Stochastic Model

9.1 General

Not any one material can claim truly constant properties both spatially and specimen specific. Variability of properties is especially high for materials produced by nature. As a natural material, wood is subject to hugely varying "production conditions" comprising a vast amount of parameters including geographic location, soil conditions, and climate. It is therefore not surprising that specific gravity, which constitutes one of the most important properties of wood because many other properties can be derived from it, can vary up to thirtyfold among species, sixfold within a species, and even threefold within a growth ring (Bodig and Jayne 1982).

MULTBOLT includes stochastic modeling to reflect the variable performance of timber joints and to allow statistical inferences about average model predictions and average test results. With the purpose to closely resemble actual conditions, statistical sampling was added at various stages in the model. But while the random effect of many parameters and their interaction may influence joint performance, it was desired to keep the number of input parameters small and rely on correlations reported in the literature.

9.2 Statistical Sampling of Basic Material Properties

Properties of interest in this study are all correlated with specific gravity of wood. Yet rather than using specific gravity as the main input variable to model variability, the parallel-to-grain modulus of elasticity (MOE) was chosen whose correlation with specific gravity also enables the determination of other properties. The reason lies in the fact that local MOE measurements are nondestructive experiments, which are by and large easier and quicker to obtain. In addition, a wealth of MOE data along with spatial distributions are reported and are widely available.

While growth characteristics were disregarded, their effect could be included by modifying correlation functions of strength properties with stiffness. In other words, the influence of growth characteristics can be accounted for by the manual input of particular strength values per segment.

9.2.1 Generation of Modulus of Elasticity Values

MULTBOLT offers the option to vary the parallel-to-grain MOE in two ways. The user can specify a *between-member* standard deviation, which describes the variation of the average MOE of each joint member when measured for many members. In addition, MULTBOLT allows MOE to vary *within* a member using lengthwise correlation based on the average value generated from the between-member variation. Since MOE can never assume negative values, it was assumed that MOE parallel to the grain varies log-normally between members. Figure 9.1 depicts the histogram of a sample generation by MULTBOLT of average parallel-to-grain MOE values for 10,000 members of southern pine.

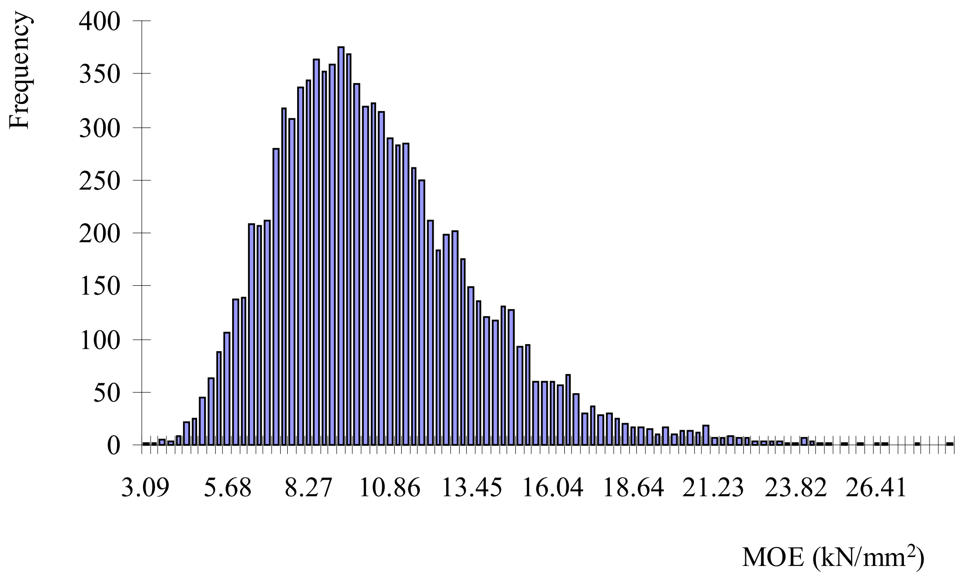

Figure 9.1: MULTBOLT generation of MOE values following a log-normal distribution.
MOE_{mean} = 10.312833 kN/mm^2, COV = 32 %.

Parallel-to-grain MOE does not vary randomly along the length of a piece of lumber. Rather, it has been established that MOE measured in one segment is correlated with the MOE measured in adjacent segments. Hence, spatial variation of MOE was modeled using the second-order Markov model as published by Kline et al. (1985). The model takes serial correlation of MOE into account and can be formulated as

$$X_{i+1} = \beta_1 \cdot X_i + \beta_2 \cdot X_{i-1} + \varepsilon_{i+1} \tag{9.1}$$

with

$$\beta_1 = \frac{\rho_1 - \rho_1 \cdot \rho_2}{1 - \rho_1^2} \tag{9.2}$$

$$\beta_2 = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} \tag{9.3}$$

and

$$\varepsilon_{i+1} = \sigma_X \cdot t \cdot \sqrt{1 - R^2} \tag{9.4}$$

		Units
X_i	MOE observation at length segment *i*	kN/mm^2
β	regression coefficient	
ε	random error	kN/mm^2
ρ	correlation coefficient	
σ_x^2	variance of observations X	(kN/mm^2)^2
R^2	coefficient of determination	
t	normally distributed random variable N(0,1)	

For a more detailed discussion of the Markov model, the reader is referred to Showalter et al. (1987), Kline at al. (1985), and Haan (1977).

With the exception of standard deviation, coefficients were used as estimated by Kline et al. It is important to realize, however, that the Markov model was developed by Kline et al. for 30-inch (762 mm) long segments of southern pine lumber. Estimated correlation coefficients as well as standard deviations and the coefficient of determination were derived from bending measurements with loading points 762 mm apart. But since bolt spacing is an input variable in MULTBOLT that can be adjusted by the user at will, the Markov model was applied in MULTBOLT on a much smaller scale. In fact, a length segment in MULTBOLT is determined by the grid spacing used to compute failure stresses (Chapter 8) and may be less than 1 mm long. **No data exist that substantiate the assumption made here that the correlation coefficients and the coefficient of determination are still valid for much smaller segments.** Therefore, the inclusion of the Markov model in MULTBOLT serves demonstration purposes only and may be used in sensitivity studies, but cannot be used reliably to predict joint performance until reliable data are available that permit the estimation of correlation coefficients for small segments.

The spatial distribution of MOE as calculated by MULTBOLT using an example 5-bolt joint member and a COV of 10 percent is demonstrated in Figure 9.2. At this point, the selected COV is arbitrary and serves to show the process. Upon generation of lengthwise MOE values, MULTBOLT computes the average MOE per segment between any two bolts and a bolt and member end, respectively. Average values are subsequently used to derive local stiffness values utilized by the structural model (Chapter 7) and local strength values employed by the failure model (Chapter 8).

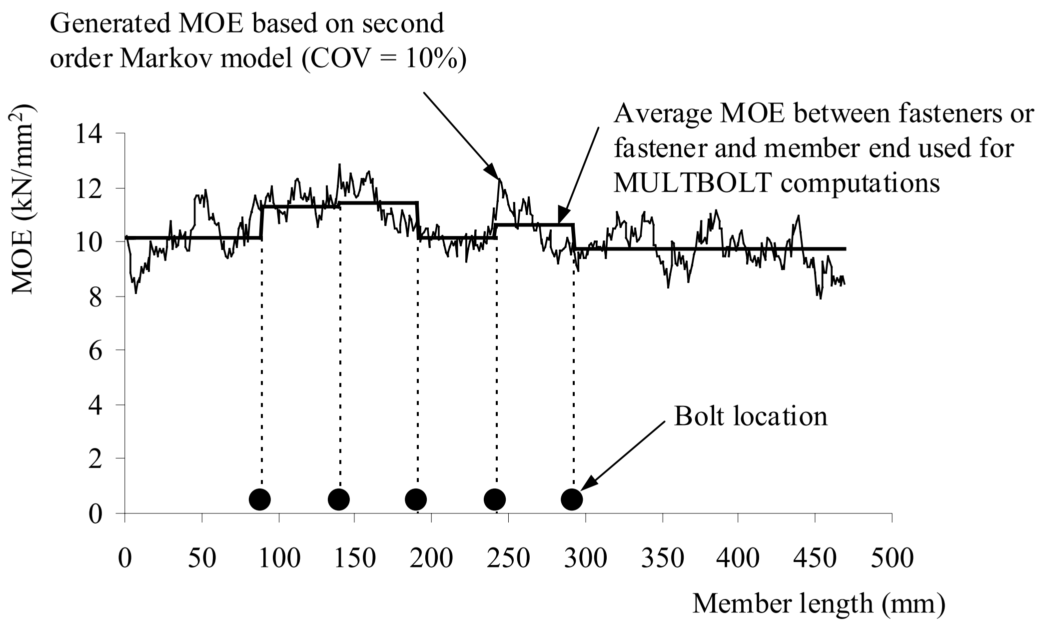

Figure 9.2: Generated MOE for 1 mm segments displayed along an example 5-bolt joint member. Computed averages for the segments between fasteners and fasteners and member end, respectively, are used to by MULTBOLT to compute joint performance. Bolt locations are depicted by black dots. $MOE_{mean} = 10.312833$ kN/mm^2.

9.2.2 Generation of Material Strength Values

Statistical sampling of material strength properties is based on regression data with specific gravity being the independent variable as published in the Wood Handbook (FPS 1999). Since MOE is positively correlated with specific gravity, all strength properties can be estimated by just knowing MOE. However, this approach does not permit the use of residuals because the correlation between residuals of any two strength properties has not been established to date. Therefore, only average estimates can be computed. To increase accuracy and insure that an average MOE yields strength values coinciding with the average strength values found in the literature for southern pine lumber (Bodig and Jayne 1982, FPS 1999)[1], the correlation relations of the Wood Handbook were offset. Thus, the resulting equations are defined as

[1] Since southern pine is a species group consisting of Loblolly Pine, Longleaf Pine, Shortleaf Pine, and Slash Pine as defined in the NDS (AF&PA 1997), values of the four species were averaged.

$$\sigma_{perp}^{T} = 11086 \cdot 10^{-8} \cdot \left(E_{para}\right)^{\frac{27.75}{21}} + \left(\hat{\sigma}_{perp}^{T} - 24203 \cdot 10^{-7}\right) \tag{9.5}$$

$$\sigma_{perp}^{C} = -58311 \cdot 10^{-9} \cdot \left(E_{para}\right)^{\frac{39.25}{21}} + \left(\hat{\sigma}_{perp}^{C} + 45688 \cdot 10^{-7}\right) \tag{9.6}$$

$$\sigma_{para}^{T} = 45186 \cdot 10^{-7} \cdot \left(E_{para}\right)^{\frac{25.25}{21}} + \left(\hat{\sigma}_{para}^{T} - 74726 \cdot 10^{-6}\right) \tag{9.7}$$

$$\sigma_{para}^{C} = -28640 \cdot 10^{-7} \cdot \left(E_{para}\right)^{\frac{24.25}{21}} + \left(\hat{\sigma}_{para}^{C} + 42382 \cdot 10^{-6}\right) \tag{9.8}$$

$$\tau^{F} = 78116 \cdot 10^{-8} \cdot \left(E_{para}\right)^{\frac{21.25}{21}} + \left(\hat{\tau}^{F} - 82828 \cdot 10^{-7}\right) \tag{9.9}$$

where

$$\hat{\sigma}_{perp}^{T} = 10667 \cdot 10^{-7} \quad \left[\frac{kN}{mm^2}\right] \tag{9.10}$$

$$\hat{\sigma}_{perp}^{C} = -20583 \cdot 10^{-7} \quad \left[\frac{kN}{mm^2}\right] \tag{9.11}$$

$$\hat{\sigma}_{para}^{T} = 325000 \cdot 10^{-7} \quad \left[\frac{kN}{mm^2}\right] \tag{9.12}$$

$$\hat{\sigma}_{para}^{C} = -88655 \cdot 10^{-7} \quad \left[\frac{kN}{mm^2}\right] \tag{9.13}$$

$$\hat{\tau}^{F} = 21236 \cdot 10^{-7} \quad \left[\frac{kN}{mm^2}\right] \tag{9.14}$$

		Units
$\sigma_{perp}{}^{T}$	tensile strength perpendicular to the grain	kN/mm^2
$\sigma_{perp}{}^{C}$	compression strength perpendicular to the grain	kN/mm^2
$\sigma_{para}{}^{T}$	tensile strength parallel to the grain	kN/mm^2
$\sigma_{para}{}^{C}$	compression strength parallel to the grain	kN/mm^2
τ^{F}	shear strength	kN/mm^2
E_{para}	modulus of elasticity parallel to the grain	kN/mm^2
$\hat{\sigma}$	published average strength	kN/mm^2

Hence, strength values are assigned the same basic distribution function as chosen for MOE. While mean and standard deviation are obviously different, the relative variation (COV) is the same.

9.3 Hysteresis Parameter Generation

The 11 hysteresis parameters are not independent and cannot be randomly generated as such. A thorough statistical approach would be to establish a covariance matrix. Here, the parameters are generated based on a data set of parameters obtained from experimental data. In other words, the program selects a parameter set from a pool of sets stored in a file similar to a lookup table. Basis for selection is the average of the four randomly generated local MOE values adjacent to each fastener (2 per member) because by nature of their computation, the hysteresis parameters "smear" the different properties of both members and around each bolt. For example, if the program generated in Member 1 a 0.75 MOE_{mean} to the left of Bolt 2 and 1.5 MOE_{mean} to the right and if for Member 2, MULTBOLT obtained 1.2 MOE_{mean} to the left and 1.3 MOE_{mean} to the right of the same bolt, the average, 1.1875 MOE_{mean}, is higher than MOE_{mean} and only hysteresis parameter sets that correlate with higher member stiffness are selected. If the Markov model is turned off, MOE values adjacent to each fastener per member equal the average MOE generated for the member.

Note that in the modified hysteresis equations (Equations 5.34 to 5.37) the product of α and ω directly determines stiffness. Thus, to select the correct set of hysteresis parameters based on variations in member stiffness, MULTBOLT computes the difference of MOE_{mean} and the average of generated MOEs adjacent to a fastener and divides the result by the between-member standard deviation. This measure is used to select a parameter set where the product of α and ω is a similar distance away from its mean in terms of standard deviations (i.e. due to discrete values of parameters stored in the data file, it is the distance that is closest to the distance computed for MOE).

9.4 Slack and Slack Variability

Unless bolt holes are precision drilled, significant variability may be introduced by manufacturing tolerances attributed to bolt holes. To facilitate assembly, holes must be drilled oversize, that is, the bolt hole diameter exceeds that of the inserted bolt. The drilling of bolt holes introduces two sources of variability. First, because of oversize, bolt location within a hole is subject to random effects. Second, bolt hole placement varies depending on the skill of the craftsman and the tools used, leading to variations in fastener spacing and consequently variability in slack. Fastener spacing and slack variability presumably affect joint performance significantly, in that individual fasteners may engage earlier with the surrounding wood than other fasteners eventually causing premature failure. It is important to point out that the assumption is made here that holes are drilled separately for each member and not, as is frequently the case, drilled through both members at once. While this practice may also introduce variability in spacing it does not yield varying bolt spacing between members, which results in the fact that total slack per bolt does not vary.

Because there are two members, each containing a hole with oversize, before the bolt touches the hole wall, *each* member can move a maximum of

$$\Delta_{\max,member} = oversize \tag{9.15}$$

in either direction. Hence, the maximum distance of translation between the two members before the bolt engages with the surrounding wood equals (Figure 9.3)

$$\Delta_{\max} = 2 \cdot oversize \tag{9.16}$$

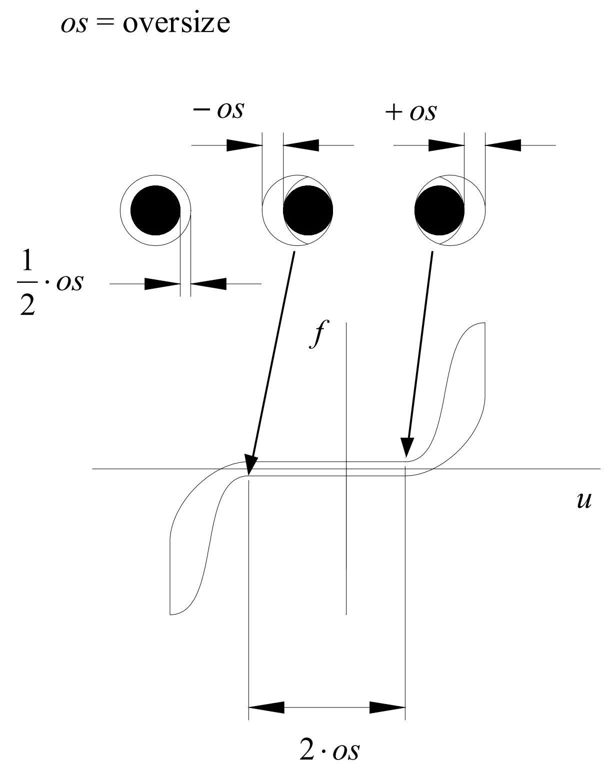

Figure 9.3: Because of relative movement, maximum slack in a two member joint, in which holes are drilled with oversize equals two times oversize.

Total slack per bolt in the direction of movement is determined by tolerances in the spacings between the bolts. The multiple-bolt model accounts for this by first allocating the spacing of member one. This is done assuming the spacing varies log-normally with a variation input by the user. The bolt locations x_{1j} for member one can then be determined by (Figure 9.4)

$$x_{1n} = \sum_{i=1}^{n} spacing_{1i} \tag{9.17}$$

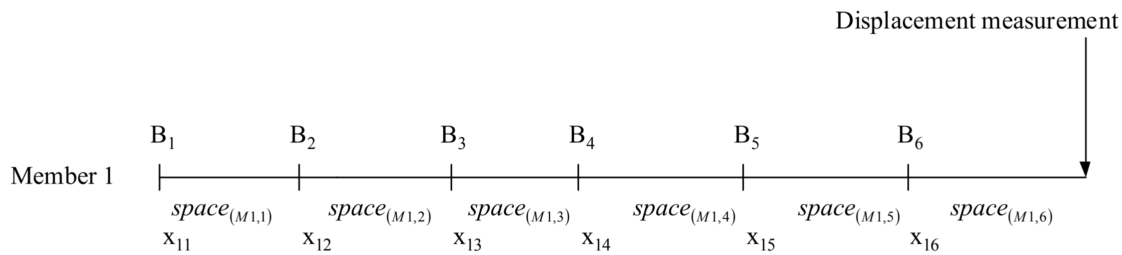

Figure 9.4: Bolt locations of Member 1 as a function of spacing tolerances.

Next, spacing of member two is computed based on the spacing of Member 1 and the maximum allowable tolerances determined by hole oversize (such that a bolt can still be inserted in both members). The slack of each bolt is then a function of the spacing tolerances (Figure 9.5) with

$$spacing_{2i} = x_{1(n+1)} - x_{1n} + \delta_n \tag{9.18}$$

and

$$\delta_n = normal\left(0, STDEV_{spacing}^2\right)\begin{Bmatrix} +oversize \\ -oversize \end{Bmatrix} \tag{9.19}$$

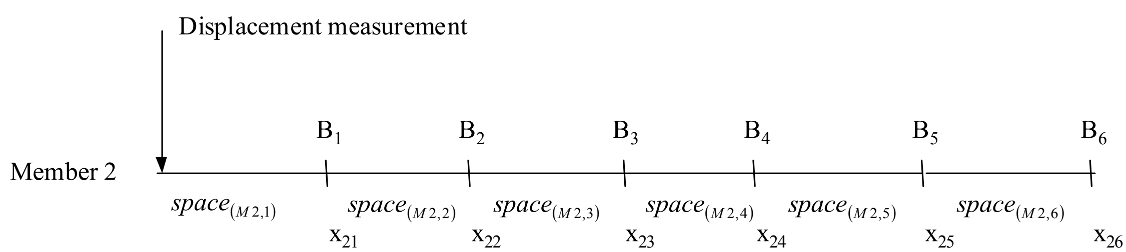

Figure 9.5: Bolt locations and spacing of Member 2 determined by bolt locations of Member 1 and normally distributed tolerance.

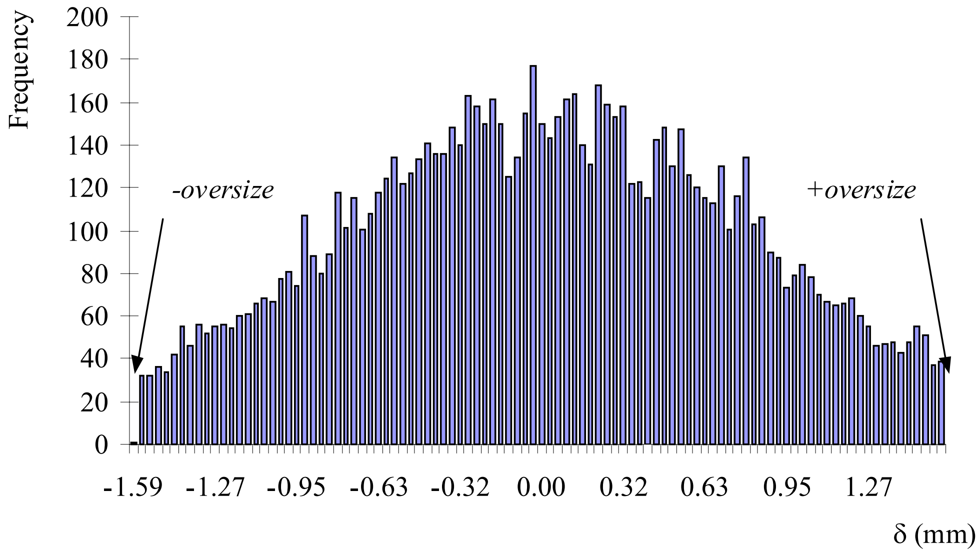

Figure 9.6: Generated random slack variable δ_n . The distribution follows a normal distribution with mean zero and standard deviation equal to that specified for spacing. To guarantee bolt penetration through both members, δ_n cannot be less than or greater than – or + *oversize*, respectively.

If the absolute location of the bolt holes in each member are measured from the free end of Member 1, then the total slack in the direction of movement can be computed no matter where the bolt is located within the two holes. This is true because total slack in any direction is the sum of the individual slacks in each member. Thus, the total slack in positive direction, $slack_p$, and total slack in negative direction, $slack_n$, are a function of (Figure 9.7)

$$slack_p = slack\_positive, Member1 + slack\_positive, Member2 = os + \left(X_2 - X_1\right) \quad (9.20)$$

$$slack_n = slack\_negative, Member1 + slack\_negative, Member2 = 2 \cdot os - slack_p \quad (9.21)$$

Where, os = oversize = d_{hole} - d_{bolt}

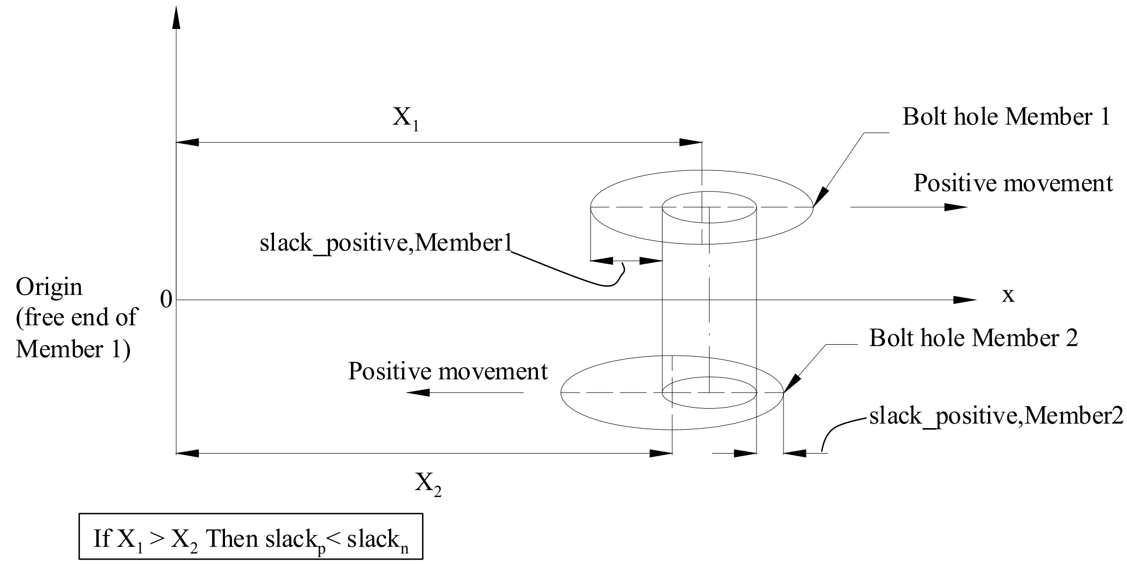

Figure 9.7: Total slack computation in the direction of movement. Total slack in any direction can be computed despite unknown exact location of the bolt within the hole.

Different slack in positive and negative directions of movement can be incorporated into the modified hysteresis model (Chapter 5) by changing the initial conditions. For example, if the bolt is located at $y(t=0) = os$ then there is a slack of

$$slack_n = -2 \cdot os \tag{9.22}$$

in the negative displacement direction. Thus, because input displacement is added, the initial displacement is set equal to

$$u(t=0) = -1 \cdot (slack_p - os)$$

In other words, to shift the slack in any direction, displacement is added in that direction. Hence, the coordinate system is not shifted but rather the input variable is modified. For example, if there is zero slack in positive direction, then the displacement starts at + *os* and the model yields an increase in load immediately. It is clear, however, that although the input variable is changed to reflect a shift in slack, load is still related to the unchanged variable. That is, model output equals

$$f(u,t) \hat{=} f(u - (slack_p - os), t) \tag{9.23}$$

which means that f is related to u not the shifted u, and when u is plotted versus t, u will start at zero.