Chapter 7

Structural Model

7.1 General

The purpose of the structural model is to account for the interaction of the individual fasteners in a multiple-bolt joint. That is, the structural model explains effects brought about by member strain and slack variation. Traditionally, material deformations were thought to be the main cause of the group action factor (unequal load sharing among bolts) and premature failure. Since all non-linearities, including slack, are explained by the hysteresis model described in Chapter 5, the structural model is an elastic stiffness model. The structural model and the modified hysteresis model are tightly interfaced and integrated into the computer model MULTBOLT.

7.2 Joint Layout and Numbering Scheme

Inherent to its asymmetry, the single shear joint must be defined both in layout and fastener numbering. The convention applied is (Figure 7.1):

- Bolt 1 in Member 1 must be at the unloaded end.
- Bolt 1 in Member 2 must be at the loaded end.
- k_{11} must be between Bolt 1 and Bolt 2.
- k_{21} must be between Bolt 1 and the applied load.
- $Space_{11}$ must be between unloaded end and Bolt 1 in Member 1.
- $Space_{21}$ must be between loaded end and Bolt 1 in Member 2.

where, k_{ij} describes the stiffness of member *i* between two bolts or a bolt and a member end, respectively. Note that the arrangement permits variation of material properties and spacing around each bolt. Variations in spacing not only influence member stiffness but also slack deviation. Positive movement is defined as “pulling” or, more specifically, when the two unloaded ends move towards each other.

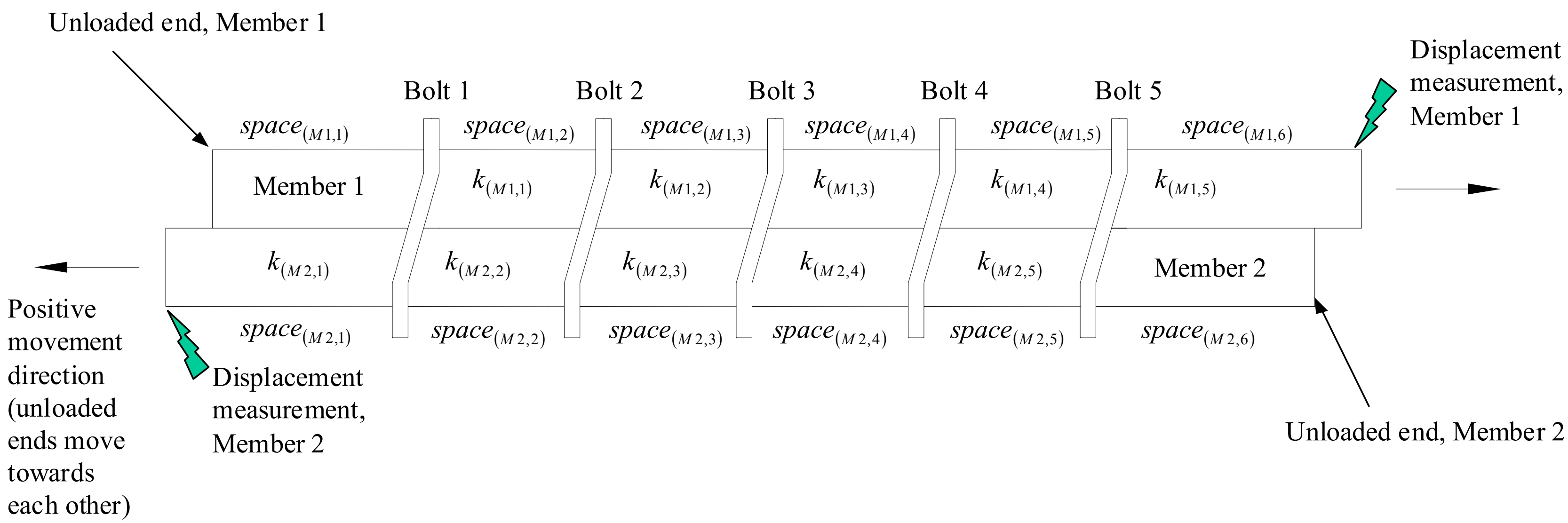

Figure 7.1: Joint numbering scheme shown at a 5-bolt joint.

Given the joint layout convention depicted in Figure 7.1, in- and output variables relevant for the structural model along with the numbering scheme used are revealed in Figure 7.2

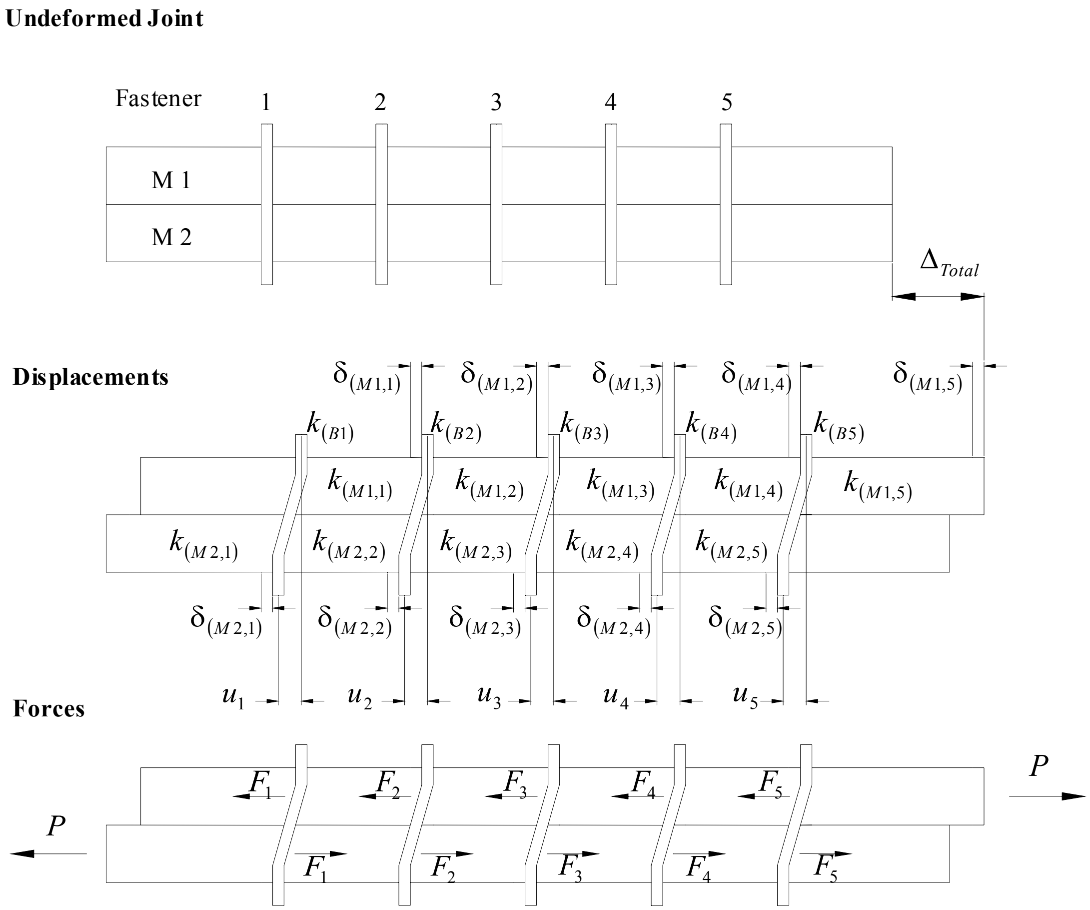

Figure 7.2: Structural model variables depicted using a five-bolt sample joint (forces and moments caused by asymmetry are neglected).

7.3 Model Development

Recall that throughout this work, the independent variable is displacement and the dependent variable is force. Also, note that due to material deformations, each bolt experiences a unique displacing function. Based on total displacement of the multiple-bolt joint (independent variable), bolt location, bolt stiffness, and member stiffness, the structural model determines the displacing function for the individual bolt within the joint, which in turn constitutes the input for the individual hysteresis model of each bolt.

The structural model is similar in principle to the models devised by Jorissen (1998) and Lantos (1969). However, substantial differences exist in the way the model is solved and how member stiffnesses are computed. Moreover, with integration of the modified hysteresis model, no assumptions are made as to linear elastic behavior of the joint. In other words, through interfacing with a non-linear model, the resulting structural model explains the true non-linear behavior of the connection. In addition, the inclusion of load reversal and slack, and the fact that single shear joints are modeled, generalization of the model was necessary to be applicable to these cases.

7.3.1 Derivation

Key to developing the structural model is the assumption that bolt stiffness is constant within a single time step. If the variation of bolt stiffness is described by the modified hysteresis model, then, for time step i, bolt stiffness $k_{(Bj)i}$, may be expressed as

$$k_{(Bj)i} = \frac{f_i\left(p_1, p_2, \cdots, p_n; u_i\right) - f_{i-1}\left(p_1, p_2, \cdots, p_n; u_{i-1}\right)}{u_i - u_{i-1}} \tag{7.1}$$

		Unit[1]
f_i	mass-normalized force computed by modified hysteresis model at time step i and input u_i.	kN/kg
p_j	estimated parameter	
u_i	bolt displacement at time step i.	mm
$k_{(Bj)i}$	stiffness of Bolt j at time step i.	kN/(mm kg)

With Equation (7.1) (local secant stiffness), bolt stiffnesses are known at each time step i based on time step i-1, and distribution of displacements among the bolts can be computed, which compose the displacement input to determine bolt force and stiffness at time step i+1 (Figure 7.3).

[1] All units used in this chapter are specified as used in MULTBOLT.

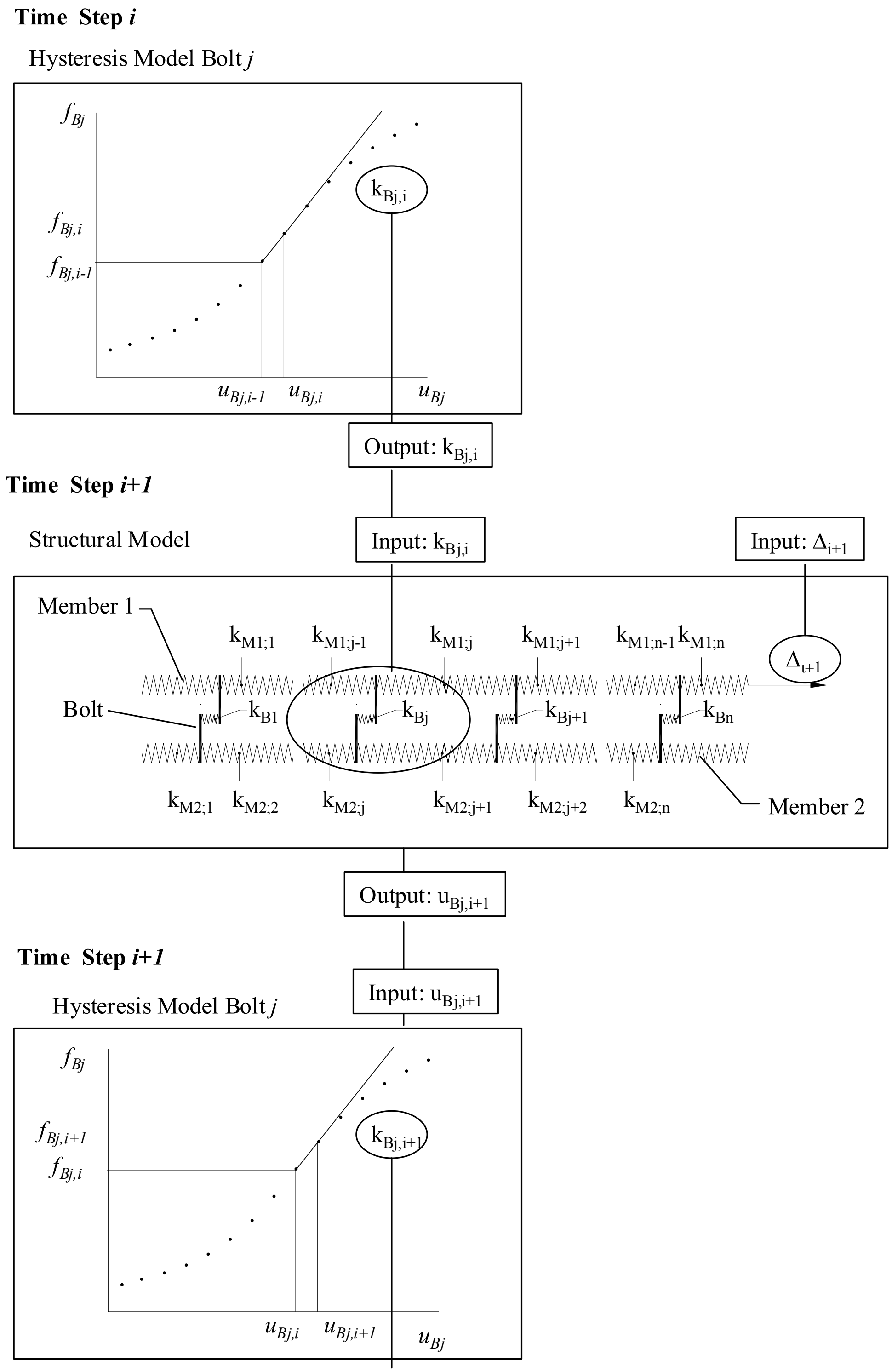

Figure 7.3: Interface between hysteresis model and structural model. At time step *i* the hysteresis model produces output bolt stiffness $k_{Bj,i}$ which is input into structural model, along with total displacement increment Δ_{i+1} of next time step. Structural model output is displacement $u_{Bj,i+1}$ which is again input in hysteresis model to compute $f_{Bj,i+1}$ and the cycle repeats.

Derivation of the structural model is best explained by means of an example joint. With reference to Figure 7.2, Section 3.3.1, and the displacement relationship expressed in Equation (3.2), assume a five-bolt joint with the unknowns

$$\{\delta_{M1,1}, \delta_{M1,2}, \delta_{M1,3}, \delta_{M1,4}, \delta_{M1,5}, \delta_{M2,1}, \delta_{M2,2}, \delta_{M2,3}, \delta_{M2,4}, \delta_{M2,5}, F_1, F_2, F_3, F_4, F_5, P, u_1, u_2, u_3, u_4, u_5\}$$

		Unit
F_j	incremental mass-normalized force of the individual bolt	kN/kg
P	incremental total resisted force of multiple-bolt joint	kN/kg
u_j	incremental displacement of the individual bolt	mm
$\delta_{M\,i,j}$	incremental strain of member *i*, adjacent to bolt *j*	

Then all unknowns can be determined by solving the system of equations

$$\frac{P}{k_{M1,1}}-\frac{F_2}{k_{M1,1}}-\frac{F_3}{k_{M1,1}}-\frac{F_4}{k_{M1,1}}-\frac{F_5}{k_{M1,1}}-\delta_{M1,1}=0 \tag{7.2}$$

$$\frac{P}{k_{M1,2}}-\frac{F_3}{k_{M1,2}}-\frac{F_4}{k_{M1,2}}-\frac{F_5}{k_{M1,2}}-\delta_{M1,2}=0 \tag{7.3}$$

$$\frac{P}{k_{M1,3}}-\frac{F_4}{k_{M1,3}}-\frac{F_5}{k_{M1,3}}-\delta_{M1,3}=0 \tag{7.4}$$

$$\frac{P}{k_{M1,4}}-\frac{F_5}{k_{M1,4}}-\delta_{M1,4}=0 \tag{7.5}$$

$$\frac{P}{k_{M1,5}}-\delta_{M1,5}=0 \tag{7.6}$$

$$\frac{P}{k_{M2,1}}-\delta_{M2,1}=0 \tag{7.7}$$

$$\frac{P}{k_{M2,2}}-\frac{F_1}{k_{M2,2}}-\delta_{M2,2}=0 \tag{7.8}$$

$$\frac{P}{k_{M2,3}}-\frac{F_1}{k_{M2,3}}-\frac{F_2}{k_{M2,3}}-\delta_{M2,3}=0 \tag{7.9}$$

$$\frac{P}{k_{M2,4}}-\frac{F_1}{k_{M2,4}}-\frac{F_2}{k_{M2,4}}-\frac{F_3}{k_{M2,4}}-\delta_{M2,4}=0 \tag{7.10}$$

$$\frac{P}{k_{M2,5}}-\frac{F_1}{k_{M2,5}}-\frac{F_2}{k_{M2,5}}-\frac{F_3}{k_{M2,5}}-\frac{F_4}{k_{M2,5}}-\delta_{M2,5}=0 \tag{7.11}$$

$$k_1\cdot u_1-F_1=0 \tag{7.12}$$

$$k_2\cdot u_2-F_2=0 \tag{7.13}$$

$$k_3\cdot u_3-F_3=0 \tag{7.14}$$

$$k_4\cdot u_4-F_4=0 \tag{7.15}$$

$$k_5\cdot u_5-F_5=0 \tag{7.16}$$

$$-P+F_1+F_2+F_3+F_4+F_5=0 \tag{7.17}$$

$$-u_1+u_2-\delta_{M1,1}+\delta_{M2,2}=0 \tag{7.18}$$

$$-u_2+u_3-\delta_{M1,2}+\delta_{M2,3}=0 \tag{7.19}$$

$$-u_3+u_4-\delta_{M1,3}+\delta_{M2,4}=0 \tag{7.20}$$

$$-u_4+u_5-\delta_{M1,4}+\delta_{M2,5}=0 \tag{7.21}$$

$$u_1 + \delta_{M1,1} + \delta_{M1,2} + \delta_{M1,3} + \delta_{M1,4} + \delta_{M1,5} + \delta_{M2,1} = \Delta_T \qquad (7.22)$$

or in matrix form

$$\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{k_{M1,1}} & -\frac{1}{k_{M1,1}} & -\frac{1}{k_{M1,1}} & -\frac{1}{k_{M1,1}} & \frac{1}{k_{M1,1}} & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{k_{M1,2}} & -\frac{1}{k_{M1,2}} & -\frac{1}{k_{M1,2}} & \frac{1}{k_{M1,2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{k_{M1,3}} & -\frac{1}{k_{M1,3}} & \frac{1}{k_{M1,3}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{k_{M1,4}} & \frac{1}{k_{M1,4}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{k_{M1,5}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{k_{M2,1}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -\frac{1}{k_{M2,2}} & 0 & 0 & 0 & 0 & \frac{1}{k_{M2,2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{1}{k_{M2,3}} & -\frac{1}{k_{M2,3}} & 0 & 0 & 0 & \frac{1}{k_{M2,3}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -\frac{1}{k_{M2,4}} & -\frac{1}{k_{M2,4}} & -\frac{1}{k_{M2,4}} & 0 & 0 & \frac{1}{k_{M2,4}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\frac{1}{k_{M2,5}} & -\frac{1}{k_{M2,5}} & -\frac{1}{k_{M2,5}} & -\frac{1}{k_{M2,5}} & 0 & \frac{1}{k_{M2,5}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} \delta_{M1,1} \\ \delta_{M1,2} \\ \delta_{M1,3} \\ \delta_{M1,4} \\ \delta_{M1,5} \\ \delta_{M2,1} \\ \delta_{M2,2} \\ \delta_{M2,3} \\ \delta_{M2,4} \\ \delta_{M2,5} \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ P \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \Delta_T \end{Bmatrix} \qquad (7.23)$$

In order to find the individual bolt displacement, the system of linear equations is solved '*incrementally*'. That is, the coordinate system must be shifted at each time step to the origin of the new stiffness line computed (Figure 7.4).

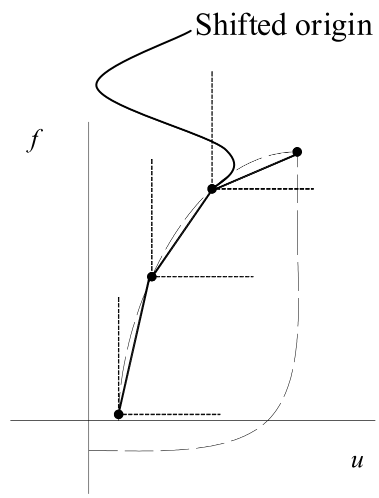

Figure 7.4: To solve the structural model, for each time step *i* the origin is shifted to point (u_i, f_i)

Hence, the displacement input of the total connection, Δ_T, at each time step is the *incremental* displacement or

$$\Delta_{T,i} = \Delta_{inputfunction,i} - \Delta_{inputfunction,i-1} \tag{7.24}$$

Obviously, the coefficient matrix changes for a different number of bolts. However, the changes follow certain rules and a subroutine can be devised that generates the correct coefficient matrix associated with a certain number of bolts. The system was solved numerically using the LAPACK driver routine (Version 3.0) developed by the University of Tennessee, University of California Berkeley, NAG Ltd., Courant Institute, Argonne National Lab, and Rice University (released on March 31, 1993). As with LSODE, the LAPACK solver is written in FORTRAN and can be downloaded free of charge from the Netlib Repository, a collection of mathematical software (www.netlib.org). Thanks to a relatively simple interface, LAPACK was easily integrated into MULTBOLT.

7.3.1.1 Load Input

To keep application as general as possible, the structural model may be extended to cases where it is desired to have load as independent variable. A change of independent variable makes it necessary to change the coefficient matrix and the subroutine which generates it based on the number of bolts. Unlike the structural model, which is based on displacement input, the structural model with load input determines the loading function for the individual bolt within the joint, which in turn constitutes the input for the individual hysteresis model of each bolt. Hence, if P denotes total load input, the system of linear equations defining the structural model for load input and a five-bolt joint becomes

$$\left[\begin{array}{ccccccccccccccccccccc}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{k_{M1,1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{k_{M1,2}} & \frac{1}{k_{M1,2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{k_{M1,3}} & \frac{1}{k_{M1,3}} & \frac{1}{k_{M1,3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{k_{M1,4}} & \frac{1}{k_{M1,4}} & \frac{1}{k_{M1,4}} & \frac{1}{k_{M1,4}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & \frac{1}{k_{M1,5}} & \frac{1}{k_{M1,5}} & \frac{1}{k_{M1,5}} & \frac{1}{k_{M1,5}} & \frac{1}{k_{M1,5}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \frac{1}{k_{M2,1}} & \frac{1}{k_{M2,1}} & \frac{1}{k_{M2,1}} & \frac{1}{k_{M2,1}} & \frac{1}{k_{M2,1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{1}{k_{M2,2}} & \frac{1}{k_{M2,2}} & \frac{1}{k_{M2,2}} & \frac{1}{k_{M2,2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \frac{1}{k_{M2,3}} & \frac{1}{k_{M2,3}} & \frac{1}{k_{M2,3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \frac{1}{k_{M2,4}} & \frac{1}{k_{M2,4}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \frac{1}{k_{M2,5}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & k_5 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\left[\begin{array}{c}
d_{M1,1} \\ d_{M1,2} \\ d_{M1,3} \\ d_{M1,4} \\ d_{M1,5} \\ d_{M2,1} \\ d_{M2,2} \\ d_{M2,3} \\ d_{M2,4} \\ d_{M2,5} \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ \Delta_T \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5
\end{array}\right]
=
\left[\begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ P
\end{array}\right]
\qquad (7.25)$$

7.3.2 Computation of Member Stiffness

According to Hooke's Law, the longitudinal stiffness, K, of a column with constant rectangular cross section and constant material-specific modulus of elasticity, E, is determined by

$$K = \frac{A \cdot E}{L} \qquad (7.26)$$

		Unit
K	Stiffness	N/mm
A	cross sectional area of beam	mm^2
L	Column length	mm

In other words, column stiffness is inversely proportional to column length. Thus, member stiffness between two fasteners is a function of fastener spacing. Equation (7.26) assumes equally distributed stresses throughout the beam by relating stiffness to total cross sectional area. In a multiple-bolt joint, however, stresses are by no means uniformly distributed, and relation of K to total cross-sectional area may introduce significant error. If a body is subject to a point load, stresses emanate from the point of application through the material. Only a large distance away from the application of the point load one may assume that stresses are uniformly distributed (St Vernant's Principle). Assume, for instance, that the member width is much larger compared to the bolt diameter. If this is the case then the stiffness between two bolts computed by Equation (7.26) would be unrealistically large. Or, if the spacing of the bolts is reduced and approaches the bolt diameter, then it becomes obvious that the stress distributions are more a local effect and the computation of stiffness should reflect that. But the true stress distribution is inherently complex and computation of stiffness reflecting the true distribution of stresses in all directions is almost impossible without numerical approximation.

To circumvent the complexities involved, but still include local effects, member stiffness between the bolts was related to an equivalent area rather than the total cross sectional area of the member. From elastic theory it can be shown that the radial stress, σ_r, in a very large isotropic plate under a concentrated load acting normal to the surface of the plate is constant along a circle of any diameter with tangent to the plate surface and center on the line of force (Ugural and Fenster 1995) (Figure 7.5). This is true except for locations close to the point of force application. Therefore, the radial stress composed of equal amounts of stress perpendicular and stress parallel to the load, lies on a line emanating at 45 degrees from the point of load application. The two 45 degree lines depicted in Figure 7.5 may be thought of as a boundary within which the influence of the stresses parallel to the applied force dominates. As rough approximation, the area of influence of stiffness parallel to the direction of the applied force lies within the 45 degree boundary. Thus, for stiffness computation, it is assumed that the stress parallel to the grain caused by a loaded bolt spreads out at 45 degrees on either side in direction of loading (Figure 7.6).

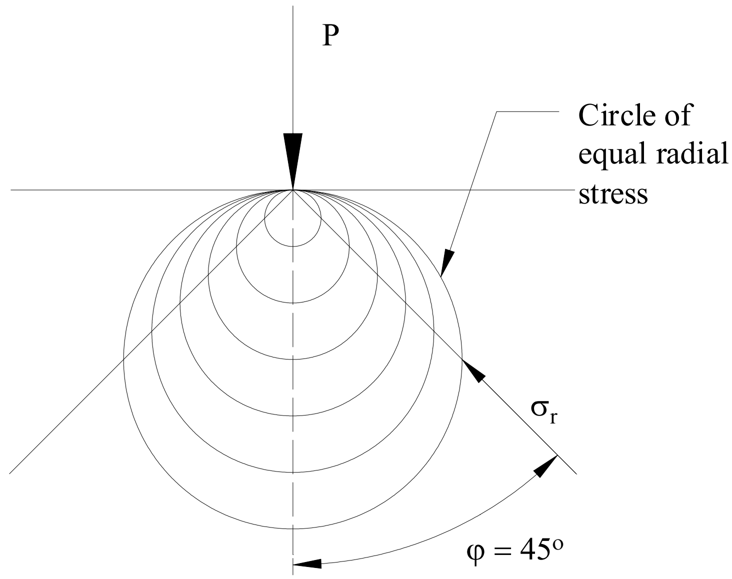

Figure 7.5: Iso-radial-stress lines (circles of equal radial stress) in a very large plate subjected to a concentrated force. Note: this is true except for the point of load application.

The equivalent area is equal to the average area under the assumed stress distribution determined by (with reference to Figure 7.6)

$$A_{equivalent,i} = spacing_i \cdot t \tag{7.27}$$

for

$$spacing_i \leq \frac{w}{2} \tag{7.28}$$

where t = member thickness and w denotes member width. However, for

$$spacing_i > \frac{w}{2} \tag{7.29}$$

the equivalent area is the weighted average area determined by (with reference to Figure 7.6)

$$A_{equivalent,i+1} = \frac{w/2}{spacing_{i+1}} \frac{w}{2} \cdot t + \frac{spacing_{i+1} - w/2}{spacing_{i+1}} \cdot w \cdot t \tag{7.30}$$

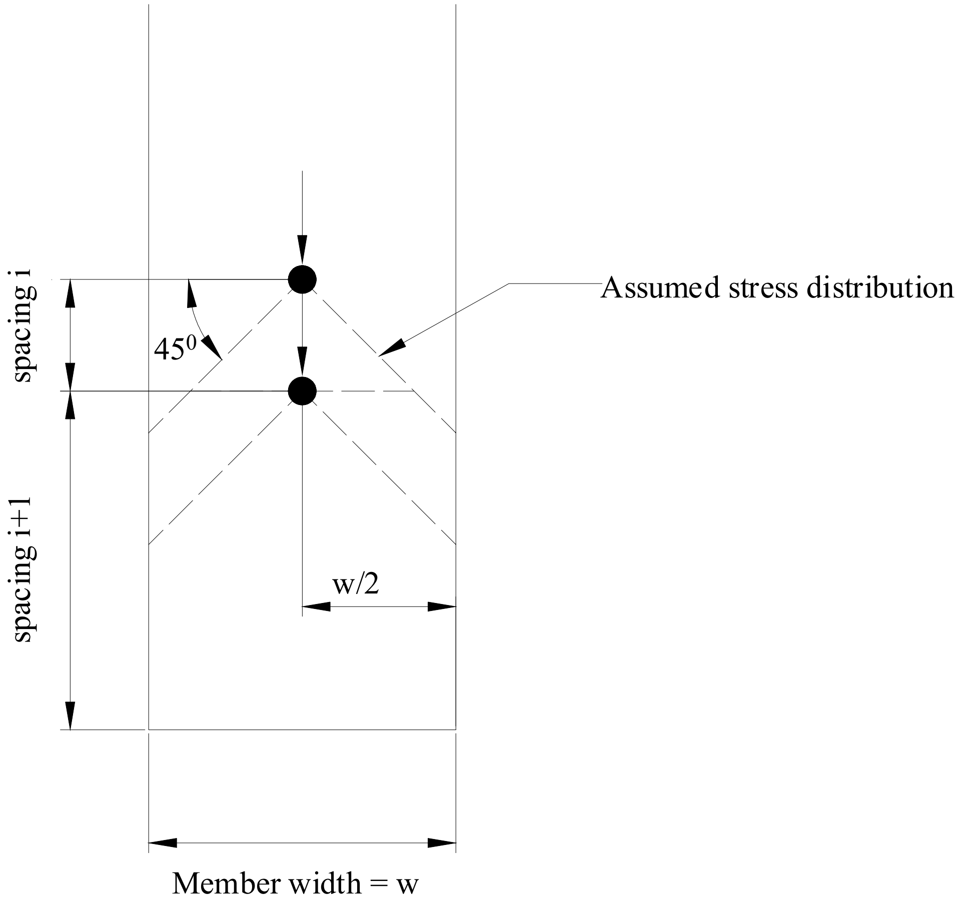

Figure 7.6: Assumed stress distribution throughout the member caused by lateral force on dowel.

With the above approach, member stiffness is a function of member width, spacing and local modulus of elasticity.