Chapter 9: Summary and Future Work

9.1 Summary of Results and Important Contributions

The Air Force Research Laboratory Directed Energy Directorate (AFRL/DE) is currently striving to place ultra-large, membrane mirrors on orbit within the next 10 – 15 years. The key to seeing such technology come to fruition is a keen intuition of the physics governing the membrane aperture, a means of modeling those physics, and novel methods for controlling any unwanted dynamics. The present work has focused primarily on developing a distributed parameter systems approach for both modeling and controlling the dynamics of such a satellite. Such an approach is the core of SPIDER technology—developing an integrated system that incorporates a level of active control to enhance, overall, the satellite's dynamic properties. As will be discussed shortly, an extension of this work is using the derived physics models for developing static shape controllers of the integrated optic or radar aperture.

In the first part of this work, an in depth study was performed on modeling the interaction between a piezoelectric bimorph and a thin strip of membrane material. By starting with a thin membrane strip, the transverse dynamics of the system could be modeled as a 1-D beam under axial loading. Modeling the combined system as a 1-D membrane, or string, would be ineffective, as string theory cannot account for the local stiffness of the added piezoelectric bimorph. However, beam theory is able to capture both the additional mass and stiffness effects of the combined system.

A rigorous mathematical framework for defining the weak form of the system dynamics for a beam under axial loading was presented. The weak form of dynamic equations, if it can be found, is a powerful tool for applying approximation techniques when analytical solutions fail. Further, by placing system dynamics into the weak form, issues that arise in many structural dynamic problems, like unbounded operators representing discrete sensors and actuators, can be avoided as the solution to the weak form of the equation requires less continuity or smoothness than the original strong form of the equations. Once the weak form was properly defined, the dynamics of the integrated membrane and piezoelectric bimorph were approximated using finite elements. In particular, cubic B-splines were chosen as the finite elements as they have been demonstrated in the literature to converge quickly due to their high level of connectivity from one element to the next.

Having established the distributed system theory behind the augmented system, an experimental validation procedure was performed. A 21.8 cm long, 1.8 cm wide Kapton membrane was augmented with a piezoelectric bimorph. The integrated system was dynamically tested within a Tenney environmental chamber. The chamber could be de-pressurized to near-vacuum, allowing for testing at both ambient and vacuous conditions. In the experiment, the piezoelectric bimorph was used as the excitation source, and the velocity response of the structure was measured directly with a Polytec laser vibrometer up to 500 Hz. A tensile load was applied to the membrane – PZT sample using a mounted lead screw, and the tensile load was measured directly with a Transducer Techniques load cell. Within the bandwidth of interest, the integrated structure demonstrated both transverse and torsional dynamics. As Euler-Bernoulli beam theory was used to describe the transverse dynamics of the augmented membrane, the torsional modes of vibration were not modeled. The transverse dynamics, on the other hand, were captured by the developed finite element model and agreement was found to within 6% error. Next, a series of experimental dynamic tests were performed over a wide range of applied axial loads, from 0.6 to 19 N. The response of the membrane – PZT structure was then compared to the simulated response of the finite element model, and less than 6% error was achieved for tensile loads between 4 and 19 N. At tensile loads less than 4 N, the “membrane” or wave term within governing dynamical equation takes over, and the finite element model has a difficult time dealing with the system’s flexible dynamics. At loads greater than 19 N, the membrane strip began to undergo noticeable deformation, so 19 N was set as the upper limit within the analysis. One of the tradeoffs that face control design is that the addition of PZT material to the membrane significantly increases the total system mass. Consequently, the active system may require high power electronics to sustain acceptable levels of vibration attenuation. Unfortunately, additional electronics also means additional overall system mass. The control system, including the dimensions of the active region within the membrane optic or aperture should be designed to maximize the effective control effort and minimize the additional system mass. Presently, a bimorph representing 36% of the total system mass was able to eliminate an initial disturbance using a maximum of 9 V in less than 0.15 s.

The focus of the present work was on designing a linear quadratic regulator (LQR) controller, as formulation of the approximate solution to the infinite dimensional control problem is a natural extension of the distributed parameter system. The state weighting matrix, Q, was formulated based on the strain (potential) and velocity (kinetic) energies of the approximated system. As a consequence of choosing the Q matrix in this manner, the functional gains of the combined membrane – PZT system could be calculated. The functional gains for a distributed parameter system are similar to the gain matrices calculated in state-space control. However, because they are based on the potential and kinetic energies of the distributed parameter system, the functional gains also contain spatial information. Therefore, throughout the domain of the structure, the computed functional gains will take on maximum, absolute values at particular spatial locations. These spatial locations correspond to ideal sensor locations, and accordingly, the functional gains provide important insight based on the governing dynamics of the system as to where to place particular sensors (i.e. strain gages or velocity measurements). In this way, the functional gains can be used as a guide to the control engineer for intelligent placement of sensors prior to designing an observer-based controller. Conclusions can be drawn from the functional gain analyses performed in the present work on the importance and location of strain and velocity measurement for membrane optic systems.

For the 1-D membrane – PZT problem, the strain functional gain took on maximum value at the edges of the attached PZT. The velocity functional gain took on maximum value on either side of the PZT but also extending into the interior domain of the structure. Where both the strain and velocity gains have values on the same order of magnitude, both states are important for measurement (according to the weighting matrices Q and R chosen for the LQR controller). Following up with work performed by Miller (1998), who suggested that the functional gains could be used in a Gaussian quadrature manner for optimally placing sensors, the optimal control signal, $u_{opt}(t)$, was reconstructed based on a discretization of the strain and velocity functional gains. Using three simulated strain gages and seven simulated velocity measurements, the optimal control signal was reconstructed using the discretized approximations with sufficient accuracy. A significant contribution from this chapter is the distributed parameter systems approach for both modeling and control of a thin strip of PZT augmented membrane material. The distributed parameter systems approach maintains the integrity of the physics of the system while providing insight into the type of sensors needed for effective control of the structure. Some researchers in the past have used the formula of beam under axial loading for capturing the effects of thin membrane strips, but the current effort is the first to address from a distributed parameter system's standpoint why such a theory is applicable, and is also the first work to incorporate PZT with the membrane and both model and experimentally validate the use of beam under axial loading theory for such a system.

The successful demonstration of using beam under axial loading theory to capture the added mass and stiffness effects of the 1-D membrane and PZT sample led to the use of thin plate theory to model a 2-D membrane with added PZT material. To demonstrate the effectiveness of thin plate theory, a series of experiments was designed. First, a Kapton membrane was pre-stressed and glued to a piece of acrylic with a rectangular cutout. The cutout section served as the taut region of the membrane. The membrane was dynamically excited using the magnetic field interaction between a small piece of permanent magnet attached to the surface of the membrane and an electromagnet. The dynamic response of the membrane was characterized using pure membrane theory. In doing so, the tension applied to the membrane in both the *x* and *y-directions* could be determined. The theoretical model matched within 6% of the measured resonant frequencies of the membrane.

Next, a piezoelectric bimorph was glued to the surface of the membrane. The dynamic response of the membrane was again measured, although in this set of tests, the PZT bimorph was used as the excitation actuator. The measured dynamic response was then compared to a developed finite element model based on thin plate theory with applied axial load. As with using beam under axial loading theory, the weak form of the thin plate dynamics was rigorously derived for approximation using cubic B-splines. Both the experimentally determined operating mode shapes and the measured resonant response of the active membrane were in good agreement with the finite element model. The simulated resonant frequencies from the finite element model were within 16% error compared to the experimental results. To further support the assertion that thin plate theory was the modeling medium of choice for the active membrane, a third finite element model was derived based on membrane theory but allowing density changes within the membrane material. The predicted resonant frequencies from this analysis were in excess of 21% in error compared to the experimental results. Conclusively, thin plate theory under applied axial loading is the modeling medium of choice for augmenting active material onto the surface of a thin film membrane. Thin plate theory is able to account for both the added mass and added stiffness of the active material. The combination of both the analytical and experimental validation of using thin plate theory to model the active membrane system is a significant contribution to the literature. Although alluded to in some previous works, the current effort is the first to demonstrate conclusively the validity of using thin plate theory for modeling the active membrane.

As was done previously in this work for the 1-D case, the LQR problem was formulated for the active 2-D membrane system. The state weighting matrix, Q, was defined based on the strain and kinetic energy of the active membrane. Choosing appropriate weightings, the response of the membrane was effectively attenuated to within a micron of displacement within 0.05 s using less than 11 V applied to the PZT bimorph. Further, as a benefit of rigorously defining the optimal control problem, the strain and velocity functional gains for the active membrane were derived and plotted. The strain velocity gain took on maximum value around the edges of the attached PZT bimorph. On the other hand, the velocity functional gain took on maximum value within the domain of the PZT bimorph. In comparing the magnitudes of the strain and velocity functional gains to each other, the strain functional gain was an order of magnitude greater than the velocity functional gain. A conclusion that can be drawn from this comparison is that strain sensory information is of greater importance in designing an observer-based controller than velocity feedback. As may be expected, the functional gain analysis from both the 1-D case and the 2-D case demonstrate a trend in collocated feedback control with piezoelectric sensors and actuators.

One of the main drawbacks of augmenting active material, like PZT, to the skin of a membrane mirror or aperture is the added mass. Not only is the effective bandwidth adversely affected by the introduction of active material, but the power electronics required in conjunction with the active materials significantly increases the total system mass. As additional mass translates immediately into additional mission costs, the idea of augmenting piezoelectric materials with membrane mirrors or apertures may not be the best answer. An experimental investigation was performed to understand the response of a membrane augmented with a thin, air-filled cavity behind it. The idea behind using a thin, air-filled cavity on the backside of a membrane was based on the design of a condenser microphone, which features a thin membrane stretched taut over a volume. The membrane is parallel to a thin plate, and a potential is placed across the two. As pressure waves are incident on the membrane, the gap between the membrane and thin plate is varied, and an amplified signal is generated based on the average displacement of the membrane. However, where condenser microphone designs attempt to create uniform response over a large bandwidth with maximum sensitivity to incident pressure waves, the opposite is desired for a membrane optic or aperture. The goal is to design the membrane optic like a condenser microphone that is quite insensitive to incident pressure waves over a large bandwidth. Such a design would be ideal.

In lines with this analogy, a series of dynamic tests were performed on a Mylar membrane mirror backed by a shallow cavity. The back plate of the cavity was manually adjusted from 5.9 mm to 1.0 mm. As the back plate neared the membrane, the dynamics of the membrane became highly damped and demonstrated an overall upward shift in the resonant frequencies of the system. The series of experiments, performed at a chamber pressure of 10 Torr, illustrate that the air trapped within the cavity acts as both a distributed spring and distributed damper. Accordingly, a simple model was developed demonstrating this phenomenon by assuming the membrane was backed by a series of springs and dampers. Although simple in nature, the simulated response was in agreement with demonstrated experimental results. Most importantly, the distributed damping effect (caused by the local viscosity of air resisting the motion of the membrane) not only attenuates the response of the membrane significantly (vibration attenuation on the order of 30 and 50 dB was demonstrated experimentally), but the localized damping also fills in regions within the frequency bandwidth of anti-resonances, thus creating a uniform, attenuated response over a large bandwidth. The ability of the trapped air to act as both a spring and damper is a significant development—it means that a purely passive means for gaining a desirable response from a membrane optic can be achieved by designing a thin, air-filled cavity of the backside of the membrane mirror. Although previous research has been presented in the literature in using a damping annulus in conjunction with a membrane optic, the present work is the first to investigate experimentally the behavior of air trapped within a small enclosed back cavity as a means for attenuating the response of a membrane optic. Further, a distributed spring and damper model has also been provided to further explain from a phenomenological perspective how the air is influencing the membrane's dynamic response.

An anticipated drawback of using an air-backed cavity with a space optic is the adverse effect on-orbit temperature fluxes would play with the internal pressure of the cavity. A back-of-the-envelope thermal analysis was carried out to show that a pressure swing of 4 Torr was possible for a Mylar mirror. The analysis assumed that the Mylar membrane behaved like a thin plate adiabatically insulated on the backside. Such an assumption provides a means of analyzing the hottest and coldest temperatures the Mylar would experience while on-orbit, from being in direct sunlight to falling behind the Earth's shadow. Despite the 4 Torr pressure swing, a properly designed thermal management system in conjunction with an external supply tank to the cavity could easily alleviate any thermal issues associated with the on-orbit temperature dynamics of the satellite.

The effective bandwidth of the membrane could also be extended by increasing the level of pre-stress within the membrane. However, doing so would have two undesirable outcomes. Firstly, the membrane would be more susceptible to fatigue and rupture at high levels of pre-stress throughout the life of the mission. Secondly, the membrane would become quite inflexible—an undesirable consequence if the intention is to use the membrane as an adaptive optic. The ability to use the membrane mirror in an adaptive optic scheme was the last subject investigated in the current work.

The inherent flexibility of a membrane optic can be used advantageously in an adaptive optics scheme. In adaptive optics, an image is focused onto a deformable mirror to remove residual image aberrations and thereby make a blurry image clear. A membrane is a desirable media for deformation due to its flexibility. Further, recent research efforts by the Air Force Research Laboratory Directed Energy Directorate and SRS Technologies (Huntsville, AL) have produced a membrane material (CP-1-DE) that has optical level surface quality. The material science is almost to the point where optical membrane mirrors could easily be created (currently, SRS Technologies has demonstrated a 1 m diameter sample of optical-quality CP-1-DE that has a 30 nm peak-to-peak undulation).

In the present work, it has been demonstrated (in collaboration with AFRL/DE) that a membrane mirror could be manipulated via a uniform back pressure and via boundary control to eliminate most incoming image aberration. However, the residual aberration is incompatible with the mechanics of the membrane. Image aberrations are usually described by Zernike polynomials, a complete set of orthogonal functions that are specifically defined over a unit circle. The boundary conditions of the Zernike polynomials do not have any restrictions, and therefore are incompatible with the clamped boundary conditions of the membrane mirror. To try and alleviate this issue, a novel basis was presented in this work known as the clamped Zernike radial polynomials.

The clamped Zernike polynomials are proportional to the dynamical mode shapes of a circular membrane. In essence, they bridge the gap between the optical and mechanical worlds. The clamped Zernike radial polynomials enforce the zero boundary condition around the edge of the membrane optic. Consequently, the novel basis provides a means of describing three methods of control for achieving a nearly 100% effective adaptive optic: uniform pressure, boundary manipulation, and a mapping for distributed actuation within the domain of the membrane to eliminate the residual error. Further, the residual aberrations within the novel clamped Zernike radial polynomial basis were then expressed as a Fourier series using the mode shapes of a circular membrane. The resulting Fourier series expansion could be used in a modal space-based control algorithm for distributed actuation or shape control of the membrane mirror.

The work presented has helped lay the groundwork for a more intensive investigation into the actual design and construction of a prototype membrane adaptive optic. The following section addresses this concept.

9.2 Future Work in Membrane Adaptive Optics

The collaborative efforts of the Air Force Research Laboratory Directed Energy Directorate and Virginia Tech have led to the proposal of a novel membrane adaptive mirror design. The concept is referred to as a Pressure-augmented, Boundary-controlled, Spatially-actuated (PBS) adaptive mirror (Marker et al., 2006). The PBS adaptive mirror will feature four main control categories. Figure 9.1 provides a conceptual drawing of the PBS design, and highlights the four control categories.

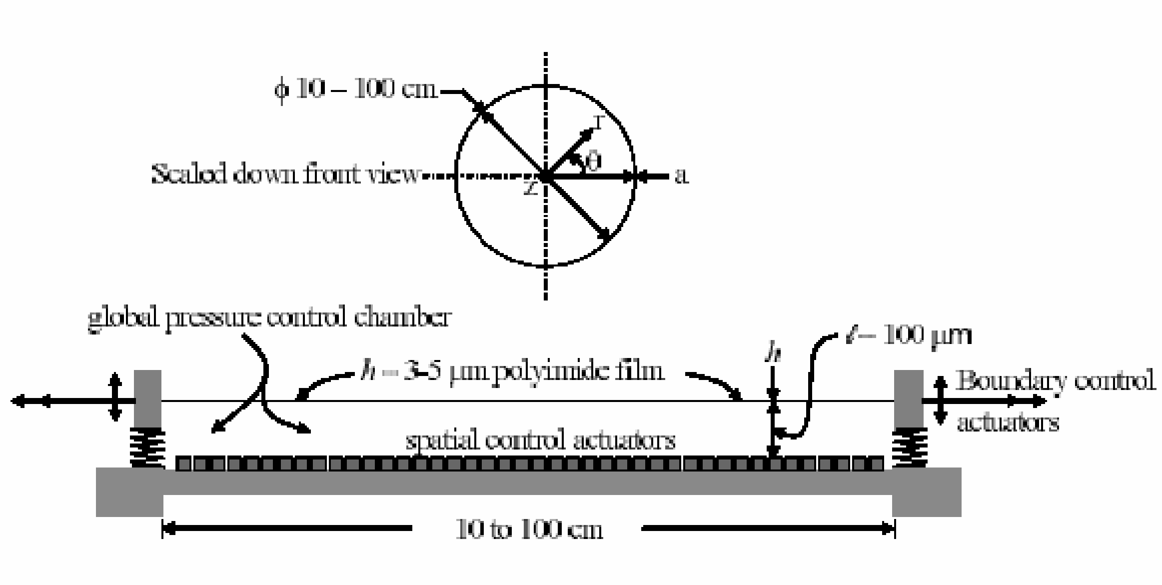

Figure 9.1. Conceptual drawing of the PBS.

The four main control categories are:

1) Piston, Tip / Tilt control (a well-established control in the adaptive optics community),
2) Hydrostatic pressure effects to identically correct for defocus (as demonstrated in Chapter 8), without any actuator influence function,
3) Boundary undulation, and
4) Spatial control actuators, like electrostatic actuators or MEMs magnetic actuators.

The main goal of the PBS design is to construct a large area, long throw adaptive optic. However, there are numerous research areas that must still be addressed before such a device can see fruition. First of all, the coupled dynamics of the fluid-filled back cavity and the membrane mirror must be well-modeled. The spring and damping effects introduced by the back cavity must be modeled so that the geometry of the cavity can be designed in such a way as to tailor the dynamic response of the membrane mirror. Another area of research is in the types of spatial actuator technologies available, or to be developed, that could take the unique mapping offered by the clamped Zernike polynomials and eliminate the residual aberration within the membrane lens. Electrostatic actuators have been used in the past, but it is envisioned that a distributed set of MEMs electromagnets could be used on the backside of a magnetically impregnated thin film layer. Another possibility would be to design the backside of the membrane mirror like a honeycomb, and use a small piston-like actuator to excite columns of air on the backside of the membrane (although such concepts may have significant actuator influence functions). The boundary control presented in Figure 9.1 is also an open area of research. From the present work, the use of piezoelectric materials may prove suitable in providing distributed control around the rim of the membrane optic.

The PBS concept is a prime example of SPIDER technology—a blend of active and passive technologies and the merging of multiple disciplines in an effort to create a new adaptive optic. Continued perseverance into proper modeling of the interaction between the membrane mirror and air-filled cavity will enable system-level design that tailors the dynamic response of the mirror. The groundwork has been laid for research into the design of the next generation of adaptive optic mirrors and radar apertures. The Air Force's goal of developing a 1 m class membrane mirror adaptive optic is on the threshold of becoming a reality.

9.3 Summary of Contributions at a Glance

In conclusion, the following contributions have been made based on the work presented in this volume:

- A distributed parameter systems approach to modeling a 1-D membrane strip augmented with a piezoelectric bimorph has been defined using beam under axial load theory and validated experimentally.
- The useful tensile loading range from the developed 1-D model was demonstrated to be effective at predicting the response of the system within 6% between 4 and 19 N.
- The LQR control problem has been formulated and demonstrated numerically to eliminate detrimental vibration to a 1-D membrane strip.

- As a consequence of formulating the LQR control problem, the functional gains of the 1-D system were defined and plotted. Strain and velocity measurements near the active material region were demonstrated to be most important when choosing sensor type and location.
- The 1-D results were then extended to 2-D via thin plate theory. In a systematic fashion, it was demonstrated experimentally that the addition of a PZT bimorph to a membrane significantly alters the system dynamics, and that the altered system can be described using thin plate theory under axial loading. The maximum error between the predicted response of the system and the experimentally measured response was less than 16%.
- Membrane theory with variable density within the structure proved inadequate for describing the dynamics of the augmented system (errors of the order of 30% were demonstrated).
- The LQR control problem and functional gains were derived for the 2-D system as an extension to the previously defined 1-D results. The functional gains demonstrated that strain sensors were more important than velocity measurements for feedback control within the region of the active material. Such a result is in agreement with the use of collocated control in structural systems.
- Experimental work on the dynamics of membranes backed by shallow cavities was performed. The response of the membrane with a 1.0 mm cavity at 10 Torr demonstrated superior damped dynamics over the bandwidth of 1 – 500 Hz. Such a system is analogous to a poorly designed microphone, as it is insensitive to incident pressure waves over a large bandwidth.
- A simplified distributed spring and damper model was used to numerically simulate the interaction between a membrane and shallow cavity, where the trapped air acted as both a spring and damper. Simulated results demonstrated a similar phenomenological response within the bandwidth of 500 Hz.
- A novel optical-mechanical basis has been proposed for use in designing a membrane deformable mirror. The clamped Zernike radial polynomials have zero displacement along the rim of a circular optic, and consequently are amiable to the structural response of a membrane lens. Such a mapping could be used for

distributed shape control of a membrane optic, and consequently create a nearly 100% useful deformable membrane mirror for an adaptive optic scheme.