Chapter 2
Public Firm Vs. Private Firm: An Agency Approach

I. Introduction

What is the difference between private and public firms? How to decide to privatize or nationalize a firm? These two questions have initiated plenty of research for decades, not only because of their intellectual challenge, but also their empirical relevance. In addition, the surge of privatization in Eastern Europe and other countries spawned an even more exigent need to clarify the welfare gains and losses from privatization or nationalization.

The legend about privatization begins with the belief that an alternation of ownership structure can resolve problems of inefficiency intrinsic in public firms. Low work motivation in the public firm in association with allocative inefficiency - resources cannot be allocated efficiently inside the firm - waste an enormous amount of resources. Yet, as the residual claimant of the firm, the owner of a privatized firm has stronger incentive to encourage effort. But such presumption about the inferiority of public firms has been shaken by reality. In main-land China, the growth rate of total factor productivity of state-owned enterprises (SOEs) between 1980 and 1992 is 2.5% which is only 1% lower than that of collective enterprises, the major non-state-owned component of China's economy. In western countries, studies with inconclusive comparisons between private and public enterprises also abound. For example, Atkinson and Halvorsen (1986) discuss some empirical studies which embrace the result that the private and public utilities are equally cost inefficient when they are subject to profit constraints imposed by the government.

So, which one is superior, the public firm or the private firm? Is privatization a panacea to cure problems of inefficiency? This paper can be considered to be an endeavor to shed light on these questions.

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2 The ownership of collective enterprises is not always well defined. For instance, people in a community can be the owner. See Jefferson, Rawski and Zheng (1996) and Li (1996).
My analysis is based on asymmetric information, assuming that the owner of a firm has cost information, but also bears the cost of production. I find that the question which type of ownership, private or public, is superior does not have a clear cut answer. Under my assumptions, private ownership may induce higher work effort but suffers from a discrepancy of private and social goals. Whereas the production distortion is less serious, an obvious disincentive to work exists in the public firm case. This conclusion is consistent with the empirical observations. Thus the agency approach proves once more a fruitful way to study issues of social institutions.

Three particular models will be developed and the respective results will be compared in the sequel. The first one is the first best (“FB” hereafter) case, the second one is a private firm (“PRF” hereafter) model and the third one is a public firm (“PUF” hereafter) model. The major assumption that distinguishes the PRF and the PUF centers on how information is allocated among parties. In the PUF case, the government is able to collect some cost information about the firm but it cannot do so in the PRF case. This vertical flow of information between the government and the firms is not only a characteristic of centrally planned economies but can also be observed in economies without central planning. For example, the government can delegate representatives in companies in which it owns shares. By doing that, the government collects, processes and distributes information to firms which can utilize it to produce. This phenomenon is indeed similar to internal hierarchical control of a modern company. Top management always reveals limited information to subordinates because they believe that the more information they collect, the better they can control the subordinates. This observation validates my above assumption regarding the more informed position of the central government in the PUF case. However, as long as the government can obtain the cost information, it is committed to be responsible for the cost of production. As is well known, it is a typical practice in socialist countries that the government will reimburse the production cost of a firm. Since auditing or monitoring is not strictly implemented, shirking activity becomes

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1 My model does not exclude the possibility that the subordinates can observe some private information. This will become clear below.

2 That is the price for being the owner of information. The commitment problem is not considered in this paper.
rather common: the public firm cheats by exaggerating the production cost. In the PRF case, the government cannot observe any cost information and has no obligation to any cost reimbursement.

The way of information being allocated is my yardstick to differentiate the PRF and PUF models. In addition, using the right to access “essential cost data” as a proxy of ownership of a firm distinguishes my study from the recent literature. In the PUF case, the government owns the data so it is responsible to subsidize the firm. In contrast, a PRF is not entitled to receive any cost reimbursement since its ownership of data implies its responsibility for profit and loss.

Tirole and Laffont (1986) come closest in spirit to my paper. In their articles, the government is assumed to be able to use cost information to regulate a firm. But the main concern of that article, the detailed form of the optimal incentive contract, is not the focus of my investigation. Instead, we will develop our analysis by contrasting the differences of the distortionary effects embodied in the PRF and PUF models. There is some other related literature which applied agency theory to analyze privatization problem. Sappington and Stiglitz (1987) is a seminal contribution. Shapiro and Willig (1990) also develop agency models for private and public firms but the number of layers of hierarchy is different in these two kinds of firms. Laffont and Tirole (1991) examine the privatization and incentive problems in a common agency framework. Schmidt (1996a, 1996b) addresses the privatization problem in a way similar to mine but with a different model set-up. Our papers are different in two major ways. First, in Schmidt's papers, the public firm manager's wage cannot be made contingent on the state of nature. Though this assumption captures one salient feature of most public firms, it makes the comparison of effort level between public firm and private firm managers less interesting. Given a fixed wage and the government not implementing penalties to punish low effort level, it is not surprising to find that the effort level exerted by the public firm manager is minimal. In my model, such wage level is contingent on the output produced by the public firm. Second, in my public firm case, the government can observe some aggregate but not detailed cost information about the firm. This assumption distinguishes my public firm model from the others. For example, in Schmidt's papers, the government is
assumed to be able to observe a parameter determined by the state of nature. But such
detailed information is not accessible to the government in my model. The justification
for this assumption is based on the observations that cheating is also prevalent among
public officials.

The following section will describe the economic environment. Subsections III.1, III.2
and III.3 set up three models: the benchmark case of full information, the PRF model,
and the PUF model. Section IV compares the results. Section V concludes and discusses
some possible extensions of my study.

II. The Environment

There is a government and a firm in the economy. Goods being produced by the firm are
of public nature and can only be distributed by the government to the people. The
government is interested in the gain and loss of privatizing or nationalizing the firm.
When the game begins, the government offers a contract \( \{x, S, e\} \) in the PRF case or
\( \{x, W, e\} \) in the PUF case to the firm. \( x \) is the output level. \( S \) is a subsidy and \( W \) is
wage. \( e \) is the effort level exerted by the firm. If the contract is rejected, the game ends.
Otherwise, the firm will carry out the production. At last, all payoffs are realized. At the
beginning, both parties do not observe the value of a productive parameter \( \alpha \). They just
share the distribution of this value. \( T_i \) is the probability that \( \alpha = \alpha_i, i = 1, 2; \ T_1 + T_2 = 1. \)
But the firm can observe its exact value at a later stage of the game: after signing the
contract offered by the government.

The timing of events is summarized as follows:
(1) The government offers a contract to the firm.
(2) The firm accepts and signs the contract. Otherwise, both parties get zero payoff.
(3) The state of nature determines the value of \( \alpha \).
(4) The value of \( \alpha \) is observed by the firm privately.
(5) The firm chooses the effort level and the output level.
(6) The total production cost is realized.
(7) All payoffs are realized.
Output $x$ is observable by all parties. Marginal cost assumes the form:

$$C = \alpha - e.$$  

Effort $e$ is used to reduce the cost of production. $\alpha$ is determined by the state of nature which can be either $\alpha_1$ or $\alpha_2$. We further assume that $\alpha_2 > \alpha_1$. Let us call $\alpha_1$ a good state and $\alpha_2$ a bad state. The value of $\alpha_i$ and $e$ are assumed to be the firm’s private knowledge. The output can be costlessly observed by the government without error. A firm which observes $\alpha_1$ (or $\alpha_2$) is called type 1 (or 2) firm. The government can regulate the firm through a subsidy ($S$) if the firm is privately owned. If the government nationalizes the firm, it will pay a wage ($W$) to the firm subsequent to the production.

We assume that the government observes the marginal cost $(\alpha - e)$ and pays the total production cost incurred by the firm in the PUF case but not in the PRF case. This ability to observe some cost information pinpoints the more favorable position of the central government in the PUF case. Although the government can observe the marginal cost level, $\alpha$ and $e$ are still the agent’s private information. The government is not able to disentangle the garbled cost information. Thus there is still room for shirking behavior. The commitment to reimburse the total production cost can be regarded as a cost of gathering information – another interpretation of the notorious "soft budget constraint" effect.

When the firm exerts labor effort $e$, it experiences a kind of disutility $\varphi(e)$ which satisfies $\varphi'(e) > 0, \varphi''(e) > 0$ and $\varphi(0) = 0$. The social welfare function from consumption $V(x)$ has the properties $V'(x) > 0$ and $V''(x) < 0$. The production function $x$ is strictly decreasing with respect to $\alpha$. The utility functions of the government and the firm are described as follows for the two different cases:

**PRF case:**

$$U^G = V(x) - S + \beta U^f$$
$$U^f = F[S - (\alpha - e)x] - \varphi(e)$$

**PUF case:**

$$U^G = V(x) - W - (\alpha - e)x + \beta U^f$$
$$U^f = F(W) - \varphi(e)$$
$U^G$ is the government’s utility function. $U^f$ is the firm’s separable utility function. $\beta$ is a parameter between 0 and 1 which measures the significance of the firm’s welfare in the government’s utility function. The function $F$ satisfies $F' > 0$ and $F'' < 0$. So, the government is assumed to be risk neutral but the firm is risk averse.

A parallel hierarchical structure is adopted: Principal = government, Agent = firm. It helps to simplify the analysis, since we only need to focus on one agency relationship at a time.

### III. Models

Most of the literature about incentive theory uses a full information model as a benchmark. Following this tradition, I commence with introducing this first best model that will serve as a benchmark case.

#### III. 1 First Best Model

In this case, the effort level $e$ and the value of $\alpha$ are known to all parties. Therefore, the program $FB$ to be solved by the government is:

$$\max_{W,x,e} V(x) - W - (\alpha - e)x + \beta U^f$$

s.t. $F(W) - \varphi(e) \geq 0$

There is no need for the government to induce truth telling and hence to choose the right effort for the firm. What it does is to ensure the non-negativity of the firm’s reservation utility level (which is normalized as zero). As can be seen from the objective function, the government is assumed to incur the total production cost in this case. It is easy to show that all conclusions derived in this section hold even if the production cost is borne by the firm. The implication of this result is that it is irrelevant who bears the production cost. Under the full information assumption, the public and the private firms are equivalent.

The corresponding Lagrangian of the program $FB$ is:
\[ L = V(x) - W - (\alpha - e)x + \beta U' + \Theta\{F(W) - \varphi(e)\} \]

with \( \Theta \) is the Lagrange multiplier. The first order conditions yield the following:

\[
\frac{\partial L}{\partial W} = 0 \Rightarrow F'(W^*) = \frac{1}{\beta + \Theta} \quad (1)
\]

\[
\frac{\partial L}{\partial x} = 0 \Rightarrow V'(x^*) = \alpha - e \quad (2)
\]

\[
\frac{\partial L}{\partial e} = 0 \Rightarrow \varphi'(e^*) = xF'(W^*) \quad (3)
\]

From (1), it is easy to see that the inequality constraint may not be binding because the government is benevolent in the sense that it will leave some rents to the firm. \( \Theta \) is greater than 0 (which implies a binding constraint) if \( \beta \) is equal to zero. Equation (2) tells us that the marginal social utility of production is equal to the marginal cost of production. Equation (3) shows that, when effort is increased by one more unit, the production cost decreases by \( x \) times \( F'(W^*) \).

### III.2 Private Firm (PRF) Model

The government does not own the firm in this case. The private firm produces and sells a public good to the government for a subsidy. It can also privately observe a technological variable, \( \alpha \in \{ \alpha_1, \alpha_2 \} \), which is determined by the state of nature after the contract is signed by the government and the firm. Though the government cannot observe the value of variable \( \alpha \) and labor effort \( e \), it knows the probability distribution of \( \alpha \). Note that the government in this case does not reimburse the production cost incurred by the firm.

Since the government cannot observe \( \alpha \), it will ask for a report from the firm about the value of \( \alpha_i \) in order to design the optimal contract. By the revelation principle, the government can concentrate on the type of contract which induces the firm to report its
own type truthfully. This contract satisfies three inequality constraints. Accordingly, the government solves the following program PRF:

\[
\begin{aligned}
\max_{(S_i,x_i,e_i)} T_1[V(x_1) - S_1 + \beta U'_1] + T_2[V(x_2) - S_2 + \beta U'_2] \\
\text{s.t. } & T_1\{F[S_1 - (\alpha_1 - e_1)x_1] - \phi(e_1)\} + T_2\{F[S_2 - (\alpha_2 - e_2)x_2] - \phi(e_2)\} \geq 0 \quad (4) \\
& F[S_1 - (\alpha_1 - e_1)x_1] - \phi(e_1) \geq F[S_2 - (\alpha_1 - e_2)x_2] - \phi(e_2) \quad (5) \\
& F[S_2 - (\alpha_2 - e_2)x_2] - \phi(e_2) \geq F[S_1 - (\alpha_2 - e_1)x_1] - \phi(e_1) \quad (6)
\end{aligned}
\]

The first inequality is the firm’s participation constraint. It ensures that the agent can achieve the expected reservation utility level which he can get elsewhere. Since both the government and the firm cannot observe the value of \( \alpha \) at the beginning of the game, it is sufficient to induce the firm’s participation when its expected utility from participation is not smaller than the reservation level, which is normalized at zero in my model. The next two inequalities are the incentive compatibility constraints which guarantee a compatible choice of firm's action in different states of nature. They are used to induce the firm to reveal its true type. We need to consider two incentive constraints because the government knows that the firm can observe the value of \( \alpha \) after signing the contract. It has to make sure that the firm will make a compatible choice regardless of the state of nature it observes. Besides, it is also noteworthy that the private firm is free to choose the effort level after observing the state of nature. But since the benefit from cost reduction belongs to the private firm, the moral hazard is not a problem. The private firm is always choosing the first best effort level. This is most easy to see by comparing the effort optimization condition of the firm: \( xF' = \phi'(e) \) and the first best effort level condition (3). They are the same.

\( \mu_1 \) and \( \mu_2 \) are the Lagrange multipliers. In appendix, I show that only constraint (6) is not binding in equilibrium. The corresponding Lagrangian is:

\[
\mathcal{L} = T_1[V(x_1) - S_1 + \beta U'_1] + T_2[V(x_2) - S_2 + \beta U'_2] + \mu_1\{F[S_1 - (\alpha_1 - e_1)x_1] - \phi(e_1)\} + T_2\{F[S_2 - (\alpha_2 - e_2)x_2] - \phi(e_2)\} + \mu_2\{F[S_1 - (\alpha_1 - e_1)x_1] - \phi(e_1) - F[S_2 - (\alpha_1 - e_2)x_2] + \phi(e_2)\}
\]
Then the first order conditions of this program are:

\[
\frac{\partial L}{\partial S_1} = 0 \Rightarrow F'(S_1) = \frac{T_1}{(T_1\beta + T_1\mu_1 + \mu_2)} \tag{7}
\]

\[
\frac{\partial L}{\partial S_2} = 0 \Rightarrow F'(S_2) = \frac{T_2}{(T_2\beta + T_2\mu_1 - \mu_2)} \tag{8}
\]

\[
\frac{\partial L}{\partial x_1} = 0 \Rightarrow V'(x_1) = (\alpha_1 - e_1) \tag{9}
\]

\[
\frac{\partial L}{\partial x_2} = 0 \Rightarrow V'(x_2) = (\alpha_2 - e_2) + \frac{\mu_2}{T_2} F'(\alpha_2 - \alpha_1) \tag{10}
\]

\[
\frac{\partial L}{\partial e_1} = 0 \Rightarrow \phi'(e_1) = x_1 F' \tag{11}
\]

\[
\frac{\partial L}{\partial e_2} = 0 \Rightarrow \phi'(e_2) = x_2 F' \tag{12}
\]

The following proposition summarizes the main properties of the optimal contract.

**Proposition 1.**

(i) \( S_1 > S_2 \).

(ii) At state 1, there is no production distortion. But at state 2, \( x_2 < x_2^* \).

(iii) There is no effort distortion at either state of nature.

Proof:

(i) If \( \mu_2 = 0 \), \( F'(S_1) = F'(S_2) = \frac{1}{\beta + \mu_1} \). If \( \mu_1, \mu_2 > 0 \), then we can rewrite

\[
F'(S_1) = \frac{1}{(\beta + \mu_1) + \frac{\mu_2}{T_1}} < \frac{1}{\beta + \mu_1} \quad \text{and} \quad F'(S_2) = \frac{1}{(\beta + \mu_1) - \frac{\mu_2}{T_2}} > \frac{1}{\beta + \mu_1}.
\]

Therefore, \( F'(S_2) > F'(S_1) \) which implies \( S_1 > S_2 \) by the property of the function \( F \).

(ii) The first half of (ii) is obvious by comparing equations (2) and (9). Since \( \alpha_2 > \alpha_1 \) and \( F' > 0 \). Therefore, \( V'(x_2) > (\alpha_2 - e_2) = V'(x_2^*) \) from equation (10). By the properties of social welfare function \( V() \), \( x_2 < x_2^* \). The second term in the left hand side of this equation
measures the proclivity of the production distortion. Note that the smaller the $T_2$, the more serious the production distortion is.

(iii) It is easy to see that by comparing equations (3) and (11) and (12). ■

$S_1$ is set higher than $S_2$ to induce effort from the type 1 private firm. In the following, I will show that the second best contract derived above is designed in such a way that the ex post profit left to the type 1 private firm is minimized.

**Proposition 2.** At the second best contract, $e_2$ and $x_2$ are designed to minimize the ex post profit earned by the type 1 private firm.

Proof:

If (4) and (5) are binding, then it is easy to find that

$$T_1F[S_2 - (\alpha_1 - e_2)x_2] + T_2F[S_2 - (\alpha_2 - e_2)x_2] = \varphi(e_2)$$

Therefore, the type 1 private firm's utility $U_{i}^{f}$ is:

$$F[S_1 - (\alpha_1 - e_1)x_1] - \varphi(e_1)$$

$$= F[S_2 - (\alpha_1 - e_2)x_2] - \varphi(e_2)$$

$$= F[S_2 - (\alpha_1 - e_2)x_2] - T_1F[S_2 - (\alpha_1 - e_2)x_2] - T_2F[S_2 - (\alpha_2 - e_2)x_2]$$

$$= T_2F[S_2 - (\alpha_1 - e_2)x_2] - T_2F[S_2 - (\alpha_2 - e_2)x_2]$$

$$= T_2 [F[S_2 - (\alpha_1 - e_2)x_2] - F[S_2 - (\alpha_2 - e_2)x_2]] > 0$$

because $\alpha_2 - e_2 > \alpha_1 - e_2$ and $S_2 - (\alpha_2 - e_2)x_2 < S_2 - (\alpha_1 - e_2)x_2$. Therefore,

$$F[S_2 - (\alpha_1 - e_2)x_2] > F[S_2 - (\alpha_2 - e_2)x_2]$$

Let $\Delta = F[S_2 - (\alpha_1 - e_2)x_2] - F[S_2 - (\alpha_2 - e_2)x_2]$. Obviously, to minimize $U_{i}^{f}$ is equivalent to minimizing $\Delta$.

Since,

$$\frac{\partial \Delta}{\partial x_2} = F'(\alpha_2 - \alpha_1) > 0$$

by $F' > 0$ and $(\alpha_2 - \alpha_1) > 0$.

Therefore, the government should set $x_2$ as low as possible. ■

The type 2 private firm will get negative utility since
\[ F[S_2 - (\alpha_2 - e_2)x_2] - \varphi(e_2) \]
\[ = F[S_2 - (\alpha_2 - e_2)x_2] - T_1 F[S_2 - (\alpha_1 - e_2)x_2] - T_2 F[S_2 - (\alpha_2 - e_2)x_2]. \]
\[ = T_1 [F[S_2 - (\alpha_2 - e_2)x_2] - F[S_2 - (\alpha_1 - e_2)x_2]] < 0 \]

It is not surprising to find that the effort level exerted by the firm is at the first best level in this model since the private firm is responsible for its own cost. Production distortion occurs due to the fact that the government cannot observe the cost of production. That makes the private firm have the incentive to misreport the cost in order to recoup part of the cost of production. Since the firm's expected utility function is binding, a type 2 private firm's negative utility is necessary to offset the positive utility earned by the type 1 private firm.

In the following section, we will develop the third model of this paper - the public firm model.

### III.3 Public Firm (PUF) Model

As the owner of the public firm, the government employs people to produce and is able to observe the marginal cost of production. But, as a price for being the owner of this information, the government needs to reimburse the total cost of production incurred by the firm. Even so, the government still cannot observe worker’s effort level and the true type of the firm. It has to design a contract to prevent workers from cheating. In this case, the probability distribution of \( \alpha \) is assumed to be the same as in the PRF model.

The government needs to solve the following program PUF:

\[
\begin{align*}
Max_{(W_i, x_i, e_i)} & \quad T_1 [V(x_1) - W_1 - (\alpha_1 - e_1)x_1 + \beta U_i^1] + T_2 [V(x_2) - W_2 - (\alpha_2 - e_2)x_2 + \beta U_2^f] \\
\text{s.t.} & \quad T_1 [F(W_i) - \varphi(e_i)] + T_2 [F(W_2) - \varphi(e_2)] \geq 0 \\
& \quad F(W_1) - \varphi(e_1) \geq F(W_2) - \varphi(\hat{e}_2) \\
& \quad F(W_2) - \varphi(e_2) \geq F(W_1) - \varphi(\hat{e}_1)
\end{align*}
\]
The first inequality is the participation constraint and the next two are incentive compatibility constraints. \( \hat{e}_1 \) and \( \hat{e}_2 \) in inequalities (14) and (15) are defined by:

\[
(\alpha_1 - e_1) = (\alpha_2 - \hat{e}_1)
\]
\[
(\alpha_2 - e_2) = (\alpha_1 - \hat{e}_2)
\]

In words, \( \hat{e}_1 \) (or \( \hat{e}_2 \)) is the effort level exerted by type 2 (or 1) firm if it masquerades type 1 (or 2) firm. Apparently, \( \hat{e}_1 > e_1 \) and \( \hat{e}_2 < e_2 \). Recall that the marginal cost can be observed by the government in this case. The firm needs to exert an effort level which can prevent revelation of its true identity through the cost observation.

By the same argument as being used in appendix, we can show that inequalities (13) and (14) are the only effective constraints at the optimal solution. The corresponding Lagrangian function, \( \tilde{L} \), of program PUF is:

\[
\tilde{L} = T_1[V(x_1) - W_1 - (\alpha_1 - e_1)x_1 + \beta U_1'] + T_2[V(x_2) - W_2 - (\alpha_2 - e_2)x_2 + \beta U_2'] +
\]
\[
\lambda_1[T_1(F(W_1) - \varphi(e_1)) + T_2(F(W_2) - \varphi(e_2))] + \lambda_2[F(W_1) - \varphi(e_1) - F(W_2) + \varphi(\hat{e}_2)]
\]

F.O.C.s:

\[
\frac{\partial \tilde{L}}{\partial W_1} = 0 \Rightarrow F'(W_1) = \frac{T_1}{(T_1 \beta + T_1 \lambda_1 + \lambda_2)} \tag{16}
\]

\[
\frac{\partial \tilde{L}}{\partial W_2} = 0 \Rightarrow F'(W_2) = \frac{T_2}{(T_2 \beta + T_2 \lambda_1 - \lambda_2)} \tag{17}
\]

\[
\frac{\partial \tilde{L}}{\partial x_1} = 0 \Rightarrow V'(x_1) = (\alpha_1 - e_1) \tag{18}
\]

\[
\frac{\partial \tilde{L}}{\partial x_2} = 0 \Rightarrow V'(x_2) = (\alpha_2 - e_2) \tag{19}
\]

\[
\frac{\partial \tilde{L}}{\partial e_1} = 0 \Rightarrow \varphi'(e_1) = x_1 F'(W_1) \tag{20}
\]

\[\text{The discussion here is similar to the one in Khalil and Lawarrée (1995).}\]
\[
\frac{\partial L}{\partial e_2} = 0 \Rightarrow \varphi'(e_2) = x_2 F'(W_2) - \frac{\lambda_2}{T_2} F'(W_2)[\varphi'(e_2) - \varphi'(\hat{e}_2) \frac{\partial \hat{e}_2}{\partial e_2}] 
\] (21)

The following proposition will characterize this optimal contract for the public firm.

**Proposition 3.**
(i) \( W_i > W_2 \)
(ii) There is no production distortion at both states of nature.
(iii) At state 1, there is no effort distortion. But at state 2, the public firm will exert a lower than the first best effort level. That is: \( e_2 < e_2^* \).

Proof:

(i) Since \( F'(W_i) = \frac{1}{(\beta + \lambda_i) + \frac{\lambda_i}{T_i}} < \frac{1}{\beta + \lambda_1} \) and \( F'(W_2) = \frac{1}{(\beta + \lambda_2) - \frac{\lambda_2}{T_2}} > \frac{1}{\beta + \lambda_1} \) if \( \lambda_1 \) and \( \lambda_2 > 0 \), \( F'(W_i) > F'(W_2) \). Thus, \( W_i > W_2 \) by the property of the function F. It is apparent that \( W_i > 0 \) but no guarantee for the non-negativity of \( W_2 \).

(ii) Straightforward by comparing equations (2) with (18), (19).

(iii) It is easy to see the first half of part (iii) by comparing (3) and (20).

Since \( \varphi'(\hat{e}_2) < \varphi'(e_2) \) and \( \frac{\partial \hat{e}_2}{\partial e_2} < 1 \), \( \varphi'(e_2) - \varphi'(\hat{e}_2) \frac{\partial \hat{e}_2}{\partial e_2} > 0 \). Therefore, we can conclude that \( \varphi'(e_2) - \varphi'(\hat{e}_2) = x_2 F'(W_2) \) from (21). By the properties of the disutility function \( \varphi(e) \), \( e_2 < e_2^* \). \( \blacksquare \)

Now, I will show that the second best \( e_2 \) is set below the first best level to minimize the ex post profit given to the type 1 public firm.

**Proposition 4.** \( e_2 \) in the second best contract is designed to minimize the ex post profit left to type 1 public firm.

Proof:

Since equations (13) and (14) are binding, it is easy to show that \( F(W_2) = T_2 \varphi(\hat{e}_2) + T_2 \varphi(e_2) \). Therefore,

\[
F(W_2) - \varphi(\hat{e}_2) = T_2 [\varphi(e_2) - \varphi(\hat{e}_2)] . \text{ Or equivalently,}
\]

\[
F(W_2) - \varphi(e_2) = T_2 [\varphi(e_2) - \varphi(\hat{e}_2)] > 0 \text{ since } e_2 > \hat{e}_2.
\]

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In order to minimize the type 1 public firm's utility: \( F(W_1) - \varphi(e_1) \), the principal needs to minimize the difference between \( \varphi(e_2) \) and \( \varphi(\hat{e}_2) \). That is why, at the second best contract, the \( e_2 \) is set below the first best level. ■

The type 2 public firm will get negative utility since

\[
F(W_2) - \varphi(e_2) \\
= T_1\varphi(\hat{e}_2) + T_2\varphi(e_2) - \varphi(e_2) \\
= -T_1[\varphi(e_2) - \varphi(\hat{e}_2)] \leq 0
\]

In this model, production distortion does not occur because the government can observe the marginal cost of production. Since the firm can save the disutility of effort by mimicking other types of firms, it is optimal for the government to distort the effort in the optimal contract to reduce such interest of shirking behavior. Just like in the PRF model, the type 2 public firm is also earning negative utility in this case.

**IV. Comparison**

Though the superiority of the private firm has been affirmed again and again, our theoretical findings - as well as empirical evidence - suggest that there are tradeoffs between private and public ownership. The following propositions 5 and 6 exhibits the crucial differences in production and effort distortions between PUF and PRF scenarios.

**Proposition 5.** At state 1, no production distortion can be observed at either kind of firm. At state 2, only the private firm experiences a production distortion. That is,

\[ x_{2}^{\text{PUF}} = x_{2}^{*} > x_{2}^{\text{PRF}}. \]

The property that all agents' behavior at state 1 complies with the first best is known as the “no distortion at the top” in the asymmetric information literature. At state 2, the private firm produces less than the optimal level because of the adverse selection problem. Recall that the government cannot observe the value of \( \alpha \) in the PRF model. Therefore, the private firm has the incentive to misreport this value to recoup part of its
cost of production. In order to minimize such incentive to cheat (type 1 firm pretends to be type 2), it will be optimal for the government to set a lower than the first best $x_2$ at the second best contract.

But in the PUF case, the production distortion does not occur because the government can observe the marginal cost of production.

**Proposition 6.** There is no effort distortion in both kinds of firms at state 1. At state 2, the PRF will exert the first best effort level but the PUF will exert effort lower than the first best, or $e_2^{PRF} = e^*_2 > e_2^{PUF}$.

This is the major finding of this paper. The type 2 private firm exerts the first best effort level because it is responsible for the cost of production. The benefits from the cost reduction totally belong to the private firm. Thus, it has incentive to exert the first best effort level. Therefore, it is not necessary for the government to distort the first best contract to induce effort from the firm.

In the PUF case, effort distortion arises because the type 1 public firm can save effort by mimicking a type 2 firm. By setting a lower than the first best $e_2$ at the optimal contract the government can reduce the ex post profit earned by the type 1 firm so that its incentive to conduct shirking behavior is reduced.

The claim that, by improving material incentives, managers' effort level of those public firms can be stimulated is supported by empirical evidence. Groves, Hong, McMillan and Naughton (1994) show that the introduction of an incentive payment mechanism, e.g. allowing firms to retain more profits, improves the productivity of state-owned enterprises in mainland China after 1978. The responsibility for the cost of production, the way I introduce to stimulate effort from the firm, can be considered as such a kind of mechanism. Regarding the production distortion, empirical studies can seldom be found. But Aharoni (1986) quoted a report by Mann about a comparative study of public and private water utilities in the United States. In that study, Mann compared 315 public and 56 private water utilities. He found that "The mean daily production of the publicly
owned utilities was much larger than privately owned utilities, .....(publicly owned utilities) served more population that is less dense.....". (P. 201 in Aharoni 1986) At last, Atkinson and Halvorsen (1986) are one of those empirical works which echo my conclusion that it is difficult to tell the relative inefficiency of private and public ownership. They conduct a study of efficiency about those private and public electric utilities in U.S. and found that these two types of firms are equally cost inefficient in a regulated environment.

Intuitively, it may be surprising to see that, at the second best contract, the private firm will exert a first best effort level but end up with a lower than first best production output. That totally hinges on the model set-up. Recall that $x$ is a function of $\alpha$ only and effort $e$ is used to reduce the cost of production rather than to increase output. That is why it is possible that a first best effort level does not induce a first best output in the private firm case.

V. Conclusion

The main conclusion of this paper is that effort and production distortions in the private or public institutions are inevitable if informational asymmetry exists. When the ownership is transformed from private to public (or vice versa), the nature of distortions which the firm encounters changes. From the above discussion, we find that the adverse selection effect arises in the private firm case. In the public firm model, the moral hazard problem dominates. The policy implication is that privatization indeed cannot reconcile all problems of inefficiency. What it does is to merely transform the nature of the agency problem. From this viewpoint, privatization cannot be a panacea to cure low efficiency problem.

There are some other promising extensions of my investigations. First, it might be interesting to see whether the conclusions persist with a continuous state space. Second, assuming that public and private firms have different attitudes towards their employees, explicit consideration of the firm's work force could alter the results. This direction of
research would require a multi-layer hierarchy model - an additional technical complication.

Appendix

The following proof is adopted from Hart and Holmstrom (1987). We can show that the inequality (6) is not binding in equilibrium by ruling out the possibility of other cases.

Case (a) Both constraints are binding. That is,

\[
F[S_1 - (\alpha_1 - e_1)x_1] - \varphi(e_1) = F[S_2 - (\alpha_1 - e_2)x_2] - \varphi(e_2)
\]

\[
F[S_2 - (\alpha_2 - e_2)x_2] - \varphi(e_2) = F[S_1 - (\alpha_2 - e_1)x_1] - \varphi(e_1)
\]

If that is the case, then the agent will be indifferent to truth telling or lying in each state of nature. So, it is optimal for the government to set \( S_1 = S_2 = S^* \). But if that is the case, it is optimal for the government to choose the first best contract which contradicts the assumption that case (a) is an equilibrium solution.

Case (b) Both constraints are not binding. That is,

\[
F[S_1 - (\alpha_1 - e_1)x_1] - \varphi(e_1) \geq F[S_2 - (\alpha_1 - e_2)x_2] - \varphi(e_2)
\]

\[
F[S_2 - (\alpha_2 - e_2)x_2] - \varphi(e_2) \geq F[S_1 - (\alpha_2 - e_1)x_1] - \varphi(e_1)
\]

Then, it is possible to make a Pareto improvement by reducing \( S_1 \) (or \( S_2 \)) to make the first (or second) constraint less slack and keep the expected participation constraint holding at the same time. The second (or first) constraint will continue to be satisfied. Therefore, case (b) is not an equilibrium.

Case (c)

\[
F[S_1 - (\alpha_1 - e_1)x_1] - \varphi(e_1) \geq F[S_2 - (\alpha_1 - e_2)x_2] - \varphi(e_2)
\]

\[
F[S_2 - (\alpha_2 - e_2)x_2] - \varphi(e_2) = F[S_1 - (\alpha_2 - e_1)x_1] - \varphi(e_1)
\]

Then, we have \( F[S_1 - (\alpha_1 - e_1)x_1] - \varphi(e_1) > F[S_2 - (\alpha_2 - e_2)x_2] - \varphi(e_2) \) since

\[
F[S_2 - (\alpha_1 - e_2)x_2] - \varphi(e_2) > F[S_2 - (\alpha_2 - e_2)x_2] - \varphi(e_2).
\]

The firm prefers state 1 to state 2. Then a Pareto improvement (from a risk sharing point of view) is possible if we reduce \( S_1 \) but increase \( S_2 \) a bit and keep the participation constraint binding. So, case (c) cannot be an equilibrium, either.
So, only case (d) is left. That is,

\[ F[S_1 - (\alpha_1 - e_1)x_i] - \phi(e_i) = F[S_2 - (\alpha_1 - e_2)x_2] - \phi(e_2) \]
\[ F[S_2 - (\alpha_2 - e_2)x_2] - \phi(e_2) \geq F[S_1 - (\alpha_2 - e_1)x_1] - \phi(e_1), \]

which is the only possible case.

References


