Chapter 5

Proposed Response Prediction Methods

Previous chapters described the experimental program, analytical prediction methods, and comparisons of the predictions with measurements. The objective of this chapter is to use the results of the previous chapters to develop practical design recommendations.

5.1 Modeling and Prediction of Modal Properties

As mentioned in Chapter 3, reasonable prediction of modal properties (natural frequencies, mode shapes, and damping) is the essential first step toward a reasonable prediction of acceleration due to walking. This section presents recommendations for creating and analyzing finite element models for this purpose.

5.1.1 Defining the Structure

Models of floor structures should be defined using a combination of the methods given in DG11, Barrett (2006), and the SCI DG.

Extent of Structure to be Modeled

Structural models used for strength and stiffness determination usually encompass the entire structure or a large portion of it. In general, this is not necessary for floor vibration analysis. Exceptions would be complicated structures such as stadiums. As shown in Section 4.5.1, it is prudent to include the entire floor being analyzed or at least a large portion of it. Inclusion of only a small area (one bay in each direction, for example) around the bay being evaluated is likely to result in very significant effective mass under-predictions and subsequent very significant acceleration over-predictions. It is anticipated, however, that relatively complicated areas (such as areas with several openings) several bays away can be modeled as more typical framing areas without producing significantly different results than those predicted by a more complicated model.

Slab Shell Elements

Floor slabs should be modeled using orthotropic shell or plate elements with flexural stiffnesses computed using basic mechanics. The concrete elastic modulus should be computed using the 1.35 increase recommended in Section 3.2 of DG11 (Murray 1997). The shear modulus should be computed using a Poisson's ratio of 0.2. As indicated by Barrett (2006), the use of orthotropic stiffnesses results in more accurate frequency and mode shape predictions.

Plate elements may be a better choice than shell elements in some cases because they have fewer DOF, resulting in a less computationally intensive model. However, if they are used, the beams are not restrained by the slab and might have lateral bending frequencies in the general range of interest for floor vibrations. The beam weak-axis moment of inertia can be artificially increased to alleviate this problem.

Cracks exist over girder lines in many composite slabs, regardless of whether the construction is shored or unshored. However, insufficient research exists to provide guidance for considering the effects of these cracks, so in the writer's opinion, it is reasonable to compute slab section properties based on gross properties.

The shell mesh should be fine enough to allow convergence of the natural frequencies, which is a good finite element modeling practice in general. For the bays included in this research, 36 in. element sizes (1/10 of most of the bay sizes) were small enough, but should be verified on a case by case basis. Automatic submeshing significantly increased the convenience of creating and manipulating models included in this research and is highly recommended in general. Of course, frame elements must be meshed to provide nodes that coincide with the shell nodes.

Steel Members

Several options exist for representing beams and girders in finite element models. The simplest approach is to place frame elements, having transformed moments of inertia computed using DG11, in the same plane as the shell elements without double-counting the bending stiffness of the shell elements (subtract the slab moment of inertia about its own axis during the transformed section moment of inertia calculation). This approach was used in four of the six floors included in this research, is consistent with the SCI DG and Barrett (2006), and is recommended for general use in systems such as the ones included in this research. If trusses are included in the floor system, then offsetting the beam from the slab centroid is recommended as described in the SCI DG Section 6.1.2. This method was used during analysis of the two open web joist footbridges included in this research and produced very accurate frequency and mode shape predictions. However, it is more cumbersome than placing the frame element in the same plane as the shells. It also requires the use of rigid links or nodal constraints in most commercially available programs, increasing the complexity and time required to create the model. Note also that this approach does not work if the slab is represented by plate elements instead of shell elements because plate elements do not have in-plane translational degrees of freedom.

Beam and girder ends should be continuously connected regardless of the type of connection that existed in reality. This is consistent with finite element modeling recommendations of the SCI DG and is reasonable because floor vibration due to human walking results in forces that are insufficient to cause bolt slip or significant deformation at shear connections.

Columns may be included in the model and extended to halfway below and above the floor being modeled. This is also consistent with the finite element modeling recommendations of the SCI DG. The assumption is that the mode shapes will have the columns in double curvature, so inflection points are usually located halfway below and above the floor being analyzed.

Spandrel members at typical curtainwall or steel stud backed brick or EIFS (Exterior Insulation and Finish System) cladding should be assumed to be 2.5 times stiffer than computed, per the recommendations of Barrett (2006). Barrett's buildings had cladding that was more complete than the ones included in this research so are more representative of service conditions. This is not consistent with the SCI DG, however, which recommends that spandrels should be fully restrained by continuous cladding. It was shown by Barrett (2006) that mode shapes are less accurately predicted if the spandrel is over-restrained. For some systems, precast panels for example, it seems advisable to directly model the cladding as frame or shell elements.

Masses

Masses from sources such as flooring, partitions, and ductwork should be defined in the model as accurately as possible, rather than over-estimated which is the typical safe practice used during strength and stiffness calculations. Masses from live loads should be extracted from DG11—typical values are around 10 psf rather than the 50 psf to 80 psf that is typically used for strength and stiffness design calculations. Modern analysis programs usually offer several equivalent options for assigning the mass. A common approach is to compute a shell material density that results in an area mass equivalent to the slab mass plus any superimposed masses that are expected to exist on or under the slab (Perry 2003, Barrett 2006). This approach has the advantage of being permanently set in the program after it is defined—any shell used will have the correct mass. The practical disadvantage, however, is that the fictitious density is usually a number that has no intuitive meaning to anyone checking the model at a later date. A second approach is to create mass-less shell elements, superimpose a uniform load (psf), and direct the program to use this load to define the mass. The advantage of this method is ease of checking the input because the load is applied in a unit with great intuitive meaning to structural engineers. This second method is recommended in general. Of course, in all cases, the analysis program should compute the steel member masses using the cross-sectional area and density. Spandrel loads should be applied as line loads or masses.

5.1.2 Prediction of Natural Frequencies and Mode Shapes

Natural frequencies and mode shapes should be predicted by solving the very well known MDOF eigenvalue problem, which is valid for undamped and proportionally damped systems. Measured and predicted natural frequencies are listed in Chapter 4 and summarized in Figure 5.1. As seen in Chapter 4, the measured vibration modes for the floor structures included in this research were mostly quasi-real, so the assumption of proportional damping is acceptable. However, if dampers are used, providing a very uneven distribution of damping through a floor, then the system might contain very significant complexity, which invalidates the use of undamped modes.

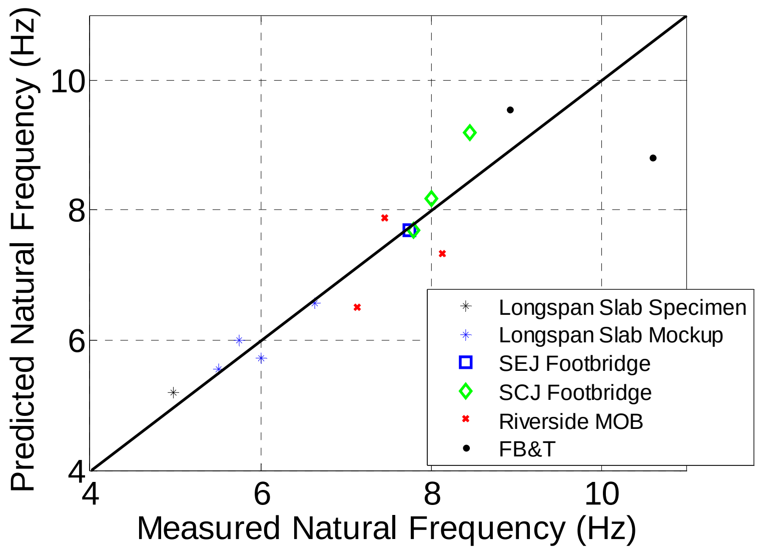

Figure 5.1: Predicted and Measured Natural Frequencies

The number of modes should be selected to provide specific frequency content in the subsequent acceleration predictions. Humans are most sensitive to vibrations in the 4-8 Hz range, with sensitivity decreasing approximately linearly outside that bandwidth (Murray et al. 1997, Smith et al. 2007). Therefore, for low frequency floors such as the ones included in this research, it seems prudent to include frequency content up to 15 Hz, a somewhat arbitrary limit. In each bay included in this research, the vast majority of the response to walking was attributed to one or two low frequency modes, so it is anticipated that analysis results will not be sensitive to the choice of upper limit. Also, as shown in Chapter 4 and reported by Pavic et al. (2007), finite element models have very limited success predicting numerous modes in the correct order and frequency spacing, so it is likely that most or all of the higher frequency modes will not correspond with modes in the actual structure anyway. Finally, the higher frequency modes in a low frequency floor are inevitably double curvature in some or all bays, so have little effect on the acceleration at midspan. Another general approach is to include many more modes and frequency weight the response as described in the SCI DG (Smith et al. 2007) Section 5.2.2. In the writer’s opinion, this approach adds significant complexity for little or no benefit as compared to simply including only the lower frequency modes.

5.1.3 Damping

All of the specimens included in this research were bare slabs, so it is not possible to provide comprehensive guidance on the subject of damping. DG11 Table 4.1 and the SCI DG Table 4.1 provide recommendations for common conditions. For bare slabs, these two documents recommend critical viscous damping ratios of 1.0% and 1.1%, respectively. However, measured damping ratios for the tested bare slabs were found to vary between 0.17% of critical and 1.5% of critical. The majority of the specimens had damping ratios somewhat close to 0.5% of critical, so this ratio is recommended for bare slabs.

5.2 Accelerance FRF Magnitude Prediction

It is usually somewhat difficult, and sometimes impossible, to determine which mode or modes will be responsive in a given bay simply by looking at the mode shapes. Therefore, an estimate of the FRF magnitude for load (1 lbf) and response at the center of the bay under consideration is also a necessity. SAP2000, in particular, provides a convenient facility for predicting the FRF: Steady State Analysis. The prediction is composed of a real and an imaginary part at each spectral line. The magnitude of this complex number (square root, sum of squares of real and imaginary parts) at each line is the FRF magnitude. The frequency bandwidth should surround all natural frequencies that can be excited by the first four harmonics of walking, typically from 3 Hz to 9 Hz. The number of intermediate increments can be large enough to provide a plot of the FRF, if desired, but more likely will include only the modal frequencies because this type of frequency domain analysis is computationally intensive. The plot can be scaled to display the FRF in intuitive engineering units such as %g/lbf. Note also that some programs may provide response history analysis capability, but not have the ability to perform steady-state analysis. It is also possible, but more time consuming, to semi-manually plot the FRF by performing response history analyses using a 1 lbf sinusoidal load with frequency matching each natural frequency.

The predicted FRFs are used for two purposes: prediction of which modes will be responsive in a given bay and subsequent prediction of the walking acceleration using the Simplified Frequency Domain Method described in Chapter 3. The predicted FRFs are very useful because finite element models of floor systems usually predict several, if not dozens, of vibration modes.

SAP2000's steady-state analysis requires the use of hysteretic damping instead of viscous damping. However, as described in Chapter 3, hysteretic damping equal to double the viscous damping results in the same response at natural frequencies. For example, if the viscous damping ratio is 0.005 at a natural mode, a hysteretic damping ratio of 0.01 results in an equivalent acceleration response at that modal frequency. In general, damping will not be known, but will have to be assumed as described in Section 5.1.3. These ratios are critical viscous damping ratios, so must be converted to hysteretic damping ratios.

Table 5.1 shows comparisons of measured and predicted accelerance FRF peak magnitudes for all specimens. The predicted magnitudes were computed using the methods and assumptions described in this section. The table indicates wide data dispersion, more so than shown in Table 4.16 due to the use of assumed 0.5% critical viscous damping instead of measured critical damping ratios. The comparisons are also shown in Figure 5.2.

Table 5.1: Comparison of Measured and Predicted FRF Magnitudes Summary of Accelerance Peak Magnitude Comparisons

		Accelerance Peak Magnitude		
Specimen	Description	Measured (%g/lbf)	Predicted (%g/lbf)	Predicted / Measured
Long Span Composite Slab Specimen	Mode 1	0.364	0.381	1.05
Long Span Composite Slab Mockup	Bay 1, Mode 1	0.153	0.165	1.08
	Bay 1, Mode 2	0.0680	0.169	2.49
	Bay 2, Mode 1	0.162	0.166	1.02
	Bay 2, Mode 2	0.0638	0.182	2.85
Square-End Joist Footbridge	Mode 1	2.48	1.33	0.536
Shear-Connected Joist Footbridge	End Bay, Mode 1	1.57	0.507	0.323
	End Bay, Mode 2	2.03	0.730	0.360
	End Bay, Mode 3	0.313	0.262	0.837
	Middle Bay, Mode 1	1.37	0.481	0.351
	Middle Bay, Mode 3	1.55	1.04	0.671
Riverside MOB	Bay 1	0.151	0.156	1.03
	Bay 2	0.128	0.240	1.88
	Bay 3	0.118	0.141	1.19
	Bay 4	0.0770	0.206	2.68
First Bank & Trust Building	Bay 1	0.181	1.20	6.63
	Bay 2	0.261	1.04	3.98
	Bay 3	0.162	1.22	7.58
			Average =	2.02
			COV =	101%
			Avg Excluding FB&T =	1.22
			COV Excluding FB&T =	67.3%

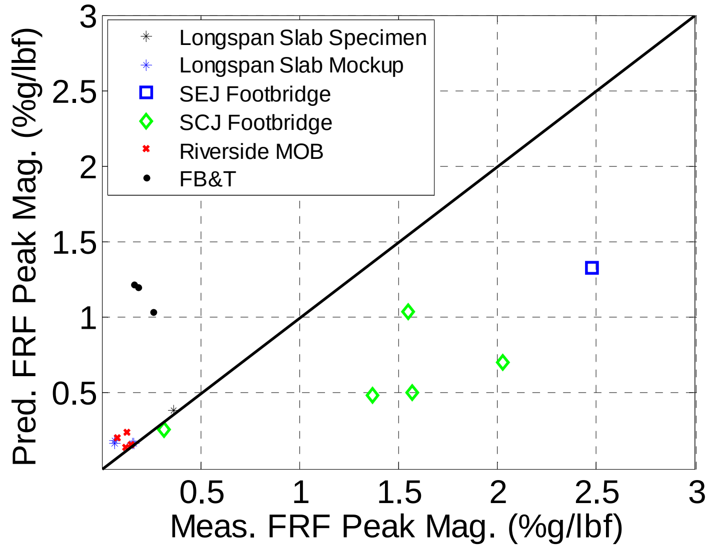

Figure 5.2: Measured and Predicted FRF Peak Magnitudes

5.3 Predicting Acceleration Due to Walking (Individual Footsteps)

Response history analysis using individual footsteps as the load, as described in detail in Chapter 3, is summarized as follows: Apply individual footsteps to the model at locations approximately where they will land at times selected to cause resonance with one of the predicted natural modes. Perform a linear response history analysis to predict the acceleration waveform that results from the individually applied footstep series. The method is conservative because the electronic walker walks precisely at resonance, as discussed in Chapter 4. An adjustment factor is proposed to bring the predictions into better agreement with measured responses. The analysis results in a prediction of the maximum acceleration response that can be reasonably expected to occur due to a person walking on the slab.

5.3.1 Loading Functions

The footsteps, as discussed in Chapter 2, were selected to provide specific frequency contents, so result in consistent acceleration predictions when applied in models. Figure 5.3 shows the footstep frequency contents presented by Willford et al. (2007). The "Design Value" DLFs represent a 25% probability of exceedance and were used to define the footsteps presented in Chapter 2. Note that footsteps corresponding to the "Mean Value" DLFs shown in Figure 5.3 were used in the Chapter 4 comparisons. Conservatism is appropriate for design use, so the "Design Value" footsteps are recommended for that purpose. Also, actual bodyweights were used in Chapter 4. For design purposes, an average bodyweight is appropriate. The SCI DG (Smith et al. 2007) recommends 168 lbf, a value that is recommended here also.

It is important that a footstep be applied at approximately its reference frequency. For example, the 120 bpm design footstep should only be applied at approximately 120 bpm and will have significantly different frequency content if applied at 110 bpm or 130 bpm. Frequency content does not significantly change over a range of approximately 2-3 bpm, so steps were defined at 5 bpm increments. For example, if the desired step frequency in the model is 127 bpm, the 125 bpm footstep would be used as the loading function. This requires several different footsteps to cover the considered range of step frequencies: 1.6 Hz to 2.2 Hz (Murray et al. 1997).

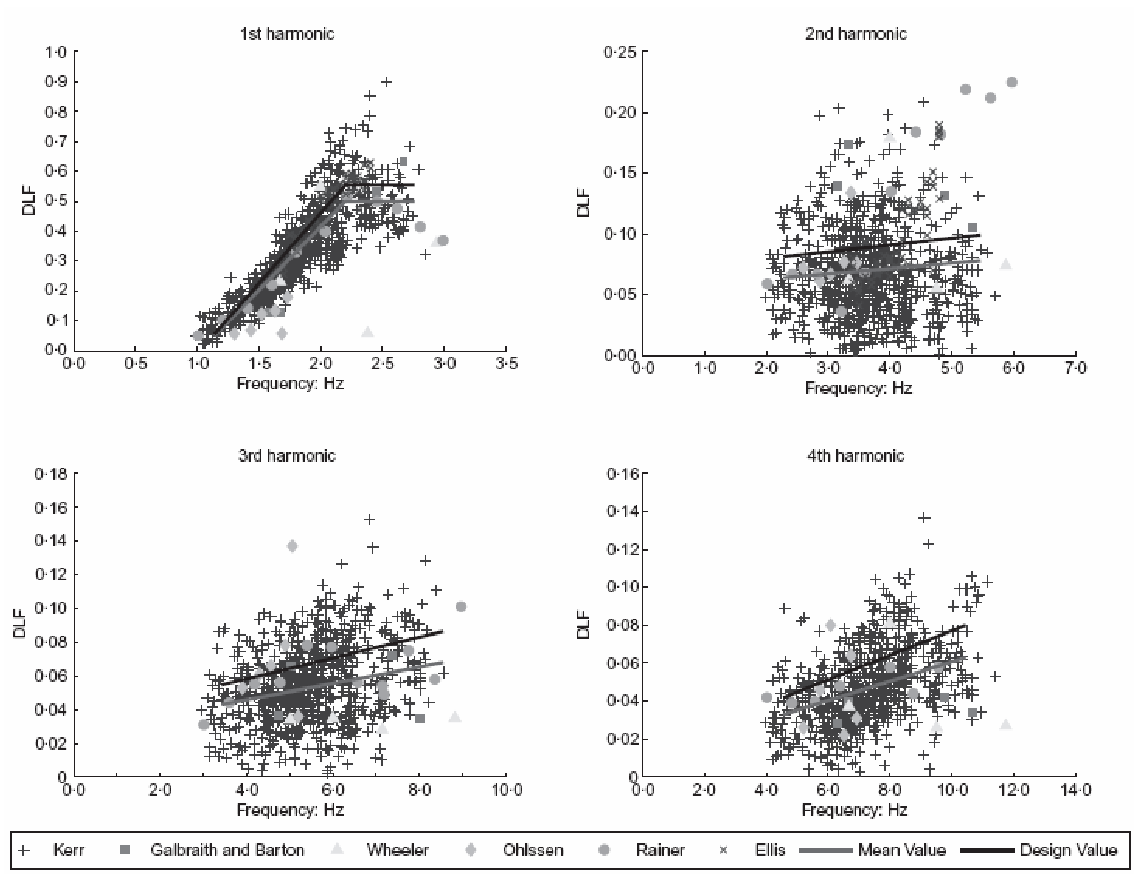

Figure 5.3: Footstep Force Harmonics (Willford et al. 2007)

5.3.2 Force Application

Footsteps should be applied at natural frequency subharmonics (integer divisions) of any modes that are responsive in the bay under consideration. The step frequency should be selected to be in the range of 1.6 Hz to 2.2 Hz (96 bpm to 132 bpm).

Consider the predicted FRF shown in Figure 5.4. In Figure 5.4(a), the 7.12 Hz and 7.34 Hz modes are approximately equally responsive in the bay under consideration. Two analyses should be performed: (1) at a step frequency equal to 7.12 Hz / 4 = 1.78 Hz, which will excite the 7.12 Hz mode using the fourth harmonic of the walking force, and (2) at a step frequency of 7.34 Hz / 4 = 1.84 Hz, which will excite the 7.34 Hz mode.

As another example, consider the predicted FRF shown in Figure 5.5. The 6.50 Hz and 7.88 Hz modes are both responsive in this bay. In this case, the third harmonic of the walking force can excite the 6.50 Hz mode whereas the fourth harmonic of the walking force is required to excite the 7.88 Hz mode. The third harmonic of the walking force is larger than the fourth harmonic. Therefore, separate analyses should be performed, exciting each mode, although it is expected that the 7.88 Hz mode will provide higher response to walking because its FRF peak magnitude is much higher than the 6.50 Hz FRF peak magnitude. The two cases should have the following step frequencies: 6.50 Hz / 3 = 2.17 Hz and 7.88 Hz / 4 = 1.97 Hz, respectively.

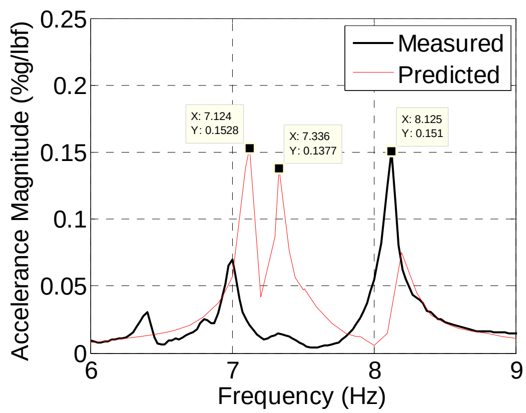

Figure 5.4: Sample FRF Magnitude

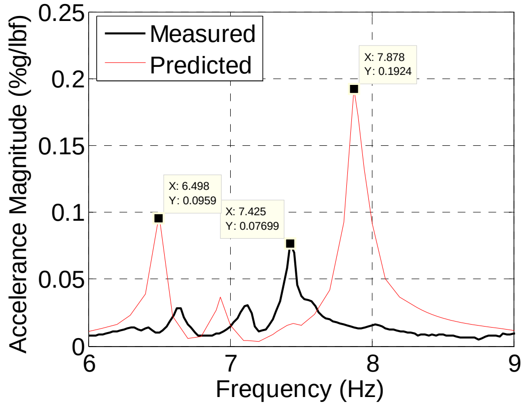

Figure 5.5: Sample FRF Magnitude

Upon selection of the step frequency, the appropriate footstep forcing function from Section 2.4.1 should be selected and applied in the models at specific arrival times to result in the appropriate step frequency. For example, to cause a 1.97 Hz (118 bpm) step frequency, steps would be applied in the model at 0.5076 sec. increments (0 sec., 0.5076 sec., 1.0152 sec., etc.).

Footsteps should be applied in the model to nodes at approximate footstep locations, which are assumed to be spaced at approximately the average human stride length. Walking paths will be obvious in some cases (corridor), but in many cases, it will not be known in advance. (It is also possible that partitions will be re-arranged during future remodeling to create unanticipated walking paths.) In these cases, the walking path should be across the bay being checked. It was observed during this research that resonant build-ups are seldom, if ever, more than a few steps long. Therefore, for very long bays, an upper limit path length of 25 ft to 30 ft is reasonable. The mean stride length is approximately 26-30 in. (Pachi and Ji 2005, Zivanovic et al. 2007). Often, nodes do not exist in the model at approximately this spacing. Four options exist for situations such as this: further subdivide the shells to provide nodes at approximately the required locations, use the coarser mesh and individually move specific nodes to appropriate locations, use line elements and apply the load as a member load instead, or apply each step at existing nodes that are as close as possible to the step's idealized location. Small step location discrepancies are not considered by the writer to be significant because the acceleration response at midspan will not be significantly different if a footstep is applied at its idealized location or a foot or two away. For further discussion of this topic, see Section 3.3.1.

5.3.3 Response History Analysis

Response history (time history) analysis should be used to predict the model's response to the series of applied footsteps. The following aspects of response history analysis are discussed in this section: choice of solution method, time step size, number of modes to include, and damping.

Two basic types of response history analysis are available in current readily available structural analysis software: modal superposition and direct integration. As discussed in Chapter 3, modal superposition is suitable for the response history analyses recommended in this research. The main advantages are computational efficiency, simplicity, and the fact that inclusion of only a certain number of modes serves as a low pass filter, excluding frequency content above the frequency of the highest computed mode. Linear analyses should be used because the amplitude of load and displacement will be very small due to walking excitation. SAP2000, in particular, offers the option of considering the load as a one-time event, including transient effects, or a repeated periodic event with all transient effects damped out. The analysis should be performed as a transient, one-time event.

For low frequency floors such as the ones included in this research, the time step size can be set to 0.005 sec, as described in Chapter 3. Coarser time domain resolutions can often be used if it is verified that waveform peaks are not missed. This will usually be possible because the vast majority of the response will almost always be attributed to a single vibration mode with a frequency far below 15 Hz. However, linear response history analysis using modal superposition is extremely computationally efficient, so refinement of the time step size will not usually result in significant time savings.

The number of output time steps should simply be selected to simply provide a long enough record for the electronic walker to traverse the walking path. For example, if 5 sec. are required for the electronic walker to traverse the walking path, then the number of output time steps should be 5 sec. / 0.005 sec. = 1000.

Constant modal viscous damping ratios, per Section 5.1.3, should be used. The choice of modal damping ratio is very important, but has far less significance for response walking than for steady-state response to sinusoidal excitation. This is due to the very incomplete resonant build-up which occurs during a small number of steps applied at resonance. For example, consider the predicted acceleration waveforms shown in Figure 5.6 These were computed using the First Bank & Trust model, assuming damping ratios ranging from 0.005 to 0.05. The ratio of steady-state acceleration response to sinusoidal load, considering a tenfold increase in damping, is 10. However, the ratio of acceleration due to walking, considering the same increase in damping, is only 4.3. In reality, the difference is probably far less extreme than that considering that resonant build-ups observed in this research only occur over a few steps rather than the approximately 10 step build-ups predicted by the model. To illustrate the consequences of inaccurate damping assumption, consider the response if the damping ratio is assumed to be 3% when in reality, it is 2%. Using 2% and 3% damping, respectively, the peak accelerations are predicted to be 4.23%g and 3.31%g, a 30% difference. Considering the coarse nature of prediction of response to walking, this is a relatively small difference. As indicated in Chapter 4, currently unavoidable errors in mode shape prediction cause much larger errors than this.

As indicated in Chapter 4, predicted accelerations are usually on the conservative side due to over-prediction of resonant build-up. Rarely was a resonant build-up of more than a few steps measured, whereas build-ups continued for the entire response history in most of the predictions. In a few cases, accelerations were very significantly over-predicted due to effective mass under-prediction (seen as accelerance FRF peak magnitude over-prediction). Excluding cases with very significant FRF peak magnitude over-prediction, the average ratio of predicted and measured peak accelerations was 1.18. Therefore, it is recommended that the peak acceleration predicted using response history analysis be multiplied by 0.85 which is approximately equal to 1/1.18.

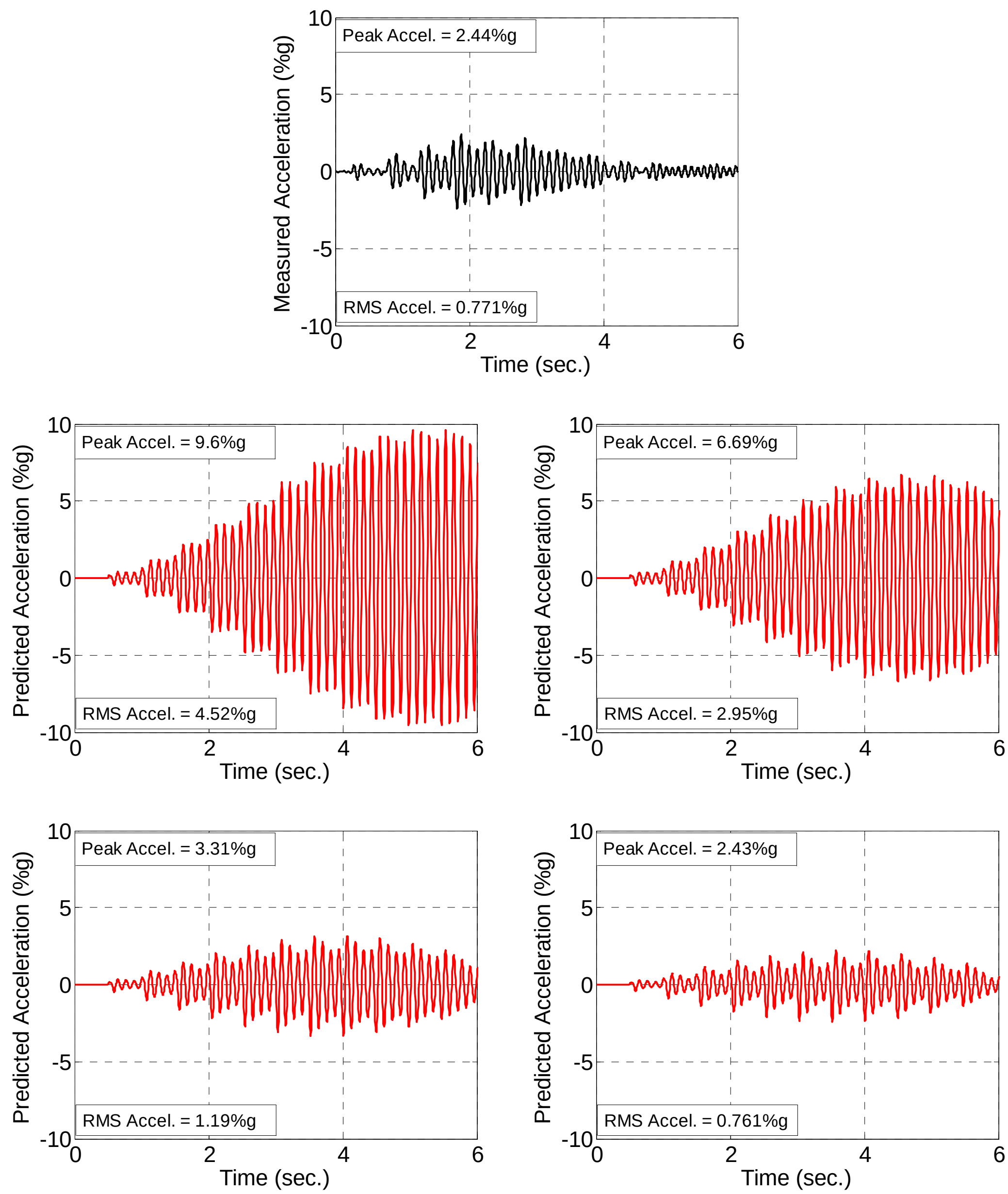

Figure 5.6: Sample Waveforms Illustrating Effect of Damping; (a) Measured, 1.3% Critical Damping, (b) Predicted, 0.5% Critical Damping, (c) 1% Critical Damping, (d) 3% Critical Damping, (e) 5% Critical Damping.

Table 5.2 shows comparisons of measured and predicted acceleration due to walking for all specimens. Figure 5.7 also summarizes the comparisons. The predicted peak accelerations were computed using the methods and assumptions described in this section. The ratios of predicted-to-measured accelerations range between 0.657 and 3.34 with an average of 1.30. The COV is 54.3%, indicating significant dispersion. Shading in the table indicates that the accelerance FRF magnitude was very significantly over-predicted in the bay under consideration. If these outlying values are excluded to illustrate the effect of inaccurate FRF magnitude prediction (stemming from inaccurate mode shape prediction), the ratios range between 0.657 and 1.61 with an average of 0.973 and COV of 30.0%. The table indicates that the proposed method reasonably accurately predicts the peak acceleration. Significant variability does exist, however, but this should be expected considering the variability in footstep forces, damping, and numerous other variables.

Interestingly, the writer's preconception was that variability in the walking force would cause the most extreme prediction errors, especially considering the dispersion shown in Figure 5.3. For example, from viewing the third and fourth harmonic DLF plots in that figure, one might expect the walking force to vary by many multiples, perhaps up to 8 or 10 times. The consistency in response appears to contradict the notion that frequency contents are this variable, however. The largest errors, by a very wide margin, were caused by incorrect prediction of mode shapes, leading to under-prediction of effective mass, which leads to very significant over-prediction of response in the bay under consideration.

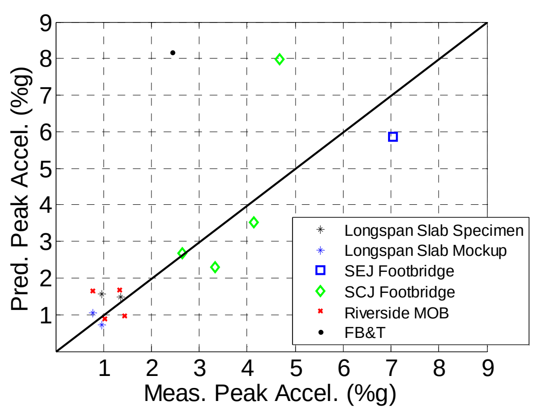

Figure 5.7: Measured and Predicted Peak Accelerations—Individual Footsteps Method

Table 5.2: Summary of Walking Acceleration Response Comparisons (Predictions Using Individual Footsteps)

		Peak Acceleration Due to Walking		
Specimen	Description	Measured (%g)	Predicted (%g)	Predicted / Measured
Long Span Composite Slab Specimen	Parallel to Deck	1.36	1.49	1.10
	Perpendicular to Deck	0.972	1.56	1.61
Long Span Composite Slab Mockup	Bay 1	0.768	1.05	1.37
	Bay 2	0.966	0.723	0.748
Square-End Joist Footbridge	-	7.03	5.87	0.835
Shear-Connected Joist Footbridge	End Bay, Mode 1	3.32	2.30	0.693
	End Bay, Mode 2	4.14	3.53	0.853
	Middle Bay, Mode 1	2.64	2.68	1.02
	Middle Bay, Mode 3	4.67	7.99	1.71
Riverside MOB	Bay 1	1.45	0.952	0.657
	Bay 2	1.33	1.67	1.26
	Bay 3	1.03	0.884	0.858
	Bay 4	0.775	1.64	2.17
First Bank & Trust Building	Bay C-1/F-2	2.44	8.16	3.34
			Average =	1.30
			COV =	54.3%
			Average Excluding Shaded Values =	0.973
			COV Excluding Shaded Values =	30.0%

5.3.4 Response History with Individual Footsteps Method—Step-by-Step Procedure

This sub-section offers a step-by-step procedure for predicting the acceleration response in a given floor bay using the method proposed above.

- Create a finite element model of the floor system using the guidance given in Section 5.1.1. The model should encompass the entire floor or at least a large portion of it.

- Perform an eigenvalue analysis to determine natural frequencies and mode shapes. These will form the basis for the subsequent analyses to predict the response to walking, so include only the modes in the frequency range of interest, usually 1 Hz to 15 Hz for low frequency floors.
- Preferably perform a frequency domain analysis to predict the FRF magnitude per Section 5.2. The FRF magnitude indicates which natural modes will provide significant response to walking excitation. If this option is not available, then the mode shapes must be used to judge which modes will be responsive in the bay under consideration.
- For each responsive mode, determine the step frequency of a walking case that will cause resonance. The step frequency must fall into the reasonable range of step frequencies: 1.6 Hz (96 bpm) to 2.2 Hz (132 bpm). Select the design footstep force waveform corresponding to the step frequency. For example, if the step frequency is 118 bpm, select the 120 bpm footstep.
- Create an analysis case with the design footstep applied at successive nodes in the model approximating actual footstep locations. Walking paths should either be along known paths such as corridors or through the middle of the bay under consideration. The maximum walking path length should be 25 ft to 30 ft. Assume an average stride length of 26 in. to 30 in., although the footstep locations can be approximately determined without causing significant errors. Apply the loads at arrival times that result in the step frequency determined in the previous step. For example, if the step frequency is 120 bpm, footsteps are applied every 0.5 sec.
- Perform a linear (modal superposition) response history analysis using the methods recommended in Section 5.3.3 to predict the acceleration waveform. Multiply the response by 0.85.

5.4 Predicting Acceleration Due to Walking (Fourier Series)

Response history analysis using Fourier series representation of the load applied by a walker is described in detail in Chapter 3 and is summarized as follows: Apply a four term Fourier series at midspan of the bay under consideration. Choose the four harmonic frequencies such that one of them matches a natural frequency to be excited, therefore causing resonance. Perform a linear response history analysis to predict the acceleration waveform that results from the applied load. This method is less direct than the one described in Section 5.3, but has the advantage of being easier to apply in some analysis programs. Because the load is applied at midspan and at resonance, this method is also expected to produce conservative solutions as shown in Chapter 4. An adjustment factor is proposed, based on the results presented in Chapter 4, to bring the predictions into better agreement with measured responses. The analysis results in a prediction of the maximum acceleration response that can be reasonably expected to occur due to a person walking on the slab.

5.4.1 Loading Functions

The four term Fourier series is defined using the "Design Value" frequency contents from Willford et al. (2007), which correspond to a 25% probability of exceedance, as shown in Figure 5.8 through Figure 5.11. Each figure shows the linear equation that defines the "Design Value" DLFs and the average over the range of frequencies that the DLF can be applied. Fortunately, the DLFs do not vary greatly over these relatively small frequency ranges. Therefore, there are two options for defining the Fourier series amplitudes: compute the DLF at the step frequency or use the average (0.4, 0.09, 0.07, and 0.06 for the four harmonics, respectively). Comparisons of results using both approaches are presented later in this section.

Phase angles are also required to fully define the four term series. These are taken from the SCI DG Table 3.1: 0, $-\pi/2$, π, and $\pi/2$ for the four harmonics, respectively.

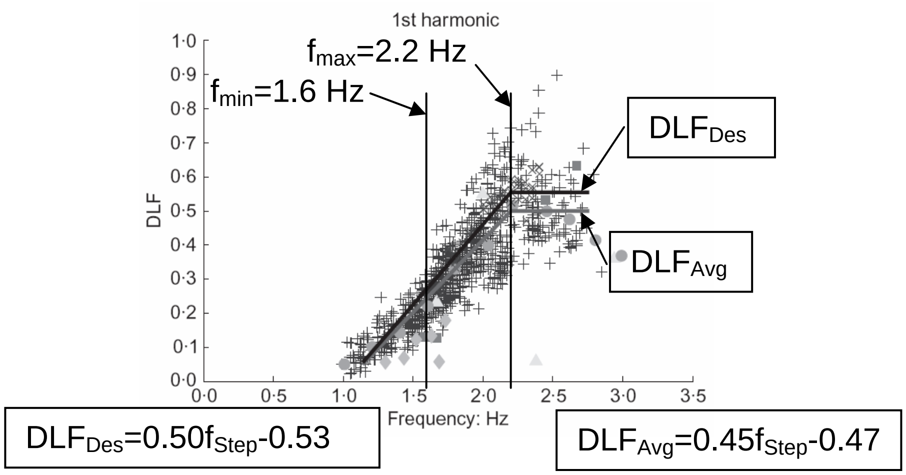

Figure 5.8: First Harmonic Amplitudes (Willford et al. 2007)

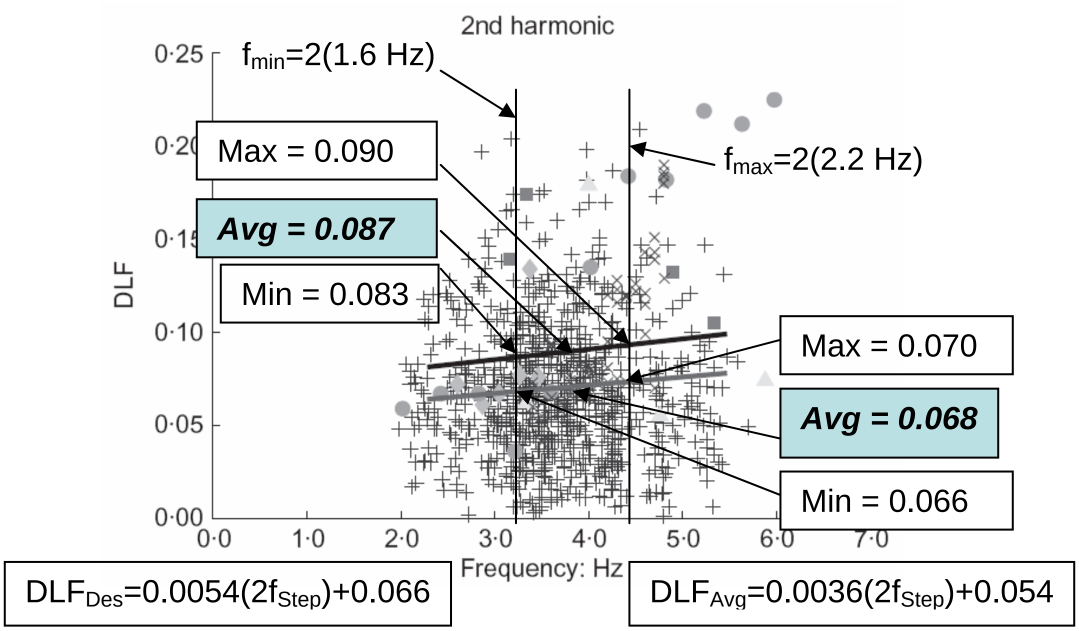

Figure 5.9: Second Harmonic Amplitudes (Willford et al. 2007)

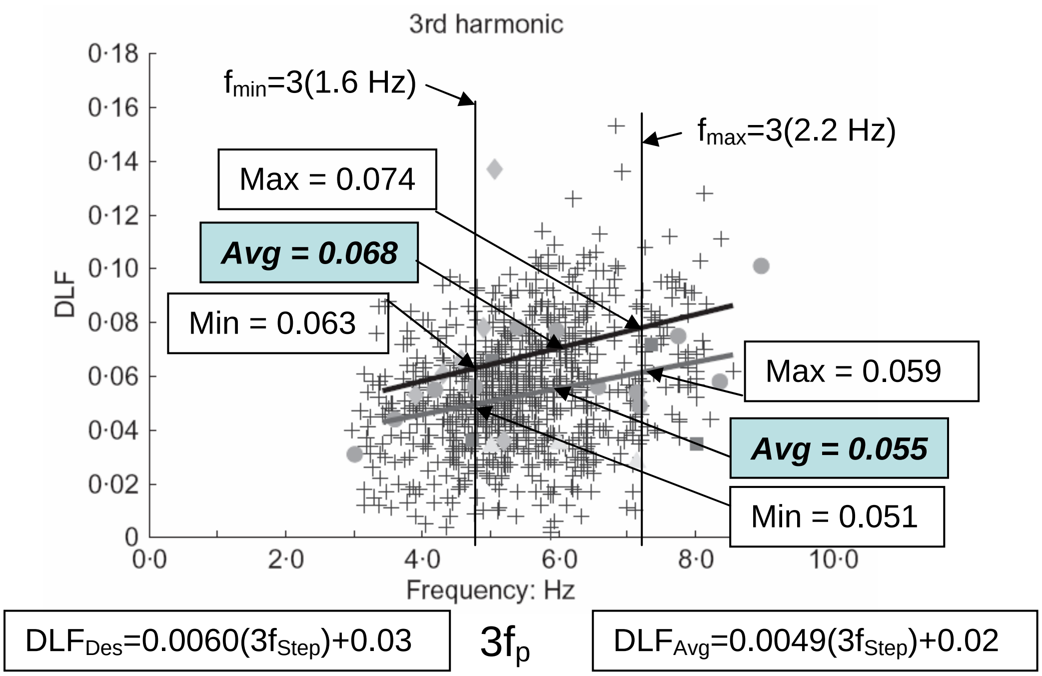

Figure 5.10: Third Harmonic Amplitudes (Willford et al. 2007)

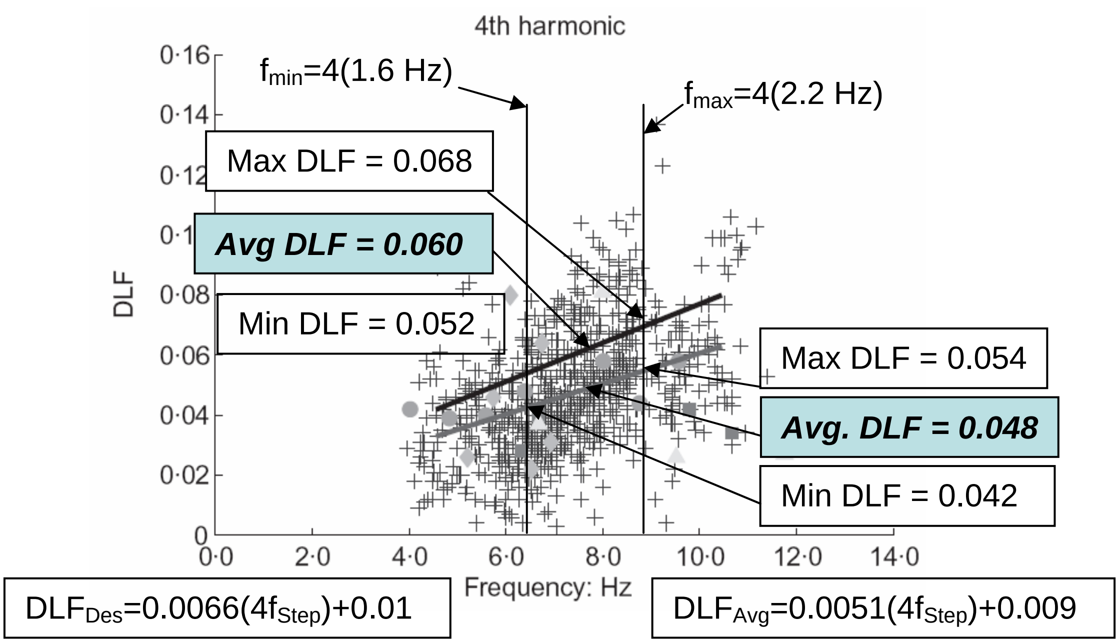

Figure 5.11: Fourth Harmonic Amplitudes (Willford et al. 2007)

5.4.2 Force Application

The Fourier series load is applied at the middle of the bay under consideration. The series is determined by first observing which harmonic can match the natural frequency to be excited. The lowest harmonic that can match the natural frequency is used to cause resonance because the harmonic forces decrease with increasing harmonic as indicated in the previous section. For example, if the natural frequency is 6.0 Hz, the third harmonic of the walking force can match the natural frequency because 6.0 Hz / 3 = 2.0 Hz which is in the considered step frequency range (1.6 Hz to 2.2 Hz). The second harmonic cannot match the natural frequency because 6.0 Hz / 3 = 3.0 Hz which is outside the range of step frequencies that are considered to be reasonable. The first through fourth harmonics of the Fourier series in this case would have the following frequencies: 2.0 Hz, 4.0 Hz, 6.0 Hz (matching the natural frequency and causing resonance), and 8.0 Hz. The next step is to determine the DLF for each harmonic. If the equations are used, the four DLFs are: 0.47, 0.088, 0.066, and 0.063. The assumed average walker weight is 168 lbf, which should be multiplied by the DLFs to determine the four sinusoidal amplitudes in the Fourier series: 79 lbf, 14.8 lbf, 11.1 lbf, and 10.6 lbf. The four sinusoidal terms should be offset relative to each other by the phase lags listed previously in this section. The first harmonic sinusoid is assumed to be zero-valued at t = 0 sec. For other harmonics, a positive phase lag indicates that the sinusoid is offset in the positive direction on a force versus time plot. Note that this Fourier series is only slightly altered by the use of average design DLFs. The amplitudes in this case would be 71 lbf, 15 lbf, 12 lbf, and 10 lbf, which are constants for any step frequency in the considered step frequency range.

5.4.3 Response History Analysis

Response history analyses should be performed for the bay under consideration using the same methodologies described in Section 5.3.3, except for the determination of the number of output time steps and final adjustment factor.

The number of output time steps was selected to terminate the analysis at the time required to walk across the specimen. This should be determined as follows. The number of steps required to traverse the specimen should be estimated to be the path length (usually the bay size) divided by 30 in. which is approximately the average stride length. The time required for one step is, of course, the reciprocal of the step frequency. For example, if the step frequency is 1.8 Hz, the time from the start of one step to the start of the next step is 0.556 sec. Multiplying the number of steps by the time required for one step gives the duration of the response history.

As mentioned previously, this procedure results in conservative acceleration predictions because the walking force is applied at midspan for the entire response history duration and because the Fourier series models walking at a perfect cadence. Therefore, an adjustment factor of 0.65 is proposed, based on the results of Chapter 4.

Table 5.3 shows comparisons of measured and predicted acceleration due to walking for all specimens. Figure 5.12 also summarizes the comparisons. The predicted peak accelerations were computed using the methods and assumptions described in this section, with the Fourier series DLFs computed using the equations shown in Figure 5.8 through Figure 5.11. The ratios of predicted-to-measured accelerations range between 0.538 and 2.80 with an average of 1.19. The COV is 47.4%, indicating significant dispersion. Shading in the table indicates that the accelerance FRF magnitude was very significantly over-predicted in the bay under consideration. If these values are excluded, the ratios range between 0.538 and 1.89 with an average of 0.999 and COV of 36.3%.

The predicted peak accelerations were also computed using the methods and assumptions described in this section, with the Fourier series DLFs set to the average values of 0.42, 0.09, 0.07, and 0.06. Table 5.4 and Figure 5.13 summarize these results. The ratios of predicted-to-measured accelerations range between 0.556 and 2.58 with an average of 1.16 and COV of 46.4%. Shading in the table indicates that the accelerance FRF magnitude was very significantly over-predicted in the bay under consideration. If these values are excluded, the ratios range between 0.556 and 2.05 with an average of 1.01 and COV of 41.1%.

Table 5.5 shows a final set of comparisons of measured and predicted acceleration due to walking for all specimens. The predicted peak accelerations were computed using the methods and assumptions described in this section, except using a single Fourier series term—the term that causes resonance. The ratios of predicted-to-measured accelerations range between 0.521 and 2.50 with an average of 1.11 and COV of 48.3%. Shading in the table indicates that the accelerance FRF magnitude was very significantly over-predicted in the bay under consideration. If these values are excluded, the ratios range between 0.521 and 2.02 with an average of 0.968 and COV of 44.3%.

Table 5.3: Summary of Walking Acceleration Response Comparisons (Predictions Using Fourier Series, 4 Terms, Computed DLFs, Average Walker Weight)

		Peak Acceleration Due to Walking		
Specimen	Description	Measured (%g)	Predicted (%g)	Predicted / Measured
Long Span Composite Slab Specimen	Parallel to Deck	1.36	1.84	1.35
	Perpendicular to Deck	0.972	1.84	1.89
Long Span Composite Slab Mockup	Bay 1	0.768	0.723	0.941
	Bay 2	0.966	0.754	0.781
Square-End Joist Footbridge	-	7.03	6.96	0.989
Shear-Connected Joist Footbridge	End Bay, Mode 1	3.32	2.74	0.826
	End Bay, Mode 2	4.14	4.07	0.984
	Middle Bay, Mode 1	2.64	2.64	1.00
	Middle Bay, Mode 3	4.67	6.35	1.36
Riverside MOB	Bay 1	1.45	0.780	0.538
	Bay 2	1.33	1.33	1.00
	Bay 3	1.03	0.702	0.682
	Bay 4	0.775	1.21	1.57
First Bank & Trust Building	Bay C-1/F-2	2.44	6.83	2.80
			Average =	1.19
			COV =	47.4%
			Average Excluding Shaded Values =	0.999
			COV Excluding Shaded Values =	36.3%

From the results of Table 5.3 through Table 5.5, it is clear that the proposed method accurately predicts the acceleration response on the average, with significant variability. It also seems that the very significant simplification of using a single Fourier series term based on the average (constants: 71 lbf, 15 lbf, 12 lbf, and 10 lbf for the four terms, respectively) design DLF results in predictions that are approximately equal to those produced using the four-term Fourier series with linearly varying DLFs. Therefore, it is considered by the writer to be an acceptable design simplification to perform most or perhaps all analyses using only the single Fourier series term that causes resonance.

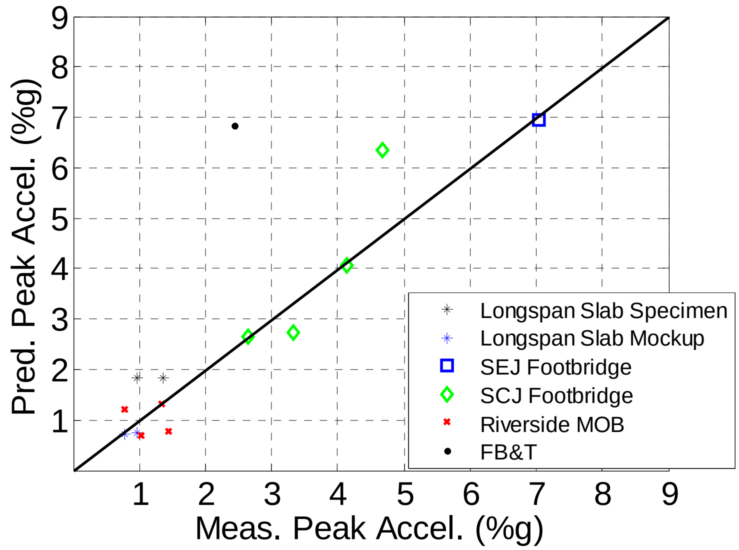

Figure 5.12: Measured and Predicted Peak Accelerations—4-Term Fourier Series, Using Computed DLFs, Average Walker Weight

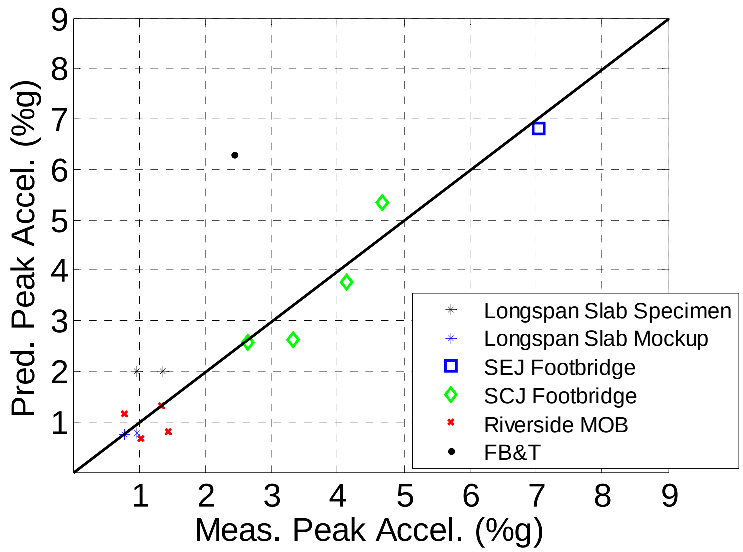

Figure 5.13: Measured and Predicted Peak Accelerations – 4 Term Fourier Using Average Design DLFs, Average Walker Weight

Table 5.4: Summary of Walking Acceleration Response Comparisons (Predictions Using Fourier Series, 4 Terms, Average Design DLFs, Average Walker Weight)

		Peak Acceleration Due to Walking		
Specimen	Description	Measured (%g)	Predicted (%g)	Predicted / Measured
Long Span Composite Slab Specimen	Parallel to Deck	1.36	1.99	1.46
	Perpendicular to Deck	0.972	1.99	2.05
Long Span Composite Slab Mockup	Bay 1	0.768	0.748	0.974
	Bay 2	0.966	0.774	0.801
Square-End Joist Footbridge	-	7.03	6.80	0.967
Shear-Connected Joist Footbridge	End Bay, Mode 1	3.32	2.62	0.789
	End Bay, Mode 2	4.14	3.77	0.911
	Middle Bay, Mode 1	2.64	2.57	0.973
	Middle Bay, Mode 3	4.67	5.33	1.14
Riverside MOB	Bay 1	1.45	0.806	0.556
	Bay 2	1.33	1.33	1.00
	Bay 3	1.03	0.663	0.643
	Bay 4	0.775	1.16	1.50
First Bank & Trust Building	Bay C-1/F-2	2.44	6.29	2.58
			Average =	1.16
			COV =	46.4%
			Average Excluding Shaded Values =	1.01
			COV Excluding Shaded Values =	41.1%

Table 5.5: Summary of Walking Acceleration Response Comparisons (Predictions Using Fourier Series, 1 Term, Average Design DLFs, Average Walker Weight)

		Peak Acceleration Due to Walking		
Specimen	Description	Measured (%g)	Predicted (%g)	Predicted / Measured
Long Span Composite Slab Specimen	Parallel to Deck	1.36	1.96	1.44
	Perpendicular to Deck	0.972	1.96	2.02
Long Span Composite Slab Mockup	Bay 1	0.768	0.701	0.913
	Bay 2	0.966	0.711	0.736
Square-End Joist Footbridge	-	7.03	6.70	0.953
Shear-Connected Joist Footbridge	End Bay, Mode 1	3.32	2.55	0.768
	End Bay, Mode 2	4.14	3.59	0.867
	Middle Bay, Mode 1	2.64	2.47	0.936
	Middle Bay, Mode 3	4.67	5.10	1.09
Riverside MOB	Bay 1	1.45	0.760	0.524
	Bay 2	1.33	1.29	0.970
	Bay 3	1.03	0.537	0.521
	Bay 4	0.775	1.09	1.41
First Bank & Trust Building	Bay C-1/F-2	2.44	6.11	2.50
			Average =	1.11
			COV =	48.3%
			Average Excluding Shaded Values =	0.968
			COV Excluding Shaded Values =	44.3%

5.4.4 Response History with Fourier Series Method—Step-by-Step Procedure

This sub-section offers at step-by-step procedure for predicting the acceleration response in a given floor bay using response history analysis with a Fourier series representation of the walking force. It is assumed that a four-term Fourier series will be used, but the step-by-step process is almost identical if a single term is used.

- Create a finite element model of the floor system using the guidance given in Section 5.1.1. The model should encompass the entire floor or at least a large portion of it.
- Perform an eigenvalue analysis to determine natural frequencies and mode shapes. These will form the basis for the subsequent analyses to predict the response to walking, so include only the modes in the frequency range of interest, usually 1 Hz to 15 Hz for low frequency floors.
- Preferably perform a frequency domain analysis to predict the FRF magnitude per Section 5.2. The FRF magnitude indicates which natural modes will provide significant response to walking excitation. If this option is not available, then the mode shapes must be used to judge which modes will be responsive in the bay under consideration.
- For each responsive mode, determine the step frequency of a walking case that will cause resonance. The step frequency must fall into the reasonable range of step frequencies: 1.6 Hz (96 bpm) to 2.2 Hz (132 bpm). Determine the four Fourier series sinusoidal amplitudes and phase angles as described in Section 5.4.1 and 5.4.2. One Fourier series harmonic should match the natural frequency to be excited.
- Create an analysis case with the Fourier series applied at midspan. The response history duration should be equal to the total time required to traverse the intended walking path, usually the bay dimension.
- Perform a response history analysis using the methods recommended in Section 5.3.3 to predict the acceleration waveform. Multiply the predicted response by 0.65.

5.5 Predicting Acceleration Due to Walking (Simplified Frequency Domain Method)

This section describes the third of three proposed acceleration prediction methods that are presented in this research: Simplified frequency domain analysis using a single Fourier series term representation of the load applied by a walker.

This method advantageously uses the fact that the vast majority of the response to walking in most bays (in the writer's experience) is due to the footstep force harmonic that matches the natural frequency of the dominant mode or another responsive mode. In a nutshell, the accelerance FRF peak magnitude is predicted and multiplied by the harmonic force amplitude. This gives the steady-state response to walking which is then reduced, using a resonance build-up factor to predict the actual response to walking. The method is extremely efficient and easy to use.

The disadvantage is that the method has no straightforward way to consider excitation of multiple modes. In the writer's opinion, however, this does not pose a serious question about the method's usefulness for the reasons articulated in Chapter 3.

The method is illustrated by example in Section 3.5. The only modifications for design use are damping estimates, recommended force amplitudes, and the inclusion of an adjustment factor. Damping should be specified using the recommendations from Section 5.1.3, keeping in mind that frequency domain analyses such as SAP2000's steady-state analysis require use of hysteretic damping. The loading function is a single term of the Fourier series described in Section 5.4.1 and 5.4.2. The simplified constant forces recommended at the end of Section 5.4.2 are used with the procedure. For the first four harmonics, the sinusoidal amplitudes should be assumed to be 71 lbf, 15 lbf, 12 lbf, and 10 lbf, respectively. Note that the phase angles are not needed because only one term of the Fourier series defines the load. As shown in Chapter 4, this method results in conservative acceleration predictions because the load is applied at midspan (whereas walking usually takes place across the bay) and because the Fourier series models perfect walking. Therefore, based on Chapter 4 comparisons, an adjustment factor of 0.65 is recommended.

Table 5.6 shows the comparisons of measured and predicted peak accelerations due to walking for every bay included in this research. Figure 5.14 also summarizes these comparisons. The predicted peak accelerations were computed using the methods and assumptions described in this section. The ratios of predicted-to-measured accelerations range between 0.549 and 2.51 with an average of 1.16 and COV of 44.4%. Shading in the table indicates that the accelerance FRF magnitude was very significantly over-predicted in the bay under consideration. If these values are excluded, the ratios range between 0.549 and 2.02 with an average of 1.03 and a COV of 39.1%. Note that the only significant under-predictions were at the Riverside MOB, due to the much heavier-than-average walker. The results in Table 5.6 indicate that the proposed method predicted the acceleration due to walking very accurately on the average, but with significant variability.

These results, like the ones shown in the previous sections clearly indicate that the largest errors in walking prediction stem from inaccurate modal predictions rather than walking force variability. In bays with reasonably accurate accelerance peak magnitude predictions, the acceleration due to walking was reasonably well predicted, with the expected variability due to walking force.

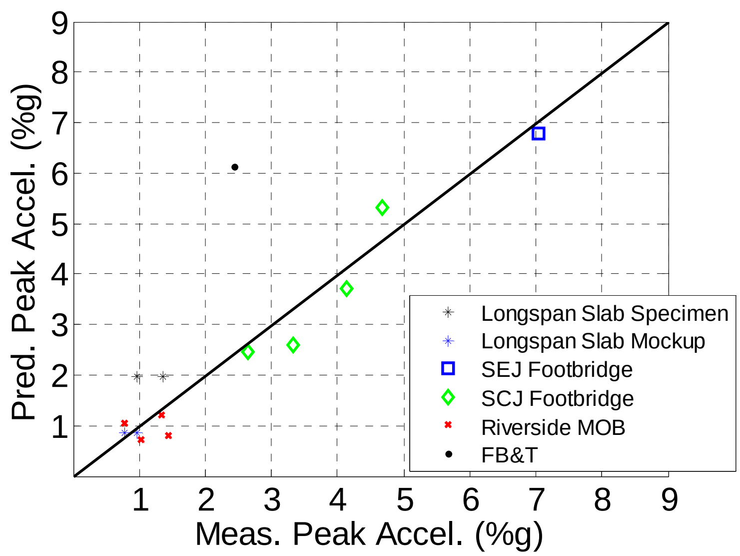

Figure 5.14: Measured and Predicted Peak Accelerations—Simplified Frequency Domain Procedure

Table 5.6: Summary of Walking Acceleration Response Comparisons (Predictions Using Simplified Frequency Domain Procedure)

		Peak Acceleration Due to Walking		
Specimen	Description	Measured (%g)	Predicted (%g)	Predicted / Measured
Long Span Composite Slab Specimen	Parallel to Deck	1.36	1.97	1.45
	Perpendicular to Deck	0.972	1.97	2.02
Long Span Composite Slab Mockup	Bay 1	0.768	0.856	1.14
	Bay 2	0.966	0.858	0.888
Square-End Joist Footbridge	-	7.03	6.78	0.964
Shear-Connected Joist Footbridge	End Bay, Mode 1	3.32	2.59	0.780
	End Bay, Mode 2	4.14	3.71	0.896
	Middle Bay, Mode 1	2.64	2.45	0.928
	Middle Bay, Mode 3	4.67	5.31	1.14
Riverside MOB	Bay 1	1.45	0.796	0.549
	Bay 2	1.33	1.22	0.917
	Bay 3	1.03	0.720	0.699
	Bay 4	0.775	1.05	1.35
First Bank & Trust Building	Bay C-1/F-2	2.44	6.12	2.51
			Average =	1.16
			COV =	44.4%
			Average Excluding Shaded Values =	1.03
			COV Excluding Shaded Values =	39.1%

The following is a step-by-step procedure for predicting the acceleration response in a given floor bay using the Simplified Frequency Domain Method.

- Create a finite element model of the floor system using the guidance given in Section 5.1.1. The model should encompass the entire floor or at least a large portion of it.

- Perform a frequency domain analysis to predict the FRF magnitude per Section 5.2. The FRF magnitude indicates which natural modes will provide significant response to walking excitation.
- For each responsive mode, determine which of the first four harmonics of the walking force can match the natural frequency to be excited. Consider the range of reasonable walking frequencies to be between 1.6 Hz and 2.2 Hz.
- Select the sinusoidal amplitude for the Fourier series term that causes resonance. The forces for the first four harmonics, respectively, are 71 lbf, 15 lbf, 12 lbf, and 10 lbf. For example, if the third harmonic causes resonance, the applied force is 12 lbf.
- Multiply the sinusoidal force by the accelerance FRF peak magnitude to determine the steady-state response to walking. This fictitious case represents the response to resonant walking for a very long time.
- Reduce the steady-state walking acceleration by multiplying it by the resonant build-up factor given in Section 3.5 and then by 0.65. The result is the predicted peak acceleration due to walking.

5.6 Probability and Sensitivity Considerations

The sensitivity of key variables in the three floor response prediction methods developed in this study are now examined. The variables fall into two general categories: those that affect prediction of modal properties and those that affect the loads imposed by human activity. The first one of these is of primary interest for this research. The loading function is very variable, but it has been and is continuing to be heavily researched by others (Brownjohn et al. 2004, Zivanovic et al. 2007).

5.6.1 Modal Properties

As has been shown clearly by comparison to measured data, reasonable predictions of FRF peak magnitude are consistently associated with reasonable predictions of maximum acceleration due to walking. Conversely, almost every inaccurate prediction of acceleration due to walking in this research was caused by drastic over-prediction of the FRF peak magnitude. The FRF is directly associated with the natural frequencies and mode shapes which are primarily affected by the distribution of mass and stiffness throughout the floor system. Of course, the FRF and acceleration predictions are also affected by damping.

Numerous parameters affect the mass and stiffness distribution in a floor system. The major ones being:

- Concrete thickness non-uniformity from bay to bay. It is well known that concrete slabs on steel decks are rarely the thickness shown on the design documents. Adjacent, nominally-identical bays undoubtedly often have different volumes of concrete. The concrete unit weight also varies from batch to batch.
- Slab orthotropic stiffnesses. These are affected primarily by the extent of concrete cracking, slab thickness, and concrete dynamic elastic modulus. These parameters naturally contain some level of variability.
- Member stiffnesses. Because the transformed section is used for floor vibration analysis, the member stiffnesses are affected by the slab thickness and concrete dynamic elastic modulus. Stiffness is also affected by the steel beam cross-section and elastic modulus, which do not vary appreciably. Spandrel beam stiffnesses are affected by the cladding which provides vertical stiffness regardless of whether the connections are rigid or designed to slip (Barrett 2006).
- Damping, which is extremely variable, does not affect the natural frequencies to an appreciable degree for lightly damped floors, but it has a strong effect on the prediction of steady-state vibration response. It also has a moderately strong effect on prediction of response to walking.

5.6.1.1 Concrete Thickness and Unit Weight

For typical composite slabs, it seems likely that the concrete thickness could range between the nominal thickness and the nominal thickness plus approximately 1.5 in. as a bay-wide average. It seems unlikely that the concrete would typically be placed thinner than the nominal thickness due to fire protection concerns. The upper bound is somewhat arbitrarily assumed considering typical deflection limits for deck, beams, and girders.

To investigate the effects of varying concrete thickness, the Riverside MOB model was modified to include various thicknesses of concrete. This bay was chosen because its FRF prediction was one of the most inaccurate. The FRFs computed for load and displacement at the center of Bay 2 are shown in Figure 5.15 for the nominal thickness of concrete, 1.5 in. extra concrete in Bay 2 only, and 1.5 in. extra concrete over the entire floor area. The natural frequency was slightly affected, but the FRF peak magnitudes are not appreciably affected by the additional concrete. They are 0.236 %g/lbf, 0.215 %g/lbf, and 0.217 %g/lbf for the three respective analyses. Thus, the FRF and walking acceleration predictions appear to be insensitive to deviations from the nominal thickness.

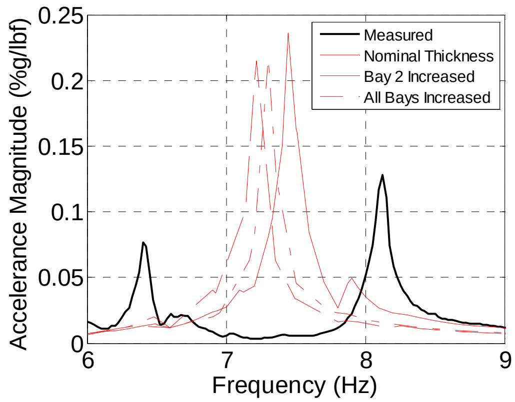

Figure 5.15: Riverside MOB Bay 2 FRFs – Various Concrete Thicknesses.

Predictions will be similarly insensitive to deviations in unit weight. According to Lamond and Pielert (2006), unit weight can be expected to vary between 140 pcf and 150 pcf for normal weight concrete. If one uses the average, 145 pcf, for the calculations, then the deviation is only expected to be approximately ±3.5%.

5.6.1.2 Concrete Slab Stiffness

Concrete Cracking

The analyses performed during this research use the typical assumption of uncracked composite slabs. However, slab cracks are almost always present in composite slabs over girders and over beams at column lines. Pavic et al. (2007) attributed some of the discrepancy between their experimental results and analytical predictions to slab cracks. Cracks were also noticed at the locations listed in Section 5.6.1 at the Longspan Composite Slab Mockup and Shear Connected Joist Footbridge.

Cracks usually initially develop in shored slabs due to negative moment. In unshored slabs, they develop due to shrinkage and the simple fact that the slab is thinnest over beams at column lines. Negative moment due to superimposed dead load and live load can also contribute to cracks in shored or unshored slabs. The typical depth of these cracks is unknown. They probably do not usually penetrate the entire depth due to restraint from the composite deck so it seems likely that cracks extend only one-half to three-fourths through the slab, although this is an assumption only.

Three models were modified to determine the effect of cracks which were simplistically modeled by reducing the shell flexural stiffness over the supports. The shells were made 5% as stiff as the surrounding shells which corresponds to a crack that is roughly 65% of the way through the slab.

Figure 5.16 shows the measured (black line) and predicted (red lines, dashed indicates cracking) FRFs for the three specimens. A consistent result can be seen: the effect of cracking depends on the curvature at the support. For example, the mode shape shown in Figure 5.17(a), which is for the Longspan Composite Slab Mockup, is only slightly affected by cracking. This mode corresponds to the first FRF peak in Figure 5.16(b). Modes such as the one shown in Figure 5.17(b) are significantly affected, however. In Figure 5.16(a), the second predicted peak shifted to the left, but the accuracy of the prediction did not improve. The accuracy of the second mode prediction (frequency and FRF peak magnitude) shown in Figure 5.16(b) was made worse by the use of reduced stiffness shell elements over supports. However, the Shear Connected Joist Footbridge FRF predictions were improved by the use of the reduced stiffness shell elements. The second and third mode predictions shown in Figure 5.16(c) and Figure 5.16(d) were significantly improved by considering cracking. Finally, the Riverside MOB Bay 2 FRF was re-computed assuming cracked elements over supports, but the FRF was not significantly affected as indicated by Figure 5.16(e).

In conclusion, modes of specific shapes (with adjacent bays displacing the same direction) are affected by cracking over the supports. However, insufficient data exists in the current study to provide definitive recommendations for how to consider the effects of cracking.

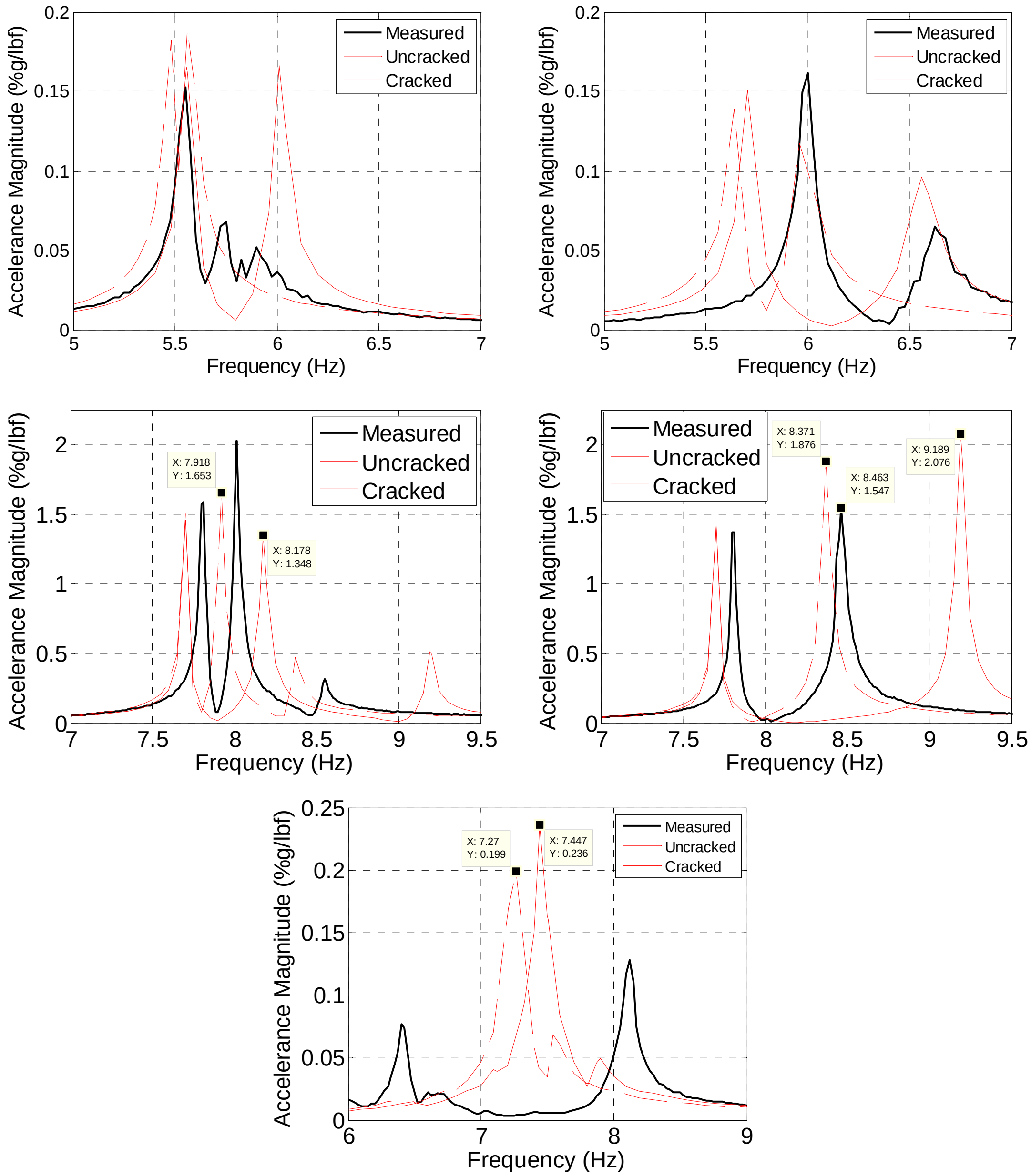

Figure 5.16: Effect of Cracking. (a) Longspan Composite Slab Mockup Bay 1; (b) Longspan Composite Slab Mockup Bay 2; (c) Shear-Connected Joist Footbridge End Bay; (d) Shear-Connected Joist Footbridge Middle Bay; (e) Riverside MOB Bay 2.

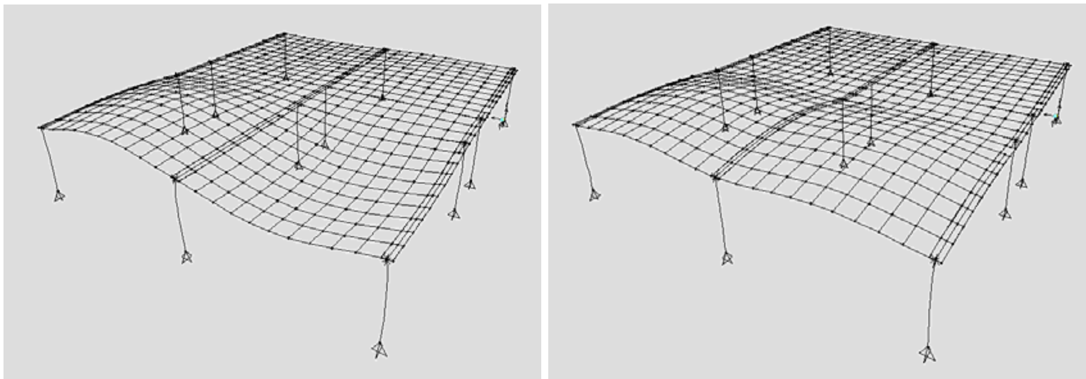

Figure 5.17: Longspan Composite Slab Mockup Mode Shapes. (a) Mode Shape Not Affected by Cracking; (b) Mode Shape Affected by Cracking.

Concrete Elastic Modulus

The concrete elastic modulus is expected to deviate approximately ±20% from the value computed using the ACI equation (Lamond and Pielert 2006). Systems that rely primarily on concrete bending, such as long span composite slabs, are directly affected by the deviation in the ACI equation. Systems that rely on composite beam bending are affected only to the extent that the transformed width is affected via the modular ratio.

Figure 5.18 shows the measured and predicted FRFs for the long span composite slab mockup which is a system that should be directly affected. As expected, the FRF magnitudes are mostly shifted to the left or right for decreases or increases in the elastic modulus, respectively, but the FRF peak magnitudes did not change appreciably. From this, it can be concluded that the predictions of peak acceleration to walking are insensitive to deviations in elastic modulus within the normal range. The exception is if a different harmonic of walking is required to cause resonance in reality versus in the model. For example, if the model predicts 6.5 Hz, then the third harmonic can cause resonance. If the actual structure has a natural frequency of 7.0 Hz, then the fourth harmonic is the lowest one that can cause resonance. However, the second through fourth harmonic forces do not vary by a large amount. Systems with typical composite framing will be less sensitive to deviations in concrete elastic modulus than the long span composite mockup whose FRFs are shown here.

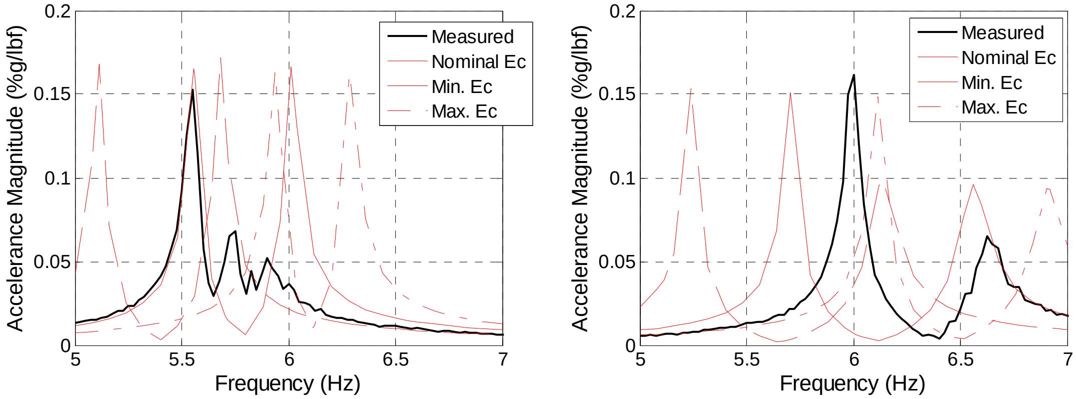

Figure 5.18: Long Span Composite Slab -- Effect of Changes in Concrete Elastic Modulus. (a) Bay 1; (b) Bay 2.

5.6.1.3 Member Stiffness

Pavic et al. (2007) determined that variation in member stiffness accounted for a large part of the discrepancy between their analytical model and experimental results. They also pointed out that member stiffnesses are affected by concrete cracking and they indicated that their member stiffnesses had to be varied by ±20% to bring their model's natural frequencies and mode shapes into agreement with their measurements. The reader is referred to their paper for more information on this sub-topic.

5.6.1.4 Damping

Damping is a parameter that exhibits significant, sometimes extreme, variability even within a given floor system. Consider the six specimens that were included in this research. Even though they were all bare slab specimens, their modal damping ratios ranged from 0.17% of critical to 1.5% of critical, a 780% difference. The Riverside MOB modal damping values ranged from 0.51% of critical to 0.61% on the same floor and the First Bank & Trust values ranged from 0.95% to 1.5% of critical. The damping ratios would undoubtedly vary by a much wider margin if non-structural elements were also included. Steady-state vibration response is directly related to damping: doubling the damping halves the steady-state response. Fortunately, walking excitation is less sensitive to deviations in damping due to incomplete resonant build-ups that take place over just a few footsteps. To illustrate this, the Long Span Composite Slab Mockup Bay 1 was analyzed for various damping ratios using response history analysis with the 4-term Fourier series. The predicted waveforms for various damping ratios ranging from

0.5% (measured) to 5% are shown in Figure 5.19. At a glance, it can be seen that the acceleration only decreased by 3.76 times for the tenfold damping increase from 0.5% to 5% (Figure 5.19(a) versus Figure 5.19(f)). Presumably, damping estimate errors in practice will not be so large. For instance, it is common to assume 3% of critical damping. If the actual damping is 2%, then the response is under-estimated by approximately 30%, which is still considered a good prediction in the world of floor vibrations due to walking. If the actual damping is 4%, then the response is over-estimated by only 17%.

It is also instructive to observe that the vast majority of resonant build-ups measured in this research took place over only a few steps even though a metronome was used and the walkers did their very best to walk at the resonant frequency for the 10 to 12 steps required to traverse each bay. Several of the tests were conducted at very natural step frequencies near 120 bpm with similar build-ups, so short resonant build-ups did not appear to be due to forced walking at unnatural cadences. Human footsteps have natural variability both in frequency and frequency content, which prevents very long resonant build-ups in most cases. The resonant build-ups practically topped out at around 4 sec. in the vast majority of cases and earlier than that in many tests. Excluding the 0.5% damping case shown in Figure 5.19, the others are within a factor of two of each other at 4 sec. even though the damping increased fivefold.

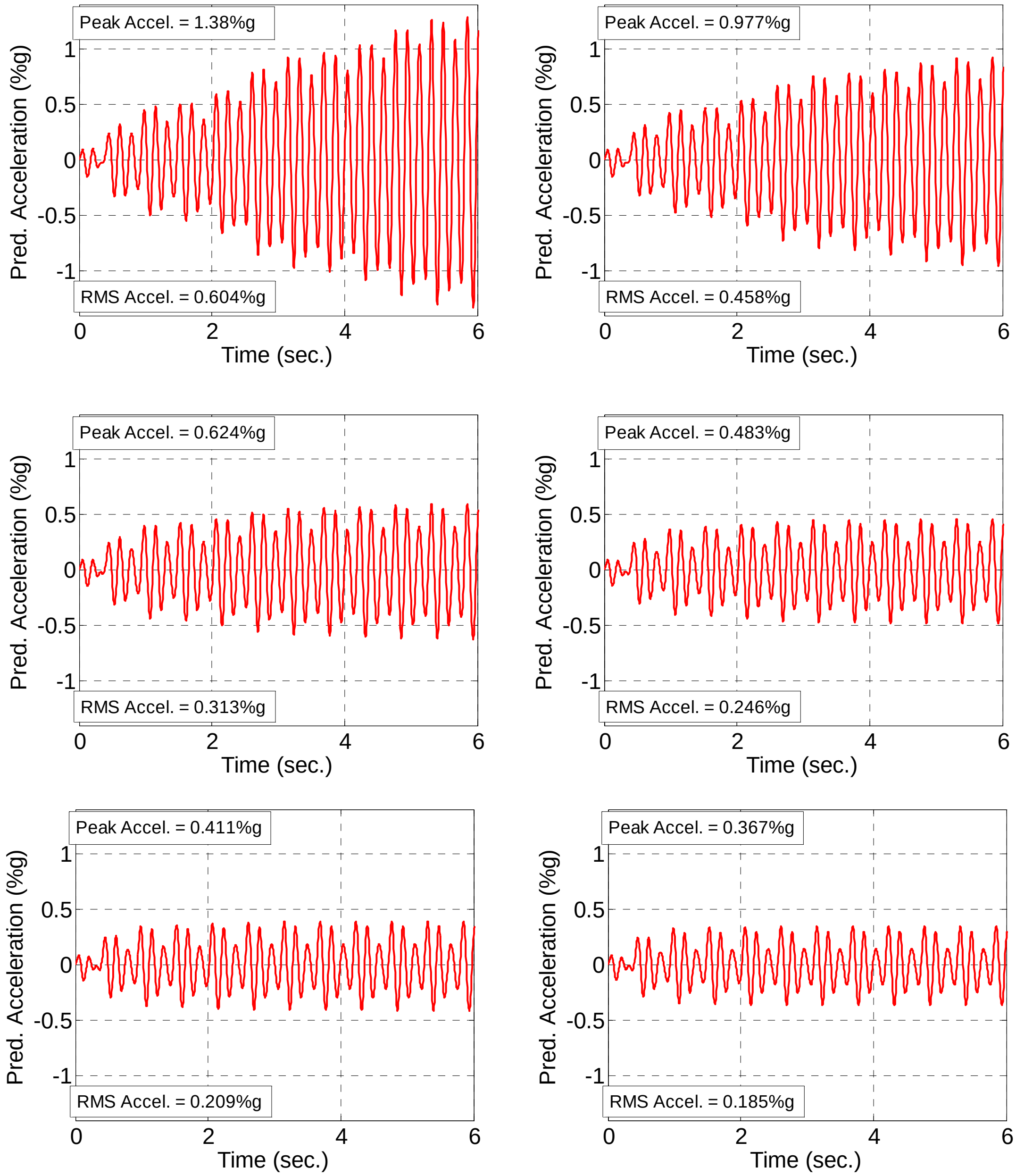

Figure 5.19: Long Span Composite Slab -- Effect of Damping. (a) 0.5% Critical Damping; (b) 1% Critical Damping; (c) 2% Critical Damping; (d) 3% Critical Damping; (e) 4% Critical Damping; (f) 5% Critical Damping.

5.6.2 Footstep Loads

It is well known that the forces applied to a floor by a walker are extremely variable in stride length, step frequency, and applied harmonic force. This variability is a natural place to start when developing a probability-based floor vibration prediction method, as has been done by Brownjohn et al. (2004) and Zivanovic et al. (2007).

Brownjohn et al. (2004) identified walking as a “narrow band random process” rather than a periodic process that can be represented accurately using a Fourier series. They developed the initial framework for a probabilistic treatment of the loading function. Zivanovic et al. (2007) extended the work of Brownjohn et al. by incorporating subharmonics (frequency contents between main harmonics of the walking force caused by differences between left and right footsteps), phase information, and extending the loading function to five harmonics. (As an interesting aside, the researchers found that the phases were approximately uniformly distributed between $-\pi$ and π.) The end result of their research is frequency domain and time domain representations of the walking force that reflect inter- and intra-subject variability. They verified their simulation procedure by comparing measurements and predictions generated using measured modal properties from a real low frequency bridge. In the writer’s opinion, this approach has great potential for generating accurate footstep loads.