APPENDIX A
CORRECTIONS TO TRIAXIAL DATA

An area correction and membrane correction were applied to all triaxial tests conducted on soil-bentonite for the laboratory testing program described in Chapter 3. A standard area correction was used, assuming that the sample deforms as a right circular cylinder. Aspects of various membrane corrections in the literature were used to develop a new membrane correction. The area correction and membrane correction are described in this appendix.

The combined area correction and membrane correction is referred to here as the Baxter and Filz correction in order to compare the correction to others in the literature. In this appendix the area correction is described, the membrane correction is described including 1) various membrane corrections found in the literature, 2) a discussion of evaluating the stress strain curve to find the Young's modulus of the membrane, 3) mechanics of the membrane 4) details of the new membrane corrections, and 5) comparisons of the new membrane corrections with those found in the literature.

<u>Area correction</u>

The cross sectional area of the specimen was corrected during consolidation and shearing phases assuming that the specimen deforms as a right circular cylinder. This correction is used in the standard test method (ASTM D4767) for CU tests and is recommended by La Rochelle et al. (1988) for bulging type failure. All soil bentonite samples exhibited a bulging type failure. The area correction is given by equation A.1:

$$A_c = \frac{A_o(1-\varepsilon_{vol})}{1-\varepsilon_a} \tag{A.1}$$

where:

A_c = corrected area of specimen

A_o = initial area of specimen

ε_{vol} = volumetric strain of sample

ε_a = axial strain of sample

Membrane Correction

It is generally well established in the literature that the membrane provides resistance to the applied loads and that it may be necessary to correct for it's contribution. The ASTM standard D4767 provides a membrane correction and recommends correcting for the strength of the membrane if the error in deviator stress due to strength of the membrane will be greater than 5%. The error due to the axial strength of the membrane during consolidation and shear phases was estimated for SB1, the soil-bentonite tested in the laboratory testing program. The error in deviator stress for CU tests was typically 8% but was as high as 22% for tests at low stresses.

Many membrane corrections, such as the one in ATSM D4767, neglect strains in the membrane that occur during the consolidation phase. For the soil bentonite testing, these strains are large since the sample is consolidated from a very high slump consistency in the triaxial cell. A new procedure to correct for the membrane was formulated based on aspects of other membrane corrections in the literature.

Literature Review of Membrane Corrections

A review of the literature indicated several main issues for membrane corrections. Henkel and Gilbert (1952) ran experiments to show that the strength of the membrane is proportional to the stiffness of the membrane. They developed two theories to estimate the strength of the membrane during undrained shear called the compression shell theory and hoop stress theory. Duncan and Seed (1967) recommend correcting for the consolidation phase and incorporating the changing area of the membrane. La Rochelle et al. (1988) recommend incorporating the initial diameter of the membrane which may be different from the initial diameter of the specimen. There are different recommendations on how to estimate the Young's modulus of the membrane.

According to Henkel and Gilbert (1952), the compression shell theory would apply if the cell pressure was able to keep the membrane up against the sample. The membrane and the sample would deform together with the deformation assumed to be a right circular cylinder. The membrane was described as a compression shell around the sample. Poisson's ratio for the sample and the membrane was assumed to be 0.5 for undrained loading. Provided that the sample and membrane deform together, no circumferential stresses were assumed to develop in the membrane. The axial stress in the membrane was estimated using elastic theory. A correction to reduce the axial stress in the sample was calculated.

According to Henkel and Gilbert (1952), the hoop stress theory would apply if the membrane "buckled', which was taken to mean to separate from the sample. If bucking occurred, the increase in diameter of the specimen would induce circumferential strains in the membrane that restrict the specimen laterally. Elastic theory was used to calculate the radial stress in the membrane caused by radial deformation of the sample. A correction to increase the lateral stress on the sample was calculated; however, there appears to be a small error in this correction.

The authors also devised a method to estimate the extension modulus of the membrane using a load-extension test. This modulus, M, is given in units of load per inch per strain and is related to the Young's modulus, E, by the following equation

$$E = M/t \qquad (A.2)$$

where t = thickness of the membrane.

Duncan and Seed (1967) modified the compression shell theory to incorporate the stresses and strains that occur during consolidation. The correction is based on previous work by

Duncan (1965). The area of the sample and the area of the membrane are both corrected for right circular cylinder deformation.

La Rochelle et al. (1988) recommend Henkel and Gilbert's compression shell correction and proposed a new hoop stress correction. They ran experiments on rubber dummies to model both theories. In both of the cases, they added a correction to the radial stress to account for the fact that the membrane typically has a different initial diameter than the sample. They compared the diameter of the sample after consolidation with the initial diameter of the membrane and recommended a correction to increase the radial confining pressure after consolidation. They suggested a new hoop stress correction to fit with their experimental results.

The correction recommended by ASTM decreases the axial stress in the sample due to the axial contribution of the membrane during shearing. It appears to be the same as the compression shell theory modified to correct for the changing area of the membrane; however, no derivations are given.

The correction use for this research is based on the compression shell theory with modifications suggested in the literature. It was observed during testing that the cell pressure kept the membrane up next to the sample and the sample and membrane deformed together. The compression shell theory was modified to 1) account for stress and strain of the membrane during consolidation, 2) correct both the sample area and the membrane area for right circular cylinder deformation, and 3) incorporate the initial diameter of the membrane. In addition, the initial tangent modulus was used instead of as average secant modulus.

It was decided that the hoop tension theory did not apply for the testing on soil-bentonite specimens, although it was observed that wrinkles in the membrane developed during the shearing phase at large strains. The wrinkles first formed as small dots where the membrane would separate from the specimen. The dots progressively expanded horizontally into thin lines that extended a quarter of the way or a half of the way around the sample. It is assumed that this is what Henkel and Gilbert (1952) and other researchers referred to as buckling of the membrane. Henkel and Gilbert assumed buckling would release the axial strain in the membrane and that the tangential strains in the sample would cause tangential stress in the membrane. The overall effect would be a lateral restraining of the sample by the membrane or an increase in the minor effective stress. Contrary to Henkel and Gilbert, it was assumed that the wrinkles released the axial strains in the membrane in a local area only. It was observed that the membrane above and below the wrinkle was held firmly next to the sample and continued to deform with the sample. At the location of the wrinkle, the membrane was observed digging into the sample and restricting the sample; however the effect of the wrinkle was neglected since the effect was seen to be a local one. As noted by Henkel and Gilbert (1952), the effect of the membrane is actually a complex combination of the two modes.

Young’s modulus of the membrane

Two issues about the modulus of the membrane were discovered from the literature review. The first is how to evaluate the modulus of the membrane and the second is whether to correct for the changing area of the membrane during the triaxial test.

Most references (ASTM D4767, La Rochelle et al. 1988, Head 1986) recommend the same procedure to measure the Young’s modulus, E, based on an extension test proposed by Henkel and Gilbert (1952). The test involves stretching a 1 inch wide loop of the membrane with weights and measuring the axial deformation. This test was performed on the membranes used for soil bentonite testing. The stress-strain relationship was found to be linear to approximately 6% axial strain after which the response became less stiff. The stress in the membrane was calculated based on the original dimensions of the membrane. A schematic of the experimental extension test is shown in Figure A.1.

Most researchers recommend using an average secant modulus from the extension test (Henkel and Gilbert 1952; La Rochelle et al. 1988), and do not correct for the changing area of the membrane during the triaxial test. It was decided that this procedure would only be appropriate for representing another extension test because the modulus was determined from an extension test and during the triaxial test the membrane undergoes compression. During the extension test, the thickness of the membrane decreases with axial strain, and the actual stress in the membrane is higher. Using the secant modulus from the extension test, incorporates the effect of the decreasing cross sectional area of the membrane. If it was possible to run a simple test with the membrane in compression without failure, the experimental curve based on the original area of the membrane area would curve upward and lie above the extension curves. A schematic of an experimental compression test is shown in Figure A.1.

Assuming the membrane is linear elastic, if the stress was corrected for the changing area of the membrane, the stress-strain curve of the extension curve would be straight. A schematic of a stress-strain curve for a constant Young's modulus is shown in Figure A.1. The curve lies above the experimental extension curve.

It was decided to use the initial tangent modulus to estimate the straight portion of the stress-strain curve, and to correct the area of the membrane during the triaxial test.

In contrast, most researchers recommend using some sort of average extension secant modulus (Henkel and Gilbert 1952; La Rochelle et al. 1988), and do not correct for the changing area of the membrane. Duncan and Seed (1967) corrects for the changing area of the membrane but does not recommend how to evaluate the modulus. It appears that the ASTM correction corrects for the changing area of the membrane, but also does not recommend where to evaluate the modulus.

It was found that the difference between using the secant modulus evaluated at 15% (216 psi) and the initial tangent modulus (256 psi) is slight as discussed below.

Baxter and Filz Membrane Correction

To analyze the membrane it was assumed that the membrane is linear elastic and is incompressible (Poisson's ratio = 0.5). It was observed during testing that the sample and the membrane behave as a composite material. The cell pressure presses the membrane up next to the sample and they deform together during both consolidation and shear. It was assumed that the sample and membrane deform together as a right circular cylinder.

During the consolidation phase, the cell pressure applies an axial stress and radial stress to the sample and membrane. Most of the pressure is taken up by the sample, but a small part of the axial and radial load is taken up by the membrane. During shearing, an additional axial load is applied to the specimen and membrane, and again the membrane takes up part of the applied axial load. To calculate the loads in the membrane, the membrane is treated as a thin walled cylinder so that the tangential stress can be considered constant throughout the thickness of the membrane. This assumption is considered valid for cylinders that have a thickness less than one tenth of the inner radius (Popov 1990). The membrane meets this criterion, since the thickness is approximately 1/200th of the radius of the specimen.

The stresses in the membrane consist of the axial stress (σ_a), tangential stress (σ_t), and radial stress (σ_r). The tangential stress is also sometimes referred to as the hoop stress or circumferential stress. Assuming that the membrane acts like a thin-walled cylinder, the tangential stress can be assumed to be constant across the thickness of the membrane.

The axial strain, tangential strain, and radial strain are defined in equations A.3 through equations A.5. The axial strain is calculated from the change in height divided by the initial height. The tangential strain is calculated from the change in the circumference divided by the initial circumference, which reduces to equation A.4. The radial strain is calculated from the change in thickness divided by the initial thickness.

$$\varepsilon_a = \frac{h_o - h}{h_o} \tag{A.3}$$

$$\varepsilon_t = \frac{d_o - d}{d_o} \tag{A.4}$$

$$\varepsilon_r = \frac{t_o - t}{t_o} \tag{A.5}$$

where:

h_o = initial height of membrane

h = height of membrane at time t

d_o = initial diameter of membrane

d = diameter of membrane at time t

t_o = initial thickness of membrane

t = thickness of membrane at time t

The governing equations at a point in the membrane, assuming the membrane is elastic, are given in equations A.6 through A.8.

$$\varepsilon_a = \frac{1}{E}\left[\sigma_a - \nu\left(\sigma_t + \sigma_r\right)\right] \tag{A.6}$$

$$\varepsilon_t = \frac{1}{E}\left[\sigma_t - \nu\left(\sigma_a + \sigma_r\right)\right] \tag{A.7}$$

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu\left(\sigma_a + \sigma_t\right)\right] \tag{A.8}$$

During consolidation, the cell pressure applies an axial and radial stress on the sample and membrane. It is convenient to assume that the cell pressure and radial stress on the membrane is zero and calculate the stresses in the membrane in excess of the cell pressure. Based on experimental observation, it was assumed that the membrane was unstretched during the forming of the sample when the soil bentonite was spooned into the mold. This was taken as the initial dimensions of the membrane. After the consolidation phase, the cell was disassembled and the height and the diameter of the specimen and membrane was measured. After consolidation, the axial strain and tangential strain of the membrane was calculated. Equations A.9 and A.10 were used to solve for the axial stress and tangential stress in the membrane, assuming that the radial stress was zero.

$$\sigma_a = E(\varepsilon_a+\varepsilon_t\upsilon)/(1-\upsilon^2) \quad \text{(A.9)}$$

$$\sigma_t = E(\varepsilon_a\upsilon+\varepsilon_t)/(1-\upsilon^2) \quad \text{(A.10)}$$

The radial strain was found from equation A.8 and the thickness of the membrane at the end of consolidation was calculated from equation A.5. Knowing the stresses in the membrane, corrections were made to the stresses on the specimen using equations A.11 and A.12. The corrections were subtracted from the major and minor principal stress in the specimen.

$$\Delta\sigma_{1\ con} = 4\sigma_a t/D \quad \text{(A.11)}$$

$$\Delta\sigma_{3\ con} = \sigma_t t/r \quad \text{(A.12)}$$

where:

t = thickness of membrane after consolidation

D = diameter of sample after consolidation

r = radius of sample after consolidation

σ_a = axial stress in membrane after consolidation

σ_t = tangential stress in membrane after consolidation

During shearing, a correction was made to the major principal stress in the specimen using equation A.13.

$$\Delta\sigma_{1,shear} = \frac{4\varepsilon_a t_o E}{D_o(1-\varepsilon_{vol})} \quad \text{(A.13)}$$

where:

ε_a = axial strain measured from the beginning of shear

ε_{vol} = volumetric strain measured from the beginning of shear

t_o = thickness of the membrane at the beginning of shear

D_o = diameter of the sample at the beginning of shear.

The area and membrane correction together is referred to here as the Baxter and Filz correction. The stress paths for three CU tests with the Baxter and Filz correction on soil-bentonite mixture SB1 are shown in Figure A.2 by the solid lines. Also shown in the figure are the uncorrected triaxial data as indicated by the dotted lines. The figure shows that a linear failure envelope that passes through the origin can be drawn from the corrected data but not from the uncorrected data.

The effect of various components of the Baxter and Filz corrections are shown in Figure A.3. In Figure A.3a, the stress path for a CU test is shown with the Baxter and Filz correction with and without the membrane correction during the consolidation phase. The correction during the consolidation phase translates the curve to the left and reduces the deviator stress, resulting in a lower strength. In Figure A.3b, the correction with and without the area correction is shown. Without the area correction, the failure line does not pass through the origin. In Figure A.3c, the correction with and without the membrane correction during the shear phase is shown. Without the correction, the failure line does not pass through the origin. In Figure A.3d, the correction using a secant modulus at

15% axial strain (E=216 psi) and an initial tangent modulus (E=256 psi) is shown. The figure shows that the effect is small.

Comparison with published corrections

A comparison of the Baxter and Filz corrections with those in the literature are shown in Figure A.4. The ASTM correction is shown with the Baxter and Filz correction in Figure A.4a. Most of the difference occurs because ASTM neglects stresses and strains in the membrane during consolidation. An average secant Young's modulus for the membrane was assumed for the ASTM correction instead of the initial tangent modulus, but as shown previously in Figure A.3d this has only a minor effect.

The Henkel and Gilbert (1952) corrections for both compression shell and hoop stress theory are shown in Figure A.4b with the Baxter and Filz correction. Again, most of the difference is because Henkel and Gilbert neglects the stresses in the membrane during consolidation. Also, the membrane area is not corrected and an average secant modulus evaluated over 15% axial strain is used for Henkel and Gilbert. The two theories by Henkel and Gilbert result in similar stress paths. The compression shell theory decreases σ_1 and the hoop stress theory increases σ_3, although the compression shell theory results in a slightly larger correction.

The Duncan and Seed (1967) correction is shown in Figure A.4c and is theoretically the closest to the Baxter and Filz correction. Both account for stresses and strains due to the consolidation phase; however, the Duncan and Seed formulation assumes that the initial diameter of the membrane is the same as the initial diameter of the sample. For the soil-bentonite tests, the initial diameter of the sample was 2.83 inches and the initial diameter of the membrane was measured as 2.58 inch. Also, according to the Duncan and Seed correction, application of the vertical load during undrained shear may cause changes in the radial stress in the membrane. Although these changes were small (0.04 psi for

CU_7), they are assumed to be zero for Baxter and Filz. The fact that they are found not be zero for the case of test CU_7 may indicate that the membrane is not completely incompressible. This effect is small and the differences in the two methods in the figure is due mostly to the assumption of the initial diameter of the membrane.

The La Rochelle et al. (1988) corrections for both compression shell and hoop stress are shown with the Baxter and Filz corrections in Figure A.4d. The La Rochelle corrections consider the influence of the initial diameter of the membrane and recommends an correction to the radial confining pressure. However, they do not address axial compression and strength of the membrane during consolidation. They do not correct for the changing area of the membrane and they recommend an average secant modulus evaluated at 10% extension.

It can be seen from Figure A.4, that the assumptions made for the Baxter and Filz corrections are slightly different from the existing corrections. It is clear that neglecting the stress and strain in the membrane during consolidation can be significant for soil bentonite if the material is consolidated in the membrane from a slurry.

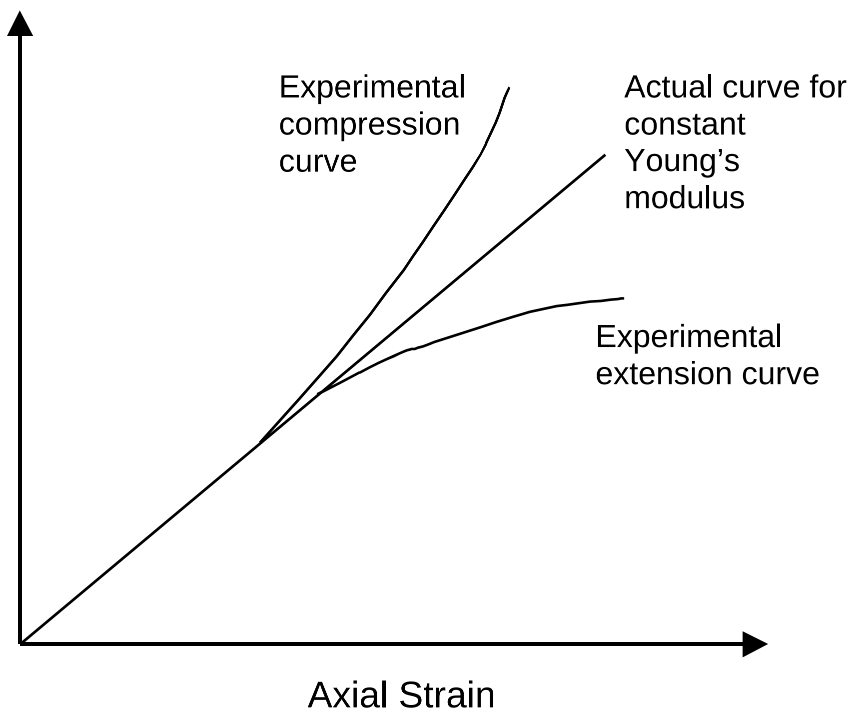

Figure A.1 Schematic of Stress Strain Curves for Membrane

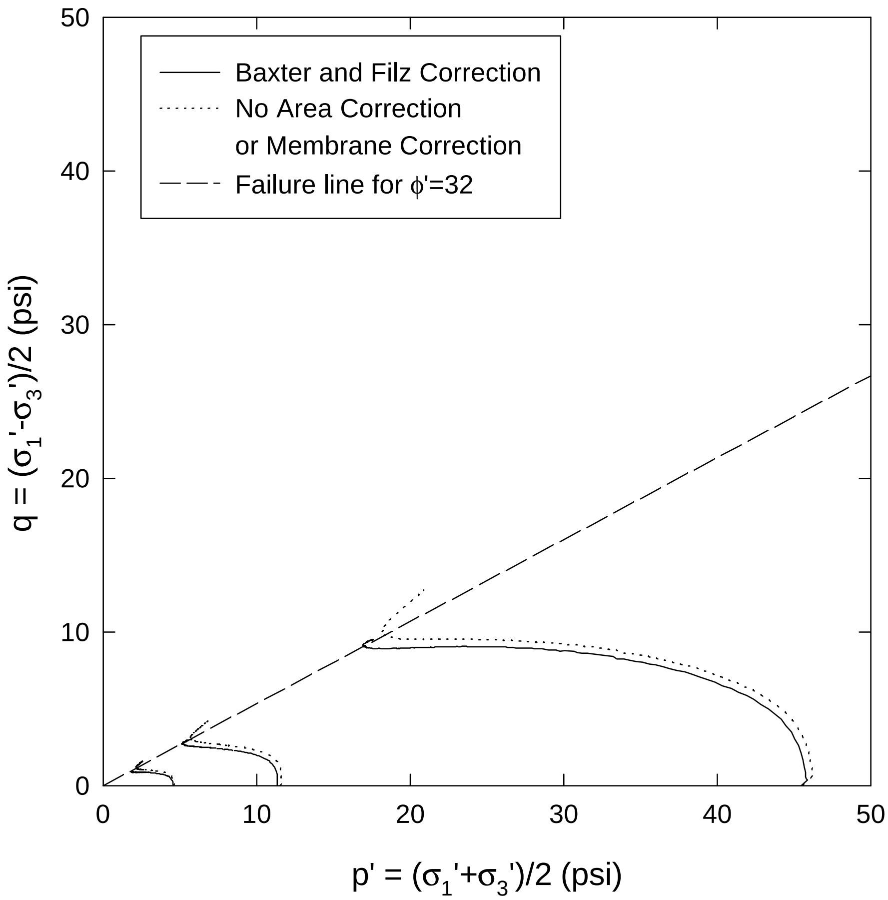

Figure A.2 Stress Paths from CU Tests on SB1 With and Without Baxter and Filz Triaxial Corrections

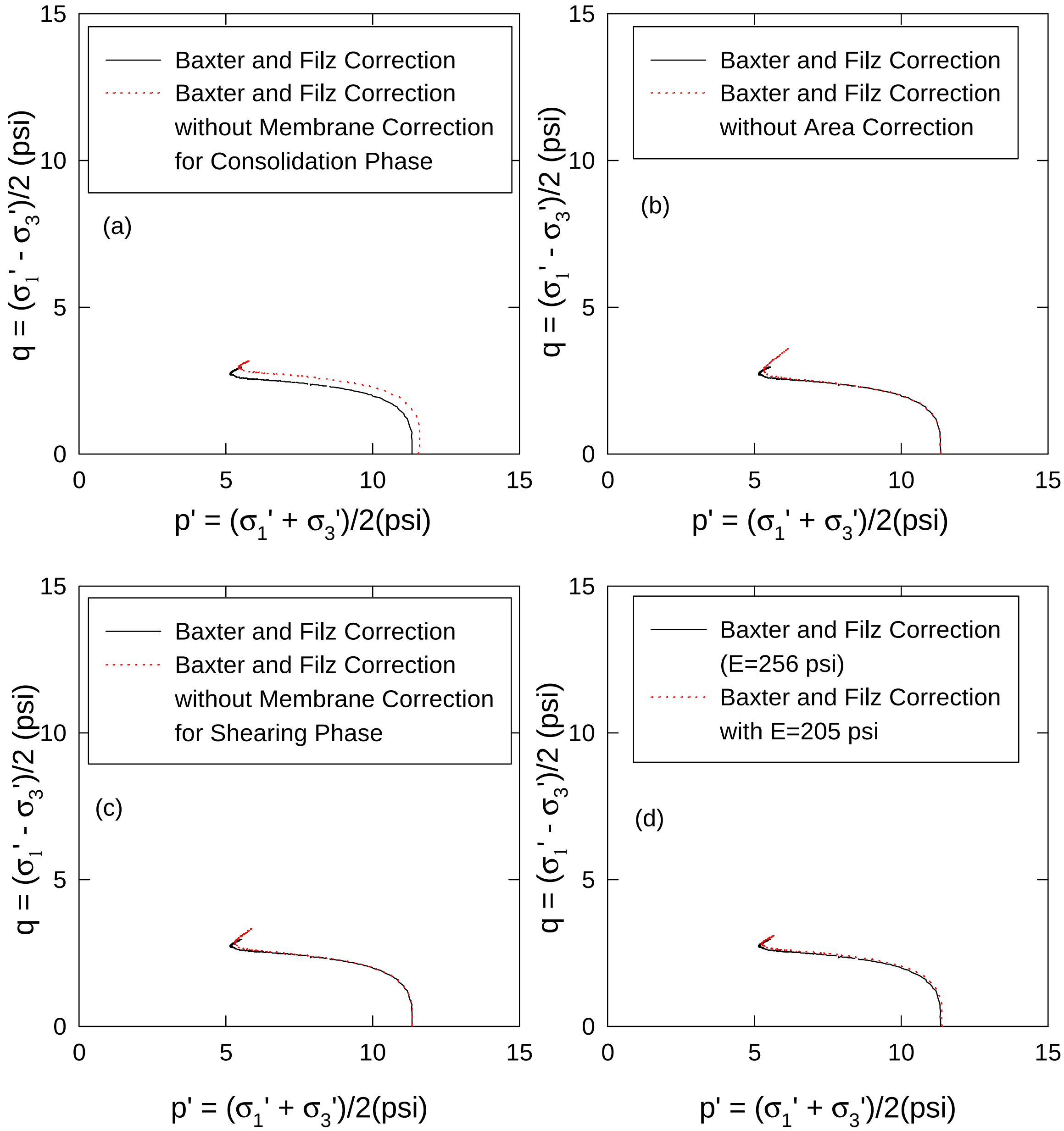

Figure A.3 Effect of Various Components of Baxter and Filz Correction

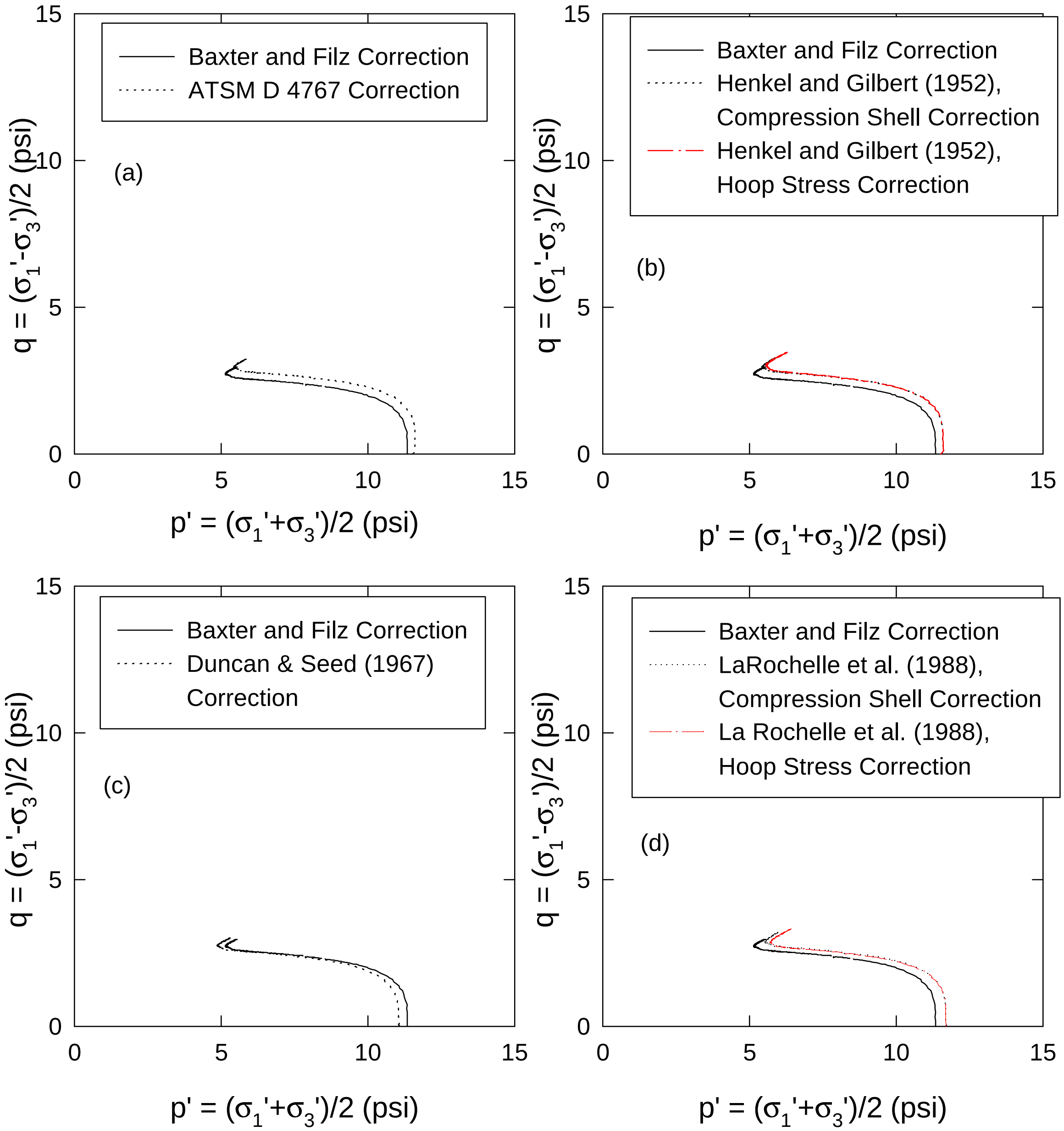

Figure A.4 Comparison of Various Triaxial Corrections