

Chapter 3

Determining Preferred Levels of Tuition and Need-Based Aid Subsidies for Colleges

3.1 Introduction

This chapter analyzes the public provision of higher education subsidies. When voters have a positive utility from the share of college-educated among the young, they may decide to vote for subsidies for students which increase the number of college students. These subsidies are paid for by income taxes that reduce the amount of consumer goods voters can purchase. There are a group of credit-constrained young who must pay high interest rates on student loans. This paper shows that even though voters do not explicitly care about the welfare of poor students, it is preferable for policymakers in the government to choose positive amounts of need-based subsidies which only benefit the credit-constrained young.

An important issue that this paper addresses is how government policy concerning education subsidies will be affected by a rise in the wage premium for a college education. This is an issue that has not been addressed much in the economics literature despite the great amount of research concerning the evidence of and causes of the rise in the wage gap. This paper finds that the tax

rate for education subsidies should fall while need-based aid may increase at the expense of general tuition subsidies.

The rest of this introductory section is devoted to a literature review covering other models of public provision of education, the methods states use to provide education subsidies, and the trends in the provision of tuition subsidies and need-based aid since the 1970s.

3.1.1 The Provision of Public Higher Education

In this paper, I do not explicitly model why voters care about the number of college graduates. This section is devoted to literature which provides rationales for the public provision of education, especially when agents in the economy differ by income.

Fernandez and Rogerson (1995) argue that the middle class may be able to use public subsidies for colleges to expropriate funds from poorer agents who will not be able themselves to attend college. The model consists of agents who live for two periods. However, this is not an overlapping generations setting, so there are no parents to provide education funds. Individuals receive an endowment in the first period equal to y_i . They must decide whether to get an education which costs E units of first-period income. In the second period, they receive $f(y_i)$ if they attend school or y_i if they do not. They must pay for the education out of first-period earnings due to market imperfections which do not allow borrowing from second-period income for education loans. Utility in the model is the sum of the individual's income in both periods.

Fernandez and Rogerson assume that $f(y_i) - E > y_i$ for all individuals, so everyone will want an education. The society may redistribute first-period income using an income tax. The revenue from the income tax is distributed equally to each college student. Redistribution is efficient if it maximizes the number of people who get educations. However, in a poor economy redistribution may reduce the number of students if the tax on the middle class agents is so high that it reduces their after-tax income below E .

In richer economies, it is likely that the middle class will choose a tax rate which excludes the poor from education but forces them to pay taxes for it. Increased inequality increases the chance that such a redistribution from the poor is obtained.

Perotti (1993) incorporates a production externality which encourages middle class agents to vote to provide income subsidies to the poor. As in Fernandez and Rogerson, every agents would like to

attend school, except there is no borrowing allowed for education loans. However, unlike Fernandez and Rogerson, the subsidies raised by a proportional income tax are distributed equally to every agent, regardless of whether the individual actually attends school. Also, there is an externality so that the income of any agent increases as the percentage of educated workers in the economy rises.

Creedy and Francois (1990) find that uneducated workers will support subsidies to workers who attend college if the wages of unskilled workers rise with the proportion of college-educated workers in the economy. In their model, young agents decide to attend college if their net lifetime income is greater than their income without education. Agents differ by an ability variable which determines how much they can earn and ensures that there will be workers who will not find getting a college education to be profitable. By allowing agents to determine whether education is a good investment, Creedy and Francois get interesting comparative statics; it is this aspect of their model which I have used in this paper. For instance, as the returns to education increase, more agents will attend college at a given subsidy; the median voter will reduce the preferred level of subsidy in this case since the number of students will rise.

Each of the papers discussed in this section share an important aspect. There are many possible equilibria in these models, depending on the initial conditions. Fernandez and Rogerson and Perotti limit their model by assuming there are only three types of agents: rich, middle class and poor. Even then, there are many possible outcomes, depending on the overall income of the economy, the relative incomes of each group, the magnitude of the externality (in Perotti), etc. Creedy and Francois consider a continuum of agents differentiated by ability, but they derive results only for simulations using specific parameter values.

3.1.2 State Government Practices in Determining Higher Education Spending

In the United States, higher education is provided both privately and publicly. This complicates the analysis of higher education policies considerably. I will concentrate the following discussion mainly on the determination of subsidies to public universities.¹

The subsidization of higher education comes from both state and the federal government sources. States generally provide a large fraction of the operating expenditures of public colleges and uni-

¹Enrollment in public colleges and universities in the United States for fall, 1999, is projected to be 11,378,000 while enrollment in private colleges will be around 3,154,000. Source: *Digest of Education Statistics 1997*.

versities while the federal government is the main provider of financial aid to families. For instance, in the 1992-93 academic year, state governments provided 53% of the revenue for public colleges.² The federal government generally provided grants and loan subsidies to college students at amounts 3 to 4 times as great as the states; these funds were distributed primarily to lower-income families.³

There are several important aspects of the debate concerning subsidies towards higher education. First, there has been a traditional desire to make the decision to attend college free of concerns regarding ability to pay. This incorporates the idea that all available subsidies should be distributed based on need and that a large amount of financial support from the federal government would be needed. Countering this idea was the initiative by states to subsidize universities so that tuitions to all students would become negligible. This was led by California, which created an education system with three levels of quality. Students would be admitted to one or the other based on ability. Sometimes these policies are characterized as “high-tuition, high-aid” versus “low-tuition, low-aid.” Neither of these policies has been fully implemented since there is a lack of funds, but the data suggest that greater emphasis seems to have been placed on lowering overall tuitions at public universities.

Hearn, Griswald and Marine (1996) attempt to empirically estimate the factors that determine whether a state will have high tuitions and high need-based aid or low tuitions and low aid. They suggest that states generally do not have an integrated process for determining tuition and aid policies. Northeast and Midwest states tend to have higher tuition and aid levels than other states. Their empirical method uses a step-wise regression on cross-sectional data. Among their results is that poorer states will have greater pressure to keep tuitions low; less-educated states have lower tuitions because citizens do not understand the tuition-aid tradeoff; and that states with a higher share of private colleges will have higher tuition levels in the public colleges.

Gold (1990) examines the role that states play in higher education financing. In the 1980s, states began offering college savings bonds and prepayment plans. Higher education is the second largest part of state budgets (after primary and secondary education). Although real spending per student rose 12% between 1983 and 1988, it does not appear that public spending on higher education has benefited from the link between higher education and economic growth in the 1980s. Between 1981 and 1988, tuitions rose an average of 92% while state appropriations rose 63%. This can be compared with the period 1973-1981 when tuitions rose 60% while appropriations rose 146%. Over

²Implicitly, this aid allows universities to charge lower tuitions.

³Source of data: McPherson and Schapiro (1998).

the 1980s, there has been a general trend to use user fees to pay for public expenditures instead of general taxes.

In the 1980s, the wage ratio between college-educated workers and high school-educated workers rose steadily. In 1979, the wage premium was around 1.45; it rose to over 1.6 by 1985 and to 1.8 by 1991. This change was paralleled by an increase in the percentage of young people who enrolled in universities and colleges. In 1979, approximately 35% of young men attended college; in 1992, this number had risen to 46%.

In the United States, government (mostly at the state level) has provided subsidies to students who attend public colleges. How has the level of subsidies changed over the 1980s as the wage premium rose? Mumber and Anderson show that for a sample of 46 states, average tuition rates at public universities rose in real terms in every state except New York. The average rise in real tuition costs was 47% between 1981 and 1991. Another component of government subsidies for higher education is need-based assistance. Mumber and Anderson show that need-based real grant dollars per full-time equivalent (FTE) college student rose in 26 of the 46 states surveyed.

The trend in the 1980s can also be compared to the trend over the 1970s when the college wage premium was falling. The share of public university revenues represented by state funding was 57% in the 1969-70 academic year. This rose to 62% for 1979-80, but fell back to 58% by 1989-90. Meanwhile, federal real spending on subsidies to students rose 96% between 1970-71 and 1980-81 and declined 4.6% between 1980-81 and 1990-91. The decline was attributed mainly to decreases in veteran and Social Security programs. The student loan program represented the largest growth of federal aid. Between 1970-71 and 1980-81, real spending in the loan program grew by 180%, while it grew 32% between 1980-81 and 1990-91. In 1993, the Clinton administration began a policy in which it was hoped that direct student loans from the federal government would issue all new student loans; this would decrease the amount of subsidies being paid to banks which make student loans. However, the federal government's programs have only managed to control 30% of the student loan market.

Other issues relating to government subsidies include the effect of subsidies on the quality of the education received by students and the effect that subsidies actually have at encouraging higher enrollment.

Hilmer (1992) explores the effects of providing public subsidies to both universities and 2-year community colleges. This paper is motivated by the idea that it may be cheaper for state governments

to encourage students to attend 2-year colleges and transfer to universities for the last 2 years instead of attending universities for all 4 years. Hilmer finds that the own-price effect of increasing fees for either type of college is negative while cross-price effects are positive. Thus, increases in university fees encourage more students to attend community colleges. But, it is also possible that fewer students will attend college at all because the increase in university fees will affect them for the last 2 years of their educations. Hilmer finds that for high-income students, increasing university fees increases the probability of attending community college while low-income students have a higher probability of not attending college at all.

Granderton (1992) argues that subsidies to public colleges may reduce enrollment in higher-quality private universities. The switching of students from private to public universities reduces social welfare, except in the case where market failures, such as borrowing constraints, are present. He finds that student ability and family income play large roles in the choice of attending lower quality public schools when tuitions are lower. Students match their ability levels with quality of the university they attend. However, students with lower family income are induced to attend low-cost public schools. College quality is measured by average SAT scores of incoming freshmen, which is more a measure of the quality of the inputs and not necessarily the output.

3.2 The Model

The model which I use in this section is inspired by Creedy and Francois (1990). Young agents use rational-based decision-making to determine whether they will attend college. This allows agents to react to education policies and other related variables such as the return to education. In turn, policies are chosen taking into account the response of the young agents. I expand the model of Creedy and Francois to include a group of credit-constrained young. In addition, I include a group of older agents, who I refer to simply as voters, who determine the level of education subsidies. Creedy and Francois assume that the proportion of educated workers affects the growth rate of the economy; I do not assume this.

3.2.1 The Voters

There are two types of people in the model: voters and the young. Further, as discussed below, the young agents are divided into two groups distinguished by whether or not they receive endowments.

The voters in the model wish to maximize the following utility function:

$$u_i^v = c_i^\alpha S^\beta \quad (3.1)$$

where u_i^v is the utility of voter i , c_i is the level of private consumption of individual i and S is the number (or share) of young agents who get college educations. This is a fairly reduced model of the behavior of voters in an education setting; however, it captures an interest in the education of young agents due to either altruistic or economic reasons and it possesses a positive income elasticity of public education provision.

Voters decide the level of the income tax rate in the economy which is used to finance education subsidies. The voters face the following budget constraint:

$$(1 - t)I_i^v = c_i \quad (3.2)$$

where t is the proportional income tax rate and I_i^v is voter i 's level of income. This income can be considered to be net of all other taxes (or transfers from government) paid by (or to) the voters.

I consider three types of education subsidies. First, the government may provide general tuition subsidies for all students. Also, there are two need-based subsidies: a student-loan subsidy which reduces the interest rate which students must pay for education loans and a need-based tuition subsidy which is directed only to the low-income students in addition to the general tuition subsidy. Below I discuss combinations of the general subsidy with each of the need-based subsidies separately.

3.2.2 The Young Agents

Although the government provides subsidies to increase the number of college students, the determination of S is a result mainly of the decisions taken by the young agents. At the time education decisions are being made, each young agent possesses a level of propensity to earn (which may be thought of as ability) equal to y_i . This is a random variable, so that the young people in the model have different levels of earning ability.

The young want to maximize the present value of their lifetime after-tax incomes. This can be viewed as an overlapping-generations model by assuming the young in this period become the voters in the second period. The utility function of the voters, (3.1), is not inconsistent with allowing agents to maximize their incomes in the first period. Income is generated over both periods of life.

In the first period, the young must decide whether to attend college. Their current and future incomes depend on this decision. If a young person does not attend college, his income in each period is given by:

$$I_1 = (1 - t)y_i \quad (3.3)$$

$$I_2 = (1 - \hat{t})y_i \quad (3.4)$$

where \hat{t} is the tax rate in the next period. Individuals do not know the value of the tax rate in the second period when they are using it to make decisions. I assume below that they simply assume that the tax rate in the next period will equal the tax rate in the first period ($\hat{t} = t$). I show in the appendix that this assumption suggests that young agents reduce their college attendance when the tax rate increases, due to the negative effect on next period's net income. This result is consistent with other work and itself provides a reason for government subsidization of college tuitions (see Johnson, 1984). When agents do not attend college, their income in both periods is simply equal to their propensity to earn.

If a young person attends college, his income is given by:

$$I_1 = 0 \quad (3.5)$$

$$I_2 = (1 - \hat{t})h(y_i) \quad (3.6)$$

The young who go to college receive no income in the first period of life. Their second-period income is a function of y_i . I will follow Creedy & Francois in using the following function for $h(\cdot)$:

$$h(y_i) = (1 + ay_i)y_i \quad (3.7)$$

where a is a positive constant. This functional form gives the return to education as a convex function of y_i . One individual who can earn twice as much as another individual without education will be able to earn more than twice as much if both individuals receive educations. In addition to foregoing first-period income when attending college, individuals also pay a direct tuition cost. The full tuition cost is represented by E . Students pay a fraction of this equal to $(1 - \rho)E$, where ρ is the percentage of tuition costs that is paid by the government.

The young are divided into two groups. Group 1 agents are endowed with the amount E in the first period.⁴ This allows them to pay for college (if they choose to attend) from their initial income.

⁴It may seem highly coincidental that the bequests equal the tuition level. It does not change the results if the bequests are greater than E . Specifically, the value of y_1^* discussed below is not affected by this.

Group 2 agents have no endowment. If they choose to attend college, they must borrow the amount E and repay it in the second period at interest rate \tilde{r} . This interest rate is equal to r^e minus an interest rate subsidy from the government. The term r^e is the interest rate on education loans. It is greater than r , where r is the interest rate on savings and lending for non-education purposes. There are several models of the banking system which imply that education loans will bear a higher interest rate than non-education loans. This is mainly due to the non-transferability of education.

As in Creedy and Francois, there is some critical level of y , y^* , for each group of agents referred to as the “educational choice margin” such that an individual with $y_i = y^*$ will be indifferent between going to college and not. This will occur when the present value of lifetime income is the same regardless of whether the person goes to college. For Group 1 agents, this condition can be written as:⁵

$$(1+r)(1-t)y_1^* + (1+r)E + (1-t)y_1^* = (1+r)\rho E + (1-t)(1+ay_1^*)y_1^*. \quad (3.8)$$

The left-hand side of this equation represents the income of the agent if he doesn't attend college. The first term on the left-hand side is the value of first-period after-tax income; the second term is the value of the endowment; and the last term is second-period net income. The right-hand side represents the income of the agent if he attends college. The first term is the remaining endowment after tuition payment and the second term is second-period income. It is straightforward to solve the above equation for y . The solution is a quadratic with a positive and a negative root. The negative root, however, can be ignored. All type 1 agents with $y_i > y_1^*$ will attend college while all those with $y_i \leq y_1^*$ will not. The number of students from Group 1 is given by:

$$S_1 = N_1 [1 - F(y_1^*)], \quad (3.9)$$

where N_1 is the number of members of Group 1 and $F()$ is the c.d.f. of y . A larger value of y_1^* implies a smaller number of students.

A similar analysis exists for deriving y_2^* , which separates the members of Group 2 who attend college from those who do not. The value of y_2^* solves the following equality:

$$(1+r)(1-t)y_2^* + (1-t)y_2^* = -(1+\tilde{r})(1-\rho)E + (1-t)(1+ay_2^*)y_2^*. \quad (3.10)$$

Again, the solution to the above equation determines the number of type 2 college students:

$$S_2 = N_2 [1 - F(y_2^*)], \quad (3.11)$$

⁵I find it more convenient to represent the second-period value of lifetime income.

Table 3.1: Simulations of y^* with Different Parameter Values

t	0.2	0.2	0.2	0.2
r	0.05	0.05	0.05	0.05
\tilde{r}	0.2	0.2	0.5	0.2
a	0.8	0.8	0.8	0.8
ρ	0.8	0.2	0.8	0.8
E	10	10	10	20
y_1^*	2.5829	4.3379	2.5831	3.3006
y_2^*	2.7012	4.5844	2.9186	3.4723
$y_2^* - y_1^*$	0.1183	0.2465	0.3355	0.1718

where N_2 is the number of members of Group 2.

The members of Group 2 face a higher cost for education as long as $\tilde{r} > r$. This ensures that $y_2^* > y_1^*$, so fewer members of Group 2 attend college. This is a type of inefficiency since second-period aggregate income can be increased by taking the education funds from some of the Group 1 students with low y_i 's and given to members of Group 2 who are on the margin of attending college. Figures 3.1 and 3.2 illustrate the differences between the 2 groups.

Figure 3.1 shows the values of y_1^* (solid line) and y_2^* (dashed line) for different values of ρ (holding all other variables constant). As ρ increases to 1, y_2^* converges to y_1^* ; when $\rho = 1$ there is no difference between the two groups because Group 2 agents do not have to borrow for their educations. Both curves are concave, and the slope of y_2^* is larger (in absolute value).

Figure 3.2 shows the values of y_1^* (solid line) and y_2^* (dashed line) as the interest rate subsidy on student loans increases. Obviously, y_1^* is unaffected by the size of \tilde{r} while the value of y_2^* decreases as the interest rate subsidy for Group 2 rises. Again, when $\tilde{r} = r$ there is no difference between the two groups.

Table 3.1 presents the values of the critical levels of y for different parameter values. The general comparative statics terms are presented in Appendix A. Column 2 of Table 3.1 presents a base parameterization. Here \tilde{r} is greater than r so y_2^* is greater than y_1^* . Although the values of y_1^* and y_2^* do not depend on any specific distribution over the young, it is helpful to compare these results

using the proportion of each group which attends college. Later in the paper, I assume that both y 's are distributed uniformly between 1 and 5. Then, the educational choice margins in Column 2 can be interpreted as 60.4% of Group 1 agents attend college and 57.5% of Group 2 agents attend college.

Columns 3–5 show results of altering some of the parameters. The parameters that are changed are shown in boldface. Column 3 suggests that as the government subsidy of education decreases, the number of students from both groups will decline. The difference between the groups widens as Group 2 agents must borrow a larger amount at the higher interest rate (16.5% of Group 1 agents and 10.4% of Group 2 agents attend college). The fourth column shows the effect of increasing \tilde{r} . This causes y_2^* to rise since the effective cost of education for members of Group 2 will rise (the percentage of agents going to college falls to 52% compared to Column 2); Group 1 is unaffected by the change. Column 5 shows that the number of students declines as the direct cost of education rises. Note that for Group 1 the decline in college attendance in Column 4 is totally due to the increase in the students' cost of education (it rises from 2 to 4, when compared with the second column) and not to the rise in their endowments.

3.3 Government Supply of General Tuition Subsidies and College Loan Subsidies

In this section, I investigate the optimal government policy when the government can provide either general tuition subsidies which pay for a certain percentage of college costs for all students or subsidies to reduce the interest rate on college loans for Group 2 agents.

3.3.1 The Government's Budget Constraint

I assume that governments must balance their budgets in each period. This condition ensures that the following equation must be satisfied:⁶

$$\rho ES + (1 - \rho) \left[\frac{r^e - \tilde{r}}{1 + r} \right] ES_2 = t \left[N_v \bar{y}_v + N_1 \int_0^{y_1^*} y_1 dF(y) + N_2 \int_0^{y_2^*} y_2 dF(y) \right] \quad (3.12)$$

⁶I assume that the government pays its share of the interest on student loans to banks in the first period. Thus, its payment is discounted by $1 + r$.

The terms on the left-hand side of this equation represent the costs to the government of the direct tuition subsidy and the interest rate subsidy, respectively. The right-hand side represents the revenue raised by the tax. The first term in the brackets is the total income of the voters, where N_v is the number of voters and \bar{y}_v is the average income of the voters. The second term is the income of the young in Group 1 who decide not to go to college, and the third term is the income of the young in Group 2 who decide not to attend college.

Voters maximize (3.1) subject to (3.2) and (3.12) by choosing t , ρ , and \tilde{r} . This problem may be simplified analytically by dividing it into a two-part problem. Given any tax rate, voters want the government to use the revenue generated by this tax to attain the largest value of S . This involves trading off the levels of ρ and \tilde{r} . To see this tradeoff from the perspective of satisfying the government budget constraint, consider a small decrease in the level of ρ , holding the tax rate constant. From the l.h.s. of (3.12), the costs of education to the government will fall for two reasons. First, there will be a decrease in the cost of subsidizing each college student. Second, the number of students will decrease as the critical values of y_1 and y_2 rise. From the r.h.s. of (3.12), as y_1^* and y_2^* rise, the number of young who work in the first period will rise. This generates more income for the government. The combination of lower costs and higher revenues implies that \tilde{r} can be lowered until (3.12) is again satisfied.

The analytical solution to this problem involves finding an equation where the following maximizing condition holds when the equalities in (3.8), (3.10) and (3.12) are satisfied:

$$\frac{\partial S}{\partial \rho} = - \frac{\partial S}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial \rho} \Big|_{t=\bar{t}}. \quad (3.13)$$

The justification for this condition can be seen by understanding the implications of changing one of the subsidy variables on the other, as discussed in the previous paragraph.⁷ Suppose the l.h.s. of (3.13) were greater than the r.h.s. Then the government has an incentive to increase ρ . This will force an increase in \tilde{r} to balance the budget. But, the increase in the number of college students from the rise in ρ would more than offset the fall in the number of college students (all from Group 2) caused by the rise in \tilde{r} , so the government has an incentive to increase ρ . Now, a similar argument holds if the l.h.s. of (3.13) were less than the r.h.s. Thus, the government will not have an incentive to alter its policies only if the equality in (3.13) holds.

⁷The problem to be solved may be constrained by the two boundary conditions $0 \leq \rho \leq 1$ and $r \leq \tilde{r} \leq r^e$. When $\rho = 1$ or $\tilde{r} = r^e$, equation (3.13) becomes $\frac{\partial S}{\partial \rho} \geq - \frac{\partial S}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial \rho}$. Alternatively, when $\rho = 0$ or $\tilde{r} = r$, equation (3.13) becomes $\frac{\partial S}{\partial \rho} \leq - \frac{\partial S}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial \rho}$.

This equilibrium condition can also be explained graphically. One can graph in $(\rho, r^e - \tilde{r})$ -space two curves: an “iso-tax-rate curve” which shows the combinations of ρ and \tilde{r} which satisfy the government’s budget constraint and a family of “iso-student curves,” each of which shows the combinations of ρ and \tilde{r} which leads to a given number of students. Greater combinations of ρ and $r^e - \tilde{r}$ cause the number of students to be larger, so the number of students is increasing as the iso-student curve shifts right. The optimal allocation will be a point of tangency of the two curves. The slope of the iso-student curve is $\frac{\partial S/\partial \rho}{\partial S/\partial \tilde{r}}$ and the slope of the iso-tax-rate curve is $-\frac{\partial \tilde{r}}{\partial \rho}\Big|_{t=\bar{t}}$. Equating these expressions and rearranging gives (3.13).

The differential terms in (3.13) can be “solved” in terms of the parameters of the model by a few applications of the implicit function theorem. Specifically, we can use the equality in (3.12)⁸ to find an expression for $\frac{\partial \tilde{r}}{\partial \rho}$:

$$\frac{\partial \tilde{r}}{\partial \rho}\Big|_{t=\bar{t}} = \frac{ES - \frac{r^e - \tilde{r}}{1+r} ES_2 + \frac{\partial S_1}{\partial \rho} [\rho E + ty_1^*] + \frac{\partial S_2}{\partial \rho} \left[\rho E + (1 - \rho) \left[\frac{r^e - \tilde{r}}{1+r} \right] E + ty_2^* \right]}{\frac{1}{1+r} (1 - \rho) ES_2 - \frac{\partial S_2}{\partial \tilde{r}} \left[\rho E + (1 - \rho) \left[\frac{r^e - \tilde{r}}{1+r} \right] E + ty_2^* \right]} \quad (3.14)$$

The derivation of the partial differentials in this equation as well as the partial differential of S with respect to the two types of subsidies can be determined by applications of the implicit function theorem to (3.8) and (3.10). This gives:

$$\frac{\partial y_1^*}{\partial \rho} = \frac{-(1+r)E}{D_1} \quad (3.15)$$

$$\frac{\partial y_2^*}{\partial \rho} = \frac{-(1+\tilde{r})E}{D_2} \quad (3.16)$$

$$\frac{\partial y_2^*}{\partial \tilde{r}} = \frac{(1-\rho)E}{D_2} \quad (3.17)$$

$$\frac{\partial S_1}{\partial y_1^*} = -N_1 f(y_1^*) \quad (3.18)$$

$$\frac{\partial S_2}{\partial y_2^*} = -N_2 f(y_2^*) \quad (3.19)$$

where $D_1 = ((1+r)^2(1-t)^2 + 4a(1-t)(1+r)(1-p)E)^{\frac{1}{2}}$ and $D_2 = ((1+r)^2(1-t)^2 + 4a(1-t)(1+\tilde{r})(1-p)E)^{\frac{1}{2}}$. The values of $\frac{\partial S_1}{\partial \rho}$, $\frac{\partial S_2}{\partial \rho}$ and $\frac{\partial S_2}{\partial \tilde{r}}$ can be determined by applying the chain rule to the appropriate terms above.

⁸equation (3.12) can be considered an identity by recognizing that S , S_2 , y_1^* and y_2^* in that equation are themselves functions of ρ and \tilde{r} .

The first step of the maximizing problem should give the optimal values of ρ and \tilde{r} for any tax rate. Then, voters must only choose the tax rate which maximizes (3.1). I solve this problem numerically by choosing different values for t between 0 and 1. This will give solutions for ρ and \tilde{r} using Equations (3.12) and (3.13).

3.3.2 The Choice of the Tax Rate with Majority Voting

This section offers a discussion of the changes in the equilibrium tax rates and subsidies when the exogenous parameters change. As discussed in Section 3.2.2, the number of college students will be a function of several variables: $S = S(t, \rho, r, \tilde{r}, E, a, F(y))$. Voters attempt to increase S by combinations of subsidies, paid for by income taxes. The question asked here is what effect do changes in the exogenous parameters (E, a, r) have on the tax rate and subsidization rates. First, however, I will discuss the conditions for choosing the tax rate. This will show the conditions on the effect of a tax rate change on the number of students to provide sufficient conditions for a maximum. Also, I show that there will be a median voter defined as the voter with median income, and how the tax rate is affected by a change in the income of the median voter.

Consider the first-order condition for voters choosing a tax rate. Although voter utility is composed of consumption spending and the number of college graduates, each of these is only a function of the tax rate as far as voters are concerned. Therefore, voters are maximizing utility by the choice of the tax rate. Of course, an increase in the tax rate leads to changes in the subsidies and changes in the educational choice margins for each of the types of young agents. But these effects are taken as exogenous by the voters. In this section, when I mention the marginal effect of an increase in the tax rate, I assume that these other variables are being affected. Although I use a partial derivative sign, I mean something a little different; only the exogenous variables in the system are held fixed.

Since both consumption and the number of college students can be defined in terms of the tax rate from the perspective of voters, the maximization of voter utility is determined by the choice of the tax rate. For the first-order condition, the change in the marginal utility of consumption due to a small increase in the tax rate should be equal to the change in the marginal utility of S due to the tax rate increase:

$$U_c \frac{\partial c}{\partial t} = -U_s \frac{\partial S}{\partial t} \quad (3.20)$$

where U_c is the marginal utility of consumption and U_s is the marginal utility of S . Since the marginal utilities are positive and $\frac{\partial c}{\partial t}$ is negative, then $\frac{\partial S}{\partial t}$ must be positive for an interior solution

to exist. If $\frac{\partial S}{\partial t}$ were negative, then voters could attain both more consumption and more students by reducing the tax rate.

The second-order condition is given by:

$$\left[U_{cc} \frac{\partial c}{\partial t} + U_{cs} \frac{\partial s}{\partial t} \right] \frac{\partial c}{\partial t} + \left[U_{cs} \frac{\partial c}{\partial t} + U_{ss} \frac{\partial s}{\partial t} \right] \frac{\partial s}{\partial t} + U_s \frac{\partial^2 s}{\partial t^2} \quad (3.21)$$

where U_{cc} , U_{ss} , and U_{cs} are the second-derivatives of the utility function and use has been made of the fact that $\frac{\partial^2 c}{\partial t^2} = 0$. Given the Cobb-Douglas utility function, the following second-order partial derivatives satisfy:⁹

$$U_{cc} < 0, \quad U_{ss} < 0, \quad U_{cs} > 0. \quad (3.22)$$

For a maximum, (3.21) must be negative. A sufficient condition for this is that $\frac{\partial^2 s}{\partial t^2}$ is negative.

In order to discuss the equilibrium voter outcome, we need to determine if the Median Voter Theorem can be applied. A median voter will exist if, for any tax rate, the voters are ordered in their preferences based on their income. This is similar to the Hierarchical Adherence concept (Roberts, 1977). If one can show that the tax rate is changing monotonically in voter income, the median voter will be the voter with median income; this is important in the simulations where utility levels are compared for a voter with fixed income.¹⁰ Interpreting the first-order condition as an identity, one can derive the relationship between the tax rate and voter income:

$$\frac{dt^*}{dI} = - \frac{U_{cc} \frac{\partial c}{\partial I} \frac{\partial c}{\partial t} + U_{cs} \frac{\partial c}{\partial I} \frac{\partial s}{\partial t} + U_c \frac{\partial^2 c}{\partial t \partial I}}{\Psi} \quad (3.23)$$

where t^* is the preferred tax rate and Ψ equals (3.21). Since the denominator is negative, the sign of this differential depends on the sign of the numerator. Given the specific functional form for the utility function, this equation will be positive if $\alpha > \beta(1-t)S^{-1} \frac{\partial s}{\partial t}$.

Now consider a change in one of the parameters (E, r, a) which causes the number of students to rise. This implies that a greater number of students will be receiving the subsidies and fewer young agents will be working in the first period. Thus, for a given tax rate, ρ must fall and/or \tilde{r} must rise to ensure the government's budget is balanced. Assume that ρ^* and \tilde{r}^* are the student-maximizing subsidies that satisfy the budget constraint at the old tax rate. It must be true that $S(\rho^*, \tilde{r}^*, t; \cdot) \geq S(\rho, \tilde{r}, t; \cdot)$.

⁹These second-order conditions are satisfied by more general utility functions. The discussion of this section extends to these functions.

¹⁰In practice, the preferred tax rate in the simulations is very robust to the choice of the voter's own income.

The increase in S will increase $\frac{\partial c}{\partial t}$ and decrease $\frac{\partial S}{\partial t}$, by (3.22). If $\frac{\partial S}{\partial t}$ is constant,¹¹ then the tax rate must fall to again satisfy the utility-maximizing equation. Since the tax rate falls and the number of students rises, then the subsidization variables must fall to satisfy the government's budget constraint. If, however, $\frac{\partial S}{\partial t}$ increases sufficiently, then the tax rate may not need to fall.

3.3.3 An Initial Simulation

This section presents a solution to the modeling procedure discussed above using a uniform distribution for the “propensities to earn” of the young workers. For both types, $f(y) = \frac{1}{b_2 - b_1}$ where b_1 and b_2 are the support of the distribution. This parametrization is used throughout this paper. The effect of a non-uniform distribution or a different support for each group may be discussed at a future time. Table 3.2 presents the results of the maximization procedure solving for t^* , ρ , \tilde{r} , y_1^* and y_2^* .¹² The second column of the table is a base parametrization. In each successive column, one of the parameters is adjusted; the changed parameter is shown in boldface.

The first thing to note in this table is that the interest rate subsidy is not used in 3 of the 5 simulations, and it is very small in the other 2. In fact, voters would prefer to increase the interest rate on student loans and use the money from this to increase the tuition subsidy. Consequently, the constraint that $\tilde{r} \leq r^e$ is binding. The preference for tuition subsidies versus interest rate subsidies can be partly understood by comparing Equations (3.16) and (3.17). First of all, a small change in the tuition subsidy will always have a larger effect than a marginal change in the interest rate subsidy. This is supported by empirical studies which show a small incentive effect of loan subsidies on encouraging students to attend college (Schwartz, 1985; Miller, 1981). Equation (3.16) suggests that the larger the interest rate on loans, the larger the effect of an increase in the tuition subsidy. Meanwhile, equation (3.17) suggests that when the tuition subsidy is larger, the effect of a decrease in the interest rate will be smaller. Therefore, once a tuition subsidy is enacted, the momentum is to continue to increase the tuition subsidy. The tuition subsidy may be preferred initially because it affects the college populations of both groups of young agents, while interest rate subsidies only affect Group 2.

A comparison of Columns 2 and 3 shows the results of increasing the non-subsidized interest rate

¹¹The term $\frac{\partial c}{\partial t}$ is equal to $-I^v$, which is constant.

¹²Other parameter values used in Tables 3.2 through 3.6: Voter preferences ($\alpha = .9$, $\beta = .1$); median voter income=20; distribution of y ($b_1 = 1$, $b_2 = 5$).

Table 3.2: Preferred Rates of ρ and \tilde{r}

r	.10	.10	.10	.10	.10
r^e	.5	1	.5	.5	.5
a	.8	.8	.8	.8	.8
E	10	10	12	10	10
$N_v \bar{y}_v$	500	500	500	600	500
N_1	10	10	10	10	5
N_2	10	10	10	10	15
t^*	.05	.058	.059	.051	.054
ρ	.443	.518	.491	.488	.483
\tilde{r}	.5	.958	.468	.5	.5
y_1^*	3.608	3.428	3.753	3.496	3.514
y_2^*	4.073	4.293	4.207	3.941	3.962

on student loans. Group 2 agents are discouraged from attending college. Voters respond to this by increasing the tax rate to increase the number of college students. The optimal policy is to use most of the tax revenue to increase the tuition subsidy, leaving the (subsidized) interest rate on student loans high. Group 1 agents benefit from the higher interest rate since the subsidy for them increases.

The effect of higher education costs is shown in Column 4. The higher costs reduce the number of young agents who want to attend college. This causes voters to raise the tax rate. The higher tax rate and the larger number of young agents in the workforce in the first period allows the subsidization rates to rise.

In Column 5, voter income is increased. This leads to a larger amount of tax revenue so that the tuition subsidy can be increased. The last column shows the effect of increasing the share of the young agents who are from Group 2 from one-half to two-thirds. Since a larger share of the young are credit-constrained, the number of students is lower than in Column 2, encouraging voters to increase the tax rate.

3.4 General Tuition Subsidies Versus Need-Based Tuition Subsidies

In this section, governments are allowed to choose between the general type of tuition subsidies discussed in the previous section and need-based tuition subsidies ρ_2 which are provided only to the students in Group 2. The combined value of the subsidies affects the choice of whether Group 2 agents attend college. The value of y_2^* is determined where y_2 solves the following equality (cf. (3.10)):

$$(1+r)(1-t)y_2^* + (1-t)y_2^* = -(1+\tilde{r})(1-\rho-\rho_2)E + (1-t)(1+ay_2^*)y_2^* \quad (3.24)$$

As this equation suggests, Group 2 agents are indifferent as to whether tuition is lowered by a general subsidy or a need-based one, so that response to either will be the same. Technically, this implies:

$$\frac{\partial y_2^*}{\partial \rho} = \frac{\partial y_2^*}{\partial \rho_2} \quad (3.25)$$

3.4.1 The Optimal Mix of Subsidies

The discussion of the optimal values of ρ and ρ_2 as well as the tax rate t^* follows that of the previous section. For any given tax rate, the government chooses the combination of ρ and ρ_2 which maximizes S subject to its budget constraint.¹³ This will be true when the following first order condition is satisfied:

$$\frac{\partial S}{\partial \rho} = -\frac{\partial S}{\partial \rho_2} \frac{\partial \rho_2}{\partial \rho} \quad (3.26)$$

The last term is determined by implicitly differentiating the budget constraint. The algebra of this problem is much simpler than that of the previous section, and the above condition can be simplified as:

$$\frac{\partial S_1}{\partial \rho} = \frac{\partial S_2}{\partial \rho} \left[\frac{ES_1 + \frac{\partial S_1}{\partial \rho} [\rho E + ty_1^*]}{ES_2 + \frac{\partial S_2}{\partial \rho} [(\rho + \rho_2)E + ty_2^*]} \right] \quad (3.27)$$

Table 3.3 presents simulations of this model to find the optimal tax rates and the corresponding values of ρ , ρ_2 , y_1^* and y_2^* . These simulations suggest that the optimal policy is a combination of

¹³The budget constraint is $\rho ES + \rho_2 ES_2 = t \left[N_v \bar{y}_v + N_1 \int_0^{y_1^*} y_1 dF(y) + N_2 \int_0^{y_2^*} y_2 dF(y) \right]$.

Table 3.3: Preferred Rates of ρ and ρ_2

r	.10	.10	.10	.10	.10
r^e	.5	1	.5	.5	.5
a	.8	.8	.8	.8	.8
E	10	10	12	10	10
$N_v \bar{y}_v$	500	500	500	600	500
N_1	10	10	10	10	5
N_2	10	10	10	10	15
t^*	.05	.055	.058	.05	.055
ρ	.365	.373	.428	.405	.373
ρ_2	.15	.252	.125	.149	.149
y_1^*	3.796	3.785	3.927	3.702	3.786
y_2^*	3.857	3.914	4.029	3.733	3.844

both general tuition subsidies and need-based subsidies. A comparison of the second column of Table 3.3 with the same column in Table 3.2 shows that Group 2 agents are relatively better off under the regime of this section while Group 1 agents are worse off. With interest rate subsidies, 34.8% of Group 1 agents and 23.3% of Group 2 agents go to college. But with need-based tuition subsidies, the proportion of Group 1 agents who attend college drops to 30.1% while the percentage of Group 2 agents who attend college rises to 28.6%.

As in Table 3.2, Columns 3–6 in Table 3.3 present simulations with changes in parameters. The results generally those of the previous section, so I will not elaborate on these. However, note that in Table 3.3, where the interest rate on student loans is increased, Group 2 agents are compensated by a large increase in the need-based tuition subsidy. The results is that these agents are not nearly as bad off as in the previous section (and Group 1 agents do not improve by as much).

3.4.2 The Response to a Rising College Wage Premium

In this section, I simulate the reaction of the government to an increase in the college wage premium. I interpret an increase in the college wage premium as an increase in the parameter a .¹⁴

As the returns to college education rise, more young will decide to go to college at the current levels of government subsidies. This puts downward pressure on the size of the subsidies that voters are willing to provide. First of all, for a given level of ρ , the government's expenses will increase as S increases. This will require an increase in the tax rate just to keep the level of ρ constant. Voters are likely to resist a tax increase, particularly if S is rising on its own. And secondly, as S rises, the marginal utility of additional students falls, so that voters reduce the tax rate in order to increase relatively their private consumption.

Table 3.4 presents simulations of the model where government chooses general tuition subsidies and need-based tuition subsidies. These simulations can be compared with Table 3.3. The difference between these tables is the increase in a from .8 to .85.

3.5 Government Maximization of Future Incomes of the Young Agents

Even if we assume that voters wish to maximize the number of college-educated young agents, given the constraints, it is possible that the government may follow a different agenda. In representative democracies, agenda-setting power can be used by a small group of bureaucrats to impress their desires upon the polity. Here I assume that such a group of individuals wish to maximize the future incomes of the young agents, perhaps because this will increase the tax base from which they fund pet projects.

Specifically, I assume that government officials choose ρ and \tilde{r} to maximize the future tax base, while voters still choose the tax rate to maximize their utility function, (3.1). The future aggregate

¹⁴If the college wage premium is defined as the average wage of a college graduate divided by the average wage of a noncollege worker, then a change in any of the parameters which leads to increases in the educational choice margins will increase the premium.

Table 3.4: Preferred Rates of ρ and ρ_2 for Larger Value of a

r	.10	.10	.10	.10	.10
r^e	.5	1	.5	.5	.5
a	.85	.85	.85	.85	.85
E	10	10	12	10	10
$N_v \bar{y}_v$	500	500	500	600	500
N_1	10	10	10	10	5
N_2	10	10	10	10	15
t^*	.043	.05	.053	.045	.048
ρ	.298	.314	.375	.347	.304
ρ_2	.164	.275	.136	.163	.163
y_1^*	3.795	3.773	3.914	3.691	3.789
y_2^*	3.863	3.906	4.017	3.725	3.855

income of the young is given by:

$$S' = N_1 \int_{\underline{y}_1}^{y_1^*} x f(x) dx + N_2 \int_{\underline{y}_2}^{y_2^*} x f(x) dx + N_1 \int_{y_1^*}^{\bar{y}_1} (1 + ax) x f(x) dx + N_2 \int_{y_2^*}^{\bar{y}_2} (1 + ax) x f(x) dx \quad (3.28)$$

An alternative assumption is that the government may wish to maximize the utility of the young agents. Under certain assumptions this is equivalent to maximizing lifetime income. This would require netting out both the direct and indirect costs of education. Since Group 2 agents face a higher cost of education than Group 1 agents (as long as $\tilde{r} > r$), then this assumption would result in less need-based subsidies than I derive in this section by maximizing equation (3.28).

The terms in equation (3.28) can be regrouped as:

$$S' = N_1 \int_{\underline{y}_1}^{\bar{y}_1} x f(x) dx + N_2 \int_{\underline{y}_2}^{\bar{y}_2} x f(x) dx + a \left[N_1 \int_{y_1^*}^{\bar{y}_1} x^2 f(x) dx + N_2 \int_{y_2^*}^{\bar{y}_2} x^2 f(x) dx \right] \quad (3.29)$$

The first two terms in (3.29) are constants, so that maximization of S' is the same as maximization of the term in square brackets in (3.29). The maximization condition for choosing ρ and \tilde{r} is (cf. equation (3.13)):

$$\frac{\partial S'}{\partial \rho} = - \frac{\partial S'}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial \rho} \quad (3.30)$$

The term $\frac{\partial \bar{r}}{\partial \rho}$ is the same as in (3.14). The only difference is the partial derivatives of S'_1 and S'_2 with respect to y_1^* and y_2^* respectively. These equations are given by:

$$\frac{\partial S'_1}{\partial y_1^*} = -aN_1 y_1^{*2} f(y_1^*) \quad (3.31)$$

$$\frac{\partial S'_2}{\partial y_2^*} = -aN_2 y_2^{*2} f(y_2^*) \quad (3.32)$$

One would expect that politicians will choose policies which generate a smaller difference between the educational choice margins of the two groups than that found in previous sections. Since y_2^* is greater than y_1^* in the equilibrium, this would imply a greater use of the interest rate subsidy (and the need-based tuition subsidy when that is used).

I simulate this model for both policy regimes considered above. Table 3.5 shows results when the government chooses between a general tuition subsidy and an interest rate subsidy, and Table 3.6 gives results when the government chooses between a general tuition subsidy and a need-based one. The results in Tables 3.5 and 3.6 can be compared with the results in Tables 3.2 and 3.3 respectively. In each column, the educational choice margins for Group 2 are smaller in Tables 3.5 and 3.6 while the margins for Group 1 are larger. This has been achieved by reducing the general tuition subsidy and increasing the subsidies which benefit only the members of Group 2.

In general, the results of Table 3.5 imply that the interest rate subsidy will be used to a much greater degree than in Table 3.2. Only in the last column of Table 3.5 is there no interest rate subsidy, and the subsidy tends to be quite large. The assumptions of this section imply a great effect on the optimal policy in this case. Table 3.6 also shows that when the government agency maximizes future incomes, there is a larger need-based tuition subsidy and a smaller general subsidy than in Table 3.3.

In this section, there is an inconsistency problem which might cause the government to set a different policy. Since the policies of the government are not set to maximize the number of students, which is what the voters care about, then the voters could set a lower tax rate. In other words, if $\frac{\partial s}{\partial t}$ in (3.20) is lower, the optimal tax rate for a given voter will decline. If the tax rate declines too much, it is possible that to maximize future incomes, the government should follow the policy of maximizing the number of students. This effect on the tax rate is not prevalent in the simulations which I have run in this section, since the chosen tax rates did not change. The difference in the number of students is not large; for example, comparing the second column of Tables 3.2 and 3.5, the number of students declines from 5.8 to 5.78.

Table 3.5: Preferred Rates of ρ and \tilde{r} when Government Maximizes Earnings

r	.10	.10	.10	.10	.10
r^e	.5	1	.5	.5	.5
a	.8	.8	.8	.8	.8
E	10	10	12	10	10
$N_v \bar{y}_v$	500	500	500	600	500
N_1	10	10	10	10	5
N_2	10	10	10	10	15
t^*	.05	.058	.059	.051	.054
ρ	.411	.462	.461	.46	.483
\tilde{r}	.354	.634	.325	.361	.5
y_1^*	3.688	3.572	3.837	3.569	3.514
y_2^*	3.999	4.17	4.13	3.875	3.962

Table 3.6: Preferred Rates of ρ and ρ_2 when Government Maximizes Earnings

r	.10	.10	.10	.10	.10
r^e	.5	1	.5	.5	.5
a	.8	.8	.8	.8	.8
E	10	10	12	10	10
$N_v \bar{y}_v$	500	500	500	600	500
N_1	10	10	10	10	5
N_2	10	10	10	10	15
t^*	.05	.055	.058	.05	.055
ρ	.359	.36	.419	.402	.365
ρ_2	.16	.272	.139	.155	.159
y_1^*	3.809	3.815	3.95	3.71	3.804
y_2^*	3.844	3.884	4.007	3.726	3.838

3.6 Conclusions

This paper has evaluated the optimal policy mix concerning college subsidies when there are a group of students who are credit-constrained. Need-based aid is used to increase the number of students without having to expend funds to subsidize all students. Simulations of several models are used to evaluate the effects of changes in certain parameters.

In Section 3, the government was allowed to use general tuition subsidies and interest rate subsidies for credit-constrained agents. The simulations suggest that the interest rate subsidy in general is not used. In addition, non-credit-constrained individuals benefit from a higher pre-subsidized interest rate on student loans due to the increase in the general tuition subsidy. In Section 4, the government was allowed to use need-based tuition subsidies and the general tuition subsidy. Simulations of this model suggest that need-based subsidies are used to a large extent. In Section 5, it is assumed that government officials are interested in maximizing the young's future aggregate income; this causes the need-based subsidies to be used to a wider extent, as expected. Section 7 shows the effect of increasing the returns to college-educated workers. This is shown to reduce the tax rate and the subsidies provided to college students. Actually, the need-based tuition subsidy is increased so that the general tuition subsidy must be reduced.

Future research in this area may include a formal test of the effect of a changing college-wage premium on college subsidies. In addition, it is possible that altering the distributions of abilities in the two groups of young agents may affect the subsidies. This could tie-in college-level policies and policies for primary and secondary education. One of the main policy issues at the lower levels is finance equalization. The model described in this paper may address the issue of how college subsidies will change if equalization is successful at increasing the distribution of ability among students in poorer households.

Appendix A

A.1 Comparative Statics Terms for y_i^* .

The following equations give the comparative statics effects on y_i^* for changes in the parameters in equations (3.8) and (3.10). Note that in the complete model when t changes, ρ and/or \tilde{r} will also be affected. This is not taken into account in the following. Also, the indirect effects of changes in ρ and \tilde{r} on one another is not taken into account here. The terms $D1$ and $D2$ are defined as $D1 = \sqrt{(1+r)^2(1-t)^2 + 4a(1-t)(1+r)(1-p)E}$ and $D2 = \sqrt{(1+r)^2(1-t)^2 + 4a(1-t)(1+\tilde{r})(1-p)E}$. Using this notation, the values of y_1^* and y_2^* can be written as:

$$y_1^* = \frac{(1+r)(1-t) + D1}{2a(1-t)} \quad y_2^* = \frac{(1+r)(1-t) + D2}{2a(1-t)}. \quad (\text{A.1})$$

Comparative statics terms:

$$\begin{aligned} \frac{\partial y_1^*}{\partial r} &= \frac{D1 + (1-t)(1+r) + 2a(1-\rho)E}{2aD1} > 0 \\ \frac{\partial y_2^*}{\partial r} &= \frac{D2 + (1-t)(1+r)}{2aD2} > 0 \end{aligned}$$

An increase in r increases y_i^* . Since r represents the returns to saving in the first period, the young are more likely to want to save (by working in the first period and not spending their endowments on tuition (type 1 agents)).

$$\begin{aligned}\frac{\partial y_1^*}{\partial \tilde{r}} &= 0 \\ \frac{\partial y_2^*}{\partial \tilde{r}} &= \frac{(1-\rho)E}{D_2} > 0\end{aligned}$$

A change in \tilde{r} has no effect on type 1 agents. An increase in \tilde{r} increases the financing costs of education for type 2 agents, so fewer type 2 agents will get educations.

$$\begin{aligned}\frac{\partial y_1^*}{\partial t} &= \frac{2a(1+r)(1-t)(1-\rho)E}{2a(1-t)^2 D_1} \\ \frac{\partial y_2^*}{\partial t} &= \frac{2a(1+\tilde{r})(1-t)(1-\rho)E}{2a(1-t)^2 D_2}\end{aligned}$$

The sign of the effect of a change in t on y_i^* is positive. Since the tax affects income in both periods, an increase in the tax rate will increase the incentive to get an education because of the first-period reduction in net income. But, the tax will reduce the incentive to get an education due to the reduction in second-period net income. The second effect always dominates in this model.

$$\begin{aligned}\frac{\partial y_1^*}{\partial a} &= \frac{2a(1-t)(1+r)(1-\rho)E}{D_1} - \frac{(1+r)(1-t)D_1 + D_1^2}{D_1} < 0 \\ \frac{\partial y_2^*}{\partial a} &= \frac{2a(1-t)(1+\tilde{r})(1-\rho)E}{D_2} - \frac{(1+r)(1-t)D_2 + D_2^2}{D_2} < 0\end{aligned}$$

Since $D_1^2 > 2a(1-t)(1+r)(1-\rho)E$ and $D_2^2 > 2a(1-t)(1+\tilde{r})(1-\rho)E$, these relations can be signed negatively. As expected, an increase in a will encourage more young to get an education.

$$\begin{aligned}\frac{\partial y_1^*}{\partial \rho} &= \frac{-(1+r)E}{D_1} < 0 \\ \frac{\partial y_2^*}{\partial \rho} &= \frac{-(1+\tilde{r})E}{D_2} < 0\end{aligned}$$

An increase in the government's share of tuition payments will increase the number of students.

$$\begin{aligned}\frac{\partial^2 y_1^*}{\partial \rho^2} &= -2a(1+r)^2(1-t)E^2 D_1^2 < 0 \\ \frac{\partial^2 y_2^*}{\partial \rho^2} &= -2a(1+\tilde{r})^2(1-t)E^2 D_1^2 < 0\end{aligned}$$

There is a concave relationship between the educational choice margins and the tuition subsidy. See Figure 3.1.

$$\begin{aligned}\frac{\partial y_1^*}{\partial E} &= \frac{(1+r)(1-\rho)}{D_1} > 0 \\ \frac{\partial y_2^*}{\partial E} &= \frac{(1+\tilde{r})(1-\rho)}{D_2} > 0\end{aligned}$$

An increase in the tuition amount reduces the number of college students.

A.2 Multi-Government Subsidization of Education.

In this appendix I analyze a model where there are two governments deciding education policies. The first government chooses between general tuition subsidies and need-based subsidies while the second government uses only a student loan subsidy. This framework is similar to the structure of the United States government where states determine tuition rates and the federal government sponsors student loan initiatives. Consequently, I call the government that provides student loan subsidies the federal government and the government that provides tuition subsidies the state government.

I present a simulation of a model of this type of regime. I assume that the federal government acts “first” by implementing an optimal student loan subsidy. This subsidy optimizes the utility of the median voter and obeys a budget constraint.¹ The federal government does not consider the reaction by the state government or even the fact that the state government will have to raise taxes to cover the increased subsidies at the current subsidization rate (student loans will increase

¹The federal government’s budget constraint is $(1-\rho-\rho_2) \left[\frac{r^e-\tilde{r}}{1+r} \right] = t \left[N_v \bar{y}_v + N_1 \int_0^{y_1^*} y_1 dF(y) + N_2 \int_0^{y_2^*} y_2 dF(y) \right]$, where t here is the combined tax rate of the federal and state governments.

the number of Group 2 students at the current tuition subsidy rate). The state then responds to the student loan subsidy by adjusting its policies on tuition subsidies. The state does not itself take into account the effect this has on the federal budget. So, I assume in the end that the federal budget will be unbalanced.

I begin the simulation with the numbers in the first column of Table 3.3. Here, the state government has determined its tuition subsidies under the assumption of no student loan subsidies. Given these numbers, the optimal federal policy is to provide an interest rate subsidy so that Group 2 students pay 17

The results of the simulation suggest that (a) the federal government will provide a generous student aid plan (compare with the first column of Table 3.2), (b) states respond to the federal government's action by reducing tuition subsidies, and (c) states reduce need-based subsidies more than general subsidies.

A.3 Figures

Figures 3.1 and 3.2 represent the “educational choice margins” for young agents for different levels of tuition subsidies and interest rates on student loans, respectively.

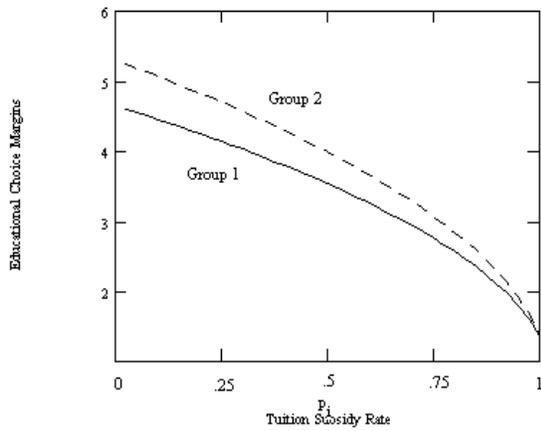


Figure 3.1. Effect of Tuition Subsidies on the Educational Choice Margin

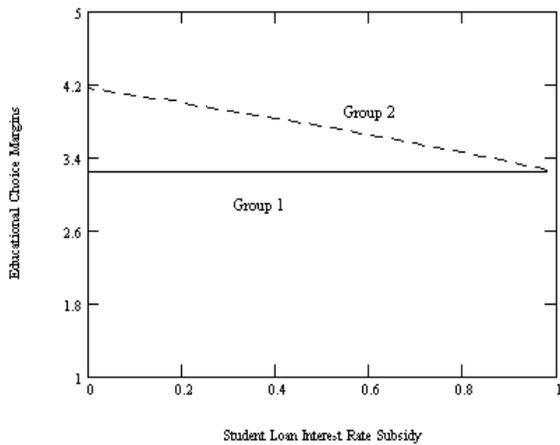


Figure 3.2. Effect of Student Loan Subsidies on the Educational Choice Margin