallest extreme value distribution. The resulting CEV weights, calculated for a smallest extreme value distribution, are

$$\begin{cases} y = \log(t) & \text{if } t \leq C \text{ (not censored)} \\ w_C & \text{if } t > C \text{ (censored)} \end{cases},$$

where

$$w_C = E(Y|Y \geq \beta_0 \log(C/\eta_0)).$$

One needs to use numerical integration to find the conditional expected values on the smallest extreme value scale. Back transforming, the CEV weights on the original Weibull scale are then

$$\begin{cases} t & \text{if } t \leq C \text{ (not censored)} \\ \eta_0 \exp\left(w_C/\beta_0\right) & \text{if } t > C \text{ (censored)} \end{cases}.$$

Again, the authors give guidelines to determine the appropriate lower control limit for different sample sizes and censoring proportions, through simulations, for a desired false alarm rate, assuming in-control parameters equal to one. The practitioner can apply a simple transformation to get control limits for different in-control parameter values.

Steiner and MacKay (2001a) recommended the use of the Extreme Value CEV chart in practice because of the balance between power and ease of application. They advocated that although the Weibull CEV has the most interpretability for production personnel (it uses the average lifetime from each sample), it is not the most desirable since its control limit depends on the shape parameter in-control value. The next best in terms of interpretability is, as the authors advocated, the Extreme Value CEV chart. The test statistic is then the geometric sample average on the original Weibull scale and the control limit determination does not depend on the in-control values of the shape parameter (considered fixed throughout their paper). Steiner and MacKay (2001a) compared the charts in terms of performance for different sample sizes and censoring proportions. They also noted that traditional Shewhart charts (that use the raw observations) gave either high false-alarm rates or low power, which results in poor ability to detect changes in the process for processes with censored data. However, replacing censored observations with their CEV weights improved the charts' performance. The authors concluded that the Weibull CEV, exponential CEV and MLE approaches are equally powerful. The Extreme Value CEV approach was less powerful. Also they not that for moderate censoring proportions, such as 50%, there is almost no loss in power to detect process mean decreases. One reason is that since the charts monitor for decreases in the mean, once the mean decreases the censoring proportion also decreases. Steiner and MacKay (2001a) also talked about the tradeoff between information loss due to censoring and data collection costs, when censoring occurs due to imposed cost restrictions. The optimal tradeoff point depends on the testing costs and the consequences of false alarms and/or missing process changes (that occur due to loss in information through censoring). Steiner and MacKay (2001a) designed these charts to detect decreases in the process mean due to decreases in the Weibull scale parameter, keeping the shape parameter fixed. They mentioned only briefly the case when both parameters can shift and process changes happen due to these simultaneous shifts. For the case considered they concluded that the Extreme Value CEV method is the most powerful. They did not give a detailed derivation of this chart for simultaneous shifts in the parameters.

Vargas and Montaño (2005) considered the Extreme Value CEV Shewhart-type chart proposed by Steiner and MacKay (2001a) and extended their performance studies. They did not change the construction of the chart. Vargas and Montaño (2005)gave a numerical example and then discussed the applicability of this chart to both high and moderate censoring proportions. They compared this chart's performance to the regular Shewhart $\bar{X}$ chart for the Weibull distribution and to the np chart, and they found that the CEV chart worked better than the traditional chart for sample sizes of three and five observations. They concluded that one should expect similar results for larger sample sizes. As expected, the efficiency of the chart decreased as the censoring proportion increased.

Zhang and Chen (2004) constructed an EWMA chart for monitoring the mean of censored Weibull lifetime data based on the CEV approach, using the exponential transformation. They constructed their charts making similar assumptions about the process as Steiner and MacKay (2001a), i.e., right-censoring, samples of n observations from the process and fixed shape parameter. With the CEV approach, censored observations are replaced by their conditional expected value (CEV), also referred to as their weights. If the random variable T follows one of the distributions preferred in the reliability practice, and C is the test stopping time, then the weights W_{ij} are determined for each observation j of each sample i put on test using

$$w = \begin{cases} t & \text{if } t \leq C \text{ (not censored)} \\ w_C & \text{if } t > C \text{ (censored)} \end{cases},$$

where

$$w_C = E(T|T \geq C).$$

Lawless (2002, pp. 223-228) gave the appropriate formulae to find these weights for several common distributions. Once again, let $X = (\frac{T}{\eta})^{\beta}$ be the failure time measured on the exponential scale, where η and β take the assumed in-control values η_0 and β_0. Zhang and Chen (2004) defined the lower-sided EWMA scaled CEV statistic, Q^L to be

$$\begin{cases} Q_i^L = min((1-\lambda)Q_{i-1}^L + \lambda\bar{W}_i, h_0), i \geq 1, \\ Q_0^L = h_0, \end{cases}$$

where $\bar{W}_i = \frac{W_{i1}+W_{i2}+\ldots+W_{in}}{n}$, λ is a smoothing parameter such that $0 < \lambda \leq 1$, and $h_0 = 1$. Here, $\bar{W}_i$ is the average weight for a sample of data. Since the determination of the CEV weights depends on the assumed distribution, different distributions result in different values for CEV. Therefore, different transformations of the random variables are of interest. Zhang and Chen (2004) included a reflecting barrier to increase the sensitivity of the chart and to avoid inertia problems. They used $h_0 = 1$ as the reflecting barrier since, in general, the expected value of a random variable following a standard exponential distribution is unity, an so the in-control mean of lifetimes in this case is 1. Zhang and Chen (2004) set the control limit as $LCL = K_L h_0$, where $0 < K_L < 1$ is a multiplier that controls the chart's performance, together with the smoothing parameter. The authors used smoothing parameter values of 0.05, 0.1 and 0.2, advocating that these are popular choices in practice. They proposed the construction of the upper limit control chart in a similar way.

Zhang and Chen (2004) compared the EWMA CEV chart with Steiner and MacKay's (2001a) Shewhart-type CEV chart in terms of average run length (ARL) and standard deviations of run lengths (SDRL). They used a simulation study to recommend combinations of (λ, K) that minimize the ARL for various out-of-control situations given a specific in-control ARL. The authors considered both the upper- and lower sided cases and studied the chart's performance for various values of the sample size, smoothing parameter, censoring proportions, and in-control average run lengths. Zhang and Chen (2004) determined that the lower-sided EWMA chart performs better than the Shewhart-type chart for detecting mean decreases. They also found that the upper-sided EWMA worked well in detecting mean increases while the Shewhart counterpart was not appropriate for this case. Steiner and MacKay (2001b) proposed CEV EWMA charts for censored data with competing risks to detect changes in the mean of a characteristic of interest. These competing risks result in a distribution for the censoring level. The authors also proposed a chart to detect changes in the mean censoring level, advocating that this should protect against possible confounding due to changes in the mean of the censoring mechanism. The CEV EWMA charts are constructed following a similar paradigm as described in Zhang and Chen (2004). In addition, a distribution was assumed for the censoring time C. Steiner and MacKay (2001b) assumed normal distributions for both the lifetime T of the characteristic of interest and for its censoring time C. The authors described the design of the charts to achieve a desired in-control average run length through simulations and then they compared the CEV EWMA chart for the primary failure mode to a Shewhart-type chart, claiming its superiority. The authors assumed that the in-control mean and the variance of the two distributions were estimated through maximum likelihood on a previous phase. In this research we assume that our products fail due to one failure mode, but our methodology can be extended to include competing risks and can be considered further in future research with this paradigm, especially for an extensive study of the out-of-control performance and for non-normal distributions.

A Chart with Left-Censoring. Schneider et al. (1995) addressed the problem of monitoring skewed and censored data in the context of detecting measurement precision in the chemical industry. Suppose we want to measure the value of a certain characteristic in a chemical reaction. Due to the limited precision of the measuring instrument, at a certain point, very low values of the characteristics cannot be observed and the data become left-censored. Schneider et al. (1995) noticed that typically the data obtained from such processes follow a Weibull or log-normal distribution, and constructed one-sided charts to detecting shifts (typically increases) in the median. They used probability plots of the data to estimate the control limits and center line parameters, for a desired probability of a false alarm p. The upper control limit and center line correspond respectively to the upper $100(p)th$ and $50th$ percentiles on the probability plot. The authors constructed the probability plots using the ranks of the data, excluding the censored points.

2.2.3 Other Applications

Other related but not overlapping areas are the monitoring of field data or the monitoring of warranty data. Wu and Meeker (2002), for example, constructed charts for early detection of reliability problems with warranty data. Batson et al. (2005) constructed charts to monitor failure trends for field failures under different sampling schemes. Although these two areas are important within reliability, they do not fall under our paradigm of n items on a test stand.

2.3 Concluding Remarks for the Literature Review

The reliability practitioner confronted with the problem of monitoring processes with censored non-normal lifetime data needs to consider several aspects when selecting an appropriate method. First of all, there are only a few developed charts considering censoring. Steiner and MacKay (2001a) proposed five competing Shewhart-type charts to detect decreases in the process mean caused by a decrease in the scale parameter with a fixed shape parameter of the assumed Weibull distribution. Zhang and Chen (2004) proposed an EWMA chart for both decreases and increases in the mean. Steiner and MacKay (2001a) compared the charts in terms of ease of construction and interpretability for the practitioner, performance, dependence on the values of in-control parameters. They also discussed the implications of the censoring proportion. They concluded that it is more economical to use a traditional Shewhart chart for a proportion,when one deals with very high censoring (more than 95%). For moderate censoring proportions of $25 - 95\%$ they concluded that the Extreme Value CEV chart should be the most appropriate, with good performance and ease of interpretation. Zhang and Chen (2004) concluded that their CEV EWMA chart had better performance than the Shewhart-type chart. The reliability practitioner must keep in mind the tradeoff between the chart performance and the censoring proportion, which are inversely related.

The work in this area should be more focused on the censoring issue. Also, researchers need to address the problem of monitoring different aspects of the lifetime data distributions, such as monitoring each parameter individually, in addition to the detection of shifts in means or percentiles of interest. Covariate information required in the monitoring of medical processes has yet to be used in industrial applications. Although right-censoring is frequently encountered in the practice of reliability, periodic inspection is quite common. The problem of monitoring interval censored lifetimes has not been clearly addressed in the relevant literature. The following chapters provide solutions to some of these problems. In Chapter 3 we construct a chart for interval censored data and we illustrate its implementation in a health-related application at Becton Dickinson. Then in Chapter 4 and 5 we develop and study the properties of charts for right-censored lifetimes with a Weibull distribution.

Chapter 3

Monitoring Interval Censored Data. A Case Study

3.1 Background

In many life tests failures are discovered only at times of inspection. Interval-censored observations consist of upper and lower bounds on failure times. Such data are also known as inspection data. If a unit has failed at its first inspection, its value is left-censored. If a unit has not failed by the time of the last inspection, its value is right-censored. The intervals can be regarded as ordered categories. This type of data can be encountered in a number of fields. For example, discrete concentrations of a drug that cures a disease define ordered categories.

We consider interval censoring due to inspection at predetermined times t_i. The inspection starts at a time t_0 and ends at a final moment t_l. We calculate the probability of one unit failing in the interval (Meeker and Escobar (1998)) (t_i, t_{i+1}) as

$$\pi_i = P(y_i < Y \leq y_{i+1}) = \Phi(y_{i+1}) - \Phi(y_i),$$

where Φ is the cumulative distribution function of the standard normal distribution, supposing that the lifetime random variable T follows a log-normal distribution. If T follows a Weibull distribution, then, from Chapter 1, $Y = \log T$ follows the smallest extreme value (SEV) distribution with parameters $b = 1/\beta$ and $u = \log(\eta)$. Regardless of the assumed underlying distribution, if one is interested in developing a monitoring technique for the mean lifetime in this circumstances, one needs to account for the nature of data, i.e., ordered categories with probabilities given by the underlying distribution. Typically, the limiting intervals are open to $+\infty$ and/or $-\infty$, to account for left and right censoring.

In this chapter we develop a monitoring method for the mean of interval censored data. We illustrate the implementation of the chart in one of the quality processes of the Diagnostic Division of Becton Dickinson. The chart monitors for changes in the average drug dosage needed to suppress an infectious organism. Although the data are not from the field of reliability, the nature of the data, i.e., ordered categories with censoring, makes the problems very similar. We develop a likelihood ratio based CUSUM chart to monitor for changes in the quality of the manufactured product, the PHOENIX panel. The panel is used in medical laboratories to help detect infectious diseases and the drugs and dosages of the drug that cure the disease. Therefore, the accurate indication of the correct reaction is highly desirable. The chart should ensure fast detection of harmful changes in these reactions. The nature of the data makes the development challenging and resembling the problem of monitoring interval censored lifetimes.

3.2 The PHOENIX AST System

PHOENIX antimicrobial susceptibility testing (AST) system is a microbroth dilution system for antimicrobial susceptibility testing and bacterial identification, currently used by a large number of laboratories across the globe. A single panel is about 6” x 2”. It displays 136 55uL wells all of which are read optically by an instrument, using red, blue, green, and UV signals.

Each panel can be inoculated with multiple drug series (typically around 20), and has circa 50 ID substrates. There are commonly 85 wells devoted to antimicrobial susceptibility testing and 51 wells devoted to bacterial identification on a panel. Each panel may be inoculated with a single organism. The drugs tested are generally antibiotics expected to suppress the organism.

Bacterial Identification. Identification systems are utilized to identify the genus and the species of an organism. These systems contain fluorogenic and chromogenic substrates. The ID portion of the panel is an extension of general biochemical identification; there are various substrates in each well, and different bacteria react and/or metabolize these substrates differently. How a given organism reacts to the entire set of ID substrates communicates to the system the identity of the strain.

Antimicrobial susceptibility testing. Susceptibility systems are used to determine what antimicrobic drugs are most effective in treating an infectious disease. The organism is tested against various concentrations of antimicrobics, determining the organism's resistance (ineffective) or susceptibility (effective) to the antimicrobics. The identification and susceptibility of an organism are information that the physician requires from the laboratory to help determine the patient treatment. The goal of antimicrobial susceptibility testing is to predict the in-vivo success or failure of antibiotic therapy. The in-vitro tests using the diagnostic panel measure the growth response of an isolated organism to a particular drug or series of drugs.

3.3 The Test Procedure

To monitor quality, samples of n panels are extracted from the manufacturing line and tested. The testing of the panel ensures that the panel has the expected reaction when used to detect a known infectious disease strain (also referred to as an organism). The panel should identify the strain correctly. Beside the strain identification, the panels should identify the right drug that suppresses the strain. A set of predetermined drugs are inoculated on the panel. The panel should produce the expected reaction for every drug applied. The manufacturer holds a list of expected reactions, also called the quality control specifications. The following example of the quality control procedure is based on fictional figures, for the sake of confidentiality. We consider that the manufacturer is monitoring the quality of a batch of panels manufactured during one month by applying three different strains in combination with four different drugs. The panel reaction to each of the strains (bacterial identification) and to each of the strain and drug combinations (susceptibility testing) should conform to the known standards. These reactions are evaluated through measurements with an optical instrument and quantified through a set of scores. The manufacturer randomly selects eight different panels to apply each strain and drug combination. The set of eight replications is usually collected during a sequence of eight days. The average score from the set of eight replications is compared to the standard. One single organism is inoculated on each panel in a growth environment, typically microbroth. During incubation, the instrument takes readings from the wells that result in raw antimicrobial susceptibility (AST) and bacterial identification (ID) data. The processing of raw measurement results in higher-level data, such as the minimum inhibitory concentration (MIC) data, which are compared to the standards given for each strain and drug combination tested. The MIC is defined as the lowest concentration of an antimicrobial agent at which bacterial growth is inhibited in in-vitro testing. Zhou et al. (2009) presents a probabilistic approach for estimating MIC data based on growth curves, approach currently used at Becton Dickinson (BD) to make an initial estimate of the MIC with PHOENIX. Each lot of panels is tested by this procedure for multiple strain and drug combinations, to ensure that the panel indicates the expected reactions.

3.4 Measurement and Data Processing

To test and determine what the MIC for a given strain/drug combination actually is, a panel is charged with drugs at different and discrete concentrations. These panels are then inoculated (and all the drug-containing wells) with a particular strain in growth media. The PHOENIX instrument reads the wells and detects the first well where the strain is not growing, and reports the concentration as the MIC, since that is the lowest tested concentration where it was inhibited and by definition the minimum inhibitory concentration.

The general assumption is that there is, for any given strain, some concentration of any given drug that suppresses it. Also, the testing assumes that all strain/drug combinations have some quantifiable MIC. However, the amount of drug required varies greatly by strain/drug combination. For any given drug, different strains could have a very wide range of MICs. Furthermore, there is limited space on a panel, implying that any given panel type can hold only a limited number of different concentrations of a drug. The manufacturer only tests the clinically relevant concentrations of the drug, meaning the laboratory does not test drug concentrations so high that the clinical side effects of those dosages become unreasonable, or that simply cannot be tolerated in vivo. There are established standard concentrations that are known to generally suppress certain types of strains. These standard concentrations are useful since they provide information about the likely presence of resistance mechanisms and the general viability of the drug as a treatment. Therefore, the range and number of discrete concentrations on a panel type is frequently small.

When a particular strain is resistant to all the concentrations on a panel, PHOENIX identifies that all the wells are growing. Thus, it cannot determine the MIC since it does not detect the inhibition of the strain. It can only indicate a lower boundary of the MIC, which would be the highest concentration well on the panel. It therefore reports this lower boundary of the MIC. This is an off-scale high MIC, resulting in right-censored data.

When a certain strain is inhibited for all the concentrations on a panel, PHOENIX detects that none of the wells are growing. PHOENIX cannot determine the MIC since it has no information on the concentrations when the strain is not inhibited and therefore cannot determine what is the actual minimum concentration. It can only indicate an upper boundary, which is the lowest concentration present on the panel. PHOENIX then reports this upper boundary of the MIC. This is an off-scale low MIC, resulting in left-censored data. The presence of off-scale MICs is an intrinsic aspect of microbroth dilution susceptibility testing, although very large drug ranges minimize the number of off-scale results.

3.5 The Monitoring Methodology

3.5.1 Real Time Monitoring Objective

A large database containing historical test data is available for the analysis. Descriptive analysis of historical data by separate strain and drug combinations led to the conclusion that on occasions MIC means shift in time and the quality control pass-fail testing might not succeed in capturing these shifts if the size of the shift is small. A real time monitoring instrument could react earlier than just common sense to shifts in the means across panel ranges. Since a significant volume of historical data are already available, the instrument design relies on the information provided in the database and on process operators' expertise. We design a CUSUM chart that detects persistent small shifts in the average MIC needed to suppress a certain strain. As we noted in Section 3.3, the data are represented by ordered categories, in some cases reflecting censoring. The chart needs to account for these complications intrinsic to MIC data.

3.5.2 A CUSUM Chart for Ordinal Categorical Data

Consider the BD MIC testing process for one particular strain and drug combination. Samples of six panels are extracted from the production line and tested. The technician records the MIC measurements or the drug dosages. The type of data is intrinsically ordinal categorical (the dosages are usually 0, 2, 4, 8, and 16). The practitioner observed that taking the base 2 logarithm of the MIC linearizes the data. For some strain and drug combinations, the technician only records the lower or upper limit for the MIC, making the data censored ordinal categorical. Consider a list of ℓ possible $\log_2 MIC$ values for a strain and drug combination: $t_1, t_2, ..., t_\ell$. We add to this list $t_0 = -\infty$ and $t_{\ell+1} = +\infty$, to include the possibility of left and right censoring.

Consider a latent continuous random variable T that takes values in the intervals (t_{j-1}, t_j), with j going from 0 to $\ell + 1$. The intervals bounded by $+\infty$ and $-\infty$ correspond to the practical situation where the observed MIC value is only an upper or lower bound for the actual data. Assume that the random variable u is normally distributed with mean μ and standard deviation σ, as observed by the practitioner in an in-control situation. Let $\pi_j(\mu)$ be the probability that an observation falls in the interval (t_{j-1}, t_j). This notation makes the explicit dependence of this probability upon μ. We can calculate this probability by

$$\pi_j(\mu) = P(t_{j-1} < T \leq t_j) = P(\frac{t_{j-1} - \mu}{\sigma} < T \leq \frac{t_j - \mu}{\sigma}) = \Phi(\frac{t_j - \mu}{\sigma}) - \Phi(\frac{t_{j-1} - \mu}{\sigma}), j = 1, \ldots, \ell+1,$$

where Φ is the standard normal cdf. Let $\boldsymbol{\pi}\ (\mu)$ be the vector of probabilities for a specific μ; thus,

$$\boldsymbol{\pi}\ (\mu)' = (\pi_1(\mu), \pi_2(\mu), \ldots, \pi_{\ell+1}(\mu))\,.$$

Let $\boldsymbol{c}_i$ be the vector of counts in each interval for the i^{th} sample, where

$$\boldsymbol{y}_i' = (c_{i1}, c_{i2}, \ldots, c_{i\ell}).$$

We note that the counts c_{ij} follow a multinomial distribution. Let C_j be the random variable associated with data falling into interval j. The probability function is

$$L_i(\boldsymbol{y}_i|\boldsymbol{\pi}\ (\mu)) = P(C_1 = c_1, \ldots C_{\ell+1} = c_{\ell+1}) = \pi_1(\mu)^{c_1}\pi_2(\mu)^{c_1} \ldots \pi_{\ell+1}(\mu)^{c_{\ell+1}}.$$

Let μ_0 be the in-control mean for the process and σ be the historically observed value for the collected data during a reasonably long interval of time (the data are available in the BD database). The standard deviation and the in-control mean are estimated from the historical data on a previous stage using an appropriate technique. The standard deviation is considered fixed to this estimated value throughout the considered monitoring process. We are interested in detecting if there has been a shift in the process mean to an alternative μ_1 value.

Hawkins and Olwell (1998) observed that the optimal diagnostic for a step change in the parameter of any distribution is a cumulative sum (CUSUM) chart with the score statistic

$$z_i = \log \frac{L(\boldsymbol{y}_i | out - of - control)}{L(\boldsymbol{y}_i | in - control)}, i = 1, 2, \ldots, \tag{3.1}$$

where $\boldsymbol{y}_i$ is the i^{th} sample of size n and L is the likelihood function, formulated according to the distribution of choice and to the censoring scheme. Thus, z_i is determined as the logarithm of the ratio between the likelihood of the data in the out-of-control and in-control scenarios. Then the CUSUM chart statistics are

$$S_i = max[0, S_{i-1} + z_i], i = 1, 2, \ldots, \tag{3.2}$$

where $S_0 = 0$ and the chart resets to 0 every time there is a signal. The chart signals an out-of-control state when $S_i > h$, where h is a constant determined by simulation to achieve a given average run length when the process is in control. In this case we can rewrite

$$z_i = \log \frac{L_i(\boldsymbol{c}_i | \boldsymbol{\pi}\ (\mu_1))}{L_i(\boldsymbol{c}_i | \boldsymbol{\pi}\ (\mu_0))}, i = 1, 2, \ldots,$$

where $\boldsymbol{\pi}$ are the probabilities associated with each interval. We calculated the likelihood of the data under each of the in-control and out-of-control state with μ shifted, using the multinomial distribution. With some algebra, we obtain

$$z_i = \sum_{j=1}^{\ell+1} c_{ij} \log \left(\frac{\pi_j(\mu_1)}{\pi_j(\mu_0)} \right).$$

The CUSUM chart is constructed for each strain and drug combination, for a negative and a positive shift of interest.

3.6 Implementation

A control chart is a sequence of index values maintained over time and which are calculated from measured data. When the index exceeds a threshold value, the chart triggers an alert. In this case alerts are sent by e-mail to individuals specified in advance. The term "chart" refers to the sequence of index values rather than an actual graphical plot, although the index values can be plotted over time or iteration number.

3.6.1 Data Processing for the Chart Implementation

One specifies in advance the strain and drug combinations of interest. A chart is maintained for each combination, and charts are treated independently. Typically there are two charts per combination, one which detects increasing trends and one which detects decreasing trends in means.

The quality control procedure results in a large database containing a substantial amount of historical data. The product is FDA-approved and the performance achieved during the clinical trial stage, which was deemed adequate, must be maintained during the life of the product. One option is to assume that the clinical trial stage data is in control and use them to estimate the in-control parameters. On the other hand the database contains a huge volume of production data, with information on what passed and failed the quality control testing or had other problems. The practitioner can use what is deemed to be good data for the in-control parameters estimation and "data with problems" to define the out-of-control situations.

In order to design a chart for each strain/drug combination used in the panel testing, data are queried from the quality control database. Each data record provides one result per combination per panel, for all panels used in the particular batch of testing. A number of variables identify each data record, indicating: the organism used for the panel testing, the drug, the current batch of panels tested, the test date, the MIC value and the MIC instrument sign. When the MIC instrument sign is $<=$ then the MIC value is left-censored (off-scale low), while if the sign is $>$ then the MIC value is right-censored (off-scale high), otherwise the MIC is on-scale. When dealing with off-scale data, in some cases, one merges the MIC levels, and update the data used for the chart set-up. Typically the merging simply narrows the range of MIC values. One or more consecutive extreme MIC values are merged into a more central MIC level. However, the on-scale data delivers more precise information on a specific strain/drug combination, so the chart construction uses predominantly on-scale data.

In order to design a CUSUM chart for a specific bacterial strain and drug combination, one needs to determine a set of characteristic parameters. These are the in-control mean, a potential shift illustrating the out-of-control state, the process standard deviation to be maintained constant for both in-control and out-of-control instances, the categories defined by the MIC levels and the interval bounds for these categories, and the chart upper threshold. One determines these parameters for the chart design based on the available historical data.

3.6.2 CUSUM Chart Design Algorithm

The determination of the chart parameters follow several steps outlined next. We exemplify the chart design for a particular strain/drug combination, with fictional names for the sake of confidentiality.

Determination of the CUSUM chart in-control and out-of-control parameters. On a first stage, one queries historical data from the database for the strain and drug combination for a period of time of reasonable length (at least 100 samples of data) when the process was considered stable. The output of this step is a subset of at least 100 samples of size n of test results for a certain strain/drug combination that are representative for an in-control state. Further, we characterize the distribution of the $\log_2 MIC$ data in the in-control state. **Example:**We illustrate the design of the chart for a combination of strain/drug for which the database contains a significant amount of data.

Table 3.1: Frequency of Data for Penicillin and Streptococcus Combination.

$\log_2 MIC$	Count
-1	1
0	5
1	205
2	2526
3	1469
4	2
N= 4208	

Based on the frequency of values the analyst should decide on the appropriate number of intervals for the chart set-up. The use of two up to five categories is recommended. We determine the interval bounds considering the frequency of different values. If a number of distinct values are poorly represented, the corresponding intervals are pooled together. In this case, the mean and standard deviation under the null hypothesis are recalculated (the maximum likelihood procedure reapplied), so that the chart construction is based on the updated data. The lower and upper bounds extend indefinitely for the extreme intervals. Another option is to define the upper and lower bounds of extreme intervals depending on how well the normal distribution fits for the limiting cells, exactly as for the other cells. A chi-squared test can serve the purpose of checking the cell fitting. In this particular example the instrument indicated off-scale low and high data on instances, i.e., left and right censoring. Therefore, the first and the last intervals are not bounded below and above respectively. **Example:** In our case there are four intervals, respectively: $(-\infty, 1.5]$, $(1.5, 2.5]$, $(2.5, 3.5)$, and $[3.5, +\infty)$.

The maximum likelihood procedure (MLE) can be applied to data using one of the standard software packages or custom designed software. The maximum likelihood estimation algorithm for the mean and standard deviation of the normal distribution consists of constructing the likelihood function and then maximizing it. The maximization consists of setting the first derivative of the log-likelihood function (score function) with respect to each parameter to zero and determining the solutions of this equation system (the system has closed-form solutions for the normal distribution). These are approximations for the mean and standard deviation from the data, since the distribution is not exactly normal. We can also approximate the in-control mean and standard deviation with the sample numerical summaries.

Afterwards, evaluating the distribution of the data, the in-control mean is selected so that about the same cumulative percent of data assuming a normal distribution with the corresponding mean and standard deviation lies to the left of a certain interval cut-off, as indicated by the tally of sample data. We adjusted the in-control mean through trial and error. We considered the standard deviation constant at this stage throughout chart design. The magnitude of standard deviation determines the sensitivity of the chart. By trial and error, a standard deviation that was 33% higher than the maximum likelihood estimate seemed optimum for most cases, as evidenced by the simulated average run length for in-control and out-of-control scenarios.

The next step after determining the mean, standard deviation, and interval cut-offs is to determine a reasonable shift size. The shift indicates the process mean under the alternative hypothesis, stating that the process is out of control. One determines the shift size based on the knowledge of the process and business or technical needs, i.e. if the shift is sufficiently large, the product does not work. One can set-up a CUSUM chart for each direction separately. Nevertheless, the expected direction of a potential shift could be determined by observing the skewness of the sample data distribution. Different shift sizes were implemented to determine the appropriateness of the results. Based on trial and error, shifts of 66% of the standard deviation provide reasonable detection performance, in the sense that the average run length for the chart for the in-control and out-of-control scenarios are reasonable. **Example:** We design the chart for an in-control mean $\mu_0 = 2.234$ and standard deviation $\sigma = 0.75$, which is about 33% higher than the standard deviation maximum likelihood estimator from data, $\sigma = 0.5636$. The potential negative shift $d = -0.5$ is established at about 66% of the standard deviation.

The CUSUM chart statistic and the threshold. The next step is to calculate the CUSUM chart statistics and determine the threshold for the chart through trial and error for a desired in-control average run length. The CUSUM chart statistic S is determined through the following algorithm. On a previous stage we determined the in-control parameters, interval delimitations, shift magnitude and potential direction. Next, following the details outlined in Section 3.5, we calculate the probability of each interval in an in-control state (using a normal distribution) as the difference between the values of the normal cumulative distribution function evaluated at the interval limits. From normal theory, we need to use numerical integrations to compute the normal cumulative distribution function, performed by a number of statistical software packages (we use IMSL). Next one calculates the probability weights in an out-of-control state, with the shifted mean (while maintaining the standard deviation constant). We obtain the weights assigned to each interval by applying the natural logarithm to the ratios of the probabilities calculated for the out-of-control and in-control states. For our example these are: $\log 1 = 1.269486$, $\log 2 = -0.01947531$, $\log 3 = -1.280822$, and $\log 4 = -2.690709$ respectively. Next, for each sample of data, we count how many observations fall in each interval and we multiply this count by the specific weight calculated at the previous step. By summing these weighted counts we obtain the z_i statistics corresponding to each sample, sorted in chronological order. Finally, we calculate the actual CUSUM chart statistic S for each sample of data using (3.2). Whenever S crosses the chart threshold, it is reset to 0 and the chart signals. We can calculate an average run length as the average number of samples until signal.

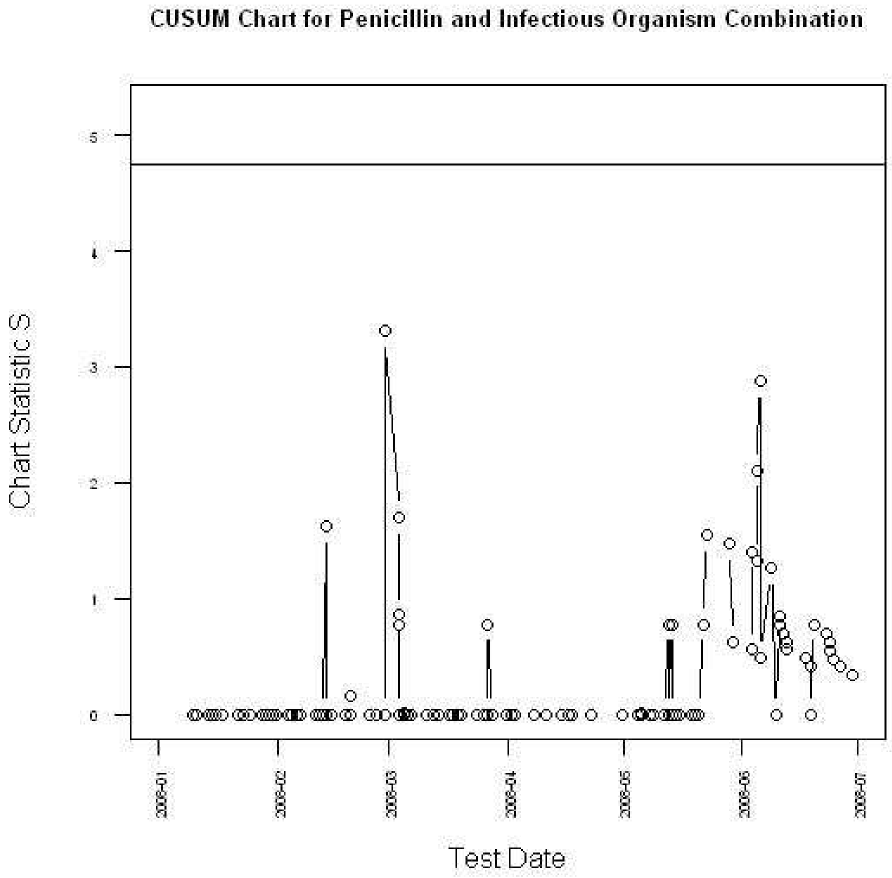
To determine the CUSUM chart threshold, we used a trial and error process. At each iteration, we simulated the control chart performance for an in-control situation. From the quality control literature, an in-control average run length between 371 and 500 runs is satisfactory. Afterwards we simulated the control chart performance for an out-of-control situation. The chart should signal as soon as possible when the mean shifted, within a reasonable number of runs. The practitioner should use the process knowledge in order to determine the desired in-control average run length.

Description of the simulation. We use C++ to perform the simulations to determine the chart threshold through trial and error to achieve a desired in-control average run length. We used the IMSL routines to generate random normally distributed data with the determined mean and standard deviation for both in-control and out-of-control cases. We generated a thousand samples of size n and we calculated the chart statistic as described. We counted the number of signals and determined the average run length. We repeated this process $10,000$ times and determined an average run length over all iterations. The simulation error is the standard deviation of the run lengths divided by the number of iterations. In our case, the simulations provided an optimum threshold value of 4.75.

3.6.3 Validation of the CUSUM Chart

The CUSUM chart is set up using the determined parameters and applied to a historical sample of data considered to be in control. The chart should only signal a reasonable number of times. For example, we have 5 false signals from a total of 391 in-control samples. If the performance is unsatisfactory, the chart is re-calibrated. The operator should adjust the parameters and redesign the chart. Additional validation is performed on a sample of historical data extracted from an out-of-control portion of the process. The chart should

Figure 3.1: CUSUM chart for Censored Ordered Categorical Data.



signal within a reasonable number of samples. Figure 3.1 illustrates the CUSUM chart applied to a sequence of historical data for the considered case, monitoring for a potential negative shift. The process is considered to be in control, and the chart is expected to signal a limited number of times. The chart displays 154 samples (tests).

3.6.4 Re-calibration and Sensitivity Analysis

Theoretical properties change (simulated in-control and out-of-control average run lengths) for different shift and standard deviation sizes. For the considered example, the simulation study showed the following properties:

Table 3.2: CUSUM ARLs for Different Shifts and Standard Deviations.

Shift	0.25	0.5	0.5	0.5	0.75	1.25	0.25	0.5	0.5
Standard Deviation	0.75	0.75	0.56	0.56	0.56	0.56	0.56	1	2
In-control ARL	531	432	381	734	675	797	563	515	704
Out-of-control ARL	12	4	2.5	2.5	1.96	1.08	11	8	28
Cut-off	4.5	4.75	4.75	6	4.75	4.75	4.75	4.75	4.75

As expected, there is a trade-off between the sensitivity of the chart (the ability to identify real shifts) and the number of false alarms (signals when there is no actual shift). The dimension of the shift should be reasonably small. The CUSUM chart implemented for the BD PHOENIX process can be adapted to monitor interval censored lifetimes.

3.6.5 Conclusions and Recommendations

In this chapter we developed and illustrated the implementation of a likelihood-ratio based CUSUM chart for censored ordinal categorical data. This type of data is similar to interval censored data frequently encountered in the practice of reliability. We presented a case study with data from the quality control process for one of the manufactured products at Becton Dickinson, the PHOENIX AST System. The chart was developed to detect increases or decreases in the mean dosage of a drug that suppresses an infectious organism. The data consist of sets of drug dosages and, on occasions, only upper or lower bounds to the actual values can be determined from the instrument. The detection of small changes in the minimum inhibitory concentration (the smallest drug dose that suppresses the disease) is important for the quality control of the manufacturing process. The pilot implementation of the chart for a few strain/drug combinations proved successful at Becton Dickinson and is in the process of extension to a greater number of combinations of drugs and organisms (200 to 1,000). The chart implementation is ongoing at Becton Dickinson, and its performance closely monitored.

Chapter 4

CUSUM Charts for Monitoring the Characteristic Life of Censored Weibull Lifetimes

4.1 Background

Consider a cumulative sum (CUSUM) chart for detecting changes in the scale parameter of the Weibull distribution, also known as the characteristic life, when the shape parameter is fixed. The charts found in the literature usually consider monitoring the mean, which is equivalent to monitoring the characteristic life. However, practitioners in the reliability field prefer to work with the actual shape and scale parameters directly, rather than with the mean and the variance, as in the normal theory. We have described our framework in Chapter 1. Samples of n products are put on a test stand at the same time and they are run to failure. Usually the test is stopped before all unit fails. The units still running are considered suspensions or right-censored data.

Steiner and MacKay (2000) gave an example of a testing process that incurs censoring. They considered the manufacturing of electrical boxes used to cover transformers in residential neighborhoods. The manufacturer is interested in maintaining the rust resistant capabilities of the paint applied to the boxes. The rusting process takes time even under accelerated conditions. To speed up the rusting process, the manufacturer performs an accelerated salt fog endurance test by cutting and scratching boxes and putting them in a warm salt spray chamber. The units are checked daily for rust, and stay in the chamber until rust appears, but not more than 20 days. After 20 days some units have no sign of rusting, and those are the right-censored units. Type I right-censoring occurs when one stops the test at a certain predetermined time and the testing of the remaining units that did not fail by that time is censored. A monitoring procedure ensures that the painting anti-rust capabilities do not deteriorate. The authors advocated that for this type of application, one can analyze the observed censored data from the testing process, ignoring the accelerated nature of the test, as long as the testing is performed in the same manner and the relationship between accelerated and normal conditions does not change. In this chapter we consider Type I right-censoring within a similar framework.

The manufacturer is usually interested in increasing the lifetime of products, so it is of interest to determine if significant changes occurred in the process, which might affect negatively the reliability characteristics of the product. We focus therefore on detecting negative shifts in the characteristic lifetime of products. In the following sections we present the technical details of our methodology, a detailed description of the simulations we undertook to evaluate the charts' properties, and the simulation results. We conclude with a few final comments and recommendations.

4.2 Monitoring Methodology

Consider a likelihood ratio based CUSUM chart with samples of size $n \geq 1$ of right-censored data following a Weibull distribution with a variable scale parameter, η, and a fixed known shape parameter, β. We evaluate the impact of different transformations to the data on the chart's design. Then we assess the chart's performance in different scenarios and compare its performance to the performance of the CEV EWMA chart proposed by Zhang and Chen (2004).

Consider the likelihood ratio based CUSUM chart discussed in Chapter 3, with score statistic given by (3.1) and chart statistic (3.2). In general, the likelihood function for any distribution including right-censored data is (Meeker and Escobar (1998, p. 175))

$$L(u,b;y) = \prod_{j=1}^{n} f(y_j)^{\delta_j}[1 - F(y_j)]^{1-\delta_j},$$

where $\delta_j = \left\{ \begin{array}{l} 1\; if\; item\; fails\; at\; time\; t \\ 0\; if\; item\; is\; censored \end{array} \right\}$, and f and F are, respectively, the probability density function (pdf) and the cumulative distribution function (cdf) of the assumed distribution. Then the statistic z_i for a single sample of n observations becomes

$$\begin{aligned}
z_i &= \log \frac{L(\boldsymbol{y}_i|(\eta_1, \beta_0))}{L(\boldsymbol{y}_i|(\eta_0, \beta_0))} \\
&= \log \frac{\prod_{j=1}^{n} f_1(y_{ij})^{\delta_{ij}}[1 - F_1(y_{ij})]^{1-\delta_{ij}}}{\prod_{j=1}^{n} f_0(y_{ij})^{\delta_{ij}}[1 - F_0(y_{ij})]^{1-\delta_{ij}}} \\
&= \log \prod_{j=1}^{n} [\frac{f_1(y_{ij})}{f_0(y_{ij})}]^{\delta_{ij}} [\frac{1 - F_1(y_{ij})}{1 - F_0(y_{ij})}]^{1-\delta_{ij}} \\
&= \sum_{j=1}^{n} \log [\frac{f_1(y_{ij})}{f_0(y_{ij})}]^{\delta_{ij}} + \sum_{j=1}^{n} \log [\frac{1 - F_1(y_{ij})}{1 - F_0(y_{ij})}]^{1-\delta_{ij}} \\
&= \sum_{j=1}^{n} \delta_{ij} \log [\frac{f_1(y_{ij})}{f_0(y_{ij})}] + \sum_{j=1}^{n} (1 - \delta_{ij}) \log [\frac{1 - F_1(y_{ij})}{1 - F_0(y_{ij})}], i = 1, 2, ...
\end{aligned}$$

We customize the score statistic z_i for a likelihood ratio based CUSUM chart with samples of n right-censored data,

$$z_i = \sum_{j=1}^{n} \delta_{ij} \log [\frac{f_1(y_{ij})}{f_0(y_{ij})}] + \sum_{j=1}^{n} (1 - \delta_{ij}) \log [\frac{1 - F_1(y_{ij})}{1 - F_0(y_{ij})}], i = 1, 2, \ldots, \tag{4.1}$$

by using the proper distributions for different monitoring goals in later sections. The actual number of failures in each generated sample is a random variable that takes values specific to the random sample and the censoring time C, $r_i = \sum_{j=1}^{n} \delta_{ij}$ for the i^{th} sample. When none of the observations are censored, $r_i = n$, and the likelihood function simplifies to the uncensored case.

4.2.1 Equivalence of Transformations

In general, log likelihood ratio tests are invariant to monotonic transformations to the random variables (see for example Dagenais and Dufour (1991)). In this section we verify that this property holds for the likelihood ratio based CUSUM chart for right-censored lifetimes with a Weibull distribution, making it invariant to the transformations typically applied to data in reliability applications. For sake of consistency we further proceed with the CUSUM chart design with the standard exponential transformation, which Zhang and Chen (2004) used for their EWMA CEV chart.

Raw Data, The Weibull Distribution

We use the usual parametrization for the two-parameter Weibull distribution described in Chapter 1. We assume that the in-control values of the parameters, η_0 and β_0, are known. In an in-control situation, T, the lifetime random variable, follows a Weibull($\beta = \beta_0, \eta = \eta_0$) distribution. In an out-of-control situation, if only η shifts and β is fixed, then T follows a Weibull($\beta = \beta_0, \eta = \eta_1$) distribution. With this approach, and (4.1) the z_i statistics become

$$z_i = r_i \beta_0 (\log(\frac{\eta_0}{\eta_1})) - \sum_{j=1}^{n} ((\frac{t_{ij}}{\eta_1})^{\beta_0} - (\frac{t_{ij}}{\eta_0})^{\beta_0}), i = 1, 2, \ldots,$$

where r_i here is the number of failures in the sample.

Smallest Extreme Value (SEV) Transformation

If $T \sim Weibull(\beta, \eta)$, then $Y = \log(T)$ follows a smallest extreme value distribution described in Chapter 1. In an in-control situation, Y follows a $SEV(b = b_0, u = u_0)$ distribution. In an out-of-control situation, with u shifting from u_0 to an out-of-control value $u_1 = \log(\eta_1)$, Y follows a $SEV(b_0, u_1)$ distribution. With the SEV transformation and (4.1) z_i become

$$z_i = r_i(\frac{(u_0 - u_1)}{b_0}) + \sum_{j=1}^{n} [\exp(\frac{y_i - u_0}{b_0}) - \exp(\frac{y_i - u_1}{b_0})], i = 1, 2, \ldots,$$

where r_i is the number of failures.

The Standard Exponential Transformation

If the random variable T follows a Weibull distribution with scale parameter η and shape parameter β, then the random variable $X = (T/\eta)^{\beta}$ follows an exponential distribution with mean 1, also called a standard exponential distribution. If $\eta = \eta_0$ and $\beta = \beta_0$ then the random variable $X = (T/\eta_0)^{\beta_0}$ follows a standard exponential distribution, with pdf

$$f(x) = e^{-x}, \tag{4.2}$$

and cdf

$$F(x) = 1 - e^{-x}. \tag{4.3}$$

The standard exponential is a particular case of the exponential distribution with parameter $\lambda = 1$. If $\beta = \beta_0$ fixed and $\eta_1 = (1 - d) * \eta_0$, then in an in-control situation $(T/\eta_0)^{\beta_0}$ follows a standard exponential distribution. Otherwise, in an out-of-control situation, $(T/\eta_1)^{\beta_0}$ follows a standard exponential distribution. We can write

$$X = (\frac{T}{\eta_0})^{\beta_0} = (\frac{T}{\eta_1})^{\beta_0} * (\frac{\eta_1}{\eta_0})^{\beta_0}.$$

We can derive that in general, if the random variable X follows an exponential distribution with parameter λ, and a is a constant, then the random variable aX follows an exponential distribution with parameter $a\lambda$. Consider the standardized random variable, $X = (T/\eta_0)^{\beta_0}$. Then in an out-of-control situation with $\eta_1 = (1-d)*\eta_0$, X follows an exponential distribution with parameter $a\lambda = (\frac{\eta_1}{\eta_0})^{\beta_0} * 1 = (\frac{\eta_1}{\eta_0})^{\beta_0}$, with pdf

$$f(x) = \frac{1}{(\frac{\eta_1}{\eta_0})^{\beta_0}} e^{[-\frac{x}{(\frac{\eta_1}{\eta_0})^{\beta_0}}]}, \tag{4.4}$$

and cdf

$$F(x) = 1 - e^{[-\frac{x}{(\frac{\eta_1}{\eta_0})^{\beta_0}}]}. \tag{4.5}$$

Starting from (4.1) and using (4.2), (4.3), (4.4), and (4.5), z_i become

$$z_i = r_i \beta_0 \log(\frac{\eta_0}{\eta_1}) - ((\frac{\eta_0}{\eta_1})^{\beta_0} - 1) \sum_{j=1}^{n} x_{ij}, i = 1, 2, \ldots. \tag{4.6}$$

As expected, after performing a simulation to implement the CUSUM chart with the three different transformations and fixed $\beta = \beta_0$, the simulation results show that for each value of β_0, $z_{Weibull} = z_{SEV} = z_{Std.Exponential}$. Also, the CUSUM chart design depends on the value of β_0 but does not depend on the value η_0. We obtain different threshold values and out-of-control performance for different values of β_0, but equivalent charts for different values of η_0.

Since transformations do not have an impact on the determination of the control statistic, we use the standard exponential transformation to design the likelihood ratio based CUSUM chart, evaluate its properties and compare it to the CEV EWMA chart designed on the standard exponential scale.

4.2.2 The Likelihood Ratio Based CUSUM for Characteristic Life of Weibull Lifetimes

We construct the CUSUM chart to monitor for shifts in the scale parameter η, the characteristic life of Weibull lifetimes. The in-control values of the scale and shape parameters are assumed to be known, having been estimated in a previous phase. We are interested in Phase II monitoring in this research. We consider the shape parameter fixed to the in-control value. We define an out-of-control situation characterized by $\eta = \eta_1$, where $\eta_1 = (1-d) * \eta_0$, where $d*100\%$ represents a percent change in η. We generate samples of n items in a testing process and T represents the lifetime of the products. We stop the test at a predetermined time C. Any product that had not failed by time C generates a censored lifetime. The lower bound C replaces the value of each censored lifetime. There is a random number of r_i censored data in each sample i, with values replaced by the censoring time C. The number of failures in each generated sample is a random variable that takes values specific to the random sample and the censoring time C. In this scenario, with a standard exponential transformation, from (4.6), the score statistics are

$$\begin{aligned} z_i &= r_i\beta_0 \log(\frac{\eta_0}{\eta_1}) - ((\frac{\eta_0}{\eta_1})^{\beta_0} - 1)\sum_{j=1}^{n} x_{ij} \\ &= r_i\beta_0 \log(\frac{1}{1-d}) - ((\frac{1}{1-d})^{\beta_0} - 1)\sum_{j=1}^{n} x_{ij}, i = 1, 2, \ldots, \end{aligned}$$

where $x_{ij} = (\frac{t_{ij}}{\eta_0})^{\beta_0}$ follow a standard exponential distribution. Note that $1 - (\frac{1}{1-d})^{\beta_0}$ is a constant and we can rescale both sides of the expression of z_i and rewrite

$$z_i^* = (1 - (\frac{1}{1-d})^{\beta_0}) \left[\sum_{j=1}^{n} x_{ij} - k_i \right], i = 1, 2, \ldots,$$

where

$$k_i = -\frac{r_i \beta_0 \log(\frac{1}{1-d})}{(1 - (\frac{1}{1-d})^{\beta_0})}$$

depends on the number of censored units in each sample. If $d > 0$, we are interested in detecting a decrease in the scale parameter and $(1 - (\frac{1}{1-d})^{\beta_0}) < 0$. To detect such a change, we can rescale the chart statistics

$$S_0 = 0$$
$$S_i = max[0, S_{i-1} + z_i], i = 1, 2, ..,$$

by $(1 - (\frac{1}{1-d})^{\beta_0}) < 0$, and use the chart statistics

$$C_0^- = 0,$$
$$C_i^- = min[0, C_{i-1}^- + \sum_{j=1}^{n} x_{ij} - k_i], i = 1, 2, \ldots,$$

where

$$C_i^- = S_i/(1 - (\frac{1}{1-d})^{\beta_0}), i = 1, 2, \ldots.$$

The CUSUM signals if $C_i^- < -h^-$, when the chart indicates a decrease in the scale parameter, η. Similarly, if we want to detect a positive shift in the scale parameter, given by $d < 0$, $(1 - (\frac{1}{1-d})^{\beta_0}) > 0$, we use the chart statistics

$$
\begin{aligned}
&C_0^+ = 0,\\
&C_i^+ = max[0, C_{i-1}^+ + \sum_{j=1}^{n} x_{ij} - k_i], i = 1, 2, \ldots .
\end{aligned}
$$

The chart signals if $C_i^+ > h^+$, when it indicates an increase in the scale parameter.

Next we evaluate the properties of the designed charts. From the expression of the score statistic z_i we can see that the chart design depends on the in-control values of the fixed shape parameter. We provide guidelines for the practitioner in selecting values for the threshold h in several scenarios. If a practitioner's scenario is significantly different, we provide a detailed description of the algorithm one should follow to design the likelihood ratio based CUSUM chart.

4.3 Properties

For the design of the CUSUM chart, we assume that the shape parameter is fixed to the in-control value and known, $\beta = \beta_0$, taking one of the values $0.5, 1, 3,$ or 5, which are commonly used in reliability. We set, without loss of generality, η_0 to 1, since the chart design is independent of the in-control value of the scale parameter. The in-control values of the parameters are considered known from a previous phase of the quality control process, although they are usually determined through various estimation methods from process data.

4.3.1 Simulation Algorithm and Diagnostic Procedures

We first evaluate the performance of the likelihood based CUSUM chart monitoring for a decrease in the characteristic life of a Weibull distribution (which is equivalent to monitoring for a decrease in the mean of lifetimes, when the shape parameter is fixed). Our data are samples of n lifetimes, with $n \geq 1$, that include Type I censoring. We compare the CUSUM chart to the CEV EWMA for mean of lifetime data proposed by Zhang and Chen (2004) in different scenarios of interest. The simulation study was designed in the following way:

- Sample size $n = 3, 5, 10$, as encountered in similar papers and in quality control sampling practice.
- If the lifetime random variable T follows a Weibull(β,η) distribution, with Type I right-censoring at a predetermined time C, then $p_C = \exp[-\frac{C}{\eta}]^{\beta}$, with $\beta = \beta_0$ and $\eta = \eta_0$. For example, a 50% censoring rate corresponds to roughly half of the products failing by the stopping time C and half of the units being censored. We generate Weibull data to correspond to theoretical censoring rates $p_C = 5\%, 30\%, 50\%, 80\%, 95\%$. We evaluated the performance of the charts in low, moderate, and high censoring scenarios. The theoretical censoring rate corresponds to an in-control situation. The actual number of censored values in a generated random sample is random. If the process mean or characteristic life increases, we expect more censored units. Alternatively, if the mean life or characteristic life decreases, we expect a lower number of censored units in the generated sample. We designed the chart for the corresponding theoretical in-control censoring rate.
- In-control values for the fixed shape parameter $\beta = \beta_0 = 0.5, 1, 3, 5$.
- In-control value for the scale parameter $\eta = \eta_0 = 1$, without loss of generality.
- Negative shift size $d = 2.5\%, 5\%, 10\%, 20\%$. The out-of-control η value is then $\eta_1 = \eta_0 * (1 - d)$. We evaluated scenarios with small, moderate, and large shifts.

- Desired in-control run length $IARL = 370$, as frequently used in quality control procedures.

- We compared the CUSUM chart to the CEV EWMA with smoothing parameters $\lambda = 0.05, 0.1, 0.2$, which Zhang and Chen (2004) recommended. We presented the details of the CEV EWMA chart design in Chapter 2.

The results of the simulations for these scenarios are displayed in Tables A.2 through A.17 in Appendix A.2 and discussed in Section 4.3.3.

4.3.2 Simulation Algorithm Description

We evaluated the performance of the likelihood ratio based CUSUM chart and compared it to the performance of the CEV EWMA chart in the listed scenarios, using Monte Carlo simulations. The steps required are the following:

- Initialize the scenario parameters with the desired values: the in-control scale $\eta = \eta_0$, the fixed in-control shape $\beta = \beta_0$, the sample size n, the censoring proportion p_C, and the shift of interest d.

- Based on the values β_0, η_0 and p_C we determined the censoring time C as $C = \eta_0 * ((\log(1/(p_C)))^{1/\beta_0})$.

- We initialized the search space for the threshold, for example, the lower limit is $h_1 = -20$ and the upper limit is $h_2 = 0$.

- We calculated the out-of-control values of the parameters as $\beta_1 = \beta_0$ and $\eta_1 = \eta_0 * (1 - d)$.

- We started searching the threshold space, using the bisection search method.

- For each intermediary threshold, we performed $10,000$ replications of the simulation.

- At each simulation step we generated samples of n observations from a Weibull distribution with parameters η_0 and β_0, we censored them according to our censoring scheme, we calculated the CUSUM statistics, C_i^-, and we obtained an in-control run length for this simulation step. In order to generate random numbers following a Weibull distribution, we used the inverse cdf rule starting from samples of uniformly distributed $(0, 1)$ random numbers. We used the Mersenne-Twister uniform random number generator implemented in a C++ class. This generator is known to have improved performance over the older generators (long series of uncorrelated random numbers).

- We averaged over the $10,000$ run lengths to calculate the in-control ARL (IARL) and we calculated a standard error of the run lengths.

- We checked if the estimated IARL was close to the desired in-control ARL of 370. We tolerated a difference between 0 and 5 from the desired value. We repeated the search in a maximum of 30 steps.

- If the estimated IARL is sufficiently close to the desired value, we retained the value of the threshold. Otherwise, we resumed the search through the bisection method and we repeated these steps.

The deliverables of this simulation process are the chart's threshold, the estimated average in-control run length (IARL), and the simulation error, calculated as the standard error divided by the square root of the number of simulation replications, for each of the considered scenarios. The next simulation evaluates the out-of-control performance of the chart, using the threshold determined in the previous simulation, in each scenario:

- We performed $10,000$ replications.

- For each replication we generated out-of-control random samples of n observations from a Weibull distribution with parameters $\eta = \eta_1$ and $\beta = \beta_0$, we censored them according to our censoring scheme, we calculated the CUSUM statistics and we obtained an out-of-control run length (OARL).

- We averaged over the $10,000$ out-of-control run lengths, and we obtained a standard deviation of the run lengths and a simulation standard error.

A C++ program for designing all the CUSUM charts and evaluating their properties is available from the author on request. We performed similar simulations for the CEV EWMA chart designed by Zhang and Chen (2004).

4.3.3 Simulation Results. Relative Performance of the CUSUM Chart and Properties

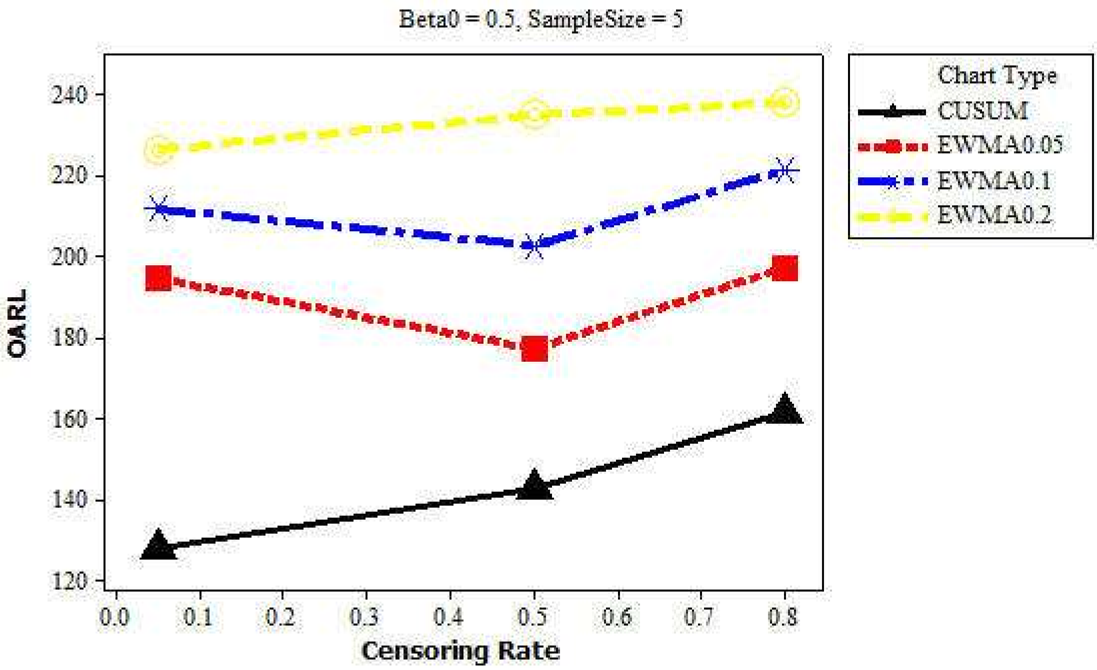
In this section we discuss the relative performance of the CUSUM and CEV EWMA charts, and the trends in CUSUM chart performance depending on sample size, censoring rate, shift size and value of the in-control shape parameter. We evaluate the charts' performance in terms of the out-of-control average run length, while the charts are designed to provide an in-control average run length of 370. The best chart gives the lowest out-of-control ARL (OARL). Appendix A.2 provides tables with the parameters of the chart design (the thresholds for various scenarios) and the estimated in-control average run lengths and simulation errors. Figures 4.1 through 4.8 summarize the information in Tables A.2 through Table A.17. Tables A.2 through A.5 display, for each considered in-control value of the shape parameter, censoring rate, sample size and the smallest shift size to be detected, the parameters of the CUSUM chart design: the threshold h, the estimated average in-control run length and the corresponding simulation error, and the estimated average out-of-control run length and the corresponding simulation error. The CEV EWMA design is independent of values of the fixed shape parameter β, as Zhang and Chen (2004) advocated in their paper. Nevertheless, we ran simulations to study these properties for different values of the fixed $\beta = \beta_0$, in different censoring situations, and to parallel for the comparison's sake the cases considered for the CUSUM chart design. Tables A.6 through A.9 contain the simulation results for the same cases considered for the CUSUM chart, but using the CEV EWMA method with smoothing parameter $\lambda = 0.05$. Tables A.10 through A.17 summarize the simulation results for the CEV EWMA chart with different smoothing parameters, $\lambda = 0.1$ and 0.2. The tables display the threshold h, the estimated average in-control run length and the corresponding simulation error, and the estimated average out-of-control run lengths and the corresponding simulation errors.

It is also interesting to study the out-of-control performance of the chart for different values of the in-control shape parameter β_0. The CEV EWMA chart design for a desired in-control average run length of 370 is independent on the β_0 values. However, in an out-of-control situation, using the determined threshold in the in-control scenario, the chart's performance varies with the various values of β_0. The dependence of the out-of-control performance of the charts on the value of β_0 was previously unrecognized in the literature. For example, if the CEV EWMA chart is designed for a sample size $n = 10$, censoring rate $p_C = 5\%$, and we target a 5% shift in the scale parameter, the out-of-control performance is different depending on the values of β_0. For $\beta_0 = 0.5$ from Table A.6, the chart signals on average in 244 samples; for $\beta_0 = 1$ from Table A.7, the chart signals on average in 116 samples; for $\beta_0 = 3$ from Table A.8, the chart signals on average in 28 samples; and for $\beta_0 = 5$ from Table A.19, the chart signals on average in 14 samples. In order to choose the superior CEV EWMA chart, we designed the chart and checked properties for two other values of the smoothing parameter, $\lambda = 0.1$ and $\lambda = 0.2$. Tables A.10 through A.17 display the corresponding charts' properties.

Tables A.2 through A.5 contain the simulation results for the CUSUM chart for a decrease in scale for each of the in-control values of the shape parameter $\beta_0 = 0.5, 1, 3$ and 5. The chart appears to perform reasonably in terms of the out-of-control average run lengths for larger shifts and for larger β_0 values. Censoring and sample size also impact the chart's performance, but the shift size appears to be the major factor. Tables A.7 through A.10 display the same patterns for the CEV EWMA chart for a scale decrease with smoothing parameter $\lambda = 0.05$. Although the charts are more sensitive for high in-control values of the shape parameter β_0, they are also more sensitive to the negative effect of high censoring. In Appendix A.3, Tables A.18 through A.21 summarize the simulation results for the CEV EWMA chart for a mean increase, while Tables A.22 through A.25 summarize the simulation results for the competing likelihood ratio based CUSUM chart for a scale increase. The impact of a high censoring rate is greater in the case of a scale parameter increase, as we discuss later in more detail.

Figures 4.1 and 4.2 display the relative performance of the likelihood ratio based CUSUM and the CEV EWMA charts with the smoothing parameters $\lambda = 0.05, 0.1, 0.2$ to detect a 10% decrease in the scale parameter, for different censoring proportions, sample sizes and values of the fixed shape parameter. It appears that the CUSUM chart performs better than the EWMA charts overall. For a moderate censoring proportion we have roughly equivalent performance. Another observation we can make here is that the EWMA chart with $\lambda = 0.05$ tends to be the best out of the three considered EWMA charts. We consider a more extensive comparison of this chart with the CUSUM chart next. In several instances, for high censoring rates, an in-control average run length of 370 could not be achieved with either the CEV EWMA chart. In these cases the OARL is also very high. These cases were not displayed. In these cases the superiority of the CUSUM chart is of increased importance.

Figure 4.1: Comparison Between Out-of-Control ARL to Detect 10% Decrease in the Scale Parameter with the Optimum CUSUM Chart, and with the CEV EWMA with Smoothing Parameter $\lambda = 0.05, 0.1$, and 0.2, and $\beta_0 = 0.5$ and 1.



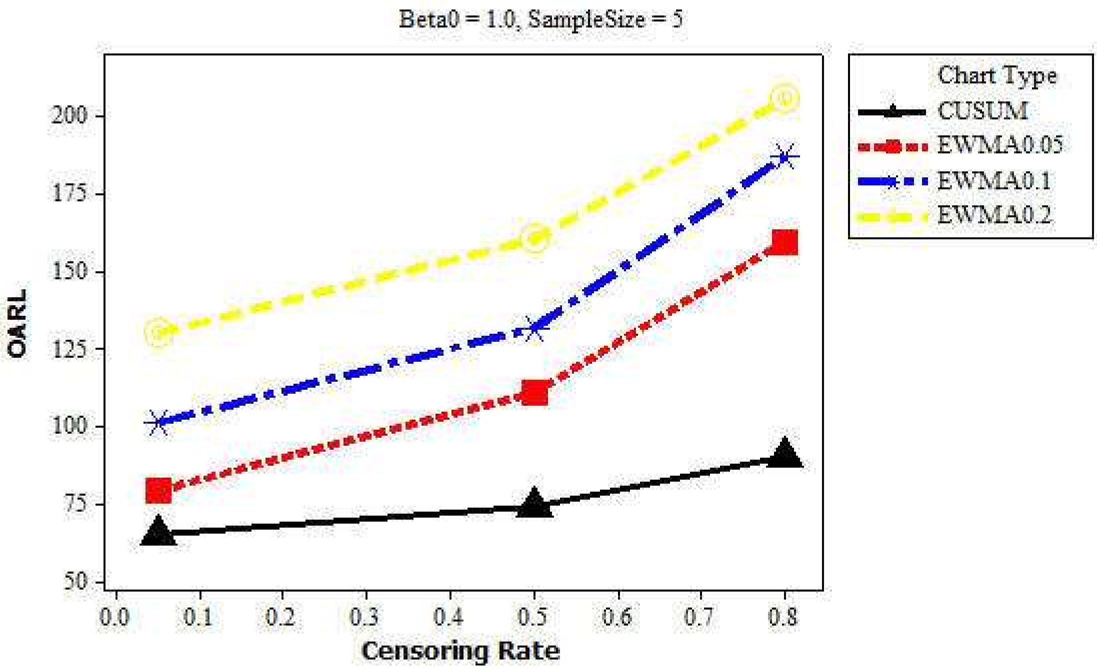
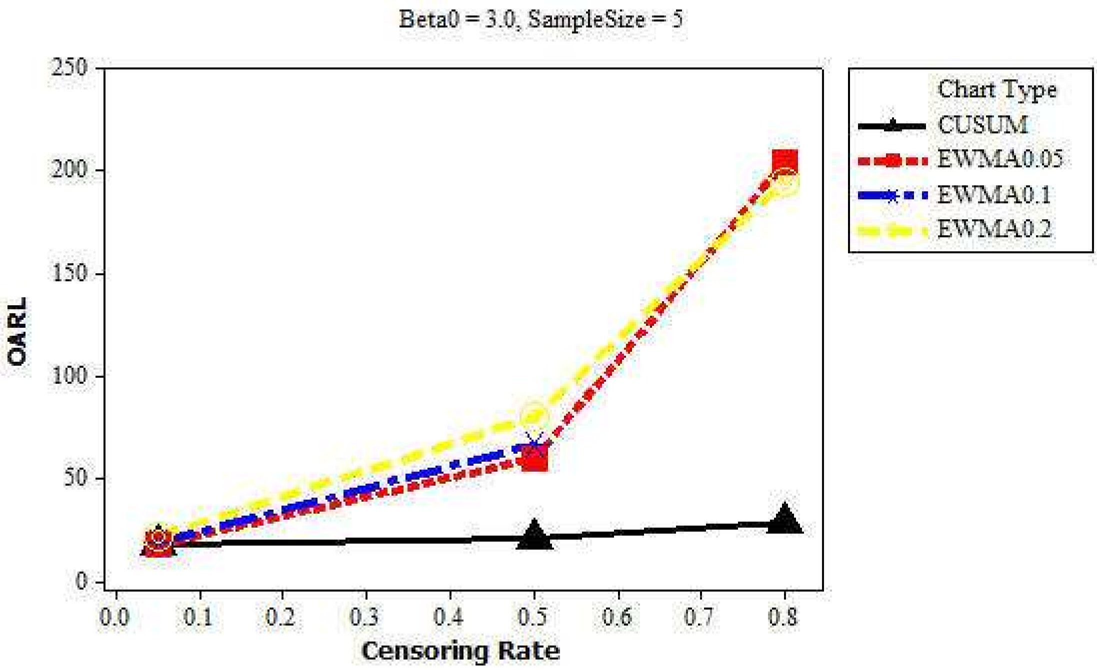
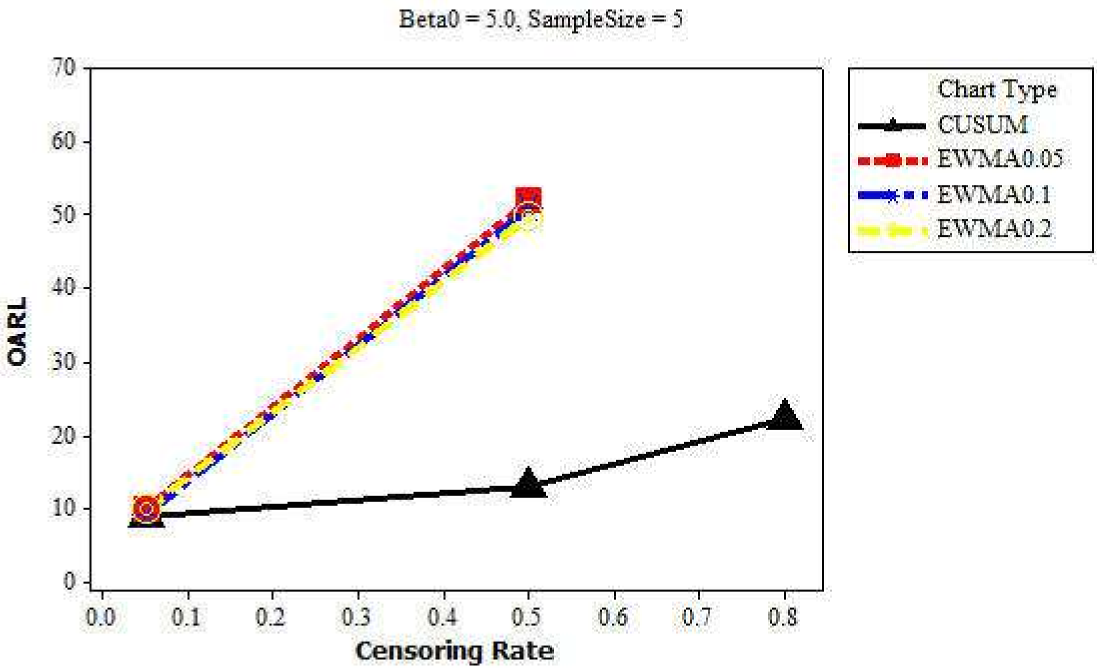


Figure 4.2: Comparison Between Out-of-Control ARL to Detect 10% Decrease in the Scale Parameter with the Optimum CUSUM Chart, and with the CEV EWMA with Smoothing Parameter $\lambda = 0.05, 0.1$, and 0.2, and $\beta_0 = 3$ and 5.

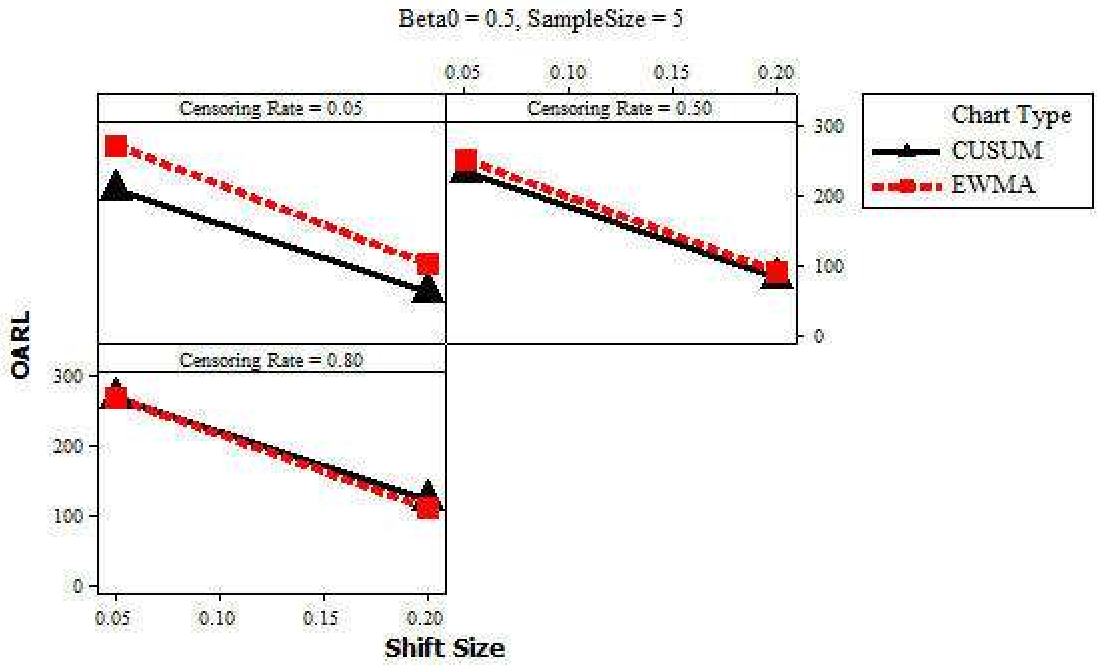




Figures 4.3 and 4.4 display the out of control ARL for the EWMA chart and CUSUM chart when detecting minimum shifts of different sizes. For $\beta_0 = 0.5$, the CUSUM chart performance is comparable. For small shifts and high censoring, the CUSUM chart is superior. The two charts' performance is comparable for $\beta_0 = 1$, with the CUSUM performance being slightly better. For $\beta_0 = 3$ and $\beta_0 = 5$ the CUSUM chart is better, however the difference in performance is lower for low censoring rates. The CUSUM chart performs much better for moderate and high censoring rates. As expected, both charts' signal faster for larger shifts. Usually, we expect small persistent shifts in the parameters, so the performance for small shifts is of more interest. Overall, the CUSUM chart outperforms the CEV EWMA chart proposed by Zhang and Chen (2004).

Next we evaluate more trends in the performance of the CUSUM chart for a scale parameter decrease. The censoring proportion has a large impact, as expected. Figure 4.3 displays the CUSUM chart's out-of-control behavior for different censoring rates, β_0 values, and sample size scenarios, for a variety of shift sizes. As expected, the chart signals faster in an out-of-control situation with lower censoring proportions. The difference in OARL values is significant, varying between zero and 370 samples. Again, for the extreme censoring cases when we cannot achieve an IARL of 370, OARL reaches poor values and we do not display OARL values larger than 370 for the EWMA chart. We do not have the same problem with the CUSUM chart. As Figures 4.5 and 4.6 show, the CUSUM chart's performance depends on the values of the fixed shape parameter, as expected. The chart's performance is best when designed for scenarios with higher values of β_0. The chart's capacity to detect shifts of different magnitudes varies with the values of the fixed shape parameter and the censoring amount. The method performs well even for very high censoring proportions. The impact of censoring seems to be reduced when the size of the shift the chart is designed to detect is larger. Another factor of interest is the sample size, since usually there are higher costs associated with higher sample sizes. In Figure 4.8 we see that, for larger sample sizes, the CUSUM chart signals sooner in an out-of-control situation. The chart performs better even with small sample sizes when β_0 is large. However, the sample size effect is not large, considering the levels of n included in this study.

Figure 4.3: Comparison Between Out-of-Control ARL for the Optimum CUSUM Chart and for the CEV EWMA Chart, with $\beta_0 = 0.5$ and 1.



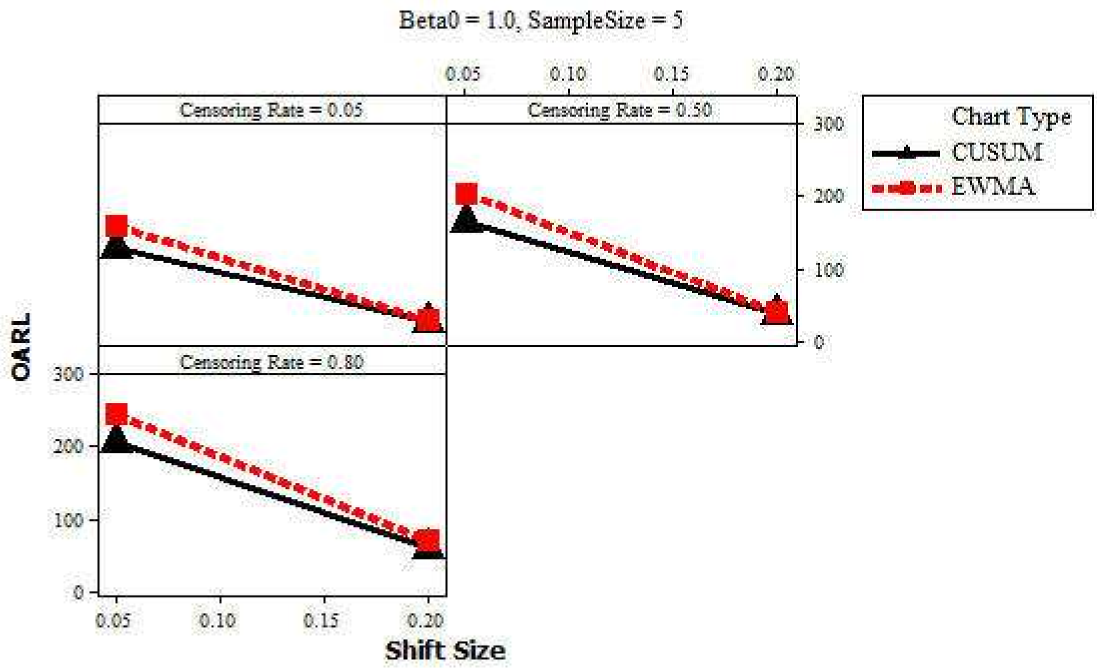
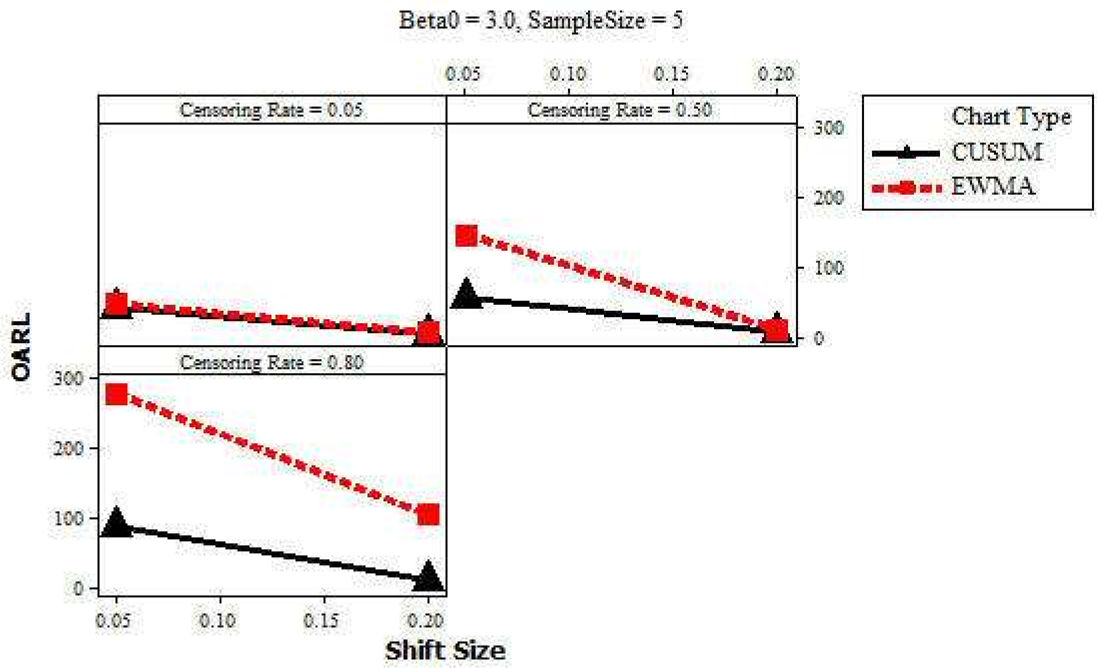


Figure 4.4: Comparison Between Out-of-Control ARL for the Optimum CUSUM Chart and for the CEV EWMA Chart, with $\beta_0 = 3$ and 5.



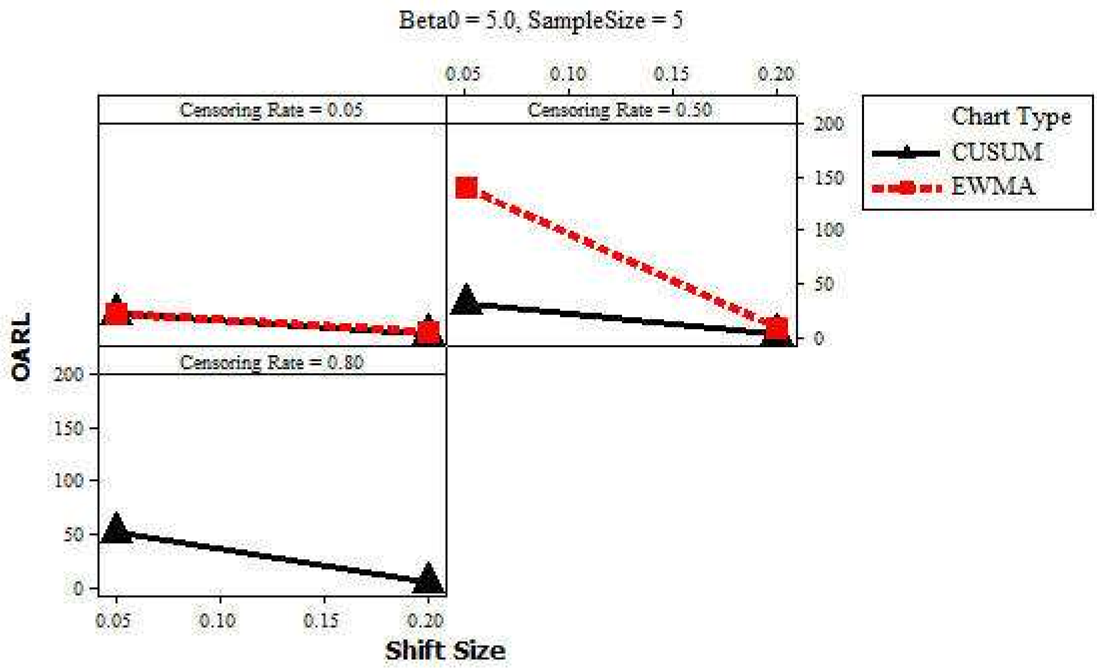
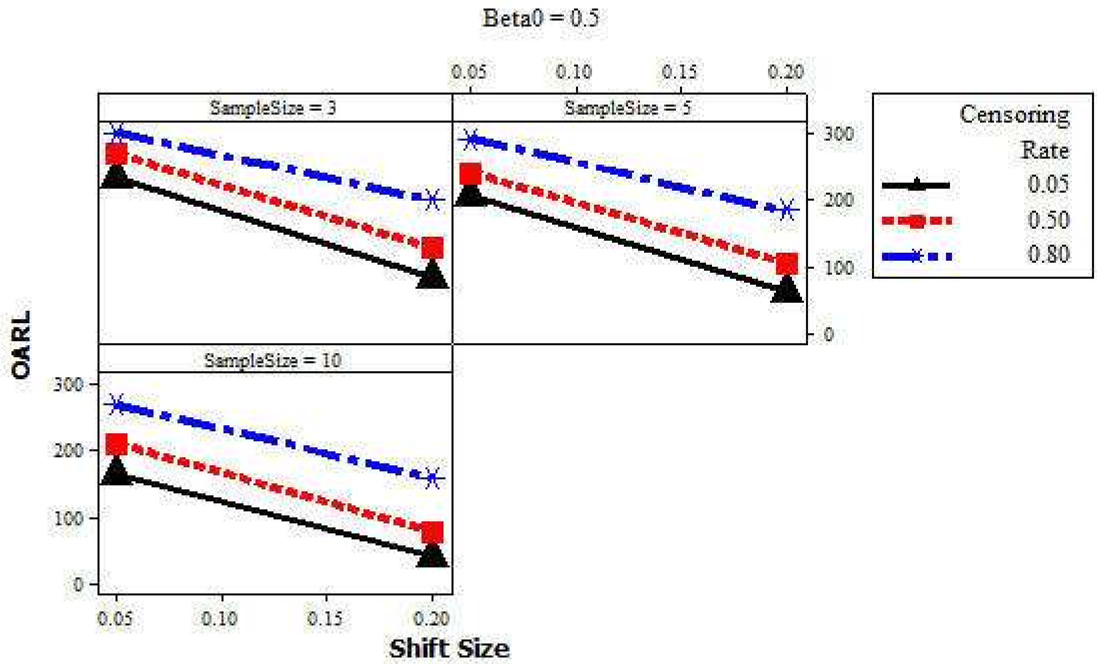


Figure 4.5: Out-of-Control Performance of the Optimum CUSUM Chart with Different Censoring Rates, for $\beta_0 = 0.5$ and $\beta_0 = 1$.



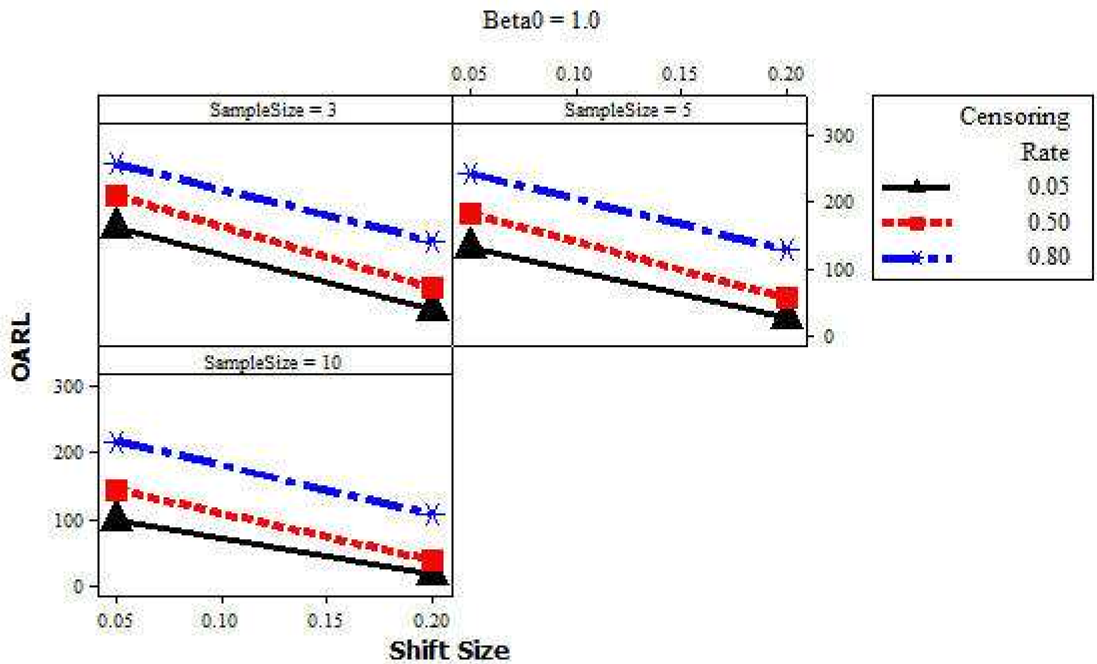
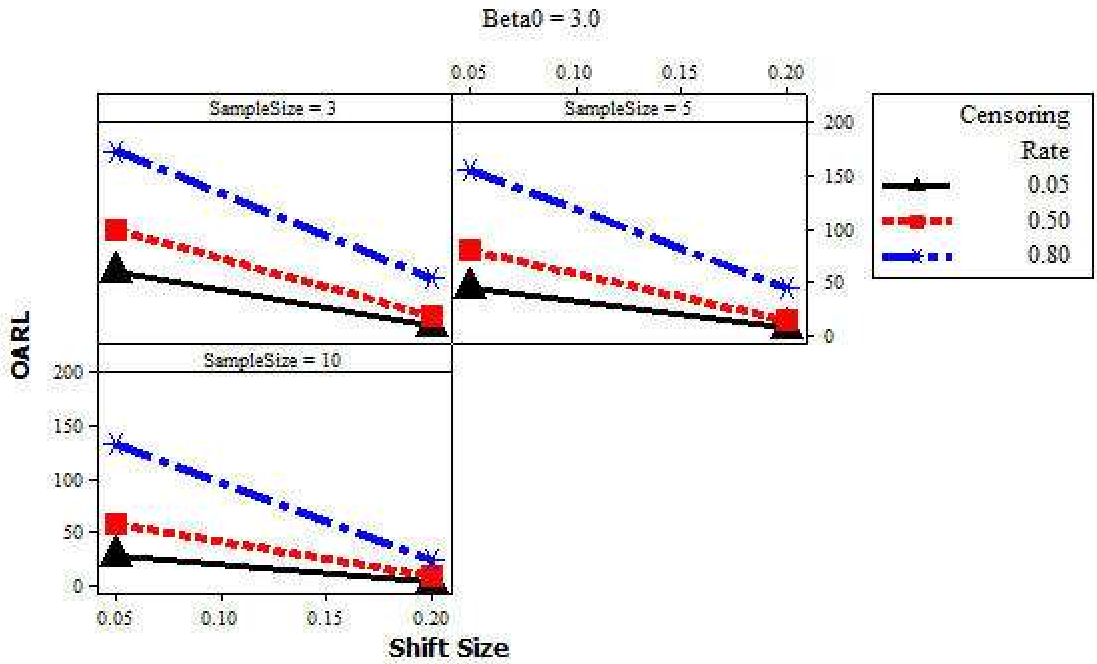


Figure 4.6: Out-of-Control Performance of the Optimum CUSUM Chart with Different Censoring Rates, for $\beta_0 = 3$ and $\beta_0 = 5$.



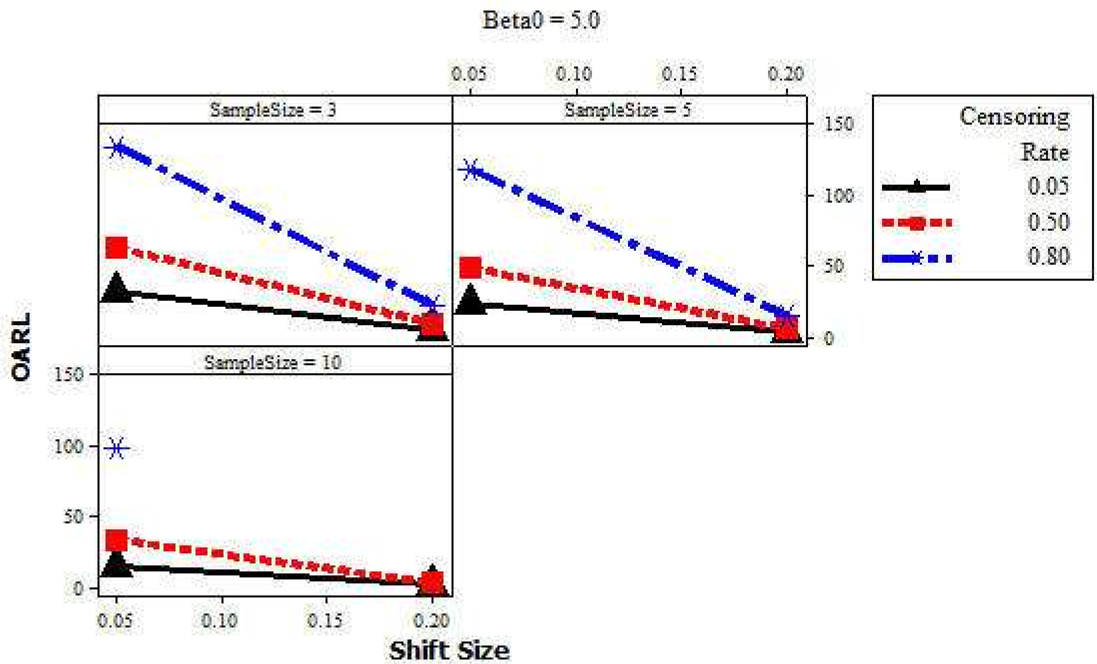


Figure 4.7: Out-of-Control Performance of the Optimum CUSUM Chart for Different Values of the Fixed Shape Parameter, $\beta_0 = 0.5, 1, 3, 5$.

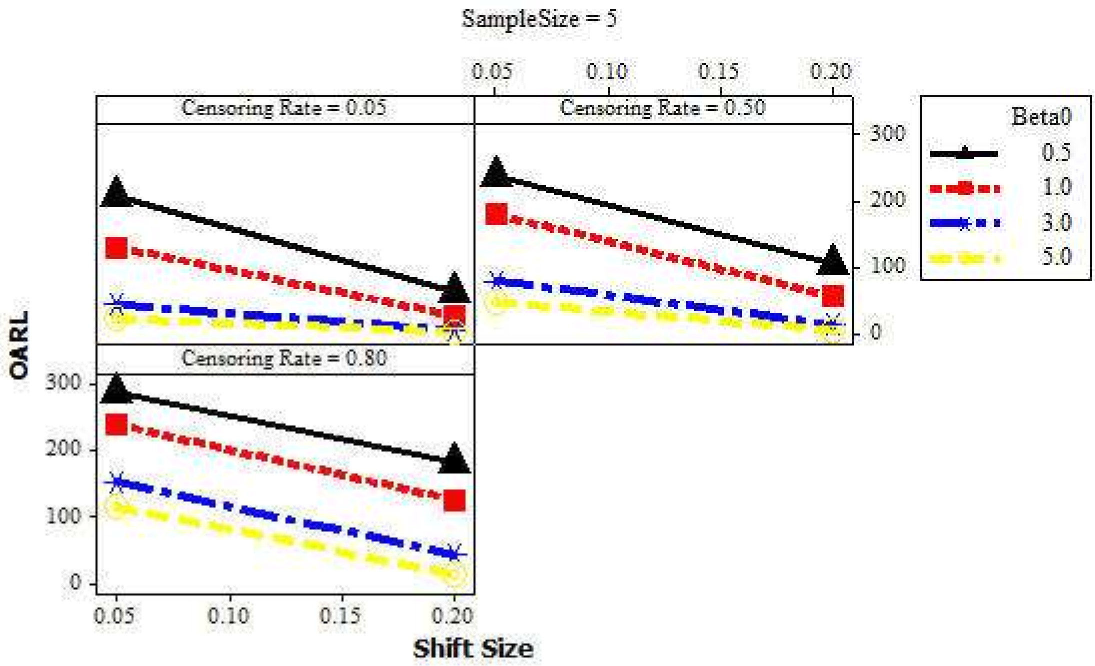
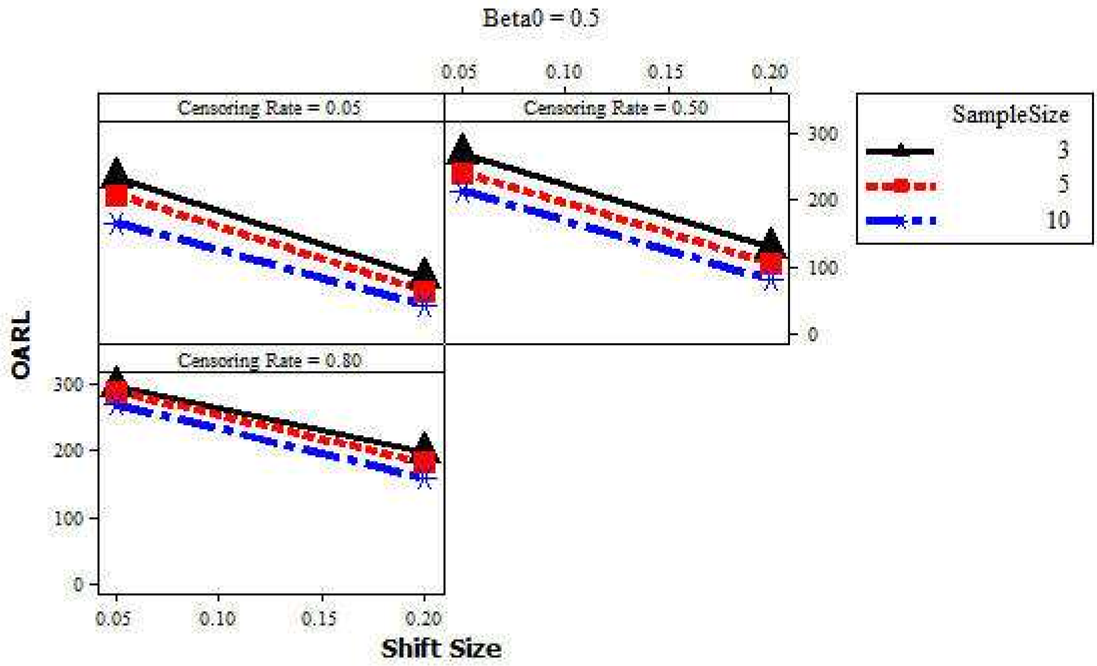
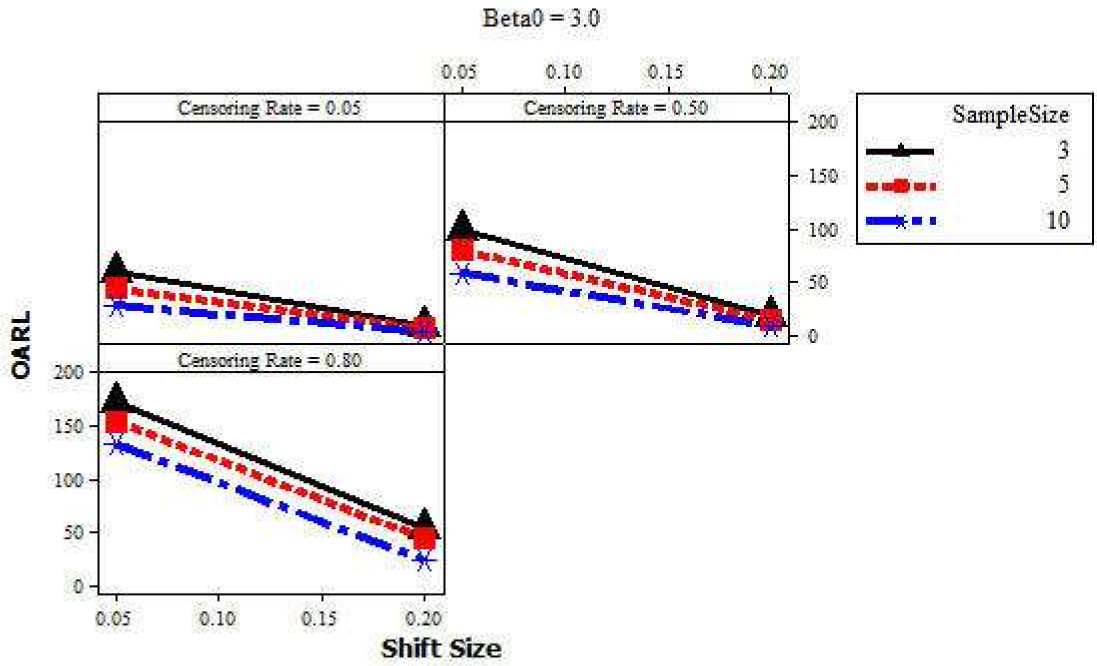


Figure 4.8: Out-of-Control Performance of the Optimum CUSUM Chart for Different Sample Sizes, $n = 3, 5, 10$.

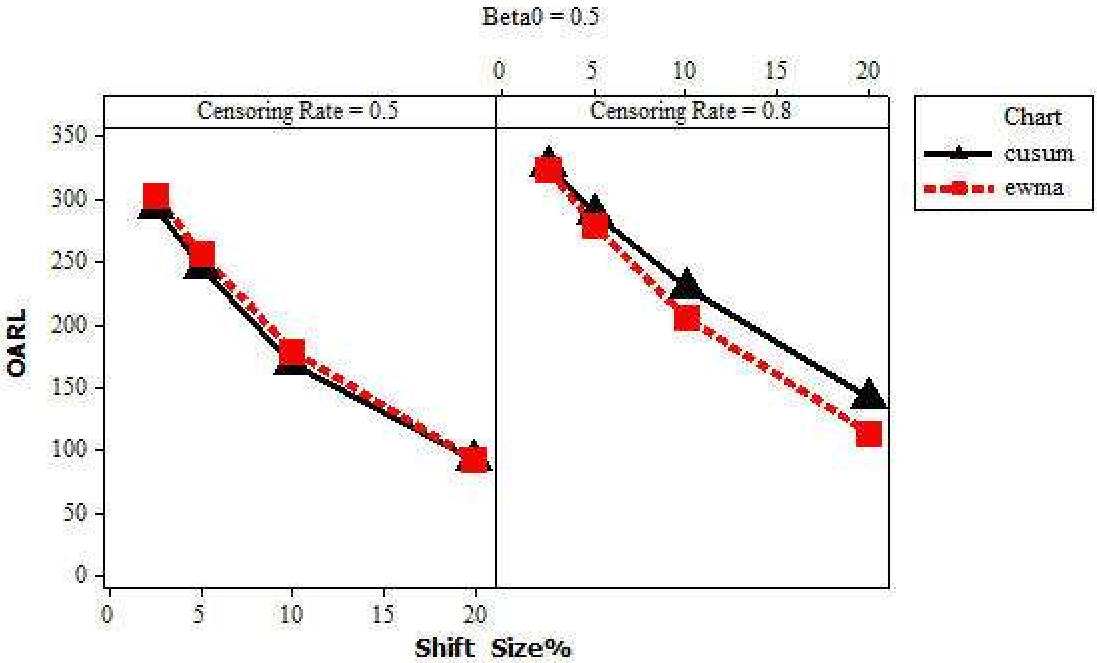


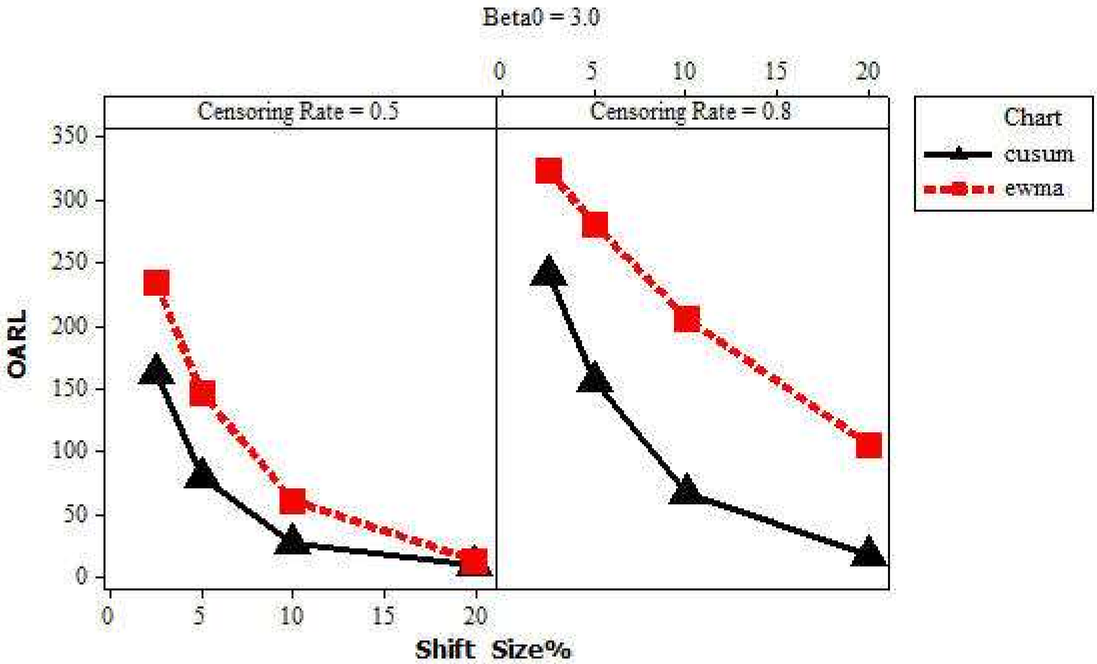


In all the considered scenarios, the simulation error follows similar patterns and has comparable magnitudes (roughly between 0 and 0.1 for most scenarios) for the CEV EWMA and the CUSUM charts. It is highly dependent on censoring rates and decreases with the increase in the shift of interest. We notice that when we have an extreme amount of censoring, both charts' performance is less than reasonable. For example in a 95% censoring rate scenario, when there is a 5% chance that a unit fails, with a sample size of 3, frequently all the units are censored, both in an in-control and in an out-of-control situation, and the z_i statistics for the CUSUM chart's design are 0 for all of these instances.

So far we have studied the performance of the CUSUM charts in an out-of-control scenario that they were designed for, evaluating their capability of detecting the smallest shift for which they were designed. However, there are circumstances in the practice of reliability when the practitioner does not have insight into what size of shift one should expect. Therefore, it is of interest to evaluate the performance of a CUSUM chart designed for a certain minimum shift in an out-of-control situation when the scale parameter decrease is of a different magnitude. We evaluate the performance of the CUSUM chart designed to detect a minimum 5% shift in the scale parameter and compare it to the CEV EWMA chart designed with a smoothing parameter $\lambda = 0.5$ in moderate and high censoring rates scenarios, for both high and low values of the fixed shape parameter $\beta = \beta_0 = 0.5$ and $\beta = \beta_0 = 3$, respectively. We assume a moderate sample size of $n = 5$. The charts were designed to achieve a desired in-control average run length of 370. Figure 4.9 illustrates the comparable performance that follows similar trends as noticed in the previous comparisons. The CUSUM chart dominates the CEV EWMA chart for all the shifts considered when β_0 takes high values. When β_0 takes low values, the comparison is not as clear cut anymore.

Figure 4.9: Relative Out-of-Control Performance of the CUSUM Chart(Designed for 5% decrease) and the CEV EWMA for Different Shift Magnitudes.





4.4 Normal Approximation

When $\beta_0 = 3$, the Weibull distribution resembles somewhat a normal distribution. A practitioner interested in detecting shifts (especially negative shifts) in the characteristic life of products (which is equivalent with detecting shifts in the mean life of products when the shape parameter β is fixed, in this case $\beta = \beta_0 = 3$) might decide to use the likelihood ratio based CUSUM chart designed using the normal theory, due to simplicity and broad use. We studied what is the loss in performance in this case, as compared to using the more accurate Weibull CUSUM chart. We compared the performance of these two charts for an uncensored data framework with sample sizes $n = 3, 5$ and 10, and for a variety of minimum shift sizes the charts were designed to detect. Without loss of generality, we considered that the in-control scale parameter $\eta = \eta_0 = 1$. We considered that the variance was fixed to the value calculated using the in-control values of the shape and scale parameters. Then the in-control sample mean and the fixed variance are, respectively

$$\mu = \mu_0 = \eta_0 \Gamma\left(1 + \frac{1}{\beta_0}\right) = 0.9$$

and

$$\sigma^2 = {\sigma_0}^2 = {\eta_0}^2 \Gamma\left(1 + \frac{2}{\beta_0}\right) - \left(\eta_0 \Gamma\left(1 + \frac{1}{\beta_0}\right)\right)^2 = 0.1.$$

Next we review the design of the CUSUM chart for a mean in the normal framework and we compare its performance with the analogous Weibull CUSUM chart for the characteristic life. Hawkins and Olwell (1998) reviewed the design of the likelihood-ratio based CUSUM for normal data for $n = 1$. We review its development and allow $n \geq 1$.

We consider the likelihood ratio based CUSUM chart from Chapter 3, with ratio statistics z_i given by (3.1) and chart statistics S_i given by (3.2). Using the normal pdf, the z_i statistics become

$$\begin{aligned}
z_i &= \log \frac{L(\boldsymbol{y}_i|(\mu_1,\sigma))}{L(\boldsymbol{y}_i|(\mu_0,\sigma))} \\
&= \log \frac{\prod_{j=1}^n f_1(y_{ij})}{\prod_{j=1}^n f_0(y_{ij})} \\
&= \log \prod_{j=1}^n \frac{\exp\left(-\frac{(y_{ij}-\mu_1)^2}{2\sigma^2}\right)}{\exp\left(-\frac{(y_{ij}-\mu_0)^2}{2\sigma^2}\right)} \\
&= \sum_{j=1}^n [(-\frac{(y_{ij}-\mu_1)^2}{2\sigma^2}) - (-\frac{(y_{ij}-\mu_0)^2}{2\sigma^2})] \\
&= \frac{1}{2\sigma^2}\sum_{j=1}^n ({y_{ij}}^2 - 2y_{ij}\mu_0 + {\mu_0}^2 - {y_{ij}}^2 - {\mu_1}^2 + 2y_{ij}\mu_1) \\
&= \frac{\mu_1-\mu_0}{2\sigma^2}\sum_{j=1}^n (y_{ij} - \frac{\mu_0+\mu_1}{2}) \\
&= \frac{\mu_1-\mu_0}{2\sigma^2}([\sum_{j=1}^n y_{ij}] - n\frac{\mu_0+\mu_1}{2}), i = 1, 2, \ldots.
\end{aligned}$$

Then, the general likelihood ratio based CUSUM chart statistics become

$$\begin{aligned}
S_0 &= 0 \\
S_i &= max[0, S_{i-1} + \frac{\mu_1-\mu_0}{2\sigma^2}([\sum_{j=1}^n y_{ij}] - n\frac{\mu_0+\mu_1}{2})], i = 1, 2, \ldots.
\end{aligned}$$

The chart signals when $S_i > h$. We notice that $\frac{\mu_1-\mu_0}{2\sigma^2}$ is a negative constant when there is a decrease in mean from μ_0 to μ_1. Then we can rewrite

$$S_0 = 0$$
$$S_i = \frac{\mu_1 - \mu_0}{2\sigma^2} min[0, \frac{S_{i-1}}{\frac{\mu_1-\mu_0}{2\sigma^2}} + \sum_{j=1}^{n} y_{ij} - n\frac{\mu_0 + \mu_1}{2}], i = 1, 2, \ldots,$$

where the max becomes a min due to the division by a negative constant. We can rescale both sides of the expression of S_i by the constant $\frac{\mu_1-\mu_0}{2\sigma^2}$, as it follows

$$\frac{S_0}{\frac{\mu_1-\mu_0}{2\sigma^2}} = 0$$
$$\frac{S_i}{\frac{\mu_1-\mu_0}{2\sigma^2}} = min[0, \frac{S_{i-1}}{\frac{\mu_1-\mu_0}{2\sigma^2}} + \sum_{j=1}^{n} y_{ij} - n\frac{\mu_0 + \mu_1}{2}], i = 1, 2, \ldots.$$

We obtain new chart statistics, $C_i^- = \frac{S_i}{\frac{\mu_1-\mu_0}{2\sigma^2}}$, with

$$C_0^- = 0,$$
$$C_i^- = min[0, C_{j-1}^- + \sum_{j=1}^{n} y_{ij} - k], i = 1, 2, \ldots.$$

The CUSUM chart that uses the rescaled chart statistics C^- issues a signal when $C_i^- < h^-$, with $h^- = \frac{h}{\frac{\mu_1-\mu_0}{2\sigma^2}}$. The chart has equivalent performance, since the only difference is rescaling by a constant. Here y_{ij} is the i^{th} observation in the j^{th} sample from the population. The constant k is $k = n\frac{\mu_0+\mu_1}{2}$, where μ_0 is the sample mean for an in-control situation, and μ_1 is the shifted mean. We assume a constant standard deviation σ equal to the in-control value. For example, if we consider an out-of-control situation where there is a negative 10% shift in scale, down to $\eta = 0.9$, then $\mu_1 = 0.81$, using the formula for the Weibull mean and that the shape parameter value is fixed at $\beta_1 = \beta_0 = 3$. We need to find the threshold h^- using trial and error simulations for a desired in-control average run length, typically set to 370 samples. Table A.1 in the Appendix A.1 presents the thresholds, estimated in-control and out-of-control average run lengths, and the corresponding simulation errors for the considered cases. Table 4.1 illustrates the relative performance of the two charts, in terms of the out-of-control average run lengths. The CUSUM chart designed according to the Weibull theory clearly dominates the chart designed according to the normal theory, in all the scenarios considered. Although the Weibull distribution with a shape parameter $\beta = 3$ resembles a normal distribution, the approximation results in clear performance loss.

Table 4.1: OARL for the Normal and the Weibull Theory. Uncensored Weibull data with $\beta = 3$ fixed.

Sample Size	Shift Size	Normal Theory	Weibull Theory
n=3	2.5	125.208	116.554
	5.0	62.986	57.976
	10.0	26.865	24.258
	20.0	10.245	9.300
n=5	2.5	101.164	91.226
	5.0	46.869	42.105
	10.0	19.082	17.125
	20.0	7.147	6.549
n=10	2.5	71.186	63.606
	5.0	30.587	27.163
	10.0	11.709	10.429
	20.0	4.545	4.141

4.5 CUSUM Chart for an Increase in the Scale Parameter

The performance of the chart is worse when monitoring for increases in the scale parameter of the Weibull distribution with right-censored data and a fixed shape parameter. As Steiner and MacKay (2001a) noticed, when the mean life of products increases, the mean life becomes closer to the censoring time C than for a decreased mean life, the censoring proportion increases, and so the chart performance deteriorates. The same is true for the characteristic life. However, the practitioners' interest relies mostly in detecting decreases in characteristic life, which could reflect negatively on the perceived quality of the product. For censoring rates higher than 50%, it is very difficult to obtain a threshold corresponding to a desired in-control average run length of 370 for both the EWMA CEV chart and the CUSUM chart. Appendix A.3 presents the simulation results for a few considered scenarios for both a likelihood ratio based CUSUM chart and CEV EWMA chart with smoothing parameter $\lambda = 0.05$, when one expects an increase in the Weibull characteristic life. The cases considered are a selection from the cases considered in the expected negative shift study. In particular, they are the following:

- Sample size $n = 3, 5, 10$.
- Theoretical in-control censoring rates $p_C = 5\%, 50\%, 80\%$. We evaluated the performance of the charts in low, moderate and high censoring scenarios.
- Values for the fixed shape parameter $\beta = \beta_0 = 0.5, 1, 3, 5$.
- In-control value for the scale parameter $\eta = \eta_0 = 1$, without loss of generality.
- Increase magnitude $d = 5\%, 20\%$. The out-of-control η would be then $\eta_1 = \eta_0 * (1 + d)$. We evaluated scenarios with small and large shifts.

- In-control average run length (IARL) of interest $IARL = 370$, as frequently used in quality control procedures.

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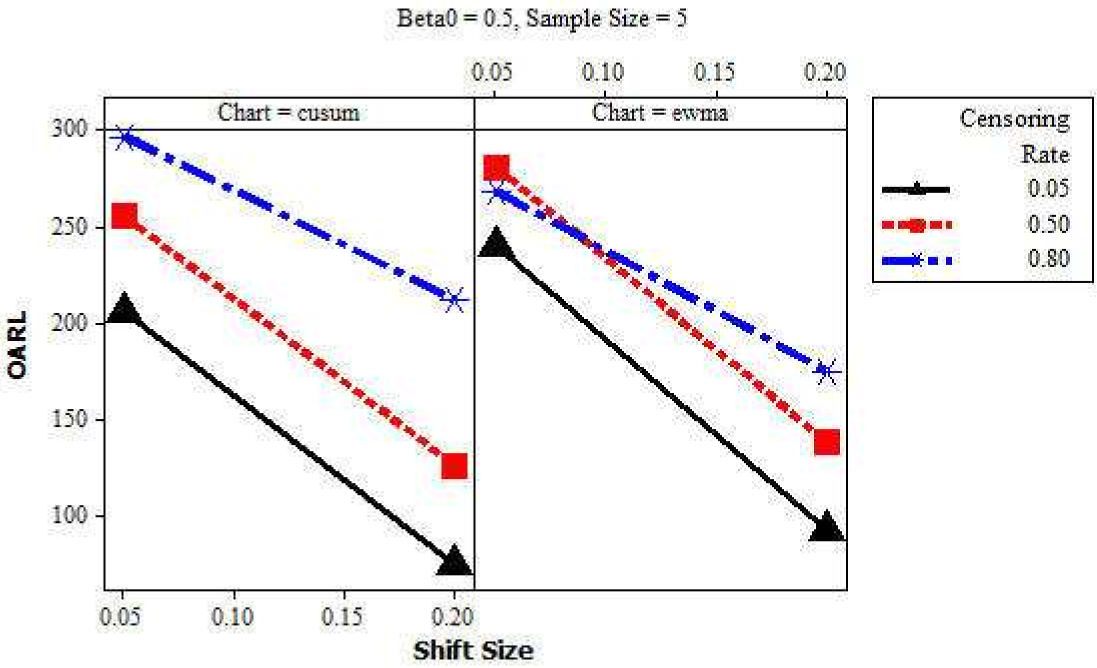
Tables A.18 through A.25 in Appendix A.2 summarize the detailed results for each scenario considered, providing a threshold h, and the estimated in-control and out-of-control average run lengths and the corresponding simulation errors for both the CUSUM and EWMA charts for detecting an increase in the scale parameter. The tables contain the same information for each considered case as for the chart for a decrease in the scale parameter. Figure 4.9 illustrates the relative performance in terms of the average out-of-control run lengths and the corresponding simulation errors of the two charts, for different censoring rates and with different sample sizes. Figure 4.10 displays the results for both a low value and a high value of the fixed shape parameter. The CUSUM and CEV EWMA charts for detecting an increase in the scale parameter are evaluated for the three different censoring rates and sample sizes, and for two different magnitudes of the increase in the scale parameter. One can notice a few patterns. The CUSUM chart performs better than the EWMA chart in every scenario. The performance of the two charts depends heavily on the censoring rate, especially the CEV EWMA chart. As discussed earlier, the impact of censoring is even heavier for an increase in scale than for a decrease. Also, in high censoring rate and large shifts scenarios, especially for high values of the fixed shape parameter, the CEV EWMA chart's threshold h converges to the base value of 1 and we cannot achieve a threshold corresponding to a desired in-control ARL of 370. The CUSUM chart is more reliable and easier to design in this case.

Figure 4.10: Out-of-Control Average Run Lengths for the Optimum CUSUM Chart and for the CEV EWMA Chart for an Increase in the Scale Parameter, $\beta_0 = 0.5$ and $\beta_0 = 3$.



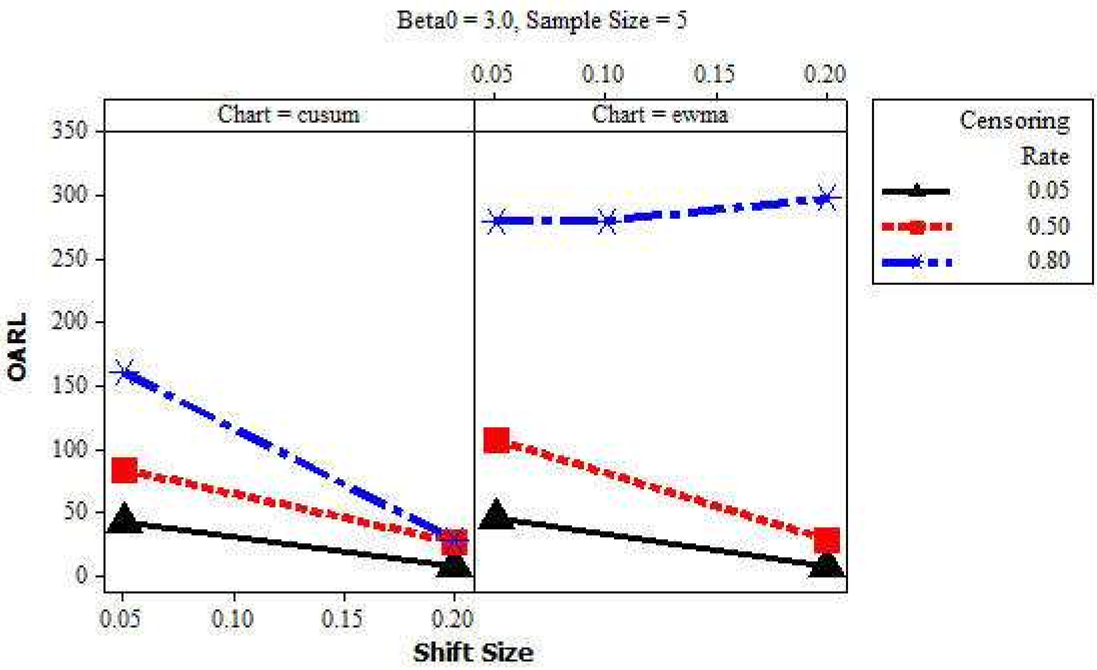
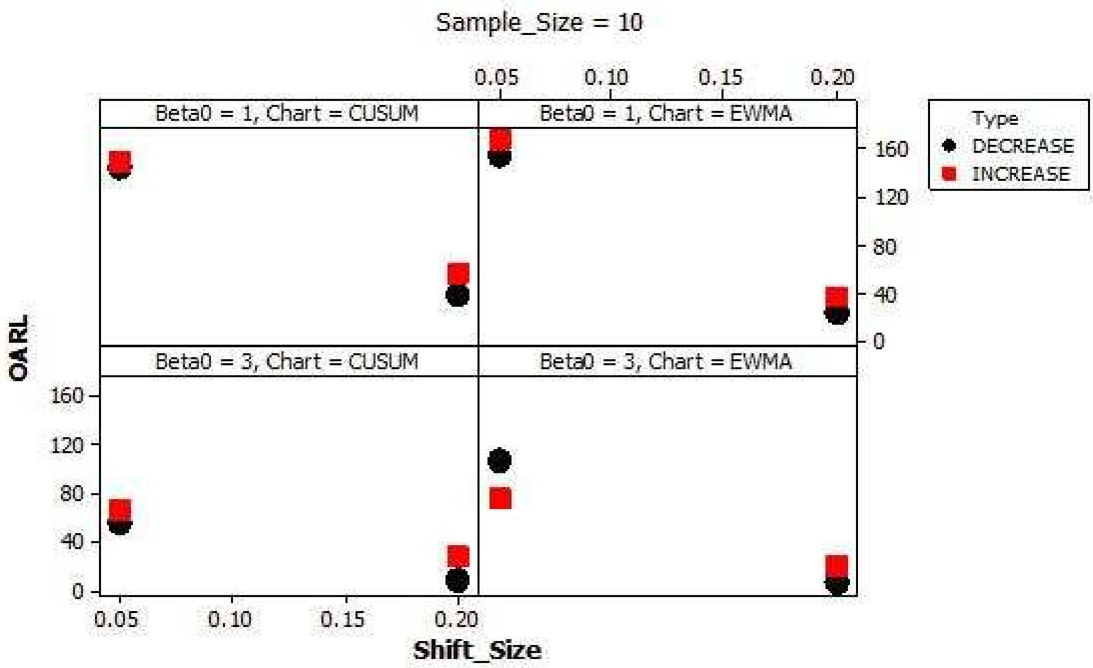


Figure 4.11 illustrates out-of-control average run lengths corresponding to a CUSUM chart and a CEV EWMA chart for both an increase and a decrease in the scale parameter. Thus we further illustrate the heavier impact of censoring on the charts for an increase in the scale parameter. One can notice that the out-of-control average run length for the detection of an increase in the scale parameter is larger than the out-of-control average run length for a decrease of the same magnitude in every considered scenario. The difference appears to be slightly larger for the EWMA chart.

Figure 4.11: Relative Performance of the Optimum CUSUM Chart and the EWMA Chart for both an Increase and for a Decrease in the Scale Parameter.

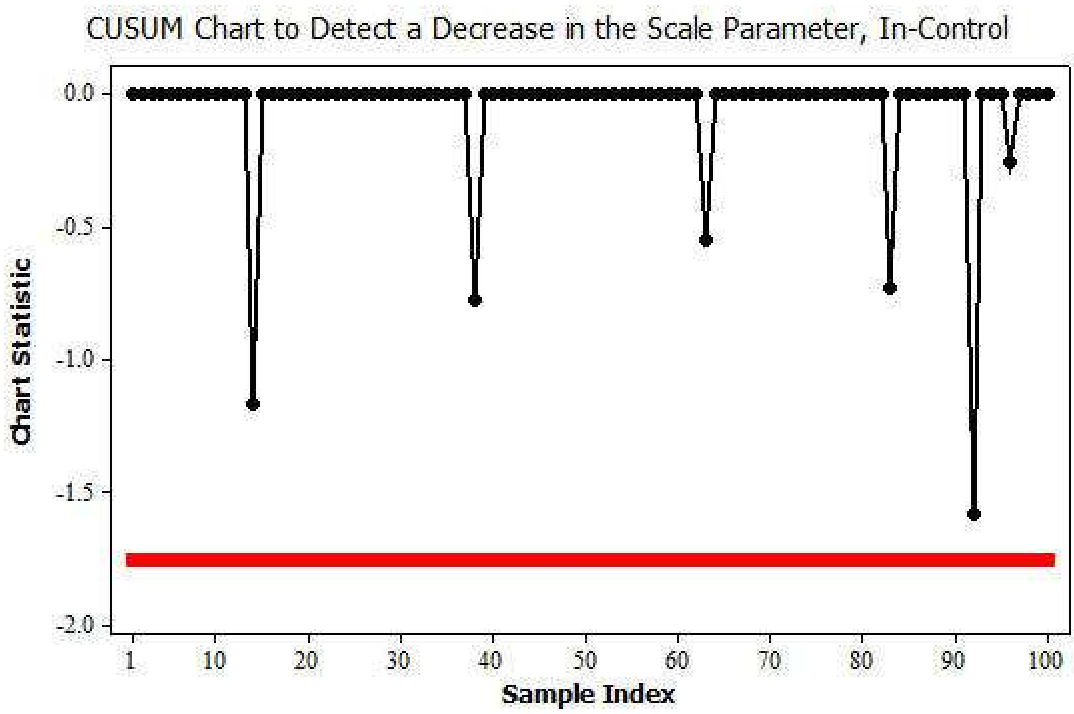


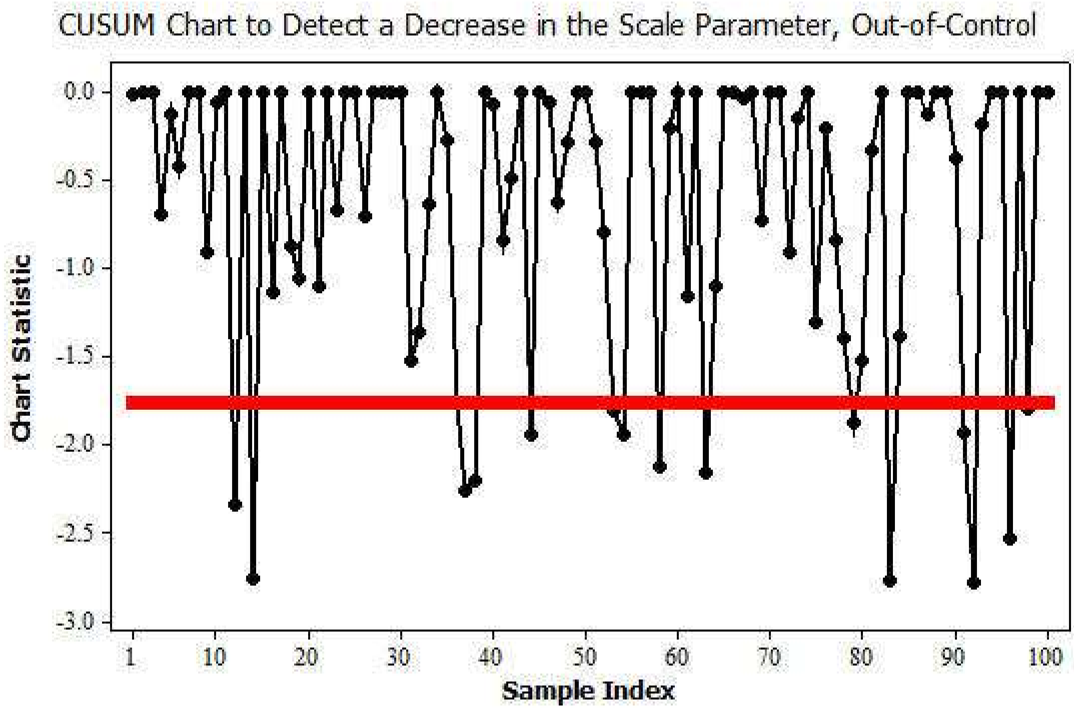
4.6 Application

We consider a fictional quality control process. Samples of ten products are put on a test stand and we are interested in detecting decreases in the characteristic life of this product.

The test stops after a predetermined number of hours since the test inception and about 30% of the products in each sample are suspended. The process owner has prior knowledge of the process and concludes that the data follow a Weibull distribution with in-control values of the scale and shape parameters of 1 and 3 respectively. For this situation, the threshold for our designed likelihood based CUSUM chart for detecting a decrease in scale is, from Table A.4, $h = -1.76$, corresponding to a desired in-control average run length of 370. We perform 100 consecutive tests. Figure 4.12 displays the CUSUM chart for 100 in-control samples, with no false signals. If the process shifts out of control, for example the scale parameter drops down to 80% of the in-control value, the CUSUM chart signals for the first time after about 12 samples put on test. The out-of-control average run length from Table A.4 is around 5 samples.

Figure 4.12: CUSUM Chart In-Control and Out-of-Control for $\beta_0 = 3$, 20% Decrease in the Scale Parameter, Censoring Rate 30%, and Sample Size $n = 10$. Threshold $h = -1.76$.





4.7 Conclusions and Recommendations

In this chapter we have studied the problem of monitoring characteristic life changes in Weibull processes with samples of $n \geq 1$ of right-censored lifetime data, which is equivalent to monitoring the mean of lifetime data when the shape parameter is fixed. We focused on the problem of decreases in the characteristic life, which signals a decrease in the process reliability and a real problem for the manufacturer. We designed a likelihood ratio based CUSUM chart and compared it to the CEV EWMA chart of Zhang and Chen (2004) in terms of the out-of-control average run lengths in different scenarios. We provided design tables and detailed guidelines for the practitioner to design similar charts under different frameworks one might encounter in the practice of reliability and quality control. The design of the CUSUM chart is dependent on the in-control value of the fixed shape parameter. However, our study illustrated through extensive simulations that the out-of-control performance of both charts is highly dependent on the in-control value of the shape parameter, β. This was previously unrecognized in the literature. We have designed the two charts to achieve a desired in-control average run length of 370 and then we have studied the properties in different out-of-control scenarios.

It appears that, although dependent on the values of the fixed shape parameter, the CUSUM chart displays improved performance in term of out-of-control average run length, as compared to the CEV EWMA method, in most cases. For $\beta_0 = 3$ and 5, the CUSUM method outperforms the CEV EWMA method, for all censoring rates, sample sizes and shift sizes considered. In this same scenario, the censoring rate effect is reduced and the chart achieves reasonable out-of-control average run lengths even for small shift sizes and high censoring rates. The out-of-control performance of the CUSUM method deteriorates for small values of β_0, as does the CEV EWMA performance. In these cases the charts sometimes compete, achieving similar performance in some scenarios. Considering the scarce resources usually available in a quality control procedure, the reduced sensitivity of the chart's out-of-control performance to sample size choices is good news. An important achievement of the CUSUM chart is its performance for high censoring rates. The inclusion of the actual number of failures in each sample in the design of the chart led to increased performance and an obvious advantage over the EWMA chart. To summarize, the chart performs best when β_0 takes high values, we have low to moderate censoring degrees and sample sizes of at least 5 units. In this case the CUSUM chart detects different sizes of shifts in the scale parameter, or equivalently in the mean, remarkably fast. We have also studied the loss in performance caused by a practitioner's choice of a CUSUM chart designed with the normal theory for lifetime data that follows a Weibull distribution with shape parameter $\beta = 3$. In this case the distribution resembles somewhat a normal distribution. However, the normal chart results in performance loss. The CUSUM chart designed according to the Weibull theory clearly dominates in an out-of-control situation.

Another advantage of using the likelihood ratio based CUSUM method is its flexibility of customization for different censoring schemes, underlying distributions, and monitoring goals- if we can formulate a likelihood, we can design a chart. We have studied the properties of the charts in a variety of scenarios to make the practitioner aware of the expected performance and drawbacks. We have also provided detailed guidelines for the practitioner to design the chart in scenarios that might be different, customized for his/her specific needs.

Chapter 5

Alternative CUSUM Charts for Censored Weibull Lifetimes

5.1 Background

Papers in the literature usually consider monitoring the mean of lifetimes, assuming that the shape parameter is fixed to some known value, as in Chapter 4. However, practitioners in the reliability field have an interest in the overall behavior of the process. However, one might wonder what happens if the shape parameter value changes. Assuming that the scale parameter does not change, a change in the shape parameter affects the process mean as well. Studying the behavior of both parameters of the Weibull distribution reveals different aspects of the process.

A sudden decrease of the shape parameter might indicate a shift in the failure mechanism, and a need for increased budget allocations for warranty service. Therefore, monitoring for a change in the shape parameter is a proactive measure. Another situation of interest is when a shift in the characteristic life is accompanied by a shift in the failure mechanism. In that case one should detect such a complex change as soon as possible. Since these two shifts may not occur independently, a simultaneous chart targeting the combined effect is appropriate.

Our framework is similar to the framework considered in Chapter 4 for the development of the CUSUM chart for the characteristic life. The manufacturer is usually interested in increasing the lifetime of products, so it is of interest to determine if significant changes occurred in the process which might affect negatively the reliability characteristics of the product. Although it became customary in the reliability theory and practice to consider that the process is characterized by a fixed failure mechanism, we do not make this assumption and we develop appropriate monitoring techniques. We then evaluate the properties of these methods and we make recommendations for their most effective use.

5.2 Monitoring Methodology

We propose two additional likelihood ratio based CUSUM charts for samples of right-censored lifetimes following a Weibull distribution. The first chart detects changes in the shape parameter β when the scale parameter η is fixed to the in-control value η_0, and the second chart detects simultaneous shifts in both the scale and the shape parameters. As in Chapter 4, the in-control values of the parameters are considered known from a previous phase of the quality control study, and we are only interested in Phase II monitoring. We consider the CUSUM chart proposed in Chapter 4 and adapt it to the current monitoring objectives. We also address the charts' design under different transformations to the data. A practitioner can consider any of the equivalent forms of the score statistic z_i at his/her convenience. If the data are already transformed due to other quality procedures when the monitoring begins, one has an appropriate chart readily designed for each instance. Then we evaluate the performance of the charts for different censoring rates, sample sizes, in-control values of the parameters and minimum shift sizes that the charts should detect quickly. Since we do not have competing previously developed charts with these monitoring objectives, we provide the practitioner with a set of guidelines for effective use of the charts.

5.2.1 The Likelihood Ratio Based CUSUM Chart for the Shape Parameter, β

We first develop the CUSUM chart that monitors for changes in the shape parameter of the Weibull distribution, while the scale parameter is fixed to the known in-control value.

Design with the Weibull Distribution. When one considers the data in its raw form, we use the usual parametrization for the two-parameter Weibull distribution. We assume that the in-control values of the parameters, η_0 and β_0, are known. In an in-control situation, T, the lifetime random variable, follows a Weibull$(\beta = \beta_0, \eta = \eta_0)$ distribution. In an out-of-control situation, if only the β parameter shifts from the value β_0 to $\beta_1 = (1-d) * \beta_0$, and η is fixed, then T follows a Weibull$(\beta = \beta_1, \eta = \eta_0)$ distribution. With this approach and (4.1), the z_i statistics become

$$z_i = r_i[\log\frac{\beta_1}{\beta_0} + (\beta_0 - \beta_1)\log\eta_0] + (\beta_1 - \beta_0)\sum_{j=1}^{n}\delta_{ij}\log t_{ij} + \\ + \sum_{j=1}^{n}[(\frac{t_{ij}}{\eta_0})^{\beta_0} - (\frac{t_{ij}}{\eta_0})^{\beta_1}], i = 1, 2, \ldots.$$

Design with the Smallest Extreme Value (SEV) transformation. Following the relationship between the SEV distribution and the Weibull distribution presented in Chapter 2, in an in-control situation, $Y = \log(T)$ follows a $SEV(b = b_0, u = u_0)$ distribution. In an out-of-control situation, with b shifting from b_0 to an out-of-control value $b_1 = log(\beta_1)$, Y follows a $SEV(b_1, u_0)$ distribution. Then, with (4.1), the statistics z_i become

$$z_i = r_i\log(\frac{b_0}{b_1}) + \sum_{j=1}^{n}[\delta_{ij}(\frac{y_{ij}-u_0}{b_1} - \frac{y_{ij}-u_0}{b_0}) \quad -[\exp(\frac{y_{ij}-u_0}{b_1}) - \exp(\frac{y_{ij}-u_0}{b_0})]], i = 1, 2, \ldots.$$

Design with the Standard Exponential Transformation. In an in-control situation the random variable $X = (T/\eta_0)^{\beta_0}$ follows a standard exponential distribution. In an out-of-control situation, when $\eta = \eta_0$ is fixed and $\beta_1 = (1-d)*\beta_0$, where $d*100\%$ represents a percentage shift in the shape parameter, we can write

$$X = (\frac{T}{\eta_0})^{\beta_0} = [(\frac{T}{\eta_0})^{\beta_1}]^{\frac{\beta_0}{\beta_1}}.$$

We can derive that in general, if the random variable X follows an exponential distribution with parameter λ, and a is a constant, then the random variable X^a follows a Weibull distribution with shape parameter $\beta = 1/a$ and scale parameter $\eta = 1$. Then the random variable $X = (T/\eta_0)^{\beta_0}$, in an out-of-control situation with $\beta_1 = (1-d)*\beta_0$, follows a Weibull distribution with scale parameter $\eta^* = 1$ and shape parameter $\beta^* = 1/(\frac{\beta_0}{\beta_1}) = \frac{\beta_1}{\beta_0} = (1-d)$. In this case the z_i statistics become

$$\begin{aligned} z_i &= r_i \log(\frac{\beta_1}{\beta_0}) + (\frac{\beta_1}{\beta_0} - 1)\sum_{j=1}^{n} \delta_{ij} \log x_{ij} + \sum_{j=1}^{n}[x_{ij} - {x_{ij}}^{\frac{\beta_1}{\beta_0}}] \\ &= r_i \log(1-d) + (-d)\sum_{j=1}^{n} \delta_{ij} \log x_{ij} + \sum_{j=1}^{n}[x_{ij} - {x_{ij}}^{1-d}], i = 1, 2, \ldots. \end{aligned}$$

The chart design does not depend on the particular values of the in-control parameters β_0 and η_0, and $z_{Weibull} = z_{SEV} = z_{Std.Exponential}$. One can use any of the three equivalent designs to implement the CUSUM chart monitoring for a shift in the shape parameter, when the scale parameter is fixed.

5.2.2 CUSUM Charts that Detect Simultaneous Shifts in the Shape Parameter, β, and in the Scale Parameter, η

We now suppose that shifts in both the scale parameter and the shape parameter can occur. We adjust the likelihood ratio based CUSUM chart to monitor for shifts in the process, caused by both a shift in the shape parameter β, from β_0 to $\beta_1 = (1-d_\beta) * \beta_0$, and a shift in the scale parameter η, from η_0 to $\eta_1 = (1-d_\eta) * \eta_0$. We again evaluate the simplified form of the statistics z_i, considering different transformations to the data.

Design with the Weibull distribution. Under the Weibull distribution, the z_i statistics become

$$
\begin{aligned}
z_i = r_i[\log \frac{\beta_1}{\beta_0} + \log \frac{\eta_0}{\eta_1} + (\beta_0 - 1) \log \eta_0 - (\beta_1 - 1) \log \eta_1] + \\
+ (\beta_1 - \beta_0) \sum_{j=1}^{n} \delta_{ij} t_{ij} + \sum_{j=1}^{n} [(\frac{t_{ij}}{\eta_0})^{\beta_0} - (\frac{t_{ij}}{\eta_1})^{\beta_1}], i = 1, 2, \ldots.
\end{aligned}
$$

Design with the Smallest Extreme Value (SEV) transformation. In an in-control situation, $Y = \log(T)$ follows a $SEV(b = b_0, u = u_0)$ distribution. In an out-of-control situation, with u shifting from u_0 to an out-of-control value $u_1 = log(\eta_1)$, and b shifting from b_0 to an out-of-control value $b_1 = log(\eta_1)$, Y follows a $SEV(b_1, u_1)$ distribution. Using the general form of the statistic z_i given by (4.1), and the SEV pdf and cdf, z_i become

$$
\begin{aligned}
z_i = r_i \log(\frac{b_0}{b_1}) + \sum_{j=1}^{n} [\delta_{ij}(\frac{y_{ij} - u_1}{b_1} - \frac{y_{ij} - u_0}{b_0}) - \\
- [\exp(\frac{y_{ij} - u_1}{b_1}) - \exp(\frac{y_{ij} - u_0}{b_0})]], i = 1, 2, \ldots.
\end{aligned}
$$

Design with the Standard Exponential transformation. In an in-control scenario the random variable $X = (T/\eta_0)^{\beta_0}$ follows a standard exponential distribution. If $\eta_1 = (1-d_\eta)*\eta_0$ and $\beta_1 = (1-d_\beta)*\beta_0$, then $(T/\eta_1)^{\beta_1}$ follows a standard exponential distribution in this out-of-control situation. We can rewrite the random variable X as

$$X = (\frac{T}{\eta_0})^{\beta_0} = [[(\frac{T}{\eta_1})^{\beta_1}]^{(\frac{\beta_0}{\beta_1})}] * (\frac{\eta_1}{\eta_0})^{\beta_0}.$$

We can derive that in general, if the random variable X follows an exponential distribution with parameter λ, and a and b are constants, then the random variable bX^a follows a Weibull distribution with shape parameter $\beta^{**} = 1/a$ and scale parameter $\eta^{**} = b$. Then, in an out-of-control situation with $\eta_1 = (1-d_\eta)*\eta_0$ and $\beta_1 = (1-d_\beta)*\beta_0$, X follows a Weibull distribution with scale parameter $\eta^{**} = (\frac{\eta_1}{\eta_0})^{\beta_0} = (1-d_\eta)^{\beta_0}$, and shape parameter $\beta^{**} = \frac{\beta_1}{\beta_0} = (1-d_\beta)$. Starting from (4.1), and using the standard exponential pdf and cdf, the z_i statistics become

$$\begin{aligned} z_i &= r_i[\log\frac{\beta_1}{\beta_0} - \beta_0\log\frac{\eta_1}{\eta_0} - \beta_0(\frac{\beta_1}{\beta_0}-1)\log\frac{\eta_1}{\eta_0}]+ \\ &+ (\frac{\beta_1}{\beta_0}-1)\sum_{j=1}^{n}\delta_{ij}\log x_{ij} + \sum_{j=1}^{n}[x_{ij} - \frac{{x_{ij}}^{\beta_1/\beta_0}}{(\eta_1/\eta_0)^{\beta_1}}] \\ &= r_i[\log d_\beta - \beta_0\log d_\eta - \beta_0(d_\beta-1)\log d_\eta] + (d_\beta-1)\sum_{j=1}^{n}\delta_{ij}\log x_{ij}+ \\ &+ \sum_{j=1}^{n}[x_{ij} - \frac{{x_{ij}}^{d_\beta}}{{d_\eta}^{\beta_1}}], i = 1,2,\ldots. \end{aligned}$$

This chart's design is also independent of the chosen transformation to the data. We note that the chart's design varies with different in-control values of the shape parameter, β_0, but it is independent of the considered in-control values of the scale parameter, η_0. Next we evaluate the properties of the two developed charts. The properties hold regardless of the considered transformation to the data.

5.3 Properties of the CUSUM Chart for Detecting Changes in the Shape Parameter, when the Scale Parameter Is Fixed

For the design of this chart we assume, without loss of generality, that the scale parameter is fixed to the in-control value, $\eta = \eta_0 = 1$. We also assume that the in-control value of the shape parameter β_0 is 1, without loss of generality, since the chart design and in-control and out-of-control performance are independent of the in-control values of the shape and scale parameters.

5.3.1 Simulation Description

We evaluate the performance of the likelihood based CUSUM chart monitoring for changes in the failure mechanism of a process modeled by the shape parameter of a Weibull distribution, in different scenarios of interest. The cases considered are the following:

- Sample size $n = 3, 5, 10$.
- Theoretical in-control censoring rates $p_C = 5\%, 50\%, 80\%$. We evaluated the performance of the charts in low, moderate, and high censoring rates scenarios. The theoretical censoring rate corresponds to an in-control situation. If the lifetime random variable T follows a Weibull(β,η) distribution, with Type I right-censoring at a predetermined time C, then $p_C = \exp[-\frac{C}{\eta}]^{\beta}$, with $\beta = \beta_0$ and $\eta = \eta_0$. If the process mean or characteristic life increases, we expect more censored units. Alternatively, if the mean life or characteristic life decreases, we expect a lower number of censored units in the

generated sample. We designed the chart for the corresponding in-control censoring rate.

- In-control value of the shape parameter $\beta = \beta_0 = 1$, without loss of generality.
- In-control value of the fixed scale parameter $\eta = \eta_0 = 1$, without loss of generality.
- Shift sizes for the shape parameter $d = +5\%, +20\%, -5\%, -20\%$, for which we design the optimum CUSUM chart. The out-of-control β value is then $\beta_1 = \beta_0 * (1 - d)$. One should notice here that $d = +5\%$ for example results in a shape parameter decrease of 5%.
- Desired in-control average run length $IARL = 370$.
- We repeat the simulations to obtain the threshold through trial and error, and to evaluate the out-of-control performance for each scenario, with $10,000$ replications.

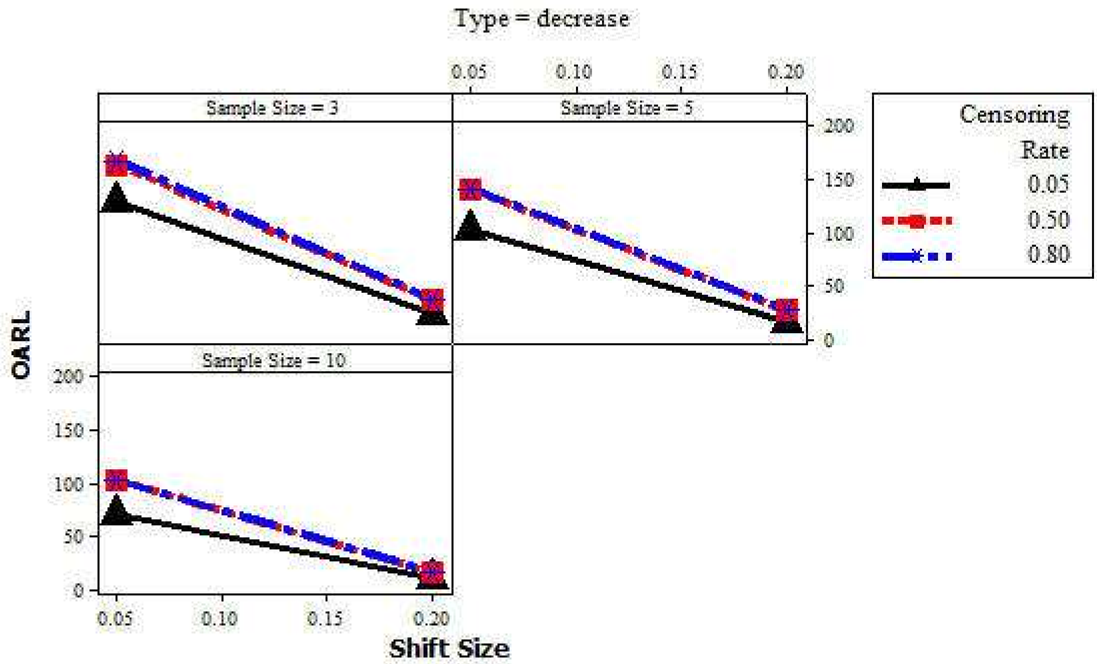
We discussed the simulation algorithm in detail when we designed the CUSUM chart for the scale parameter in Chapter 4. We follow a similar approach here. The deliverables of this simulation study are the threshold, the estimated average in-control ARL (IARL), the estimated average out-of-control ARL (OARL) and the corresponding simulation standard errors, calculated as the standard deviations of the run lengths divided by the square root of the number of simulation replications.

5.3.2 Simulation Results

In this section we discuss the CUSUM chart's performance for various sample sizes, censoring proportions, and shift sizes and directions. We evaluate the charts' performance in terms of the out-of-control average run length, while the chart is designed to provide an in-control average run length of 370. The desired out of control ARL (OARL) value should be low. Tables B.1 and B.2 from Appendix B provide the thresholds, the estimated in-control and out-of-control average run lengths, and the corresponding simulation errors, for the scenarios listed in the simulation plan, for detecting a decrease, and respectively, an increase in the shape parameter.

We evaluate the chart's out-of-control performance and we study the effect of different parameters, such as censoring amounts and sample sizes, for the shifts for which the charts are designed to be optimal. In a later paragraph, we evaluate the chart's out-of-control performance for various sizes of shifts, different from the shift for which the chart was designed to be optimal. As expected, the magnitude of the shift size together with the censoring rate and the sample size, dictate the performance of the chart to detect either a positive or a negative change in the shape parameter. Tables B.1 and B.2 from Appendix B display reasonable out-of-control performance corresponding to a desired in-control average run length of approximately 370 samples, for shifts sizes of 20%, either positive or negative, especially for sample sizes of $n = 10$, even for moderate and high censoring rates of 50% and 80%. For example, when there is a 20% shift in the shape parameter β, from $\beta_0 = 1$ down to $\beta_1 = 0.8$, the likelihood based CUSUM chart signals on average with 17 samples, in a scenario with a moderate censoring rate and a sample size of $n = 10$. It is expected that the performance of the chart is poorer for very small shifts, which we clearly see. The out-of-control average run length for the chart designed to detect shifts as small as 5% in magnitude is about 100 samples.

Figure 5.1: Optimum CUSUM Charts Designed to Detect an Increase or a Decrease in the Shape Parameter. Impact of Censoring Rates.



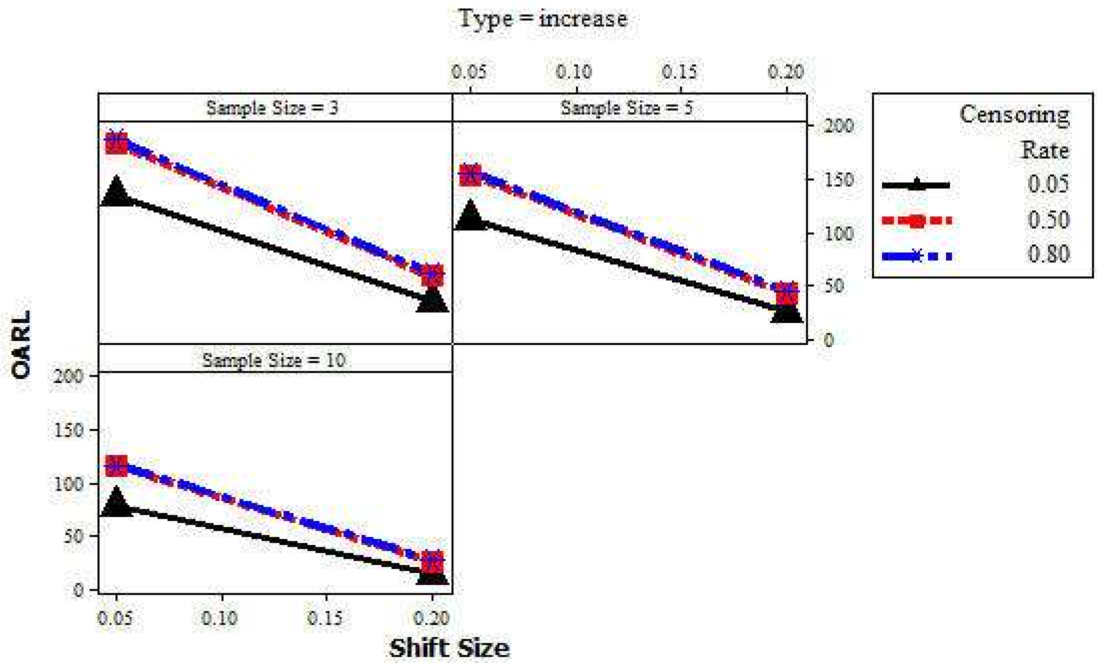
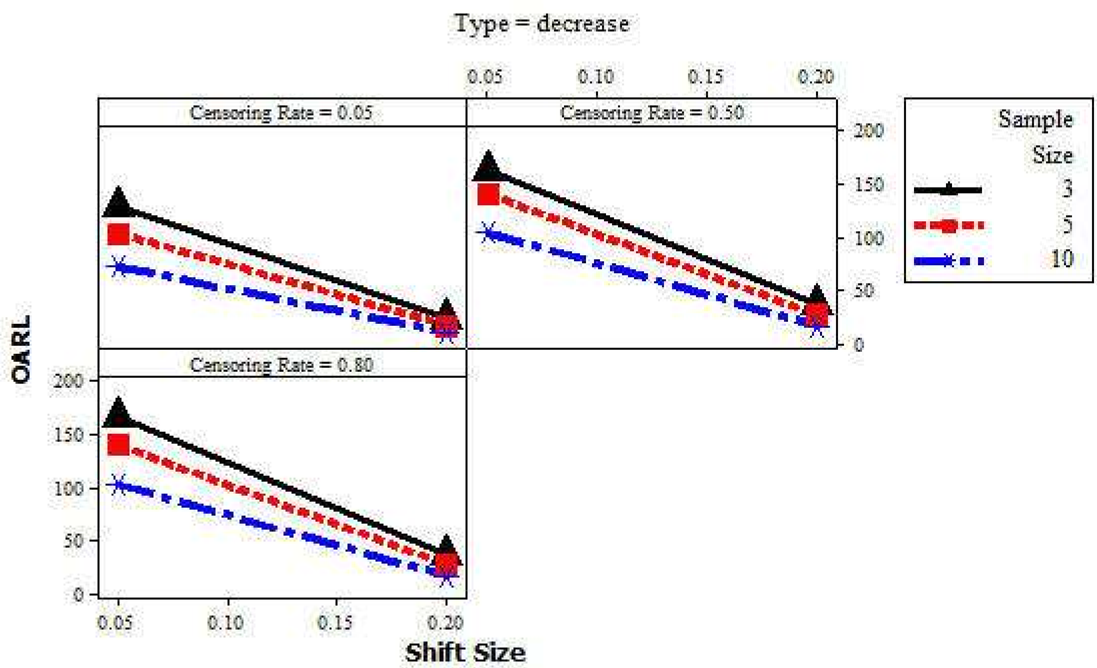


Figure 5.2: Optimum CUSUM Charts Designed to Detect an Increase or a Decrease in the Shape Parameter. Impact of Sample Sizes.



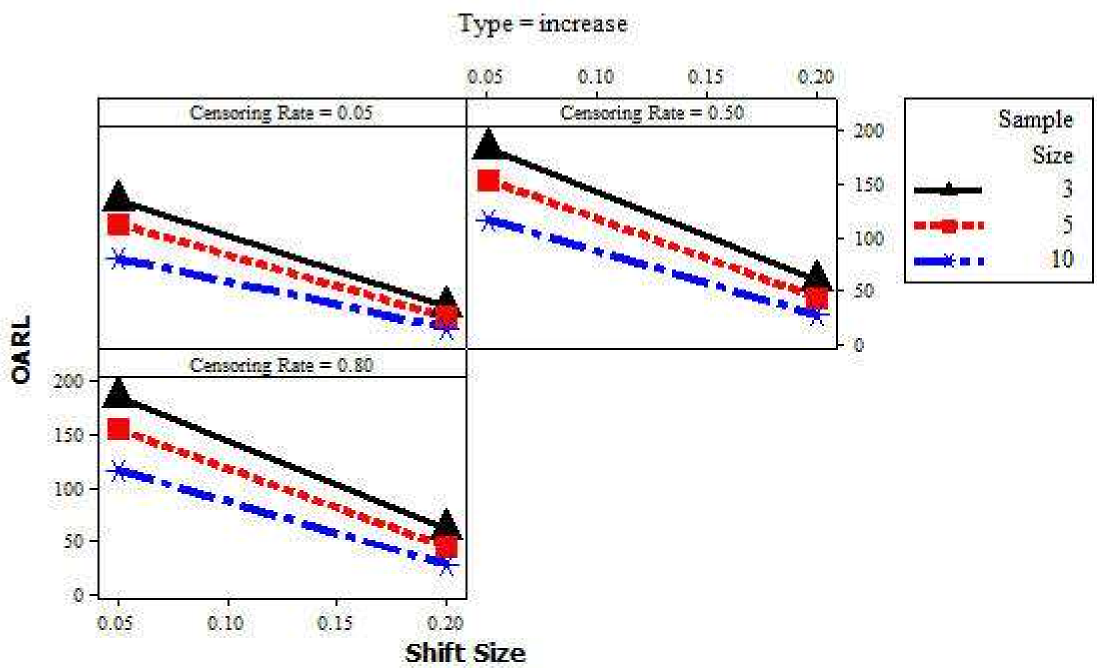


Figure 5.1 displays the relative performance of the CUSUM chart for the shape parameter when the scale parameter is fixed to the in-control value for different censoring rates. Figure 5.1 displays in each panel the out-of-control average run lengths for the three censoring rates considered. Each panel corresponds to a sample size. The top figure correspond to the CUSUM chart that detects decreases in the shape parameter, while the bottom figure corresponds to the chart designed to detect increases in the shape parameter. Figure 5.1 shows that the difference in performance between moderate or high censoring rate scenarios is not significant. Low censoring rates result in better performance. Figure 5.2 shows the sample size impact on the chart's performance. Figure 5.2 displays in each panel the out-of-control average run lengths for the three sample sizes considered. Each panel corresponds to a censoring rate. The top figure correspond to the CUSUM chart that detects decreases in the shape parameter, while the bottom figure corresponds to the chart designed to detect increases in the shape parameter. It appears that the out-of-control behavior of the chart improves with the sample size increase, and the impact is greater for small shifts.

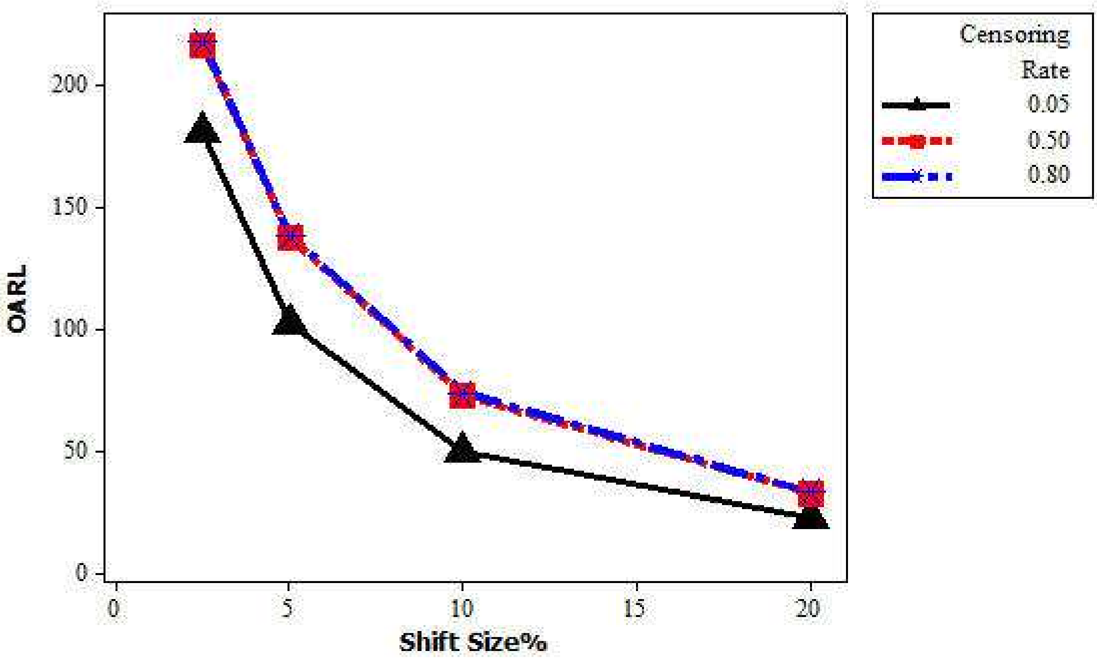
The design of the CUSUM chart for the shape parameter is independent of the in-control values of both β and η parameters. The in-control and out-of-control performance is unchanged for any values of β_0 and η_0. Table 5.1 illustrates this independence for a particular scenario. One can attribute the differences to the simulation errors inherent in the study.

Table 5.1: Relative performance of CUSUM chart to detect a 5% increase in the shape parameter for different β_0 values and η_0, illustrating the invariance of the chart to the parameters' in-control values.

Censoring Rate	Shift Size	Sample Size	β_0	η_0	Threshold	OARL
p_C=50%	d=5%	n=5	0.5	1	1.28418	153
			1.0	1	1.289	153
			1.0	5	1.289	153
			1.0	10	1.289	153
			3.0	1	1.285	152
			5.0	1	1.279	152

So far we have studied the performance of the CUSUM chart for the shape parameter in an out-of-control scenario corresponding to a shift for which the chart was designed to be optimal. We have evaluated the impact of different factors on the out-of-control performance. However, there are circumstances in the practice of reliability when the practitioner does not have insight into what size of shift one should expect. Therefore, it is of interest to evaluate the performance of a CUSUM chart designed for a certain minimum shift in an out-of-control situation when the actual shape parameter decrease is of a different magnitude. We evaluate the performance of the CUSUM chart designed to detect a minimum 5% shift in the shape parameter in low, moderate and high censoring rates scenarios. We assume a moderate sample size of $n = 5$. The charts were designed to achieve a desired in-control average run length of 370. Figure 5.3 illustrates the study results. We notice that moderate and high censoring rates of 50% and 80% respectively result in very similar performance. When only around 5% of the data are censored, the chart's performance improves for all shift magnitudes.

Figure 5.3: Performance of the CUSUM Chart for the Shape Parameter Designed for a 5% Decrease to Detect Decreases of Different Magnitudes.



5.4 CUSUM Chart for a Simultaneous Shift in the Scale and the Shape Parameters of the Weibull Distribution

In this section we study the properties of the simultaneous chart that detects changes in both the scale and the shape parameters through simulations and we discuss the results.

5.4.1 Simulation Description

We evaluated the performance of the likelihood based CUSUM chart monitoring for simultaneous shifts in both parameters of a Weibull distribution in the following scenarios:

- Sample size $n = 3, 5, 10$.
- Theoretical in-control censoring rates $p_C = 5\%, 50\%, 80\%$.
- In-control values for the shape parameter $\beta = \beta_0 = 0.5, 1, 3, 5$.
- In-control value for the scale parameter $\eta = \eta_0 = 1$, without loss of generality, since the chart's design and performance is independent of the in-control values of the scale parameter.
- Shift size for the scale parameter $d_\eta = +5\%, +20\%, -5\%, -20\%$. The out-of-control η value is then $\eta_1 = \eta_0 * (1 - d_\eta)$.
- Shift size for the shape parameter $d_\beta = +5\%, +20\%, -5\%, -20\%$. The out-of-control β value is then $\beta_1 = \beta_0 * (1 - d_\beta)$.
- Desired in-control average run length (IARL) $IARL = 370$.
- We repeated the simulations to obtain the threshold and then to evaluate the out-of-control performance for each scenario with $10,000$ replications.

5.4.2 Simulation Results

We explore the behavior of the simultaneous chart for different censoring rates, sample sizes and in-control values of the shape parameter, and for small and large positive and negative shifts in both the shape parameter and the scale parameter. The practitioner can find the chart's design details in the Appendix C, Tables C.1 through C.12. Each table presents for each scenario considered the threshold h found through trial-and-error, the estimated in-control and out-of-control average run lengths, and the corresponding simulation errors. We evaluate the impact of different factors, such as the sample size, the censoring rate, the in-control value of the shape parameter and the specific combination of shift sizes and directions, in an out-of-control situation for which the chart is designed to be optimal. In a later paragraph we evaluated the chart's out-of-control performance for various shifts, other than the combination for which the chart was designed to be optimal. Inspecting the out-of-control average run lengths in the various scenarios, one can notice a few patterns. The chart's performance is mainly driven by the shifts' direction and magnitude. Also, as expected, high censoring rates and small sample sizes adversely impact the chart's behavior. An important point is that the various values of β_0 result in different effects of shift size, censoring rate, and sample size. Table 5.2 illustrates the large impact of the β_0 value on the simultaneous chart's performance. The chart signals a simultaneous decrease of 20% for the scale parameter and of 5% for the shape parameter on average in 4 samples when $\beta_0 = 5$, as compared to 61 samples when $\beta_0 = 0.5$, when 50% of the data are censored in a sample of five items.

Table 5.2: Effect of different in-control values of the shape parameter on the performance of the simultaneous chart in an out-of-control situation when η drops 20% and β drops 5%. Moderate and high censoring rates of 50% and 80% and sample size $n = 5$.

Censoring Rate	β_0	Average No. of Samples to Signal
p_C=50%	0.5	61.5728
	1.0	32.0428
	3.0	7.8147
	5.0	4.0291
p_C=80%	0.5	72.5815
	1.0	42.7840
	3.0	11.3768
	5.0	5.3975

Next we evaluated the impact of censoring rate on the out-of-control performance in Figures 5.4 and 5.5 and Tables 5.3 and 5.4. Figure 5.4 and 5.5 display in each panel the out-of-control average run lengths as a function of the size of the shift in the scale parameter, either positive or negative. Each panel corresponds to a simultaneous shift in the shape parameter, either positive or negative. Figure 5.4 and Table 5.3 correspond to $\beta_0 = 0.5$. Figure 5.5 and Table 5.4 correspond to $\beta_0 = 5$. Tables 5.3 and 5.4 support with examples the information displayed in Figures 5.4 and 5.5. Figure 5.4 and Table 5.3 show that the impact of different censoring rates on the out-of-control performance of the chart depends highly on the size and direction of the shift for which the chart was designed. Figure 5.5 and Table 5.4 illustrate that the interaction behaves differently in different in-control failure mechanism scenarios. While in infantile mortality scenarios ($\beta_0 = 0.5$) combinations of large positive shifts in the scale parameter and small negative shifts in the shape parameter are highly affected by the amount of censoring, this is not a significant problem in rapid wear-out in-control scenarios (when $\beta_0 = 5$ in Figure 5.5). High in-control β_0 values reduce the impact of the amount of censoring on the chart's out-of-control performance overall.

Figure 5.4: Impact of Censoring Rates on the Out-of-Control Performance of the Simultaneous Chart, with Various Shifts in Both Parameters for which the Chart was Designed, with $\beta_0 = 0.5$ and $n = 5$.

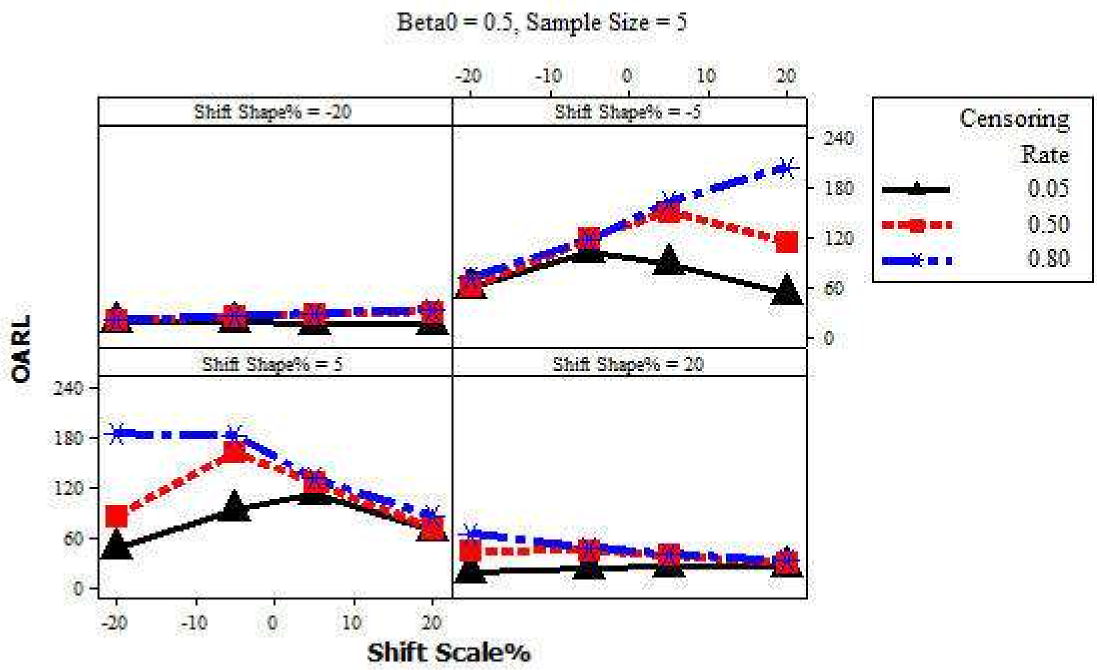


Table 5.3: Example for Figure 5.4. Impact of censoring on the out-of-control performance of the simultaneous CUSUM chart, for $\beta_0 = 0.5$, sample size $n = 5$, in a scenario with a small negative shift in the shape parameter and in a scenario with a large positive shift in the shape parameter, together with a 20% positive shift in the scale parameter.

Censoring Rate	OARL 5% Shape Decrease	OARL 20% Shape Increase
0.05	51.932	26.1937
0.50	115.150	31.5966
0.80	205.635	34.1805

Figure 5.5: Impact of Censoring Rates on the Out-of-Control Performance of the Simultaneous Chart, with $\beta_0 = 5$ and $n = 5$.

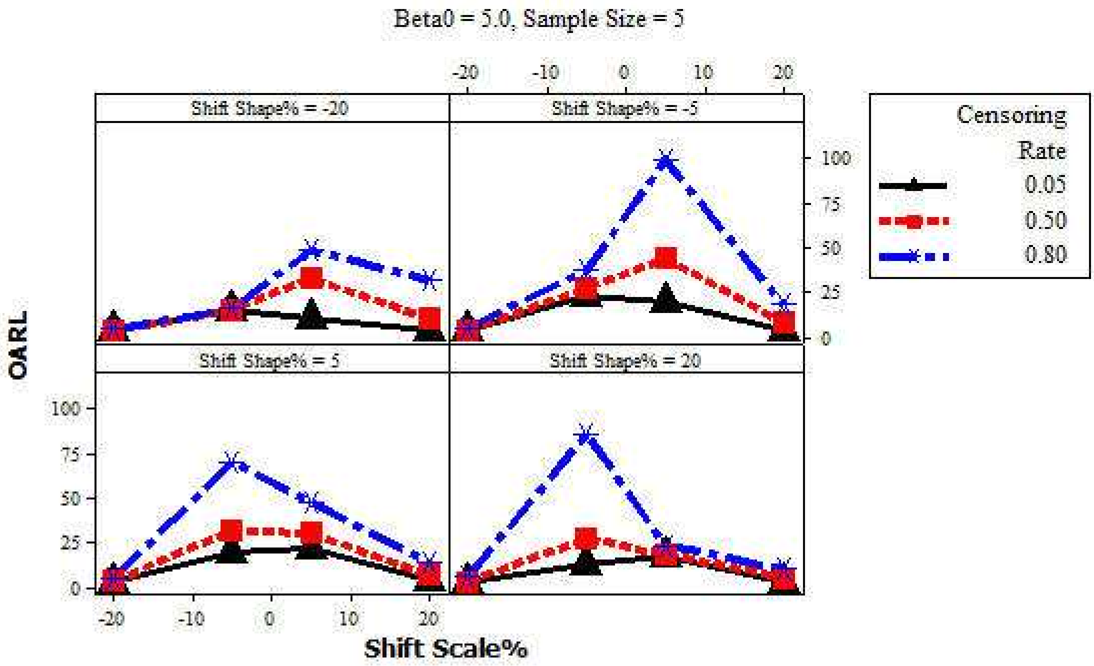


Table 5.4: Example for Figure 5.5. Impact of censoring on the out-of-control performance of the simultaneous CUSUM chart for $\beta_0 = 5$, sample size $n = 5$, with a 5% decrease in the shape parameter.

Censoring Rate	OARL 5% Scale Decrease	OARL 5% Scale Increase	OARL 20% Scale Increase
0.05	23.3314	19.5986	3.9410
0.50	27.2733	43.7410	8.2809
0.80	37.2494	98.3245	18.6344

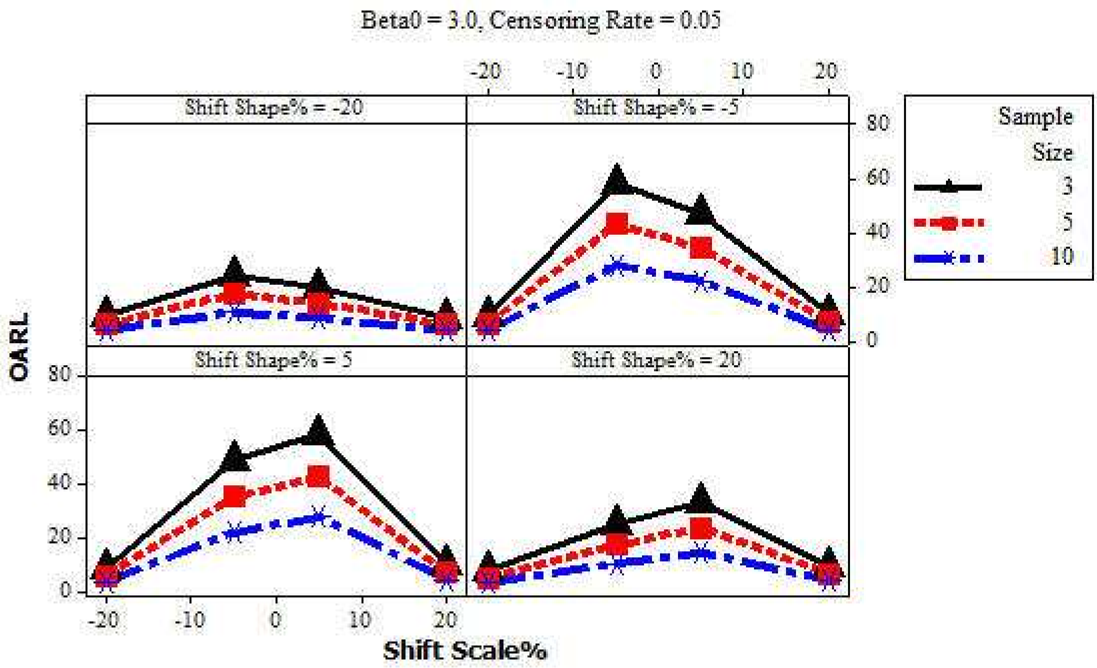
Figure 5.6 illustrates the impact of different choices of sample sizes on the out-of-control performance of the simultaneous CUSUM chart for a process with normal wear out when

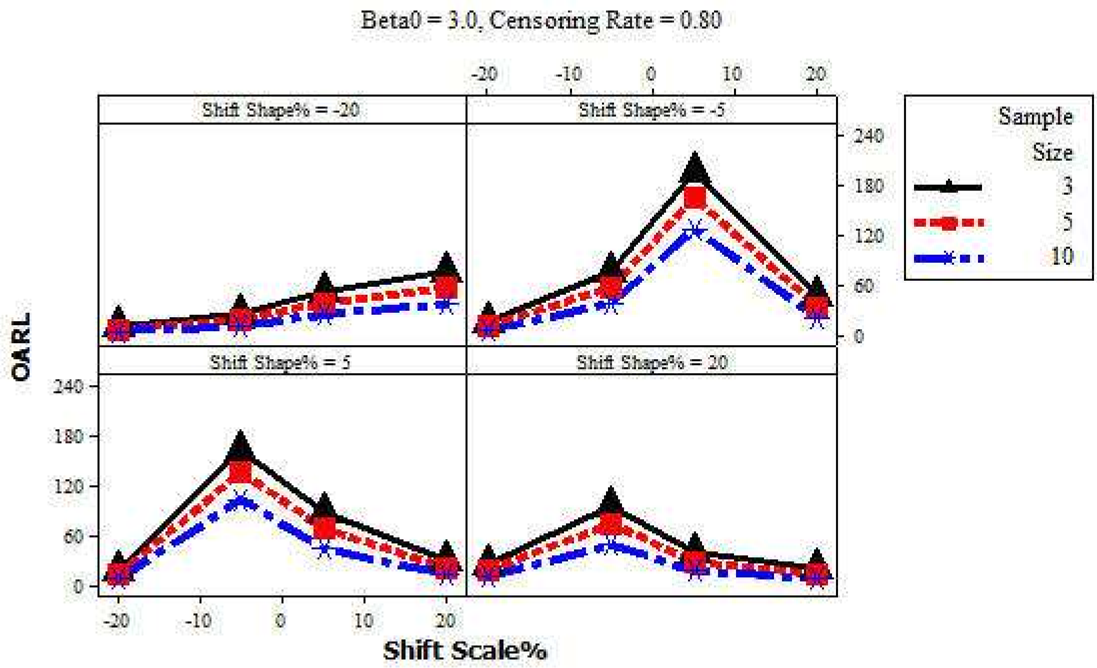
in-control ($\beta_0 = 3$), for low, moderate and high censoring rates. Each panel of the figure illustrates the impact of the sample size in a different scenario. Each panel display the out-of-control average run lengths as a function of the magnitude of the shift in the scale parameter for which the chart was designed. Each panel corresponds to a different simultaneous shift in the shape parameter. The top four panels illustrate the impact of the sample size when we have a low censoring rate. The bottom four panels illustrate the impact of the sample size when we have a high censoring rate. Table 5.5 supports the information displayed in Figure 5.6 with a numerical example. The simulation results show that increasing the size of the sample helps the chart's performance, especially in high censoring rates scenarios. The impact of the sample size choice is also influenced by the type of in-control failure mechanism. The impact is larger for smaller β_0 values, as we can see in Table 5.5.

Table 5.5: Example for Figure 5.6. Impact of sample size on the out-of-control performance of the simultaneous CUSUM chart in low, moderate and high censoring rate scenarios, for $\beta_0 = 0.5$ and $\beta_0 = 5$, when there is a 5% increase in the scale parameter and a 5% decrease in the shape parameter.

	Censoring Rates					
	$p_C = 0.05$		$p_C = 0.5$		$p_C = 0.8$	
	OARL $\beta_0 = 0.5$	OARL $\beta_0 = 5$	OARL $\beta_0 = 0.5$	OARL $\beta_0 = 5$	OARL $\beta_0 = 0.5$	OARL $\beta_0 = 5$
n=3	111.978	27.2051	177.882	58.2476	189.488	122.775
n=5	89.624	19.5986	151.754	43.7410	163.374	98.325
n=10	62.792	12.3279	114.586	27.6969	127.956	68.545

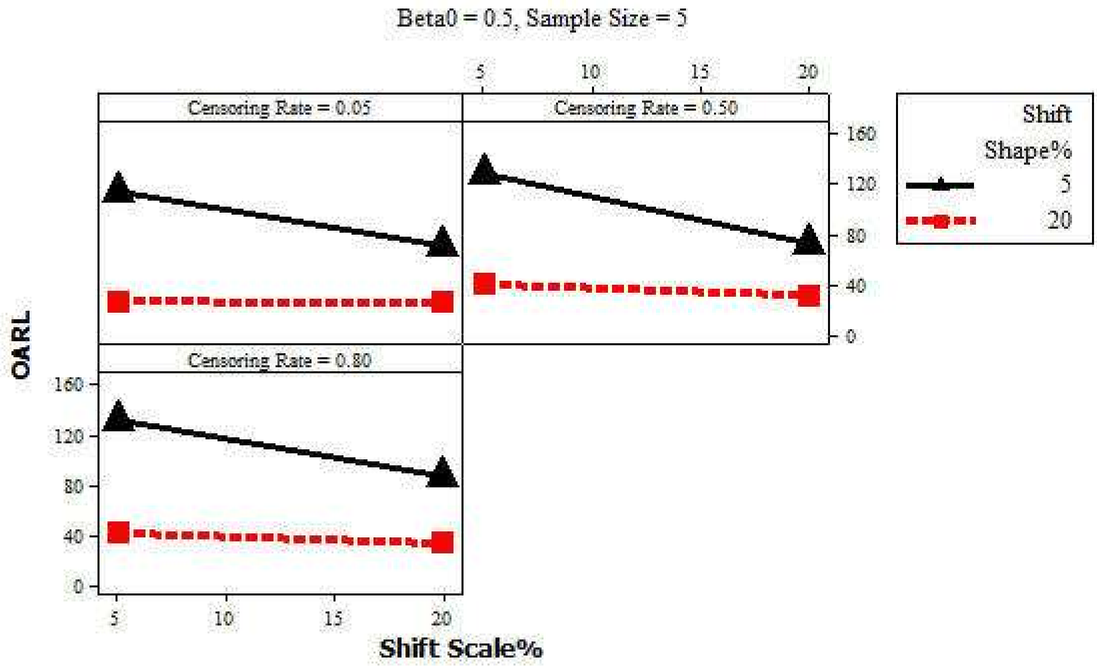
Figure 5.6: Impact of the Sample Size on the Out-of-Control Performance of the Simultaneous Chart Both in a Low and in a High Censoring Rate Scenario, when $\beta_0 = 3$.





Next, Figures 5.7 through 5.10, as well as Tables 5.5 through 5.8, illustrate the performance of the simultaneous CUSUM chart for specific combinations of shifts in the scale and the shape parameters. Each panel of each figure displays the impact of different combinations of shifts in the scale parameter and in the shape parameter on the out-of-control performance of the charts designed for those combinations. Each panel corresponds to a certain censoring rate. All panels are based on a sample size of $n = 5$. For each figure, the top three panels correspond to a low value of β_0, while the bottom three panels correspond to a high value of β_0. The tables 5.6 through 5.8 support the information from the figures with numerical examples. Figure 5.7 considers combinations of a positive shift in the scale parameter, accompanied by a positive shift in the shape parameter. Figure 5.8 considers a positive shift in the scale parameter and a negative shift in the shape parameter. Figure 5.9 summarizes the results for combinations with negative shifts in the scale parameter and positive shifts in the shape parameter. Figure 5.10 considers combinations with a negative shift in the scale parameter and a negative shift in the shape parameter. Tables 5.6, 5.7, and 5.8 illustrate numerically the shift effects by β_0 value and for different censoring amounts.

Figure 5.7: Out-of-Control Performance of the Simultaneous Chart to Detect an Increase in the Scale Parameter and an Increase in the Shape Parameter for $\beta_0 = 0.5$ and $\beta_0 = 5$.



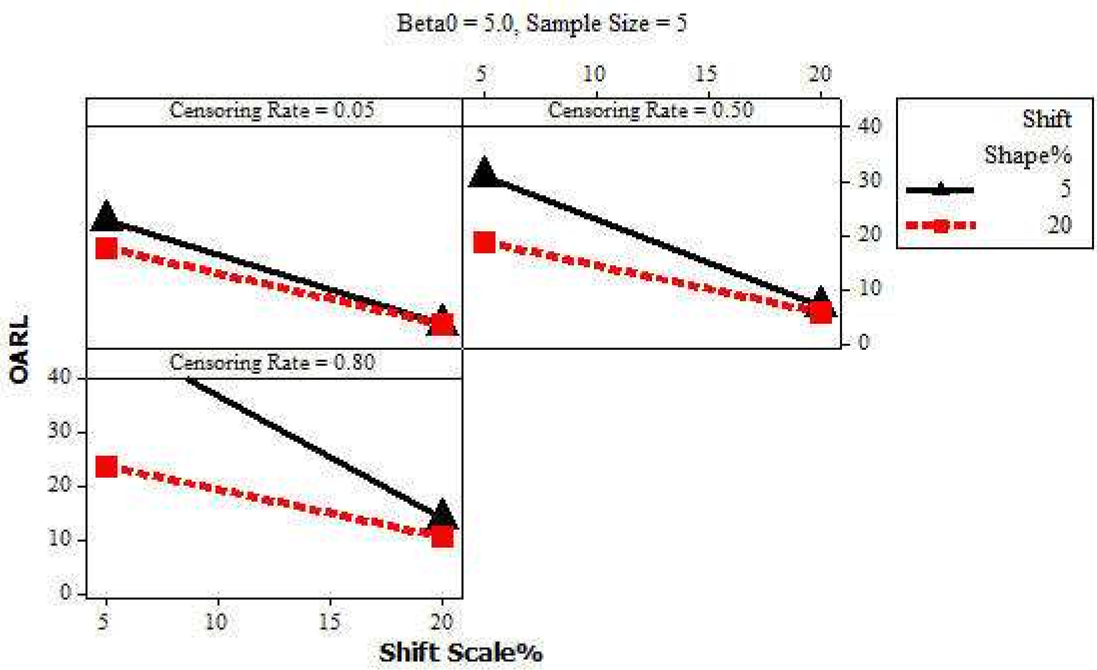
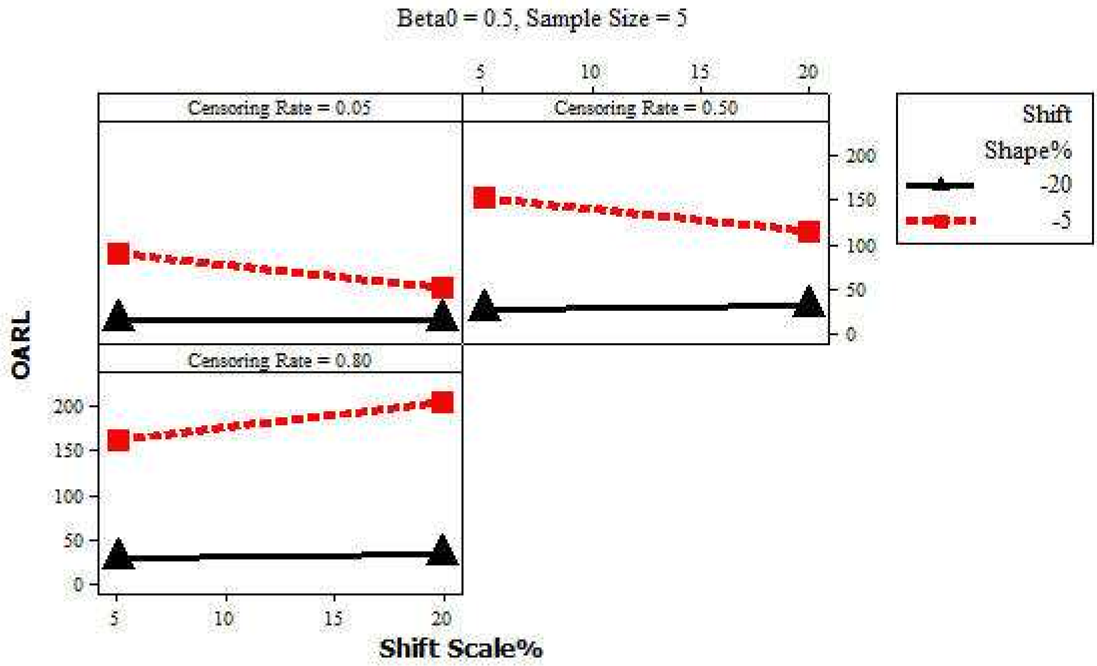


Figure 5.8: Out-of-Control Performance of the Simultaneous Chart to Detect an Increase in the Scale Parameter and a Decrease in the Shape Parameter for $\beta_0 = 0.5$ and $\beta_0 = 5$.



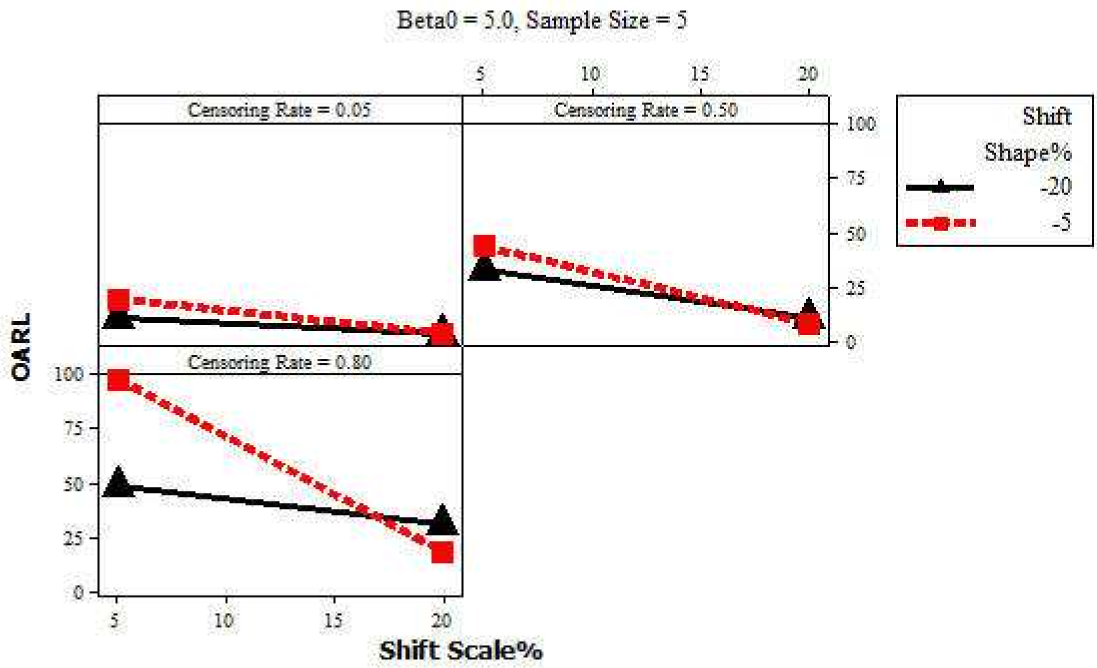
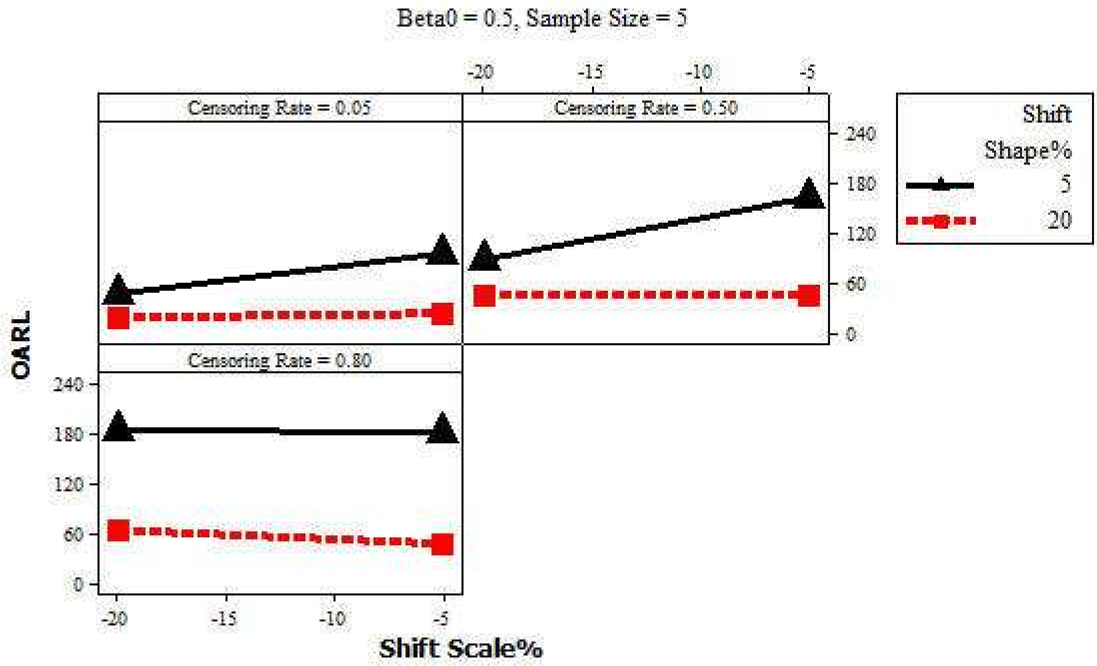


Figure 5.9: Out-of-Control Performance of the Simultaneous Chart to Detect a Decrease in the Scale Parameter and an Increase in the Shape Parameter for $\beta_0 = 0.5$ and $\beta_0 = 5$.



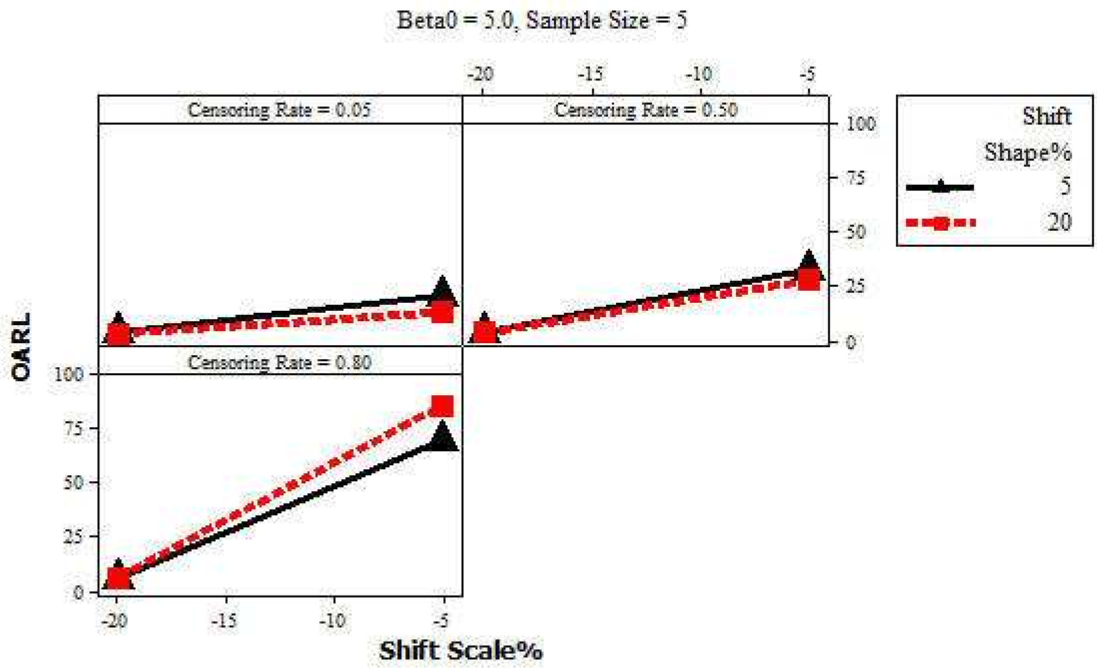
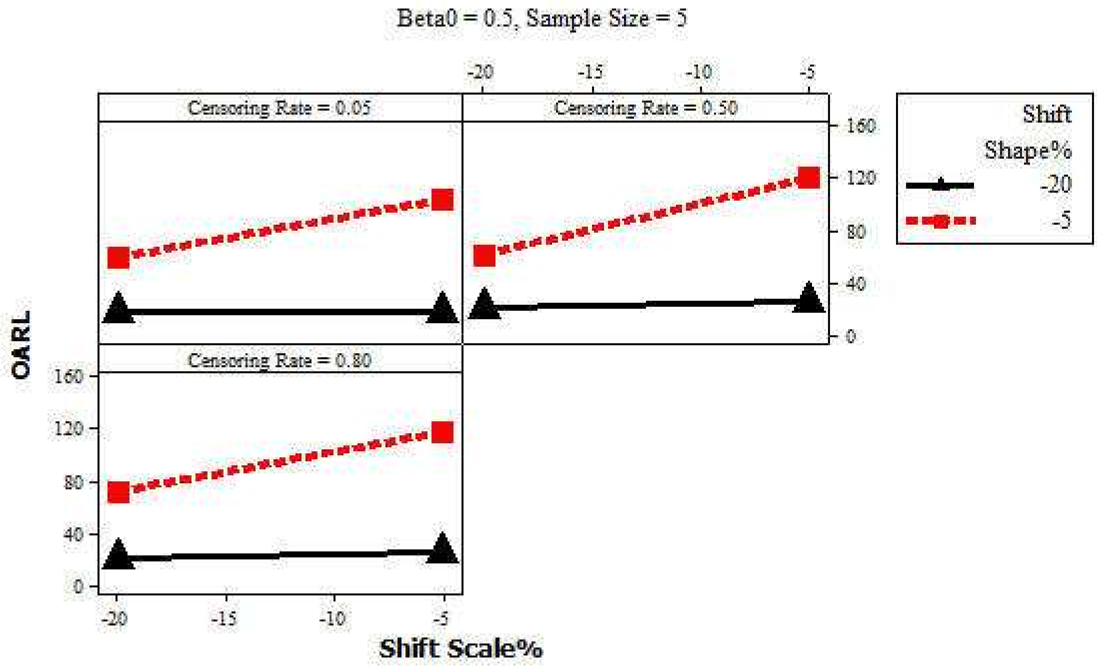


Figure 5.10: Out-of-Control Performance of the Simultaneous Chart to Detect a Decrease in the Scale Parameter and a Decrease in the Shape Parameter for $\beta_0 = 0.5$ and $\beta_0 = 5$.



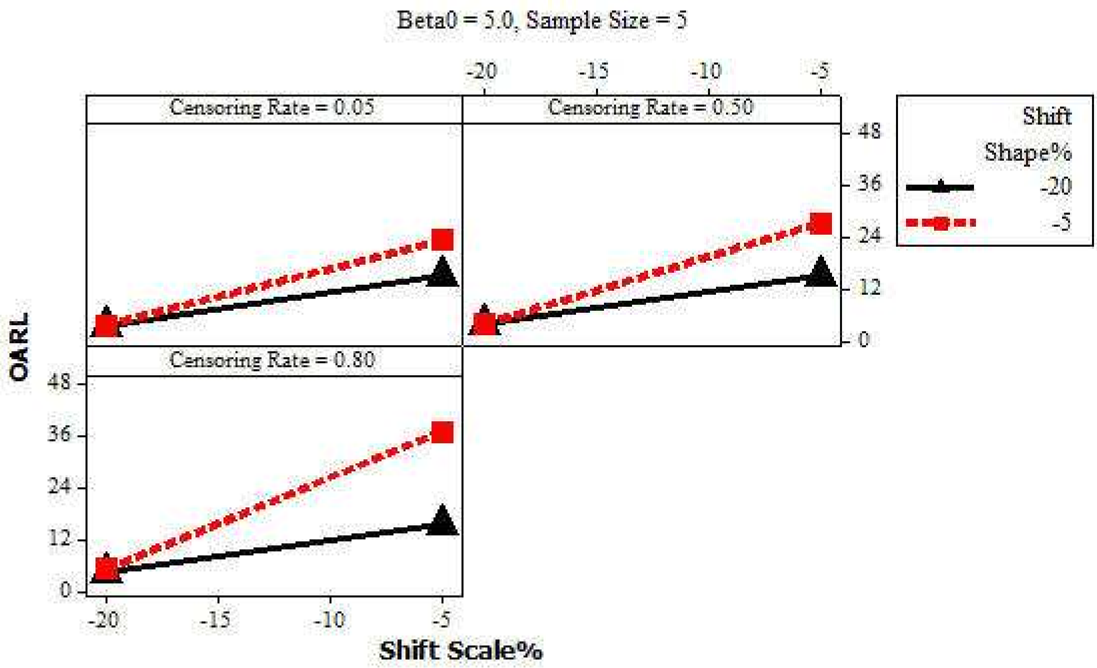


Table 5.6: Out-of-control performance of the simultaneous CUSUM chart for $\beta_0 = 0.5$ and $\beta_0 = 5$. Low censoring rate of 5%. Sample size $n = 5$.

%Scale	%Shape	OARL $\beta_0 = 0.5$	OARL $\beta_0 = 5$
−20	−20	18.106	3.8719
20	−20	15.909	3.9362
−5	−20	18.190	14.9775
5	−20	17.293	11.3504
−20	−5	59.468	3.7673
20	−5	51.932	3.9410
−5	−5	103.643	23.3314
5	−5	89.624	19.5986
−20	5	47.868	3.6098
20	5	70.390	3.8533
−5	5	95.610	20.2309
5	5	113.948	22.7200
−20	20	19.831	3.3847
20	20	26.194	3.6654
−5	20	24.647	13.0233
5	20	26.661	17.6769

Table 5.7: Out-of-control performance of the simultaneous CUSUM chart for $\beta_0 = 0.5$ and $\beta_0 = 5$. Moderate censoring rate of 50%. Sample size $n = 5$.

%Scale	%Shape	OARL $\beta_0 = 0.5$	OARL $\beta_0 = 5$
-20	-20	21.360	4.0075
20	-20	31.825	10.7782
-5	-20	26.258	15.2991
5	-20	28.467	32.7919
-20	-5	61.573	4.0291
20	-5	115.150	8.2809
-5	-5	119.664	27.2733
5	-5	151.754	43.7410
-20	5	88.273	3.8877
20	5	73.091	7.1408
-5	5	164.330	32.1944
5	5	127.676	30.7676
-20	20	46.050	3.6498
20	20	31.597	5.8151
-5	20	46.414	27.6787
5	20	40.226	18.8139

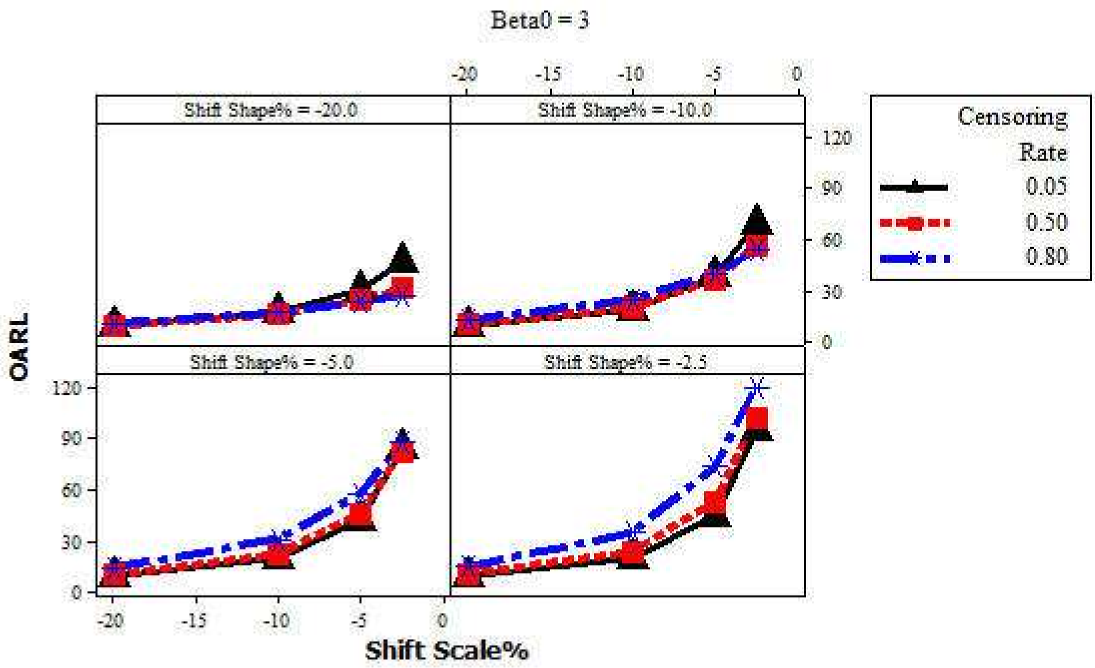
Table 5.8: Out-of-control performance of the simultaneous CUSUM chart for $\beta_0 = 0.5$ and $\beta_0 = 5$. High censoring rate of 80%. Sample size $n = 5$.

%Scale	%Shape	OARL $\beta_0 = 0.5$	OARL $\beta_0 = 5$
-20	-20	21.681	4.7379
20	-20	34.567	31.7944
-5	-20	26.432	16.0090
5	-20	29.516	48.5238
-20	-5	72.582	5.3975
20	-5	205.635	18.6344
-5	-5	118.415	37.2494
5	-5	163.374	98.3245
-20	5	187.385	5.8330
20	5	87.966	14.1085
-5	5	185.470	70.5211
5	5	133.308	47.9661
-20	20	65.957	6.2912
20	20	34.181	10.7123
-5	20	49.674	86.3485
5	20	42.248	23.8755

In these figures and tables we examine the out-of-control performance in low, moderate and high censoring scenarios and for different in-control failure mechanisms (modeled by β_0). Since we have seen that the sample size is not one of the major drivers of the out-of-control performance, we detail these behaviors for the intermediate sample size $n = 5$. We notice that a 20% shift in the shape parameter, either positive or negative, in combination with any of the considered shifts in the scale parameter, result in the best chart performance, with any censoring rate. Here $\beta_0 = 0.5$. It appears that the magnitude of the shift in the shape parameter is an important performance driver. On the other hand, a 5% positive or negative shift in shape results in considerably poorer performance in the same scenario. A large in-control β value $\beta_0 = 5$ mitigates the impact of the combination of shifts sizes and directions, especially for low and moderate censoring rates. The chart's out-of-control performance is better for higher values of β_0. There is a significant interaction effect between the in-control β values, the censoring rates and the shift size and direction combinations. For example, from Table 5.8, when we expect about 80% data to be censored, the chart signals on average in 187 samples a simultaneous 20% negative shift in the scale parameter and 5% positive shift in the shape parameter when the in-control β is $\beta_0 = 0.5$. On the other hand, the chart signals on average in 5 samples in the same scenario when $\beta_0 = 5$.

So far we have studied the performance of the simultaneous CUSUM charts in the out-of-control scenarios for which they were designed, evaluating the impact of different factors on the out-of-control performance. However, in most circumstances, the practitioner does not know what size of shift one should expect. Therefore, it is of interest to evaluate the performance of a CUSUM chart designed for certain minimum shifts when the actual shifts that occur are of different magnitudes. First, we evaluate the performance of the simultaneous CUSUM chart designed to detect a minimum 5% decrease in the shape parameter and a minimum 5% decrease in the scale parameter in low, moderate and high censoring rates scenarios. We assume a moderate sample size of $n = 5$. The charts were designed to achieve a desired in-control average run length of 370.

Figure 5.11: Out-of-Control Performance of the Simultaneous Chart, for Decreases of Various Magnitudes in Both the Scale and the Shape Parameters. Chart Designed for a 5% Decrease in the Scale Parameter and a 5% Decrease in the Shape Parameter to Achieve 370 In-Control ARL.



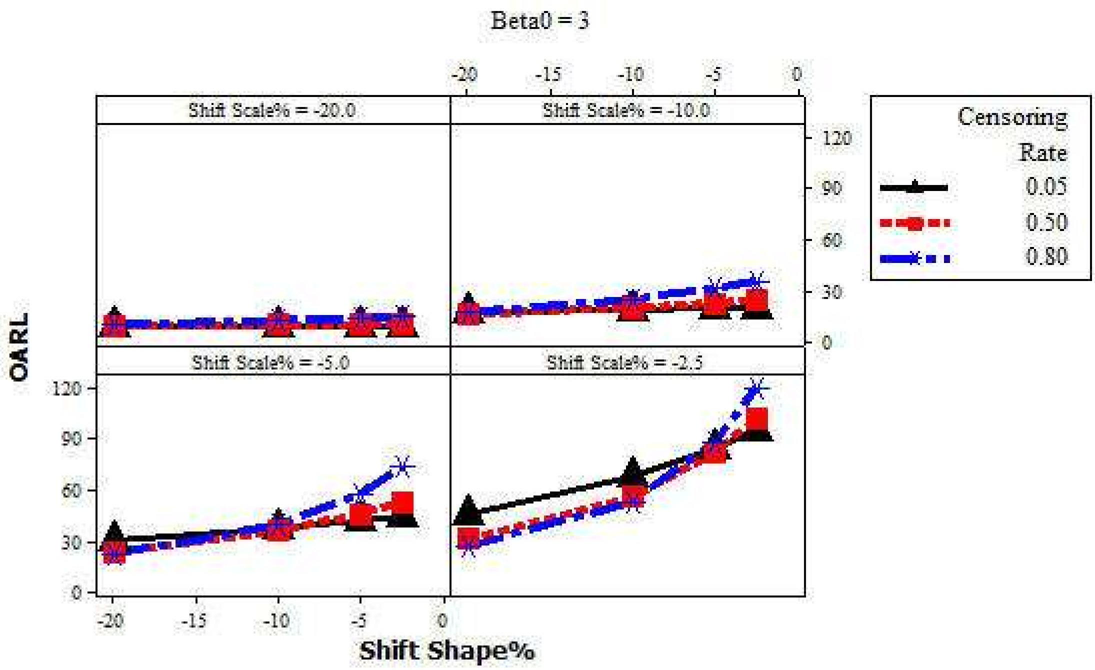
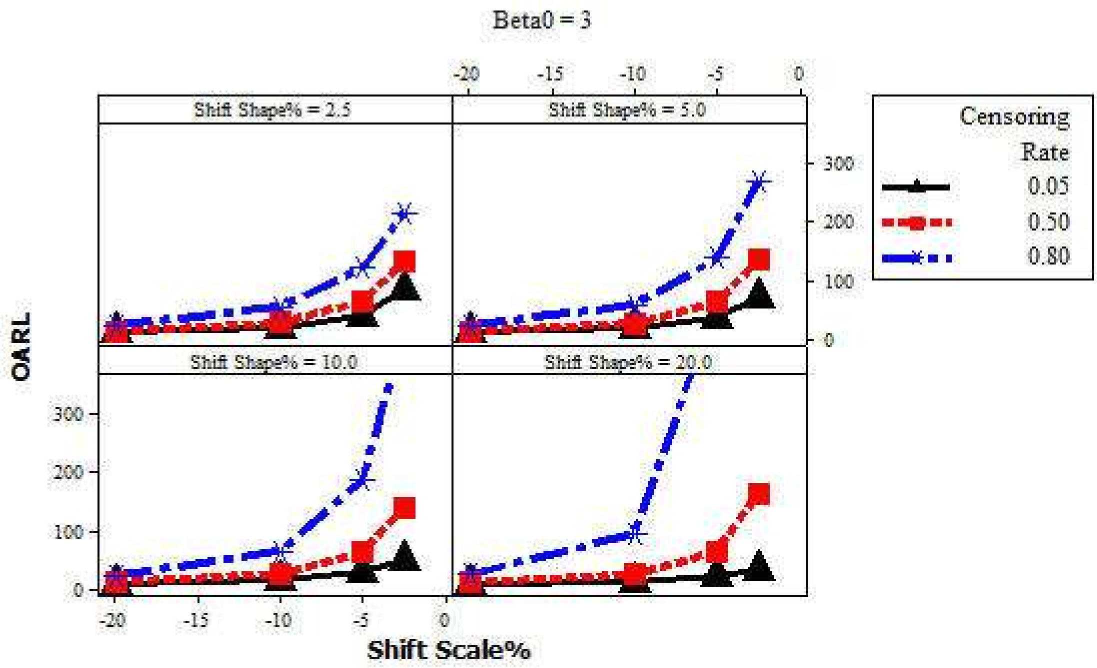


Figure 5.11 displays the variation of out-of-control average run lengths for different combinations of decreases in the scale and the shape parameter, for low, moderate and high censoring proportions. We exemplify the behavior for the scenario with $\beta_0 = 3$, specific to the normal wear-out failure mechanism, and $n = 5$. Each panel from the top chart displays the out-of-control average run lengths for a certain decrease in the shape parameter (for example 20% in the first panel) and various decreases in the scale parameter (2.5%, 5%, 10%, and 20%). Each panel from the bottom chart displays the out-of-control average run lengths for a certain decrease in the scale parameter (for example 20% in the first panel) and various decreases in the shape parameter (2.5%, 5%, 10%, and 20%). We can see in the top chart, as expected, that the chart's performance deteriorates the smaller the magnitude of the decrease in scale becomes. A larger magnitude of the shift in shape mitigates this effect (panel 1 from the top chart). The censoring rates play a more important role in the fourth panel, when the decrease in the shape parameter is very small. From the bottom chart we can see that the out-of-control performance of the chart is relatively insensitive to the magnitude of the decrease in the shape parameter, in all panels, except perhaps when the decrease in the scale parameter is very small.

Figure 5.12: Out-of-Control Performance of the Simultaneous Chart, for Decreases of Various Magnitudes in the Scale Parameter and Increases in the Shape Parameter. Chart Designed for a 5% Decrease in the Scale Parameter and a 5% Increase in the Shape Parameter to Achieve 370 In-Control ARL.



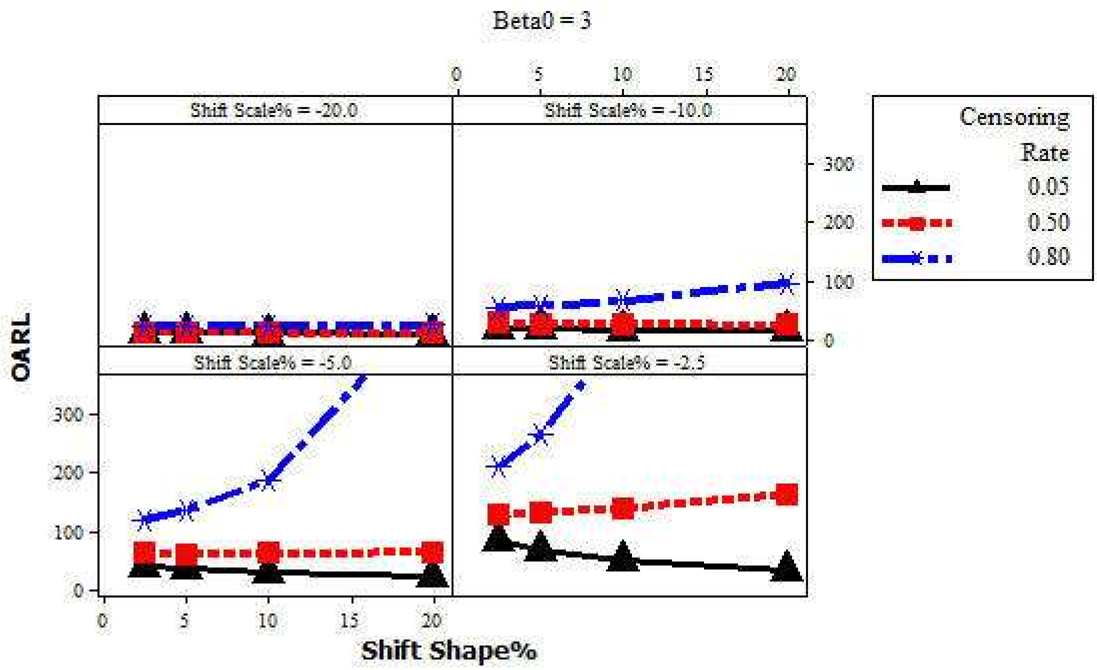


Figure 5.12 displays the variation of out-of-control average run lengths for different combinations of decreases in the scale and the shape parameter, for low, moderate and high censoring proportions. In this case we assumed that the characteristic life of the product deteriorated, while the shape parameter increased, but we do not know the magnitude of the shifts. We assumed that the simultaneous CUSUM chart was designed to detect a 5% decrease in the scale parameter, together with a 5% increase in the shape parameter. Each panel from the top chart displays the out-of-control average run lengths for a certain increase in the shape parameter (for example 2.5% in the first panel) and various decreases in the scale parameter (2.5%, 5%, 10%, and 20%, respectively). Each panel from the bottom chart displays the out-of-control average run lengths for a certain negative shift in the scale parameter (for example 20% in the first panel) and various positive shifts in the shape parameter (2.5%, 5%, 10%, and 20%). We can see in both charts that a small decrease in the scale parameter, combined with a large increase in the shape parameter, result in serious deterioration of the chart's performance, especially when we have large amounts of censoring. The chart's performance is best when a large decrease in the scale parameter occurs simultaneously with an increase in the shape parameter. The magnitude of the shift in the scale parameter appears to drive the out-of-control performance of the chart.

5.5 Recommended Use of the Three Designed CUSUM Charts

We stated in the introduction of this chapter that our goal is to detect any potentially harmful change in a process. The reliability tests are assumed to generate right-censored data following a Weibull distribution with scale parameter η and shape parameter β. For that purpose, we have constructed three likelihood ratio based CUSUM charts with different monitoring objectives: a CUSUM chart monitoring for shifts in the characteristic life of products while the failure mechanism is set (the CUSUM chart for scale designed in Chapter

4), a CUSUM chart that detects changes in the failure mechanism, while the characteristic life does not change (the CUSUM chart for detecting a shift in the shape parameter), and a simultaneous CUSUM chart that targets changes in both the failure mechanism and the characteristic life (the simultaneous CUSUM chart for detecting shifts in the shape and the scale parameters). We recommend that the practitioner should use any insight into the process they might have from historical data at the inception of the monitoring to select the proper chart. It is highly desirable that the practitioner uses the correct chart, i.e., one should only use the simultaneous chart if one has a reasonable belief that both parameters are expected to change in the specified ways.

The papers in the literature usually only consider monitoring the mean of the process, or equivalently the characteristic life (the scale parameter), assuming that the shape parameter is fixed. Nevertheless, if a different change occurs, and one only uses the chart for the scale parameter, one incurs a loss in performance. For example, if there is a 5% decrease in the scale parameter and a 20% decrease in the shape parameter, with a censoring rate of 50%, in a scenario with sample size $n = 5$ and $\beta_0 = 1$, the simultaneous chart detects such a combined shift on average in 25 samples, when the chart is designed for this scenario to achieve an in-control average run length of 370. If one ignores the possibility that both parameters can change and only monitors for changes in the characteristic life, assuming a fixed shape parameter, the corresponding individual chart designed to detect a 5% shift in the scale parameter signals on average in 134 samples. The loss in performance is significant if one does not use the appropriate chart. Similarly, if one uses only the chart for the scale parameter, one can miss changes in the shape parameter, if these occur. The chart for the shape parameter, designed for a situation with fixed scale parameter signals a 20% decrease on average in 43 samples. One such change might indicate for example the failure mode shifting from normal wear-out toward infantile mortality. The most effective use of the charts is for the scenarios for which they were designed. However one may not have complete information on what type of changes one might expect. Therefore, an interesting topic for future research is the joint use of the charts designed to achieve a certain desired in-control average run length when used jointly.

5.6 Conclusions and Recommendations

In this chapter we have studied the problem of monitoring different aspects of the Weibull processes with right-censored lifetime data. The practitioner is interested in fast detection of potentially harmful changes to the process, as well as in understanding the nature of the changes in order to take the appropriate business actions. For this purpose, we recommend the appropriate use of three charts, each of them with different monitoring objectives: the CUSUM chart for characteristic life (η) when the failure mechanism is known and fixed ($\beta = \beta_0$), which we developed and studied in Chapter 4; the CUSUM chart monitoring for changes in the failure mechanism (β), when we assume a fixed and known in-control η value; and the CUSUM chart monitoring for simultaneous shifts in both parameters of the Weibull distribution. The use of the appropriate chart is of even more importance when the resources are scarce, meaning high censoring rates and small sample sizes, and when detecting small shifts is vital to the quality control process. After studying the properties of the three developed chart, we conclude that a practitioner should benefit from using the charts monitoring separately for changes in the two parameters of the Weibull distribution jointly.

We studied the properties of the CUSUM chart for detecting shifts in the scale parameter in Chapter 4. In this chapter we designed and studied the other two charts. The CUSUM chart for the shape parameter, given a known and fixed value of the scale parameter, is invariant to the in-control values of both parameters. The simultaneous CUSUM chart displays best performance for combinations with larger positive or negative shifts in the scale parameter, signaling on average in 4 samples in an out-of-control situation, while targeting an in-control average run length of 370. The simultaneous CUSUM chart's performance is highly dependent on the values of β_0, and on the interaction between these and the censoring rates and shift sizes. High values of β_0 result in good performance of the simultaneous chart and mitigate the effect of the other performance drivers: the censoring rate, the sample size, and the types of shifts. The appropriate use of the three charts enhances the detection capabilities of the charts in different scenarios. This research introduces a more comprehensive monitoring procedure for the Weibull process with right-censored data, meant to bring insight into what type of changes can occur and what is the out-of-control performance of the monitoring procedures in each circumstance. We have also provided detailed guidelines for the practitioner to develop customized charts.

Chapter 6

Concluding Remarks

The goal of this work was to address a number of challenges in the area of monitoring reliability characteristics of lifetime data. As discussed in Chapter 1, lifetime data resulting from lifetime tests usually are censored, involve multiple items on test, and follow non-normal distributions. The Weibull distribution enjoys popularity within reliability practice, given its flexibility to model a variety of failure mechanisms. We focused on the paradigm of samples of $n \geq 1$ products on a test stand. Another very important characteristic of lifetime data that imposes additional challenges for the monitoring methods is that data are predominantly censored. In Chapter 1 we discussed different censoring patterns and we included censoring in the developed methods. In Chapter 2 we presented the existing relevant work in the area of "Quality Control" at the crossroads with "Reliability". A significant share of the previous research focused on the less realistic case of uncensored data, while in most reliability applications, and not only, censoring is a frequent characteristic. In very few cases censoring is considered in the existing literature, and in those cases the conditional expected value approach for Type I right-censoring was the most common.

In order to address these challenges, in this research we constructed likelihood-ratio based cumulative sum (CUSUM) charts developed for censored data with some non-normal distributions. The monitoring objectives can be diverse and customized for the needs of the practitioners. In the literature review we presented monitoring techniques developed for censored or uncensored lifetimes or data from other fields with similar characteristics. One can notice that the relevant literature focuses on the uncensored case, while most reliability applications involve censoring, sometimes with extreme degrees and various patterns. In Chapter 3 we illustrated a case with interval censoring. Most of the existing monitoring procedures for Weibull lifetimes focused on detecting shifts in the process mean. We have taken a comprehensive approach and allowed for changes in β, which was previously considered fixed. An advantage of using the likelihood ratio based CUSUM method is its flexibility of customization for different censoring schemes, underlying distributions, and monitoring goals, i.e., if we can formulate a likelihood, we can design a chart. For the developed methodologies, we studied their in-control and out-of-control properties extensively, through simulations. Different transformations of the data do not impact the performance of the charts, making the implementation more flexible. A very important finding of this research is that there is a high dependence of the out-of-control performance of the control charts on the in-control value of the shape parameter, β_0. This dependence was unrecognized and not studied for the methodologies existent in the literature. Chapter 4 and 5 addressed these different goals and illustrated the properties of the developed methods.

In Chapter 3 we presented the development and implementation of a CUSUM chart for censored ordinal categorical data, frequently encountered in the practice of reliability when data are collected periodically at predetermined inspection times. The goal of this chapter is to illustrate the implementation of a methodology for interval censored data. Becton Dickinson implemented this chart on similar data in a case study for the PHOENIX process, in a manufacturing setting. The pilot implementation of the chart for a few bacterial strain/drug combinations proved successful.

Chapters 4 and 5 focused on a comprehensive monitoring approach for Weibull processes with samples of $n \geq 1$ of right-censored data, designed to bring insight into what type of changes can occur and what the out-of-control performance of the monitoring procedures is in each circumstance. The practitioner is interested in fast detection of potentially harmful changes to the process, as well as in understanding the nature of the changes in order to take the appropriate business actions. For this purpose, we proposed three charts, each of them having different monitoring objectives. In Chapter 4 we developed and studied the CUSUM chart for characteristic life of a Weibull distribution, when the failure mechanism is known and fixed. In Chapter 5 we developed CUSUM charts monitoring for changes in the failure mechanism, when we assume a fixed and known in-control value of the scale parameter, and CUSUM charts monitoring for simultaneous shifts in both parameters of the Weibull distribution.

The chart developed in Chapter 4 is equivalent to a chart monitoring the mean of lifetime data when the shape parameter is fixed. An important achievement of this chapter is the extensive study of the average run length properties of the charts. We focused on the problem of decreases in the characteristic life, which signals a decrease in the process reliability and a real problem for the manufacturer. We compared the chart to the CEV EWMA chart, previously determined as superior to Shewhart-type charts, in terms of the out-of-control average run lengths, in different scenarios, and concluded that the CUSUM chart outperforms the EWMA chart in the majority of the considered cases. For high censoring rates, the difference in performance is large, especially for high β_0. The chart sensitivity to the amount of censoring was significantly reduced as compared to the CEV EWMA chart, due to the fact that the number of censored units in each sample was explicitly included in the chart's design. This is an important achievement for a reliability practitioner, since censoring is one of the main and frequent problems in such applications. Our study illustrated through extensive simulations that the out-of-control performance of both charts is highly dependent on the in-control value of the shape parameter, which was previously unrecognized in the literature. Even when one uses a pivotal quantity with a known distribution that is invariant to the in-control values of the parameters, this changes in an out-of-control state. The out-of-control performance of the charts is superior for higher values of the shape parameter. The relevant papers in the surveyed literature involved the assumption of a fixed shape parameter.

We allowed for changes in the shape parameter and studied the performance of the appropriate monitoring techniques in Chapter 5. A reliability practitioner should be aware that when one suspects that changes in both parameters of the Weibull distribution occur in specific directions, a chart targeting such a simultaneous shift displays remarkable out-of-control performance in most instances. The gain in performance over the use of a traditional chart for monitoring the mean of lifetimes while assuming a fixed shape parameter is worthwhile. In this research we considered Phase II monitoring, assuming that the in-control shape parameter was estimated at a previous stage. Future research should address the problem of estimation error in the previous phase and the impact on the charts' performance in Phase II. Furthermore, future research is needed to study the combined use of the three CUSUM charts designed to achieve a desired in-control average run length. Also, different censoring schemes should be included in the development of the charts, and their properties studied in both in-control and out-of-control scenarios.

We have studied the properties of the developed charts in a variety of scenarios to make the practitioner aware of the expected performance and drawbacks. We have also provided detailed guidelines for the practitioner to design the chart in scenarios that might be different, customized for his/her specific needs. The goal of this research was to provide the reliability practitioner with a toolbox of monitoring techniques that he/she can use in a variety of scenarios, for different monitoring goals, underlying distributions, censoring schemes, and assumptions about the in-control parameters. We also characterized the techniques' performance in different in-control and out-of-control scenarios, to provide a realistic assessment of the expected performance.

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Appendix A

Monitoring the Scale Parameter

A.1 Normal Approximation Results

Table A.1: CUSUM chart for a decrease in the mean (or equivalently in the characteristic life), using the normal and the Weibull theory. Uncensored Weibull data with $\beta = 3$ fixed.

Sample Size	Shift Size	Const k	Threshold	IARL	ISimErr	OARL	OSimErr	CUSUM Chart
3	2.5	2.66625	−8.7656	370.803	3.15347	125.208	0.80004	Normal
5	2.5	4.44375	−10.8740	372.283	3.23014	101.164	0.60385	Normal
10	2.5	8.88750	−14.4766	370.300	3.23149	71.186	0.37194	Normal
3	5.0	2.63250	−6.4453	370.479	3.37506	62.986	0.36986	Normal
5	5.0	4.38750	−7.5586	373.695	3.45687	46.869	0.25925	Normal
10	5.0	8.77500	−9.0967	372.174	3.44930	30.587	0.15246	Normal
3	10.0	2.56500	−4.2355	373.428	3.56886	26.865	0.13787	Normal
5	10.0	4.27500	−4.6826	373.864	3.57912	19.082	0.09522	Normal
10	10.0	8.55000	−5.2432	372.479	3.60537	11.709	0.05210	Normal
3	20.0	2.43000	−2.4707	374.770	3.66061	10.245	0.04195	Normal
5	20.0	4.05000	−2.5893	371.544	3.63625	7.147	0.02731	Normal
10	20.0	8.10000	−2.6797	371.999	3.71805	4.545	0.01506	Normal
3	2.5	2.88800	−20.9110	374.526	3.31900	116.554	0.80100	Weibull
5	2.5	4.81300	−24.7460	374.909	3.41600	91.226	0.60900	Weibull
10	2.5	9.62500	−30.4880	373.878	3.47900	63.606	0.40600	Weibull
3	5.0	2.77500	−15.3130	371.988	3.43900	57.976	0.34800	Weibull
5	5.0	4.62500	−17.3630	372.839	3.50800	42.105	0.24500	Weibull
10	5.0	9.25000	−20.1520	370.420	3.57900	27.163	0.15300	Weibull
3	10.0	2.55100	−9.5160	374.703	3.59900	24.258	0.12700	Weibull
5	10.0	4.25100	−10.3590	370.503	3.62300	17.125	0.08300	Weibull
10	10.0	8.50300	−11.2500	371.235	3.63400	10.429	0.04800	Weibull
3	20.0	2.10700	−4.5940	373.347	3.70400	9.300	0.03600	Weibull
5	20.0	3.51200	−4.7340	372.113	3.66900	6.549	0.02400	Weibull
10	20.0	7.02400	−4.7400	371.774	3.62100	4.141	0.01300	Weibull

A.2 Simulation Results for the CUSUM and CEV EWMA Charts for a Decrease in the Scale Parameter

Table A.2: CUSUM Chart for a Decrease in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 0.5$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.5	0.05	3	0.025	−28.4407	373.904	3.07813	289.952	2.26669
0.5	0.05	5	0.025	−35.7917	371.824	3.14844	268.346	2.08429
0.5	0.05	10	0.025	−48.8396	373.059	3.14000	243.795	1.90900
0.5	0.05	3	0.050	−26.2788	374.408	3.20357	231.072	1.76286
0.5	0.05	5	0.050	−32.5630	373.392	3.23574	208.293	1.56760
0.5	0.05	10	0.050	−42.0644	370.751	3.24504	164.956	1.23257
0.5	0.05	3	0.100	−23.3973	374.448	3.33268	154.240	1.14603
0.5	0.05	5	0.100	−28.1379	374.445	3.39542	127.943	0.95156
0.5	0.05	10	0.100	−35.2489	373.250	3.36928	94.341	0.67781
0.5	0.05	3	0.200	−17.8873	371.734	3.40440	83.017	0.57446
0.5	0.05	5	0.200	−20.4690	373.420	3.50579	62.696	0.42867
0.5	0.05	10	0.200	−24.2504	371.731	3.59481	41.790	0.27696
0.5	0.30	3	0.025	−44.1316	374.202	3.12904	297.887	2.48235
0.5	0.30	5	0.025	−55.1847	370.385	3.16617	276.586	2.33310
0.5	0.30	10	0.025	−74.0058	371.406	3.20139	255.093	2.14540
0.5	0.30	3	0.050	−39.9149	374.742	3.27054	244.055	2.06178
0.5	0.30	5	0.050	−48.2600	373.274	3.27756	218.660	1.80255
0.5	0.30	10	0.050	−62.4644	371.635	3.32191	187.686	1.55603
0.5	0.30	3	0.100	−42.1054	372.341	3.39800	143.075	1.45411
0.5	0.30	5	0.100	−49.6768	372.233	3.43588	120.579	1.22072
0.5	0.30	10	0.100	−60.2806	373.407	3.49242	91.970	0.95081
0.5	0.30	3	0.200	−31.1570	374.969	3.51003	81.939	0.82904
0.5	0.30	5	0.200	−34.9699	370.198	3.53325	64.242	0.64822
0.5	0.30	10	0.200	−39.9811	370.997	3.63875	45.097	0.45273
0.5	0.50	3	0.025	−23.6364	373.576	3.21071	307.273	2.63311
0.5	0.50	5	0.025	−29.2614	372.461	3.18999	296.678	2.59693
0.5	0.50	10	0.025	−38.7145	371.995	3.23890	265.706	2.27163
0.5	0.50	3	0.050	−20.8523	373.067	3.33760	260.646	2.27588
0.5	0.50	5	0.050	−25.0000	370.598	3.35013	233.880	2.07951
0.5	0.50	10	0.050	−31.5909	370.733	3.39093	206.480	1.81544
0.5	0.50	3	0.100	−21.4976	371.928	3.41514	164.282	1.79724
0.5	0.50	5	0.100	−24.9837	372.729	3.53629	142.720	1.55154
0.5	0.50	10	0.100	−29.7407	374.944	3.59422	113.872	1.24733
0.5	0.50	3	0.200	−15.3969	374.816	3.60191	101.605	1.12707
0.5	0.50	5	0.200	−17.0310	372.300	3.55794	83.548	0.91943
0.5	0.50	10	0.200	−19.1009	370.707	3.67234	63.383	0.70966
0.5	0.80	3	0.025	−16.7778	371.021	3.27677	317.376	2.91378
0.5	0.80	5	0.025	−20.2881	372.885	3.40760	311.271	2.93435
0.5	0.80	10	0.025	−25.9550	371.323	3.34279	289.566	2.79878
0.5	0.80	3	0.050	−14.0112	370.113	3.47762	279.785	2.70784
0.5	0.80	5	0.050	−16.4060	371.452	3.52136	271.325	2.61930
0.5	0.80	10	0.050	−19.8716	373.318	3.50286	253.207	2.48023
0.5	0.80	3	0.100	−16.6423	370.949	3.59915	171.282	2.35931
0.5	0.80	5	0.100	−18.6483	370.509	3.55280	161.310	2.27887
0.5	0.80	10	0.100	−21.2143	370.327	3.62700	146.081	2.06398
0.5	0.80	3	0.200	−11.6680	370.497	3.60094	133.922	1.87636
0.5	0.80	5	0.200	−12.5000	373.067	3.69886	123.183	1.77373
0.5	0.80	10	0.200	−13.3869	374.458	3.70291	107.395	1.54099
0.5	0.95	3	0.025	−1.7388	372.868	3.51733	330.054	3.25821
0.5	0.95	5	0.025	−2.0526	371.916	3.49337	325.263	3.23972
0.5	0.95	10	0.025	−2.5108	374.924	3.58141	324.062	3.26453
0.5	0.95	3	0.050	−1.4160	374.613	3.56995	311.248	3.24228
0.5	0.95	5	0.050	−1.6123	371.750	3.57881	309.844	3.23031
0.5	0.95	10	0.050	−1.8593	373.277	3.62926	299.948	3.17756
0.5	0.95	3	0.100	−1.7008	374.748	3.69168	217.666	3.13289
0.5	0.95	5	0.100	−1.8471	370.569	3.70185	202.591	2.93236
0.5	0.95	10	0.100	−2.0117	371.374	3.64469	200.869	2.94367
0.5	0.95	3	0.200	−1.2939	373.945	3.69079	192.026	2.81392
0.5	0.95	5	0.200	−1.3351	372.995	3.72313	184.795	2.79213
0.5	0.95	10	0.200	−1.2995	370.023	3.74013	172.606	2.49986

Table A.3: CUSUM Chart for a Decrease in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 1$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
1	0.05	3	0.025	−27.1520	373.232	3.13533	235.813	1.82233
1	0.05	5	0.025	−33.2800	371.315	3.19158	205.472	1.59120
1	0.05	10	0.025	−43.6694	372.885	3.24288	166.383	1.27309
1	0.05	3	0.050	−23.2335	372.753	3.30072	158.015	1.17736
1	0.05	5	0.050	−27.8590	372.996	3.28400	129.434	0.91900
1	0.05	10	0.050	−35.1655	374.002	3.37773	97.239	0.70086
1	0.05	3	0.100	−18.2364	370.138	3.38394	84.334	0.61023
1	0.05	5	0.100	−21.2040	373.550	3.46183	64.882	0.45338
1	0.05	10	0.100	−25.0811	372.586	3.52835	44.181	0.29522
1	0.05	3	0.200	−12.2294	370.392	3.50235	37.113	0.23338
1	0.05	5	0.200	−13.5060	370.369	3.55300	26.845	0.14700
1	0.05	10	0.200	−14.9757	370.725	3.62203	16.783	0.09921
1	0.30	3	0.025	−44.3496	373.682	3.27972	227.723	2.05894
1	0.30	5	0.025	−54.1152	373.004	3.34205	205.755	1.84499
1	0.30	10	0.025	−69.4537	372.141	3.39577	172.130	1.52937
1	0.30	3	0.050	−36.7737	371.011	3.36681	163.541	1.46933
1	0.30	5	0.050	−43.4862	372.931	3.45197	138.742	1.25753
1	0.30	10	0.050	−53.3257	371.903	3.45645	109.008	0.96209
1	0.30	3	0.100	−34.6166	371.870	3.53156	71.400	0.87674
1	0.30	5	0.100	−39.3084	374.589	3.51631	57.176	0.70473
1	0.30	10	0.100	−44.9057	374.546	3.61174	39.879	0.48331
1	0.30	3	0.200	−22.6506	373.335	3.63965	34.125	0.40691
1	0.30	5	0.200	−24.4445	372.641	3.63612	24.967	0.28979
1	0.30	10	0.200	−26.2779	373.403	3.68127	16.571	0.19327
1	0.50	3	0.025	−23.2322	374.387	3.34223	244.079	2.25608
1	0.50	5	0.025	−28.0933	374.506	3.40394	225.318	2.14249
1	0.50	10	0.025	−35.3218	372.843	3.42000	195.228	1.88405
1	0.50	3	0.050	−18.9393	371.616	3.46236	188.696	1.76169
1	0.50	5	0.050	−21.9380	374.676	3.28700	163.963	1.22900
1	0.50	10	0.050	−26.1204	374.219	3.58313	131.324	1.26535
1	0.50	3	0.100	−17.1937	372.814	3.56717	87.354	1.14969
1	0.50	5	0.100	−19.2205	370.603	3.61083	74.218	1.00087
1	0.50	10	0.100	−21.4875	371.094	3.64326	54.897	0.74323
1	0.50	3	0.200	−11.1881	373.732	3.71539	47.049	0.62886
1	0.50	5	0.200	−11.8280	372.121	3.55600	37.314	0.23200
1	0.50	10	0.200	−12.4194	373.269	3.66337	26.052	0.34796
1	0.80	3	0.025	−17.2304	370.908	3.38498	259.610	2.73732
1	0.80	5	0.025	−20.1509	370.797	3.45308	247.093	2.58101
1	0.80	10	0.025	−24.5371	372.566	3.44800	235.055	2.51800
1	0.80	3	0.050	−13.4248	371.678	3.56773	221.305	2.39392
1	0.80	5	0.050	−15.0310	372.668	3.19000	208.767	1.60900
1	0.80	10	0.050	−17.1655	370.165	3.62900	188.064	2.06100
1	0.80	3	0.100	−13.8432	372.494	3.61857	100.547	1.99524
1	0.80	5	0.100	−14.8264	372.617	3.66200	90.166	1.83500
1	0.80	10	0.100	−15.9885	372.502	3.65000	80.030	1.59300
1	0.80	3	0.200	−9.1780	373.867	3.72045	68.386	1.34095
1	0.80	5	0.200	−9.3200	370.061	3.49900	61.907	0.42800
1	0.80	10	0.200	−9.2464	372.939	3.70478	53.080	1.08199
1	0.95	3	0.025	−1.7414	372.022	3.65459	289.148	3.25101
1	0.95	5	0.025	−1.9867	374.129	3.66244	285.951	3.23676
1	0.95	10	0.025	−2.2925	373.393	3.64983	280.951	3.20809
1	0.95	3	0.050	−1.3668	370.441	3.57166	276.115	3.15017
1	0.95	5	0.050	−1.4951	370.324	3.67335	265.224	3.06961
1	0.95	10	0.050	−1.6295	373.339	3.63873	259.864	2.96339
1	0.95	3	0.100	−1.5214	373.087	3.71460	138.666	2.79288
1	0.95	5	0.100	−1.5676	371.138	3.66173	128.389	2.57192
1	0.95	10	0.100	−1.5457	373.320	3.68612	125.889	2.62150
1	0.95	3	0.200	−1.0802	370.223	3.68550	113.400	2.32124
1	0.95	5	0.200	−1.0592	371.680	3.69322	108.209	2.20835
1	0.95	10	0.200	−0.7482	374.529	3.76022	95.447	1.95497

Table A.4: CUSUM Chart for a Decrease in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 3$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
3	0.05	3	0.025	−20.8376	372.704	3.39000	118.786	0.86300
3	0.05	5	0.025	−24.4957	371.280	3.38400	91.199	0.65000
3	0.05	10	0.025	−29.7811	374.632	3.43200	64.528	0.44700
3	0.05	3	0.050	−15.2443	370.817	3.48900	58.166	0.39300
3	0.05	5	0.050	−17.2480	371.791	3.52600	43.162	0.25900
3	0.05	10	0.050	−19.7209	374.728	3.62500	27.483	0.17200
3	0.05	3	0.100	−9.2066	371.628	3.58700	24.566	0.14300
3	0.05	5	0.100	−9.9267	372.086	3.65300	17.366	0.09800
3	0.05	10	0.100	−10.7184	372.029	3.68171	10.690	0.05598
3	0.05	3	0.200	−4.6056	370.272	3.62099	9.334	0.04198
3	0.05	5	0.200	−4.7260	373.878	3.68500	6.550	0.02400
3	0.05	10	0.200	−4.6621	374.145	3.69300	4.170	0.01600
3	0.30	3	0.025	−38.6225	374.640	3.46803	102.132	1.12885
3	0.30	5	0.025	−44.7584	374.936	3.47610	85.011	0.94013
3	0.30	10	0.025	−52.6066	371.727	3.56230	60.660	0.66983
3	0.30	3	0.050	−27.4567	370.033	3.60005	54.997	0.59897
3	0.30	5	0.050	−30.2036	370.837	3.57635	42.498	0.47083
3	0.30	10	0.050	−33.7174	371.543	3.69326	28.459	0.30738
3	0.30	3	0.100	−17.0697	372.880	3.60141	18.262	0.25788
3	0.30	5	0.100	−17.9822	370.561	3.64726	13.151	0.18402
3	0.30	10	0.100	−18.6058	374.007	3.62043	8.501	0.11551
3	0.30	3	0.200	−9.0147	370.991	3.73372	6.757	0.08442
3	0.30	5	0.200	−8.9013	371.165	3.66845	4.705	0.05654
3	0.30	10	0.200	−8.5115	371.446	3.65197	3.047	0.03501
3	0.50	3	0.025	−19.4064	372.557	3.52339	122.061	1.46204
3	0.50	5	0.025	−21.9178	370.700	3.56047	101.331	1.20503
3	0.50	10	0.025	−25.4495	372.195	3.59105	79.156	0.95495
3	0.50	3	0.050	−13.5464	374.266	3.66952	72.262	0.86510
3	0.50	5	0.050	−14.6880	373.541	3.50400	58.194	0.37900
3	0.50	10	0.050	−15.9817	370.129	3.60309	42.336	0.52033
3	0.50	3	0.100	−8.4442	372.031	3.57227	26.901	0.43722
3	0.50	5	0.100	−8.7312	371.895	3.70830	20.904	0.33171
3	0.50	10	0.100	−8.8446	372.235	3.71688	14.501	0.23502
3	0.50	3	0.200	−4.5074	374.052	3.75650	10.193	0.15436
3	0.50	5	0.200	−4.4790	373.213	3.68700	7.851	0.03500
3	0.50	10	0.200	−3.6994	374.936	3.73594	5.043	0.07452
3	0.80	3	0.025	−15.2691	371.586	3.58790	135.593	2.19389
3	0.80	5	0.025	−16.7544	373.856	3.56216	125.720	1.99963
3	0.80	10	0.025	−18.5072	372.283	3.68214	113.002	1.82652
3	0.80	3	0.050	−10.6647	370.350	3.73451	101.755	1.63622
3	0.80	5	0.050	−11.1250	374.147	3.39500	91.100	0.64600
3	0.80	10	0.050	−11.5557	373.047	3.66824	78.583	1.30621
3	0.80	3	0.100	−8.9735	373.322	3.73984	32.569	1.11554
3	0.80	5	0.100	−8.9375	372.875	3.73040	28.242	0.94860
3	0.80	10	0.100	−8.1119	373.161	3.71734	23.458	0.79140
3	0.80	3	0.200	−4.8278	371.387	3.67411	15.378	0.51390
3	0.80	5	0.200	−4.0470	374.423	3.71100	12.822	0.07200
3	0.80	10	0.200	−4.3360	372.151	3.67300	8.033	0.04100
3	0.95	3	0.025	−8.8202	370.237	3.64209	186.549	2.91686
3	0.95	5	0.025	−9.3852	370.272	3.61909	179.629	2.81216
3	0.95	10	0.025	−9.8146	373.351	3.71247	178.511	2.87061
3	0.95	3	0.050	−6.7369	370.160	3.58315	160.761	2.56869
3	0.95	5	0.050	−6.7970	373.295	3.30900	158.047	1.23200
3	0.95	10	0.050	−5.9917	370.041	3.65267	144.842	2.33790
3	0.95	3	0.100	−4.3934	374.069	3.65685	68.976	1.95030
3	0.95	5	0.100	−4.0842	371.553	3.75300	66.885	1.91576
3	0.95	10	0.100	−3.7950	373.911	3.77499	53.720	1.51586
3	0.95	3	0.200	−2.7730	370.875	3.57700	40.265	0.28900
3	0.95	5	0.200	−3.1450	373.660	3.59600	29.653	0.19900
3	0.95	10	0.200	−3.6040	370.349	3.61100	19.241	0.12100

Table A.5: CUSUM Chart for a Decrease in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 5$

β_0	Censoring Proportion	Sample Size	Shift Size	k	Threshold	IARL	ISimErr	OARL	OSimErr
5	0.05	3	0.025	−16.8403	370.769	3.41207	71.795	0.50355	
5	0.05	5	0.025	−19.0606	370.297	3.49960	52.603	0.35854	
5	0.05	10	0.025	−22.3837	372.774	3.56362	35.360	0.22974	
5	0.05	3	0.050	−11.1537	370.017	3.56656	31.381	0.19348	
5	0.05	5	0.050	−12.1850	373.607	3.60400	22.663	0.13551	
5	0.05	10	0.050	−13.3691	374.497	3.65772	13.983	0.07885	
5	0.05	3	0.100	−5.7127	374.055	3.63310	12.628	0.06230	
5	0.05	5	0.100	−5.9472	373.879	3.63847	8.884	0.04181	
5	0.05	10	0.100	−6.0622	370.688	3.68702	5.499	0.02326	
5	0.05	3	0.200	−2.2700	372.244	3.71134	5.070	0.01743	
5	0.05	5	0.200	−2.1970	374.479	3.66461	3.682	0.01115	
5	0.05	10	0.200	−2.0532	371.063	3.63194	2.486	0.00680	
5	0.30	3	0.025	−31.9115	372.973	3.52707	57.791	0.72632	
5	0.30	5	0.025	−35.6612	374.139	3.63100	45.849	0.57120	
5	0.30	10	0.025	−40.2085	374.385	3.65641	31.269	0.38438	
5	0.30	3	0.100	−11.1223	370.716	3.59834	10.948	0.12000	
5	0.30	5	0.100	−11.2348	370.859	3.64893	7.714	0.08562	
5	0.30	10	0.100	−11.1690	374.464	3.76235	5.068	0.05328	
5	0.30	3	0.200	−4.7947	370.450	3.69949	3.808	0.03157	
5	0.30	5	0.200	−4.8228	370.409	3.67402	2.849	0.02277	
5	0.30	10	0.200	−2.6357	373.122	3.75797	1.761	0.01075	
5	0.50	3	0.025	−15.7771	373.762	3.62000	75.009	1.02000	
5	0.50	5	0.025	−17.4211	371.653	3.62600	60.883	0.83800	
5	0.50	10	0.025	−19.2337	372.010	3.68700	44.937	0.61700	
5	0.50	3	0.050	−10.3712	373.984	3.64083	40.062	0.54468	
5	0.50	5	0.050	−10.8200	371.373	3.59124	30.816	0.42562	
5	0.50	10	0.050	−11.2189	373.175	3.68318	21.591	0.29665	
5	0.50	3	0.100	−5.5511	372.969	3.68800	17.052	0.22700	
5	0.50	5	0.100	−5.6085	372.014	3.73300	12.916	0.17100	
5	0.50	10	0.100	−5.1874	374.582	3.73100	8.749	0.11600	
5	0.50	3	0.200	−2.7277	370.425	3.64500	5.929	0.06800	
5	0.50	5	0.200	−2.1630	373.396	3.71300	3.963	0.04400	
5	0.50	10	0.200	−2.0960	370.463	3.65500	2.691	0.00800	
5	0.80	3	0.025	−12.8402	373.735	3.65489	84.940	1.81784	
5	0.80	5	0.025	−13.5463	371.203	3.70318	76.946	1.69883	
5	0.80	10	0.025	−14.3810	374.512	3.64150	66.690	1.47561	
5	0.80	3	0.050	−8.6122	372.611	3.72700	59.318	1.31000	
5	0.80	5	0.050	−8.6560	371.153	3.77100	52.182	1.14100	
5	0.80	10	0.050	−8.3711	372.743	3.65300	43.731	0.96200	
5	0.80	3	0.100	−5.7708	371.341	3.77900	24.758	0.68500	
5	0.80	5	0.100	−5.4432	372.120	3.71000	22.355	0.60900	
5	0.80	10	0.100	−2.8476	371.090	3.62900	14.396	0.38700	
5	0.80	3	0.200	−2.0160	371.127	3.64000	7.858	0.03900	
5	0.80	5	0.200	−2.0860	374.605	3.70000	5.606	0.02500	
5	0.80	10	0.200	−2.0214	373.000	3.50000	3.807	0.02000	
5	0.95	3	0.025	−8.4282	370.053	3.61071	136.088	2.52677	
5	0.95	5	0.025	−8.6036	370.278	3.70173	128.721	2.37842	
5	0.95	10	0.025	−8.0286	370.703	3.65367	119.630	2.21802	
5	0.95	3	0.050	−5.9018	370.610	3.64769	109.540	2.13221	
5	0.95	5	0.050	−5.6860	372.336	3.75181	100.991	0.76300	
5	0.95	10	0.050	−3.4010	374.656	3.69481	90.982	1.84297	
5	0.95	3	0.100	−5.5769	370.956	3.63579	45.943	1.31312	
5	0.95	5	0.100	−3.7930	371.199	3.69814	43.048	0.29700	
5	0.95	10	0.100	−2.2687	371.500	3.68000	38.782	0.29000	
5	0.95	3	0.200	−1.6730	373.970	3.65400	17.285	0.11300	
5	0.95	5	0.200	−1.8280	372.802	3.64800	12.509	0.07500	
5	0.95	10	0.200	−1.0934	370.463	3.65500	11.269	0.07000	

Table A.6: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.05$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 0.5$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.5	0.05	3	0.025	0.74309	370.09	3.6460	310.10	3.0030
0.5	0.05	5	0.025	0.81136	374.19	3.7324	320.84	3.1845
0.5	0.05	10	0.025	0.88506	372.70	3.6795	313.10	3.0657
0.5	0.05	3	0.050	0.74309	374.87	3.6588	268.73	2.5932
0.5	0.05	5	0.050	0.81139	374.70	3.6739	271.35	2.6260
0.5	0.05	10	0.050	0.88555	374.94	3.7410	244.99	2.4215
0.5	0.05	3	0.100	0.74219	372.67	3.6384	210.73	2.0129
0.5	0.05	5	0.100	0.81176	370.63	3.6384	194.53	1.8530
0.5	0.05	10	0.100	0.88530	374.11	3.6950	168.95	1.6464
0.5	0.05	3	0.200	0.74219	371.47	3.6686	123.44	1.1502
0.5	0.05	5	0.200	0.81172	370.25	3.6358	102.69	0.9755
0.5	0.05	10	0.200	0.88545	371.68	3.6362	74.18	0.7033
0.5	0.30	3	0.025	0.74261	371.78	3.6300	318.11	3.0819
0.5	0.30	5	0.025	0.80326	370.86	3.5985	309.79	3.0237
0.5	0.30	10	0.025	0.86719	372.41	3.6229	275.47	2.6390
0.5	0.30	3	0.050	0.74189	374.74	3.6070	283.57	2.7376
0.5	0.30	5	0.050	0.80469	370.86	3.5985	255.21	2.4776
0.5	0.30	10	0.050	0.86586	373.69	3.6458	229.60	2.2581
0.5	0.30	3	0.100	0.74180	373.61	3.6230	213.34	2.0322
0.5	0.30	5	0.100	0.80469	373.69	3.6426	176.95	1.6946
0.5	0.30	10	0.100	0.86719	372.34	3.7026	137.38	1.2801
0.5	0.30	3	0.200	0.70000	373.61	3.6230	121.60	1.1000
0.5	0.30	5	0.200	0.80000	373.69	3.6426	92.30	0.8000
0.5	0.30	10	0.200	0.90000	372.34	3.7026	60.00	0.5000
0.5	0.50	3	0.025	0.72109	372.03	3.5660	316.03	3.0120
0.5	0.50	5	0.025	0.77109	370.85	3.5235	302.88	2.8725
0.5	0.50	10	0.025	0.82148	374.42	3.6112	283.81	2.6814
0.5	0.50	3	0.050	0.72026	371.56	3.5858	279.65	2.6874
0.5	0.50	5	0.050	0.77109	374.55	3.5821	250.70	2.3550
0.5	0.50	10	0.050	0.82168	370.25	3.5592	219.02	2.0195
0.5	0.50	3	0.100	0.72109	370.83	3.5473	205.52	1.9146
0.5	0.50	5	0.100	0.77148	370.51	3.5631	177.38	1.6127
0.5	0.50	10	0.100	0.82148	372.34	3.5881	133.75	1.1833
0.5	0.50	3	0.200	0.72090	370.01	3.5993	121.17	1.0912
0.5	0.50	5	0.200	0.77109	371.35	3.5511	91.34	0.7880
0.5	0.50	10	0.200	0.82168	371.02	3.5546	59.33	0.4519
0.5	0.80	3	0.025	0.68612	372.20	3.6000	321.05	2.9785
0.5	0.80	5	0.025	0.72305	372.27	3.5209	316.32	2.9279
0.5	0.80	10	0.025	0.75928	370.94	3.4157	297.54	2.7131
0.5	0.80	3	0.050	0.68612	372.20	3.6000	285.65	2.6259
0.5	0.80	5	0.050	0.72269	374.24	3.5443	270.24	2.4242
0.5	0.80	10	0.050	0.75928	374.06	3.4636	239.16	2.1086
0.5	0.80	3	0.100	0.70000	372.20	3.6000	224.70	2.0000
0.5	0.80	5	0.100	0.72305	374.75	3.4648	197.22	1.7585
0.5	0.80	10	0.100	0.75945	370.70	3.4383	158.83	1.3219
0.5	0.80	3	0.200	0.68612	372.20	3.6000	137.79	1.1950
0.5	0.80	5	0.200	0.72270	374.87	3.5046	111.85	0.9017
0.5	0.80	10	0.200	0.75930	371.12	3.4514	80.02	0.5701
0.5	0.95	3	0.025	0.73359	373.93	3.5546	345.99	3.2733
0.5	0.95	5	0.025	0.76309	372.60	3.4580	329.45	3.1227
0.5	0.95	10	0.025	0.79299	370.62	3.4099	312.19	2.8419
0.5	0.95	3	0.050	0.73398	370.02	3.5487	299.84	2.8078
0.5	0.95	5	0.050	0.76309	372.43	3.4579	291.47	2.6513
0.5	0.95	10	0.050	0.79336	371.12	3.4380	258.32	2.3114
0.5	0.95	3	0.100	0.73398	370.73	3.5202	248.69	2.3073
0.5	0.95	5	0.100	0.76309	372.55	3.5123	225.82	2.0354
0.5	0.95	10	0.100	0.79336	371.42	3.3756	190.13	1.6699
0.5	0.95	3	0.200	0.73340	373.43	3.5136	173.55	1.5255
0.5	0.95	5	0.200	0.76279	372.66	3.4993	144.83	1.2307
0.5	0.95	10	0.200	0.79285	370.32	3.4675	109.52	0.8466

Table A.7: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.05$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 1$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
1.0	0.05	3	0.025	0.70457	374.99	3.6318	271.91	2.5803
1.0	0.05	5	0.025	0.76411	370.18	3.5880	242.17	2.3148
1.0	0.05	10	0.025	0.82827	372.58	3.6546	210.76	1.9902
1.0	0.05	3	0.050	0.70421	370.49	3.6167	200.55	1.8822
1.0	0.05	5	0.050	0.76606	370.22	3.5901	158.22	1.5094
1.0	0.05	10	0.050	0.82969	370.03	3.5858	116.56	1.0472
1.0	0.05	3	0.100	0.70547	371.04	3.5696	108.66	0.9531
1.0	0.05	5	0.100	0.76484	370.30	3.6053	79.14	0.6817
1.0	0.05	10	0.100	0.82871	372.01	3.6307	50.55	0.4033
1.0	0.05	3	0.200	0.70405	372.27	3.6155	42.73	0.3136
1.0	0.05	5	0.200	0.76375	370.87	3.5891	28.90	0.1886
1.0	0.05	10	0.200	0.82969	372.46	3.6407	17.14	0.0865
1.0	0.30	3	0.025	0.72730	371.28	3.6213	274.31	2.6516
1.0	0.30	5	0.025	0.78672	371.21	3.6588	254.14	2.4483
1.0	0.30	10	0.025	0.84678	373.01	3.6579	219.97	2.0756
1.0	0.30	3	0.050	0.72658	372.94	3.6500	212.18	1.9998
1.0	0.30	5	0.050	0.78672	371.21	3.6588	175.22	1.6406
1.0	0.30	10	0.050	0.84844	370.19	3.5512	132.23	1.2162
1.0	0.30	3	0.100	0.72658	372.94	3.6500	121.51	1.0974
1.0	0.30	5	0.100	0.78672	371.21	3.6588	93.42	0.8348
1.0	0.30	10	0.100	0.84844	370.19	3.5512	59.14	0.4930
1.0	0.30	3	0.200	0.72658	372.94	3.6500	48.83	0.3910
1.0	0.30	5	0.200	0.78672	371.21	3.6588	33.45	0.2385
1.0	0.30	10	0.200	0.84844	370.19	3.5512	19.65	0.1153
1.0	0.50	3	0.025	0.75889	371.34	3.5811	288.11	2.7670
1.0	0.50	5	0.025	0.81231	371.43	3.5954	275.37	2.6732
1.0	0.50	10	0.025	0.86797	372.05	3.6006	232.81	2.2196
1.0	0.50	3	0.050	0.75889	371.34	3.5811	226.35	2.1195
1.0	0.50	5	0.050	0.81231	371.43	3.5954	203.04	1.9259
1.0	0.50	10	0.050	0.86797	372.05	3.6006	154.55	1.4376
1.0	0.50	3	0.100	0.75889	371.34	3.5811	140.02	1.3074
1.0	0.50	5	0.100	0.81231	371.43	3.5954	110.89	1.0142
1.0	0.50	10	0.100	0.86797	372.05	3.6006	73.50	0.6296
1.0	0.50	3	0.200	0.75898	372.70	3.6258	59.51	0.5015
1.0	0.50	5	0.200	0.81245	370.22	3.5518	40.92	0.3161
1.0	0.50	10	0.200	0.86733	373.18	3.6125	24.31	0.1576
1.0	0.80	3	0.025	0.83594	373.03	3.6423	297.80	2.8678
1.0	0.80	5	0.025	0.87295	370.99	3.6459	300.67	2.9334
1.0	0.80	10	0.025	0.91201	373.97	3.6373	274.93	2.6566
1.0	0.80	3	0.050	0.83433	372.13	3.6328	261.18	2.5517
1.0	0.80	5	0.050	0.87280	373.27	3.6049	246.06	2.3992
1.0	0.80	10	0.050	0.91179	373.99	3.6124	210.16	2.0074
1.0	0.80	3	0.100	0.83537	370.52	3.6390	184.08	1.7600
1.0	0.80	5	0.100	0.87302	370.46	3.6431	159.41	1.5235
1.0	0.80	10	0.100	0.91187	372.58	3.6373	120.35	1.0919
1.0	0.80	3	0.200	0.83537	370.56	3.5799	97.40	0.8895
1.0	0.80	5	0.200	0.87330	371.31	3.6416	71.93	0.6417
1.0	0.80	10	0.200	0.91211	373.88	3.6741	45.62	0.3701
1.0	0.95	3	0.025	0.90399	372.16	3.6803	350.60	3.4673
1.0	0.95	5	0.025	0.92891	370.02	3.7024	335.90	3.2965
1.0	0.95	10	0.025	0.95254	374.66	3.6886	315.84	3.0775
1.0	0.95	3	0.050	0.90399	372.16	3.6803	312.69	3.0467
1.0	0.95	5	0.050	0.92891	370.02	3.7024	292.52	2.9047
1.0	0.95	10	0.050	0.95254	374.66	3.6886	265.62	2.5880
1.0	0.95	3	0.100	0.90399	372.16	3.6803	258.21	2.4490
1.0	0.95	5	0.100	0.92891	370.02	3.7024	236.12	2.2409
1.0	0.95	10	0.100	0.95254	374.66	3.6886	193.45	1.8889
1.0	0.95	3	0.200	0.90399	372.16	3.6803	171.24	1.6393
1.0	0.95	5	0.200	0.92891	370.02	3.7024	141.29	1.3433
1.0	0.95	10	0.200	0.95254	374.66	3.6886	103.21	0.9560

Table A.8: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.05$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 3$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
3.0	0.05	3	0.025	0.70469	372.89	3.5666	142.00	1.2989
3.0	0.05	5	0.025	0.76251	370.68	3.6130	114.42	1.0214
3.0	0.05	10	0.025	0.82656	370.00	3.6076	77.54	0.6650
3.0	0.05	3	0.050	0.70469	372.89	3.5666	68.53	0.5673
3.0	0.05	5	0.050	0.76251	370.68	3.6130	48.19	0.3774
3.0	0.05	10	0.050	0.82656	370.00	3.6076	28.48	0.1906
3.0	0.05	3	0.100	0.70469	372.89	3.5666	24.87	0.1431
3.0	0.05	5	0.100	0.76251	370.68	3.6130	17.71	0.0870
3.0	0.05	10	0.100	0.82656	370.00	3.6076	11.30	0.0408
3.0	0.05	3	0.200	0.70469	372.89	3.5666	10.54	0.0283
3.0	0.05	5	0.200	0.76251	370.68	3.6130	8.12	0.0173
3.0	0.05	10	0.200	0.82656	370.00	3.6076	5.87	0.0093
3.0	0.30	3	0.025	0.68306	373.74	3.5714	145.79	1.3064
3.0	0.30	5	0.025	0.73359	370.63	3.5701	112.91	0.9592
3.0	0.30	10	0.025	0.78272	373.37	3.5411	80.24	0.6165
3.0	0.30	3	0.050	0.68306	373.74	3.5714	71.05	0.5680
3.0	0.30	5	0.050	0.73359	370.63	3.5701	50.38	0.3652
3.0	0.30	10	0.050	0.78272	373.37	3.5411	33.20	0.1930
3.0	0.30	3	0.100	0.68306	373.74	3.5714	27.69	0.1550
3.0	0.30	5	0.100	0.73359	370.63	3.5701	19.79	0.0889
3.0	0.30	10	0.100	0.78272	373.37	3.5411	14.28	0.0457
3.0	0.30	3	0.200	0.68306	373.74	3.5714	11.59	0.0310
3.0	0.30	5	0.200	0.73359	370.63	3.5701	9.29	0.0196
3.0	0.30	10	0.200	0.78272	373.37	3.5411	7.32	0.0112
3.0	0.50	3	0.025	0.87116	374.33	3.6979	241.36	2.4387
3.0	0.50	5	0.025	0.91613	373.12	3.7291	233.36	2.2992
3.0	0.50	10	0.025	0.95883	371.30	3.7100	179.43	1.7299
3.0	0.50	3	0.050	0.87116	374.33	3.6979	162.03	1.5666
3.0	0.50	5	0.050	0.91613	373.12	3.7291	146.87	1.4214
3.0	0.50	10	0.050	0.95883	371.30	3.7100	107.73	1.0618
3.0	0.50	3	0.100	0.87116	374.33	3.6979	72.67	0.6758
3.0	0.50	5	0.100	0.91613	373.12	3.7291	59.81	0.5743
3.0	0.50	10	0.100	0.95883	371.30	3.7100	38.07	0.3577
3.0	0.50	3	0.200	0.87116	374.33	3.6979	18.28	0.1417
3.0	0.50	5	0.200	0.91613	373.12	3.7291	13.03	0.0993
3.0	0.50	10	0.200	0.95883	371.30	3.7100	7.57	0.0512
3.0	0.80	3	0.025	0.98202	370.07	3.7043	331.03	3.2540
3.0	0.80	5	0.025	0.99623	370.08	3.7434	323.53	3.2103
3.0	0.80	10	0.025	0.99802	373.73	3.7263	294.37	2.8922
3.0	0.80	3	0.050	0.98202	370.07	3.7043	298.24	2.9158
3.0	0.80	5	0.050	0.99623	370.08	3.7434	280.01	2.7436
3.0	0.80	10	0.050	0.99802	373.73	3.7263	251.86	2.5304
3.0	0.80	3	0.100	0.98202	370.07	3.7043	228.63	2.2522
3.0	0.80	5	0.100	0.99623	370.08	3.7434	204.00	2.0274
3.0	0.80	10	0.100	0.99802	373.73	3.7263	195.68	1.9136
3.0	0.80	3	0.200	0.98202	370.07	3.7043	137.08	1.3705
3.0	0.80	5	0.200	0.99623	370.08	3.7434	105.77	1.0287
3.0	0.80	10	0.200	0.99802	373.73	3.7263	93.07	0.9001
3.0	0.95	3	0.025	1.00000	2485.09	25.0809	2319.30	23.2545
3.0	0.95	5	0.025	1.00000	1496.42	14.6601	1384.13	13.9052
3.0	0.95	10	0.025	1.00000	744.12	7.3397	688.23	6.8233
3.0	0.95	3	0.050	1.00000	2485.09	25.0809	2145.55	21.6483
3.0	0.95	5	0.050	1.00000	1496.42	14.6601	1270.42	12.5147
3.0	0.95	10	0.050	1.00000	744.12	7.3397	638.17	6.4013
3.0	0.95	3	0.100	1.00000	2485.09	25.0809	1833.39	18.2521
3.0	0.95	5	0.100	1.00000	1496.42	14.6601	1092.45	10.8771
3.0	0.95	10	0.100	1.00000	744.12	7.3397	541.28	5.3771
3.0	0.95	3	0.200	1.00000	2485.09	25.0809	1271.23	12.6074
3.0	0.95	5	0.200	1.00000	1496.42	14.6601	761.51	7.5845
3.0	0.95	10	0.200	1.00000	744.12	7.3397	383.44	3.8396

Table A.9: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.05$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 5$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
5.0	0.05	3	0.025	0.70469	372.89	3.5666	86.15	0.7405
5.0	0.05	5	0.025	0.76875	371.32	3.5946	57.21	0.4634
5.0	0.05	10	0.025	0.82633	371.98	3.6349	38.15	0.2772
5.0	0.05	3	0.050	0.70469	372.89	3.5666	34.34	0.2324
5.0	0.05	5	0.050	0.76875	371.32	3.5946	22.05	0.1279
5.0	0.05	10	0.050	0.82633	371.98	3.6349	14.40	0.0640
5.0	0.05	3	0.100	0.70469	372.89	3.5666	13.35	0.0478
5.0	0.05	5	0.100	0.76375	371.32	3.5946	9.92	0.0279
5.0	0.05	10	0.100	0.82633	371.98	3.6349	7.01	0.0147
5.0	0.05	3	0.200	0.70469	372.89	3.5666	7.20	0.0099
5.0	0.05	5	0.200	0.76875	371.32	3.5946	5.63	0.0064
5.0	0.05	10	0.200	0.82633	371.98	3.6349	4.37	0.0049
5.0	0.30	3	0.025	0.67773	372.38	3.5530	87.80	0.7252
5.0	0.30	5	0.025	0.72617	370.61	3.5356	64.47	0.4873
5.0	0.30	10	0.025	0.77478	371.64	3.6246	42.44	0.2697
5.0	0.30	3	0.050	0.67773	372.38	3.5530	36.55	0.2317
5.0	0.30	5	0.050	0.72617	370.61	3.5356	25.94	0.1345
5.0	0.30	10	0.050	0.77478	371.64	3.6246	18.12	0.0683
5.0	0.30	3	0.100	0.67773	372.38	3.5530	15.02	0.0520
5.0	0.30	5	0.100	0.72617	370.61	3.5356	11.75	0.0313
5.0	0.30	10	0.100	0.77478	371.64	3.6246	9.07	0.0173
5.0	0.30	3	0.200	0.67773	372.38	3.5530	7.90	0.0106
5.0	0.30	5	0.200	0.72617	370.61	3.5356	6.64	0.0072
5.0	0.30	10	0.200	0.77478	371.64	3.6246	5.47	0.0053
5.0	0.50	3	0.025	0.91184	372.81	3.7236	253.21	2.5103
5.0	0.50	5	0.025	0.96031	373.66	3.7003	228.16	2.2588
5.0	0.50	10	0.025	0.98785	370.34	3.6383	213.99	2.1235
5.0	0.50	3	0.050	0.91184	372.81	3.7236	169.69	1.6494
5.0	0.50	5	0.050	0.96031	373.66	3.7003	139.18	1.3834
5.0	0.50	10	0.050	0.98785	370.34	3.6383	118.33	1.1687
5.0	0.50	3	0.100	0.91184	372.81	3.7236	74.07	0.7085
5.0	0.50	5	0.100	0.96031	373.66	3.7003	51.91	0.5030
5.0	0.50	10	0.100	0.98785	370.34	3.6383	39.42	0.3762
5.0	0.50	3	0.200	0.91184	372.81	3.7236	14.43	0.1230
5.0	0.50	5	0.200	0.96031	373.66	3.7003	8.87	0.0689
5.0	0.50	10	0.200	0.98785	370.34	3.6383	5.75	0.0405
5.0	0.80	3	0.100	1.00000	599.92	5.9797	362.59	3.5949
5.0	0.80	5	0.100	0.99594	370.84	3.7281	221.04	2.1932

Table A.10: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.1$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 0.5$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.5	0.05	3	0.1	0.60620	374.249	3.67546	226.694	2.19119
0.5	0.05	5	0.1	0.69794	371.146	3.64029	211.771	2.06856
0.5	0.05	10	0.1	0.80400	374.392	3.63900	176.555	1.74500
0.5	0.30	3	0.1	0.61621	373.701	3.71988	232.038	2.27324
0.5	0.30	5	0.1	0.70093	373.049	3.74871	206.802	2.00314
0.5	0.30	10	0.1	0.79131	371.120	3.70415	164.419	1.58345
0.5	0.50	3	0.1	0.60642	373.203	3.64638	227.724	2.20099
0.5	0.50	5	0.1	0.68060	372.708	3.59916	202.682	1.97227
0.5	0.50	10	0.1	0.75700	373.779	3.65100	161.415	1.51800
0.5	0.80	3	0.1	0.59735	372.860	3.59270	240.870	2.27550
0.5	0.80	5	0.1	0.65390	370.469	3.57322	221.469	2.10716
0.5	0.80	10	0.1	0.71200	372.926	3.65900	175.042	1.62600
0.5	0.95	3	0.1	0.65503	374.040	3.67585	267.856	2.60073
0.5	0.95	5	0.1	0.70606	372.295	3.58752	232.091	2.22613
0.5	0.95	10	0.1	0.75500	372.711	3.60900	196.518	1.87700

Table A.11: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.1$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 1$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
1.0	0.05	3	0.1	0.58496	370.220	3.65039	130.250	1.20971
1.0	0.05	5	0.1	0.66199	371.782	3.62348	101.134	0.93948
1.0	0.05	10	0.1	0.75006	372.982	3.61355	63.948	0.56353
1.0	0.30	3	0.1	0.60327	373.213	3.70843	143.125	1.35856
1.0	0.30	5	0.1	0.68530	370.415	3.65470	112.586	1.06563
1.0	0.30	10	0.1	0.77240	370.483	3.60504	75.727	0.69015
1.0	0.50	3	0.1	0.64014	372.949	3.66823	165.061	1.59447
1.0	0.50	5	0.1	0.71918	373.975	3.67165	131.818	1.24591
1.0	0.50	10	0.1	0.80084	371.162	3.63340	91.708	0.85153
1.0	0.80	3	0.1	0.73989	374.103	3.69468	210.502	2.06481
1.0	0.80	5	0.1	0.80399	372.775	3.65890	187.007	1.79974
1.0	0.80	10	0.1	0.86609	372.677	3.79320	132.012	1.27204
1.0	0.95	3	0.1	0.84648	372.264	3.71409	241.955	2.39308
1.0	0.95	5	0.1	0.88628	370.259	3.67103	245.008	2.39757
1.0	0.95	10	0.1	0.92386	373.295	3.71523	218.357	2.10731

Table A.12: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.1$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 3$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
3.0	0.05	3	0.1	0.58408	371.324	3.68641	29.638	0.22096
3.0	0.05	5	0.1	0.66172	370.095	3.68748	18.877	0.11864
3.0	0.05	10	0.1	0.74863	372.591	3.67538	11.019	0.05347
3.0	0.30	3	0.1	0.57349	372.049	3.54698	31.045	0.22685
3.0	0.30	5	0.1	0.64347	371.645	3.61993	20.558	0.12803
3.0	0.30	10	0.1	0.71802	372.071	3.65587	12.512	0.05655
3.0	0.50	3	0.1	0.77330	371.976	3.66596	79.254	0.75644
3.0	0.50	5	0.1	0.84404	372.856	3.74161	66.546	0.63940
3.0	0.50	10	0.1	0.92002	372.373	3.62110	43.659	0.41483

Table A.13: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.1$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 5$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
5.0	0.05	3	0.1	0.58350	374.867	3.74605	13.564	0.06920
5.0	0.05	5	0.1	0.66211	372.945	3.71034	9.170	0.03524
5.0	0.05	10	0.1	0.74922	372.097	3.67852	6.124	0.01624
5.0	0.30	3	0.1	0.57007	374.284	3.66751	14.119	0.06909
5.0	0.30	5	0.1	0.63855	372.264	3.61782	10.079	0.03701
5.0	0.30	10	0.1	0.71128	373.306	3.66141	7.135	0.01834
5.0	0.50	3	0.1	0.82881	372.862	3.68488	75.609	0.74447
5.0	0.50	5	0.1	0.93938	373.418	3.68401	50.802	0.48373
5.0	0.50	10	0.1	0.97636	374.633	3.74169	38.490	0.37681
5.0	0.80	3	0.1	1.00000	599.906	6.04664	360.944	3.62293

Table A.14: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.2$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 0.5$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.5	0.05	3	0.1	0.45391	372.94	3.7355	224.47	2.1918
0.5	0.05	5	0.1	0.55422	373.65	3.6924	226.45	2.2039
0.5	0.05	10	0.1	0.68648	370.33	3.6464	195.81	1.9229
0.5	0.30	3	0.1	0.45903	374.29	3.7012	257.82	2.5275
0.5	0.30	5	0.1	0.56602	371.08	3.6626	215.81	2.1372
0.5	0.30	10	0.1	0.68711	371.43	3.6476	190.88	1.8717
0.5	0.50	3	0.1	0.45903	373.50	3.6590	247.81	2.4196
0.5	0.50	5	0.1	0.55713	374.08	3.6781	234.99	2.3538
0.5	0.50	10	0.1	0.66538	374.11	3.6346	190.84	1.8652
0.5	0.80	3	0.1	0.46860	374.44	3.6739	268.70	2.6395
0.5	0.80	5	0.1	0.55439	370.72	3.5943	238.04	2.3369
0.5	0.80	10	0.1	0.64043	373.33	3.6419	203.90	1.9517
0.5	0.95	3	0.1	0.53833	374.99	3.6581	284.30	2.8561
0.5	0.95	5	0.1	0.61643	371.54	3.6501	270.69	2.6890
0.5	0.95	10	0.1	0.69336	372.41	3.6774	237.22	2.3146

Table A.15: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.2$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 1$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
1.0	0.05	3	0.1	0.44043	370.04	3.6459	160.06	1.5539
1.0	0.05	5	0.1	0.53418	370.02	3.6580	130.02	1.2329
1.0	0.05	10	0.1	0.64682	371.44	3.6979	85.71	0.8273
1.0	0.30	3	0.1	0.45238	373.00	3.6798	173.34	1.6801
1.0	0.30	5	0.1	0.55234	370.03	3.6274	138.64	1.3217
1.0	0.30	10	0.1	0.66972	373.71	3.6742	99.93	0.9639

Table A.16: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.2$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 3$

Beta0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
3.0	0.05	3	0.1	0.43984	370.29	3.6078	41.81	0.3726
3.0	0.05	5	0.1	0.54688	374.73	3.7400	21.81	0.1697
3.0	0.30	3	0.1	0.43654	370.16	3.5993	42.12	0.3694
3.0	0.30	5	0.1	0.52637	371.70	3.6020	25.70	0.2076
3.0	0.50	3	0.1	0.61404	373.38	3.7240	95.09	0.9189
3.0	0.50	5	0.1	0.72609	371.41	3.6824	79.76	0.7663
3.0	0.80	10	0.1	0.99267	370.50	3.6010	197.08	1.9584
3.0	0.95	10	0.1	1.00000	749.15	7.4312	542.96	5.4035

Table A.17: CEV EWMA Chart for a Decrease in the Scale Parameter, $\lambda = 0.2$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 5$

β_0	Censoring Proportion	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
5.0	0.05	3	0.1	0.43950	374.07	3.7199	16.65	0.1167
5.0	0.05	5	0.1	0.53516	374.30	3.6857	9.84	0.0534
5.0	0.05	10	0.1	0.64502	370.48	3.6584	5.88	0.0219
5.0	0.30	3	0.1	0.43906	372.69	3.6411	16.19	0.1096
5.0	0.30	5	0.1	0.52226	374.50	3.6650	10.52	0.0580

A.3 Simulation Results for CUSUM and CEV EWMA Charts for an Increase in the Scale Parameter

Table A.18: CEV EWMA Chart for an Increase in the Scale Parameter, $\lambda = 0.05$, In-control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 0.5$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.5	0.05	3	0.05	1.68184	374.101	3.59373	265.498	2.50760
0.5	0.05	5	0.05	1.57139	373.209	3.49196	240.102	2.24514
0.5	0.05	10	0.05	1.46434	370.302	3.48076	204.190	1.82512
0.5	0.05	3	0.20	1.68184	374.101	3.59373	120.479	1.06976
0.5	0.05	5	0.20	1.57139	373.209	3.49196	92.799	0.78429
0.5	0.05	10	0.20	1.46434	370.302	3.48076	63.617	0.46545
0.5	0.50	3	0.05	1.19263	373.250	3.65987	302.108	2.97697
0.5	0.50	5	0.05	1.14186	372.786	3.66906	280.943	2.76160
0.5	0.50	10	0.05	1.09106	372.968	3.68031	256.903	2.45124
0.5	0.50	3	0.20	1.19263	373.250	3.65987	168.425	1.60780
0.5	0.50	5	0.20	1.14186	372.786	3.66906	137.976	1.31483
0.5	0.50	10	0.20	1.09106	372.968	3.68031	100.316	0.95145
0.5	0.80	3	0.05	1.07511	599.376	5.93245	508.089	5.01888
0.5	0.80	5	0.05	1.04453	370.261	3.66759	268.490	2.62181
0.5	0.80	10	0.05	1.02471	372.710	3.72475	301.307	3.03850
0.5	0.80	3	0.20	1.07511	599.376	5.93245	328.479	3.21864
0.5	0.80	5	0.20	1.04453	370.261	3.66759	174.887	1.72459
0.5	0.80	10	0.20	1.02471	372.710	3.72475	175.970	1.76257

Table A.19: CEV EWMA Chart for an Increase in the Scale Parameter, $\lambda = 0.05$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 1$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
1	0.05	3	0.05	1.36738	370.462	3.64790	190.058	1.82967
1	0.05	5	0.05	1.27656	370.320	3.62052	159.121	1.49481
1	0.05	10	0.05	1.19258	372.594	3.62342	122.931	1.13740
1	0.05	3	0.20	1.36738	370.462	3.64790	50.481	0.42288
1	0.05	5	0.20	1.27656	370.320	3.62052	34.299	0.26397
1	0.05	10	0.20	1.19258	372.594	3.62342	21.190	0.13857
1	0.50	3	0.05	1.23730	374.409	3.58148	233.757	2.26042
1	0.50	5	0.05	1.18398	374.149	3.66129	198.591	1.88605
1	0.50	10	0.05	1.13163	371.476	3.61168	167.531	1.57918
1	0.50	3	0.20	1.23730	374.409	3.58148	86.523	0.76053
1	0.50	5	0.20	1.18398	374.149	3.66129	58.593	0.49887
1	0.50	10	0.20	1.13163	371.476	3.61168	37.512	0.29026
1	0.80	3	0.05	1.13453	374.219	3.60929	282.465	2.71603
1	0.80	5	0.05	1.10801	370.449	3.60752	255.798	2.41014
1	0.80	10	0.05	1.07852	373.165	3.62464	216.992	2.05872
1	0.80	3	0.20	1.13453	374.219	3.60929	141.661	1.28753
1	0.80	5	0.20	1.10801	370.449	3.60752	109.274	0.96803
1	0.80	10	0.20	1.07852	373.165	3.62464	72.959	0.63076

Table A.20: CEV EWMA Chart for an Increase in the Scale Parameter, $\lambda = 0.05$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 3$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
3	0.05	3	0.05	1.38320	371.612	3.62371	61.017	0.53171
3	0.05	5	0.05	1.29224	372.226	3.57142	44.772	0.37128
3	0.05	10	0.05	1.20094	371.418	3.65105	27.756	0.20129
3	0.05	3	0.20	1.38320	371.612	3.62371	9.176	0.04835
3	0.05	5	0.20	1.29224	372.226	3.57142	6.948	0.03002
3	0.05	10	0.20	1.20094	371.418	3.65105	4.961	0.01630
3	0.50	3	0.05	1.45850	372.409	3.52022	134.378	1.11470
3	0.50	5	0.05	1.41499	371.929	3.52148	106.915	0.82038
3	0.50	10	0.05	1.37090	372.431	3.38642	77.328	0.51761
3	0.50	3	0.20	1.45850	372.409	3.52022	35.734	0.18182
3	0.50	5	0.20	1.41499	371.929	3.52148	28.104	0.11273
3	0.50	10	0.20	1.37090	372.431	3.38642	21.032	0.05779
3	0.80	3	0.05	1.22312	370.926	2.94541	281.203	2.04151
3	0.80	5	0.05	1.22300	373.748	3.06703	279.776	2.16703
3	0.80	3	0.20	1.20000	373.700	3.10000	279.800	2.20000
3	0.80	5	0.20	1.22312	373.748	3.06703	297.537	2.17994
3	0.80	10	0.20	1.22300	373.700	3.10000	358.330	2.87001

Table A.21: CEV EWMA Chart for an Increase in the Scale Parameter, $\lambda = 0.05$, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 5$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
5	0.05	3	0.05	1.38578	374.907	3.64926	29.763	0.22926
5	0.05	5	0.05	1.29196	372.010	3.62412	20.799	0.14257
5	0.05	10	0.05	1.20105	372.744	3.58457	13.105	0.07262
5	0.05	3	0.20	1.38578	374.907	3.64926	4.895	0.01992
5	0.05	5	0.20	1.29196	372.010	3.62412	3.960	0.01305
5	0.05	10	0.20	1.20105	372.744	3.58457	3.071	0.00773
5	0.50	3	0.05	1.60660	371.988	3.41969	125.328	0.92796
5	0.50	5	0.05	1.57786	371.156	3.38616	97.786	0.66121
5	0.50	10	0.05	1.54380	374.695	3.39277	71.451	0.40677
5	0.50	3	0.20	1.60660	371.988	3.41969	39.068	0.14179
5	0.50	5	0.20	1.57786	371.156	3.38616	31.711	0.08776
5	0.50	10	0.20	1.54380	374.695	3.39277	25.421	0.04560

Table A.22: CUSUM Chart for an Increase in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 0.5$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.5	0.05	3	0.05	26.9375	370.901	3.16389	227.513	1.80165
0.5	0.05	5	0.05	33.6836	374.045	3.23345	205.749	1.59844
0.5	0.05	10	0.05	44.7656	373.791	3.28510	170.637	1.29541
0.5	0.05	3	0.20	20.7031	370.763	3.41997	94.210	0.69060
0.5	0.05	5	0.20	24.3750	373.026	3.49996	73.777	0.51686
0.5	0.05	10	0.20	29.5430	374.595	3.54507	50.132	0.32985
0.5	0.50	3	0.05	20.5000	373.952	3.11408	262.666	2.05512
0.5	0.50	5	0.05	25.7500	373.252	3.16118	241.962	1.87633
0.5	0.50	10	0.05	34.5000	372.458	3.16175	207.096	1.59687
0.5	0.50	3	0.20	16.6797	374.883	3.37538	129.108	0.92289
0.5	0.50	5	0.20	19.9219	372.241	3.37905	105.787	0.74523
0.5	0.50	10	0.20	24.7578	372.288	3.40803	74.392	0.50623
0.5	0.80	3	0.05	13.3125	370.870	3.01308	295.112	2.30170
0.5	0.80	5	0.05	16.7813	371.561	3.05044	279.646	2.16895
0.5	0.80	10	0.05	23.0781	373.800	3.16405	253.790	1.95655
0.5	0.80	3	0.20	11.7500	374.056	3.17646	181.552	1.32719
0.5	0.80	5	0.20	14.2813	374.104	3.20318	157.288	1.12880
0.5	0.80	10	0.20	18.3750	371.323	3.32131	115.659	0.81200

Table A.23: CUSUM Chart for an Increase in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 1$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
1	0.05	3	0.05	24.0838	370.671	3.27862	147.664	1.19755
1	0.05	5	0.05	29.4298	370.936	3.34383	128.704	1.00836
1	0.05	10	0.05	37.4188	373.418	3.41953	94.765	0.74347
1	0.05	3	0.20	15.6088	370.561	3.58207	42.343	0.31854
1	0.05	5	0.20	17.6071	372.600	3.61466	31.539	0.22605
1	0.05	10	0.20	20.1434	374.112	3.60712	19.983	0.13482
1	0.50	3	0.05	22.2151	371.912	3.40073	164.807	1.78302
1	0.50	5	0.05	26.7551	374.704	3.51414	142.211	1.59511
1	0.50	10	0.05	33.9496	372.998	3.53518	112.689	1.30687
1	0.50	3	0.20	13.7331	371.640	3.59609	63.351	0.83017
1	0.50	5	0.20	15.5991	370.954	3.58977	52.339	0.70222
1	0.50	10	0.20	18.1285	374.118	3.68172	35.024	0.52191
1	0.80	3	0.05	13.0395	370.612	3.45785	233.077	2.42224
1	0.80	5	0.05	15.7497	370.216	3.48441	220.705	2.30851
1	0.80	10	0.05	20.5458	371.513	3.54135	199.556	2.15699
1	0.80	3	0.20	10.2344	373.880	3.34805	112.063	0.75194
1	0.80	5	0.20	11.9824	374.243	3.40154	88.154	0.58930
1	0.80	10	0.20	14.6875	372.435	3.43590	60.093	0.38088

Table A.24: CUSUM Chart for an Increase in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 3$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
3	0.05	3	0.05	17.4130	372.541	3.57485	56.368	0.42807
3	0.05	5	0.05	20.0539	371.477	3.58160	42.467	0.30639
3	0.05	10	0.05	23.1082	370.341	3.62294	27.038	0.18747
3	0.05	3	0.20	8.2002	373.246	3.68125	10.084	0.06578
3	0.05	5	0.20	8.5187	370.619	3.66567	7.341	0.04338
3	0.05	10	0.20	8.7323	370.059	3.67015	4.724	0.02493
3	0.50	3	0.05	9.5190	370.207	3.61896	104.059	0.95422
3	0.50	5	0.05	10.6333	371.556	3.59259	83.359	0.80314
3	0.50	10	0.05	12.2536	374.119	3.67266	61.297	0.61349
3	0.50	3	0.20	1.9355	370.896	3.69988	31.263	0.34347
3	0.50	5	0.20	1.8239	373.634	3.71889	26.270	0.29993
3	0.50	10	0.20	2.0127	372.875	3.66902	21.485	0.27160
3	0.80	3	0.05	2.8020	374.999	3.61311	177.144	1.85210
3	0.80	5	0.05	3.1137	370.325	3.62192	159.767	1.65958
3	0.80	10	0.05	3.7775	371.229	3.71485	143.488	1.53566
3	0.80	3	0.20	7.3242	371.246	3.53381	39.036	0.22009
3	0.80	5	0.20	8.0928	374.401	3.56384	27.718	0.15294
3	0.80	10	0.20	9.0938	374.997	3.70404	22.232	0.08740

Table A.25: CUSUM Chart for an Increase in the Scale Parameter, In-Control and Out-of-Control ARL and Simulation Errors, $\beta = \beta_0 = 5$

Beta0	Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
5	0.05	3	0.05	14.0131	374.336	3.65964	29.4802	0.22093
5	0.05	5	0.05	15.5469	371.905	3.61472	22.0003	0.13518
5	0.05	10	0.05	17.2225	371.305	3.65888	13.5110	0.09031
5	0.05	3	0.20	5.5065	372.365	3.68204	5.2212	0.03112
5	0.05	5	0.20	5.4531	371.764	3.72413	3.9172	0.02030
5	0.05	10	0.20	4.7383	373.379	3.70878	2.6945	0.01173
5	0.50	3	0.05	6.3242	372.184	3.60899	68.3579	0.65357
5	0.50	5	0.05	6.7642	372.895	3.65376	54.2388	0.54205
5	0.50	10	0.05	7.3687	373.559	3.70259	40.6594	0.42701
5	0.50	3	0.20	6.8711	372.150	3.62053	11.0288	0.05044
5	0.50	5	0.20	7.0625	372.872	3.65419	7.6045	0.03202
5	0.50	10	0.20	7.3750	373.173	3.64929	4.8255	0.01825
5	0.80	3	0.05	9.4531	370.220	3.34350	85.9542	0.55433
5	0.80	5	0.05	10.8750	370.795	3.47143	66.5393	0.42137
5	0.80	10	0.05	13.0625	370.167	3.49841	44.4069	0.26848
5	0.80	3	0.20	6.1719	371.104	3.55011	23.0768	0.11224
5	0.80	5	0.20	6.6944	374.758	3.59606	16.2592	0.07648
5	0.80	10	0.20	7.2313	372.355	3.64391	9.8256	0.04333

Appendix B

Simulation Results for the CUSUM Chart for the Shape Parameter

Table B.1: CUSUM Chart for a Decrease in the Shape Parameter- Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta_0 = 1$, $\eta_1 = \eta_0 = 1$

Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.05	3	0.05	1.44043	370.999	3.28729	129.048	0.98222
0.05	5	0.05	1.71875	374.385	3.46826	103.425	0.75472
0.05	10	0.05	2.13867	370.518	3.38078	72.150	0.49190
0.05	3	0.20	3.21289	371.261	3.63656	24.980	0.15934
0.05	5	0.20	3.51563	370.569	3.57277	17.802	0.10722
0.05	10	0.20	3.88672	371.353	3.60748	11.121	0.06032
0.50	3	0.05	1.06934	374.392	3.35864	162.879	1.28417
0.50	5	0.05	1.32080	374.404	3.37983	139.693	1.06428
0.50	10	0.05	1.67969	370.439	3.40848	103.969	0.75228
0.50	3	0.20	2.69531	372.083	3.56745	37.423	0.26408
0.50	5	0.20	3.04443	374.374	3.56477	27.425	0.18278
0.50	10	0.20	3.49670	374.253	3.65359	17.367	0.10765
0.80	3	0.05	1.05469	373.998	3.36111	166.824	1.31613
0.80	5	0.05	1.28906	373.701	3.36971	140.546	1.07901
0.80	10	0.05	1.65039	372.388	3.42348	103.540	0.76565
0.80	3	0.20	2.66602	371.530	3.59968	38.279	0.27080
0.80	5	0.20	3.00781	372.466	3.55924	28.355	0.18881
0.80	10	0.20	3.44727	371.897	3.66615	17.755	0.11069

Table B.2: CUSUM Chart for an Increase in the Shape Parameter- Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta_0 = 1$ and $\eta_1 = \eta_0 = 1$.

Censoring Rate	Sample Size	Shift Size	Threshold	IARL	ISimErr	OARL	OSimErr
0.05	3	0.05	1.40625	370.369	3.25343	134.734	0.93642
0.05	5	0.05	1.69434	373.306	3.32255	112.408	0.77860
0.05	10	0.05	2.11914	374.231	3.39826	80.054	0.53095
0.05	3	0.20	3.13721	371.129	3.53831	35.951	0.20071
0.05	5	0.20	3.46680	372.693	3.54951	25.535	0.13863
0.05	10	0.20	3.87695	372.126	3.63368	15.986	0.08106
0.50	3	0.05	1.04492	372.239	3.18098	182.466	1.32590
0.50	5	0.05	1.28906	372.401	3.23799	153.362	1.08827
0.50	10	0.05	1.65527	373.403	3.32044	116.442	0.80574
0.50	3	0.20	2.57813	373.687	3.43023	59.026	0.34895
0.50	5	0.20	2.92480	373.907	3.49097	43.362	0.25260
0.50	10	0.20	3.39844	372.715	3.58074	27.328	0.14632
0.80	3	0.05	1.03027	370.904	3.17541	185.632	1.33194
0.80	5	0.05	1.25977	370.138	3.19067	155.664	1.09014
0.80	10	0.05	1.62109	371.001	3.33628	117.539	0.82369
0.80	3	0.20	2.55981	374.423	3.40664	61.644	0.35204
0.80	5	0.20	2.89063	370.563	3.52039	45.242	0.25774
0.80	10	0.20	3.37891	371.506	3.50401	28.686	0.15435

Appendix C

Simulation Results for the Simultaneous Chart for the Shape and the Scale Parameters

Table C.1: Simultaneous CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 0.5$, $\eta_0 = 1$, Censoring Rate 5%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	3.17383	372.546	3.60458	25.101	0.16350
5	−20	−20	3.49610	374.052	3.59770	18.106	0.11100
10	−20	−20	3.86690	370.919	3.67570	11.223	0.06240
3	−20	−5	2.07031	372.163	3.49898	77.501	0.53404
5	−20	−5	2.39750	373.464	3.49650	59.468	0.39720
10	−20	−5	2.86130	373.813	3.47210	39.187	0.24920
3	−20	5	2.45117	373.243	3.38028	65.317	0.39594
5	−20	5	2.81250	372.702	3.53529	47.868	0.27751
10	−20	5	3.27390	372.220	3.53990	30.599	0.16760
3	−20	20	3.45703	370.591	3.53941	28.273	0.14227
5	−20	20	3.78910	374.872	3.61860	19.831	0.09560
10	−20	20	4.15040	374.841	3.57600	12.222	0.05470
3	−5	−20	3.20313	374.876	3.61902	25.395	0.16203
5	−5	−20	3.49610	373.506	3.62840	18.190	0.11030
10	−5	−20	3.88670	374.863	3.59350	11.417	0.06270
3	−5	−5	1.40625	373.635	3.36959	129.167	0.96407
5	−5	−5	1.68950	373.522	3.49020	103.643	0.74460
10	−5	−5	2.10940	372.847	3.43560	74.902	0.52310
3	−5	5	1.59668	374.054	3.28685	120.769	0.81882
5	−5	5	1.89449	372.940	3.37517	95.610	0.63297
10	−5	5	2.34380	370.522	3.48810	66.889	0.42920
3	−5	20	3.18359	371.240	3.49540	34.666	0.19107
5	−5	20	3.53520	374.438	3.60950	24.647	0.12890
10	−5	20	3.93560	374.862	3.66530	15.203	0.07610
3	5	−20	3.23242	370.301	3.62741	24.089	0.15518
5	5	−20	3.55470	373.568	3.70080	17.293	0.10350
10	5	−20	3.90630	371.938	3.70040	10.893	0.06010
3	5	−5	1.56250	370.027	3.39020	111.978	0.84780
5	5	−5	1.87260	374.878	3.41580	89.624	0.65330
10	5	−5	2.30470	371.863	3.49140	62.792	0.43490
3	5	5	1.38672	374.867	3.30787	141.695	0.99524
5	5	5	1.65771	373.785	3.35398	113.948	0.78375
10	5	5	2.08980	372.934	3.43900	81.541	0.54570
3	5	20	3.09570	372.188	3.47186	36.872	0.20815
5	5	20	3.43750	372.350	3.60980	26.661	0.14280
10	5	20	3.83790	370.474	3.57160	16.239	0.08140
3	20	−20	3.31055	370.299	3.52854	21.945	0.13987
5	20	−20	3.63280	374.074	3.67810	15.909	0.09360
10	20	−20	3.95510	372.733	3.64270	9.863	0.05490
3	20	−5	2.14844	374.636	3.43903	69.334	0.49115
5	20	−5	2.48050	372.237	3.56720	51.932	0.35340
10	20	−5	2.94920	370.725	3.61040	33.994	0.22000
3	20	5	1.91895	371.043	3.38751	91.038	0.61790
5	20	5	2.24365	371.188	3.48889	70.390	0.46114
10	20	5	2.69530	373.138	3.50080	47.074	0.29420
3	20	20	3.07617	371.184	3.54311	36.963	0.21109
5	20	20	3.41800	371.827	3.62710	26.194	0.14260
10	20	20	3.82810	370.590	3.58580	16.273	0.08440

Table C.2: Simultaneous CUSUM Chart for the Shape and the Scale Parameters -Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 0.5$, $\eta_0 = 1$, Censoring Rate 50%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	2.96875	370.857	3.57251	29.490	0.20189
5	−20	−20	3.31060	373.697	3.69920	21.360	0.14060
10	−20	−20	3.70120	374.937	3.65740	13.489	0.07990
3	−20	−5	2.03125	372.677	3.36597	80.886	0.57678
5	−20	−5	2.34390	371.690	3.51220	61.573	0.41380
10	−20	−5	2.81250	370.314	3.53390	41.382	0.26270
3	−20	5	1.63086	372.179	3.34936	113.496	0.81147
5	−20	5	1.92630	372.884	3.43120	88.273	0.61380
10	−20	5	2.37790	374.229	3.52020	62.295	0.41240
3	−20	20	2.42188	371.483	3.41560	62.848	0.39762
5	−20	20	2.75390	371.778	3.55650	46.050	0.27670
10	−20	20	3.24220	371.631	3.55940	29.679	0.16900
3	−5	−20	2.75391	373.914	3.59276	36.055	0.25692
5	−5	−20	3.09450	370.250	3.60250	26.258	0.17510
10	−5	−20	3.53520	373.606	3.69310	16.578	0.10060
3	−5	−5	1.27075	373.451	3.31651	144.009	1.08815
5	−5	−5	1.54300	373.969	3.41810	119.664	0.88230
10	−5	−5	1.93600	374.848	3.47630	84.775	0.60740
3	−5	5	0.95215	371.825	3.15455	189.483	1.38113
5	−5	5	1.17190	373.913	3.28280	164.330	1.18840
10	−5	5	1.50390	370.631	3.32720	123.986	0.86520
3	−5	20	2.49023	373.769	3.48708	62.909	0.37360
5	−5	20	2.85160	374.174	3.53890	46.414	0.27250
10	−5	20	3.33980	374.904	3.50830	29.397	0.15860
3	5	−20	2.65625	373.527	3.60069	39.088	0.27688
5	5	−20	3.00780	371.300	3.55510	28.467	0.18760
10	5	−20	3.43750	370.701	3.61230	18.254	0.11380
3	5	−5	0.97656	373.012	3.32089	177.882	1.37736
5	5	−5	1.19630	370.785	3.29420	151.754	1.15790
10	5	−5	1.55270	374.409	3.43160	114.586	0.83810
3	5	5	1.25000	371.308	3.16604	155.052	1.10122
5	5	5	1.51860	374.408	3.31140	127.676	0.88270
10	5	5	1.92630	373.987	3.26470	94.273	0.64170
3	5	20	2.65625	370.193	3.38806	54.884	0.31969
5	5	20	3.01330	371.724	3.52360	40.226	0.22450
10	5	20	3.47660	374.454	3.58250	25.465	0.13510
3	20	−20	2.54883	372.380	3.54063	43.341	0.30541
5	20	−20	2.90040	374.942	3.56850	31.825	0.21480
10	20	−20	3.37910	371.373	3.64590	20.356	0.12590
3	20	−5	1.32813	371.478	3.28303	142.121	1.02233
5	20	−5	1.59180	370.290	3.28610	115.150	0.82260
10	20	−5	2.01660	374.988	3.45370	84.233	0.56840
3	20	5	1.87500	373.522	3.40738	95.419	0.64289
5	20	5	2.20700	372.485	3.44280	73.091	0.48190
10	20	5	2.65630	370.922	3.53060	49.137	0.30880
3	20	20	2.91992	374.063	3.49383	44.320	0.25022
5	20	20	3.26170	370.532	3.51440	31.597	0.17440
10	20	20	3.71090	373.920	3.59430	19.760	0.10440

Table C.3: CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 0.5$, $\eta_0 = 1$, Censoring Rate 80%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	2.94922	372.719	3.67604	29.894	0.20481
5	−20	−20	3.28130	371.186	3.65060	21.681	0.13860
10	−20	−20	3.69140	371.714	3.62210	13.612	0.07970
3	−20	−5	1.82129	374.749	3.49458	93.041	0.67573
5	−20	−5	2.12400	373.090	3.52900	72.582	0.51460
10	−20	−5	2.58790	371.621	3.50000	49.734	0.33170
3	−20	5	0.76660	374.526	3.23322	215.347	1.67625
5	−20	5	0.94230	370.366	3.23750	187.385	1.41740
10	−20	5	1.24760	370.845	3.29740	147.492	1.08200
3	−20	20	2.10938	373.087	3.32009	86.539	0.51420
5	−20	20	2.46950	370.891	3.35120	65.957	0.38150
10	−20	20	2.94920	374.862	3.52140	43.292	0.24770
3	−5	−20	2.72949	372.920	3.65381	36.555	0.25360
5	−5	−20	3.08590	373.775	3.58880	26.432	0.17640
10	−5	−20	3.51560	370.301	3.63430	16.775	0.10130
3	−5	−5	1.23535	371.710	3.32062	142.550	1.10941
5	−5	−5	1.49900	370.948	3.40120	118.415	0.91220
10	−5	−5	1.89210	370.720	3.37730	86.433	0.60710
3	−5	5	0.83008	370.398	3.03964	212.010	1.50828
5	−5	5	1.02540	371.172	3.14350	185.470	1.35360
10	−5	5	1.35000	370.038	3.22050	145.404	1.01580
3	−5	20	2.46094	373.176	3.44228	67.275	0.38536
5	−5	20	2.81250	371.569	3.43780	49.674	0.28570
10	−5	20	3.27880	374.814	3.56260	31.364	0.16880
3	5	−20	2.59766	371.101	3.55421	39.896	0.28751
5	5	−20	2.94920	373.001	3.62330	29.516	0.20070
10	5	−20	3.37890	370.706	3.59720	19.044	0.12080
3	5	−5	0.87891	372.417	3.34193	189.488	1.54587
5	5	−5	1.08400	372.944	3.31290	163.374	1.28470
10	5	−5	1.42580	374.359	3.41290	127.956	0.96910
3	5	5	1.22070	372.984	3.22467	160.551	1.13703
5	5	5	1.48440	374.191	3.28110	133.308	0.93200
10	5	5	1.88480	372.505	3.35020	97.281	0.64170
3	5	20	2.61719	371.306	3.43061	57.135	0.32739
5	5	20	2.98580	372.896	3.52590	42.248	0.23610
10	5	20	3.44240	372.466	3.61190	26.745	0.14220
3	20	−20	2.42676	373.305	3.58877	46.860	0.34299
5	20	−20	2.77340	372.393	3.56600	34.567	0.24700
10	20	−20	3.24220	372.496	3.54860	22.503	0.14660
3	20	−5	0.62500	371.186	3.25032	224.472	1.80887
5	20	−5	0.79100	370.917	3.25670	205.635	1.62500
10	20	−5	1.06450	370.289	3.31230	168.266	1.29900
3	20	5	1.69922	374.866	3.32619	114.329	0.76564
5	20	5	1.98240	372.227	3.31320	87.966	0.58310
10	20	5	2.44140	372.535	3.49740	61.632	0.39350
3	20	20	2.85156	372.420	3.56515	47.744	0.26924
5	20	20	3.19340	372.619	3.50530	34.181	0.18490
10	20	20	3.63530	372.307	3.58620	21.620	0.11100

Table C.4: CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 1$, $\eta_0 = 1$, Censoring Rate 5%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	3.28125	372.427	3.55025	22.072	0.14095
5	−20	−20	3.60350	373.054	3.62640	15.718	0.09520
10	−20	−20	3.94410	372.368	3.64980	9.896	0.05430
3	−20	−5	2.92664	372.973	3.50303	38.305	0.23589
5	−20	−5	3.26660	371.667	3.58120	28.208	0.16680
10	−20	−5	3.69140	371.411	3.62080	17.583	0.09480
3	−20	5	3.25195	373.933	3.59587	33.713	0.18345
5	−20	5	3.57910	373.048	3.61024	23.726	0.12134
10	−20	5	3.98440	373.940	3.62260	14.630	0.06990
3	−20	20	3.81592	371.469	3.61397	20.447	0.09206
5	−20	20	4.11620	374.754	3.66170	14.344	0.06160
10	−20	20	4.39450	373.629	3.58330	8.761	0.03550
3	−5	−20	3.17627	372.717	3.54573	25.625	0.16500
5	−5	−20	3.49120	370.307	3.58360	18.307	0.11110
10	−5	−20	3.86720	371.533	3.60270	11.559	0.06530
3	−5	−5	1.52344	370.710	3.31601	116.662	0.86360
5	−5	−5	1.82370	372.327	3.42240	93.490	0.67860
10	−5	−5	2.24610	372.134	3.52200	65.149	0.44630
3	−5	5	1.85547	370.564	3.36119	101.346	0.65579
5	−5	5	2.18750	371.305	3.38453	77.672	0.49535
10	−5	5	2.63670	373.578	3.50310	53.239	0.32900
3	−5	20	3.27148	372.852	3.52568	32.718	0.17374
5	−5	20	3.59380	374.495	3.62700	22.953	0.11680
10	−5	20	3.97950	374.163	3.63180	14.219	0.06780
3	5	−20	3.26660	373.252	3.63412	23.744	0.15039
5	5	−20	3.57420	373.835	3.68650	16.648	0.09940
10	5	−20	3.90630	372.792	3.69420	10.434	0.05800
3	5	−5	1.77734	371.665	3.41588	94.512	0.67997
5	5	−5	2.08980	372.478	3.47540	74.823	0.53410
10	5	−5	2.53910	374.004	3.56500	50.538	0.34100
3	5	5	1.52466	373.655	3.32718	127.785	0.88081
5	5	5	1.81152	372.074	3.39167	100.687	0.68237
10	5	5	2.24610	374.233	3.41080	71.024	0.46300
3	5	20	3.07617	374.741	3.53211	37.802	0.21649
5	5	20	3.40820	371.064	3.59570	26.802	0.14560
10	5	20	3.83300	374.412	3.63260	16.735	0.08520
3	20	−20	3.45703	372.791	3.60140	18.177	0.11391
5	20	−20	3.75000	374.693	3.66900	13.103	0.07600
10	20	−20	4.04790	370.756	3.57440	8.223	0.04380
3	20	−5	2.79297	371.101	3.53205	37.784	0.25764
5	20	−5	3.12990	371.591	3.59050	27.266	0.17790
10	20	−5	3.55470	373.061	3.77040	17.363	0.10350
3	20	5	2.70508	374.973	3.58706	45.545	0.29344
5	20	5	3.03711	372.806	3.58148	33.212	0.20676
10	20	5	3.48330	374.836	3.68100	21.135	0.12100
3	20	20	3.27148	373.577	3.54485	30.696	0.17232
5	20	20	3.58890	372.470	3.65170	21.660	0.11410
10	20	20	3.96480	370.848	3.61350	13.450	0.06690

Table C.5: CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 1$, $\eta_0 = 1$, Censoring Rate 50%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	3.24219	370.120	3.65408	23.022	0.15033
5	−20	−20	3.55470	371.044	3.62470	16.454	0.10090
10	−20	−20	3.90630	372.187	3.62110	10.309	0.05630
3	−20	−5	2.74292	370.446	3.51372	43.518	0.27902
5	−20	−5	3.08840	371.502	3.54910	32.043	0.19760
10	−20	−5	3.53520	374.896	3.61000	20.468	0.11810
3	−20	5	2.57813	371.866	3.53213	54.035	0.34671
5	−20	5	2.91500	370.182	3.49520	39.835	0.24460
10	−20	5	3.35940	373.265	3.58450	25.437	0.14670
3	−20	20	2.78809	373.544	3.51063	43.435	0.27739
5	−20	20	3.12010	372.633	3.58590	31.590	0.19340
10	−20	20	3.55960	374.468	3.58650	19.982	0.11490
3	−5	−20	2.81250	373.617	3.60024	33.736	0.23530
5	−5	−20	3.16410	374.886	3.70250	24.536	0.16200
10	−5	−20	3.57420	372.223	3.60570	15.444	0.09320
3	−5	−5	1.49414	370.078	3.34927	117.417	0.86221
5	−5	−5	1.78710	370.457	3.44040	95.712	0.68520
10	−5	−5	2.21680	371.104	3.49850	68.079	0.46920
3	−5	5	1.04980	373.446	3.22420	175.759	1.30203
5	−5	5	1.28420	372.942	3.30670	147.181	1.09000
10	−5	5	1.65040	370.724	3.30390	111.040	0.76850
3	−5	20	2.42676	373.076	3.41390	64.724	0.38974
5	−5	20	2.79050	374.223	3.52450	47.546	0.27310
10	−5	20	3.28130	373.332	3.55050	30.947	0.17030
3	5	−20	2.61232	370.427	3.54569	40.331	0.28779
5	5	−20	2.95790	372.176	3.59420	29.885	0.20300
10	5	−20	3.40820	371.773	3.56890	18.916	0.11890
3	5	−5	1.02585	374.065	3.24800	175.017	1.34601
5	5	−5	1.25980	374.297	3.24720	146.150	1.08460
10	5	−5	1.62110	373.888	3.36760	112.947	0.80960
3	5	5	1.48926	374.199	3.33407	130.372	0.92362
5	5	5	1.77250	372.457	3.36430	103.521	0.72060
10	5	5	2.20700	371.296	3.40990	72.927	0.47100
3	5	20	2.77344	371.555	3.49562	50.981	0.29675
5	5	20	3.10550	374.720	3.58540	37.195	0.20950
10	5	20	3.55470	373.739	3.54730	23.303	0.12070
3	20	−20	2.55859	374.628	3.57156	45.724	0.31304
5	20	−20	2.90040	371.383	3.54790	33.557	0.21870
10	20	−20	3.37720	373.911	3.61860	21.420	0.13150
3	20	−5	2.07031	372.170	3.38777	79.874	0.53318
5	20	−5	2.41210	373.911	3.48330	61.577	0.40390
10	20	−5	2.85650	372.675	3.52750	40.332	0.25340
3	20	5	2.53418	371.253	3.53753	56.447	0.36176
5	20	5	2.88090	371.499	3.52880	41.507	0.25680
10	20	5	3.33010	371.105	3.54940	26.479	0.15250
3	20	20	3.24219	373.067	3.55520	31.824	0.17257
5	20	20	3.57420	373.793	3.57620	22.544	0.11970
10	20	20	3.96730	370.163	3.62940	13.969	0.06900

Table C.6: CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 1$, $\eta_0 = 1$, Censoring Rate 80%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	3.18359	373.481	3.60292	23.829	0.15582
5	−20	−20	3.49300	373.938	3.62210	17.165	0.10560
10	−20	−20	3.85740	374.038	3.61470	10.666	0.06010
3	−20	−5	2.39746	372.850	3.54821	58.120	0.40201
5	−20	−5	2.71480	371.541	3.52670	42.784	0.28560
10	−20	−5	3.17380	374.864	3.65630	27.819	0.17360
3	−20	5	1.64246	373.711	3.46009	109.990	0.80981
5	−20	5	1.92380	374.644	3.47180	85.234	0.60570
10	−20	5	2.38280	373.781	3.52700	59.892	0.40640
3	−20	20	1.73828	374.764	3.27056	110.700	0.69862
5	−20	20	2.08010	374.271	3.42230	86.454	0.52950
10	−20	20	2.55860	371.101	3.43100	58.448	0.34470
3	−5	−20	2.81250	373.617	3.56732	34.478	0.23927
5	−5	−20	3.14450	372.802	3.60570	24.976	0.16450
10	−5	−20	3.55870	371.802	3.64350	15.933	0.09840
3	−5	−5	1.42578	370.954	3.33515	127.818	0.96710
5	−5	−5	1.69920	373.551	3.41520	101.451	0.75360
10	−5	−5	2.12890	370.991	3.46190	72.303	0.51030
3	−5	5	0.66895	372.442	3.02585	241.051	1.74559
5	−5	5	0.83980	372.885	3.10020	214.536	1.52580
10	−5	5	1.12120	372.743	3.12250	172.913	1.23350
3	−5	20	2.36328	371.942	3.33889	72.738	0.43392
5	−5	20	2.71000	374.201	3.54200	53.196	0.30680
10	−5	20	3.18360	374.761	3.64630	34.313	0.18460
3	5	−20	2.51953	372.738	3.62419	42.933	0.31363
5	5	−20	2.89060	372.567	3.57320	31.686	0.21910
10	5	−20	3.33980	374.453	3.60770	20.199	0.12790
3	5	−5	0.72067	372.836	3.31267	211.334	1.74156
5	5	−5	0.89840	372.019	3.36370	185.637	1.46230
10	5	−5	1.19870	373.891	3.31800	148.766	1.14720
3	5	5	1.40625	371.724	3.25269	140.930	0.97904
5	5	5	1.67970	374.504	3.33550	114.024	0.77640
10	5	5	2.09900	372.131	3.38850	81.352	0.53060
3	5	20	2.71362	371.550	3.48799	53.371	0.30182
5	5	20	3.06640	373.257	3.51860	38.941	0.21660
10	5	20	3.51560	372.043	3.58780	24.685	0.12970
3	20	−20	2.18750	373.617	3.58988	57.479	0.43828
5	20	−20	2.51950	373.609	3.62860	42.398	0.31080
10	20	−20	2.99810	372.013	3.53190	28.059	0.18910
3	20	−5	1.18164	374.709	3.22374	165.269	1.20038
5	20	−5	1.42580	371.999	3.27860	138.561	0.97460
10	20	−5	1.81640	374.113	3.37730	101.902	0.69770
3	20	5	2.18750	372.490	3.39951	78.056	0.50282
5	20	5	2.51220	372.404	3.47970	58.287	0.35460
10	20	5	2.98830	373.481	3.45070	38.915	0.23050
3	20	20	3.11523	371.722	3.50923	38.368	0.20666
5	20	20	3.43750	374.207	3.59600	27.482	0.14230
10	20	20	3.84770	371.461	3.57760	17.026	0.08380

Table C.7: Simultaneous CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 3$, $\eta_0 = 1$, Censoring Rate 5%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	4.07951	372.175	3.68509	9.3909	0.04760
5	−20	−20	4.25049	370.079	3.67920	6.6335	0.03036
10	−20	−20	4.33105	371.174	3.69296	4.2776	0.01716
3	−20	−5	4.28906	374.811	3.67586	9.5987	0.04097
5	−20	−5	4.44141	371.703	3.72090	6.7357	0.02679
10	−20	−5	4.50000	370.264	3.63080	4.2809	0.01461
3	−20	5	4.43555	373.207	3.67136	9.0180	0.03345
5	−20	5	4.54688	373.838	3.68862	6.2622	0.02161
10	−20	5	4.52344	374.125	3.68319	4.0011	0.01205
3	−20	20	4.60022	371.864	3.58985	7.9169	0.02570
5	−20	20	4.65332	370.652	3.75472	5.5115	0.01615
10	−20	20	4.50195	371.343	3.67601	3.5613	0.00892
3	−5	−20	3.20801	374.130	3.54387	24.3821	0.15596
5	−5	−20	3.52051	371.546	3.68410	17.5103	0.10596
10	−5	−20	3.89648	372.387	3.59290	10.9280	0.06045
3	−5	−5	2.43103	371.382	3.47511	58.0870	0.38206
5	−5	−5	2.77344	370.659	3.48936	43.2842	0.27313
10	−5	−5	3.24220	373.600	3.55010	27.9193	0.16820
3	−5	5	2.80762	373.724	3.47732	49.0231	0.28508
5	−5	5	3.14453	371.568	3.55775	35.3722	0.19800
10	−5	5	3.60350	373.445	3.64800	22.1755	0.11590
3	−5	20	3.60107	372.371	3.61715	24.9822	0.12090
5	−5	20	3.92578	374.566	3.59226	17.4574	0.07924
10	−5	20	4.25781	374.619	3.69483	10.6263	0.04536
3	5	−20	3.39844	372.824	3.61296	20.0169	0.12619
5	5	−20	3.70117	373.136	3.57822	14.2111	0.08425
10	5	−20	4.01367	372.535	3.69264	8.8220	0.04688
3	5	−5	2.55829	370.752	3.59511	46.9007	0.33173
5	5	−5	2.91016	371.837	3.62247	34.3710	0.23212
10	5	−5	3.35940	374.538	3.66440	22.0860	0.13740
3	5	5	2.43896	370.126	3.49157	58.2844	0.37560
5	5	5	2.77344	370.852	3.49374	42.7233	0.27053
10	5	5	3.23790	374.569	3.58330	27.9926	0.17020
3	5	20	3.18359	372.156	3.55060	33.6575	0.18892
5	5	20	3.49609	373.573	3.65208	23.9760	0.12734
10	5	20	3.89648	374.083	3.73677	14.7788	0.07423
3	20	−20	3.96240	373.200	3.73885	8.7085	0.04865
5	20	−20	4.13086	371.606	3.69559	6.3578	0.03276
10	20	−20	4.21875	373.955	3.72968	4.1554	0.01838
3	20	−5	3.92578	373.366	3.70848	9.9997	0.05620
5	20	−5	4.10156	373.152	3.66634	7.1523	0.03672
10	20	−5	4.24219	372.120	3.69727	4.6149	0.02060
3	20	5	3.93750	370.279	3.65089	10.0914	0.05460
5	20	5	4.13818	371.532	3.70566	7.1910	0.03612
10	20	5	4.26563	373.238	3.70347	4.6217	0.01987
3	20	20	4.08203	371.006	3.70625	9.7907	0.04919
5	20	20	4.25537	373.954	3.69543	6.8919	0.03185
10	20	20	4.37500	373.355	3.72500	4.4662	0.01807

Table C.8: Simultaneous CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 3$, $\eta_0 = 1$, Censoring Rate 50%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	4.02344	371.725	3.61951	9.6576	0.05054
5	−20	−20	4.20898	370.690	3.61613	6.8742	0.03293
10	−20	−20	4.31890	374.237	3.69400	4.4450	0.01840
3	−20	−5	4.06348	372.915	3.69808	11.0365	0.05480
5	−20	−5	4.26563	373.792	3.67563	7.8147	0.03531
10	−20	−5	4.37109	371.386	3.75473	4.9168	0.01975
3	−20	5	4.10156	373.068	3.63216	11.0703	0.05250
5	−20	5	4.27734	370.757	3.63829	7.7289	0.03466
10	−20	5	4.40625	372.673	3.68059	4.9216	0.01961
3	−20	20	4.18213	372.967	3.63262	10.4011	0.04899
5	−20	20	4.33594	370.047	3.65347	7.2310	0.03150
10	−20	20	4.41410	372.422	3.69790	4.6559	0.01800
3	−5	−20	3.06641	373.677	3.60202	27.0762	0.18241
5	−5	−20	3.39355	374.553	3.62442	19.2896	0.12248
10	−5	−20	3.78910	372.916	3.66960	12.1650	0.06980
3	−5	−5	2.32422	374.662	3.48255	62.6540	0.42045
5	−5	−5	2.66602	373.748	3.54017	47.0976	0.30719
10	−5	−5	3.11770	370.622	3.55160	30.1018	0.18560
3	−5	5	2.03125	371.438	3.49456	82.6103	0.55910
5	−5	5	2.34375	371.714	3.48924	62.8744	0.41952
10	−5	5	2.80750	373.816	3.51770	41.8487	0.25910
3	−5	20	2.50000	370.794	3.47373	55.7839	0.35335
5	−5	20	2.87109	374.998	3.52055	41.8123	0.25986
10	−5	20	3.33040	372.159	3.57130	26.3865	0.15200
3	5	−20	2.53906	374.366	3.53728	44.5647	0.30986
5	5	−20	2.89063	373.918	3.60722	33.6160	0.22819
10	5	−20	3.33980	373.655	3.60360	21.2080	0.13270
3	5	−5	1.80664	374.307	3.39732	99.0622	0.67309
5	5	−5	2.11914	370.631	3.41349	78.5549	0.53323
10	5	−5	2.55860	370.503	3.44970	52.3262	0.33390
3	5	5	2.31445	372.560	3.41924	67.9965	0.43653
5	5	5	2.65625	372.950	3.49429	50.7378	0.31396
10	5	5	3.11520	374.462	3.57190	32.9710	0.19720
3	5	20	3.12500	372.762	3.54664	36.4032	0.20195
5	5	20	3.45703	373.309	3.60090	25.5385	0.13617
10	5	20	3.85730	371.465	3.67450	15.9601	0.08180
3	20	−20	3.30078	374.833	3.59253	27.4458	0.15627
5	20	−20	3.62061	371.830	3.61127	19.6736	0.11021
10	20	−20	3.99410	370.530	3.62760	12.0812	0.06210
3	20	−5	3.53760	370.738	3.56643	21.8358	0.12234
5	20	−5	3.78516	371.764	3.70125	15.4708	0.08111
10	20	−5	4.14844	370.708	3.66928	9.5405	0.04652
3	20	5	3.75000	371.320	3.58886	17.4397	0.09088
5	20	5	4.01953	373.601	3.70295	12.2867	0.06090
10	20	5	4.26544	373.315	3.76103	7.6279	0.03399
3	20	20	4.04419	371.698	3.66676	13.2008	0.06268
5	20	20	4.23828	371.288	3.60691	9.2042	0.04106
10	20	20	4.42380	373.847	3.71420	5.7313	0.02270

Table C.9: Simultaneous CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 3$, $\eta_0 = 1$, Censoring Rate 80%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	3.84308	374.324	3.77402	11.164	0.06385
5	−20	−20	4.07227	372.074	3.65830	8.110	0.04265
10	−20	−20	4.25780	372.906	3.69930	5.154	0.02340
3	−20	−5	3.63281	370.183	3.65165	16.176	0.09390
5	−20	−5	3.91406	373.957	3.64737	11.377	0.06173
10	−20	−5	4.17188	374.201	3.69231	7.156	0.03503
3	−20	5	3.46875	373.844	3.61740	20.016	0.11792
5	−20	5	3.76465	372.851	3.62526	14.131	0.08020
10	−20	5	4.10156	370.553	3.61375	8.973	0.04679
3	−20	20	3.24219	374.337	3.53152	26.167	0.16391
5	−20	20	3.52051	374.832	3.67130	18.924	0.11283
10	−20	20	3.88430	370.241	3.67940	11.818	0.06550
3	−5	−20	3.04688	370.653	3.55911	27.577	0.18349
5	−5	−20	3.37402	370.734	3.58210	19.838	0.12737
10	−5	−20	3.74970	374.086	3.64100	12.436	0.07220
3	−5	−5	2.05566	371.000	3.43067	77.059	0.54887
5	−5	−5	2.37427	374.731	3.47768	58.287	0.40164
10	−5	−5	2.83200	371.940	3.57690	38.897	0.25330
3	−5	5	1.10046	372.039	3.35057	164.192	1.25766
5	−5	5	1.32813	370.375	3.27138	138.432	1.03264
10	−5	5	1.71880	371.714	3.39090	105.273	0.76630
3	−5	20	1.95313	371.401	3.26243	97.132	0.57608
5	−5	20	2.30469	374.799	3.40637	75.955	0.44841
10	−5	20	2.76860	370.922	3.45830	50.201	0.29150
3	5	−20	2.28516	374.587	3.58433	53.647	0.40542
5	5	−20	2.61719	370.617	3.61192	38.945	0.27747
10	5	−20	3.08590	370.240	3.57120	25.975	0.17340
3	5	−5	0.93750	374.998	3.23792	198.195	1.46272
5	5	−5	1.13281	371.793	3.20318	166.355	1.21405
10	5	−5	1.47220	371.937	3.28280	128.406	0.90630
3	5	5	2.00684	373.199	3.41512	88.942	0.57975
5	5	5	2.32910	372.619	3.44496	69.975	0.44691
10	5	5	2.79300	372.764	3.52170	46.135	0.27870
3	5	20	3.02734	373.768	3.49441	41.788	0.23071
5	5	20	3.35938	370.119	3.54345	29.675	0.15421
10	5	20	3.78910	373.827	3.57770	18.600	0.09320
3	20	−20	2.06787	370.786	3.36936	75.818	0.52433
5	20	−20	2.40234	374.921	3.51585	57.552	0.38796
10	20	−20	2.87600	372.828	3.54240	37.541	0.23770
3	20	−5	2.83008	372.491	3.49103	48.342	0.28546
5	20	−5	3.14063	372.259	3.55952	34.547	0.19689
10	20	−5	3.60938	373.228	3.54843	21.902	0.11668
3	20	5	3.32813	374.634	3.61126	32.198	0.16889
5	20	5	3.62695	371.797	3.55382	22.830	0.11937
10	20	5	3.98438	371.464	3.63383	14.129	0.06822
3	20	20	3.75000	373.177	3.53639	21.482	0.09612
5	20	20	4.10889	372.950	3.60229	15.008	0.06754
10	20	20	4.33590	371.541	3.69220	9.110	0.03720

Table C.10: Simultaneous CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 5$, $\eta_0 = 1$, Censoring Rate 5%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	4.42993	374.346	3.69130	5.2903	0.02067
5	−20	−20	4.43359	374.144	3.75166	3.8719	0.01368
10	−20	−20	4.23828	373.124	3.74216	2.6711	0.00793
3	−20	−5	4.60938	372.352	3.67745	5.2046	0.01699
5	−20	−5	4.50928	370.449	3.69438	3.7673	0.01084
10	−20	−5	4.33594	374.598	3.74302	2.5681	0.00676
3	−20	5	4.66797	370.826	3.64919	4.9783	0.01451
5	−20	5	4.50134	372.294	3.64285	3.6098	0.00889
10	−20	5	4.37500	370.295	3.63600	2.4667	0.00605
3	−20	20	4.74609	374.235	3.69787	4.6907	0.01181
5	−20	20	4.41406	372.581	3.67351	3.3847	0.00687
10	−20	20	4.33594	370.495	3.70013	2.3184	0.00509
3	−5	−20	3.34961	374.141	3.64448	20.5192	0.12785
5	−5	−20	3.66211	370.739	3.63142	14.9775	0.08832
10	−5	−20	3.99410	372.184	3.61510	9.2679	0.04980
3	−5	−5	3.11523	370.726	3.54188	33.1632	0.19780
5	−5	−5	3.43750	373.255	3.66728	23.3314	0.12937
10	−5	−5	3.85740	373.958	3.65680	14.7978	0.07820
3	−5	5	3.41797	372.976	3.60356	28.6759	0.14431
5	−5	5	3.74023	372.758	3.59725	20.2309	0.09898
10	−5	5	4.11130	374.828	3.67530	12.4196	0.05670
3	−5	20	3.92578	374.941	3.65845	19.0032	0.08177
5	−5	20	4.19922	373.528	3.62767	13.0233	0.05340
10	−5	20	4.45310	371.202	3.74880	7.8692	0.02990
3	5	−20	3.57422	372.128	3.64192	16.0061	0.09918
5	5	−20	3.82813	373.763	3.69375	11.3504	0.06578
10	5	−20	4.09670	370.569	3.57070	7.2224	0.03790
3	5	−5	3.09570	371.544	3.55186	27.2051	0.17710
5	5	−5	3.42773	370.192	3.60839	19.5986	0.12081
10	5	−5	3.80620	372.690	3.67970	12.3279	0.06890
3	5	5	3.08105	371.863	3.60574	31.2401	0.19290
5	5	5	3.39844	372.146	3.61121	22.7200	0.13589
10	5	5	3.78420	372.636	3.65880	14.0019	0.07670
3	5	20	3.45703	373.333	3.52542	25.1143	0.13852
5	5	20	3.75977	372.692	3.65161	17.6769	0.09288
10	5	20	4.10160	371.905	3.63710	10.8756	0.05230
3	20	−20	4.17969	372.620	3.70285	5.3122	0.02542
5	20	−20	4.21875	370.884	3.68006	3.9362	0.01736
10	20	−20	4.01367	372.237	3.75369	2.7965	0.00964
3	20	−5	4.20898	374.705	3.73408	5.2906	0.02485
5	20	−5	4.23706	372.051	3.65770	3.9410	0.01659
10	20	−5	4.04300	373.035	3.71880	2.7596	0.00910
3	20	5	4.22852	374.704	3.66620	5.1704	0.02416
5	20	5	4.25781	371.858	3.69223	3.8533	0.01595
10	20	5	4.06250	371.667	3.66733	2.6973	0.00874
3	20	20	4.29688	374.909	3.71770	4.9382	0.02126
5	20	20	4.30176	374.817	3.71711	3.6654	0.01435
10	20	20	4.05273	373.906	3.76264	2.5865	0.00782

Table C.11: Simultaneous CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 5$, $\eta_0 = 1$, Censoring Rate 50%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	4.35547	370.782	3.61919	5.4899	0.02280
5	−20	−20	4.38477	373.037	3.65862	4.0075	0.01506
10	−20	−20	4.22120	371.972	3.70160	2.7526	0.00850
3	−20	−5	4.48242	372.691	3.70886	5.5451	0.02155
5	−20	−5	4.45313	372.156	3.65549	4.0291	0.01376
10	−20	−5	4.29690	373.160	3.68320	2.7310	0.00820
3	−20	5	4.51172	370.222	3.62154	5.2821	0.01897
5	−20	5	4.43848	373.134	3.67244	3.8877	0.01213
10	−20	5	4.31640	374.357	3.70120	2.6486	0.00760
3	−20	20	4.57031	373.799	3.69051	4.9931	0.01654
5	−20	20	4.37500	370.167	3.74926	3.6498	0.00967
10	−20	20	4.27730	370.498	3.65800	2.5012	0.00660
3	−5	−20	3.32031	372.408	3.66596	21.5617	0.13772
5	−5	−20	3.63280	372.841	3.61430	15.2991	0.09190
10	−5	−20	3.97460	373.216	3.69180	9.4633	0.05050
3	−5	−5	2.92969	372.409	3.54361	38.0395	0.24310
5	−5	−5	3.25200	370.287	3.53310	27.2733	0.16380
10	−5	−5	3.67190	374.150	3.59970	17.2013	0.09790
3	−5	5	2.77832	373.604	3.50600	45.0071	0.28784
5	−5	5	3.10540	371.933	3.54560	32.1944	0.19260
10	−5	5	3.55960	373.671	3.60830	20.4802	0.11430
3	−5	20	2.92969	371.204	3.47151	37.9245	0.23827
5	−5	20	3.26170	372.979	3.55000	27.6787	0.16620
10	−5	20	3.70120	372.838	3.61920	17.3918	0.09690
3	5	−20	2.63672	370.124	3.51504	44.5623	0.29806
5	5	−20	2.99070	373.437	3.59190	32.7919	0.20980
10	5	−20	3.43750	372.757	3.63870	20.8676	0.12500
3	5	−5	2.46094	370.770	3.51043	58.2476	0.37332
5	5	−5	2.81250	372.473	3.55300	43.7410	0.27390
10	5	−5	3.24220	371.179	3.53850	27.6969	0.16450
3	5	5	2.86133	372.482	3.50901	42.2450	0.25390
5	5	5	3.20313	373.401	3.62652	30.7676	0.18210
10	5	5	3.63280	374.726	3.62100	19.1611	0.10430
3	5	20	3.43750	374.218	3.55235	26.0322	0.13896
5	5	20	3.75980	373.242	3.65060	18.8139	0.09730
10	5	20	4.10160	374.101	3.65910	11.4714	0.05640
3	20	−20	3.85742	370.565	3.60663	15.3700	0.07812
5	20	−20	4.08932	370.106	3.68175	10.7782	0.05204
10	20	−20	4.29690	372.226	3.73100	6.6455	0.02870
3	20	−5	4.04297	374.906	3.68467	11.7814	0.05522
5	20	−5	4.24805	371.052	3.63282	8.2809	0.03681
10	20	−5	4.45310	370.402	3.66590	5.2093	0.02070
3	20	5	4.18823	374.267	3.70848	10.2273	0.04574
5	20	5	4.31832	374.591	3.69371	7.1408	0.02920
10	20	5	4.37500	371.581	3.76990	4.5343	0.01610
3	20	20	4.37500	373.495	3.74483	8.3507	0.03410
5	20	20	4.60938	373.560	3.70623	5.8151	0.02315
10	20	20	4.60940	372.868	3.71880	3.7815	0.01310

Table C.12: Simultaneous CUSUM Chart for the Shape and the Scale Parameters-Threshold, In-control and Out-of-Control ARL and Simulation Errors for $\beta = \beta_0 = 5$, $\eta_0 = 1$, Censoring Rate 80%.

Sample Size	Shift Scale %	Shift Shape %	Threshold h	IARL	ISimErr	OARL	OSimErr
3	−20	−20	4.16016	370.098	3.67966	6.472	0.03154
5	−20	−20	4.27246	373.570	3.74514	4.738	0.02059
10	−20	−20	4.21880	371.138	3.64420	3.186	0.01170
3	−20	−5	4.15039	374.035	3.69767	7.510	0.03744
5	−20	−5	4.27734	374.066	3.72297	5.398	0.02361
10	−20	−5	4.29690	374.694	3.74790	3.609	0.01390
3	−20	5	4.10675	373.737	3.70427	8.197	0.04115
5	−20	5	4.25781	373.416	3.71790	5.833	0.02642
10	−20	5	4.35060	374.995	3.70450	3.820	0.01510
3	−20	20	4.08203	372.025	3.68264	8.869	0.04519
5	−20	20	4.24316	371.149	3.59621	6.291	0.02940
10	−20	20	4.37990	374.088	3.75430	4.088	0.01660
3	−5	−20	3.25195	370.806	3.62536	22.320	0.14507
5	−5	−20	3.56450	373.846	3.65210	16.009	0.09930
10	−5	−20	3.91600	373.157	3.68840	10.032	0.05530
3	−5	−5	2.53906	370.529	3.53624	51.086	0.34869
5	−5	−5	2.87110	370.494	3.54420	37.249	0.24240
10	−5	−5	3.30810	374.503	3.63990	23.858	0.14540
3	−5	5	1.86462	373.707	3.41295	90.159	0.64054
5	−5	5	2.16720	371.481	3.43740	70.521	0.49420
10	−5	5	2.62700	372.554	3.52900	47.660	0.31650
3	−5	20	1.68945	370.613	3.26822	109.972	0.71287
5	−5	20	2.03250	370.202	3.33460	86.349	0.55370
10	−5	20	2.50000	370.291	3.51010	58.500	0.36090
3	5	−20	2.03186	374.809	3.55095	64.519	0.49036
5	5	−20	2.38280	374.102	3.60440	48.524	0.35750
10	5	−20	2.85690	372.553	3.58120	32.286	0.22310
3	5	−5	1.58203	370.797	3.29331	122.775	0.84689
5	5	−5	1.87500	373.261	3.34470	98.325	0.66480
10	5	−5	2.30470	373.088	3.43430	68.545	0.44150
3	5	5	2.45117	371.778	3.43959	62.872	0.38586
5	5	5	2.79300	373.259	3.52850	47.966	0.29030
10	5	5	3.25200	370.747	3.53130	30.256	0.17010
3	5	20	3.27148	370.437	3.55030	34.284	0.18291
5	5	20	3.57420	374.591	3.67420	23.876	0.11930
10	5	20	3.98440	371.719	3.64170	14.810	0.07020
3	20	−20	2.91016	374.825	3.53305	44.252	0.25764
5	20	−20	3.26172	370.806	3.52179	31.794	0.17469
10	20	−20	3.67190	370.564	3.60700	19.891	0.10410
3	20	−5	3.53760	370.415	3.56158	26.525	0.13524
5	20	−5	3.85742	373.514	3.58591	18.634	0.09016
10	20	−5	4.17970	370.736	3.70240	11.409	0.05190
3	20	5	3.87865	396.344	3.84607	20.782	0.09480
5	20	5	4.10156	370.566	3.59556	14.109	0.06254
10	20	5	4.35060	372.212	3.70900	8.632	0.03500
3	20	20	4.10156	370.283	3.62613	15.569	0.06200
5	20	20	4.33594	372.458	3.61459	10.712	0.04035
10	20	20	4.45310	373.480	3.67800	6.552	0.02360