CHAPTER 9

RESISTANCE FACTOR FOR COMPOSITE BEAMS

9.1 General

In Chapter 6, a new stud strength prediction method was formulated based on push-out test results. The results of three new beam tests were presented in Chapter 7. These, along with many other beam tests, were evaluated in Chapter 8 and were found to agree with the strengths that were predicted using the new stud strength model. The purpose of this chapter is to present a resistance factor and its development for the AISC flexural model in Section 8.3.2. The AISC specification currently uses a resistance factor of 0.85 for composite beam flexural strength. This value was calculated using the stud strength model and beam test results from Grant et al (1977).

9.2 Development of Resistance Factors

The method for computing the resistance factors for the stud strength model and the beam strength flexural model was derived and presented by Hansell et al (1978). These criteria were presented in a series of articles in the September 1978 issue of the *ASCE Journal of the Structural Division*. They formed the basis of the AISC LRFD specification. The resistance factors, presented herein, for the stud strength model were found from the results of the push-out tests at VT, and for the beam strength flexural model from the results of the beam tests presented in Chapter 8. These formulas, when used to calculate the resistance factor for beam strength in Section 9.3.2, do not account for variations in the stud strength or the reliability of the new stud strength model.

Modified resistance factors for beam strength, that are similar to the ones presented by Galambos and Ravindra (1976), that include components for the new stud strength model reliability will be presented in Sections 9.4 and 9.5.

The development of a resistance factor for use in LRFD begins with the assumption that the load effects *Q* and the resistance *R* are statistically independent random variables. Fig. 9.1 shows frequency distributions for *Q* and *R*, assuming that a

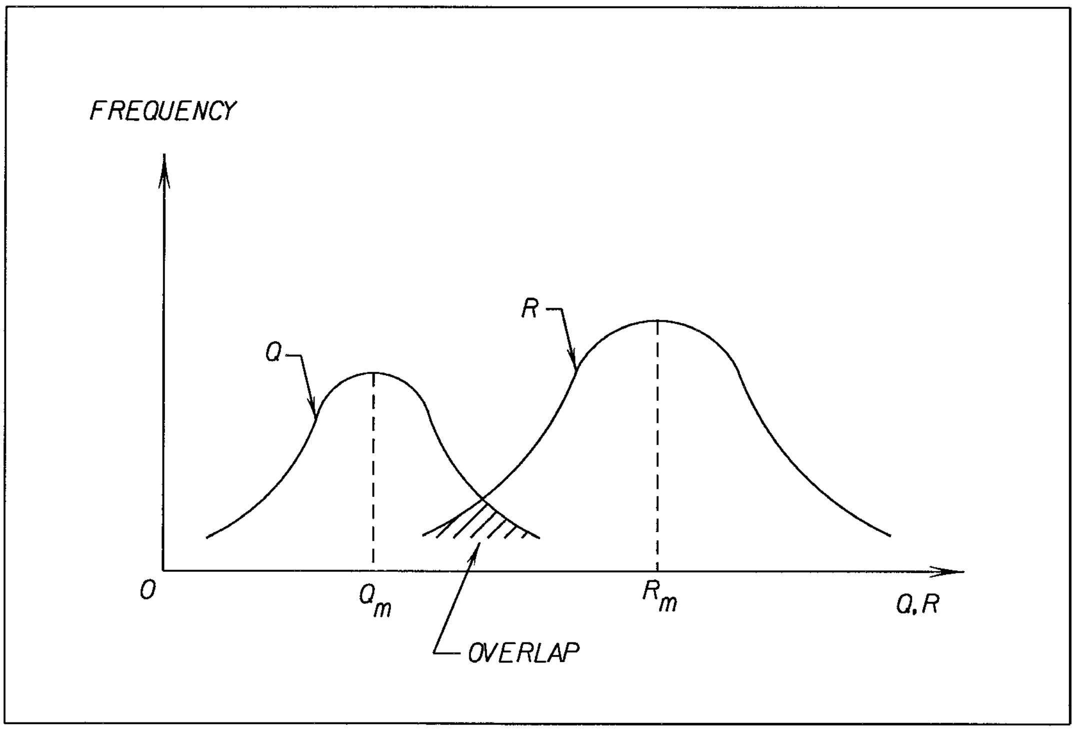

Fig. 9.1 Frequency Distribution of Load Effect Q and Resistance R (from Load 1993)

margin of safety exists (i.e., *R* is greater than *Q*). The section on the graph where *Q* and *R* overlap represents the small probability that *R* may be less than *Q*. The probability of failure is the probability that *R* is less than *Q*,

$$p_f = 1 - p_s = P(R < Q) \tag{9.1}$$

where p_s is the probability of survival, $P(R>Q)$. A single combined probability density function can be used to represent the margin of safety (as reported by Barker and Puckett 1997). Assuming that Q and R are lognormally distributed, this function is

$$g(R,Q) = \ln(R) - \ln(Q) = \ln\left(\frac{R}{Q}\right) \tag{9.2}$$

This graph is shown in Fig. 9.2.

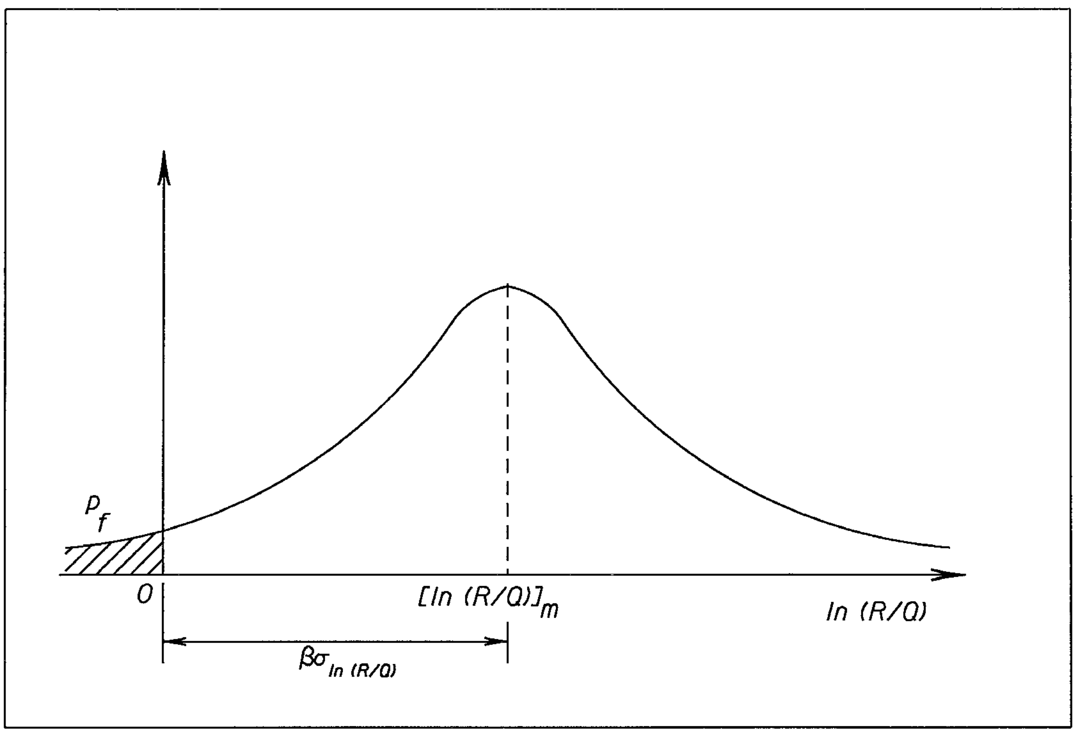

Fig. 9.2 Definition of Reliability Index, β (from Load 1993)

This lends another expression for p_f,

$$p_f = 1 - F_u\left(\frac{\ln(R_m/Q_m)}{\sqrt{V_R^2 + V_Q^2}}\right) \tag{9.3}$$

where R_m = mean value of resistance

Q_m = mean value of load effects

V_R = coefficient of variation of resistance

V_Q = coefficient of variation of load effects

$F_u(\ )$ = standard normal cumulative distribution function

The probability of failure is given by $ln(R/Q) < 0$, which is the area under the curve to the left of the vertical axis in Fig. 9.2.

The safety index, β, is the number of standard deviations, σ_g, that the mean value of $ln(R_m/Q_m)$ of the limit state function $ln(R/Q)$ is greater than the value defining the failure condition $ln(R/Q) = 0$ (as reported by Barker and Puckett 1997). The mean value of the function $g(R,Q)$ is

$$\bar{g} = \ln\left(\frac{R_m}{Q_m}\right) \tag{9.4}$$

with a standard deviation of approximately

$$\sigma_g = \sqrt{V_R^2 + V_Q^2} \tag{9.5}$$

which makes

$$\beta = \frac{\ln(R_m/Q_m)}{\sqrt{V_R^2 + V_Q^2}} \qquad \text{or} \qquad \ln(R_m/Q_m) = \beta\sqrt{V_R^2 + V_Q^2} \tag{9.6}$$

The simplest method of calculating the resistance factor, ϕ, is to first separate Q and R in the above equation. This is done by using the approximation, suggested by Lind (1971),

$$\sqrt{V_R^2 + V_Q^2} \approx \alpha(V_R + V_Q) \tag{9.7}$$

so that

$$\ln R_m - \ln Q_m = \ln\left(\frac{R_m}{Q_m}\right) = \beta\alpha\left(V_R + V_Q\right) \tag{9.8}$$

Taking the exponential of both sides of the equation,

$$\frac{R_m}{Q_m} = e^{\alpha\beta\left(V_R + V_Q\right)} = e^{\alpha\beta V_R} e^{\alpha\beta V_Q} \tag{9.9}$$

Moving the effects of R to the left-hand side of the equation and the effects of Q on the right-hand side,

$$\frac{R_m}{e^{\alpha\beta V_R}} = Q_m e^{\alpha\beta V_Q} \quad \text{or} \quad R_m e^{-\alpha\beta V_R} = Q_m e^{\alpha\beta V_Q} \tag{9.10}$$

Introducing another term called the bias factor, λ,

$$\lambda = \frac{x_m}{x_n} \tag{9.11}$$

where x_m = mean value

x_n = nominal value

The mean values R_m and Q_m can be written as $\lambda_R R_n$ and $\lambda_Q Q_n$, respectively, so that

$$\lambda_R R_n e^{-\alpha\beta V_R} = \lambda_Q Q_n e^{\alpha\beta V_Q} \tag{9.12}$$

In design, ϕ is multiplied by the nominal value of R, R_n. From the resistance side of the above equation, ϕ is found to be

$$\phi = \lambda_R e^{-\alpha\beta V_R} = \frac{R_m}{R_n} e^{-\alpha\beta V_R} \tag{9.13}$$

The value for α, that was introduced when the effects of R and Q were first separated, is taken as 0.55 (Hansell et al 1978). A β of 3.0 for composite members, which gives a probability of failure of 0.00115 or about 1:1000 is used in the development of the AISC specification. The coefficient of variation for the resistance, V_R, includes variations of the yield stress, V_{Fy}, fabrication, V_F, and the test-to-prediction ratio, V_P, also called the "professional factor". V_R is expressed as

$$V_R = \sqrt{V_{F_y}^2 + V_F^2 + V_P^2} \tag{9.14}$$

Values for V_{Fy} and V_F are taken to be 0.10 and 0.05, respectively, in the LRFD specifications development (Hansell et al 1978).

The expression for the ratio of the mean-to-nominal resistance is given as

$$\frac{R_m}{R_n} = \frac{(F_y)_m}{(F_y)_n}\left(\frac{Test\ capacity}{\Pr ediction}\right)_m \tag{9.15}$$

where the subscripts *m* and *n* denote mean and nominal values.

The ratio of mean-to-nominal yield stress is taken as:

$$\frac{(F_y)_m}{(F_y)_n} = 1.07\ . \tag{9.16}$$

Finally, ϕ can be found from the following equation:

$$\phi = 1.07\left(\frac{Test\ capacty}{Prediction}\right)_m e^{-0.55(3.0)V_R} \tag{9.17}$$

The value for *(Test capacity/Prediction)$_m$* is the mean value of the ratios of experimental strength-to-predicted strength. The value for V_P is found by dividing the standard deviation of these ratios by the mean value of these ratios.

9.3 Calculation of Resistance Factors

9.3.1 Resistance Factor for Stud Strength

Phi-factors for stud strengths were found by comparing the experimental stud strengths ("*Test capacity*") from push-out tests to the predicted stud strengths ("*Prediction*"). The equations presented in Section 9.2 were used to find the phi-factors for the stud strengths, where the ratio *Test capacity/Prediction* and the coefficient of variation V_P were found from push-out test data. Phi-factors were calculated for both the AISC stud strength model and the new stud strength model, as discussed in the following sections.

9.3.1.1 AISC Stud Strength Model

Phi-factors for the AISC stud strength model are given in Tables 9.1 and 9.2. Phi-factors are calculated according to deck height, stud diameter, and stud position. For all VT tests combined, ϕ is 0.467, with an average Q_e/Q_{AISC} of 0.653, a standard deviation of 0.142, and a coefficient of variation V_P of 0.217.

9.3.1.2 New Stud Strength Model

Phi-factors for the new stud strength model are given in Tables 9.1 and 9.2. Phi-factors are calculated according to deck height, stud diameter, and stud position. For all VT tests combined, ϕ is 0.802, with an average Q_e/Q_c of 1.027, a standard deviation of 0.159, and a coefficient of variation V_P of 0.155.

Table 9.1 Phi Factors for Stud Strength from Push-Out Tests

	Deck Height							
	1"		1.5"		2"		3"	
	30 tests		2 tests		130 tests		40 tests	
NEW METHOD	0.943	0.085	1.109	0.096	1.047	0.149	1.021	0.210
	0.090	**0.796**	0.086	**0.940**	0.142	**0.831**	0.206	**0.742**
AISC METHOD	0.523	0.113	0.514	0.044	0.663	0.119	0.727	0.164
	0.217	**0.374**	0.086	**0.435**	0.180	**0.500**	0.226	**0.513**

LEGEND	
No. Tests in Categ.	
ave Q_e/Q_c	st dev
cov	ϕ

	Stud Diameter							
	3/8"		1/2"		5/8"		3/4"	
	21 tests		27 tests		29 tests		125 tests	
NEW METHOD	1.121	0.241	0.985	0.142	0.978	0.159	1.032	0.139
	0.215	**0.804**	0.144	**0.780**	0.162	**0.756**	0.134	**0.828**
AISC METHOD	0.752	0.144	0.563	0.112	0.635	0.216	0.660	0.112
	0.192	**0.558**	0.198	**0.414**	0.340	**0.376**	0.169	**0.506**

	Stud Position											
	S		W		2S		Stag		M		2M	
	63 tests		43 tests		34 tests		30 tests		26 tests		6 tests	
NEW METHOD	1.020	0.164	0.996	0.181	1.143	0.147	1.033	0.113	0.945	0.101	0.990	0.040
	0.161	**0.790**	0.182	**0.749**	0.129	**0.924**	0.109	**0.854**	0.106	**0.784**	0.040	**0.871**
AISC METHOD	0.732	0.122	0.571	0.112	0.712	0.106	0.679	0.127	0.523	0.111	0.518	0.116
	0.167	**0.562**	0.195	**0.422**	0.149	**0.560**	0.188	**0.507**	0.212	**0.377**	0.225	**0.366**

Table 9.2 Stud Strength Phi Factors Using All Push-Out Tests

LEGEND	
No. Tests	
ave Q_e/Q_c	st dev
cov	ϕ

NEW METHOD	
All VT Tests	
1.027	0.159
0.155	**0.802**

AISC METHOD	
All VT Tests	
0.653	0.142
0.217	**0.467**

9.3.2 Resistance Factor for Beam Strength

Phi-factors for the bending strength of composite beams were found by comparing the experimental flexural strengths ("Test capacity") from beam tests to the calculated flexural strengths ("*Prediction*"). The method for calculating the calculated flexural strengths was discussed in Section 8.3.2. The equations presented in Section 9.2 were used to find the phi-factor for the beam strength, where the ratio *Test capacity/Prediction* and the coefficient of variation V_P were found from beam test data.

Data was separated into different sets: tests performed by Grant et al (1977), tests performed before Grant et al (1977), tests performed by and before Grant et al (1977), tests performed since Grant et al (1977), and a combination of all of these tests. Phi-factors were calculated for the AISC flexural model using several stud strength prediction methods, as discussed in the following sections.

9.3.2.1 Beam Strength Using AISC Stud Strength Model

The phi-factors shown in Table 9.3 are for the beam strength predictions made using the AISC stud strength model and AISC flexural model. From all beam tests reported in Chapter 8, ϕ is 0.817, with an average M_e/M_{AISC} of 0.958, a standard deviation of 0.0770, and a coefficient of variation V_P of 0.0804.

The statistics reported above seem reasonable (i.e., ϕ is between 0.8 and 1.0 and M_e/M_{AISC} is somewhat close to 1.0). However, if ϕ is to be equal to 0.85, as currently given by AISC for composite beam flexural strength, and M_e/M_{AISC} and V_P are equal to 0.958 and 0.0804, respectively, then the safety index, β, is 2.47, which is a probability of

Table 9.3 Phi Factors for Beam Strength Using AISC Stud Strength Model

(a) Using AISC Flexural Model

AISC STUD STRENGTH MODEL		
Tests	ave. M_e/M_{AISC}	st dev
	cov	ϕ
Grant et al (1977)	0.946	0.073
17 tests	0.077	**0.809**
Before Grant et al (1977)	0.980	0.078
34 tests	0.079	**0.836**
Grant et al (1977) and Before	0.969	0.077
51 tests	0.080	**0.826**
Since Grant et al (1977)	0.918	0.064
13 tests	0.070	**0.791**
ALL	0.958	0.077
64 tests	0.080	**0.817**

(b) Using Flexural Model from Grant et al (1977)

AISC STUD STRENGTH MODEL		
Tests	ave. M_e/M_{AISC}	st dev
	cov	ϕ
Grant et al (1977)	0.970	0.071
17 tests	0.074	**0.832**
Before Grant et al (1977)	1.036	0.077
34 tests	0.074	**0.889**
Grant et al (1977) and Before	1.014	0.081
51 tests	0.079	**0.865**
Since Grant et al (1977)	0.956	0.058
13 tests	0.061	**0.829**
ALL	1.002	0.080
64 tests	0.080	**0.855**

failure of approximately 1:89, where the probability of failure, $p_f = 460e^{-4.3\beta}$ (Rosenblueth and Esteva 1972). This is much less than the desired value of β, given in AISC, of 3.0, which is a probability of failure of approximately 1:869.

It is interesting to note that the phi-factors are different if the beam strength predictions are made using the AISC stud strength model and the beam strength flexural model from Grant et al (1977). This flexural model assumes that the resultant force in the concrete slab acts at mid-depth of the total slab thickness. From all beam tests reported in Chapter 8, ϕ is 0.855, with an average M_e/M_{AISC} of 1.002, a standard deviation of 0.080, and a coefficient of variation V_P of 0.080. It is clear that this flexural model is conservative in that it predicts a lower beam strength than the AISC flexural model, and therefore increases ϕ to 0.855 from 0.817.

9.3.2.2 Beam Strength Using New Stud Strength Model

The phi-factors shown in Table 9.4 are for the beam strength predictions made using the new stud strength model developed in Chapter 6 and the AISC flexural model. From all beam tests reported in Chapter 8, ϕ is 0.861, with an average M_e/M_c of 1.017, a standard deviation of 0.0890, and a coefficient of variation V_P of 0.0875. The new stud strength model is compatible with AISC, in that the phi-factor is based on an assumed value for the safety index, β, of 3.0 and the AISC flexural model is used to calculate M_c.

Table 9.4 Phi Factors for Beam Strength Using New Method

NEW STUD STRENGTH MODEL		
Tests	ave. M_e/M_c	st dev
	cov	ϕ
Grant et al (1977)	0.961	0.099
17 tests	0.103	**0.800**
Before Grant et al (1977)	1.050	0.082
34 tests	0.078	**0.897**
Grant et al (1977) and Before	1.020	0.097
51 tests	0.095	**0.857**
Since Grant et al (1977)	1.003	0.047
13 tests	0.047	**0.878**
ALL	1.017	0.089
64 tests	0.088	**0.861**

9.4 Modified Resistance Factor for Composite Beams

In Sections 9.2 and 9.3, resistance factors are calculated on the basis of the LRFD composite beam criteria (Hansell et al 1978). Note that the expression for the coefficient of variation for the resistance V_R included only variations in the yield stress of the steel beam V_{FY}, fabrication V_F, and test-to-prediction ratio V_P. Also, the expression for R_m/R_n included only the ratio of mean-to-nominal yield stress of the steel beam and the experimental strength-to-predicted strength of the composite beam. The uncertainty of the stud strength was not included in the development of the resistance factor. In this section, the method for calculating the resistance factor is modified to include the variation of, the ratio of the mean-to-nominal, and the ratio of the experimental-to-predicted stud strength.

Galambos and Ravindra (1976) used a first-order reliability method that included these variations. They calculated resistance factors for composite beams with solid slabs by analyzing the expression for the flexural strength. They used statistical data that was available for the variables, which included the yield strength of steel, the concrete strength, and the shear connector strength, and on the reliability of the expression (i.e., the prediction-to-capacity ratio).

The expression for the composite beam strength varies depending on the location of the neutral axis and whether or not the beam is fully composite. The expression is simplified by assuming that the neutral axis is at the concrete slab-steel beam interface. To further simplify the expression, it can be assumed that the resultant force in the slab acts at mid-depth of the total slab thickness. This simplification eliminates variables such as the concrete strength and the effective width of the slab. First, Eqn. 9.15 is modified to include the ratio of the mean-to-nominal stud strength. The expression for the composite beam mean strength is, therefore,

$$\left(M_u\right)_m = \left[\left(\sum Q_u\right)_m \frac{t_s}{2} + A_s\left(F_y\right)_m \frac{d}{2}\right]\left[\frac{Test\ capacity}{Prediction}\right]_m \tag{9.18}$$

where $\sum Q_u = NQ_u$ if N = number of studs.

Because $$\left(\sum Q_u\right)_m = \left(\frac{\left(Q_u\right)_m}{\left(Q_u\right)_n}\right)\left(\sum Q_u\right)_n \tag{9.19}$$

and $$\left(F_y\right)_m = \left(\frac{\left(F_y\right)_m}{\left(F_y\right)_n}\right)\left(F_y\right)_n, \tag{9.20}$$

$$\left(M_u\right)_m = \left[\frac{\left(Q_u\right)_m}{\left(Q_u\right)_n}\left(\sum Q_u\right)_n \frac{t_s}{2} + \frac{\left(F_y\right)_m}{\left(F_y\right)_n}\left(F_y\right)_n A_s \frac{d}{2}\right]\left[\frac{Test\ capacity}{Prediction}\right]_m . \tag{9.21}$$

The composite beam nominal strength is

$$\left(M_u\right)_n = \left[\left(\sum Q_u\right)_n \frac{t_s}{2} + A_s\left(F_y\right)_n \frac{d}{2}\right]. \tag{9.22}$$

Assuming a fully composite section with $\left(\sum Q_u\right)_n = A_s\left(F_y\right)_n$, (9.23)

$$\left(M_u\right)_m = \left[\frac{\left(Q_u\right)_m}{\left(Q_u\right)_n}\left(\sum Q_u\right)_n \frac{t_s}{2} + \frac{\left(F_y\right)_m}{\left(F_y\right)_n}\left(\sum Q_u\right)_n \frac{d}{2}\right]\left[\frac{Test\ capacity}{Prediction}\right]_m \tag{9.24}$$

which reduces to $\left(M_u\right)_m = \left[\frac{Test\ capacity}{Prediction}\right]_m \frac{\left(\sum Q_u\right)_n}{2}\left[\frac{\left(Q_u\right)_m}{\left(Q_u\right)_n} t_s + \frac{\left(F_y\right)_m}{\left(F_y\right)_n} d\right].$ (9.25)

The composite beam nominal strength becomes

$$\left(M_u\right)_n = \left[\left(\sum Q_u\right)_n \frac{t_s}{2} + \left(\sum Q_u\right)_n \frac{d}{2}\right] = \frac{\left(\sum Q_u\right)_n}{2}\left(t_s + d\right). \tag{9.26}$$

As previously noted, $\frac{\left(F_y\right)_m}{\left(F_y\right)_n} = 1.07$ (9.27)

and from Section 9.3.2.2, $\left[\frac{Test\ capacity}{Prediction}\right]_m = 1.017$. (9.28)

The nominal stud strength is given as

$\left(Q_u\right)_n = R_p R_n R_d A_s \left(F_u\right)_n$ in Section 6.13 (9.29).

The mean stud strength is

$$\left(Q_u\right)_m = \left(\frac{Q_{Test}}{Q_{Pred}}\right)_m \left[R_p R_n R_d A_s \left(\frac{\left(F_u\right)_m}{\left(F_u\right)_n}\right)\left(F_u\right)_n\right]. \tag{9.30}$$

Using $\left(\frac{Q_{Test}}{Q_{\text{Pred}}}\right)_m = 1.027$ from Section 6.14 (9.31)

and assuming $\frac{\left(F_u\right)_m}{\left(F_u\right)_n} = 1.07$ (9.32)

results in $\frac{\left(Q_u\right)_m}{\left(Q_u\right)_n} = \frac{1.027\left\lfloor R_p R_n R_d A_s (1.07)\left(F_u\right)_n\right\rfloor}{R_p R_n R_d A_s \left(F_u\right)_n} = (1.027)(1.07) = 1.099$ (9.33)

so that $\left(M_u\right)_m = (1.017)\frac{\left(\sum Q_u\right)_n}{2}\left[1.099 t_s + 1.07 d\right]$ (9.34)

and $\frac{\left(M_u\right)_m}{\left(M_u\right)_n} = \frac{1.017\left(1.099 t_s + 1.07 d\right)}{t_s + d}.$ (9.35)

Dividing the numerator and denominator by *d*,

$$\frac{\left(M_u\right)_m}{\left(M_u\right)_n} = \frac{1.017\left(1.099\frac{t_s}{d} + 1.07\right)}{\frac{t_s}{d} + 1}. \tag{9.36}$$

To account for the variation in the stud strength, the coefficient of variation for the resistance V_R can be taken as

$$V_R = \sqrt{V_M^2 + V_F^2 + V_P^2} \tag{9.37}$$

where V_M includes material strength variations of both the studs Q_u and steel beam F_y. Using a first-order reliability method, where partial derivatives of the composite beam strength are taken,

$$V_M^2 = \frac{\left(\frac{\partial M_u}{\partial Q_u}\right)_m^2 \sigma_{Q_u}^2 + \left(\frac{\partial M_u}{\partial F_y}\right)_m^2 \sigma_{F_y}^2}{\left(M_u\right)_m^2} \tag{9.38}$$

where $\left(\frac{\partial M_u}{\partial F_y}\right)_m = \frac{dA_s}{2}$, (9.39)

$$\left(\frac{\partial M_u}{\partial Q_u}\right)_m = \frac{t_s N}{2}, \tag{9.40}$$

$$\sigma_{F_y} = V_{F_y}\left(F_y\right)_m, \tag{9.41}$$

and $\sigma_{Q_u} = V_{Q_u}\left(Q_u\right)_m$. (9.42)

This results in

$$V_M^2 = \frac{\left(\frac{t_s}{2}N\right)^2 V_{Q_u}^2 \left(Q_u\right)_m^2 + \left(\frac{dA_s}{2}\right)^2 V_{F_y}^2 \left(F_y\right)_m^2}{\left(M_u\right)_m^2}. \tag{9.43}$$

From AISC, $V_{F_y} = 0.1$, (9.44)

and $\left(F_y\right)_m = 1.07\left(F_y\right)_n$ (9.45)

and from Eqn. 9.33, $\left(Q_u\right)_m = 1.099\left(Q_u\right)_n$. (9.46)

The coefficient of variation of the stud is expressed as

$$V_{Q_u} = \sqrt{V_{\frac{Test}{Pred}}^2 + V_{F_u}^2}. \tag{9.47}$$

From Section 6.14, $V_{\frac{Test}{\mathrm{Pred}}} = 0.155$. (9.48)

Also, $$V_{F_u}^2 = \frac{\left(\frac{\partial Q_u}{\partial F_u}\right)_m^2 \sigma_{F_u}^2}{(Q_u)_m^2} = \frac{\left[(1.027)(1.07)(R_p R_n R_d A_s)\right]^2 \left[(0.05)(1.07)(F_u)_n\right]^2}{\left[(1.027)(1.07)(R_p R_n R_d A_s (F_u)_n)\right]^2} \tag{9.49}$$

which results in $V_{Q_u} = 0.1397$. (9.50)

Therefore,

$$V_M^2 = \frac{\left(\frac{t_s}{2}N\right)^2 (0.1397)^2 \left(1.099(Q_u)_n\right)^2 + \left(\frac{dA_s}{2}\right)^2 (0.1)^2 \left(1.07(F_y)_n\right)^2}{\left[\left(\frac{t_s}{2}N\right)\left(1.099(Q_u)_n\right) + \left(\frac{dA_s}{2}\right)\left(1.07(F_y)_n\right)\right]^2}. \tag{9.51}$$

Assuming a fully composite section with $\left(\sum Q_u\right)_n = N(Q_u)_n = A_s(F_y)_n$, (9.52)

$$V_M^2 = \frac{\left(\frac{t_s}{2}\right)^2 (0.1397)^2 (1.099)^2 \left(A_s(F_y)_n\right)^2 + \left(\frac{d}{2}\right)^2 (0.1)^2 (1.07)^2 \left(A_s(F_y)_n\right)^2}{\left[\left(\frac{t_s}{2}\right)(1.099)\left(A_s(F_y)_n\right) + \left(\frac{d}{2}\right)(1.07)\left(A_s(F_y)_n\right)\right]^2} \tag{9.53}$$

which reduces to

$$V_M^2 = \frac{0.005893t_s^2 + 0.0028623d^2}{(0.5495t_s + 0.535d)^2} \tag{9.54}$$

or $$V_M = \frac{\sqrt{0.005893t_s^2 + 0.0028623d^2}}{0.5495t_s + 0.535d}. \tag{9.55}$$

Dividing the numerator and denominator by *d*,

$$V_M = \frac{\sqrt{0.005893\left(\frac{t_s}{d}\right)^2 + 0.0028623}}{0.5495\frac{t_s}{d} + 0.535}. \tag{9.56}$$

The coefficient of variation for the resistance is expressed as

$$V_R = \sqrt{V_M^2 + V_F^2 + V_P^2} \tag{9.57}$$

where $V_F = 0.05$ from AISC (9.58)

and $V_P = 0.0875$ from Section 9.3.2.2 (9.59)

which makes $V_R = \sqrt{V_M^2 + (0.05)^2 + (0.0875)^2}$. (9.60)

The expression for the resistance factor in Eqn. 9.13 can be used, except Eqn. 9.15 is substituted with

$$\frac{R_m}{R_n} = \frac{(M_u)_m}{(M_u)_n}. \tag{9.61}$$

Therefore, the resistance factor is found by

$$\phi = \frac{(M_u)_m}{(M_u)_n} e^{-0.55(3.0)V_R}. \tag{9.62}$$

The resistance factor, which varies with the ratio t_s/d, is as follows:

for $t_s/d = 0.6$, $\phi = 0.887$

for $t_s/d = 0.5$, $\phi = 0.887$

for $t_s/d = 0.4$, $\phi = 0.885$

for $t_s/d = 0.3$, $\phi = 0.883$

for $t_s/d = 0.2$, $\phi = 0.878$

for $t_s/d = 0.1$, $\phi = 0.871$

If t_s/d is taken as 0, $\phi = 0.861$. This is the same value as the resistance factor calculated in Section 9.3.2.2, where the variation in the stud strength is not accounted for. The average value for t_s/d from all beam tests is approximately 0.34, which results in ϕ = 0.884.

9.5 New Modified Resistance Factor for Composite Beams

In Section 9.4, the resistance factor was calculated by including the variation of, the ratio of the mean-to-nominal, and the ratio of the experimental-to-predicted stud strength. The calculations were simplified by assuming that the neutral axis is at the concrete slab-steel beam interface and that the resultant force in the slab acts at mid-depth of the total slab thickness.

In this section, ϕ is calculated for each beam test. The expression for the flexural strength is analyzed individually for the parameters of each beam test, accounting for the location of the neutral axis and the resultant force in the slab that is assumed by the AISC flexural theory. The values of ϕ for all beam tests are then averaged. The expression for the flexural strength, given by AISC, is as follows:

$$(M_u)_m = \left[\left(\sum Q_u\right)_m (d_1 + d_2) + A_s (F_y)_m (d_3 - d_2)\right]\left[\frac{Test\ capacity}{Prediction}\right]_m \tag{9.63}$$

where $\sum Q_u = NQ_u$ if N = number of studs.

Because $$\left(\sum Q_u\right)_m = \left(\frac{(Q_u)_m}{(Q_u)_n}\right)\left(\sum Q_u\right)_n \tag{9.64}$$

and $$(F_y)_m = \left(\frac{(F_y)_m}{(F_y)_n}\right)(F_y)_n, \tag{9.65}$$

$$(M_u)_m = \left[\frac{(Q_u)_m}{(Q_u)_n} \left(\sum Q_u\right)_n (d_1 + d_2) + \frac{(F_y)_m}{(F_y)_n} (F_y)_n A_s (d_3 - d_2) \right] \left[\frac{Test\ capacity}{Prediction} \right]_m .(9.66)$$

The composite beam nominal strength is

$$(M_u)_n = \left\lfloor \left(\sum Q_u\right)_n (d_1 + d_2) + A_s (F_y)_n (d_3 - d_2) \right\rfloor . \quad (9.67)$$

Assuming a partially composite section with $\left(\sum Q_u\right)_n = \rho A_s (F_y)_n$, (9.68)

Where ρ = percent composite action/100

$$(M_u)_m = \left[\frac{(Q_u)_m}{(Q_u)_n} \left(\sum Q_u\right)_n (d_1 + d_2) + \frac{(F_y)_m}{(F_y)_n} \left(\sum Q_u\right)_n \frac{(d_3 - d_2)}{\rho} \right] \left[\frac{Test\ capacity}{Prediction} \right]_m (9.69)$$

which reduces to

$$(M_u)_m = \left[\frac{Test\ capacity}{Prediction} \right]_m \left(\sum Q_u\right)_n \left[\frac{(Q_u)_m}{(Q_u)_n} (d_1 + d_2) + \frac{(F_y)_m}{(F_y)_n} \frac{(d_3 - d_2)}{\rho} \right] . \quad (9.70)$$

The composite beam nominal strength becomes

$$\begin{aligned} (M_u)_n &= \left[\left(\sum Q_u\right)_n (d_1 + d_2) + \left(\sum Q_u\right)_n \frac{(d_3 - d_2)}{\rho} \right] \\ &= \left(\sum Q_u\right)_n \left[(d_1 + d_2) + \frac{(d_3 - d_2)}{\rho} \right] . \end{aligned} \quad (9.71)$$

From the AISC Criteria, $\frac{(F_y)_m}{(F_y)_n} = 1.07$ (9.72)

and $\left[\frac{Test\ capacity}{Prediction} \right]_m$ is calculated for each beam test. (9.73)

The nominal stud strength is given as

$(Q_u)_n = R_p R_n R_d A_s (F_u)_n$ in Section 6.13 (9.74).

The mean stud strength is

$$(Q_u)_m = \left(\frac{Q_{Test}}{Q_{Pred}}\right)_m \left[R_p R_n R_d A_s \left(\frac{(F_u)_m}{(F_u)_n}\right)(F_u)_n\right]. \tag{9.75}$$

Using $\left(\frac{Q_{Test}}{Q_{\text{Pred}}}\right)_m = 1.027$ from Section 6.14 (9.76)

and assuming $\frac{(F_u)_m}{(F_u)_n} = 1.07$ (9.77)

results in $\frac{(Q_u)_m}{(Q_u)_n} = \frac{1.027\lfloor R_p R_n R_d A_s (1.07)(F_u)_n \rfloor}{R_p R_n R_d A_s (F_u)_n} = (1.027)(1.07) = 1.099$ (9.78)

so that $(M_u)_m = \left[\frac{Test\ capacity}{\text{Prediction}}\right]_m \left(\sum Q_u\right)_n \left[1.099(d_1 + d_2) + 1.07\frac{(d_3 - d_2)}{\rho}\right]$ (9.79)

and $\frac{(M_u)_m}{(M_u)_n} = \frac{\left[\frac{Test\ capacity}{\text{Prediction}}\right]_m \left(1.099(d_1 + d_2) + 1.07\frac{(d_3 - d_2)}{\rho}\right)}{(d_1 + d_2) + \frac{(d_3 - d_2)}{\rho}}.$ (9.80)

To account for the variation in the stud strength, the coefficient of variation for the resistance V_R can be taken as

$$V_R = \sqrt{V_M^2 + V_F^2 + V_P^2} \tag{9.81}$$

where V_M includes material strength variations of both the studs Q_u and steel beam F_y. Using a first-order reliability method, where partial derivatives of the composite beam strength are taken,

$$V_M^2 = \frac{\left(\frac{\partial M_u}{\partial Q_u}\right)_m^2 \sigma_{Q_u}^2 + \left(\frac{\partial M_u}{\partial F_y}\right)_m^2 \sigma_{F_y}^2}{(M_u)_m^2} \tag{9.82}$$

where $\left(\frac{\partial M_u}{\partial F_y}\right)_m = A_s(d_3 - d_2)$, (9.83)

$$\left(\frac{\partial M_u}{\partial Q_u}\right)_m = d_1 + d_2, \tag{9.84}$$

$$\sigma_{F_y} = V_{F_y}(F_y)_m, \tag{9.85}$$

and $\sigma_{Q_u} = V_{Q_u}(Q_u)_m$. (9.86)

This results in

$$V_M^2 = \frac{((d_1 + d_2)N)^2 V_{Q_u}^2 (Q_u)_m^2 + (A_s(d_3 - d_2))^2 V_{F_y}^2 (F_y)_m^2}{(M_u)_m^2}. \tag{9.87}$$

From AISC, $V_{F_y} = 0.1$, (9.88)

and $(F_y)_m = 1.07(F_y)_n$ (9.89)

and from Eqn. 9.78, $(Q_u)_m = 1.099(Q_u)_n$. (9.90)

The coefficient of variation of the stud is expressed as

$$V_{Q_u} = \sqrt{V_{\frac{Test}{Pred}}^2 + V_{F_u}^2}. \tag{9.91}$$

From Section 6.14, $V_{\frac{Test}{\mathrm{Pred}}} = 0.155$. (9.92)

Also, $V_{F_u}^2 = \frac{\left(\frac{\partial Q_u}{\partial F_u}\right)_m^2 \sigma_{F_u}^2}{(Q_u)_m^2} = \frac{[(1.027)(1.07)(R_p R_n R_d A_s)]^2 [(0.05)(1.07)(F_u)_n]^2}{[(1.027)(1.07)(R_p R_n R_d A_s(F_u)_n)]^2}$ (9.93)

which results in $V_{Q_u} = 0.1397$. (9.94)

Therefore,

$$V_M^2 = \frac{((d_1 + d_2)N)^2(0.1397)^2(1.099(Q_u)_n)^2 + (A_s(d_3 - d_2)^2(0.1)^2(1.07(F_y)_n)^2}{[((d_1 + d_2)N)(1.099(Q_u)_n) + (A_s(d_3 - d_2)(1.07(F_y)_n)]^2}. \tag{9.95}$$

Assuming a partially composite section with $\left(\sum Q_u\right)_n = N(Q_u)_n = \rho A_s (F_y)_n$, (9.96)

$$V_M^2 = \frac{\rho^2(d_1 + d_2)^2(0.1397)^2(1.099)^2(A_s(F_y)_n)^2 + (d_3 - d_2)^2(0.1)^2(1.07)^2(A_s(F_y)_n)^2}{[(d_1 + d_2)(1.099)(A_s(F_y)_n) + (d_3 - d_2)(1.07)(A_s(F_y)_n)]^2} \tag{9.97}$$

which reduces to

$$V_M^2 = \frac{0.02357\rho^2(d_1 + d_2)^2 + 0.01145(d_3 - d_2)^2}{(1.099\rho(d_1 + d_2) + 1.07(d_3 - d_2))^2} \tag{9.98}$$

or $$V_M = \frac{\sqrt{0.02357\rho^2(d_1 + d_2)^2 + 0.01145(d_3 - d_2)^2}}{1.099\rho(d_1 + d_2) + 1.07(d_3 - d_2)}. \tag{9.99}$$

Dividing the numerator and denominator by $(d_3 - d_2)$,

$$V_M = \frac{\sqrt{0.02357\left(\frac{\rho(d_1 + d_2)}{d_3 - d_2}\right)^2 + 0.01145}}{1.099\left(\frac{\rho(d_1 + d_2)}{d_3 - d_2}\right) + 1.07}. \tag{9.100}$$

Note that if full composite action is assumed, where ρ = 1.0, and the neutral axis is assumed to act at the slab-steel interface, so that $d_2 = 0$, $d_1 = t_s/2$, $d_3 = d/2$, then Eqn. 9.100 gives the same results as the modified method in Section 9.4, Eqn. 9.56.

The coefficient of variation for the resistance is expressed as

$$V_R = \sqrt{V_M^2 + V_F^2 + V_P^2} \tag{9.101}$$

where $V_F = 0.05$ from AISC (9.102)

and $V_P = 0.0875$ from Section 9.3.2.2 (9.103) which makes $V_R = \sqrt{V_M^2 + (0.05)^2 + (0.0875)^2}$. (9.104)

The expression for the resistance factor in Eqn. 9.13 can be used, except Eqn. 9.15 is substituted with

$$\frac{R_m}{R_n} = \frac{(M_u)_m}{(M_u)_n} . \tag{9.105}$$

Therefore, the resistance factor is found by

$$\phi = \frac{(M_u)_m}{(M_u)_n} e^{-0.55(3.0)V_R} . \tag{9.106}$$

The resistance factor, which is a function of d_1, d_2, d_3, ρ, and [*Test capacity/Prediction*], was calculated individually for each test reported in Chapter 8. Instead of using the mean value from all tests for [*Test capacity/Prediction*], the ratio for the particular test was used. An average ϕ from all tests is 0.881. This is very close to ϕ = 0.884, which was calculated in Section 9.4. This demonstrates that assuming the neutral axis is at the concrete slab-steel beam interface and the resultant force in the slab acts at mid-depth of the total slab thickness has little effect on the calculations for ϕ.

9.6 Summary

The resistance factor for the AISC stud strength model is 0.467. For the new stud strength model, the resistance factor improves to 0.802.

The resistance factor, using the AISC approach to flexural strength for composite beams, is between 0.86 and 0.89 for all beam tests reported in Chapter 8 when the new stud strength prediction method is used. From Section 9.3.2.2, if the method for calculating resistance factors from Hansell et al (1978) is used, the resistance factor is 0.861. From Section 9.4, if the method is modified, as was done in Galambos and Ravindra (1976), to include the reliability of the shear studs, then the resistance factor is 0.884 for an average t_s/d of 0.34. This modified method, however, assumes that the resultant force in the concrete slab acts at mid-depth of the total slab thickness and that the beam is fully composite. From Section 9.5, if the actual locations of the neutral axis and resultant concrete force are used, then the resistance factor is 0.881.

The resistance factor, using the AISC approach to flexural strength for composite beams, is about 0.817 for all beam tests reported in Chapter 8 when the AISC stud strength prediction method and a safety index, β, of 3.0 (probability of failure = 1:869) are used. Currently, AISC uses a resistance factor of 0.85, which equates to a safety index of only 2.47 (probability of failure = 1:89).