

## CHAPTER 3

### ANTENNA ARRAYS AND BEAMFORMING

Array beam forming techniques exist that can yield multiple, simultaneously available beams. The beams can be made to have high gain and low sidelobes, or controlled beamwidth. Adaptive beam forming techniques dynamically adjust the array pattern to optimize some characteristic of the received signal. In beam scanning, a single main beam of an array is steered and the direction can be varied either continuously or in small discrete steps.

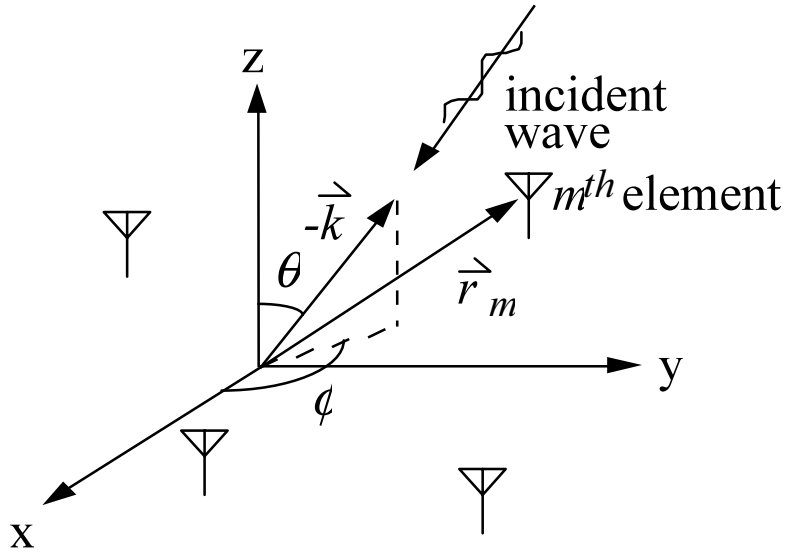
Antenna arrays using adaptive beamforming techniques can reject interfering signals having a direction of arrival different from that of a desired signal. Multi-polarized arrays can also reject interfering signals having different polarization states from the desired signal, even if the signals have the same direction of arrival. These capabilities can be exploited to improve the capacity of wireless communication systems. This chapter presents essential concepts in antenna arrays and beamforming.

An array consists of two or more antenna *elements* that are spatially arranged and electrically interconnected to produce a directional radiation pattern. The interconnection between elements, called the feed network, can provide fixed phase to each element or can form a *phased array*. In optimum and adaptive beamforming, the phases (and usually the amplitudes) of the feed network are adjusted to optimize the received signal. The geometry of an array and the patterns, orientations, and polarizations of the elements influence the performance of the array. These aspects of array antennas are addressed as follows. The pattern of an array with general geometry and elements is derived in Section 3.1; phase- and time-scanned arrays are discussed in Section 3.2. Section 3.3 gives some examples of fixed beamforming techniques. The concept of optimum beamforming is introduced in Section 3.4. Section 3.5 describes adaptive algorithms that iteratively approximate the optimum beamforming solution. Section 3.6 describes the effect of array geometry and element patterns on optimum beamforming performance.

#### 3.1 Pattern of a Generalized Array

A three dimensional array with an arbitrary geometry is shown in Fig. 3-1. In spherical coordinates, the vector from the origin to the  $n$ th element of the array is given by  $\vec{r}_m = (\rho_m, \theta_m, \phi_m)$  and  $-\hat{k} = (1, \theta, \phi)$  is the vector in the direction of the source of an

incident wave. Throughout this discussion it is assumed that the source of the wave is in the far field of the array and the incident wave can be treated as a plane wave. To find the array factor, it is necessary to find the relative phase of the received plane wave at each element. The phase is referred to the phase of the plane wave at the origin. Thus, the phase of the received plane wave at the  $n$ th element is the phase constant  $\beta = \frac{2\pi}{\lambda}$  multiplied by the projection of the element position  $\vec{r}_m$  on to the plane wave arrival vector  $-\hat{k}$ . This is given by  $-\vec{k} \cdot \vec{r}_m$  with the dot product taken in rectangular coordinates.



**Figure 3-1.** An arbitrary three dimensional array

In rectangular coordinates,  $-\hat{k} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} = \hat{r}$  and  $\vec{r}_m = \rho_m \sin \theta_m \cos \phi_m \hat{x} + \rho_m \sin \theta_m \sin \phi_m \hat{y} + \rho_m \cos \theta_m \hat{z}$ , and the relative phase of the incident wave at the  $n$ th element is

$$\begin{aligned} \zeta_m &= -\vec{k} \cdot \vec{r}_m \\ &= \beta \rho_m (\sin \theta \cos \phi \sin \theta_m \cos \phi_m + \sin \theta \sin \phi \sin \theta_m \sin \phi_m + \cos \theta \cos \theta_m) \quad (3.1) \\ &= \beta (x_m \sin \theta \cos \phi + y_m \sin \theta \sin \phi + z_m \cos \theta) \end{aligned}$$

### 3.1.1 Array factor

For an array of  $M$  elements, the array factor is given by

$$AF(\theta, \phi) = \sum_{m=1}^M I_m e^{j(\zeta_m + \delta_m)} \quad (3.2)$$

where  $I_m$  is the magnitude and  $\delta_m$  is the phase of the weighting of the  $m$ th element.

The normalized array factor is given by

$$f(\theta, \phi) = \frac{AF(\theta, \phi)}{\max\{|AF(\theta, \phi)|\}} \quad (3.3)$$

This would be the same as the array pattern if the array consisted of ideal isotropic elements.

### 3.1.2 Array pattern

If each element has a pattern  $g_m(\theta, \phi)$ , which may be different for each element, the normalized array pattern is given by

$$F(\theta, \phi) = \frac{\sum_{m=1}^M I_m g_m(\theta, \phi) e^{j(\zeta_m + \delta_m)}}{\max\left\{\left|\sum_{m=1}^M I_m g_m(\theta, \phi) e^{j(\zeta_m + \delta_m)}\right|\right\}} \quad (3.4)$$

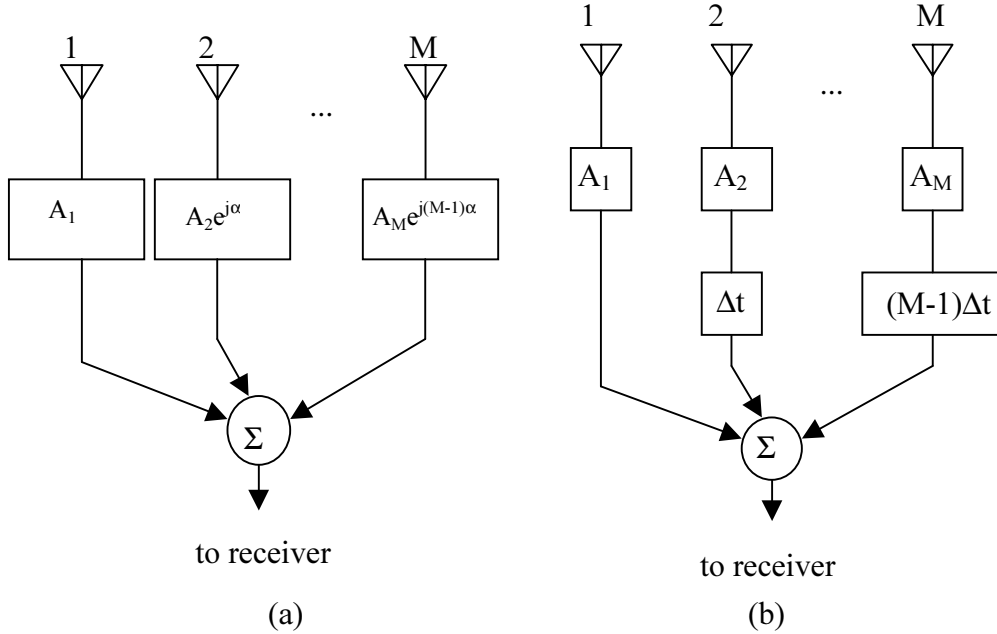
In (3.4), the element patterns must be represented such that the pattern maxima are equal to the element gains relative to a common reference.

## 3.2 Phase and Time Scanning

Beam forming and beam scanning are generally accomplished by phasing the feed to each element of an array so that signals received or transmitted from all elements will be in phase in a particular direction. This is the direction of the beam maximum. Beam forming and beam scanning techniques are typically used with linear, circular, or planar arrays but some approaches are applicable to any array geometry. We will consider techniques for forming fixed beams and for scanning directional beams as well as adaptive techniques that can be used to reject interfering signals.

Array beams can be formed or scanned using either phase shift or time delay systems. Each has distinct advantages and disadvantages. While both approaches can be used for other geometries, the following discussion refers to equally spaced linear arrays

such as those shown in Fig. 3-2. In the case of phase scanning the interelement phase shift  $\alpha$  is varied to scan the beam. For time scanning the interelement delay  $\Delta t$  is varied.



**Figure 3-2.** (a) a phase scanned linear array (b) a time-scanned linear array

### 3.2.1 Phase scanning

Beam forming by phase shifting can be accomplished using ferrite phase shifters at RF or IF. Phase shifting can also be done in digital signal processing at baseband. For an  $M$ -element equally spaced linear array that uses variable amplitude element excitations and phase scanning the array factor is given by [3.1]

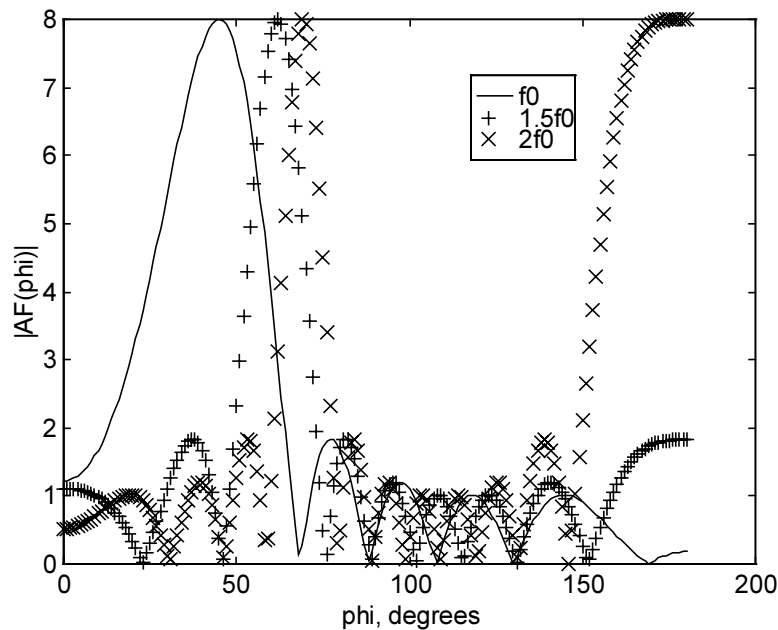
$$AF(\phi) = \sum_{m=0}^{M-1} A_m e^{jm\left(\frac{2\pi d}{\lambda} \cos\phi + \alpha\right)} \quad (3.5)$$

where the array lies on the x-axis with the first element at the origin. The interelement phase shift is

$$\alpha = -\frac{2\pi d}{\lambda_0} \cos\phi_0 \quad (3.6)$$

and  $\lambda_0$  is the wavelength at the design frequency and  $\phi$  is the desired beam direction. At a wavelength of  $\lambda_0$  the phase shift  $\alpha$  corresponds to a time delay that will steer the beam to  $\phi$ .

In narrow band operation, phase scanning is equivalent to time scanning, but phase scanned arrays are not suitable for broad band operation. The electrical spacing ( $d/\lambda$ ) between array elements increases with frequency. At different frequencies, the same interelement phase shift corresponds to different time delays and therefore different angles of wave propagation, so using the same phase shifts across the band causes the beam direction to vary with frequency. This effect is shown in Fig 3-3. This beam squinting becomes a problem as frequency is increased, even before grating lobes start to form.



**Figure 3-3.** Array factor of 8-element phase-scanned linear array computed for three frequencies ( $f_0$ ,  $1.5f_0$ , and  $2f_0$ ), with  $d=0.37\lambda$  at  $f_0$ , designed to steer the beam to  $\phi=45^\circ$  at  $f_0$ .

### 3.2.2 Time scanning

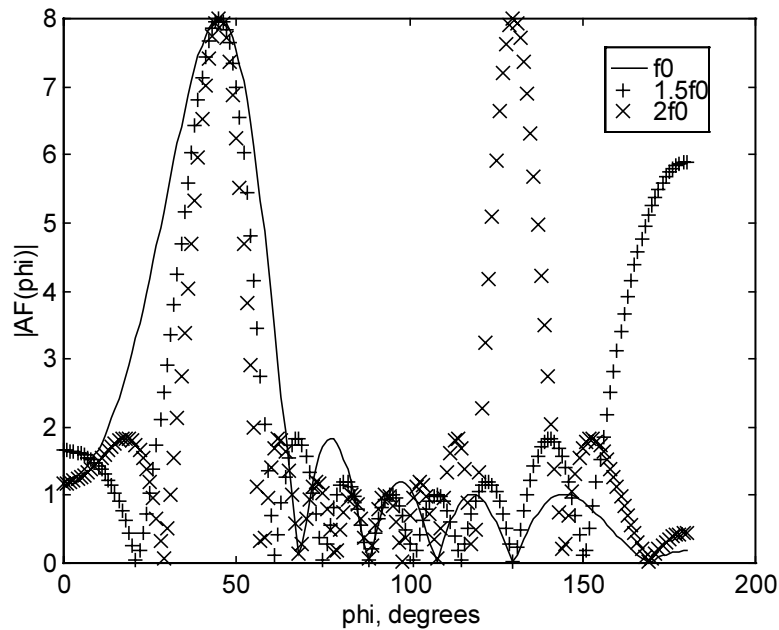
Systems using time delays are preferred for broadband operation because the direction of the main beam does not change with frequency. The array factor of a time-scanned equally spaced linear array is given by

$$AF(\phi) = \sum_{m=0}^{M-1} A_m e^{jm\left(\frac{2\pi d}{\lambda} \cos\phi + \omega\Delta t\right)} \quad (3.7)$$

where the interelement time delay is given by

$$\Delta t = -\frac{d}{c} \cos\phi_0 \quad (3.8)$$

Time delays are introduced by switching in transmission lines of varying lengths. The transmission lines occupy more space than phase shifters. As with phase shifting, time delays can be introduced at RF or IF and are varied in discrete increments. Time scanning works well over a broad bandwidth, but the bandwidth of a time scanning array is limited by the bandwidth and spacing of the elements. As the frequency of operation is increased, the electrical spacing between the elements increases. The beams will be somewhat narrower at higher frequencies, and as the frequency is increased further, grating lobes appear. These effects are shown in Fig. 3-4.

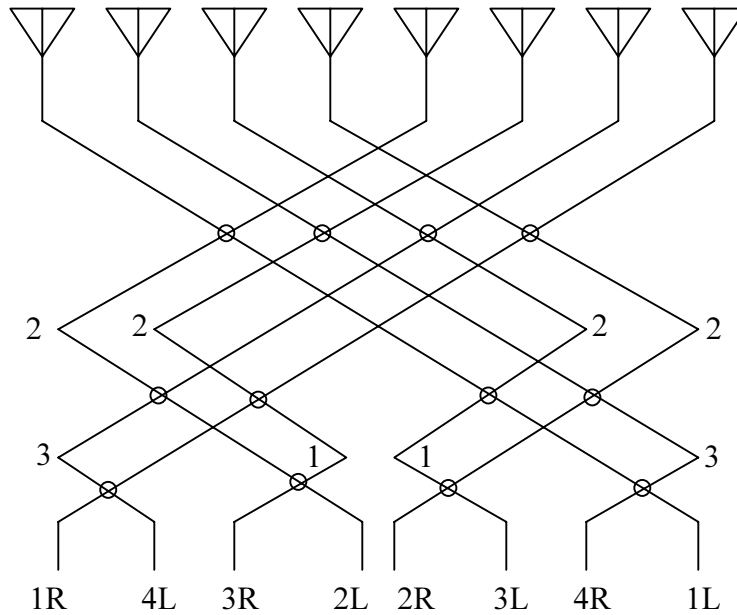


**Figure 3-4** Array factor of 8-element time-scanned linear array computed for three frequencies ( $f_0$ ,  $1.5f_0$ , and  $2f_0$ ), with  $d=0.37\lambda$  at  $f_0$ , designed to steer the beam to  $\phi_0=45^\circ$  at  $f_0$ .

### 3.3 Fixed Beam Forming Techniques

Some array applications require several fixed beams that cover an angular sector. Several beam forming techniques exist that provide these fixed beams. Three examples are given here.

**3.3.1 Butler matrix** The Butler matrix [3.2] is a beam forming network that uses a combination of  $90^\circ$  hybrids and phase shifters. An  $8 \times 8$  Butler matrix is shown in Fig 3-5. The Butler matrix performs a spatial fast Fourier transform and provides  $2^n$  orthogonal beams. These beams are linearly independent combinations of the array element patterns.



**Figure 3-5.** An 8x8 Butler matrix feeding an 8-element array. Circles are 90° hybrids and numbers are phase shifts in units of  $\pi/8$

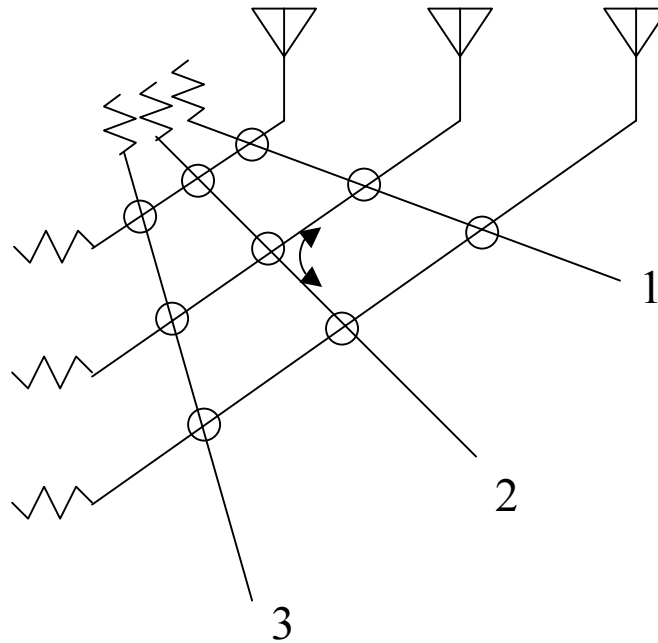
When used with a linear array the Butler matrix produces beams that overlap at about 3.9 dB below the beam maxima. A Butler matrix-fed array can cover a sector of up to 360° depending on element patterns and spacing. Each beam can be used by a dedicated transmitter and/or receiver, or a single transmitter and/or receiver can be used, and the appropriate beam can be selected using an RF switch. A Butler matrix can also be used to steer the beam of a circular array by exciting the Butler matrix beam ports with amplitude and phase weighted inputs followed by a variable uniform phase taper.

### 3.3.2 Blass Matrix

The Blass matrix [3.3] uses transmission lines and directional couplers to form beams by means of time delays and thus is suitable for broadband operation. Figure 3-6 shows an example for a 3-element array, but a Blass matrix can be designed for use with any number of elements. Port 2 provides equal delays to all elements, resulting in a broadside beam. The other two ports provide progressive time delays between elements and produce beams that are off broadside. The Blass matrix is lossy because of the resistive terminations. In one recent application [3.4] a three-element array fed by a



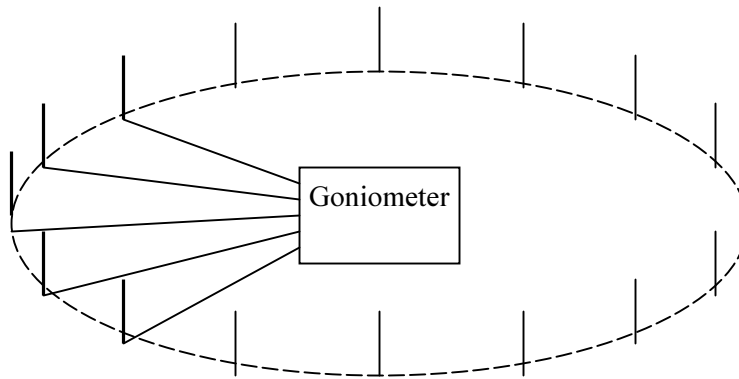
Blass matrix was tested for use in an antenna pattern diversity system for a hand held radio. The matrix was optimized to obtain nearly orthogonal beams.



**Figure 3-6.** A Blass matrix (The circles are directional couplers.)

### 3.3.3 Wullenweber Array

A Wullenweber array [3.5] is a circular array developed for direction finding at HF frequencies. An example is shown in Fig. 3-7. The array can use either omnidirectional elements or directional elements that are oriented radially outward. The array typically consists of 30 to 100 evenly spaced elements. About a third of the elements are used at a time to form a beam that is oriented radially outward from the array. A switching network called a goniometer is used to connect the appropriate elements to the radio, and may include some amplitude weighting to control the array pattern. Advantages of the Wullenweber array are the ability to scan over  $360^\circ$  with very little change in pattern characteristics. At lower frequencies the Wullenweber array is much smaller than the rhombic antennas that might be used otherwise. Time delays are used to form beams radial to the array, enabling broad band operation. The bandwidth of a Wullenweber array is limited by the bandwidth and spacing of the elements.



**Figure 3-7.** A Wullenweber array [3.5]

### 3.3.4 Other fixed beam forming techniques

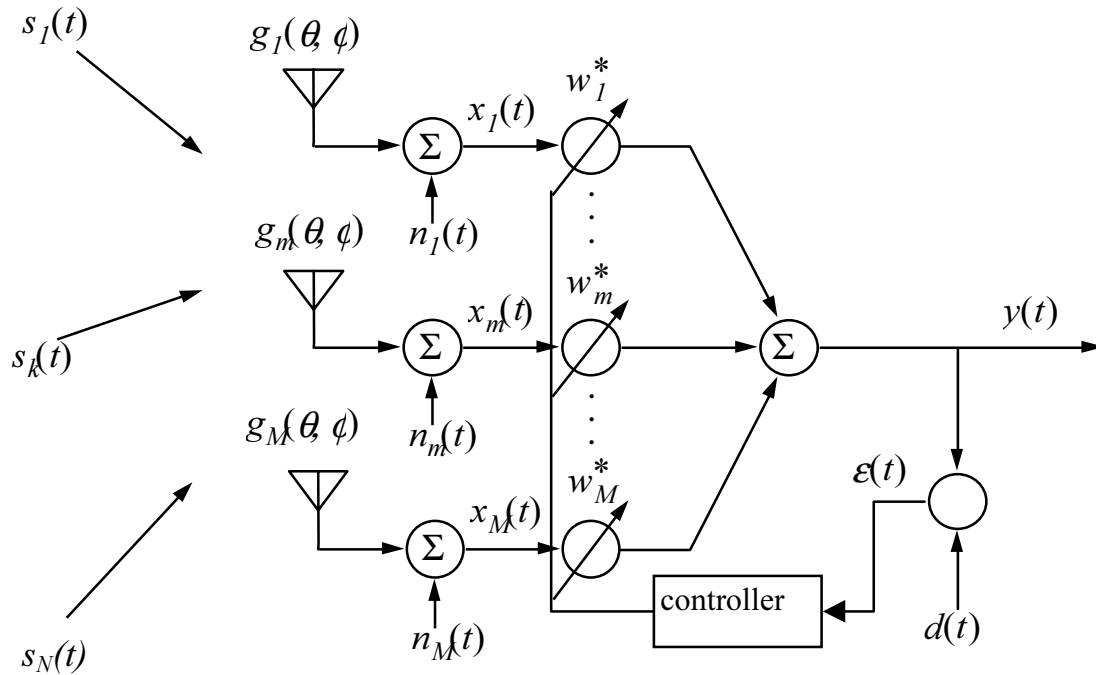
Fixed beams can also be formed using lens antennas such as the Luneberg lens or Rotman lens with multiple feeds. Lenses focus energy radiated by feed antennas that are less directive. Lenses can be made from dielectric materials or implemented as space-fed arrays. Multi-beam arrays can be used to feed reflector antennas as well.

### 3.4 Optimum Beamforming

Complex weights for each element of the array can be calculated to optimize some property of the received signal. This does not always result in an array pattern having a beam maximum in the direction of the desired signal but does yield the optimal array output signal. Most often this is accomplished by forming nulls in the directions of interfering signals. Adaptive beamforming is an iterative approximation of optimum beamforming.

A general array with variable element weights is shown in block-diagram form in Fig. 3-8. The output of the array  $y(t)$  is the weighted sum of the received signals  $s_i(t)$  at the array elements having patterns  $g_m(\theta, \phi)$  (the patterns include gain) and the thermal noise  $n(t)$  from receivers connected to each element. In the case shown,  $s_1(t)$  is the desired signal, and the remaining  $L$  signals are considered to be interferers. In an adaptive system, the weights  $w_m$  are iteratively determined based on the array output  $y(t)$ , a reference signal  $d(t)$  which approximates the desired signal, and previous weights. The

reference signal is assumed to be identical to the desired signal. In practice this can be achieved or approximated using a training or synchronization sequence or a CDMA spreading code, which is known at the receiver.



**Figure 3-8.** An adaptive antenna array

Here we will find the optimum weights that minimize the mean squared error  $\epsilon(t)$  between the array output and the reference signal. A desired signal  $s_1(t)$ ,  $L$  interfering signals, and additive white gaussian noise are considered in the derivation. Rather than the usual implicit assumption of isotropic elements, general directional element patterns are considered. The element patterns need not be the same for all elements.

The array output is given by

$$y(t) = w^H x(t) \quad (3.9)$$

where  $w^H$  denotes the complex conjugate transpose of the weight vector  $w$ .

### 3.4.1 Array response vector

The *array response vector* for a signal with direction of arrival  $(\theta, \phi)$  and polarization state  $P$  can be written as follows

$$a(\theta, \phi, P) = \begin{bmatrix} e^{j\zeta_1} g_1(\theta, \phi, P) \\ e^{j\zeta_2} g_2(\theta, \phi, P) \\ \vdots \\ e^{j\zeta_M} g_M(\theta, \phi, P) \end{bmatrix} \quad (3.10)$$

The phase shifts  $\zeta_m$  represent the spatial phase delay of an incoming plane wave arriving from angle  $(\theta, \phi)$ . The factor  $g_m(\theta, \phi, P)$  is the antenna pattern of the  $m^{\text{th}}$  element.

### 3.4.2 Spatial-polarization signature

The *spatial-polarization signature* is the total response of the array to a signal with  $N$  multipath components and is expressed as

$$v = \sum_{n=1}^N \alpha_n a(\theta_n, \phi_n, P_n) \quad (3.11)$$

where  $\alpha_n$  is the amplitude and phase of the  $n^{\text{th}}$  component. The angle of arrival and polarization state of the  $n^{\text{th}}$  component are given by  $\theta_n$ ,  $\phi_n$ , and  $P_n$ .

### 3.4.3 Spatial-polarization signature matrix

The response of the array to multiple signals (in this case a desired signal and  $L$  interfering signals) can be written using a spatial-polarization signature matrix. The columns of the matrix are the spatial-polarization signatures of the individual signals. The matrix is written as

$$U = [v_1 \mid v_2 \mid \cdots \mid v_{L+1}] \\ = [U_d \mid U_i] \quad (3.12)$$

where  $U_d$  is the response to the desired signal  $s_1(t)$  and  $U_i$  is the response to the interfering signals.

The output of the  $M$  receivers prior to weighting is

$$x(t) = Us(t) + n(t) \quad (3.13)$$

### 3.4.4 Signals and noise

The incident signals (excluding direction of arrival and polarization information) are given by

$$s(t) = [s_1(t) \quad s_2(t) \quad \cdots \quad s_{L+1}(t)]^T = [s_d(t) \quad | \quad s_i(t)]^T \quad (3.14)$$

where  $s_d(t)=s_1(t)$  is the desired signal and  $s_i(t)$  consists of the remaining, interfering signals. In this case all signals are considered to be uncorrelated and to have the form  $s_k(t) = \sqrt{S_k} u_k(t) e^{j\omega_0 t}$  where  $\sqrt{S_k}$  is the amplitude of the signal and  $u_k(t)$  is a normalized baseband modulating signal. The noise in the  $M$  receivers is given by

$$n(t) = [n_1(t) \quad n_2(t) \quad \cdots \quad n_M(t)]^T \quad (3.15)$$

and the noise in different receiver branches is uncorrelated.

### 3.4.5 Optimum weights

To optimize the element weights, we seek to minimize the mean squared error between the array output and the reference signal  $d(t)$ . Optimizing SINR will lead to weights that differ by a scalar multiplier from the weights shown here [3.6]. The derivation proceeds as for the case of omnidirectional elements, and the solution for the optimum weights is

$$w_{opt} = R_{xx}^{-1} r_{xd} \quad (3.16)$$

where  $R_{xx}=x(t)x^H(t)$  is the signal covariance matrix and  $r_{xd}=d^*(t)x(t)$ . This is the same as the expression for the optimum weights for an array with isotropic elements (see [3.6]). In this case, however,  $R_{xx}$ ,  $r_{xd}$ , and hence  $w_{opt}$  are functions of the angles of arrival of the  $L+1$  signals, and of the element patterns.

### 3.5 Adaptive Algorithms

Adaptive beamforming algorithms iteratively approximate these optimum weights. Adaptive beamforming began with the work of Howells [3.7] and Applebaum [3.8]. Since then many beamforming algorithms have been developed. Several algorithms are briefly described below. This closely follows the discussion in [3.6].

#### 3.5.1 Least mean squares (LMS)

This algorithm uses a steepest-descent method and computes the weight vector recursively using the equation

$$w(n+1) = w(n) + \mu x(n) [d^*(n) - x^H(n)w(n)] \quad (3.17)$$

where  $\mu$  is a gain constant and controls the rate of adaptation. The LMS algorithm requires knowledge of the desired signal. This can be done in a digital system by periodically transmitting a training sequence that is known to the receiver, or using the spreading code in the case of a direct-sequence CDMA system. This algorithm converges slowly if the eigenvector spread of  $R_{xx}$  is large.

#### 3.5.2 Direct sample covariance matrix inversion (DMI)

In this algorithm (3.14) is used to obtain the weights, but with  $R_{xx}$  and  $r_{xd}$  estimated from data sampled over a finite interval. The estimates are given by

$$\hat{R}_{xx} = \sum_{i=N_1}^{N_2} x(i)x^H(i) \quad (3.18)$$

and

$$\hat{r}_{xd} = \sum_{i=N_1}^{N_2} d^*(i)x(i) \quad (3.19)$$

The DMI algorithm converges more rapidly than the LMS algorithm but it is more computationally complex. The DMI algorithm also requires a reference signal.

### Recursive least squares (RLS) algorithm

The RLS algorithm estimates  $R_{xx}$  and  $r_{xd}$  using weighted sums so that

$$\tilde{R}_{xx} = \sum_{i=1}^N \gamma^{n-1} x(i)x^H(i) \quad (3.20)$$

and

$$\tilde{r}_{xd} = \sum_{i=1}^N \gamma^{n-1} d^*(i)x(i) \quad (3.21)$$

The inverse of the covariance matrix can be obtained recursively, and this leads to the update equation

$$\hat{w}(n) = \hat{w}(n-1) + q(n)[d^*(n) - \hat{w}^H(n-1)x(n)] \quad (3.22)$$

where

$$q(n) = \frac{\gamma^{-1} R_{xx}^{-1}(n-1)x(n)}{1 + \gamma^{-1} x^H(n) R_{xx}^{-1}(n-1)x(n)} \quad (3.23)$$

and

$$R_{xx}^{-1}(n) = \gamma^{-1} [R_{xx}^{-1}(n-1) - q(n)x(n)R_{xx}^{-1}(n-1)] \quad (3.24)$$

The RLS algorithm converges about an order of magnitude faster than the LMS algorithm if SINR is high. It requires an initial estimate of  $R_{xx}^{-1}$  and a reference signal.

### 3.5.3 Decision directed algorithms

In decision-directed algorithms, the weights can be updated using any of the above techniques, but the reference signal is obtained by demodulating  $y(t)$ . This means that no external reference is required, but convergence is not guaranteed because  $y(t)$  may not correspond to  $d(t)$ .

### 3.5.4 Constant modulus algorithm (CMA)

The constant modulus algorithm is a blind adaptive algorithm proposed by Goddard [3.9] and by Treichler and Agee [3.10]. That is, it requires no previous

knowledge of the desired signal. Instead it exploits the constant or nearly constant-amplitude properties of most modulation formats used in wireless communication. By forcing the received signal to have a constant amplitude, CMA recovers the desired signal. The weight update equation is given by

$$w(n+1)=w(n)-\mu x(n)\varepsilon^*(n) \quad (3.25)$$

where

$$\varepsilon(n)=[1-|y(n)|^2]y(n)x(n) \quad (3.26)$$

When the CMA algorithm converges, it converges to the optimal solution, but convergence of this algorithm is not guaranteed because the cost function  $\varepsilon$  is not convex and may have false minima. [3.6] Another potential problem is that if there is more than one strong signal, the algorithm may acquire an undesired signal. This problem can be overcome if additional information about the desired signal is available. Variations of CMA exist that use different cost functions.

The least-squares CMA (LSCMA) is a variation of CMA that uses a direct matrix inversion. The weights are calculated as follows:

$$w = R_{xx}^{-1}r_{xd} \quad (3.27)$$

where  $R_{xx}$  and  $r_{xd}$  are as described in Section 3.4.5 except that a constant-modulus estimate of the desired signal given by  $d = \frac{y}{|y|}$  is used.

Multitarget versions of CMA use a Graham-Schmidt orthogonalization process to produce two or more orthogonal sets of weights. A multitarget CMA algorithm can separate a number of signals equal to the number of array elements. Soft orthogonalization [3.11] or hard orthogonalization [3.12] can be used. Hard orthogonalization is described here. Initially, for an  $N$ -element array,  $N$  orthogonal weight vectors are used. Each weight vector is updated independently using the CMA as in (3.25) or (3.27). All but the first weight vector are periodically reinitialized as follows to prevent more than one weight vector from converging to the same value.

$$w^k = w^k - \sum_{i=1}^{k-1} \frac{w^{iH} R_{xx} w^k}{w^{iH} R_{xx} w^i} w^i, \quad k=2, 3, \dots, M \quad (3.28)$$



### **3.5.6 Other techniques**

Other adaptive beamforming approaches include spectral self-coherence restoral (SCORE) a blind adaptive algorithm that uses the cyclostationary property of a signal. Neural networks and maximum likelihood sequence estimators can also be used to perform adaptive beamforming. In partially adaptive arrays, only some of the elements are weighted adaptively. This technique is useful for large arrays. Partial adaptivity allows an array to cancel interfering signals but requires less computational complexity than adapting all the element weights.

**Table 3-1** Summary of adaptive beamforming algorithms

Algorithm	Weight update equations	Advantages	Disadvantages
<b>Least mean squares (LMS)</b>	$w(n+1) = w(n) + \mu x(n)[d^*(n) - x^H(n)w(n)]$	Always converges	Requires reference signal
<b>Direct matrix inversion (DMI)</b>	$\hat{R}_{xx} = \sum_{i=N_1}^{N_2} x(i)x^H(i)$ $\hat{r}_{xd} = \sum_{i=N_1}^{N_2} d^*(i)x(i)$ $w = \hat{R}_{xx}^{-1} \hat{r}_{xd}$	Always converges  Faster than LMS	Requires reference signal Computationally complex
<b>Recursive least squares (RLS)</b>	$\hat{w}(n) = \hat{w}(n-1) + q(n)[d^*(n) - \hat{w}^H(n-1)x(n)]$ $q(n) = \frac{\gamma^{-1} R_{xx}^{-1}(n-1)x(n)}{1 + \gamma^{-1} x^H(n) R_{xx}^{-1}(n-1)x(n)}$ $R_{xx}^{-1}(n) = \gamma^{-1} [R_{xx}^{-1}(n-1) - q(n)x(n)R_{xx}^{-1}(n-1)]$	Always converges, ~10 x faster than LMS	Requires reference signal and initial estimate of $R_{xx}^{-1}$
<b>Constant modulus algorithm (CMA)</b>	$w(n+1) = w(n) - \mu x(n)\varepsilon^*(n)$ $\varepsilon(n) = [1 -  y(n) ^2]y(n)x(n)$	Does not require reference signal	Theoretically may not converge

### 3.6 Effect of Array Geometry and Element Patterns

Most literature on adaptive beamforming considers linear arrays of omnidirectional elements. Other geometries and element patterns are possible, and these factors influence array performance. Ishide and Compton [3.13], and later Compton [3.14] showed that using elements with directional patterns can introduce grating nulls in patterns. More recently, Liang and Paulraj performed computer simulations to compare the diversity performance of several array configurations in a multipath channel [3.15]. Transmission from a single mobile to a base station was considered. The simulation used 20 multipath components, with the directions of arrival uniformly distributed over an arc of 10 to 60 degrees. All multipath components had equal power. The array configurations included the following: (a) single circular array with  $d=0.5\lambda$  element spacing; (b) single circular array with large inter-element spacing that was varied to maintain a  $5\lambda$  diameter regardless of the number of elements; (c) single small ( $d=0.5\lambda$ ) circular array plus one sensor; (d) double small ( $d=0.5\lambda$ ) circular arrays spaced  $5\lambda$  apart; (e) single linear array ( $d=0.5\lambda$ ); and (f) double linear arrays (two colinear arrays, each with  $d=0.5\lambda$ , spaced  $7.5\lambda$  apart). The circular arrays used cardioid element patterns, with the element pattern maxima oriented radially outward. The linear arrays used omnidirectional elements. The range extension that could be obtained with the six topologies, using equal numbers of elements, with maximal ratio combining, was compared. The number of elements considered ranged from 2 to about 22. The dual circular arrays (d) performed better than the small circular array (a) for small multipath angle spreads. For larger angle spreads the large circular array (b) performed slightly better than (a) or (d). The single circular array plus one sensor (c) also performed better than the small circular array and was considered to be more suitable for downlink beamforming than (b) and (d), since the small circular array could be used with the extra sensor for diversity on the uplink but without the extra sensor for downlink beamforming. Because of the small interelement spacing in the circular array in configuration (c) there would not be severe grating lobes in the downlink pattern. The performance of the two linear array configurations (e) and (f) was very similar and slightly better than that of the circular array configurations. This may have been due to the use of omnidirectional element patterns as much as the array geometry. For all arrays compared, the differences

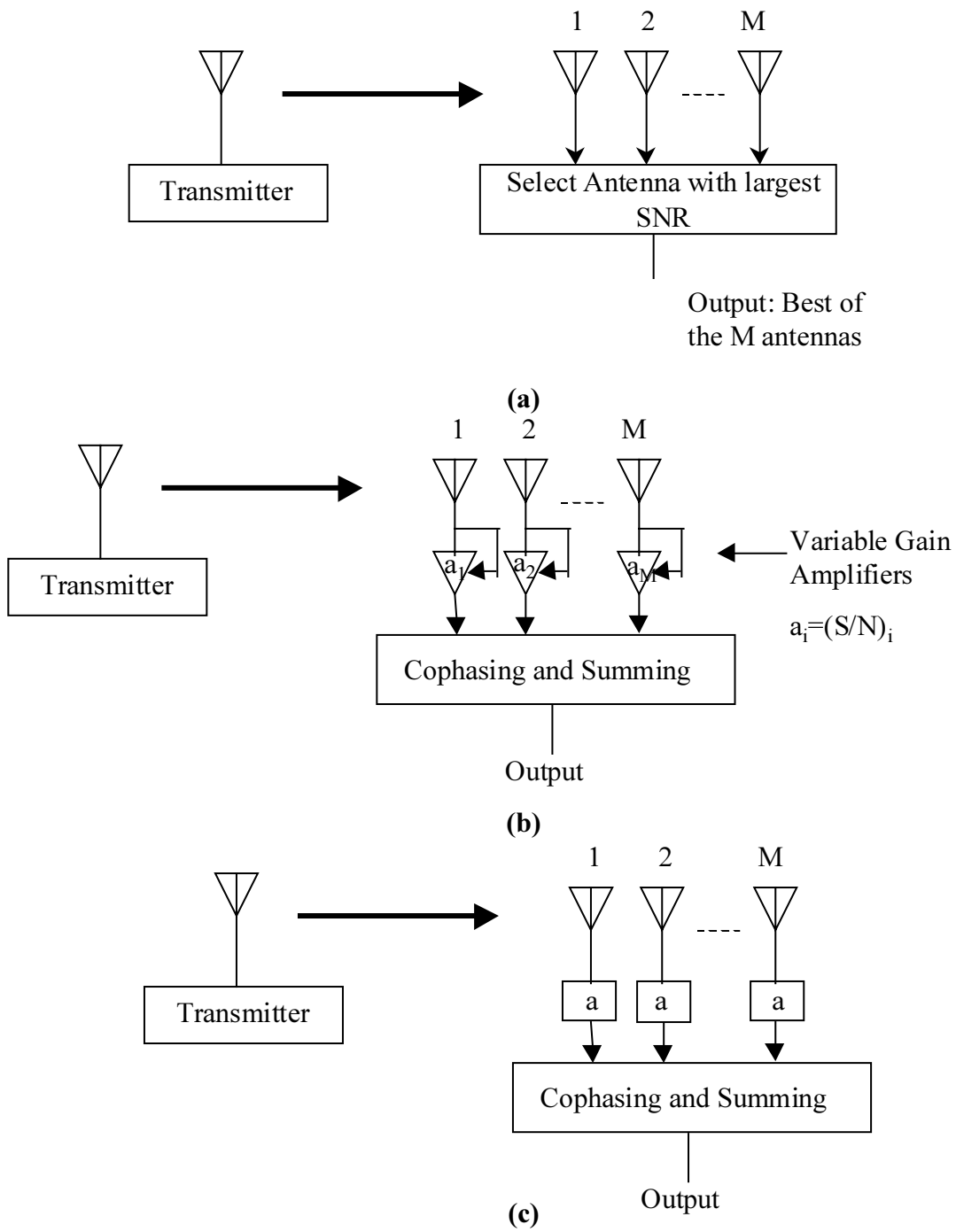
between configurations decreased with increasing multipath angle spread and increasing number of elements.

Computer modeling was used in [3.16] to compare triangular, square, and cylindrical array configurations using directional elements. The arrays were evaluated in a TDMA system that resembled GSM/DCS 1800 with the addition of spatial division multiple access (SDMA) which allows a frequency to be reused even in the same cell if the spatial separation between users is sufficient. "Spatial reference" algorithms that compute direction of arrival for desired and interfering signals and then form directional beams were used. A midamble data sequence used for equalizer training was used as an identification code for each signal. Only the uplink was considered. Triangular and square arrays provided similar SDMA capabilities, but the circular array performed very poorly. This is because the UCA-ESPRIT algorithm used for the circular array did not take into account the directional antenna patterns caused by the supporting mast. In contrast, the unitary ESPRIT direction finding algorithm used with the planar array faces of the triangular and square arrays performed despite the directional elements used in those arrays. This is likely because the element patterns in the planar arrays are aligned and could be factored out of the array pattern.

### **3.7 Diversity Combining [3.17],[3.18]**

In addition to phased and adaptive arrays, signals from multiple antennas can be combined to improve performance in fading channels. Figure 3-9 depicts the block diagrams of three diversity combining techniques. Selection diversity, shown in Fig. 3-9 (a), is the simplest of these methods. From a collection of  $M$  antennas the branch with the largest signal to noise ratio at any time is selected and connected to the receiver. As one would expect, the larger the value of  $M$  the higher the probability of having a larger signal to noise ratio (SNR) at the output. Maximal ratio combining takes a better advantage of all the diversity branches in the system. Fig. 3-9 (b) shows this configuration where all  $M$  branches are weighted with their respective instantaneous signal voltage to noise ratios. The branches are then cophased prior to summing in order to ensure that all branches are added in phase for maximum diversity gain. The summed signals are then used as the received signal. Maximal ratio has advantages over selection

diversity but is more complicated; proper care has to be taken in order to ensure that signals are cophased correctly and gain coefficients have to be constantly updated. A variation of maximal ratio combining is equal gain combining (see Fig. 3-9 (c)). In this scheme the gains of the branches are all set to the same value and are not changed thereafter. As with the previous case, the output is a cophased sum of all the branches.



**Figure 3-9.** Diversity combining techniques [3.18]: (a) selection diversity, (b) maximal-ratio combining, (c) equal-gain combining

### 3.8 Conclusion

This chapter has addressed several aspects of antenna arrays and beamforming. These include the pattern of an array with arbitrary geometry and elements, phase- and time-scanned arrays, and fixed-beam forming techniques. Optimum beamforming and adaptive algorithms are also discussed.

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