Appendix D
Determination of Interface Hyperbolic Parameter Values

This appendix describes the procedure developed by Clough and Duncan (1971)[1] for the determination of hyperbolic parameter values for interfaces. Hyperbolic parameter values of the interfaces between concrete and dense Density Sand, medium-dense Density Sand, and dense Light Castle Sand are determined based on the results of the initial loading tests presented in Appendix C. Example calculations of hyperbolic parameter values for the interface between dense Light Castle Sand and concrete are presented in the last section of this appendix.

D.1 Transformed Plots

The procedure for determination of hyperbolic parameters is illustrated using the data from initial loading tests on dense Density Sand against concrete interface. The data from the initial loading tests, which is shown in Figure C1c of Appendix C, is represented in the transformed diagram of Figure D1 following the procedure described by Clough and Duncan (1971). In this transformed diagram, the value of interface displacement[2] Δ_s measured during the test is divided by the corresponding value of shear stress τ and plotted against the interface displacement. If the shear stress-displacement relationship measured during the triaxial test is hyperbolic, the transformed plot is a straight line. The intercept a of this straight line on the Δ_s/τ axis is the reciprocal of the initial shear stiffness K_{si} of the interface. The slope b of the line is the reciprocal of the asymptotic shear stress τ_{ult}.

The shear stress-displacement relationship of an interface usually differs from a hyperbola. Clough and Duncan (1971) indicated that the values of parameters a and b can be determined from a straight line passing through the points in the transformed plot that correspond to 70 and 95 percent of the interface shear strength. The transformed plot in Figure D1 shows the 70 and 95 percent strength data points for each of the tests performed on the dense Density Sand against concrete interface. Straight lines are drawn through each pair of these points.

[1] References cited in this Appendix are included in the References at the end of the main text

[2] For convenience, symbols are listed and defined in the Notation (Appendix F)

It can be seen in Figure D1 that the lines drawn through the 70 and 95 percent data points match closely the transformed data sets. This type of comparison may be useful for avoiding errors in the determination of hyperbolic parameter values that can arise from inconsistencies in the data. Transformation of the entire set of test data is easily achieved in an electronic spreadsheet. The use of a spreadsheet also facilitates modification of the values of parameters *a* and *b* to obtain the best possible fit to the data.

D.2 Hyperbolic Parameter Values

D.2.1 Determination of K_I and n_j

The values of the parameters *a* and *b* determined from the transformed plots are presented in the table included in Figure D1. The values of initial interface shear stiffness and asymptotic shear stress are determined using the following equations:

$$K_{si} = \frac{1}{a} \qquad \text{(D1)}$$

$$\tau_{ult} = \frac{1}{b} \qquad \text{(D2)}$$

The values of K_{si} and τ_{ult} for each of the tests performed on the interface between dense Density Sand and concrete are presented in the table in Figure D1. It can be seen that the value of K_{si} increases with increasing normal stress σ_n. The following relationship is used to relate the value of initial interface stiffness to the magnitude of normal stress:

$$K_{si} = K_I \cdot \gamma_w \cdot \left(\frac{\sigma_n}{p_a} \right)^{n_j} \qquad \text{(D3)}$$

where

K_I = stiffness number

γ_w = unit weight of water

p_a = atmospheric pressure

n_j = stiffness exponent

This relationship implies that there is a linear relationship between the logarithm of the initial interface shear stiffness and the logarithm of the normal stress. Figure D2 is a logarithmic diagram that shows the values of normalized initial stiffness K_{si}/γ_w represented against the values of normalized normal stress σ_n/p_a. A best-fit straight line is drawn through the data points. The value of the stiffness number K_I is equal to the value of normalized initial stiffness given by this best-fit line for a normal stress of 1 atm. The slope of the line is the stiffness exponent n_j.

D.2.2 Determination of R_{fj}

The table in Figure D1 shows the values of shear stress at failure τ_f, determined from the shear stress-displacement data from the tests, which are presented in Figure C1c of Appendix C. It can be seen in the table that the values of τ_{ult} are larger than the values τ_f in all the tests. The value of the failure ratio R_{fj} for each of the tests is determined from the following expression:

$$R_{fj} = \frac{\tau_f}{\tau_{ult}} \qquad \text{(D4)}$$

The table contains the values of R_{fj} determined for each of the tests. For modeling, an average value of R_{fj} is determined from the test results as shown at the bottom of the table. According to Table 4-11 in Chapter 4 of this report, typical values of R_{fj} range between 0.4 and 0.95.

D.3 Comparison of Model to Test Data

Once the hyperbolic parameter values are determined, it is necessary to compare the model response to the test data. The shear stress-displacement response from the model is calculated using the following expression:

$$\tau = \frac{\Delta_s}{\dfrac{1}{K_I \cdot \gamma_w \cdot \left(\dfrac{\sigma_n}{p_a}\right)^{n_j}} + \dfrac{R_{fj} \cdot \Delta_s}{\sigma_n \cdot \tan\delta}} \qquad \text{(D5)}$$

where δ is the interface friction angle. This expression is valid for frictional interfaces with zero adhesion intercept.

The shear stress-displacement response of the interface between dense Density Sand and concrete was calculated using Equation D5 and the hyperbolic parameters determined following the procedure described previously. Figure D3 compares the test data and the calculated hyperbolic response. In the figure, the shear stress-displacement hyperbola is interrupted at the value of shear stress at failure τ_f. A horizontal shear stress-displacement relationship, i.e., zero interface stiffness, is used to model the response of the soil after failure is attained.

An identical procedure was followed for the determination of the hyperbolic parameters of the interfaces between concrete and medium dense Density Sand and dense Light Castle Sand, as illustrated in Figures D4 through D9. It can be seen that the hyperbolic model provides an accurate approximation to the interface response measured during each of the tests.

Further adjustments of these parameter values may be required for the application of the extended hyperbolic model. These adjustments are fully described in Chapter 4 of this report.

D.4 Example Calculations of Hyperbolic Parameter Values

This section presents an example of the determination of hyperbolic parameter values. The data from the initial loading tests performed on the interface between dense Light Castle Sand and concrete are used for this example. The presentation of the example calculations is based on the procedure described by Duncan et al. (1980) for the determination of hyperbolic parameter values in soils.

The first step in the determination of hyperbolic parameter values is checking for inconsistencies in the data from the interface tests. Figure D10 shows the results of the tests performed on the dense Light Castle Sand against concrete interface. The data points shown in the figure are identical to those shown in Figures C5 and D9. Frequently, data from interface tests may present some inconsistencies that can be minimized by developing an assumed shear stress-displacement response. The response assumed for the determination of the hyperbolic parameters in this example is represented as a solid line in Figure D10. Because of the relatively large precision of displacement and shear stress measurements, few inconsistencies can be seen in the data and the assumed response is practically identical to the measured response.

The next step is the determination of the shear stress at failure τ_f for each normal stress. The values of τ_f can be determined from the shear stress-displacement plots of the tests. Column (2) in the table presented in Figure D11 contains the values of shear stress at failure determined from Figure D10. The values of shear stress corresponding to 70 and 95 percent of τ_f are calculated as shown in columns (3) and (6), respectively.

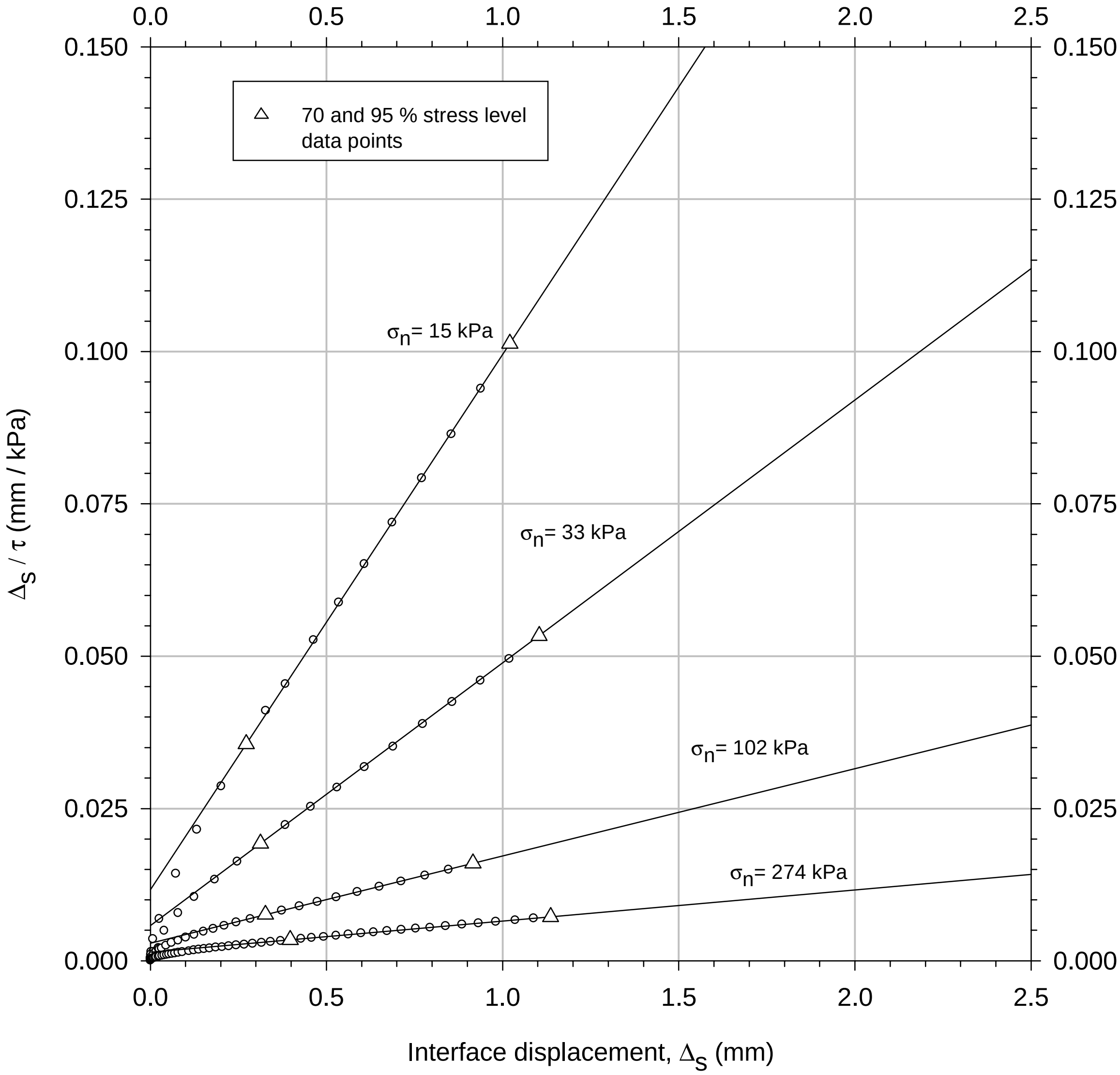
The values of interface displacement corresponding to 70 and 95 percent of the shear stress at failure are determined from the shear stress-displacement plots. Columns (4) and (7) in Figure D11 contain the displacement values determined as shown in Figure D10.

The values in columns (2), (3), (4), (6), and (7) are the basis for the determination of the values of initial interface stiffness K_{si} and failure ratio R_{fj}. The sequence of calculations leading to the determination of the values of K_{si} and R_{fj} is shown in Figure D11, and corresponds to the procedure presented in the previous sections.

It must be noted that, in the method presented in Figure D11, the data are not plotted in transformed coordinates. Only the two data points corresponding to 70 and 95 percent of the shear strength are transformed as shown in columns (5) and (8). Although not strictly necessary, it is recommended to plot always the complete data set in transformed coordinates. The transformed plots are useful to check the data for inconsistencies and to verify the values of initial interface stiffness and failure ratio determined from the procedure presented in Figure D11. Transforming the data following the procedure described previously in this chapter can be accomplished easily with electronic spreadsheets.

The value of failure ratio R_{fj} to be used for modeling is the average of the values determined in Figure D11. The values of K_I and n_j are determined by plotting the normalized values of initial interface stiffness against the normalized normal stress in logarithmic scale as shown in Figure D12.

It must be noted that none of the values presented in Figure D11 was determined graphically. They were obtained directly or by interpolation of data in an electronic spreadsheet. Although graphical determination of values of shear stress and displacement from a figure such as D10 may provide fewer significant decimal places than their numerical determination, the overall precision of the values of the hyperbolic parameters is similar using both procedures. The use of an electronic spreadsheet is recommended, not for increased precision, but for ease in the calculations and verification of the results.



σ_n (kPa)	a (mm/kPa)	K_{si} (kPa/mm)	b kPa^{-1}	τ_{ult} (kPa)	τ_f (kPa)	R_{fj}
15	0.01169	86	0.0879	11.4	9.2	0.804
33	0.00579	173	0.0431	23.2	19.9	0.858
102	0.00291	343	0.0143	69.8	61.3	0.878
274	0.00143	700	0.0051	195.6	164.9	0.843

Average Rfj = 0.846

Figure D1. Transformed plots for initial loading tests on dense Density Sand-to-concrete interface

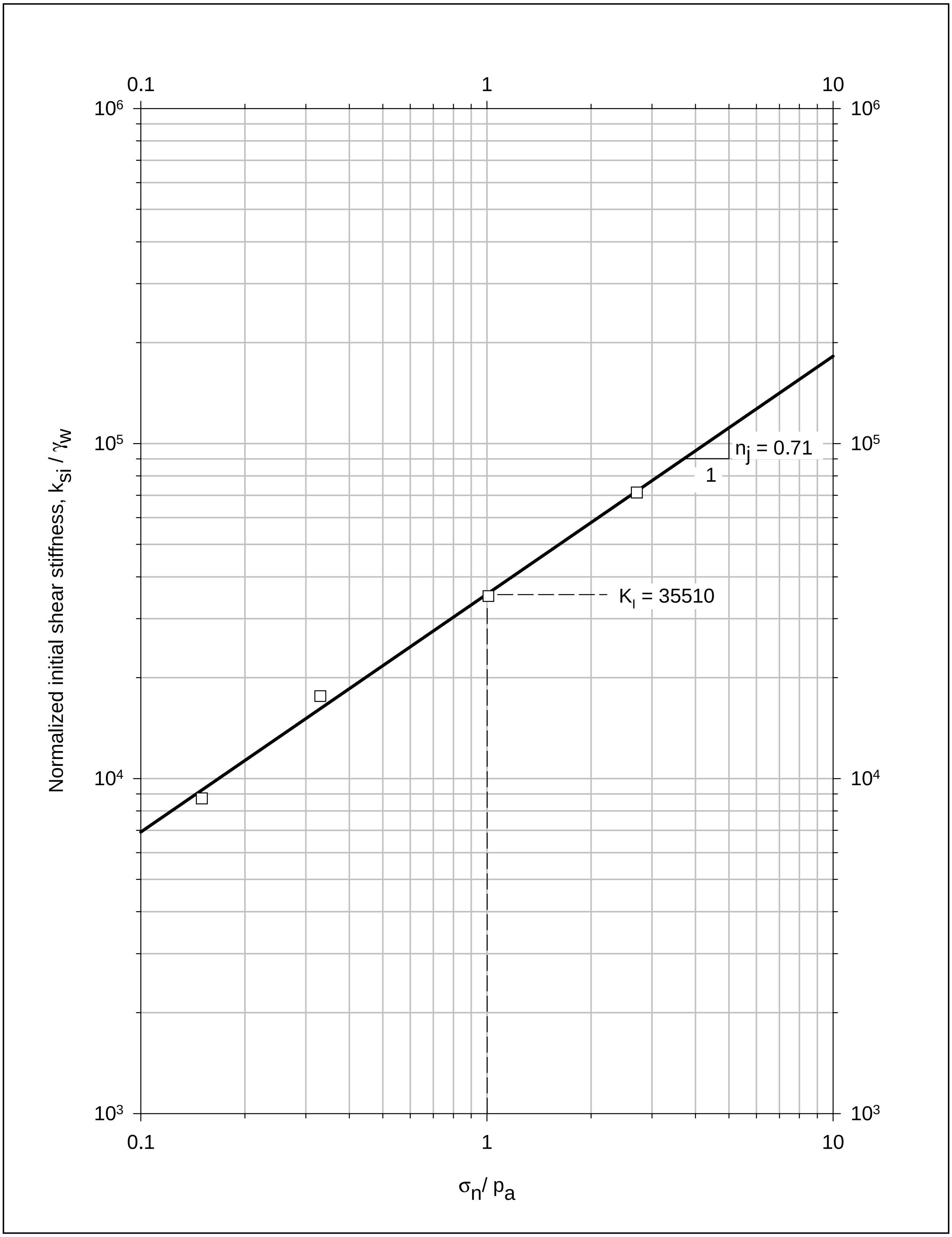


Figure D2. Determination of Hyperbolic Parameters K_I and n_j for dense Density Sand-to-concrete interface

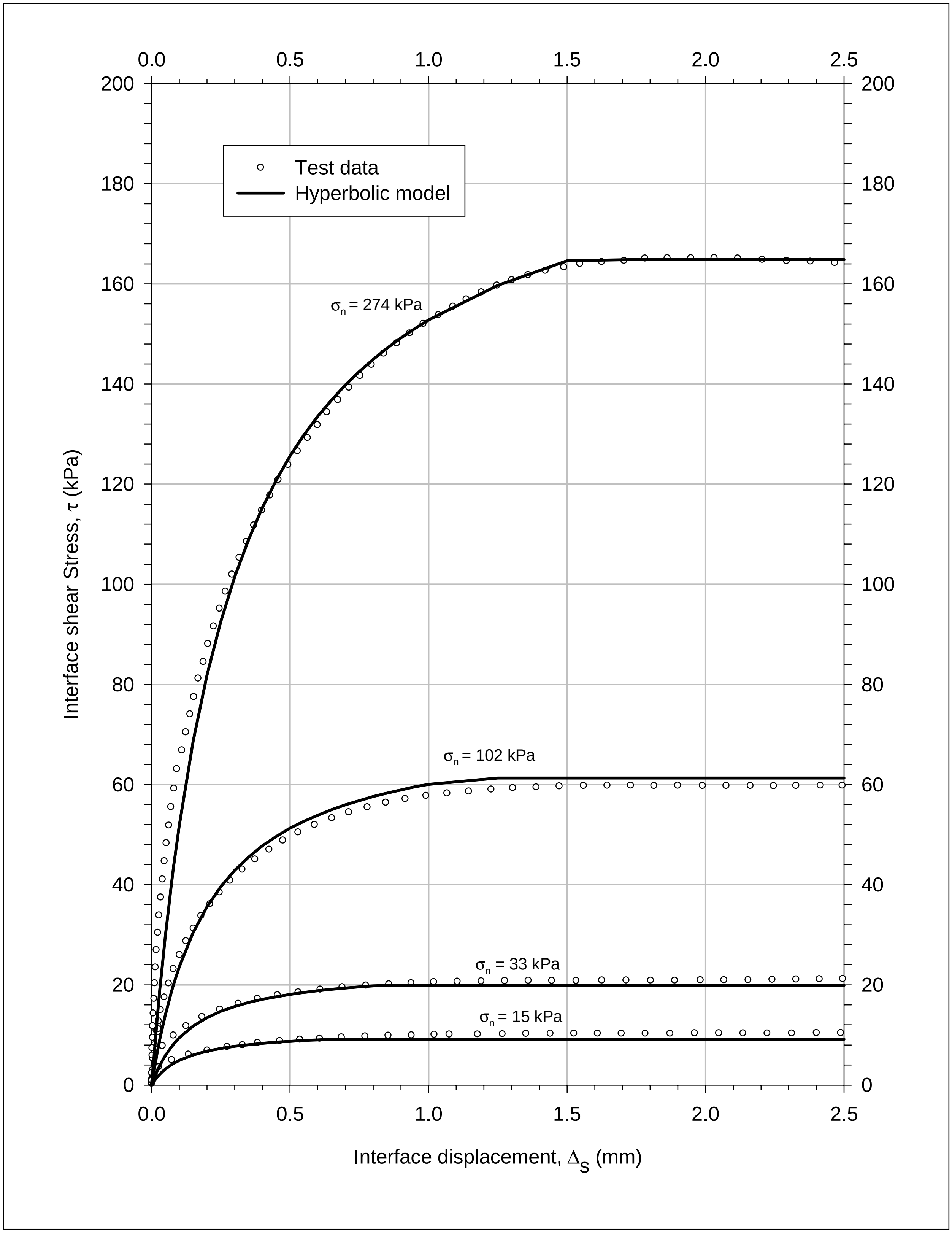
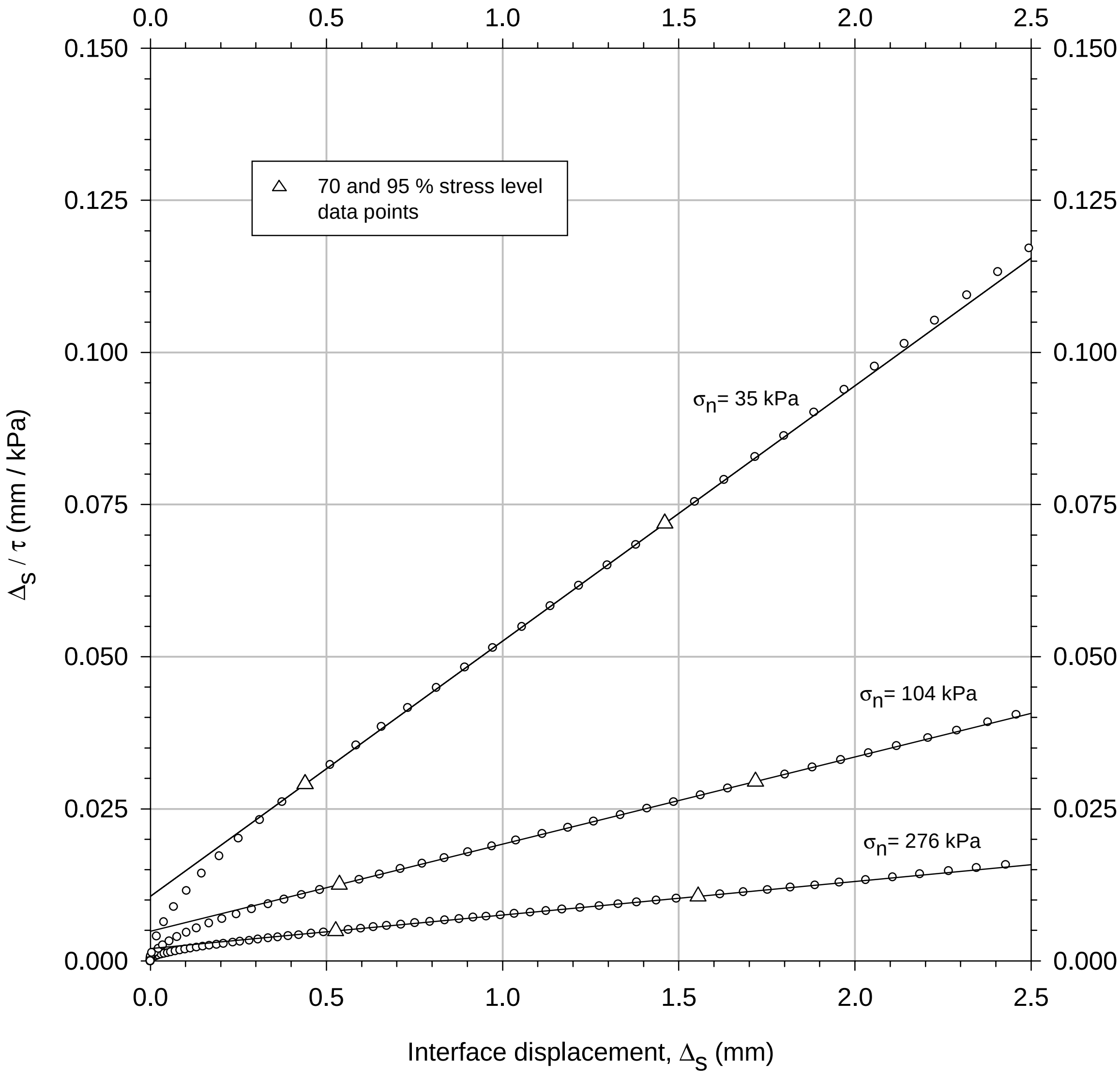


Figure D3. Comparison between the hyperbolic model and data from initial loading tests on dense Density Sand-to-concrete interface



σ_n (kPa)	a (mm/kPa)	K_{si} (kPa/mm)	b kPa^{-1}	τ_{ult} (kPa)	τ_f (kPa)	R_{fj}
35	0.01064	94	0.0419	23.8	21.4	0.898
104	0.00485	206	0.0143	69.7	61.4	0.88
276	0.00203	494	0.0055	181.1	153.8	0.849

Average R_{fj} = 0.876

Figure D4. Transformed plots for initial loading tests on medium dense Density Sand-to-concrete interface

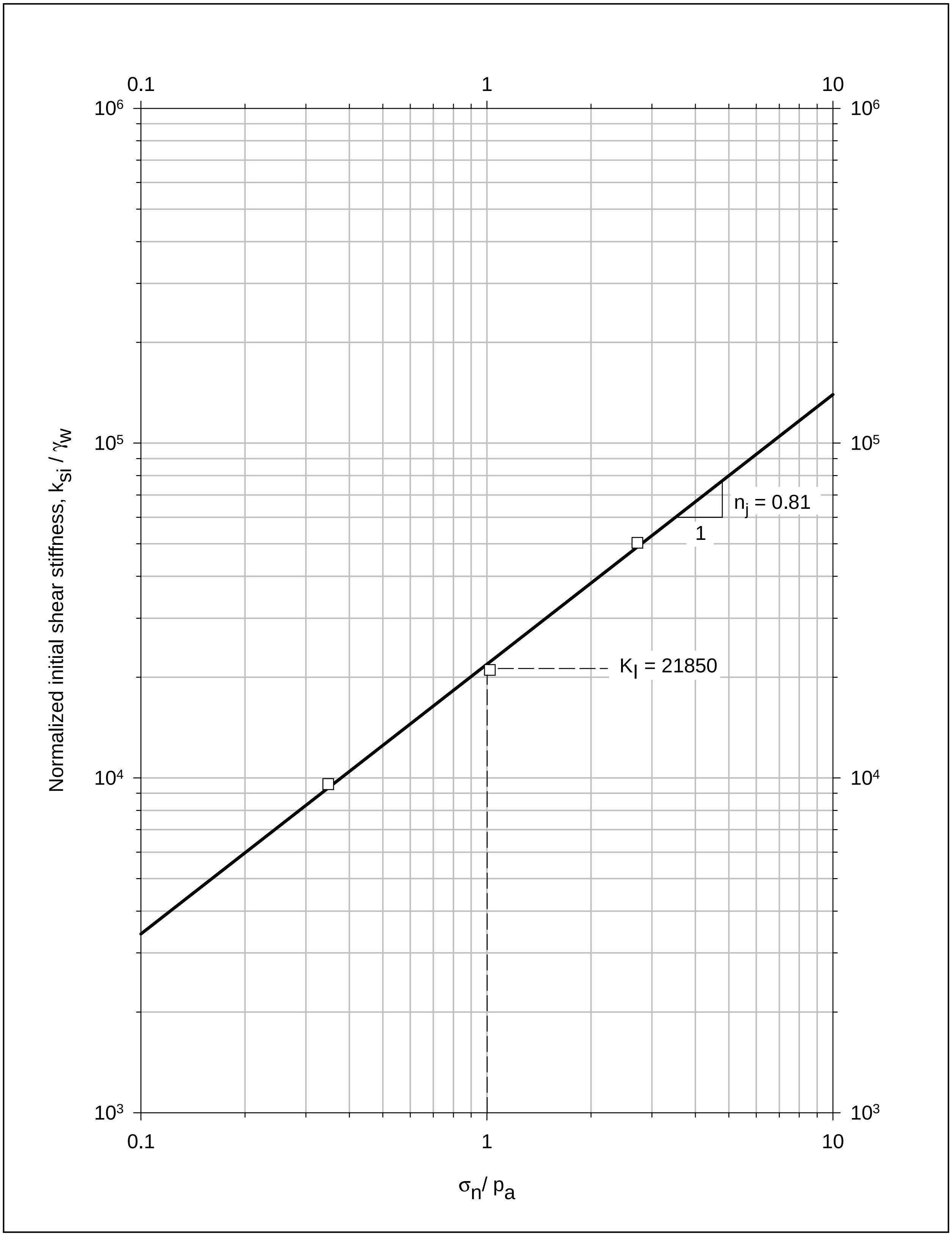


Figure D5. Determination of hyperbolic parameters K_I and n_j for medium dense Density Sand-to-concrete interface

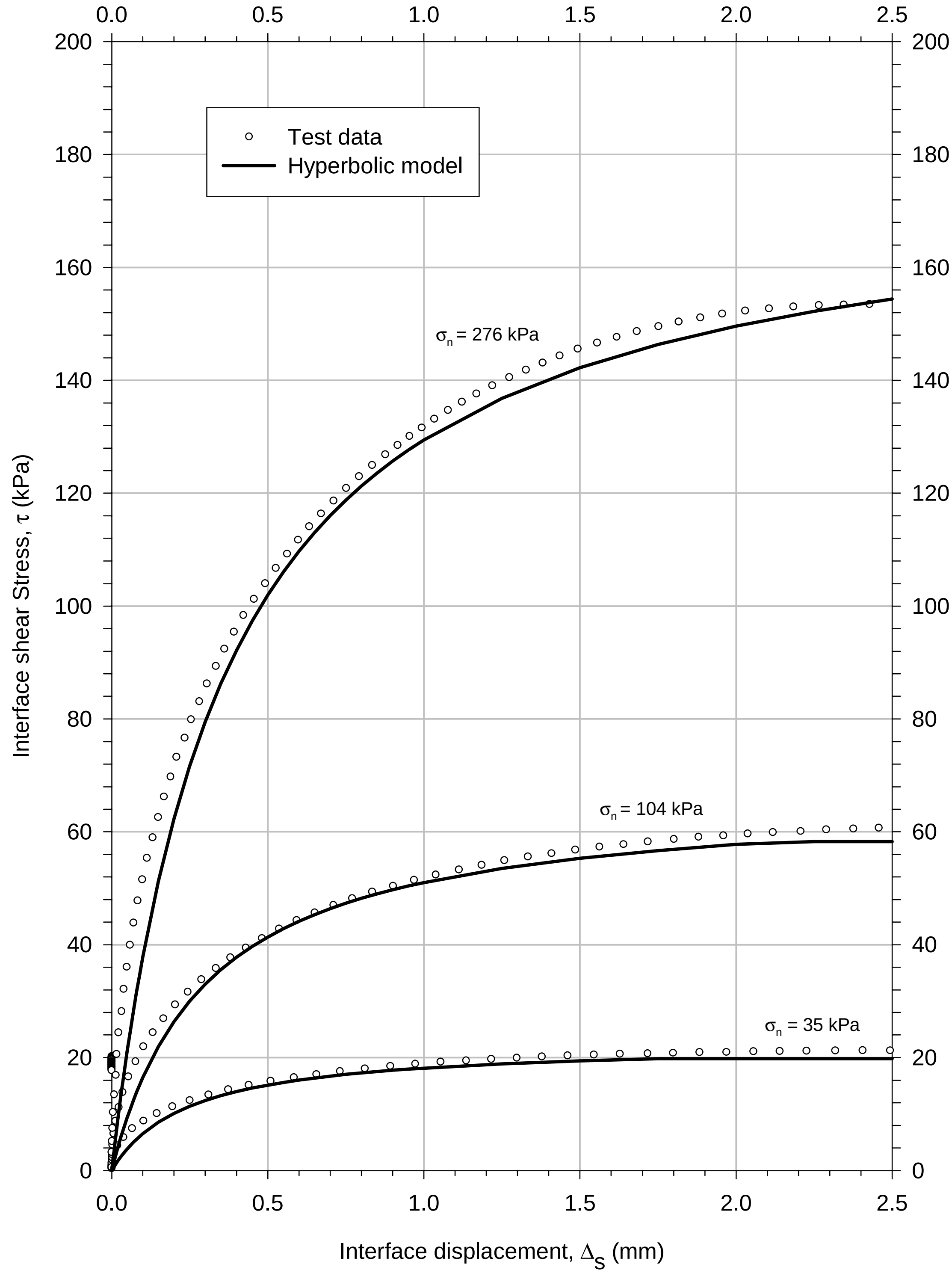
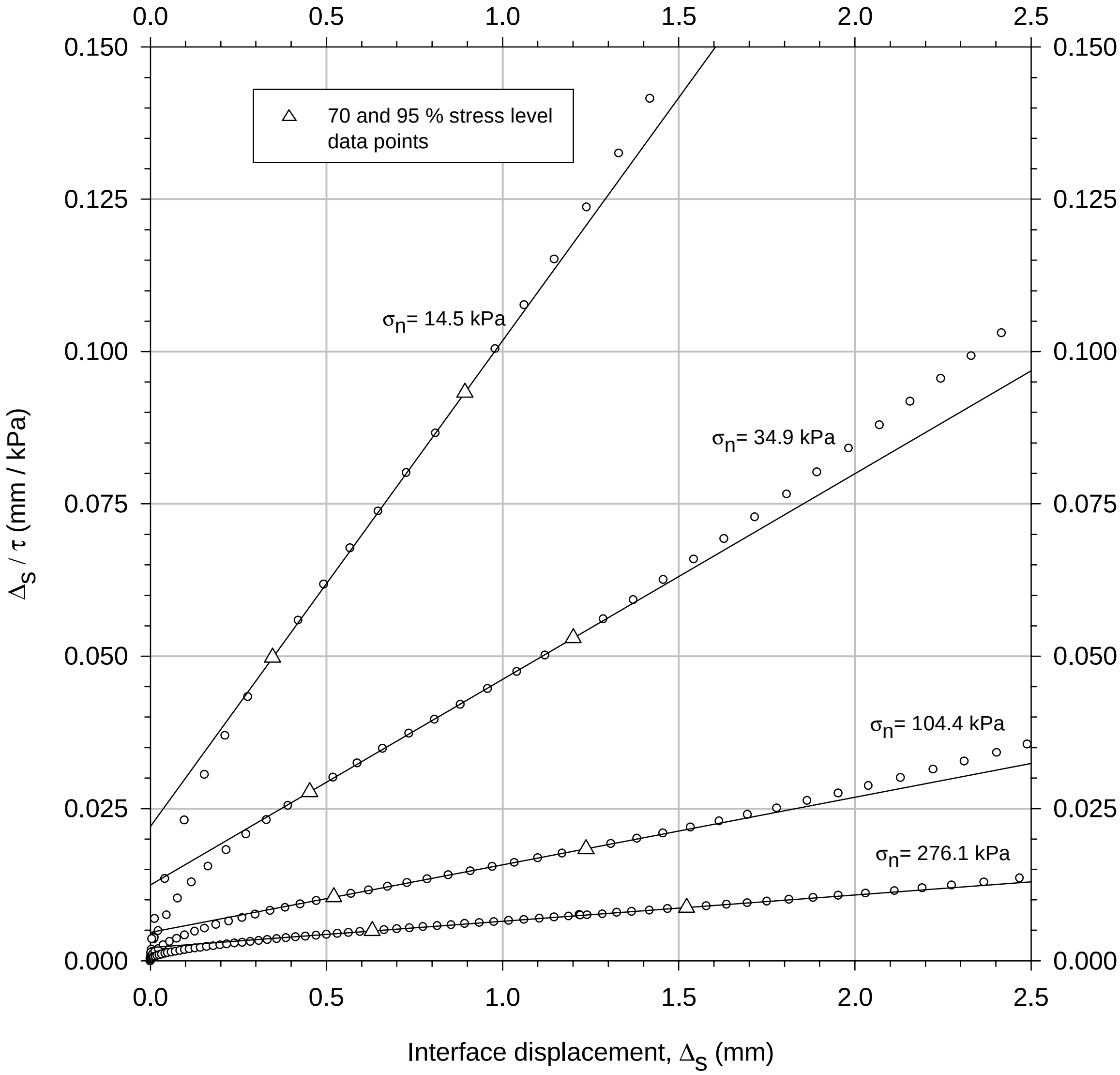


Figure D6. Comparison between the hyperbolic model and data from initial loading tests on medium dense Density Sand-to-concrete interface



σ_n (kPa)	a (mm/kPa)	K_{si} (kPa/mm)	b kPa^{-1}	τ_{ult} (kPa)	τ_f (kPa)	R_{fj}
14.5	0.0221	45	0.0797	12.5	10.0	0.800
34.9	0.01246	80	0.0338	29.6	23.6	0.796
104.4	0.00468	214	0.0111	90.2	71	0.787
276.1	0.00222	451	0.0043	233	183.7	0.789

Average R_{fj} = 0.793

Figure D7. Transformed plots for initial loading tests on dense Light Castle Sand-to-concrete interface

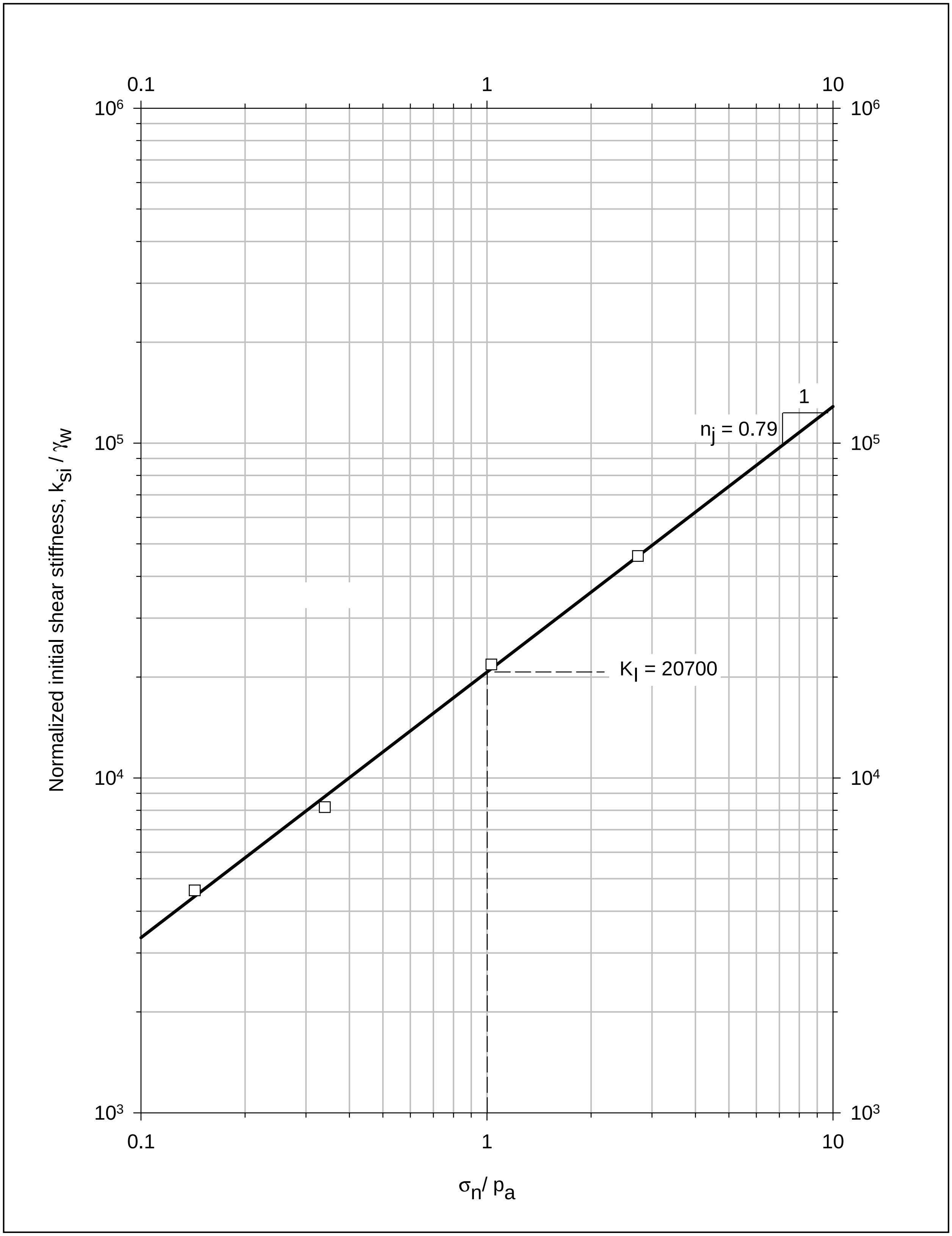


Figure D8. Determination of hyperbolic parameters K_I and n_j for dense Light Castle Sand-to-concrete interface

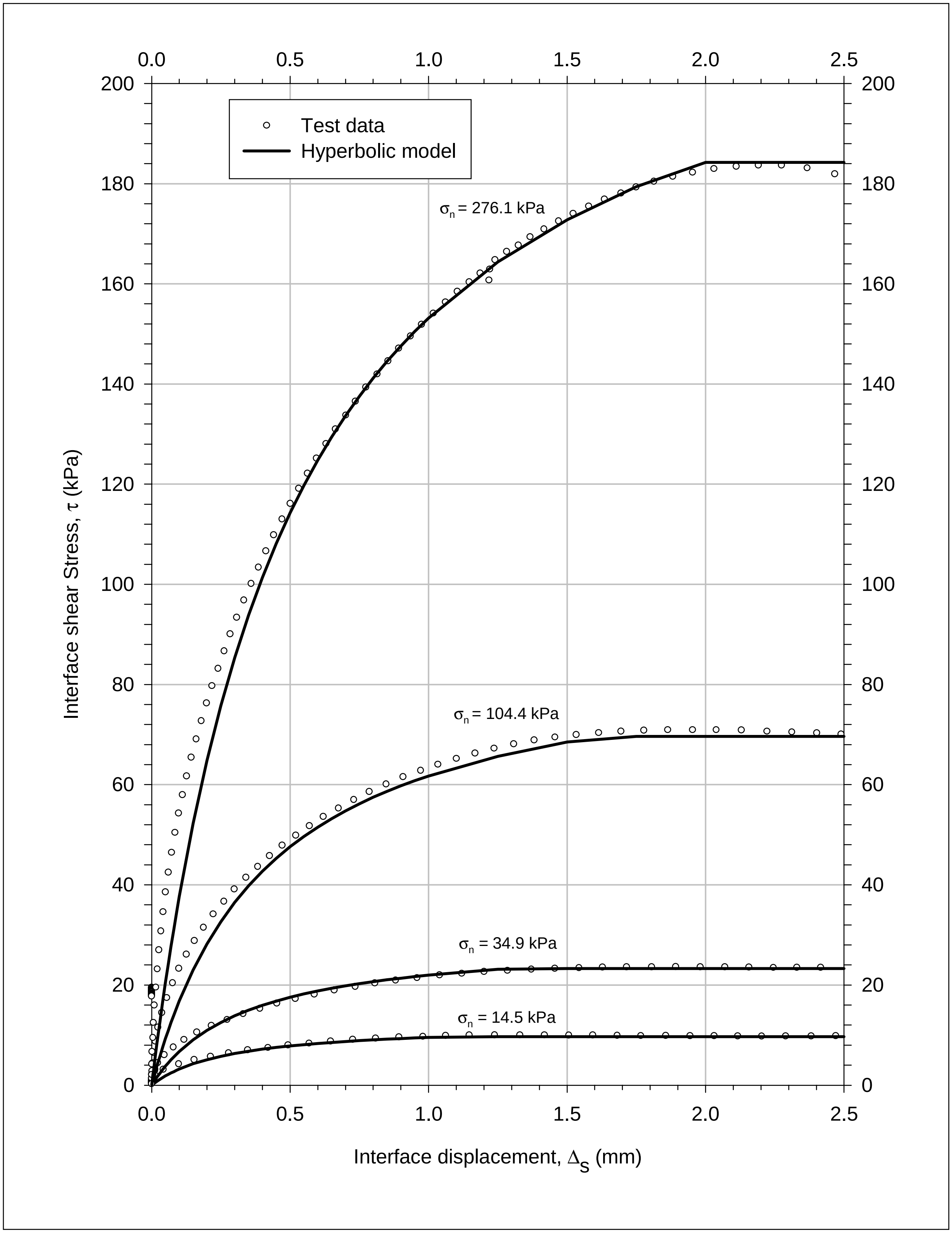


Figure D9. Comparison between the hyperbolic model and data from initial loading tests on dense Light Castle Sand-to-concrete interface

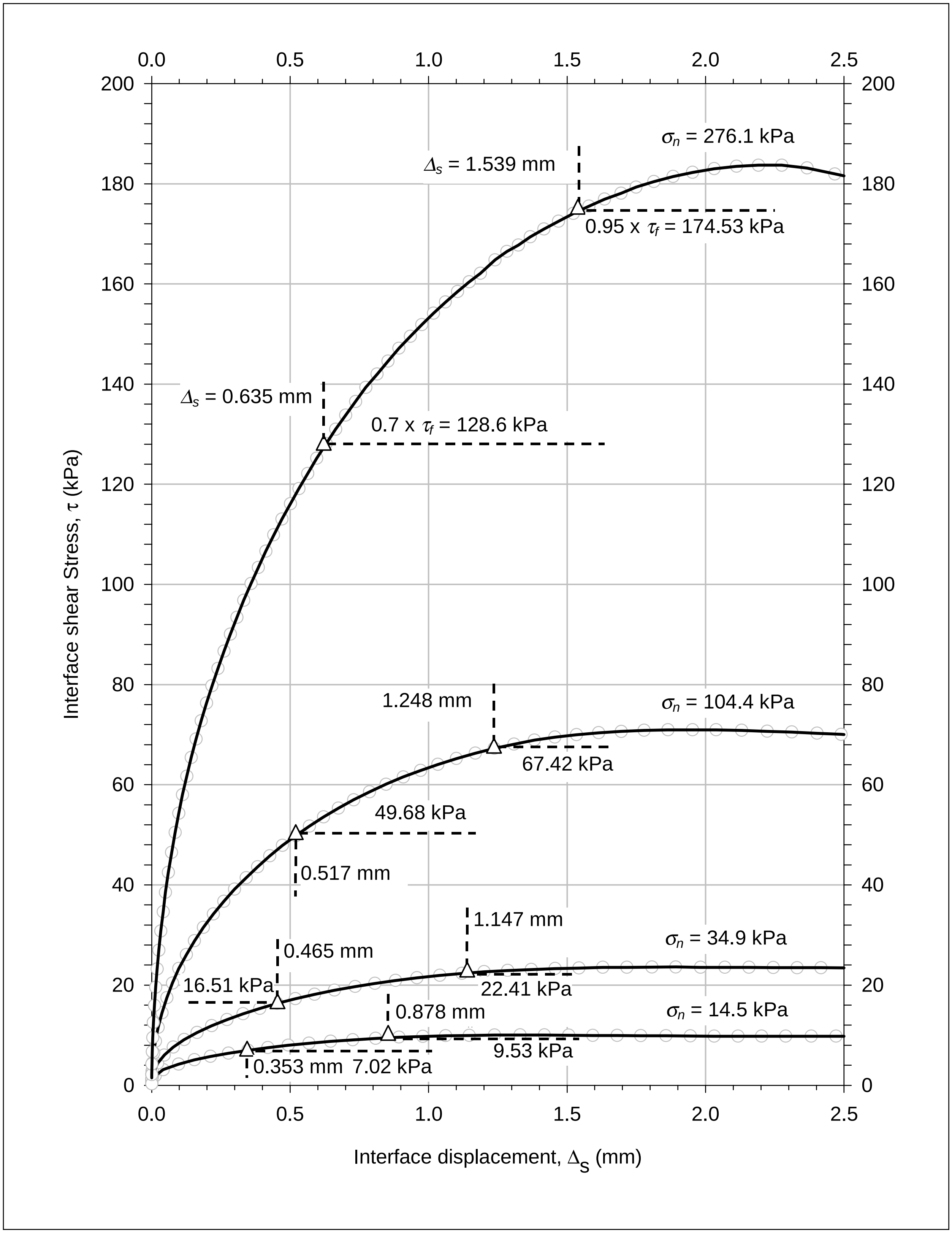


Figure D10. Example determination of interface displacements at 70 and 95 percent of strength. Data from interface tests on dense Light Castle Sand

σ_n (kPa) (1)	τ_f (kPa) (2)	Data for Determination of Hyperbolic Parameters K_I and n_j						Determination of Normalized Initial Stiffness			
		70% Stress Level			95% Stress Level						
		τ (kPa) (3)	Δ_s (mm) (4)	$\frac{\Delta_s}{\tau}$ (mm kPa^{-1}) (5)	τ (kPa) (6)	Δ_s (mm) (7)	$\frac{\Delta_s}{\tau}$ (mm kPa^{-1}) (8)	$\frac{\sigma_n}{p_a}$ (9)	$\frac{1}{\tau_{ult}}$ (kPa)$^{-1}$ (10)	R_{fj} (11)	$\frac{K_{si}}{\gamma_w}$ (12)
14.5	10.03	7.02	0.353	0.0503	9.53	0.878	0.0921	0.143	0.0796	0.798	4595
34.9	23.59	16.51	0.465	0.0282	22.41	1.147	0.0512	0.345	0.0337	0.795	8138
104.4	70.97	49.68	0.517	0.0104	67.42	1.248	0.0185	1.031	0.0111	0.787	21924
276.1	183.71	128.6	0.635	0.0049	174.53	1.539	0.0088	2.726	0.0043	0.789	46895

(1) Normal stress. Use effective normal stress, σ'_n, for effective stress analyses. Use total normal stress, σ_n, for total stress analyses
(2) Shear stress at failure. Determined from the shear stress-displacement plots of the data of each interface test.

(3) 70 percent of τ_f **(4)** Interface displacement corresponding to (3) determined as illustrated in Figure D10 **(5)** $\frac{\Delta_S}{\tau} = \frac{(4)}{(3)}$

(6) 95 percent of τ_f **(7)** Interface displacement corresponding to (6) determined as illustrated in Figure D10 **(8)** $\frac{\Delta_S}{\tau} = \frac{(7)}{(6)}$

(9) $\frac{\sigma_n}{p_a} = \frac{(1)}{101.3kPa}$ **(10)** $\frac{1}{\tau_{ult}} = \frac{(8)-(5)}{(7)-(4)}$ **(11)** $R_{fj} = (2)\cdot(10)$ **(12)** $\frac{K_{si}}{\gamma_w} = \frac{2\cdot 1000\frac{mm}{m}}{(5)+(8)-(10)\cdot[(4)+(7)]}\cdot\frac{1}{\gamma_w}$

Notes:

- p_a = 101.3 kPa
- γ_w = 9.8 kN/m^3
- Figure D10 illustrates the procedure for the determination of the values of shear stress and interface displacement used for this example
- See Chapter 2 and Appendix D for a complete explanation of the interface hyperbolic model and procedure for determination of hyperbolic parameter values
- The data from the initial loading tests performed on dense Light Castle Sand against concrete interface were used for this example

Figure D11. Determination of the normalized values of K_{si} for each of the initial loading tests performed on the dense Light Castle Sand-to-concrete interface

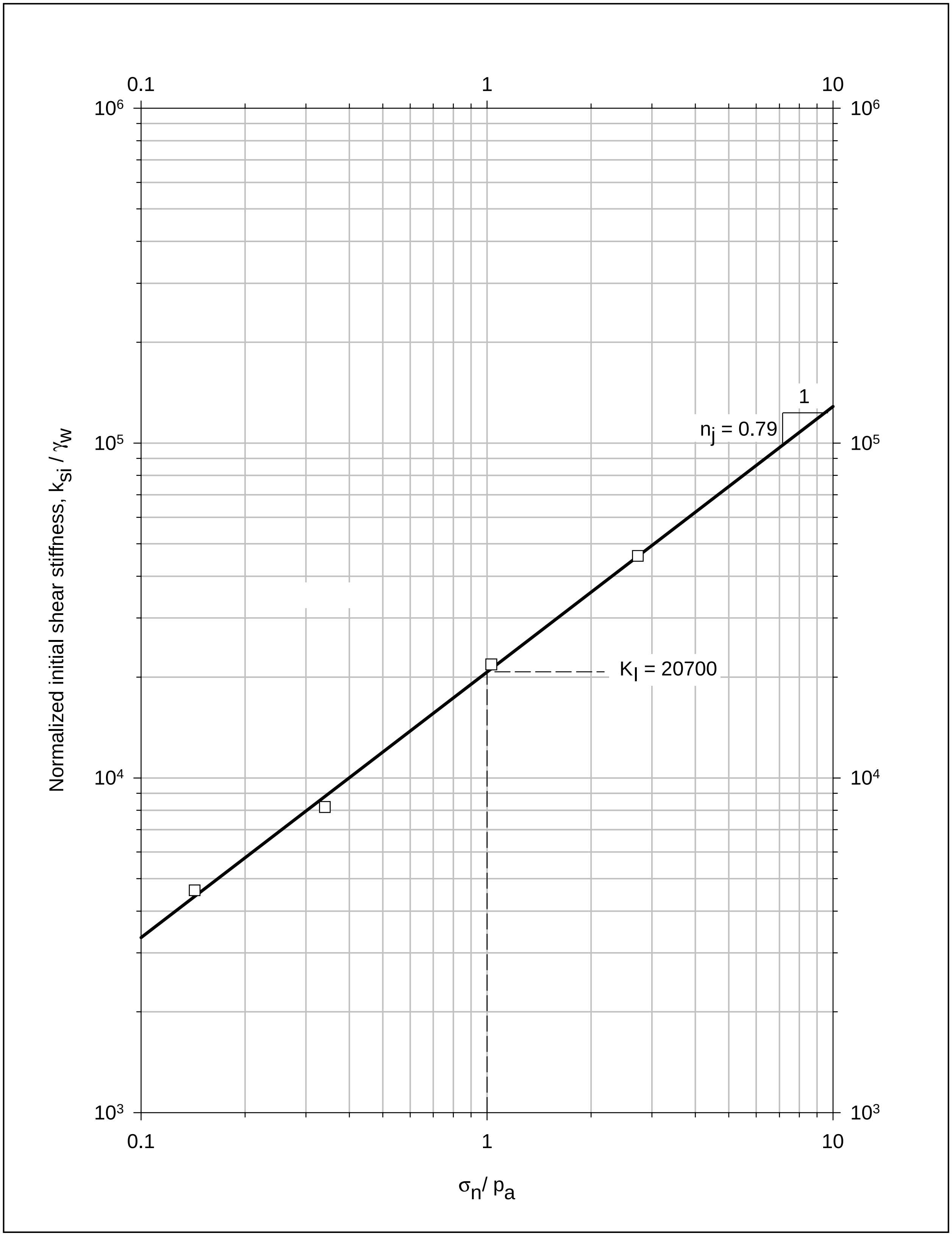


Figure D12. Determination of hyperbolic parameters K_I and n_j from the K_{si}/γ_w values determined in column (12) of Figure D11