

CHAPTER 6

NUMBER OF STOPS AND QUEUE LENGTH

ESTIMATION AT SIGNALIZED APPROACHES

Traffic signals create vehicle queues at signalized approaches that are typically dissipated during the green interval. In order to characterize traffic performance at signalized approaches, several measures of effectiveness (MOEs) can be computed, including delay, number of stops, fuel consumption, emissions and queue length. In the previous two chapters, vehicle delay was the primary subject of discussion. However, the number of stops and the length of queues are two other important MOEs. For example, the number of vehicle stops is important in vehicle fuel consumption and emissions, while the queue length is important in designing the length of left turn pocket lanes. Estimates for the number of stops and the maximum queue length are the focus of this chapter. Specifically, the objective of the chapter is to assess the consistency between existing analytical and simulation models, and more particularly to compare estimates from analytical models based on queuing theory and horizontal shock wave analysis, the Canadian Capacity Guide Model, the Catling Model and the Cronje Model, and from the INTEGRATION traffic simulation model.

6.1 INTRODUCTION

The measurement of the level of performance of signalized intersections has been an area of concern in transportation planning almost since the birth of the profession. Although there have long been many interests in evaluating the level of performance at signalized intersections, this interest has been limited to estimates based on vehicle delay only by the necessity to ensure that existing transportation systems operate at peak efficiency. However, other performance measures of particular interest are the total number of vehicle stops and the extent of queues. In the control and design of signalized intersections, so-called "measures of effectiveness", such as number of stops and queue lengths, play an important role, for they are measures not only of the

level of service that is offered to the drivers but also of the fuel consumption and air pollution associated with traffic operations.

Although many traffic models that provide performance measures such as number of stops and queue lengths, have been developed, the definition, validity and applicability of their stop and queue measurements is not well understood. Some of the models are complex and theoretical, while others are more general and simplistic. In addition, the procedures used by these models are typically based on different stop and queue definitions and have different computational approaches that lead to different results. Here, the problem is not the need to develop another method to estimate these parameters but to fully understand how they are currently estimated.

Many authors have dealt with these measures of performance. An important contribution is attributed to Webster (1958), who generated stop and delay relationships by simulating road traffic flow on a one-lane approach to an isolated signalized intersection. In particular, the curve he fitted to his simulation results has been fundamental to traffic signal setting procedures since its development.

The predominant equations for estimating the number of vehicle stops and queue length for undersaturated conditions have been developed by Webster and Cobbe (1966), Catling (1977) and Cronje (1983). Specifically, Webster and Cobbe (1966) developed a formula for estimating vehicle stops assuming random vehicle arrivals. However, this formula, which was adopted by the Canadian Capacity Guide (ITE, 1995), does not consider multiple stops. Consequently, this formula cannot be applied to oversaturated conditions. Catling (1977) adapted equations of classical queuing theory to oversaturated traffic conditions and developed a comprehensive queue length estimation procedure that captured the time-dependent nature of queues. Cronje (1986) treated traffic flow through a fixed-time signal as a Markov process and developed equations for estimating the queue length and number of vehicle stops. Both the Catling model and the Cronje model were developed for both undersaturated and oversaturated conditions.

The focus of this chapter is to develop a classification framework for the existing models, and compare their behavior to the INTEGRATION microscopic traffic simulation model. The scope of the analysis is not only limited to undersaturated conditions, i.e., those conditions in which the demand volumes are less than the approach capacity but also includes oversaturated conditions. However, as it will be shown, the analysis of the number of stops and maximum queue lengths in oversaturated conditions is a much more complex process than a similar analysis under undersaturated traffic condition.

6.2 OBJECTIVES AND LAYOUT OF THE CHAPTER

In this chapter, simulation is used to compare the various stop and queue estimation models. In particular, the INTEGRATION microscopic traffic simulation software is used as the main simulation tool. A first objective of this chapter is to compare the number of stops and queue length estimation produced by the INTEGRATION microscopic traffic simulation model with the estimates provided by the models found in the Canadian Capacity Guide 1995, by Catling, by Cronje, and with analytical models developed from deterministic queuing analysis and from horizontal shock wave analysis. This comparison is specifically made considering both uniform arrivals and random arrivals. A second objective is to assess the consistency of the number of vehicle stops and maximum queue length estimates among the various analytical approaches and the INTEGRATION software for both undersaturated and oversaturated conditions.

6.3 ESTIMATION OF NUMBER OF STOPS AT SIGNALIZED APPROACHES

6.3.1 MICROSCOPIC COMPUTATION OF VEHICLE STOPS

Analytical approaches compute vehicle stops based on the number of arrivals when the traffic signal is red or when a queue exists at the approach stopline. These models record a single stop for oversaturated conditions, contrary to what is observed in the field.

It is noteworthy that INTEGRATION will often report that a vehicle has experienced more than one complete stop along a link. Multiple stops arise from the fact that a vehicle may have to stop several times before ultimately clearing the link stop line.

The estimation of stops in the INTEGRATION software is computed every second as the ratio of the instantaneous speed reduction to the free-speed, as indicated by Equation 6.1. As shown, a reduction in speed from the free-speed to a speed of zero would constitute a complete stop while a reduction in speed from half the free-speed to a speed equal to one quarter the free-speed would constitute 0.25 of a stop. The total number of stops is finally computed as the sum of all partial stops over the entire trip.

In case of undersaturated traffic conditions, Figure 6.1 shows the graphical illustration of a partial stops with a vehicle speed profile that was generated using the INTEGRATION microscopic simulation software. The profile indicates that the partial stop starts at 935 seconds of simulation and ends at 942 seconds. For this scenario, partial stops are computed for each second between time 935 and time 942. Table 6.1 indicates how the partial stops are computed for each one-second interval by the INTEGRATION model using Equation 6.1. By summing the partial stops in Table 6.1, the estimated total number of stops is 0.848 stops.

$$S_i = \frac{u_{i-1} - u_i}{u_f} \quad \forall i, \quad \exists u_i < u_{i-1} \quad (6.1)$$

where:

S_i = estimated partial Stops,

u_i, u_{i-1} = speed of vehicle at time i and time $i-1$ (kilometers/hour),

u_f = free speed on traveled link (kilometers/hour).

Table 6.1: INTEGRATION Output of a Vehicle for Computing Partial Stops

Time	Distance	Speed	Partial Stop
935	1.877	60.0	0
936	1.893	58.0	0.033
937	1.907	52.2	0.097
938	1.918	43.1	0.152
939	1.926	32.4	0.178
940	1.931	22.3	0.168
941	1.934	12.5	0.163
942	1.936	9.1	0.057
943	1.938	10.4	0
Total Stops			0.848

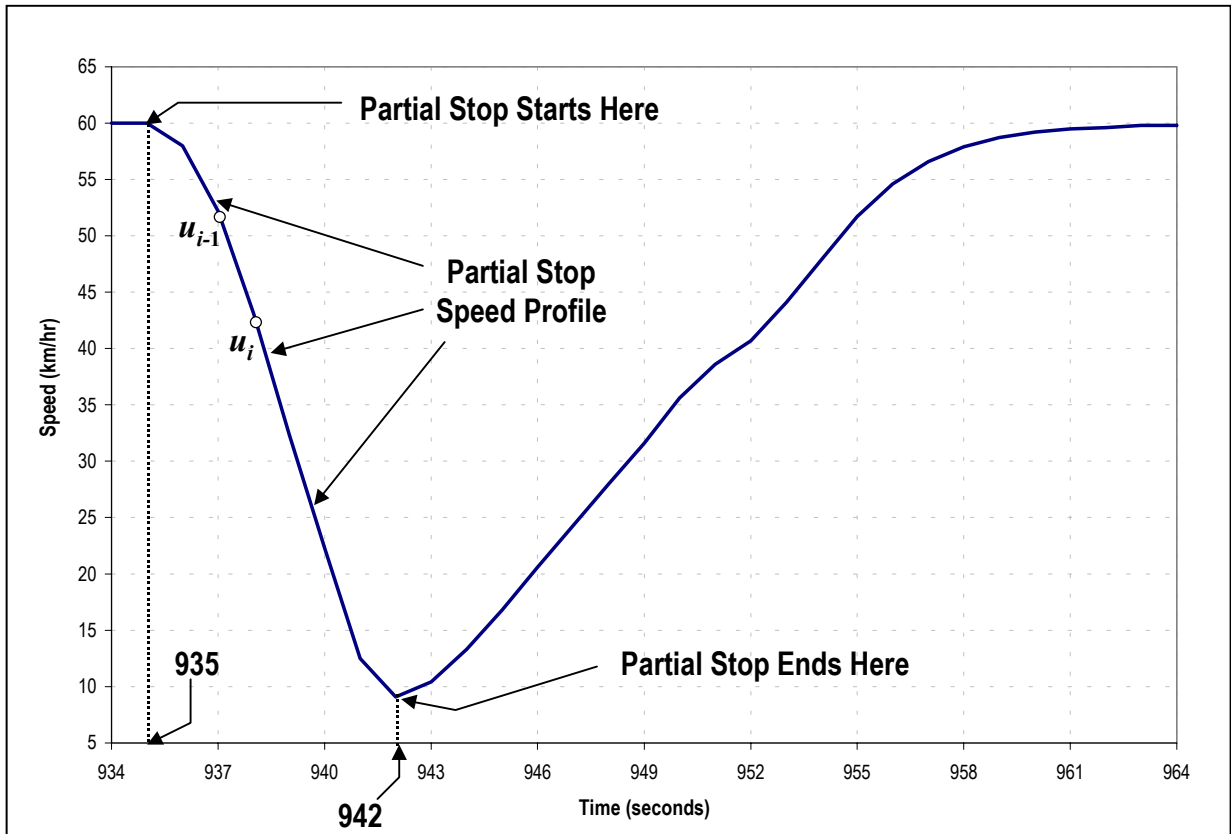


Figure 6.1: Graphical Illustration of Partial Stops for Undersaturated Condition

Figure 6.2 illustrates the speed profile of a vehicle attempting to cross a signalized intersection when the approach is oversaturated. Again, this figure was generated using the INTEGRATION microscopic software. The figure illustrates acceleration and decelerations (oscillations) along the signalized approach. For each oscillation the number of stops are computed using Equation 6.1. For example, the partial stop associated with the fourth deceleration starts at 1269 seconds of simulation and ends at 1290 seconds. By summing the second-by-second estimated partial stops over the 21-second interval, a total of 0.17 stops are made during the fourth deceleration measure. Finally, by summing the partial stop estimate obtained over all seven decelerations, it is found that the simulated vehicle made 2.24 stops to reach the intersection. Table 6.2 shows the results of each partial stop calculation, as well as the resulting total number of partial stops made by the simulated vehicle on the intersection approach.

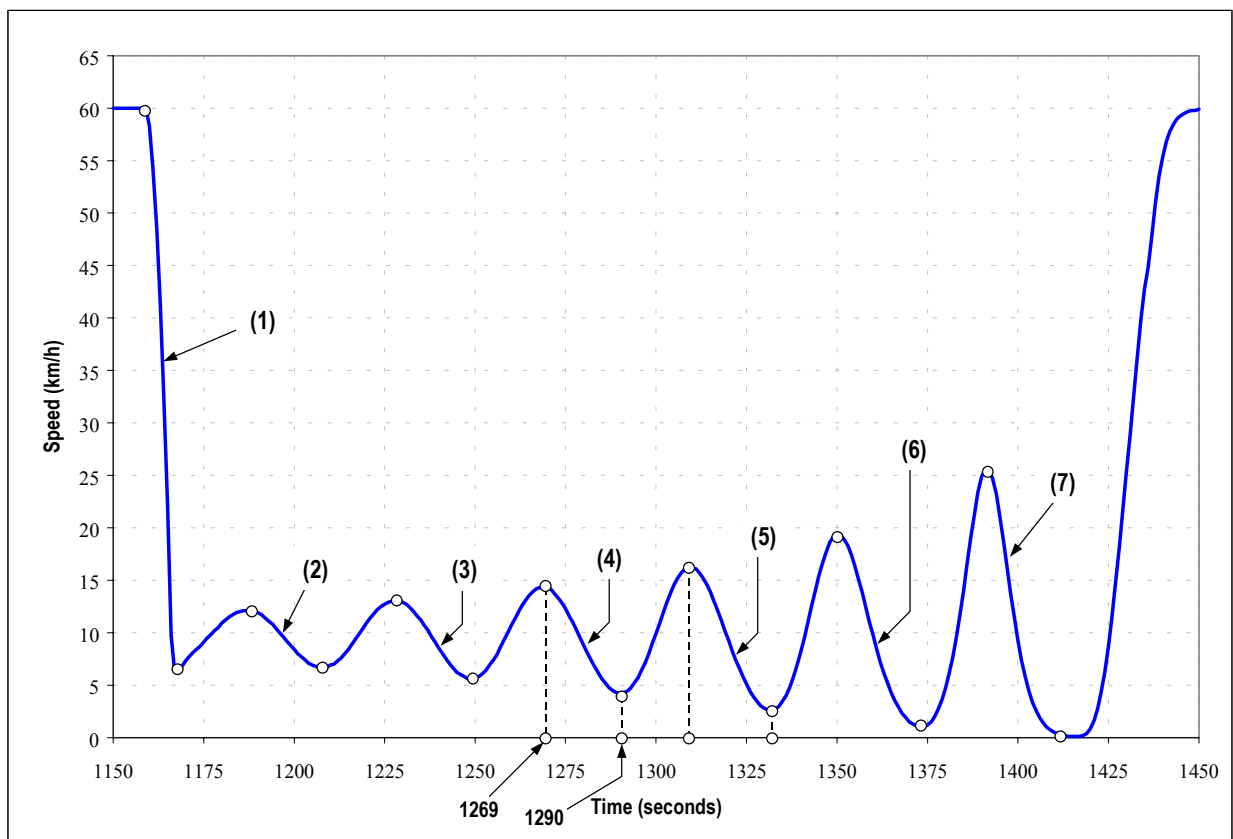


Figure 6.2: Graphical Illustration of Partial Stops for Oversaturated Conditions

Table 6.2: Each Sub-Partial Stop Results by Decelerations

Deceleration Number	Sub-Partial Stop
1	0.8967
2	0.0900
3	0.1250
4	0.1700
5	0.2267
6	0.3067
7	0.4250
Grand Total Partial Stop	2.2400

6.3.2 MACROSCOPIC COMPUTATION OF VEHICLE STOPS

6.3.2.1 NUMBER OF STOPS USING QUEUING ANALYSIS

The starting point for computing the number of vehicle stops is the idealized concept of intersection behavior used in queuing theory. Specifically, vehicles are typically considered as being stored in a vertical queue at the downstream end of an approach link. For example, consider the passage of a vehicle through a fixed-time signalized intersection, as illustrated in Figure 6.3. When traffic demand is undersaturated, vehicle A arrives during the green period and proceeds through the intersection without stopping. Vehicle B then arrives during the red period, decelerates, and comes to a complete stop. When the signal turns green, the vehicle accelerates instantaneously to its cruising speed and leaves the intersection. The idealized approximation differs from what happens in the field in a number of aspects. First, vehicles that queue upstream an intersection consume space and consequently vehicles arriving at the intersection should encounter the queue earlier than is predicted using queuing theory. Second, vehicles do not accelerate instantaneously, consequently vehicles experience numerous partial stops in many cases as opposed to complete stops. Alternatively, the queuing analysis considers only complete vehicle stops. For example, Figure 6.3 illustrates that of the three vehicle arrivals (A, B and C) only vehicle B comes to a complete stop.

The number of vehicle stops is computed as all vehicle arrivals when the traffic signal is red or when a queue exists at the approach stopline. The number of stops per vehicle can be calculated using Equation 6.2

$$N_s = \frac{s}{C(s - q)} \cdot r \quad (6.2)$$

where:

N_s = number of stops per vehicle (stops/vehicle),

s = saturation flow rate (vehicles/second),

C = cycle length (seconds),

q = arrival flow rate (vehicles/seconds),

r = red interval (seconds).

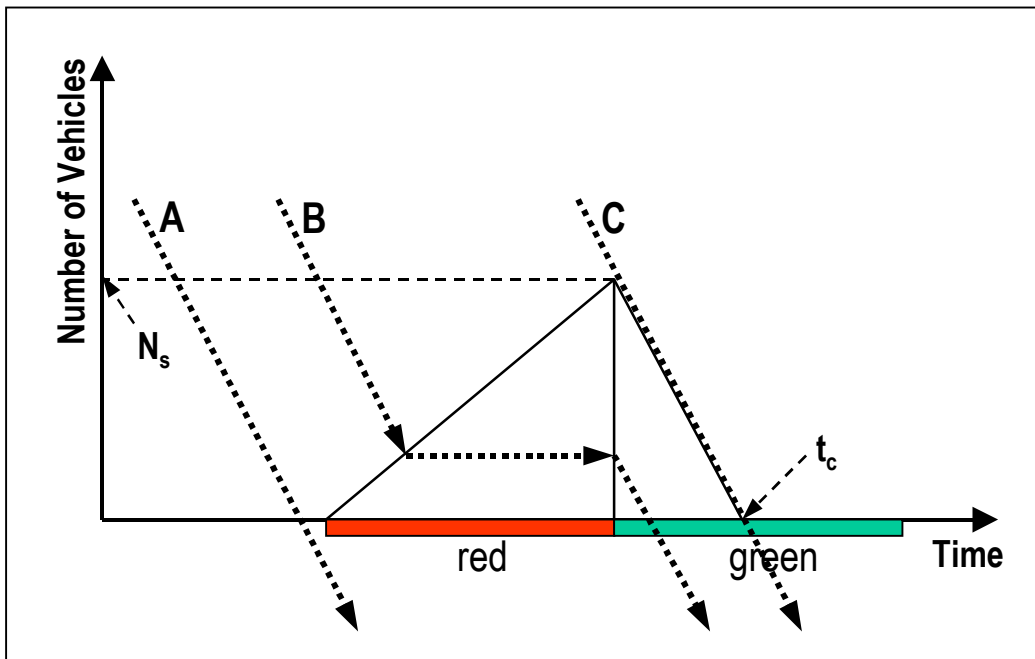


Figure 6.3: Typical Trajectory Diagram for Three Vehicles through Signal.

6.3.2.2 CANADIAN CAPACITY GUIDE MODEL

In the 1995 Canadian Capacity Guide, the number of vehicles that are stopped at least once by the signal operation during the evaluation time can be derived with the assumption of a random arrival pattern as using Equation 6.3 (Webster and Cobbe, 1966).

$$N_s = k_f t_e q (C - g_e) / [60C(1 - y)] \quad (6.3)$$

where:

N_s = number of passenger car units stopped at least once during the evaluation time (pcu). The resulting value must be capped at a maximum of:

$$N_s \leq q t_e / 60$$

representing the number of passenger car units arriving during the evaluation time,

k_f = adjustment factor for the effect of the quality of progression from delay formula,

q = arrival flow (passenger car unit/hour),

g_e = effective green interval (seconds),

C = cycle length (seconds),

y = lane flow ratio; $y = q / S$ capped at $y \leq 0.99$, with

S = saturation flow (passenger car unit/hour),

t_e = evaluation time (minutes).

The formula of Equation 6.3 does not consider multiple stops. The derived number of stops must therefore not exceed the volume during the evaluation time. This condition applies similarly to any other time period used. For higher degrees of saturation characterized by a significant overflow delay component, some vehicles must stop more than once.

Furthermore, with the k_f factor, the formula approximates the effect of signal coordination and progressive movement of vehicles through the intersection space. This factor is essential when the number of stops is subsequently used in the determination of fuel consumption and pollutant emissions. The resulting number of stops may range from zero, for excellent progression, to 2.6 times greater than the number of stops calculated by the formula for random arrivals. This effect of the k_f factor, however, may result in the number of stops exceeding the volume during the evaluation time, and must therefore be constrained to a maximum of 1 stop per vehicle.

6.3.2.3 CRONJE MODEL

Cronje model is based on a Markov process and the geometric probability distribution. The model treats traffic flow through a fixed-time signal as a Markov process for estimating the number of vehicle stops. The properties of the geometric probability distribution were applied to the equation to obtain a simple equation, thus reducing computing time. The equation was then modified to further reduce computing time without sacrificing too much accuracy. The equation for number of stops is expressed in Equation 6.4.

$$N = q[(q \cdot r + Q_0)/(s - q) + r] + Q_0 \quad (6.4)$$

where:

- N = number of stops per cycle (stops/cycle),
- q = average arrival rate (vehicles per second),
- r = effective red interval (second),
- $Q_0 = I \cdot H(\mu)x / 2 \cdot (1 - x)$,
- I = ratio of variance to the average of the arrivals per cycle,
= 1 for Poisson distribution,
- s = saturation flow rate (vehicles per second),
- $H(\mu) = e^{-(\mu + \mu^2/2)}$,
- $\mu = (1 - x)(s \cdot g)^{1/2}$,
- x = v/c ratio; degree of saturation.

6.3.2.4 UPPER BOUND FOR NUMBER OF STOPS

So far, the analysis has assumed that all vehicles that form a queue are cleared across the stop-line before the next red phase starts. However, this is not the situation in heavy flow conditions, where vehicles may be forced to stop more than once before clearing the intersection. For oversaturated conditions, specially with stochastic arrivals, it is almost impossible to find a general equation for the number of stops using queuing analysis. Instead, this research effort establishes upper bounds for the number of vehicle stops.

The derivation of the upper bound for the number of stops can be obtained using queue length estimation equations and computing the number of stops as the number of vehicle arrivals at an intersection while a queue exists plus the overflow of vehicles that are not served during the previous cycle. As an example, Figure 6.4 illustrates the first two cycles of operation at an oversaturated signalized intersection. In this diagram, the maximum number of stops for the second cycle is equal to the sum of all vehicle arrivals during the second cycle, plus the demand that did not discharge during the previous cycle. The volume that is not served in the first cycle is computed as the difference in the arrival and departure rate multiplied by the cycle length, i.e., $(q - C)c$. Similarly, the maximum number of stops for the third cycle is equal to all arriving vehicles during that cycle plus the volume that remains to be served from previous cycle. The generalized equation for computing the number of stops upper bound is shown in Equation 6.5. This upper bound is valid for uniform arrivals during oversaturated conditions.

$$N_{ub} = \frac{nqC + \sum_{i=1}^{n-1} i \cdot (q - c) \cdot C}{q \cdot t_e} \quad (6.5)$$

where:

- N_{ub} = upper bound average number of stops (stops/vehicle),
- n = number of cycle within t_e ,
- q = arrival rate (vehicles/seconds),
- C = cycle length (seconds),
- c = capacity (vehicles/second),
- t_e = evaluation time (seconds).

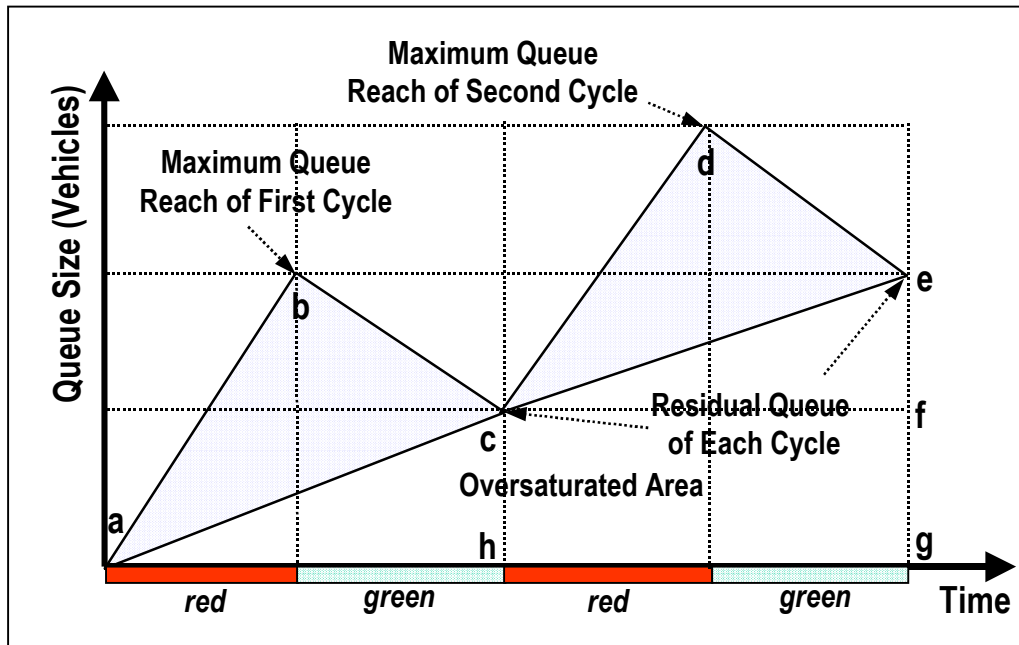


Figure 6.4: Queuing Diagram for number of stops for oversaturated condition

6.3.2.5 PROPOSED MODEL FOR OVERSATURATED CONDITIONS

In this section, a proposed model is developed to compute the number of vehicle stops at oversaturated signalized approaches. The proposed model is based on the upper bound model that described in the previous section. Using the upper bound model and the INTEGRATION stop estimates, an adjustment factor is derived that adjusts that the upper bound number of stops to compute the actual number of stops. Regression analysis is performed to derive the formula of the adjustment factor based on the v/c ratio and expressed in Equation 6.6, as illustrated in Figure 6.5. Figures 6.5 and 6.6 illustrate an excellent fit between the observed adjustment factor and the regression line. The regression line indicates that the adjustment factor reduces from 1.0 at a v/c ratio of 1.0 to 0.5 as the v/c ratio tends to 2.0. The regression summary results demonstrate a highly explanatory regression fit ($R^2 = 0.99$) with all terms significant.

$$AF = 2.352 - 1.731x + 0.405x^2 \quad (6.6)$$

where:

AF = upper bound adjustment factor,

x = v/c ratio.

Once the adjustment factor is found, the proposed model computes the number of vehicle stops based on the product of the upper bound stop estimate and the adjustment factor, as is expressed in Equation 6.7.

$$N_s = N_{ub} \times AF \quad (6.7)$$

where:

N_s = average number of stops (stops/vehicle),

N_{ub} = upper bound for number of stops (Equation 6.5),

AF = adjustment factor (Equation 6.6).

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.996789					
R²	0.993588					
Adjusted R²	0.991985					
Standard Error	0.015704					
Observations	11					
ANOVA						
	df	SS	MS	F	Significance F	
Regression	2	0.305721	0.152860	619.8653	1.69E-09	
Residual	8	0.001973	0.000247			
Total	10	0.307694				
	Coefficients	Standard Error	t Statistic	P-value	Lower 95%	Upper 95%
Intercept	2.351587	0.117527	20.00889	4.06E-08	2.080569	2.622605
X Variable 1	-1.73100	0.161529	-10.7163	5.05E-06	-2.10348	-1.35851
X Variable 2	0.405369	0.053611	7.561287	6.54E-05	0.281742	0.528997

Figure 6.5: Regression Analysis for Adjustment Factor

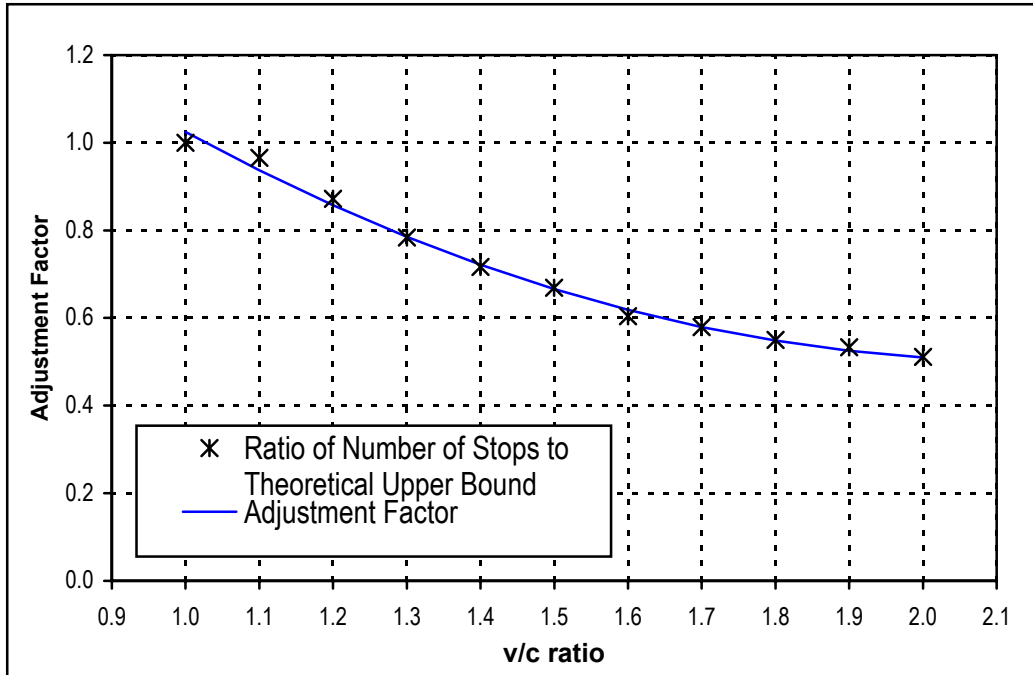


Figure 6.6: Comparison of Ratio and Adjustment Factor

6.3.3 COMPARISON OF NUMBER OF STOPS ESTIMATES

This section compares the number of stops estimates from the different analytical models to the results of INTEGRATION for both undersaturated and oversaturated conditions.

6.3.3.1 NUMBER OF STOPS ESTIMATES FOR UNDERSATURATED CONDITIONS

Table 6.3, Figure 6.7 and Figure 6.8 provide the results of the number of stops estimations in scenarios considering both uniform and stochastic arrivals for undersaturated conditions. Within INTEGRATION, Equation 6.1 was used to compute the number of vehicle stops with both uniform and random arrivals. For the analytical and existing models, Equations 6.2, 6.3 and 6.4 were used with the analytical model, CCG 1995 model, and Cronje model, respectively

Table 6.3: Number of Stops per Vehicle Estimates for Undersaturated Conditions

Models	v/c Ratio									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Analytical Model	0.526	0.556	0.588	0.625	0.667	0.714	0.769	0.833	0.909	1.000
INTEGRATION (Uniform)	0.500	0.500	0.500	0.600	0.600	0.600	0.700	0.700	0.800	0.800
INTEGRATION (Random)	0.540	0.500	0.560	0.600	0.660	0.670	0.720	0.820	0.940	1.220
CCG 1995 Model	0.526	0.556	0.588	0.625	0.667	0.714	0.769	0.833	0.909	1.000
Cronje Model	0.526	0.556	0.588	0.625	0.667	0.716	0.772	0.837	0.913	1.000

The results of Table 6.3 indicate that the theoretical, analytical, CCG 1995 and Cronje models, produce similar number of stops estimates when applied to undersaturated pretimed signalized intersections. In this case, almost identical results were expected between the analytical model and the CCG 1995 and Cronje models since all the theoretical models assume uniform traffic flows and were derived from queuing analysis. For the INTEGRATION model, it is observed that the number of stops is slightly underestimated when considering uniform arrivals when compared to the theoretical models, as illustrated in Figure 6.7. These differences are attributed to two factors. The first of these factors is the fact that vehicle within the INTEGRATION model decelerate as they approach a queue. Consequently, they do not experience a full stop as the theoretical models would indicate. Second, the simulation model is a discrete vehicle departure model while the theoretical models consider average hourly flow rates that often yield fractional average vehicle arrivals within a single cycle length. In summary, the lower number of vehicle stops that are estimated by the INTEGRATION model appear to be more consistent with traffic behavior.

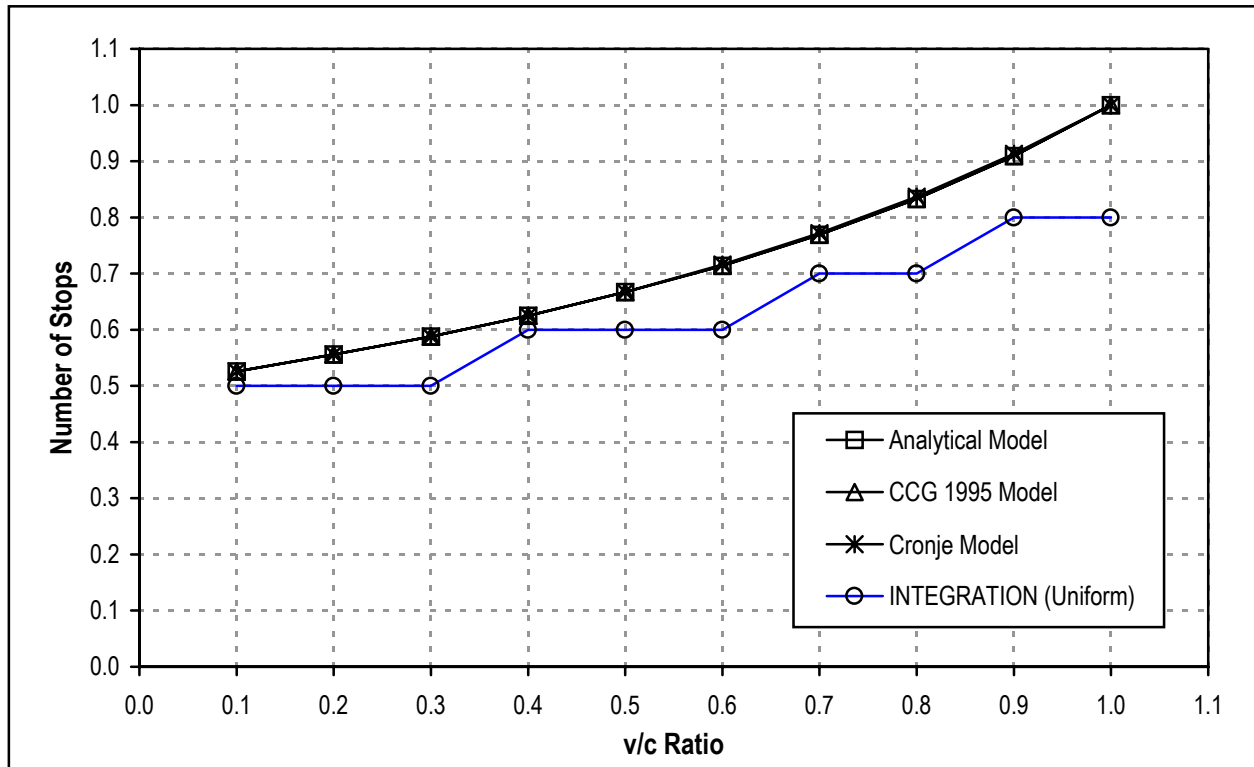


Figure 6.7: Number of Stops Estimates under Uniform Arrivals for Undersaturated Conditions

Figure 6.8 shows the results of the number of stops estimations that were carried out for the example of Figure 6.7 using the random arrival flow scenarios. For this set of scenarios, the same equations, Equation 6.2, 6.3 and 6.4, were used to compute the number of stops for the different theoretical models. The number of stops reported for the INTEGRATION simulation model are in this case the average of the number of stops produced by ten replications of each test scenario. Replications were made for this set of scenarios to account for the stochastic variability of the simulation processes within the INTEGRATION model. Figure 6.8 further superimposes the number of stops predicted by the theoretical models to those obtained with each of the ten replications that were conducted with the INTEGRATION model.

The results of Figure 6.8 indicate that there is a general agreement between the INTEGRATION stop estimates and the number of stops predicted by the analytical, CCG 1995, and Cronje models. As it can be observed, all three models and the INTEGRATION model agree at low v/c ratios, i.e., ratios of less than 0.9. However, as traffic demand approaches saturation (v/c ratio of

1.0), it can be observed that there is a general disagreement between all the three models and the INTEGRATION simulation model. This disagreement was expected, as the analytical model, CCG 1995 model and Cronje model only assume uniform vehicle arrivals. Figure 6.7 also indicates that the number of stops estimates predicted by the INTEGRATION model are generally consistent to those predicted by the theoretical models.

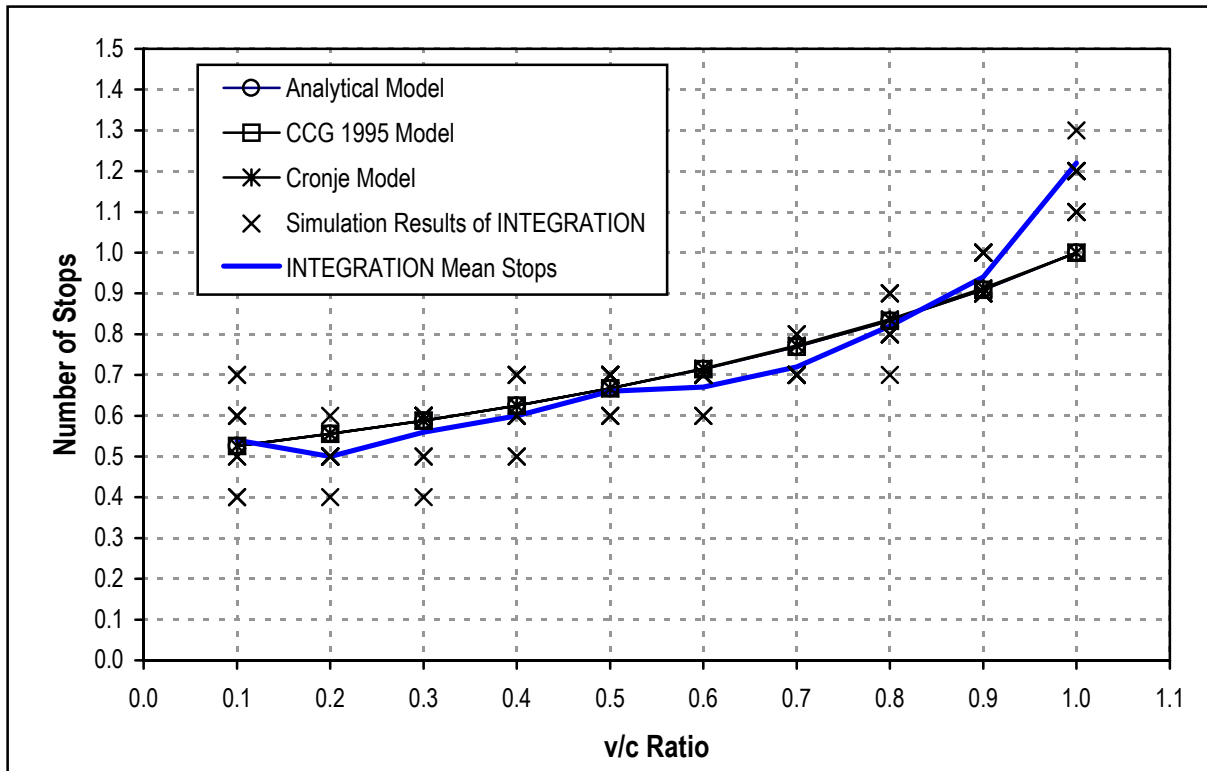


Figure 6.8: Number of Stops Estimates under Stochastic Arrivals for Undersaturated Conditions

6.3.3.2 NUMBER OF STOPS ESTIMATES FOR OVERSATURATED CONDITIONS

Table 6.4 and Figures 6.9, 6.10 provide the results of the number of stops estimates that were carried out for the scenarios considering uniform and stochastic vehicle arrivals only for oversaturated conditions. Equations 6.2, 6.4, 6.5 and 6.7 are used for estimating the number of stops for the analytical model, Cronje model, upper bound model and proposed model, respectively. In addition, CCG 1995 could not be applied in this analysis to estimate the number of stops since the model is not valid for multiple stops, as indicated earlier.

Table 6.4: Overall Number of Stops Estimates for Oversaturated Conditions

Models	v/c Ratio									
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Analytical Models	1.111	1.250	1.429	1.667	2.000	2.500	3.333	5.000	10.000	∞
INTEGRATION (Uniform)	1.500	1.800	2.000	2.100	2.200	2.200	2.200	2.200	2.200	2.200
INTEGRATION (Random)	1.580	1.890	2.050	2.150	2.230	2.190	2.250	2.260	2.300	2.300
Cronje Model	1.101	1.223	1.382	1.604	1.929	2.431	3.274	4.951	9.955	∞
Theoretical Upper Bound	1.636	2.167	2.615	3.000	3.333	3.625	3.882	4.111	4.316	4.500
Proposed Model	1.532	1.856	2.052	2.163	2.219	2.241	2.247	2.251	2.264	2.293

For the uniform vehicle arrival scenarios, Table 6.4 and Figure 6.9 show that there is a disagreement in the results. Specifically, the analytical models estimate the number of vehicle stops in the v/c ratio range from 1.1 to 1.5. These models overestimate the number of vehicle stops for v/c ratios in excess of 1.6. Furthermore, these models estimate stops that are higher than the derived upper limit. However, Table 6.4 and Figure 6.9 indicate that there is a significant consistency between the INTEGRATION model and the proposed model that is more reliable method to find the number of stops when compared to other analytical models

These analytical models were derived as steady state queuing models, which means that the stop curves tend to infinity as the volume v/c ratio increases. This result arises from the unrealistic assumption that the system is in a perpetual steady state. In reality, any peak period ends at a point in time and the arrival flow rate thus decreases long before a steady state is reached. It is interesting to note that the number of stops estimated by the INTEGRATION model follow a similar trend as the theoretical upper bound. Furthermore, the INTEGRATION results never exceed the upper bound. Finally, it should be noted that the INTEGRATION results indicate that the number of stops tend to a steady state of approximately 2.3 because vehicles tend to oscillate between similar speeds, as illustrated earlier in Figure 6.2.

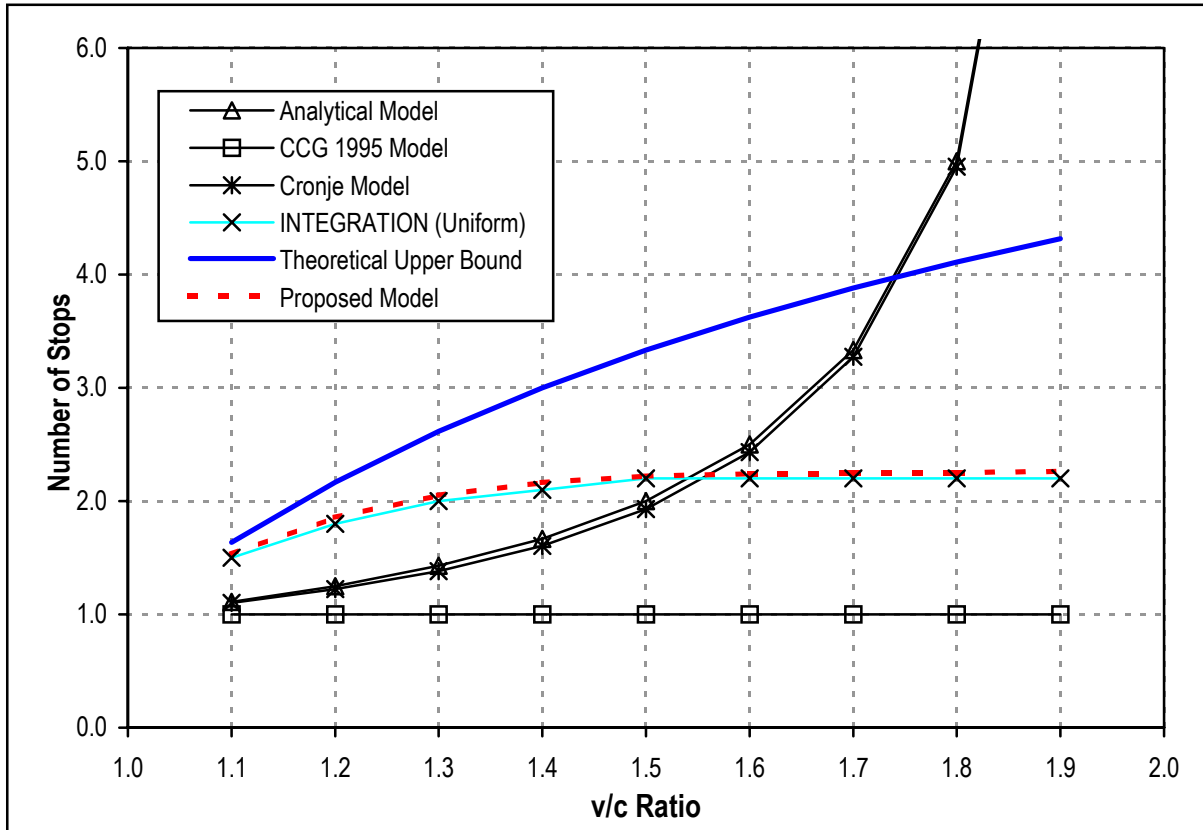


Figure 6.9: Number of Stops Estimates with Uniform Arrivals for Oversaturated Conditions

The results for random arrivals are similar to the uniform arrivals, as illustrated in Figure 6.10. The randomness of the arrivals has a minor impact on the results because the system is already experiencing over congestion. Consequently, the existence of queues dilutes the effects of randomness. In addition, the results of Figure 6.10 indicate that there is a significant agreement between the INTEGRATION stop estimates and the number of stops predicted by the proposed model for random arrivals.

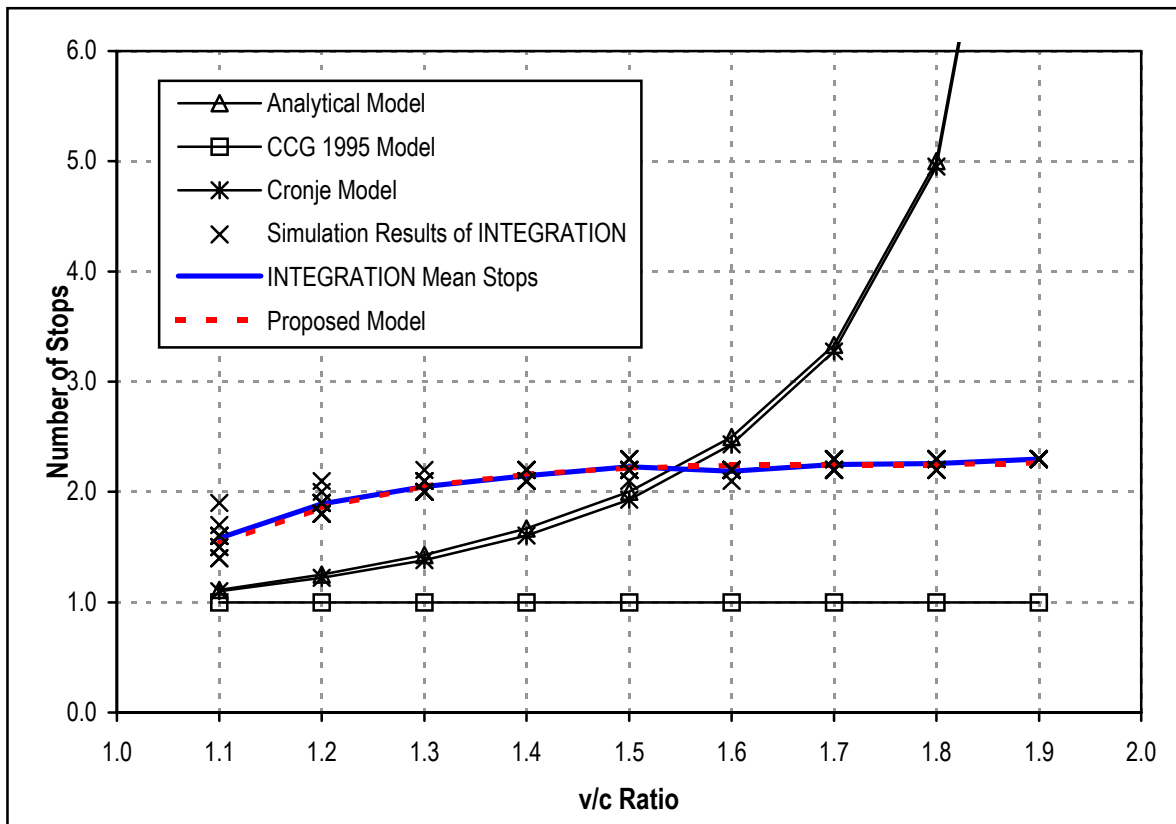


Figure 6.10: Number of Stops Estimates with Stochastic Arrivals for Oversaturated Conditions

6.4 QUEUE LENGTH ESTIMATION AT SIGNALIZED APPROACHES

6.4.1 QUEUE LENGTH ESTIMATION

The focus of this section is to compare the queue length estimates based on analytical procedures to those observed with the INTEGRATION environment. Unfortunately, several terms are used in the literature to describe queues. These include the average queue length, maximum queue length, and maximum extent of queue. Very often, the procedure employed to estimate queue length and an account of how these estimates vary is not readily available. In this section, one MOE, the maximum spatial extent of a queue, is examined in detail.

The modeling of queues involves an accounting of the accumulation and discharge of vehicles that arrive at a bottleneck, like for example an approach to an isolated fixed-time signalized

intersection. The queue accumulation is important for the determination of the delay and level of service. While many analytical concepts are common to all of the existing queuing models, each has unique characteristics that distinguish it from other models. A wide variation in terminology has evolved, and the same term may mean different things to different models.

Figure 6.11 illustrates the queuing concept and distinguishes between the maximum vehicles in the queue and the maximum queue extent. Specifically, Figure 6.12 illustrates the maximum queue length calculated by most models and the maximum extent of a queue. As shown in Figure 6.12, the maximum queue length is measured at the beginning of the effective green time, while the maximum queue extent accounts for vehicle arrivals that join the back of queue after the green time starts, even though vehicles begin to depart from the front of the queue. The distinction between these two estimates is extremely important in the design of left turn pocket lanes. Specifically, a pocket lane that extends beyond the maximum queue extent ensures that the left turners do not spillback on other lanes. The computation of the queue extent in the field or in a simulation environment is difficult to quantify especially during oversaturated conditions. For example, when is the profile that is illustrated in Figure 6.2 considered to be in queue? Consequently, the INTEGRATION model does not report the maximum queue as part of its standard output.

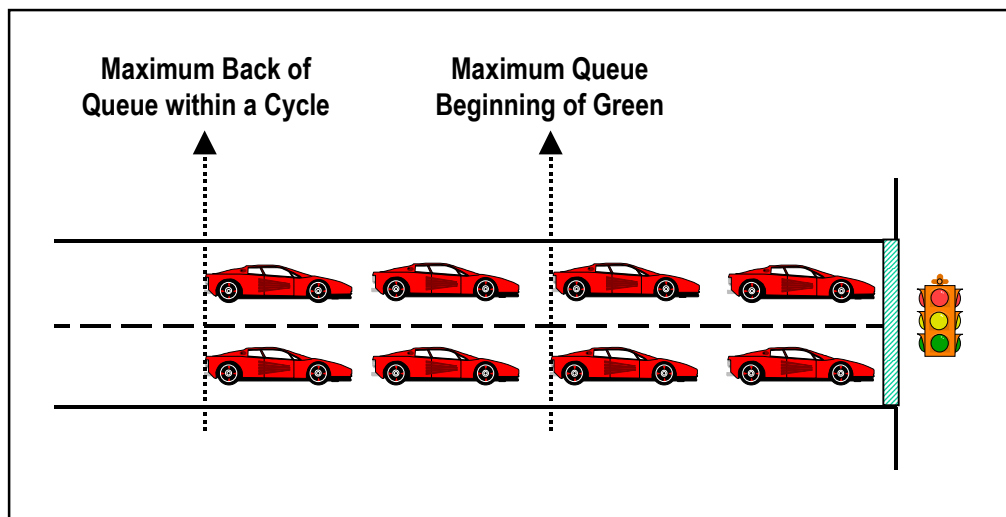


Figure 6.11: Illustration of Queue Description.

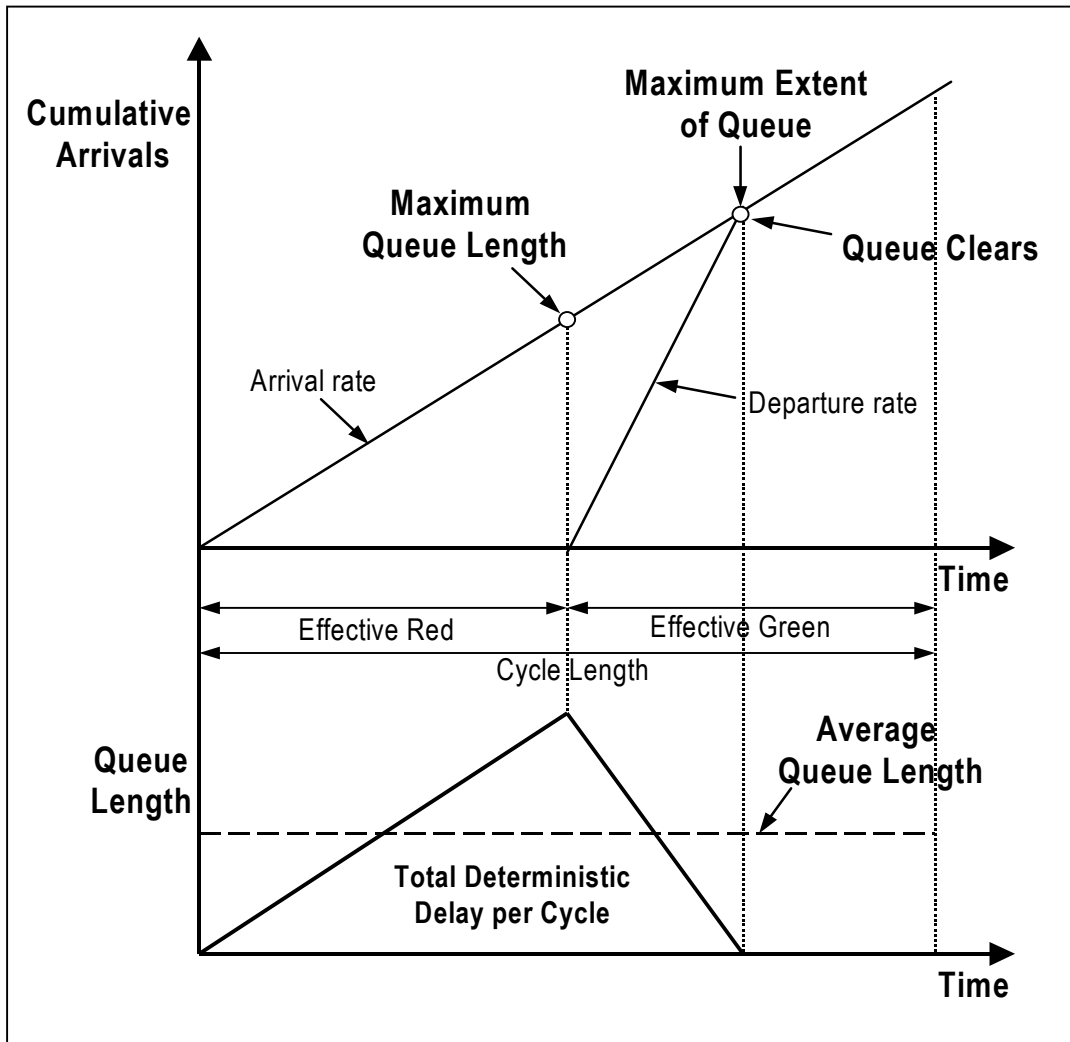


Figure 6.12: Queue Length Comparison

6.4.2 ANALYTICAL MODELS

6.4.2.1 VERTICAL DETERMINISTIC QUEUING ANALYSIS

The deterministic queuing analysis serves as one of the most commonly utilized approaches for estimating queue lengths at the macroscopic level because it is relatively simple. In this analysis, both undersaturated and oversaturated conditions are considered. Furthermore, the queue discipline is assumed to be a "first in, first out (FIFO) system.

The estimates of queue length at signalized intersections can be computed using deterministic queuing theory. Assuming a vertical queue, the maximum queue that forms upstream a traffic

signal would be equal to the number of arrivals during the red interval plus dissipation time, as illustrated in Figure 6.13. In this case, the maximum queue length can be found using Equation 6.8.

However, during the green interval, the vehicles discharge at the saturation flow rate at the front of the queue while vehicles join the queue at a rate equal to the arrival rate at the back of the queue. The time required to dissipate the queue is computed using Equation 6.9. If traffic demand is undersaturated, the time required to dissipate the queue is less than the green interval, and the maximum queue length becomes the summation of the red interval and dissipation time multiplied by the arrival flow rate. Therefore, the maximum queue size can be computed using Equation 6.10. Dividing Equation 6.10 by the jam density it is possible to compute the spatial extent of the queue.

$$Q_{red} = r \cdot q \quad (6.8)$$

$$t = \frac{Q_{red}}{s - q} \quad (6.9)$$

$$Q_{max} = \frac{s}{s - q} \cdot r q \quad (6.10)$$

where:

- t = time to dissipate queue (seconds),
- Q_{red} = queue length at end of red interval (vehicles),
- Q_{max} = maximum queue length during a cycle (vehicles),
- r = red interval (seconds),
- s = saturation flow rate (vehicles/second),
- q = arrival flow rate (vehicles/second).

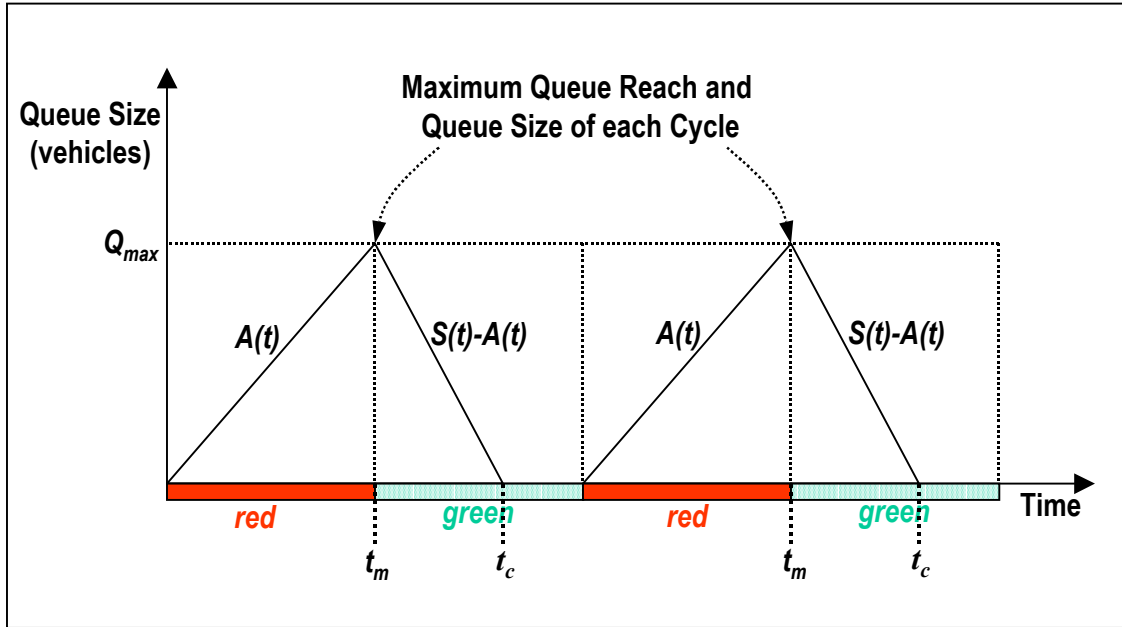


Figure 6.13: Vertical Deterministic Queuing Diagram for Undersaturated Condition

Alternatively, when traffic demand is oversaturated, a residual queue remains at the end of the green interval, as illustrated in Figure 6.14. The figure shows only three cycle times for illustration purposes. In this diagram, points a, c, and e illustrate the queue length at the conclusion of the red interval within each cycle, while points b, d, and f show the length of the residual queue at the end of each cycle. The difference between the arrival flow rate and the capacity of a signalized intersection gives the residual queue of vehicles at the end of a cycle. When the residual vehicles of a first cycle are known, the residual vehicles of N^{th} cycle can be computed as the residual vehicles of the first cycle multiplying by number of cycle (N). Based on this observations, Equations 6.11 and 6.12 can be derived to calculate the residual queue length at the end of each cycle and provide an approximate estimate of the maximum queue length for each cycle at an oversaturated fixed-time signalized approach. The computation of the maximum queue length is not possible using queuing theory, instead shock wave analysis should be considered as discussed in the forthcoming section.

$$Q_{res} = N \cdot [qr - (s - q)g] \quad (6.11)$$

$$Q_{max} = Nqr - (N - 1)(s - q)g \quad (6.12)$$

where:

Q_{res} = residual queue length of end of each cycle (vehicles),

N = number of cycle,

q = arrival flow rate (vehicles/second),

r = red interval (seconds),

g = green interval (seconds),

s = saturation flow rate (vehicles/second),

Q_{max} = approximate estimate of maximum queue length during a cycle (vehicles).

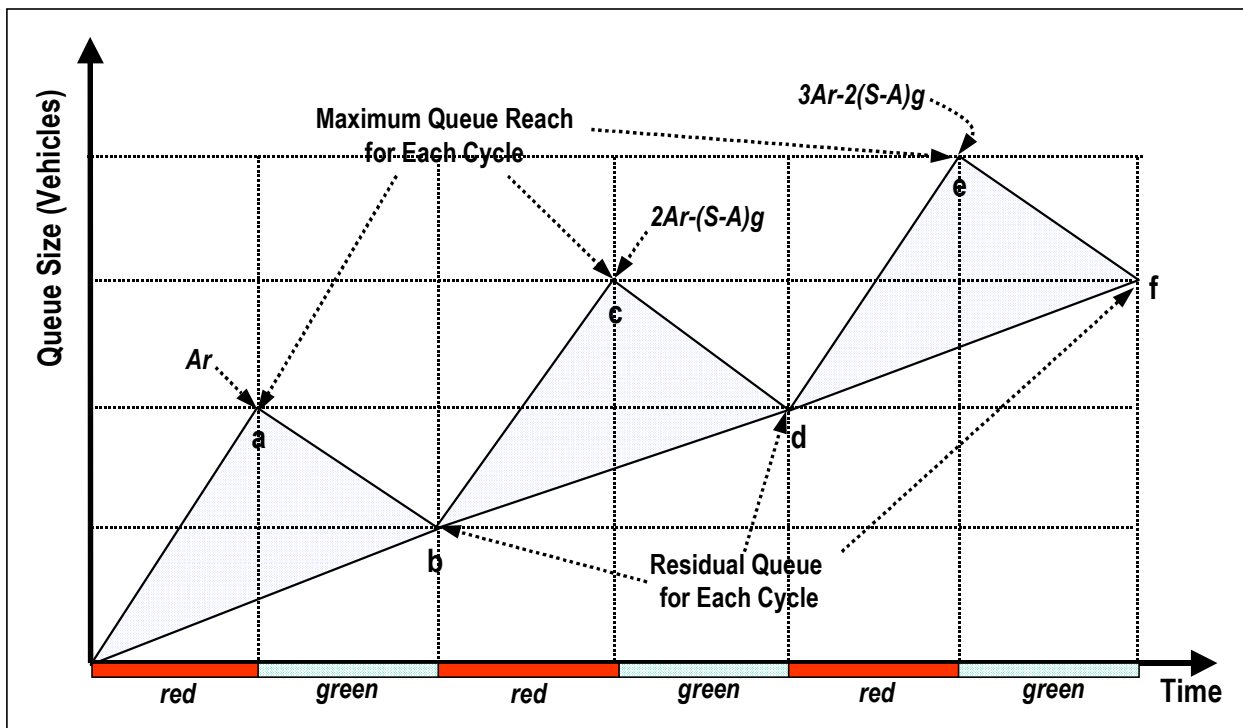


Figure 6.14: Vertical Deterministic Queuing Diagram for Oversaturated Condition

6.4.2.2 HORIZONTAL SHOCK WAVE ANALYSIS

Shock wave theory can also be used to estimate queue lengths at signalized intersections. The main difference between shock wave and queuing analysis models is in the way vehicles are assumed to queue at the intersection stop line. As mentioned in Chapters 4 and 5, while queuing analysis assumes vertical queuing, i.e., stacks of vehicles, shock wave analysis considers that

vehicles are queued horizontally one behind each other, i.e., that each vehicle occupies a physical space. This treatment allows shock wave analysis to capture more realistic queuing behavior and to more realistically compute the extent of the queue.

Consider for example the diagram of Figure 6.15, which illustrates the formation and dissipation of a queue of vehicles through shock wave analysis for undersaturated conditions. In this figure, it can be observed that while the maximum number of queued vehicles still occurs at the end of the red interval, the maximum reach of the back of queue is now correctly modeled to occur later, as is observed in the field. Table 6.5 shows the estimated values to compute the maximum extent of queue for the horizontal shock wave. Based on this analysis, Equation 6.13 and 6.14 can be derived for determining the maximum extent of queue and maximum number of queued vehicles caused by the fixed-time signalized intersection, respectively.

$$x_m = \frac{qsr}{s(k_j - k_a) - q(k_d - k_j)} \quad (6.13)$$

$$N_Q = x_m \cdot k_j \quad (6.14)$$

where:

- x_m = distance of maximum extent queue (kilometers),
- q = arrival rate (vehicles/second),
- s = saturation flow rate (vehicles/second),
- k_d = density of discharge flow (vehicles/kilometer),
- k_j = jam density (vehicles/kilometer),
- k_a = density of approach flow (vehicles/kilometer),
- N_Q = number of vehicles in maximum queue (vehicles).

Table 6.5: Estimated Values to Compute Maximum Extent of Queue using Shock Wave Analysis for Undersaturated Condition

v/c	k_a	U_{AB}	x_r	t_m	t_c	mbq	mnv
0.1	1.5	-0.759	-0.006	1.184	1.579	0.007	0.789
0.2	3.0	-1.538	-0.013	2.500	3.333	0.014	1.667
0.3	4.5	-2.338	-0.019	3.971	5.294	0.022	2.647
0.4	6.0	-3.158	-0.026	5.625	7.500	0.031	3.750
0.5	7.5	-4.000	-0.033	7.500	10.000	0.042	5.000
0.6	9.0	-4.865	-0.041	9.643	12.857	0.054	6.429
0.7	10.5	-5.753	-0.048	12.115	16.154	0.067	8.077
0.8	12.0	-6.667	-0.056	15.000	20.000	0.083	10.000
0.9	13.5	-7.606	-0.063	18.409	24.545	0.102	12.273
1.0	15.0	-8.571	-0.071	22.500	30.000	0.125	15.000

k_a = density of approach flow (vehicles/km),
 U_{AB} = speed of shock wave between area A and area B (km/hr),
 x_r = queue length at end of red interval (km),
 t_m = time at maximum extent of queue (seconds),
 t_c = time to clear queue (seconds),
 mbq = maximum extent of queue (km),
 mnv = maximum number of vehicles (vehicles).

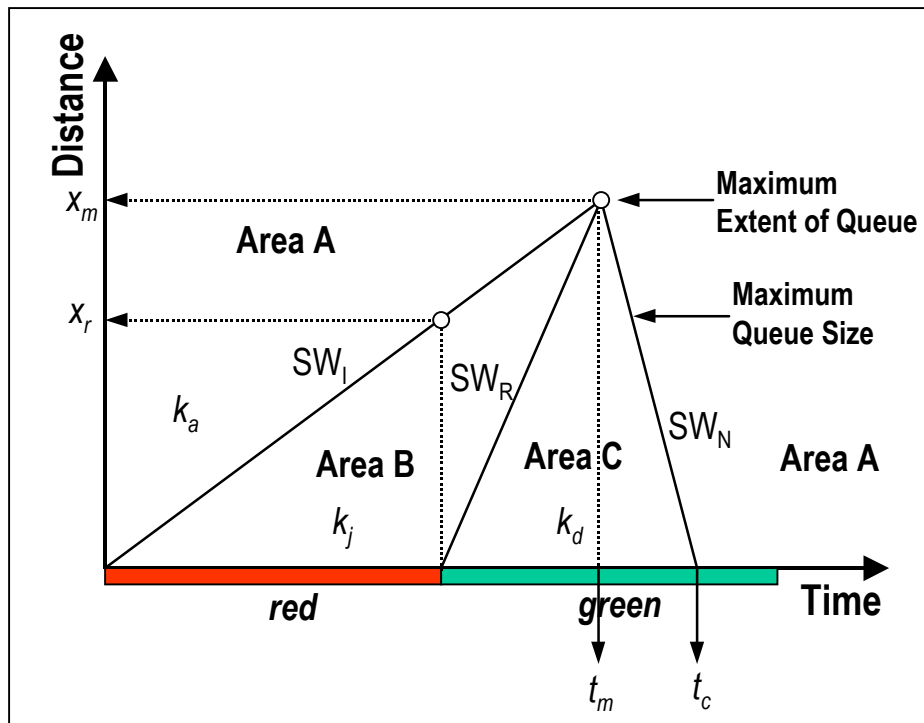


Figure 6.15: Horizontal Shock Wave Diagram for Undersaturated Condition

Unlike the analysis for undersaturated conditions, the analysis of oversaturated conditions involves a more complex process to calculate the maximum spatial extent of the queue. In particular, storage length requirements are seldom based on the excessive queues that accumulate over periods of oversaturated operation. The derivation of the maximum queue extent is demonstrated in Table 6.6 and illustrated in Figure 6.16 using horizontal shock wave analysis diagrams. In this figure, the locations of each value shown in Table 6.4 and used to compute the maximum extent of queue and the maximum number of queued vehicles is illustrated.

Table 6.6: Estimated Values to Compute Maximum Extent of Queue using Shock Wave Analysis for Oversaturated Condition

v/c	K_a	U_{AB}	x_{r1}	t_{m1}	x_{m1}	t_{c1}	x_{c1}	tt	noc	mbq	mnv
1.1	16.5	-9.56	-0.080	27.5	0.153	5.0	0.028	62.50	14	0.389	46.67
1.2	18.0	-10.59	-0.088	33.75	0.188	7.5	0.063	71.25	12	0.750	90.00
1.3	19.5	-11.64	-0.097	41.79	0.232	7.5	0.107	79.29	11	1.179	141.43
1.4	21.0	-12.73	-0.106	52.50	0.292	7.5	0.167	90.00	10	1.667	200.00
1.5	22.5	-13.85	-0.115	67.50	0.375	7.5	0.250	105.00	8	2.000	240.00

- k_a = density of approach flow (vehicles/km),
- U_{AB} = speed of shock wave between area A and area B (km/hr),
- x_{r1} = queue length at end of red interval (km),
- t_{m1} = time at maximum extent of queue (seconds),
- x_{m1} = maximum extent of queue during the first cycle length (km),
- t_{c1} = time at end of the first cycle (seconds),
- tt = total time spend of each cycle (seconds),
- noc = number of cycle during evaluation time,
- mbq = maximum extent of queue during evaluation time (km),
- mnv = maximum number of vehicles during evaluation time (vehicles).

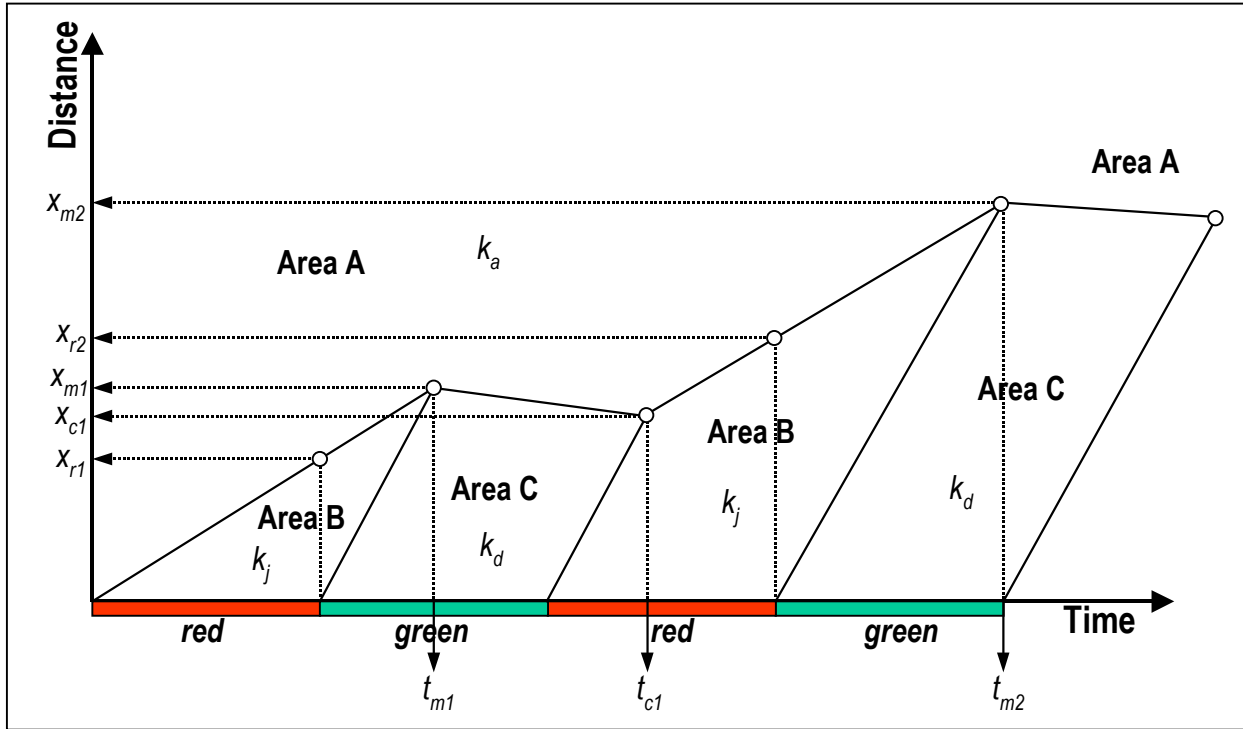


Figure 6.16: Horizontal Shock Wave Diagram for Oversaturated Condition

6.4.3 MICROSCOPIC SIMULATION MODEL

As mentioned earlier, the INTEGRATION traffic simulation software does not produce maximum extent of queue length estimations. However, the INTEGRATION model can be compared to theoretical models to assess the consistency of maximum extent of queue estimation using the model's output files. In particular, File 16 provides vehicle probe listing which chronicles trip arrival and departure statistics of each probe vehicle on a second-by-second basis.

To assess the consistency of the INTEGRATION model relative to the horizontal shock wave analysis model, the estimated queue length values obtained by the horizontal shock wave analyses are superimposed on the simulated time-space trajectories that are derived from INTEGRATION, as illustrated in Figure 6.17. There are two underlying assumptions to consider when comparing the INTEGRATION and horizontal shock wave analysis models for both undersaturated and oversaturated conditions. First, each cycle of operation is identical and the arrivals and departures are completely uniform, i.e., all vehicles arrive and depart at exactly the

same rate. Second, there is no residual queue from the previous cycle at the beginning of the red interval.

Figure 6.17 illustrates that the estimated values from the horizontal shock wave analysis are consistent with the plotted time space profiles. In this diagram, the speed of the initial shock wave is 6.667 km/h and the maximum extent of queue, which stretches up to 0.083 km from the stop line, is not achieved until the instant before the queue has completely dissipated. The time at which the maximum extent of queue is achieved is 15 seconds after the green interval starts. In addition, it is observed that the maximum queue length at the end of the red interval is 0.056 km. The main reason for the difference between the maximum extent of queue and the maximum queue length is that vehicles continue to join the back of queue after the end of the red interval, as vehicles begin to depart from the front of the queue. In addition, the maximum number of vehicles in a queue can be found by multiplying the length of maximum extent of queue by the jam density. The plotted diagram in Figure 6.17 clearly shows that the maximum extent of queue estimates using the INTEGRATION model are consistent with the horizontal shock wave analysis.

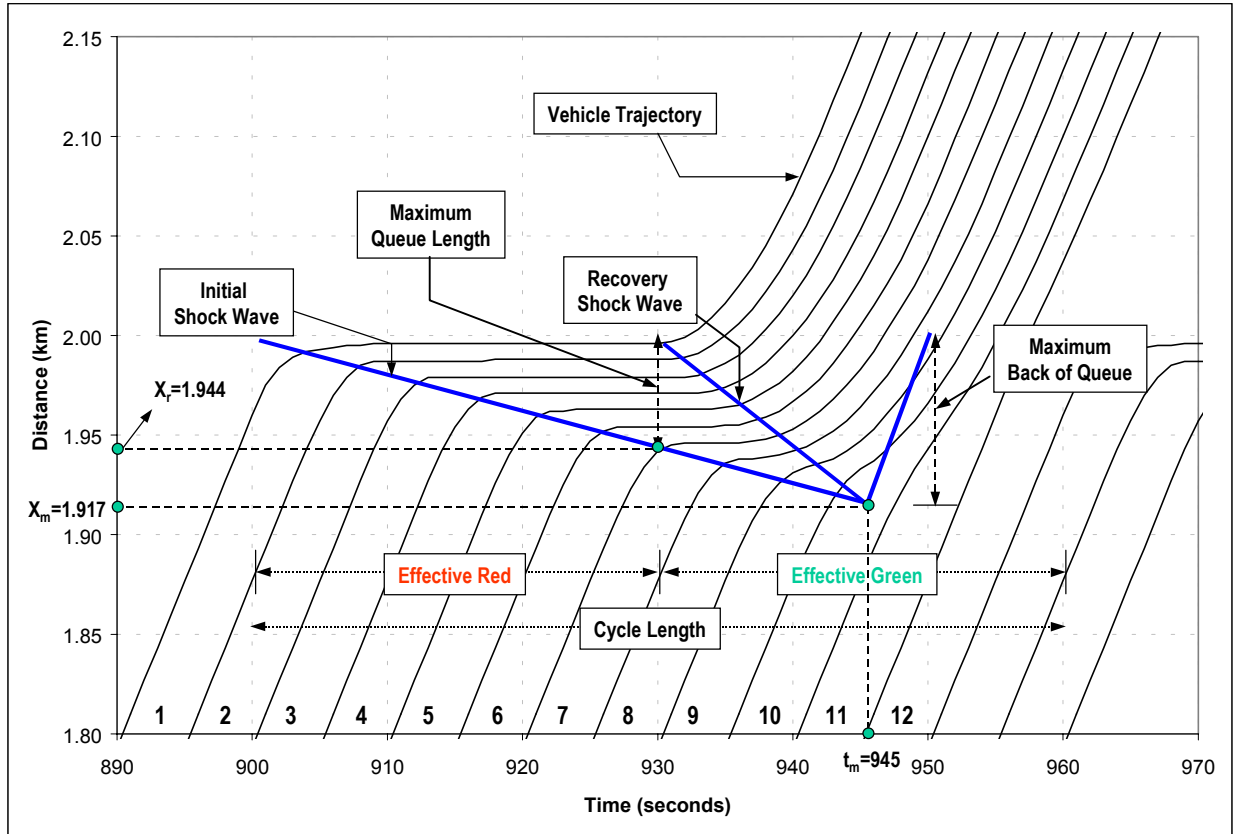


Figure 6.17: Simulated vs. Shock Wave Diagram for Undersaturated Condition

When the degree of saturation is high, i.e., $v/c > 1$, a residual queue remains at the end of each cycle, causing faster initial shock wave speeds to occur when compared to undersaturated conditions. Similar to Figure 6.17, Figure 6.18 illustrates that the estimated values from the horizontal shock wave analysis are consistent with the time-space profiles of vehicles from the INTEGRATION model for oversaturated conditions when the v/c ratio equals to 1.5. From the diagram, the maximum extent of queue for the first cycle is found to stretch up to 0.375 km upstream of the intersection stop line as the effect of the residual queues propagates from one cycle to the other. Table 6.6 shows all necessary values to compute the maximum extent of queue from the horizontal shock wave analysis for oversaturated conditions. The plotted diagram in Figure 6.18 also shows that the maximum extent of queue estimates from the INTEGRATION model are consistent with the horizontal shock wave analysis for oversaturated conditions.

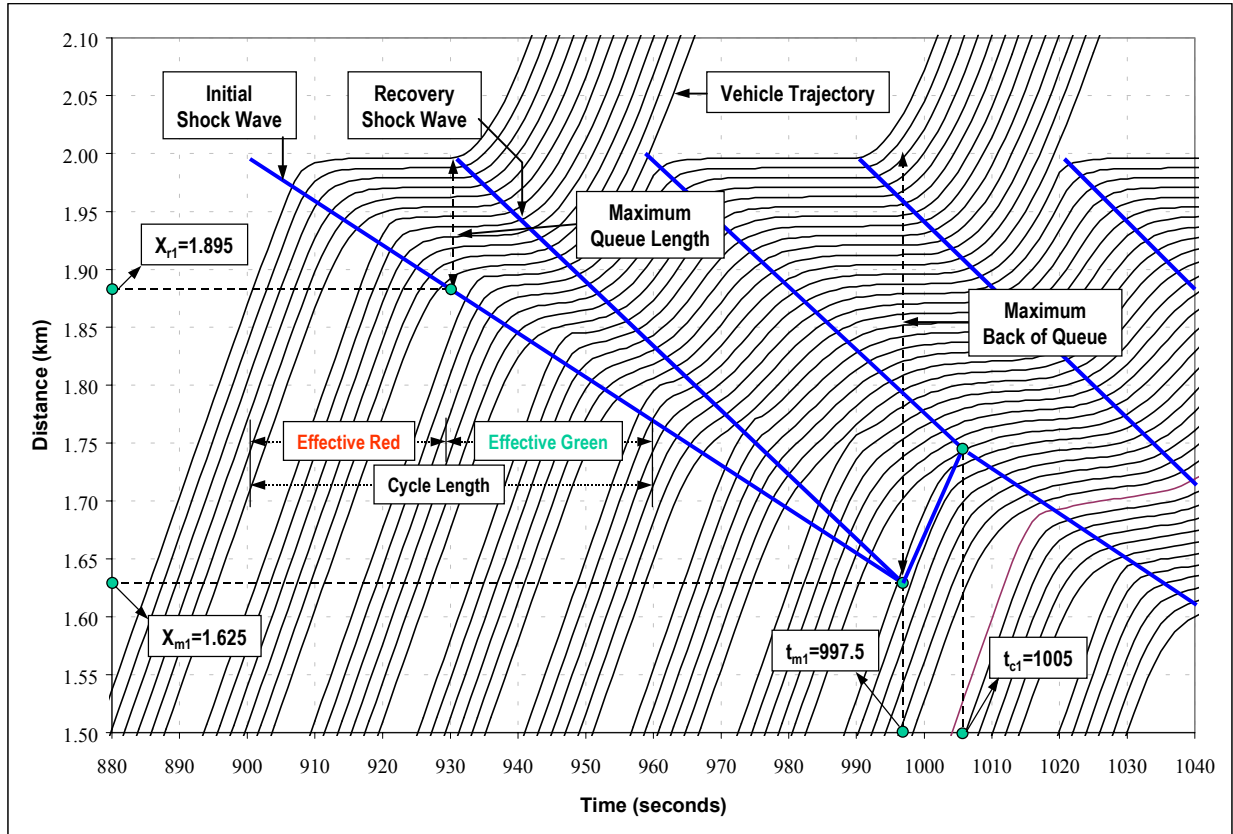


Figure 6.18: Simulated vs. Shock Wave Time-Space Diagram for Oversaturated Condition

6.4.4 EXISTING MODELS

6.4.4.1 CANADIAN CAPACITY GUIDE MODEL

The Canadian Capacity Guide for Signalized Intersections, published in 1995, contains evaluation criteria for estimating queue lengths. These criteria recognize all three queue definitions: Uniform Average Queue Accumulation, Uniform Maximum Queue Reach, Uniform Conservative Approach as illustrated in Figure 6.19. The Uniform Maximum Queue Reach is represented as a liberal approach to determining the required storage space and the Uniform Conservative Approach is represented as a conservative approach. These two parameters converge at high levels of saturation.

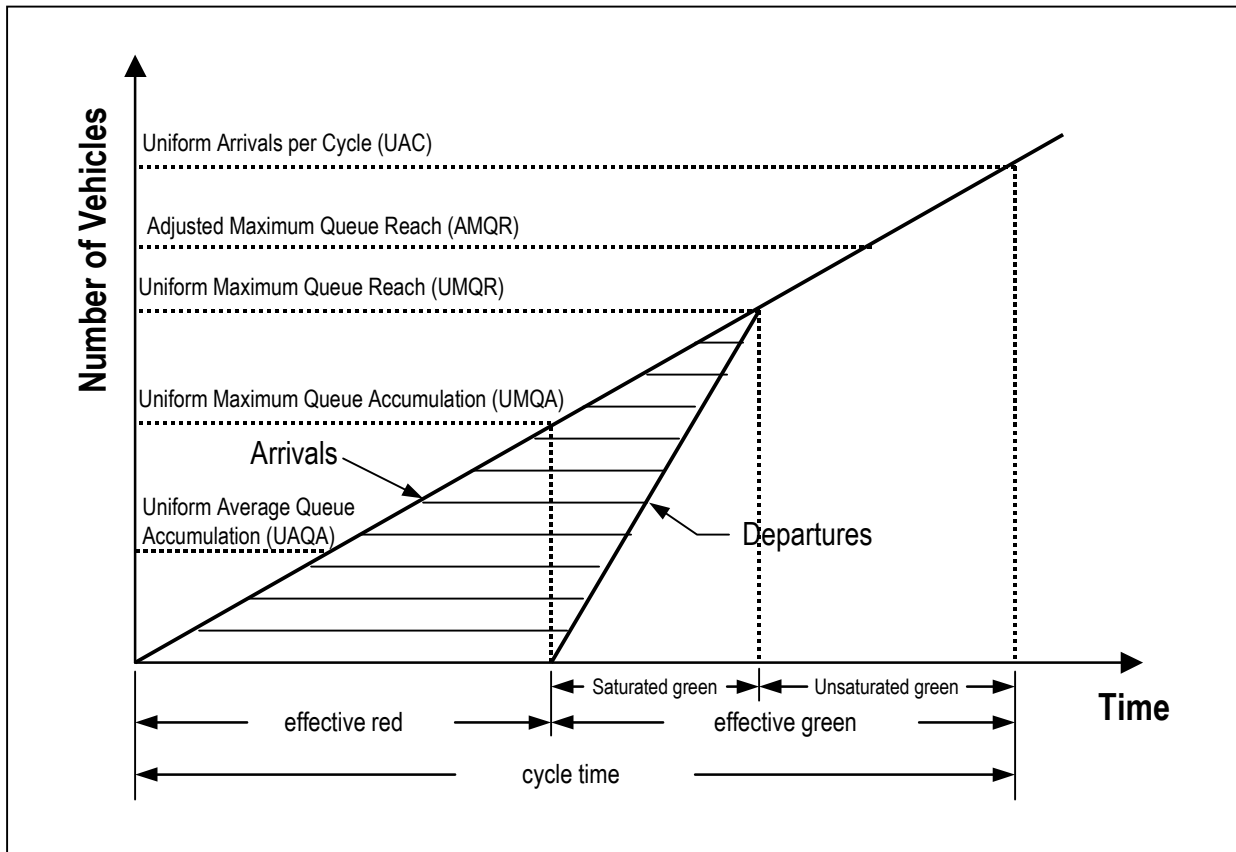


Figure 6.19: Graphical Representation of the Queue Accumulation and Discharge Process*

* Source: Canadian Capacity Guide for Signalized Intersections (1995)

The CCG model deals with the random and overflow effects in a unique manner. Most models treat the probability of spillover in a mathematically rigorous manner, as shown in Equation 6.15.

$$P(\text{no spillover}) = 1 - P(\text{spillover}) \quad (6.15)$$

However, the CCG model formulates this probability using Equation 6.16.

$$P(\text{no spillover}) = [1 - P(\text{spillover})]^2 \quad (6.16)$$

This formulation, which expresses the probability of no spillover on either of two consecutive cycles, is an appropriate surrogate for the explicit treatment of the random and overflow phenomena. The method of the CCG model also includes a separate procedure for treating

oversaturated conditions. The equations of queue length for undersaturated conditions and oversaturated conditions in the CCG model are described by Equation 6.17 and Equation 6.18, respectively.

$$Q1_{CCG} = qC/3600 \quad (6.17)$$

$$Q2_{CCG} = [t_e(q - c)]/60 + Q1_{CCG} \quad (6.18)$$

where:

$Q1_{CCG}$ = estimates of average queue reach (vehicles),

q = arrival flow rate (vehicle/hour),

C = cycle length (seconds),

$Q2_{CCG}$ = maximum queue reach during the congestion period (vehicles),

t_e = evaluation time (minutes),

c = capacity (vehicles/hour)

6.4.4.2 CATLING MODEL

Catling (1977) adapted existing equations of classical queuing theory to oversaturated traffic conditions and developed a comprehensive queue and number of stops estimates that can represent the effects of intersections in a time-dependent traffic assignment model. The time-dependent assignment model requires division of a peak period into time intervals that are chosen so that the mean arrival rate remains approximately constant within an interval. It is therefore obviously important to determine the sensitivity of estimated queue length on oversaturated traffic signal approaches to the method of dividing the peak period.

Consider an approach to a traffic signal which has cycle time c . When the length of queue is increasing with increasing time, Catling's formula for computing the mean queue length is described by Equation 6.19.

$$G(t) = [(\beta^2 + 2 \cdot x^2 \cdot c^2 \cdot t^2 \cdot \alpha \cdot z)^{1/2} - \beta] / \alpha \quad (6.19)$$

where:

$G(t)$ = the mean queue length at time t (vehicles),

$$\begin{aligned}
\alpha &= 2(c \cdot t - z), \\
\beta &= c \cdot t[c \cdot t(1-x) + 2 \cdot z \cdot x], \\
c &= s \cdot g / C, \text{ capacity (vehicles/hour)}, \\
z &= 0.55, \\
g &= \text{effective green time (seconds)}, \\
q &= \text{average arrival rate of traffic (vehicles/hour)}, \\
s &= \text{saturation flow rate (vehicle/hour)}, \\
x &= qc/gs, \text{ the degree of saturation.}
\end{aligned}$$

In fact, when $x < 1$ the steady-state is represented by a limiting value of $G(t)$ as $t \rightarrow \infty$. This limiting value is denoted by $G1(x)$ and can be expressed by Equation 6.20.

$$G1(x) = \lim G(t) = \frac{Cx^2}{1-x} \quad (6.20)$$

These expressions can be applied only for an interval with stationary mean arrival rate and starting with zero queue length. If there is a different demand during subsequent time intervals the formulas cannot be applied since the queue formed at the end of the first interval will cause extra delay to the following arrivals.

6.4.5 RESULTS AND COMPARISON OF QUEUE LENGTH ESTIMATIONS

Table 6.7 and 6.8 provide the results of the maximum extent of queue length estimations for undersaturated and oversaturated conditions, respectively. Figure 6.20 illustrates the results of the maximum extent of queue length estimations for the scenarios considering both undersaturated and oversaturated conditions. For the vertical deterministic queuing analysis, Equation 6.8 and Equation 6.12 were used to compute the maximum extent of queue length for undersaturated and oversaturated conditions. Equation 6.13 was used to compute the maximum extent of queue using the horizontal shock wave analysis for only undersaturated conditions since the horizontal shock wave analysis does not provide a general formula for oversaturated conditions. For the CCG 1995 models, Equation 6.17 for undersaturated conditions and

Equation 6.18 for oversaturated conditions were used to compute the maximum queue length and the maximum extent of queue, respectively. Finally, Equation 6.19 was used for the Catling model.

For both undersaturated conditions, the maximum queue length determined by the vertical deterministic queuing analysis corresponds to the queue size at the end of the red interval with both uniform and random arrivals. As indicated in Table 6.7, the vertical queuing analysis provides lower results since it does not include vehicles that continue to queue at the back of queue after the end of the red interval. On the contrary, the horizontal shock wave analysis and the CCG 1995 models identify how far upstream a queue will stretch. This quantity is measured from the stop line to the rear of the last vehicle joining the queue at the moment when the backward recovery shock wave intersects the backward formation shock wave created by vehicles joining the queue. However, the two models have a different concept in undersaturated conditions. The CCG 1995 model assumes that all vehicles arriving during a cycle must stop and join the queue, while the horizontal shock wave analysis finds the exact times that a queue of vehicles dissipates. In this case, arriving vehicles after a queue is assumed to have cleared are considered not to join the queue. Therefore, this caused the CCG 1995 model to predict higher maximum extent of queue estimation in the undersaturation domain. The formula for the Catling model produced a mean queue length estimate, not maximum extent of queue.

Table 6.7: Overall Maximum Extent of Queue Length Estimates for Undersaturated Condition

Models	v/c Ratio									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Vertical Queuing Analysis	0.75	1.50	2.25	3.00	3.75	4.50	5.25	6.00	6.75	7.50
Horizontal Shock Wave Analysis	0.79	1.67	2.65	3.75	5.00	6.43	8.08	10.00	12.27	15.00
CCG 1995 Model	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50	15.00
Catling Model	0.01	0.03	0.07	0.14	0.26	0.46	0.82	1.56	3.47	10.23

For oversaturated conditions, the maximum queue length estimates from the vertical deterministic queuing analysis correspond to the queue length at the end of the red interval with both uniform and random arrivals. As indicated in Table 6.8, this model produces lower estimates as it does not count vehicles joining the queue after the end of the red interval. The maximum extent of queue for the CCG 1995 model during the evaluation or congestion period may be approximated as the sum of the continuous overflow queue at the end of the evaluation period. As it can be observed, the CCG 1995 model produced lower maximum extent of queue lengths when compared with the horizontal shock wave analysis. This result is not surprising since the first part of Equation 6.18 multiplies the residual queue of each cycle by the evaluation time. The Catling model, finally, produced the lowest queue length when compared to the other models since the model estimates the mean queue length. Figure 6.20 illustrates in graphically the results obtained by the models in both undersaturated and oversaturated domains.

Table 6.8: Overall Maximum Extent of Queue Length Estimates for Oversaturated Conditions

Models	v/c Ratio				
	1.1	1.2	1.3	1.4	1.5
Vertical Queuing Analysis	29.25	51.00	72.75	94.50	116.25
Horizontal Shock Wave Analysis	46.67	90.00	141.43	200.00	240.00
CCG 1995 Model	39.00	63.00	87.00	111.00	135.00
Catling Model	26.61	47.32	69.07	91.17	113.42

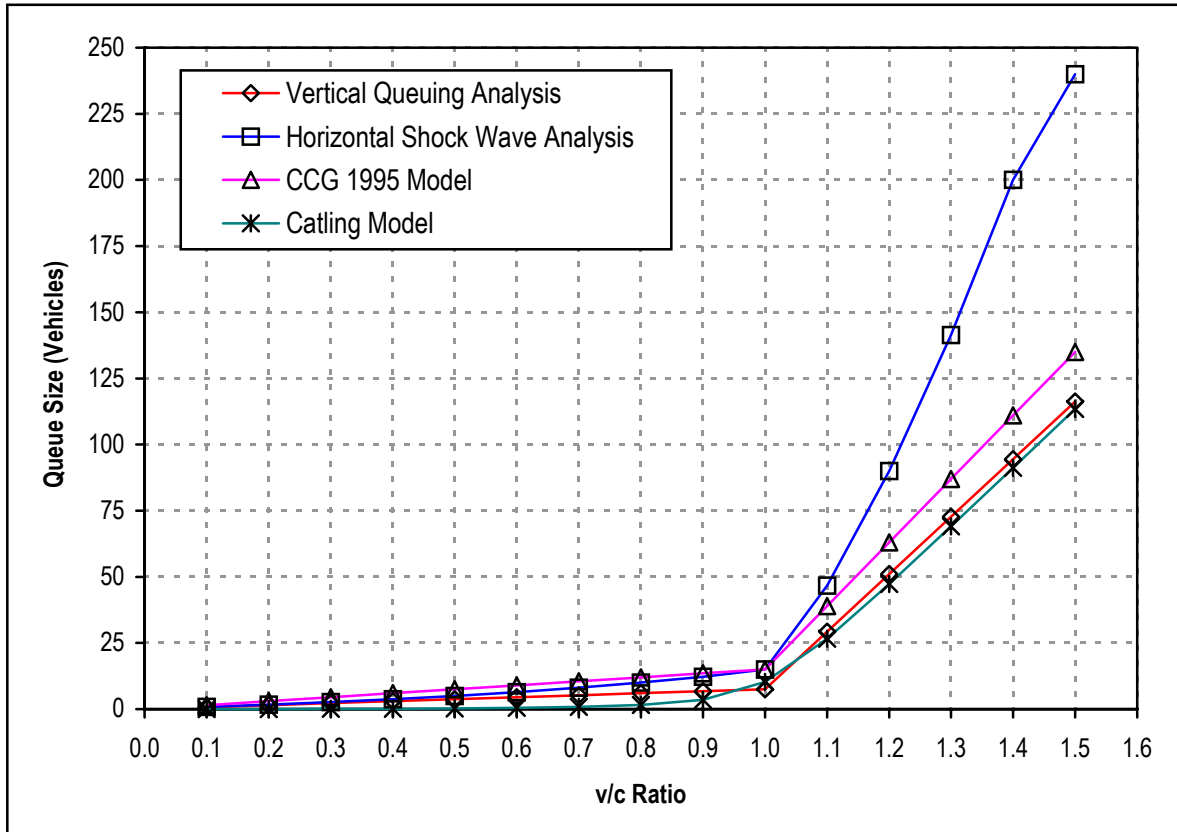


Figure 6.20: Maximum Extent of Queue Length Estimates at an Isolated Traffic Signal

6.5 SUMMARY AND CONCLUSIONS

This chapter compared the number of stops and the maximum extent of queue estimates using analytical procedures and the INTEGRATION simulation model for both undersaturated and oversaturated signalized intersections. This chapter further presented an overview of the most widely used and relevant models available to transportation engineers to estimate both the number of stops and the queue length. The number of stop estimates and the maximum extent of queue predicted by each model under uniform and random arrivals, in addition to undersaturated and oversaturated conditions were compared to assess their consistency and to analyze their applicability.

For the number of stops estimates, it is found that there is a general agreement between the INTEGRATION microscopic simulation model and the analytical models for undersaturated

signalized intersections. The differences in number of stops estimates between the analytical models and the INTEGRATION simulation results in uniform arrival scenarios can be explained by the fact that the model allows vehicles to decelerate as they approach stopped vehicles rather than come to a complete stop. In addition, the INTEGRATION simulation model only allows integer number of arrivals to go across an intersection while the analytical models consider average hourly flow rates that often yield real number of arrivals over a certain number of cycles. The random arrivals demonstrated consistency between the INTEGRATION model and the analytical procedures, however, at a v/c ratio of 1.0 the analytical models underestimate the number of stops.

This chapter also developed an upper limit for the number of vehicle stops for oversaturated conditions. Furthermore, using the upper limit formula, a proposed model for estimating the number of vehicle stops for oversaturated conditions was developed. It was demonstrated that the analytical models can provide stop estimates that far exceed the upper bound. On the other hand, the INTEGRATION model and the proposed model were found to be consistent with the upper bound and demonstrated that the number of stops converge to 2.3 as the v/c ratio tends to 2.0. In addition, there was a significant agreement between the INTEGRATION model and the proposed model that is more reliable method to find the number of stops estimates.

For the maximum extent of queue estimates, it was observed that the estimated maximum extent of queue predicted from horizontal shock wave analysis was higher than the predictions from vertical deterministic queuing analysis. The higher estimates were caused by the fact that the vertical deterministic queuing analysis measured the queue length at the end of the red interval and not at the time a queue truly dissipates. The horizontal shock wave model predicted lower maximum extent of queue than the CCG 1995 model because it assumed that all vehicles arriving at an undersaturated signalized intersection during a cycle must stop and join the queue. For oversaturated conditions, it was, similarly, observed that the vertical deterministic queuing model underestimated the maximum queue length. It was further found that the CCG 1995 predictions were lower than those from the horizontal shock wave model. These differences were attributed to the fact that the CCG 1995 model estimates the remaining residual queue at the

end of evaluation time. Finally, as mentioned in section 6.4.3, a consistency was found between the INTEGRATION model and the horizontal shock wave model predictions with respect to the maximum extent of queue for both undersaturated and oversaturated signalized intersections.