Chapter 7

Concluding Remarks

7.1 Summary of the Presented Work

Inflated structures possess many advantageous properties such as lightweight, small stowage volume, moderate strength, and on-orbit deployability. Therefore, usage of inflatables as large space structures will incur lower cost compared to the conventional metal or composite space structures. An inflated torus is an integral part of several such proposed inflatable space structures. The goal of this research was to understand the dynamics of an inflated torus and to develop a methodology for its vibration suppression using piezoelectric actuators and sensors. In the following paragraphs, we present a summary of what was done in this research:

1) An inflated structure can be characterized by a very thin wall and internal pressure. Hence, it is appropriate to model it using a shell theory. We used Sanders' shell theory and derived the static and dynamic equations for a thin shell under pressure using Hamilton's principle. To take into account the prestress effects of internal pressure, we used geometric nonlinearity, and to model the follower action of pressure force, we

considered the work done by internal pressure during the vibration of the shell. Finally, the governing equations were specialized to obtain approximate equations presented by other researchers.

2) Continuing the theoretical formulations for static and dynamic analyses, we derived mathematical models of actuators and sensors made of piezoelectric patches attached to a shell under pressure. Besides, the passive effects of patches were included in the dynamic equations of motion. The equivalent forces were obtained for both unimorph and bimorph configurations. The equations were derived using Hamilton's principle by including the constitutive equations of the piezoelectric material. The derivation was made for Macro-Fiber Composite^{TM} Actuators. However, the methodology presented here could well be extended to more complicated actuators. The sensor equations were obtained using an open-circuit configuration. The actuator and sensor equations, along with the equations of motion including the mass and stiffness effects of the patches, were used later in deriving the modal forces and modal sensing constants.

3) Using the governing equations for a thin shell under pressure, we performed a free vibration analysis of the inflated torus. We used Galerkin's method along with the separation of variables technique to find the analytical solution for the natural frequencies and mode shapes. Due to the axisymmetric nature of the torus, deflection functions in the circumferential direction were represented by one-term sine and cosine functions. However, the displacement function in the meridional direction was represented using Fourier series. The torus had a circular cross-section and it was symmetric around a plane perpendicular to the axis of rotation that divides the total volume in half. This made it possible to divide the mode shapes into the so-called symmetric and antisymmetric modes. A circular cylinder can be seen as a special case of an inflated torus. At first, the natural frequencies and mode shapes of a circular cylinder with no internal pressure were presented and verified with the exact solution. The agreement was found to be perfect. Thereafter, the analysis was carried out for an inflated torus. Again, we compared the natural frequencies with published results and found good agreement. An extensive study of the vibration characteristics including the rigid-body modes was made. We also

studied how different parameters (aspect ratio, internal pressure, and wall-thickness) of the inflated torus affect the natural frequencies and mode shapes. The natural frequencies were found to be mostly decreasing with an increase in thickness and aspect ratio and increasing with the internal pressure. Changes in these parameters affected the deformation as well as the order of modes. It was noticed that at low aspect ratio, the lower order modes mostly consist of ring-type modes. Finally, we compared the results obtained from the theory used in this research with the results from different approximate theories and commercial finite element codes. It was noted that the commercial finite element codes fail to take into account the direct effect of pressure force and produce nonzero rigid-body natural frequencies. Besides, the rigid-body frequencies and mode shapes also become inaccurate. Similarly, a wide discrepancy was noted among the results of different approximate shell theories and the one used here. Note that the rigid-body frequencies from this analysis are exactly zero. Moreover, compared to results from approximate theories, results of this analysis compare better with the experimental ones. This suggested that the considerations of accurate geometric nonlinearity and pressure force are very significant in the vibration analysis of an inflated torus.

4) Numerical analysis was continued to quantify the modal interactions of piezoelectric actuators and sensors. Using the results from the free vibration analysis and the mathematical formulations of modal force and modal sensing constant, behaviors of piezoelectric actuators and sensors were analyzed. It was found that due to small wall-thickness of the torus, a bimorph actuator/sensor does not provide any better actuation/sensing capability than a unimorph and hence for all the subsequent analysis we used a unimorph configuration. A detailed study was done in order to understand how the size and location of actuator and sensor affect the modal forces, the modal sensing constants, and the overall performance on all the considered modes. The performances of actuators and sensors were determined using the eigenvalues of the controllability and observability grammians. It was seen that actuator and sensor authorities on various modes are very sensitive to the locations and sizes of the patches. It was observed that the sensor structure considered in this study could not detect either purely torsional or rigid body modes. Since control of rigid-body modes is not possible with the piezoelectric

actuators and the purely torsional modes occur at relatively high frequencies, the sensor used here should be sufficient. A genetic algorithm was used to obtain optimal actuators and sensors. Results of this optimization process show that the actuator size tends to be larger compared to the sensor size for higher performance indices. An inflated torus has in general close and repeated frequencies. This implies that a large number of actuators and sensors are needed to have good controllability and observability. It was observed that one needed at least five actuators and five sensors for good controllability and observability. Taking into account the first nine modes of the inflated torus, optimal sizes and locations of the five actuators and sensors were determined. Natural frequencies and mode shapes were calculated considering the passive effects of actuators and sensors. It was seen that the MFC^{TM} actuators have significant effect on the natural frequencies and mode shapes of the inflated torus. These natural frequencies and mode shapes with five optimal actuators and sensors were later used in the vibration control problem.

5) Finally, we attempted the vibration control of the inflated torus using the optimal actuators and sensors found by the optimization process. The inflated torus, when used as a part of a satellite structure, will have to undergo different environmental conditions and disturbances. To encounter these changes, the controller and observer should have robustness properties. To this end, we designed a sliding mode controller and a sliding mode observer. At first, the controller and observer were implemented without considering the residual modes. Numerical simulations showed good performance. Thereafter, the effects of residual modes were taken into account. The observation spillover made the controller incapable of reducing the vibration of the torus. The observation spillover effects were reduced by including the residual modes in the observer. This technique significantly improved the performance of the controller. Finally, the robustness properties of the controller and observer were verified against the parameter uncertainties caused by the change in internal pressure and external disturbances.

7.2 Main Contributions

This research work focused on the vibration analysis and control of an inflatable torus. Though the basic elements such as shell theory and vibration control of distributed structures using piezoelectric materials are available in the literature, only a very few researchers have studied their applications to an inflatable structure. Moreover, the methods adapted in past suffer from inaccuracies due to simplification in the governing equations, as well as the previous work being far from complete. In the following paragraphs, we note the main contributions of the work in this dissertation:

1) We derived the static and dynamic equations for a shell under pressure using the line-of-curvature approach. These equations were derived before by Budiansky (1968) using tensors. As felt by the author, the tensorial approach is not commonly understandable. This could be the one of the reasons why most of the researchers used incorrect theories for such analysis even if a more accurate theory was available. As explained in this work, these approximate/inaccurate theories lead to misleading and sometime confusing results. Therefore, the present derivation should have significant impact on future studies in the area of inflatable structures due to its simplicity.

2) We presented a thorough investigation on why we need an accurate theory for the vibration analysis of an inflated torus. In the author's opinion, only Lipins (1965) used an accurate theory and presented a detailed analysis of natural frequencies and mode shapes. The present work differs from that work in mostly the adopted approach. While Lipins (1965) used a finite difference method and found natural frequencies using a trial and error method, this analysis uses Galerkin's method and the mode shapes are calculated using Fourier series. Also, some parametric studies presented in this study cannot be found elsewhere.

3) We showed that the vibration analysis using commercial finite element codes as done by previous researchers (Lewis, 2001; Leigh et al., 2001) did not take into account the pressure force. Their studies only included the prestress effects of the internal pressure. We showed how these inaccurate finite element analyses produce non-zero rigid-body frequencies for an inflated torus with free boundary conditions. It was clearly shown what difference these analyses bring in the natural frequencies and mode shapes of the rigid-body modes. For the first time, a study has brought these issues to light, and pointed out the missing elements of the analysis.

4) The actuator and sensor models presented in the literature were directly extended for a shell under pressure. All the techniques used here were taken from the work of other researchers. While the extension in the theory may not be considered as a contribution, their solution for the case of an inflated torus is completely new. The effects of a piezoelectric actuator on different modes, as well as studies on the effects of sizes and locations of the actuators and sensors, were never attempted before for the case of a torus. It should be pointed out that a correct solution of this problem is important so that the actuators and sensors interactions with the structure do not become misleading. For example, the sensor study presented by Tzou et al. (2001) for the case of a torus uses incorrect mode shapes and hence the results cannot be relied upon.

5) Other contributions of this research come from the application point of view. A vibration control of an inflatable torus was never attempted before. Similarly, finding optimal sizes and placements of actuators and sensors for a toroidal structure is new.

7.3 Recommendations for Future Work

There could be several ways in which the present research can be advanced. Below, we summarize a few of such possibilities.

1) Thus far, researchers in the area of experimental modal analysis have been able to show only a very few modes. Moreover, these modes are either the in-plane or the out-of-plane bending modes. In reality, as shown by the analytical studies, an inflated torus has a very rich dynamics. A comprehensive experimental analysis by considering a more realistic space condition would be a significant contribution. Similarly, the actuator and sensors models of piezoelectric materials presented in this study have not been verified experimentally. Before these techniques can be applied, these models need to be verified. Experimental demonstration of the vibration control using all the significant modes will be an important step towards realizing the uninterrupted functioning of the inflatable antennas.

2) The present study focused only on an inflated torus. In reality, an inflated space structure will consist of several other types of structures, such as inflatable struts, inflatable mirror, etc. Vibration analysis of the combined structure will be quite difficult using the techniques presented in this work, and a finite element method can be employed for such cases. However, as we saw in this work, the commercial finite element codes suffer from inaccuracies and the results obtained from these analyses cannot be relied upon. Therefore, it would be quite useful to develop a finite element code based upon an accurate shell theory and correct models of the internal pressure effect. This will provide a designer the capability to analyze even a complicated inflatable structure.

3) The presented vibration analysis was confined to a linear domain. A few static analyses considering geometric nonlinearity have been done previously, suggesting its importance. For example, in an inflated torus, the difference between the circumferential stresses

calculated using nonlinear theory and linear theory can be as high as 18% (Jordan, 1962). Static analysis of a toroidal shell shows that the differences in the deflections and strains calculated using nonlinear theory and linear theory are around 35% (Maksimyuk and Chernyshenko, 1999). We considered the geometric nonlinearity only with prestresses so as to keep the governing equations linear. One direct extension will be to consider geometric nonlinearity with the vibratory stresses also. Moreover, the nonlinearity could be considered in the bending and twisting strains as well.

4) In the present study, the constitutive laws were assumed to follow the Hookean law. The materials (e.g., Kapton, Mylar) used in the inflatable structures are well known for their nonlinear behavior (Palisoc and Huang, 1997). Salama et al. (1994) analyzed an inflated circular membrane made of Mylar using a finite element analysis and an experiment. Their pressure vs. deflection plot showed clearly a nonlinear behavior. This is also due to the internal pressure, which causes high initial deflection. This, in turn, could bring nonlinearity in the material property. Another evidence showing material nonlinearity is that the elastic moduli of these materials vary with excitation frequency, temperature, and the level of excitations (Tinker, 1998). These facts suggest that one could consider material nonlinearity for a more accurate vibration analysis.

5) One complexity in the analysis of an inflatable structure arises from wrinkling. In a dynamic analysis, wrinkling can be viewed as a result of negative stresses or singularities. Analysis can be carried out to understand the phenomenon of wrinkling and ways to avoid it. Note that the wrinkling in inflatable mirrors and reflectors could produce detrimental effects, since a high surface accuracy is necessary for their proper functioning.

6) Since the inflatable structures considered in this study will be floating in space, their boundary conditions must be considered as free, which gives rise to six rigid-body motions. Due to the high flexibility of the inflatable structure, the elastic deformation may not be neglected while considering the overall rigid-body motions (Seo and Yoo, 2002). Studies on the coupling between the elastic deformations and the rigid-body

motions will have significant contribution in understanding the dynamics of inflatables. This study becomes especially important in a high-precision structure, such as an inflatable solar concentrator or antennas.

7) Since nonlinearity is important in the dynamic analysis, its effects should also be studied in the actuator and sensor models. To date, most researchers have considered either linear theory or, at most, the geometric nonlinearity of the host structure. In this respect, including the nonlinearities of the structure and the smart material in the actuator and sensor models will advance the present state-of-the-art.

8) In the present study, we only considered the pressure due to the inside air. The other effects of air can be due to its mass and damping. Jordan (1967) found that the acoustic resonance due to the air inside an inflated torus plays an important role in its vibration. Giraudeau, Pierron, and Chambard (2002) studied the air effects on a thin vibrating plate using experiments. The natural frequencies and modal damping were found to be less in the case of in-air compared to the in-vacuum condition. Since an inflatable structure is very light, a study is needed to understand the effects of air on its dynamic properties.