Chapter 5

OPPORTUNITY COST BASED MODELS FOR TRAFFIC INCIDENT RESPONSE PROBLEMS

5.1 Introduction

Traffic incidents annually account for nearly sixty percent of the delay (in vehicle-hours) on the roads. These incidents are non-recurrent random events that cause disruptions and reductions in road capacities. Several methods have been proposed in the literature that aim to reduce the duration of the incident by separately analyzing the various factors that contribute to incident delay. The total delay due to an incident can be partitioned into four components, namely, incident detection/verification time, dispatch time, response vehicle travel time, and incident clearance time (Zografos et al.,1993). Of these, the sum of the dispatch and travel times for response vehicles is defined as the incident response time. Another important observation on routing was made by Carter et al. (1972). They noted that when service to anticipated future demands is considered, dispatching the nearest available vehicle may not be an optimal choice.

This viewpoint of Carter et al. is echoed in our study. The present study extends this idea to the actual development of an on-line incident response model that explicitly incorporates anticipated future demand within the objective function. Our model differs from previous incident response models in that the expense for serving future demand is explicitly addressed as an opportunity cost. An immediate effect of such an objective function on the solution is that the dispatchment of the closest available response vehicle to serve the current set of incidents is no longer necessarily an optimal choice. Previous research efforts in this area attempt to *locate* vehicle depots that minimize overall allocation costs in the long run. This study assumes that vehicle locations (possibly decided upon by using such location methods) are *given* and we are primarily interested in developing real-time incident management strategies by optimally *allocating* response vehicles to serve the current set of incidents, while taking into consideration near future demands. (The corresponding joint location-allocation model will be treated in a subsequent study.) Also, although the dynamic repositioning of response vehicles may not always be a feasible option, our model could be used in conjunction with such an on-line repositioning algorithm to further reduce allocation costs for future demands, if possible.

The remainder of this chapter is organized as follows. The next section discusses modeling considerations and motivates the proposed mixed-integer programming model for the general *m*-incident, *n*-response problem, and presents an illustrative example. Following this, some special cases of the incident-response problem are discussed that lend themselves to polynomial-time enumerative solution approaches. An alternative formulation for the general mixed-integer programming model is suggested in next along with some comparative computational results. Finally, a realistic implementation procedure for integrating this model into an area-wide incident management decision support system is described, and the study concludes with a summary of results and recommendations for future research.

5.2 The *m*-Incident, *n*-Response Problem

The m-incident, n-response, or, the multiple-incident, multiple-response (*MIMR*) problem may be stated as follows.

Given: Road network $G(N, A)$ having a node set N and an arc set A, along with the following information.

(a) Response vehicle node location set L with r_i vehicles available at node $i \in L$.

(b) Set of m incidents having occurred at some node set F, $|F| = m$, with a requirement of n_f service vehicles at node $f \in F$.

(c) $\lambda_{iv} \equiv$ minimum response time for a vehicle dispatched from node i to node v in G, for each $i \in L$ and $v \in N$. (Note that these might be determined based on time-dependent link travel time functions - see Ziliaskopoulos and Mahmassani (1993), for example.)

(d) $\lambda_v \equiv \underset{i \in L}{\text{minimum}} \{\lambda_{iv}\} \ \forall v \in N$.

Find: An optimal assignment of response vehicles to incidents, considering both service and opportunity costs for any assignment as described below.

As far as the service cost related to any such assignment is concerned, defining the decision variables

$$x_{if} = \text{number of vehicles dispatched from } i \in L \text{ to } f \in F, \tag{5.1}$$

we can denote this cost as $\sum_{i \in L} \sum_{f \in F} \lambda_{if} x_{if}$. As mentioned above, we will also include in our objective function a new modeling construct related to an opportunity cost for serving an additional incident that might occur probabilistically on G (for simplicity and as typically used in practice, we assume that the occurrence of incidents follows a Poisson distribution over G), subsequent to the commitment of vehicles according to (5.1). Toward quantifying this cost, consider the following. Let

P_v = probability of an incident occurring at any node (or vertex) $v \in N$, given (conditioned on) the set of incidents on F.

Accordingly, let us define the additional response decision variables

$$y_{iv} = \left\{ \begin{array}{l} 1 \text{ if a vehicle is dispatched from } i \in L \text{ to } v \in N \text{ in} \\ \quad \text{response to an additional incident, and} \\ 0 \text{ otherwise.} \end{array} \right\} \tag{5.2}$$

The opportunity cost associated with the decision y_{iv} = 1, provided that this is feasible after having made the allocations for serving F, is taken as (λ_{iv} - λ_v). The corresponding opportunity cost related term in the objective function is then given by $\sum_i \sum_v P_v(\lambda_{iv} - \lambda_v) y_{iv}$. Note that this opportunity cost is being computed with respect to the requirement of a single vehicle being needed to serve an additional incident at each node in the network. (The (spatial) Poisson assumption implies that only one such additional incident could occur simultaneously.) However, a direct extension of the proposed model is possible for considering more complex scenarios involving multiple response vehicles or even multiple secondary incidents.

To illustrate the possible suboptimality in simply dispatching the closest available vehicle when future anticipated demand is treated, consider the example shown in Figure 5.1. Two vehicle depots (each with a capacity of a single vehicle) are located at nodes i_1 and i_2. The current incident site is located at f. Hence, $F = \{f\}$ with $m = 1$, and we assume that $n_f = 1$. Additional incidents might occur at f with a probability of $P_f = 0.1$, but might also occur at another potential demand node v with a probability of $P_v = 0.5$. Assume that there is no other potential demand node in the network.

The minimum response times for response vehicles to attend to incidents at these nodes are shown in Figure 5.1 against the arcs connecting the depots to the nodes. By inspection, we can see that the closest available response vehicle that can serve the

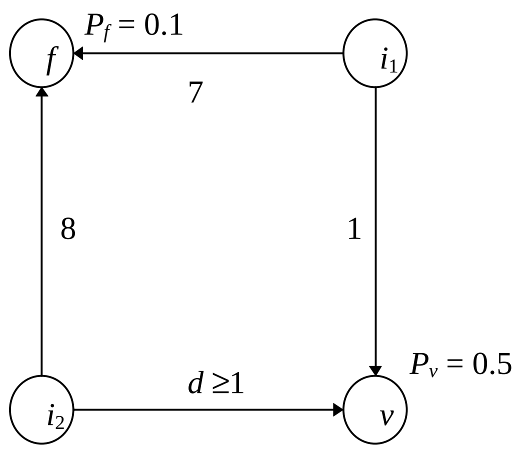

Figure 5.1. Example for illustrating the effect of opportunity costs.

incident at node f is located at i_1. If this vehicle is dispatched to node f, then any future incident occurring at node f or v has to be attended to by the response vehicle at i_2. The total cost of this assignment (call this c_1) equals the cost of serving the current incident plus the opportunity cost of serving a future incident on the network. Assuming that the response time from node i_2 to node v is $d \geq 1$, this is given by

$$c_1 = 7 + 0.1(8 - 7) + 0.5(d - 1) = 6.6 + 0.5d.$$

Similarly, we can calculate the cost (c_2, say) of assigning the vehicle located (8 time units away) at i_2 to serve the current incident at node f, as follows:

$$c_2 = 8 + 0.1(7-7) + 0.5(1-1) = 8.0.$$

Hence, for values of $d > 2.8$, assigning the closest vehicle to serve the current incident is not an optimal decision. Observe the nature of the compromise made here. Dispatching the closest available response vehicle was not a good choice in this instance (when $d > 2.8$)

because another node (v) having a relatively high expected service cost with respect to the resulting available response facility suffered a significant loss in coverage. On the other hand, if the number of response vehicles available at i_1 is increased to two, then assigning one of these vehicles to serve the current incident would yield an optimal choice for the current scenario. In general, for more complex situations involving multiple incidents and multiple response vehicle requirements, a mixed-integer programming model can be developed to solve this problem. The proposed incident response (MIMR-1) model can be formulated as follows.

MIMR-1:

$$\text{Minimize} \quad \sum_{i\in L}\sum_{f\in F}\lambda_{if}x_{if} + \sum_{i\in L}\sum_{v\in N}P_v(\lambda_{iv}-\lambda_v)y_{iv} \tag{5.3a}$$

subject to

$$\sum_{f\in F}x_{if} + s_i = r_i \quad \forall i\in L \tag{5.3b}$$

$$\sum_{i\in L}x_{if} = n_f \quad \forall f\in F \tag{5.3c}$$

$$\sum_{i\in L}y_{iv} = 1 \quad \forall v\in N \tag{5.3d}$$

$$y_{iv} \le s_i \quad \forall i\in L,\ v\in N \tag{5.3e}$$

$$y \text{ binary},\ x\ge 0 \text{ (and integer)},\ s\ge 0. \tag{5.3f}$$

The objective function is comprised of the sum of the primary response and the opportunity cost related terms. Note that this is equivalent to minimizing the sum of the primary response costs given by the first term in (5.3a) and the expected secondary response cost given by $\sum_i\sum_v P_v\lambda_{iv}y_{iv}$, because from (5.3d), the remaining term $-\sum_i\sum_v P_v\lambda_v y_{iv} = -\sum_v P_v\lambda_v$ is a constant. Hence, this is an alternative equivalent interpretation of the model. Constraint (5.3b) states that the number of vehicles dispatched from any depot must not exceed the number of response vehicles available at the depot, with s_i being the resident slack at node $i \in L$. Constraint (5.3c) stipulates that the number of vehicles dispatched to any current incident node should meet the response vehicle requirements at that node. Constraint (5.3d) requires a response vehicle to be ready for dispatch to serve a future incident at any (demand) node in the network, while (5.3e) states that such an additional vehicle in (5.3d) can be dispatched from a depot only if it is available at that depot, given the current response decision. Constraint (5.3f) imposes binary and integrality restrictions on the *y* and *x* variables, respectively. However, note that due to the underlying network structure in the variables (*x*, *s*) for a fixed (feasible) choice of binary variables *y*, the *x*-variables can be treated as continuous and integrality will hold automatically at a vertex optimum to the foregoing network problem. Some special cases of this incident response problem can be solved using polynomial-time enumeration methods. These instances are described in the next section along with their specialized solution approaches.

5.3 Polynomially Solvable Special Cases Of The MIMR Problem

To present certain special cases of the MIMR problem that admit polynomial-time enumerative solution procedures, consider the following classification scheme. Denote by Problem $\{m/n/r\}$ an instance of MIMR wherein some *m* incidents are being considered, and where the number of response vehicles required per incident is equal to some fixed value *n*, and the number of response vehicles available at the depots are all equal to some fixed value *r*. The three special cases of MIMR treated in this section are $\{1/1/r\}$, $\{1/2/r\}$, and $\{2/1/r\}$. The solution approaches described can be easily modified to solve these special case problems without the assumption of a fixed value for r for all $i \in L$. The main task in the proposed specialized solution approach for these cases is the calculation of the opportunity cost associated with vehicle-to-incident assignments. Some expedients for computing this are described next.

Expedients for Deriving Expressions for the Opportunity Costs

Let $E[S_L]$ denote the minimal expected cost of serving an additional incident on G with all vehicles available. Similarly, let $E[S_{L-i}]$ denote the minimal expected cost of serving an additional incident on G with all but a single vehicle at node i being available. Since the realization of an incident at any node f precludes the consideration of any other simultaneous incident in the network, we can write (based on the mutually exclusive random event property)

$$E[S_L] = \sum_v P_v \lambda_v \tag{5.4a}$$

$$E[S_{L-i}] = \sum_v P_v \lambda_{v|i} \tag{5.4b}$$

where $\lambda_{v|i}$ is the smallest response time to node v from a node in L that has an available vehicle, given that a single vehicle at node i is now unavailable.

We can similarly define $E[S_{L-i-j}]$ as the minimal expected cost of serving an additional incident on G, given all but two vehicles, one at each of nodes i and j, being available. (Note that we permit i and j to possibly be the same.) This can be mathematically represented as

$$E[S_{L-i-j}] = \sum_v P_v \lambda_{v|(i,j)} \tag{5.4c}$$

where $\lambda_{v|(i,j)}$ is defined similar to $\lambda_{v|i}$.

Further, denote

$$O_i = E[S_{L\text{-}i}] - E[S_L] \tag{5.5a}$$

as the "opportunity cost" associated with serving an additional incident on G, given the prior assignment of a vehicle from location i to serve a primary incident on G, and

$$O_{ij} = E[S_{L\text{-}i\text{-}j}] - E[S_L] \tag{5.5b}$$

as the "opportunity cost" associated with serving an additional incident on G, given the prior assignment of one vehicle from each of locations i and j (possibly $i = j$) to serve some primary incidents on G.

We now derive expressions for the foregoing opportunity cost components. Note that as observed earlier, we could have equivalently considered the expected secondary incident service costs as opposed to the opportunity costs, and this would have entailed neglecting the subtraction of $E[S_L]$ from (5.5a) and (5.5b). However, as we shall see, this subtraction eases the necessary computations by canceling common terms.

Let $\alpha_{\rho v}$ be the ρ^{th} smallest response value from some node in L to node v (so that α_{1v} is the smallest, α_{2v} is the second smallest, …, and $\alpha_{|L|v}$ is the largest such value for each $v \in N$). These values can be conveniently conceptualized by a matrix $\text{SP}_{|L|\times|N|}$, where each individual entry $\text{SP}_{(\rho,v)} = (i_{\rho v},\ \alpha_{\rho v})$ represents the two-tuple given by the ρ^{th} closest vehicle's location ($i_{\rho v} \in L$) from node v, (with ties broken arbitrarily), and the corresponding response time value ($\alpha_{\rho v}$). Hence, the vector $\{\alpha_{\rho v}\}$ for a given node v is the set of second components in the v^{th} column in the matrix. Note that this matrix SP is not actually computed; it is only necessary to obtain the information that is required by the formulation (5.6) and (5.7) below.

Now, define the set $N_{\rho i}$ to be comprised of all the column indices v for which the first component entries in the ρ^{th} row have $i_{\rho v} = i$. Hence, $N_{\rho i}$ is comprised of all nodes v for which the ρ^{th} nearest vehicle is located at node $i \in L$ (in some given rank ordering). Similarly, define $N_{12,i,j}$ to be the set of nodes that have vehicles at locations i and j as the closest two vehicles in the given rank ordering. Using these relations, we can re-write the opportunity cost expressions as shown below. Note that these computations are unaffected by the breaking of ties while ranking the vehicles located at the nodes in L with respect to their response time from any node v. Also, multiple vehicles at any given node in L are only identified by their common location i and are not ascribed a specific identity.

$$O_i = \sum_{v \in N_{1i}} P_v (\alpha_{2v} - \alpha_{1v}) \tag{5.6}$$

$$O_j = \sum_{v \in N_{1j}} P_v (\alpha_{2v} - \alpha_{1v})$$

$$O_{ij} = \sum_{v \in \{(N_{1i} \cup N_{1j}) - N_{12,ij}\}} P_v (\alpha_{2v} - \alpha_{1v}) + \sum_{v \in N_{12,ij}} P_v (\alpha_{3v} - \alpha_{1v})$$

If $i = j$, then noting that $\alpha_{1v} = \alpha_{2v} \forall v \in N_{12,ij}$, we have

$$O_{ii} = O_i + \sum_{v \in N_{12,ii}} P_v (\alpha_{3v} - \alpha_{2v}). \tag{5.7a}$$

If $i \neq j$, then

$$O_{ij} = O_i + O_j + \sum_{v \in N_{12,ij}} P_v [(\alpha_{3v} - \alpha_{1v}) - (\alpha_{2v} - \alpha_{1v})]$$

Hence, if $i \neq j$, then

$$O_{ij} = O_i + O_j + \sum_{v \in N_{12,ij}} P_v (\alpha_{3v} - \alpha_{2v}). \tag{5.7b}$$

Intuitively, the last expression for O_{ij} above asserts that in addition to the opportunity costs due to the unilateral absence of vehicles at i and j as a first-choice selection, this cost component must consider cases where the first two choices involve vehicles at locations i

and j, hence requiring the corresponding nodes in $N_{12,i,j}$ to be served by the third closest vehicle location. This excess is precisely the third term in the final expression. This expression also reveals that the opportunity cost associated with dispatching a pair of vehicles from distinct locations is greater than or equal to the sum of the opportunity costs of dispatching these vehicles individually. Using these expressions for computing the opportunity costs, the following special cases of the MIMR problem can be solved in polynomial-time using an efficient enumeration scheme.

Analysis for Problem {1/1/*r*}

The case $\{1/1/r\}$ refers to the single incident-single vehicle response problem. Note that when $r \geq 2$, this instance is solved by simply assigning the least response time vehicle from a node $i \in L$ to node f at which the incident has occurred. This involves solving a shortest path problem from all nodes to node f which can be accomplished in $O(|N|^2)$ time. For the instance $\{1/1/1\}$, Proposition 1 provides an efficient solution procedure.

Proposition 5.1. Consider the special case $\{1/1/1\}$ of Problem MIMR-1. An optimal solution to this problem is given by finding $\min_{i \in L}\{\lambda_{if} + O_i\}$, where O_i is given by (5.6). Moreover, this can be accomplished in time $O(|L||N|^2)$.

Proof. Problem MIMR-1 can be decomposed in this case as follows, noting that given a primary response assignment determined by the x-variables, Problem MIMR-1 decomposes into separable secondary response problems, one for each $v \in N$. Hence, we can write

$$\text{MIMR-1:} \quad \underset{i \in L}{\text{minimize}} \quad \{\lambda_{if} + \sum_{v \in N} \underset{p \in L-\{i\}}{\text{minimum}} \quad [P_v(\lambda_{pv} - \lambda_v)]\}$$

$$\equiv \quad \underset{i \in L}{\text{minimize}} \quad \{\lambda_{if} + \sum_{v \in N} P_v \lambda_{v|i} - E[S_L]\} \text{ via (5.4a) and (54c)}$$

$$\equiv \quad \underset{i \in L}{\text{minimize}} \quad \{\lambda_{if} + O_i\} \text{ via (5.4a), (5.4b) and (5.5a),} \tag{5.8}$$

where O_i is computed as in (5.6). To evaluate (5.8), suppose that we find the shortest paths to all nodes in G from each of the nodes in L. This can be accomplished in $O(|L||N|^2)$ time (see Bazaraa et al., 1990, noting that travel times are all nonnegative). The resulting information can be arranged in $O(|N||L| \log |L|)$ time to determine the matrix SP defined above, whence O_i can be computed using (5.6) for each $i \in L$. Noting that the dominating time in these steps is $O(|L||N|^2)$, this completes the proof. □

Analysis for Problem {1/2/*r*}

The case {1/2/r} refers to the single incident-double vehicle response problem. Note that when $r \geq 3$, this instance is solved by simply assigning the least response time vehicles from a node $i \in L$ to node f at which the incident has occurred. For the instance {1/2/r} where r equals one or two, an efficient solution procedure can be devised using Proposition 2 below.

Proposition 5.2. Consider the special case {1/2/r} of Problem MIMR-1, where r equals one or two. An optimal solution to this problem is given by finding $\min_{i, j \in L, i \leq j} \{\lambda_{if} + \lambda_{jf} + O_{ij}\}$, where O_{ij} is given by (5.7a) and (5.7b). Moreover, this can be accomplished in time $O(|L||N|^2)$.

Proof. The problem MIMR-1 can be decomposed in this case as follows, similar to the analysis of the {1/1/1} case.

$$\text{MIMR -1}: \underset{\substack{i,j\in L, i\le j \\ \{i\neq j \text{ if } r=1\}}}{\text{minimize}} \{\lambda_{if} + \lambda_{jf} + \sum_{v\in N} \underset{p\in L-\{i,j\}}{\text{minimum}} [P_v(\lambda_{pv} - \lambda_v)]\}$$

$$\equiv \underset{\substack{i,j\in L, i\le j \\ \{i\neq j \text{ if } r=1\}}}{\text{minimize}} \{\lambda_{if} + \lambda_{jf} + \sum_{v\in N} P_v \lambda_{v/(i,j)} - E[S_L]\} \text{ via (5.4a) and (5.4c)}$$

$$\equiv \underset{\substack{i,j\in L, i\le j \\ \{i\neq j \text{ if } r=1\}}}{\text{minimize}} \{\lambda_{if} + \lambda_{jf} + O_{ij}\} \text{ via (5.4a) and (5.5b),} \tag{5.9}$$

where O_{ij} is computed as in (5.7a) and (5.7b). Again, the dominating time step is to construct the matrix SP which can be accomplished in time $O(|L||N|^2)$. □

Analysis for Problem {2/1/*r*}

The case {2/1/r} refers to the double incident-single vehicle response problem. When $r\geq 3$, this instance is solved as in the {1/2/r} case. This involves finding the shortest path from all nodes in G to nodes f_1 and f_2 which can be accomplished in $O(|N|^2)$ time, as before. When r equals one or two, Proposition 3 below provides an efficient solution procedure for the problem {2/1/r}.

Proposition 5.3. Consider the special case {2/1/r} of Problem MIMR-1, where r equals one or two. Let F = {f_1, f_2} be the current set of incidents. An optimal solution to this problem is given by finding $\min\limits_{\substack{i,j\in L, i\le j\\ i\ne j \text{ if } r=1}} \{\min\{\lambda_{if_1}+\lambda_{jf_2}, \lambda_{if_2}+\lambda_{jf_1}\}+O_{ij}\}$, where O_{ij} is given by (5.7a) and (5.7b), and this can be accomplished in time $O(|L||N|^2)$.

Proof. The problem MIMR-1 can be decomposed in this case as follows, similar to the analysis of the case $\{1/2/r\}$.

$$\text{MIMR -1:}\quad \underset{\substack{i,j\in L\\ (i\ne j \text{ if } r=1)}}{\text{minimize}}\ \{\lambda_{if_1}+\lambda_{jf_2}+\sum_{v\in N}\underset{p\in L-\{i,j\}}{\text{minimum}}\ [P_v(\lambda_{pv}-\lambda_v)]\}$$

$$\equiv\quad \underset{\substack{i,j\in L\\ (i\ne j \text{ if } r=1)}}{\text{minimize}}\ \{\lambda_{if_1}+\lambda_{jf_2}+\sum_{v\in N}P_v\lambda_{v/(i,j)}-E[S_L]\}$$

$$\equiv\quad \underset{\substack{i,j\in L, i\le j\\ (i\ne j \text{ if } r=1)}}{\text{minimize}}\ \{\min\{\lambda_{if_1}+\lambda_{jf_2}, \lambda_{jf_1}+\lambda_{if_2}\}+O_{ij}\} \tag{5.10}$$

where O_{ij} is computed as in (5.7a) and (5.7b). As before, the time-complexity is dominated by the time required to construct the matrix SP, which is $O(|L||N|^2)$. □

5.4 Computational Experience And An Alternative Model And Heuristic

Test Results for the Specialized Procedures and for Model MIMR-1

The model MIMR-1 was tested and verified for the particular cases of $\{1/1/r\}$, $\{1/2/r\}$ and $\{2/1/r\}$. These latter three scenarios are special cases for which efficient search procedures have been developed in above. Note that in these approaches, the computation of the opportunity cost for a particular assignment can be omitted whenever the response time itself exceeds the incumbent minimum cost solution value. For these test instances, the specialized solution procedures were much faster than using a commercial MIP package (CPLEX-MIP) for solving MIMR-1. Table 5.1 presents some comparative computational results using randomly generated instances of $\{1/1/r\}$, $\{1/2/r\}$, and $\{2/1/r\}$, where Time (Special) and Time (MIP) are the CPU execution times in seconds for the specialized and MIP approaches, respectively, on a SunSparc 1000 computer.

From the results shown in Table 5.1, the efficiency of using the specialized approaches for these particular classes of problems is clearly evident. While such methods can be extended to handle larger values of *m* and *n*, the time-complexity of the opportunity cost computation increases exponentially with the problem size, and an explicit enumeration approach becomes increasingly burdensome. Hence, for more general instances, a model such as MIMR-1 can be used to solve the problem more efficiently.

For further testing MIMR-1, hypothetical data were generated using random number streams. The λ values were taken to be integers, and probabilities P_v were generated such that $\sum_{v \in N} P_v = 1$ (this is not required, but was used simply for generating the data). Model MIMR-1 was then solved using the CPLEX-MIP package.

For almost all problems (including all cases in Table 5.1), the linear programming relaxation of Model MIMR-1 yielded integer results. For problems involving less than or equal to four incidents, no fractional solution has as yet been observed. In addition, when the response times were sufficiently diverse, the relaxed solution almost always yielded integer results. These results are shown in Table 5.2, where in each case, the LP relaxation was sufficient to obtain an optimal integer solution. When the response times were restricted to a smaller range of values (see Table 5.3), the continuous relaxation of the model yielded some fractional results. The branch-and-bound procedure for such problem instances resulted in the enumeration of a large number of nodes because of the low diversity in the solution space. In many cases, the procedure was prematurely terminated after having enumerated more than 1000 branch-and-bound nodes.

An Alternative Model and Comparative Results

To remedy the shortcoming of Model MIMR-1 on certain classes of problems, we derive an alternative formulation MIMR-2 of this problem below. While this alternative model is equivalent to MIMR-1 in its LP relaxation, the branching decisions for MIMR-2 are induced to be different via a redefinition of integer restricted variables. As illustrated in the sequel, this yields a significant savings in computational time by greatly reducing the number of branch-and-bound nodes enumerated.

MIMR-2: Minimize $\sum_{i\in L}\sum_{f\in F}\lambda_{if}x_{if} + \sum_{i\in L}\sum_{v\in N}P_v(\lambda_{iv}-\lambda_v)y_{iv}$ (5.11a)

subject to

$$\sum_{f\in F}x_{if} + s_i = r_i \quad \forall i\in L \tag{5.11b}$$

$$\sum_{i\in L}x_{if} = n_f \quad \forall f\in F \tag{5.11c}$$

$$\sum_{i\in L}y_{iv} = 1 \quad \forall v\in N \tag{5.11d}$$

$$z_i \le s_i \quad \forall i\in L \tag{5.11e}$$

$$y_{iv} \le z_i \quad \forall i\in L,\ v\in N \tag{5.11f}$$

$$y \ge 0,\ (x,s)\ge 0,\ z \text{ binary.} \tag{5.11g}$$

The constraints (5.3e) and (5.3f) in (5.3) have been replaced by constraints (5.11e) – (5.11g) in MIMR-2. Note that (5.11) has only one set of binary variables, namely z, with the rest of the variables being declared as continuous. Because of the underlying network structure, it is readily seen that a vertex optimal solution to MIMR-2 for any fixed feasible, binary value of z will automatically yield integer values for x and y. (Note that for a fixed z, MIMR-2 decomposes into separable problems in the (x, s) and the y variables.)

Table 5.4 presents comparative computational results for models MIMR-1 and MIMR-2. The number of nodes enumerated by MIMR-2 have been drastically reduced over those required by MIMR-1, thereby enabling the exact solution of all problem instances with a reasonable computational effort. Additional computational tests were conducted for model MIMR-2 based on the desired accuracy of the solution. This was done by varying the optimality tolerance parameter (δ) available in CPLEX-MIP, whereby the branch-and-bound procedure is terminated when the maximum possible (percentage) improvement in the objective function is less than some user-specified tolerance value (δ), as follows:

$$\frac{|Z_{\text{best integer}} - Z_{\text{best node}}|}{1 + |Z_{\text{best node}}|} < \delta$$

where $Z_{\text{best node}}$ represents the best lower bound over the active nodes, $Z_{\text{best integer}}$ represents the objective value of the incumbent integer-feasible solution, and δ is an user-specified optimality tolerance value.

Tables 5.5, 5.6, and 5.7 exhibit the results obtained for δ = 1%, 3%, and 5%, respectively, in comparison with δ = 0.01% (default optimality tolerance used in Table 5.4). These results indicate that for some instances, a significant savings in time can be obtained by setting δ = 3% rather than solving MIMR-2 to optimality, while not impairing the quality of the best found solution. Furthermore, there does not seem to be any significant gain in time-savings by increasing the optimality tolerance to 5%. Hence, an optimality tolerance value of 3% seems to be a good choice for model MIMR-2.

LP-Based Heuristic Procedure (LPH)

While the computational performance of MIMR-2 is acceptable for small sized problems or for problems having diverse response times, it may not be quick enough for real-time implementation in other situations. Toward this end, we present an LP-based heuristic that can be used to obtain a quick, initial integer feasible upper bound on the optimal objective value. The procedure **LPH** operates as follows.

Step 1. Solve the LP relaxation of Model MIMR-2. If an integer solution is obtained, stop; the optimal solution is at hand. Otherwise, proceed to Step 2.

Step 2. Let $\bar{x}$ be the fractional LP solution obtained. Consider the transformation $x = \lfloor \bar{x} \rfloor + \xi$, where $0 \leq \xi \leq \hat{e}$, and where $\hat{e}$ is a suitable vector of ones. Solve the residual transportation problem TP as shown below.

TP: Minimize $\sum_{i \in L} \sum_{f \in F} \lambda_{if} \xi_{if}$

subject to :

$$\sum_{f} \xi_{if} \leq r_i - \sum_{f} \lfloor \bar{x}_{if} \rfloor \quad \forall i \in L$$

$$\sum_{i} \xi_{if} = n_f - \sum_{i} \lfloor \bar{x}_{if} \rfloor \quad \forall f \in F$$

$$0 \leq \xi \leq \hat{e}.$$

Problem TP has a feasible solution given via the solution to the LP relaxation of Model MIMR-2, and therefore, has an optimal 0-1 solution ξ^* by the total unimodularity property of the constraint set of Problem TP (Bazaraa, et al., 1990).

Let $x^* = \lfloor \bar{x} \rfloor + \xi^*$ be the prescribed x solution with s^* being the corresponding value of the slack variable s in (5.11b). Proceed to Step 3.

Step 3. For each $v \in N$, find $p \in L$ such that:

$$p = \underset{\substack{i \in L \\ \ni S^*_i \geq 1}}{\operatorname{argmin}} \{(\lambda_{iv} - \lambda_v)\}$$

and set $y^*_{pv} = 1$, and $y^*_{iv} = 0$ $\forall i \neq p$. Proceed to Step 4.

Step 4. Evaluate the objective function for the resulting heuristic solution (x^*, y^*) to obtain the corresponding heuristic objective value Z_{LPH}.

The heuristic procedure LPH was applied to some of the test cases presented in Table 5.5 using the Simplex and Network Optimization routines available in the CPLEX software package. A comparison of the computational performance and the objective value of the solution obtained by the heuristic procedure, with the corresponding statistics for the exact solution of MIMR-2 is presented in Table 5.8. It was observed that for large problem sizes and diverse response times, the heuristic procedure yielded an objective value within 5% of the optimal solution, while being nearly three to four times faster than MIMR-2. In general, the computational times of the heuristic procedure obtained were much lower than that of MIMR-2, while the upper bounds obtained were quite close to the optimal values.

5.5. Practical Implementation Issues: Integration Within WAIMSS

The incident response model proposed in this study is intended for real-time implementation as part of the *Java*-based version of the area-wide incident management software system (WAIMSS) (Jonnalagadda, 1996; Dhingra, 1996) developed at the Center for Transportation Research (CTR) at the Virginia Polytechnic Institute and State University. A Geographical Information System (GIS) constitutes the core of WAIMSS.

This incident management software can be used to graphically input incident locations and characteristics on the road network. The “Smart” Incident Database developed over the years at the CTR has details of over 6,000 incidents that have been recorded in the Northern Virginia area. These details have been used to develop a multiple-regression model, in tandem with a Knowledge-Based Expert System (KBES) that can be used to predict response requirements for future incidents as well as to recommend the necessary response agencies to contact in each case. In the previous version of WAIMSS, the incident response time was assumed to be fixed at a value of ten minutes. Deterministic queuing was then used to calculate the incident clearance time and the closest response vehicles were assigned the task of clearing incidents, with the actual requirements decided upon using the NEXPERT knowledge base.

The MIMR computer program is an embedded, object-oriented module written in C++ within an ARC/INFO AML (Arc Macro Language) module. This AML module contains a list of instructions that are executed by the ARC/INFO engine. ARC/INFO stores the network data and its attributes in a GIS database that can be displayed on the monitor. The user can visually input the locations of the incidents on the displayed map and then select a suitable window that captures the affected portion of the network. Also, other incident details such as the nature of the incident(s) (hazmat/non-hazmat related, number of vehicles involved, time-of-day, location of incident(s), extent and nature of damage, etc.) are also entered as input. The user can also either use the default location of the incident response teams or specify their current locations. Upon completion of these interactive input directives, during the second stage, the AML module writes these incident, link, and node attributes from the ARC/INFO database to the input file for the

MIMR module, and for NEXPERT. During the third stage, the embedded MIMR module is used to run the CPLEX-based optimization procedure. Here, the CPLEX module that solves the mixed-integer program, as well as the network optimization routines used by the heuristic LPH, are part of the CPLEX-PROBLEM object (C++ class). The network data structures and (time-dependent or time-independent) shortest path routines form the NETWORK class. The shortest path module uses a label processing double-ended queue object, coupled with a label-correcting algorithm. This was found to be empirically efficient for sparse transportation networks (Subramanian 1997). This network object is called to read the network data and compute the shortest paths from all the response vehicle locations to the nodes in the network. These results are then used to generate the objective function coefficients and constraints for the MIMR module which develops an optimal incident response plan for each response vehicle type. The specialized procedures are used to solve the particular instances $\{1/1/r\}$, $\{2/1/r\}$, and $\{1/2/r\}$, whereas for more general problems, the LP-based heuristic (LPH) is used to generate a quick initial solution, and MIMR-2 is then used to improve this solution within a prescribed fixed time-limit. Once the optimal assignments are determined, the corresponding shortest path-based dispatch routes are regenerated. (Note that because of the dynamic nature of the network, the shortest path between any pair of nodes generated as input for MIMR, and that computed as output need, not coincide.) In addition, NEXPERT is used to generate other relevant information regarding any qualitative action that is needed, such as the various emergency response agencies to contact and the steps to be initiated by the appropriate response teams regarding these incidents. At the fourth stage, the AML module reads the results from MIMR and NEXPERT in the form of dispatch routes from response unit locations to incident nodes, and in regard to textual data such as the relevant agencies to contact, response vehicle requirements, etc. Finally, these details are displayed on the monitor. Currently, WAIMSS is a UNIX-based software package for Workstations and is being ported to a Java-based Windows Environment on PCs.

To illustrate, a subset of the road network of Seminole, Florida (having about 100 nodes and 250 arcs) was used to test the MIMR procedure for various incident scenarios (1-5 incidents and 10 response locations). In all cases, the linear programming relaxation of Model MIMR-2 yielded integer results. Since dynamic network data was unavailable, static (time independent) shortest path and disjoint path based dispatch routes were used to generated the input and output.

5.6 Summary And Discussion

This study has addressed the *m*-incident, *n*-response (MIMR) problem that attempts to optimally allocate response vehicles to serve a current set of incidents on a network, while considering service requirements for anticipated future demands. The proposed model introduces the concept of an opportunity cost that is incorporated into the objective function to reflect a measure for the expense of serving future incidents, subsequent to the commitment of vehicles in response to a current set of incidents. Efficient polynomial-time specialized approaches have been developed for the particular instances of $\{1/1/r\}$, $\{1/2/r\}$ and $\{2/1/r\}$, while two mixed-integer programming models (MIMR-1 and MIMR-2) have been formulated for the general case. Based on comparative computational results, the special case procedures along with model MIMR-2 are recommended for solving the incident response problem. An algorithmic module composed of these procedures, and used in concert with the LP-based heuristic scheme, has been incorporated into an area-wide incident management decision support system (WAIMSS).

The concepts and models introduced in this study can be enhanced by considering various other link travel time characteristics and dispatch policies. For example, we can incorporate stochastic link delays of the type used in Daskin (1987). Note that in this case, given a selection of a depot location from which more that one vehicle is to be dispatched to serve a particular incident, the routing of such multiple vehicles along the expected shortest path may be suboptimal.

Alternative dispatch policies can also be considered. For example, when dispatching a pair of vehicles, one might require that the two response vehicles use disjoint paths or selectively disjoint paths, as discussed in the previous chapter. The motivation is to prescribe effective paths that diversify link usage under stochastic delays. It is possible that such a dispatch policy might yield a lower expected arrival time of the first vehicle under certain types of stochastic delay functions when compared with shortest path routing. A dispatch policy of this type can also serve as a quick heuristic means of pre-determining input routes when compared to the more exact method described in Daskin (1987).

Another feature not considered in most papers is the effect of the incident on the travel patterns in the vicinity of the incident. In the aftermath of an incident, one can expect the travel times for nearby links to vary with time, and this may considerably increase incident response times. In the Intelligent Transportation Systems (ITS) scenario, such variations in travel times may be predicted with greater accuracy and can be more frequently updated using automated traffic data collection methods. Using this information, time-dependent (single) shortest path methods, or the shortest pair of (time or space) disjoint or selectively disjoint path algorithms (Sherali et al., 1998), could be used to determine optimal routes for response vehicles, and to simultaneously develop a time schedule for dispatching response vehicles from depots. Routing approaches of this type are recommended for future research.

TABLE 5.1. Comparison of the Specialized and MIP Approaches.

Case	$\|(N, F, L)\|$	r	*Time (Special)	*Time (MIP)
$\{1/1/r\}$	(500, 1, 20)	3	0.01	3.71
$\{1/1/r\}$	(500, 1, 250)	2	0.02	49.57
$\{1/2/r\}$	(100, 1, 100)	3	0.02	3.04
$\{1/2/r\}$	(1000, 1, 100)	2	0.05	43.11
$\{2/1/r\}$	(50, 2, 20)	2	0.01	0.23
$\{2/1/r\}$	(200, 2, 30)	2	0.01	1.44

*Seconds on a SunSparc 1000 computer.

TABLE 5.2. CPLEX-MIP Results for MIMR-1 for Various Ranges of Response Times.

$\|(N,F,L)\|$	max (r_i, n_f)	Range for λ_v	Z_{LP} = Z_{MIP}	Time (Sec)
(300, 3, 50)	(3, 1)	[0, 75]	9.04	3.63
(150, 10, 20)	(2, 4)	[0, 30]	48.83	4.42
(100, 20, 60)	(2, 6)	[0, 600]	1855.02	20.83
(500, 5, 10)	(2, 4)	[0, 5000]	7706.12	20.87
(200, 10, 20)	(2, 4)	[0, 4]	73.99	29.48
(1000, 4, 5)	(4, 5)	[0, 5000]	2370.54	42.67
(200, 20, 50)	(2, 5)	[0, 100]	266.34	44.01
(200, 25, 50)	(2, 4)	[0, 800]	205.60	244.12
(100, 25, 50)	(3, 6)	[0, 500]	3595.14	288.43
(200, 15, 45)	(3, 9)	[0, 90]	97.88	437.35

. **TABLE 5.3. CPLEX-MIP Results for MIMR-1 for Low Variances in Response Times.**

$\|(N,F,L)\|$ max (r_i, n_f)	Range for λ_v	Z_{LP}	LP Sec	Z_{MIP}	MIP Sec	Nodes Enum.	$\frac{(ZMIP - ZLP).100\%}{Z_{MIP}}$
(50,5,15) (2,6)	[0,8]	1.16	0.34	1.18	0.36	12	1.69
(50,10,15) (2, 3)	[0, 4]	0.017	0.45	0.024	0.80	13	29.17
(100,20,60) (2, 6)	[0, 18]	18.73	9.17	18.74	4.97	16	0.05
(100,5,20) (2, 8)	[0, 3]	4.38	5.75	4.45	15.17	56	1.57
(100,10,15) (2,3)	[0, 3]	0.25	6.07	0.32	91.14	287	21.88
(200,15,20) (6, 8)	[0,4]	0.39	44.99	inf.*	>913.58	1000+	—
(150,15,30) (2, 4)	[0, 4]	1.18	26.69	inf.*	>1093.73	1000+	—
(100,25,30) (5, 6)	[0,6]	3.35	19.13	3.46	>1274.47	1000+	3.18
(150,20,50) (2, 5)	[0, 11]	0.11	46.42	0.13	>1597.92	1000+	15.38
(200,30,40) (3, 4)	[1, 24]	31.46	81.20	31.56	>2006.17	1000+	0.32
(100,25,50) (2, 4)	[0, 5]	0.42	85.54	inf.*	>2582.56	1000+	—

* inf. = integer infeasible 1000+ = more than 1000 nodes enumerated.

Z_{MIP}, Z_{LP} = optimal value (or best known value) of MIMR-1 and its continuous relaxation, respectively.

TABLE 5.4. Comparative Results for Models MIMR-1 and MIMR-2.

$\|(N,F,L)\|$ max (r_i, n_f)	Range for λ_v	Z_{LP}	Z_{MIP} (5.3)	MIP (5.3) Sec	MIP (5.3) Nodes Enum.	Z_{MIP} (5.11)	MIP (5.11) Sec	MIP (5.11) Nodes Enum.
(50,5,15) (2,6)	[0,8]	1.16	1.18	0.70	12	1.18	0.83	3
(50,10,15) (2, 3)	[0, 4]	0.017	0.024	1.25	13	0.024	0.96	2
(100,20,60) (2, 6)	[0, 18]	18.73	18.74	14.14	16	18.74	13.07	2
(100,5,20) (2, 8)	[0, 3]	4.38	4.45	20.92	56	4.45	12.28	8
(100,10,15) (2,3)	[0, 3]	0.25	0.32	97.21	287	0.32	16.69	11
(200,15,20) (6, 8)	[0,4]	0.39	inf.	>958.57+	1000+	0.44	110.28	20
(150,15,30) (2, 4)	[0, 4]	1.18	inf.	>1120.42+	1000+	1.26	430.11	185
(100,25,30) (5, 6)	[0,6]	3.35	3.46	>1293.60+	1000+	3.45	103.23	39
(150,20,50) (2, 5)	[0, 11]	0.11	0.13	>1644.54+	1000+	0.13	218.95	44
(200,30,40) (3, 4)	[0, 24]	31.46	31.56	>2087.37+	1000+	31.56	139.56	29
(100,25,50) (2, 4)	[0, 5]	0.42	inf.	>2668.10+	1000+	0.55	974.58	203

Legend: Same as for Table 5.3, with Models MIMR-1 and MIMR-2 being referred to by their equation numbers (5.3) and (5.11), respectively.

TABLE 5.5. 1% Optimality Results for MIMR-2.

$\|(N,F,L)\|$ max (r_i,n_f)	$Z_{optimal}$	MIP Sec	MIP Nodes Enum.	$Z_{best\ integer}$ $\delta = 0.01$	Time (sec) $\delta = 0.01$	Nodes Enum. $\delta = 0.01$
(50,5,15) (2,6)	1.18	0.83	3	1.18	0.76	3
(50,10,15) (2, 3)	0.024	0.96	2	0.024	0.96	2
(100,20,60) (2, 6)	18.74	13.07	2	18.74	12.51	2
(100,5,20) (2, 8)	4.45	12.28	8	4.45	11.78	6
(100,10,15) (2,3)	0.32	16.69	11	0.32	16.57	11
(200,15,20) (6, 8)	0.44	110.28	20	0.44	110.28	20
(150,15,30) (2, 4)	1.26	430.11	185	1.26	368.83	112
(100,25,30) (5, 6)	3.45	103.23	39	3.45	101.40	35
(150,20,50) (2, 5)	0.13	218.95	44	0.14	211.14	42
(200,30,40) (3, 4)	31.56	139.56	29	31.58	118.86	17
(100,25,50) (2, 4)	0.55	974.58	203	0.55	933.43	157

Legend: Same as for Table 5.4, with $Z_{optimal}$ = optimal objective value attained by model (5.11), and $Z_{best\ integer}$ = δ-optimal integer feasible objective value attained by (5.11).

TABLE 5.6. 3% Optimality Results for MIMR-2.

$\lvert(N,F,L)\rvert$ max (r_i,n_f)	$Z_{optimal}$	MIP Sec	MIP Nodes Enum.	$Z_{best\ integer}$ $\delta = 0.03$	Time (sec) $\delta = 0.03$	Nodes Enum. $\delta = 0.03$
(50,5,15) (2,6)	1.18	0.83	3	1.18	0.76	3
(50,10,15) (2, 3)	0.024	0.96	2	0.024	0.96	2
(100,20,60) (2, 6)	18.74	13.07	2	18.74	12.51	2
(100,5,20) (2, 8)	4.45	12.28	8	4.51	9.40	2
(100,10,15) (2,3)	0.32	16.69	11	0.32	15.46	10
(200,15,20) (6, 8)	0.44	110.28	20	0.44	108.41	18
(150,15,30) (2, 4)	1.26	430.11	185	1.29	330.71	96
(100,25,30) (5, 6)	3.45	103.23	39	3.51	81.51	27
(150,20,50) (2, 5)	0.13	218.95	44	0.14	194.31	37
(200,30,40) (3, 4)	31.56	139.56	29	31.86	93.58	11
(100,25,50) (2, 4)	0.55	974.58	203	0.56	817.89	113

Legend: Same as for Table 5.5.

TABLE 5.7. 5% Optimality Results for MIMR-2.

$\|(N,F,L)\|$ max (r_i,n_f)	$Z_{optimal}$	MIP Sec	MIP Nodes Enum.	$Z_{best\ integer}$ $\delta = 0.05$	Time (sec) $\delta = 0.05$	Nodes Enum. $\delta = 0.05$
(50,5,15) (2,6)	1.18	0.83	3	1.18	0.76	3
(50,10,15) (2, 3)	0.024	0.96	2	0.024	0.96	2
(100,20,60) (2, 6)	18.74	13.07	2	18.74	12.51	2
(100,5,20) (2, 8)	4.45	12.28	8	4.51	9.40	2
(100,10,15) (2,3)	0.32	16.69	11	0.32	11.99	5
(200,15,20) (6, 8)	0.44	110.28	20	0.45	96.55	12
(150,15,30) (2, 4)	1.26	430.11	185	1.31	208.81	58
(100,25,30) (5, 6)	3.45	103.23	39	3.55	34.10	7
(150,20,50) (2, 5)	0.13	218.95	44	0.14	194.31	37
(200,30,40) (3, 4)	31.56	139.56	29	31.86	93.58	11
(100,25,50) (2, 4)	0.55	974.58	203	0.58	648.70	66

Legend: Same as for Table 5.5.

TABLE 5.8. Comparative Results for the LP-based Heuristic (LPH) and Model MIMR-2.

$\|(N, F, L)\|$ max (r_i, n_f)	Range for λ_v	Z_{LP}	Z_{MIP} (5.11)	MIP (5.11) Sec	Z_{LPH}	*LPH* Sec
(50,5,15) (2,6)	[0,8]	1.16	1.18	0.83	1.183	0.50
(100,20,60) (2, 6)	[0, 18]	18.73	18.74	13.07	18.894	12.32
(100,5,20) (2, 8)	[0, 3]	4.38	4.45	12.28	4.563	7.57
(100,10,15) (2,3)	[0, 3]	0.25	0.32	16.69	0.413	5.71
(200,15,20) (6, 8)	[0,4]	0.39	0.44	110.28	0.605	31.96
(100,25,30) (5, 6)	[0,6]	3.35	3.45	103.23	3.646	13.83
(150,20,50) (2, 5)	[0, 11]	0.11	0.13	218.95	0.259	27.19
(200,30,40) (3, 4)	[0, 24]	31.46	31.56	139.56	32.05	41.89

Legend: Same as in Table 5.5, and additionally,

Z_{LPH} = objective value obtained by the heuristic procedure, and

LPH (Sec) = CPU time in seconds for the heuristic procedure.