Chapter 1

Introduction

1.1 Motivation

Inflatable structures possess special properties like lightweight, minimal stowage volume, and high strength-to-mass ratio. These remarkable properties make them suitable for a cost effective large space antenna, which provides high resolution and large frequency bandwidth. Figure 1.1 shows a schematic of the Inflatable Antenna Experiment (IAE) conducted by NASA in 1996. The structure, which was carried in a container of size $7' \times 3' \times 1.5'$ by the Space Shuttle, when inflated attained the shape of an inflatable reflector of diameter 50 feet attached to three 92-foot long struts (Dornheim and Anselmo, 1996). Except for a few difficulties in the inflation and deployment phase, the flight was successful overall (Freeland, 1997). There are several possible applications of inflatable structures, e.g., optical mirrors, communication antennas, entry and landing systems, habitats, solar sails, and solar collectors. An inflated torus is an integral part of these structures. It can be seen in Fig. 1.1, which shows a 50-foot diameter inflated torus surrounding the reflector of IAE, and in Fig. 1.2, which shows an inflatable torus surrounding a space vehicle for possible use as a landing system in an interplanetary mission.

Fig. 1.1: Inflatable antenna experiment conducted by NASA in 1996.

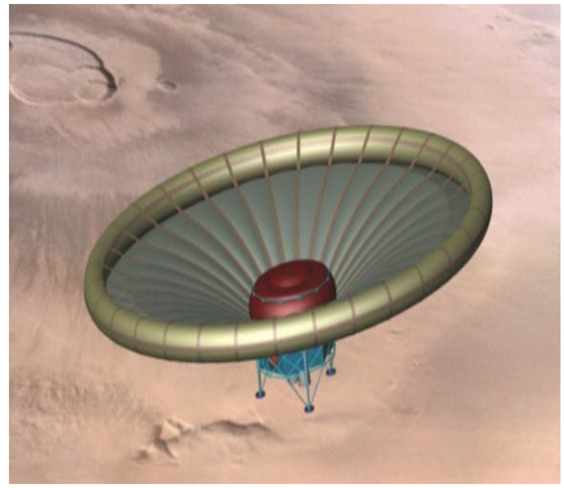

Fig. 1.2: An artist's concept of a landing device made of inflatables.

The inflated antenna, which attains the shape of a paraboloid of revolution due to its internal pressure, must maintain a desired surface accuracy so as to focus all the incoming light and microwave on a receiver. Like any other satellite structure, inflatables are also subjected to vibrations, which could be caused by meteoroid impacts, thermal shock, satellite repositioning, unbalance of onboard rotating gyroscope, etc. This necessitates an understanding of the dynamics of an inflated torus. The adverse effect of satellite vibration is that it distorts the surface of the antenna, making it dysfunctional for a certain amount of time. Apart from the mechanisms of maintaining the surface accuracy of the antenna, one possible way to reduce the vibration would be to control the vibration of its main support structure - the inflated torus. Motivated by these facts, our goal in this research is to perform vibration analysis of an inflated torus and to present a methodology for its vibration control.

Traditional methods of active or passive vibration reduction will increase the weight and maintenance cost of the inflatable structures. Actuators and sensors made of coupons of piezoelectric materials are lightweight and conformable to curved surfaces. For this reason, they do not bring any significant change in the dynamic properties of the host actuators. Moreover, the piezoelectric actuator and sensor do not involve any internal mechanical or electric components for their functioning; rather, actuation and sensing capabilities come from the very nature of the material. This has the potential for reducing the maintenance cost of the inflatable satellites. One of the limitations of the thin film piezoelectric materials is the amount of force it can exert. Since the inflatable structures are relatively less stiff compared to the conventional composite and metal structures, piezoelectric materials could be sufficient to excite them.

From the above discussion, one can conclude that vibration reduction of an inflated torus will help in reducing the overall vibration of the inflatable antenna. Furthermore, for inflatable structures, piezoelectric material coupons can be used as actuators and sensors due to their lightweight, conformability to the curved surfaces, and low maintenance requirements. Therefore, we devote this study towards understanding the dynamic characteristics of an inflated torus and its active vibration control using piezoelectric actuators and sensors.

1.2 History of Inflatable Space Structures

The origin of inflatable structure, also called gossamer structures, dates from the late 1950's to the mid-1960's, during which Goodyear Corporation developed the idea of inflatable structures for search radar antennas, radar calibration spheres, and lenticular inflatable parabolic reflectors (Freeland, 2000). Each of these three projects contributed in the overall gossamer structure effort. The inflatable search radar antenna used rigidizable truss members and a metallic mesh to form the surface of the aperture. From the project, developed concepts included: the fabrication, assembly, and alignment of inflatable elements; techniques for attaching inflatable support members to precision mesh surfaces; and deployment techniques for a complex, inflatable structure. The radar calibration sphere, six meters in diameter, was constructed from multiple hexagonal-shaped membrane panels. It contributed to gossamer spacecraft by demonstrating: thin-film processes; high-precision panel assembly; inflation of a large, thin-film structure; and metalization of thin film (for high radio frequency reflectivity). Finally, Goodyear Corporation's lenticular inflatable parabolic reflector consisted of a reflector supported by a torus. Concurrent with Goodyear

Corporation's work, NASA Langley Research Center and NASA Goddard Space Center were developing the first passive space-based communication reflector, Echo I. On August 12, 1960, a Delta rocket deployed the Echo I communications sphere at an altitude of 1610 km. After Echo I's successful deployment, emphasis was placed on very large baseline interferometry (VLBI) axisymmetric reflector antennas, offset reflectors, and sunshade supports for large sensors and telescopes. During the development of these technologies in the early 1970's through the late 1980's, L'Garde, Inc. made important contributions. These technological achievements throughout the 70's and 80's led to the NASA sponsored In-Space Technology Experiments Program (IN-STEP) Inflatable Antenna Experiment (IAE) that flew on May 29, 1996 (Fig. 1.3).

Fig. 1.3: NASA's Inflatable Antenna Experiment fully deployed in orbit.

According to Freeland (2000), the IAE was a success in that it (1) demonstrated the fabrication of a large, space-worthy inflatable structure for about 1 million dollars, (2) demonstrated very efficient mechanical packaging by stowing a 14 x 28 m inflatable structure in a container of an office desk size, (3) produced a reflective surface accurate to a few millimeters (rms), and (4) demonstrated the robustness of deployment. A timeline emphasizing the major contributions to gossamer technology is shown in Fig. 1.4. It is expected in 2002 and 2003 that NASA will attempt experiments with the Solar Orbital Transfer Vehicle, equip Deep Space 5 satellite with a solar sail, and launch the NOAA Geostorm, which will use a solar sail to remain at sub-orbital speeds between the earth and sun. Further, by 2008, NASA expects to launch ARISE (the Advanced Radio Interferometer between Space and Earth), which will be equipped with a 25 m inflatable antenna (Dornheim, 1999).

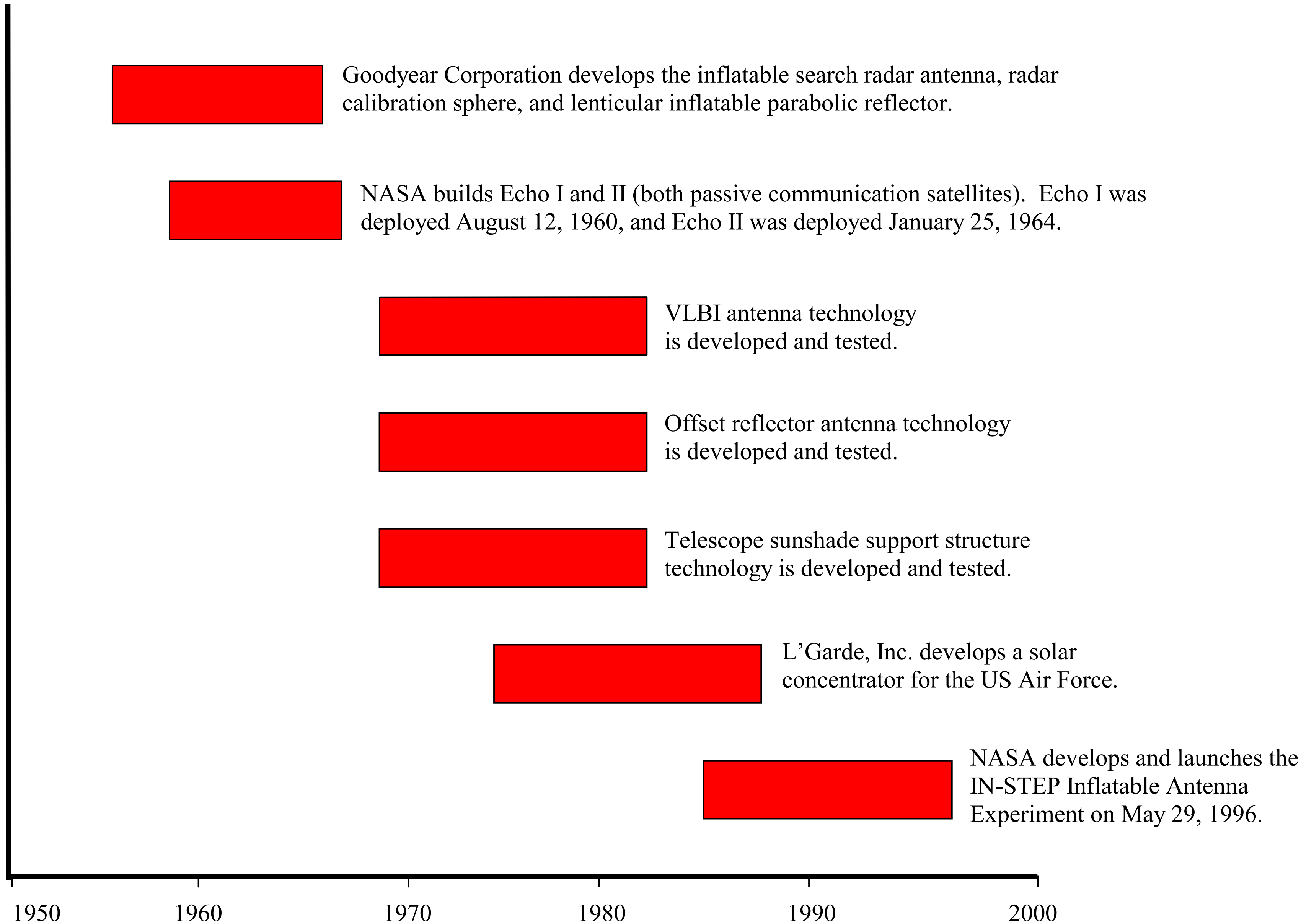

Fig. 1.4: A timeline showing the years of research contributing to the growth of gossamer technology

1.3 Objectives

As discussed earlier, an inflated torus serves as an important structural component for inflatable structures (Figs. 1.1, 1.2). It was also pointed out that the vibration of these structures must be suppressed for their proper functioning, and piezoelectric materials could be used as actuators and sensors. This sets the goal of this research as to develop a methodology for the vibration control of an inflated torus using piezoelectric actuators and sensors. The following paragraphs point out the relevant issues to be resolved in order to achieve this final goal.

An understanding of the vibration characteristics of the inflated torus is needed before attempting the control problem. This leads to finding the natural frequencies and the mode shapes of the structure via a free vibration analysis. The major source of strength of inflatables lies in its internal pressure, and hence it is important to include the effects of pressure on the vibration of the inflatable torus. Along with this, the effects of pressure and geometry changes on the natural frequencies and mode shapes are to be understood.

In order to use the piezoelectric actuators and sensors, it is important to quantify the actuation and sensing behaviors of piezoelectric films attached to the inflatable torus. In the same line, it is also important to know how well the actuators can affect the vibrations in different modes and how well the sensors can sense vibrations in different modes. Since inflatable structures are very light and flexible, the inertia and stiffness effects of actuators and sensors are required to be understood as well. Several types (e.g., Lead Zirconate Titanate (PZT) and Polyvinylidene Fluoride (PVDF), Macro-Fiber Composite Actuator (MFC™)) and several configurations (e.g., unimorph, bimorph) of actuators and sensors are possible. To this end, a study is needed to quantify the capabilities of different kinds of actuators and sensors and to select the most suitable one.

One of the limitations of a piezoelectric material film is the amount of force it can exert. Hence, it is important to optimize the locations and sizes of the actuators so that the required control effort is minimum. Similarly, to obtain good signal to noise ratio, sensors should be chosen to provide maximum output for the vibration in the modes of interest. These problems become more critical as the number of actuators and sensors and the complexities of mode shapes increase as expected in the case of an inflated torus. The goal here would be to find optimal locations and sizes of the actuators and sensors to maximize control and sensing authorities.

After finding the natural frequencies, mode shapes, actuator and sensor models, and their proper sizing and placements, the vibration control problem can be attempted. Given the complexities of the inflated structure and its interaction with actuators and sensors, mathematical models may not capture the phenomena very well. Moreover, the satellites are often subjected to varying environmental conditions, e.g., changing temperature due to different amount of sunlight. This causes differences in the mathematical model and the real system, known as model uncertainty. There could also be some unknown disturbances acting on the structure. To this end, the designed controller should be robust so that the vibration suppression could be achieved in the presence of model uncertainties and disturbances.

In this research, we try to resolve the above-mentioned issues that will lead to understanding of a series of related problems regarding the vibration control of an inflated torus.

1.4 Literature Survey

1.4.1 Vibration Analysis of Inflated Toroidal Shell

Toroidal shells have curvatures in two principal directions. This causes coupling between bending and stretching actions of a load and gives rise to a smaller deflection when compared to beams, plates, and cylinders. This makes the toroidal shells capable of carrying higher loads. Due to internal pressure, an inflated torus possesses moderately high strength even if its wall thickness is very small (of the order of a few microns). These properties make an inflatable toroidal shell (torus) a practical and interesting research topic. In the following paragraphs, we review analytical and experimental studies on the vibration of an inflated torus.

Timoshenko (1940) presented static analysis of an inflated torus using linear membrane theory. While this classical solution gave an acceptable stress distribution, the solutions for displacements contained singularities. In an effort to remove the displacement singularities and to find a more accurate analysis, Jordan (1962) used nonlinear membrane theory. The results obtained by Jordan eliminated the incompatibility of the displacements. He also showed that while the linear stress resultant in the meridional direction (hoop stress) is similar to that obtained from the nonlinear membrane analysis, the circumferential stress resultant may contain significant error. Reissner (1963) suggested that either one needs to consider bending stiffness or nonlinear membrane theory or both to resolve this problem. He used a more general form of equations of which the linear bending theory and the nonlinear membrane theory can be derived as a special case. He also suggested the ranges of parameters in which these modifications are valid. In 1963, Sanders and Liepins presented an alternative solution using nonlinear membrane theory. They used asymptotic integration and arrived at conclusions similar to those presented earlier by Jordan. Colbourne and Flügge (1967) resolved the issue of the displacement discontinuities by considering it as a singular perturbation problem involving the so-called prestress parameter that depends on the internal pressure, Young's modulus, shell thickness, and the internal radius of the torus.

Some studies regarding pressurized arches, which resemble incomplete inflated tori, are worth mentioning here. Dietz et al. (1969) tested various inflatable arch models and demonstrated the effects of wrinkling on the deformation of the structure. Kawaguchi et al. (1972) performed experiments on pressurized arches and studied the effect of internal pressure on the bending stiffness of the fabric skin members. Steeves (1978) analyzed inflatable arches and found the critical loading before the onset of wrinkling as well as the resultant deformation of the arch under external loads. Mohan (1997) developed a three-noded flat triangular shell element and applied it to membranes and arches stiffened by internal pressure. As a shell with applied pressure load deforms, the direction of the force changes since the pressure tends to act normal to the surface during the deformation. This is called the follower action of the pressure force. He included this effect in the formulation of the triangular shell element and calculated the deflected shapes of inflatable arches due to concentrated and distributed loadings. Molloy (1998) analyzed a pair of inflatable arches subjected to different wind and snow loading using finite element code ABAQUS. Using Sanders' shell theory and the Rayleigh-Ritz method, Plaut et al. (2000) calculated the displacements of a pressurized arch due to various types of snow and wind loading. While the cross-section was assumed circular, the centerline of the cross-section was chosen to be arbitrary.

Buckling and stability of inflated tori have also drawn a considerable amount of attention. Lukasiewicz and Balas (1990) examined the collapse of an inflated toroidal membrane under concentrated loads. Li and Steigmann (1995) studied wrinkling of finitely deformed toroidal membranes using tension-field theory. The strain energy function was modified in order to account for wrinkling and to avoid unstable compressive stresses. Papargyri-Pegiou (1995) studied the stability of a compressible, nonlinear, elastic toroidal membrane based on finite deformation. He assumed that the radius of the tube is much smaller than the radius of the torus in order to simplify the equations of equilibrium, and found the internal pressure of the torus when it collapses. Using geometric nonlinearity, Raouf and Palazotto (1996) analyzed the effects of centrifugal force arising from the spinning of an inflated torus. They found that the differences in the in-plane stresses from linear and nonlinear analyses are larger for internal pressure than for external pressure. They also found significant differences in the stresses when compared to those of Timoshenko (1940) for the case of a stationary torus. Combescure and Galletly (1999) studied plastic bifurcation buckling of an inflated torus with an elliptical cross-section. They also studied the post-buckling behavior of the inflated torus by adding an initial imperfection, and found the post-bucking behavior to be stable. Studies of the buckling of a toroidal shell under external pressure can be found in Blachut and Jaiswal (2000) and the references mentioned therein.

Study of dynamic behavior of satellite structures is particularly important as they are subjected to a variety of time-varying loadings. A free vibration study is needed in order to obtain the natural frequencies and the mode shapes, which becomes the basis for a forced vibration analysis. Liepins (1965) presented an extensive study of the free vibration analysis of a toroidal membrane subjected to an internal pressure. He solved the governing equations using a finite difference method, and obtained the natural frequencies and mode shapes by a trial and error method in the Holzer fashion. He divided the modes into four groups depending upon the dominant characteristics, and showed that only lower frequencies are affected by the prestresses due to internal pressure. Another important observation was that with a low aspect ratio (radius of the tube of the torus divided by the radius of the torus) and high prestress, the torus vibrates as a ring. Jordan (1966) predicted a few lower vibratory frequencies using the Rayleigh quotient. Later, he presented experimental results for the vibration testing of inflated tori of free and fixed boundary conditions (Jordan, 1967). He also noticed the acoustic resonance due to the inside air. Saigal et al. (1986) found a closed-form solution for the natural frequencies and mode shapes of a pre-stressed toroidal membrane with fixed boundary conditions. He assumed that the in-plane displacement components are zero and that the torus has a very small aspect ratio. The results are valid only for special cases due to the strong assumptions made to simplify the governing equations. Lewis (2000) analyzed an inflated toroidal structure using the finite element software package ANSYS™. He calculated natural frequencies and mode shapes of an inflated torus with a circular cross-section and free boundary conditions for different aspect ratios and internal pressures. Leigh et al. (2001) used the finite element code MSC/NASTRAN™ to model an inflated torus along with three struts, and compared the results to test data. Lewis (2000) and Leigh et al. (2001) only included the prestress effects due to the internal pressure but not the follower action of pressure force. As will be shown in this study, neglecting this effect leads to incorrect natural frequencies and mode shapes. A literature survey on the experimental studies related to modal analysis of an inflated torus can be found in Ruggiero (2002).

1.4.2 Actuator/Sensor Models and Vibration Control of a Shell

Coupons made of piezoelectric materials possess properties like lightweight, conformity, and actuation/sensing capabilities. For these reasons, they are suitable candidates for vibration testing, control, and health monitoring of different types of structures. Since the discovery of piezoelectric phenomena in 1880 by the Curie brothers, there has been a great amount of research activity in the area of piezoelectricity theories, its applications in various engineering problems, and the development of new kinds of materials and technologies for better actuation and sensing. Given the amount of literature in this area, we focus this brief survey only on the analytical modeling of piezoelectric actuators and sensors and vibration control related to a shell-type structure. The research activities regarding finite element formulations are not mentioned here. Extensive reviews on different types of smart materials (PVDF, PZT, shape memory alloys, magnetostrictive materials, magnetorhealogical fluids, etc.) and their applications for sensing and control of flexible structures (Tzou, 1998; Rao, 1999; Chopra, 2001)

Dökmeci (1990) presented the derivation of electroelastic equations for a shell by means of the unified variational principles. The governing equations were provided in a complete Lagrangian description for perturbed and unperturbed states of piezoceramic shell coated with very thin electrode. He included the effects of biasing stress, which could be caused by external perturbations like thermal, mechanical, electrical, and magnetic fields, and presented a nonlinear theory of piezoceramic shell. In special cases, equations were given for a biased plate of arbitrary shape and piezoceramic shells. Tzou (1993) presented a general piezoelectric shell vibration theory for both thick and thin shells. He also provided sensor equations for a distributed sensor patch attached to a shell. The derivations were specialized for the case of beams, plates, rings, cylinders, and spherical shells. A general shell, within the framework of linear theory, with different preliminary polarization was considered by Rogacheva (1994). The generalized equations of piezoelectric shells were derived using an asymptotic method and three-dimensional electroelasticity theory. The boundary conditions were derived using the generalized Saint Venant's principle for electroelasticity. Pletner and Abramovich (1997) presented modeling of spatially discrete piezoelectric actuators and sensors attached to an anisotropic piezolaminated shell. They took into account the stiffness and mass of the piezoelectric patches.

In the special cases, cylindrical and spherical shells are studied most. Lester and Lefebvre (1993) developed analytical models of in-plane and bending actuators, made of small piezoelectric patches, attached to a cylindrical shell. They derived the line moment (due to the out-of-phase actuations) and line force (due to in-phase actuations) acting on the perimeter of the actuator. The major drawback of their models was that they ignored the curvature effects on the actuator forces. They concluded that, compared to the induced bending moment, the induced in-plane forces couple more efficiently with the low order modes. Birman and Simonyan (1994) studied a cylindrical sandwich shell with piezoelectric elements bonded to the outer and inner surfaces. Thermal effects and sensor equations were also presented in this study. Lalande, Chaudhry, and Rogers (1995) derived an impedance-based model to calculate the response of cylindrical shells subjected to surface-bonded induced strain actuators. They included the mass and stiffness effects of the piezoelectric actuators in the derivation but neglected it in the final calculations. They compared the out-of-phase and in-phase actuation and found that out-of-phase actuation couples better in the lower order bending modes. They also compared the results with those of a finite element analysis and found good agreement. Banks, Smith, and Wang (1996) presented the dynamics of structures bonded with piezoelectric patches. They considered both active and passive effects of the piezoelectric patches. The resulting equations were presented in strong and weak forms. The spatial distribution of a piezoelectric patch is usually described by the Heaviside function, which is nonzero for the coordinates where the patch is located and zero everywhere else. The strong forms of the equations contain the first and the second order derivatives of the Heaviside function and hence make the solution procedure very difficult. In order to remove this difficulty, they presented the weak forms of the equations, which do not involve differentiations of the forces and moments produced by the patches. Sonti and Jones (1996) derived a composite differential equation for the vibration of a circular cylinder attached to piezoelectric actuators, considering stiffness and mass of the actuators. They also derived approximate analytical expressions for the equivalent forces exerted by the actuators. Vibration suppression of a cylindrical shell was attempted by Qiu and Tani (1996) using Flügge's shell theory, μ-synthesis control, and disturbance cancellation method. The cylinder was excited by the random horizontal movement of its base. They concluded that the piezoelectric actuators can be successfully utilized in the vibration control problem and that the hybrid control (μ-synthesis control and disturbance cancellation method) gives better control performance compared to either of the two. del Rosario and Smith (1998) applied a linear quadratic regulator (LQR) scheme for the vibration control of a circular cylinder using distributed piezoceramic patches. The shell equations were formulated using Donnell-Mushtari theory and the solutions were obtained using the Galerkin method with the basis functions constructed from tensored Fourier polynomials and modified cubic splines. Active control of vibration and noise transmission of a double wall cylinder was investigated by Wang and Vaicaitis (1998). They used random excitations and performed parametric excitation to show the sensitivity of actuator size, placement, and voltage gain on the performance of the controller.

The problem of control and observation spillover is quite common in the vibration control of a distributed structure. In order to prevent control spillover, Tzou, Zhong, and Hollkamp (1994) suggested using spatially shaped distributed actuators orthogonal to unwanted modes. They applied the method for a ring problem and showed the filtering capability of the actuator. Tsai and Wang (1996) modeled multiple active-passive hybrid piezoelectric actuators attached to a ring structure. They derived the governing equations using Hamilton's principle and took into account the external R-L circuits used for passive damping. They estimated the states using piezoelectric sensors and a Kalman filter. Their methodology presented a way to simultaneously optimize the control gains and the values of inductors and resistors for active-passive control. They showed that, compared to a purely active control, the active-passive control scheme can deliver better performance in reducing the vibration and acoustic emissions. Jayachandran and Sun (1998) investigated actuation of a shallow-spherical-shell actuator, called RAINBOW (Reduced and Internally Biased Oxide Wafer) actuator, using a shell theory. In order to increase the linear stroke of the actuator and to tune the first resonance frequency, they added a concentrated mass on the apex of the actuator. The inertia terms in the equations of motion were modified so as to account for the concentrated mass. They performed several parametric studies regarding the effects of curvature, mount stiffness, and mount mass on the natural frequencies, linear stroke, and volume velocity. Ghaedi and Misra (1999) modeled piezoelectric actuators attached to a spherical shell. They obtained the expression for actuator forces by considering the shell-actuator interaction and using the principle of virtual work. They used the linear quadratic regulator (LQR) and velocity feedback control scheme and concluded that both of these methods can be successfully applied for vibration control. They observed that the system could be modeled by only the first few modes (4-10) for a good controller performance. Their study included the performance of a controller under changes of different parameters of the shell. Jayachandran et al. (1999) performed experiments in order to show the use of RAINBOW and THUNDER actuators in noise reduction. The enclosure was made of a cylinder and the speaker was driven by the RAINBOW and THUNDER (pre-curved piezoceramic) actuators. They used multiple error filtered-x LMS feed-forward algorithm for the real time control of the low frequency noise. They concluded that these devices are well capable of achieving significant attenuation. Zhou and Tzou (2000) applied an active control technique to a shallow spherical shell, considering the geometric nonlinearity. Piezoelectric layers were assumed to be distributed uniformly over the top and the bottom surfaces of the shell. The nonlinearity was modeled using von Karman formulations and the solutions were obtained using the Galerkin method and a perturbation technique. They showed that the deformation and frequencies of the shell could be changed using high control voltage across the piezoelectric layers. Granger, Washington, and Kwak (2000) studied a THUNDER actuator, using Hamilton's principle and composite curved beam theory. The results were also experimentally verified.

Sensor application of piezoelectric films for cylindrical shell was considered by Birman and Simonyan (1994). The derivation was made for open-circuit configuration in which the electric displacement is taken to be zero. They used axial sensors to detect the delamination of the sandwich shell by calculating the voltages generated by the sensors near the delaminated and intact regions. Callahan and Baruh (1999) followed a closed-circuit approach, in which the electric field across the thickness of the sensor is zeroed by short-circuiting the electrode on the surfaces. They showed that piezoelectric sensors with hexagonal symmetric structure cannot sense axisymmetric torsional modes of the circular cylinder and the rigid-body modes cannot be sensed by any choice of piezoelectric material. They calculated the modal coordinates using the inverse of the charge/modal coordinate matrix and then extracted the modal velocities by designing an observer. Application of a piezoceramic cylindrical shell as a gyroscope, i.e., rotation sensor, was considered by Yang, Fang, and Jiang (2000). Gyroscopic usage of piezoelectric material involves two vibration modes of perpendicular motions. When the piezoelectric gyroscope is attached to a rotating body and excited by an alternating voltage in one of the two modes, the other mode is automatically excited due to the Coriolis force. In order to achieve good sensitivity, the frequencies of the two modes should be close enough. They used torsion and horn modes of a cylinder as the frequencies of these modes can be tuned almost independently by changing the length and the radius of the cylinder. Distributed sensing characteristics for a toroidal shell were studied by Tzou, Wang, and Hagiwara (2001) using Donnel-Mushtari-Vlasov theory and the geometric nonlinearity of von Karman type. However, the results cannot be relied upon, as the mode shapes considered for the calculation are incorrect.

One of the drawbacks of the piezoelectric film actuators is that they provide small forces. It is also known that the traditional piezoelectric materials, such as PZT and PVDF, have induced strain coefficients in the vertical directions (d_{33}) higher (approximately double) than the induced strain coefficients in the longitudinal direction (d_{31}, d_{32}). Since the actuation effect is due to longitudinal coefficients, it is more useful to use the d_{33} coefficient for the in-plane actuation. Motivated by this fact, two high performance piezoelectric actuators have been developed recently. They are Macro-Fiber Composite Actuator (MFC™), developed by NASA Langley Center (Wilkie et al., 2000), and Active Fiber Composite Actuator (AFC), developed by MIT (Hagood and Bent, 1993; Bent, 1997). Apart from the stronger induced coefficient in the longitudinal direction, these actuators are directional, and more conformable and flexible compared to the traditional monolithic piezoceramic wafers that exhibit in-plane isotropy. The research activities on the analytical modeling of AFC/MFC actuation have been very limited. Bent, Hagood, and Rodgers, (1995) investigated structural actuation with the active fiber composite actuator using Classical Laminated Plate Theory. They showed how twist action could be imparted to a host structure due to these types of actuations. They also compared the results with experiments and found good agreement. Wilkie, Belvin, and Park (1998) applied directional properties of the composite actuator for the twist control of a helicopter rotor blade.

1.4.3 Optimal Placement of Piezoelectric Actuators/Sensors

It is a known fact that actuators and sensors placed inappropriately can lead to a loss of observability and controllability, which, in turn, may render the actuators and sensors ineffective. This necessitates the right placement of actuators and sensor for distributed structures. There have been several studies on the optimal selection of conventional actuators and sensors for different structural applications (Padula and Kincaid, 1999). Also, a number of researchers have studied this problem for piezoelectric actuators and sensors applied to different kinds of distributed structures. In this section, we briefly summarize the research activities regarding the optimal sizes and placements of piezoelectric actuators and sensors applied to beams, plates, trusses, and cylindrical shells.

Finding the optimum locations of actuators and sensor for a beam was undertaken by Yang and Lee (1993) by simultaneously optimizing the placement and feedback gain for collocated actuator and sensor. They used two types of controller – optimal active damping control and optimal output feedback control. They found that in the case of optimal active damping control, the optimal location of the collocated actuator and sensor is not at the root of the beam. Another important finding was that a collocated actuator/sensor does not necessarily guarantee stability. Main, Garcia, and Howard (1994) provided a design guide to choose piezoelectric actuator thickness and location based upon the modulus ratio between the materials of the actuator and the host structure. Using simulated annealing, Chattopadhyay and Seeley (1995) found actuator location, ply angle, and the cross-section for a composite beam by maximum natural frequency and minimum vibration and control effort. The method was implemented using the finite element method, Kreisselmeier-Steinhauser function, and optimal control theory. The equations of motion considered the bending and torsion coupling as well as rotation of the composite box beam. The authors found a significant improvement in the objective functions with the optimized actuators. Chen and Cao (2000) tried finding the best locations of actuators and sensors by keeping/removing the finite elements based upon the element sensitivity to the singular values of controllability and observability. The method was illustrated using the examples of a beam and a solar panel by comparing the tip transient displacements for best and worse locations. Wang and Wang (2001) proposed a controllability index based upon the amount of control energy supplied to the host structure by the actuators for a given control input. The controllability index was again obtained using a singular value analysis.

Optimal design of a piezoactuator for a plate was attempted by Kim and Jones (1991). They derived an expression for the effective bending moment induced by a pair of piezoelectric actuators attached to a plate. Their derivation gave results different from those by other researchers (Crawley and de Luis, 1987; Dimitriadis and Fuller, 1989). They performed several parametric studies relating the bending moment and the thickness ratio of the actuators and host structure. It was found that, for a steel plate, an actuator of the half-plate thickness and, for an aluminum plate, an actuator of the quarter-plate thickness provide the maximum bending moment. They also considered the effect of bonding layer thickness on the bending moment. Clark and Fuller (1992) found optimal placements of an actuator and a sensor for active structural acoustic control using linear quadratic optimal control theory. They found that optimal design of the single PZT actuator and PVDF sensor significantly enhanced the global power reduction compared to the three arbitrarily placed actuators and three microphones located in the acoustic field as error sensors. Tzou and Fu (1994) compared a single-piece symmetrically distributed actuator and sensor layer with the quarterly segmented actuators and sensors. They found that quarterly segmented actuators/sensors improved the controllability and observability of even modes without degrading the performances for the odd modes of a simply supported plate. However, it was found that quarterly segmented actuators/sensors cannot control/sense the quadruple modes of the plate. They suggested that a finer segmentation could further improve the controllability/observability. In order to reduce the radiated noise from a plate, Varadan, Kim, and Varadan (1997) provided a finite element method along with automatic mesh generation algorithm to optimally design two circular disk-shaped piezoelectric actuators. They observed significant noise reduction near and away the resonant frequencies. They considered different patterns of pressure loading conditions and varying excitation frequency. However, they noted that the method takes a very long time to converge. Vibration control of a distributed structure is usually done in the modal domain. Hence, it is important to quantify the interaction of actuators and sensors with different modes. Several methods have been suggested to quantify the overall performances of actuators and sensors (Moore, 1981; Longman et al., 1982; Hamdan and Nayfeh, 1989; Hac and Lui, 1993). These methods have been successfully used in finding suitable actuators and sensors. Han and Lee (1999) used a genetic algorithm to find efficient locations of piezoelectric actuators and sensors for vibration control of a cantilevered composite plate. Based upon the definition given in Hac and Lui (1993), They used the eigenvalues of steady state controllability and observability grammian to define the objective function, which also tries to suppress the spillover effects. They used a positive position feedback controller and targeted the first three modes. They demonstrated the vibration reduction using the optimal actuators and sensors using experiments. They found that the actuators and sensors obtained using this objective function give very small residual effects. Moreover, the closed-loop system was found to be robust against the uncertainty in the system parameters. Sadri, Wright, and Wynne (1999) used modal controllability (similar to Hamdan and Nayfeh, 1989) and the controllability grammian to find the optimal locations of actuators in an isotropic plate. They used a genetic algorithm to search for the optimal location and showed vibration control using the closed-loop frequency response of the plate. Kim and Soong (1999) presented a performance index for actuators based upon the balancing of modal actuator forces for controlled and uncontrolled modes. A similar definition was used for finding optimal sensor locations. They used a genetic algorithm along with linear quadratic Gaussian controller to find the optimum locations, and showed a global noise reduction. Leleu, Abou-Kandil, and Bonnassieux (2000) slightly modified the approach suggested by Hac and Lui (1993) by including the standard deviation of the eigenvalues of the controllability/observability grammian. This was done in order to discard the location where there are larger differences among the eigenvalues. They applied a modified performance measure to find actuator and sensor locations for a plate.

Unlike the case of beams and plates, there have been very few studies on the optimal piezoelectric actuators/sensors for a shell-type structure. Moreover, all the studies, which are relatively recent, are done for a cylindrical shell. Sun and Tong (2001) used the criterion of control and observation spillover to design optimal actuators and sensors, and presented simulation studies of vibration reduction. The sensing and control effectiveness of an arbitrary patch and quarterly segmented patches attached to a cylindrical shell were studied by Tzou, Bao, and Venkayya (1996). They performed parametric studies to evaluate the sensitivities of the sensors to different strains. They found that the quadruple modes could not be sensed using the quarterly segmented sensor, and the membrane and bending sensitivities increase with an increase in the sensor thickness. In general, the membrane effect in the cylindrical shell was observed to be dominating compared to the bending. However, in a zero-curvature structure, the bending effect was found to be higher in both actuator and sensor applications. They found that the membrane action of an actuator is more effective for lower order modes, while the bending actuation is more effective in the higher-order modes. The modal control effect was greater for a thicker actuator and a thinner shell.

1.4.4 Vibration Control of Inflatable Structures

In the previous section we saw the applications of piezoelectric actuators and sensors for different types of metallic and composite shell structures. However, there seems to be very few such studies done on inflatables. Salama et al. (1994) tried controlling an inflated circular membrane using piezoelectric material and found that the method is suitable only for making small local adjustment in the shape of the antenna. Maji and Starnes (2000) used a PVDF film actuator to control the shape of a membrane. They concluded that piezoelectric actuation could control the shape at an optical level. Williams (2000) modeled the interaction of piezoelectric patches attached to an inflated torus using prestressed flat membrane theory. While he was able to model the actuation behavior of the piezoelectric patches, the modeling of the sensing behavior seemed infeasible with the method he used. Park, Kim, and Inman (2002) demonstrated experimentally the effectiveness of an MFC actuator in the vibration control of an inflated torus using a positive position feedback controller.

1.5 Contributions of This Research

The above section surveyed the work done in the static and dynamic analyses of an inflatable torus. It also overviewed the state-of-the-art in different aspects of vibration control of distributed structures using piezoelectric actuators and sensors. Next, we summarize the contributions of this research in the dynamic analysis and vibration control of an inflated torus.

In the literature survey regarding the dynamic analysis of an inflated torus, we saw that the work of Jordan (1967) was far from comprehensive and predicted only a few natural frequencies using Rayleigh quotient method. The closed-form solution presented by Saigal et al. (1986) suffered from the strong assumptions made regarding zero in-plane displacement and very small aspect ratio. Certainly, these assumptions are very strong for a general doubly curved toroidal shell. Moreover, the solutions were obtained for an inflated torus with fixed boundary conditions. The other studies used numerical techniques – finite difference method (Lipins, 1965) and finite element method (Lewis, 2000; Leigh et al., 2001). While the analysis and results of Lipins (1965) are very thorough and commendable, the solution was obtained using a finite difference method and the natural frequencies and mode shapes were predicted by a trial and error method. The finite element analysis by Lewis (2000) and Leigh et al. (2001) suffered from the inaccuracy by ignoring the follower action of pressure force. This study will try to find a solution using the Galerkin method with the mode shapes given by Fourier series, which is more suitable for a circular structure like the inflated torus. There will be no approximation made regarding the in-plane deflection and aspect ratio as done in Saigal et al. (1986). The natural frequencies and mode shapes will be exact within the framework of the Galerkin method. The equations of motion for a shell under pressure considering an accurate geometric nonlinearity and the follower action of pressure force will be derived and used here. This derivation was presented before by Budiansky (1965) using tensors, which is relatively more difficult to understand. Most of the researchers did not take into account these factors, which led to some serious problems regarding the free vibration analysis of rigid-body and non-rigid-body modes. We discuss these issues in this study to clarify the existing confusion regarding the use of shell theory for an inflatable structure. To the best of the author's knowledge, this study will be the first of its kind. Taking cues from several researchers, a comprehensive derivation of the shell theory for an inflated structure is presented using the line of curvature coordinates. From this general derivation, we obtain equations presented by other researchers as special cases and in due course point out the omissions. The drawbacks of these omissions have been noted for the vibration analysis of an inflated torus.

While the equations for analytical modeling of actuators and sensors already exist in the literature for a general shell, their solutions have been limited to beams, plates, cylinders, and spherical shells. In this study, we provide a solution for an inflated torus. The sensor modeling for a toroidal shell without internal pressure was presented by Tzou, Wang, and Hagiwara (2001). But, the results were not reliable because of the inaccuracy in the mode shapes. To the best of the author's knowledge, no study has been performed regarding the application of piezoelectric actuators and sensors for an inflated toroidal shell. In this respect, this study will have an important contribution. Both active and passive effects of piezoelectric actuators/sensors will be clearly outlined by solving the equations in the two cases.

Similar voids can be seen in the optimal placement and control problem of distributed structures. In the past, studies have been limited to simple structures like beams, plates, and cylinders. The present study will extend the previous studies and methodologies to an inflated toroidal shell, which is fairly complicated due to its double curvature and internal pressure. A detailed analysis of the effects of sizes and locations of actuators and sensors is presented, which will be of a direct aid to a designer. For the first time, it will provide a study on the vibration control of an inflated torus using piezoelectric materials. The controller used in this case will be shown to be robust against the parameter uncertainties and external disturbances, which is often the problem with a complicated structure working in a harsh environment.

1.6 Outline of Dissertation

In this chapter, we introduced the inflatable satellite technology, its advantages and disadvantages. We briefly summarized the past and recent NASA space missions using inflatable structures. Thereafter, objectives of this study were outlined and the state-of-the-art on different aspects of this research were surveyed. In this context, we also pointed out the contributions of this research. Now, we provide brief descriptions of the remaining chapters.

In Chapter 2, equations of motion for a general shell under pressure are derived. First, we derive second order nonlinear strain-displacement relations from the three-dimensional elasticity theory. Thereafter, using Hamilton's principle, we derive the governing equations of motion and boundary conditions for a shell under pressure. Along with the dynamic equations of motion, we also present the static equations from which the prestresses can be obtained from the applied pressure. All the derivations are based upon Sanders' shell theory (Sanders, 1959). These equations of motion are also specialized to obtain the equations presented in Sanders (1963), Soedel (1986), and Plaut et al. (2000).

Chapter 3 deals with the derivations of mathematical models for the actuation and sensing behaviors of piezoelectric material patches attached to a shell under pressure. Expressions are developed for a piezoelectric material with in-plane anisotropy so that it can be used for both monolithic actuators (PZT, PVDF) and directional actuators (MFC™). Both unimorph and bimorph types of actuators and sensors are considered. The actuators and sensors also change the stiffness and mass of the host structure. These effects are taken into account in the derivations. In order to control the vibration of a structure, it is important to know how the actuators and sensors are responding to the vibration in different modes. To this end, expressions for the modal forces and modal sensing constants are presented.

Unlike the previous two chapters, which deal with a general shell under pressure, Chapter 4 concentrates upon the specific case of an inflated toroidal shell. It presents a free vibration analysis, which serves as the basis of all the subsequent analysis regarding vibration control of the inflated torus. The governing equations of motion, presented in Chapter 2, are used. The solutions (natural frequencies and mode shapes) are obtained using Galerkin method and compared with published results, finite element analysis, and experiments. The natural frequencies are also obtained for a circular cylinder with shear diaphragm boundary condition as a special case of the toroidal shell and verified with the closed-form solution. The effects of aspect ratio, pressure, and thickness on the natural frequencies of the inflated torus are studied. In order to show the importance of geometric nonlinearity and the pressure force, natural frequencies are compared with those obtained from the simpler theories derived in Chapter 2.

Chapter 5 presents the modal forces and the modal sensing constants for the inflated torus. Using the expressions developed in Chapter 3, modal forces and modal sensing constants are obtained for the inflated torus. The effects of sizes and locations of the actuators and sensors on the modal forces and modal sensing constants are presented. To obtain a cumulative performance measure of all the controlled modes, controllability and observability indices are used. Using these performance indices, optimal locations and sizes of the actuators and sensors are determined using a genetic algorithm.

Chapter 6 is devoted to the vibration control problem. Using the optimal locations and sizes of the actuators and sensors obtained in the previous chapter, a robust controller is designed in order to suppress the vibration of the inflated torus. We use sliding mode controller and sliding mode observer, as they are known to be robust to the matched uncertainty and disturbance. First, the theoretical background of the sliding mode controller and observer is described and then the results from the numerical simulations are presented.

Chapter 7 presents a summary, conclusions, and contributions of this research. It also provides recommendations for further work in this direction.