Chapter 4

The Returns to Aggregated Factors of Production when Labor Is Measured by Education Level

4.1 Introduction

The goal of this paper is to provide an estimate of the productivity of different types of factors of production. A considerable debate concerning the productivity of public capital[1] has brewed since the latter part of the previous decade. Several articles have found that public capital stocks increase the productivity of the economy to such an extent that the decrease in public spending in the 1970s can explain (at least in part) the decline in the growth rate of the US economy since that time. There is definitely a correlation between public spending on infrastructure and the overall growth rate of the economy, but the causation could easily run either way. Indeed, much of the more recent literature soundly finds an empirically insignificant (and often negative) elasticity of public capital in state-wide and in industry-wide production. For a brief summary of this debate, see Holtz-Eakin (1994).

This paper will also attempt to measure the different returns to labor with different levels of educational attainment. One motivation for this is to provide evidence of whether state governments should alter their current spending on education. It has been argued that governments should engage in education finance for both efficiency and equity reasons.[2] For dynamic efficiency, it is required that resources should be invested in activities that will provide a higher return (in present value) than if these resources were

[1] Public capital is generally defined as the amount of publicly-financed infrastructure in an economy such as transportation, sewer and buildings.

[2] For reasons why public education supports equity, see Stiglitz (1974) and Glomm & Ravikumar (1992). But others question the equity of certain public education programs. See Fernandez & Rogerson (1995), Edlin (1993), and Hoxby (1996) for arguments against equity in college tuition subsidies, financial aid programs and school finance centralization, respectively.

consumed today. The students who receive the education will benefit in the long run; this is especially true for post-secondary education since individuals who choose to continue their education are supposedly basing this decision on a cost-benefit analysis. But the other citizens may also benefit from the additional future tax revenues that will be collected even though they bear a burden in financing the education subsidies.

In this study, I use an aggregate production function based on data for the period 1980-1992 for the contiguous 48 states to estimate the marginal returns to workers from different education groups. I assume that an economy has an aggregate production function where the amount of inputs--private capital, public capital, and labor--determine the total private output (GDP) of the economy. I base this method on recent research that has investigated the returns to public infrastructure spending using an aggregate production function.

Holtz-Eakin (1994) and Evans & Karras (1994) use the approach which I follow. Both studies use aggregate production functions of the following form:

$$y_{st} = a_0 + a_1 k_{st} + a_2 l_{st} + \sum_{n=1}^{N} b_n g_{nst} + \varepsilon_{st} \tag{4.1}$$

where y_{st} is the logarithm of private output in state s in period t, k_{st} is the logarithm of private capital in state s in period t, l_{st} is the logarithm of the number of workers in state s in period t, g_{nst} is the logarithm of public capital of type n in state s in period t. In Holtz-Eakin, all public capital is aggregated into one variable, the value of the stock of public capital.[3] Evans & Karras use different levels of aggregation and include current spending variables.

One of the innovations of these two papers compared with similar research is the way that the error term is specified. The error term can be written as:

$$\varepsilon_{st} = f_s + \gamma_t + \mu_{st} \tag{4.2}$$

where f_s represents a state-specific fixed or random effect, γ_t represents a fixed time effect, and μ_{st} is an IID error term. This decomposition of the error term into state-specific and time-specific effects

[3] Holtz-Eakin(1994) also uses a measure of public capital which includes only infrastructure capital such as roads and sewers. This does not have much effect on the results.

highlights the fact that ε is unlikely to be IID with mean zero for a sample of US states. The states differ in the levels of unmeasured productivity, and productivity across states is likely to change over time due to either technological progress or business cycle shocks. Holtz-Eakin points out that the state-specific effect may be either a fixed effect or a random effect. For a fixed effect, the data can be differenced over time for each state;[4] for a random effect, GLS should be used since the error terms in the regression would be heteroscedastic and autocorrelated. Evans & Karras suggest that μ should be modeled as autoregressive.

The results of Holtz-Eakin and Evans & Karras suggest that public capital is not productive. The elasticities of public capital represented by the estimates of the b_n's are consistently negative and insignificant. Evans & Karras find that the only significant public spending variable is current education spending. But, the modeling of the returns to public education spending is somewhat different from the modeling of the returns to public physical capital. While spending on physical capital increases the stocks of different types of infrastructure which can be considered factors of production in their own right, spending on education helps to create private inputs, i.e. educated workers embodied with human capital. Current education spending would most likely give a positive return under two circumstances: first, public spending on education is positively correlated with current output, which would mean that the estimates are biased due to endogeneity; second, current education spending is correlated with past education spending, which has created an increase in the number of high-educated workers, which increases the output of the state.

Mulligan & Salai-i-Martin (1995) attempt to measure human capital in use in a state by taking the ratio of wages of workers to the wages of zero-schooling workers. This ratio represents the relative amount of productive human capital assuming that all labor types are perfect substitutes and that relative changes in wages are not due to any factor other than the change in human capital levels. The first assumption will be discussed further below. The second assumption may seem difficult to accept given the large amount of theoretical and empirical work which has explained the change in the wage premium as a product of changing returns to skill due to technological innovations or changes in terms of trade.

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[4] I use two methods of calculating the fixed effects model. In one method, I transform the variables x_{st} into $x_{st} - \bar{x}_s + \bar{\bar{x}}$, where $\bar{x}_s$ is the average value of the variable in state s and $\bar{\bar{x}}$ is the average value of the variables over the entire sample. The second method involves adding dummy variables for each state. The two methods give the same estimates.

Mulligan & Sala-i-Martin argue that schooling in itself is not perfectly correlated with the stock of human capital since technological shifts alter the role of education as a means of directly increasing the market productivity of workers. In addition, they argue that one cannot compare the education quality of workers in different states because the education production function will have different inputs in different states, i.e. states with larger stocks of human capital will produce additional human capital more efficiently. They cite Card & Krueger (1992) which shows that workers who have moved from high-income states with large expenditures on education have higher wages than workers who move from low-income states. However, I do not believe Card & Krueger have taken into account sufficiently the incentives for workers to migrate. Workers from high-income states are likely to move only if their wages will be high, while workers from lower income states will migrate in response to jobs with relatively lower wages. This endogeneity may explain Card & Krueger's result, which is not generally supported in the vast literature (Hanushek).

4.2 Aggregate Output and Education-Grouped Workers

In models of aggregate output using equation (4.1), all types of labor are treated alike and lumped together into one variable. A more general model of aggregate production would disaggregate labor into different skill groups. Specifically, workers with different levels of education should be allowed to have different effects on output. In the following estimation models, I follow this approach. I divide the work force into 3 education groups: (1) workers who have not completed high school, (2) workers who completed high school but did not complete 4 years of college, and (3) workers who have completed 4 years of college or more. See the section on data sources for the derivation of the data on these variables. I label the groups H1, H2 and H3 respectively.

In equation (4.1) where only aggregate labor is considered, one implicitly constrains the elasticities of the different education groups with respect to output to be equal and constrains the groups to be perfect substitutes in production. (The technical rate of substitution between different education groups is 1). That is,

$$Y_{st} = A_{st} K^{a1} G^{a2} (H_1 + H_2 + H_3)^{a3} \tag{4.3}$$

Below, I assume the aggregate production function takes the following form:

$$Y_{st} = A_{st} K_{st}^{\alpha 1} G_{st}^{\alpha 2} H_{1st}^{\beta 1} H_{2st}^{\beta 2} H_{3st}^{\beta 3} \tag{4.4}$$

Taking the logarithm of the above equation gives a linear equation in the parameters:

$$y_{st} = \alpha_0 + \alpha_1 k_{st} + \alpha_2 g_{st} + \beta_1 h_{1st} + \beta_2 h_{2st} + \beta_3 h_{3st} + \varepsilon_{st} \tag{4.5}$$

where the h_i's represent the logarithm of the number of workers of different education levels in state s at time *t*. With this functional form, the technical rate of substitution is constrained to be bi/bj hj/hi for any two groups of workers. However, the elasticity of substitution between groups is constrained to be equal to 1. This constraint may be too strong given that many empirical studies have found that skilled labor is less easily substitutable for physical capital than is unskilled labor (Hamermesh, 1986, p. 461). I investigate other functional forms below.

The use of equation (4.1) instead of equation (4.5) in a regression can lead to biased and inconsistent estimates of all the parameters. To see this, consider a simplified example where there is only one capital variable and two subdivisions of labor. Suppose the true model of production is given by:

$$y = \beta_1 k + \alpha_1 h_1 + \alpha_2 h_2 + u$$

but the estimated equation is:

$$y = \beta_1 k + \beta_2 l + \upsilon$$

where $L = H_1 + H_2$, and the small letters in the formulas dentoe logarithms. The estimate of β_1 is equal to $\frac{\sum ky \sum u - \sum kl \sum yl}{\sum kk \sum ll - \left[\sum kl\right]^2}$. Replacing *y* with the r.h.s. of (7) and simplifying (assuming both *k* and *y* are orthogonal to the error term *u*) gives:

$$\hat{\beta}_1 = \beta_1 + \frac{\alpha_1\left[\sum kh_1 \sum ll - \sum h_1 l \sum kl\right] + \alpha_2\left[\sum kh_2 \sum ll - \sum h_2 l \sum kl\right]}{\sum kk \sum ll - \left[\sum kl\right]^2}$$

The direction of the bias will be determined by the signs of the terms in the brackets. It is unlikely that the bias will converge to zero as the number of observations increases since the terms in the summations will converge to the respective variances and covariances of the terms. Thus, the estimated parameters will be inconsistent.

4.3 Deriving Marginal Products of Labor

The estimated parameters in equation (4.5) will give the elasticities of the different inputs. However, one is usually more interested in the structure of the marginal returns to labor since this should correspond, in a competitive economy, to the wages received by workers.

Assuming that the production function is accurate, we may derive the imputed marginal products of workers in each state based on the elasticity estimates. For a given education group, the marginal product in state s is:

$$mp_{ist} = \beta_i Y_{st} \;/\; H_{ist} \tag{4.6}$$

Wages in different states differ by either (a) an error term which affects output or (b) differences in levels of human capital. Wages within states differ over time due to either (a) changes in output (b) changes in levels of human capital or (3) changes in β.

The differences in mean incomes across education groups is often reported. The numbers in Table 4.1 show the mean yearly incomes of different education groups for 1990:[5]

Table 4.1: Mean Yearly Income in 1990 (1992 $)

not high school graduate	high school graduate	associate degree	bachelor's degree	professional	doctorate
10,974	17,397	24,090	31,910	71,205	58,269

[5] These data are from the *Statistical Abstract of the United States, 1992/93*. The data were converted from monthly income by multiplying by 12; the GDP deflator was used to transform the data into 1992 dollars.

4.4 Data Sources

The data used in this study consists of observations of the following variables for each of the 48 contiguous states for the years 1980 to 1992: gross state private output, prvate capital stocks, public (state and local) capital stocks, private employment, and employment of workers with certain educational attainments. The gross state product data were taken from the Bureau of Economic Analysis's Survey of Current Business. Private capital stocks were constructed from information primarily from the Economic Censuses following the procedure described in Munnell (1990). Private employment by state was taken from the STAT-USA files of the US Department of Commerce.

4.4.1 Construction of Aggregate Variables

Public capital stocks were constructed following closely the procedure in Holtz-Eakin (1993). Data on the value of state and local capital outlays were collected from issues of *Governmental Finances* for the years 1959-1992. These data were deflated using the deflator for state and local government fixed capital provided by the BEA. Aggregate data on state and local government capital stocks were also provided by the BEA. The capital stocks for each state were calculated using the following perpetual inventory accounting equation:

$$K_t = \sum_{s=0}^{t-1959} (1-\mathrm{d})^s I_{t-s} \tag{4.7}$$

where I represents the capital outlays in each year and δ represents a discount rate. The discount rate was imputed by equating the growth rate of aggregate capital stocks over the 1980-90 period using the above equation to the growth rate of aggregate capital stocks over the same period using the data from the BEA. The discount rate represents the combined effects of service lives, discard rates and depreciation, with the latter being downplayed.[6]

[6] Capital outlays represents net investments in public capital stocks. Specifically, capital outlays is classified such that it "Includes amounts for additions, replacements and major alterations to fixed works and structures. However, expenditure for repairs to such works and structures is classified as current operation expenditures as are payments on operating leases." (Governmental Finances: 1991-92)

The educational attainment variables were constructed for each state using data from the March supplements of the Current Population Survey (CPS). Five variables were extracted from the data: employment status, state of residence, age, educational attainment, and an individual weighting variable. Age was restricted to be between 16 and 70. For the years 1980-1991, educational attainment was constructed based on questions as to the last year of school attended and whether the individual completed the last year. I divide observations into the educational attainment categories in the following manner. Observations which reported education levels of less than 12 years were assigned to the "less than high school" group; observations with 12 to 15 years of schooling were classified as "high school graduate or some college"; observations with 16 or more years of schooling were classifed as "4-year college graduates." Obviously, there is some potential for error in translating reported years of schooling into levels of schooling completed. In 1992, the CPS questionnaire was altered so that individuals reported their highest level of schooling completed. The work force was comprised of all observations who reported being either working or "with job, not at work."

4.4.2 Summary Statistics for Education Group Variables

Table 4.2a summarizes the growth rates of the three education worker groups over the 1980-1992 period. On average, the growth rate in the number of workers with less than high school has declined 2.68% between 1980 and 1992, with the number of workers with high school diplomas or 4-year degrees has risen 2.17% and 3.975%, respectively. There is, however, much deviation from this mean, especially within states. Large changes may be due to 3 reasons: (1) changes in the number of workers of each type due to matriculation, retirement, or migration, (2) changes in the employment of workers over the business cycle, and (3) measurement error in the variables. The "between min" and "between max" give the minimum and maximum state averages respectively. The "within min" and "within max" give the smallest and largest deviations between an observation and the difference between the state mean and the total sample mean.

Table 4.2b shows the correlations between the growth rates of real private gross state product and the growth rates of the labor groups. There are positive correlations between the growth rate of state GDP and the growth rates of the labor variables.

Table 4.3a presents statistics on the share of the work force that is represented by each education group. On average, 61.4% of workers had between 12 and 15 years of education. This ranges from a low of 50.93% in North Carolina in 1980 (30% of the work force had less than 12 years of schooling) to a high of 72.04% in Wyoming in 1993. On average, 21.7% of the work force had 16 or more years of education. West Virginia had the lowest percentage in 1980 (9.54%), while Massachusetts had the highest percentage of college-educated workers in 1991 (35.28%).

Table 4.3b presents the growth rates in the share of the work force that is represented by each education group. On average, the share of workers with less than high school has been declining, while the shares of the other groups have risen.

4.5 Empirical Results

I begin by comparing the results I obtained by following the methods of Holtz-Eakin (1994). Table 4.4 summarizes Holtz-Eakin's results and Table 4.5 presents my results using similar methods. The difference between the two tables is due to the different years of the panel data. Holtz-Eakin uses 1969-1986 data, and I use 1980-1992 data.

The columns in Table 4.4 refer to different empirical procedures. OLS gives the results of a least-squares regression on all the data in the panel. FIX refers to a fixed effects regression where state-specific fixed effects are controlled. The LONG model transforms the variables by taking the difference between the values in the last year in the sample and the first year. Then OLS is run on these transformed variables (there are 48 observations). GLS stands for generalized least squares, where state-specific effects are random, affecting the variance of the error term. IV is an instrumental variables model; it uses a measure of the neighboring states' levels of public spending to instrument for the state's own public spending, which is likely endogenous.

Holtz-Eakin finds the GLS model to be inconsistent because a Hausman test can reject the hypothesis that the error term and the independent variables are correlated. Note that the main contribution of Holtz-Eakin is the finding that the empirical model can greatly affect the estimated elasticity of public capital on output. The OLS model gives a large positive elasticity of public capital, while the fixed effects model (as well as others) gives an insignificant (negative) elasticity.

Table 4.5 presents my results based on following the procedures used in Holtz-Eakin with data from 1980 to 1992. For the IV model, I instrument for public capital using the average public capital of the states in the same geographic subregion as defined by the Census instead of neighboring states. The last column gives the results of fixed effects regressions using the IV model.

Comparison of the standard OLS regressions and the fixed effects models in Table 4.5 suggests that the assumption of state fixed effects increases the elasticity of output with respect to private capital, but greatly reduces the elasticity of output with respect to public capital. The high elasticity with respect to private capital in the fixed effects models is consistent with the results on convergence among states in Barro and Sala-i-Martin (1991). They find that the rate of convergence across the states is approximately 2 percent which would imply an elasticity of .8 on capital in a Cobb-Douglas production function.

4.5.1 Pooled OLS Estimation

Column 1 of Table 4.6 provides the estimates of the parameters in equation (4.5) under the simplifed assumptions that the disturbance term is distributed normally and is homoskedastic with no serial correlation:

$$\mathbf{ee'} = \sigma^2 \mathbf{I} \tag{4.8}$$

where $\boldsymbol{I}$ is a nT x nT identity matrix.

The marginal products of the education groups implied by these estimated parameters are given in Table 4.7. This model suggests that the returns to workers with less than high school education is greater than the returns to workers with high school degrees and some college. The returns to college educated workers is much greater than the other two types.

As mentioned in the introduction, the specification of the disturbance term used in this model may be incorrect. I discuss changes in the estimation model in the next two sections. In the rest of this section, I continue to assume the disturbance structure in equation (4.8). In the remaining columns of Table 4.6, I present estimates of the parameters assuming different models for aggregate output. Each of these models is shown to have greater predictive power than the simple log-linear model. (The comparisons of these models are made using F-tests, so that the assumption in equation (4.8) is still being implied).

4.5.2 Fixed Effect and Random Effect Models of Aggregate Output

In this section, I present estimates of the elasticities and marginal products of the inputs into aggregate output using the assumption that there are state-specific fixed or random effects. These estimates are provided in Tables 4.8 and 4.9.

For the fixed effect model, the error term is specified as in equation (4.2). The fixed effect model predicts a much smaller estimate of the marginal product of college-educated workers than the pooled OLS model, while the marginal products of the other labor groups are not very different. The fixed effect model assigns a much greater return to private physical capital than does the OLS model.

The random effect model assumes the disturbance term takes the following form:

$$e_{it} = u_i + \upsilon_{it} \tag{4.9}$$

where u_i is a state-specific component of the error term. The random effect model is similar to the groupwise heteroscedastic model discussed above, except that the expectation of the disturbance in the heteroscedastic model is assumed to be zero while it is equal to the expectation of u_i in the random effect model. The fixed effect model's estimates of elasticities and marginal products are given in Column 2 (labeled GLS) of Tables 4.8 and 4.9 respectively. The imputed marginal product of the college-educated workers is greater than the estimate of the fixed effect model, while the marginal products of the other groups is slightly larger.

One problem with the random effect model is that if the state-specific effect is correlated with any of the right-hand side variables in the regression model then the error term is correlated with an independent variable and the estimates will be inconsistent. This is not true of the fixed effect model where the state-specific effect is not part of the error structure. A Hausman test rejects the similarity of the parameters in the fixed effect and random effect models (the chi-squared statistic is 499.89, with 5 degrees of freedom), so that the random effect estimates are are assume to be inconsistent (assuming the model itself is not misspecified).

There is also concern using the fixed effect model that certain independent variables may be correlated with past values of state output. For instance, it is likely that the level of public capital spending in a given state is a function of output (or lagged output) since tax revenues to fund spending will increase when output is high. Since in the fixed effects model we transform each variable *x* into $x_{it} - \bar{x}_i$, it is likely that g_{it} and $\bar{g}_i$ are both correlated with $\bar{e}_i$. This produces a bias in the estimated parameters. Since government capital should be positively correlated with $\bar{e}_i$, we expect the estimated elasticity of public capital to be less than the true value. The parameters of the other independent variables will be biased upward if they are positively related to public capital (see Nickell, 1983).

It is also likely that the number of workers in each education category will be affected by past levels of state output. This is because state and local education subsidies may encourage citizens to extend their education. We expect that the number of workers with 4-year college degrees or more will rise and the number of workers with less than high school will decline if more resources are used in education programs. A positive correlation between the number of higher educated workers and past levels of output will bias the estimate of highly educated labor's elasticity in the fixed effect model downwards, while the negative correlation between low-educated workers and past output will bias the estimated elasticity of low-educated workers upwards.

One way to take into account this endogeneity is to instrument for the right-hand-side variables. However, finding good instrumental variables proves difficult. One candidate for an instrumental variable is the average levels of the right-hand-side variables of each state's neighbors. It is likely that public infrastructure spending is correlated across neighboring states. In addition, neighboring states may have similar education policies. The actual instruments I use are the average levels of each variable in the states in the same (Census) subregion. For example, the instrument for the log of public capital in New York in year t is the average level of the log of public capital in New Jersey and Pennsylvania in year t. Another instument for the labor variables is the average of the number of each type of workers in other states with similar proportions of college-educated members of the labor force in 1980. However, neither of these instruments give significant results.

Another way of taking advantage of the state-specific fixed effect specification without having to worry about the transformation used above is to take a long difference of all the variables by subtracting the first year's observation from the last year's. This removes the fixed effect and the span of the difference is great enough to reduce the inconsistencies caused by the presence of serial correlation between the right-hand side variables and past values of GSP. In addition, if the error term follows an AR1 model, the long difference will be enough to reduce the correlation between the two end-point error terms significantly. However, this reduces the number of observations considerably (from 624 to 48) along with all the information that those observations carried.

The third column of Tables 4.8 and 4.9 (labeled LONG) presents the estimated elasticities and marginal products, respectively, of the long-differencing model. The marginal products of the two less-educated worker groups are smaller than in the fixed effect model. The marginal product of the college-educated workers is slightly greater.

4.5.3 Groupwise Heteroscedastic and Autocorrelated Errors

In this section, I consider two revisions of the structure of the disturbance term. The first of these revisions allows the disturbance term to have different variances for each state. I test whether this assumption is valid and present estimates of the elasticities and marginal products under this assumption. The second revision allows the disturbance term to be autocorrelated. A third likely assumption is that the error term is spatially correlated; that is, in any given year, the shock to one state's economy may be correlated with the shock to a neighboring state's economy. However, I do not have enough time periods to model this type of serial correlation.

It is possible that the error variance-covariance matrix exhibits groupwise heteroscedasticity. This is likely given that the dependent variable (the logarithm of real private gross state product) takes on very different values across the states. Table 4.10 shows some distributional statistics of the predicted standard errors for each state using the estimated errors from the pooled OLS regression. It seems that states with smaller GSPs have greater standard errors of the disturbance term. The disturbance term in this model represents a random percentage change in output. It could be that smaller, more specialized economies are more affected by any given shock to the economy.

The first column of Table 4.11 presents the estimated elasticities of the log-linear aggregate production model (see equation (4.5) above) allowing for an error variance-covariance structure of the following form (cf. equation (4.8)):

$$\mathbf{ee}' = \Sigma \otimes \mathbf{I} \tag{4.10}$$

where Σ is the *n x n* diagonal matrix of state-specific error variances and **I** is a *T x T* identity matrix . Otherwise, the assumptions of the error terms are similar to the Classical model; there is no serial correlation over time within each state and there is no cross-correlation between the states. This model is estimated using a two-step Feasible Generalized Least Squares (FGLS). In the first step, the state-specific error variances are estimated from OLS residuals. The second step is application of GLS.

The first column of Table 4.12 shows the marginal products and their standard errors using the delta method for the heteroscedastic error model. These marginal products suggest a larger return to high-school educated workers and a lower return to college-educated workers than the pooled OLS model.

In much of the empirical work on aggregate output, it is assumed that the error term has an autoregressive structure (see, e.g., Evans & Karras, 1994). This will lead to biased estimates of the parameters if lagged values of the dependent variable appear as right-hand side variables. In the model I consider here, lagged values of aggregate output do not appear directly as explanatory variables. However, it is possible that some (if not all) of the right-hand side variables are related to lagged values of output. Specifically, public capital stocks may be correlated with lagged values of output if higher levels of output generated a larger government budget, which was used to produce more capital. Likewise, larger budgets may be used to increase education expenditures. This may affect the number of workers in each education group. For the simplest case, consider the following model:

$$\begin{aligned} y_t &= \alpha x_t + \varepsilon_t \\ x_t &= \beta y_{t-1} + \upsilon_t \\ \varepsilon_t &= \rho \varepsilon_{t-1} + u_t \end{aligned} \tag{4.11}$$

Given this structure and assuming stationarity, the covariance of x_t and ε_t will be given by:

$$\operatorname{cov}(x_t, \varepsilon_t) = \frac{\rho\sigma_u^2}{(1-\beta\alpha\rho)(1-\rho^2)} \tag{4.12}$$

and the probability limit of the estimate of α will be biased:

$$\operatorname{plim}\hat{\alpha} = \alpha + \frac{\operatorname{cov}(x_t, \varepsilon_t)}{\operatorname{var}(x_t)} \tag{4.13}$$

From the covariance term, the bias increases with α and β.

The third column of Table 4.11 gives estimates of the model in equation (4.5) allowing for both groupwise heteroskedasticity and an autoregression of order 1. The predicted marginal products shown in Table 4.12 exhibit a slightly larger marginal product for workers with high school degrees and a significantly smaller marginal product for workers with 4-year college degrees than the pooled OLS model. The marginal product of workers with less than high school is about 50% lower than the marginal product given in the pooled OLS model and slightly smaller than the model with only groupwise heteroscedastic disturbances. Also the elasticity of public capital becomes significantly positive, which is different than the other models.

4.6. Conclusions

This paper has investigated the returns to different types of inputs in aggregate production functions incorporating a production function that distinguishes between workers with different education levels. I use several different assumptions regarding the error term. For each, I derive the marginal products of each group of workers. Since the wages that workers receive should be closely related to their marginal product in a competitive economy, I can compare the estimates of marginal products to actual wages for each group; the best model should give estimates which are close to actual wages. I find that the model in which the error term is assumed to be state-wise heteroscedastic with autocorrelated errors does the best job of fitting the pattern of marginal products of the education groups. In addition, this model suggests a significantly positive elasticity for public capital. I also offer reasons why the fixed effects model which has been used in a number of other studies of this issue may be biased.

Table 4.2a: Summary Statistics for Worker Groups -- Growth Rates

variable		mean	st dev	min	max
growth rate of workers with less than HS	overall	-.0268	.1135851	-.3306335	.4846642
	between		.018173	-.0571517	.0234593
	within		.11215	-.3074991	.46819
growth rate of workers wit HS or some college	overall	.0217	.0498	-.1275	.2216
	between		.0113	.00511	.0521
	within		.0485	-.1173	.2036
growth rate of workers with 4-year college or more	overall	.03975	.0933	-.2565	.4885
	between		.0150	-.00205	.0773
	within		.0921	-.2941	.4510

Table 4.2b: Summary Statistics for Worker Groups -- Correlations of Growth Rates

	growth rate of real private gross state product	growth rate of workers with less than HS	growth rate of workers with HS or some college	growth rate of workers with college
growth rate of real private gross state product	1.000			
growth rate of workers with less than HS	.2347	1.000		
growth rate of workers with HS or some college	.2607	-0.0490	1.000	
growth rate of workers with college	.0405	-.1580	-.0651	1.000

Table 4.3a: Summary Statistics for Worker Groups -- Share of Each Group in the Work Force

variable		mean	st.dev	min	max
share of workers with less than HS	overall	.1693	.04874	.0732	.3227
	between		.0364	.1117	.2458
	within		.0331	.0771	.2909
share of workers with HS or some college	overall	.6136	.0408	.5093	.7204
	between		.0336	.5460	.6717
	within		.0234	.5369	.6958
share of workers with 4-year college	overall	.2171	.0420	.0954	.3528
	between		.0358	.1588	.2899
	within		.0228	.1423	.2879

Table 4.3b: Summary Statistics for Worker Groups -- Growth Rates of the Share of Each Group in the Work Force

variable		mean	st.dev	min	max
share of workers with less than HS	overall	-.0407	.1046	-.3293	.4092
	between		.0142	-.0717	-.01177
	within		.1037	-.3027	.3802
share of workers with HS or some college	overall	.006181	.0319	-.0818	.1173
	between		.00509	-.0036	.0181
	within		.0315	-.0808	.1157
share of workers with 4-year college	overall	.0243	.0857	-.2756	.4471
	between		.0119	-.0056	.0544
	within		.0849	-.276	.4170

Table 4.4: Estimated Parameters Derived in Holtz (1994) for 1969-86

	OLS	FIX	LONG	GLS	IV
log labor	0.497 (.0144)	0.691 (.0262)	.643 (.137)	.659 (.0225)	0.759 (.0821)
log private capital	0.359 (0.0112)	0.301 (.0302)	.504 (.142)	.361 (.0233)	0.500 (.0454)
log public capital	0.203 (.0190)	-0.0517 (.0267)	-0.115 (.126)	.0077 (.0235)	-0.0218 (.131)

Table 4.5: Estimated Parameters Using Data for 1980-92

	OLS	FIX	LONG	GLS	IV
log labor	.811 (.0212)	.754 (.0489)	.439 (.1555)	.845 (.0375)	.846 (.1345)
log private capital	.181 (.0168)	.603 (.0277)	.847 (.0758)	.528 (.02275)	.201 (.07557)
log public capital	.0808 (.0936)	-.278 (.0312)	-.0237 (.1077)	-.275 (.0297)	.0242 (.2135)

Table 4.6: Pooled OLS Estimation

	pooled OLS	OLS with time trends	OLS with regional dummies
physical capital	.251 (.0167)	.250 (.0169)	.114 (.0623)
public capital	.0166 (.0279)	.0159 (.0284)	.0473 (.0258)
worker group 1	.125 (.0152)	.0794 (.0279)	.157 (.0405)
worker group 2	.202 (.0372)	.377 (.0546)	.07365 (.0981)
worker group 3	.473 (.0229)	.331 (.0340)	.700 (.0433)
(worker group 1)*time		.0067 (.00369)	
(worker group 2)*time		-.0259 (.00592)	
(worker group 3)*time		.0215 (.00502)	
(worker group 1)*Midwest			-.00277 (.0512)
(worker group 1)*South			-.118 (.0472)
(worker group 1)*West			6.48e-06 (.0539)
(worker group 2)*Midwest			.370 (.119)
(worker group 2)*South			.0562 (.112)
(worker group 2)*West			.335 (.122)

Table 4.6: Pooled OLS Estimation (continued)

	pooled OLS	OLS with time trends	OLS with regional dummies
(worker group 3)*Midwest			-.416 (.0661)
(worker group 3)*South			-.281 (.0522)
(worker group 3)*West			-.585 (.0661)
Midwest			-.656 (.302)
South			1.433 (.307)
West			-.487 (.322)
no. of obs.	624	624	624
adj. R^2	.9875	.9879	.9928
RSS (df)	659.386 (5)	659.653 (8)	663.01 (20)
ESS (df)	8.246 (618)	7.979 (615)	4.626 (603)

Includes interaction between region and *k*.

Table 4.7: Imputed Marginal Products Based on Estimated Elasticities in Table 4.6

		workers with less than HS	workers with HS degree and/or some college	workers with 4-year college or more
pooled OLS		28,704	25,156	47,311
OLS with time trends	1980	56,959	25,186	73,006
	1992	14,640	4,561	110,442
OLS with regional dummies	Northeast	49,120	5,973	133,211
	Midwest	42,067	28,794	56,808
	South	7,682	9,109	84,073
	West	47,835	31,995	21,430

Table 4.8: Estimated Elasticities for State-Specific Effects

	Fixed-Effect	Random-Effect	LONG
workers with less than high school	.1259 (.0154)	.1412 (.0127)	.0283 (.0589)
workers with high school, some college	.2508 (.0331)	.3103 (.0335)	.1712 (.1063)
workers with 4-year college or more	.1111 (.0221)	.1943 (.0224)	.1318 (.0803)
private capital	.7967 (.0237)	.6740 (.0228)	.9080 (.0677)
public capital	-.1572 (.0379)	-.2008 (.00842)	-.0004 (.1122)

Table 4.9: Imputed Marginal Products Based on Elasticities in Table 4.8

	Fixed-Effect	Random-Effect	LONG
workers with less than high school	29,121	32,670	7,119
workers with high school, some college	16,209	20,053	11,012
workers with 4-year college or more	21,739	38,017	25,379

Table 4.10: Statistics on the Predicted Standard Errors of the Pooled OLS Regression

	state	predicted standard error
minimum	PA	.000712
median	ND	.005452
maximum	DE	.111250

Table 4.11: Estimated Elasticities with Groupwise Heteroscedasticity and Autoregression of the Disturbances

	GLS, heteroscedastic Σ	GLS, heteroscedastic Σ, AR(1)	GLS, heteroscedastic Σ, panel-specific AR(1)
workers with less than high school	.0944 (.00874)	.0849 (.0105)	.0732 (.00916)
workers with high school, some college	.3321 (.0215)	.3745 (.0262)	.3724 (.0217)
workers with 4-year college or more	.4081 (.0143)	.2120 (.0162)	.2252 (.0157)
private capital	.2535 (.0151)	.2830 (.0215)	.3121 (.0213)
public capital	-.0053 (.0157)	.134 (.0285)	.1136 (.0205)

Table 4.12: Imputed Marginal Products Based on Estimated Elasticities in Table 4.11

imputed marginal products of	GLS, heteroscedastic Σ	GLS, heteroscedastic Σ, AR(1)	GLS, heteroscedastic Σ, panel-specific AR(1)
workers with less than HS	23,570	21,204	18,282
workers with HS degree, some college	23,957	27,016	26,868
workers with 4-year college or more	78,229	40,643	43,158