Appendix A

Derivation of Stress-dilatancy Relationship

Denoting Δ as the angle between the direction of major principal stress and that of major principal strain increment, the dissipated energy in 2D conditions can be calculated as:

$$W = \sigma_1 d\varepsilon_1 \cos^2\Delta + \sigma_1 d\varepsilon_2 \sin^2\Delta + \sigma_2 d\varepsilon_2 \cos^2\Delta + \sigma_2 d\varepsilon_1 \sin^2\Delta \tag{A-1}$$

The energy ratio K can be calculated as

$$\frac{\sigma_1 d\varepsilon_1 \cos^2\Delta - \sigma_1 d\varepsilon_2 \sin^2\Delta}{-\sigma_2 d\varepsilon_2 \cos^2\Delta + \sigma_2 d\varepsilon_1 \sin^2\Delta} = K \tag{A-2}$$

The left hand side of Equation (A-2) represents the ratio of input energy to output energy. Noting that $d\varepsilon_2$ is always negative, so both the numerator and denominator are positive. From the Mohr's circle of stress and strain increment, we have

$$\frac{\sigma_1}{\sigma_2} = \tan^2\left(\frac{\pi}{4} + \frac{\phi_{ps}}{2}\right) \tag{A-3}$$

$$-\frac{d\varepsilon_2}{d\varepsilon_1} = \tan^2\left(\frac{\pi}{4} + \frac{\psi}{2}\right) \tag{A-4}$$

Substituting Equation (A-3), (A-4) into Equation (A-2), one obtains

$$K = \tan^2\left(\frac{\pi}{4} + \frac{\phi_{ps}}{2}\right) \frac{1 + \tan^2\Delta \tan^2\left(\frac{\pi}{4} + \frac{\psi}{2}\right)}{\tan^2\Delta + \tan^2\left(\frac{\pi}{4} + \frac{\psi}{2}\right)} \tag{A-5}$$

Equation (A-5) can be transformed into

$$K = \tan^2\left(\frac{\pi}{4} + \frac{\phi_{ps}}{2}\right) \frac{1 - \sin\psi \cos 2\Delta}{1 + \sin\psi \cos 2\Delta} \tag{A-6}$$

Let $$\sin A = c \sin \psi \tag{A-7}$$

where $c = \cos 2\Delta$ is the Gutierrez-Ishihara non-coaxiality parameter, and A is the nominal dilation angle considering the non-coaxiality effect, then we have

$$K = \tan^2\left(\frac{\pi}{4} + \frac{\phi_{ps}}{2}\right)\frac{1-\sin A}{1+\sin A} = \tan^2\left(\frac{\pi}{4} + \frac{\phi_{ps}}{2}\right)\Bigg/\tan^2\left(\frac{\pi}{4} + \frac{A}{2}\right) \tag{A-8}$$

Now, Rowe's principle of minimum energy ratio will be used to find the relationship between ϕ_{ps} and A, which is

$$\phi_{ps} = f(A) \qquad \text{(When } K \to \min \text{)}. \tag{A-9}$$

Introducing the following replacements:

$$B = \frac{\pi}{4} + \frac{A}{2} \ , \tag{A-10}$$

$$g(B) = \frac{\pi}{4} + \frac{\phi_{ps}}{2} = \frac{\pi}{4} + \frac{f(A)}{2}, \tag{A-11}$$

we get

$$\sqrt{K} = F(B, g(B)) = \tan g(B) / \tan B \, . \tag{A-12}$$

For F to take a minimum value, the following condition is required

$$\frac{dF}{dB} = \frac{1}{\tan^2 B}\left(\sec^2 g(B)\frac{dg}{dB}\tan B - \tan g(B)\sec^2 B\right) = 0 \tag{A-13}$$

Rearranging Equation (A-13) one obtains:

$$\frac{dg(B)}{\sin 2g(B)} = \frac{dB}{\sin 2B} \, . \tag{A-14}$$

The solution to the differential equation above is expressed as:

$$\lambda \tan B = \tan g(B) \ , \tag{A-15}$$

where λ is an integration constant. Substituting Equation (A-10), (A-11) into Equation (A-15) yields:

$$\lambda \tan\left(\frac{\pi}{4}+\frac{A}{2}\right)=\tan\left(\frac{\pi}{4}+\frac{\phi_{ps}}{2}\right). \tag{A-16}$$

In the critical state deformation ($\phi=\phi_{crit}$), we have $d\nu=0, (\psi=0)$. From Equation (A-7) we also obtain $A=0$. Thus the integration constant λ is:

$$\lambda=\tan\left(\frac{\pi}{4}+\frac{\phi_{crit}}{2}\right) \tag{A-17}$$

Therefore, the extended Rowe's stress-dilatancy equation (flow rule) takes the form of

$$\tan\left(\frac{\pi}{4}+\frac{\phi_{ps}}{2}\right)=\tan\left(\frac{\pi}{4}+\frac{\phi_{crit}}{2}\right)\tan\left(\frac{\pi}{4}+\frac{A}{2}\right). \tag{A-18}$$

Equation (A-18) can also be written as:

$$\sin\phi_{ps}=\frac{\sin\phi_{crit}+\sin A}{1+\sin\phi_{crit}\sin A} \tag{A-19}$$

or

$$\frac{1+\sin\phi_{ps}}{1-\sin\phi_{ps}}=\frac{1+\sin\phi_{crit}}{1-\sin\phi_{crit}}\frac{1+\sin\psi\cos 2\Delta}{1-\sin\psi\cos 2\Delta} \tag{A-20}$$

Equations (A-18) through (A-20) are the extended Rowe's stress-dilatancy relationships taking into account the non-coaxiality behavior. Different from the conventional Rowe's equation, the nominal dilation angle A takes the place of the actual dilation angle ψ and is usually smaller than the actual dilation angle ψ due to the non-coaxiality effects.

Appendix B

Plane Strain and Direct Shear Angles of Friction

Assuming coaxiality and horizontal zero linear extension direction, the relationship between plane strain and direct shear angles of friction is (Davis 1968):

$$\tan\phi_{ds} = \frac{\sin\phi_{ps}\cos\psi}{1-\sin\phi_{ps}\sin\psi} \quad . \tag{B-1}$$

Where non-coaxiality exists and the zero linear extension direction deviates from the horizontal, the following relation is found from the geometry of Mohr's stress circle (Figure 3a):

$$\sin\phi_{ps} = \frac{t}{s} = \frac{\tan\phi_{ds}}{\cos\beta + \sin\beta\tan\phi_{ds}} \tag{B-2}$$

where s and t are stress invariants defined as

$$s = \frac{1}{2}(\sigma_1 + \sigma_2) \quad , \quad t = \frac{1}{2}(\sigma_1 - \sigma_2) \tag{B-3}$$

and β is the geometrical angle shown in Figure 3a with

$$\sin\beta = (\sigma_{xx} - \sigma_{yy})/2t \tag{B-4}$$

Note that β is equal to dilation angle ψ with the assumptions of coaxiality and horizontal zero linear extension direction. In that case, Equation (B-2) is identical to Equation (B-1).

On the other hand, we have the following relationships from Mohr's circle of strain increment (Figure 3b):

$$\sin\psi = -\frac{d\nu/2}{d\gamma/2} \tag{B-5}$$

$$\sin\alpha = \frac{d\nu/2 - d\varepsilon_{yy}}{d\gamma/2} \tag{B-6}$$

where $d\nu$ and $d\gamma$ are invariants of volumetric and shear strain increment; and α is another geometrical angle shown in Figure 3b. If we denote

$$\sin\delta = -\frac{d\varepsilon_{yy}}{d\gamma/2} \tag{B-7}$$

we have

$$\sin\alpha = \sin\delta - \sin\psi \tag{B-8}$$

In small values of δ and ψ, Equation (B-8) can be approximated as

$$\alpha \approx \delta - \psi \tag{B-9}$$

Noting that with the non-coaxial angle Δ and geometries of Mohr's circles, we can show

$$\Delta = \frac{\alpha - \beta}{2} \quad \text{or} \quad \beta = \alpha - 2\Delta \approx \delta - \psi - 2\Delta \tag{B-10}$$

Substituting Equation (B-10) into Equation (B-2) yields

$$\sin\phi_{ps} = \frac{\tan\phi_{ds}}{\cos(\alpha - 2\Delta) + \sin(\alpha - 2\Delta)\tan\phi_{ds}} \tag{B-11}$$

$$\sin\phi_{ps} \approx \frac{\tan\phi_{ds}}{\cos(\delta - \psi - 2\Delta) + \sin(\delta - \psi - 2\Delta)\tan\phi_{ds}} \tag{B-12}$$

Equation (B-11) and (B-12) describes the relationship between plane strain and direct shear angles of friction in the most general case where the principal stress direction does not coincide with the principal strain increment direction and the zero linear extension direction deviates from horizontal.

Appendix C

Simulation Data of Parametric Study

Table C-1. Simulation data for Group 1 with $S_w/D_{50} = 2$, $S_r/D_{50} = 0$, uniformly-graded material ($D_{max}/D_{min} = 1.1$), particle to surface friction coefficient $\tan\phi_\mu = 0.05$

R_t/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg) $\theta \geq 90^\circ$	θ_a (deg) $\theta > 92^\circ$	R_n	R_{nc}
0.1	5.7	0.8	100	0.5	0.3	0.1192	0.111	0.06	0.21	0.18	3.5	6.4	5.6	5.6	0.10	0.10
0.2	11.3	0.8	100	0.5	0.3	0.1198	0.2	0.1	0.37	0.29	8.5	11.3	10.8	10.9	0.20	0.19
0.5	26.6	0.8	100	0.5	0.3	0.12	0.48	0.31	0.89	0.91	24.4	26.7	23.9	24.2	0.50	0.44
1	45.0	0.8	100	0.5	0.1	0.1215	0.5	0.32	0.93	0.94	42.3	42.1	39.0	39.3	1.00	0.79
1.2	50.2	0.8	100	0.5	0.3	0.1209	0.483	0.32	0.89	0.94	54.2	53.1	43.3	43.7	1.20	0.91
1.5	56.3	0.8	100	0.5	0.3	0.1201	0.477	0.32	0.88	0.94	70.0	67.9	48.8	49.1	1.50	1.09
2	63.4	0.8	100	0.5	0.1	0.1182	0.473	0.31	0.88	0.91	80.7	80.1	55.7	55.9	2.00	1.37
3	71.6	0.8	100	0.5	0.1	0.118	0.52	0.35	0.96	1.03	84.7	84.0	64.6	64.8	3.00	1.90
4	76.0	0.8	100	0.5	0.1	0.1182	0.488	0.33	0.90	0.97	87.6	87.0	70.0	70.2	4.00	2.41
5	78.7	0.8	100	0.5	0.1	0.119	0.53	0.32	0.98	0.94	88.0	87.8	73.6	73.7	5.00	2.92
10	84.3	0.8	100	0.5	0.05	0.1227	0.49	0.32	0.91	0.94			81.5	81.5	10.00	5.41

Table C-2. Simulation data for Group 2 with $R_t/D_{50} = 1$, $S_w/D_{50} = 2$, uniformly-graded material ($D_{max}/D_{min} = 1.1$), particle to surface friction coefficient $\tan\phi_\mu = 0.05$

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1208	0.45	0.33	0.83	0.97	41.0	40.3	40.0	40.0	0.73	0.66
0.5	45.0	0.8	100	0.5	0.3	0.1211	0.48	0.33	0.89	0.97	34.5	35.0	38.5	40.5	0.69	0.68
1	45.0	0.8	100	0.5	0.1	0.117	0.52	0.33	0.96	0.97	34.5	35.6	30.0	40.5	0.67	0.65
2	45.0	0.8	100	0.5	0.1	0.1176	0.47	0.31	0.87	0.91	29.2	29.9	19.0	40.5	0.50	0.48
3	45.0	0.8	100	0.5	0.3	0.1206	0.45	0.32	0.83	0.94	26.7	27.9	13.6	40.5	0.40	0.39
5	45.0	0.8	100	0.5	0.3	0.1195	0.45	0.32	0.83	0.94	23.9	25.8	8.4	40.5	0.29	0.28
8	45.0	0.8	100	0.5	0.3	0.1206	0.43	0.3	0.80	0.88	21.0	23.4	5.3	40.5	0.20	0.19
10	45.0	0.8	100	0.5	0.3	0.1192	0.4	0.3	0.74	0.88	18.5	21.2	4.2	40.5	0.17	0.16

Table C-3. Simulation data for Group 3 with R_t/D_{50} = 1, S_r/D_{50} = 0, uniformly-graded material (D_{max}/D_{min} = 1.1), particle to surface friction coefficient $\tan\phi_\mu = 0.05$

S_w/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	84.3	0.8	100	0.5	0.1	0.1224	0.068	0.053	0.13	0.16	1.0	3.9	5.8	6.7	1.00	0.01
0.5	76.0	0.8	100	0.5	0.1	0.1223	0.323	0.16	0.60	0.47	17.8	20.4	15.0	16.0	1.00	0.07
1	63.4	0.8	100	0.5	0.3	0.1213	0.428	0.32	0.79	0.94	81.3	79.6	37.7	38.6	1.00	0.38
2	45.0	0.8	100	0.5	0.5	0.1215	0.5	0.32	0.93	0.94	42.3	42.1	39.0	39.3	1.00	0.79
3	33.7	0.8	100	0.5	0.3	0.1203	0.47	0.31	0.87	0.91	27.6	29.7	31.0	31.3	0.55	0.52
4	26.6	0.8	100	0.5	0.3	0.1198	0.43	0.28	0.80	0.82	22.7	25.1	25.0	25.4	0.49	0.47
5	21.8	0.8	100	0.5	0.3	0.121	0.36	0.26	0.67	0.76	17.9	20.7	21.0	21.2	0.36	0.35
10	11.3	0.8	100	0.5	0.3	0.1212	0.207	0.13	0.38	0.38	9.1	11.9	11.0	11.2	0.20	0.20

Table C-4. Simulation data for Group 4 with uniformly-graded material (D_{max}/D_{min} = 1.1), particle to surface friction coefficient $\tan\phi_\mu$ = 0.05

A_u/D_{50}	A_r/D_{50}	R_{tmax}/D_{50}	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
														$\theta \geq 90^\circ$	$\theta > 92^\circ$		
1	1	2	0.8	100	0.5	0.3	0.1197	0.416	0.32	0.77	0.94	25	26.8	28.8	30.3	0.54	0.42
2	1	2	0.8	100	0.5	0.3	0.1205	0.45	0.31	0.83	0.91	31.2	33.0	28.7	30.4	0.56	0.48
2	0.5	2	0.8	100	0.5	0.3	0.1206	0.35	0.28	0.65	0.82	17.2	18.8	18.5	20.2	0.32	0.29
4	2	4	0.8	100	0.5	0.3	0.1198	0.43	0.34	0.80	1.00	67.3	65.8	46	46.2	1.07	0.95
2	0.25	1	0.8	100	0.5	0.3	0.1203	0.29	0.21	0.54	0.62	13.3	15.4	10.2	12.3	0.16	0.15
4	0.25	2	0.8	100	0.5	0.3	0.1188	0.3	0.24	0.56	0.71	11.9	14.1	11.7	14	0.21	0.20
4	4	4	0.8	100	0.5	0.3	0.1196	0.47	0.35	0.87	1.03	70	68.6	50.3	50.5	1.25	1.09

Table C-5. Simulation data for Group 5 with uniformly-graded material (D_{max}/D_{min} = 1.1), particle to surface friction coefficient $\tan\phi_\mu$ = 0.05

Material surfaces	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
												$\theta \geq 90^\circ$	$\theta > 92^\circ$		
Geomembrane surface No. 1	0.8	100	0.5	0.3	0.1212	0.313	0.2	0.58	0.59	14.2	17.0	3.3	12.7	0.15	0.14
Geomembrane surface No. 2	0.8	100	0.5	0.3	0.1235	0.397	0.28	0.74	0.82	18.7	21.3	3.8	19.7	0.17	0.15
Geomembrane surface No. 3	0.8	100	0.5	0.3	0.1204	0.43	0.33	0.80	0.97	19.8	21.7	11.9	32.1	0.32	0.23
Wood surface	0.8	100	0.5	0.3	0.1233	0.145		0.27		5.2	8.0	8.1	9.4		
Stone surface	0.8	100	0.5	0.3	0.1233	0.0805		0.15		1.6	4.5	4.8	6.3		
Concrete surface	0.8	100	0.5	0.3	0.1227	0.137		0.25		4.8	7.6	6.2	7.7		

Table C-6. Simulation data for Group 1 with $S_w/D_{50} = 2$, $S_r/D_{50} = 0$, well-graded material ($D_{max}/D_{min} = 3.0$), particle to surface friction coefficient $\tan\phi_\mu = 0.05$

R_t/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.1	5.7	0.8	100	0.5	0.3	0.1173	0.119	0.06	0.19	0.14	4.0	6.8	5.6	5.6	0.10	0.10
0.2	11.3	0.8	100	0.5	0.3	0.1181	0.214	0.1	0.34	0.23	9.3	12.2	10.8	10.9	0.20	0.19
0.5	26.6	0.8	100	0.5	0.3	0.1187	0.476	0.31	0.76	0.70	24.0	26.2	23.9	24.2	0.50	0.44
1	45.0	0.8	100	0.5	0.3	0.1187	0.579	0.34	0.92	0.77	42.0	41.4	39.0	39.3	1.00	0.79
1.2	50.2	0.8	100	0.5	0.3	0.1191	0.552	0.32	0.88	0.73	53.0	52.0	43.3	43.7	1.20	0.91
1.5	56.3	0.8	100	0.5	0.3	0.1209	0.544	0.35	0.86	0.80	64.0	61.8	48.8	49.1	1.50	1.09
2	63.4	0.8	100	0.5	0.1	0.1202	0.525	0.33	0.83	0.75	76.0	74.3	55.7	55.9	2.00	1.37
3	71.6	0.8	100	0.5	0.1	0.1227	0.546	0.35	0.87	0.80	84.0	82.5	64.6	64.8	3.00	1.90
4	76.0	0.8	100	0.5	0.1	0.1245	0.571	0.36	0.91	0.82	87.0	86.0	70.0	70.2	4.00	2.41
5	78.7	0.8	100	0.5	0.05	0.1272	0.57	0.33	0.90	0.75	88.0	87.3	73.6	73.7	5.00	2.92
10	84.3	0.8	100	0.5	0.05	0.138	0.552	0.34	0.88	0.77	89.0	88.6	81.5	81.5	10.00	5.41

Table C-7. Simulation data for Group 2 with $R_t/D_{50} = 1$, $S_w/D_{50} = 2$, well-graded material ($D_{max}/D_{min} = 3.0$), particle to surface friction coefficient $\tan\phi_\mu = 0.05$

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^{\circ}$	$\theta > 92^{\circ}$		
0.2	45.0	0.8	100	0.5	0.3	0.1196	0.51	0.36	0.81	0.82	39.0	39.7	40.0	40.0	0.73	0.66
0.5	45.0	0.8	100	0.5	0.3	0.1248	0.56	0.34	0.89	0.77	39.0	38.8	38.5	40.5	0.69	0.68
1	45.0	0.8	100	0.5	0.3	0.1196	0.544	0.33	0.86	0.75	35.0	35.4	30.0	40.5	0.67	0.65
2	45.0	0.8	100	0.5	0.1	0.1199	0.546	0.36	0.87	0.82	31.0	31.8	19.0	40.5	0.50	0.48
3	45.0	0.8	100	0.5	0.3	0.1186	0.52	0.33	0.83	0.75	28.0	28.9	13.6	40.5	0.40	0.39
5	45.0	0.8	100	0.5	0.3	0.1177	0.487	0.33	0.77	0.75	24.0	26.6	8.4	40.5	0.29	0.28
8	45.0	0.8	100	0.5	0.3	0.1189	0.45	0.31	0.71	0.70	22.0	24.2	5.3	40.5	0.20	0.19
10	45.0	0.8	100	0.5	0.3	0.1188	0.435	0.28	0.69	0.64	22.0	23.5	4.2	40.5	0.17	0.16

Table C-8. Simulation data for Group 3 with $R_t/D_{50} = 1$, $S_r/D_{50} = 0$, well-graded material ($D_{max}/D_{min} = 3.0$), particle to surface friction coefficient $\tan\phi_\mu = 0.05$

S_w/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	84.3	0.8	100	0.5	0.1	0.1247	0.068	0.055	0.11	0.13	1.1	3.8	5.8	6.7	1.00	0.01
0.5	76.0	0.8	100	0.5	0.1	0.124	0.322	.23	0.51	0.52	16.0	17.0	15.0	16.0	1.00	0.07
1	63.4	0.8	100	0.5	0.3	0.1224	0.53	0.34	0.84	0.77	71.0	70.0	37.7	38.6	1.00	0.38
2	45.0	0.8	100	0.5	0.5	0.1187	0.579	0.34	0.92	0.77	42.0	41.4	39.0	39.3	1.00	0.79
3	33.7	0.8	100	0.5	0.3	0.1189	0.535	0.34	0.85	0.77	29.0	30.9	31.0	31.3	0.55	0.52
4	26.6	0.8	100	0.5	0.3	0.1192	0.421	0.28	0.67	0.64	22.0	24.4	25.0	25.4	0.49	0.47
5	21.8	0.8	100	0.5	0.3	0.1177	0.384	0.26	0.61	0.59	19.0	21.5	21.0	21.2	0.36	0.35
10	11.3	0.8	100	0.5	0.3	0.1192	0.215	0.15	0.34	0.34	9.6	12.3	11.0	11.2	0.20	0.20

Table C-9. Simulation data for Group 4 with well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.05

A_u/D_{50}	A_r/D_{50}	R_{tmax}/D_{50}	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
														$\theta \geq 90^\circ$	$\theta > 92^\circ$		
1	1	2	0.8	100	0.5	0.3	0.1224	0.45	0.34	0.71	0.77	25	27	28.8	30.3	0.54	0.42
2	1	2	0.8	100	0.5	0.3	0.1217	0.509	0.33	0.81	0.75	32	32.5	28.7	30.4	0.56	0.48
2	0.5	2	0.8	100	0.5	0.3	0.1211	0.48	0.31	0.76	0.70	23	25.5	18.5	20.2	0.32	0.29
4	2	4	0.8	100	0.5	0.3	0.1225	0.503	0.33	0.80	0.75	57	56	46	46.2	1.07	0.95
2	0.25	1	0.8	100	0.5	0.3	0.1205	0.326	0.2	0.52	0.45	15	17.2	10.2	12.3	0.16	0.15
4	0.25	2	0.8	100	0.5	0.3	0.1209	0.3	0.22	0.48	0.50	14	16.3	11.7	14	0.21	0.20
4	4	4	0.8	100	0.5	0.3	0.1229	0.47	0.37	0.75	0.84	66	64.5	50.3	50.5	1.25	1.09

Table C-10. Simulation data for Group 5 with well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.05

Material surfaces	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg) $\theta \geq 90^\circ$	θ_a (deg) $\theta > 92^\circ$	R_n	R_{nc}
Geomembrane surface No. 1	0.8	100	0.5	0.3	0.1215	0.3	0.2	0.48	0.45	13	15.2	3.3	12.7	0.15	0.14
Geomembrane surface No. 2	0.8	100	0.5	0.3	0.1211	0.425	0.28	0.67	0.64	20	22.3	3.8	19.7	0.17	0.15
Geomembrane surface No. 3	0.8	100	0.5	0.3	0.1205	0.485	0.35	0.77	0.80	24	26	11.9	32.1	0.32	0.23
Wood surface	0.8	100	0.5	0.3	0.1213	0.133	0.074	0.21	0.17	4.5	7.4	8.1	9.4		
Stone surface	0.8	100	0.5	0.3	0.1211	0.0988	0.06	0.16	0.14	2.5	5.4	4.8	6.3		
Concrete surface	0.8	100	0.5	0.3	0.1217	0.185	0.07	0.29	0.16	7.4	10.3	6.2	7.7		

Table C-11. Simulation data for Group 1 with $S_w/D_{50} = 2$, $S_r/D_{50} = 0$, well-graded material ($D_{max}/D_{min} = 3.0$), particle to surface friction coefficient $\tan\phi_\mu = 0.2$

R_t/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.0	0.0	0.8	100	0.5	0.3	0.1181	0.2	0.2	0.32	0.45	0	11.0	0	0	0	0
0.2	11.3	0.8	100	0.5	0.3	0.118	0.355	0.25	0.56	0.57	8.7	19.6	10.8	10.9	0.20	0.19
0.5	26.6	0.8	100	0.5	0.3	0.1187	0.533	0.32	0.85	0.73	23.0	29.0	23.9	24.2	0.50	0.44
1	45.0	0.8	100	0.5	0.3	0.1196	0.615	0.39	0.98	0.89	43.0	41.0	39.0	39.3	1.00	0.79
1.2	50.2	0.8	100	0.5	0.3	0.1191	0.633	0.36	1.03	0.82	51.6	47.2	43.3	43.7	1.20	0.91
1.5	56.3	0.8	100	0.5	0.3	0.1209	0.58	0.36	0.88	0.82	62.4	58.0	48.8	49.1	1.50	1.09
3	71.6	0.8	100	0.5	0.1	0.126	0.59	0.37	0.94	0.84	83.8	83	64.6	64.8	3.00	1.90

Table C-12. Simulation data for Group 2 with R_t/D_{50} = 1, S_w/D_{50} = 2, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu = 0.2$

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5									40.0	40.0	0.73	0.66
0.5	45.0	0.8	100	0.5	0.3	0.1249	0.594	0.37	0.94	0.84	38.4	38.4	38.5	40.5	0.69	0.68
1	45.0	0.8	100	0.5	0.3	0.1197	0.604	0.38	0.96	0.86	36.8	38.6	30.0	40.5	0.67	0.65
2	45.0	0.8	100	0.5	0.3	0.1199	0.587	0.36	0.93	0.82	29.6	33.1	19.0	40.5	0.50	0.48
3	45.0	0.8	100	0.5	0.3	0.1186	0.546	0.38	0.87	0.86	24.5	29.6	13.6	40.5	0.40	0.39
5	45.0	0.8	100	0.5	0.3	0.1176	0.517	0.37	0.82	0.84	20.4	28.8	8.4	40.5	0.29	0.28
8	45.0	0.8	100	0.5									5.3	40.5	0.20	0.19
10	45.0	0.8	100	0.5	0.3	0.1188	0.5	0.33	0.79	0.75	17.7	26.1	4.2	40.5	0.17	0.16

Table C-13. Simulation data for Group 3 with $R_t/D_{50} = 1$, $S_r/D_{50} = 0$, well-graded material ($D_{max}/D_{min} = 3.0$), particle to surface friction coefficient $\tan\phi_\mu = 0.2$

S_w/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	84.3	0.8	100	0.5	0.1	0.1248	0.217	0.2	0.34	0.45	0.8	12.0	5.8	6.7	1.00	0.01
0.5	76.0	0.8	100	0.5	0.3	0.1237	0.431	0.3	0.68	0.68	60.0	57.0	15.0	16.0	1.00	0.07
1	63.4	0.8	100	0.5	0.3	0.1223	0.62	0.42	0.98	0.95	71.6	70.0	37.7	38.6	1.00	0.38
2	45.0	0.8	100	0.5	0.3	0.1196	0.615	0.39	0.98	0.89	43.0	41.0	39.0	39.3	1.00	0.79
3	33.7	0.8	100	0.5	0.3	0.1189	0.587	0.34	0.93	0.77	30.0	33.0	31.0	31.3	0.55	0.52
4	26.6	0.8	100	0.5									25.0	25.4	0.49	0.47
5	21.8	0.8	100	0.5	0.3	0.1176	0.487	0.31	0.77	0.70	17.0	27.0	21.0	21.2	0.36	0.35
10	11.3	0.8	100	0.5	0.3	0.1191	0.34	0.28	0.54	0.64	8.0	18.8	11.0	11.2	0.20	0.20

Table C-14. Simulation data for Group 4 with well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_{\mu}$ = 0.2

A_u/D_{50}	A_r/D_{50}	R_{tmax}/D_{50}	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg) $\theta \geq 90^{\circ}$	θ_a (deg) $\theta > 92^{\circ}$	R_n	R_{nc}
1	1	2	0.8	100	0.5	0.3	0.1227	0.5	0.34	0.79	0.77	36	37	28.8	30.3	0.54	0.42
2	1	2	0.8	100	0.5	0.3	0.1218	0.505	0.35	0.80	0.80	29.2	30.9	28.7	30.4	0.56	0.48
2	0.5	2	0.8	100	0.5	0.3	0.1213	0.497	0.32	0.79	0.73	21.1	26.8	18.5	20.2	0.32	0.29
4	2	4	0.8	100	0.5	0.3	0.1236	0.468	0.36	0.74	0.82	61.2	57.6	46	46.2	1.07	0.95
2	0.25	1	0.8	100	0.5	0.3	0.1206	0.393	0.3	0.62	0.68	11.4	21.3	10.2	12.3	0.16	0.15
4	0.25	2	0.8	100	0.5	0.3	0.1211	0.372	0.3	0.59	0.68	10	20	11.7	14	0.21	0.20
4	4	4	0.8	100	0.5	0.3	0.1241	0.48	0.36	0.76	0.82	69.3	66.1	50.3	50.5	1.25	1.09

Table C-15. Simulation data for Group 5 with well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.2

Material surfaces	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
												$\theta \geq 90^\circ$	$\theta > 92^\circ$		
Geomembrane surface No. 1	0.8	100	0.5	0.3	0.1217	0.424	0.31	0.67	0.70	11.7	21.8	3.3	12.7	0.15	0.14
Geomembrane surface No. 2	0.8	100	0.5	0.3	0.1211	0.486	0.34	0.77	0.77	15.6	25.2	3.8	19.7	0.17	0.15
Geomembrane surface No. 3	0.8	100	0.5	0.3	0.1206	0.544	0.42	0.86	0.95	21.1	28.4	11.9	32.1	0.32	0.23
Wood surface	0.8	100	0.5	0.3	0.1213	0.29		0.46	0.00	5	16.1	8.1	9.4		
Stone surface	0.8	100	0.5	0.3	0.1212	0.24	0.2	0.38	0.45	2	13.3	4.8	6.3		
Concrete surface	0.8	100	0.5	0.3	0.1217	0.305		0.48	0.00	6	16.8	6.2	7.7		

Table C-16. Simulation data for Group 1 with $S_w/D_{50} = 2$, $S_r/D_{50} = 0$, well-graded material ($D_{max}/D_{min} = 3.0$), particle to surface friction coefficient $\tan\phi_\mu = 0.5$

R_t/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.0	0.0	0.8	100	0.5	0.3	0.1181	0.46	0.3	0.73	0.68	0.0	24.7	0	0	0	0
0.1	5.7	0.8	100	0.5	0.3	0.1173	0.509	0.25	0.81	0.57	3.2	27.0	10.8	10.9	0.20	0.19
0.5	26.6	0.8	100	0.5	0.3								23.9	24.2	0.50	0.44
1	45.0	0.8	100	0.5	0.3	0.1196	0.6	0.37	0.95	0.84	42.7	42.0	39.0	39.3	1.00	0.79
1.2	50.2	0.8	100	0.5	0.3	0.1191	0.65	0.35	1.03	0.80	51.9	51.5	43.3	43.7	1.20	0.91
1.5	56.3	0.8	100	0.5	0.3	0.1208	0.557	0.36	0.88	0.82	64.4	62.7	48.8	49.1	1.50	1.09
3	71.6	0.8	100	0.5	0.3								64.6	64.8	3.00	1.90

Table C-17. Simulation data for Group 2 with R_t/D_{50} = 1, S_w/D_{50} = 2, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu = 0.5$

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5									40.0	40.0	0.73	0.66
0.5	45.0	0.8	100	0.5	0.3	0.1249	0.594	0.37	0.94	0.84	35.6	38.7	38.5	40.5	0.69	0.68
1	45.0	0.8	100	0.5	0.3	0.1197	0.597	0.36	0.95	0.82	34.0	40.5	30.0	40.5	0.67	0.65
2	45.0	0.8	100	0.5	0.3	0.1199	0.592	0.37	0.94	0.84	26.9	34.7	19.0	40.5	0.50	0.48
3	45.0	0.8	100	0.5	0.3	0.1186	0.568	0.37	0.90	0.84	21.8	31.0	13.6	40.5	0.40	0.39
5	45.0	0.8	100	0.5	0.3	0.1176	0.548	0.35	0.87	0.80	16.6	30.8	8.4	40.5	0.29	0.28
8	45.0	0.8	100	0.5									5.3	40.5	0.20	0.19
10	45.0	0.8	100	0.5	0.3	0.1188	0.57	0.33	0.90	0.75	10.1	29.6	4.2	40.5	0.17	0.16

Table C-18. Simulation data for Group 3 with $R_t/D_{50} = 1$, $S_r/D_{50} = 0$, well-graded material ($D_{max}/D_{min} = 3.0$), particle to surface friction coefficient $\tan\phi_\mu = 0.5$

S_w/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	84.3	0.8	100	0.5	0.1	0.1248	0.466	0.25	0.74	0.57	1.3	25.4	5.8	6.7	1.00	0.01
0.5	76.0	0.8	100	0.5	0.3								15.0	16.0	1.00	0.07
1	63.4	0.8	100	0.5	0.3	0.1222	0.548	0.36	0.87	0.82	79.3	79.2	37.7	38.6	1.00	0.38
2	45.0	0.8	100	0.5	0.3	0.1196	0.6	0.37	0.95	0.84	42.7	42.0	39.0	39.3	1.00	0.79
3	33.7	0.8	100	0.5	0.3	0.1189	0.608	0.36	0.97	0.82	29.1	34.6	31.0	31.3	0.55	0.52
4	26.6	0.8	100	0.5									25.0	25.4	0.49	0.47
5	21.8	0.8	100	0.5	0.3	0.1176	0.562	0.32	0.89	0.73	13.2	30.2	21.0	21.2	0.36	0.35
10	11.3	0.8	100	0.5	0.3	0.1191	0.495	0.28	0.79	0.64	5.7	26.4	11.0	11.2	0.20	0.20

Table C-19. Simulation data for Group 4 with well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.5

A_u/D_{50}	A_r/D_{50}	R_{tmax}/D_{50}	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg) $\theta \geq 90^\circ$	θ_a (deg) $\theta > 92^\circ$	R_n	R_{nc}
1	1	2	0.8	100	0.5	0.3	0.1227	0.504	0.34	0.80	0.77	35.2	40.1	28.8	30.3	0.54	0.42
2	1	2	0.8	100	0.5	0.3	0.122	0.48	0.36	0.76	0.82	27.2	34.2	28.7	30.4	0.56	0.48
2	0.5	2	0.8	100	0.5	0.3	0.1213	0.495	0.34	0.79	0.77	19.7	27.9	18.5	20.2	0.32	0.29
4	2	4	0.8	100	0.5	0.3	0.1239	0.485	0.36	0.77	0.82	58.2	49.5	46	46.2	1.07	0.95
2	0.25	1	0.8	100	0.5	0.3	0.1206	0.44	0.33	0.70	0.75	11.5	23.8	10.2	12.3	0.16	0.15
4	0.25	2	0.8	100	0.5	0.3	0.1213	0.39	0.32	0.62	0.73	4.6	21.5	11.7	14	0.21	0.20
4	4	4	0.8	100	0.5	0.3	0.1244	0.503	0.35	0.80	0.80	66.5	63.7	50.3	50.5	1.25	1.09

Table C-20. Simulation data for Group 5 with well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.5

Material surfaces	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
												$\theta \geq 90^\circ$	$\theta > 92^\circ$		
Geomembrane surface No. 1	0.8	100	0.5	0.3	0.1217	0.439	0.28	0.70	0.64	4.9	24.5	3.3	12.7	0.15	0.14
Geomembrane surface No. 2	0.8	100	0.5	0.3	0.1211	0.43	0.33	0.68	0.75	8	24.1	3.8	19.7	0.17	0.15
Geomembrane surface No. 3	0.8	100	0.5	0.3	0.1206	0.53	0.37	0.84	0.84	20.3	41.2	11.9	32.1	0.32	0.23
Wood surface	0.8	100	0.5	0.3	0.1213	0.454		0.72	0.00	4	25.5	8.1	9.4		
Stone surface	0.8	100	0.5	0.3	0.1212	0.43		0.68	0.00	2.3	23.8	4.8	6.3		
Concrete surface	0.8	100	0.5	0.3	0.1218	0.477		0.76	0.00	3.4	25.5	6.2	7.7		

Table C-21. Simulation data for Group 6 with R_t/D_{50} = 1, S_w/D_{50} = 3, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu = 0.05$

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1265	0.43		0.68		28.8	30.5	31.3	31.3		
0.5	45.0	0.8	100	0.5	0.3	0.1254	0.5		0.79		28.9	30.5	28.9	31.3		
1	45.0	0.8	100	0.5	0.3	0.1198	0.514		0.82		26.2	27.8	23.7	31.3		
2	45.0	0.8	100	0.5	0.1	0.1193	0.571		0.91		26.6	28.6	17.0	31.3		
3	45.0	0.8	100	0.5	0.3	0.1208	0.526		0.83		23.3	25.9	13.0	31.3		
5	45.0	0.8	100	0.5	0.3	0.1195	0.528		0.84		21.9	24.0	8.4	31.3		
8	45.0	0.8	100	0.5												
10	45.0	0.8	100	0.5	0.3	0.1183	0.534		0.85		18.5	20.9	4.5	31.3		

Table C-22. Simulation data for Group 6 with R_t/D_{50} = 1, S_w/D_{50} = 3, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu = 0.2$

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.122	0.557		0.88		28.2	33.3	31.3	31.3		
0.5	45.0	0.8	100	0.5	0.3	0.1207	0.57	0.35	0.90	0.80	27.1	33.0	28.9	31.3		
1	45.0	0.8	100	0.5	0.3	0.1212	0.546	0.33	0.87	0.75	25.4	31.1	23.7	31.3		
2	45.0	0.8	100	0.5									17.0	31.3		
3	45.0	0.8	100	0.5	0.3	0.1186	0.539	0.31	0.86	0.70	20.5	29.9	13.0	31.3		
5	45.0	0.8	100	0.5	0.3	0.1183	0.493		0.78		16.4	26.6	8.4	31.3		
8	45.0	0.8	100	0.5												
10	45.0	0.8	100	0.5	0.3	0.1182	0.472		0.75		16.0	25.1	4.5	31.3		

Table C-23. Simulation data for Group 6 with R_t/D_{50} = 1, S_w/D_{50} = 3, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu = 0.5$

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1219	0.558	0.37	0.89	0.84	26.5	34.1	31.3	31.3		
0.5	45.0	0.8	100	0.5	0.3	0.1207	0.578	0.36	0.92	0.82	24.0	33.5	28.9	31.3		
1	45.0	0.8	100	0.5	0.3	0.1212	0.564		0.90		22.9	31.4	23.7	31.3		
2	45.0	0.8	100	0.5									17.0	31.3		
3	45.0	0.8	100	0.5	0.3	0.1186	0.568	0.36	0.90	0.82	15.0	31.3	13.0	31.3		
5	45.0	0.8	100	0.5	0.3	0.1183	0.547	0.35	0.87	0.80	13.0	28.8	8.4	31.3		
8	45.0	0.8	100	0.5										31.3		
10	45.0	0.8	100	0.5	0.3	0.1182	0.55		0.87		10.6	28.9	4.5	31.3		

Table C-24. Simulation data for Group 7 with R_t/D_{50} = 1, S_w/D_{50} = 5, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_{\mu}$ = 0.05

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^{\circ}$	$\theta > 92^{\circ}$		
0.2	45.0	0.8	100	0.5	0.3	0.1193	0.388		0.62		19.2	21.9	20.0	21.2		
0.5	45.0	0.8	100	0.5	0.3	0.1191	0.37	0.26	0.59	0.59	18.7	21.2	19.2	21.2		
1	45.0	0.8	100	0.5	0.3	0.1182	0.372	0.26	0.59	0.59	18.4	21.1	16.7	21.2		
2	45.0	0.8	100	0.5												
3	45.0	0.8	100	0.5	0.3	0.1177	0.352	0.23	0.56	0.52	17.0	19.8	10.8	21.2		
5	45.0	0.8	100	0.5	0.3	0.1184	0.33		0.52		15.8	18.6	7.6	21.2		
8	45.0	0.8	100	0.5												
10	45.0	0.8	100	0.5	0.3	0.1178	0.317		0.50		14.6	17.4	4.5	21.2		

Table C-25. Simulation data for Group 7 with R_t/D_{50} = 1, S_w/D_{50} = 5, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.2

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1193	0.514	0.31	0.82	0.70	17.6	28.0	20.0	21.2		
0.5	45.0	0.8	100	0.5	0.3	0.1191	0.5	0.31	0.79	0.70	17.1	27.3	19.2	21.2		
1	45.0	0.8	100	0.5	0.3	0.1182	0.464	0.3	0.74	0.68	15.6	25.6	16.7	21.2		
2	45.0	0.8	100	0.5												
3	45.0	0.8	100	0.5	0.3	0.1177	0.463	0.3	0.73	0.68	14.7	25.3	10.8	21.2		
5	45.0	0.8	100	0.5	0.3	0.1184	0.414	0.29	0.66	0.66	12.4	22.8	7.6	21.2		
8	45.0	0.8	100	0.5												
10	45.0	0.8	100	0.5	0.3	0.1179	0.4	0.29	0.63	0.66	10.8	21.8	4.5	21.2		

Table C-26. Simulation data for Group 7 with R_t/D_{50} = 1, S_w/D_{50} = 5, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.5

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1193	0.58	0.32	0.92	0.73	16.5	31.0	20.0	21.2		
0.5	45.0	0.8	100	0.5	0.3	0.119	0.583	0.35	0.93	0.80	14.5	31.1	19.2	21.2		
1	45.0	0.8	100	0.5	0.3	0.1182	0.544	0.34	0.86	0.77	12.9	29.3	16.7	21.2		
2	45.0	0.8	100	0.5												
3	45.0	0.8	100	0.5	0.3	0.1177	0.53	0.32	0.84	0.73	11.8	28.3	10.8	21.2		
5	45.0	0.8	100	0.5	0.3	0.1184	0.504	0.3	0.80	0.68	8.5	27.5	7.6	21.2		
8	45.0	0.8	100	0.5												
10	45.0	0.8	100	0.5	0.3	0.1179	0.5		0.79		7.6	26.8	4.5	21.2		

Table C-27. Simulation data for Group 8 with R_t/D_{50} = 1, S_w/D_{50} = 10, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.05

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1188	0.21	0.15	0.33	0.34	9.1	11.9	11.0	11.2		
0.5	45.0	0.8	100	0.5	0.3	0.1185	0.22		0.35		9.7	12.6	10.5	11.2		
1	45.0	0.8	100	0.5	0.3	0.1186	0.219		0.35		9.6	12.4	9.6	11.2		
2	45.0	0.8	100	0.5										11.2		
3	45.0	0.8	100	0.5	0.3	0.119	0.2	0.16	0.32	0.36	8.5	11.4	7.2	11.2		
5	45.0	0.8	100	0.5	0.3	0.1191	0.211		0.33		9.1	12.0	5.7	11.2		
8	45.0	0.8	100	0.5										11.2		
10	45.0	0.8	100	0.5	0.3									11.2		

Table C-28. Simulation data for Group 8 with R_t/D_{50} = 1, S_w/D_{50} = 10, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.2

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1187	0.35		0.56		8.2	19.5	11.0	11.2		
0.5	45.0	0.8	100	0.5	0.3	0.1185	0.345		0.55		8.2	19.1	10.5	11.2		
1	45.0	0.8	100	0.5	0.3	0.1186	0.367	0.25	0.58	0.57	9.0	20.2	9.6	11.2		
2	45.0	0.8	100	0.5										11.2		
3	45.0	0.8	100	0.5	0.3	0.1191	0.338	0.27	0.54	0.61	7.8	18.8	7.2	11.2		
5	45.0	0.8	100	0.5	0.3	0.1191	0.354		0.56		8.3	19.5	5.7	11.2		
8	45.0	0.8	100	0.5										11.2		
10	45.0	0.8	100	0.5	0.3									11.2		

Table C-29. Simulation data for Group 8 with R_t/D_{50} = 1, S_w/D_{50} = 10, well-graded material (D_{max}/D_{min} = 3.0), particle to surface friction coefficient $\tan\phi_\mu$ = 0.5

S_r/D_{50}	Δ_a (deg)	D_r	σ_n (KPa)	μ_p	μ_{pcon}	P_{ini}	$(\tau/\sigma)_p$	$(\tau/\sigma)_{ss}$	E_p	E_{ss}	θ_n (deg)	θ_r (deg)	θ_a (deg)		R_n	R_{nc}
													$\theta \geq 90^\circ$	$\theta > 92^\circ$		
0.2	45.0	0.8	100	0.5	0.3	0.1187	0.505		0.80		4.3	26.9	11.0	11.2		
0.5	45.0	0.8	100	0.5	0.3	0.1185	0.555		0.88		4.7	29.2	10.5	11.2		
1	45.0	0.8	100	0.5	0.3	0.1186	0.505	0.27	0.80	0.61	5.6	27.2	9.6	11.2		
2	45.0	0.8	100	0.5										11.2		
3	45.0	0.8	100	0.5	0.3	0.1191	0.481	0.27	0.76	0.61	5.4	25.7	7.2	11.2		
5	45.0	0.8	100	0.5	0.3	0.1191	0.455	0.26	0.72	0.59	4.0	24.6	5.7	11.2		
8	45.0	0.8	100	0.5										11.2		
10	45.0	0.8	100	0.5	0.3									11.2		

Notation

The following symbols are used in this appendix:

A_u = asperity unit

A_r = asperity ratio

D_{50} = median particle diameter (mm)

D_{max} = maximum particle diameter (mm)

D_{min} = minimum particle diameter (mm)

D_r = relative density

E_p = peak efficiency

E_{ss} = steady state efficiency

P_{ini} =initial porosity before shearing starts

R_n = normalized roughness parameter

R_{nc} = normalized roughness parameter based on profiles of median particle centroid trace

R_t = asperity height (mm)

R_{tmax} = maximum accumulated asperity height (mm)

S_r = spacing between asperities (mm)

S_w = asperity width (mm)

Δ_a = asperity slope (deg)

ϕ_μ = particle to surface friction angle (degrees)

μ_p = interparticle friction coefficient during shear

μ_{pcon} = interparticle friction coefficient during consolidation

θ = angle between surface normal and the horizontal based on profiles of median particle centroid trace (degrees)

θ_a = principal direction of surface normal distribution based on profiles of median particle centroid trace (degrees)

θ_n = principal direction of average contact normal force anisotropy measured at the interface between the rough surface and the particles touching the surface (degrees)

θ_r = principal direction of average contact resultant force anisotropy measured at the interface between the rough surface and the particles touching the surface (degrees)

σ_n = normal stress (KPa)

$(\tau/\sigma)_p$ = peak stress ratio measured at the rough surface

$(\tau/\sigma)_{ss}$ = steady state stress ratio measured at the rough surface