# APPENDIX A

# APPROACH FOR EVALUATION OF DIVERSITY AND ADAPTIVE BEAMFORMING PERFORMANCE

As stated earlier, the objective of our investigation is to investigate antenna configurations for diversity combining and adaptive beamforming at handheld radio terminals. To accomplish this, it is necessary to evaluate the performance of measured and simulated diversity and adaptive beamforming systems using various antenna configurations. This appendix discusses the relevant statistics for diversity and adaptive combining and includes the procedures used to calculate these statistics.

## A1. Relevant Statistics for Diversity Combining

In a diversity combining system, two or more receiver branches are connected to different antennas, and the envelopes of the various branch signals experience different propagation conditions and will fade differently with time. Signals from one or more branches are selected or combined in a way that improves the instantaneous received signal-to-noise ratio (SNR). Diversity combining techniques include selection, equal-gain combining, and maximum-ratio combining. Appendix A contains more detailed background information on diversity combining.

Statistics that are relevant to diversity combining performance include diversity gain, envelope cross-correlation between branches, and power imbalance between branches. These statistics are discussed in the following sections.

### A1.1 Envelope Cross-Correlation

The cross-correlation coefficient of the $m^{th}$ and $n^{th}$ received signal envelopes is given by

$$\rho_{e_{mn}} = \frac{E\left[(e_m - \mu_{e_m})(e_n - \mu_{e_n})\right]}{\sigma_{e_m}\sigma_{e_n}} \tag{A1}$$

where $e_m = |s_m|$ is the envelope of the $m^{th}$ signal and varies with time and/or position of the receiver or transmitter. Also, $\mu_{e_m}$ and $c_{e_m}$ are the mean and standard deviation of $e_m$, respectively. The corresponding quantities for the $n$th signal are defined similarly. The cross-correlation coefficient $\rho_{e_{mn}}$ is calculated over a distance or time interval. A cross-correlation near zero indicates that fading on the branches is for the most part independent and that significant diversity gain is possible. An envelope cross-correlation of 0.7 or less is generally considered sufficient for satisfactory diversity combining performance. The cross-correlation is often used as the sole measure of diversity performance, but this can be misleading because the diversity gain also depends on the balance between branches.

### 3.1.1 Branch Imbalance

If the time-average SNR on the branches of a diversity combining system is different, the branches are said to be unbalanced. In a typical system, the noise power in all branches is very nearly equal, and the SNR imbalance is approximately equal to the power imbalance. For a two-channel system the branch imbalance is given by

$$\begin{aligned}\Delta_{branch} &= \frac{\langle \gamma_1 \rangle}{\langle \gamma_2 \rangle} \\ &\approx \frac{\langle s_1^2 \rangle}{\langle s_2^2 \rangle}\end{aligned} \tag{A2}$$

The diversity gain that is achieved for a given envelope cross-correlation is highest if the branches are balanced. The diversity gain can be much lower if the branches are unbalanced.

### 3.1.2 Diversity gain

Diversity gain is the ultimate measure of how much the SNR of a received signal can be improved through the use of multiple inputs of the same signal from different receiver branches. The actual amount of diversity gain realized with a system depends on the imbalance between the branches in addition to the envelope cross-correlation between the branches. This relationship is shown empirically in [Turkmani] and analytically in [Dietze]. For a true comparison of diversity measurements, it is necessary to calculate the diversity gain that can be achieved with an antenna configuration in a particular channel.

Consider a diversity system with $M$ branches. The instantaneous SNR on the $m^{th}$ branch is denoted by $\gamma_m(t) = s_m^2(t)/2\sigma_{N_m}^2$, where $\sigma_{N_m}^2$ is the noise power on the $m^{th}$ diversity branch, $s_m(t)$ is the envelope of the signal on the $m^{th}$ branch, and the instantaneous signal power on the $m^{th}$ branch is $s_m^2(t)/2$. To avoid an overly optimistic result, diversity gain is calculated relative to the strongest branch. For all diversity combining techniques the input signal-to-noise ratio $\gamma_{in}$ is defined as the SNR of the branch that has the highest time average SNR. That is,

$$\begin{aligned}\gamma_{in}(t) &= \gamma_{m_{\max}} \\ &= \frac{s_{m_{\max}}^2(t)}{2\sigma_{N_{m_{\max}}}^2} \\ &= \frac{s_{m_{\max}}^2(t)}{2\sigma_N^2}\end{aligned} \tag{A3}$$

where

$$m_{\max} = m : \langle \gamma_m \rangle = \max(\langle \gamma_1 \rangle, \cdots \langle \gamma_M \rangle) \tag{A4}$$

Diversity gain is defined as the ratio of the output SNR after combining ($\gamma_{out}$) to the input SNR on the strongest branch, $\gamma_{in}$, and is calculated based on cumulative probabilities. For a given cumulative probability of $X$, the diversity gain is

$$G_{div}(X) = \frac{\gamma_{out}(X)}{\gamma_{in}(X)} \tag{A5}$$

Diversity gain can be read from the cumulative distribution function of the SNR before and after combining. Figure 3-1 is an example CDF from a measurement in a multipath channel with an obstructed line of sight.

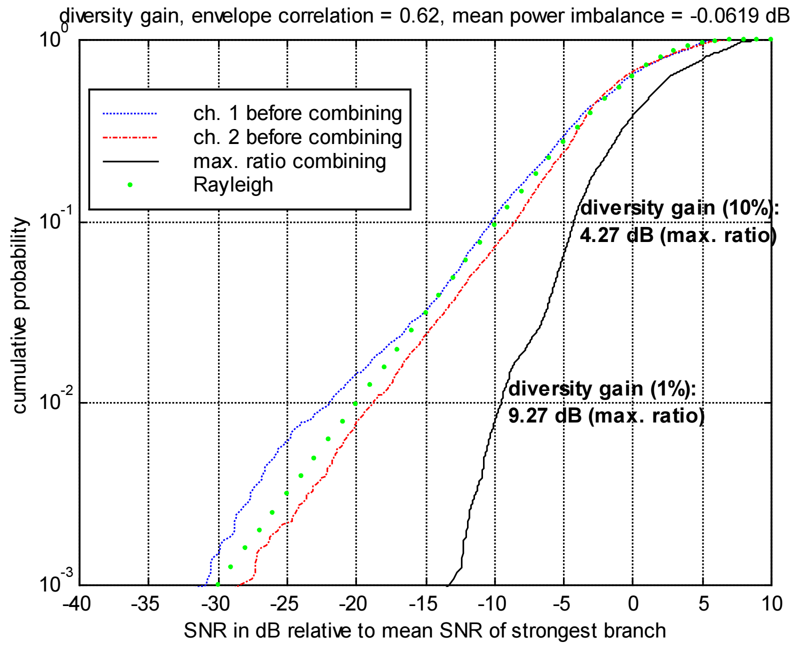


**Figure A1.** Diversity gain measured from cumulative distribution functions

In Fig. A1, CDFs are shown for the signals on each branch, as well as for the combined signal. All CDFs are normalized to the time average SNR of the stronger branch. The diversity gain for a given cumulative probability (read from the y-axis) is the horizontal distance between the curve for the stronger branch (channel 2 in this case) and the curve for the combined signal. The diversity gain is approximately 4.3 dB for 10% cumulative probability and 9.3 dB for a 1% cumulative probability.

For a cumulative probability $X$, the SNR after combining is

$$\begin{aligned}\gamma_{out}(X) &= \arg\left(P\left[\gamma_{out} \leq \gamma\right] = X\right) \\ &= \frac{1}{2\sigma_{N_{out}}^2} \arg\left(P\left[s_{out}^2 \leq s^2\right] = X\right) \\ &= \frac{s_{out}^2(X)}{2\sigma_{N_{out}}^2}\end{aligned} \qquad \text{(A6)}$$

where $\sigma_{N_{out}}^2$ is the noise power in the combined signal and may be different from $\sigma_N^2$ depending on the combining technique. The input SNR is

$$\begin{aligned}\gamma_{in}(X) &= \arg\left(P[\gamma_{in} \le \gamma] = X\right) \\ &= \frac{1}{2\sigma_N^2}\arg\left(P[s_{in}^2 \le s^2] = X\right) \\ &= \frac{s_{in}^2(X)}{2\sigma_N^2}\end{aligned} \tag{A7}$$

## A2 Approximate Expressions for Diversity Gain

Consider a diversity system with $M$ branches. It is desirable to measure the diversity gain of the system, but SNR information is not always available. Expressions are developed here for diversity gain that do not require explicit knowledge of the SNR of the branches or the combined signal. These expressions are useful for efficient data analysis. The approximate expressions for diversity gain are based on the following assumptions:

1. $\sigma_{N_m}^2 = \sigma_N^2 \quad \forall m$
2. $s_m^2(t) >> 2\sigma_N^2 \quad \forall m,t$

where $\sigma_{N_m}^2$ is the noise power on the $m^{th}$ diversity branch, $s_m(t)$ is the envelope of the signal on the $m^{th}$ branch, and the instantaneous signal power on the $m^{th}$ branch is $s_m^2(t)/2$. If assumption 2 holds, then the signal power can be approximated by the signal-plus-noise power, and

$$\begin{aligned}s'^2 &= s^2 + 2\sigma_N^2 \\ &\approx s^2\end{aligned} \tag{A8}$$

Based on these definitions, approximate diversity gain calculations are given for three diversity combining techniques: selection diversity, equal gain combining, and maximum ratio combining.

### A2.1. Selection diversity

In selection diversity, two or more receivers are used, with each connected to a different antenna. The output of the receiver that has the highest SNR (averaged over some short period) is selected for demodulation. This is the configuration shown in Fig. 1 (a) of Appendix A. For selection diversity, the output SNR is given by

$$\begin{aligned}\gamma_{sel}(t) &= \max\left(\frac{s_m^2(t)}{2\sigma_{N_m}^2}\right)\\ &= \frac{1}{2\sigma_N^2}\max\left(s_m^2(t)\right)\\ &= \frac{s_{sel}^2(t)}{2\sigma_N^2}\end{aligned} \tag{A9}$$

The diversity gain is

$$\begin{aligned}G_{sel}(X) &= \frac{\gamma_{sel}(X)}{\gamma_{in}(X)}\\ &= \frac{s_{sel}^2(X)}{s_{m_{\max}}^2(X)}\end{aligned} \tag{A10}$$

The raw signal includes noise, so from a measurement standpoint it is more direct to use a signal that includes noise. The approximate diversity gain (if the assumptions hold) is found by substituting (A8) into (A10).

$$G_{sel}(X) \approx \frac{s_{sel}'^2(X)}{s_{m'_{\max}}'^2(X)} \tag{A11}$$

where

$$m'_{\max} = m : \left\langle s_m'^2 \right\rangle = \max\left(\left\langle s_1'^2 \right\rangle, \cdots \left\langle s_M'^2 \right\rangle\right) \tag{A12}$$

If the assumptions hold then $m_{\max} = m'_{\max}$

### A2.2. Equal gain combining

Equal gain combining is achieved by co-phasing and summing signals from two or more receiver branches. This is shown in Fig. 1 (c) of Appendix A. The signals add coherently but the noise does not (assuming noise on different branches is uncorrelated). This results in an SNR after combining that is higher than either branch SNR unless one branch SNR is consistently low. For equal gain combining, the instantaneous SNR of the combined signal is

$$\gamma_{eg} = \frac{\left(\sum_{m=1}^{M} s_m\right)^2}{2\sum_{m=1}^{M}\sigma_{N_m}^2} = \frac{s_{eg}^2}{2M\sigma_N^2} \tag{A13}$$

The diversity gain is

$$G_{eg}(X) = \frac{\gamma_{eg}(X)}{\gamma_{in}(X)} = \frac{s_{eg}^2(X)}{Ms_{m_{\max}}^2(X)} \tag{A14}$$

The approximate diversity gain is

$$G_{eg}(X) \approx \frac{s_{eg}'^2(X)}{Ms_{m'_{\max}}'^2(X)} \tag{A15}$$

where

$$s'_{eg} = \sum_{m=1}^{M}\sqrt{s_m^2 + 2\sigma_{N_m}^2} \tag{A16}$$

**A2.3. Maximal-ratio combining**

In maximal-ratio combining, the signals from all receiver branches are co-phased, weighted, and summed. The amplitude weighting of each branch is proportional to the SNR (power ratio) on that branch. This results in the highest possible SNR for the signal after combining.

For maximal-ratio combining, the instantaneous SNR of the combined signal is given by the sum of the SNRs on the *M* branches.

$$\gamma_{mr} = \sum_{m=1}^{M}\gamma_m = \frac{\sum_{m=1}^{M} s_m^2}{2\sigma_N^2} = \frac{s_{mr}^2}{2\sigma_N^2} \tag{A17}$$

The diversity gain for maximal-ratio combining is

$$G_{mr}(X) = \frac{\gamma_{mr}(X)}{\gamma_{in}(X)} = \frac{s_{mr}^2(X)}{s_{m_{\max}}^2(X)} \tag{A18}$$

The approximate diversity gain is

$$G_{mr} \approx \frac{s_{mr}'^2(X)}{s_{m'_{\max}}'^2(X)} \tag{A19}$$

where

$$s'_{mr} = \sqrt{\sum_{m=1}^{M}(s_m^2 + 2\sigma_{N_m}^2)} \tag{A20}$$

Note that $G_{mr} \geq G_{eg}$.

### 3.3 Relevant Statistics for Adaptive Beamforming

Adaptive beamforming algorithms can achieve diversity gain and also reject interfering signals. The performance of systems using these algorithms can be evaluated using the signal to interference-plus-noise ratio (SINR). SINR is the equivalent of SNR for a channel with interference. This is a power ratio and is calculated as follows:

$$SINR = \frac{s_d^2}{\sum_{\ell=1}^{L} s_\ell^2 + 2\sigma_N^2} \tag{A21}$$

#### 3.3.1 Time average SINR

One measure of performance is the time-average SINR over the duration of a measurement or simulation. This is simply the time average of (A21).

#### 3.3.2 Cumulative probability of SINR

Another useful measure of the performance of an adaptive beamformer in a dynamic environment is the net improvement in the SINR that is achieved with a given cumulative probability. This is similar to the definition of diversity gain in (A5)

$$G_{SINR}(X) = \frac{SINR_{out}(X)}{SINR_{in}(X)} \tag{A22}$$

### 3.4 Measurement of SINR

A technique for measuring SINR in a narrowband system is outlined here. This was implemented in the HAAT data processing software software described in Chapter 6. The desired and interfering transmitters transmit CW signals that are slightly offset in frequency within the bandwidth of the receiver. The relative strength of the signals is determined by taking an FFT of a block of the received signal. The power in the bins containing the desired and interfering signals is compared. Windowing can be used before taking the FFT in order to reduce frequency sidelobes. This technique provides the signal-plus-noise to interference-plus-noise ratio, which is very close to the SINR for large SNR. An analogous measurement approach has been used in the time domain for a TDMA system, where the desired and interfering users transmitted during different time slots [Biedka, et al.]. It is also possible to use different PN spreading codes to distinguish between transmitters to enable measurements for wideband systems that use direct sequence spread spectrum signals.