Game-Theoretic Analysis of Topology Control

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(ABSTRACT)

Ad hoc networks are emerging as a cost-effective, yet, powerful tool for communication. These systems, where networks can emerge and converge on-the-fly, are guided by the forward-looking goals of providing ubiquitous connectivity and constant access to information. Due to power and bandwidth constraints, the vulnerability of the wireless medium, and the multi-hop nature of ad hoc networks, these networks are becoming increasingly complex dynamic systems. Besides, modern radios are empowered to be reconfigurable, which harbors the temptation to exploit the system. To understand the implications of these issues, some of which pose significant challenges to efficient network design, we study topology control using game theory.

We develop a game-theoretic framework of topology control that broadly captures the radio parameters, one or more of which can be tuned under the purview of topology control. In this dissertation, we consider two parameters, *viz.* transmit power and channel, and study the impact of controlling these on the emergent topologies.

We first examine the impact of node selfishness on the network connectivity and energy efficiency under two levels of selfishness: (a) nodes cooperate and forward packets for one another, but selfishly minimize transmit power levels and; (b) nodes selectively forward packets and selfishly control transmit powers. In the former case, we characterize all the Nash Equilibria of the game and evaluate the energy efficiency of the induced topologies. We develop a better-response-based dynamic that guarantees convergence to the minimal maximum power topology. We extend our analysis to dynamic networks where nodes have limited knowledge about network connectivity, and examine the tradeoff between network performance and the cost of obtaining knowledge. Due to the high cost of maintaining knowledge in networks that are dynamic, mobility actually helps in information-constrained networks. In the latter case, nodes selfishly adapt their transmit powers to minimize their energy consumption, taking into account partial packet forwarding in the network. This work quantifies the energy efficiency gains obtained by cooperation and corroborates the need for incentivizing nodes to forward packets in decentralized, energy-limited networks.

We then examine the impact of selfish behavior on spectral efficiency and interference minimization in multi-channel systems. We develop a distributed channel assignment algorithm to minimize the spectral footprint of a network while establishing an interference-free connected network. In spite of selfish channel selections, the network spectrum utilization is shown to be within 12% of the minimum on average. We then extend the analysis to dynamic networks where nodes have incomplete network state knowledge, and quantify the price of ignorance. Under the limitations on the number of available channels and radio interfaces, we analyze the channel assignment game with respect to interference minimization and network connectivity goals. By quantifying the interference in multi-channel networks, we illuminate the interference reduction that can be achieved by utilizing orthogonal channels and by distributing interference over multiple channels. In spite of the non-cooperative behavior of nodes, we observe that the selfish channel selection algorithm achieves load balancing.

Distributing the network control to autonomous agents leaves open the possibility that nodes can act selfishly and the overall system is compromised. We advance the need for considering selfish behavior from the outset, during protocol design. To overcome the effects of selfishness, we show that the performance of a non-cooperative network can be enhanced by appropriately incentivizing selfish nodes.

Dedication

In the loving memory of my little sister, Sita

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Part I

Introduction to Topology Control and Game Theory

Chapter 1

Introduction

With the growing impetus to deploy next generation ad hoc wireless networks, there is an immediate need to address several outstanding issues. Protocols have to be redesigned and new interfaces have to be introduced to effectively cope with the volatile dynamics of the environment. The desire to embrace an "open" network paradigm—by distributing network control to autonomous, distributed, and independent radios—can foster abberant behavior among nodes. By not complying to protocols, nodes can act selfishly, sometimes even maliciously, and "game" the system. A thorough analysis of existing protocol robustness and design of new protocols to contend with these additional challenges is warranted.

Taking into account the distributed and autonomous traits of self-organization, nodes' proclivity for non-cooperation, and the lack of complete information in distributed wireless systems, this dissertation analyzes topology control with a network-centric approach to enhance the end-to-end performance of ad hoc networks. In particular, we advance the need for considering selfish behavior from the outset, in the network design stage, when optimizing wireless network systems.

1.1 Overview of Ad hoc Networks

The resounding success of the Internet, its unprecedented growth and evolution, and the convergence of data and voice services has sparked a paradigm shift in networking technologies. Owing to the widespread proliferation of mobile, portable communication and computing devices, continuing advancements in data processing capabilities, and increasing demand for ubiquitous connectivity, research in ad hoc networks is of particular interest. Ad hoc networks hold great promise not only because of their immense potential to aid military applications, emergency disaster rescue-and-relief operations, and pervasive computing, but also because of their ease and speed of deployment. Automotive (vehicular) networks, home networks, networks of sensors and actuators, mesh networks, underwater acoustic networks and RFID systems are just a few variants of ad hoc networks, some of which are perhaps more promising than others.

An ad hoc network can be established on-the-fly when heterogeneous devices (such as PDAs, laptops, cellular phones etc.) distributed over a geographical region communicate with each other wirelessly, possibly on different parts of the spectrum. In addition to supporting various node functionalities and capabilities, the network heterogeneity also extends to allow the co-existence of devices accessing and operating on different channels and serving different types of traffic. These networks can be envisioned to operate as stand-alone entities or as stubs connecting to fixed access networks to reach hosts outside the ad hoc network. These networks differ from cellular networks, in that there is no fixed pre-existing infrastructure. As devices move around the network or change their transmission parameters, a dynamic network topology emerges; this necessitates that the topology be self-configuring with an intrinsic ability to self-adapt and be resilient. The lack of infrastructure and fluctuations in node density implies that distributing information across the network is a complex task. Even if this is achieved, the dynamic nature of the network quickly makes this information obsolete. Thus, it is more practical for nodes to carry out the functions of the network in a distributed way. With no controlling station, each node must perform the necessary networking tasks using some distributed optimization algorithms. Furthermore, each node must rely on information exchange with its local neighborhood and act on limited information while making optimal decisions. This approach is not only effective with regard to power management, as each node typically has a limited power supply, but it is also immune to changes far away in the network, thereby obviating the need for revising decisions constantly.

In general, ad hoc networks are maintained and run by every node that participates in the network. Unlike in cellular networks, where it is sufficient that each node be connected to a base station, communication between two nodes in an ad hoc network is multi-hop and may go through many intermediary nodes. Thus, every node, in addition to being a source or a destination, is also a relay node, forwarding packets for other nodes. The reliance on intermediate nodes for packet forwarding naturally calls for cooperative routing mechanisms. Multi-hop and distributed constructs alleviate issues such as single-point-of-failures, increase fault tolerance and generally improve load balancing in the network. However, with network devices typically being autonomous and under complete control of their agents, multi-hop routing makes the network more vulnerable and susceptible to selfish optimizations and malicious attacks.

The complexity of ad hoc networks resembles that of complex networks; thus, it is desired that the architecture of these multi-hop networks be entirely decentralized to facilitate self-organization, better scalability and higher fault-tolerance. Caution must be exercised by developing network protocols that exhibit robustness to exploitation by selfish users in decentralized autonomous systems. Finally, the decentralized and multi-hop nature of ad hoc networks can make the task of information access and exchange extremely challenging. Nodes in these networks are generally information-limited and must often act in the face of unreliable and incomplete information. In the wake of such uncertainties nodes must exhibit certain amount of intelligence, through observation, awareness, learning, and reasoning, to fuse multiple pieces of information and then make appropriate decisions.

1.2 Motivation

The philosophy of ad hoc networks is "anywhere, anytime communication". The goal of providing ubiquitous connectivity and constant access to information about the world make such networks increasingly complex dynamic systems – self-configuring, multi-hop, power- and bandwidth-constrained, and mobile. Consequently, the opportunities offered by these systems come with a myriad of challenges. Perhaps the most significant challenge of all is to provide acceptable Quality of Service (QoS) guarantees. Some factors that pose the QoS problem at different layers are time-varying wireless channel characteristics, pronounced medium-access contention due to limited bandwidth (because of the shared nature of the medium), node mobility, and frequent variations in network topology. In order to meet the QoS requirement and achieve performance levels close to that of their wired counterparts, it is crucial to address these issues at all layers and optimize the system. Our research focuses on improving QoS from the Medium Access Control (MAC) and network layers' perspective.

Ad hoc networks inherit all the traditional problems of wireless networks; besides, the dynamic topology and multi-hop traits of these networks introduce additional challenges in network design, management, and optimization. The network design problem manifests in many forms depending on how the problem is posed and what metric is being optimized. Nonetheless, efficient network design to achieve a certain network-level objective (or objectives) and to improve global network performance is of paramount importance in a Mobile Ad Hoc Network (MANET). *Topology Control* is a protocol that specifies how to efficiently configure the topology of a MANET so as to enhance the end-to-end network performance.

Power control and topology control are considered distinct corpi of work, yet significant parallels exist between the two. Power control is often understood as a physical layer problem; however, the impact of power control may span all layers of the protocol stack [2]. Transmitter power affects the reliability of a communication link through Signal to Interference and Noise Ratio (SINR) at the respective receiver(s). Topology control is a natural extension of power control to multi-hop networks: the topology of such networks can be controlled by adjusting per-node transmission power levels.

Conventionally, regulating transmit power is construed as the primary mode of performing topology control. While this is true in large part, we develop a more generalized framework that encompasses other radio parameters to achieve topology control objectives; we denote this set of parameters by $\mathcal{P}$. Transmit power, antenna pattern, spreading code length, energy state, cooperation level, and channel selection are some examples of the radio parameters (elements that belong to $\mathcal{P}$), one or more of which can be tuned under the purview of topology control. *Topology control and management is a deliberate, judicious, and an iterative decision-making process of maintaining certain well-chosen communication links, employed at every node, in order to optimize the global network performance.* Unlike in wired networks, where the topology is fixed by the physical infrastructure, the topology of a wireless network is not pre-defined. Hence, it is desirable to have some knowledge of the topology—perhaps incomplete, imprecise or even local—to facilitate packet routing. In essence, the purpose of topology control is to determine where to place the "wires" in a wireless network.

The rationale for topology control is simple and evident: to enhance the overall network performance. It is important to underscore that topology control is a network-oriented solution paradigm. Network connectivity is the most fundamental requirement for effective communication to take place between any two nodes in the network. Therefore, a topology design protocol must accommodate some degree of connectivity in its design space. The technique of topology control can also be drawn upon to extend the duration of network operation. For applications such as sensor networks in 2-D planes, improving the network lifetime is central to such systems; lately, the network lifetime problem for networks in 3-D spaces have also been considered [3]. Mitigating the overall network interference by means of topology control is another overarching objective. The amount of interference impacts end-to-end network throughput capacity; a good network topology is one that is able to deliver as much capacity as any other topology. In practice, 802.11 ad hoc mode is the de facto standard MAC for MANETs, wherein nodes use maximum power for transmitting data and control packets. Employing appropriate power control to reduce interference can improve the spatial reuse and thereby the network capacity. Constructing network topologies for QoS provisioning—such as meeting the delay and bandwidth specifications of the system—can be accomplished through load-balancing. Lastly, a key feature of ad hoc networks is node mobility; thus, some amount of redundancy must be admitted for increased fault-tolerance, self-configuration, and system robustness during topology design. While it is desirable to have one or more of the above characteristics when developing a topology control algorithm, it is to the detriment of the system if the algorithm is not energy efficient. Mobile nodes are typically portable and are equipped with radios, memory, and processors, all of which are often powered by a battery. Hence, it is imperative that every node be energy efficient: this not only increases the node's own operational lifetime but also contributes to an overall increase in the network lifetime. Thus, energy is a limiting factor for desirable network performance. Given these goals of topology control and attributes of a good topology design, it is common to cast the topology control problem as a constrained multi-objective optimization problem on a communication graph. The intrinsic combinatorial complexity associated with graph-theoretic problems often renders efficient heuristics as the most viable alternative.

Given the volatility of ad hoc network configuration, networks that are not optimized by topology control are likely to suffer from poor performance. On one hand, a topology that is too dense can lead to high interference and limited spatial reuse, which in turn leads to reduced network capacity. On the other hand, a topology that is too sparse runs the risk of network partitioning, *e.g.*, in the event of a link or a node failure. Figure 1.1 visually illustrates the impact of topology control.

Research in topology control has been steady for nearly two decades. Yet, some ambiguity exists as to where the topology control functionality should belong in the protocol stack. Topology control inherits features from, and interacts with, both MAC and network layers; including topology control either at MAC or network layer or creating a new interface for it has been the general norm in literature.

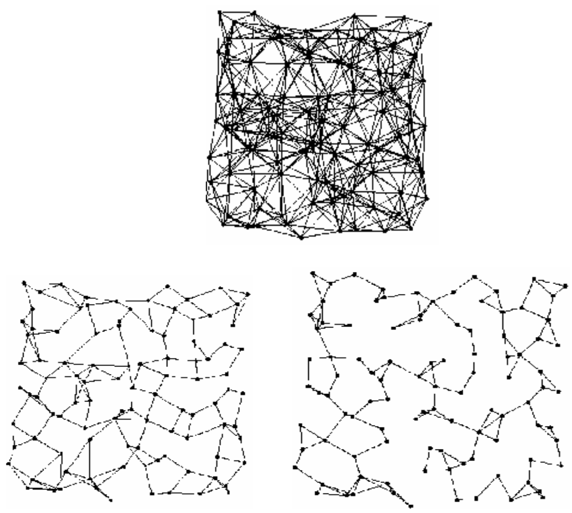


Figure 1.1: From a maximum power topology (top), two possible topology controlled configurations (bottom) are derived. Figures adapted from [1] (© IEEE 2005)

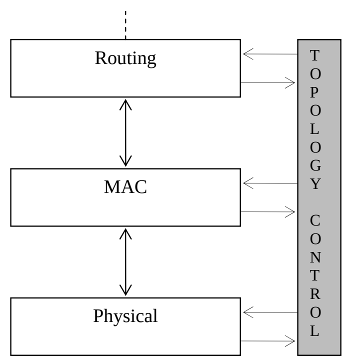


Figure 1.2: Topology Control in the protocol stack.

We believe that topology control takes on a "cross-layer" functionality and as such parameters of network, MAC, and PHY layers can be tuned appropriately through the topology control interface (such control is closely related to the vertical calibration process defined in [4]). For instance, the selection of power level (and hence the topology) clearly affects the MAC layer (which emits control beacons at a certain power level for channel contention); on the other hand, if the MAC interface detects varied responses from its neighborhood, it can trigger topology control for a possible neighborhood change. Likewise, if the topology control layer discovers a neighborhood change, the network layer can respond by performing route updates; and if the routing algorithm returns a break in a route, this can trigger topology recomputation for a possible topology change. Figure 1.2 illustrates our position on the location of topology control with respect to the protocol hierarchy.

1.3 Scope & Contributions

Topology control is a prescription to enhance end-to-end network performance. Barring a few proposals, literature in this field has been dominated by theories and algorithms that rely on cooperation among nodes. In large part, the community is guided by the expectation that node cooperation—through cooperative sensing and information sharing, packet forwarding, and collective decision making—is critical to meet the end-to-end goals of the system. However, ad hoc networks are not monolithic structures; these networks are transitory and emerge when a few migrant radio nodes congregate on an ad hoc basis. These networks are inherently distributed and controlled by end-users. From a user's perspective, it is difficult to justify the cooperative assumption because nodes are either competing for network resources (*e.g.* bandwidth) or conserving their own limited resources (*e.g.* battery lifetime). For instance, why would a node choose to forward packets along its next hop interface and drain its battery resource when it has no incentive for doing so[1]? In this scenario, nodes may behave in exactly the opposite way: conserve their resources and act in their self-interest. In some sense, the problem is further exacerbated by network heterogeneity, where user objectives may conflict. These issues pose a serious question on the applicability of cooperative algorithms for ad hoc networks and render the solution they provide infeasible.

The use of game theory in wireless networks has attracted a great deal of interest lately. (For a concise treatment of the subject, see [6]). Our research is at the nexus of game theory and wireless networking; the primary contribution of this dissertation is in the application

[1] From a broader perspective, it may be in the best interest of nodes to cooperate with each other and do well; however, each node faces a temptation to "defect" and increase its payoff at the expense of other nodes and network performance, in general. Such problems are classified as social dilemma games [5].

of game theory to topology control—specifically, for designing efficient wireless network topologies. As centralized communication networks make way to distributed systems, there is a greater urgency to make these systems work and withstand this transition. The loss of control over networks may diminish the overall network performance; therefore, bridging this performance loss is of paramount importance. Game-theoretic approaches are innately distributed as entities seek to maximize their individual performance. Surprisingly, though, applying game theory to distributed topology control has received little attention.

The recent spate of proposals on cross-layer design and cognitive radio networks show that current networking paradigms are inadequate in dealing with complexities and dynamics of modern wireless communication networks. Modern networks tout an open network architecture due in part to the increasing reconfigurability and programmability of communicating devices. This, coupled with the network heterogeneity, network dynamics, and the shared nature of wireless medium, opens up the possibility of an extremely complex interactive system between the various network elements. Additionally, the network elements in distributed systems are often limited in scope and must contend with limited and partial knowledge about the network operating state. These limitations — the selfish agent optimizations and their interdependencies, and the lack of complete information — may drastically affect the end-to-end system performance. Because topology control is a prescription for improving the overall network efficiency, understanding the implications of the aforementioned constraints in an ad hoc network is key. Game theory provides a natural and flexible framework to analyze and predict the outcome of the interactions between rational selfish network agents that are optimizing their performance in the face of limited network knowledge. Moreover, game-theoretic analysis is valuable in understanding which network protocols are vulnerable to selfish exploitation. This analysis can also provide deep insights into the design of robust protocols.

The focus of this dissertation is on analyzing topology control in selfish multi-channel multi-radio wireless systems. The literature is dominated by proposals of topology control in single channel networks whose access if regulated by MAC protocols. We begin by considering the impact of selfish node behavior on the topology control of single channel systems and study how to overcome the effects of selfishness. We then extend our study to multi-channel network domains. In both studies, and in this dissertation in general, analyzing the interplay between stability and efficiency is one of the significant contributions. From a network viewpoint, both objectives are desirable, but are often in conflict. Here too, game theory can illuminate the tradeoffs between the two objectives, and sometimes even mitigate the conflict. In the first half of the dissertation, we examine the impact of selfish behavior on power and energy efficiency in ad hoc networks. We investigate the problem under both static and dynamic network conditions, as nodes operate along the continuum of knowledge. The second half of the dissertation focuses on spectral efficiency and channel allocation for interference minimization. As before, we examine the effects of incomplete information and network dynamics.

This dissertation can be grouped into two distinct, yet related, studies:

- *Impact of selfish power control on power and energy efficiency*

  We examine the following question: if nodes behave in a selfish manner, how does it impact the overall connectivity and energy consumption in the resulting topologies? We study the problem under two levels of selfishness: (a) nodes cooperate and forward packets, but selfishly minimize power levels and; (b) nodes selectively forward packets and selfishly control transmit powers.

  In the former case, nodes are myopically selfish and minimize energy consumption by reducing their transmit powers. Once nodes selfishly determine their power levels, and therefore the links in the topology, they forward packets for one another at the chosen power. We characterize all the Nash Equilibria and evaluate the efficiency of the induced topologies when nodes employ a greedy best response algorithm. We show that even when the nodes optimize using complete network state knowledge, the steady state topologies are sub-optimal. To mitigate the sub-optimality of the induced NE topologies, we propose a better-response-based dynamic. We show that this algorithm reconciles the selfish objectives of nodes with the overall network goals, and converges to energy-efficient topologies. Besides, the node transmit powers are more evenly distributed, suggesting a fair power level allocation.

We extend our analysis of static networks to consider the effect of network dynamics. Additionally, we analyze dynamic topology control when nodes have limited knowledge about network connectivity. We examine the trade-off between network performance and the cost of obtaining knowledge (by exchanging control information). We show that when networks are more dynamic, those operating under certain amounts of partial knowledge consume least energy due to the high cost of maintaining knowledge. Not surprisingly, this means thats mobility actually helps in information-constrained networks.

By considering the second order effects of selfishness, nodes may take a more holistic approach of minimizing their energy consumption by limiting not only their transmit powers but also by regulating their packet forwarding levels. These two issues—selfish power control and packet forwarding—have largely been studied in isolation. The novel contribution of the latter case lies in analyzing the effect of partial packet forwarding and power control on energy-efficient topology design. Here, nodes selfishly adapt transmit powers to minimize their energy consumption taking into account partial packet forwarding in the network. We show that when nodes forward a small percentage of packets directed through them, the resulting NE topologies that minimize energy are more densely connected and consume more energy than the topologies that emerge when nodes forward a large portion of incoming packets. From the energy viewpoint, this result is particularly interesting as it quantifies the energy efficiency gains obtained by cooperation and corroborates the need for encouraging nodes to forward packets in a decentralized network.

- *Spectral efficiency and channel allocation in multi-channel systems*

  Traditionally the field of topology control has examined power control problems that disregard spectral efficiency or vice-versa. Given the scarcity of spectrum availability, minimizing spectral footprint of a network through topology control is one way of alleviating the spectrum under-utilization problem. We develop a distributed channel assignment algorithm to ensure interference-free network connectivity. We extend our analysis on static networks to consider the impact of restricted node mobilities and incomplete network state knowledge. For all scenarios, we develop core strategies to ensure interference-free connectivity while maintaining low spectral impact.

In realistic scenarios, the number of available channels are far fewer than the number of devices in the network, causing users to share channels and minimize interference. Reducing interference enhances the spatial reuse in the network, and therefore improves the available capacity. The novelty of this study lies in performing topology control of a network purely through channel assignment in multi-channel systems. If there are fewer number of available channels than necessary to support all links in the network, nodes "switch off" some of their radios by not assigning any channels to the corresponding links, causing the topologies to be a lot sparser. We evaluate the performance of NE topologies with respect to network connectivity and interference minimization goals, and examine the tradeoff between the two objectives. Having more channels for a given level of network connectivity naturally leads to lower interference topologies. Likewise, for a given number of channels, supporting a larger portion of the network on those channels results in increased levels interference in the network. Furthermore, in spite of the non-cooperative node behavior, the number of radios on each available channel are evenly distributed, suggesting the load balancing effect of NE

The connecting thread between the above studies reveals how topology control can be implemented in two different ways: (a) by assigning transmit powers to nodes or, (b) by assigning channels to links in the network. Clearly, the outcome of the two approaches will be different as former case examines energy-aware topology control whereas the latter case examines interference-aware topology control. Nevertheless, there exists some overlap when one considers the tradeoff between interference minimizing and energy minimizing topology control: multi channel operation reduces the observed interference, but it also draws significantly more power. We outline our solution approach in the next section.

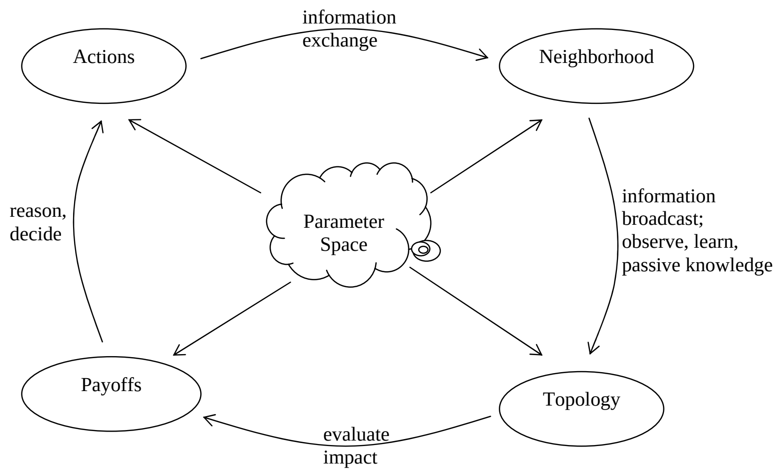


Figure 1.3: A visual illustration of a possible non-cooperative topology control process.

1.4 Our Approach

The solution methodology we adopt to analyze the topology control problem for non-cooperative networks is depicted in Figure 1.3, which specifies how a topology control game might unfold. This dissertation focuses on $\mathcal{P} = \{\text{transmit power, channel}\}$. Each of the remaining chapters focus on the dynamics of topology control games, when nodes tune the parameters of $\mathcal{P}$ above. The analysis is carried out along the dimension of knowledge level, as nodes operate along the continuum of knowledge.

We begin with a brief overview of game theory and literature review of topology control in Chapter 2. In Chapters 3, 4 and 5 we study the game-theoretic models of energy-efficient topology control through transmit power adaptation. In Chapters 6 and 7 we examine the game-theoretic models of interference-aware topology control through channel assignment.

We conclude in Chapter 8 by providing a summary of our contributions, examining ties between this work and cognitive networks, and discussing future research directions.

Chapter 2

Preliminaries

The primary contribution of this research is applying game-theoretic tools to understand topology control from a network design and optimization perspective. Game theory is a distributed optimization technique that analyzes the outcomes of a multi-agent interactive system. Topology control is the study and design of wireless network structures that emerge from multi-agent interaction. Through this research, we strive to understand this interplay between design and analysis for creating efficient network topologies consisting of selfish nodes.

2.1 Game Theory Fundamentals

We begin by presenting basic tenets of game theory as applicable to our research.

2.1.1 A Brief Introduction to Game Theory

Game theory is a mathematical study of conflict and cooperation between rational and intelligent entities. Game-theoretic models can be applied whenever the actions, and therefore the payoffs, of these entities are interdependent[1]. A game models an interactive-decision-making situation, wherein the "rules of engagement" are well-defined. In essence, game theory provides a suite of modeling tools for analyzing interactive scenarios and strives to predict their possible outcomes.

The main object of game-theoretic study is the *game*. At this point, purists in game theory often introduce two distinct classes of games: cooperative and non-cooperative. A cooperative game analyzes the outcomes that result when players come together in different combinations. A main feature of cooperative games is to allow the players to form binding agreements; there is a strong incentive for players to work together to obtain the largest total payoff. Each coalition—a group of players—creates a value or worth for itself. A specific problem addressed by cooperative game theory is how to split or allocate the value generated by coalitions, among their members. Intuitively, the way these values are allocated provides what incentives are available to players and determines what coalitions can form. Without further discussion on this topic, we refer to [7], for a primer on cooperative game theory and its applications.

Unlike cooperative game theory which deliberately abstracts many player-level details of a game, non-cooperative game theory postulates a more detailed model of all potential "moves" available to players. A non-cooperative game establishes a precise framework by laying out the *players* of a game, the *actions* available to the players, and the *preferences* of the players. In the simplest form, such games are called normal or strategic form games. In many cases, however, games unfold over time and players may encounter one another repeatedly, during the course of the game. Such games are classified as extensive form games, which admit a temporal dimension in their construct. These games provide a richer description and specify which outcomes result when players make their moves in various combinations at various points in time. In other words, the timing (and therefore, the order) of the moves are key and the actions may change over time; a repeated game is one such example, where a particular normal form game—the stage game—is repeatedly played and the outcome of previous stages

[1]When the actions are independent, the problem reduces to one of decision theory.

(history) are known in the current stage. For our purposes, the knowledge of strategic form games is sufficient, to develop and analyze games in topology control. For a comprehensive treatise of these and other topics in game theory, we refer to the text by Fudenberg and Tirole [8].

Formally, a game[2] has the following three components:

1. Player set N : $N = \{1, 2, \ldots, n\}$ where n is the number of players in the game.

2. Action set A : $A = \times_{i=1}^{n} A_i$ is the space of all action vectors (tuple), where each component, a_i, of the vector $a \in A$ belongs to the set A_i, the set of actions of player i. Often the action profile is denoted by $a = (a_i, a_{-i})$ where a_i is player i's action and a_{-i} denotes the actions of the other $n - 1$ players. Similarly, $A_{-i} = \times_{j \neq i} A_j$ is used to denote the set of action profiles for all players except i.

3. Utility u: For each player $i \in N$, utility function $u_i : A \to \mathbb{R}$, captures the player's preferences over outcomes determined by the action profiles. $u = (u_1, \ldots, u_n) : A \to \mathbb{R}^n$ denotes the vector of such utility functions.

In an ad hoc network setting, the individual radios are often the players of a game. When the radios are performing topology control, recall that some of the parameters under its control are specified by $\mathcal{P}$. Thus, a possible action may include choosing one of the alternatives in $\mathcal{P}$, *e.g.*, transmit power level. Together, the collection of all radios' actions determines the outcome of the game. Radios may prefer outcomes with, *e.g.*, more reliable connectivity, low power expenditure, etc.; in practice though, these outcomes often conflict with each other.

One of the goals of game theory is to predict the likely outcome(s) of a game. The Nash Equilibrium (NE) is perhaps the most well-studied and generalized solution concept in game theory. Most other equilibrium concepts are, in one form or another, refinements, extensions or derivatives of the NE concept. An NE is a stable point from which no rational player[3] has

[2]Henceforth in this document, a game refers to a non-cooperative strategic form game.

[3]Here, rationality signifies strict adherence to strategies that maximize utility. In some sense, the notion of

any incentive to unilaterally alter his action. Therefore, in some sense, an NE is a consistent predictor of the outcome of a game. Formally,

Definition 2.1.1. *An action profile* $a^* = \left(a_i^*, a_{-i}^*\right)$ *is a* Nash Equilibrium *if,* $\forall i \in N$ *and* $\forall a_i \in A_i$*,*

$$u_i\left(a^*\right) \geq u_i\left(a_i, a_{-i}^*\right) \tag{2.1}$$

A technical note before we move on: we restrict ourselves only to *pure* strategy spaces A, *i.e.*, players select one action or another with certainty from A_i. Pure strategies are a degenerate case of *mixed* strategies, which represent a probability distribution over pure strategies. For instance, in a packet forwarding game, a pure strategy could be either to drop or to forward packets, *i.e.*, $a_i \in \{0, 1\}$; but a mixed strategy could be to forward packets with some probability $0 < q < 1$.

An alternate interpretation of NE is that it specifies a systematic *modus operandi* for rational players. From the players' perspective, the most logical course of action is to eliminate inferior strategies and play the "best" strategy. When multiple alternatives exist, it defines a *best response (BR)* set—a collection of possible BR actions—$\mathcal{B}_i$, for each player, given the strategies of other players. Formally,

Definition 2.1.2. *The best response set of player* i *is given by* $\mathcal{B}_i\left(a_{-i}^{'}\right) = \left\{b_i \in A_i \mid u_i\left(b_i, a_{-i}^{'}\right) \geq u_i\left(a_i, a_{-i}^{'}\right); \forall a_i \in A_i\right\}$. *The action vector* a^* *is an NE, if* $\forall i \in N$, $a_i^* \in \mathcal{B}_i\left(a_{-i}^*\right)$.

Definition 2.1.2 illustrates why NE are often referred to as the fixed points of the BR function $\mathcal{B}$. Under certain restrictions on the game, existence of pure strategy NE can be shown using Kakutani's fixed point theorem [9]. For arbitrary games, however, an NE solution may not exist; even when it does, identifying it may be a non-trivial exercise. For some games, multiple equilibria may exist; the task of eliminating the undesirable ones can be particularly

rationality is more justifiable in wireless networks (than in social settings), where radios can be programmed to "obey" the protocol specifications to select a rational strategy.

challenging. When one considers convergence properties of the game, the problem is further compounded when multiple NE exist.

Fortunately, there exist a special class of games for which both existence of, and convergence to, NE is assured. Notice how Definition 2.1.2 offers guidelines for simple BR-based algorithm constructs to find some NE of the game. If players start the game at an arbitrary non-NE strategy profile, reason according to a BR process, and select strategies from $\mathcal{B}$, then the game *may* end up in an NE profile. Again, for arbitrary games, such a dynamic may cycle and therefore not converge. We pursue some of these issues in greater details, and introduce a promising class of games that are particularly well-suited for analyzing wireless systems.

2.1.2 Potential Games

Nash Equilibria can be considered to be the absorbing states of a game. In real systems, one can not expect to play according to NE because of the limited information or imperfect knowledge about the game. Nevertheless, *a priori* knowledge about NE can aid in designing systems so that NE outcomes are more likely. When players revise their strategies constantly (until they can no longer improve their utilities), as in a repeated game construct, a dynamic process emerges. Unlike in repeated games, we assume players are *myopic* with no memory of past or no foresight of future. For a deeper understanding of how a dynamic process evolves over time in a game-theoretic setting, we introduce two such processes: best response and better response dynamics.

Definition 2.1.3. *A sequence of actions* $\left\{a^{(0)}, a^{(1)}, a^{(2)}, \ldots, a^{(T)}\right\}$ *defines a* best response dynamic *if* $a^{(k+1)} = \left(a_{i_{k+1}}^{(k+1)}, a_{-i}^{(k)}\right)$ *and* $a_{i_{k+1}}^{(k+1)} \in \mathcal{B}_i\left(a_{-i}^{(k)}\right)$ *for some* $i_{k+1} \in N$ *in every round* $k = 0, 1, 2, \ldots, T-1$. *In other words, given the strategies of other players in a given round, each player chooses a best response strategy at every given opportunity.*

A best response dynamic requires that all players have precise knowledge of the best strategy over his entire strategy space A_i. In contrast, a better response dynamic only requires that players be able to pick improving strategies in every round. In terms of both implementation and complexity, this is a much simpler process as players can observe their current payoffs and select any random strategy as long it improves their payoffs.

Definition 2.1.4. *A sequence of actions* $\left\{a^{(0)}, a^{(1)}, a^{(2)}, \ldots, a^{(T)}\right\}$ *defines a* better response dynamic *if if* $a^{(k+1)} = \left(a_{i_{k+1}}^{(k+1)}, a_{-i}^{(k)}\right)$ *and* $a_{i_{k+1}}^{(k+1)} \in \left\{b_i \in A_i \mid u_i\left(b_i, a_{-i}^{(k)}\right) > u_i\left(a^{(k)}\right)\right\}$*, for some* $i_{k+1} \in N$*, in every round* $k = 0, 1, 2, \ldots, T-1$*. If no such* b_i *exists, the player reverts to his previously chosen action.*

In the semantics of game theory, both best and better response dynamics define an improvement path (IP)—a sequence of improving action profiles—in which only one player changes his action between any two contiguous action profiles. *Potential Games* are a special class of games guaranteed to converge to an NE because the IPs are finite[4]. If the game admits many NE, which NE is reached may depend on the initial action profile, the dynamic process, and the player update function (whether asynchronous, random, or deterministic order or some combination thereof).

Definition 2.1.5. *A strategic game* $\Gamma = \langle N, A, u\rangle$ *is an* Exact Potential Game (EPG) *if there exists a function* $V : A \rightarrow \mathbb{R}$ *such that* $\forall i \in N, \forall a_{-i} \in A_{-i}$*, and all* $a_i, b_i \in A_i$

$$V\left(a_i, a_{-i}\right) - V\left(b_i, a_{-i}\right) = u_i\left(a_i, a_{-i}\right) - u_i\left(b_i, a_{-i}\right) \tag{2.2}$$

V *is called the Exact Potential Function (EPF) of* Γ*.*

Definition 2.1.6. *A strategic game* $\Gamma = \langle N, A, u\rangle$ *is an* Ordinal Potential Game (OPG) *if there exists a function* $V : A \rightarrow \mathbb{R}$ *such that* $\forall i \in N, \forall a_{-i} \in A_{-i}$*, and all* $a_i, b_i \in A_i$

$$V\left(a_i, a_{-i}\right) - V\left(b_i, a_{-i}\right) > 0 \Leftrightarrow u_i\left(a_i, a_{-i}\right) - u_i\left(b_i, a_{-i}\right) > 0 \tag{2.3}$$

V *is called an Ordinal Potential Function (OPF) of* Γ*.*

[4]also referred to as the finite improvement path (FIP) property of potential games

By definition, an EPG is also an OPG with the same potential function. Potential games with compact action spaces are known to possess at least one NE in pure strategies [10]. The following lemma due to [10] establishes how Nash equilibria of the game can be identified.

Lemma 2.1.1. *Let Γ be an OPG and V its corresponding OPF. If $a^* \in A$ maximizes V, then it is an NE.*

Thus, potential maximizers form a subset of the NE of a potential game. If we can identify potential functions for a game, we can immediately identify some NE of the game by solving for the potential maximizers. In addition to their NE existence property, potential games have remarkable convergence properties of simple, yet, selfish adaptations. Using the FIP property, it can be shown that both the best and better response dynamics defined in Definitions 2.1.3 and 2.1.4 are assured to converge to a pure strategy NE. The convergence property is particularly attractive in topology control problems, which mostly involve design of algorithms that generate certain desirable topologies. It is this property that allows yet another interpretation of potential games: As a system designer, if the social welfare function coincides with potential function, then by appropriately designing the game, it may be possible to align (and therefore, reconcile the inherent conflict between) the network-level goals with the individual selfish objectives of the players.

The tension between stability and efficiency is fundamental to any dynamic system, more so in an interactive system of independent selfish agents. This is because stability (in an NE sense) is based on self-interest whereas system efficiency is based on communal interest. The Prisoners' Dilemma [11] is a classic example that illustrates the inefficiency of the stable outcomes. The efficiency concept used here and in most game-theoretic studies is called *Pareto optimality*.

Definition 2.1.7. *An action profile $\hat{a}$ is* Pareto Optimal *if there does not exist an $a \in A$, such that $u_i(\hat{a}) \leq u_i(a)\ \forall i \in N$ and $u_j(\hat{a}) < u_j(a)$ for at least one $j \in N$.*

The above definition illustrates how, from a Pareto Optimal (PO) state, it is impossible to move to another state and improve the utility of some player without reducing the utility of some other player. Just as multiple NE can exist for a game, many PO profiles can also exist for a given game; the set of all PO states is often called the Pareto frontier. Concepts such as *price of anarchy* and *price of stability* quantify how close the NE is to the *social optimum* [12, 13]. The former measures the ratio of aggregate user payoffs in the worst NE state as compared to the optimum aggregate payoff; the latter gives a measure of the best-case NE performance instead of the worst case.

In general, establishing the efficiency of NE states is a non-trivial problem in game theory. Repeated game theory is often used to eliminate certain inefficient NE. In a single stage game, some NE may be Pareto-dominated; repetition of same stage game can eliminate those NE and support NE that are PO. Potential games also offer strong efficiency properties of some stable states (the potential maximizers). This can be easily verified in cases where efficiency of a system is cast as sum of user payoffs. In games where potential functions happen to be sum of individual utility functions, it is straightforward to see that potential maximizers are efficient NE states. In Chapter 3, we present an instantiation of this scenario and provide a detailed characterization of potential maximizers and their relationship to network performance, in the context of power efficiency.

Before we conclude this section, it is important to underscore that potential games, while attractive, are somewhat rare. It is a handy tool, nevertheless, especially when modeling situations where improving the overall system performance takes precedence, as in topology control problems. For this reason, it might be worthwhile to investigate if, starting from a system objective as a potential function, one could "design" a potential game with individual payoffs that make practical sense. Notice how this approach is in stark contrast to conventional game theory, which begins with an appropriate utility function and then tries to understand the emergent behavior of a system. This "reverse" game-theoretic approach belongs to the area of *Mechanism Design* that is commonly used in analyzing topology control problems (see discussion in the following section). The idea here is to engineer incentive mechanisms that will steer distributed autonomous agents towards a pre-determined system-wide optimal operating state. Recall our discussion earlier on the inefficiency of NE states; mechanism design alleviates this problem, if an appropriate incentive structure exists. Besides, this approach is also amenable from a networking viewpoint, where we may be able to program nodes with some utility function to maximize so as to improve the network performance. For readers interested in further discussion on potential games, see [6, 10, 14].

2.2 Related Work

Despite its origin in pure mathematics, the term topology, used in the context of wireless networks, has a striking similarity to its mathematical counterpart. In an abstract sense, topology refers to the study of neighborhoods or open sets. Topology is sometimes referred to as “rubber sheet” geometry, where the actual shapes of neighborhoods do not hold much bearing. Likewise, in case of wireless networks, topology control is about defining neighborhoods where the radio antenna patterns and propagation characteristics of the environment define the shapes of the neighborhood; neighbors under omni-directional transmission may not be neighbors under directional transmissions and vice-versa.

Over the past decade the field of topology control has evolved from taking a single dimensional, transmission-power-control-based viewpoint, to a more holistic viewpoint. In the most general case, Topology Control (TC) encompasses aspects from physical layer to application layer. Because TC primarily takes on a network layer view, most of the details from underlying layers are either abstracted or ignored to make the problem more tractable. In Figure 2.1 we present our view of TC taxonomy: the broad scope of the TC problems addressed and the general TC techniques employed, in literature. We broadly focus on the essence of the literature in TC-related areas – the solutions they provide and their limitations and drawbacks. Our intention is purely to provide an overview that will serve as a guideline to explore unsolved problems in this subject.

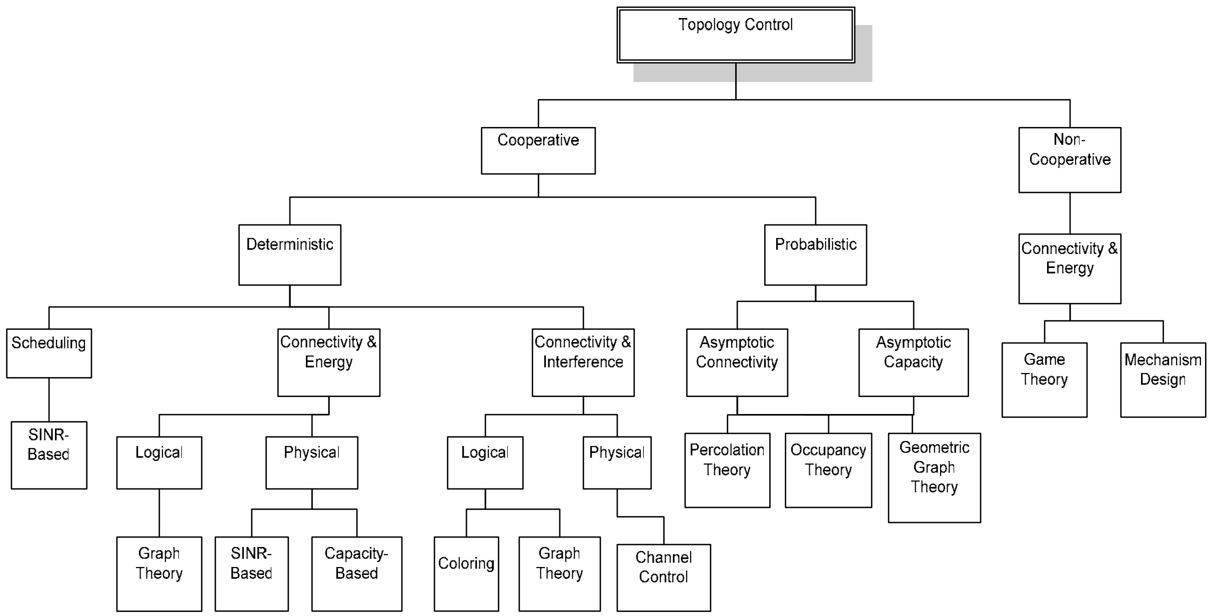


Figure 2.1: Taxonomy of Topology Control.

2.2.1 Review of Topology Control

The topology of wireless networks is unstructured and vulnerable to volatilities. To cope with such transient characteristics, which in some ways are inevitable in a wireless environment, a TC algorithm must transcend the variability in topologies.

Power control is perhaps the most fundamental aspect of a topology design process. The literature is fraught with TC algorithms that use transmit power control. Naturally, different applications have different objectives and hence there is no single, unified, accepted power-control-based TC algorithm. The origin of TC can be traced back to the 80s; first Takagi and Kleinrock and later Hou and Li [15, 16] were among the first to study TC in Packet Radio Networks (PRNs). Not surprisingly, the authors show that transmitting in the forward direction, towards the destinations, at optimal power level yields the best throughput performance; optimal power assignments for various multiaccess schemes were derived.

With the inception of the Survivable Radio Network (SURAN) project, sponsored by DARPA, and with the advent of low cost portable communication devices, such as IEEE 802.11 network interface cards (NICs), the PRNs evolved into ad hoc networks in the 90s. To date, low energy communication has been the primary thrust of ad hoc network research. With the work of [17, 18], there has been a renewed interest in the topology control community, for designing energy efficient ad hoc network topologies. In [18], the authors propose a distributed algorithm for constructing minimum energy topologies on stationary networks. The work of Hu [17] was perhaps the first to draw upon geometric techniques in constructing energy efficient topologies. The specific technique used in [17] is the Delaunay Triangulation (DT), which forms a convex hull of a set of points under certain constraints. In many ways, the synergy between the geometric structures and graph-theoretic techniques yielded the most compelling basis for design of efficient topologies. Among the well-known computational constructs are DT, Yao Graph (YG), Relative Neighborhood Graph (RNG) and Gabriel Graph (GG)—proximity graphs satisfying geometric properties such as optimum equi-angularity, uniform edge length distribution, maximum internal area: attributes useful in minimizing overall energy of a network. YGs, in particular, are well-suited for modeling network of radios with directional antennas. The use of these techniques in efficient topology design can be found in [19, 20]. The mathematical basis for these graphs can be found in [21, 22, 23].

Ad hoc networks are often abstracted, in one form or another, as static graphs, for the sheer simplicity offered by graph-based representations. As radios communicate at some power level, it is convenient to represent the induced graph as $G = (N, E, \Omega)$, with radios as the vertex set $N = \{1, \ldots, n\}$ and all feasible communication links as the edge set $E \subseteq N^2 = N \times N$. $\Omega = [\omega_{ij}]$ denotes the matrix of edge-weights with the weight function $\omega : E \to \mathbb{R}^+$, where $\omega(i, j)$ may signify the power required to close a link $(i, j) \in E$. We hasten to add that the interpretation of ω depends on the context. For instance, if the goal of the system is to establish reliable connectivity in the network, $\omega(i, j)$ may be specified by the channel characteristics of the medium, such as attenuation and fading. In other situations, as in interference-reducing networks, $\omega(i, j)$ may signify the amount of interference caused by having the edge (i, j) in the network. Nevertheless, the problem of connectivity is fundamental to most TC problems.

Network connectivity manifests in two distinct forms: deterministic and probabilistic; the former can be argued to be a degenerate case of the latter. In any event, it is the connectivity at the link level that governs end-to-end connectivity. The simplest and by far the most well-studied link connectivity is the disk model or the protocol model [24]. This is a non-interference model, where transmission conflicts are ignored; therefore, connectivity problem reduces to one of power assignment. In essence, the disk model is boolean: a link (i, j) exists between transmitter i and all receivers j only if transmitter power level $p_i \geq \omega(i, j)$, otherwise not. Typically, the antenna gain patterns are assumed isotropic[5]. Under the purview of this deterministic model, several power efficient TC algorithms have been proposed, *e.g.* [1, 25, 26]. Elegant generalization of YGs is proposed in [1]; Ramanathan *et al.* pioneered the use of transmission power control for topology design [25] ; a localized version of optimization scheme in [25] is developed in [26]. The merit of all three works lies in the development of precise, adaptive, distributed, and localized power control algorithms. The overarching feature of most deterministic TC algorithms can be simply stated as an optimization problem. The objective, predominantly, is to determine a power assignment vector $p : N \rightarrow \mathbb{R}^+$, such that $\sum_i p(i)$ or $\max_i p(i)$ is minimized, while maintaining network connectivity, subject to one or more desirable properties: bounded node degree, spanner, k-connectivity.

In all the aforementioned models, and in most TC problems with energy minimization goals in general, interference is neglected. The explanation offered is that low power assignment and sparse connectivity automatically reduce interference. This myth was first exposed in [27], which underscores the importance of explicitly modeling interference. Following [27], several interference-reducing static graph-based approaches have been studied [28, 29, 26]. In all these studies, a common theme can be discerned: an interference "number"—the number of radios or transmissions that are affected—per active transmission is evaluated and the

[5]In this case, $\omega(i, j) = k \cdot d^{\alpha}(i, j)$, where d is the distance metric, α is the path loss coefficient and k is a constant specified by receiver sensitivity and ambient noise strength.

topology that minimizes some aggregate interference measure is constructed. All the above models of TC rely on a MAC protocol to set apart the conflicting transmissions in time.

While it is common to cast connectivity as a single channel problem, the problem can be augmented by considering the time dimension to establish a multi-channel, multi-radio scenario. The use of multiple channels effectively reduces channel contention (and therefore interference) and improves spatial reuse. Reducing interference through channel selection is tantamount to solving the classical coloring problem, which is known to be NP-hard in many instances. Most research efforts exploit multi-channel capability by proposing new MAC and routing protocols to support multi-channel operation, *e.g.*, [30]. In [31], the authors provide a self stabilizing local channel assignment algorithm for improving throughput of wireless mesh networks. Lately, smart channel allocation strategies have also been considered for dynamic spectrum access, in the context of cognitive radio networks, for improving spectrum utilization; see [32] and references therein. The underlying theme in all the aforementioned papers is to provide efficient algorithms for channel allocation in multi channel domains. The work of [33] is among the first to cast channel assignment as a TC problem that opened up a novel way of controlling topologies through channel allocation in multi-channel networks. When the connectivity of a network is fixed and there are fewer number of channels than needed, [33, 34] present heuristics to minimize the interference. Xue and Ganz provide a temporal TC protocol in [35] to cope with multi channel capabilities taking into account the implementation considerations. They too take a TC viewpoint in which different topologies emerge from different channel assignments.

While the disk model provides an elegant framework for connectivity, the underlying channel condition is not truly taken into consideration; thus, it abstracts away the important physical layer aspects of the problem. The all-or-nothing nature of the model prohibits any amount of interference, however small it may be. Technically, two nodes are said to be connected if data can be exchanged at some non-zero rate. In an information-theoretic sense, any non-zero SINR enables communication. Of course, the quality of reception and achievable rates depend on the SINR at the receiver. Gupta and Kumar propose a more realistic model of connectivity, the physical model [24]: communication is feasible as long as the SINR is above a pre-defined threshold. Under the physical model, (i, j) exists if:

$$\frac{p_i g\,(i,j)}{\sigma + \sum_{k\in N} p_k g\,(k,j)} \geq \gamma_j \tag{2.4}$$

here, $[g]$ is the link gain matrix, γ_j the SINR threshold that depends on the receiver sensitivity and σ is the noise power. In [36, 37] the authors extend the SINR model and advocate the notion of capacity-based connectivity. In their model, two nodes are said to be connected if the bit error rate (BER) or symbol error rate (SER) is below a certain threshold. SER depends on the underlying SINR, modulation, framelength etc.; therefore, it is a more accurate indicator of connectivity.

Unlike the disk model, the physical model allows for a greater spatial reuse. The disk model precludes simultaneous transmissions even when the disks marginally overlap and the interference is negligible. Allowing concurrent transmissions entails scheduling at the link layer. For arbitrary topologies formed by exercising power control while satisfying (2.4), Moscibroda *et al.* [38] derive bounds on the scheduling complexity: the minimum number of time slots required to legally schedule a set of valid requests.

The issue of optimal scheduling and power control when combined with optimal routing leads us to characterize capacity of arbitrary topologies. Gupta and Kumar pioneered the research in analyzing capacity of multi-hop wireless networks in their landmark paper [24]. Specifically, the authors show that the traffic carrying capacity of arbitrarily designed networks grows vanishingly small, asymptotically, as the node density increases. This influential result spurred an enormous amount of research; Grossglauser and Tse followed up in [39] by developing a 2-hop relay model and showing that mobility improves per-node throughput for random networks, possibly at the cost of inordinately large delays; analogous results to that of [24] (with constant factor improvements) are presented in [40] for radios equipped with directional antennas. The capacity region—the set of achievable rates—under some specific transmission strategies are developed in [41]. In all these studies the network capacity is defined in an information-theoretic sense. The information-theoretic definition of network capacity is developed using Shannon's notion of link capacity; naturally, this definition favors "shorter" links over "longer" ones and thus suffers from fairness and link starvation issues. Network information theory, a whole new research thread in itself, deals with developing Shannon-like capacity regions for wireless networks [42].

From a network perspective, one would like to know what input traffic rates the network can support such that average delay is bounded and no queue grows without bound. This network layer notion of capacity region is developed by Neely *et al.* [43] following the work of [44]; queue stability is guaranteed as long as the input rate matrices are within the capacity region (note that this is a different notion of capacity region than the aforementioned one). Besides, the authors also develop a joint power control, routing and scheduling algorithm that stabilizes the system whenever the rates are interior to the capacity region. Similar multi-commodity flow control approach is adopted in [45] to characterize achievable rates, but in a much restricted setting. Along similar lines, Kumar *et al.* develop centralized and distributed algorithms to maximize throughput capacity [46]; their work, in some sense, is an algorithmic version of [24].

Despite immense advancements in the theoretical study of network capacity regions and in understanding its fundamental limits, there is little evidence in the actual benefits of topology control on improving network capacity. Interestingly, the contrary was shown to be true in [47], which was verified independently in [48] for sensor networks. Thus, the role of conventional TC for capacity improvements seems to be unclear. In fact, characterizing capacity-optimal topologies that can transport data across the network at rates close to capacity is still an open question. And there seems no single answer to this question; the achievable capacity depends on many parameters: network size and node distribution, traffic patterns, power and bandwidth constraints, and routing strategies. Evaluation of the performance of well-known TC algorithms is carried out in [49] with mixed results. Use of TC to improve throughput capacity is shown in [50] via simulations. In [51], extensive experimental studies on the impact of various network structures on end-to-end throughput is performed.

We would be remiss if we fail to mention the large body of TC research that exists for analyzing asymptotic network behavior. The inherent complexity involved in designing precise, efficient topologies renders distributed implementation infeasible; in many cases, TC problems are known to be NP-hard (for instance, minimizing the aggregate power). Furthermore, most approaches assume perfect information in one way or another, *e.g.*, knowledge of exact node locations. To make the analysis more meaningful and practical, probabilistic models of connectivity and capacity are developed. Unfortunately, probability models are more tractable under large-scale conditions, and hence much of the analysis is developed in an asymptotic sense. These models answer questions of the following nature: what is the minimum transmission range (likewise, the minimum node degree) that ensures (asymptotic) connectivity with high probability (w.h.p) under a given node distribution; what is the node density for a given transmitter range, that ensures connectivity w.h.p (this is a dual to the previous problem); what is impact of (random) SINRs and random traffic patterns on the outage probability (and hence the connectivity), and where do the percolation phase transitions lie; what is the minimum number of time-slots needed to restore connectivity of an interfering network (this is useful in establishing bounds on the capacity)? Some well-known tools that are commonly used in asymptotic analysis is the Geometric Random Graph (GRG) theory (here, typically, the deployment region is a square and node distribution is uniform) and the Percolation theory (used for varying shapes of the region and for varying node distributions) and Occupancy theory. In all cases, the goal is to study the "giant component" phenomena (sometimes, also referred to as the "0-1" law): the critical threshold regions around which network is operating in some degree between full connectivity and no connectivity. For a more comprehensive study, refer to [52] and references contained therein.

2.2.2 Non-Cooperative Topology Control

Next generation wireless networks and ad hoc networks in particular will be inherently decentralized. Besides, it is expected that these networks will eventually include software-defined cognitive radios, which have the ability to control and adapt their parameters such as waveform, power, channel, etc., at will [53]. The transfer of control from a centralized entity (such as a base station in cellular networks) to these individual radio agents is a paradigm shift in network control and operation. From a topology control perspective radios can selfishly compete for resources, which can result in gaining unfair advantage at the expense of overall network performance. To analyze the possible conflict of interests, either amongst the individual radios or between radios and the network, game theory can be applied. In some ways, game theory—the study of interactive decision making between rational agents—is more suitable to radio networks (than social networks) because radios, once programmed, are expected to behave rationally. Despite the far-reaching consequences of decentralization and deregulation, surprisingly little work exists for topology control in non-cooperative networks. The research efforts to address the problem of topology control in the presence of selfish nodes are fairly recent. Game theory and mechanism design are the commonly used approaches to address this problem.

It is generally perceived that even if nodes act selfishly, some amount of cooperation is required to sustain an autonomous ad hoc network (see [54] and references therein). The crux of the problem is how to stimulate the nodes to cooperate—by using reputation-based or pricing-based frameworks—when they are driven by self-interested objectives. The need for cooperation is a fundamental problem, which manifests in various forms at all layers [55]. Eidenbenz *et al.* are the first to pose the TC problem as a non-cooperative game and study connectivity properties [56]. Much of their work is devoted to the analysis of algorithmic complexity in finding the NE, when it exists, and deriving bounds on the price of anarchy—the ratio of worst NE to the globally optimal state. Three connectivity games are proposed but the existence of NE is not guaranteed. Furthermore, the authors do not provide energy-efficiency characteristics of the topologies that emerge.

Mechanism design seeks to achieve global efficiency by aligning the selfish objectives of individual users with the socially desirable outcome. In the context of TC, mechanism design is employed to provide adequate incentives to users so that they maximize their objective function when the network minimizes total energy consumption, subject to connectivity constraints. This approach has been adopted in [57] and [58] by engineering a payment system that leads selfish nodes to forward packets for others. The utility function proposed in [58] requires that each node declare the per-edge price that it intends to charge in exchange for forwarding packets.

Despite the limited attention on game-theoretic analysis of TC, there have been some notable contributions in the application of game theory to TC related areas. Use of game theory to analyze the power control problem at the physical layer has received enormous attention; similarly, selfish contention for channel access has been studied at the MAC layer; see [55] and references therein. Channel selection has been examined using game theory, but only in the context of dynamic spectrum access and cognitive radio networks *e.g.* [59, 60, 61]. All these studies analyze the problem from a radio viewpoint at the link level. Extending these techniques to meet end-to-end goals of the system is imperative; for instance, network-aware channel selection can greatly enhance the network performance through topology designs that incorporate interference-minimization. Game-theoretic analysis of network layer issues such as routing and packet forwarding is perhaps the most mature, see [12, 62, 63, 64, 65]. However, this viewpoint too lacks a holistic perspective because much of the analysis is based on a pre-defined topology. Dynamically building topologies by exploiting the synergy between power control, packet forwarding and route selections stems the possible loss of performance through its multi-objective optimization construct.

2.3 Positioning Our Research

Our research belongs to the body of literature in TC for non-cooperative ad hoc networks. Our framework, discussed in Section 1.4, adopts neither the radio viewpoint nor the network viewpoint. On one hand, the radio viewpoint lacks the scope to make network-aware decisions; on the other hand, the network viewpoint acts on the pretext of a pre-defined topology. Both perspectives have a pre-conceived notion of the network and therefore may result in sub-optimal performance.

Our research contribution differs from all the studies discussed in Section 2.2. Our study of non-cooperative networks is fundamentally different from all cooperation-based studies. As next generation networks become more and more decentralized and deregulated (controlled by end-users), issues such as selfish node behavior, lack of complete and precise information, must be confronted with, perhaps by means of a distributed, localized, cross-layer, multi-objective, game-theoretic framework. In contrast to the works that consider non-cooperation, our work (in Chapters 3, 4 and 5) guarantees the existence of equilibrium states and develops distributed algorithms that converge to desirable topology states, from an efficiency standpoint. To the best of our knowledge, we are the first to consider and examine the effect of partial packet forwarding on TC (in Chapter 5). Our study on spectral footprint minimization (in Chapters 6) gives a feel for the lower bound on the bandwidth consumption – a useful metric for network operators and providers administering bandwidth-limited networks. Finally, performing TC purely through channel selection has received little attention. This is a novel contribution of our work (in Chapter 7), which specifies conditions on channel selection for radio interfaces for reducing interference and improving spatial reuse.

One of the goals of this research is to tread the middle ground between the polar perspectives of radio and network by taking a topological viewpoint and understanding the emergent characteristics—one of the traits of self-organization—of the steady state topologies (see discussion in Chapters 4 and 7). Another overarching objective of our research is to study the role of information on the efficiency of the final outcomes; making network-aware decisions requires more information (knowledge) than making radio-level decisions, but more information also leads to more efficient outcomes. We believe that both these issues—the interplay between efficiency and information and the dissonance between radio vis-à-vis network goals—have received little attention in literature and both problems are intrinsically coupled.

Part II

Selfish Topology Control in Single Channel Networks

Chapter 3

Non Cooperative Power Minimization – Static Case

In this chapter, we examine the energy-efficient topology control problem. The parameter space is $\mathcal{P} = \{p\}$, where *each node has the ability to tune only its transmission power settings* according to its self-interested objectives. All nodes share a common spectrum which is regulated by a MAC protocol. Besides, nodes do not possess the faculty to drop packets. In other words, nodes selfishly minimize their energy consumption by controlling their transmit power levels, but are assumed to forward packets for one another.

This chapter addresses an important question related to distributed topology control: in selecting their operating parameters, should radios be programmed to optimize their own "selfish" utilities or a network-wide objective function? We evaluate the network performance under two scenarios: (a) nodes have global topology state knowledge; and (b) nodes have only local topology state knowledge. Under this framework, we study the tradeoffs associated with the antithetical viewpoints of network, which strives to minimize energy globally, and those of the nodes, whose goal is to minimize their energy consumption locally.

3.1 Framework and Assumptions

3.1.1 System Model

The wireless medium is subject to losses like fading and multipath effects; therefore, it is desirable to have link-level acknowledgments for packets received. Link bidirectionality is also crucial for proper functioning of MAC protocols such as 802.11 [66]. Hence, we assume that links in our ad hoc network model must be bidirectional in order to be useful. Also, our focus is on single channel networks. Wireless channels are characterized by interference between nearby transmissions. We suppose that a MAC protocol ensures temporal separation of conflicting transmissions and disregard interference in our model. In Part III of this dissertation, we explicitly model interference and design efficient, interference-aware topologies.

For our model, let the network consist of heterogeneous nodes embedded in a 2-D planar region $\mathbb{R}^2$. Each node may have different maximal power $p_i^{\max}$, allowing asymmetries in the network. It is then convenient to represent the network as a graph, $H = (N, E, \Omega)$, consisting of a set of nodes $N = \{1, \ldots, n\}$ and an edge set $E \subseteq N^2 = N \times N$. An edge between any two nodes represents an abstraction of the communication link between them. Let $\Omega = [\omega_{ij}]$ be a matrix of edge-weights with the weight function $\omega : E \to \mathbb{R}^+$, where $\omega(i, j)$ is the power required to close a link $ij \equiv (i, j) \in E$. The exact threshold $\omega(i, j)$ is a function of channel attenuation and inter-nodal separation; as such our model is generalized to accommodate varying channel characteristics. We, however, assume that Ω is a symmetric matrix. Following an adjustable power model, each node can adapt its transmission power appropriately and select a set of neighbors. The transmit level determines the transmission range of a node; a necessary (but not sufficient) condition for node j to hear node i is that j be within the range of i. In other words, the transmission level $p : N \to \mathbb{R}^+$ such that $p(i) = p_i \geq \omega(i, j)$, determines the subset of edges $E' \subseteq E$ that are supported. Likewise, given Ω, a bidirectional link (i, j) exists if and only if $p_i \geq \omega(i, j)$ and $p_j \geq \omega(j, i)$ The collection of all such bidirectional links results in a subgraph $G = (N, E')$ of H, called a *transmission graph* that contains edges (i, j) if j is present in i's transmit range and vice-versa. We use $\mathcal{G}$ to represent the set of all possible graphs generated by various power assignments $\mathbf{p} = (p_1, \ldots, p_n)$, and; $G(\mathbf{p})$ is a typical element in $\mathcal{G}$.

More precisely, for each node i, define a link state variable variable l_{ij} as:

$$l_{ij}(p_i) = \begin{cases} 1 & \text{iff } p_i \geq \omega(i, j); \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

When node i broadcasts with a transmission power p_i, it forms a neighborhood containing every node that is within its transmission range. Due to the broadcast nature of the wireless medium, each node can obtain its neighborhood information by broadcasting "`Hello`" beacon messages at a certain power level and by gathering the ACK replies. The `Hello` messages should at least include the node's identification, current transmission power, and maximal transmission power. Let $\mathcal{N}_i(p_i) = \{j \mid l_{ij}(p_i) \cdot l_{ji}(p_j) = 1\}$ be the set of (direct) neighbors of node i. The collection of such neighborhoods forms a topology on N. In other words, the joint transmit power level profile $\mathbf{p}$ induces a network, given by:

$$G(\mathbf{p}) = \{ij \mid l_{ij}(p_i) \cdot l_{ji}(p_j) = 1; i \neq j \in N\} \tag{3.2}$$

We denote the above network, in short, by G_p. Also, note that the inclusions $ij \in G_p$, $j \in \mathcal{N}_i$ and $i \in \mathcal{N}_j$ are all equivalent. If every node i transmits at $p_i^{\max}$, we call the induced topology $G_{\max}$ a *maximum power network*. Because our model acknowledges only bidirectional links, G_p is connected if and only if there exists a bi-directed path—a collection of contiguous bidirectional links—between every node pair $i, j \in N$.

Assumption 1. *$G_{\max}$ is a connected network.*

The objective of our distributed TC algorithm is then to derive a subgraph G_p of $G_{\max}$ that is energy efficient and preserves the connectivity of $G_{\max}$.

In the literature, energy efficiency has been defined in different ways: minimizing the maximum transmission power, minimizing the sum of radii, or maximizing energy stretch factor (for a definition, see [67]). In this chapter, we use the following definitions of energy efficiency:

Definition 3.1.1. *A connected network G_p is said to be* locally energy efficient *if no node can reduce its transmit power level without disconnecting the network.*

Definition 3.1.2. *A connected network G_p is said to be* minmax energy efficient *if* $\max_{i \in N} p_i$ *is minimized.*

Definition 3.1.3. *A connected network G_p is said to be* globally energy efficient *if* $\sum_{i \in N} p_i$ *is minimized.*

3.1.2 Game Model

Here, we formally describe the TC process as a normal form game. Individual nodes form the player set, $N = \{1, 2, \ldots, n\}$, of the game. Each node can autonomously set its transmit power level $p_i \in [0, p_i^{\max}]$. The individual power levels can be collected into a power vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$, which forms the action space, A, for the game. The power vector induces a topology G_p, which is a collection of feasible links as defined by equations (3.1) and (3.2).

Let $\mathcal{G} = \{G_p \mid \mathbf{p} \in \times_{i=1}^{n} [0, p_i^{\max}]\}$, denote the collection of all possible networks which can be generated by power vectors $\mathbf{p}$. Note that, for all $G_p \in \mathcal{G}$, $G_p \subseteq G_{\max}$. Each node perceives a trade-off between the benefit it derives from a connected topology G_p and the cost it incurs in establishing G_p. A utility function captures these tradeoffs and maps the power vector to a payoff for each node. For every $i \in N$, the utility function u_i is expressed by:

$$u_i(\mathbf{p}) = \varphi_i(G(\mathbf{p})) - \chi_i(p_i) \tag{3.3}$$

Here, $\varphi_i : \mathcal{G} \to \mathbb{R}$ represents the benefit node i derives from network G and χ_i is the cost incurred. In the context of network connectivity, each node perceives a benefit in being connected and, therefore, in being able to establish communication sessions, with other nodes in the network. The specific utility function we adopt is discussed in the next section.

3.2 A Topology Control Game

Consider a multi-hop network consisting of independent and autonomous nodes that distributively adapt their transmit power levels according to their connectivity and energy consumption preferences. Such adaptations could potentially affect the performance of other nodes, and thereby, influence their decisions. This kind of an interactive and distributed power control process impacts the topology of the network. In the context of this chapter, such an interactive and iterative process defines our TC game. For a visual illustration of the game, see Figure 1.3

We address the problem of designing energy-efficient topologies that preserve network connectivity, in the presence of complex interactions among nodes in a network. A network designer may prefer to minimize the total power consumption (global energy efficiency) of the network, or minimize the maximum power consumption of a node (minmax energy efficiency) in the network, and seek to design an efficient topology. On the other hand, individual nodes may choose to reduce their own power consumption, regardless of the network performance. More often than not, such myopic behavior may lead to an undesirable equilibrium from a network viewpoint. As discussed in Chapter 2, this inherent conflict can sometimes be reconciled if the system designer's objective function (social welfare function) is a potential function for the game. A potential game also offers strong convergence properties of simple selfish dynamic strategies; we develop three such algorithms in the next section.

3.2.1 Utility Function with Complete Information

We first consider the case when nodes are omniscient, having full knowledge of the topology state information. Using the general utility function given by (3.3), a specific utility for each node is given by:

$$\bar{u}_i(\mathbf{p}) = M_i f_i(\mathbf{p}) - p_i \tag{3.4}$$

Here, $f_i(\mathbf{p})$ is the number of the nodes that can be reached (possibly over multiple hops) by node i via bidirectional links and paths[1]. Naturally, f is non-decreasing, *i.e.*, $f_j(p_i, p_{-i}) \geq f_j(q_i, p_{-i}), \forall j \in N$ and $q_i < p_i$. The scalar benefit multiplier M_i signifies the value each node places on being connected to other nodes; we assume $M_i \geq p_{i,max}\ \forall i$. *In other words, the preference of nodes are in a lexicographic order.* This means, nodes regard connectivity more importantly that their power consumption.

Network connectivity is a basic requirement in TC as it provides the means for nodes to establish communication sessions with their destinations. The benefit component in (3.4) signifies the reachability of a node. It implicitly assumes that each node has some traffic for every other node in the network. This is a reasonable assumption because traffic load and selection of destinations are typically not available during topology formation. This necessitates that the underlying topology be connected.

Connectivity is a function of transmit power of all nodes in the network. Each node chooses a transmit level based on its objective function and not for the objectives of other nodes. However, we make a slight distinction here and emphasize that once a node decides the power level to transmit at, it continues to forward packets at its chosen transmit power. The validity of node cooperation for packet forwarding in ad hoc networks is a research thread in itself; we refer interested readers to [54, 62, 63, 64].

[1] In other words, a node places the same "value" whether it can reach another node in one hop or in multiple hops. From a connectivity standpoint, this assumption is reasonable since at the topology formation level, we only need to know whether there exists a path to any given destination. In reality though, we may prefer shorter paths to longer ones depending on the QoS (*e.g.* minimum latency) requirement of the traffic, which may alter the benefit structure.

A quick note before we move on: our utility function given in (3.4) is quite generic and works even without the knowledge of exact node locations, so long as the threshold power levels, ω, required to establish links are estimated accurately. Certainly, many other utility functions can be used to model the specific systems under study. An example, for instance, is one in which each node views benefit from covering a given area (instead of connecting to certain number of nodes as considered in (3.4)). Such a construct models applications such as sensor networks well, but requires the knowledge of node locations in assessing the sensor field coverage. For instance, two nodes that are within proximity of each other do not add to the individual coverage areas of each other because both nodes more or less "observe" the same information. However, if the two nodes are distant from each other, each node, with location information of the other, can improve its utility by connecting to the other node and thereby increasing the coverage area.

3.2.2 Utility Function under Incomplete Information

In a more practical setting nodes are expected to have only a partial and incomplete picture of the network. Because nodes have to contend with limited information during decision-making, we modify (3.4) and develop its localized version. A possible utility function that could be conceived is one in which utilities are functions of neighborhood connectivity, and not entire network connectivity as in (3.4).

$$\tilde{u}_i^{(k)}(\mathbf{p}) = M_i f_i^{(k)}(\mathbf{p}) - p_i \tag{3.5}$$

where $f_i^{(k)}$ is the number of nodes within i's k-neighborhood, *i.e.*, the nodes that can be reached in at most k hops from i via bidirectional connections (ideally k must be as low as possible). The members of each successive k hop neighbor of i can be described recursively:

$$\mathcal{N}_i^k = \begin{cases} \{i\} & k = 0 \\ \mathcal{N}_i^{k-1} \cup \Big\{ j \mid jl, lj \in E,\ l \in \mathcal{N}_i^{k-1}, j \neq i \Big\} & k > 0 \end{cases} \tag{3.6}$$

The cost component in (3.4) and (3.5) suggests that transmission power is the primary source of energy consumption. Transmission costs may include energy consumed in sourcing or in forwarding packets in a given session between two consecutive executions of the TC protocol. We ignore all additional energy consumed when receiving, storing and processing packets. It is important to underscore that we make these assumptions to keep the utility function simple; the essence of the problem is nonetheless still preserved.

Consider the case of neglecting the reception power, which may be unrealistic in certain applications. From a game-theoretic viewpoint, the present cost function can be easily extended and modified to incorporate the received power as well. The number of incoming edges in the topology that terminate at i specifies which other nodes can be heard by i. According to (3.1), an incoming edge to i from j is defined by the power level of j; thus, nodes in general have little control on their reception costs. In the semantics of game theory, the received cost component of each node i can be modeled by $C_i(p_{-i})$, a "dummy" function that depends on the power levels of all nodes except i. The addition of a dummy function does not alter the potential game property of the TC game. Consequently, the subsequent analysis of the TC game such as its convergence properties and the efficiency of NE topologies, discussed in the following sections, are unaffected. Nonetheless, the topologies that minimize the total cost (*i.e.* the sum of data transmission and reception powers) may, in fact, be different from those that minimize transmission power alone. As we shall see in Section 3.5, the steady-state topologies are quite sparse with very few extraneous unidirectional edges on average; thus, we believe that the reception costs will be comparable to those in optimal topologies. Additionally, the received cost can further be reduced by decoding a few header bits and turning off the receiver for the rest of the transmission period, in case the transmission was intended for some other receiver.

More aggressive energy consumption models can be used to create energy-efficient networks. For instance, a protocol where nodes turn their radio off and go to "sleep" mode if their participation is not mandated by the network, can significantly save energy. Likewise, a node may choose to selectively forward packets in order to conserve energy. Study of such energy models is beyond the scope of this chapter; we refer the readers to [63] and [68] for further discussion.

To study the TC games specified by utility functions (3.4) and (3.5), we develop TC algorithms for selfish nodes in the following section.

3.3 Distributed Topology Control Algorithms

We propose three TC algorithms for wireless ad hoc network formation in presence of self-interested nodes: Max Improvement Algorithm (MIA), δ-Improvement Algorithm (DIA), and Local-DIA. All the three algorithms consist of three phases: an initialization phase, an adaptation phase, and an update phase. The algorithms primarily differ in how the adaptation phase is implemented and how much information nodes have. In the MIA, nodes adapt their transmit levels according to a "greedy" best response process. Under the DIA, nodes adapt their transmit levels according to a "restrained" better response process. Both these algorithms rely on complete topology state information. Under Local-DIA too, nodes act according to a better response dynamic. Local-DIA works on the premise that nodes have only a partial and incomplete view of the network. In this sense, some or all nodes may be unaware of a portion of the network state. Because nodes must act in the face of incomplete information, the better response dynamic under Local-DIA is different from the one under DIA (though they share some similarities).

Given these preliminaries, we formalize the initialization, adaptation, and update phases as follows:

1. (Initialization) Each node i transmits at its maximum power level $p_i^{\max}$ and discovers its neighborhood $\mathcal{N}_i\left(p_i^{\max}\right)$; the induced topology is $G\left(\mathbf{p}_{\max}\right) = G_{\max}$.

2. (Adaptation) Node i, selected via some sequential order, improves its utility (given by (3.4) and (3.5)) by adjusting its power setting from $p_i^{\max}$—according to a best or better response adaptation process—to a value $p_i \leq p_i^{\max}$.

3. (Update) Neighborhood of i, $\mathcal{N}_i\left(p_i\right)$, is recomputed and the induced topology $G(p_i, p_{-i})$ is updated for the new power setting.

4. Repeat steps 2 and 3 until no node revises its power setting in a given round.

We study our distributed algorithms under two settings: (a) nodes have global topology state information; (b) nodes have only local topology state information. Consequently, the adaptation phase for these two cases is implemented differently. We now elaborate on each of the three phases.

3.3.1 Initialization Phase

Every node initializes its power setting to $p_i^{\max}$. Each node then discovers its neighborhood by broadcasting neighbor request messages at $p_i^{\max}$ and collecting the responses provided by the receivers at $p_j^{\max}$. Upon successful reception of ACKs from each responding neighbor j, node i sets its link state variable l_{ij} to 1 according to equation (3.1). Collection of all such individual neighborhoods defines the initial topology, $G_{\max}$.

3.3.2 Adaptation Phase using Complete Information

In this phase, nodes are assumed to have complete information about the overall topology state information (e.g. network connectivity). Each node is chosen from a permutation—round-robin or random—to determine its transmission power. All nodes execute either the

MIA or the DIA during the course of the game. We emphasize that only one node adapts its power setting at a time[2]. If a node alters its power setting, other nodes are made aware of this adaptation through control messages. In Sections 3.4.1 and 3.4.2, we discuss the outcome of the TC game when these strategies are implemented.

3.3.2.1 Max Improvement Algorithm

Each iteration of the game can be viewed as a normal form game, wherein, every node chooses to maximize its utility in that iteration. This iterative process allows the network topology to evolve dynamically. In this best-response-based algorithm, whenever a node has an opportunity to revise its power setting, it chooses a transmit level that maximizes its utility (3.4), given the transmit levels of all other nodes, according to:

$$\hat{p}_i = \arg\max_{q_i \in A_i} \bar{u}_i \left(q_i, p_{-i}\right) \tag{3.7}$$

The formal algorithm can be state as follows:

Algorithm 1 MIA$(G_{\max}) \rightarrow (G_{mia}, \hat{\mathbf{p}})$

while $\hat{\mathbf{p}}$ is not an NE **do**
  for all $i \in N$ **do**
    $\hat{p}_i = \arg\max_{q_i \in [0, p_i^{\max}]} \bar{u}_i \left(q_i, \hat{p}_{-i}\right)$
  end for
end while

3.3.2.2 δ-Improvement Algorithm

For the ease of exposition, we discretize the action space. Intuitively, it is sufficient to search for the optimum action over those power values that correspond to the power threshold entries

[2]This is reasonably justified because, in a practical setting, the probability of any two nodes updating their strategies at the same time instant is zero. To realize this restriction, one can imagine nodes embedded by a random timer; nodes update their strategies whenever the timer goes off. Alternately, a token passing scheme, as part of the protocol, can also serve the purpose.

of Ω. This requires each node to maintain per-neighbor power levels, and may necessitate modifications at the MAC layer. Instead of introducing additional complications, we form a modified set $\bar{A}$ that consists of a finite number of power levels, common for all nodes. We envision the network interface card hardware to only be capable of power control in such discrete steps.

For each node $i \in N$, define a modified action set as:

$$\bar{A}_i = \left\{p_{\max} = p^{(0)}, p^{(1)}, \ldots, p^{(\zeta)} = p_{\min}\right\} \tag{3.8}$$

where $\bar{A}_i$ is an ordered set, *i.e.*, $p^{(k)} < p^{(k-1)}$. (In the next section, we show that maintaining network connectivity is always a better response strategy; therefore, $\exists\zeta$ (and thus a $p_{\min} \neq 0$) such that $p_i \geq p^{(\zeta)}\ \forall i$ is a necessary (though not sufficient) condition to ensure connectivity.) One way to construct $\bar{A}_i$ is to let transmit level of all nodes be initialized to $p_{\max}$ that guarantees connectivity with sufficiently high probability [69], and decrement power in steps of a predefined step-size, δ. (In the next section, we show that when a sufficiently small δ is chosen, DIA converges to a desirable NE state. Because A is a compact set, such a $\delta > 0$ (as a function of node density) can always be chosen.)

Under DIA, each node i chooses a power level one level lower[3] than its current level if the chosen action gives a better payoff than its current action. Otherwise, the node reverts to the power level it was currently transmitting at. More concisely, let $p_i^{(k)}$ be the current level at which node i is transmitting, $k = 0, 1, \ldots, \zeta - 1$. Given the transmit level of all other nodes, each node chooses to transmit next at a level given by:

$$\bar{p}_i = \underset{q_i \in \left\{p_i^{(k+1)}, p_i^{(k)}\right\}}{\arg\max}\ \bar{u}_i\left(q_i, p_{-i}\right) \tag{3.9}$$

Note that the utility at $p_i^{(k)}$ will be greater than that at $p_i^{(k+1)}$ if the network is partitioned

[3]Given the ordering of $\bar{A}_i$, note that if the current power is $p_i^{(k)}$, the node makes a switch to $p_i^{(k+1)}$ at the next opportunity.

at $p_i^{(k+1)}$ (see the argument in Proposition 3.4.3). Alternately, if network connectivity is preserved at $p_i^{(k+1)}$, then $p_i^{(k+1)} < p_i^{(k)}$ ensures that the utilities are unequal. In either case, the utilities will never be the same. Algorithm 2 formalizes the description of DIA.

Algorithm 2 DIA($G_{\max}$) $\rightarrow$ ($G_{dia}, \hat{\mathbf{p}}$)

```
m = 0
p̂_i = p^(m) ∈ Ā_i ∀i ∈ N
while p̂ is not an NE do
    m = m + 1
    for all i ∈ N do
        choose p_i = p^(m) ∈ Ā_i
        p̂_i = arg max_{p'_i ∈ {p_i, p̂_i}} ū_i (p'_i, p̂_{-i})
    end for
end while
```

Notice that the action set for DIA is constructed from information about the minimum power required by every node to reach every other node, and action choices are chosen synchronously and sequentially. Synchronous action selection requires nodes to know the network's current action choice. Furthermore, both MIA and DIA require the evaluation of the f_i function, which requires global knowledge of network connectivity.

In some sense, nodes are more aggressive when following the MIA; whereas, nodes following a DIA adaptation process are more restrained when improving their payoffs. These contrasting selfish behaviors lead to significantly different steady-state outcomes. In the context of potential games, these two simple adaptive processes are assured to converge; the latter goes one step further and aligns node-centric objectives to network-level goals (we formally prove this in the next section).

3.3.3 Adaptation Phase using Incomplete Information

To quantify partial or incomplete information, we use the idea of k hop neighborhood described in Section 3.2.2. When nodes have k hop information, nodes have full knowledge of the network state in their k hop neighborhood and no more. Furthermore, we assume that k hop knowledge is not transitive; nodes do not share information beyond their k hop neighborhood, and there is no passive learning, meaning that nodes only use information explicitly shared with them and do not utilize information overheard in the wireless medium. These assumptions allow k hop knowledge to be an experimental parameter that can be tuned to study the role of partial knowledge in network design. Note that if $k \geq dia(G)$ (where $dia(G)$ is the diameter of the topology, the maximum number of hops between any two nodes in the network) then the network is said to be operating under global knowledge since every node's k hop neighborhood includes the full network. For all $k < dia(G)$, there is some degree of partial knowledge, in the sense that some or all nodes may be unaware of some portion of the network. Also note that the fraction of the total network that the nodes are aware of is a function of their k hop knowledge and the topological connectivity; we explicitly examine this relationship in Section 3.4.3.

3.3.3.1 Local δ-Improvement Algorithm

As described in the previous subsection, the original DIA algorithm is global in its scope; it utilizes full knowledge of the network to determine the connectivity of the network, define the possible transmission powers and synchronize the power selection across the network. Local-DIA, described in Algorithm 3, has been generalized from DIA so that it can operate without global knowledge of network connectivity, required transmission power, or synchronization state. As in the case of DIA, Local-DIA operates on an action set wherein each node searches for the optimum action over those power values that correspond to the power thresholds for each reachable neighbor. Under full knowledge, Local-DIA becomes functionally the same as DIA. Generalizing (3.8), we define the action set for each node as follows:

$$\tilde{A}_i = \left\{ p_i^{\max} = p_i^{(0)}, p_i^{(1)}, p_i^{(2)}, \ldots, \right\} \tag{3.10}$$

such that one connection is dropped by node i when the power is adapted from $p_i^{(m)}$ to $p_i^{(m+1)}$. One way to construct $\tilde{A}_i$ is to initialize node i to $p_i^{\max}$ and decrement its power by a variable step size δ_i^m. Because $p_i^{(m)}$ represents the connection power thresholds, the first step-size is given by $\delta_i^0 = p_i^{\max} - \max_{j \in N} \{\omega_{ij} \mid \omega_{ij} < p_i^{\max}\}$ and subsequent step-sizes are given by $\delta_i^m = p_i^{(m)} - p_i^{(m+1)}\ \forall m > 0$.

Algorithm 3 LOCAL-DIA$(G_{\max}) \rightarrow (G_{ldia}, \hat{\mathbf{p}}_i)$

1: $m = 0$
2: $\hat{p}_i = p_i^{(m)} \in \tilde{A}_i\ \forall i \in N$
3: **while** $\hat{\mathbf{p}}$ is not an NE **do**
4: $\quad m = m + 1$
5: $\quad$ choose $p_i = p_i^{(m)} \in \tilde{A}_i$
6: $\quad \hat{p}_i = \arg\max_{p'_i \in \{p_i, \hat{p}_i\}} \tilde{u}_i^{(k)} (p'_i, \hat{p}_{-i})$
7: **end while**

3.3.4 Update Phase

Under global algorithms (MIA and DIA), nodes' choice of power level in each iteration redefines its neighborhood; this, in turn, modifies the overall topology. Once a particular node changes its power level to the current topology state, other nodes are made aware of this change by means of some optimized flooding technique. Under a local algorithm (like LOCAL-DIA), each node broadcasts its neighborhood table every time there is a change in its 1 hop neighborhood. Note that it is sufficient for nodes to send updates only to those nodes that are within their k-neighborhood, and not to all nodes in the network (as done in DIA or MIA). It is also sufficient for each node to broadcast its 1 hop neighbor table and not the entire k hop neighbor information. By propagating the control updates only to small neighborhoods, LOCAL-DIA greatly reduces the overhead cost. Besides, this idea of k hop neighborhood prevents the overhead cost from growing with network size. Upon receiving these control messages, other nodes update their respective link state tables. In turn, these nodes respond to the topology change by choosing an appropriate power level.

If none of the nodes update their power level setting from its current level, the algorithm is said to have to converged to a steady-state (NE). Since the TC game we consider is a potential game, the network is assured of converging to an NE steady state when nodes selfishly update their power settings in a sequential manner (see proofs of Proposition 3.4.3 and Lemma 3.4.4).

3.4 Game-Theoretic Analysis

We begin by showing that the game $\bar{\Gamma} = \langle N, A, \bar{u} \rangle$ with the objective function of each node given by (3.4), is a potential game.

Theorem 3.4.1. *The game* $\bar{\Gamma} = \langle N, A, \bar{u} \rangle$ *where the individual utilities are given by* (3.4) *is an OPG. An OPF is given by* (3.11)

$$V(\mathbf{p}) = M_i \sum_{i \in N} f_i(\mathbf{p}) - \sum_{i \in N} p_i \tag{3.11}$$

Proof. We prove by applying the asserted OPG in (3.11). First we have,

$$\begin{aligned} \Delta \bar{u}_i &= \bar{u}_i(p_i, p_{-i}) - \bar{u}_i(q_i, p_{-i}) \\ &= M_i \left[f_i(p_i, p_{-i}) - f_i(q_i, p_{-i}) \right] - (p_i - q_i) \end{aligned} \tag{3.12}$$

Similarly,

$$\begin{aligned} \Delta V &= V(p_i, p_{-i}) - V(q_i, p_{-i}) \\ &= M_i \left[f_i(p_i, p_{-i}) - f_i(q_i, p_{-i}) \right] - (p_i - q_i) \\ &+ M_i \left[\sum_{j \in N; j \neq i} \left\{ f_j(p_i, p_{-i}) - f_j(q_i, p_{-i}) \right\} \right] \end{aligned}$$

Thus, we have

$$\Delta V = \Delta \bar{u}_i + M_i \left[\sum_{j \in N; j \neq i} \left\{ f_j(p_i, p_{-i}) - f_j(q_i, p_{-i}) \right\} \right] \tag{3.13}$$

Since $f_i(\mathbf{p})$ is monotonic and $M_i \geq p_i^{\max}\ \forall i$, it follows from (3.12) that

$$\Delta \bar{u}_i = \begin{cases} \geq 0 & \text{if } p_i > q_i \text{ and } f_i(p) > f_i(q_i, p_{-i}); \\ \leq 0 & \text{if } p_i < q_i \text{ and } f_i(p) < f_i(q_i, p_{-i}); \\ < 0 & \text{if } p_i > q_i \text{ and } f_i(p) = f_i(q_i, p_{-i}); \\ > 0 & \text{if } p_i < q_i \text{ and } f_i(p) = f_i(q_i, p_{-i}) \end{cases} \tag{3.14}$$

The sign of the second term in (3.13) is the same as the sign of $\Delta \bar{u}_i$ for the first two cases of (3.14). For the last two cases of (3.14), the second term in (3.13) is zero, because the connectivity profile of every node remains unchanged; therefore, $\Delta V = \Delta \bar{u}_i$. In general, $\operatorname{sgn}(\Delta V) = \operatorname{sgn}(\Delta \bar{u}_i) \Rightarrow V$ is an OPF and $\bar{\Gamma}$ an OPG. □

As noted in Chapter 2, one of the overarching consequences of being a potential game is the possible relationship between a potential function and a social welfare function. In the context of our TC game, the social welfare function is the energy-efficiency metric. Alternately, potential maximizing NE of the TC game can be interpreted as the optimal power assignment vectors, *i.e.*, steady-state topologies that are globally energy efficient.

Theorem 3.4.2. *For the game $\bar{\Gamma} = \langle N, A, \bar{u} \rangle$, the class of global potential maximizers coincide exactly with the class of topologies that are globally energy efficient.*

Proof. Let $\mathbf{p}$ belong to the set of potential maximizers. For a given $\mathbf{p}$, we show that $G(\mathbf{p})$ is connected and globally energy efficient. We prove this by contradiction:

Case 1 Say $G(\mathbf{p})$ is not connected. Then $f_i(\mathbf{p}) = k_i < n$, the number of nodes in the network, $\forall i$. In other words, $k_i \leq n - 1$. Since $\mathbf{p}$ is a potential maximizer, $V(\mathbf{p})$ must be greater than the value $V(\mathbf{p}^*)$ generated by another (connected) network, say, $G(\mathbf{p}^*)$. Note that, since $G(\mathbf{p}^*)$ is connected, $f_i(\mathbf{p}^*) = n$, $\forall i$ and $V(\mathbf{p}^*) = M_i \cdot n^2 - \left(\sum_{i \in N} p_i^*\right)$. In other words,

$$\begin{aligned} V(\mathbf{p}) &= M_i\left(\sum_{i\in N} k_i\right) - \left(\sum_{i\in N} p_i\right) > M_i \cdot n^2 - \left(\sum_{i\in N} p_i^*\right) \\ &\Rightarrow M_i\left(n^2 - \sum_{i\in N} k_i\right) < \left(\sum_{i\in N} p_i - \sum_{i\in N} p_i^*\right) \end{aligned} \tag{3.15}$$

Since $k_i \le n-1$, the LHS of (3.15), $M_i(n^2-\sum_{i\in N} k_i) \ge M_i(n^2-n\cdot(n-1)) = n\cdot M_i \ge n\cdot p_{i,max}$. On the other hand, the RHS of (3.15), $\left(\sum_{i\in N} p_i - \sum_{i\in N} p_i^*\right) \le n \cdot p_{i,max}$. Thus, (3.15) is a contradiction. Hence, $G(\mathbf{p})$ is always connected when $\mathbf{p}$ is a potential maximizer.

Case 2 Now, suppose $G(\mathbf{p})$ is connected but p_i is not minimum for some i. This implies, $(\sum_{i\in N} p_i) > (\sum_{i\in N} p_i^*)$. However, since $\mathbf{p}$ is the potential maximizer, $V(\mathbf{p}) = M_i \cdot n^2 - \left(\sum_{i\in N} p_i\right) > M_i \cdot n^2 - \left(\sum_{i\in N} p_i^*\right) \Rightarrow \left(\sum_{i\in N} p_i\right) < \left(\sum_{i\in N} p_i^*\right)$, a contradiction to our assumption.

Combining cases 1 and 2, we conclude that $G(\mathbf{p})$ is always globally energy efficient, when $\mathbf{p}$ is a potential maximizer.

We now prove the reverse direction. Let $G(\mathbf{p})$ be globally energy efficient. We show that $\mathbf{p}$ is a potential maximizer.

Since G is globally energy efficient, $\forall i,\ f_i(\mathbf{p}) = n$ and $\left(\sum_{i\in N} p_i\right)$ is minimal. Thus, $V(\mathbf{p}) = M_i \cdot n^2 - \left(\sum_{i\in N} p_i\right)$ is maximal. Thus, $\mathbf{p}$ is indeed a potential maximizer.

Thus, we conclude that the network $G(\mathbf{p})$, resulting from the game $\bar{\Gamma}$, is globally energy efficient if and only if $\mathbf{p}$ is a potential maximizer of (3.11). □

3.4.1 Analysis of Max Improvement Algorithm

An immediate upshot of Theorem 3.4.1 is that both MIA and DIA are guaranteed to converge to an NE [10]. Consider the MIA: in every round, each node plays a best response to the power setting of other nodes. This defines a sequence of action profiles, where contiguous action vectors differ in exactly one element. Using the FIP property of potential games discussed in Chapter 2, it can be shown that this sequence always converges to an NE profile. Besides, the topology induced by the NE has some desirable properties, as shown in the following proposition.

Proposition 3.4.3. *The MIA algorithm converges to an NE of the game $\bar{\Gamma}$ that is locally energy efficient and preserves connectivity of $G_{\max}$.*

Proof. From Theorem 3.4.1 we have that $\bar{\Gamma}$ is an OPG. From [10], it follows that the MIA will converge to an NE. However, we are interested in only those NE that preserve connectivity in the final topology. Recall that the input to the MIA is the topology $G_{\max}$, with every node transmitting at $p_i^{\max}$. The best response for each node is to reduce its transmission power (and maximize its utility) to a value p_i so that the resulting topology is remains connected. We prove this by contradiction. Suppose node i maximizes its utility at $q_i < p_i$, given p_{-i}, and the network is not connected. This implies that $u_i\left(q_i, p_{-i}\right) = M_i \cdot k_i - q_i > M_i \cdot n - p_i$, where $k_i < n$, the total number of nodes in the network. This implies, $M_i \cdot (n - k_i) < p_i - q_i$, an impossible inequality, because the term on the LHS is larger than $p_i^{\max}$ and the term on the RHS is smaller than $p_i^{\max}$. Thus, in every round, the topology is always connected.

Since the topology is always connected in every iteration, $\forall i,\ f_i\left(\mathbf{p}\right) = n$, a constant. The utility maximization problem now becomes a power minimization problem. Thus, the final steady-state topology is also locally energy efficient. □

3.4.2 Analysis of δ-Improvement Algorithm

We have shown that the MIA is guaranteed to converge to locally efficient topologies, by Proposition 3.4.3. Under the dynamics of this process, any initial state $\mathbf{p}_{\max}$ forms the basin of the attraction and the system converges to the local maxima of the potential function.

Theorem 3.4.2 identifies the existence of globally energy-efficient states; thus, if the global maxima of the potential function are the attractors of a dynamical system, convergence to efficient topologies can be assured. Recall that the outcome of MIA depends on the order in which nodes take turn in updating their actions. Additionally, the problem of minimizing the total sum power in a network has been shown to NP-hard [70]. Hence, one needs to resort to developing efficient heuristics to closely approximate a global solution, at best. We instead adopt an alternate approach and develop a polynomial time DIA algorithm that is guaranteed converge to minmax energy-efficient topologies (that minimize the maximum power of a node in the network). Through simulations we show that, on average, the performance of minmax topologies is comparable to that of globally efficient topologies (see Figure 3.8).

In the DIA process, each node selects a power setting with a higher payoff than its current payoff. Given that each node transmits at $p_{\max}$ and the induced $G_{\max}$ is connected at the start of the algorithm, any $p_i < p_{\max}$ that preserves connectivity, is sufficient to improve i's payoff. For achieving efficiency, we first make the following assumption:

Assumption 2. *The matrix Ω is symmetric. In addition, the threshold powers ω_{ij}'s are all distinct for all $i > j$.*

As described in Section 3.3.2.2, each user adapts by decrementing her transmission power, albeit one level at a time, as long as it improves her payoff; otherwise, the user continues transmitting at her current level. In order to guarantee convergence of DIA to the minmax energy-efficient states, the step-size δ—the amount by which power levels are decremented in each step—should be sufficiently small.

Assumption 3. *Step-size δ is chosen so that at most one connection (link) is dropped from the network when the powers are adapted from $\mathbf{p}^{(k)}$ to $\mathbf{p}^{(k+1)}$, where $p^{(k)}, p^{(k+1)} \in \bar{A}_i$ from equation* (3.8).

Similar to the MIA, the DIA dynamic also specifies an improvement path—a sequence of improving action profiles. The improvement path is finite, and as a result, the DIA dynamic converges to an NE [10].

The following result is the cornerstone of this chapter: when nodes employ the DIA, the process converges to an NE that induces a minmax energy-efficient topology. The proof of this theorem is based on an MST property. Recall from Section 3.1.1 that we adopt a network model where the edge weights of the underlying graph are the power thresholds. Taking into account the wireless broadcast property, we first define a PMST as follows:

Definition 3.4.1. *A graph G is a* PMST *if it is an MST and contains any additional edges induced by wireless broadcast property.*

We present the following two lemmas which are essential in proving our main result.

Lemma 3.4.4. *Consider the game $\bar{\Gamma} = \langle N, A, \bar{u} \rangle$ where nodes employ the DIA under Assumption 3. Starting with an initial topology $G_{\max}$ induced by the power vector $\mathbf{p}_{\max}$, the algorithm converges to a subgraph, G_{dia}, of the PMST.*

Proof. Proof is by induction. For the ease of presentation, we suppose, without loss of generality, that $G_{\max}$ is a complete network. Consider a $G_{\max}$ comprising of 3 nodes: A, B, C; suppose $\omega_{AB} > \omega_{AC} > \omega_{BC}$ be the corresponding relationships. Based on Assumption 3, nodes start at power level ω_{AB} and keep decreasing their power in steps of δ, till ω_{AC}. At this point, nodes A and C will not reduce their power any further; otherwise, the network would disconnect and the nodes' payoff would decrease[4]. Because $p_A = \omega_{AC}$ and $\omega_{AB} > \omega_{AC}$, link AB is severed as a result. Thus, the DIA algorithm converges to a topology containing links AC and BC, the shortest two bidirectional links needed to connect the three nodes.

Now consider a fully connected topology with 4 nodes: A, B, C, D; without loss of generality, let $\omega_{DA} > \omega_{DC} > \omega_{DB} > \omega_{AB} > \omega_{AC} > \omega_{BC}$ (otherwise, the indices can be rearranged). All nodes keep decreasing their power from ω_{DA} till ω_{DB}. Node D now has only a single link, DB, that is bidirectional. The problem then reduces to a 3 node topology as before. Thus, the algorithm converges to a topology containing the three shortest bidirectional links AC, BC and BD (and possibly some extraneous unidirectional links as well).

[4] According to the argument in Proposition 3.4.3, which also applies for a DIA dynamic, network connectivity is preserved at every stage of the game.

The above line of reasoning can be generalized to any arbitrary network of size n. Therefore, the algorithm always hits a state that consists of the shortest $n-1$ bidirectional links needed to maintain connectivity. Note that, at this point the network is a PMST by Definition 3.4.1.

If the PMST contains a bidirected cycle (a cycle with all bidirectional links), atleast one node in the cycle may still reduce its power level further and still maintain connectivity. Otherwise, PMST contains exactly all the bidirectional links of MST. In either case, the steady-state topology G_{dia} is a subgraph of PMST (the subgraph may not be proper). This completes the proof. $\square$

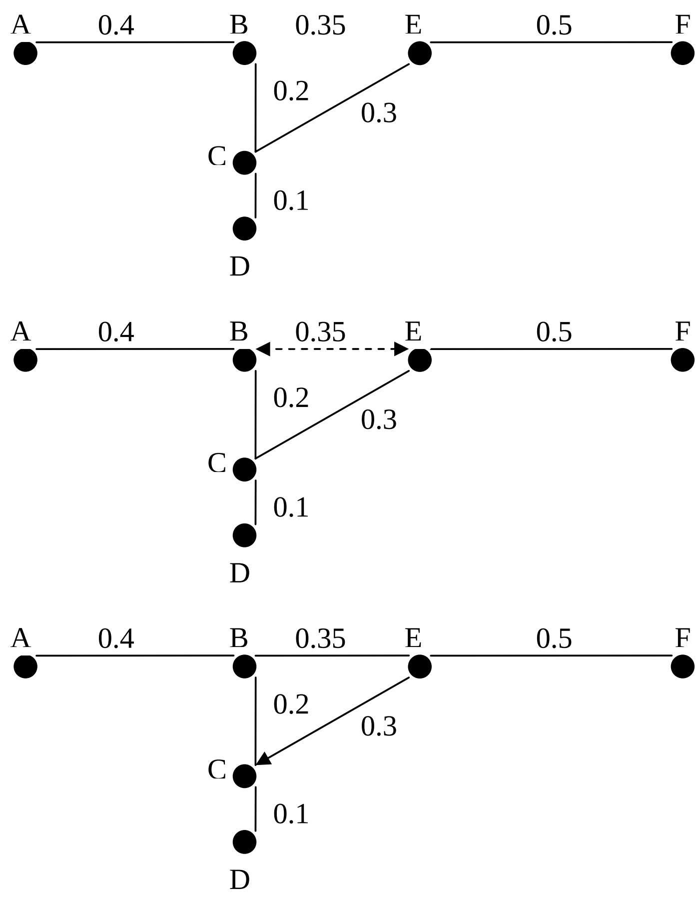


Figure 3.1: The MST (top), the induced PMST (middle) and the G_{dia} (bottom) derived over the course of DIA. Note that, due to the incidental link BE in PMST, node C is able to lower its power further (as shown in G_{dia}).

For a visual illustration on the difference between a typical MST, PMST and G_{dia} obtained over the course of the DIA, see Figure 3.1. Note that, because the bidirected link BE in the PMST is incidental, node C can lower its power level further.

Lemma 3.4.5. *MST minimizes maximum power of any given node in the network.*

Proof. The principle idea behind the proof is the fact that MST minimizes the maximum edge-weight of the network. We show this by contradiction.

Let us assume, on the contrary, that there exists another tree T that minimizes the maximum edge-weight. Let $e_t^{\max} = \arg\max_{ij \in T} \omega(ij)$ and $e_{mst}^{\max} = \arg\max_{ij \in MST} \omega(ij)$, where ω is the edge-weight function. By our contradiction, $\omega(e_t^{\max}) < \omega(e_{mst}^{\max})$. Introduce a cut—and partition the nodes into two sets N_1 and N_2—in MST by removing $e_{mst}^{\max}$ from the graph. Since T is a tree, we can find an edge $\tilde{e} \in T$ to join N_1 and N_2 and create an new tree $\tilde{T}$. Because $e_t^{\max}$ is the edge in T with the maximum weight, we have $\omega(\tilde{e}) \leq \omega(e_t^{\max}) < \omega(e_{mst}^{\max})$. Since $\tilde{T}$ is essentially created from the MST, $\sum_{e \in \tilde{T}} \omega(e) < \sum_{e \in MST} \omega(e)$; we obtain a contradiction. Therefore, MST is indeed the tree with the minimum maximum edge-weight.

The edge, $e_{mst}^{\max}$, with the maximum weight determines the node with the maximum power. Therefore, it follows that MST minimizes the maximum power of any node in the network.

□

Using the above two lemmas, the following main theorem of the chapter is an immediate consequence.

Theorem 3.4.6. *DIA converges to a minmax energy-efficient topology—one that minimizes the maximum power of any given node.*

Proof. We know that PMST contains MST and all the additional induced edges. Because none of the induced edges increase the maximum edge-weight of the graph, PMST preserves Lemma 3.4.5. From Lemma 3.4.4, the steady-state topology G_{dia} is a subgraph of PMST;

therefore, every edge in G_{dia} is contained in PMST. It follows immediately that Lemma 3.4.5 still holds for G_{dia}. Hence, the result follows. $\square$

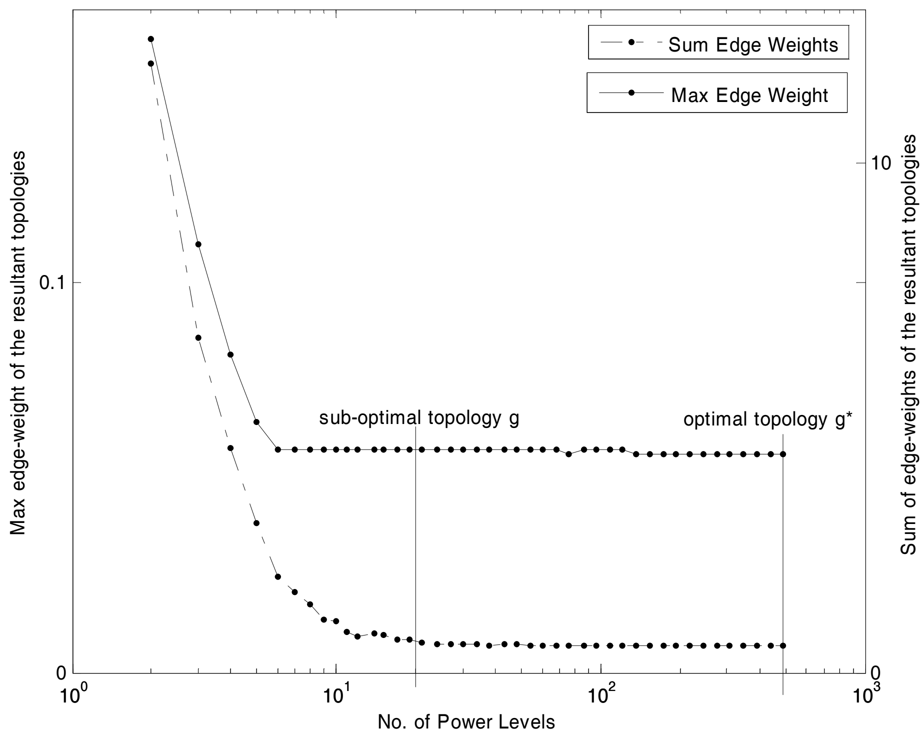


Figure 3.2: *Impact of δ on the steady-state outcome*: The higher the number of power levels in the search space, the closer the margin between optimal and sub-optimal convergent states.

We have shown that if a sufficiently small δ is chosen, DIA converges to the minmax energy-efficient topologies. As a general rule, δ decreases with increasing network density. Because δ specifies the number of power levels in the search-space $\bar{A}_i$, it requires fine granularity in power adaptations in order to converge to efficient topologies; in real applications using such small δ can be prohibitive. In Figure 3.2, we consider a random topology with a density of 30 nodes/unit^2 and quantify the impact of various δ values on the efficiency of the steady-state network. Minmax energy-efficient topology, G^*, is identified using a δ^* value that satisfies Assumption 3; then the maximum edge-weight and sum of edge-weights of G^* are computed. Using this optimal δ^* as the reference, several different δ values are chosen leading to this optimal value. For each δ, the DIA converges to (a possibly) different steady state G. We compare the energy metrics (maximum edge-weight and sum of edge-weights) for these sub-optimal topologies to those of the optimal topology G^*. For the sake of clarity, we plot the number of power levels in the search-space $\bar{A}$ in log-scale, along the x-axis in the figure below.

From the figure, we note that as the size of the search-space increases (*i.e.* as δ decreases), the resulting steady-state topologies approach the optimal topology configuration. As an engineering tradeoff, one can choose a sub-optimal topology with performance comparable to that of the optimal one while reducing the algorithm complexity. For the particular 30 node network considered in Figure 3.2, choosing a δ that corresponds to 20 power levels (size of $\bar{A}$) reduces the search space to a more practical value while still generating a good approximation of the optimal solution, which requires a search space of 500 power levels. Note that the optimal size of $\bar{A}$ depends on the node density.

3.4.3 Analysis of Local δ-Improvement Algorithm

Both MIA and DIA algorithms, though distributed, are global in scope. Each node, as it makes power adaptations, needs to know whether or not it is connected to all other nodes in the network. Local-DIA, on the other hand, is a local algorithm that relies only local neighborhood information. In Local-DIA, each node observes its current k-neighborhood and strives to maintain connectivity with every node in its k-neighborhood while making power adaptations. Local-DIA being a better response algorithm, each node only chooses powers that increase their utility; hence, nodes never choose a power level that reduces the size of their set of k hop neighbors.

Lemma 3.4.7. *In* Local-DIA*, for all $k > 0$, the better response strategy for every node is to preserve its k hop neighborhood.*

Proof. We prove this by contradiction. Suppose node i reduces its power level from p_i to q_i to reduce its k hop neighborhood (from, say, $\mathcal{N}_i^k$ to $\mathcal{N}_i'^k$) and increases its utility. This implies that $\tilde{u}_i(q_i, p_{-i}) = M_i|\mathcal{N}_i'^k| - q_i > M_i|\mathcal{N}_i^k| - p_i$, where $|\mathcal{N}_i'^k| < |\mathcal{N}_i^k|$. This implies, $M_i\left(|\mathcal{N}_i^k| - |\mathcal{N}_i'^k|\right) < p_i - q_i$, an impossible inequality, because the term on the LHS is larger than $p_i^{\max}$ and the term on the RHS is smaller than $p_i^{\max}$. □

Recall that under partial knowledge nodes cannot ensure synchronization and do not have knowledge of the values of ω_{ij} for the network. For this reason, LOCAL-DIA, under partial knowledge, does not necessarily converge to an optimal network power configuration. While each node maintains its k hop neighborhood, according to Lemma 3.4.7, its decision may reduce another node's k hop neighbor set; this happens if i drops a connection with one of its current 1 hop neighbors that belongs to the k-neighborhood of j. Unless i broadcasts its new neighborhood, the nodes in the k-neighborhood of i may be unaware of the changes in their respective k-neighborhoods. For an illustration of this fact, see Figure 3.3. The following theorem ensures that, following LOCAL-DIA, every NE still preserves the overall network connectivity.

Theorem 3.4.8. *If initial topology G is connected, then* LOCAL-DIA *converges to an NE that is also connected for all $k > 0$.*

Proof. A connected network can be disconnected in two ways:

1. if a node (say i) disconnects itself from another node, while executing LOCAL-DIA or,

2. if node i disconnects two previously connected nodes, say j and m, in the process of reducing its power during the course of LOCAL-DIA.

We know that case 1 violates Lemma 3.4.7. The latter case is not possible unless j and m are connected to each other through i. If j and m are k-neighbors of i, i will not lose connection with either j or m by virtue of Lemma 3.4.7. If j and m are beyond the k hop neighborhood of i, the only way for i to lose connectivity with either of them is to disconnect with an existing member of its k-neighborhood, which is disallowed by Lemma 3.4.7. Thus, in either case, the network will remain connected. □

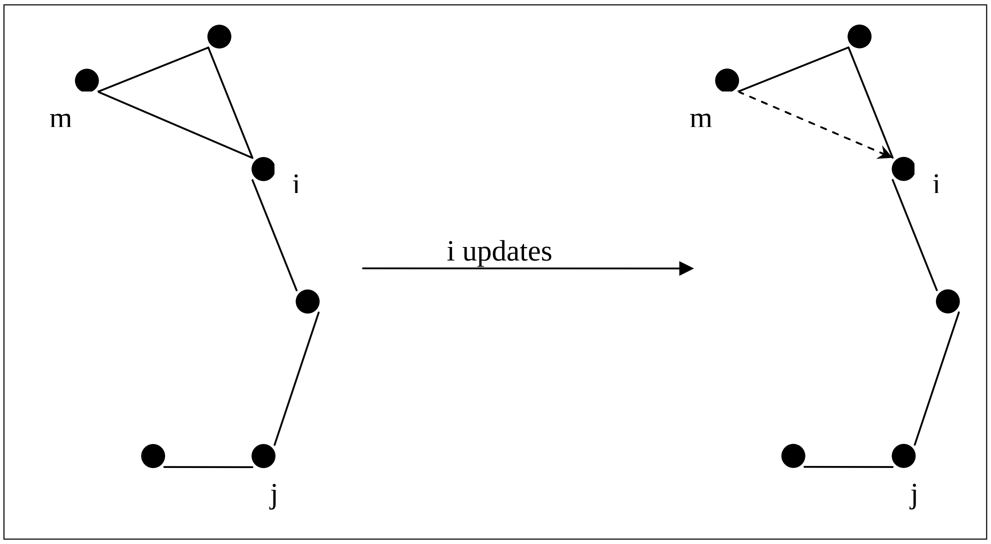


Figure 3.3: *Illustrating the* LOCAL-DIA *process*: For $k = 3$, node i can maintain connectivity with node m at reduced power level (by going through two hops), but this affects node j, which loses m from its 3 hop neighborhood.

The sub-optimality of the resultant LOCAL-DIA topology is exacerbated as the amount of knowledge decreases. The resultant topologies are over-connected under partial knowledge, given that nodes will not remove any connection that decreases the size of their k hop neighborhood. Furthermore, unlike DIA, in which nodes synchronously change power in lock-step to remove connections, there is a "first mover advantage" inherent in LOCAL-DIA. The first node to act has the most actions available to it; subsequent nodes have their action spaces reduced by previous nodes' action choices. Although Theorem 3.4.6 of DIA no longer holds (*i.e.* minimizing the maximum transmission power) in case of LOCAL-DIA, LOCAL-DIA still generates topologies that are Pareto efficient (we show this in Section 3.4.4.1).

The worst case message complexity DIA is on the order of $O(n^2)$ (for proof, see Section 3.4.4.2), where n is the number of nodes in the network. The performance benefits of increasing the amount of knowledge available to the nodes in the network are clear; having such information helps DIA to converge to maxmin efficient topologies as shown in Theorem 3.4.6. This, however, comes at a cost (overhead), requiring more transactions as the amount of knowledge increases. Analyzing this tradeoff between the cost of control information and the steady-state network optimality is an important problem, from a protocol designer viewpoint. To understand the impact of acquiring knowledge, we need a metric that allows a comparison between the network performance (with respect to the power objectives) and the cost of acquiring knowledge.

We have shown that DIA globally (and Local-DIA locally) minimizes the maximum transmission power in the topology. These strategies also are local minima for the sum power in the network. If the total power used in the network decreases, we also expect to see improvement for both objectives. As a proxy measurement for these objectives, we can measure the *total packet energy*. The total packet energy for data is calculated as the amount of energy required to transmit, via unicast, a data packet from every node to every other node, using the least-power route between every pair of nodes in the network. The power used by the node at each hop along the route in the topology is summed and this value is totaled for every node pair. To convert from power to energy, we multiply this power total by a constant equal to the length of time for a packet transmission, which assumes that all packets are of equal transmission length.

To measure the cost of maintaining the network topology, we measure the total packet energy required for knowledge by calculating the amount of energy needed to transmit an update message from every node to each of its k hop neighbors. As with the data measurement, this is calculated by determining the least power route from each node to each of its k hop neighbors. The power used by each node to reach, via unicast, every k hop neighbor is summed. We use the same time constant as with the data packets to convert from power to energy.

Figure 3.4 shows the average total packet energy required for just data and also for the sum of the data and control packets (those used to disseminate knowledge), if data packets are sent at the same frequency as control updates. This shows that increasing knowledge decreases the total packet energy required for data. It also briefly decreases the total packet energy required for knowledge, but then this begins to climb. There is a “sweet spot” for energy around 5 hop knowledge, in which the sum total of packet energy is lower than at full knowledge. This is the point at which the total energy cost of knowledge is minimized. To make a better sense of the amount of partial knowledge nodes have, we calculated the average fraction of network a node is aware of. For 5 hop knowledge, nodes have, on average, awareness of 70% of all network nodes. We also noted that this value remains same across various network sizes, with the density kept fixed. This indicates that our algorithm scales well with network size.

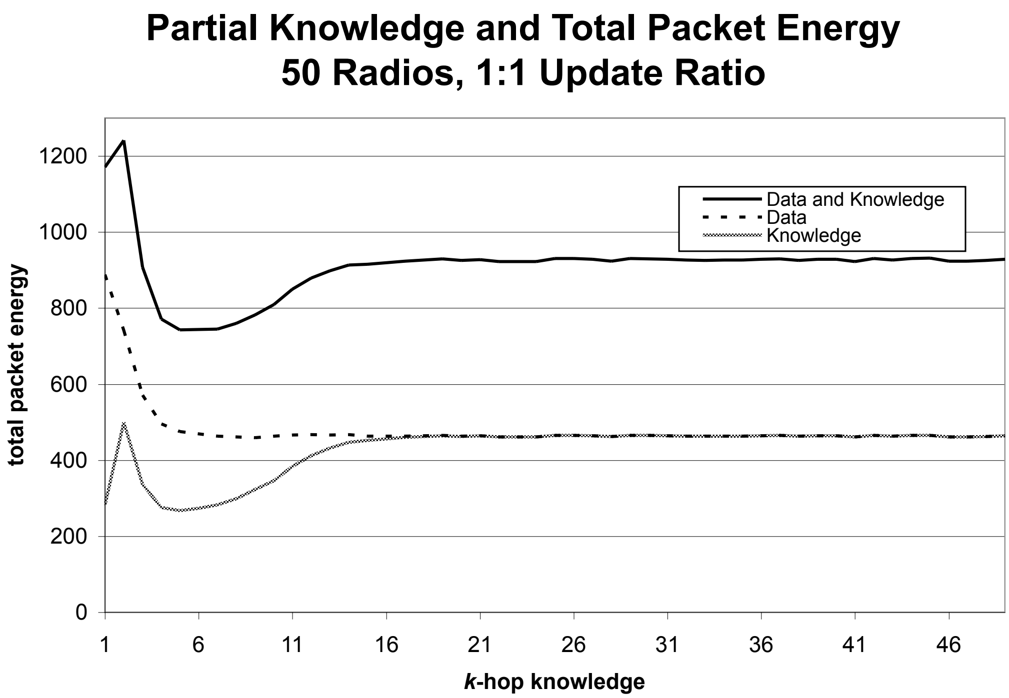


Figure 3.4: Average total packet energy required for data, and control packets along with overall energy consumed (assuming 1:1 dissemination of data and control packets) in 50 node network.

3.4.4 A Comparative Discussion

The difference between MIA and DIA can perhaps be best explained by a simple example. Consider a three node topology consisting of nodes A, B, C; for the sake of illustration, assume identical and symmetric channel states. The dynamics of the game when nodes

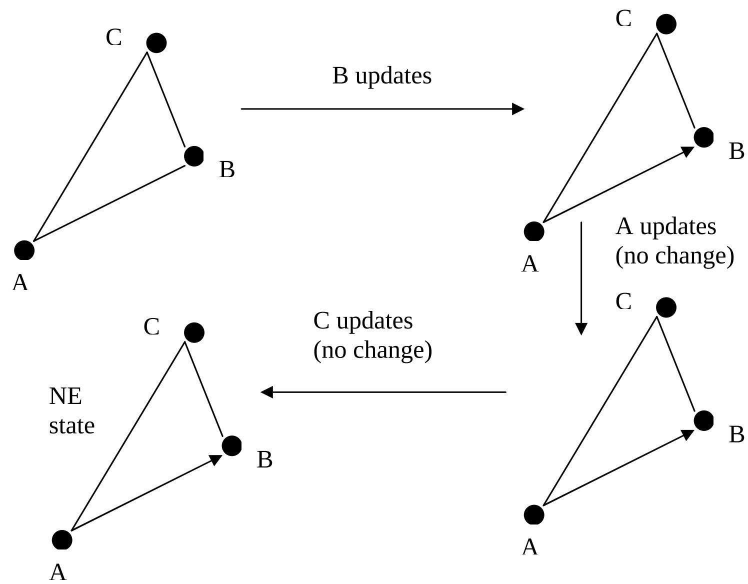


Figure 3.5: *Dynamics under MIA*: Choose benefit factor $M = \omega_{AC}$. Node B updates first and chooses power ω_{BC}; this necessitates A and C to not lower their power in order to preserve connectivity.

employ MIA is shown in Figure 3.5. We note that different steady-state outcomes emerge depending on the order in which nodes update their actions. For instance, if the order is $\{C, A, B\}$ or $\{A, C, B\}$ instead of $\{B, A, C\}$ as in Figure 3.5, then the outcome would be a topology containing links AB, BC (same as that obtained from DIA).

The dynamics of the game when nodes employ DIA is shown in Figure 3.6. Unlike in MIA, the outcome of the DIA is a unique PMST, regardless of the order in which players update their power setting.

In all the discussions above we assume that nodes are "programmed" to follow the rules specified by DIA or MIA. Both DIA and MIA are selfish algorithms, each at two extremes on the "selfishness scale"; MIA is extremely selfish, allowing nodes to their minimize power consumption in one shot, whereas DIA is more moderate, mitigating the first mover advantage by restricting the amounts by which each node can reduce its power. The DIA algorithm we developed is essentially a protocol for selfish nodes that, if they follow, is as-

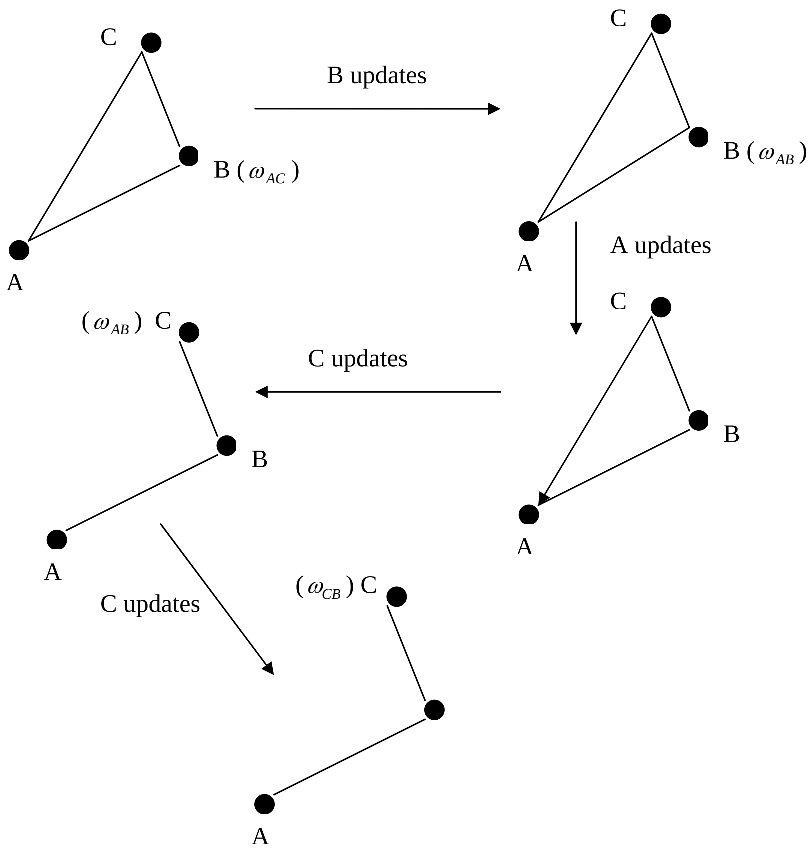


Figure 3.6: *Dynamics under DIA*: In round 1, A, B, C decrement their power level to ω_{AB}. In round 2, only C minimizes its power further, to ω_{CB}. Power levels in the parenthesis indicate a change from its previous state.

sured of converging to efficient NE states. The algorithm, though conservative, is certainly true to the non-cooperative theory and adheres to the rationality principle. Given this, it is nonetheless worthwhile investigating the outcomes when nodes disobey the selfish rules. Specifically, we study what NE states are likely to emerge in systems where nodes are selfish but not programmed to behave strictly according to some selfish algorithm (like MIA or DIA). In such systems some nodes may behave more selfishly than others (perhaps because the more selfish nodes have stricter energy conservation requirements).

We simulate one version of the above scenario by considering a non-cooperative network in which certain percentage (q) of selfish nodes employs MIA and the remaining employs DIA. Observe that when $q = 0\%$, the steady-state topologies are minmax efficient (by

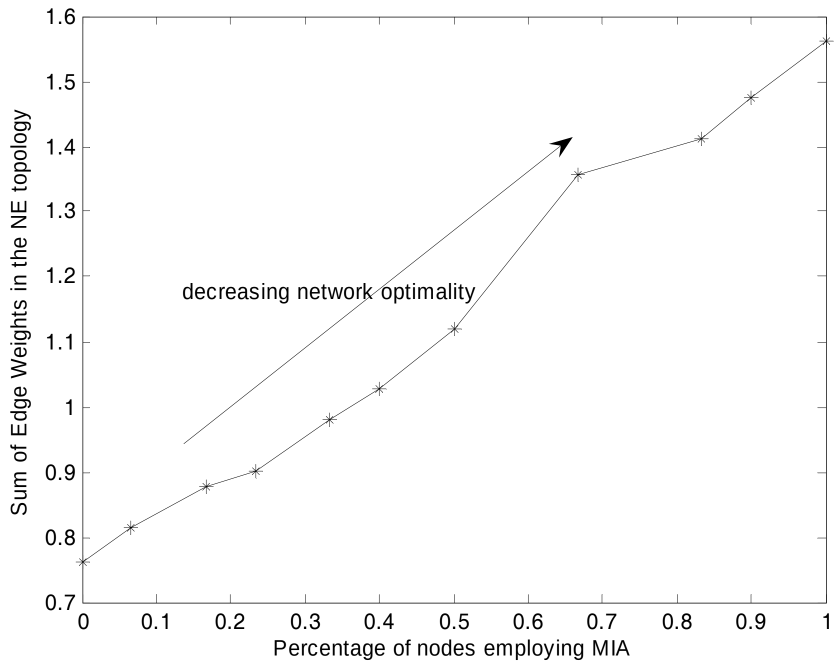


Figure 3.7: Variation in the performance of NE topologies with fraction of nodes employing MIA (and remaining nodes employing DIA).

Theorem 3.4.6), whereas when $q = 100\%$, the topologies are locally efficient (by Proposition 3.4.3). Note that, for every value q, the resulting topologies are locally efficient; however, we distinguish each of them by evaluating their aggregate edge weights. Thus, for any value of $q \in (0, 100)\,\%$, we expect that the resultant NE topologies are efficient in some degree between the efficiency of MIA topologies and that of DIA topologies. Figure 3.7 shows the variation in efficiency of NE topologies with q. The figure displays the loss in network efficiency due to the greedy nature of the MIA. DIA overcomes this first mover advantage inherent in MIA, and thus the NE topologies are more efficient as q decreases.

3.4.4.1 Fairness and Pareto Optimality

The MIA converges to one of the many NE of the game $\bar{\Gamma}$; which NE state emerges depends on the order in which nodes update their power. While all NE states satisfy Proposition 3.4.3, the power assignment vectors that define these states maybe substantially different. The greedy nature of the algorithm immediately suggests that the nodes that update their actions earlier, in a given round, choose the minimum power necessary to preserve connectivity. Consequently, the nodes that update later are forced to transmit at a higher power in order to maintain connectivity (recall that maintaining connectivity is always a best response for all nodes). Thus, the "first mover advantage" inherent in the MIA algorithm results in a biased and unfair power allocations. To a certain extent, updating in a randomized ordering alleviates this bias in power assignment.

In the DIA, power levels are more evenly distributed across all nodes. In some sense, the node with maximum power is the "weakest link" of the network; therefore minimizing the maximum power may prolong the network operability in certain situations. Distribution of steady-state power levels is comparable to that obtained from a centralized algorithm such as [25, 26]. We conjecture that the loss of network performance due to the presence of selfish nodes in the network is significantly small; in other words, the price of anarchy is close to 1.

Theorem 3.4.9. *Any algorithm that starts at $G_{\max}$, and implements a selfish strategy—such as MIA or DIA—converges to a Pareto optimal NE. Alternately, every NE that preserves network connectivity is PO.*

Proof. Any selfish algorithm that starts at $G_{\max}$ converges to a (locally-efficient) NE that preserves connectivity of $G_{\max}$; see proof of Proposition 3.4.3. Firstly, By Definition 3.1.1, no node can reduce its power any lower; otherwise the network would be disconnected and hence violate Proposition 3.4.3. Secondly, no m node (where $m \geq 2$) reduction in power levels can preserve the network connectivity either. This is because, if some node reduces its power (and therefore, disconnects the network), some other node must increase its power to re-connect the network. It follows that the new configuration is not PO. □

As a corollary to Theorem 3.4.9, observe that the NE topology, G_{dia}, obtained by DIA is PO: Suppose, on the contrary, that there exists another topology G_p which is a Pareto improvement over G_{dia}. This implies, every edge in G_p has a lower or equal weight, ω, than the edge-weights in G_{dia}. This suggests the $\sum_{e \in G_p} \omega(e) < \sum_{e \in G_{dia}} \omega(e)$; we obtain a contradiction because G_{dia} is a subgraph of PMST.

Proposition 3.4.10. *For any random topology, the steady-state power assignment vector under DIA is unique.*

Proof. Note that, MST (and therefore PMST) is unique if the edge-weights are distinct because the edges can be uniquely ordered by their weights. Thus, the results follows immediately. □

To get a feel for the performance of the topology that results from DIA, we generate NE topologies for a 30 node topology, using various permutations of the order in which nodes update their power settings under the MIA, and compare against the topology generated by DIA. To demonstrate this, we plot of a distribution of total power consumed by an arbitary NE state in Figure 3.8. The figure corroborates the fact that DIA performs much better than an average NE state generated by any other selfish algorithm; in addition, the plot also suggests that G_{dia} performs significantly close to the globally efficient topology.

3.4.4.2 Convergence

As in any engineering algorithm, there is a trade-off between efficiency and convergence rate of the algorithm. While the topologies that emerge from MIA are only locally (and not globally) efficient, the algorithm convergence speed is linear in network size.

Proposition 3.4.11. *For the TC game given by $\bar{\Gamma}$, MIA converges at a rate $O(n)$, where n is the number of nodes in the topology. More specifically, the algorithm converges in exactly n steps.*

Proof. As shown in the proof of Proposition 3.4.3, at each step, the best response for each node is to choose the minimum power level required to remain connected — no node can

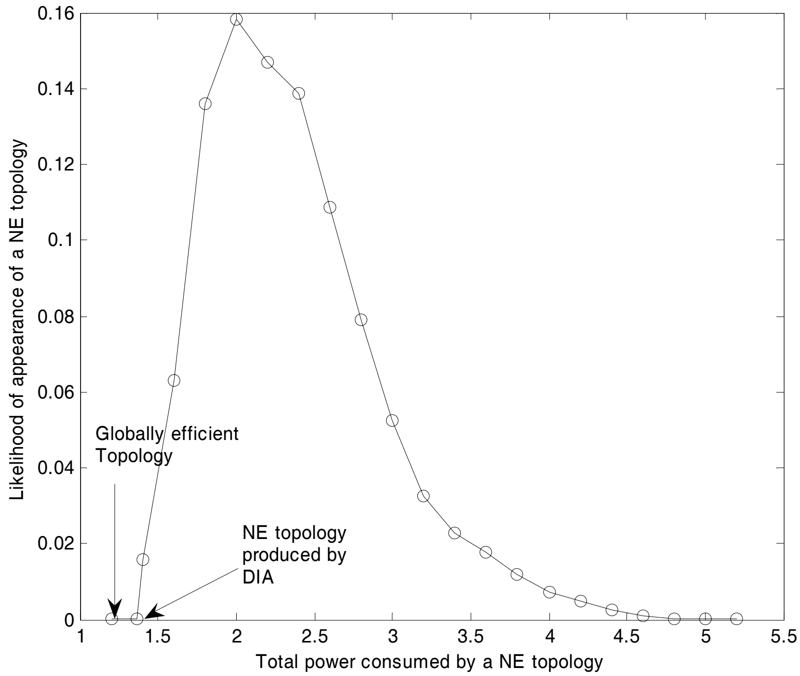


Figure 3.8: Illustrating the efficiency of G_{dia} amongst $26,000$ other locally efficient NE topologies.

reduce its power level any lower and still get a higher payoff. After the first round (when every player has updated her strategy), the payoff of each player i is given by $u_i(\hat{\mathbf{p}}) = M_i.n - \hat{p}_i$. In the second round, no player i can choose a power level $p_i < \hat{p}_i$ and still be connected. If this was possible, then $\hat{p}_i$ would not be the best response of player i in the first iteration. Thus, after n steps, MIA converges to the NE given by $\hat{\mathbf{p}} = \{(\hat{p}_1, \ldots, \hat{p}_n) \mid f_i(\hat{\mathbf{p}}) = n \ \forall i\}$. □

The following proposition formalizes the convergence speed of DIA. The step-size, δ, of the algorithm should be sufficiently small to assure convergence to the minmax energy-efficient NE. On the other hand, the small step-size also reduces the rate of convergence significantly. The choice of δ depends on the inter-node separation, or more generally, is a function of the network size.

Proposition 3.4.12. *For the TC game given by $\bar{\Gamma}$, DIA converges at a rate $O(n^2)$, where n is the number of nodes in the topology.*

Proof. The initial topology $G_{\max}$, induced by $\mathbf{p}_{\max}$, is at most a complete graph and therefore contains at most $n(n-1)/2$ bidirected edges. According to Assumption 3, in each iteration of DIA, at most one edge is severed when nodes revise their power levels. Consider the extreme case: the node j that chooses minimum power (at the end of the algorithm) is located at the periphery of the topology. In this case, the algorithm converges only after j chooses its minimum power. This means, j severs all its links except the smallest one. Under this scenario, the algorithm traverses through maximum number of iterations, *i.e.*, through $n(n-1)/2-1$ steps. Therefore, the convergence rate of the DIA algorithm is $O(n^2)$. $\square$

Note that in each iteration of DIA, at most one edge is severed; therefore, in terms of message complexity, one update message is required each time there is a change in topology state. Therefore, the maximum number of updates required is $n(n-1)/2-1$. For the example topology shown in Figure 3.6, DIA requires 2 updates (powers are first reduced to ω_{AB} and then C reduces its power level further to ω_{CB}) and the convergence is achieved in 2 rounds.

3.5 Simulation Results

In this section, we present simulation results to demonstrate the validity of our results. In the simulation study, nodes are placed randomly on 2-D plane in a $[-1,1]\times[-1,1]$ grid. We assume omnidirectional antenna gain patterns with a path loss exponent of 2 (however, the results hold for any exponent); the power required to support a link ij is $\omega_{ij} = p_{ij} = d_{ij}^2$ where d_{ij} is the euclidean distance between nodes i and j (again, all results hold even when Ω is asymmetric). The simulation is implemented in C++ and GUI in Gnuplot. Nodes are chosen to update their transmission level in a round-robin manner. Under MIA, each node carries out the best response update scheme given by (3.7). Under DIA, each node carries out the better response update scheme given by (3.9).

Consider the initial state topology, $G_{\max}$, containing 75 nodes, with each node transmitting at $p_{\max} = 2$ units. We let $G_{\max}$ be the input to the two TC algorithms, MIA and DIA. A

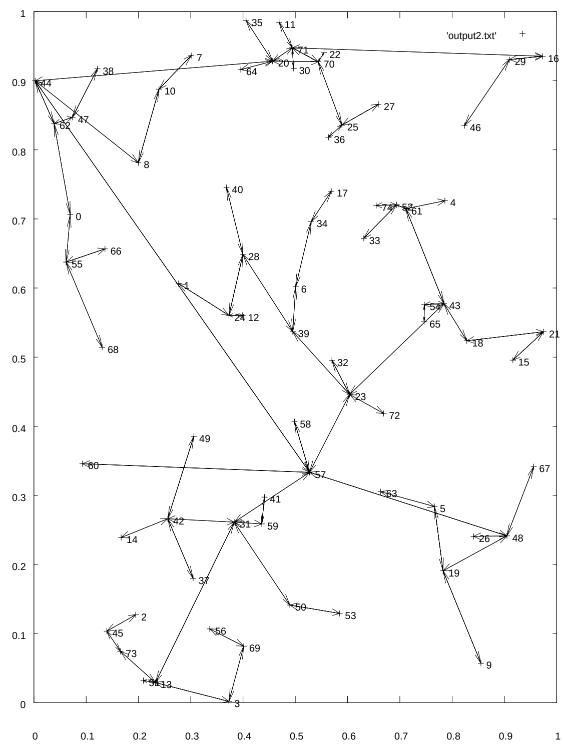


Figure 3.9: *Output of MIA*: A steady-state topology that emerges when nodes implement the MIA (Average power= 0.041 units, Maximum power= 0.596 units).

possible steady-state when nodes implement MIA is shown in Figure 3.9. The topology is much sparser as nodes operate at power levels significantly lower than their maximum levels.

The steady-state topology, when nodes implement DIA, is shown in Figure 3.10. As expected, the topology is much sparser than that produced by MIA algorithm. This topology is a subgraph of PMST and contains a few induced cycles[5]. As evident from the figures, both MIA and DIA preserve network connectivity as there exists a bidirectional path between any two nodes; besides, DIA produces a minmax topology: no other topology configuration can reduce the maximum power of any node in the network.

[5]For the sake of clarity, we suppressed the unidirectional links from the two figures above and depicted only bidirectional links.

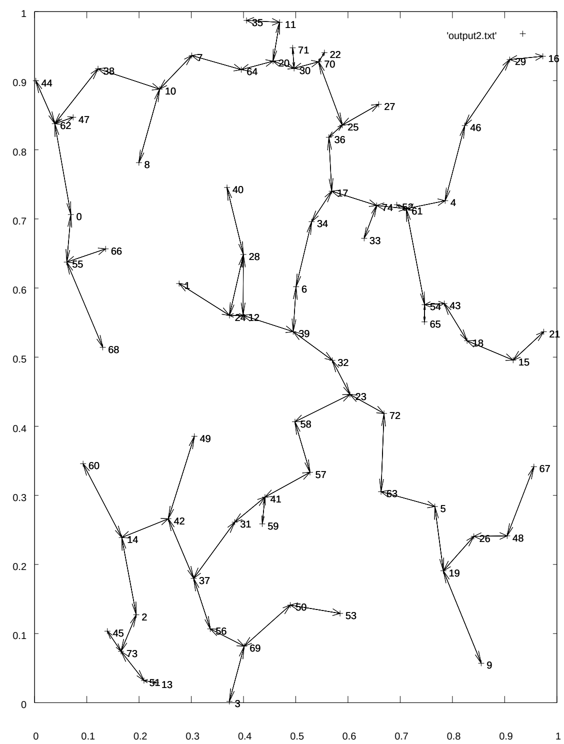


Figure 3.10: *Output of DIA*: The steady-state topology that emerges when nodes implement the DIA (Average power= 0.001 units, Maximum power= 0.023 units).

3.6 Summary and Contributions

Nodes in an ad hoc network have restricted communication radius and limited battery capacity. This forces them to rely on intermediate nodes, not only to extend their reach but also to conserve their energy consumption. This gives rise to a conflicting dynamics in the network, where nodes to try selfishly maximize their own performance.

We show that a particular instance of TC game can be viewed as a potential game. Using potential game theory, we show that the game $\bar{\Gamma}$ admits many locally efficient NE, a subset of which are also globally efficient. We develop three algorithms that deal with selfish nodes: MIA, DIA, and LOCAL-DIA. MIA converges to topologies that preserve network connectivity but are inefficient from an energy consumption standpoint. In contrast, DIA algorithm guarantees convergence to minmax efficient and connected topologies. Both MIA and DIA are full knowledge algorithms, requiring complete topology state information. Local-DIA generalizes DIA by allowing nodes to operate under incomplete or partial knowledge. We examine the trade-off between network performance and the cost of having knowledge (by exchanging control information): more information exchange makes the nodes more network-aware, and hence leads to more efficient networks, but control information itself is costly. By taking the cost of acquiring knowledge into account, we show that networks operating under certain amounts of partial knowledge consume the least energy due to the high cost of maintaining knowledge.

The present work considers relatively static topologies, where the TC algorithm converges faster than the changes in network. Analyzing TC games in presence of network dynamics is a natural extension of this work and forms the subject matter of Chapter 4.

The original contributions in this chapter are as follows:

- A framework for topology control in presence of selfish nodes having limited network state knowledge is developed.
- Two distributed algorithms, MIA and DIA, that rely on global network knowledge are developed. We also developed a local TC algorithm Local-DIA to model cases when nodes only have partial network knowledge.
- The analysis of the TC game under MIA reveals that while the algorithm convergence rate is linear (in network size), the final NE outcome can be highly sub-optimal in total power consumed.
- DIA mitigates the first mover advantage of MIA to balance the global network goals and local node objectives.
- We characterize all the NE of the TC game in terms of their energy efficiency.
- The analysis of Local-DIA examines the trade-off between network performance and

the cost of having knowledge. We show that when the cost of maintaining knowledge is high, Local-DIA can perform significantly better than DIA.

The work presented in this chapter has resulted in the following publications:

1. R. S. Komali and A. B. MacKenzie, "Distributed topology control in ad hoc networks: A game theoretic perspective," in *IEEE Consumer and Communication Networking Conference*, vol. 1, pp. 563–568, January 2006.

2. R. S. Komali, A. B. MacKenzie, and R. P. Gilles, "Effect of selfish node behavior on efficient topology design," *IEEE Transactions on Mobile Computing*, June 2008.

3. R. S. Komali, R. W. Thomas, L. A. DaSilva, and A. B. MacKenzie, "Selfishness and knowledge in dynamic topology control: A cognitive network approach," *IEEE Transactions on Mobile Computing*, Under Review.

Chapter 4

Non Cooperative Power Minimization – Dynamic Case

This chapter examines the problem of energy efficiency considered in Chapter 3 and extends the analysis, applying it to dynamic networks. The dynamics in the network is due to node mobilities which is modeled in a discrete sense, with nodes adding and removing themselves to/from the network. As before, the parameter space is $\mathcal{P} = \{p\}$, and nodes selfishly minimize their energy consumption by controlling only their transmission powers.

We address an important question related to distributed topology control: how much knowledge about network state must be available to each node to enable it to make adaptation decisions that are efficient in a network-wide sense? We evaluate the steady-state network performance considering that nodes have only limited network knowledge when making their adaptation decisions. We analyze the trade-off between network performance and the cost of having knowledge (by exchanging control information): more information exchange makes the nodes more network-aware, and hence leads to more efficient networks, but control information itself is costly.

4.1 Framework and Assumptions

We adopt the network model discussed in Section 3.1.1 and represent the network topology as a graph $G = (N, E)$, where N is the set of nodes and E is a set of directed arcs that represent the unidirectional connections. Equation 4.1 shows the set of connections: a connection e_{ij} exists if the transmission power (p_i) is greater than minimum power required from i to j. We assume a MAC protocol that regulates channel access and avoids transmission conflicts.

$$E = \left\{ e_{ij} \mid p_i \geq \omega(ij) \right\} \tag{4.1}$$

The value $\omega(ij)$ is the transmission power required to form a connection from node i to node j. As in Chapter 3, we assume ω to be symmetric, so that $\omega(ij) = \omega(ji)$.

We assume that topological connectivity comes from bidirected connections, which occur when connections between nodes exist in both directions. The set of bidirected connections consists of members of E that also have their reverse in E. G is said to be connected if and only if there exists a bidirected path—a collection of contiguous bidirectional links—between every node pair $i, j \in N$.

We extend the above static model to include dynamic changes through the addition or removal of a node from the network. Under discrete updates, mobility will appear to the network as nodes appearing and disappearing. This also represents dynamic changes that do not involve node mobility. In particular, in Wireless Local Area Networks (WLANs) nodes often drop in or out as users turn their machines on or off. Wireless sensor networks also consist of nodes that wake up or go to sleep periodically, adding or removing themselves to/from the network.

Recall the idea of partial knowledge introduced in Chapter 3. To examine the impact of partial knowledge and network dynamics, we use the following procedure: initial network is given by the model described above. Nodes then employ a distributed power control utilizing full knowledge and Local-DIA described in Algorithm 3 in Chapter 3 to arrive at the G_{dia} topology. Under the dynamic network settings, a node is then added to or removed from the network, at which point Local-DIA-Add or Local-DIA-Remove (presented in Section 4.2) are used under various degrees of ignorance to arrive at the final topology. Note that, because we are considering the addition and removal of nodes after the initial topology construction, it makes sense to study how much knowledge is required to overcome the effects of network dynamics and reconstruct the original topology configuration. This would then provide us a way to compare the initial and final (reconstructed) topology. In this way we can determine the sub-optimality of the reconstructed topology (due to partial knowledge), and evaluate purely the performances of Local-DIA-Add and Local-DIA-Remove and validate them. This experimental procedure is outlined in Figure 4.1.

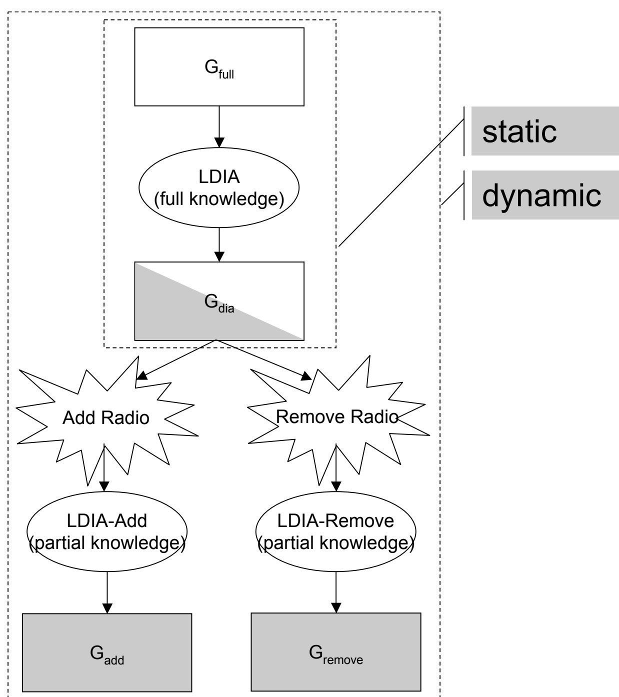


Figure 4.1: Experimental procedure for examining static and dynamic networks.

To reconstruct the topology, nodes utilize partial knowledge together with distributed power control strategies, and "fix" the network. This approach is in contrast to some existing approaches in the literature, where automatic protocol refreshment is triggered every time there is even a small change in topology. We show that, in some cases, the overhead cost of a “full restart” is prohibitive. We present a formal description of our approach to dealing with dynamic networks in Section 4.4.

We examine the performance of nodes and their strategies with an eye towards how knowledge affects performance. Knowledge is information combined with context and is used to appropriately apply learned data. For many problems, without knowledge, even the cleverest algorithm will not have any way to select one decision over another, while in contrast, full knowledge can potentially allow an algorithm to select the best decision.

Because we are interested in the effect of ignorance (taken to mean partial knowledge of network conditions) on the performance of this cognitive network, we utilize the metric *expected price of a ignorance* first introduced in [71]. The price of ignorance is the relative change in performance achieved by the network under ignorance as compared to that of the network under full knowledge. Values greater than 0 indicate that the network has worse performance under ignorance than with full knowledge; values less than 0 indicate better performance.

4.2 Distributed Topology Control Algorithms

Dynamic networks, as described earlier, are networks that change in a way that requires the power settings to be re-adapted and the topology re-formed. This change can take on many forms. Nodes can move or the environment can change, which affects the required power needed to connect nodes. We model all these as changes resulting from the addition or removal of nodes from the network.

4.2.1 Local-DIA-Add Algorithm

When adding nodes to the topology, the new node needs to connect into the existing topology and the existing topology needs to add and remove bidirectional connections such that the MST properties of G_{dia} are retained (see discussion in Section 3.3.2.2).

This strategy for maintaining the MST properties of G_{dia} after the addition of a node can be summarized as follows: the new node forms a least-power bidirectional connection with some existing node in the topology. Under full knowledge, each node transmits at the current maximum power in the network and begins DIA (without loss of generality, this maximum power can be denoted by $p^{(0)}$ and from our discussion of DIA in Section 3.3.2.2, we assume such a common $p^{(0)}$ exists). Under partial knowledge, we approximate this by having each node with k-hop knowledge of the new connection set its power to the current maximum power in its k-hop neighborhood, then beginning LOCAL-DIA.

LOCAL-DIA-ADD, described in Algorithm 4 (from a node perspective), contains two operations that can be triggered. First, "`Hello`" is triggered when a new node, x, joins and attempts to connect into the topology. The node closest to x (this is denoted by node i in Line 2) responds by forming a bidirectional connection with it, then notifies its k-hop neighbors to perform a "`Local Restart`".

Algorithm 4 LOCAL-DIA-ADD$(x) \rightarrow \hat{p}_i$

1: **while** $\langle \texttt{Hello} | x \rangle$ **do**
2: $\quad p_x = \min_{y \in N}\{\omega(yx)\}; i = \arg\min_{y \in N}\{\omega(yx)\}$
3: $\quad p_i = \max\{\omega(ix), p_i\}$
4: $\quad$ `Local Restart` $\rightarrow \mathcal{N}_i^k$
5: **end while**
6: **while** $\langle \texttt{Local Restart} \rangle$ **do**
7: $\quad p_i = \max_{j \in \mathcal{N}_i^k} p_j$
8: $\quad \hat{p}_i =$ LOCAL-DIA$(\mathbf{p})$
9: **end while**

This second operation instructs each k-hop neighbor to increase power to the maximum transmission power in its k-hop neighborhood, and then run LOCAL-DIA. Note that because nodes have knowledge of their k-hop neighborhood, they can determine this maximum. We explicitly consider the overhead cost of obtaining this knowledge and evaluate their impact on the overall network performance in Section 4.4.

4.2.2 Local-DIA-Remove Algorithm

The other network dynamic we consider is the case where nodes leave the network. The removal of a node from the network can potentially split the existing connected topology into multiple partitions.

With this in mind, we develop a localized strategy, called LOCAL-DIA-REMOVE that captures the properties of the full knowledge topology, not removing any connections or k-hop neighbors from the partitioned topology when reconstructing. This is described in Algorithm 5 from the node perspective, using $\mathcal{N}_i^k$ (see definition in 3.6) to denote the original k-hop neighborhood (before the node removal) and $\mathcal{N}_i'^k$ to denote the current k-hop neighborhood (after the node removal).

Algorithm 5 LOCAL-DIA-REMOVE$(x, m) \rightarrow \hat{p}_i$

1: $K = \mathcal{N}_i^k \setminus \left(\mathcal{N}_i'^k \cup \{x\}\right)$
2: **while** $K \nsubseteq \mathcal{N}_i'^k$ **do**
3:     $m = m - 1$
4:     $p_i = p_i^m \in \tilde{A}_i$
5: **end while**
6: $\hat{p}_i =$LOCAL-DIA$(\mathbf{p})$

In LOCAL-DIA-REMOVE, each node sequentially increases its transmission power one level higher (as specified by line 4) in $\tilde{A}_i$ until its k-hop neighborhood is recovered (as specified by the while loop in line 2). These power increases will create unidirectional connections, eventually to be complemented with their reverse, creating bidirectional connections that add at least one more node into the k-hop neighborhood. By following a random ordering, eventually all nodes will have recovered their original k-hop neighborhood. In the expected sense, this sequential stepwise strategy will reconnect the topology with fewer connections.

LOCAL-DIA-REMOVE is not guaranteed to converge to the G_{dia} topology, and in most cases it will not. Unlike LOCAL-DIA-ADD, which resets the entire network to a common power level under full knowledge, LOCAL-DIA-REMOVE resets the network to differing power levels and because of the order of action updates, it may not re-connect the least-power connections between partitions. This means even under full knowledge, LOCAL-DIA will operate in an unsynchronized manner.

4.3 Analytical Results

In this section, we discuss some properties of LOCAL-DIA-ADD and LOCAL-DIA-REMOVE and comment on the outcomes of these algorithms under full and partial network knowledge.

4.3.1 Adding Nodes

We first prove that under full knowledge, LOCAL-DIA-ADD restores G_{dia}. We begin with a lemma that identifies the bidirectional connections that make up an MST:

Lemma 4.3.1. *Cut node set N into two subsets M and O such that $M \subset N$, $O = N \backslash M$. Let F be the set of all possible connections between M and O. If $e = \arg\min_{f \in F} \omega(f)$ then e is part of the MST.*

Proof. Assume MST T does not contain e. Let $e = ij$, with i in M and j in O. This means i and j must be connected in T via some other bidirected path. This path will include some connection f that connects M to O. The tree $U = T \cup \{e\} \setminus \{f\}$ is also a spanning tree, and its sum power is smaller than T, since $\omega(e) < \omega(f)$. Thus T is not an MST, and we have proven the lemma by contradiction. □

From the information exchanged and knowledge gained over the course of LOCAL-DIA, the node closest to the new node correctly reasons to elect itself to increase its power and

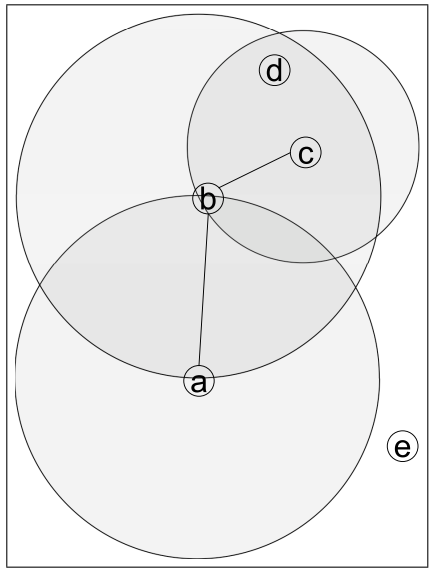


Figure 4.2: Nodes a, b and c represent an existing G_{dia} topology. Nodes d and e represent the cases where a node is added within, and a node is added outside the current transmission ranges respectively.

re-connect the topology. To verify this, we examine the impact of adding a node on the maximum transmission power of the redefined topology, using the above lemma. Adding a new node gives rise to one of two scenarios: the added node falls inside the region covered by the transmission ranges of the existing nodes, or the added node falls outside this region. These two cases are illustrated in Figure 4.2.

Lemma 4.3.2. *If node i is the least transmission power node to node x and $\omega(xi) \leq \max_{e \in G_{dia}} \omega(e)$, then the maximum transmission power in the new topology G'_{dia} is less than or equal to that in the initial G_{dia} topology:*

$$\max_{e \in G'_{dia}} \omega(e) \leq \max_{e \in G_{dia}} \omega(e)$$

Proof. From Lemma 4.3.1, in an MST, node x will be connected via bidirectional connection xi, which requires less power than all other bidirectional connections from x. From our assumptions, $\omega(xi) \leq \max_{e \in G_{mst}} \omega(e)$, so this has not increased the transmission power in G'_{dia}.

We now examine the remaining bidirectional connections in G'_{dia}. Let E be the set of all possible connections in G and let E' be the set of all possible connections in G' after adding node x. Note that $E' \supset E$, meaning we have not removed any possible connections, only added possible connections. Returning to the notation of Lemma 4.3.1, for all cutting subsets of $N \cup \{x\}$, the set of possible connections $F' \supset F$. Thus

$$\min_{f \in F'} \omega(f) \leq \min_{f \in F} \omega(f)$$

and we have proven the addition of x will not increase the maximum connection power in the network. □

Next we examine the case when node x falls outside the region covered by the maximum transmission range of the nodes in the existing topology.

Lemma 4.3.3. *If node i is the least transmission power node to node x and $\omega(xi) > \max_{e \in G_{dia}} \omega(e)$, then the maximum power connections in the new topology G'_{dia} is the connection between x and i:*

$$\omega(xi) = \max_{e \in G'_{dia}} \omega(e)$$

Proof. From the assumption that $\omega(xi) > \max_{e \in G_{mst}} \omega(e)$ and Lemma 4.3.1 we see that all cutting subsets of $N \cup \{x\}$ that include x (except that with only x) will not select a connection originating from x, because it is not the minimum power connection between the cut. Thus adding node x will not change the connections in the topology, with the exception of the min-power connection between x and i. □

Following lemmas 4.3.2 and 4.3.3, Local-DIA-Add ensures that, in the event a new node joins the network, optimal topology G_{dia} can still be re-constructed with enough network awareness (*i.e.* no edge in G_{dia} is ever removed by Local-DIA-Add). Using these two lemmas, the following theorem is an immediate consequence, and establishes the correctness of Local-DIA-Add under full knowledge.

Theorem 4.3.4. *Under full knowledge,* Local-DIA-Add *converges to the G_{dia} topology.*

Proof. Lemmas 4.3.2 and 4.3.3 prove that regardless of the amount of power required to connect node x, Local-DIA-Add will set all nodes to the maximum required connection power, $\max_{e \in G_{dia} \cup \{xi\}} \omega(e)$. Theorem 3.4.6 proves that this connected topology will converge to the G'_{dia} topology. □

4.3.2 Removing Nodes

As mentioned in Section 4.2.2, removal of a node can partition the network into in several components in the worst case. Under full knowledge, an MST construction algorithm can be used to minimize the maximum transmission power and reconnect the network. This is accomplished by treating each partition as a "meta" node, consisting of all the nodes in a partition. The required connection powers between any two partitions then becomes the minimum $\omega(ij)$ between all nodes i in the first partition and nodes j in the second.

The resultant topology G'_{dia} has several interesting properties as compared to the initial topology:

Lemma 4.3.5. *If $\mathcal{N}_i^k$ is the set of k-hop neighbors in the G_{dia} topology for node i before the removal of the node, $\mathcal{N}_i'^k$ is the set of neighbors in the G'_{dia} topology for node i after the removal of the node, and x is the removed node, then no neighbors will be removed besides x in the topology transformation,* i.e.*:*

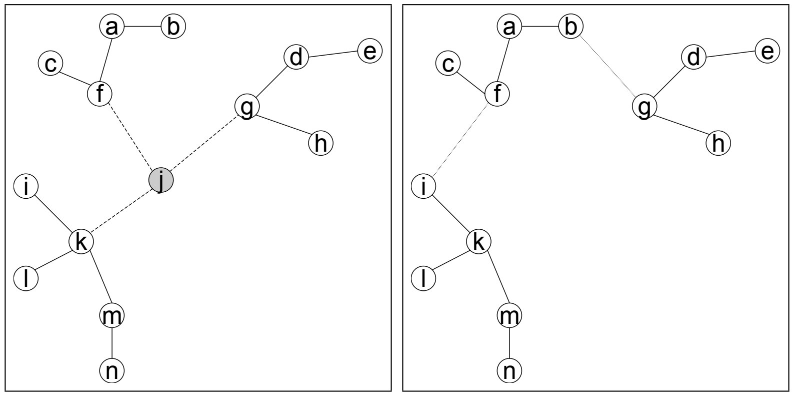
$$\mathcal{N}_i^k \backslash \{x\} \subseteq \mathcal{N}_i'^k \; \forall i \in N$$

Proof. When removing node x, the set of all possible connections in the network changes from E to E'. It is easy to see that $E' \subset E$. Therefore from Lemma 4.3.1, for every i and j not equal to x, if $ij, ji \in G'_{dia}$, then $ij, ji \in G_{dia}$ and all neighbors are retained by the nodes in topology G' with the exception of x. □

Theorem 4.3.6. *If the removal of node x creates n partitions $\{X_1, X_2, \ldots, X_n\}$ then the new G'_{dia} topology contains all bidirectional connections (except those including x) in the original topology G_{dia} plus the minimum weight bidirectional connections between the partitions.*

Proof. From Lemma 4.3.5, we see all bidirectional connections that do not include x are present in G'_{dia}. Furthermore, from Lemma 4.3.1, it is apparent that the minimum weight bidirectional connections between X_m and $N \backslash X_m$ are part of the MST and thus part of G'_{dia}. □

The previous two lemmas discuss the reconstruction of G_{dia} using LOCAL-DIA-REMOVE with full knowledge, utilizing properties of MST. Unfortunately, under partial knowledge it is not possible to utilize an MST algorithm. The remove case does not have the same foundational property that the add case does: the guarantee that the network is fully connected with the exception of the added node. The removal of a node may create, in the worst case, as many partitions as the degree of the removed node. Without full knowledge of the required power for all possible connections in the network and the members of all partitions, it is not possible to guarantee that all nodes know when the topology is fully connected and what optimal connections to use. As an example, see Figure 4.3, which illustrates G_{dia} reconstruction after a node is removed from the network. In this figure, nodes experience the loss of node j. To correctly form the G_{dia} topology, the partitions should use connections if, fi and gb, bg. Under 3 hop knowledge, nodes in the partitions do not know these are the correct connections, nor do they know that these connections reconnect the topology. For instance, node k initially had node g in its 3 hop neighborhood and under this reconnection does not. In fact, under this reconnection no node $\{i, k, l, m\}$ in the partition has all of their initial 3 hop neighbors in their new 3 hop neighborhood. Nodes will not know the topology is reconnected unless all initial k hop neighbors (with the exception of the node that was removed) become k hop neighbors again. Due to the aforementioned reasons, LOCAL-DIA-REMOVE can perform arbitrarily worse under partial knowledge.



(a) Before removal of node (b) After removal of node

Figure 4.3: Reconstructing G_{dia} topology, after removing a node from the network.

4.4 Information and Performance Tradeoff

We next discuss the performance of Local-DIA-Remove and Local-DIA-Add and evaluate the steady state topologies that emerge from these algorithms.

4.4.1 Price of Ignorance

We investigate the price of ignorance for dynamic networks, knowing that under full knowledge the network can use strategies that successfully minimize the maximum transmission power. In Figure 4.4, the price of ignorance is measured for the maximum transmission power objective for a 50 node network[1]. In this figure, a price of ignorance of, for instance, 1.5, means that the network objective performed 150% worse under that degree of ignorance than full knowledge.

Figure 4.4 shows that the price of ignorance is low for the maximum power objective, regardless of the amount of knowledge. This is not unexpected, as most of the time, little in the topology needs to change to incorporate a new node. Recall from Lemma 4.3.2 that if the least power connection for a new node is less than the current maximum power in

[1]Similar results were observed for network sizes varying from 5 to 100 nodes.

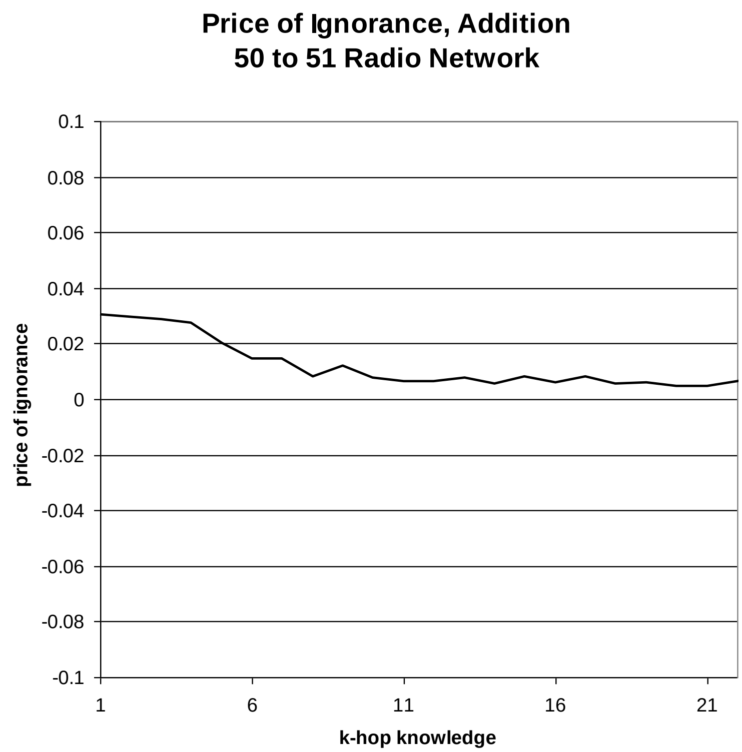


Figure 4.4: Price of ignorance for the maximum transmission power objective after a node is added in a 50 node network.

the network, the maximum power will not increase in the new topology. Furthermore, from Lemma 4.3.3, if this least power connection is greater than the current maximum, this connection will be the new maximum. Local-DIA does not increase any node's transmission power beyond these limits and only in very special cases will the addition of a node reduce the maximum transmission power. Particularly as the number of nodes in the network increases, the probability that adding a single node will reduce the maximum transmission power decreases.

Figure 4.5 shows the price of ignorance under node removal for maximum transmission power in a 50 node network. Unlike the results in Figure 4.4, the remove case shows that increased knowledge has a negative effect on the network objectives, performing worse under knowledge than ignorance. This surprising and counter-intuitive result can be explained by the oft heard expression *"what you don't know can't hurt you."* Under ignorance (such as 3 hop knowledge),

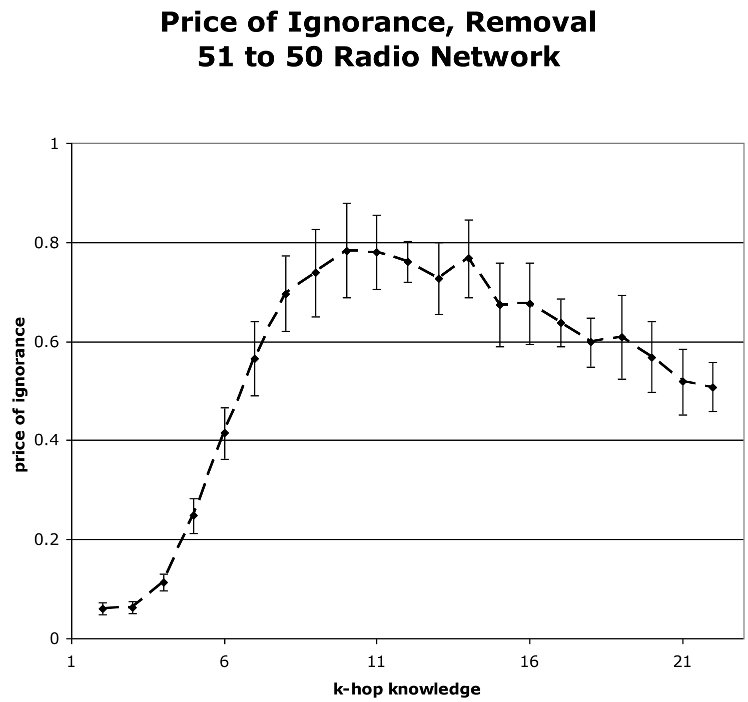


Figure 4.5: Price of ignorance for the maximum transmission power objective after a node is removed in a 50 node network.

most nodes are not aware that a partitioning has occurred, and as a result do not react in any way to the node removal, causing little change to the original topology. Under larger k hop knowledge, more nodes have more k hop neighbors that they are attempting to reconnect with, thus skewing the initial G_{dia} topology. As discussed earlier, Local-DIA combined with Local-DIA-Remove may result in many additional connections over G_{dia}. Due to the sub-optimal effects of Local-DIA-Remove, we observe an increasing trend in the price of ignorance until $k = 11$ hop knowledge. Beyond this point, the effect of Local-DIA having more knowledge now dominates. This allows Local-DIA to offset the effects of the high power connections retained by Local-DIA-Remove. Nevertheless, the sub-optimal effects are still not fully negated. Thus, in most topologies, having more knowledge degrades the overall performance and a little knowledge is in fact sufficient to select low-power connections for use in re-connecting the network.

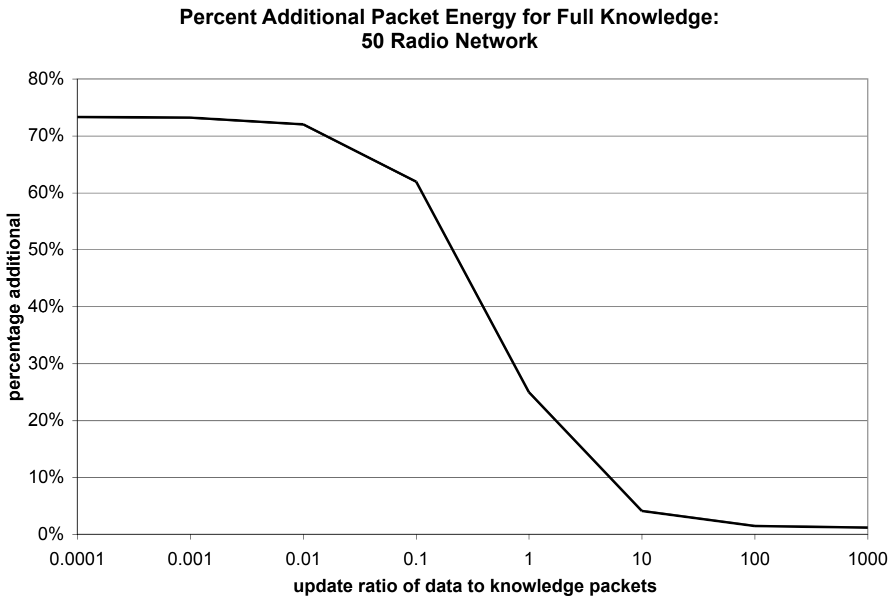


Figure 4.6: Percent additional total packet energy required under full knowledge as compared to minimum total packet energy under partial knowledge for 50 node network

4.4.2 Impact on Dynamic Networks

The study conducted in Figure 3.4 is for a low ratio of data to updates that model relatively static networks. We extend these results to more dynamic networks that admit node mobilities. Assuming the amount of data stays constant, as a network becomes more mobile, the number of updates required to maintain k hop knowledge increases proportionally to the data. Figure 4.6 illustrates the performance of dynamic networks that are constructed with the optimum k hop knowledge (the sweet spot) determined in Figure 3.4, for different ratios of data to updates. It shows the percent difference between the minimum total packet energy and the full knowledge total packet energy. As expected, when the network is relatively stable, and the ratio of data to updates is high, having full knowledge gives the best performance. When the network is dynamic, and the ratio of data to updates is low, having partial knowledge gives a lower total packet energy. In some sense, these results suggest that mobility actually helps in information-constrained networks.

4.5 Summary and Contributions

The network performance achieved by distributing the core networking functions to selfish node elements depends greatly on the amount of knowledge they have about the network. While having full knowledge can allow for optimal decisions, the lack of knowledge may lead to some degree of sub-optimality. Depending on the problem to be solved and the strategy employed, sometimes having more knowledge illuminates better solutions, while other times it may just add redundancy to the system. Regardless of the network benefit these partial knowledge solutions provide, there is always a network cost to acquiring, communicating and maintaining knowledge. Both of these factors must be taken into account to determine how much knowledge the network nodes need.

We employ a game-theoretic model and assess its effectiveness in solving the multi-dimensional topology control problem under dynamic conditions, where nodes are selfish and have partial network state information. We develop two core strategies — LOCAL-DIA-ADD and LOCAL-DIA-REMOVE — to handle the conditions of a dynamic network, where nodes join and leave the network. Both these strategies are investigated under various degrees of knowledge, spanning from local to global knowledge of network conditions. Under full knowledge, the selfish algorithms become functionally equivalent to DIA developed in Chapter 3 and thus minimize the maximum transmission power while maintaining network connectivity. When the nodes can operate along the continuum of knowledge, from 1 hop to omniscience, we show that ignorance has very little effect on the maximum transmission power of the network. Furthermore, we show that due to the high cost of maintaining network knowledge for highly dynamic networks, the cost-performance tradeoff makes it advantageous for nodes to operate under partial knowledge, rather than full knowledge.

The original contributions in this chapter are as follows:

- We provide a rich framework, in the context of dynamic topology control, that evaluates the interplay between the impact of partial knowledge, cost of obtaining knowledge, and overall network performance.

- Two distributed algorithms, Local-DIA-Add and Local-DIA-Remove, that handle the dynamics in the network are developed.

- Using the price of ignorance metric, we show that additional knowledge does not help network optimality when nodes are added or removed from the network. Specifically, we show that ignorance has little impact on the maximum transmission power in the network.

- We show how mobility can actually help in information-constrained networks. Due to the high cost of maintaining knowledge, having partial knowledge is beneficial when network is changing rapidly.

The work presented in this chapter has resulted in the following publication:

1. R. S. Komali, R. W. Thomas, L. A. DaSilva, and A. B. MacKenzie, "Selfishness and knowledge in dynamic topology control: A cognitive network approach," *IEEE Transactions on Mobile Computing*, Under Review.

Chapter 5

Non Cooperative Energy Minimization

The previous two chapters examine energy efficiency using distributed power control but disregard the effects of selfish packet forwarding in the analysis. In contrast, this chapter addresses energy minimization by jointly considering packet forwarding and packet sourcing—two main sources of energy consumption in multi-hop networks. Specifically, we examine how energy-efficient topologies can be established through non-cooperative power control taking into account the possibility of selective and partial packet forwarding in the network.

As in Chapters 3 and 4, the parameter space is $\mathcal{P} = \{p\}$. Unlike in the earlier chapters, here we let nodes select a fixed packet forwarding level between $[0, 1]$ exogenously; a fractional value signifies the proportion of traffic directed through nodes that is forwarded (remaining packets are dropped). Note that packet forwarding is modeled as a node behavior purely for notational convenience; our analysis applies even when packets are dropped due to other reasons (e.g. some nodes are less reliable than others). Under these settings, we evaluate the energy efficiency of topologies that emerge in steady state and quantify the impact of selfish packet forwarding. By quantifying the energy consumption in partially forwarding networks, our study illuminates the tradeoff between performance gains achieved through cooperation, and the overhead cost of stimulating node cooperation (for instance, by means of reputation schemes). Using these results, an optimum level of forwarding can be determined as design decision.

5.1 Framework and Assumptions

5.1.1 System Model

We model the network as consisting of nodes equipped with omnidirectional antennas with isotropic transmission patterns; the transmission range is modeled as a disc. The topology created by the connections is modeled as a communication graph $G = (N, E)$ where N is the set of nodes and E is a set of directed arcs representing unidirectional connections. Note that, unlike in previous two chapters, G is a directed network containing directional links from E. We disregard interference by assuming the existence of a MAC protocol to de-conflict transmissions.

$$E = \left\{ \vec{ij} \mid p_i \geq \omega(i, j) \right\} \tag{5.1}$$

The term $\omega(i, j)$ is the threshold transmission power required to close a connection from radio i to radio j. As the exact threshold $\omega(i, j)$ is a function of channel attenuation and inter-nodal separation, our model is quite generalized to accommodate varying channel characteristics. Once ω is specified, the condition $p_i \geq \omega(i, j)$ determines all the feasible transmissions. The induced topology G is connected if and only if there exists a path—a collection of contiguous edges from E—between every node pair in N.

5.1.2 Energy Model

In order to select an appropriate power level for data transmission, nodes utilize the per-packet power control approach. Appropriating powers on a per-packet basis has been shown to be an effective power control strategy in networks where nodes are non-homogeneously scattered [72]. Additionally, several variants of 802.11 MAC protocols that use per-packet power control have been proposed, *e.g.*, PCM [73], MACA [74], PCMA [75]. The basic idea is that RTS-CTS packets are sent at the highest power levels whereas DATA-ACK packets are transmitted at much lower power levels (see [76]). A similar scheme is employed in [77], where each node maintains a table that stores the minimum transmit powers needed to reach each of its neighbors. In all these schemes, the use of per-packet technique is shown to be effective in reducing energy consumption and improving network throughput. Besides, minimum energy routing based on the power threshold metric is also common among routing protocols, *e.g.*, PARO [76].

In addition to data transmission energy costs, nodes also consume energy due to all additional overhead traffic such as the periodic exchange of `Hello` and Topology Control messages. We assume that control traffic makes up a significant portion of total traffic in the network and therefore cannot be neglected. This is particularly true in ad hoc networks where network conditions are dynamic due to the frequent disruptions and link instabilities. Similar to the RTS-CTS philosophy, we assume that these control messages are transmitted at the highest power level necessary for a node to communicate with its "farthest" neighbor. The rationale behind such an assumption is evident: Topology Control messages are typically transmitted as broadcast packets that are usually exchanged by neighbors to collect information about any changes in the topology. Given this, control energy consumed by a node becomes a function of its power level p_i; we represent such energy costs by $\chi_i^c(p_i)$. For the purpose of our analysis, we only require that χ_i^c be a monotonic function.

Using the per-packet model of data transmission, we now derive an expression for the energy consumed by a node in transmitting data packets to its destination. In all subsequent discussions, we base our analysis on the assumption that all transmissions are unicast. Given that the traffic flows between all source-destination pairs may traverse multiple hops, intermediate nodes may choose to forward only a portion of the packets directed through them. We assume that nodes do not differentiate between flows when forwarding; this allows us to represent the fraction of traffic forwarded by an intermediate node i simply as a probability q_i. Thus, nodes randomly forward packets of other flows through them according to its chosen q_i.

To examine energy-efficient topologies when nodes forward packets sporadically, *i.e.*, the forward levels $q_i \in (0, 1]$, it makes sense to consider the expected energy consumption metric. Consider a typical path s_{ij} containing a set of intermediate nodes between source i and destination j; without loss of generality, let the node ids be ordered, *i.e.*, $s_{ij} = (i, i+1, \ldots, j-1, j)$. Then, the expected energy consumed in transporting messages from i to j along s_{ij} is given by:

$$\chi\left(s_{ij}\right) = \sum_{\substack{k \in s_{ij} \\ k \neq j}} \bar{\chi}_k \tag{5.2}$$

Here, $\bar{\chi}_k$ is the expected energy consumed by node k. To determine $\bar{\chi}_k$, we suppose that packets that are dropped at a certain node are retransmitted by its previous hop node until they are successfully forwarded on[1]. This assumption parallels the strategies used in medium access transmission schemes that are often used in literature. We believe that triggering a retransmission due to packet drops can easily be implemented at the MAC without much modification to it; this is because, the packet drops due to selfish behavior are functionally similar to the packet losses due to channel errors. The link-level retransmission strategy is also well-motivated from an energy efficiency standpoint [78]. Due to the additive construct of (5.2), the model also displays a distributed structure: the energy of each segment can be separately evaluated and then summed up.

We derive an expression for $\bar{\chi}_k$, by calculating the expected number of transmissions at k. Because k retransmits only when its next hop node drops packets, it is sufficient to consider a two-hop path, as shown in Figure 5.1, to calculate $\bar{\chi}_k$. For simplicity of analysis, we set packet transmission durations to 1 time unit assuming that all data packets are of equal

[1]Because we are studying the effect of selfish packet forwarding, we consider all other network conditions to be ideal. Therefore, packets are dropped only due to selfish node behavior and not because of errors due to channel conditions, collisions due to contentions at the MAC layer, or due to congestion issues.

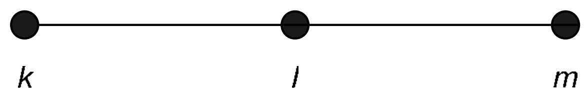


Figure 5.1: An arbitrary route from k to m containing one intermediate node (l).

length. Thus, given that l forwards with probability q_l, the expected energy consumed by k can be obtained as:

$$\bar{\chi}_k = p_k E[T] \tag{5.3}$$

where p_k is the transmission power of node k, and $E[T]$ is the expected number of transmissions at k. In the above example, $p_k = \omega(k,l)$ under the per-packet transmission model. Since T is a geometric random variable, the probability that we have t transmissions at node k is given by:

$$P[t] = (1 - q_l)^{t-1} q_l \tag{5.4}$$

and $E[T] = \frac{1}{q_l}$.

Thus, the expected energy consumed by k becomes,

$$\bar{\chi}_k = \omega(k,l)/q_l \tag{5.5}$$

Combining (5.5) and (5.2), we obtain the expected energy consumed by an arbitrary route s_{ij}:

$$\chi(s_{ij}) = \omega(j-1,j) + \sum_{\substack{k \in s_{ij} \\ k < j-1}} \omega(k,k+1)/q_{k+1} \tag{5.6}$$

As a special case, when $\mathbf{q} = \mathbf{1}$, the energy consumed by s_{ij} becomes:

$$\chi(s_{ij}) = \sum_{\substack{k \in s_{ij} \\ k \neq j}} \omega(k,k+1) \tag{5.7}$$

s_{ij} is called a minimum energy path if it consumes the least amount of energy to transport packets from i to j, among all such paths $\mathcal{S}_{ij}$ between i and j. This minimum energy is given by:

$$\chi_{ij} = \min_{s_{ij} \in \mathcal{S}_{ij}} \chi\left(s_{ij}\right) \tag{5.8}$$

An energy-efficient protocol is said to have the *minimum energy property* if it preserves the minimum energy paths between every source-destination pair [79]. Using this definition, the objective of our distributed topology control algorithm is to derive a subgraph $G_{\min}$ of $G_{\max}$ that has the minimum energy property. $G_{\min}$ is induced by a power vector which we denote by $\mathbf{p}^{\min} = (p_1^{\min}, \ldots, p_n^{\min})$, where n is the number of nodes in the networks.

Our model for energy consumption (χ_i) takes into account the energy consumed in transmitting data packets (χ_i^d) and in transmitting control packets (χ_i^c). In the following section, we introduce the node utility function that specifies the exact contributions from these two components and, how χ_i^d can be determined from (5.6) and (5.8).

5.1.3 Game Model

In our framework, each node takes a selfish view of minimizing its energy expenditure, possibly at the expense of other nodes', and even the network, performance. One way of modeling these interactions between selfish network nodes is as a non-cooperative game. Specifically, a topology control process can be viewed as normal form game: individual nodes form the player set, $N \equiv \{1, 2, \ldots, n\}$, of the game. Each node can autonomously set its transmit power level $p_i \in [0, p_i^{\max}]$. The individual power levels can be collected into a power vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$, which forms the action space, A, for the game. Each node perceives a trade-off between the benefit it derives from a connected topology and the cost incurred in establishing it. A utility function captures these tradeoffs and maps the power vector to a payoff for each node.

To enable multi-hop routing, the underlying topology must not only be connected, but also contain paths where intermediate nodes are willing to forward packets for others. In this work, we assume that packet forwarding levels, q_i, are selected exogenously. A node can decide to cooperate intermittently, forwarding packets for others with some probability $q_i \in (0, 1]$. The subject of how to spur nodes to cooperate and forward packets for others is a research thread in itself; we refer interested readers to [54, 55] (and references contained therein). Given the packet forwarding levels, we are interested in analyzing what topologies emerge in steady state when nodes optimize their performance with respect to connectivity and energy minimization goals.

Because of their energy constraint (let $\chi_i^{\max}$ be the maximum energy of each node), each node faces a temptation to conserve its total energy χ_i by reducing its power level and selecting only closeby neighbors for relaying transmissions. Additionally, because certain nodes forward packets with low probabilities, it may necessitate other nodes to increase their transmission power to circumvent these non-cooperating nodes. It stands to reason that an appropriate power level selection depends not only on other nodes' power level but also on their packet forwarding levels. We cast these interdependencies in a utility function, which for each node is given by:

$$u_i(\mathbf{p}) = \alpha_i f_i(\mathbf{p}) - \beta_i \chi_i^d(\mathbf{p}) - \kappa_i \chi_i^c(p_i) \tag{5.9}$$

where f_i is the number of nodes that can be reached (possibly over multiple hops) by node i. The last two terms collectively represent χ_i, the total energy consumed by i. *The preferences of nodes are cast in a lexicographic order.* Nodes choose to establish network connectivity over energy minimization; given a connected network, nodes then choose to minimize their data energy over control energy. By picking $\alpha_i \geq \chi_i^{\max}$, we cast network connectivity as a constraint in our model. The terms β_i and κ_i are fractional scalars and are chosen such that no node will try to lower its control energy at the cost of increased data energy. Thus, no node reduces its power lower than that required to keep all its minimum energy routes (we prove this in Section 5.2.1). In some sense, β_i is the dominant term and the total energy is dictated by the data energy term.

Without *a priori* knowledge of traffic requirements, each node acts under the premise that all destinations are equally likely. This allows us to express the energy consumed for data transmissions as $\chi_i^d(\mathbf{p}) = \sum_{j \neq i} \chi_{ij}(\mathbf{p})$, where χ_{ij} is given by (5.8). Note that, even though (5.6) does not contain the p_i terms explicitly, $\chi(s_{ij})$ is specified only for the existing routes, which are defined by the power assignment vector $\mathbf{p}$. The expression for χ_i^d implicitly suggests that nodes that initiate a packet forwarding request are responsible for the entire cost associated with transporting messages to the destination. This technique is similar to mechanism design approaches and credit/debit schemes used in energy-efficient topology design [57, 58]. Additionally, the model also allows us to reduce the complexity by decoupling the problems of regulating packet forwarding and power control for energy minimization[2].

Henceforth, the above game-theoretic framework will be referred to as the Topology Control Game (TCG). In the following sections, we study the impact of packet forwarding levels on transmit power selections in the context of energy-efficient TC. In particular, we analyze what topologies emerge in a TCG, and how well they perform in terms of energy efficiency.

5.2 A Topology Control Game

We begin by identifying some useful properties that analyze the effect of power adaptations in the TCG $\Gamma = \langle N, A, u \rangle$, where the individual utilities are given by (5.9). For the sake of brevity, we represent $\omega(i,j)$ by a more compact notation ω_{ij} in the following discussions.

Lemma 5.2.1. *If node i lowers (increases) the energy of a route to k by choosing a different route, then i also lowers (increases) the energy of all other routes to other destinations that go through k via the newly selected route.*

[2] Under this energy model, it may seem counter-intuitive to consider the $\mathbf{q} \neq \mathbf{1}$ cases. However, because of the bandwidth limitations posed by ad hoc networks, nodes may choose not to share such resources and therefore not forward packets. Note that forwarding levels are chosen exogenously; the rationale for choosing such levels in the context of energy minimization is a subject of future work. We show in the next section, however, that the steady-state topologies under $\mathbf{q} = \mathbf{1}$ case consume the least energy among all other topologies that emerge under various $\mathbf{q}$'s.

Proof. The proof follows from the additive structure of (5.6). Without loss of generality, consider two paths $s_{ik} = \{i, j, k\}$ and $s'_{ik} = \{i, l, m, k\}$. Let i lower the energy of its route to k by switching its route from s to a different route s'. This means, $\omega_{il}/q_l + \omega_{lm}/q_m + \omega_{mk} < \omega_{ij}/q_j + \omega_{jk}$, implying, $\omega_{il}/q_l + \omega_{lm}/q_m - \omega_{ij}/q_j < \omega_{jk} - \omega_{mk} \leq (\omega_{jk} - \omega_{mk})/q_k$. Rearranging the above inequality (and adding ω_{kd} to both sides) we get, $\omega_{il}/q_l + \omega_{lm}/q_m + \omega_{mk}/q_k + \omega_{kd} < \omega_{ij}/q_j + \omega_{jk}/q_k + \omega_{kd}$. This means, any other path going through k, $s'_{id} = \{i, l, m, k, d\}$ also has lower energy than $s_{id} = \{i, j, k, d\}$.

Now, let i increase the energy of its route to k. Using the same argument as above, $\omega_{ij}/q_j + \omega_{jk} < \omega_{il}/q_l + \omega_{lm}/q_m + \omega_{mk}$. This implies, $\omega_{ij}/q_j < \omega_{il}/q_l + \omega_{lm}/q_m + \omega_{mk} - \omega_{jk} \leq \omega_{il}/q_l + \omega_{lm}/q_m + \omega_{mk}/q_k - \omega_{jk}/q_k$. Rearranging the terms and adding ω_{kd} to both sides, we get $\omega_{ij}/q_j + \omega_{jk}/q_k + \omega_{kd} < \omega_{il}/q_l + \omega_{lm}/q_m + \omega_{mk}/q_k + \omega_{kd}$. This means, energy of route s'_{id} through k also increases. □

For the reverse direction of the above lemma, the following lemma does holds.

Lemma 5.2.2. *Every node i that increases the energy of a route to d by switching to a new route, also increases the energy of all its sub-routes to intermediate nodes on the new route to d.*

We omit the proof, which is similar to the one given above.

Lemmas 5.2.1 and 5.2.2 underscore the coupling between the route costs $\chi(s_{ij})$ associated with a particular node i. Lemma 5.2.3 specifies how a node's decision may impact another node's route cost.

Lemma 5.2.3. *If node i increases (decreases) the energy of a route from j to k by selecting a new route to k, it also increases (decreases) energy of its new route to k. Alternately, if node i reduces (increases) the energy of its route to k by selecting a new route, it also reduces (increases) the energy of the route from j to k that goes via the new route.*

Proof. Consider the case when $\mathbf{q} = \mathbf{1}$: First, i must be an intermediate node on the route from j to k. Let $s_{jk} = \{j, i, k\}$ and let $s'_{jk} = \{j, i, m, k\}$. Also, let i change its route to k from $s_{ik} = \{i, k\}$ and select a new route $s'_{ik} = \{i, m, k\}$. From the additive cost structure, it follows that if energy of s'_{jk} increases then $\omega_{ji} + \omega_{im} + \omega_{mk} > \omega_{ji} + \omega_{ik}$, which implies, $\omega_{im} + \omega_{mk} > \omega_{ik}$. Thus, $\chi(s'_{ik}) > \chi(s_{ik})$.

Now consider the case when $\mathbf{q} \neq \mathbf{1}$: Using the routes from above, it follows that if energy of s'_{jk} increases then, $\omega_{ji}/q_i + \omega_{im}/q_m + \omega_{mk} > \omega_{ji}/q_i + \omega_{ik}$. In turn, this implies $\omega_{im}/q_m + \omega_{mk} > \omega_{ik}$, meaning that $\chi(s'_{ik}) > \chi(s_{ik})$. For the decrease part the inequalities are reversed, and the result follows. $\square$

The following lemma quantifies the cause-effect relationship specified by Lemma 5.2.3.

Lemma 5.2.4. *If node i changes the energy of a route to k (by selecting a new route) by δ_i, the energy of route from j to k via this new route also changes by δ_i.*

Proof. Let i increase (likewise, decrease) energy of a route to k from $\chi(s_{ik})$ to $\chi(s'_{ik}) = \chi(s_{ik}) + \delta_i$ (by changing routes from s_{ik} to s'_{ik}). From the additive structure of energy consumption, the energy of a route from j to k going through i, is given by: $\chi(s_{jk}) = \sum_{m \in s_{ji}; m \neq i} \bar{\chi}_m + \chi(s_{ik})$. Therefore, $\chi\left(s'_{jk}\right) = \chi(s_{jk}) + \delta_i$. $\square$

5.2.1 Game-Theoretic Analysis

Having discussed the dynamics and implications of altering routes and changing routing costs, we now analyze the TCG Γ and its NE outcomes. We first show that when a node unilaterally decreases (or increases) its utility, the utility of other nodes either remains unaffected or decreases (or increases). For the ease of exposition of the following results, we recast (5.9) as:

$$u_i(\mathbf{p}) = \alpha_i f_i(\mathbf{p}) - \chi_i(\mathbf{p}) \tag{5.10}$$

Theorem 5.2.5. *The TCG Γ, where the individual utilities given by* (5.9) *and* (5.10)*, is an*

OPG. The OPF is given by:

$$V(\mathbf{p}) = \sum_i u_i(\mathbf{p}) \tag{5.11}$$

Proof. We prove by applying the definition of OPGs. First,

$$\begin{aligned} \Delta V(\mathbf{p}) &= \textstyle\sum_j \Delta u_j(\mathbf{p}) \\ &= \Delta u_i(\mathbf{p}) + \textstyle\sum_{j \neq i} \Delta u_j(\mathbf{p}) \end{aligned}$$

- Consider the case $\Delta u_i(\mathbf{p}) > 0$. This implies $\Delta f_i(\mathbf{p}) > 0$ or $\Delta f_i(\mathbf{p}) = 0$ and $\Delta \chi_i(\mathbf{p}) < 0$ (or both hold true). When the former holds, it is straightforward to see that $\Delta f_j(\mathbf{p}) \geq 0$ (equality holds when j is not connected to i), which then implies that, for every j, $\Delta u_j(\mathbf{p}) \geq 0$. The latter case is more interesting to analyze. Observe that $\Delta f_i(\mathbf{p}) = 0 \Rightarrow \Delta f_j(\mathbf{p}) = 0$, $\forall j$. From our discussion on the choice of β_i and κ_i in Section 5.1.3, $\Delta \chi_i(\mathbf{p}) < 0 \Rightarrow \Delta \chi_i^d(\mathbf{p}) \leq 0$, which in turn implies $\exists k$ for which $\chi_{ik}(\mathbf{p})$ is reduced. (This may occur if a new minimum energy route is created by i by virtue of increasing its p_i. Adding a new route is also consistent with Lemma 5.2.1.) From Lemma 5.2.3, it follows that the energy of path from j to k $\chi_{jk}(\mathbf{p})$ either decreases (if j routes through i) or remains the same (if j does not route through i). This implies, $\chi_j^d(\mathbf{p})$ and therefore, $\chi_j(\mathbf{p})$ does not increase. Thus, $\Delta \chi_i(\mathbf{p}) < 0 \Rightarrow \Delta \chi_j(\mathbf{p}) \leq 0$ $\forall j$. We have therefore shown that, $\Delta u_i(\mathbf{p}) > 0 \Rightarrow \Delta V(\mathbf{p}) > 0$.

- Now consider the case $\Delta u_i(\mathbf{p}) < 0$. In this case, $\Delta \chi_i(\mathbf{p}) > 0$ or $\Delta f_i(\mathbf{p}) < 0$ (or both hold). Similar to the reason mentioned in the previous case, $\Delta \chi_i(\mathbf{p}) > 0 \Rightarrow \Delta \chi_i^d(\mathbf{p}) > 0$. If j is connected to i (before and after the power level change), $\Delta \chi_i^d(\mathbf{p}) > 0$ implies $\chi_{ik}(\mathbf{p})$ increases for at least one k by definition, which in turn implies $\chi_{jk}(\mathbf{p})$ increases by virtue of Lemma 5.2.3, meaning that $\chi_j^d(\mathbf{p})$ does not decrease. Thus, $\Delta \chi_j(\mathbf{p}) > 0 \Rightarrow \Delta u_j(\mathbf{p}) < 0$. In the latter case when $\Delta f_i(\mathbf{p}) < 0 \Rightarrow \Delta f_j(\mathbf{p}) \leq 0$, meaning that $\Delta u_j(\mathbf{p}) \leq 0$. Thus, we have shown that $\Delta u_i(\mathbf{p}) < 0 \Rightarrow \Delta V(\mathbf{p}) < 0$.

Combining the above two cases, we have that $\mathrm{sgn}(\Delta u_i) = \mathrm{sgn}(\Delta V)$, $\forall i$, meaning that the game is an OPG. □

Chapter 2 discusses how a potential function may be interpreted as a social welfare function, especially in network design problems. In the context of our TCG, the social welfare function is an energy-efficiency metric. Alternately, potential maximizing NE of the game can be interpreted as the optimal power assignment vectors, *i.e.*, steady-state topologies that minimize the aggregate network energy consumption. This result builds upon the following lemma.

Lemma 5.2.6. *The potential maximizing NE of the TCG preserves network connectivity.*

Proof. We prove by contradiction. Let $\mathbf{p}^*$ be a potential maximizer and $\mathbf{p}$ any other NE that induces a connected network. Because $V(\mathbf{p}^*) > V(\mathbf{p})$, we have $\sum_i [\alpha_i f_i(\mathbf{p}^*) - \chi_i(\mathbf{p}^*)] > \sum_i [\alpha_i f_i(\mathbf{p}) - \chi_i(\mathbf{p})]$. Since $\mathbf{p}^*$ results in a network that is not connected, $f_i(\mathbf{p}^*) = k_i < n-1$ and $f_i(\mathbf{p}) = n-1$. Thus we obtain, $\sum_i \alpha_i(n-1-k_i) < \sum_i [\chi_i(\mathbf{p}) - \chi_i(\mathbf{p}^*)]$. Clearly, LHS is larger than $n \cdot \chi_i^{\max}$ and RHS is smaller than $n \cdot \chi_i^{\max}$. We thus obtain a contradiction. □

Theorem 5.2.7. *Minimum energy topologies $G_{\min}$ are the potential maximizing NE states.*

Proof. From Lemma 5.2.6, the potential maximizing NE topology is always connected. Therefore, the potential function in the NE state becomes $V(\mathbf{p}^*) = M_i \cdot n(n-1) - \sum_i \chi_i(\mathbf{p}^*)$. This implies,

$$\mathbf{p}^* = \arg\min_{\mathbf{p}} \left\{ \sum_i \left[\beta_i \chi_i^d(\mathbf{p}) + \kappa_i \chi_i^c(p_i) \right] \right\} \tag{5.12}$$

From the choice of β_i and κ_i, it can be deduced that every node keeps all the minimum energy links and therefore minimizes χ_i. Thus, $\mathbf{p}^{\min}$ satisfies (5.12). Hence, the minimum energy topologies maximize the potential function. □

Through potential game formulation we have established the existence of at least one NE—the potential maximizer. Assuming that the threshold powers ω_{ij} are all distinct for distinct node pairs, the NE for the TCG is unique. We establish this via the following theorem, noting that the minimum energy topology is uniquely determined when ω_{ij}'s are distinct.

Theorem 5.2.8. *Every NE of the TCG* Γ *is a minimum energy topology.*

Proof. Suppose that the NE topology does not contain a minimum energy path. This means, either some node has removed one of its minimum energy link or has removed a link on a minimum energy path for some other node. The former case violates the rationality principle, because a node can only increase its total energy cost by removing a minimum energy link. In the latter case, again, rationality principle is violated due to Lemma 5.2.3. Thus, in every iteration of a selfish algorithm[3], all minimum energy paths are preserved. Hence, every NE is a minimum energy topology. □

Corollary 5.2.9. *In NE, the power level of every node is at the minimum level required to maintain* $G_{\min}$*.*

Proof. We prove by contradiction. Suppose p_i^* in NE is not at the minimum required to maintain $G_{\min}$. This means, node i can further reduce its power without disconnecting any of the minimum energy connections (if it does, then Theorem 5.2.8 is violated). Therefore, the original state is not an NE and we obtain a contradiction. The NE is given by $\mathbf{p}^* = \{(p_1^*, p_2^*, \ldots, p_n^*) \mid p_i^* = \arg\min_{p_i \geq p_i^{\min}} \kappa_i \chi_i^c(p_i)\}$. □

5.2.2 Impact of Selfish Forwarding

An NE is a consistent predictor of the likely outcomes of a game. For the TCG, the NE states are stable, efficient, and unique. Consequently, the task of constructing a selfish algorithm that will converge to the NE is greatly simplified. This algorithm specifies which actions are rational from a node's perspective, and thus are likely to be chosen, given the state of the network. Because the TCG is a potential game, a simple BR construct ensures convergence

[3]Without loss of generality, we may assume all nodes initialize their power levels to $p_i^{\max}$, such that the induced topology $G_{\max}$ contains the minimum energy topology $G_{\min}$.

to the NE. For the ease of exposition, we assume that all nodes initialize their power levels to $p_i^{\max}$ at the start of the algorithm such that the induced topology is $G_{\max}$ (this assumption, however, is not necessary for the correctness of the algorithm). In a BR algorithm, nodes make selfish adaptations, revising their selections according to:

$$p_i^* = \underset{p_i \in \left[0, p_i^{\max}\right]}{\arg\max}\ u_i(\mathbf{p}) \tag{5.13}$$

When each node executes (5.13), a BR dynamic evolves, with nodes taking turns in making their optimum selections. We assume that only one node makes a selection (in a round-robin manner) at any given instant[4]. The BR dynamic defines an improvement sequence; because each node essentially has to optimize over a finite set of power level choices (one for each of its potential neighbor) to select its minimum power, the improvement sequence is finite and guaranteed to converge to NE [10]. In the next section, we analyze and characterize the topologies that emerge when nodes employ the greedy BR strategy.

To determine the efficacy of our model, we developed a simulation consisting of $|N|$ radios placed according to a uniform random distribution within a unit square. The power thresholds $\omega(i, j)$ required to close a link between nodes i and j were assumed to be equal to $d^2(i, j)$ (we choose a path loss exponent of 2, although our basic conclusions remain the same for other channel models as well), where d is the euclidean distance metric. Based on our system model, a node can transmit reliably to all neighbors within its transmission range (determined by the power level).

The initial topologies of the network are connected, meaning that there exists a directed path from every radio to every other radio. The initial power $p_i^{\max}$ was chosen such that the induced network was 1-connected with 90% probability, adjusting the value for finite networks (see [69] for this formula). We consider only the connected instances of $G_{\max}$ in our simulations. The packet forwarding levels were exogenously selected; for the ease of

[4]As discussed in Chapter 3, this can be implemented by embedding a random timer within each node, which can then make its selection every time its timer expires.

exposition, the forwarding levels were kept constant across all nodes. We varied the levels from 0.1 to 1, in steps of 0.1. Similar results can be drawn for non-uniform forwarding levels across the nodes.

Each node implemented the BR algorithm (5.13), selecting power levels that maximized (5.9). Nodes were selected in a round robin manner to make their decisions in each iteration. Other nodes are made aware of this adaptation through control messages. The traditional Dijkstra's algorithm was modified to a minimum energy routing algorithm for selecting optimal paths between node pairs. The modification is needed because the energy expression in (5.6) is not strictly additive because of q term in the denominator, *i.e.* $\chi(s_{ik}) \neq \chi(s_{ij}) + \chi(s_{jk})$. However, Dijkstra's algorithm can still be applied in principle by storing and updating both the energy cost terms in (5.6) in every iteration of the minimum cost path computation.

We begin by evaluating the effect of packet forwarding levels on the emergent topologies. At low forwarding levels, the expected energy consumed by long routes would be high, owing to greater number of retransmissions. To circumvent the high forwarding cost, we would expect nodes to transmit at higher powers in order to reduce the number of hops to reach a destination. Consequently, the resultant steady-state topologies will be highly connected, with nodes densely connected by a greater number of routes. Figure 5.2 illustrates this result for a 25 node network by computing the path lengths of the resulting NE topologies, averaged over 1000 different instances, with nodes randomly placed at different locations in each case.

To determine the energy efficiency performance, we evaluated the total energy consumed by the NE topology obtained for various forward levels. At each level, the topologies were optimized to minimize the expected energy consumption. We compare each of the NE topology in Figure 5.3.

A few insights can be drawn from Figure 5.3. The total energy consumed decreases monotonically with increasing forwarding levels. This is because, at low forward levels, nodes use higher transmit power to offset the high cost of retransmission incurred by using the relay

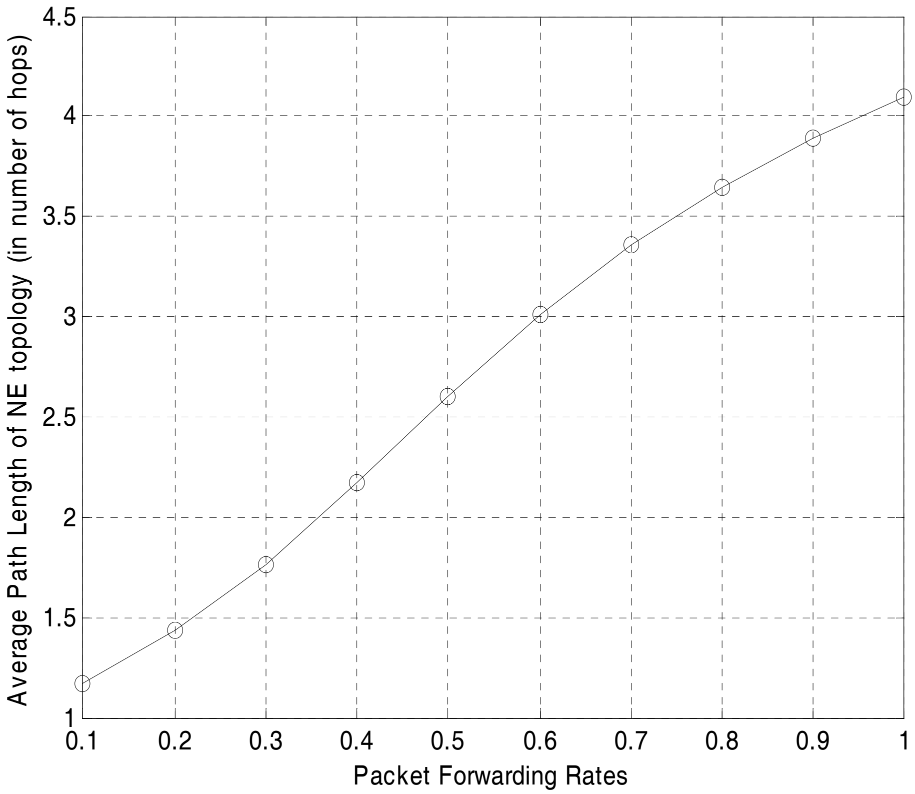


Figure 5.2: Illustrating the impact of packet forwarding levels on the average path length of NE topologies in a 25 node network.

services. We see a gradual decrease in energy consumed because, with increasing forward levels, it costs less to transmit via a relay node than transmitting directly (even if retransmissions are involved). At high forward levels, the cost of direct transmissions dominates; therefore, it is more cost effective to transmit via intermediate nodes.

By quantifying the energy consumption in partially forwarding networks, Figure 5.3 also illuminates the performance gains achieved through cooperation in such networks. This result is particularly important when using reputation-based constructs to stimulate nodes to cooperate by forwarding packets for one another. Such schemes typically incur large overhead costs as they strive to steer the network towards cooperation. Choosing an optimum level of cooperation by balancing the cost-performance tradeoff then becomes a design decision.

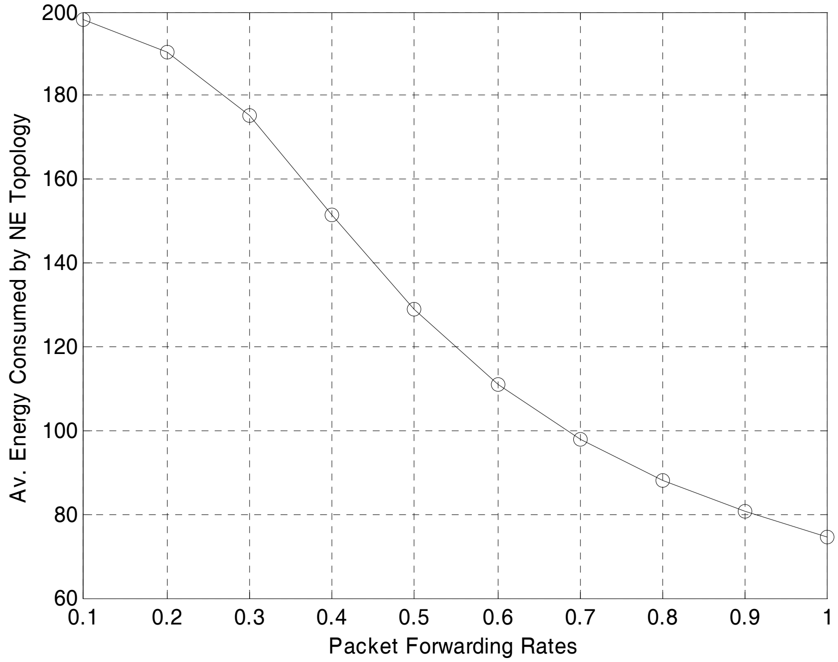


Figure 5.3: Average energy consumed by an NE topology under various packet forwarding levels for a 25 node network.

These results corroborate the well-established fact that it is better to relay messages when nodes are forwarding packets because of the exponential radio signal attenuation property. From a node viewpoint, we observe that if nodes act under the pretext that all destinations are equally likely, they benefit most by forwarding. In some sense, if one could exchange packet forwarding services for real currency, nodes would incur minimum cost when they all forward packets for one another. These observations provide compelling evidence that aligning the individual selfish goals to the network objective is an important characteristic of a self-organizing network.

5.3 Summary and Contributions

Selfish node behavior forms an additional layer of complexity in existing wireless ad hoc network problems. Conventional solutions that are based on cooperation and truthful compliance need to be more robust to effectively cope with this new dimension in the problem. This chapter addresses one such problem: minimizing the overall energy consumed in a network of selfishly motivated nodes.

We quantify the impact of selfish nodes on the overall network energy efficiency. In multi-hop networks, nodes expend energy in sourcing as well as in forwarding packets. This may induce nodes to conserve their own energy and not adhere to a protocol that optimizes the overall network energy consumption. We cast this problem as a non-cooperative game, where nodes attempt to select an optimum power level taking into account partial packet forwarding in the network; the objective of each node is to minimize its energy consumption while maintaining network connectivity.

We show that the above TCG is a potential game. Through the potential game formulation we show the uniqueness of the NE, which helps align the selfish objectives of the nodes with the overall network objective. In other words, when nodes selfishly minimize their energy expenditure, the network minimizes the aggregate energy consumption. We show that a simple BR algorithm is sufficient to steer the nodes towards these globally efficient minimum energy topologies. Our simulation studies reveal that when nodes forward packets with high probabilities, the minimum energy topology is much sparser and consumes lesser energy than the NE topology containing low forwarding nodes. From an energy perspective, this result supports the generally accepted view of stimulating nodes to forward packets for one another in a decentralized, multi-hop network.

A lingering question from this chapter is what happens when packet forwarding levels are also regulated, in addition to power control? As we have seen, our work presupposes that forwarding probabilities are selected exogenously. Generalizing our analysis requires solving a multi-dimensional optimization problem where the objective function is a joint function of both transmit powers and packet forwarding levels.

The original contributions of this chapter are as follows:

- We generalize the framework introduced in Chapter 3 and develop a more holistic framework that jointly considers energy minimization, topology control, and selfish node behavior.
- This chapter addresses two questions related to distributed topology control: in selecting their operating parameters, should nodes be programmed to optimize their own "selfish" objectives, or a network-wide objective function?; and, how much packet forwarding is necessary to make a decentralized selfish network energy-optimal?
- We propose a notion of *expected energy consumption* to characterize the energy consumed by routes containing nodes that selectively forward packets with a certain probability.
- We establish that the NE minimizes the aggregate energy consumed by the network.
- We provide a detailed characterization the NE topologies for various packet forwarding levels. The energy efficiency results substantiate the need for incentivizing selfish nodes to cooperate and forward packets in energy-limited networks. Note that, incentivizing and stimulating nodes to forward packets by means of a reputation-based mechanism often involves large overhead costs. Using our analysis on network performance in partially forwarding networks and accounting for the overhead costs associated with stimulating node cooperation, an optimum level of forwarding can be determined as a design decision.

The work presented in this chapter has resulted in the following publication:

1. R. S. Komali and A. B. MacKenzie, "Impact of selfish packet forwarding on energy-efficient topology control," in *6th Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, April 2008.

Part III

Selfish Topology Control in Multi-Channel Networks

Chapter 6

Channel Minimization for Interference-Free Connectivity – Static and Dynamic Cases

Traditionally, the field of topology control has examined power control problems that disregard spectral efficiency or vice-versa. In this chapter, we examine the objectives of power minimization and spectral efficiency jointly. Our goal is to establish a distributed framework for minimizing the spectral footprint while achieving interference-free connectivity in multi-channel networks.

We show that minimizing spectral usage through joint power control and channel assignment is tantamount to decoupling the problem into two single optimization problems – power control first, then channel selection. Drawing upon the power objective results from Chapters 3 and 4, it is then sufficient to consider the parameter space $\mathcal{P} = \{c\}$, where nodes selfishly select multiple non-interfering channels for data transmission to improve their own performance. We show that even in presence of selfish individual optimizations, the network achieves an overall spectrum usage that is close to the absolute minimum required to maintain network connectivity. We then analyze the impact of incomplete network state information on the spectral footprint. We extend the above analysis to dynamic networks, and quantify the price of ignorance when nodes are added or removed from the network. Finally, we examine the drift-from-the-optimal-performance when nodes are continually added to the network.

6.1 Framework and Assumptions

We adopt the same network model from Sections 3.1.1 and 4.1 for an ad hoc network consisting of a set of $|N|$ wireless nodes. Signal reception is modeled as being binary – depending on the transmission power, either a node can connect to another node, or it cannot. Node transmission powers induce a directed connectivity multi-graph $G = (N, E)$, where a directed arc in E represents connection from a transmitter to a receiver. We assume that all data links in E have to be bidirectional in order to be useful; unidirectional links contribute only to the interference set.

A full-duplex topology is formed out of G when the nodes in the network are assigned non-conflicting channels using Frequency Division Multiple Access (FDMA)[1]. Every node can receive on all channels simultaneously if necessary; each node can only transmit on a single channel, although it can use any from the pool of available channels (accessible to all nodes). To model possible conflicts, G is mapped to an undirected conflict graph G_{I}. This conflict graph is created by placing an edge between each pair of possible conflicting nodes.

In our model, conflicting nodes include those that share a bi-directed connection or those that share an intermediary node that is within the transmission range of both nodes, and has a bi-directed connection with at least one of the nodes. Specifically, a topology G is transformed into a modified distance-2 conflict graph G_{I} by placing undirected edges between all conflicting one-hop and two-hop neighbors, where two-hop neighbors share a common intermediary node. If $G_{\mathrm{I}} = (N, E')$, this transformation is given by $\mathfrak{T} : E \to E'$,

[1]This analysis also applies to Time Division Multiple Access (TDMA); see [80] for a justification.

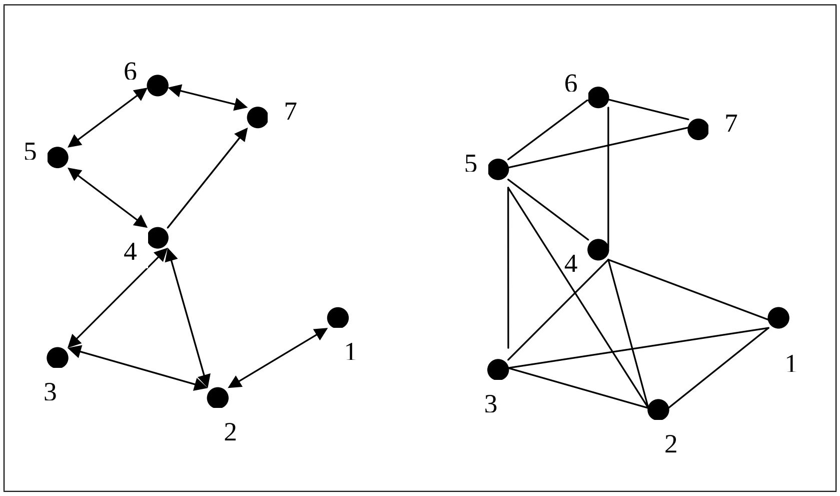


Figure 6.1: From a connectivity graph, G (left), conflict graph, G_{I}, is derived (right).

where

$$\mathfrak{T}(E) = \{e'_{ij} \mid e_{ij}, e_{ji} \in E \text{ or } e_{ik}, e_{ki}, e_{jk} \in E, \text{ for some } k \in N\}. \tag{6.1}$$

For a visual illustration of this transformation, see Figure 6.1. Our modified distance-2 conflict model is reasonable both from an implementation and interference point of view, as it only requires nodes to communicate with their bi-directionally connected nodes and only makes conflict neighbors those nodes that would cause meaningful interference.

We extend the static network model to include dynamic changes through the addition or removal of a node from the network; this dynamic model is also discussed in Chapter 4. We utilize the experimental setup illustrated in Figure 4.1 to deal with dynamics in the network and to evaluate the impact of partial information. As before, k-hop knowledge models the partial information and knowledge, whose impact is measured by the price of ignorance metric, both introduced in Chapter 3.

Given these preliminaries, we analyze the channel minimization problem as a two-phase process: the first phase is a pure power control game where nodes attempt to minimize their transmit power level and maintain network connectivity. The topologies are constructed using selfish algorithms with partial information, as described in Chapters 3 and 4. The output of the first phase is a power-efficient topology, which is fed into the second phase, where nodes selfishly play the channel selection game. Our claim is that this two-phase process minimizes the spectral usage of a network without partitioning it (during the course of the channel assignment game).

6.2 A Channel Assignment Game

Conventional MAC protocols (such as 802.11) perform poorly in multi-hop networks due to their inefficient spatial reuse. This motivates the channel allocation problem, to avoid primary and secondary collisions, and thereby to improve spatial reuse. To this end, our goal is to assign the fewest non-interfering channels to a set of nodes. Minimum channel allocation problems have a striking resemblance to the classic graph coloring problems, which are NP-hard solve for arbitrary graphs, *e.g.* [81]. For the particular problem we consider, allocating channels optimally depends on the order in which radios are assigned channels, which is not easily solvable (though we make no claims regarding membership of our channel allocation algorithm in the NP class).

Formally a topology control game, in the context of this chapter, can be described as follows: individual nodes form the player set, N. Each node can autonomously select a non-interfering channel $c_i \in \{0, 1, \ldots, |N| - 1\}$, which collectively forms the action space A for the game[2]. Note that although we allow at most $|N|$ channels in the network, one objective is to minimize the total number of channels employed (the *spectral footprint*). All nodes share a common palette, *i.e.*, $A_i = A \ \forall i$. To enable non-interfering channel selection, a simple objective function for individual nodes may be given by:

$$u_i\left(\mathbf{c}\right) = \begin{cases} 1 & \text{if } c_i \notin \{c_j \mid e_{ij} \in G_{\mathrm{I}}\}; \\ 0 & \text{otherwise.} \end{cases} \tag{6.2}$$

[2]It is trivial to prove that any coloring scheme can determine a legal coloring with a palette size equal to the number of vertices in the graph.

6.2.1 Analytical Results

We begin by showing that the channel assignment game is a potential game. This follows from an intuitive observation: when a node improves its utility (6.2) by selecting a non-interfering channel, it never reduces another node's utility. Thus, the overall network utility (in this case, the potential function) and the individual node utility move in the same direction.

Theorem 6.2.1. *The game* $\Gamma = \langle N, A, u \rangle$ *where the individual utilities are given by* (6.2) *is an OPG. The OPF is given by* (6.3)*:*

$$V(\mathbf{c}) = \sum_{i \in N} u_i(\mathbf{c}) \tag{6.3}$$

Proof. We need to show that $\forall i \in N$ and $\forall \tilde{c}_i$:

$$V(c_i, c_{-i}) - V(\tilde{c}_i, c_{-i}) > 0 \Leftrightarrow u_i(c_i, c_{-i}) - u_i(\tilde{c}_i, c_{-i}) > 0$$

Note that the change in potential function can be rewritten as:

$$\Delta V(\mathbf{c}) = \Delta u_i(\mathbf{c}) + \sum_{j \in \mathcal{N}_i; j \neq i} \Delta u_j(\mathbf{c})$$

The utilities of the nodes outside the neighborhood of i remain unaffected when node i selects a new color.

Now, when i changes its action from c_i to $\tilde{c}_i$ (and thereby changes its utility from u_i to $\tilde{u}_i$), three possibilities arise:

- $u_i = 1$ and $\tilde{u}_i = 0$: In this case, i changes its color to one that matches at least one of its neighbor's color $c_j \in c_{\mathcal{N}_i}$. Therefore, the utility of j decreases (if it had utility of 1 to begin with) or remains the same (if it had a utility of 0). It follows that, $\Delta u_i(\mathbf{c}) < 0$ implies $\sum_{j \in \mathcal{N}_i; j \neq i} \Delta u_j(\mathbf{c}) \leq 0$. Hence, $\Delta u_i(\mathbf{c}) < 0$ implies $\Delta V(\mathbf{c}) < 0$.
- $\Delta u_i = 0$: In this case, the old and the new color chosen by i are identical. Therefore,

the utility of every node j remains unaffected. It follows that, $\Delta u_i(\mathbf{c}) = 0$ implies $\sum_{j\in\mathcal{N}_i; j\neq i} \Delta u_j(\mathbf{c}) = 0$. Hence, $\Delta u_i(\mathbf{c}) = 0$ implies $\Delta V(\mathbf{c}) = 0$.

- $u_i = 0$ and $\tilde{u}_i = 1$: In this case, i originally has the same color as that of at least one of its neighbors. When i selects a new color distinct from that of all its neighbors, it obtains a utility $\tilde{u}_i = 1$. Therefore, the utility of every node $j \in \mathcal{N}_i$ would increase (if j's color matched only with that of i to begin with) or would remain same (if j's color matched with that of i and/or with that of some other neighbor in $\mathcal{N}_j$). In either case, $\Delta u_j(\mathbf{c}) \geq 0$. Therefore, $\Delta u_i(\mathbf{c}) > 0$ implies $\Delta V(\mathbf{c}) > 0$.

For all three cases, $\operatorname{sgn}(\Delta V) = \operatorname{sgn}(\Delta u_i)$. Therefore, V is an OPF and the game is an OPG. □

Lemma 6.2.2. *Every NE state is a conflict-free channel assignment.*

Proof. Proof by contradiction is immediate. If a node conflicts with another node, it can improve its utility by choosing a color different from its neighbors (this is possible because there are enough colors in the palette). This contradicts the supposition that the state is an NE. □

The previous two results reveal that possibly many conflict-free NE states exist. We, however, are interested in the ones that utilize the fewest channels.

Recall that the network objective for channel assignment is to minimize the spectral footprint. We claim that assigning channels to a power-minimized topology performs reasonably close to the minimum number of channels required by a given topology while maintaining interference-free connectivity. To verify our claim, we take the output of the power control game phase and then analyze it with respect to its spectral footprint, *i.e.*, the number of channels utilized by such a power-optimized topology.

6.2.2 Local-RS Algorithm on Static Networks

The nodes employ channel control using a selfish best response strategy during the course of the game. This strategy is examined first with respect to static networks, and then under dynamic conditions. The objective here is the same as that of graph coloring; thus there are many different heuristic strategies in the literature. In order to minimize the total number of colors, a possible best response strategy is for each randomly ordered node to choose the lowest non-conflicting channel index; we call this strategy Local-RS, as it is a localized version of the Random Sequential coloring algorithm described in [82]. A formal description of the operation of Local-RS is contained in Algorithm 6 (from the network perspective).

Algorithm 6 Local-RS(G_{I}, π) $\rightarrow \hat{\mathbf{c}}$

```
while ĉ is not an NE do
    for i in π do
        ĉ_i = min {A \ {c_j | e_ij ∈ G_I}}
        c_i = ĉ_i
    end for
end while
```

Local-RS works by randomly assigning a backoff to each node in the network within a fixed window. When the node's backoff ends, it selects the lowest channel that does not conflict with its neighbors to transmit on. These backoff periods induce an ordering to the coloring problem, represented by π in Algorithm 6. This repeats until every channel has a non-conflicting channel assignment and an NE is reached.

One advantage of Local-RS is that there exist no hard-to-color topologies[3]. A disadvantage of Local-RS is that it does not assign any priority to the nodes, and so misses out on simple optimizations such as allowing highly connected nodes to select channels earlier. Because Local-RS is a best response strategy to the potential game described in Theorem 6.2.1, we know that it will converge to an NE.

[3] A hard-to-color topology means that no implementation of the algorithm can exactly color the topology.

6.2.3 Joint Power and Channel Assignment

Unlike the previous section, this section handles the power and channel selection decision-making jointly. We show that analyzing the joint game is tantamount to decoupling the problem into two single objective optimizations – power control first (DIA), then channel assignment (Local-RS)). This demonstrates that the outcome of the joint game achieves the same spectrum usage as that obtained in the two-phased game.

In the Joint Power and Channel Control (JPCC) game, each node takes a simultaneous decision on their power and channel selections to improve its utility. We extend the utility given in (3.4) to include channel dependency as follows:

$$u_i^{\mathrm{PC}}(\mathbf{p}, \mathbf{c}) = M_i f_i(\mathbf{p}, \mathbf{c}) - p_i \tag{6.4}$$

The connectivity term f_i is now a function of both power and channel assignment and is the number of nodes i is connected to via *interference-free paths*.

Each node makes its power selection from the set $\bar{A}_i$ given in (3.8). The power adaptation process (DIA) as described in Section 3.3.2.2, allows power changes in steps of δ such that at most one link is removed per round. In addition, nodes also make channel selections $c_i \in \{0, 1, \ldots, |N| - 1\}$ according to a best response process. Algorithm 7 presents the formal description of the joint power and channel adaptation process.

Note that $G_{\max}$ is physically connected (by virtue of power assignments) but may not necessarily be interference-free or even connected after the initial channel assignment. To avoid network partitioning due to channel assignment, channel selections are initialized to the individual node id's (Line 3). The following theorem proves the correctness of the algorithm and shows that it is self-stabilizing.

Theorem 6.2.3. *When nodes make their power and channel selections according to Algorithm 7, the iterative process converges to an NE.*

Algorithm 7 JPCC$(G_{\max}) \rightarrow \left(\hat{G}, \hat{\mathbf{p}}, \hat{\mathbf{c}}\right)$

$m = 0$
$\hat{p}_i = p^{(m)} \in \bar{A}_i \ \forall i \in N$
$\hat{c}_i = i \ \forall i \in N$
while $(\hat{\mathbf{p}}, \hat{\mathbf{c}})$ is not an NE **do**
    $m = m + 1$
    for all $i \in N$ **do**
        choose $p_i = p^{(m)} \in \bar{A}_i$
        $(\hat{p}_i, \hat{c}_i) = \arg\max_{\substack{p'_i \in \{p_i, \hat{p}_i\} \\ c'_i \in \{0,1,\ldots,|N|-1\}}} u_i^{\text{PC}}\left(p'_i, c'_i, \hat{p}_{-i}, \hat{c}_{-i}\right)$
    end for
end while

Proof. In each round of Algorithm 7, nodes decrement their power levels according the DIA process and select best channels. Such a process requires us to examine three possible scenarios in any given round:

1. $p_i^{\texttt{new}} < p_i^{\texttt{old}}$; $c_i^{\texttt{new}} = c_i^{\texttt{old}}$
2. $p_i^{\texttt{new}} < p_i^{\texttt{old}}$; $c_i^{\texttt{new}} \neq c_i^{\texttt{old}}$
3. $p_i^{\texttt{new}} = p_i^{\texttt{old}}$; $c_i^{\texttt{new}} \neq c_i^{\texttt{old}}$

In all three cases, it is clear that $u_i^{\text{PC}}(\texttt{new}) > u_i^{\text{PC}}(\texttt{old}) \Rightarrow u_j^{\text{PC}}(\texttt{new}) = u_j^{\text{PC}}(\texttt{old}) \ \forall j \neq i$. This is because each node i adapts its settings while maintaining network connectivity (both of the power-assigned and channel-assigned graphs); see discussion in Proposition 3.4.3. Maintaining network connectivity does not alter the utilities of other nodes j (as f_j remains same). Due to this monotonicity property, and because of the finite number of power and channel selections, Algorithm 7 converges in a finite number of iterations. □

An as immediate corollary, notice that Algorithm 7, dictated by the power adaptations allowed by $\bar{A}_i$, proceeds identical to the DIA algorithm and hence converges to G_{dia}. Since the power-minimized graph determines the spectral footprint of a network, the joint power and channel control game performs identical to the two-phased process described earlier, in terms of spectral usage performance. For this reason, we continue our analysis of channel minimization using the two-phase approach, given the relative simplicity of implementing such a distributed process.

6.2.4 Extensions to Dynamic Networks

To understand the implications of lack of network knowledge, note that Local-RS being a strictly local algorithm, does not have the same knowledge requirements as Local-DIA, with the conflict neighborhood consisting of channel information from itself and 1-hop neighbors. Thus, there is no concept of partial knowledge. Having less than 1-hop knowledge will prevent any strategy from avoiding interference, while having more than 1-hop knowledge provides no advantage to Local-RS.

There are several possible strategies to deal with dynamic changes in the network due to the addition and removal of nodes. The simplest is to restart Local-RS after a dynamic event, and re-assign channels. Since this is the same as the original operation of Local-RS the expected performance of this restart strategy will be the same as the performance of Local-RS when run on any topology.

Another strategy is to continue utilizing Local-RS after the dynamic event. The topology of the network will have changed, changing the conflict neighborhood of some nodes. Those nodes that share channel assignments with their new conflict neighbors will update their channel selections according to the same rules and order of selection as used under Local-RS initially.

More complex strategies than these can be devised, incorporating such sub-strategies as trading channels with neighboring nodes, restarting channel selection for subsets of the topology, or identifying potential spectrum trouble spots in the topology for alternate coloring. However, in Section 6.3 we will compare the restart to the continuation strategy and show that the continuation strategy is sufficient.

6.3 Performance Analysis

We conduct several simulation studies in this section to evaluate the performance of Local-RS applied first to static networks and then to dynamic networks.

6.3.1 Static Networks

Unlike DIA, which can be proven to minimize the maximum power (see Theorem 3.4.6), the spectral optimality of Local-RS is impossible to guarantee. Determining $\kappa(G_{\mathrm{I}})$, the minimum number of channels required by the conflict graph, is an NP-hard problem for arbitrary graphs. There are well-known upper and lower bounds: $\mu(G_{\mathrm{I}}) \leq \kappa(G_{\mathrm{I}})$ and $\kappa(G_{\mathrm{I}}) \leq \Delta(G_{\mathrm{I}}) + 1$ where $\mu(G_{\mathrm{I}})$ is the size of largest clique in G_{I} (a clique is a group of vertices that are fully connected) and $\Delta(G_{\mathrm{I}})$ is the largest degree in G_{I}. However, determining $\kappa(G_{\mathrm{I}})$ using these bounds often yields loose bounds for arbitrary graphs.

To get a more meaningful assessment of the spectral performance, we evaluated DIA and each of the six other interference-reducing topology control algorithms on the same sets of scenarios ranging from 5 to 100 node networks. We then transformed the resultant connectivity graphs for each algorithm to exact distance-2 conflict graphs and ran an exact coloring algorithm on them to determine the minimum number of legal channels. The exact-coloring algorithm we used, [83], implements a branch-and-bound approach, halting if it finds a solution equal to $\mu(G_{\mathrm{I}})$. While this algorithm can require a non-polynomial amount of time to complete, only at 100 nodes did we encounter scenarios with exceptionally long termination times.

The results of this comparison are found in Figure 6.2, which shows the average minimum number of channels required by each power control algorithm. DIA has a mean channel count lower than or comparable to that of all other interference avoidance algorithms we considered, including Interference Minimum Spanning Tree (IMST) and Minimize the Average Interference Cost while Preserving Connectivity (MAICPC), which were shown in the

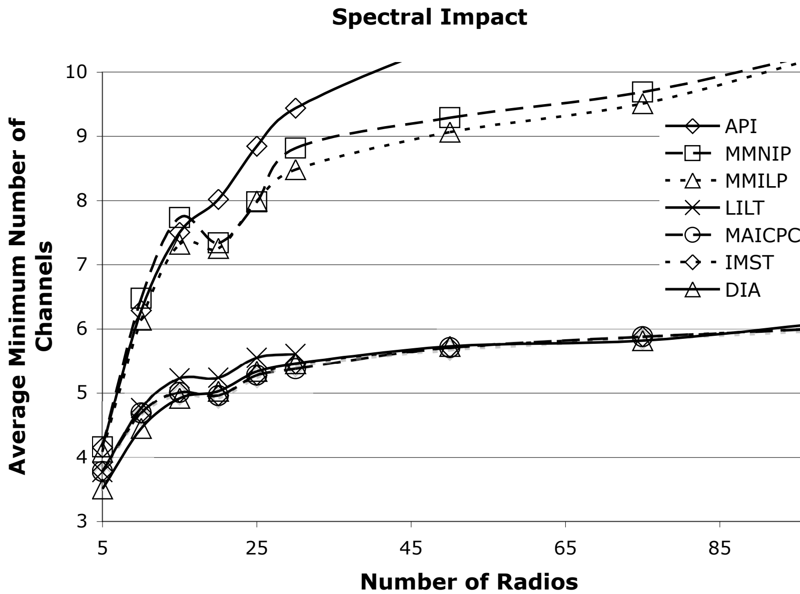


Figure 6.2: Comparison of optimal minimum average channel count for various topology control algorithms.

literature to perform well. The standard deviation of the channel count for DIA is between $10 - 16\%$ of the mean. As a comparison, IMST and MAICPC have between $10 - 17\%$ standard deviation and the other algorithms have up to 25% standard deviation. It is worth noting that the Low Interference-Load Topology (LILT) algorithm, which compared favorably at low node counts, does not scale well as the number of nodes is increased; so we do not have results beyond the 30 node case.

These results indicate that, compared to other interference-reducing algorithms, DIA produces topologies with low conflicts. To determine the performance impact of Local-RS we compare the average additional channels required over the minimum number of channels, in Figure 6.3. This was accomplished by averaging at least 100 random permutations of π for each topology generated by DIA. The exact minimum number of channels required was determined by creating a conflict graph that represented the modified distance-2 conflict used by Local-RS. Although in the worst case Local-RS can perform arbitrarily poorly, on average the algorithm requires less than 12% additional spectrum over the optimal. This shows that although there are more complex coloring strategies possible for channel assignment,

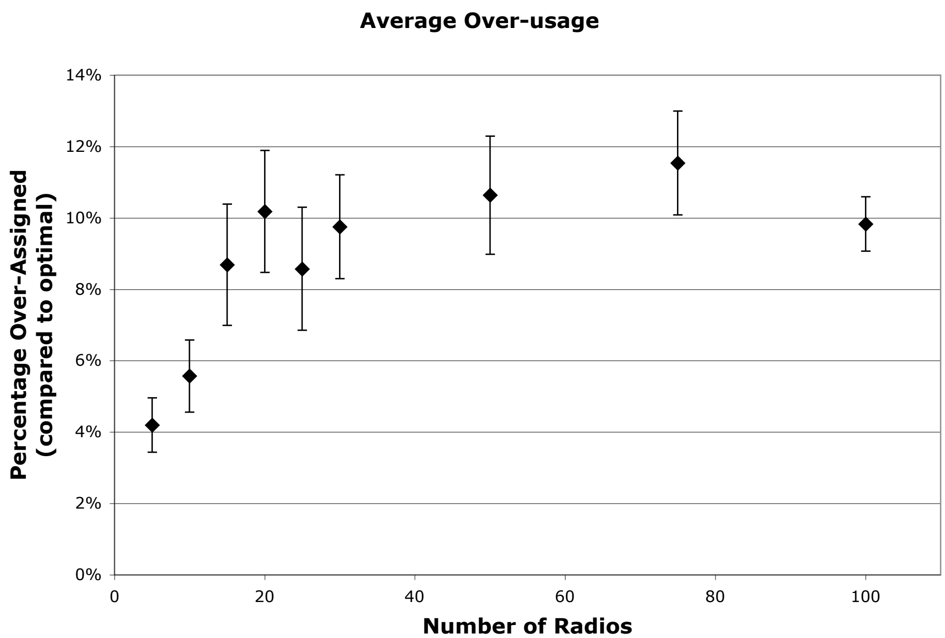


Figure 6.3: Average percentage channels over-assigned (as compared to the optimal) by LOCAL-RS (along with 95% confidence intervals).

the cost of the complexity may make them poor candidates in comparison to a localized, selfish strategy such as LOCAL-RS.

6.3.2 Dynamic Networks

While the objective of minimizing the maximum transmission power is relatively unaffected by ignorance, as observed in Figure 4.4, the spectral impact objective does not fare so well. Figure 6.4 shows the price of ignorance for the average number of channels used by 50 node network after a node is added. The average channel usage is calculated using the exact coloring algorithm. The large price of ignorance at small k hop knowledge is correlated to the fact that the total power under partial knowledge is much greater than that under full knowledge, creating more connections. In the expected sense, increasing the connectivity increases the maximum degree and clique size of the conflict graph. This in turn increases the number of channels required for a conflict-free operation.

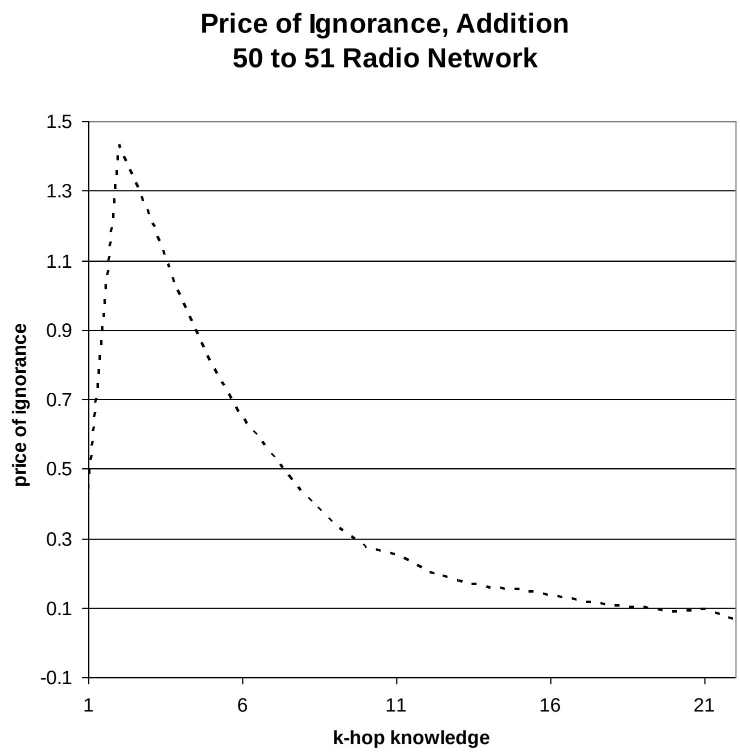


Figure 6.4: Price of ignorance for the average number of channels used in 50 node network after a node is added.

Figure 6.5 shows the price of ignorance under node removal for spectral efficiency in a 50 node network. As mentioned in Section 4.4.1, the remove case does not have the same foundation as the add case, and can perform worse with increasing knowledge. For explanation on the trend observed in Figure 6.5, see the discussion in Section 4.4.1.

The impact on the spectral performance due to partial knowledge and dynamic changes is caused by Local-DIA rather than Local-RS. Partial knowledge has no direct effect on Local-RS, which requires only 1-hop knowledge to operate. We investigate the effect of adding a node on Local-RS by comparing the average minimum number of channels required by restart and continuation strategies from Section 6.2.4 in Figure 6.6. The figure shows that there is no significant difference between the two. Since the restart strategy is an acceptable baseline (with less than 12% average additional spectrum usage over the minimum), this means that the continuation strategy is also acceptable. Any other strategy

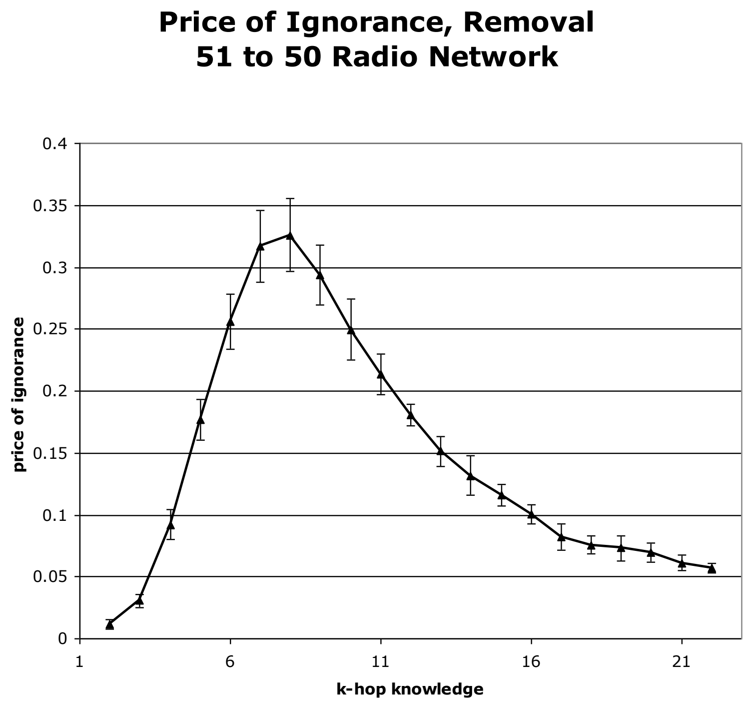


Figure 6.5: Price of ignorance for the average number of channels used in 50 node network after a node is removed (along with 95% confidence intervals).

that works better than the continuation strategy is an improvement on LOCAL-RS, and should be considered as the baseline strategy. Since the continuation strategy requires less overhead than the restart strategy, it should be used.

We now study the impact of adding nodes, one after another, on the network optimality when using LOCAL-DIA-ADD followed by LOCAL-RS. Intuitively, we expect that after each addition, the network will "drift" away from its optimal operating point. Figure 6.7 corroborates this result, and illustrates the drift in the price of ignorance with varying levels of knowledge in 50 node network, as nodes are added one at a time. At low knowledge, the network drifts away much faster from its optimal state, than at high knowledge. Notice from Figure 6.4 that at low knowledge, the resultant steady state networks are highly sub-optimal. This coupled with the cascading effects pushes the low knowledge network scenarios further away from the optimal; the less knowledge, the more rapidly this occurs.

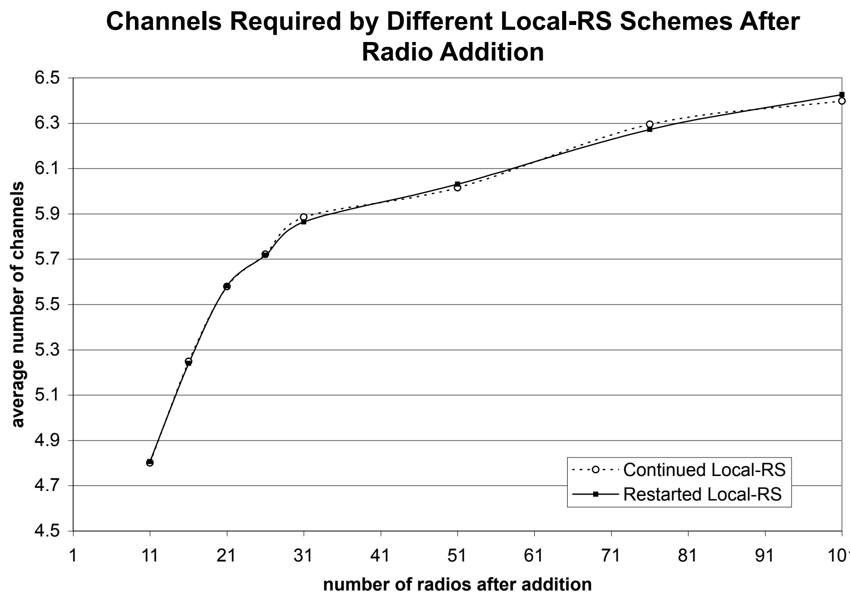


Figure 6.6: Average minimum required channels for Local-RS restart and Local-RS continuation after adding a node.

As the drift continues to grow, it is probably beneficial to restart the topology control algorithm at some point, rather than "fix" the network after each topology change event. Naturally, the restart period is longer with more network awareness. For instance, the restart period with 2 hop knowledge should be shorter than with 10 hop knowledge, for a given level of drift tolerance. However, the exact restart period is a design decision that is based on the tradeoff between the performance benefit of restarting (which depends on level of sub-optimality that can be tolerated) and the associated cost of restart.

6.4 Summary and Contributions

The problem of channel allocation has been well-studied in the domain of cellular networks, where the objective is to maximize frequency reuse by minimizing the number of channels required to cover all cells in a network. With the growing popularity of multi-hop networks and the increasing efforts in deploying such networks, analyzing the same problem in such domains is of immediate interest, especially from a network administration viewpoint.

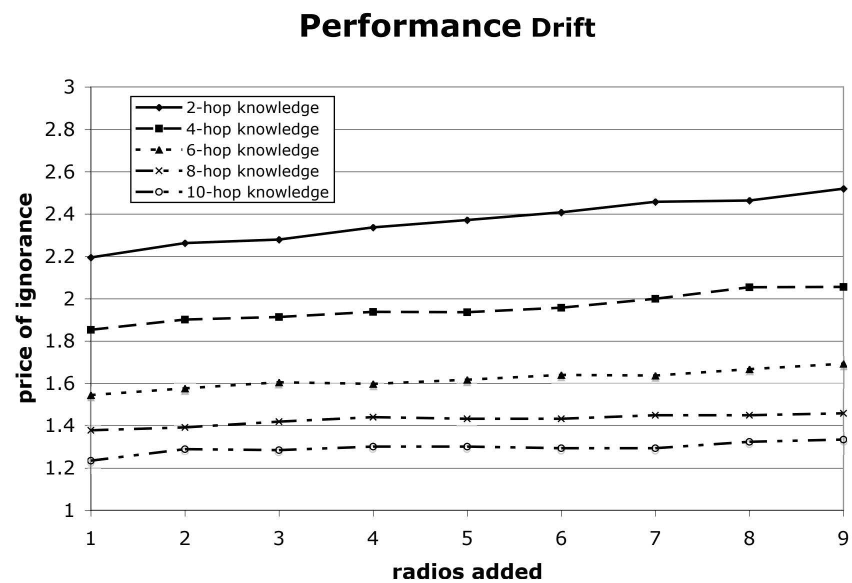


Figure 6.7: *Illustrating the drift performance at different levels of knowledge*: Price of ignorance of channel usage in a 50 node network as nodes are added, one at a time.

We analyze the problem of spectral footprint minimization from a topology control perspective. Conventionally, topology control problems examine ways to minimize transmission powers while maintaining network connectivity. We show that minimizing node power levels also improves the overall spectral footprint of the network, due to increased spatial reuse. We cast the channel selection process as a non-cooperative game and show that this selfish approach is a potential game, meaning that network convergence and stability are guaranteed. We develop a channel assignment algorithm, Local-RS, which ensures interference-free channel selection while maintaining low spectrum usage. When nodes operate under full knowledge, they create a topology whose spectral impact is comparable to those from other interference-minimizing schemes. Furthermore, the average spectrum usage of this topology is within 12% of the absolute minimum. For dynamic networks, we show that as nodes join the network, more knowledge provides better spectral performance; on the contrary, when nodes leave the network, some ignorance in the network achieves better performance. We also show that as nodes are continously added to the network, the network performance drifts away from the optimal, with the rate of drift depending on the amount of knowledge available to the nodes. Quantifying the amount of drift enables the network designer to determine the level of sub-optimality that can be tolerated, by taking into account the cost of restarting the topology control algorithm as an alternative.

The original contributions in this chapter are as follows:

- We analyze the channel minimization problem using properties of potential games. This proves the existence and convergence of stable channel assignment strategies in NE.
- A distributed channel assignment algorithm, LOCAL-RS, is developed and shown to work well under partial network knowledge.
- The spectral impact of the resulting NE topologies are comparable to that obtained from other interfence-reducing schemes, and shown to be within 12% of the optimal, on average.
- For dynamic networks, when radios join the network, having ignorance is shown to have a significant impact on the spectral performance. On the other hand, when nodes leave the network, having ignorance in the network achieves better performance.
- We quantify the drift-from-optimal-performance as network continually changes. This result is especially useful in determining the restart period of an algorithm in the face of network dynamics, using the level of sub-optimality that can be tolerated.

The work presented in this chapter has resulted in the following publications:

1. R. W. Thomas, R. S. Komali, A. B. MacKenzie, and L. A. DaSilva, "Joint power and channel minimization in topology control: A cognitive network approach," *IEEE International Conference on Communications (ICC)*, pp. 6538–6543, June 2007.
2. R. S. Komali, R. W. Thomas, L. A. DaSilva, and A. B. MacKenzie, "Selfishness and knowledge in dynamic topology control: A cognitive network approach," *IEEE Transactions on Mobile Computing*, Under Review.

Chapter 7

Multi-Radio Channel Assignment for Interference Minimization

Traditionally, and in all previous chapters, topology control is performed primarily by adjusting the transmit power of nodes to achieve certain global objective. The objective of interference minimization is generally disregarded by TC studies, which argue that the sparseness of the topologies that result from TC algorithms improves the interference performance. This myth, however, was exposed in [27] which advocates interference modeling explicitly in TC problems. In this chapter, we examine the channel allocation problem in non-cooperative multi-radio multi-channel wireless mesh networks. Specifically, we exploit the synergy between topology control and channel allocation to reduce the overall interference in such networks.

First, we show how the topology of a network can be controlled purely by assigning channels to multiple radio interfaces. Second, we formulate channel assignment as a non-cooperative game, with nodes selecting low interference channels while maintaining some degree of network connectivity; the parameter space is given by $\mathcal{P} = \{c\}$. Third, we evaluate the performance of the NE topologies with respect to interference and connectivity goals. By quantifying the impact of channel availability on interference performance, we illuminate the tradeoff between interference reduction that can be achieved by distributing interference over multiple channels, and the cost of having additional channels. Finally, we study the spectral occupancy of steady state topologies, and show that in spite of the non-cooperative behavior these topologies achieve load balancing.

7.1 Framework and Assumptions

7.1.1 System Model

Consider a wireless mesh network formed by a set of nodes $N = \{1, 2, \ldots\}$ distributed over some geographical region. Each node i may be equipped with multiple transceiver radios. Let k_i be the number of radios on node i; the j^{th} radio on node i is indexed by r_i^j. All radios on nodes can transmit omnidirectionally at a fixed common power level, meaning that power control is not allowed in our model. All transmissions are unicast and each transmitter is capable of communicating with only one other neighboring receiver on a single channel at any given time instant. Limitations on the number of radios on each node may also necessitate each radio to communicate with multiple neighboring radios on a single channel using time-sharing techniques such as Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) or TDMA. To get a feel for our model, one may view nodes as laptop devices that are equipped with multiple radios such as 802.11 NICs, and simultaneous channel access may be regulated by some MAC protocol, e.g. [84]. Given the power levels of all radios, the induced network is commonly modeled by a communication graph $G = (N, E)$ over N. The set E contains all feasible directional links e_{ij} between nodes i and j; the feasibility is dictated by node power levels. Each directed link e_{ij} corresponds to the communication between a single transmitter interface on node i and a single receiver interface on node j. Additionally, links may be shared by radios; a single radio may be assigned to multiple links e_{ij}, e_{ik} and so on.

In addition to the communication graph G, we also have an interference graph G_{I} that specifies the set of transmissions that can potentially interfere with each other if those transmissions occur simultaneously on the same channel. Interference is generally modeled using a weighted conflict graph that may be derived from G. The link weights are usually specific to the problem under consideration and the underlying communication model, and weights typically signify the relative importance or the effect of the edges in the conflict graph; see *e.g.* [81], [85]. Transmissions from one node may interfere with transmissions from every other node in the network (as in a physical network model), or with only a subset of those transmissions (as in a protocol model). We model the conflicts in the network by an undirected graph $G_{\mathrm{I}} = (N, E_{\mathrm{I}}, W)$. Here E_{I} represents the set of edges between all pairs of conflicting nodes. The weights $w_{ij} \in W$ of edges $e_{ij} \in E_{\mathrm{I}}$ specifies interference contribution of node i in the total interference level perceived by node j. Typically, these weights are associated with interference powers. Our model of conflict graph works because all nodes transmit at the same power level. We assume that the channel gains are symmetric, which gives rise to symmetric edge weights, *i.e.* $w_{ij} = w_{ij}\ \forall i, j \in N$. Note that while it is common to consider interference terms only from the strongest interferers as in the protocol model [24], our model is general enough to work with any undirected weighted conflict graph (that is not necessarily complete) where W is symmetric. In single collision domains as considered in [61], all non-diagonal entries of W are non-zero, meaning that G_{I} is a complete graph where every nodes interferes with every other node in the network. In other situations, all weights in W may be equal to 1, meaning that all interferers are treated equal and the total interference is determined by the total number of users sharing a given channel as considered in [32]. In general though, some entries of W maybe 0 while others non-zero. Obviously, a link weight of 0 in G_{I} is equivalent to that link being absent from G_{I}. For our purpose, we leave the form of G_{I} unspecified in our model, except that we require it to be symmetric. In some sense, W may be considered exogenous, and our model works with any symmetric G_{I}.

We assume that radios can access multiple channels but can only operate on a single channel at a time both while transmitting as well as while receiving. This multi-channel capability extends across the entire spectrum that can be sensed. The sensed spectrum is divided into orthogonal channels using techniques such as FDMA. The transmitter and receiver interfaces of a link must be tuned to the same channel for meaningful communication to take place. It is possible for the forward and reverse links (e_{ij} and e_{ji}) to be on different channels, thus allowing for full-duplex mode operation. A full-duplex topology is formed out of G when radios in the network are assigned to non-conflicting channels using FDMA[1]. In some sense, our multi-channel network may be viewed as a series of overlaid single channel "sub-networks"; see Figure 7.1, which also illustrates the communication links between radio interfaces on nodes. Let $\mathcal{C}$ be the aggregate set of available orthogonal channels in the net-

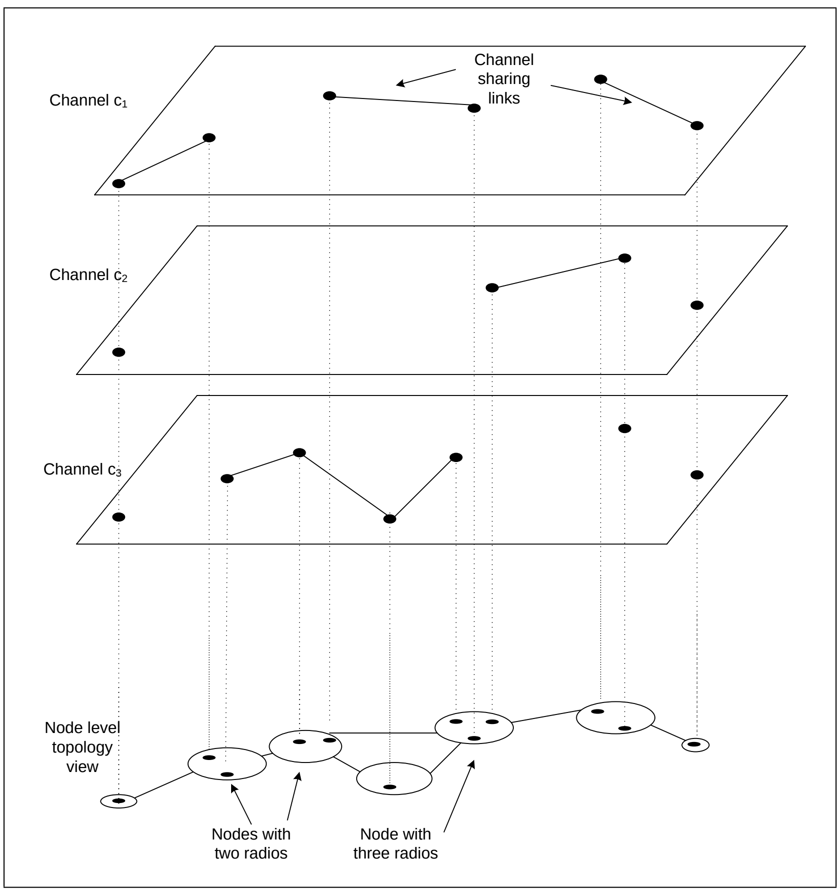


Figure 7.1: A simple illustration of our multi-radio multi-channel network model.

work. We suppose that $k_i < |\mathcal{C}|$ and distinct channels are assigned to links corresponding to

[1]This analysis also applies to TDMA; see [80] for a justification.

different radio interfaces on a node to benefit from channel diversity. Owing to limitations in number of orthogonal channels that are available, typically $\sum_{i \in N} k_i > |\mathcal{C}|$, which gives rise to potential conflicts and channel sharing. We further note that while channel diversity must be employed to minimize multiple access interference, it is also important to maintain network connectivity as part of a topology control process. To avoid network partitioning, neighboring nodes must always share some common channels. Using a common control channel is one possible mechanism for assuring network connectivity and manageability in the face of multi-channel, multi-radio operation. Under these constraints, the channel assignment problem can be viewed as a constrained optimization problem with the objective of minimizing some measure of interference across the network, subject to network connectivity.

We assume that at the pre-specified power levels, the underlying "physical" topology G is connected. However, some links in the physical topology may be unrealized if radios at the two ends of the links are on different channels. Assigning channels to links in the topology, subject to channel availability, induces a "logical" topology $G_c = (N, E_c) \subseteq G$. The channel-assigned topology G_c contains edges $e_{ij} \in E_c$ between i and j on channel c, if $e_{ij} \in E$ is assigned channel c.

7.1.2 Game Model

We formulate the multi-radio channel selection in the context of topology control as a non-cooperative game. Each node has some traffic for all other nodes, which necessitates nodes to communicate with each other over a set of channels. This, coupled with the limitations on the number of distinct orthogonal channels available, naturally gives rise to *the tragedy of commons* problem. The "tragedy" in our case is the amount of multiple access interference caused by conflicting transmissions sharing the same channel.

Channel assignment is on done a per-link basis where two ends of a link must be tuned to the same channel for any meaningful communication to take place. Technically, transmitter-receiver pairs coordinate on which channel to communicate because interference is receivercentric. Such coordination gives rise to two possibilities: the transmit channel is unavailable at the receive side, in which case the transmitter either determines a different channel or does not assign any channel to the corresponding link until a channel becomes available; or, the transmit channel is available at the receiver, in which case the transmitter decides whether or not the selected channel is an appropriate one by examining the level of interference on that channel. In any event, because the transmitter sides make decisions based on channel availability or interference estimates from receivers, and assign appropriate channels to links, we treat the transmit nodes as the primary decision-makers. Thus, each transmitting node in the network is a player in the game that assigns channels to each of its outgoing links.

Each node determines a channel assignment for its links and selects a vector of channels $\mathbf{c}_i = \left(c_i^1, \ldots, c_i^{k_i}\right)$ from its action set $A_i = \mathcal{C}$; we denote the channel assigned to outgoing links from node i by c_i^j. For simplicity, we enumerate the channels available for each link in a set $c_i^j = \{0, 1, 2, \ldots\}$. Again, for ease of exposition, $c_i^j = 0$ means that no channel is assigned to the outgoing link from r_i^j. When no channel is allocated to a link, the link is disabled, in the sense that it does not exist in G_c.

Each node determines appropriate channels to select for transmission by considering the level of interference perceived on those channels. Using the weighted conflict graph G_{I}, each node evaluates the total interference on a given channel by summing up the interference contributions from all nodes sharing the same channel. The interference contribution terms are denoted by the weights of the conflict graph. Note that, it is sufficient for each node to know the sum of all interference weights instead of the individual contributions from each interferer, which may be difficult to measure precisely.

Based on the channel-assigned graph G_c and conflict graph G_{I}, each node evaluates the total interference cost χ_i: the sum total of interference weights (obtained from W) over all interferers of i over all channels c_i^j that i is operating on. In a multi-hop network, each node i is able to communicate with their immediate neighbors on the same channels as those of i, and with other nodes (≥ 2 hops away) which may be on different (non-zero) channels than those i is operating on. The benefit in being able to communicate with other nodes is captured by f_i, which specifies the number of nodes i is able to reach (directly or over multiple hops) over all channels. Given these objectives, the utility of each node is given by:

$$u_i(\mathbf{c}) = \begin{cases} \alpha_i f_i(\mathbf{c}) - \chi_i(\mathbf{c}) & \text{if } \mathbf{c}_i \neq \mathbf{0}; \\ -\infty & \text{if } \mathbf{c}_i = \mathbf{0}. \end{cases} \tag{7.1}$$

Because we are studying the impact of channel availability on the topology outcomes, we impose the condition that nodes must communicate on at least one channel. The term α_i is a constant that specifies the relative preferences of nodes: improving network connectivity vis-a-vis selecting low interference channels.

7.2 Game-Theoretic Analysis

In this section, we analyze the dynamics of the game with utilities given by (7.1). Both terms of (7.1) are interactive terms which together determine the course of the game. When the second term (the interference term) dominates (for instance, when $\alpha_i = 0\ \forall i$) each node selects channels so as to minimize χ_i. In pursuit of minimizing local interference, such a channel assignment may increase the interference observed by some other node. The following lemma claims otherwise, from a single radio perspective.

Lemma 7.2.1. *If the interference χ_i^j of a radio r_i^j increases (or decreases) by δ owing to a channel switch, the aggregate interference of all radios affected by this switch also increases (or decreases) by δ.*

Proof. The proof is a direct consequence of the fact that interference graph is symmetric. Let $\mathcal{I}_i^j$ (likewise, $\mathcal{I}'^j_i$) represent the set of radios whose transmissions interfere with that of r_i^j on channel c_i^j (similarly on c'^j_i). Let $\chi_i^j = \kappa$ before channel switch and be $\chi'^j_i = \kappa + \delta$ after channel switch. It is obvious that the sets $\mathcal{I}$ and $\mathcal{I}'$ are distinct because a radio can not interfere with another radio on two different channels (we are assuming that all channels are orthogonal and do not overlap). Because the link weights in the conflict graph are symmetric, the aggregate interference weight for all radios in $\mathcal{I}$ decreases by κ. Likewise, the aggregate interference of radios in the set $\mathcal{I}'$ increases by $\kappa + \delta$. The net increase (or decrease) in the total interference generated by all radios (except r_i^j) becomes δ. $\square$

We can extend the result of Lemma 7.2.1, which analyzes the impact of channel assignment on a per-radio basis, to consider the impact on a per-node basis. Note that, if nodes reduce their interference cost χ_i, they may increase the per-node interference cost χ_j of some other nodes j individually.

Lemma 7.2.2. *If the interference cost χ_i increases (or decreases) by δ upon channel selection by i, the aggregate interference cost of all nodes affected by this assignment also increases (or decreases) by δ.*

Proof. For each node, we have $\chi_i = \sum_{j=1}^{k_i} \chi_i^j$. For each $j = 1, \ldots, k_i$, let $\chi_i^j = \kappa_j$ and $\chi'^j_i = \kappa_j + \mu_j$, where $\sum_{j=1}^{k_i} \mu_j = \delta$. Because channels corresponding to radios on a node are distinct, the sets χ'^j_i across radios of i do not overlap. As argued in the previous lemma, the aggregate interference cost over all members of the set $\mathcal{I}$ decreases by κ_j. Likewise, the total interference cost of the members of $\mathcal{I}'$ increases by $\kappa_j + \mu_j$. Thus, when each radio r_i^j switches channel, the net increase (or decrease) in interference cost becomes μ_j. Because the sets $\mathcal{I}'^j_i$ are distinct across r_i^j (these sets indicate the radios and nodes that are affected after channel assignment), we can sum the interference terms across all k_i radios, giving us the change in interference cost of $\sum_{j=1}^{k_i} \mu_j = \delta$. $\square$

The above lemma shows that while the interference reducing channel selections by a node may increase the interference cost observed by other nodes individually, the aggregate interference level (from a network viewpoint) still improves; this suggests a self-stabilizing effect of channel assignment.

If there are enough orthogonal channels available (say $\mathcal{I}_i = \phi \ \forall i$), nodes can maximize their utility by utilizing all their radios. On the contrary, when α_i is low and if all available channels are shared, nodes can improve their utility by not allocating channels to any link, except one with least channel interference. Also, note that assigning the same channel across multiple radios on a node always gives a lower payoff than assigning it to a single radio; this justifies our assumption of assigning distinct channels across radio interfaces on a node.

Using the previous two lemmas, we show that for extreme values of α_i, the channel assignment game becomes a potential game.

Theorem 7.2.3. *When $\alpha_i = 0 \ \forall i$, the game $\Gamma = < N, A, u >$ with payoffs given by* (7.1) *is an EPG. The EPF is given by*

$$\mathcal{P}(\mathbf{c}) = -\frac{1}{2} \sum_{i \in N} \chi_i(\mathbf{c}) \tag{7.2}$$

Proof. From Lemma 7.2.2, it is clear that the change in χ_i is:

$$\begin{aligned} \Delta\chi_i &= \sum_{j \in \mathcal{N} \setminus i} \Delta\chi_j \\ &\Rightarrow \sum_{i \in \mathcal{N}} \Delta\chi_i = 2\Delta\chi_i \end{aligned}$$

Thus, $\mathcal{P}$ given by (7.2) is indeed the EPF of Γ.

□

It is clear that the value of α and number of channels available determine how nodes utilize the channels and how many radios are assigned non-zero channels. The following theorem considers the other extreme case when $\alpha_i \geq \chi_i^{\max}$. Under this scenario, we first show that the original topology G can never be partitioned by channel assignment. (We henceforth denote the value of α that preserves network connectivity of G_c by $\alpha_{\max}$.)

Lemma 7.2.4. *When $\alpha_i \geq \chi_i^{\max}$ $\forall i$, then starting from any initial network for which G is connected, every NE achieves connectivity of G_c.*

Proof. Without loss of generality, let all nodes be initialized to the same (non-zero) channel assignment. We then need to show that in every iteration, nodes maintain network connectivity (obtaining $f_i = n-1$ $\forall i$) of G_c. This can be shown by contradiction: Suppose a node improves its utility by selecting a $c_i^j = 0$ for some of its interfaces r_i^j and disconnecting some portion of the network (thereby obtaining a revised benefit $f_i' = k_i < n-1$). Then,

$$\begin{aligned} u_i &= \alpha_i k_i - {\chi'}_i > \alpha_i \left(n-1\right) - \chi_i \\ &\Rightarrow \alpha_i \left(n-1-k_i\right) < {\chi'}_i - \chi_i \end{aligned}$$

Because $\alpha_i \geq \chi_i^{\max}$, the term on the left is greater than $\chi_i^{\max}$, whereas the term on the right is less than $\chi_i^{\max}$ and we obtain a contradiction. Thus, in every iteration, nodes maintain network connectivity while selecting interference reducing channels. Because the process of pure interference reduction is a potential game (by Theorem 7.2.3), such an adaptation process will converge to an NE. Thus, every NE preserves connectivity of G_c. □

Using this lemma, we next show that for $\alpha_i \geq \chi_i^{\max}$ the corresponding channel assignment game is also an EPG.

Theorem 7.2.5. *When $\alpha_i \geq \chi_i^{\max}$ $\forall i$, the game $\Gamma = <N, A, u>$ with payoffs given by* (7.1) *is an EPG. The EPF is given by*

$$\mathcal{P}\left(\mathbf{c}\right) = \frac{1}{2}\sum_{i\in N}\left(\alpha_i f_i\left(\mathbf{c}\right) - \chi_i\left(\mathbf{c}\right)\right) \tag{7.3}$$

Proof. From Lemma 7.2.4, if u_i increases by, say, δ in each round, then χ_i increases by δ (because $f_i = n-1$, a constant). From Lemma 7.2.2, when χ_i increases by δ, $\sum_i \chi_i$ increases by 2δ. Therefore $\sum_i u_i$ increase by 2δ (because f_i does not change). Thus, $\mathcal{P}\left(\mathbf{c}\right) = \frac{1}{2}\sum_i u_i\left(\mathbf{c}\right)$ is indeed the EPG of the game. □

As discussed in Chapter 2, potential games ensure that at least one NE exists for the game. It is fairly obvious that the channel assignment game admits many NE; depending on the order in which nodes update, different NE topologies will emerge. For both games considered above, the potential maximizing NE minimizes the total interference in the network.

Theorem 7.2.6. *For $\alpha_i = 0\ \forall i$, the potential maximizing NE minimizes $\sum_i \chi_i$, whereas for $\alpha_i = \alpha_{\max}$, the potential maximizing NE minimizes $\sum_i \chi_i$ while maintaining network connectivity of G_c.*

Proof. The proof is straightforward: in the first case $\mathcal{P}(\mathbf{c}) = -\frac{1}{2}\sum_{i\in N}\chi_i(\mathbf{c})$, which immediately suggests that the potential maximizer minimizes $\sum_{i\in N}\chi_i(\mathbf{c})$.

In the latter case, $\mathcal{P}(\mathbf{c}) = \frac{1}{2}\sum_{i\in N}(\alpha_i f_i(\mathbf{c}) - \chi_i(\mathbf{c}))$. Because $f_i(\mathbf{c}) = n-1$, a constant, in NE, the potential maximizer again minimizes $\sum_{i\in N}\chi_i(\mathbf{c})$ subject to network connectivity. □

The above theorems may not generalize for arbitrary values of α. While this does not preclude the existence of NE for the channel assignment game, the game may or may not possess an NE. For these cases, we examine the existence of NE through simulations. For the cases where NE exists, we evaluate the NE topologies with respect of interference and connectivity performance in the following section. Any selfish algorithm that nodes adopt to improve their performance objective is guaranteed to converge to some NE. We propose a better-response-based channel selection algorithm that is simple to implement, and evaluate the performance of NE topologies that emerge in steady state.

7.3 Performance Analysis

To determine the efficacy of our model, we develop a simulation consisting of $|N|$ nodes placed according to a uniform random distribution within a unit square. The power thresholds $\omega(i,j)$ required to close a link between nodes i and j were assumed to be equal to $d^2(i,j)$

(we choose a path loss exponent of 2, although our basic conclusions remain the same for other channel attenuation factors as well), where d is the euclidean distance metric. The initial node transmit power level was chosen using the formula from [69] (and adjusting the value for finite networks), such that the induced network was 1-connected with 85% probability. We consider only the connected instances of G in our simulations (meaning that there exists a path from every node to every other node in the network). Each node has a fixed number of radios capable of operating on different channels.

The connectivity graph G was transformed to an undirected weighted conflict graph G_{I} that is derived from G. In our simulation setup, conflicting pairs are chosen according to the distance-2 interference [86]: conflicting radios include both one and two hop neighbors in the undirected graph G. The links weights in the conflict graph are determined using the free-space propagation model. Thus, weights associated with nodes in the conflict graph are proportional to the channel gain between them and therefore are a decreasing function of the corresponding inter-nodal separation (with path loss factor of 2). Our conflict model is reasonable both from an implementation and interference point of view, as it only requires radios to communicate with their bidirectionally connected radios and only makes conflict neighbors those radios that would cause meaningful interference. Figure 6.1 is a simple illustration of our interference model.

Each node is a selfish player in the channel assignment game, selecting channels that improves its utility. The channel selection algorithm that nodes adopt is based on a random better response strategy. All nodes initialize their channel selections to the default non-zero channel before adapting their channel selections. Each node in the network is assigned a random backoff within a fixed window. The backoff periods induce an ordering that represents a random permutation. When the backoff ends, nodes randomly select an action $\mathbf{c}_i^{(k)}$ in every round $k = 0, 1, 2, ...$, from the set:

$$\mathbf{c}_i^{(k+1)} \in \left\{ \mathbf{c}_i \in A_i \mid u_i \left(\mathbf{c}_i, \mathbf{c}_{-i}^{(k)} \right) > u_i \left(\mathbf{c}^{(k)} \right) \right\}, \forall i \in N \tag{7.4}$$

When no such improving action exists, nodes revert to their previous action.

To study the impact of channel availability on the topologies that emerge in steady state, we vary the number of channels available in the network, keeping the number of radios on each node fixed. Each network node is equipped with four radios, and for a given set of available channels, we evaluate the total interference in the steady state NE topology, and average it over 1000 different scenarios, with nodes randomly placed at different locations in each case. In each case, we let our selfish algorithm run for a sufficiently long time to closely approximate the NE; we use a termination criteria that the payoffs of every node must change from one round to the next by less than 0.1%. Figure 7.2 illustrates the interference performance of the steady state topologies; we use a sufficiently large α value ($\alpha_{\max}$) to examine the case where network connectivity of G_c was to be supported by channel assignment. As expected, we observe that with increasing channel availability, channels are shared by fewer interfering transmissions, causing the aggregate interference in the network to decrease. By quantifying the interference in multi-channel networks, Figure 7.2 illuminates the interference reduction that can be achieved by utilizing orthogonal channels and by distributing interference over multiple channels. This result is particularly important when making the design decision on the optimal number channels to use, by examining the tradeoff between performance gains achieved and the cost of having additional channels.

We compare the average interference performance of NE topologies that result from our better response algorithm with Subramanian's centralized Tabu search approach [34] for the 25 node network. As observed in Figure 7.2, both algorithms perform comparably when the number of channels are low because of the relatively small search space. At higher channel availability, Tabu search outperforms by a small margin. Minimizing interference through multi-channel assignment can be mapped to a graph coloring problem with additional constraints on the number of interfaces and number of channels available. This problem is known to NP-hard [34]. For this reason, we compare the performance of our algorithm with the global optimum (obtained using depth first search) for a smaller sized 10 node network. Although in the worst case an NE topology can perform arbitrarily poorly, from Figure 7.3

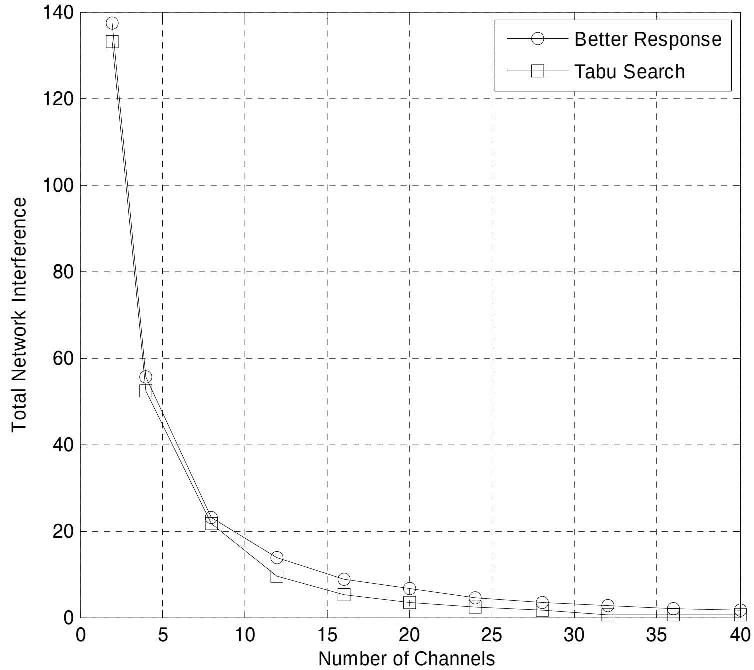


Figure 7.2: Illustrating the impact of number of available channels on the total interference in NE topologies and Tabu search based topologies of a 25 node network (4 radios per node).

we observe that on average its total interference is less than 10% of the optimum for a 10 node network.

For the same set of scenarios used in determining the network interference above, we examine the interference dependency on the benefit factor α. The term α indicates the relative preference between low interference and high connectivity. Higher values of α indicate that nodes prefer to maintain connectivity with greater number of nodes. Because we don't have the potential game results for $\alpha \in (0, \alpha_{\max})$, we first check whether or not the better response algorithm converges. We choose values of $\alpha = 0.1\alpha_{\max}$ and $0.5\alpha_{\max}$ and for each α, we examine 1000 randomly generated topologies of a 10 node network. In every case, we observed convergence within $5 - 10$ iterations. With this knowledge of convergence, we then evaluate the connectivity of the NE topologies across various α values for a 25 node network. Figure 7.4 examines the connectivity of the resulting NE topologies as a function of α. For each steady state topology, we evaluate its connectivity fraction: the fraction of nodes

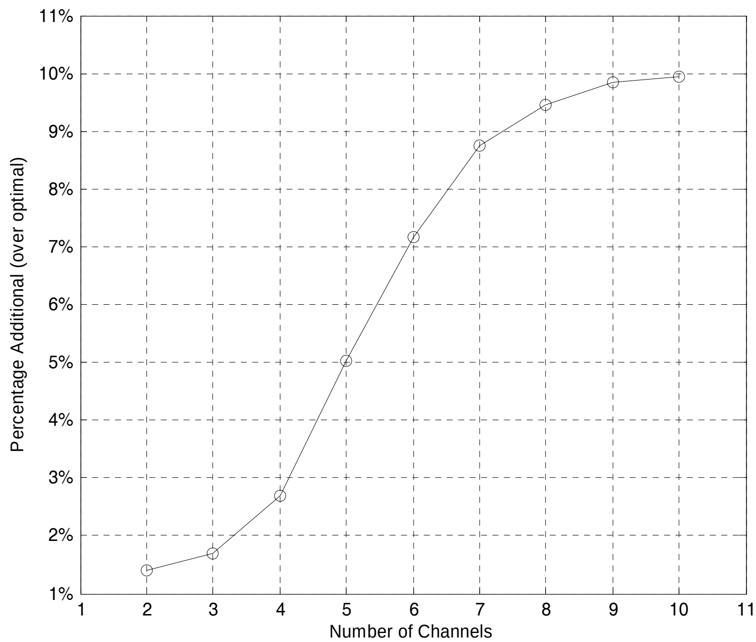


Figure 7.3: Average additional interference as compared to optimal in a 10 node network (2 radios per node).

belonging to the largest connected component of the network. For low values of α, certain links in the network are not assigned any channel and thus are disabled, so as to reduce interference. Thus, for low values α, the topologies become sparsely connected. Higher values of α indicate greater network connectivity even if supporting these connections over different channels come at the cost of high interference levels. Figure 7.5 validates this fact, and shows that, for a given number of available channels, accommodating more transmissions on various channels naturally leads to increased levels of co-channel interference.

To make a meaningful assessment of how radios share channels, we evaluate the spectral occupancy of the steady state topologies. Unlike in the above studies, we assign equal weights to the contributions from all interferers. Thus, the level of interference observed on each channel is equal to the total number of interferers sharing that channel. We fix the number of available channels at 25, and determine a typical spectral occupancy profile of NE topologies for the scenarios where network connectivity is to be supported ($\alpha = \alpha_{\max}$) by

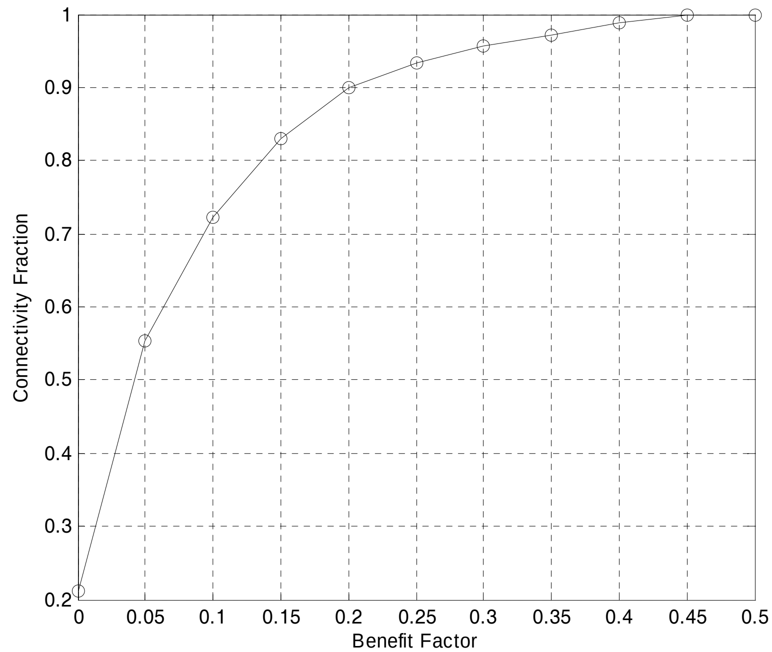


Figure 7.4: Variation in network connectivity with benefit factor (fraction of $\alpha_{\max}$) for a 25 node (4 radios per node), 25 channel network.

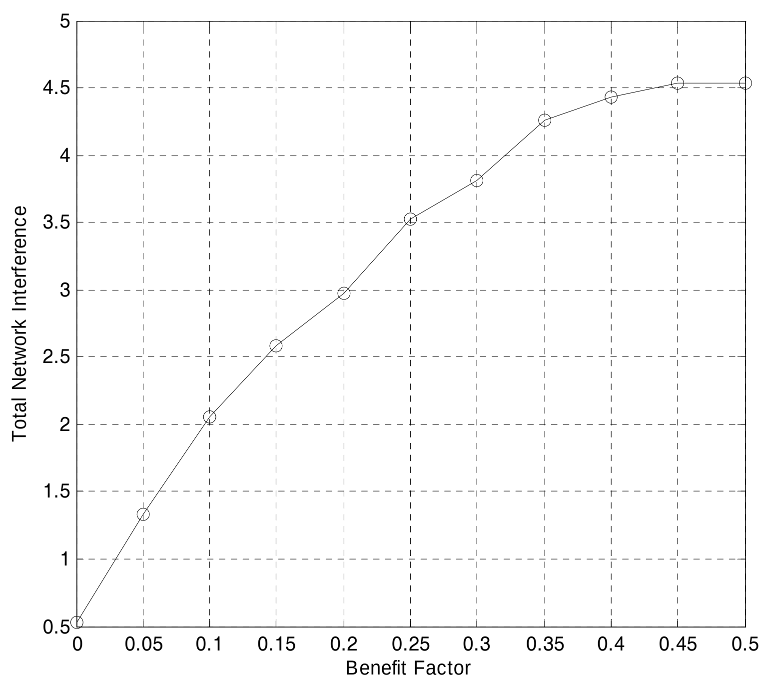


Figure 7.5: Variation in network interference with benefit factor (fraction of $\alpha_{\max}$) for a 25 node (4 radios per node), 25 channel network.

channel assignment. Interestingly, we observe that some radios are not assigned any channel in NE. This is because each node's strategy is to minimize interference while just about ensuring network connectivity. The resulting minimum interference topologies are quite sparse; hence, nodes do not need to utilize all their radios to ensure network connectivity. We also observe that radios share channels fairly evenly across channels, suggesting a load balancing effect as shown in Figure 7.6. In NE, nodes tend to minimize their interference number by utilizing all available channels and selecting channels with minimum number of interferers.

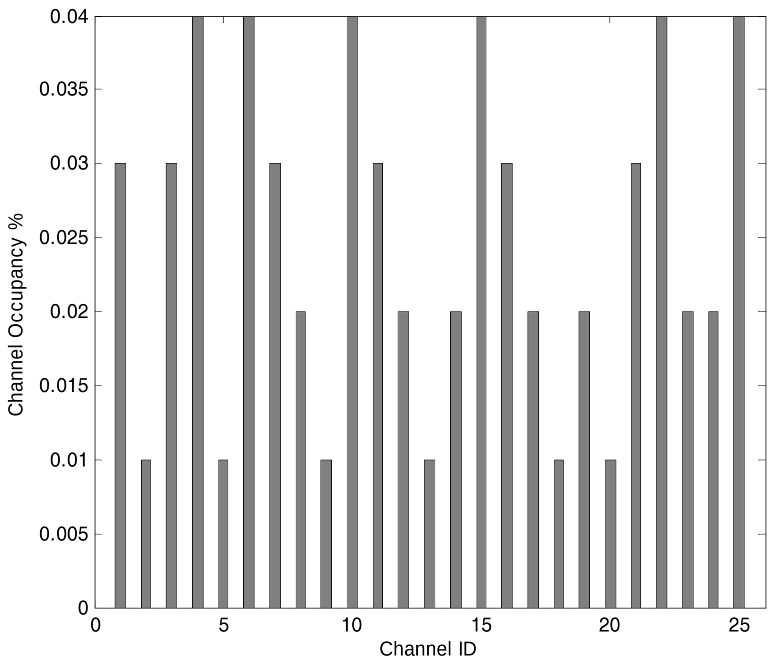


Figure 7.6: Typical channel occupancy profile of a 25 node (4 radios per node), 25 channel network.

Figure 7.7 illustrates the spectral occupancy performance in an expected sense, averaged over 1000 randomly generated topologies. We observe that the load balancing trend holds in general, with every non-zero channel supporting 1-4% of radios in the network in equilibrium. Utilizing all channels evenly indicates efficient channel reuse, therefore such channel strategies are expected to perform well in improving throughput performance of the network.

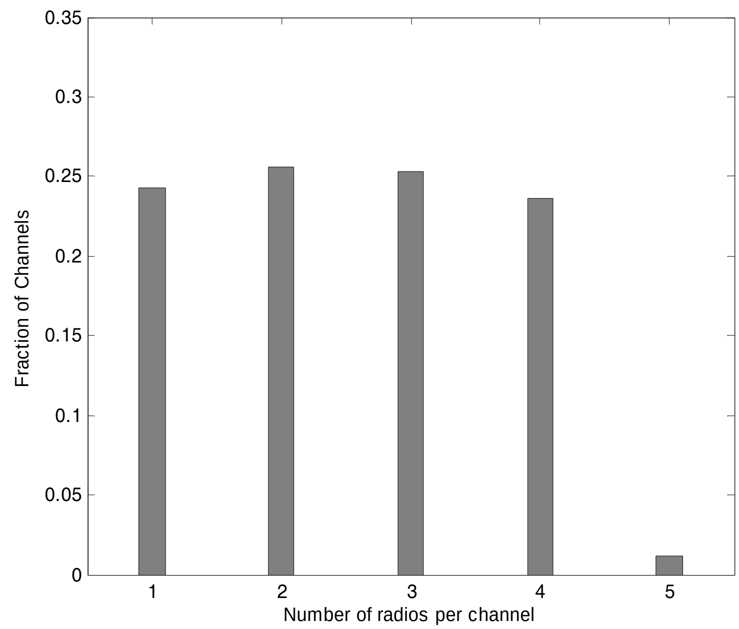


Figure 7.7: Average spectral occupancy in a 25 node network (4 radios per node) with 25 channels.

7.4 Summary and Contributions

We analyze the problem of interference minimization through multi-channel allocation in non-cooperative networks using game theory. We show how channel assignment in multi-hop networks can be viewed as a topology control problem with the goal of minimizing aggregate interference while maintaining some degree of network connectivity. Nodes selfishly select the best channel to improve their own performance. In some cases, nodes may choose not to assign any channel to links, thus disabling links in the channel-assigned topology. We show that this channel selection game is a potential game, which ensures the stability of any selfish channel selection dynamic process.

We analyze the NE topologies with respect to interference and connectivity performance. With increasing channel availability, a larger portion of the network can be supported, thus leading to more connected topologies. In addition, the total network interference decreases with increasing number of available channels in order to support a connected network. By quantifying the interference in multi-channel networks, we illuminate on the interference reduction that can be achieved by utilizing orthogonal channels and by distributing interference over multiple channels. This result is particularly important when making the design decision on the optimal number of channels to use, by examining the tradeoff between performance gains achieved and the cost of having additional channels. The performance of the distributed better response algorithm is comparable to the centralized Tabu search algorithm, and is observed to be within 10% of the optimum for a 10 node network. Finally, the selfish algorithm achieves load balancing: the number of radios on each non-zero channel in steady state are almost equal.

The work presented in this chapter has resulted in the following publication:

1. R. S. Komali and A. B. MacKenzie, "Analyzing selfish multi-radio multi-channel topology control," To be submitted.

Chapter 8

Conclusions

This dissertation broadly focuses on the design and analysis of multi-hop wireless communications systems. Here, we propose a game-theoretic framework for non-cooperative topology control to improve the end-to-end network performance by taking into account the distributed, localized, and autonomous traits of self-organization, lack of complete information in distributed systems, and users' natural disposition toward self-improving alternatives. By breaking down the overall networking tasks into node-level objectives, we strive to transcend the complexities of multi-hop wireless networks, while at the same time, mitigate the sub-optimality of distributed solutions.

Within the aforementioned framework, we identified and analyzed two classes of problems pertaining to resource management in ad hoc networks: power and energy efficiency and, spectral efficiency and interference minimization. Both power and spectrum management have broad impacts in determining the future course of wireless networks, and are of immediate interest to the military and industry research community. By quantifying the effects of non-cooperation and limited network state knowledge, and illuminating the interplay between the cost of network resources, partial information, and end-to-end network performance, this dissertation formally advances the need for considering selfish behavior and information constraints at the outset, in the network design stage, when optimizing distributed wireless systems.

For all the problems considered, we show that, under certain mild conditions, the efficiency of the operating points that can be achieved by non-cooperative users may be comparable to those achieved by cooperative algorithms.

8.1 Research Summary and Contributions

We studied the topology control of ad hoc networks in presence of self-motivated nodes. The coupling between selfish node behavior and the lack of complete information in ad hoc network design was cast as an information-constrained non-cooperative topology control process. Under this framework, we identified several parameters, one or more of which can be tuned to accomplish the topology control task. Our holistic framework provides enough scope to study the impact of non-cooperation and information constraints under the purview of distributed topology control.

Two distinct, yet related, studies form the core contributions of this dissertation. First we examined the impact of selfish behavior on power and energy efficiency in ad hoc networks. In both analyses, the fundamental questions we addressed are: (a) if nodes are programmed to maximize their local objectives, what is the impact on the overall network performance? and, (b) how do we mitigate the conflicting objectives of the nodes and those of the network in a non-cooperative setting? We begin by identifying the selfish traits with respect to energy minimization. In one setting, nodes can only control their transmit power levels, whereas in more selfish environments, where nodes have greater degree of controllability over their transmission parameters, nodes may selectively forward packets in addition to selfishly controlling their power levels.

In the former case, we characterized all the steady state NE of the power control game and determined that most NE states result in sub-optimal network performance. Besides these local minima, there exist other NE which, though not globally optimal, are reasonably close to the global network performance maximizers. To ensure that the system converges to these desirable NE, we developed a better response dynamic, DIA, that reconciles the selfish objectives of the nodes with the overall network goals. We extended this analysis on static networks to dynamic networks where network dynamics was cast as nodes adding and removing themselves from the network. In addition to selfish behavior, we also considered the effect of partial knowledge that nodes may possess about the state of the network. We examined the tradeoff between the cost of available information and the energy efficiency performance of the NE topologies that emerge. We showed that in networks that are more dynamic, the cost of maintaining knowledge can become prohibitive. In some sense, this means that mobility can actually improve the performance of information-constrained networks.

In the latter case, we assessed the impact of selfish node behavior where nodes take a holistic view of energy minimization by selectively forwarding packets in addition to selfishly adapting their transmit power levels. Analyzing the impact of partial packet forwarding on energy-minimal topologies is the main contribution of this work. By quantifying the energy efficiency gains that can be achieved through cooperation, our study corroborates the need for incentivizing nodes in energy-limited networks.

Interference is a fundamental problem intrinsic to wireless networks. Unlike the aforementioned studies which considered single channel networks, the second part of the dissertation examined multi-channel networks in the context of interference minimization. Dividing the available spectrum into multiple blocks of channels and using them opportunistically to limit multiple access interference has been shown to provide orders of magnitude improvement in the available network capacity. Controlling the topology efficiently enhances the spectral usage efficiency, which in turn leads to increased spatial reuse. We first examined the problem of minimizing the spectral footprint — the minimum number of channels required to establish an interference-free network connectivity. We showed that minimizing spectral usage through joint power control and channel assignment is tantamount to channel allocation on an already power-optimized topology. We showed that in spite of selfish channel selections, the network spectrum utilization is within 12% of the optimum on average. We then extended the analysis to dynamic networks where nodes have incomplete network state knowledge, and quantified the price of ignorance.

In the final study of this dissertation, we showed how topology control can be performed purely by means of assigning channels to links in a communication graph. Unlike in the previous study, the number of available channels and radio interfaces on a node are limited, causing users to share spectrum efficiently and minimize interference by distributing it over multiple channels. We evaluated the performance of NE topologies with respect to network connectivity and interference minimization goals, and examined the tradeoff between the two objectives. Having more channels for a given level of network connectivity naturally leads to lower interference topologies. Likewise, for a given number of channels, supporting a larger portion of the network on those channels results in increased levels interference in the network. Furthermore, in spite of the non-cooperative node behavior of nodes, radios are evenly distributed on each available channel, suggesting the load balancing effect of NE.

In all studies considered in this dissertation, we have shown how the performance of a non-cooperative network can be enhanced through topology control by appropriately incentivizing selfish nodes. While the specific contributions of this work pertain to energy efficiency and interference minimization, other performance objectives can also be considered (such as delay, network lifetime, throughput capacity etc.), some of which may require joint optimization in multiple dimensions over several parameters in $\mathcal{P}$.

8.2 Ties to Cognitive Networks

Our preliminary studies provide a strong foundation for future research: understanding the cohesion between topology control and cognitive networks. Significant parallels exist between the two: both processes are cross-layer, distributed, multi-objective, self-adaptive, self-resilient and are driven by the end-to-end network goals. Using the above examples of autonomous control and distributed resource management, we advance the idea that topology control, in general, is a good case study for cognitive networks.

We believe that the problem of topology control has natural relevance to the field of cognitive radios and dynamic spectrum access. For instance, our study on spectral footprint minimization jointly through power control and channel assignment shares similarities with the problems addressed by cognitive radios and dynamic spectrum access. Topology control is also a natural fit for Cognitive Networks (CNs), as it attempts to adapt the usable connections in the network to meet a network-wide objective such as connectivity or energy efficiency. Thomas *et al.* [53, 87] first defined *Cognitive Networks* formally as:

> "...a network with a cognitive process that can perceive current network conditions, and then plan, decide and act on those conditions. The network can learn from these adaptations and use them to make future decisions, all while taking into account end-to-end goals."

The distinguishing aspect of this definition is the end-to-end scope of its cognitive process. Without it, a system is perhaps a Cognitive Radio (CR) or a cross-layer design, but not a CN. *End-to-end*, in this definition, denotes network objectives that transcend those of the individual network elements.

The CN framework consists of multiple independent, autonomous cognitive elements distributed across the network [87]. A common thread between the various definitions of cognition [88] is the idea of a feedback loop. A feedback loop models cognition by describing how past interactions with the environment guide current and future interactions, capturing the characteristics of learning and reasoning. This idea of feedback loop has been used in this dissertation in context of selfish adaptations that nodes make in the face of limited network knowledge. These adaptations are based on simple learning and reasoning mechanisms and can be used to mimic cognition (see Figure 1.3). For the specific problems we considered, incorporating more sophisticated forms of machine learning is perhaps not necessary, as it is not yet clear that potential performance gains from such techniques will offset the increase in processing cost and reaction time.

The decision-making aspect of the CN may be construed as one or more cognitive elements operating in some degree between autonomy and full cooperation. If there are multiple elements, they may be distributed over a subset of nodes in the network, on every node in the network, or several cognitive elements may reside on a single node. In the context of Chapter 6, the cognitive elements can be envisioned as performing an important task of a CN, the cognitive cycle: the elements select (and possibly revise) their optimum settings based on the perceived topology state; these revised actions induce a change in topology configuration, either in the connectivity or in the channel index profile; the modified topology affects the utility of individual elements, which in turn update their power or channel settings, and the cycle starts all over again. Recall Figure 1.3, for a visual illustration of the cognitive cycle.

The cognitive network framework also accommodates our model of network dynamics, where nodes join or leave the network. Specifically, the cognitive elements learn from their past encounters, remember their previous transmission parameters, and then act accordingly to "fix" the network in the event a change in the network configuration is detected. This approach is in contrast to some existing approaches in the literature, where automatic protocol refreshment is triggered every time there is even a small change in topology.

Power and spectrum control are especially appropriate for cognition and operation under partial knowledge. Minimizing the number of channels, even under full knowledge, is similar to the well-known graph coloring problem, which is NP-hard to solve in the worst case. Hard problems such as these are well-suited for cognition, since there are no polynomial time solutions to the problem and all current solutions are heuristic in nature. Cognition may provide an edge in approximating the optimal solution. Minimizing the maximum transmission power (as in Chapter 3) is closely related to the formation of the MST, which requires full knowledge to guarantee the correct formation. Thus, a cognitive process that operates under some degree of ignorance in achieving this objective is a useful improvement.

Given the complexity of the problems addressed by TC, and the similarities they share with CNs (distributed, cross-layer, end-to-end scope etc.), we believe that TC is a good example where the use of CN paradigm seems justified.

8.3 Future Research Directions

Much of the work in this dissertation is developed under static network settings or using simple mobility models. Given the intractability of most TC problems, this approximation certainly eases the analysis while still providing many insightful results. A direction for future work is to expand our results by considering application-dependent mobility models (e.g. vehicular networks). It would also be worthwhile to investigate if our results generalize under more sophisticated models of communication, such as the physical model.

Optimizing topologies for maximizing network capacity is perhaps the least understood problem in TC. In part, this is because the study of theoretical and algorithmic aspects of capacity are still in their infancy. Designing the capacity-optimal topologies is still an open problem and understanding the design- and trade-space is, in some sense, the “final frontier” in TC research.

From a game-theoretic viewpoint, it is important to consider games based on imperfect information to accurately model real world systems. Lastly, we identified striking similarities between potential game and mechanism design formulations. Both models, in principle, can be used to steer selfishly motivated nodes to system-optimal states. This technique is particularly useful in situations where individual utilities are unknown; a utility function can then be appropriately designed to maximize network performance. Unlike in mechanism design, the space of available mechanisms in potential games could be algorithms. A system designer, with knowledge of optimal network operating points, can develop protocols that converge to these network-wide optimal solutions.

Appendix A

Acronyms

BER bit error rate

BR best response

CR Cognitive Radio

CN Cognitive Network

CSMA/CA Carrier Sense Multiple Access with Collision Avoidance

DIA δ-Improvement Algorithm

DT Delaunay Triangulation

EPF Exact Potential Function

EPG Exact Potential Game

FDMA Frequency Division Multiple Access

FIP finite improvement path

GG Gabriel Graph

GRG Geometric Random Graph

IP improvement path

IMST Interference Minimum Spanning Tree

JPCC Joint Power and Channel Control

LILT Low Interference-Load Topology

MAC Medium Access Control

MAICPC Minimize the Average Interference Cost while Preserving Connectivity

MANET Mobile Ad Hoc Network

MIA Max Improvement Algorithm

MST Minimum Spanning Tree

NE Nash Equilibrium

NIC network interface card

OPF Ordinal Potential Function

OPG Ordinal Potential Game

PMST power-based MST

PO Pareto Optimal

PRN Packet Radio Network

QoS Quality of Service

RNG Relative Neighborhood Graph

SER symbol error rate

SINR Signal to Interference and Noise Ratio

SURAN Survivable Radio Network

TC Topology Control

TCG Topology Control Game

TDMA Time Division Multiple Access

YG Yao Graph

w.h.p with high probability

WLAN Wireless Local Area Network

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