GEOMETRIC MODELING OF MANUFACTURING PROCESSES VARIATIONS FOR MODEL-BASED TOLERANCE ANALYSIS

by

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(ABSTRACT)

In product design, tolerances are specified due to the inherent variabilities of manufacturing processes. Tolerance specifications have significant implications on the quality and cost of the product. For proper tolerance specification, tolerance analysis must be performed. Prototyping is the the only method available for the analysis of the product geometric variations. For the automation of the analysis procedure, the part tolerance information must be represented in a format suitable for computer interpretation. Previously proposed tolerance representation schemes have suffered either from inadequate variational coverage or departure from the established ANSI tolerancing standards.

Toward this end, a tolerance representation scheme capable of modeling the range of tolerances defined in the ANSI Y14.5 standard in a format suitable for automated tolerance analysis has been proposed. One unique feature of this representation scheme is the use of B-splines for the modeling of form variations. The representation scheme can also take into account the distribution characteristics of the manufacturing processes used to enable

statistical tolerance analysis. To provide an accurate characterization of the variational form characteristics of the manufactured part features, the use of process capability templates was introduced.

For assembly tolerance analysis, a relative positioning scheme capable of modeling the interaction between mating splines was developed to propagate the individual part variations within the assembly. This enabled the tolerance stackup on the assembly design function(s) to be computed automatically without the need to formulate any tolerance functions. A prototype software, written in the C++ programming language and running from within CATIA, has been developed to demonstrate the integration of the above concepts.

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CHAPTER 1
INTRODUCTION

In mechanical design, tolerances are specified due to the inherent variabilities of manufacturing processes. The geometry of the manufactured components may deviate in size, shape, and form from the ideal. Tolerance specifications have significant implications on the quality and cost of the product. While tight tolerances can assure the product quality, they generally lead to higher manufacturing cost, manifested in additional manufacturing operations, scraps, inspection, and processing time. To ensure that the proper tolerance values which take into account the product functional requirements as well as manufacturability considerations are specified, tolerance analysis must be performed.

The need for tolerance analysis is especially prevalent in assemblies where some assembly features are more critical to the functioning of the product than others. An example of a critical design feature is the clearance gap DF in the gear box assembly shown in Figure 1.1 [Bjorke 1978]. In order for the gear assembly to function properly, DF must be larger than zero to prevent jamming, and smaller than a specified value to prevent axial motions of the gears. Typical of design features, this gap is not a manufactured feature, i.e., the actual size and shape of this gap is not directly controllable in manufacturing. Rather it is an aggregate property of the assembly which result from the interaction between the mating features of the components when assembled. The size of the gap DF can be expressed in terms of the component dimensions X1 through X5 as shown in equation (1.1),

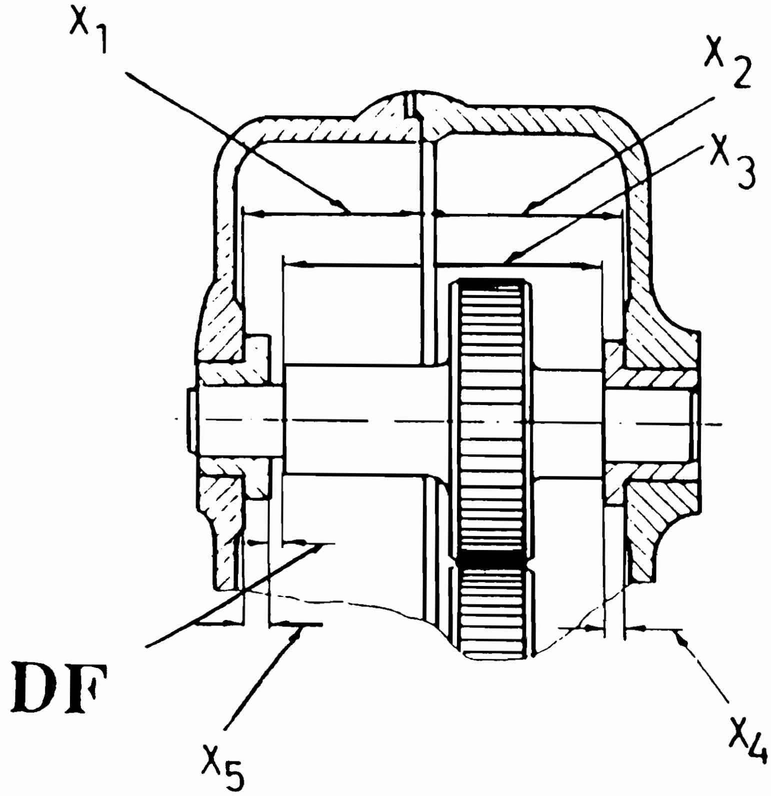


Figure 1.1 Design Function [Bjorke 1978]

$$DF = X1 + X2 - X3 - X4 - X5 \quad (1.1)$$

and the variation or tolerance of DF is the sum of the variations or tolerances of the component dimensions X1 through X5, regardless of whether the component dimensions are added or subtracted (refer to equation (1.2)).

$$Tol_{DF} = Tol_{X1} + Tol_{X2} + Tol_{X3} + Tol_{X4} + Tol_{X5} \quad (1.2)$$

Based on either experience or adopted practices, the product designer will assign appropriate tolerance values to DF. However, as far as the function of the assembly is concerned, tolerances can be assigned arbitrarily to the component dimensions X1 through X5 as long as the variations of DF do not violate the design requirements. Tolerance analysis can thus ensure the product function while allowing the widest allowable tolerances to be assigned to the component dimensions/features for economic production.

1.1 Types of Tolerances

The permissible product variations can be expressed in the form of conventional and geometric tolerances. Conventional tolerances allow the designer to specify the desirable surface condition and size of a part feature within an upper and a lower limit. The advantages of this tolerancing method are that it is simple to use and part conformance can be easily verified using direct measurement tools such as a caliper and a surface indicator. This practice, however, is lacking in many respects [Requicha 1977]. Form tolerances and other geometric constraints needed to express more complex functional and/or assembly requirements are not supported. For example, even though the shaft shown in Figure 1.2 satisfies the part size tolerance specification, it is uncertain if it will be able to fulfill its functions satisfactorily.

Geometric tolerances such as form, orientation, and true position are used when conventional tolerances alone cannot guarantee the functionality of the products designed. These tolerances constrain the part features within regions of space called tolerance zones. The use of explicit datum and functional constraints allow the designer to better convey the design intent in a clear and unambiguous manner. In the United States, the currently acceptable industrial practices of geometric tolerancing for mechanical design are embodied in the Y14.5M-1982 ANSI Standard [ASME 1983].

1.2 Tolerance Analysis

The procedure of tolerance analysis can be performed either on a worst case (WC) or statistical basis. In a WC analysis, each component's variations are assumed to be at their extreme limits. This invariably leads to tighter than necessary component tolerances [Chase & Greenwood 1988]. In a statistical analysis, the very low probability of the occurrence of the WC condition is taken into account. Wider component tolerances can be used, resulting in lower manufacturing costs.

1.2.1 Analysis Of Conventional Tolerances

In an assembly involving two mating parts, such as between a hole and a shaft, the type of fit desired is determined based on the intended product

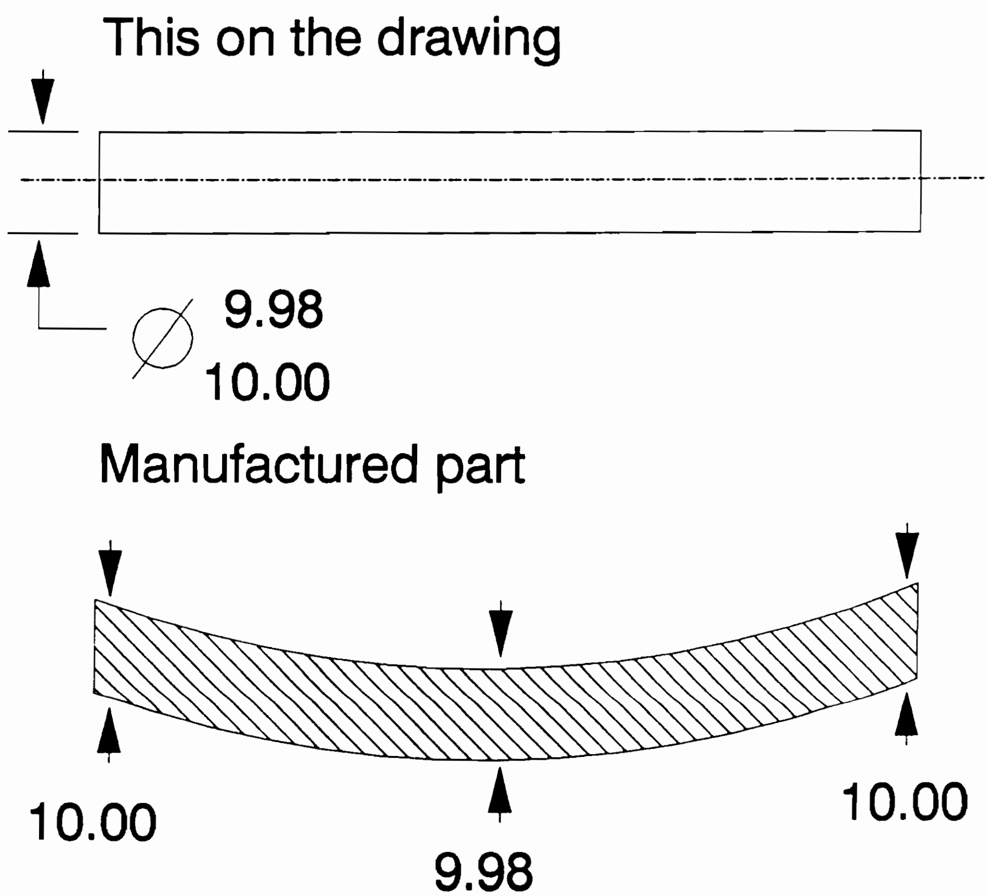


Figure 1.2 Ambiguity of Conventional Tolerance

function. If rotational or sliding motions are desired, for instance, a clearance fit would be used. The nominal sizes of the hole and the shaft are then calculated using either the Basic Hole or Basic Shaft Systems (BHS/BSS) [Zeid 1991]. Utilizing the BHS, the nominal hole size is the Basic size and the size of the shaft is adjusted to achieve the desired fit. Utilizing the BSS, the size of the shaft is the Basic size. In either case, the same tolerance value (a function of Basic size) is applied to both the shaft and the hole. From a manufacturing, and ultimately a cost standpoint, this is a poor practice since variations to internal features are much harder to control as compared to those of the external types. The same desired fit can be achieved at significant cost savings if such a simple manufacturing consideration is incorporated into the design.

For assemblies consisting of three or more components, the tolerance chains relating the variations of the assembly design functions (DF) to the variations of the components making the assembly must be identified (see equation (2)). The most widely used method for the formulation of the tolerance chains is the manual inspection of the assembly drawing [Bjorke 1978]. In practical problems, however, the design function is usually a complicated non-linear function of many dimensions whose derivation is usually a difficult analytical geometry problem [Martino & Gabriele 1989]. In addition, a design may have to satisfy several functional requirements simultaneously. Consequently, several, and usually interrelated, tolerance chains will need to be formulated and solved. For the simple mechanism shown in Figure 1.3, twelve design functions have to be formulated just to ensure that the holes in the handle and the T-shaped sliders line up closely enough for pins to be inserted through them [Martino & Gabriele 1989]. Methods for the automatic derivation

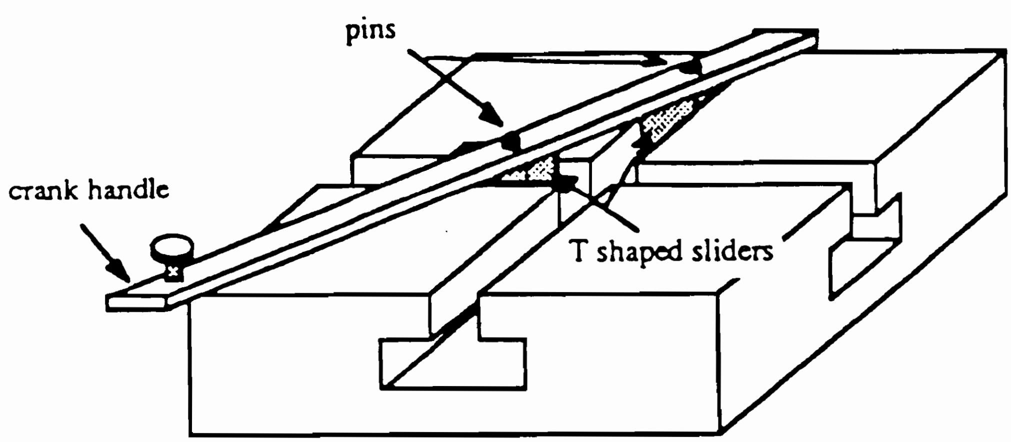


Figure 1.3 T-Shaped Slider Assembly [Martino & Gabriele 1989]

of the relevant tolerance chains have been proposed by several researchers [Chase et al 1989] [Treacy et al 1991]. Therefore, to a certain extent, the problem of conventional tolerance analysis has been resolved.

1.2.2 Analysis Of Geometric Tolerances

The problem of geometric tolerance analysis, on the other hand, remains. Geometric tolerances cannot be incorporated into the above conventional tolerance analysis method since they are associated with part features (i.e., surfaces) which have no associated dimensions. No manual analytical methods exist for the analysis of geometric tolerances.

The common procedures used for analyzing both the conventional and geometric variability of assemblies are prototyping and failure mode analysis. Prototyping, however, is a costly and time consuming trial and error procedure [Weiss et al 1990]. The product time to market constraint will often prohibit a thorough verification of all the potential variations in the product design. In addition, the machines used for prototyping are usually not the same machines used in the actual production of the product. Consequently, the shape and characteristics of the parts manufactured will be different than those of the prototypes.

In failure mode analysis, causes of product failure in the field are analyzed and traced to identify and correct problems in the product design or manufacturing. Although this is a good practice for product improvement, it is a very poor means for design analysis.

1.3 Previous Geometric Tolerance Representation Schemes

Since tolerances have no direct significance on the component geometry, they have been almost exclusively regarded as annotation on part prints. Historically, this has proved acceptable since production methods have been based on the information originating from design in the form of paper production drawings. In current commercial Computer Aided Design (CAD) systems, tolerance information is still represented in the form of notes, symbols, and labels similar to what is normally found on engineering drawings. Very limited computer interpretation of tolerances is supported by these systems.

In the effort to automate geometric tolerance analysis, several researchers have attempted to geometrically model the allowable parts variations, assemble them, and query the relevant design functions from this assembly model [Turner 1987] [Scott & Gabriele 1989]. To enable computer modeling and analysis of the allowable parts variations, the part tolerance information must be represented in a format suitable for computer interpretation [Requicha 1983]. Previous attempts to provide a computer representation of the tolerance information can be classified into parametric and non-parametric approaches.

1.3.1 Parametric Approaches

In the parametric approaches, the part tolerances are expressed as variations to the parameters defining the nominal part geometry. A rectangle, for instance, can be parametrized based on its dimensions (length and width), or the positions of its vertices. Parametrization based on the object dimensions closely relates to conventional tolerances. The variations coverage of this representation is very limited because only perfect rectangles of different sizes can be represented (see Figure 1.4). Parametrization based on the object vertices positions allows the modeling of a larger class of variational objects (see Figure 1.5). By allowing each part vertex to vary independently, rectangles with skewed sides can be modeled. However, the edges of the objects that can be represented remain perfect in form.

The inherent limitation of the parametric approaches to tolerance representation is that the range of geometric variations that can be represented is dependent on the object parametrization scheme used. Different researchers have proposed different object parametrization schemes in the attempt to increase the range of variations that can be represented. Turner [1993], for instance, suggests the introduction of additional vertices to break the edges of the part into segments. The edge segments between these vertices are, however, still perfect in form. In essence, geometric tolerances such as flatness and roundness can not be represented since they are not describable by a finite number of parameters.

1.3.2 Non-Parametric Approaches

The Offset boundary approach to tolerance representation was advanced by Requicha [1983]. In this formulation, a tolerance specification is a collection of geometric constraints associated with the object surface features. An object is in tolerance if its surface features lie within the part tolerance zones which are regions of space constructed by offsetting (expanding and shrinking) the object's

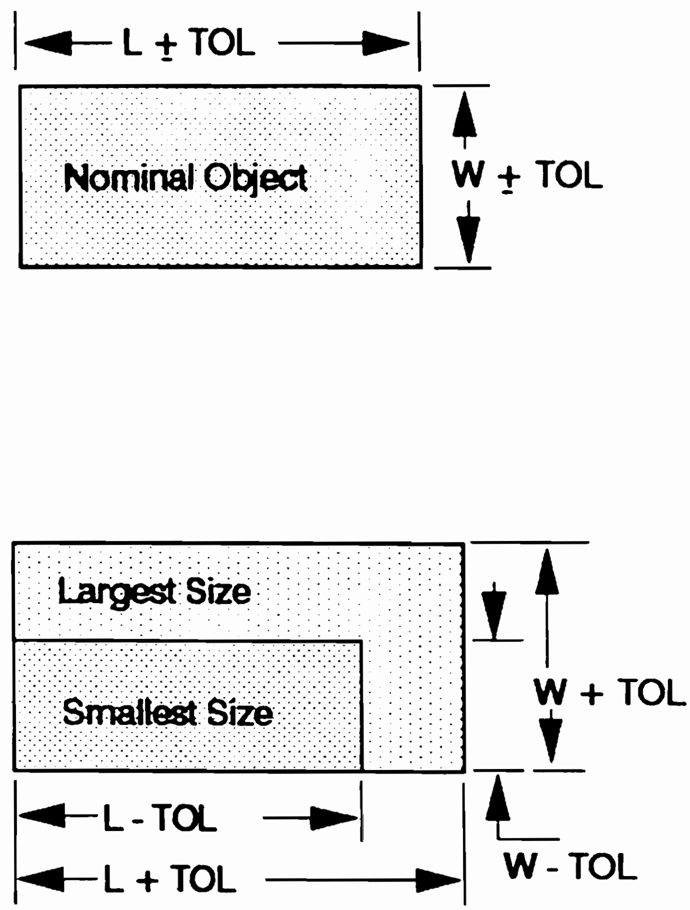


Figure 1.4 Direct Parametrization

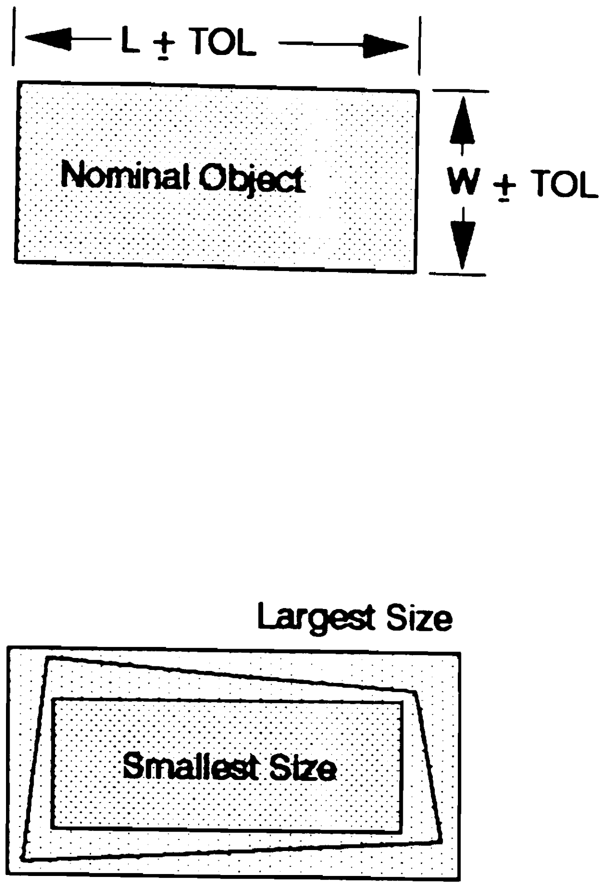


Figure 1.5 Indirect Parametrization

nominal boundaries (see Figure 1.6). Consequently, the tolerance zones generated are independent of the object parametrization scheme.

This representation captures the essence of ANSI tolerances which are defined in terms of tolerance zones. However, the major deficiency of this theory is that tolerance assertions can only be applied onto surface features [Etesami 1987]. Geometric tolerances applied to the feature-derived entities, such as the axis or center plane, cannot be represented.

Taking a different approach, Jayaraman and Srinivasan [1989] represent tolerances in terms of the spatial relationships to be satisfied by each part with respect to a collection of virtual boundaries determined from the product assembly and material bulk requirements. Assembly requirements between two mating features, for instance, dictate that the material of each feature remains on the respective sides of their virtual boundary. Individual tolerances are then derived from these virtual boundary requirements. The part virtual boundary here is synonymous to the virtual condition defined in ANSI as the theoretical boundary limit of a feature when all of the associated feature tolerances are taken into account. However, as pointed out by Turner [1993], this virtual condition is not an adequate representation of the part functional requirements which require separate and independent tolerance specifications. In addition, generalized techniques for deriving the individual part/feature tolerances from these virtual boundary requirements are not available.

It is evident that a tolerance representation scheme capable of representing the range of tolerances defined by ANSI, the currently accepted geometric tolerancing practice in industry, is yet to be realized.

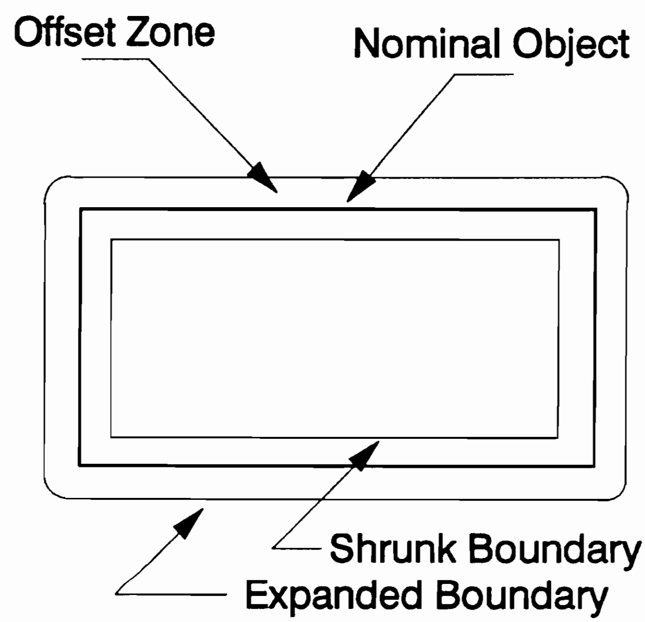


Figure 1.6 Offset Zone

1.4 Problem Definition

Tolerance specifications have significant implications on the quality and manufacturing costs of the product. Currently, the analysis of geometric tolerances can only be performed through prototyping. Previous attempts to automate this procedure have suffered either from inadequate variational coverage or departure from the established tolerancing standards.

For the automation of geometric tolerance analysis, the following must be addressed:

(1) The part tolerance information must be represented in a format suitable for computer interpretation. The representation scheme must be capable of modeling the range of geometric variations defined in the ANSI Y14.5 standard.

(2) The tolerance representation scheme should also model the variational characteristics of the manufacturing processes used. Implicit here is the capability to incorporate statistical distributions since manufacturing processes are stochastic in nature.

(3) For assembly tolerance analysis, the interaction between component parts variations must be modeled and propagated so that their effect on the assembly design functions can be analyzed.

1.5 Research Objectives

The primary thrust of this dissertation was to represent mechanical tolerance information as embodied in the ANSI Y14.5 standard in a computer comprehensible format to facilitate automated statistical tolerance analysis. A 2D model-based approach to tolerance analysis has been adopted. In this approach, instances of manufactured parts incorporating all the allowable parts variations as specified by the part tolerance information are generated based on the characteristics of the manufacturing processes to be used in fabrication. Templates representing the variational characteristics of manufacturing processes are provided for this purpose. Different process capability distributions can be incorporated. The 2D part models are represented using B-spline curves. Models of the manufactured parts generated represent instances of the parts which satisfy the part tolerance specifications.

To support assembly analysis, the product assembly models are constructed. A relative positioning scheme is used to position individual parts relative to adjacent parts so that individual parts variations can be accurately propagated, and their effect on the product design functions analyzed. The geometric characteristics of the relevant design functions can be queried directly from the assembly model constructed. A prototype software for 2D tolerance analysis has been implemented to demonstrate the concepts presented.

1.6 Dissertation Outline

In Chapter 2, available methods for the solution of the tolerance analysis problem are presented. The strength and weakness of available computer representation of the part tolerance information are also discussed. Chapter 3 presents the details of the developed tolerance representation scheme, together with the assembly modeling technique used. In Chapter 4, the details of the prototype software is presented. Chapter 5 concludes with a discussion on the advantages and limitations of the work presented and provides suggestions for further research.

CHAPTER 2
LITERATURE REVIEW

In this chapter, the various methods proposed for the solution of the tolerance analysis problem are reviewed. These methods can be categorized into analytical and model-based approaches. The analytical methods make use of the dimensional equations which relate the assembly design functions to the component dimensions affecting these design functions. Considerable efforts have been dedicated to model how component tolerances stack, and how they affect the assembly design functions.

The model-based approaches, on the other hand, attempt to model the allowable variations to the part geometry and assemble these variational parts together. The relevant assembly design functions are then queried from this assembly model directly. To enable the computer modeling of the part variations as allowed by the part tolerance specifications, however, the tolerance information must be represented in a computer comprehensible format. Consequently, the adequacy of these model-based tolerance analysis techniques is directly dependent on the efficacy of the tolerance representation scheme used.

2.1 Analytical Approaches Of Tolerance Analysis

The widely used method for the formulation of the product dimensional equation is through the manual inspection of the part assembly drawing. Since the part tolerances are traditionally viewed as small variations to the part dimensions, these dimensional equations are then used to derive the tolerance functions which express the tolerances of the design functions in terms of the tolerances of the component dimensions. Utilizing these analytical functions, a number of models with various levels of sophistication have been formulated in an attempt to predict how the tolerances of each of the component dimensions affect the assembly design functions.

The worst-case (WC) model assumes that all the component dimensions occur at their worst limit simultaneously [Fortini 1967]. The total variation of the design function is determined by summing the individual component variations, resulting in the worst possible design function limits. This approach invariably leads to tighter than necessary component tolerances. The statistical model takes into account the low probability of the occurrence of the worst case combination by assuming a stochastic distribution of component variations. This approach is justifiable based on the observation that the dimensions of manufactured parts will be stochastically distributed due to the inherent variations in the processes used. The distribution of the design function is given by the sum of the individual component distributions. By allowing a small fraction of the assemblies to be out of tolerance, the tolerance limits on the design function can be relaxed, permitting the use of looser component tolerances.

The summation of general probability distributions, however, involves a convolution process which is both time-consuming and complex. A variety of different techniques have been used to estimate the resultant distribution of the design function. The Root Sum Square (RSS) model attempts to simplify the computation by assuming that all component dimensions including the design functions, are normally distributed [Fortini 1967]. Consequently, the variance of the design function is simply given by the sum of the component variances, as shown in equation (2.1).

$$Var\left(\Sigma X_i\right) = \Sigma\left(Var X_i\right) \qquad (2.1`)$$

It has been shown, however, that the assumption of component normality can lead to significant error in computation because most manufacturing processes do not produce parts with normally distributed variations [Bjorke 1978]. The component distributions may be shifted or skewed due to setup errors or tool wear and truncated due to inspection. To account for these distribution biases, various models or guidelines have been proposed, ranging from models that suggest the use of a correction (safety) factor to models that estimate the design functions as a weighted ratio of the design function value given by a WC and the RSS models. Typical correction factor values used range from 1.4 to 1.8 [Gladman 1980]. Spotts [1983] suggested that the value of the design function should be the simple average of the result given by the WC and the RSS models in order to account for distribution truncation due to inspection. To account for the degree of uncertainty associated with individual process distributions, Greenwood and Chase [1988] introduced the use of a mean shift factor (a value between 0 and 1) which quantifies the expected mean shift as a fraction of the tolerances specified by the WC and RSS models.

To account for non-normal or skewed component distributions, advanced statistical models have been used. Details on these advance statistical models can be found in [Evans 1975] [Chase & Greenwood 1988]. In the Taylor series approach, the tolerance function is expanded in the form of a Taylor series and truncated at some point. The low order moments of the design function distribution, is computed by summing the low order moments of the component distributions. The Method of Moments uses the statistical moments of the component distributions and the first and second derivatives of the tolerance function to find the first four moments of the design function distribution. These moments can be used to find the parameters of general distributions such as the Pearson, the Johnson, or the Lambda distribution. In the Quadrature method, the low order moments of the design function is estimated using numerical integration. This method offers a precision similar to the Taylor series method, but does not require the calculation of high order derivatives of the design functions. It also requires far fewer samples of the design function than the Monte Carlo method.

The Monte Carlo simulation method can be used to generate a wide variety of component distributions. The sample of component dimensions generated is used to obtain a corresponding sample of the design function values. The yield of the assembly can be estimated by generating a sufficient number of assemblies and determining the number of rejects based on the specified tolerance limits. An alternate method is to plot a histogram of the design function and fit it to a distribution. The fitted distribution function can then be used to calculate the percentage of rejects. The most common distribution used for the fitting procedure is the Normal distribution. This approach relies on the central limit theorem which assumes that the sum of arbitrary distributions will approach Normal if a large enough number of distributions are involved.

All the advanced statistical methods are, in general, computationally complex, and require intensive CPU time due to the need to perform derivatives and series summations. The higher moment terms computed in the Quadrature methods are superfluous since current quality control methods cannot predict out-of-control conditions if only higher moments are changing [Chase & Greenwood 1988]. The main disadvantage of the Monte Carlo simulation technique is the need for the generation of large sample sizes. A more comprehensive review of the various tolerance accumulation models is given in [Chase & Parkinson 1991].

The analytical approach to tolerance analysis, above, suffers from two inherent weaknesses. First, the manual identification of the dimension chains used in the derivation of the tolerance functions is cumbersome and error prone. There may be more than one critical design function that needs to be analyzed in an assembly. A tolerance function has to be formulated for each of the design functions to be analyzed. A new set of design functions may need to be reformulated every time changes are done to the part design. In addition, design functions involving geometric variations are generally non-linear in nature, involving trigonometric and square-root functions [Requicha 1984]. Only when variations in the size of the dimensions are considered, and perfect form parts are assumed, will a linear tolerance function result. Consequently, the tolerance analysis approach presented here is only applicable in situations of limited geometric complexity. It is usually applied in one-dimensional or two-dimensional problems in which all the relevant tolerance variables act in the same direction. Naturally, the designer will only formulate the tolerance functions that he or she suspects may cause problems in the design.

Consequently, it is highly probable that some tolerance stack-up problems may go unforeseen until production has begun [Turner 1987].

Secondly, since the tolerance functions used in the above models are derived from the dimension equations, only the conventional plus/minus tolerances which are associated with the component dimensions can be analyzed. Geometric tolerances are not addressed because they are associated with part surfaces with no associated dimensions [ASME 1982]. The formulation of these functions can be prohibitive in practical problems where the component tolerance may affect the assembly tolerance in an unpredictable manner. This is especially true of part geometric characteristics, such as feature orientation or perpendicularity, which may interact in more than one direction. In addition, the use of the dimensional loop equation assumes that the position of the parts in the assembly are fixed in space and the form of these components is perfect.

Chase, et al [1989] and Treacy, et al [1991] have attempted to automate the derivation of the dimensional loop equations and, thus, the assembly design functions from a CAD model. In Treacy, et al's approach, the tolerance function is generated from the hierarchical relationships and mating conditions between components contained within the assembly's data structure. The assembly model in Chase's approach is constructed graphically by overlaying a vector loop on the CAD drawing. Each vector in the chain represents a component dimension. To incorporate geometric tolerances, each component dimension is represented by a product of two matrices: a rotation and a translation matrix. Kinematic constraints are included to assure that manufacturing variations propagate through the assembly model correctly. Both of these attempts can address size and a limited form of the orientation and position tolerances.

2.2 Model-Based Approaches To Tolerance Analysis

To enable a truly automated approach to tolerance analysis of mechanical assemblies, three requirements must be met [Turner 1987]. First, an unambiguous representation of the product model, equipped with tolerance information, must be available. Secondly, the part tolerance information must be represented in a computer comprehensible format, and capable of modeling the different types of tolerance specifications used in mechanical design. Third, the geometric modeler must be equipped with assembly modeling capabilities.

Solid modelers have been shown to have the basic properties to support automatic production activities since they have the capability to provide unambiguous representation of the product geometric data. Today there are many well-known methods of representing solids, including constructive solid geometry, boundary representation, octree, and others. Considerable effort has been undertaken by CAM-I [Ranyak & Fridshall 1988] to address the problem of attaching tolerancing syntax to a solid-modeling system. CAM-I considers a solid-modeling system as a virtual solid modeler and an application interface (AIS). The virtual modeler may be based on Boundary Representation (BRep), Constructive Solid Geometry (CSG), wireframe, surface-modeling, or any combination of these [Requicha & Voelcker 1982]. The AIS is the CAM-I standard method of interfacing the modeler to the user and external applications. They propose the creation of an auxiliary Dimensioning and Tolerancing (D&T)

structure that can be related to the primary geometric model. The D&T model is composed of three entity node types: features, tolerances, and datum reference frames (DRFs). The implementation of the D&T modeler references one or more geometric faces, allowing the tolerancing of D&T feature entities. Some geometric tolerances apply implicitly to the feature resolved-entities, while others require a qualifier. Also, if used as datum features, the D&T feature resolved-entities are arguments for the datum reference frame transformation matrix. More than one tolerance node may constrain one feature and, conversely, a single tolerance node may constrain more than one feature. The modeler categorizes tolerances as location, orientation, size, form, and surface finish. Applications needing tolerancing information can interrogate the tolerance structure using templates. The semantics or interpretation of the object's tolerance constraints is left to the applications using the information. Similarly, the effort by PDES/STEP only focuses on the modeling of tolerance information in the effort to define a complete product definition data.

The above work only provides the conceptual framework for incorporating tolerance information into solid modelers. No interpretation of the tolerance information is provided. For the automation of tolerance analysis, the semantics of the attached tolerances must be in a form appropriate for computer interpretation. The remaining portion of this section will review the various tolerance representation schemes that have been proposed by different researchers in the field. To provide a framework for the discussion on the efficacy of these proposed representation schemes and the definition of the terminology used, a brief summary of the different tolerance classes and their semantics, as documented in the ANSI Y14.5 1982 Standard, is presented first.

The subject of relative positioning as pertaining to assembly modeling is discussed in the section which follows.

2.2.1 ANSI Y14.5 1982

ANSI Y14.5M-1982, like other available standards, is a description of current industrial tolerancing practices which are not mathematically defined [ASME 1990]. Four basic types of geometric tolerances are defined. They are the form, orientation, location, and profile tolerances. A geometric tolerance applied to a feature defines the tolerance zone within which the feature or element of the feature is to be contained. Tolerance assertions can also be associated with geometric entities derived from certain features referred to as Features-of-Size (FOS). FOSs are so designated because a measure of size can be associated with them. Examples of derived entities and their respective FOSs are the center point of a spherical feature, the axis of a cylindrical feature, or the centerplane of two parallel planar features.

The geometry of the tolerance zones implied by a tolerance constraint is dependent on both the geometry of the toleranced feature and the type of tolerance applied. For instance, form tolerances of straightness, flatness, circularity, and cylindricity, when applied to different features will result in tolerance zones of different shapes. Straightness tolerance when applied to the axis of a cylindrical feature implies a cylindrical tolerance zone within which the feature axis must be contained. When applied to a surface, the relevant feature is the line elements of the toleranced surface feature. The resultant tolerance zone in this case is given by two parallel lines separated by the distance of the given straightness tolerance. The flatness and cylindrical tolerances can be applied to planar and cylindrical surface features, respectively. The orientation and position of this zone is unconstrained.

Orientation tolerances cater to form tolerances which require a datum reference. They are used to control angularity, parallelism, and perpendicularity of related features. They usually define the orientation between a tolerance feature and datum or system of datums. The orientation of the tolerance zone is fixed, although its location is not.

A location tolerance includes tolerances of position, concentricity, and symmetry. Position tolerance defines a zone within which the derived entity of a FOS is permitted to vary from true or theoretically exact position. The position of this tolerance zone is established by Basic dimensions specified from datum features, or between interrelated features. This type of tolerance can also be applied to groups of features.

In addition to the above basic tolerance types, composite tolerances, such as runout, can be used to control the functional relationship of one or more features of a part to a datum axis. The types of features controlled include those surfaces constructed around a datum axis and those constructed at right angles to a datum axis.

For the tolerancing of an FOS, material condition modifiers can be used. The three material condition modifiers are: Maximum Material Condition (MMC), Least Material Condition (LMC), and Regardless of Feature Size (RFS). MMC is the condition in which an FOS contains the maximum amount of material within the stated limits of size. LMC is the condition in which the FOS contains the least amount of material. RFS is used to indicate that a geometric tolerance or datum reference applies independent of the actual size of the feature.

Datums are the theoretically perfect counterpart of a part datum feature used as the reference for the origin of dimensions and tolerance specifications. They provide the means by which components can be located consistently during manufacturing and inspection. Any part surface feature, or the derived entities of an FOS's, can be a datum feature. A virtual condition exists for a datum feature associated with a MMC modifier. The virtual boundary of the feature is given by the collective effect of the allowable geometric variations and the MMC condition of the feature.

There are certain overlaps between the different geometric tolerances. For instance, the limits of size of the feature prescribe the extent to which variations in its geometric form, as well as size, are allowed. This is specified by Rule #1 of ANSI. Dictated by the functional requirements of the design, additional form or parallelism tolerances can be defined as a refinement of the existing tolerances. The so-called envelope principle states that the tolerance zone implied by the additional tolerances must lie within the size limits. This also implies that the form of the feature must be perfect if the hole or shaft is manufactured at their extreme sizes.

When used in conjunction with the modifiers and datums, the above geometric tolerances provide a rich environment for the specification of tolerances to satisfy functional and assembly requirements. The size, position, and orientation tolerances, together with the material condition qualifiers MMC and LMC, provide a rich class of geometric constraints for satisfying various assembly needs. Position tolerances applied to FOS, with the modifiers MMC or LMC, permits the increase of the feature position tolerance equivalent to the amount the actual manufactured size deviates from MMC. This, together with the envelope principle, enables the assignment of the largest possible tolerance value while ensuring the assemblability of the mating components. The enlargement of the feature position tolerance effectively results in more functionally equivalent parts being accepted by inspection. By incorporating information about how parts assemble and function, ANSI ensures that the design intent can be preserved without over-tolerancing.

ANSI provides a compatible specification language for conveying tolerance information from design to manufacturing and inspection. Tolerancing conventions can be attached to manually or computer drafted drawings in the form of numerical values, symbols, and notes that must be interpreted to understand the implied meaning. The semantics of this language poses no difficulty for a human machine operator or inspector, but formalization is required when computers and automatic machines are to interpret such a language. Ideally, a tolerance representation scheme must allow the modeling of the widest class of functionally useful tolerances in a manner the designer can relate to. At the very least, it must be able to represent the tolerances incorporated in ANSI, since this is the currently accepted tolerancing practice [Farmer & Gladman 1986].

2.2.2 Proposed Tolerance Representations Schemes

The profound need for a formal (mathematical) definition of tolerances was first recognized by Requicha [1977]. In the context of sets, he defines a solid to be a regular (closed) point-set in three-dimensional Euclidean space (E3). The set of all the regular subsets of E3 constitutes all well-defined objects. Tolerances are viewed as defining a class of objects that are similar to the nominal object, and are interchangeable in assembly, and functionally equivalent. Such a class is called a variational class. A variational class is thus a set within the universe of all the regular subsets of E3.

The above interpretation suggests that a variational class should include the nominal part, and supports the stipulation that a tolerance specification should not force any portion of the object's boundary to be perfect, or in an exact position [Requicha 1984]. Unfortunately, the nature of such a variational class of objects is largely unknown. For a tolerance specification, a mathematical description of the corresponding in-tolerance collection is desired. The preceding framework is not useful for this purpose. As noted by Turner and Wozny [1988], the in-tolerance collection, so defined, cannot be viewed as an equivalence class in the mathematical sense because there is no meaningful partitioning of the universe. The variational class consisting of in-tolerance objects is not a countable set, thus does not constitute a space. Consequently, the in-tolerance collection cannot be characterized as constituting some neighborhood of the nominal part. Nevertheless, the above formalization provides an abstract view of tolerances and establishes the theoretical foundations for the modeling of toleranced objects. Effort to define such a variational class has been reported [Boyer & Stewart 1992].

In the following sections, the different tolerance representation schemes proposed for the modeling of the variational class are reviewed, and their efficacy in supporting automated tolerance analysis is discussed. The different parametric tolerance representation schemes that have been proposed are presented first, followed by the non parametric approaches. Five parametric representation schemes (parametric variancing, variational geometry, vector space, vectorial, and constraint propagation) and two non-parametric approaches (offset zone and virtual boundary requirement) are reviewed here. All the parametric approaches interpret tolerances as allowable variations to some parameter values associated with the nominal part geometry. Consequently, the range of variations which they can model is dependent on the parametrization scheme used.

2.2.2.1 Direct Parametrization

In the direct parametrization approach, tolerances define the range of allowable variations to the natural parameters defining the part [Requicha 1984]. A cube, for instance, can be parametrized based on its length, width, and height, and a cylinder based on its diameter and height. A parametrically defined object is given in Figure 1.4 (see page 11). In a CSG-based system, such parameters are used in the explicit construction of the primitives solids and are called the natural parameters of the solids.

Direct object parametrization is widely used in parametric and feature-based CAD systems. In parametric systems [Requicha 1984], solid shapes may be constructed from primitive half-spaces and other low-level primitive solids, such as cylinders, cuboids, cones, and torus. Half-spaces are unbounded surfaces that partition the space into a material side and non-material side [Requicha 1977]. These half-spaces may be infinite planes or infinite cylindrical surfaces. Each half-space is represented by a half-space equation in a local coordinate system (LCS). The parameters used in the half-space equations, and in the primitive solids definition, are called the configuration parameters. These primitive half-spaces and simple solids can be combined to form composite solids through Boolean operations.

Feature-based systems [Shah & Miller 1990] [Mullins & Anderson 1991] differ from parametric systems in that they provide a library of higher-level functional features for shape construction. These predefined features may be constructed from primitive solids similar to those in parametric systems, or based on some other solid modeling scheme. The functional features used are usually defined to suit a particular application. Systems are available that allow users to define their own features using primitive solids. An example of a commercial CAD software implementing the parametric and feature-based approach is Pro/Engineer.

In both the parametric and feature-based systems, only the configuration parameters of primitive constructs are accessible to the system user for dimensioning and tolerancing purposes. Values assigned to these parameters uniquely define the size of the primitive constructs used. The position and the configuration parameters of the primitive constructs, along with the order of the Boolean operations, uniquely define a solid. To provide variational modeling capability, tolerances are reflected as numerical limits on the configuration and position parameters of the constituent features of the part.

This approach of representing part variations in terms of the parameters defining the part is closely related to the concept of conventional or plus/minus tolerancing. Tolerances expressed as the range of allowable variation to these parameters define a family of objects having different sizes. The set difference between the largest and smallest acceptable instance, called a maximum material condition (MMC) object and a least material condition (LMC) object, defines a perfect-form tolerance zone, within which the imperfect-form features of the actual object must lie.

Since tolerances can only be applied to key dimensions defining the solid, this tolerancing scheme does not provide an environment for controlling some useful dimensions generated as the result of Boolean operations, nor resolved entities such as axes or centerplanes [Etesami 1987]. Furthermore, the class of objects that can be represented consist of only perfect form objects (see Figure 1.4 on page 11). From a tolerancing point of view, this dimensioning and tolerancing scheme is inadequate because objects of imperfect form and shape cannot be represented. Shah and Miller [1990] made the observation that Datum Reference Frames (DRF) cannot be supported either. In view of the limitation in the variational coverage, this representation scheme is not suitable for the purpose of tolerance analysis.

2.2.2.2 Inverse Parametrization

The variational coverage of the parametric variancing approach discussed in the previous section is limited due to the parametrization scheme used. Since tolerance information is associated with the model parameters, a cube that is parametrized based on the length, width, and height only allows three degrees of freedom in variational modeling. Higher degrees of freedom for variational modeling is achievable by parametrizing a solid based on the positions of its vertices [Requicha 1984]. Variational information may be represented by the displacements to the nominal vertex positions. In a BRep system, these vertices are used to explicitly construct the part model, and are called the natural parameters of the solid. However, vertex parametrization is not useful for interactive CAD because part vertices do not have intrinsic geometric meaning. This limitation can be remedied using inverse parametrization. The model dimensions can be expressed as functions of the natural parameters. The length of an edge can be defined in terms of the vertex coordinates. Inverse parametrization provides the user with a more natural way of expressing a design idea or function because dimensions such as distances and angles have obvious geometric meanings.

Tolerance representation based on inverse parametrization was initially proposed by Hillyard and Braid [1978a]. Tolerance specifications are represented as linear and angular constraints on the length and angles in the boundary models of planar polyhedrals. The edges and vertices of an object are regarded as an engineering frame structure whose members and joints correspond to the edges and vertices of the object. The members are initially unconstrained in length, and the joints are pinpointed. Dimensions are then added to constrain the length and the angular relationship between the frame members. Each added dimension constrains one or more degree of freedom of the mechanism. With the addition of the proper number of dimensions, a rigid frame structure results. Each dimension added can be represented in the form of a dimensional constraint equation. The set of dimensional constraint equations defining the frame structure can be formulated in the form of a Rigidity Matrix as shown in equation (2.2),

$$R\,d = u \qquad (2.2)$$

where R is the rigidity matrix, d the displacement vector and u the variation vector. For a nominal part where all the tolerances are zero, u will be null. Note that the displacement vector here refers to the displacement of the vertex positions. Provided that the above equations can be inverted, the variations vector can be expressed in terms of the displacements to the model vertices:

$$d = F\,u \qquad (2.3)$$

where $F = R^{-1}$.

By examining the elements of F, the contribution of each variation u_i to the displacement of any vertex can be found. By treating tolerances as small changes to the model dimensions, the individual part tolerance constraint equations can be solved for the vertex coordinates, and the geometry resulting from specific maximum dimensional deviations can be determined, .

For the solution of tolerance analysis problems, the design function is treated as an indirect function of the dimensions by formulating it in terms of the model vertex coordinates. Tolerance information is represented as small variations to these dimension values. By varying the individual part dimensions, the effect of each possible part geometric variation on the design function can be assessed. An extension of this technique for assembly tolerance analysis based on forming a composite rigidity matrix for the assembled parts, has been proposed by Minnichelli [1983].

More recently, inverse parametrization has been studied as a design paradigm in which the object parameters are used as handles to modify object geometries [Lin et al 1981] [Light & Gossard 1982]. Thus, these works are more related to the subject of variational geometry rather than tolerance analysis.

Using the approach outlined above, the designer must specify exactly *n* independent constraints or tolerances on the model variables. Consequently, geometric tolerances supported by ANSI cannot be used. Geometric tolerances, such as form and parallelism, used as refinements to the existing size and position tolerances cannot be accommodated.

The most general mathematical result applicable to the solution of the model variables, in terms of the tolerances, is the inverse function theorem [Juster 1991]. For a part with *n* dimensions, the solution entails the computation of a Jacobian of *n* dimensions [Hillyard & Braid 1978b]. This yields a problem that increases rapidly in *n*, rendering calculations impractical. The storage for the Jacobian matrix of the tolerance function grows as the square of the number of model variables, and the time to invert the Jacobian matrix grows proportionally to its cube [Turner 1987]. Minichelli [1983] suggests that it would not be unusual to find as many as 10,000 model variables in a typical assembly.

The application of variational geometry to tolerance analysis has been reported by Scott and Gabriele [1989]. Their approach for modeling geometric tolerances is based on defining a set of conventional dimensions, and constraints on these dimensions, which are equivalent to the geometric tolerances. The constraints on these dimensions define the limits of the dimensions associated with the geometric tolerance zones. Equivalent dimensions and associated constraints for 3D angularity, size, parallelism, and position have been developed. Form tolerances are not represented because they are not a function of the model variables used.

2.2.2.3 Vectorial Tolerancing

Turner and Anderson [1988] represent tolerance as limits on the components of vectors that relate the "handle" of a given toleranced feature to the "reference handle" of a given reference feature. Point and line handles are characteristics geometric elements of features, representing points and lines of interest, respectively. Point handles are used for positioning and orienting feature geometry, and for establishing relationships between two or more features. These relationships between features are represented by offset vectors which express the hierarchy between features. Line handles are used as vectors to represent the private information of features, such as the depth of a hole or the length of a slot. An approach similar to this has been taken by Wirtz [1991].

This tolerancing scheme provides a means for specifying tolerances in a convenient way. By relating one feature relative to another using the offset vectors, a hierarchy chain of features, which ultimately terminates at the workpiece, is created. Through vector manipulation, variations to the position and orientation of a feature can be propagated and analyzed. It is conceivable that such representation of tolerance can be useful in process control since the actual deviation in the magnitude and direction of a part dimension can be used for process adjustment.

Nonetheless, this approach describes a different set of constraints from those given in the current tolerancing standards. It seems to correspond to conventional dimensioning and tolerancing practices. Since only the handles of the features are available for tolerancing purposes, limited variancing capability is provided. The manner in which form tolerances can be represented is not apparent.

2.2.2.4 Constraint Propagation

Bernstein and Preiss [1989] represent a dimensioned and toleranced model of an object utilizing a constraint network approach. The part boundary is decomposed into free, rigid, topological entities. The part tolerance constraints are represented as a constraint network over the degrees of freedom of these topological entities. The nominal location of the boundary entities is defined by the nominal location of the nominal solid model. The root of the constraint network represents the parts primary datum reference frame. Each arc represents a directed constraint from a source feature to a destination feature's associated degree of freedom nodes. Tolerances can be propagated within the network to compute the resultant virtual condition of any given portion of the assembly. A manufactured part conforms to the tolerance specification if its measured dimension falls within the resultant virtual condition.

The representation is based on a boundary model, which leads to a graph theory-based approach. This approach facilitates the evaluation of the validity and the well-formness of a given dimensioning and tolerancing scheme. However, it does not address the need of assembly tolerance analysis. While the location and orientation of a geometric entity can be propagated through the concatenation of the respective transformation matrices, its form can not. This is because the form of the geometric entity is independent of the location and orientation of the entity, and, thus, is not encapsulated in the entity transformation matrix.

2.2.2.5 Vector Space Approach

Hoffman [1982] defined tolerances as functions of the sum of the manufacturing errors. He proposed a set of parameter vectors that model the machining, setup, and positioning errors in manufacturing processes. A tolerance specification corresponds to a set of inequalities that constrain the tolerance functions associated with each parameter vector within an upper and lower limit. The resulting theory is well-suited for selecting the appropriate sequence of processes to be used in order that the tolerance specifications are met. By the same token, information from inspection can be fed back to the manufacturing processes for process control activities.

The parameter vectors here are similar to the configuration and position parameters used in the parameter variancing theory. The formulation again suffers from the limited variational information it can represent. Only conventional plus/minus tolerances are represented.

Refining Hoffman's formulation, Turner [1987] expressed tolerance specifications as constraints that define a feasible region in a Cartesian space of model variables. The model variables used here are a generalization of the parameter vectors used by Hoffman. By associating these model variables with the coefficients defining the part surfaces, each of the bounding surfaces of the part can be varied from their nominal shape and location. The edges and vertices of the part are defined at the intersection of the perturbed part boundary surfaces.

With the choice of proper and independent model variables as the basis vectors, a Cartesian space of model variables may be constructed. By expressing tolerance variables as functions of the model variables, a tolerance specification may be expressed as the constraint on the range of these model variables. Tolerances expressed as constraints on these model variables effectively define the extent of the allowable surface variations from nominal. The set of tolerance specifications specified on a part boundary effectively defines an in-tolerance region in the model (variable) space. Therefore, each point in the feasible region corresponds to an instance of the variational part that satisfies the specified tolerance constraints.

The model variables associated with the size tolerance of the feature in Figure 2.1 is given in Figure 2.2. The two model variables M1 and M2 have

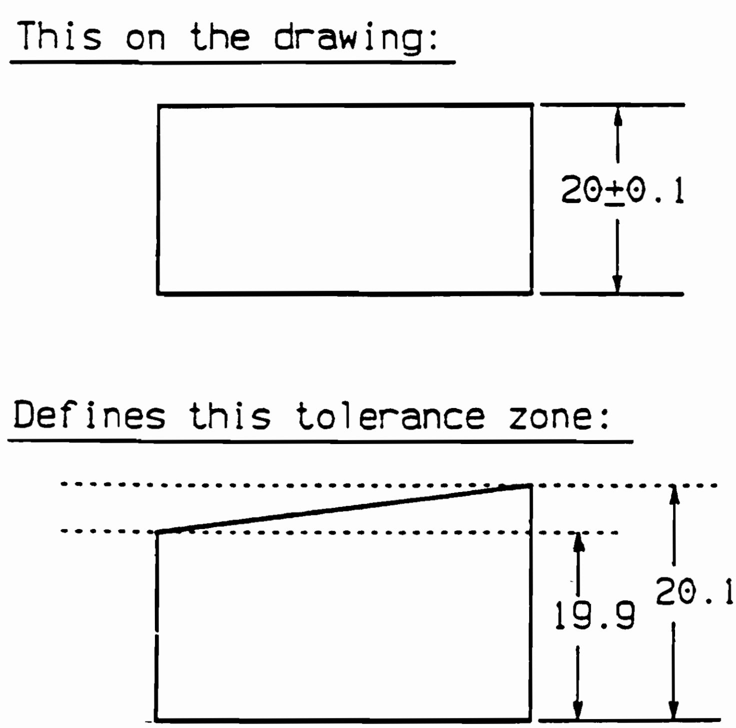


Figure 2.1 Specification of Size Tolerance

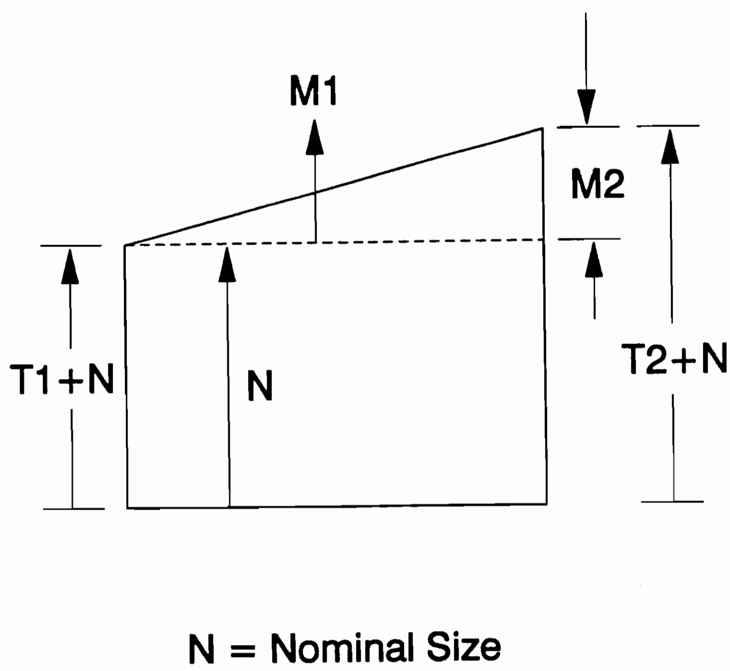


Figure 2.2 Interpretation of Size Tolerance

been introduced to apply variations to the location and slope of the planar feature. The size and implied orientation tolerance of the part can be expressed in terms of these model variables by introducing two auxiliary variables representing variations to the bounding edges of the part.

It can be seen that

$$T1 = M1 \quad (2.4)$$

$$T2 = M1 + M2 \quad (2.5)$$

The specified tolerance limits shown in Figure 2.1 is equivalent to the following constraints on the auxiliary variables:

$$-.1 <= T1 <= +.1 \quad (2.6)$$

$$-.1 <= T2 <= +.1 \quad (2.7)$$

The feasible region corresponding to the tolerance constraints is shown in Figure 2.3.

For modeling form tolerances, three different approaches have been proposed. Initially, Turner [1987] proposed to model form tolerance by substituting a higher degree polynomial function for the nominal part feature. For instance, a nominal surface would be substituted with a higher degree polynomial surface. Tolerance specifications are expressed as constraints on the surface coefficients. In addition to the highly non-linear relationship between the surface coefficients and tolerances constraints, the characteristics of the surface can become unpredictable as it's degree increases.

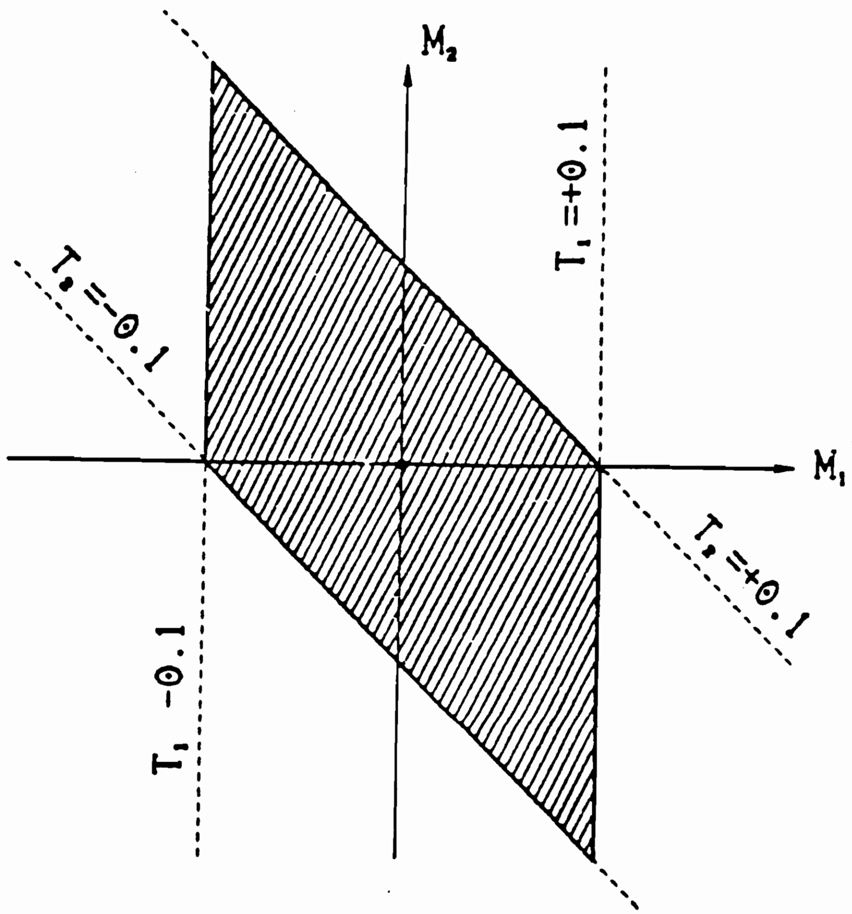


Figure 2.3 Feasibility Space of Size Tolerance

In the interest of mathematical simplicity and elegance, Turner and Wozny [1990] then proposed to model form tolerance by breaking up the part boundary by introducing additional vertices into the linear segments. The number of these interior vertices determines the maximum frequency of the form variations represented. The form variations permitted by the straightness tolerance specified in Figure 2.4 are interpreted, as shown in Figure 2.5. The vector space representation is given in Figure 2.6. The additional model variables M3 and M4 associated with these vertices allows these vertices to move in the direction normal to the line determined by M1 and M2 introduced previously. The weakness of this approach is that the segment of the boundary between the vertices still assumes perfect forms.

Recognizing the deficiencies in the previous two approaches, Gupta and Turner [1991] proposed to break a planar surface into triangular patches and fit Bezier triangles to each patch. A quadratic Bezier patch is constructed by determining three control points, in addition to the vertices of the triangle. Model variables are associated with all six points associated with each triangular patch.

This tolerance representation scheme has been implemented in a solid-based, experimental, variational geometric modeler [Turner 1987]. An "in-spec" instance of a part is generated by randomly selecting a point in the feasible vector space. A line is then extended from this point in a random direction until it intersects the boundary of the feasible region. A statistical distribution is then associated with the length of this line, and a Monte Carlo simulation procedure is applied to select a specific point along this line. The values of the model variables corresponding to this point are used to perturb the nominal part

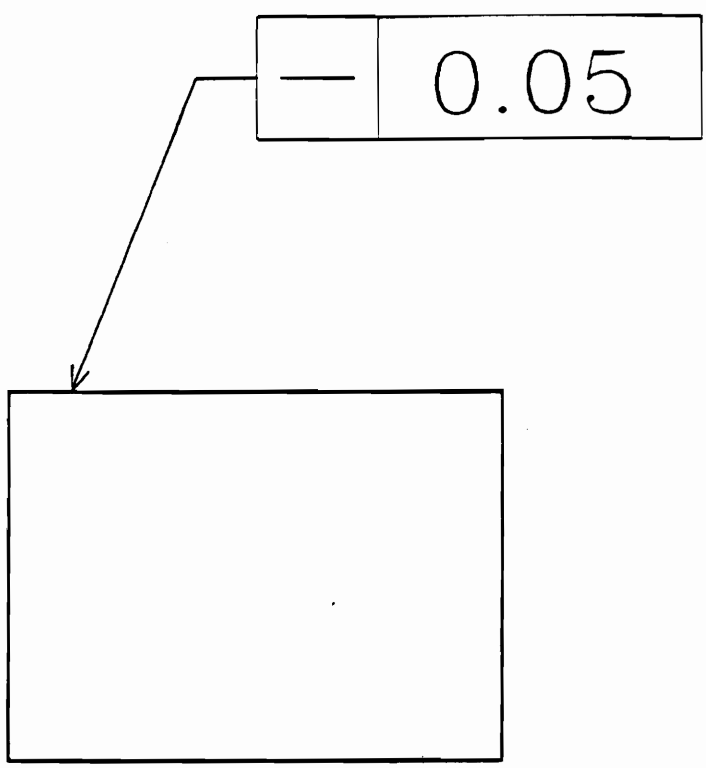


Figure 2.4 Form Tolerance Specification

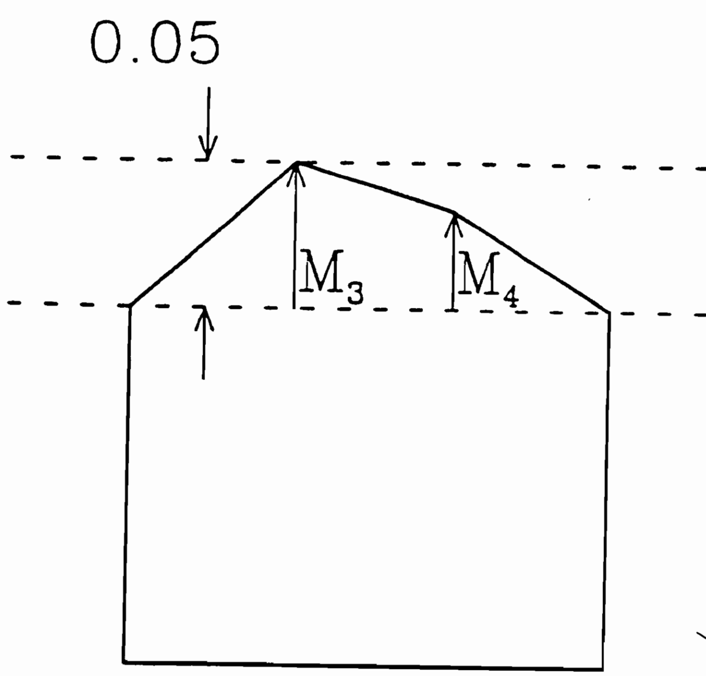


Figure 2.5 Representation of Form Tolerance Specification

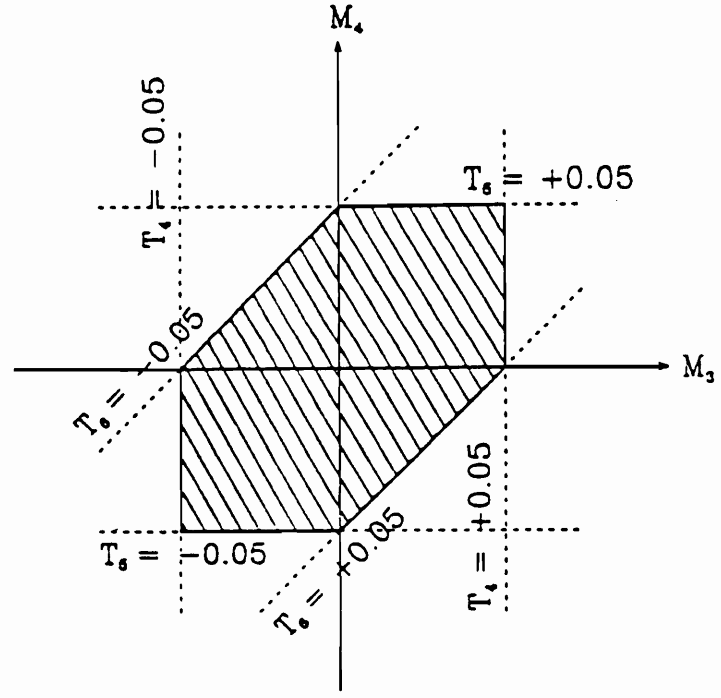


Figure 2.6 Feasibility Space of Form Tolerance Specification

geometry to generate a variational part. These variational parts are then assembled and the sum dimension is measured from the assembly. This procedure is repeated, and the statistics of the sum dimension are collected.

The use of model variables for variational modeling makes the tolerance representation scheme independent of the nominal geometry representation and allows a wider range of variational coverage. The association of the tolerances to a vector space is mathematically elegant. This allows the tools of vector analysis to be applied to tolerance related problems. Redundant constraints pose no problems since a constraint either reduces the feasible region or leaves it intact. Conflicting, or infeasible, constraints can be identified by a diminished feasible region. An under-constrained condition can be identified by an unbounded feasible region.

As the representation is surface-based, tolerances associated with derived entities cannot be represented. In addition, geometric form tolerances, such as flatness and roundness, cannot be handled by this approach because they are not describable by a finite number of independent dimensions. However, the third approach to form tolerance representation proposed by Gupta and Turner [1991] is potentially befitting. Nevertheless, the procedure for the derivation of the control points introduced for each patch is not outlined. Note that a Bezier curve, does not interpolate it's control polygon except at the first and the end point (see Figure 2.7). Consequently, form variations generated will not assume the width of the tolerance zone allowed. If variations within the extent of the zone are to be allowed, the control points generated must be allowed to extend beyond this zone. One obvious way of deriving these

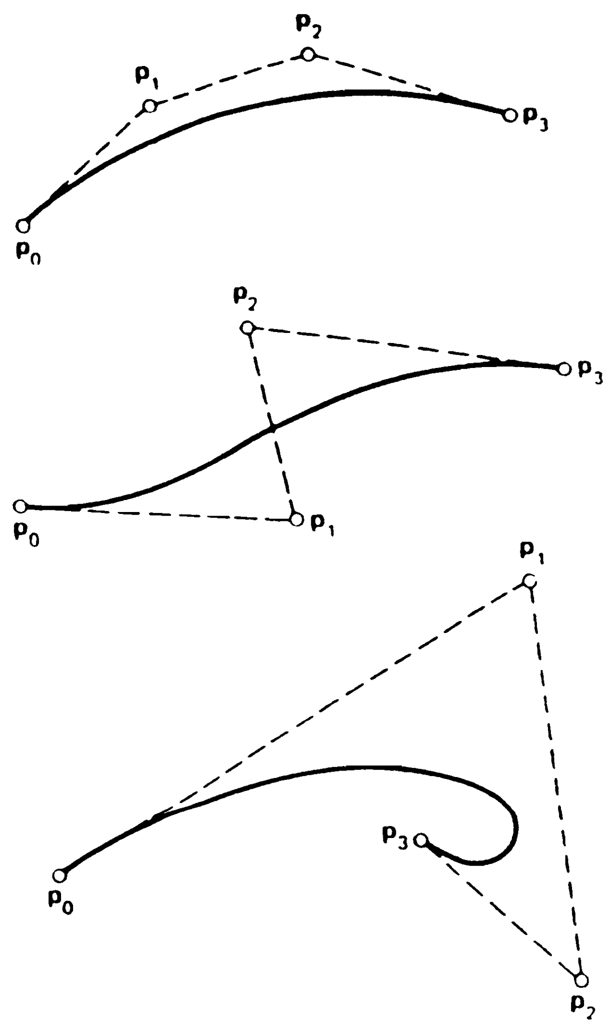


Figure 2.7 Bezier Curve

control points is through trial and error. The points can be selected randomly, and the resultant curve or surface can be tested if it violates the boundaries of each of the tolerance zones. Needless to say, this is a very tedious approach.

As can be seen from the two-dimensional example above, several model variables are needed just to represent a single tolerance variable. Since several types of geometric tolerances may be associated with a boundary feature, a large number of model variables will be needed to model all the allowed part variations. In addition, special procedures must be performed to preserve the tangency conditions between adjacent features. This is a direct consequence of the surface-based approach used. The intersection between the variational surfaces generated may occur beyond the part tolerance zones (see Figure 2.8).

2.2.2.6 Offset boundary

To augment the weakness of the parameter variancing theory, Requicha [1983] proposed a more elegant and robust tolerancing theory based on offset boundaries. In this formulation, a tolerance specification is a collection of geometric constraints associated with the object surface features. An object is in tolerance if its surface features lie within tolerance zones constructed by offsetting (expanding or shrinking) the object's nominal boundaries. The tolerance zones depend only on the numeric values of the offsets and the geometry of the nominal features. Consequently, the tolerance zone generated is independent of the object parametrization scheme used.

In this theory, Requicha rejected the notion of a measured size. Instead, mathematical rules are provided for deciding whether a feature satisfies a size

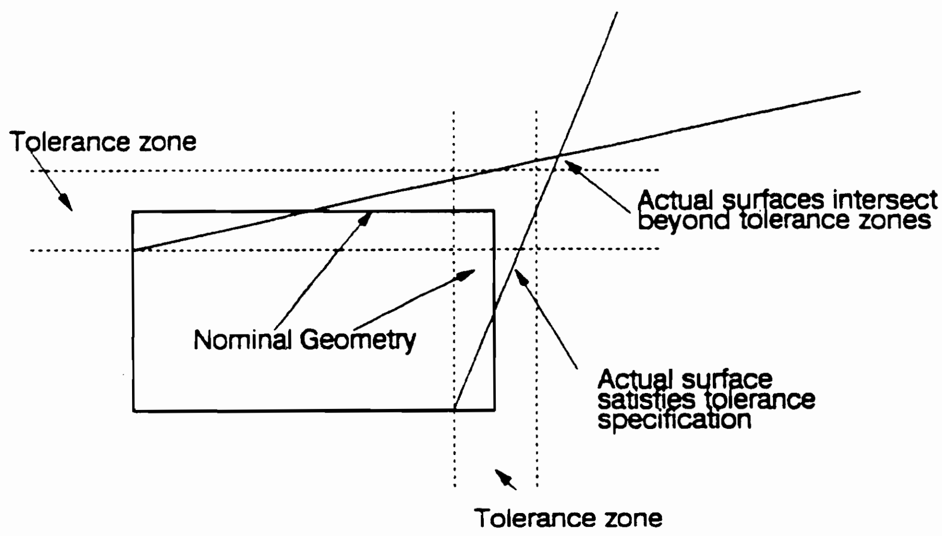


Figure 2.8 Violation of ANSI Tolerance Specification

tolerance specification. A hole satisfies a size tolerance if its boundary lies entirely within an annular tolerance zone defined by two concentric circles of radii, $r + Ts/2$ and $r - Ts/2$, corresponding to the MMC and LMC boundaries of the feature, respectively (see Figure 2.9). The location and orientation of this zone is arbitrary. Similarly, the position tolerance is defined by an annulus defined by two concentric circles of radii, $r + Tp/2$ and $r - Tp/2$, correctly located and oriented with respect to the given coordinate system, as shown in Figure 2.10.

Requicha proposed to replace various special-case form tolerances used in current practice with a single-form tolerance that applies to all features. For a circular feature, a form tolerance zone is an arbitrarily positioned annulus defined by two concentric circles of radii, $r1$ and $r2$, that are unrelated to the part radius r, but satisfy $r1 - r2 = Tf$. Subsequently, a feature that satisfies a size tolerance, Ts, also satisfies a form tolerance of $Tf = Ts$. A feature satisfying a position tolerance of Tp will also satisfy a form and size tolerance of the same value.

Requicha's theory captures the spirit of modern tolerancing scheme which is based on the concept of tolerance zones. Mathematical descriptions of offset solids, which are implicit in many ANSI standard specifications but not formally defined, are also given [Rossignac & Requicha 1986]. The representation scheme proposed here has been incorporated into a solid modeler based on CSG representation. Features and tolerance attributes are represented in the modeler in the form of a variational graph structure [Requicha & Chan 1986].

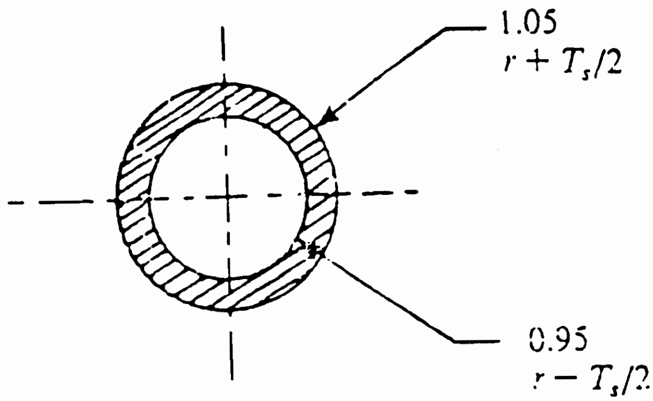


Figure 2.9 Offset Zone of Size Tolerance [Requicha 1983]

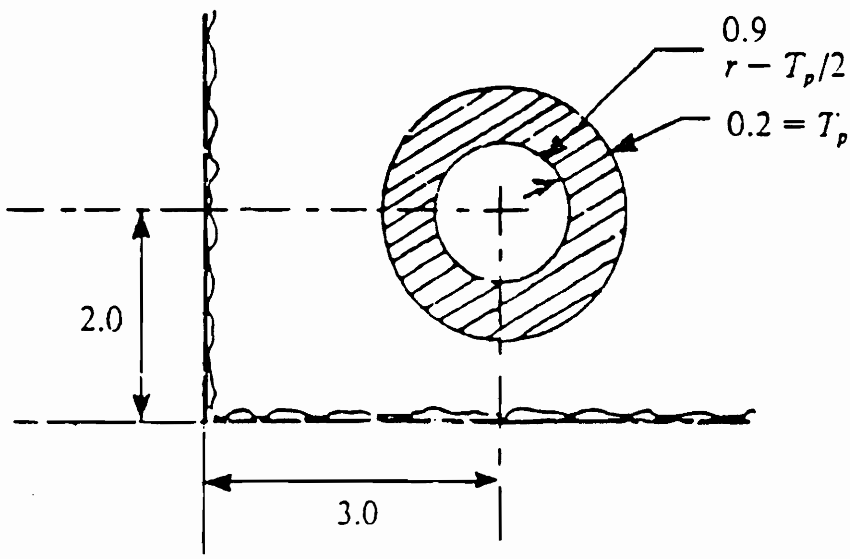


Figure 2.10 Offset Zone of Position Tolerance [Requicha 1983]

A framework for solid-based tolerance representation and analysis based on CSG, involving the creation of a tolerance shell similar to the offset zone concept, has been proposed by Elgabry [1986].

The major deficiency of the offset theory is that tolerance assertions can only be applied onto surface features. The theory cannot be used to represent geometric tolerances on derived entities, such as the axis of a feature. The generalized form tolerance can powerfully accommodate many of the current practices in ANSI standards. However, some ANSI form specifications that call for constraining axes and curves directly, rather than surfaces, have different semantics. As pointed out by Etesami [1987], although the specifications of axis straightness and circularity both reflect a form control on the surface of the cylinder, their semantics, as defined by ANSI, are quite different. While a straightness tolerance controls the surface deformation with respect to the feature axis independent of the feature length, a cylindricity tolerance controls the whole surface profile of the part irrespective to any internal axes. This leads to considerable difference in the inspection procedure used for part verification. The position tolerance defined in this theory also implies size and form tolerances. This too is inconsistent with the principle of independence between individual tolerances adopted by ANSI.

Although this theory is useful in describing individual tolerance requirements, it is not possible to combine these tolerance zones to determine a least material and a maximum material boundary between which all acceptable part boundaries must lie. Defined as a composite of multiple offset zones, individual tolerances cease to be independent constraints. For instance, form tolerance information used as a refinement of size tolerance will be lost since it only comes into effect when the feature size is smaller than MMC.

One of the assumptions of this theory is that the part surface can only have high frequency, low amplitude form imperfection. A direct consequence of this assumption is that the offset solid generated may fail to characterize the geometry of the object features. For instance, as pointed out by Etesami [1987], the offset zones generated here differ from the tolerance zones of ANSI, which seems to preserve the shape of the part (see Figure 2.11).

It has been pointed out by Farmer and Gladman [1986] that the size specification, as defined here, is too stringent because it also implies the requirement of perfect form at LMC. Although the ANSI prescribed the requirement of perfect form at MMC, perfect form at LMC is not a requirement. This criteria is adopted by ANSI in light of the assembly requirement of mating components, which requires that no part of the features should violate the envelope of perfect form of the mating features. At LMC, however, the form of the feature is not critical for assembly purposes.

Clement et al. [1991] proposed an extension of the solid offsets approach in which a single set of offset boundaries is defined for each part with the intention of establishing a control on the relationships between functional surfaces. The use of reference datums as defined in the standards is abandoned. Consequently, the approach proposed here departs considerably from standard practices.

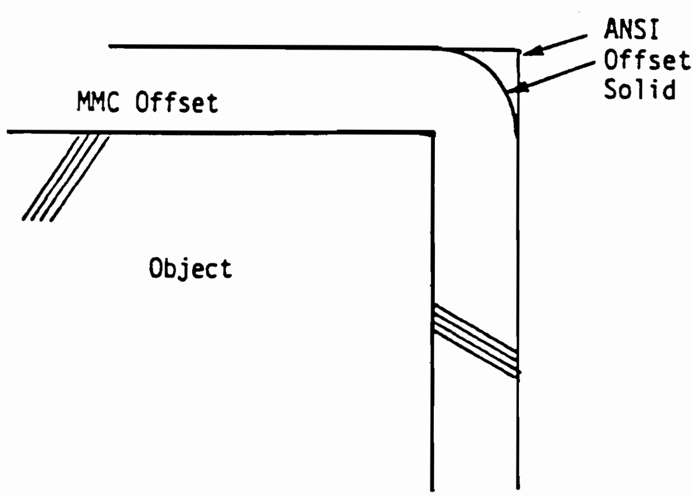


Figure 2.11 Offset Zone vs. ANSI Zone [Etesami 1987]

2.2.2.7 Virtual Boundaries Requirements

Jayaraman and Srinivasan [1989] addressed the problem of representing geometric tolerances that arise from assembly and material bulk requirements. They called such requirements the part Virtual Boundary Requirements. The virtual boundary here is synonymous to the virtual condition defined in ANSI. The virtual condition of a feature is defined in ANSI to be the theoretical limit boundary of an FOS when the combined effects of all associated tolerances are taken into account [ASME 1983]. The product functional requirements are formulated in terms of the spatial relationships to be satisfied by each part with respect to a rigid collection of virtual half-spaces (see Figure 2.12). The virtual half-spaces are an abstraction of the relevant surface features of the mating parts. The virtual half-space of a part will be the complement of the virtual half space associated with the part mating component. The surfaces of each part are to be as close in contact as possible to this rigid collection of virtual half-spaces without violating its boundary. Such close contact requirements are informally referred to in ANSI as datum requirements. The physical solid constructed in terms of the virtual half-spaces is equivalent to the part functional gages. The material bulk in critical portions of the part is maintained in a similar manner by ensuring the volume occupied by the part boundaries.

Since the tolerance values assigned to the assembly components are derived from this virtual boundary requirements which is formulated based on the product functional requirements, the proper functioning of the final assembly is assured. Part mating is also assured since each part is constrained to be on its own side of the virtual boundary.

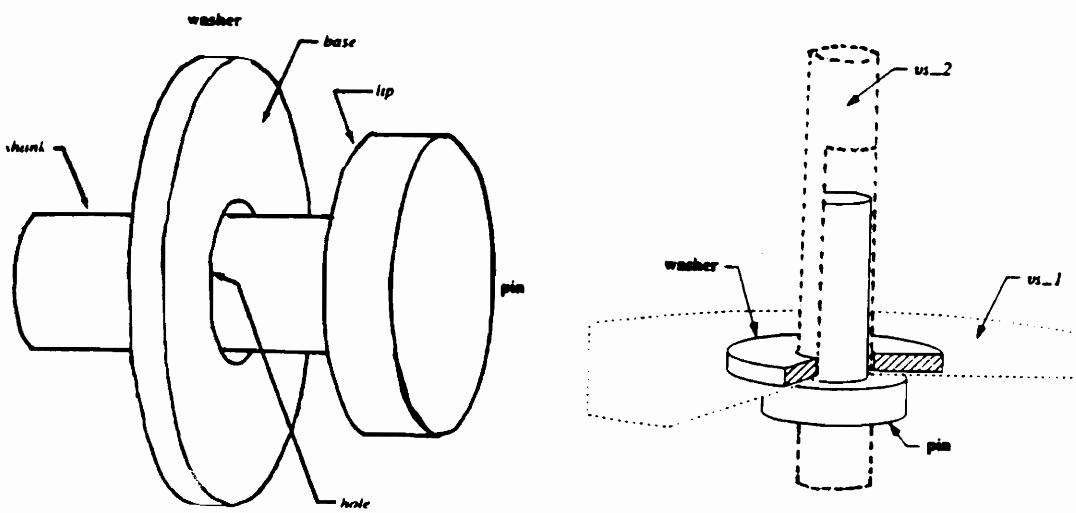


Figure 2.12 Virtual Surfaces [Srinivasan & Jayaraman 1989]

The main difficulty encountered here is in the derivation of the individual part tolerances from these virtual boundary requirements [Srinivasan & Jayaraman 1989]. The conversion procedure is complex, and no general conversion rules are available. Extensive research would be needed to derive the different types of tolerances associated with different types of product functional requirements. In cases where a part mates with two or more parts in the assembly, the spatial relationships of the part features with the corresponding virtual half-spaces will have to be satisfied simultaneously. Turner [1991] pointed out that the part virtual condition, which is a composite tolerance, is not an adequate representation of the part functional requirements that dictate separate independent tolerances of size, orientation, position, and form.

2.2.3 Relative Positioning For Assembly Modeling

The extensive literature pertaining to the subject of relative positioning can be found in the area of mechanisms design and robotics assembly research [Lee & Gossard 1985] [Liu & Nnaji 1991]. User specified spatial relationships that express the desired relationship between component parts are used to derive the final position and orientation of the components in the assembly [Lee & Andrews 1985] [Sodhi & Turner 1994]. Each spatial relationship can be interpreted as a constraint imposed on the degree of freedom between relative mating or interacting features. If enough constraints are specified, a rigid assembly results. A mechanism results if any of the degrees of freedom are left unconstrained. Typical spatial constraints are the against relationship, which constraints one planar feature to be coplanar with another, and fits, which constrains one cylindrical feature to fit inside another, aligning the two features along their common axis. These spatial constraints are used to compute the relationship between the component local coordinate system to the assembly reference frame. This information can be used to analyze the functioning of the mechanism or to plan the sequence of the product assembly [Liu & Nnaji 1991].

The significance of relative positioning for tolerance analysis has been discussed by several researchers. Minnichelli [1983] models the position relationships of one part relative to another as an algebraic relationship among the model variables of two parts. Fleming [1988] represents the geometric constraint on the relative positions of the tolerance zones of part features in terms of algebraic constraints in the form of equality and inequality functions. In Turner's [1987] approach to tolerance analysis, the exact constraints of the assembly are specified in terms of the contact between two surfaces of adjoining parts. For two planar features, the against assembly constraint is translated into the requirement that the two surfaces be coplanar, two edges, one from each surface to be collinear, and two vertices to be coincident.

The above relative positioning scheme deals primarily with parts with perfect form geometry. Some are able to cope with the variations in the orientation of part surfaces or features, but not with surfaces of imperfect form [Scott & Gabriele 1989]. With actual parts possessing inherent variations in size and form, the theoretically exact constraints on the nominal parts may not be exactly constrained. It is theoretically impossible to manufacture parts which have surfaces that are identically coplanar or axes that are exactly aligned.

Implying that a point on one part remains in contact with a point on another part is unreal, particularly when variation of the part features does not allow the two points to meet.

2.3 Commercial Systems

A good review of the capabilities of commercially available, model-based tolerance analysis software packages can be found in Turner and Gangoiti [1991]. The Variational Simulation Analysis package [VSA 1987], marketed by Applied Computer Solutions, is a wireframe-based system. This package allows the user to develop a 3D part model by specifying the coordinates for a set of points on the part. A point may be specified at an absolute spatial location, or it may be specified relative to other points. Variational models are constructed by associating probability distributions with the model variables associated with the coordinates of the points.

The user specifies an assembly sequence in terms of mating relationships between the defining points of the parts. The user must then provide a procedure to compute the design function of interest as a function of the coordinates of the specified points of the assembled parts. All of this information is specified in a textual format without the use of an interactive CAD system. At each iteration, a Monte Carlo simulation technique is used to generate values for the model variables directly from specified probability distributions. These are used to compute new coordinates for the part-defining points. The assembly procedure specified by the designer is used to simulate the parts assembly process. Finally, the design variables are measured, and statistics are computed.

In defining the model, the user must have a good understanding of how the actual manufacturing processes will affect the location of each modeled point and how the variations will be distributed. The manner in which model variables are associated with the model are determined by the user. Consequently, the software cannot generate variations, except as directed by the user. Each variation of interest must be modeled by the user explicitly. Thus, the system can only detect tolerance problems anticipated by the user. Another drawback is that it may not be easy for the user to model an assembly directly in terms of relationships between point coordinates or to specify appropriate probability distributions. Since the data structure is essentially wire-frame, the system is limited to problems involving simple geometric design constraints. The Assembly Variation Simulation System (AVSS) [Turner & Gangoiti 1991] marketed by John Deere and Company, adopts essentially the same approach as the VSA package.

The feature-based solid modeler, Pro-Engineer, marketed by Parametric Technology [Pro/Engineer 1989], is equipped with a 3D geometric tolerance analysis module. The tolerance representation scheme is based on the parametric representation scheme discussed earlier in Section 2.2.2.1.

2.4 Discussions

The analytical approach to tolerance analysis is tedious, and error prone. In addition, since the tolerance functions are derived from the dimensional loops, geometric tolerances with no associated dimensions cannot be accommodated. The model-based approach relieves the user from the difficult procedure of formulating the necessary tolerance functions for each of the design functions that needs to be analyzed. However, there is a need for the part tolerance specifications to be represented in a computer comprehensible format. In addition, an assembly model is needed to enable the propagation of the individual part variations. Proposed tolerance representation schemes have been reviewed, and their strengths and weaknesses have been identified.

A tolerance representation scheme should be able to model the whole range of tolerance specifications embodied in the ANSI standard Y14.5 since this represents the currently accepted industrial tolerancing practice. In addition, since manufacturing is statistical in nature, it is desirable that the tolerance representation scheme can support statistical tolerance analysis. However, it is apparent from the discussions in the preceding sections that proposed tolerance representations have not been successful in fulfilling these criteria. They either fail to provide the necessary variational coverage, violate the semantics of ANSI tolerances, or do not support statistical tolerance analysis. In all but the feasibility space approach proposed, the tolerance representation schemes attempt to represent only the zones implied by the tolerance specifications. No representation of the actual part or feature surfaces is available.

For inspection purposes, the modeling of tolerances based on the tolerance zone defined by perfect form geometry is sufficient. A part conforms to the tolerance specification if the dimension of the measured feature falls within this tolerance zone. For the purpose of assembly tolerance analysis, however, the modeling of the characteristics of the actual surface geometry is needed. Tolerance analysis based on the virtual condition, or the MMC and LMC part envelope, is essentially a worst-case analysis. In addition, perfect form representation will not be able to accurately model the mating conditions between actual parts within the assembly.

Since the whole concept of tolerancing is based on the observation that parts produced by existing manufacturing processes are imperfect in nature, a model of the manufactured part should not be constructed using perfect form geometry. A geometric modeling scheme capable of representing the range of possible manufacturing process variations is needed. The generated variational part models should reflect the characteristics of the manufacturing processes used while satisfying the part tolerance specifications. Consequently, the modeling scheme should provide appropriate means for geometric manipulation and evaluation to constrain the geometry of the variational part models within the appropriate ANSI tolerance zones.

The part assembly model is needed to model the effect of the interaction and the accumulation of the variations of the individual components on the assembly design functions. In order that the variation of each components is propagated properly, parts/features must be positioned relative to adjacent mating parts/features. This will assure that their positions and orientations are compliant to the variations in adjacent parts.

CHAPTER 3
RESEARCH METHODOLOGY

This chapter describes the methodology used in developing a model-based approach to two-dimensional (2D) statistical tolerance analysis. Geometric variations as allowed by the part tolerance specifications are modeled based on the characteristics of the manufacturing processes to be used in their fabrication. The part tolerance specifications define the zone within which individual part features must lie. Instances of the manufactured parts are generated using the templates of user specified processes and then assembled using a relative positioning scheme. Relevant assembly tolerance information can be obtained by querying the assembly model directly.

The developed model-based approach to 2D statistical tolerance analysis was realized following four major tasks:

(1) Design of data model with relevant feature information for the attachment of tolerance information. Since ANSI tolerances are feature based, appropriate part features must be available for tolerancing purposes. In addition, the data model must also support the use of datum reference frames and tolerance qualifiers.

(2) Modeling of instances of 'in spec' manufactured parts using user specified processes templates. The geometry of the manufactured parts are constrained within the tolerance zones constructed per the ANSI Y14.5 standard specifications.

(3) Creation of an assembly model to propagate the effects of the interaction between individual component variations within the assembly. In this model, each of the assembly components is positioned relative to the adjacent component(s) features based on the user specified mating requirements and assembly sequence.

(4) Measurement and tabulation of the user indicated design functions.

Details of each of these tasks undertaken are presented in subsequent sections. Section 3.1 describes the developed data structure. Section 3.2 details the tolerance representation scheme developed. The assembly modeling technique used and the assembly data structure are described in section 3.3. Section 3.4 describes the user design functions measurement procedures and the system output.

3.1 Data Structure

The ANSI Y14.5 standard provides a rich class of tolerances which can be specified on low level geometric elements such as part edges, and on higher level geometric elements such as Feature Of Sizes (FOSs). FOSs are defined as features of a part with size parameter and tolerance, such as cylinders and parallel planes. In 2D representation, these features can be reduced to parallel lines. To accommodate the different types of tolerances defined by ANSI, a boundary model of the nominal part is used. Tolerance information is attached to relevant geometric features as attributes. Tolerance information, such as orientation and position, which requires the specification of appropriate Datum

Reference Frames (DRFs) have these attached as attributes. DRFs are a set of reference features identified on the part and are used for feature location.

The proposed part data structure is given in Figure 3.1. Each part node has pointers to its bounding loops. Each loop node, in turn, points to the part edges and each edge node has pointers to the vertex nodes defining its end points. Analogously, each vertex and edge node has pointers to the edges and the part it belongs to, respectively. FOS nodes have pointers to the lower level geometric elements and vice versa. Tolerance information can be attached to both the edges and FOSs. Geometric tolerances specified on FOSs are implicitly applied to the feature-derived entities. For instance, orientation and position tolerances applied to a slot feature are interpreted as the allowable orientation and position error on the feature axis. These feature-derived entities are treated as properties of their respective FOSs. A size tolerance specified on a FOS of two parallel line segments applies to the separation between these lines. Each datum node has pointers to the appropriate datum feature(s) which can either be an edge or an FOS.

3.2 Manufactured Part Modeling

For the modeling of the manufactured parts/features, two requirements were taken into consideration:

(1) Models of the manufactured parts must satisfy the parts tolerance specifications. This requires that each of the manufactured parts

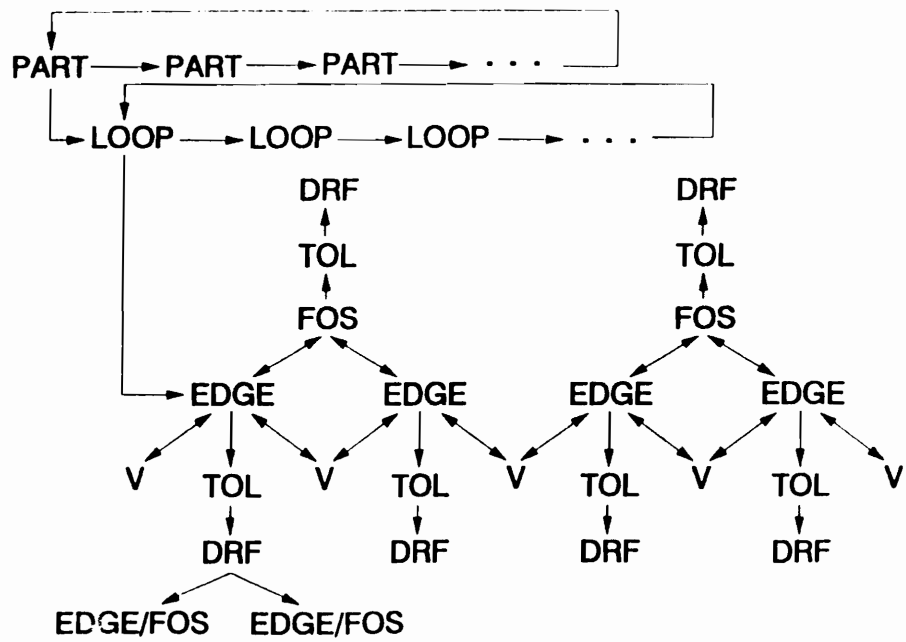


Figure 3.1 Data Structure

features be contained within the tolerance zones defined by the feature tolerance specifications.

(2) The modeling scheme used must be able to represent the range of geometric variations characteristics of the manufacturing processes used. In addition, since manufacturing processes are stochastic in nature, the modeling scheme should be able to accommodate the different process distributions as well.

In subsequent sections, the approach taken to model instances of the manufactured components satisfying the component tolerance specifications is presented.

3.2.1 Modeling of ANSI Tolerance Zones

Consistent with ANSI definition, tolerance specification is interpreted here as constraints that define the zones within which the manufactured part features must be constrained. The width, location, and orientation of these zones are dependent on the geometry of the features being toleranced, the type and value of the tolerances, and any applicable tolerance qualifiers (ASME 1982). The method used in modeling the zones implied by the major types of ANSI tolerances, the tolerance modifiers, and datums, are discussed in the following sections.

3.2.1.1 Size Tolerance

The tolerance zone associated with the allowable size variation is defined by the largest and smallest value of the parameter associated with the FOS (see Figure 3.2). For a circular feature, size variations applied on the magnitude of the radius will define a tolerance zone consisting of two concentric circles. The location and the orientation of the zones are unconstrained. For the set of two parallel edges, variation is applied to the separation between the two edges. For an internal FOS, the largest and smallest size parameter value corresponds to the Least Material Condition (LMC) and the Maximum Material Condition (MMC) of the feature. For an external FOS, the opposite is true. The feature MMC and LMC defines the feature envelope, or the extent of the feature.

3.2.1.2 Geometric Tolerances

The tolerance zone implied by a form tolerance specification applied to an edge is defined by two perfect form edges separated by the magnitude of the tolerance value (see Figure 3.3). The position and orientation of this zone are constrained only by the tolerance zones defined by the orientation tolerance and the part envelope (given by the size tolerance of associated FOS). This zone is computed by offsetting the nominal part feature. When specified on a FOS, these tolerances apply to the feature-resolved entities. The feature resolve entities are computed from their corresponding FOSs, and the tolerance zones associated with these entities (center planes, axes, and centers) are constructed in a similar manner.

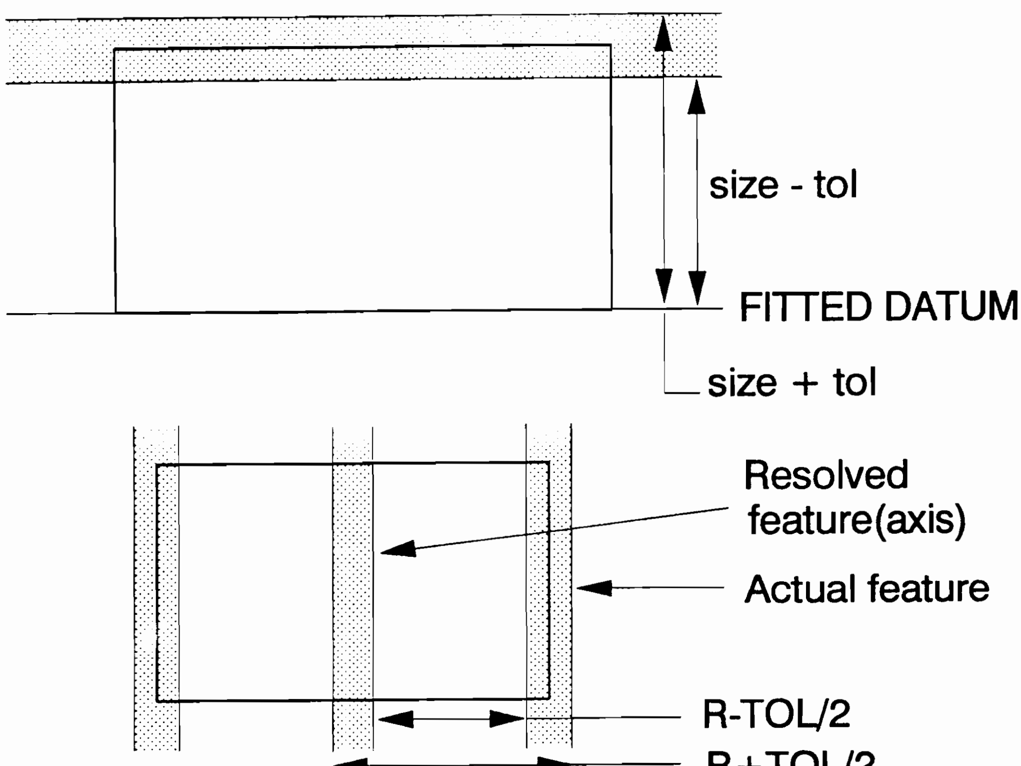


Figure 3.2 Size Tolerance Zone

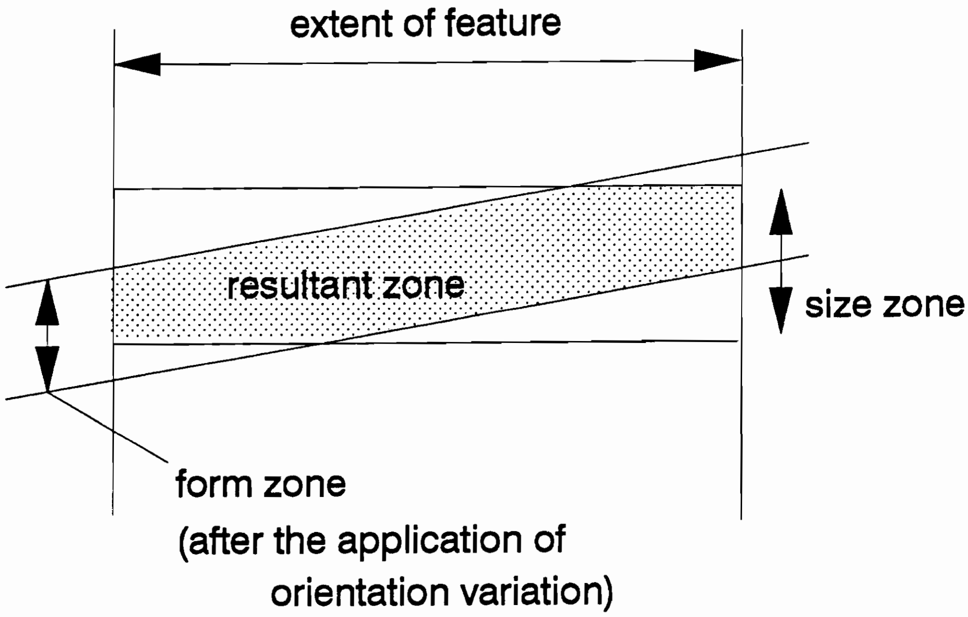


Figure 3.3 Geometric Tolerance Zone

The zone associated with orientation tolerances is similar to those of form tolerances, with the exception that the orientation of this zone is fixed with respect to the specified datum. For position tolerances, both the orientation and location of the tolerance zones are fixed relative to the specified DRF.

3.2.1.3 Modifiers and Datums

The material condition modifiers are treated as special attributes of a tolerance specification. The effective tolerance value specified is computed based on the actual size of the FOS. The theoretically perfect datums which constitute a DRF are derived from the appropriate non-perfect datum features through fitting procedures. The use of fitting procedures to simulate datums is consistent with the manner components are set up and manufactured. These fitted datums simulate the machine or fixture surfaces used in the component fabrication or inspection. The fitting procedure used here is presented in Section 3.2.3.2.

3.2.2 Modeling of Manufacturing Variations

To fulfill the variational modeling needs outlined in Section 3.2, B-splines are used. A B-spline curve is the continuous map of a collection of intervals $U_0<\ldots<U_L$ into three-dimensional Euclidean space, where each interval $[U_i, U_{i+1}]$ is mapped onto a polynomial curve segment. Each real number in U_i is called a breakpoint or a knot. The collection of all U_i are called the knot sequence. The degree of the curve can be arbitrarilly determined to satisfy different modeling needs. The shape and position of this curve can be controlled in a predictable

manner by manipulating the degree of the curve, the knot sequence, and the positioning of the curve control points. The ensuing discussions will be limited to the use of B-spline curves for the modeling of manufactured part geometries in 2D. The same concept is extendible to the use of B-spline surfaces for the modeling of 3D geometries.

3.2.2.1 Quadratic B-Spline Curves

The characteristics and flexibility of B-spline curves lend themselves well to the modeling of manufactured part features. Curves of different orders can be used to model the different degrees of waviness or other forms of cyclic deformations due to the inherent variations in manufacturing. A "three lobe" condition often encountered in external cylindrical grinding, for instance, can be represented using piecewise curves.

In addition, it is characteristic of B-spline curves to approximate the control polygon, and be contained within the convex polygon defined by these control points (see Figure 3.4). Consequently, the location and form of these curves can be easily constrained within the tolerance zones implied by the individual tolerance specifications by constraining their convex polygons within these zones. A quadratic, or second order, B-spline curve, in particular, interpolates the end points of its control polygon and is tangent to successive edges joining the curve control points.

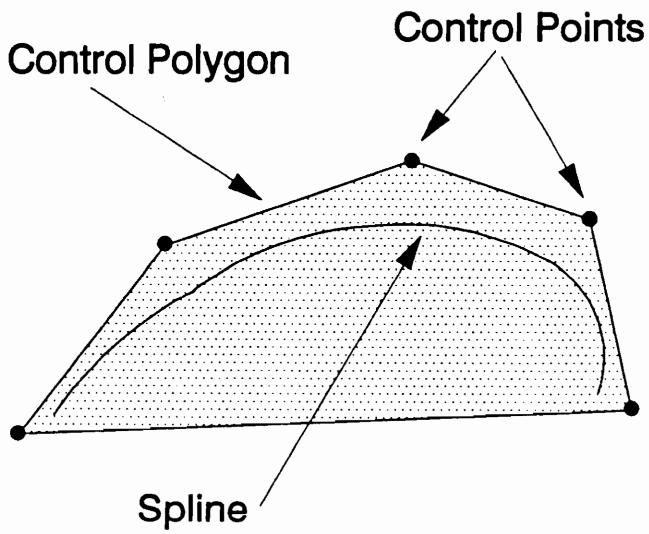


Figure 3.4 B-spline

A quadratic B-spline is shown in Figure 3.5. The tangent of the curve at the end point is given by the slope of the line connecting the last two points of the control polygon. This property allows the tangent and continuity conditions between adjoining curves to be handled conveniently. The perpendicularity or tangency condition between two edges can be maintained by constraining the end point of the control polygon to vary only perpendicular to, or along, the adjacent edge, respectively. The cylindrical surface of a shaft or a hole can be conveniently modeled using closed curves, or by joining several curve segments together. By making the successive edges of the control polygon, which the resulting curve touches, to lie on the extent of the feature tolerance zone, the defined curve will occupy the extent of the feature tolerance zone. In the next section, the use of B-splines for the modeling of manufacturing process variations is discussed.

3.2.2.2 Modeling Of Processes Characteristics

Each manufacturing process possesses characteristic variabilities which impart specific types of variations to the geometry of the parts manufactured. For example, besides the variations in the diameter, a manufactured shaft may be deformed as shown in Figure 3.6. To facilitate the modeling of the different manufacturing processes variational characteristics, process templates are used. These templates are defined by characteristic points which correspond to the control points of the B-spline curves. The template for a barreled shaft, for instance, is given in Figure 3.7. Based on the anticipated part deformation, the appropriate process template can be selected and used in the part/feature instantiation. The manufactured part features are created by instantiating these

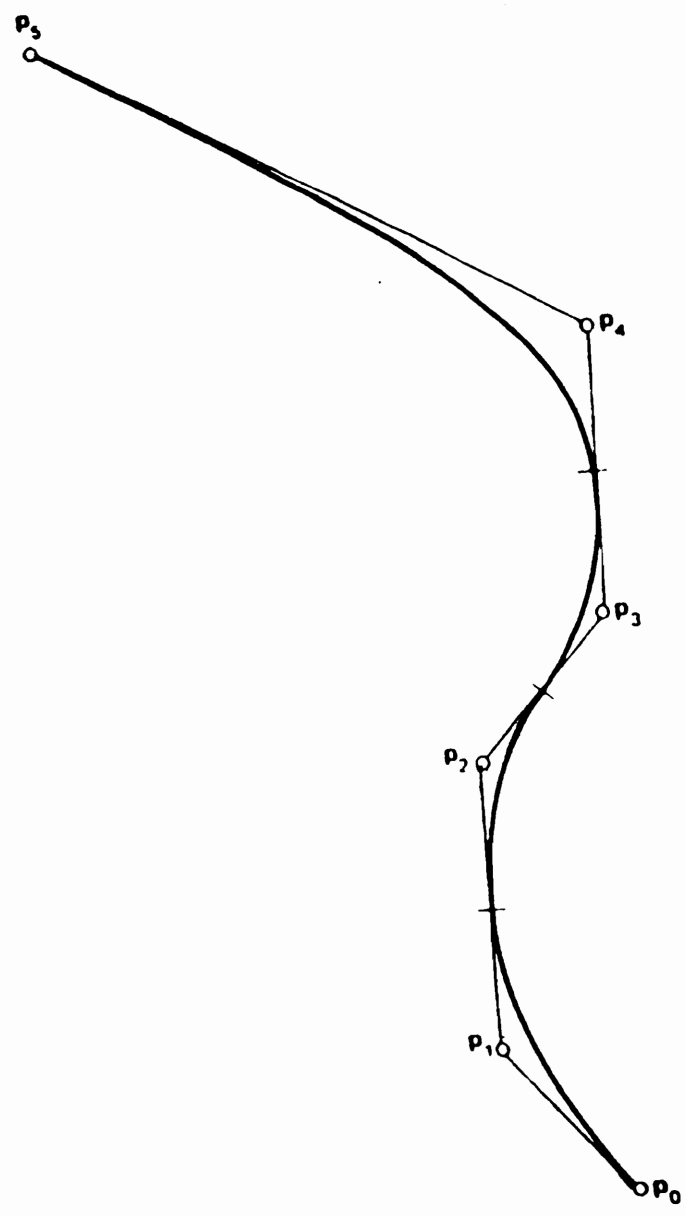


Figure 3.5 A Quadratic B-spline Curve

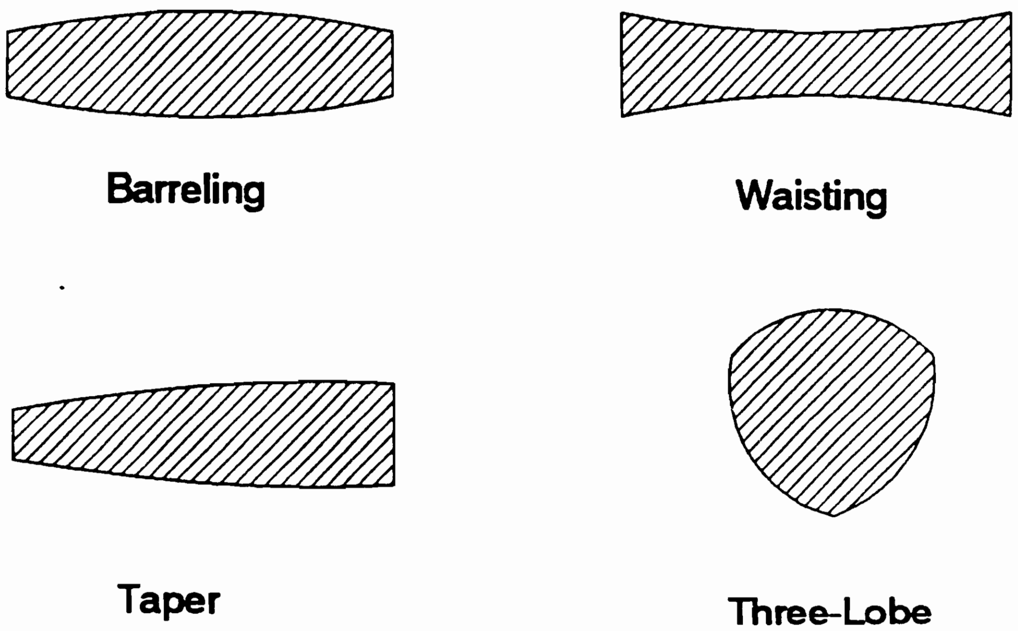
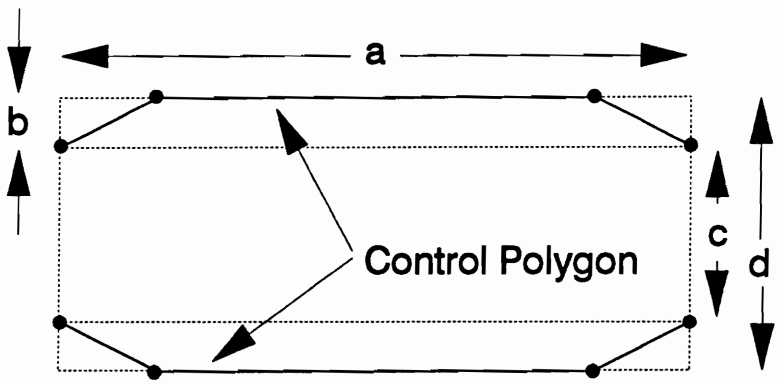


Figure 3.6 Manufacturing Form Errors



a = Extent of feature

b = Form tolerance value

c $\geqslant$ Basic size - size tolerance (LMC)

max d = Basic size + size tolerance (MMC)

min d = Basic size - size tolerance + 2*form tolerance

Figure 3.7 Template for a Barrelled Shaft templates, and positioning them within the feature tolerance zones. Where applicable, the part DRFs and the material condition modifiers are used to modify and locate the resultant tolerance zone implied by the tolerance specification. The construction of the feature tolerance zone and the procedure used in the feature instantiation is outlined in the following sections.

3.2.3 Modeling of an "In-Spec" Component

To generate an instance of a manufactured component, each feature of the component is simulated individually. The procedure outlined below is used:

(1) The tolerance zone associated with the feature is constructed. The shape of this zone is based on the shape of the nominal feature geometry. The location and orientation of this zone is determined relative to the appropriate DRFs. Any tolerance modifiers and process capability distributions are taken into account here to compute the resultant tolerance values.

(2) Based on the manufacturing process to be used or the anticipated form errors, the appropriate feature template is selected. The template parameters are then instantiated to the feature nominal parameters and tolerance values.

The order in which the different types of tolerances are taken into account, and the order in which the different features are instantiated are considered next.

3.2.3.1 Tolerance Precedence

The order in which the different types of tolerances are taken into account are based on the tolerance precedence. Size tolerance specifications will have precedence over form tolerance since form tolerance is a refinement of size. These are followed by position tolerance which has precedence over orientation tolerance since orientation tolerance is a refinement of position. The resultant or composite tolerance zone obtained defines the region within which the manufactured part feature is constrained.

For features of size controlled by tolerances applied to both its actual and derived feature entities (i.e., straightness tolerance applied to the axis and cylindricity tolerance to the surface of a hole), the tolerances on the actual feature are applied first. The actual size of the manufactured feature is then used to determine the allowable tolerance on the derived feature entities. For example, the position tolerance of a hole specified with a MMC qualifier is increased by the amount the actual hole size deviates from the MMC. The actual allowable position tolerance can only be ascertained after the actual feature size is known. This is consistent with the procedure outlined by ANSI for the computation of bonus tolerances.

3.2.3.2 Feature Precedence

Consistent with the manner in which components are manufactured, the component datum features are simulated first. The appropriate DRFs are then constructed by fitting a perfect form geometry (duplicate of the nominal feature) to the datum feature. Theoretically, a component surface will only be in a three point contact with the machine or fixture surfaces. It follows that, in 2D, a spline will only be in a two point contact with a straight line. The datum features here consist of spline curves. The primary datum will have a two point contact with the primary datum feature/spline. To identify these contact points the datum line is rotated into the primary datum spline as shown in Figure 3.8. Each new contact point identified will be used as the new pivot of rotation until the end point of the spline is brought in contact with the datum. The two consecutive pivot points that are spaced the farthest apart will be the points of contact between the datum and the datum features/spline.

The secondary datum is perpendicular the primary datum, and forms a single point of contact with the secondary datum feature/spline. Is is constructed by sliding a line perpendicular to the primary datum line into the secondary datum feature of the part. The primary and secondary datums constructed are then used as the reference frame for the positioning of other feature tolerance zones. The procedure used their construction are consistent with ANSI specification.

Using the above feature instantiation process, instances of the manufactured parts generated will reflect the characteristics of the processes used. The conformance of these parts to the tolerance specifications is assured. The whole range of tolerance classes defined in ANSI can be modeled. The use of fitting procedures to simulate the datum is consistent with the manner parts are setup and manufactured.

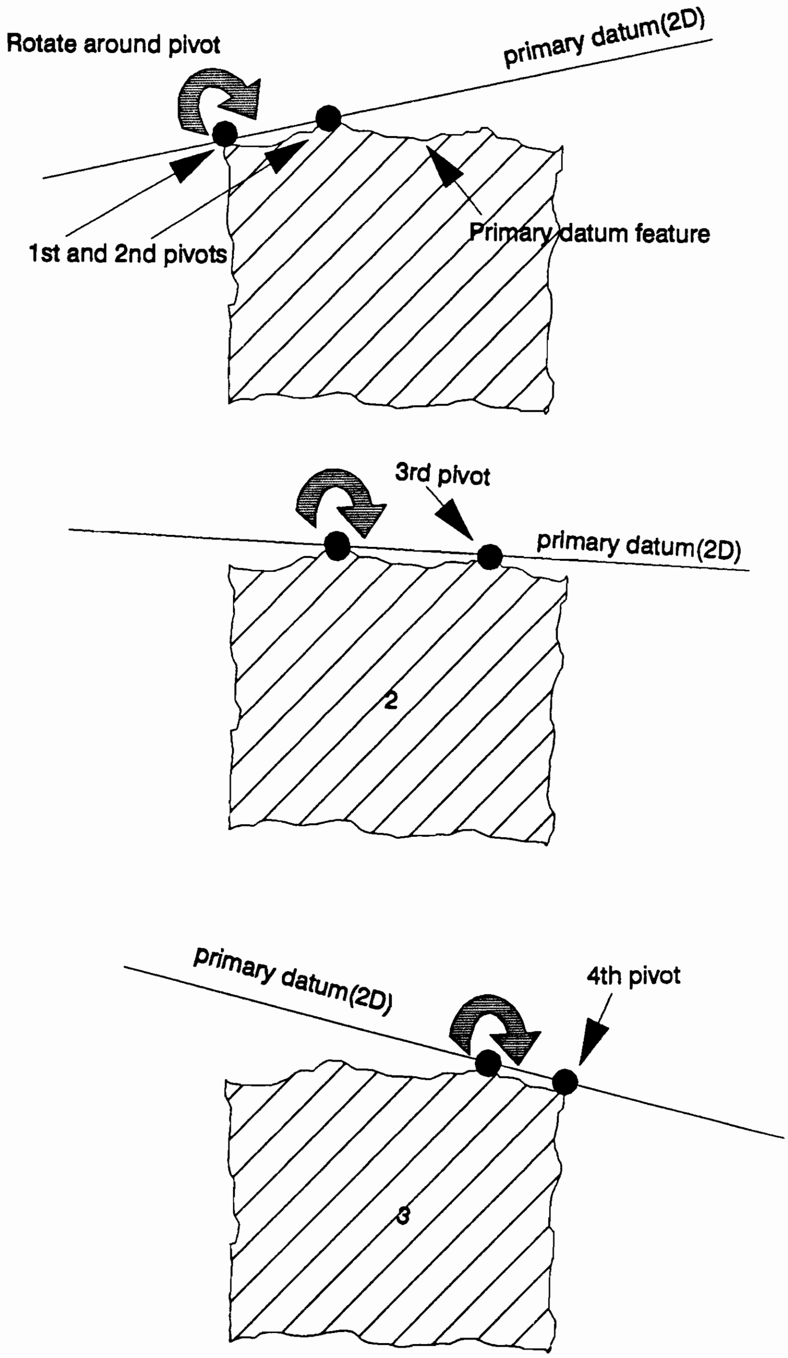


Figure 3.8 Primary Datum Requirement

For assembly tolerance analysis, the product assembly model must be constructed. The assembly model must be able to model the interaction between the mating part features, and propagate the individual part variations within the assembly. In the next section, the assembly modeling scheme used is presented.

3.3 Assembly Modeling

In constructing the product assembly model, a relative positioning scheme is used to allow the position and orientation of a component to be compliant to the variations of adjacent components. This relative positioning constraint, together with a non-interference requirement, resembles the physical mating conditions within an actual assembly. The assembly data structure is shown in Figure 3.9. Mating nodes have pointers pointing to the mating feature pairs. In addition, each mating node points to the next mating node. The list of mating nodes defines the assembly sequence of the components making up the assembly. The use of a sequential assembly process is implicit in this representation.

In 2D representation, each mating node has pointers to two pairs of mating features, designated as primary and secondary mating feature pairs. The primary mating requirement designates the two features from adjacent parts which should be in close contact with each other. The secondary mating requirement designates the features which should be brought together while preserving the relationship between the primary mating feature pairs. These

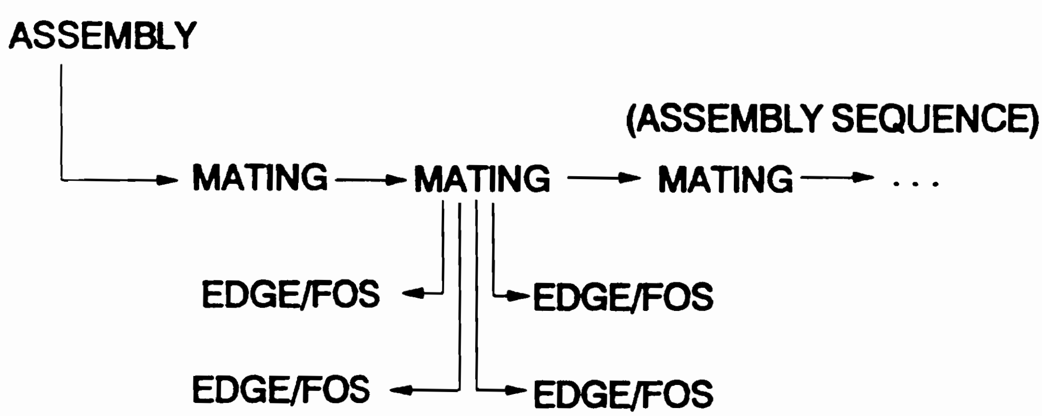


Figure 3.9 Assembly Data Structure

mating requirements resemble the primary and secondary datum requirements of ANSI.

With respect to the geometric modeling scheme used here, an 'against' relationship between two parts entails the mating between arbitrary B-spline curves. For the top block in Figure 3.10 to rest on the bottom block, the two mating splines will touch each other at the minimum of two points. The against relationship here corresponds to a primary mating constraint between the two assembly components. In a secondary mating relationship, only a single point of contact between the mating splines need to be established.

It should be apparent that the points of contact between the splines are functions of the relative positions (in the horizontal direction) of the two splines. A change in the relative position will affect the location and orientation of the target block since the contact points between the two mating splines would have changed. In establishing the secondary datum requirement, the target part may need to be translated into, or away, from the secondary datum feature of the base part. In preserving the primary mating requirement established earlier, the target part has to be translated along the spline of the base part primary datum. The contact points between the two primary datum features must be re-established following each translation along the primary mating datums.

Since no analytical method for the solution of these points of contacts is available, an iterative solution procedure is used. In the procedure used here, the top block is designated the target while the bottom is designated the base. The points of contact between the two mating splines can be identified by rotating the target spline into the base spline, as shown in Figure 3.11. Each

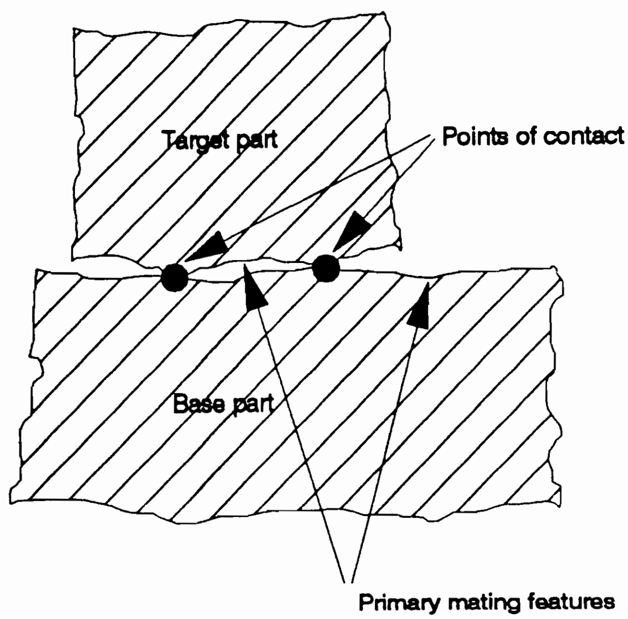


Figure 3.10 Primary Mating Requirement

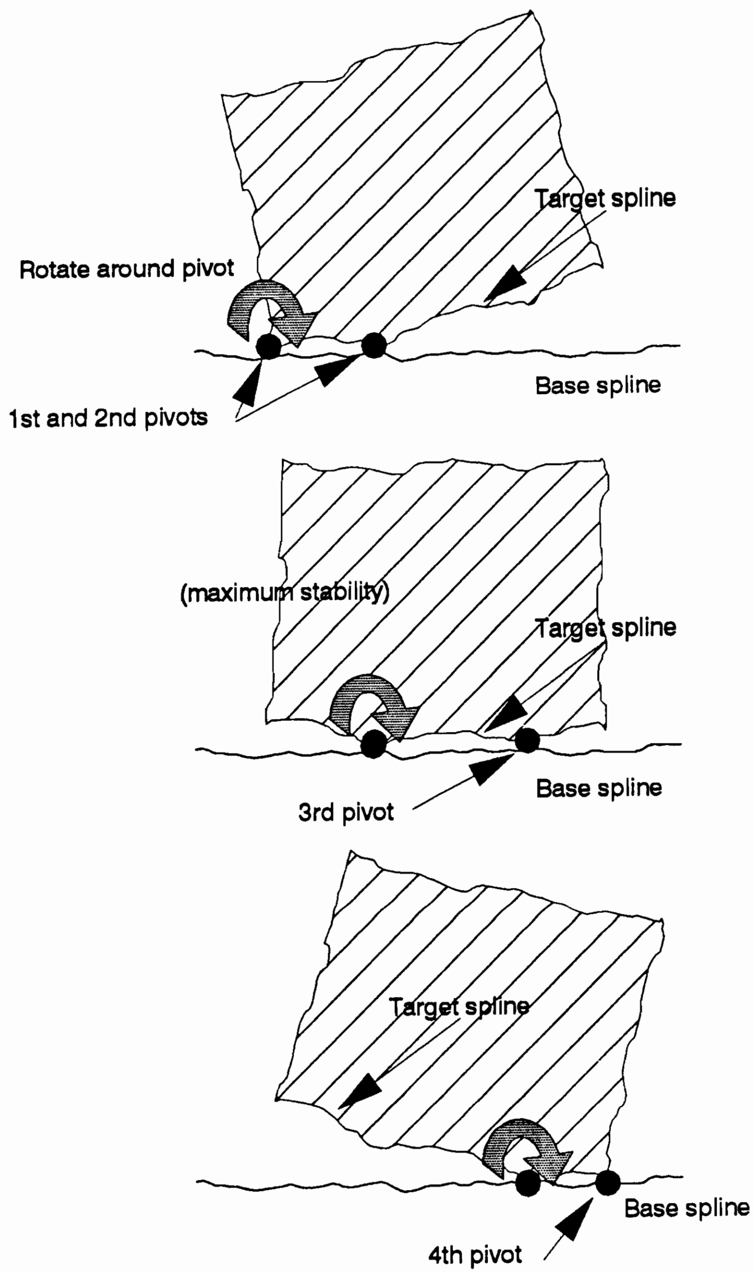


Figure 3.11 Identification of Primary Mating Points

new contact point identified is used as the pivot point of rotation until the end point of either curve is brought in contact with the other. To provide greatest stability to the target block, the two consecutive pivot points that are spaced the furthest apart are the contact points between the parts. Based on this information, the actual location and orientation of the target block are computed. Relevant assembly design functions can then be queried from this assembly model. Multiple design functions may be assessed simultaneously without the need for the formulation of any tolerance functions.

3.4 Measurement of Design Functions

The magnitude of the user defined product design function(s) is measured directly from the assembly model without the need for the formulation of any tolerance chains. These measurement results are then tabulated and relevant statistics presented to the user.

For the purpose of measurement, the first edge selected by the user is designated the reference edge. Measurements are always taken perpendicular to the nominal edge of the reference edge. The nominal edge is used here since the direction of the actual edge is ill-defined.

For design functions resembling a FOS of two parallel lines, the overlapping segment is computed. Measurements are then taken perpendicular to the reference edge at the limits of the overlap. For design functions consisting of parallel but non-overlapping edges, the distance between the closest edge endpoints is measured perpendicular to the nominal reference edge. If the edges involved are not parallel, the angle between them is computed instead.

In the following chapter, the details of the prototype software implementing the concepts outlined here are presented.

CHAPTER 4
SOFTWARE PROTOTYPE

In this chapter, the implementation details of the prototype 2D statistical tolerance analysis package, STOLA, are presented. STOLA is developed to aid designers in the specification of functional and manufacturable tolerances. Thus, it is imperative that the tool is integrated into a CAD environment, interactive, and able to accept the existing CAD model as input.

In Section 4.1, the overall system architecture is presented. The operational structure of STOLA is presented in Section 4.2. Section 4.3 takes the reader through a step by step analysis of an example assembly and is intended to be a guide for users of the software.

4.1 System Architecture

STOLA was designed to integrate with CATIA through the Interactive User Access (IUA) interface. CATIA is a commercially available CAD software package marketed by Dassault Systems, Inc. The CATIA software used here runs on the IBM RISC/6000 workstation, under the AIX operating system. The CATIA/IUA interface is a programming environment which supports a FORTRAN-like interpretive programming language to allow quick development of user applications. The actual library functions used in the development of all the CATIA interactive functions can be accessed through this interface. The CATIA CATGEO and mathematical routines, and other user routines developed in high level languages can also be interfaced with programs developed in this language. CATGEO and CATIA mathematical routines are FORTRAN library functions which can be called to manipulate the CATIA database. Further details about the CATIA IUA, CATGEO, and mathematical subroutines can be found in the CATIA User Manuals.

The overall system architecture is shown in Figure 4.1. The input to STOLA is a 3D CATIA geometric model created in the X-Y plane (i.e., the z-coordinates of all the geometric elements equal zero). In effect, the part geometries are two-dimensional. Three-dimensional geometries are used here in anticipation of STOLA being extended to address 3D analysis. The CATIA CATGEO and mathematical library functions are used extensively to perform geometric entities creation, transformations, intersections, and other model database manipulations.

The software user interface is developed by invoking the appropriate IUA routines and utilities. For instance, the PANEL utility was used to create the windows for accepting user input. In addition, the system locator and keypad are also utilized. User input, in addition to the part models, is needed because the part models, in this case, only consist of low level geometric entities such points and lines. Higher level information, such as features, topology, assembly and tolerances, are needed in the analysis. The nature and use of this information will be discussed in the next section.

STOLA has been implemented in the C++ programming language. In order for it to be integrated with the CATIA IUA, the program object code must first be linked with the appropriate CATIA libraries using a utility provided to

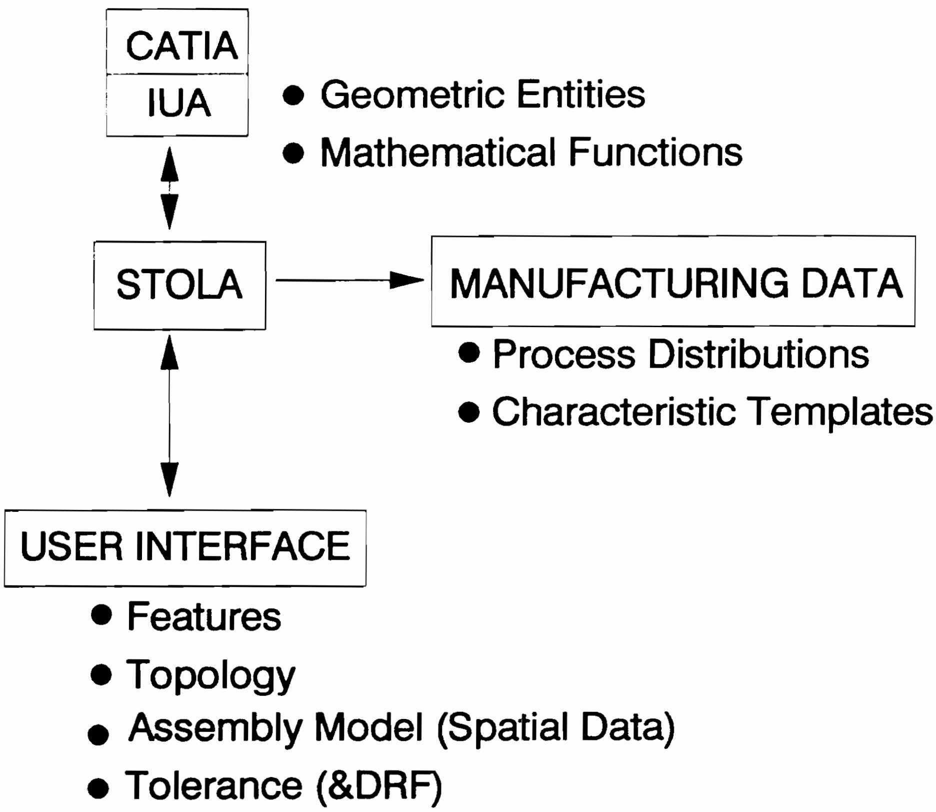


Figure 4.1 Software Architecture

create a Dynamically Linked Library (DLL) function. The details of the program structure are given in the next section.

The manufacturing datafile contains information on the different process templates and distributions which can be used in the simulation of the manufactured part features. This information is contained in external data files accessible to the main analysis module. In the present implementation, two different process templates and two different processing capability distributions (Normal and Uniform) are provided. Additional process templates can be developed as needs arise.

4.2 Prototype Software Operational Structure

The software flow diagram is shown in Figure 4.2. Upon the activation of STOLA, the user is prompted to provide all the non-geometric information needed but unavailable in the part model. Appropriate messages are displayed in the message display area to aid the user during these interactive sessions. Panels are provided to accept user input. In addition to the system keyboard, the keypad and locator are also used. The keypad is used to input YES/NO information in response to the software prompts. The locator is used for selecting the model geometric features on the screen. It should be noted that the word ‘feature’ is used here, and in the software, to refer either to an edge or an FOS. In the step-by-step process which follows, the type of user input information needed, its input format, why it is needed, and other applicable instructions are presented.

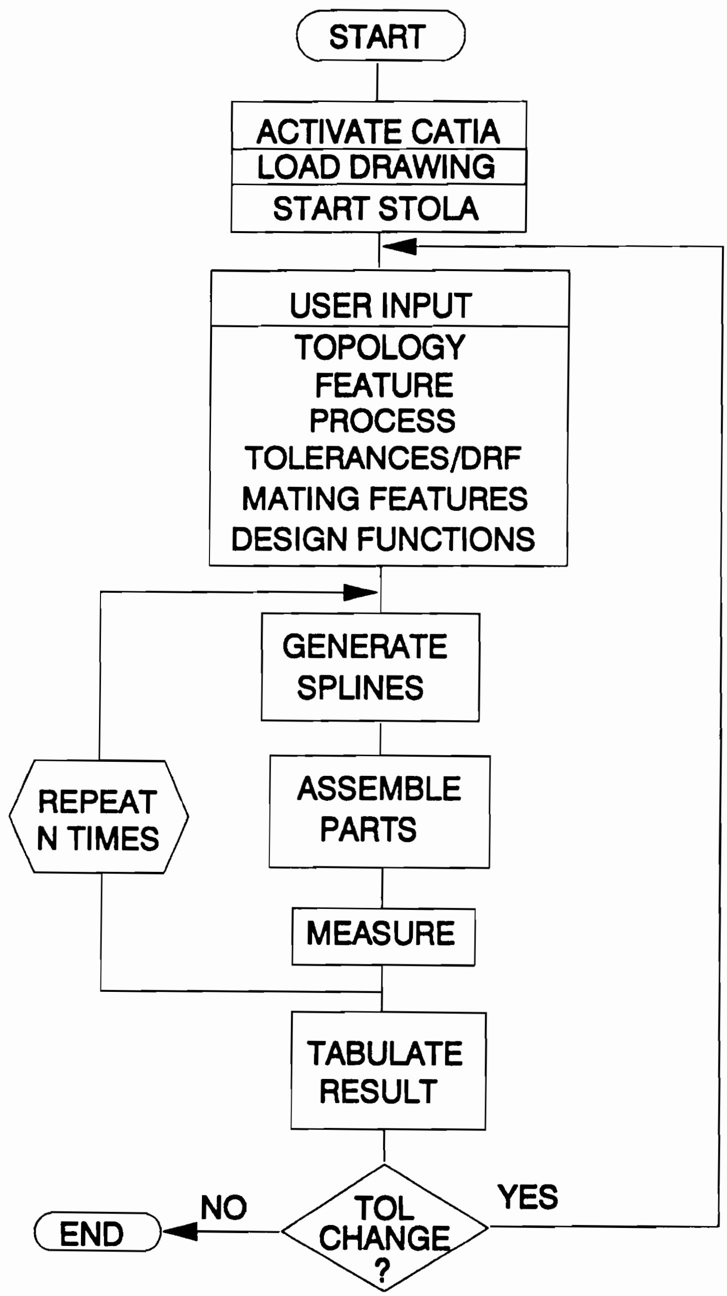


Figure 4.2 Flow Diagram

(1) *Activate STOLA*. At this time the part model should have been created or loaded and displayed on the screen. The software will proceed to automatically identify all the lines present in the part model. For checking purposes, the number of lines found will be recorded.

(2) *Topology Information*. The part topology information is need to establish the material side of the component parts. This information helps to determine if a feature is an internal or external FOS. The user is required to select two consecutive edges of each part in the counter clockwise (CCW) order. Each of the selected edges will be highlighted.

(3) *Default Datums, Tolerances, and Process Templates*. The ANSI Y14.5 standard specifies that each feature of a part must be fully constrained (toleranced). Since the user may only be interested in tolerancing certain aspects of the design which are critical to the functioning of the design, default tolerances, datums, and process templates are provided and applied to all the other part features. The default tolerances, modifiers and process templates are presented in a panel. This allows the user to change these values as needed. For the default datum specifications, the user is prompted to select two features (edges or FOSs) to be designated as the primary and secondary default datums. After the selection of the first edge, the software will prompt the user for the second parallel edge in the attempt to identify an FOS to be designated as the primary datum (similarly for the secondary datum). If the datum is just an edge, the select mode should be terminated by pressing the NO key on the keypad.

Consistent with ANSI definition, all primary and secondary datums selected must be perpendicular to each other

(4) *Edge Tolerances.* This step is optional since all the edges identified in step (1) have been assigned the default information specified in step (3). To re-specify this information, individual edges must be selected and relevant information changed. The same interactive session as described in step (3) will ensue.

(5) *FOS Tolerances.* To tolerance an FOS, two parallel edges must be selected. The procedure used here is similar to the tolerancing of edges. In addition to the form and orientation tolerances specified on edges, size and position tolerances can also be specified.

(6) *Mating Features and Assembly Sequence.* To construct an assembly model, the component parts must be assembled. To assemble two parts, a primary and a secondary features pairs must be selected. The primary mating requirement signifies a two point contact between the feature pair, and the secondary mating requirement signifies a single point contact between the feature pair. In this implementation, only mating pairs of edges are supported. Two pairs of mating features must be specified for each part to be assembled.

(7) *Design Function(s).* Lastly the user can select the pair(s) of edges designating the design function(s) they want measured. For parallel edge pairs, the separation between them will be measured. For non parallel edge pairs, the angle between them will be measured. Theoretically, any

number of design functions can be specified. The current implementation allows the maximum of two design functions to be specified at any one time.

(8) *Design Function Measurement*. Measurements of the user-indicated design functions are perform automatically. The design function will be highlighted and applicable statistics (mean and range) of the measurement results presented. If more than one user design function has been specified, the user will be prompted to press a key to display the next measurement statistics.

After all the information collected in the steps above has been processed, instances of the parts satisfying the tolerance information are generated utilizing the appropriate process templates and distributions. Instances of these parts are then assembled based on the mating information provided. The user specified product design functions are measured directly from the assembly model constructed. The steps of manufactured part modeling, assembling, and measuring of the assembly design functions can be repeated a specified number of times. The user can then re-iterate through the analysis procedure with different tolerance values, if so desired.

The software structure and algorithms have been validated by comparing the software outputs with manual calculations. Without the inclusion of geometric variations, the analysis result corresponds to results from conventional tolerance analysis. When geometric variations were allowed, disparity in the results were observed.

In the following section, the procedure presented here is applied toward the analysis of an example assembly.

4.3 A Sample Analysis

The analysis of the sample assembly shown in Figure 4.3 is outlined here. The part components are to be assembled as shown in Figure 4.4. The assembly design function to be measured is indicated by DF. To follow the steps taken in performing this sample analysis, it will be helpful if the procedure outlined in Section 4.2 is consulted.

To establish the default primary and secondary datums, the user is prompted to select two edges which are perpendicular to each other. In this exercise, E1 and E8 are selected. For the establishment of the default tolerances, process template, capability distribution, and modifiers, default values will be provided by the system. A panel displaying all the system defaults are presented to the user. At this point the user has the option of accepting the system defaults or specifying new information. The user can change any of these values by selecting the corresponding panel cells using the locator. Appropriate messages are then displayed at the bottom of the screen to aid the user.

To tolerance the individual edges, the edge must be selected. A panel is then presented, displaying the current values of the edge tolerances and process information. This information is just the default values established in the previous step. With the use of default values, there is no need for the user to specify all the information needed. The user only needs to provide the

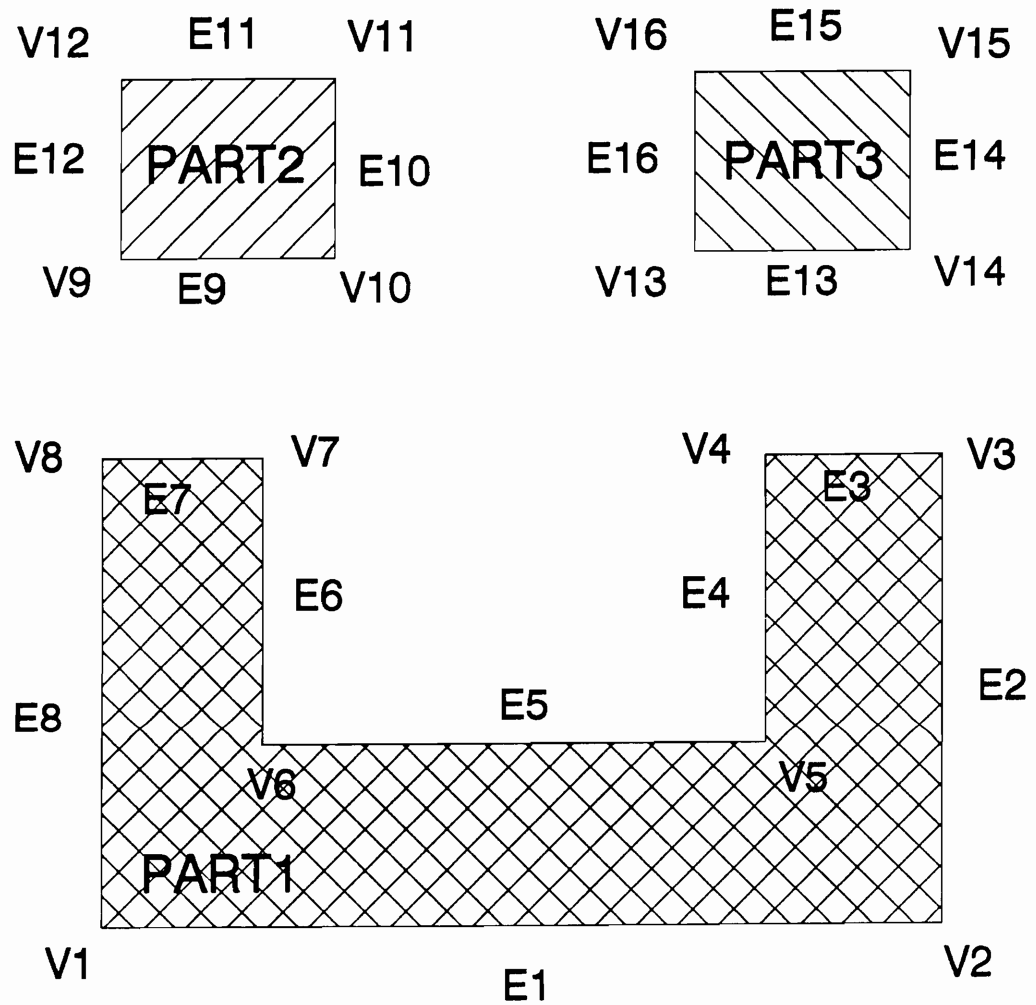


Figure 4.3 Component Drawing

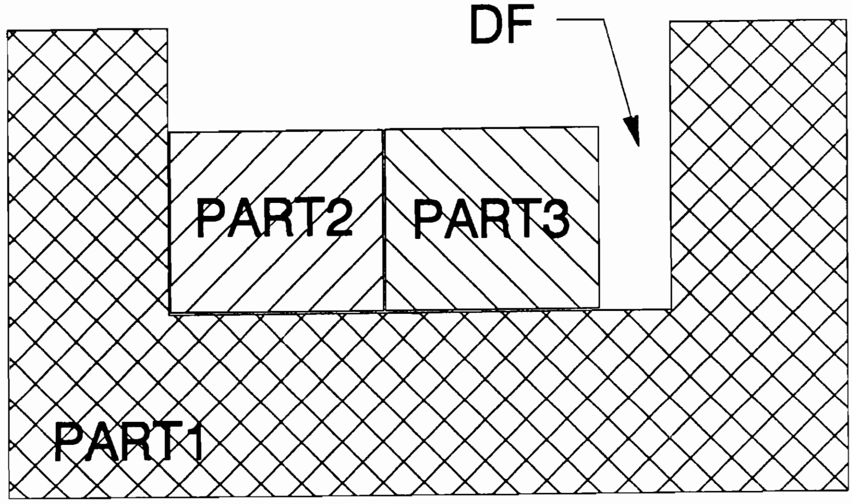


Figure 4.4 Assembly Drawing

information tolerance values or other information which differs from the default. For this exercise, only the tolerances of the edges which mate with adjacent parts are modified (refer to Figure 4.4). These edges are E6, E5, E12, E9, E10, E16, and E13. The form tolerances of these edges are reduced to 0.03 from the 0.04 default value. This was done by selecting the form tolerance field of the tolerance panel using the locator, and entering the new value of 0.03. Their orientation tolerances, the applicable datums, and the process templates were left unchanged.

The same circumstances apply to FOSs. For the example here, three FOSs are selected and re-toleranced. These are FOSs consisting of E6 and E4, E12 and E10, and E16 and E14. The size tolerances of these FOSs were changed to 0.04 from the default of 0.05. The other default information was left unchanged.

To construct the assembly model shown in Figure 4.4, two mating objects are defined. In the first, the primary mating feature pair consisted of E5 and E9, and the secondary mating feature pair consisted of E6 and E12. In the second, the primary mating feature pair consisted of E5 and E13, and the secondary mating feature pair consisted of E10 and E16. Based on this mating information, PART1 and PART2 will be assembled first. The spline of E9 is brought into a two point contact with the spline of E5, and then a one point contact between the splines of E12 and E6 is established (while preserving the two point contact between E5 and E9). Similarly, in assembling PART3, a two point contact between the splines of E5 and E13 was established, followed by a one point contact between the splines of E10 and E16 (while preserving the two point contact between E5 and E13).

The design function (DF) to be measured here is the gap between E4 and E14. In this case, the overlap between the two splines generated will approximate the extent of the E14 spline. The measurements taken are the average horizontal distance between vertices V14 and V15 and the spline of E4.

CHAPTER 5
CONCLUSIONS AND FUTURE WORK

5.1 Concluding Remarks

The ability to perform tolerance analysis provides designers with a rational means for assigning component tolerances. Tighter than necessary tolerance values can be avoided, resulting in products which are both functional and manufacturable. Potential manufacturability problems can be detected early, thus eliminating the high cost associated with design changes performed late in the product life cycle.

Currently, the only method available for the analysis of the product geometric variations is prototyping. For the automation of the analysis procedure, the part tolerance information must be represented in a format suitable for computer interpretation. Previously proposed tolerance representation schemes have suffered either from inadequate variational coverage or departure from the established ANSI tolerancing standards.

Toward this end, a tolerance representation scheme capable of modeling the range of tolerances defined in the ANSI Y14.5 standard in a format to facilitate automated tolerance analysis has been proposed. One unique feature of this representation is the use of B-splines for the modeling of form variations. To provide an accurate characterization of the geometric characteristics of the manufactured part features, the use of process templates was introduced.

The representation scheme can also take into account the distribution characteristics of the manufacturing processes used to enable statistical tolerance analysis. The capability for statistical tolerance analysis is critical in probabilistic design methods which attempt to develop robust designs that lessen the product sensitivity of performance to variations in environment, as well as manufacturing. The ability to perform statistical tolerance analysis is thus central to the effort to develop cost-effective and high-quality products.

For assembly tolerance analysis, a relative positioning scheme capable of modeling the interaction between the part mating splines was used to propagate the individual part variations within the assembly. This enables the tolerance stackup on the assembly design function(s) to be measured automatically, without the need to formulate any tolerance functions.

5.2 Recommendations for Future Work

It is assumed here that the product nominal geometry is valid and unambiguous, and that the tolerances specified are complete and non redundant. Techniques and tools for verifying the validity and unambiguity of a part model and the well-formness of the tolerance specifications are well documented [Mortenson 1985] [Requicha 1980] [Bernstein & Preiss 1989]. The check for the validity and unambiguity of the part models are provided by most solid modelers available in the market today. Tools for the specification and verification of the part tolerances per ANSI standards are also commercially available. Consequently, the integration of the developed tolerance analysis routine with such systems is desirable.

In the present prototype, parts with circular features are not addressed. This is not a severe limitation in 2D analysis since circular features such as bores and shafts can be reduced to FOSs consisting of two parallel edges. Nevertheless, this limitation needs to be addressed to increase the software robustness. Moreover, the analysis procedure proposed here should be extended to incorporate three-dimensional geometries. For manufactured parts modeling purposes, this extension should not pose to be a problem. B-spline surfaces can be used in place of the curves used here for 2D analysis. The manufacturing process templates can be modeled in a similar manner. The part data structure can be easily expanded to incorporate Face (surface) entities.

However, further research would be needed to extend the assembly modeling technique used here to address three-dimensional part models. Instead of the two point contact between the primary mating features, a three point contact is needed, together with a two point and one point contact between the secondary and tertiary mating features. Approximation techniques may need to be used to establish the relative positions between mating parts [Sodhi & Turner 1994].

A datafile of process templates and capability distributions representing the variational characteristics of a larger class of manufacturing processes should be made available to enable a more accurate modeling of the actual manufactured parts and assembly. Conceivably, such information could be synthesized from the inspection data obtainable from Coordinate Measuring Machines. At the present time, very limited information on such process models is reported in the literature. It is the author's hope that the demonstrated need of such information in this work will encourage their compilation.

Such information will be increasingly demanded as researchers strive to provide more accurate and realistic product models within the CAD environment to achieve true software prototyping capabilities. The developed tolerance representation scheme is intrinsically well suited to support such undertaking.

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