APPENDIX B
PRELIMINARY FINITE ELEMENT ANALYSES WITH SAGE

This appendix contains preliminary finite element analyses that were performed with SAGE including 1) verification of the RS model in SAGE, 2) analysis of arching in a soil-bentonite cutoff wall, and 3) shearing of a thin 2-D element of soil-bentonite. These analyses were performed prior to development of the finite element model described in Chapter 6.

B.1 Verification of the RS Model in SAGE

As part of this research effort, the RS constitutive model was created to represent the behavior of soil-bentonite. The RS model was incorporated into the existing finite element computer program SAGE. Several simple problems were run using the RS model to ascertain that the model was properly implemented. The following loading conditions were simulated: axisymmetric drained compression, axisymmetric undrained compression, one-dimensional consolidation, isotropic consolidation, and plane strain compression. The finite element analyses were verified with other numerical or analytical methods and/or checked for reasonableness. This section contains a description of each loading condition, presentation of the results, and discussion of the results.

The following parameter values, which are similar to the values used for SB1, were used to represent soil-bentonite in all problems except where noted: M= 1.3, λ=0.07, κ= 0.0048, ν=0.37, N=2.044 (for pressure in psf), R=4.0, S=2.5, and k=6x10^{-8} cm/sec.

Axisymmetric Drained Compression

A consolidated drained (CD) triaxial test was modeled with SAGE by running an axisymmetric drained compression problem. An uncoupled analysis was used. The finite element mesh consisted of one 2-D, axisymmetric element. An initial isotropic stress of 2030 psf was assigned. The top of the element was displaced in a series of steps to simulated vertical loading. Each step was divided into a number of substeps. The analysis was refined by increasing the number of substeps and decreasing the allowable tolerances until the results converged on a solution. Convergence was achieved in 230 substeps and the following tolerances: 0.01 for displacement, 0.001 for residual forces, and 0.001 for stress integration.

The stress-strain results using SAGE are shown as open circles in Figure B.1. Theoretically, an axisymmetric test will have an effective stress path with a slope of 3:1 if the effective radial stress is constant, as in a CD test. According to critical state theory, as a soil approaches critical state, it should undergo unlimited distortional strains and zero volume change. The stress-strain curves in Figure B.1 exhibit these trend.

A coupled analysis performed with SAGE was also used to simulate the CD test. Zero initial pore pressures were assigned and drainage was allowed along the bottom of the element. Again, the top of the element was displaced in steps. The excess pore pressures were allowed to dissipate between each step. The results are the same as the uncoupled analysis. This verifies that, in the limiting case of complete drainage, the coupled analysis procedures in SAGE produce the correct results.

The results of the finite element analyses were verified with a spreadsheet written to simulate a CD compression test using the RS model. The spreadsheet uses a finite difference approach and assumes constant radial stress. The user inputs the RS model parameter values and the initial condition of the soil-bentonite. The user applies increments of mean effective stress, and the spreadsheet calculates the resulting stresses and strains. Convergence of the spreadsheet solution was achieved by increasing the number of increments until the results were constant. 250 increments were used in the converged solution. The results of the spreadsheet are shown in Figure B.1 by the solid line. The spreadsheet and SAGE give the same solution.

Axisymmetric Undrained Compression

A consolidated undrained (CU) triaxial test was modeled with SAGE by running an axi-symmetric undrained compression problem. A coupled analysis was performed without drainage. An initial isotropic stress of 2430 psf was assigned. The nodes at the top of the mesh were displaced in a series of steps to simulate the undrained loading.

A finite element mesh of one 2-D element did not give realistic results. The stress stain response was too soft; i.e., the predicted strains were too great. A finite element mesh of four 2-D elements gave satisfactory results. In the one element problem, all of the nodes were specified with a no-flow boundary condition. In the four element problem, all of the exterior nodes were specified with a no-flow condition, but one node in the middle did not have a pore pressure constraint. Since the one element problem didn't work, it appears that SAGE will not give good results if all of the nodes in the mesh have a no-flow constraint.

The converged results from SAGE are shown as open circles in Figure B.2. The stress path and two stress stain curves are shown. The effective stress path looks reasonable because it has a shape roughly similar to the yield surface. As the soil approaches critical state, the sample undergoes large axial strains at constant effective stress, as expected. For the effective stress path plotted, the pore pressures were verified by subtracting the difference between the effective stress path and the total stress path. The total stress path is not shown, but it has the same initial point as the effective stress path and rises at a 3:1 slope. The initial slope of the distortional stress versus axial strain curve was verified using the elastic parameters to estimate the undrained Young's modulus (Wood, 1990).

The finite element analyses were verified with a spreadsheet written to simulate a CU compression test using the RS model. The spreadsheet uses a finite difference approach. Constant radial stress and zero volume change are assumed. The converged results of the spreadsheet are shown in Figure B.2 by the solid line. The spreadsheet and SAGE give the same solution.

<u>One-Dimensional Consolidation</u>

A one-dimensional consolidation problem was modeled with SAGE. A one foot thick layer was used with an initial stress state that corresponds to roughly 100 feet of overburden. The water table was specified at the top of the layer. The average vertical effective stress in the layer was approximately 12,000 psf. An initial value of K_o was assumed. The layer was first loaded vertically using two increments of 1000 psf each. Next the layer was unloaded vertically by 1000 psf. A finite element mesh of 10 elements stacked in a column was used. Drainage was allowed at the top. The problem was set up to approximate a homogeneous layer with a constant coefficient of volume change so that the results could be compared to Terzaghi's consolidation theory.

The theoretical effective stress path for one-dimensional loading is a straight line from the origin due to a constant ratio of horizontal to vertical effective stress. The theoretical value of the slope of the stress path can be determined from the condition of no lateral strain (Wood 1990; Kutter and Sathialingam 1992). If the elastic strains are neglected (assuming they are much smaller than the plastic strains), K_o is a function of the M and S parameter values for the RS model. For SB1 (M=1.3 and S=2.5), the RS model predicts a K_o value of 0.76. This corresponds to an effective stress path with a slope of 0.28.

The stress-strain results from SAGE are shown in Figure B.3. The initial K_o value that is assumed for the initial stress state in SAGE greatly affects the stress-strain outcome. The results are shown for initial K_o values of 0.57 and 0.76. These values were used because 0.57 is the value that was measured in the lab for SB1 and 0.76 is the theoretical value predicted by the RS model for SB1. Using an initial value of K_o equal to 0.76, the SAGE results match the theoretical prediction. The stress path continues at a constant slope and

K_o remains essentially constant. For an initial value of K_o equal to 0.57, the stress path veers towards the theoretical curve.

The void ratio versus vertical effective stress is also shown in Figure B.3. The SAGE output using an initial K_o value of 0.76 matches the theory. The theoretical curve can be determined from the RS parameter values and the K_o value that corresponds to 1-D consolidation. When the soil is under virgin loading, it consolidates according to the consolidation coefficient lambda (λ) for both 1-D and isotropic consolidation (Wood 1990). It can also be seen from this graph that when the layer is unloaded, the soil rebounds according to the unload/reload coefficient kappa (κ). If an initial K_o value of 0.57 is used, the stress-strain curve does not match the RS theory. The void ratio corresponding to this stress path is lower, which agrees with the model's theory; the stress state corresponds to a larger yield surface and consequently the void ratio is lower.

The SAGE time-deformation results for the first applied load (initial K_o=0.76) are presented in Figure B.4. The consolidation ratio, U_z, throughout the layer is shown for several different times. The distance from the top of the layer is represented by the variable z and the variable H represents the height of the layer. The SAGE results were compared with Terzaghi's consolidation theory by estimating a value of c_v for the soil-bentonite. A coefficient of volume change, m_v, of 4.0×10^{-6} 1/psf was estimated from the void ratio versus vertical effective stress curve for the stress range that was experienced. A coefficient of consolidation, c_v, of 0.681 ft^2/day was calculated. Two of the steps from SAGE, labeled in the figure, correspond to time factors, T=0.10 and T=0.78. The closed form solution by Terzaghi for T=0.10 and T=0.8 are also shown in the figure by the dotted lines (Taylor 1948). The results from SAGE are consistent with the theoretical solution.

The average consolidation ratio for the whole layer is also shown versus time in Figure B.4. The SAGE output for average consolidation was compared to the theoretical closed form solution (Taylor 1948) at 20%, 50%, and 90% consolidation. The estimated c_v

value was used to convert the theoretical solution from time factor to time. The theoretical values are shown in the figure with the open circles. The analytical and theoretical results are consistent.

Isotropic Consolidation

An isotropic consolidation problem was modeled with SAGE. A one foot thick layer of soil-bentonite was loaded isotropically. The layer was assigned a homogeneous isotropic stress of 10,000 psf, which corresponds to roughly 80 feet of overburden. The layer was then loaded isotropically in two increments of 2000 psf. The pore pressures were allowed to fully dissipate between loads. A finite element mesh of 10 axisymmetric elements stacked in a column was used. Drainage was allowed at the top.

The stress-strain results are shown as open circles in Figure B.5. The theoretical predictions are shown in the figure by the thick solid lines. The theoretical stress path for isotropic consolidation should follow along the mean effective stress axis. Theoretically, the void ratio should decrease according to the parameter lambda, which relates the void ratio and the mean effective stress. The SAGE output matches the theory. The dimensionless pore pressure data is not presented, as it is the same as the 1-D results.

Plane Strain Compression

Plane strain compression problems were modeled with SAGE assuming normally consolidated and overconsolidated samples of soil-bentonite. Uncoupled analyses were performed. The finite element mesh consisted of one 2-D element with beam elements on either side. These problems were run because they were used to verify the Modified Cam Clay model during the original coding of SAGE (Morrison 1995).

The constitutive model parameter values used by Morrison (except R and S) were used for these problems to compare results. The parameter values are listed as follows: $M=0.882$, $\lambda=0.088$, $\kappa=0.031$, $\nu=0.36$, $N=2.36$, R=various, S=various.

Normally Consolidated Bentonite

The following loading sequence was used for plane strain compression on normally consolidated soil-bentonite. The sequence is described by Morrison (1995) in greater detail. An initial isotropic stress of 50 psf was assigned. Next 450 psf was applied in the vertical and horizontal directions. The load is not isotropic because the load in the out-of-plane direction cannot be specified in plane strain analysis. Next the top of the mesh was displaced vertically downward to simulate vertical compression. The mesh was displaced until the soil-bentonite reached critical state.

Various R and S parameters were used. The RS model becomes the same as the Modified Cam Clay model when R and S are equal to 2. Numerical difficulties arise if the parameters are exactly equal to 2; however, the problem was run with R=2.01 and S=2.01 in order to compare with the Modified Cam Clay model. The problem was also run with R=4.0 and S=2.01.

The results in Figure B.6 show that the Modified Cam Clay and RS model give the same results if R=2.01 and S=2.01, as expected. These results are similar to those reported by Morrison (1995). The figure also shows the results of the RS model if R=4.0 and S=2.01. The stress path is the same, but there is more volume change if R=4.0 as shown in the plot of void ratio versus mean effective stress. This is consistent with the model's theory. The parameter R is related to the distance between the isotropic consolidation line and the critical state line. The critical state line in the p'-q space is the same for any value of R for a given value of M. However, the critical state line in the void ratio versus p' space is a function of R. The critical state lines for R=2 and R=4 are shown on the figure. As the value of the parameter R increases, for a given value of mean effective stress, the void ratio must be lower at critical state.

Overconsolidated Soil-Bentonite

Plane strain compression was also run on overconsolidated soil-bentonite. The overconsolidation was created by loading and unloading the sample in several steps. The following loading sequence was used as described by Morrison (1995). An initial isotropic stress of 50 psf was assigned. A pressure of 3950 psf was applied in the vertical and horizontal directions. Next 3500 psf was removed in the vertical and horizontal directions. After unloading, the top of the mesh was displaced until the soil-bentonite reached critical state.

The results are shown in Figure B.7. The yield surface created from the initial loading to 4000 psf vertical stress is shown. During the unloading, the specimen rebounds and travels inside the yield surface. Due to the plane strain condition, the out-of-plane stress remains large with respect to the vertical and horizontal, and the distortional stress increases during unloading. Eventually, the sample yields at the left of the critical state line during unloading at point A in the enlargement. Also during the unloading, the void ratio travels along the unload/reload curve (url) shown in the void ratio versus mean effective stress graph. This url is associated with the yield surface shown in the p-q graph. During subsequent vertical loading, the distortional stress first reduces slightly and then increases as the vertical stress becomes greater than the out-of-plane stress. The sample intersects the yield surface again just after point B in the enlargement. The sample then approaches critical state at point C. The results appear reasonable given the model's theory.

Conclusions

The RS model produced reasonable and verifiable results using SAGE. The following situations were tested: coupled analysis, uncoupled analysis, 1-D consolidation, isotropic consolidation, plane strain conditions, axisymmetric conditions, overconsolidated stress states, and normally consolidated stress states. The model was shown to behave the same as the Modified Cam Clay model if R=2.01 and S=2.01, as intended. The stress-strain and consolidation behavior of the RS model in SAGE is consistent with the model's theory, and the model performs as intended in SAGE.

B.2 Analysis of Arching in a Soil-Bentonite Cutoff Wall

A finite element analysis to simulate arching theory in a soil-bentonite cutoff wall was performed. The arching theory is described in more detail in Section 2.3. A 10 foot deep and 5 foot wide trench was analyzed. The trench walls were fixed in the lateral direction, as assumed in arching theory. Five columns of 2-D elements were used to represent soil-bentonite backfill. Interface elements were used at the trench walls to represent filter cake. The RS model was used to represent the soil-bentonite and properties from soil-bentonite mixture SB1 were used. An interface element described in the sage manual that uses an elastic hyperbolic relationship between shear stress and displacement was used. Properties for the interface were found from direct shear tests on filter cakes from a related research project performed by Laura Henry, who was a graduate research assistant at Virginia Tech at the time. Plots of shear stress versus displacement were used to find properties that were a good fit with tests that were run at four different normal stresses.

The soil-bentonite backfill was assumed to be in place in the trench with an assumed initial stress state. The initial effective vertical stress was assumed to be very low and constant with depth. Excess pore pressures were assigned equal to the total weight of the soil-bentonite minus the vertical effective stress. Horizontal effective stresses were assumed to be equal to the vertical effective stresses. The excess pore pressure were allowed to dissipate with time during the analyses. The initial and final effective vertical stress distribution in the center of the trench is shown in Figure B.8a. Geostatic stresses are also shown. The initial and final pore pressures are shown in Figure B.8b. Hydrostatic pressures are also shown. The final effective vertical stresses calculated with SAGE are compared to arching theory and geostatic stresses in Figure B.8a. The final displaced mesh is shown in Figure B.10

The results indicate the following: a) over time, the pore pressures dissipate and reach hydrostatic conditions, b) vertical effective stresses in the center of the trench approximate stresses predicting using arching theory, c) the trend of settlement in the trench approximate the trend assumed for arching theory, i.e. there is very little deformation at the trench walls and the most settlement occurs in the center of the trench.

The analyses show that arching theory can be simulated using finite element analyses with SAGE, and that dissipation of initial excess pore pressures that exist in the soil-bentonite backfill can be modeled.

B.3 Shearing of a Thin 2-D Element of Soil-Bentonite

One element problems were run with SAGE on a thin 2-D soil-bentonite element to observe the deformation behavior of long, thin elements. An eight-noded element was used. One long side of the element was pinned and the other long side of the element was displaced in increments to simulate simple shear. It was found that thin elements with aspect ratios of 1:20 were well behaved in shear, and showed reasonable stress-strain behavior. Element with aspect ratios of 1:5 were also tested in shear and showed reasonable stress strain behavior. Stress-strain results of the elements with 1:5 and 1:20 aspect ratios are shown in Figure B.10.

The SAGE analysis of a long thin 2-D element of soil-bentonite in simple shear is compared with data of a direct simple shear test on a filter cake (Henry et al. 1998). The figure shows that the SAGE analysis is a reasonable approximation of the shearing of the filter cake.

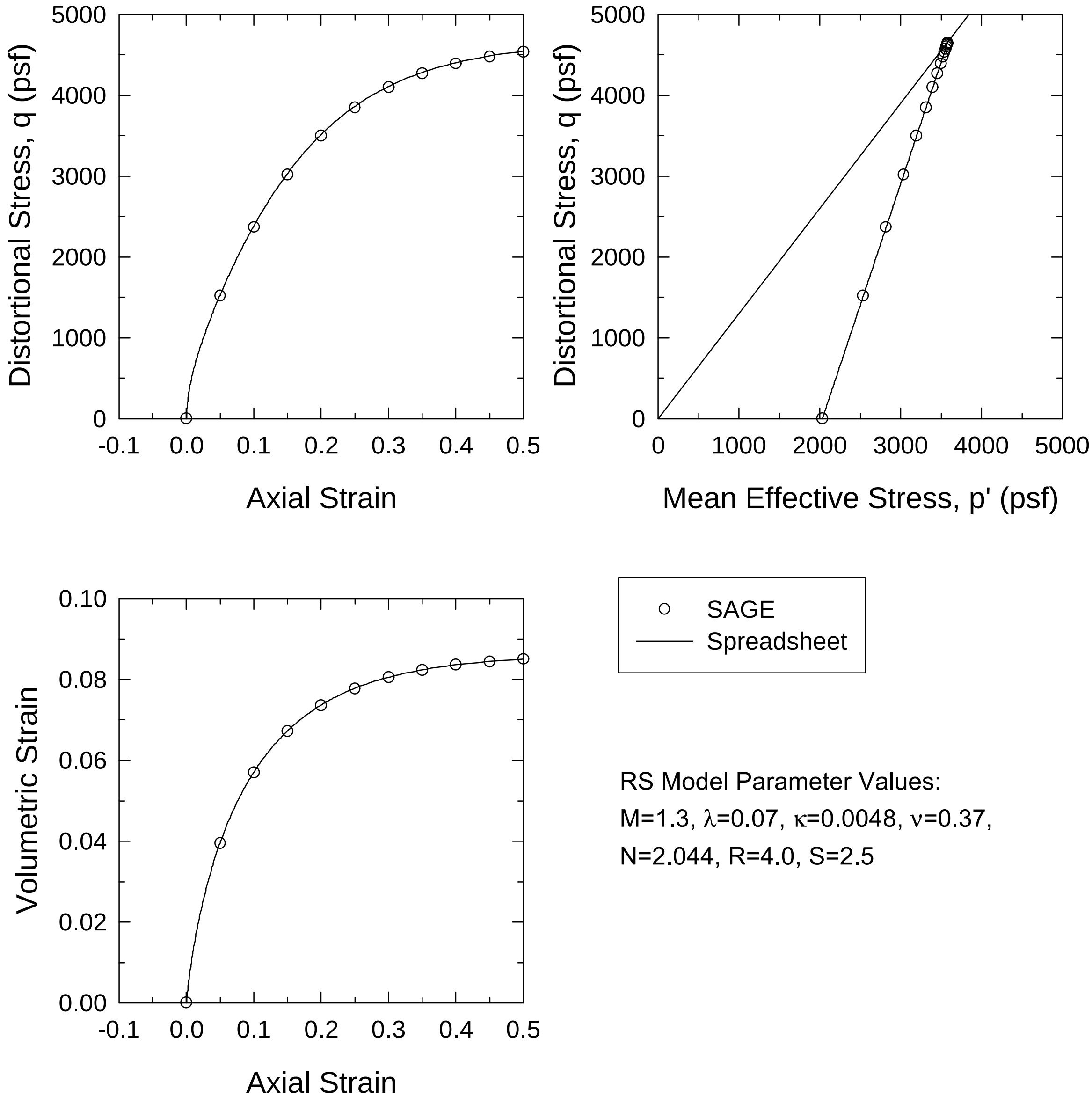

Figure B.1 Numerical Modeling of a CD Triaxial Test Using the RS Model

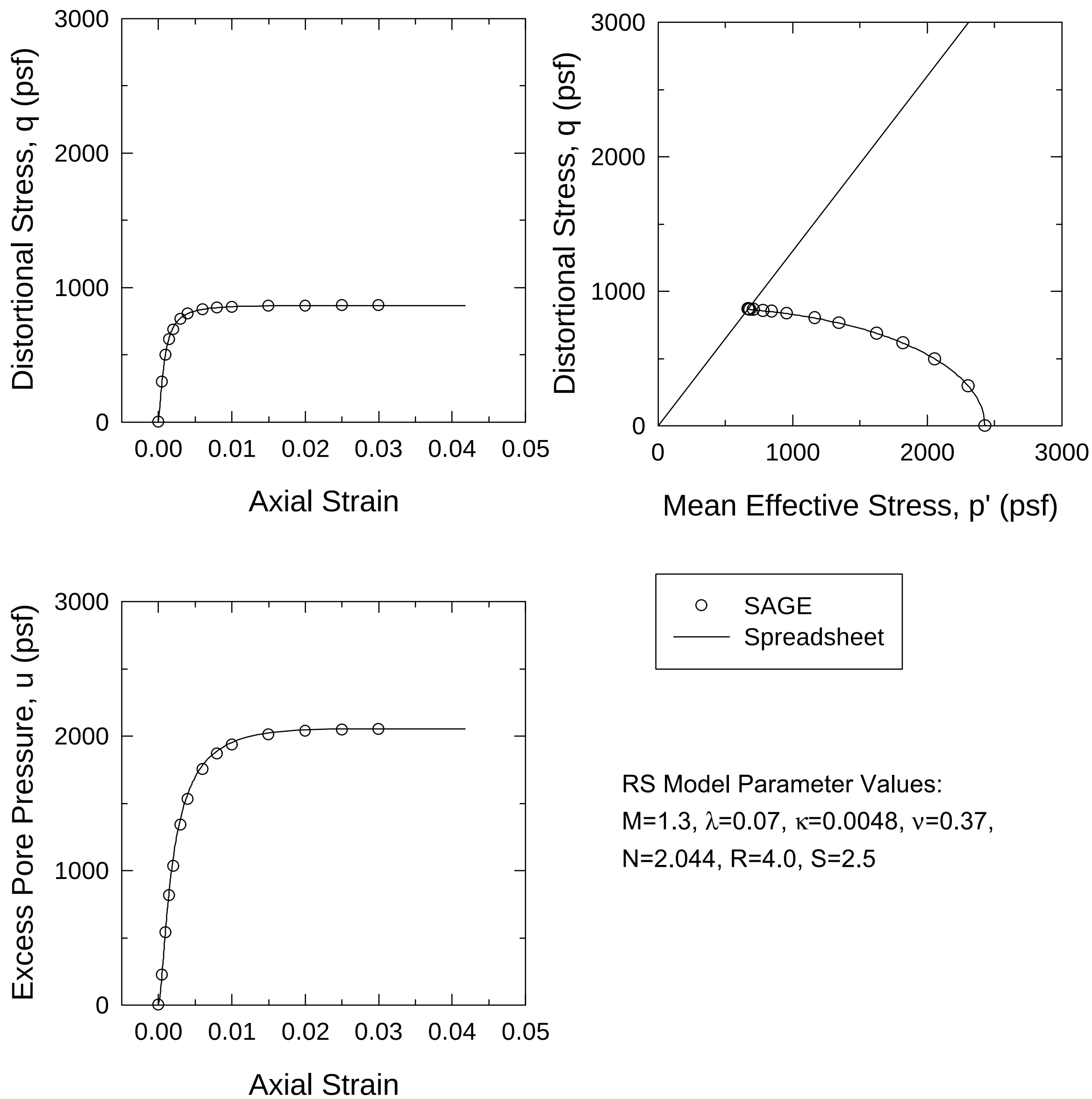

Figure B.2 Numerical Modeling of a CU Triaxial Test Using the RS Model

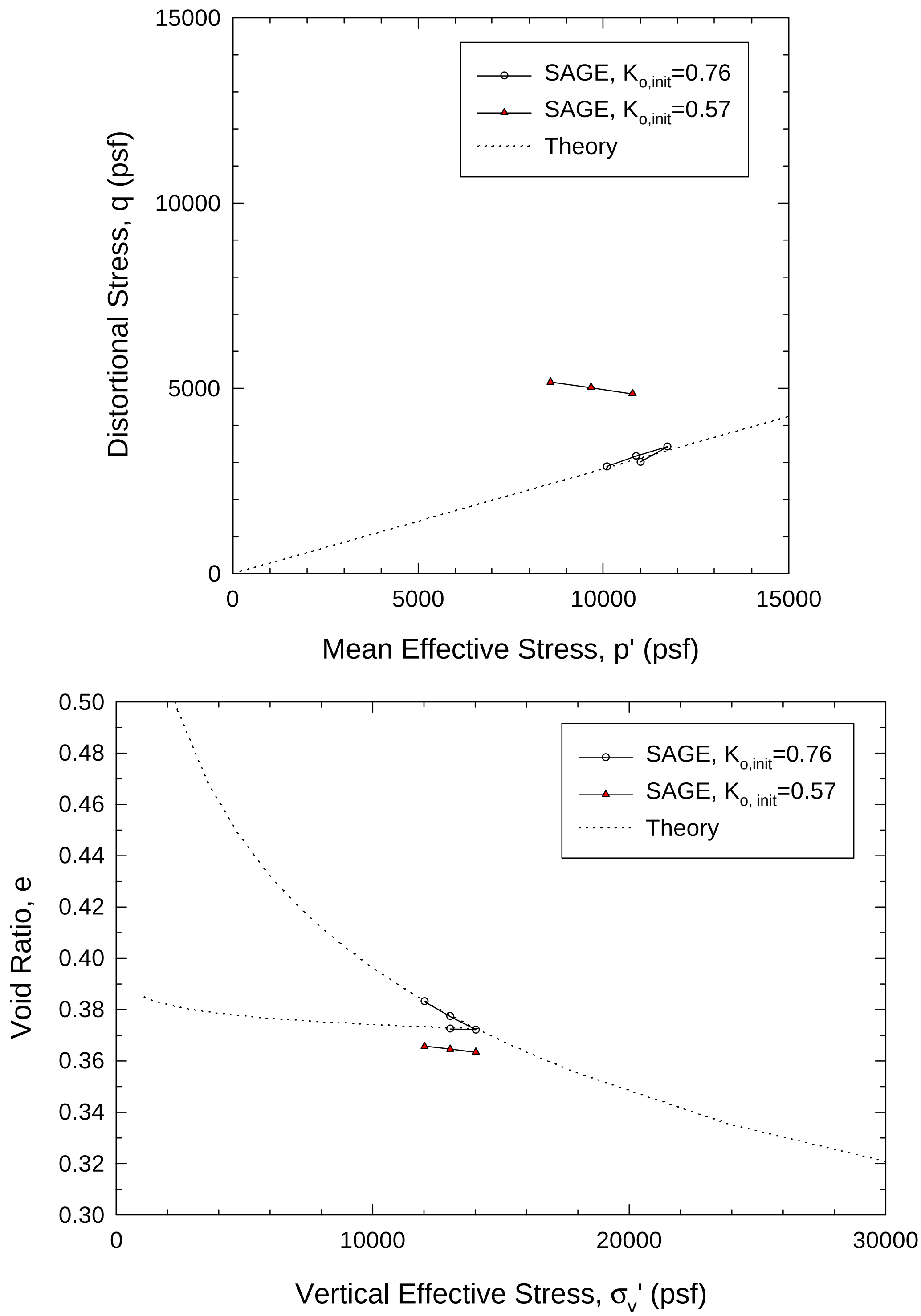

Figure B.3 Stress-Strain Predictions of a One-Dimensional Consolidation Test Using the RS Model

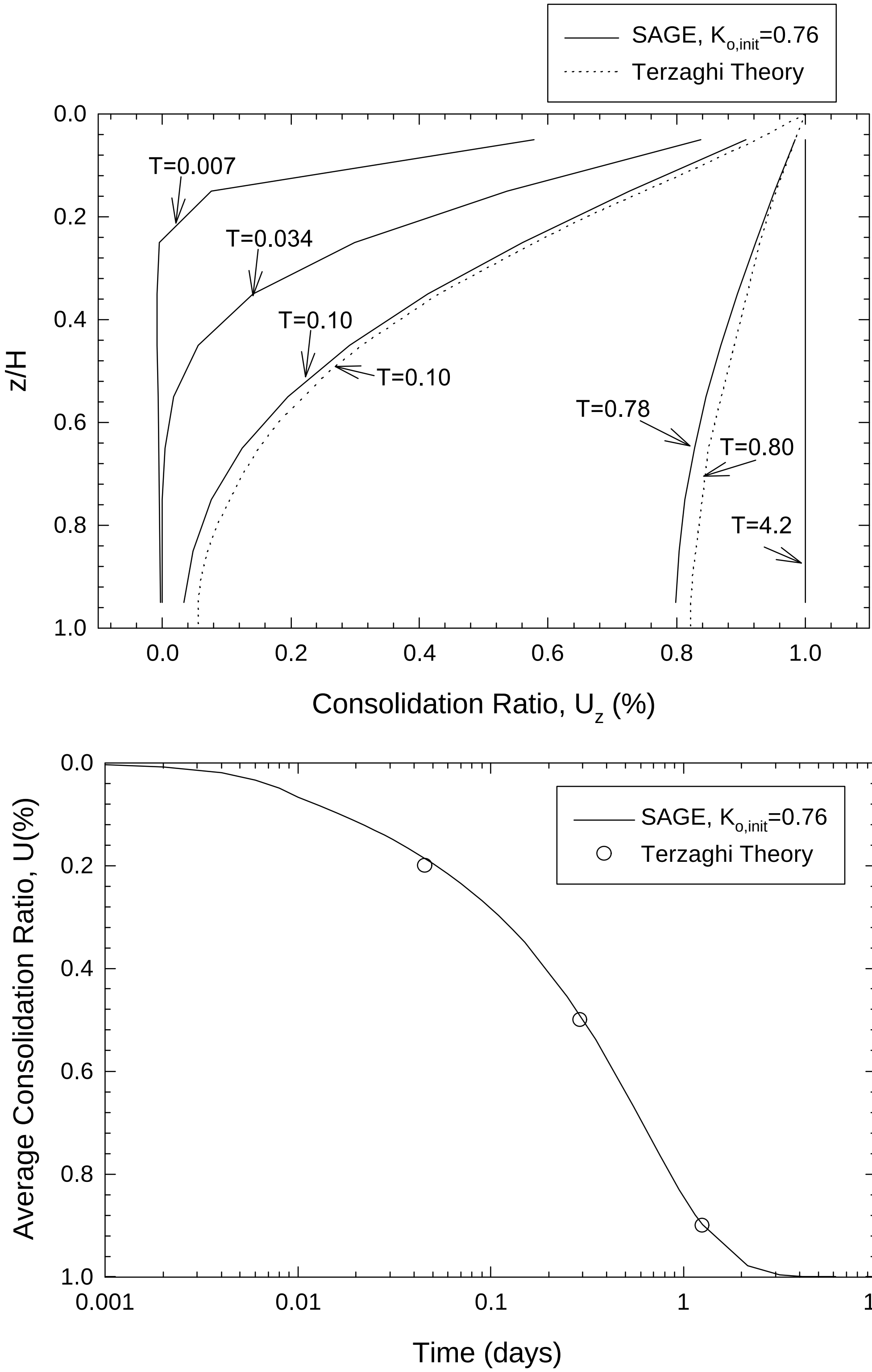

Figure B.4 Settlement versus Time Predictions of a One-Dimensional Consolidation Test Using the RS Model

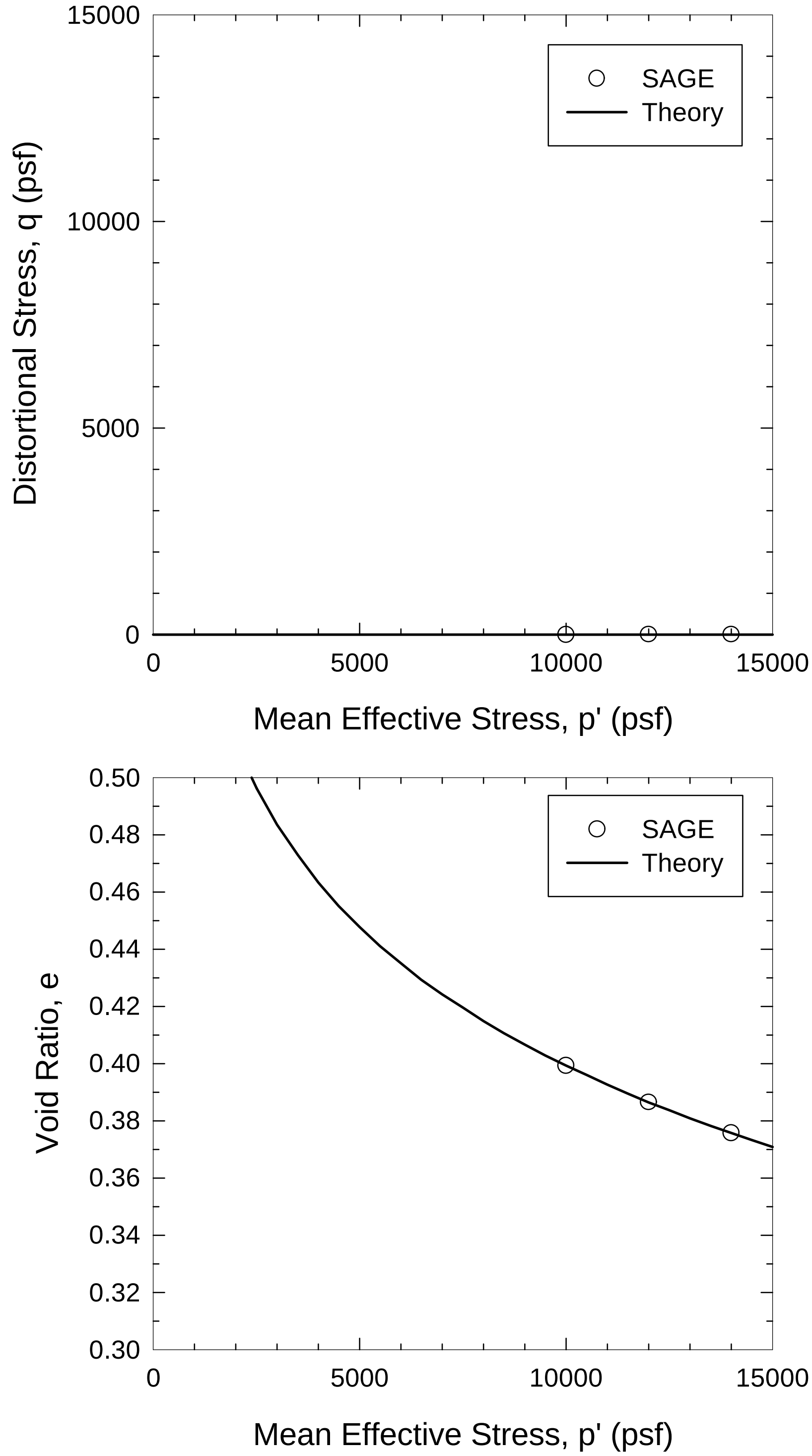

Figure B.5 Numerical Modeling of an Isotropic Consolidation Test Using the RS Model

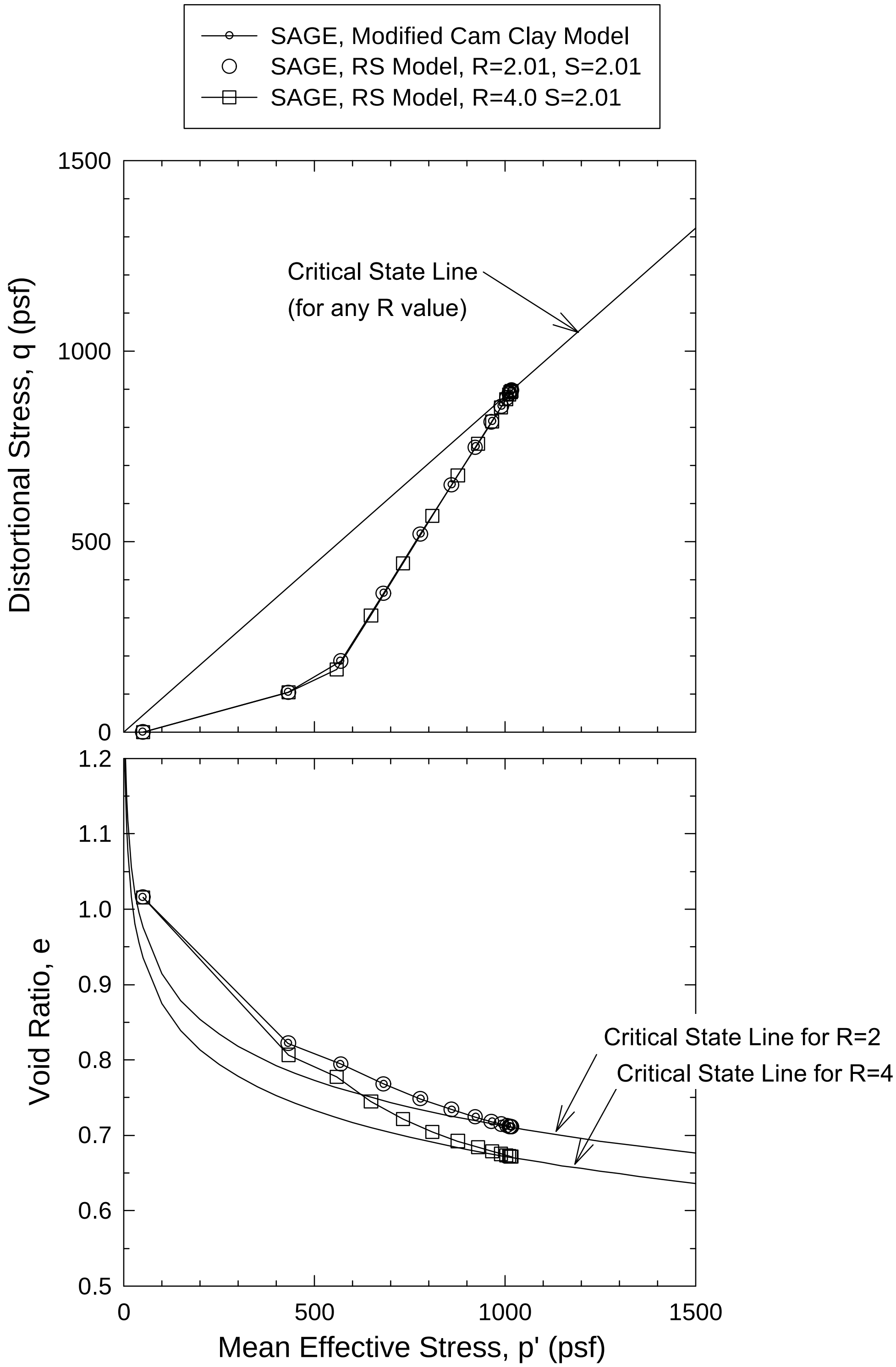

Figure B.6 Predictions of Plane Strain Compression of Normally Consolidated Soil-Bentonite Using SAGE

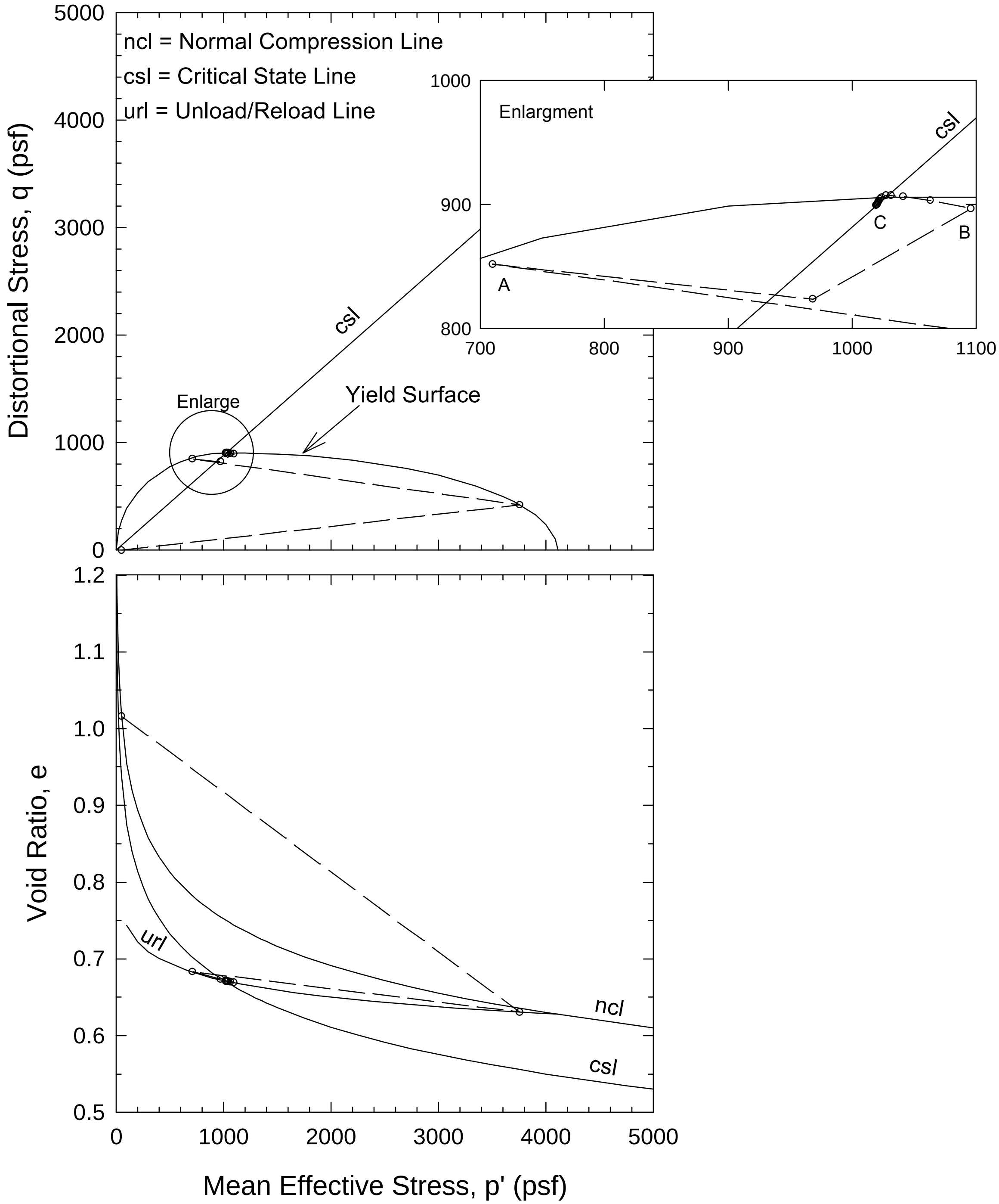

Figure B.7 Predictions of Plane Strain Compression of Over Consolidated Soil-Bentonite Using SAGE

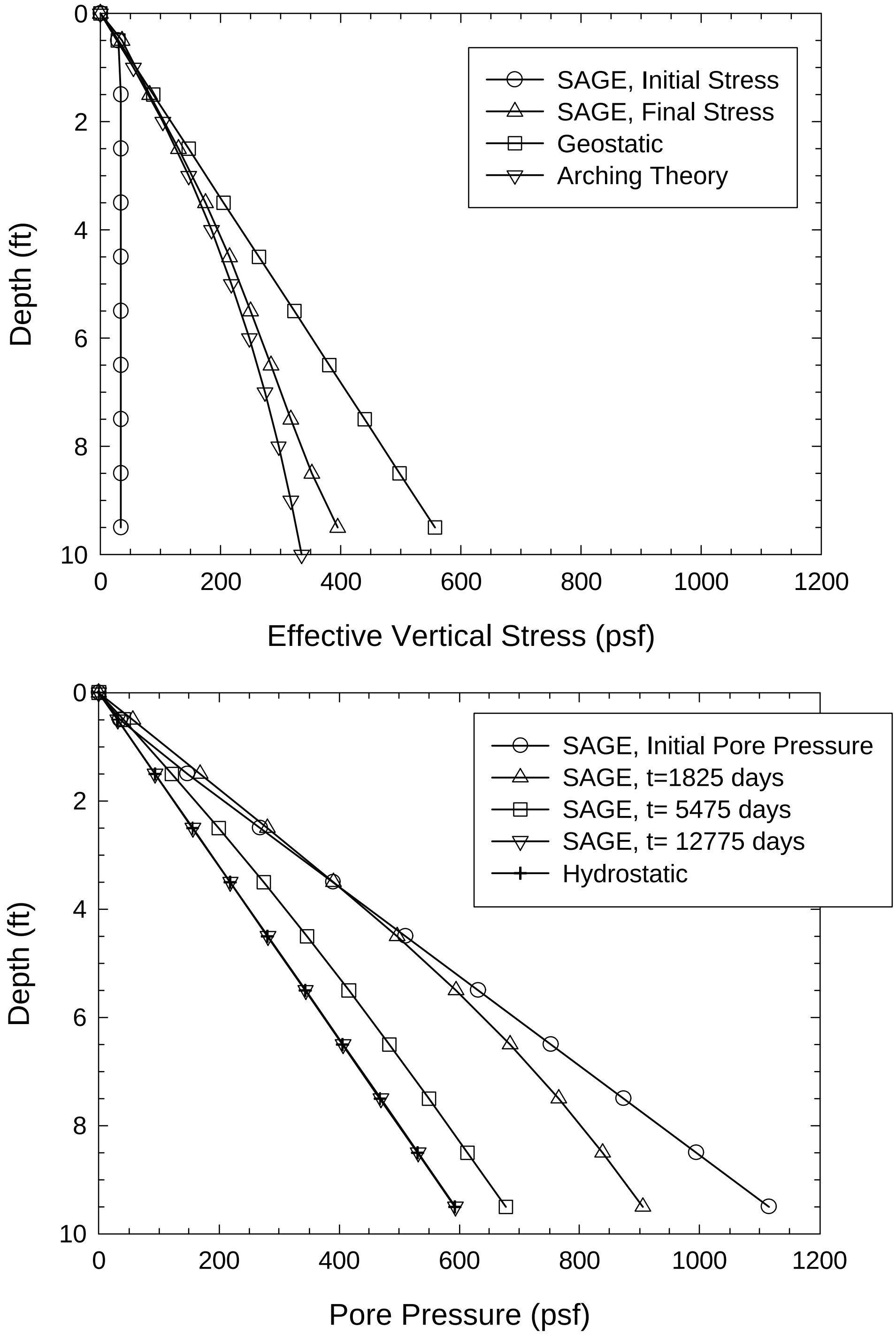

Figure B.8 Stresses in Soil-Bentonite at Center of Trench for Arching Analysis

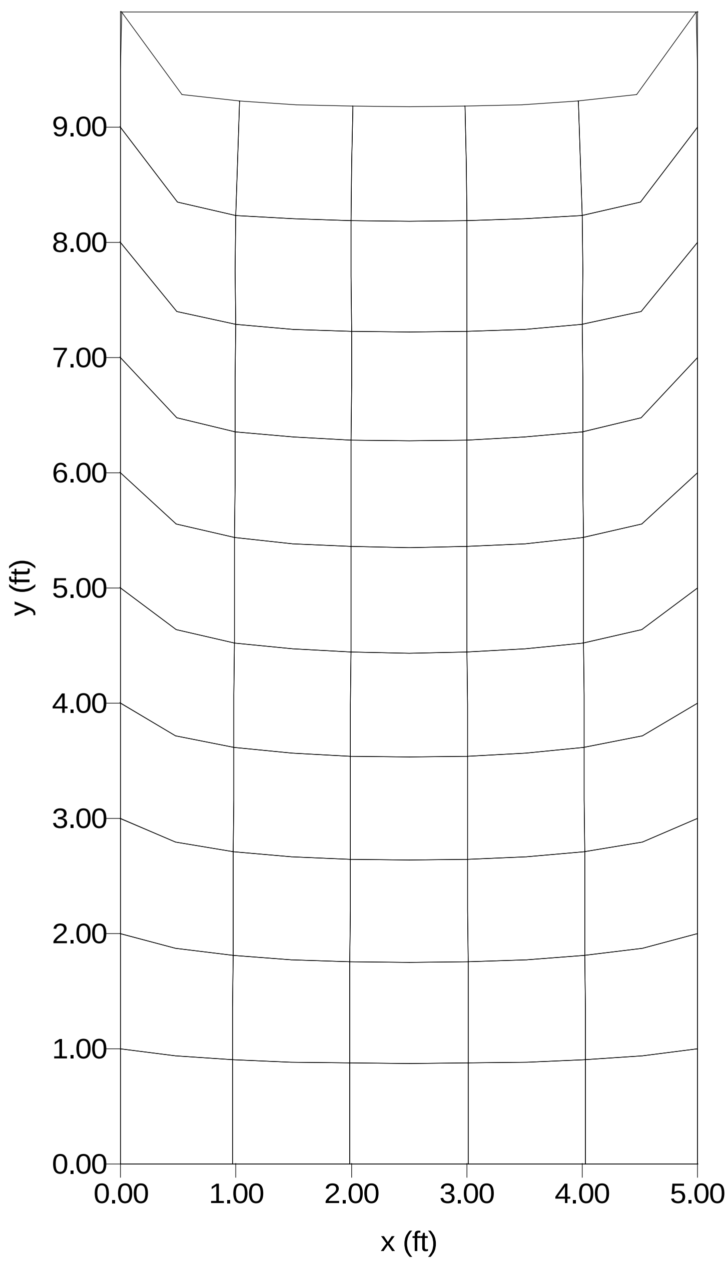

Figure B.9 Displaced Mesh for Arching Analysis

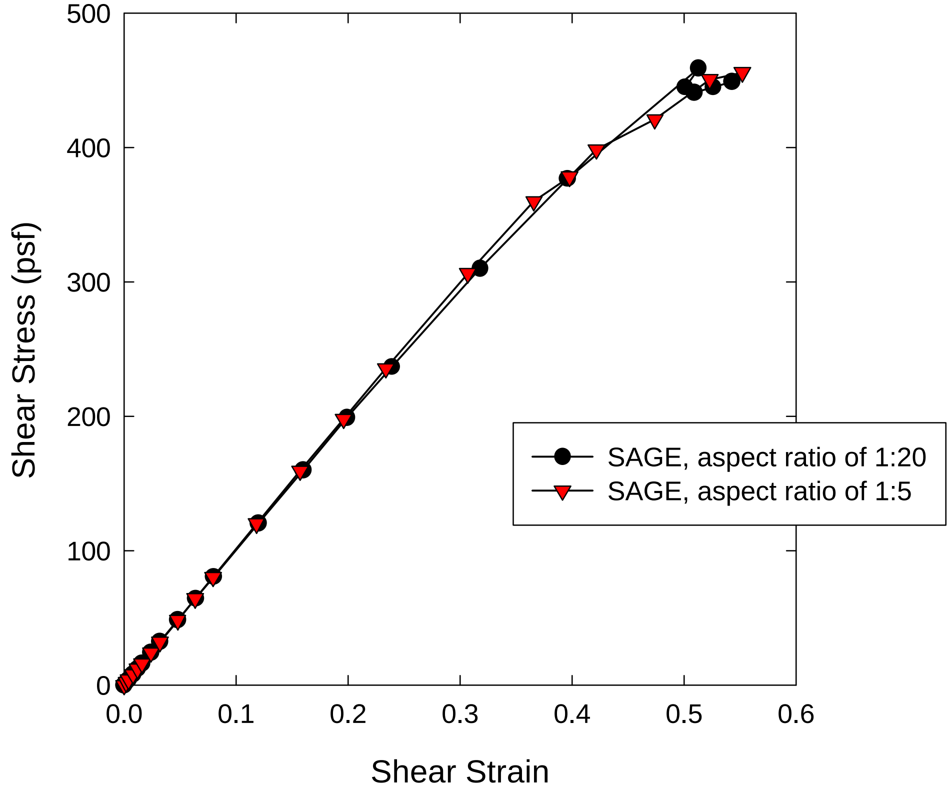

Figure B.10 Stress-Strain Behavior of a Long Thin 2-D Element of Soil-Bentonite Using RS Model

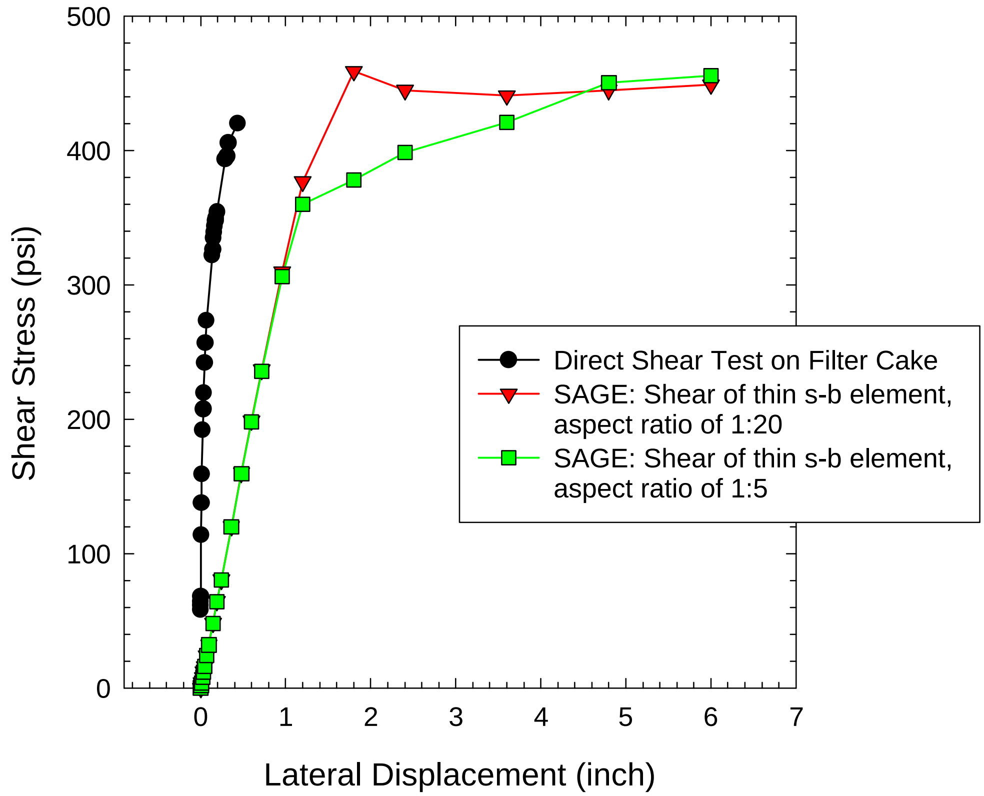

Figure B.11 Comparison of Behavior of Direct Shear Test on Filter Cake and SAGE Analysis of Shearing a Long Thin 2-D Soil-Bentonite Element