Stochastic Methods for Dilemma Zone Protection at Signalized Intersections

By

Pengfei Li

Dissertation submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

In

Civil and Environmental Engineering

Montasir M. Abbas, Chair
Raghu Pasupathy, Member
Hesham A. Rakha, Member
Antonio A. Trani, Member
Linbing Wang, Member

August, 2009

Blacksburg, Virginia Tech

Key word: Traffic Engineering, Traffic Signal Control, Dilemma zone, dilemma hazard, traffic safety, Optimization, Simulation.

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Abstract

Dilemma zone (DZ), also called decision zone in other literature, is an area where drivers face an indecisiveness of stopping or crossing at the yellow onset. The DZ issue is a major reason for the crashes at high-speed signalized intersections. As a result, how to prevent approaching vehicles from being caught in the DZ is a widely concerning issue. In this dissertation, the author addressed several DZ-associated issues, including the new stochastic safety measure, namely dilemma hazard, that indicates the vehicles' changing unsafe levels when they are approaching intersections, the optimal advance detector configurations for the multi-detector green extension systems, the new dilemma zone protection algorithm based on the Markov process, and the simulation-based optimization of traffic signal systems with the retrospective approximation concept. The findings include: the dilemma hazard reaches the maximum when a vehicle moves in the dilemma zone and it can be calculated according the caught vehicles' time to the intersection; the new (optimized) GES design can significantly improve the safety, but slightly improve the efficiency; the Markov process can be used in the dilemma zone protection, and the

Markov-process-based dilemma zone protection system can outperform the prevailing dilemma zone protection system, the detection-control system (D-CS). When the data collection has higher fidelity, the new system will have an even better performance. The retrospective approximation technique can identify the sufficient, but not excessive, simulation efforts to model the true system and the new optimization algorithm can converge fast, as well as accommodate the requirements by the RA technique.

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Acknowledgements

I would like to express my appreciation to my advisor, Dr. Montasir Abbas, for his help while I was pursuing my Ph.D. degree. He guided me from a rookie engineer, who was often lost in the work, to a professional with various skill sets as well as extensive knowledge. He helped me obtain more courage and fortitude while facing difficulties. Many times, I felt like I was talking with an older brother rather than my advisor. I could not be what I am today without him and I consider my time studying/working with him as one of the most memorable periods in my entire life.

I am also indebted to the assistance of the other committee members: Dr. Raghu Pasupathy, Dr. Hesham Rakha, Dr. Antonio, Trani and Dr. Linbing Wang. Each of them provided me with valuable suggestions and tremendous support that expedited my research. They, besides Dr. Abbas, were the lighthouse that navigated me through the dark sea in my research.

I am grateful to my family who supported me in many ways. I believe they are happy with what I have achieved.

I would also like to give special thanks to my friend, Mr. Joerg Rosenbohm. We have known each other since we worked for the same company in China. He illustrates for me the definition of friendship by helping me in many ways. When I got frustrated, he encouraged me; when I had troubles, he helped me out. I am blessed to have such a sincere and supportive friend.

I would thank my lab mates, Zain, Sherif, Yatish and Yan for their help in the research. I am especially thankful to Dr. Vicki Neal and Dr. Zac Doerzaph at Virginia Tech Transportation Institute for providing accessibility to their intersection vehicle trajectory data. Finally I would thank Mr. Steve Sawyer for helping me edit my dissertation.

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Chapter 1 Introduction

1.1 Problem Statement

Dilemma zone (DZ), also called decision zone in other literature, is an area at high-speed signalized intersections, where the drivers face an indecisiveness of stopping or passing at the yellow onset. Some previous research [1] reported that more than one million rear-end and right-angle collisions occur at intersections in U.S each year, at least 10% of which are caused by red light runners [1], which results in numerous property loss and casualties. Consequently, how to effectively protect the dilemma zone has been a widely concerning issue in transportation community.

The issues surrounding dilemma zone protection have been studied for decades. Although the traditional dilemma zone protection systems implicitly cover the randomness residing in traffic systems, they usually ignore or partially ignore drivers' individual randomness. As a result, the effectiveness of the dilemma zone protection may be limited and thus there is a need to introduce the stochastic methods into the dilemma zone issues.

1.2 Dissertation Objective

In this dissertation, the author investigated the recent development of stochastic optimization and simulation and then applied such knowledge to several dilemma zone protection issues. The objective is to explore the potential of applying stochastic methods to dilemma zone protection. Meanwhile, the issues addressed in this dissertation are commonly concerning and the author hopes the research deployed here can help better understand the stochastic properties of dilemma zone issues and further improve the effectiveness of dilemma zone protection.

1.3 Research Approach

The stochastic approaches used in the dissertation include Monte Carlo simulation technique to model the random driving behaviors of those vehicles caught in the dilemma zone at the yellow onset; the genetic algorithm to optimize the traditional green extension system; the Markov process to design an innovative dilemma zone protection algorithm and the retrospective approximation framework to deploy simulation-based optimization. All of these methods reflect the recent development of driving behavior studies and optimal dilemma zone system designs in the stochastic scheme. They will be explained in detail in the following chapters.

1.4 Dissertation Layout

In Chapter 2, the author reviewed the definitions of two types of the dilemma zone and several state-of-art dilemma zone protection systems on how the prevailing dilemma zone protection systems work and how their performances are evaluated. Based on those reviews, in Chapter 3, the author analyzed the possible problems brought about by using the number of vehicles in the DZ as the measurement of the DZ-associated safety. Then the author designed a stochastic crash-potential-based hazard model, namely dilemma hazard model, to reflect the changing DZ-associated unsafe level of vehicles. This dilemma hazard model was calibrated with vehicle trajectory data collected in Christiansburg, VA.

In Chapter 4, the author applied the new dilemma hazard model to optimizing the detector configuration of the multi-detector green extension system (GES). GES provides the dilemma zone protection using multiple advance detectors. While vehicles are moving in the DZ, they place calls on these advance detectors and extend the green. When there are no vehicles in the

DZ (and therefore no calls), the green ends. The prevailing detection configurations are partially based on engineering judgment and therefore they cannot guarantee to provide the best designs. To address this issue, the author first developed a simulation engine for GES to evaluate the dilemma hazard under different detector configurations and then used the *Genetic Algorithm* to obtain the optimal detector configuration, which caused the minimum dilemma hazard.

In Chapter 5, the author designed a new dilemma zone protection algorithm using the Markov process (MP). Although the Markov process has been used to describe a wide range of systems, its application to dilemma zone issues has been very limited. In light of the discrete Markov process concept, the author considers the number of vehicles in the DZ (state) as a Markov process and therefore the future probabilities for a traffic system to change to other states are only determined by its current state. The new algorithm was also compared with another prevailing DZ protection system, the detection-control system (D-CS). Both the new MP-based algorithm and D-CS were deployed and evaluated in VISSIM and the results showed that the new algorithm performs better than D-CS and the new algorithm will become even better if the vehicle trajectories can be predicted with higher fidelity (e.g., use vehicle-infrastructure-integration technique).

Given that most traffic signal systems cannot be modeled in a close form, optimization through simulation sampling is better than through analytical models. In Chapter 6, the author introduced the retrospective approximate (RA) concept into the optimization of traffic signal systems. RA reflects the latest theoretical development of simulation-based optimization. Rather than use one single approximate function, RA uses a sequence of approximate problems with increasing sample sizes and decreasing error tolerances. In this way, RA can leverage the approximate efforts and optimization efforts. Specifically, when the sample sizes (i.e., approximate efforts) are low, the optimization efforts will be also low because the optimization of the low-fidelity approximate problem is primarily to better understand the problem; with the sample size increasing, more optimization efforts are needed to obtain the global optimum. To fully address this issue, the author first reviewed the theoretical development of the simulation-based optimization and its application to traffic signal systems. Then the author designed a variant of the Markov Monotonic Search Algorithm (a stochastic global optimization algorithm), namely inheritable Markov Monotonic Search Algorithm (IMMSA), which can converge fast and accommodate the requirements by the RA framework as well. The author also developed the RA-based optimization engine using VC.NET and VISSIM and applied it to optimizing the maximum green of the multi-detector green extension systems. The results show that a sample size of 4 replications is sufficient to approximate GES at an isolated intersection and the optimal maximum green can significantly reduce the composite cost (delay cost plus safety cost) compared to the traditional maximum green value.

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Chapter 2 Review of Previous Work

2.1 Previous Efforts in the Dilemma Zone Definitions and Traffic Conflict Techniques

2.1.1 Type I Dilemma Zone

Gazis, Herman and Maradudin (GHM) defined dilemma zone based on the difference between the distance from the stop line to the nearest vehicle that can completely stop (x_c) and the distance (x_0) from the stop line to the farthest vehicle that can cross an intersection at the yellow onset [2]. Their definition is illustrated in Fig. 2-1 and is often referred to as the GHM model. These two critical distances are calculated as:

$$x_0 = v_0\tau_0 + \frac{1}{2}a_1^*(\tau_0 - \delta)^2 - L - w \qquad \text{(For crossing vehicles)} \qquad (2\text{-}1)$$

And

$$x_c = v_0\delta + \frac{v_0^2}{2a_2^*} \qquad \text{(For stopping vehicles)} \qquad (2\text{-}2)$$

Where:

- v_0 : Initial speed when the yellow interval begins (feet/s);
- a_1^* : Maximum acceleration (feet/s^2);
- a_2^* : Maximum deceleration (feet/s^2);
- δ : Perception-Reaction (P-R) time (sec);
- L: Vehicle length (feet);
- w: The width of intersection (feet), and

- τ_0 : The yellow duration (sec).

When x_c is greater than x_0, Type I DZ exists with a length (x_c-x_0). Based on the assumption that a crossing vehicle does not accelerate, the DZ can be eliminated by adjusting the clearance interval to set (x_c-x_0) to zero as in equation (2-3):

$$\tau_0 = \delta + \frac{v_0}{2a_2^*} + \frac{L+w}{v_0} \tag{2-3}$$

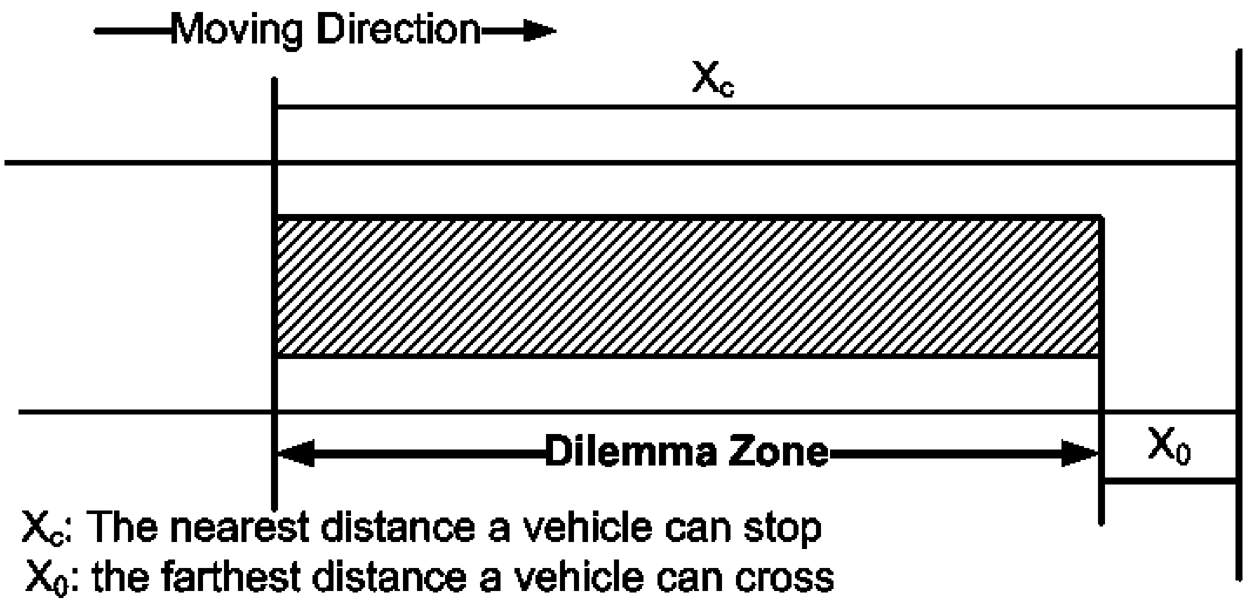


Figure 2-1 Illustration of Type I Dilemma Zone

Researchers now divide the yellow duration (τ_0), originally defined in the GHM model, as:

The permissive yellow interval $y = \delta + \dfrac{v_0}{2a_2^*}$ and the all-red clearance interval: $r = \dfrac{L+w}{v_0}$

The GHM model was commonly used to define the yellow interval and all-red clearance [3]. Liu and Herman extended the GHM model by taking the decreasing relationship between acceleration and instantaneous speed into account [4]. In addition, several other notable calibrations were reported [5, 6, 7, 8, 9].

The GHM model and its extensions are essentially kinematic models that assume that all vehicles will stop, if they can, at the yellow onset. However, Olson and Rothery observed that some vehicles used the yellow interval as the green extension [6]. May concluded that some vehicles accelerated/decelerated heavily to escape DZ [10] and Liu et al. observed that driver behaviors were considerably different from the theoretical assumptions [11]. These studies imply that vehicle approaching speeds and driver behaviors have a wide distribution. Lack of capability to tackle the randomness in the driving behaviors is the main disadvantage of the GHM model.

There were limited research efforts to address the above issue. For example, Easa attempted a stochastic approach by assuming that Type I DZ length is normally distributed and derived a solution to lower the probability of DZ existence to a low level such as α =10% [12]. The recommended yellow time by Easa's model was considerably longer than that by the GHM model, possibly impairing traffic efficiency. Meanwhile, the consideration of randomness is incomplete since it did not cover the driver's decision randomness. As a result, its application has been limited.

2.1.2 Type II Dilemma Zone

Parsonson et al. defined Type II DZ. The Type II DZ begins where 90% of vehicles will stop, and ends where only 10% of vehicles will stop when the yellow is presented [13]. Many researchers attempted to identify the Type II DZ area under different conditions. Some reported values include 2.5s~5s [14], 2.0s~4.5s [8], 3.0s~5.0/6.0s [15]; and 1.7s~4.7s [16]. These values were reported based on different local observations. The Type II DZ definition is widely used in DZ protection systems, such as the green extension systems (GES) [14], LHOVRA [17], the green termination system [18], the detection control system(D-CS) [19], and the Platoon Identification and Accommodation System [20].

2.1.3 Traffic Conflict Techniques

The dilemma hazard model is based on the analysis of traffic conflicts (TC), which is an alternative proxy to evaluate the safety at intersections. The idea of TC is that drivers experience many traffic conflicts, some of which might become real accidents. Analyzing the traffic conflicts helps the researchers investigate potential crashes and near-misses and help improve intersection safety. The traffic conflict technique was first used by Perkins and Harris to model the crash potentials due to aggressive driving behaviors, such as heavy braking or eruptive weaving, which pushes drivers into an impending accident situation or traffic violation [21]. The TC concept is widely accepted because it can be more easily observed than real accidents and it helps traffic engineers improve the safety before the accidents occur. Cooper and Feguson reported the ratio between a traffic conflict occurrence and a real accident occurrence as 2000:1 [22]. Given that the TC techniques have many advantages over the actual accident studies, many research efforts have been dedicated to them [22, 23, 24, 25, 26, 27].

2.2 Previous Efforts in Dilemma Zone Protection

2.2.1 Multi-detector Green Extension System (GES)

The multi-detector green extension system is the most common dilemma zone protection system. By placing several advance detectors, GES extends the green to allow fast vehicles to clear the dilemma zone and ends the green before slow vehicles enter the dilemma zone. When a vehicle hits the advance detector, a call will be placed and the green will be extended. If this vehicle is fast enough to reach the next advance detector, the extension timer will be reset; otherwise, the timer will be expired and the green will end before this vehicle enters the DZ. In this way, GES ensures no vehicles are caught in the DZ at the yellow onset.

2.2.2 Microprocessor Optimized Vehicle Actuation System (MOVA)

MOVA was developed in Britain and has been used for more than 15 years [28]. There are three kinds of detectors in MOVA, IN detector, EXIT detector (X detector) and OUT detector. However, in most cases, only IN and EXIT detectors are activated. Figure 2-2 shows the typical layout of MOVA detectors.

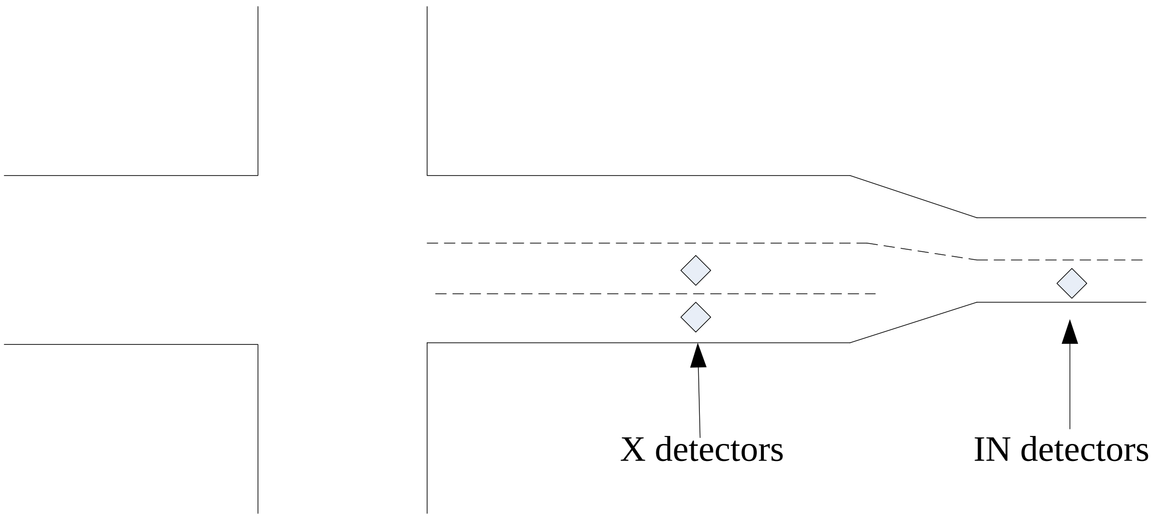


Figure 2-2 Detector Layout of MOVA

Via IN and EXIT detectors, MOVA can count the vehicles between the stop bar and the X-detector, and the vehicles between the IN-detector and the X-detector in each lane. The end-green decision in MOVA is made based on the comparison of the "benefits" of preventing vehicles from being caught in the DZ and the possible delay caused by ending the green. MOVA takes into account both safety and efficiency. However, tracking vehicles based on their presence rather than their locations also makes MOVA less valid when unexpected situations happen. For example, the through traffic after the X-detectors cannot be interrupted by right-turning traffic or parking vehicles. If it is, MOVA will be confused by such maneuvers.

2.2.3 LHOVRA and TTI Truck Priority System

LHOVRA is the acronym of six function modules, truck priority (L), primary road priority (H), accident reduction (O), variable amber (V), control of red-light running (R) and changes of mind at all-red (A) [29].

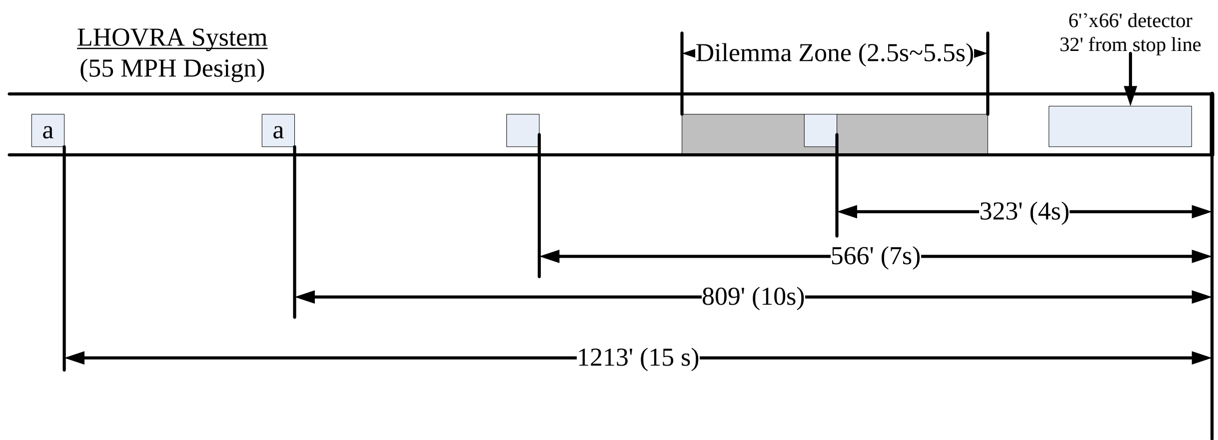


Figure 2-3 LHOVRA design layout for 55-mph

Among these modules, Module O and L aim to improve safety. Module O minimizes the number of vehicles caught in the DZ by extending the green. The highlight of LHOVRA is to use upstream detectors and a vehicle classifier to identify the presence and the vehicle type. Once a truck is detected, Module L will be activated and it will hold the green until the truck leaves the dilemma zone. In this way, dynamic dilemma zone protection according to vehicle type is achieved.

Another similar system is the *Truck Priority System* developed by TTI [30]. The Truck Priority System can even override the controller phasing and hold the phase longer than maximum green - until the trucks leave the dilemma zone.

2.2.4 Self-Organized Signal System (SOS)

SOS, also called "green termination" system, is similar with LHOVRA except Module O [18]. The objective of SOS is to improve the safety as well as reduce the delay on conflicting approaches. Like MOVA, SOS can improve the safety but still keep a proper level of service at intersections. Detectors are used to detect vehicle presence at different locations (Fig. 2-4). Then the controller collects these detector data, estimates vehicle trajectories and then calculates the safety benefit and the delay cost accordingly. The decision whether to end the green is made by comparing the possible DZ-associated benefits and the possible control delay (Fig. 2-5). Like MOVA, SOS identifies and predicts vehicle trajectories using advance detectors.

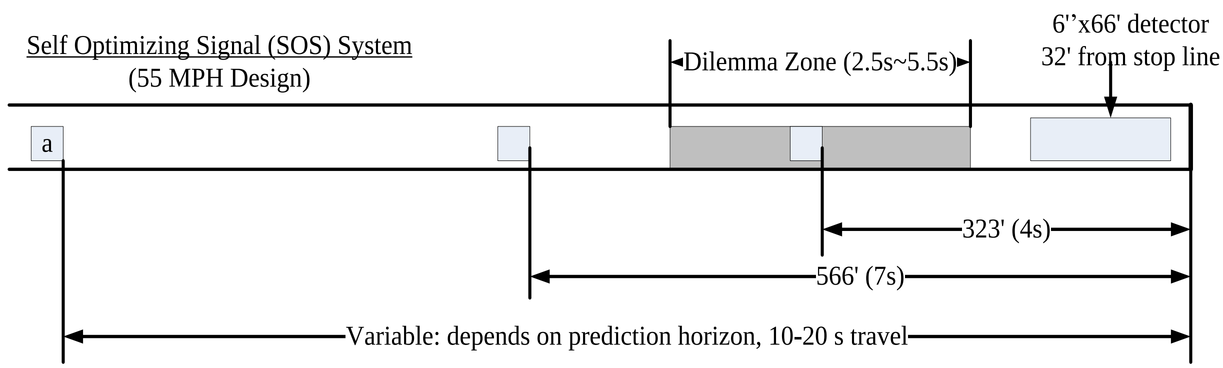


Figure 2-4 Typical detector layout of SOS system

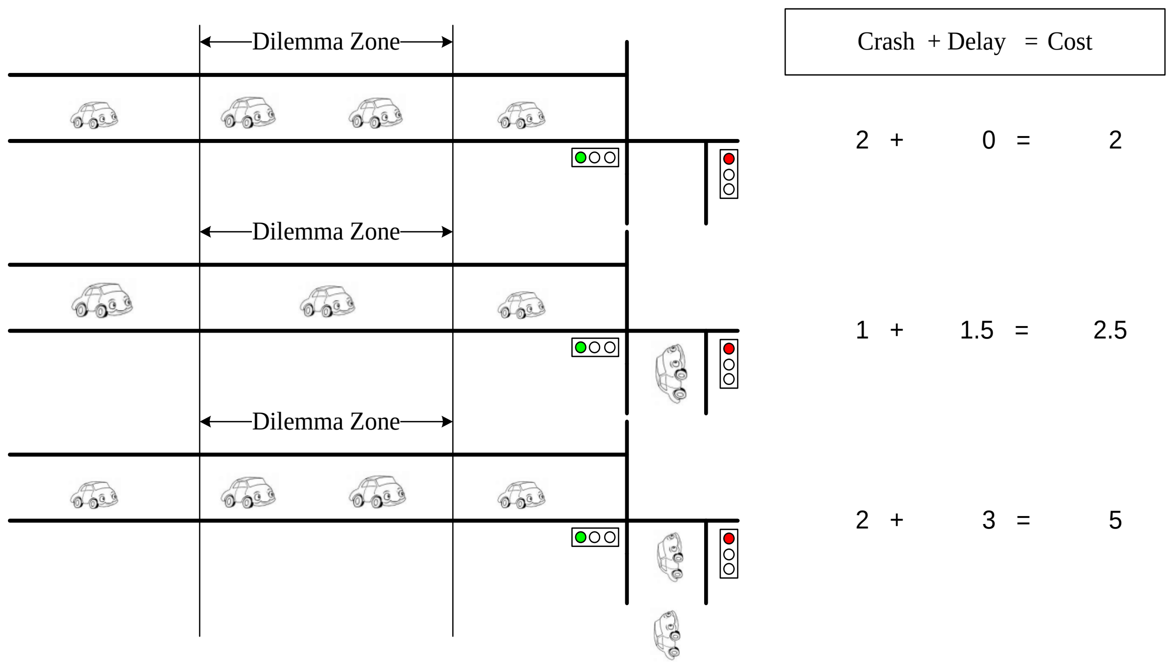


Figure 2-5 Illustration of end-green strategy in SOS

2.2.5 Advance Detection with Advance Warning Flasher (AWF)

As in Fig. 2-6, when the green duration is close to the maximum green and an approaching vehicle is detected, the warning flashers AWF begin to flash after three seconds [31]. If the vehicle is slow, its driver will see the flashing and decelerate, which can prevent the vehicle from being caught in the DZ. Otherwise, if the vehicle is fast, the driver will have passed the AWF before it is activated. As a result, the driver will keep the same speed and cross the intersection.

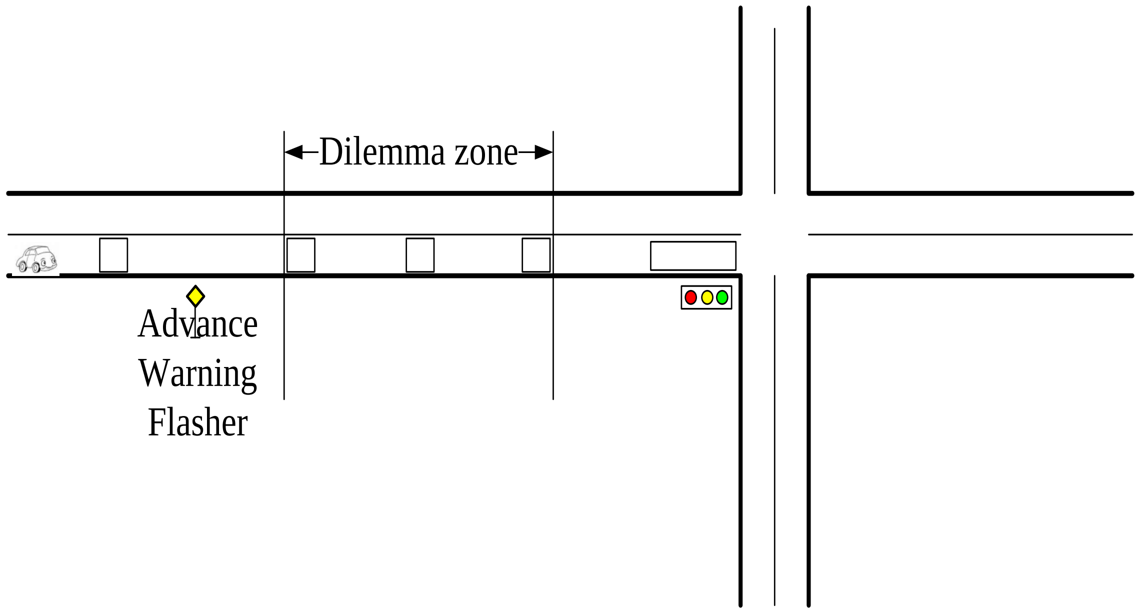


Figure 2-6 Dilemma-Zone protection system with advance-warning flashers

The advantage of AWF is to alarm approaching drivers to slow down even before they are detected. McCoy *et al* reported [31] that the AWF system could provide the dilemma zone protection for those vehicles at the speed up to 81 MPH at 65-MPH intersections. AWF's disadvantage, however, is to assume that all drivers will decelerate on seeing the AWF's flashing, which is not always true. In practice, some drivers may even accelerate trying to cross the intersection before the green ends.

2.2.6 Advance Warning Systems for End of Green Phase System (AWEGS)

AWEGS is designed primarily to supplement the existing green extension systems. It uses additional "speed trap" (a pair of advance detectors) and works with other existing detectors [1, 32].

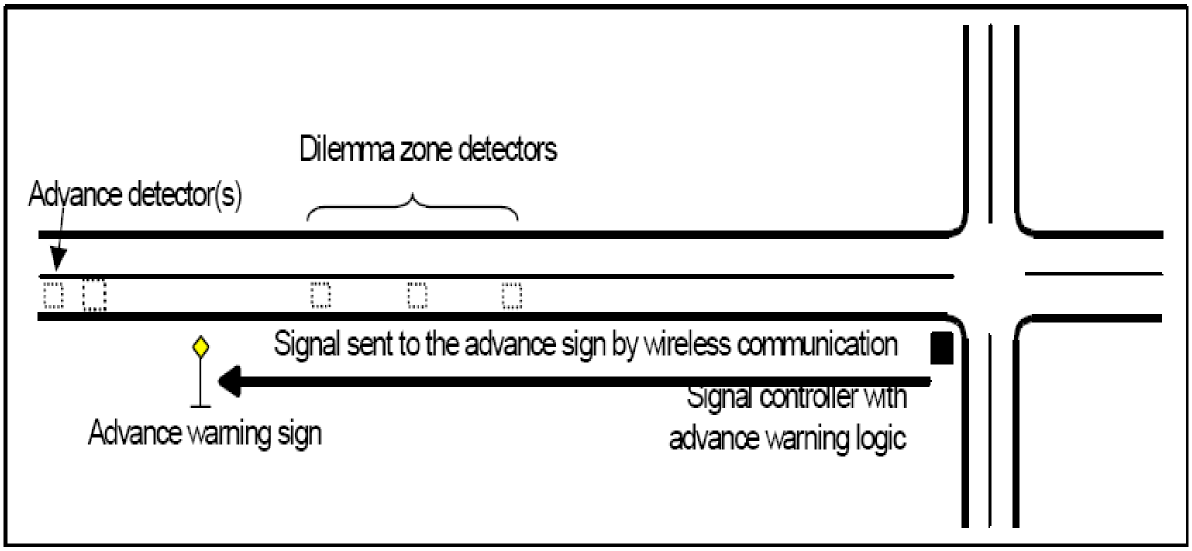


Figure 2-7 Typical detector configurations of AWEGS

AWEGS has a fail-safe design, which divides all possible scenarios into three levels:

- Level 0: if both detectors of the speed trap fail, AWEGS will degenerate to a regular green extension system with advance warning flasher
- Level 1: If one of the time-trap detectors fails, the remaining detector will be used an additional advance detector of GES.
- Level 2: If both detectors work, AWEGS can identify vehicle speeds and types, predict the vehicle trajectories and then decide when to end the green. Specifically, AWEGS predicts the vehicle trajectories and then holds the green as well as activate the flasher to ensure no vehicles are caught in the DZ. After all the vehicles in the DZ are cleared, AWEGS drops off the phase and lets the controller determine when to gap out.

2.2.7 Detection-Control System (D-CS)

D-CS developed by TTI has two functional modules, the vehicle module and the phase module. The vehicle module is to predict vehicle trajectories and the phase module is to determine when to end the green according to the vehicle trajectories [33]. D-CS is equipped with two advance detectors to detect approaching vehicle speeds and types. The vehicle module uses this information to predict vehicle trajectories and when they will enter/leave the dilemma zone. The phase module then uses this information to decide the best time to end the green to minimize the vehicles caught in the DZ.

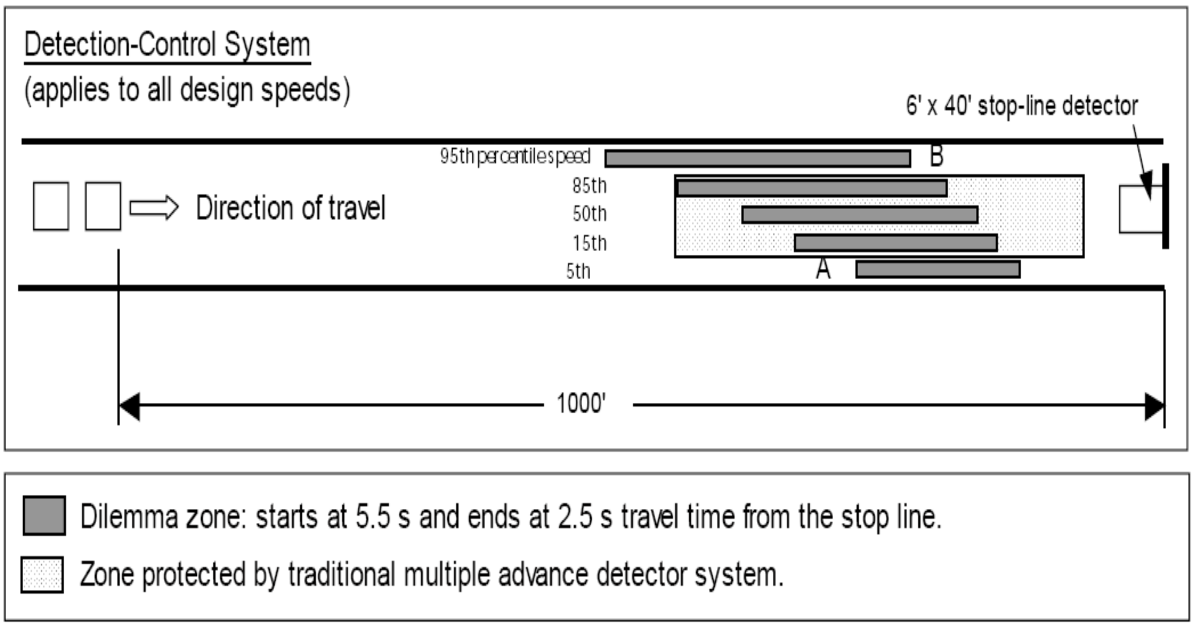


Figure 2-8 Illustration of the Detection-Control System

D-CS introduces the car-following model into the vehicle module. If the fast following vehicle is catching with the slow lead vehicle, the following vehicle will decelerate to keep a safe headway from its lead vehicle. This assumption is very conservative in light traffic with multiple lanes since fast vehicles can change lanes to keep their desire speeds.

Another highlight of D-CS is that it divides the green into two phases and, in the second stage, D-CS allows up to one car caught in the DZ to avoid max-out. The rationale is that max-out will likely catch 2-3 vehicles in the DZ per lane and therefore it will be better to end the green with only one vehicle in the DZ rather than let the max-out occur.

2.2.8 Platoon Identification Accommodation System (PIA)

PIA was also developed by TTI and it provides “green preemption” for approaching traffic platoons [20, 33]. With the advance detectors and PIA program installed in an industrial computer, PIA monitors approaching vehicles all the time and, if certain conditions are satisfied, it will send a low-priority preemption request to the controller to end the red or extend the green. When the preemption request is accepted, PIA will override the controller and provide the approaching platoon with green preemption, whose duration is dependent on the platoon length. PIA also takes into account the dilemma zone protection issue by extending the green preemption to ensure that the last vehicle of the platoon can leave the dilemma zone. The green preemption also has a maximum to avoid too much delay on the conflicting approaches.

2.3 Driving Behaviors Associated With the Dilemma Zone

When drivers are caught in the dilemma zone, they will make several sequential decisions to stay safe. First, they will determine whether to stop or not; second, they will determine the acceleration/deceleration rates according to their first decision. In addition, the drivers have different desire speeds and perceptions about the traffic conditions. As a result, while the drivers are approaching intersections, they will have random speeds, random stop-or-go decisions and random acceleration/deceleration rates. The research literature on these associated driving behaviors is rich, and the author analyzed the most commonly accepted ones in this dissertation.

2.3.1 Approaching Speed at Intersections

Vehicle approaching speeds at signalized intersections are usually considered normally distributed. ITE recommends that the 85th percentile approaching speed, typically 5 mph faster than the posted speed limit, is used for the yellow interval design [34].

2.3.2 Drivers' Stopping Decision at The Yellow Onset

Intuitively, the farther from stop line a vehicle is, the more likely it is to choose to stop at the yellow onset. This phenomenon was supported by several studies. For example, Yosef and Mahmassani used Probit model, a cumulative normal distribution function, to present the relationship between TTI/distance-to-the-intersection (DTI), and stop decision [34]. Other studies used the logistic regression model and concluded that the stopping probability has an increasing relationship with the distance to stop line while a decreasing one with the vehicle instantaneous speeds at the yellow onset [8, 16, 35].

2.3.3 Drivers' Maximum Acceleration/Deceleration Rates

Drivers' will select the appropriate acceleration/deceleration rates as perceived to make their maneuvers safe. Meanwhile, the desired acceleration/deceleration rates are limited by the laws of physics and therefore acceleration/deceleration has limits. Gazis et al. recommended 0.3g~0.5g as the maximum deceleration [2]; ITE recommends 10 feet/s^2 [36]; and American Association of State Highway and Transportation Officials (AASHTO) uses 11.2 feet/s^2 [37]. Other reported maximal deceleration rates for the 85th percentile speed include: 14 feet/s^2 [7]; 10.5 feet/s^2 [8]; 12.9 feet/s^2 [35] and 10.7 feet/s^2 [38]. As for acceleration, Gazis et al. provided a constant acceleration model according to a vehicle's instantaneous speed at the yellow onset and suggested 0.5g~0.8g ft/s^2 as maximum rates [2]. Liu and Herman used a linearly decreasing model between acceleration and instantaneous speed to calculate the yellow and the all-red interval [4].

2.3.4 Perception-Reaction Time

P-R time is a "time-lag" phenomenon in drivers. It is also an indicator of driver aggressiveness. It is commonly accepted that the more aggressive a driver is, the shorter the P-R time he/she will have. Some reported P-R time values include 1.0s [3]; 1.13s [36]; 0.9s [35]; 1.86s~2.32s [11] and 2.5s [16].

2.4 Traditional Designs of The Green Extension System

The multiple-detector green extension system is the most commonly used dilemma zone protection system. It uses a couple of advance detectors to extend the green until either there are no vehicles in the dilemma zone or the green time reaches the maximum. The simplest GES uses only one advance detector. A 6 x 6 feet^2 advance detector is placed where the DZ begins. The single-detector design can only tackle one prevailing speed. As a result, when the gap-out occurs, vehicles will still be caught in the DZ.

Sackman et al. recommended another design for GES which requires placing a series of detectors with uniform one-second extension [36]. The first detector is placed where a vehicle at the design speed can stop safely; the second detector is placed where a vehicle at a speed 10 mph lower than the design speed can safely stop, and so on. These advance detectors cover the entire dilemma zone. The Texas Department of Transportation (TxDOT) modified this approach and used the stopping distance criterion recommended by the American Association of State Highways and Transportation Officials (AASHTO).

The Southern District of the Institute of Transportation Engineers (SDITE) recommended a two-detector design [13, 37]. With the SDITE method, the traffic signal controller operates in a non-locking mode. The green gaps out before the slow vehicles reach the dilemma zone. The first detector is placed five seconds upstream of the stop line and has a 3-second extension; the second detector is placed at 2.5 seconds upstream of the stop line with a 2-second extension.

Another prevailing GES configuration was developed By Bonneson et al. [15]. The features in the Bonneson design include providing multiple levels of protection for vehicles (70% and 95%)

and two types of goals (clear the dilemma zone and reach the stop line). The number of advance detectors is determined by the protection level (e.g., 2 detectors for 70% protection and 3 detectors for 95% protection). The first detector is placed where the fastest vehicles (V_U) enter its dilemma zone. The second and third detectors are designed using one of the following two methods:

- *Constant-speed method:* The second detector is placed where vehicles traveling with 10 MPH less than V_U enter the DZ. If the protecting ratio is 95%, the third detector is placed where the vehicles with 20 mph less than V_U enter the DZ. After the detector locations are determined, the extension time for each detector is calculated using Eq.(2-5):

  $$t_i = (x_i - x_{i+1} - L_d - L_v) / v_{i+1} \tag{2-5}$$

  Where:

  - t_i : the detector *i*'s extension;
  - x_i : the distance from the detector *i*’s leading edge to the stop line;
  - L_d: the detector length;
  - L_v: the average vehicle length;
  - v_{i+1}: the design speed for the detector *i+1*

- *Constant-time method:* The detectors have the same extension and the first detector is placed as with the constant-speed method. The following detectors are placed and configured with equation group (2-6):

$$x_i = (x_{i\text{-}1} - t * v_i - L_d - L_v)$$
$$x_i = A * v_i + \frac{v_i^2}{2 * B} \qquad (2\text{-}6)$$

Where

- t: constant extension time;
- x_i : the distance from the detector i's leading edge to the stop line;
- L_d: the detector length;
- L_v: the average vehicle length;
- v_i: the design speed for the detector i;
- A, B: the constants corresponding to the dilemma zone boundaries

Fig. 2-9 illustrates several GES designs for the 45 MPH speed limit.

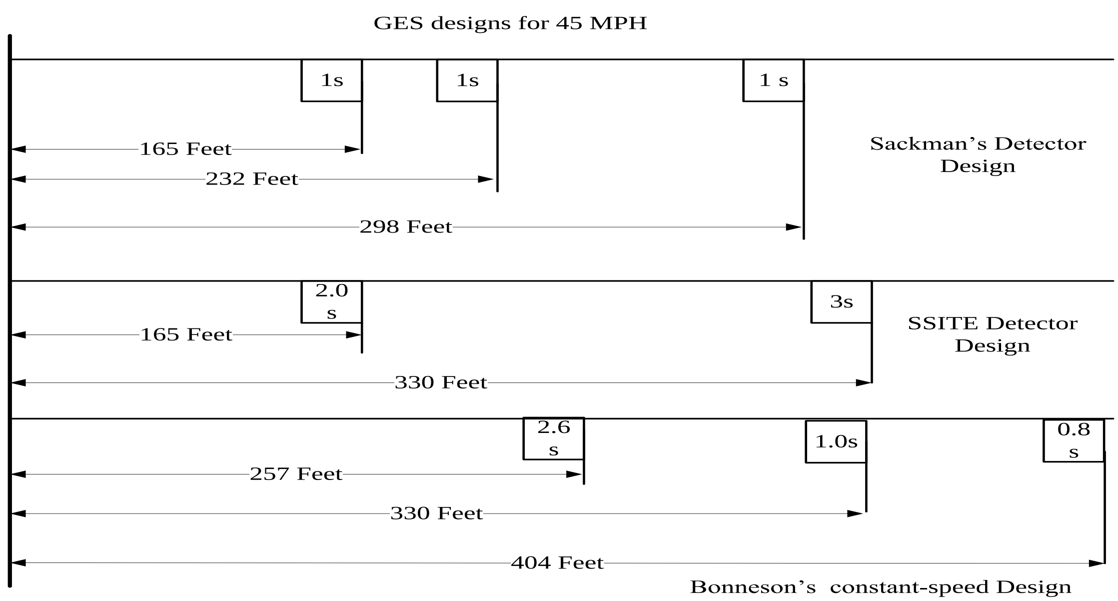


Figure 2-9 Different GES designs for the 45 MPH design speed

2.5 Application of the Markov Process to Transportation Studies

The Markov process has been proved capable of modeling highly stochastic, non-linear traffic systems. A typical Markov control model is composed of four items: state space, control actions, states transition probabilities and reward matrix.

The state space X: it is a *Borel*[2] *Space*, a topological space that can be formed from open sets through the operations of countable union, and each element in the space is called *state*. In the context of the traffic system, the state space is defined to reflect traffic dynamics and it can be, for example, the queue length, the number of vehicles in the DZ or the control delay. Previous studies on the stochastic traffic dynamics primarily focused on how to improve mobility and therefore they usually used the number of vehicles (e.g., the queue length or the platoon length) to define the state spaces. Taken as examples, Botma used the number of vehicles in a queue to model the stochastic traffic [38]; Botma's model was later used in Hoogendoorn's study on the robust control of stochastic traffic networks [39]; Alfa and Neuts used the number of vehicles in a platoon to define the state space for the random traffic arrival profile [40]; Viti and Zuylen used the number of vehicles in queues as the state space to re-cast the queuing models at signalized intersections [41]; Geroliminis and Skabardonis used the queue length as the state space to model the stochastic traffic dynamics along signalized arterials [42]; Cascetta modeled traffic assignment evolution as a stochastic process using the Markov process. Obviously, the more detailed a state space is modeled, the closer to the reality it is. Excessive details, however, may also dramatically increase a problem's dimension and result in excessively long computing time. For this reason, in their Markov-process-based adaptive signal control framework, Yu and Recker used a binary state space, congested vs. uncongested, rather than the queue length on each approach to ensure that the optimizing algorithm can work fast. When the number of vehicles in a link is greater than a threshold value, that link is marked as “congested”, otherwise it is considered “uncongested” [43]. Similarly, Kim *et al.* used “congested” and “uncongested” to mark the states for each node of the network in their vehicle routing studies [44].

The possible control actions A: it is a *Borel Space* and defined as the set of all possible controls (or alternatives). Each element x in the state space X is associated with, or results from, a subset of A. From the perspective of the traffic signal control, it is usually translated into control strategies. Taken as examples, some well-known control strategies include: TRANSYT-7F [45], SCOOT [46], SCATS [47], OPAC [48], RHODES [49] and TRPS [50]. However, although most signal control strategies implicitly incorporate the traffic’s stochastic features, they are in general based on deterministic models rather than on stochastic models. As a result, those traditional strategies sometimes may not function well under highly dynamic traffic patterns. Another issue about the control strategies is how to efficiently optimize the strategies on-line or off-line. Off-line optimization works well when a traffic pattern is repeatable. Some well-known studies include: Robertson used the *Hill Climbing* optimizing technique in the package of TRANSYT-7 [45]; Nakatsuji and Kaku applied neural networks in their self-organizing traffic control strategies [51]; Park et al. used the *Genetic Algorithm* to optimize the signal control strategies in TRANSYT-7F (version 8.1) [52] and Abbas et al. used a revised *Multiple objective Genetic Algorithm* to optimize their traffic responsive signal control framework [53].

On-line adaptive optimal signal control strategies have a higher requirement for computing efficiency. There are two optimizing approaches for adaptive control strategies:

a) *Binary choice logic,* in which time is divided into successive small steps. Between the minimum green and the maximum green, a binary decision (i.e., end vs. extend current green) is made at each time step. Examples of the binary control logics include the modern vehicle actuated signal control strategies, and the stepwise adjustment of timing plans [54, 55]. The binary choice logic considers a short-term future (10sec~30sec) and can be usually optimized fast.

b) *Sequential approach*, in which the decision-making window is longer (50sec~100sec) than the binary logic. *Dynamic programming* (DP) is commonly used here. The original dynamic programming may need excessively long time with the increase of the problem dimensions and therefore it is often revised in practice. For example, the well-known adaptive signal control strategies, OPAC [48] and RHODES [49] use the *rolling-horizon* and iterative dynamic programming technique to increase the optimizing speed. Yu et al. used a rapid-calculate version of dynamic programming to obtain the optimal control strategies [43].

The probability measure space: it is typically described in a matrix P and each element p^k_{ij} in P stands for a transition probability from state i to state j under control measure k. A stochastic process is called *Markov Process* if its future probabilities are only determined by its most recent states. In the field of the traffic signal systems, most previous related studies considered the traffic dynamics (e.g., traffic arrival profile, queue length) as the Markov process. Viti and Zuylen designed a mesoscopic Markov model for queues at signalized intersections [56]; Yu and Recker used the *Poisson Process* to analyze the queue length changing probabilities in their MP-based adaptive signal control [43]; Geroliminis and Skabardonis used MP to model arrival profiles and queue lengths along signalized arterials [42]; Alfa and Neuts used a discrete time Markovian arrival process to model traffic platoons [40]; Adam et al. used Markov transition matrix in their dilemma zone protection policy studies [57, 58]; Kim et al. used travel time estimation to calculate the transition probabilities between states [44]; Sun et al. used Gaussian Mixture Model (GMM) model and Maximum Likelihood Estimation (MLE) to estimate the transition matrix [59]; Sherlaw-Johnson et al. used the *Maximum Likelihood Estimation* (MLE) to estimate the transition matrix [60]; Hazelton and Walting used linear exponential learning filter [61]; Hazelton and Walting Gaussian process to derive the transition matrix their traffic equilibrium distribution studies [61].

One-step reward R is the immediate result from the control action. Reward is important for on-line optimization in adaptive signal control strategies. Reward is defined according to the problems. Taken as examples, Yu and Recker assigned a high reward for a control policy if the traffic state transitioned from the congested to the uncongested [43]; Adam et al used the number of vehicles in the dilemma zone [57, 58]; Kim *et al.* estimated the cost proportional to the travel times [44].

2.6 Review of Simulation-Based Optimization

Stochastic optimization methods, also called stochastic root-finding methods in other literature, are a family of optimization algorithms which incorporate random elements either in the problem structure (objective function, constraints, etc) or in the algorithms (random choice of parameters, etc). There are two major issues in the stochastic optimization issues: (1) how to approximate a stochastic system that can only be observed with random errors; (2) how to search optimal solutions to the random but observable systems. In this section, we briefly discuss the development of stochastic optimization theories and their applications in traffic signal systems.

The first stochastic optimization algorithm was developed by Robbins and Munro [62] to solve stochastic equations. It will be relevant to stochastic optimization if the equations are interpreted as the gradient of objective functions. The simple iterative structure is

$$X_{k+1} = X_k - a_k(\overline{Y_k} - \gamma)\,, \tag{2-7}$$

Where $k = 0\,1\,...$; X_0 is an initial guess of the root x^*, $\overline{Y}_k = \frac{Y_j(X_k)}{m}, \{Y_1(x),...,Y_m(x)\}$ is a set of random samples from the distribution of $Y(x)$, and $\{a_k\}_{k=0}^{\infty}$ is a predetermined sequence of positive constants which satisfies $\sum_{k=0}^{\infty} a_k = \infty, \sum_{k=0}^{\infty} a_k^2 < \infty$. γ is a given constant This method is also called *Class Stochastic Approximation* (CSA). Much of the modern literature in stochastic optimization is a variant of CSA, which addressed such issues as how to increase convergence rates or relax convergence conditions (*3, 4, 5, 6).*

Sample Average Approximation (SAA) is another technique to solve stochastic optimization problems. The SAA technique was first suggested by Healy and Schruben [63] and later referred to by Rubinstein and Shapiro [64] and Shapiro and Homem-de-Mello [65] to solve stochastic optimization problems. The idea of SAA is: for a given set of parameters *x*, we may estimate the performance measure by taking *m* samples. Let $\overline{Y}_m(x)$ denote the estimator of the performance measure (e.g., control delay, queue length in traffic signal systems) and $y_m(x;\underline{\omega})$ denote one realization of the estimator with independent random numbers $\underline{\omega} = \{\omega_1, \omega_2, ..., \omega_m\}$. Obviously, the equation, $\overline{Y}_m(x) = \frac{\sum_{i=1}^{m} y_m(x, \omega_i)}{m}$, still holds. Given the true problem *G* is often impossible to know, we may solve *G*'s approximate problem *S*. The performance measures for *S* can be estimated with $\overline{Y}_m(x)$. In light of SAA, instead of optimizing the original stochastic problem (*G*), the approximate deterministic problem (*S*) generated with a sufficient sample size can be solved to optimality with proper optimization algorithms. The solution to *S* will converge to the solution to *G* with probability 1 when $m \rightarrow \infty$.

Nearly all the research efforts on the optimization of the traffic signal systems followed the SAA concept. Taken as examples, Park et al. first applied the SAA concept by coupling the genetic algorithms with the simulation-based optimization to provide best signal timings under oversaturated traffic conditions [66] . Later Park et al. applied this method to a couple of related studies [67, 68, 69]. Stevanovic et al. integrated the genetic algorithm with VISSIM to optimize the signal system and transit priority systems in Park City, UT and Albany, NY [70].

2.7 Summary of the Literature Review

Although there is rich literature on driving behaviors at intersections, the application of these findings to traffic signal control has been limited. Meanwhile, although many signal control algorithms implicitly cover traffic's randomness, the stochastic nature of a traffic system and the complex feedback mechanism of traffic signal control have not been addressed well. As a result, there is a need to devote more research efforts to traffic signal control issues in the stochastic scheme. The remainder of the dissertation addresses several widely concerning issues in the area of dilemma zone protection. The innovation of this research is that it utilizes the recent development of random driving behaviors and stochastic optimization techniques to solve the traffic problems.

Chapter 3 A Stochastic Dilemma Hazard Model at High-speed Signalized Intersections

3.1 Introduction

A recent survey conducted by the National Safety Council reports that the crashes associated with signalized intersections constitute up to 30 percent of all crashes [71]. Besides driver errors, indecisiveness in the dilemma zone is another leading cause, especially at high-speed signalized intersections. Consequently, preventing approaching vehicles from being caught in the DZ has witnessed an increasing interest by the scientific community.

The objective of this chapter is to develop a model to quantify the hazard level of vehicles in the DZ as a function of its remaining Time-To-Intersection (TTI) at the yellow onset. The analysis begins with the single-vehicle scenario to conceptually illustrate how to calculate the dilemma hazard. Next, this model is expanded to a more realistic scenario—multiple vehicles in the DZ. The first vehicle is the leading vehicle and the second vehicle is the subject vehicle. In addition to the hazard generated as in the single-vehicle scenario, the subject vehicle is associated with additional dilemma hazard situations due to its interaction with the first vehicle (e.g., rear-end-collision hazard).

The dilemma hazard model calculates the dilemma hazard based on the comparison of driver decisions and their actual driving capability. The driver decision probability at the yellow onset is calculated based on field observation; the drivers' actual driving capabilities are decided based on vehicle kinematics; and the vehicles' maximum acceleration/deceleration is based on previous studies [72, 73].

Next, we developed a Monte-Carlo simulation framework to account for the variability in actual vehicle movements. The system input for the Monte Carlo simulation was obtained from field observation. The TTI-dependent dilemma hazard model obtained from the Monte-Carlo simulation was verified using field observation data.

3.2 The Dilemma Hazard Model

A traffic conflict is generated when vehicles make unsafe maneuvers, such as running a red-light or following too closely (causing rear-ends collisions). Therefore, the proposed model analyzes the DZ-associated red-light-running incidents potential as well as the DZ-associated rear-end collision potential. Drivers in the DZ will first decide to stop or cross, then select a perceived safe acceleration/deceleration rate. However, due to their different perceptions, the drivers select acceleration/deceleration randomly, making their decisions either safe or unsafe. When a driver decides to cross but is incapable of clearing the intersection during the clearance interval, it becomes a candidate for a right-angle collision and generates a dilemma hazard. In the multiple vehicle condition, if the lead vehicle decides to stop but the second vehicle decides to cross the intersection, a rear-end collision dilemma hazard is generated. We start with the single-vehicle condition then expand it to a more general two-vehicle condition to illustrate this concept.

3.2.1 Single-Vehicle Scenario

In this scenario, only the right-angle collision hazard is possible. The driver will make two sequential decisions at the yellow onset:

- Decide to cross or to stop; and
- Choose the acceleration or deceleration rate.

The stopping decision is random and follows a certain probability distribution. In addition to the well-known probit and logistic models, other distributions such as lognormal and gamma distributions can also be used to represent the stopping probability.

Drivers randomly select the acceleration/deceleration that they consider safe. Previous research reported that the drivers' acceleration/deceleration is normally distributed and that the distribution varies over time and locations [74, 75], which can be interpreted as:

$$a \sim N(\mu(TTI), \sigma(TTI)) \tag{3-1}$$

Where:

- a : Acceleration/deceleration at the yellow onset;
- $\mu(TTI), \sigma(TTI)$: The mean and standard deviation of the normal distribution as functions of TTI;

When the subject vehicle decides to cross an intersection and is still in the intersection after the all-red interval expires, it will generate a right-angle-collision hazard. Mathematically, this situation can be expressed as:

$$v_0(y+r)+\frac{1}{2}a_{1(TTI)}(y+r-\delta)^2 \le d+w+L \tag{3-2}$$

Where:

- TTI: Time to intersection at yellow onset;
- $a_{1(TTI)}$: TTI-dependent Acceleration (feet/s^2);
- y : Yellow duration in seconds;
- r : All-red clearance duration in seconds;

- δ : Perception-Reaction time in seconds;
- v_0 : Instantaneous speed at the yellow onset (feet/s) and;
- *d, w and L*: Distance from stop line, intersection width and vehicle length respectively (feet).

The dilemma hazard probability of a single vehicle not being able to clear the intersection can be expressed as:

$$P_1\left\{v_0(y+r)+\frac{1}{2}a_{1(TTI)}(y+r-\delta)^2 \le d+w+L|cross\right\} \tag{3-3}$$

In the stopping situation, the vehicles in the DZ can use both the yellow interval and the following red time to safely decelerate. When a driver decides to stop but cannot stop completely before the stop line, it will generate a right-angle hazard. Mathematically, the equation to express this situation is (3-4):

$$\frac{1}{2}v_0^2 > a_{2(TTI)}*d \tag{3-4}$$

Where:

- $a_{2(TTI)}$: TTI-dependent deceleration; and
- Other terms: Defined as before.

The dilemma hazard probability of a single vehicle not able to safely stop before the intersection is:

$$P_2\left\{\frac{1}{2}v_0^2 > a_{2(TTI)}*d|stop\right\} \tag{3-5}$$

In summary, the dilemma hazard for the single vehicle scenario is:

$$H_s=P\{cross\}*P\left\{\left(v_0(y+r)+\frac{1}{2}a_{1(TTI)}(y+r-\delta)^2\leq d+w+L\right)|cross\right\}$$
$$+P\{stop\}*P\left\{\frac{1}{2}v_0^2>a_{2(TTI)}*d|stop\right\} \quad (3\text{-}6)$$

Where: P{stop},P{cross} are the stopping/crossing probabilities

3.2.2 Multiple Vehicle Scenario

May concluded that the headway between vehicles is exponentially distributed in light or moderate traffic volumes, while it is normally distributed in heavy traffic volume [76]. Drivers typically keep a minimum safe headway to their immediate lead vehicles. This safe headway, however, is often violated by aggressive drivers. As a result, the number of vehicles per lane in a Type II DZ can be 1, 2, 3 or even more with respect to various Type II DZ boundaries.

When multiple vehicles are in a DZ, another type of accident potential, namely the rear-end collision hazard, can be generated if the first vehicle of two consecutive vehicles decides to stop but the second vehicle decides to cross.

We limit our study scope to two consecutive vehicles in a DZ. The second vehicle is the subject vehicle and the first vehicle is considered not to be influenced by the second vehicle's behavior. Following this approach, three or more vehicles in DZ can be modeled as several two-vehicle units as in Fig. (3-1).

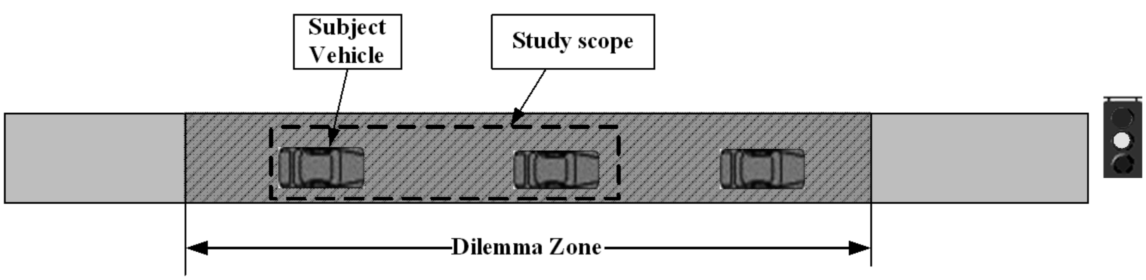


Figure 3-1 Study object of the dilemma hazard model

The subject vehicle has totally four conditions that can possibly generate a dilemma hazard:

1. The subject vehicle has no lead vehicle in the DZ (single vehicle condition);
2. Both the subject vehicle and its lead vehicle decide to stop;
3. The subject vehicle decides to cross while its lead vehicle decides to stop;
4. Both the subject vehicle and its lead vehicle decide to cross.

These four exclusive conditions are explained below:

Case 1: If the lead vehicle has left the DZ, the situation will be the same as in the single-vehicle case. The subject vehicle's remaining time in the DZ is ($TTI\text{-}DZ_L$) seconds, where DZ_L is the time a vehicle travels from the near end of the DZ to the stop line. If the headway between the subject and the lead vehicle is less than ($TTI\text{-}DZ_L$), the lead vehicle is also in the DZ. As a result, the probability that the lead vehicle exists in the DZ can be expressed as:

$$P\left\{ \text{headway} \le (TTI - DZ_L) \right\} \tag{3-7}$$

And the expected dilemma hazard for Case 1 is:

$$P_1 = H_s * \left(1 - P\left\{ \text{headway} \le (TTI - DZ_L) \right\}\right) \tag{3-8}$$

Where:

- H_s : The dilemma hazard on the single-vehicle condition (Eq. (3-6));

Case 2: If the lead vehicle is in the DZ and the subject vehicle decides to stop, the subject vehicle will not have a rear-end-collision, but might still have a right-angle-collision hazard. The expected dilemma hazard for this case is:

$$\begin{aligned} P_2 = {} & P\{\text{lead in DZ}\} * \\ & P\{\text{subject stops}|\text{lead in DZ}\} * \\ & P\{\text{right-angle-collision hazard}|\text{subject stops}\} \end{aligned} \tag{3-9}$$

Case 3: If the subject vehicle decides to cross but its lead vehicle, which is also in the DZ, decides to stop, the subject vehicle will have a rear-end-collision hazard. Please note the subject vehicle may change their decision to avoid an accident once it perceives its decision is incompatible with its front vehicle. Nonetheless, a traffic conflict (rear-end-collision hazard) is already generated given their instantaneous responses at the yellow onset.

The conditional probability that the lead vehicle decides to stop is:

$$
\begin{aligned}
&P\{\text{lead vehicle stops}|\text{lead vehicle in DZ}\} \\
&=P\{\text{lead vehicle stops}|\text{headway}<=TTI\text{-}DZ_L\} \\
&=\frac{\int_0^{TTI\text{-}DZ_L} P\{\text{lead vehicle stops, headway}=x\}f(x)dx}{\int_0^{TTI\text{-}DZ_L} P\{\text{headway}=x\}.f(x)dx}
\end{aligned}
\tag{3-10}
$$

Where:

- x : Headway between the two vehicles;
- f(x): Probability density function of the headway distribution;
- DZ_L : Near end of DZ;

$$
\begin{aligned}
P_3=&P\{\text{lead vehicle in DZ}\}* \\
&P\{\text{lead vehicle stops}|\text{lead vehicle in DZ}\}* \\
&P\{\text{rear-end hazards}|\ \text{lead vehicle stops}\}
\end{aligned}
\tag{3-11}
$$

Case 4: If the lead vehicle is in the DZ and both vehicles decide to cross, the subject vehicle will have no rear-end-collision hazard caused by the stop-or-cross decision but will still have a right-angle collision hazard.

$$
\begin{aligned}
P_4 = P&\{\text{lead vehicle in DZ}\}* \\
&P\{\text{ both vehicles cross |lead vehicle in DZ}\}* \qquad (3\text{-}12)\\
&P\{\text{right-angle-collision hazard| both vehicles cross}\}
\end{aligned}
$$

Note that any of the four cases exist only if all of their conditions are met. This guarantees that these four cases are mutually exclusive.

3.3 Model Calibration Using the Field Data

The dilemma hazard model is validated and calibrated using the field data collected by Virginia Tech Transportation Institute. The importance of this task is twofold: (1) to estimate the mean and standard deviation of driver acceleration and deceleration for each TTI value, and (2) to validate the proposed dilemma hazard model by comparing the simulation-based and the observed dilemma hazard values.

With a high performance data acquisition system (DAS) provided by Virginia Tech Transportation Institute, we analyzed the vehicle trajectory at the four-leg Peppers Ferry intersection in Christiansburg, Virginia. The intersection width for the through lanes is approximately 70 feet (5 lanes+10 feet buffer) as shown in Fig. 3-2.

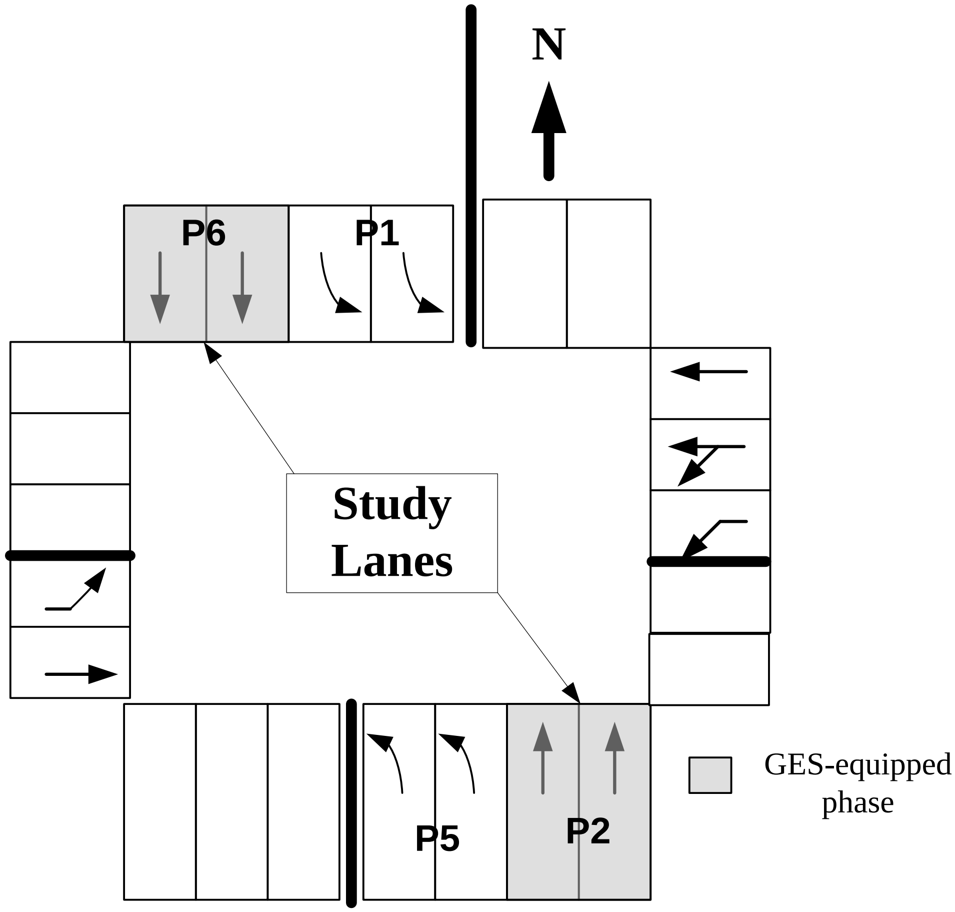


Figure 3-2 Geometry of the study intersection

The data were choreographed and recorded by a custom hardware package. Parametric data (vehicle trajectories, signal phase states, error messages) were stored at 20 HZ to a binary file. The sensing system was composed of radar, signal sniffer and video subsystems. The signal phase was correlated with one or more lanes on a given approach. At the same time, the signal sniffer provided the current light status and timing information for up to eight different signal phases [77].

Raw data were filtered and only those vehicles in the dilemma zone of study lanes (shown in Fig. 3-2) at the yellow onset were analyzed. In order to guarantee the vehicles' perception-reaction time and driving behaviors were fully collected. Two weeks of trajectory data were used for this study and we found the off-peak traffic was light and the peak traffic was moderate. Like most studies on drivers' stopping probability at the yellow onset, drivers were assumed to make a one-time stopping decision and the aggregated acceleration/deceleration was measured with their speed changes during the yellow and all-red clearance times.

We plotted all the vehicle trajectories to identify the criteria distinguishing whether a vehicle decided to cross the intersection or stop at the yellow onset. Fig. 3-3A illustrates the detected vehicle trajectories during the yellow and all-red clearance and Fig. 3-3B illustrates the vehicles' instant speeds when they passed the stop line. The negative distance means a vehicle has passed the stop line. According to the vehicle trajectories, the stopping vehicles, which have no trajectories after the stop line, have much lower speeds than crossing vehicles when they reach the stop line. From Fig. 3-3A and 3-3B, one can clearly see that a threshold of 8 meters/second speed (when vehicles reached the stop line) can be used as an effective dividing criterion. If a vehicle passed the stop line at greater than 8 meters/ second, this vehicle can be classified as a vehicle of the crossing group; otherwise the vehicle belongs to the stopping group. Please note that this threshold is high due to the offset between the stop line of the DAS and the actual stop line. After deleting those unclassified vehicles, 441 vehicles were classified as the crossing vehicles and 865 vehicles were classified as the stopping vehicles. All of those vehicles were grouped again according to their time to intersection at the yellow onset.

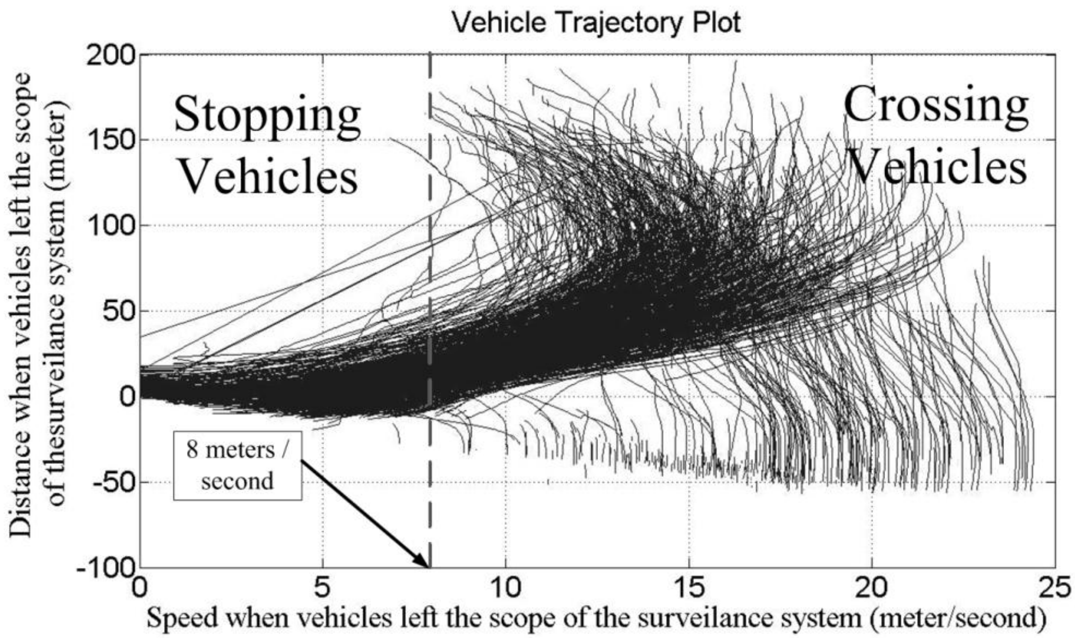


Figure 3-3 Vehicles trajectories recorded by DAS

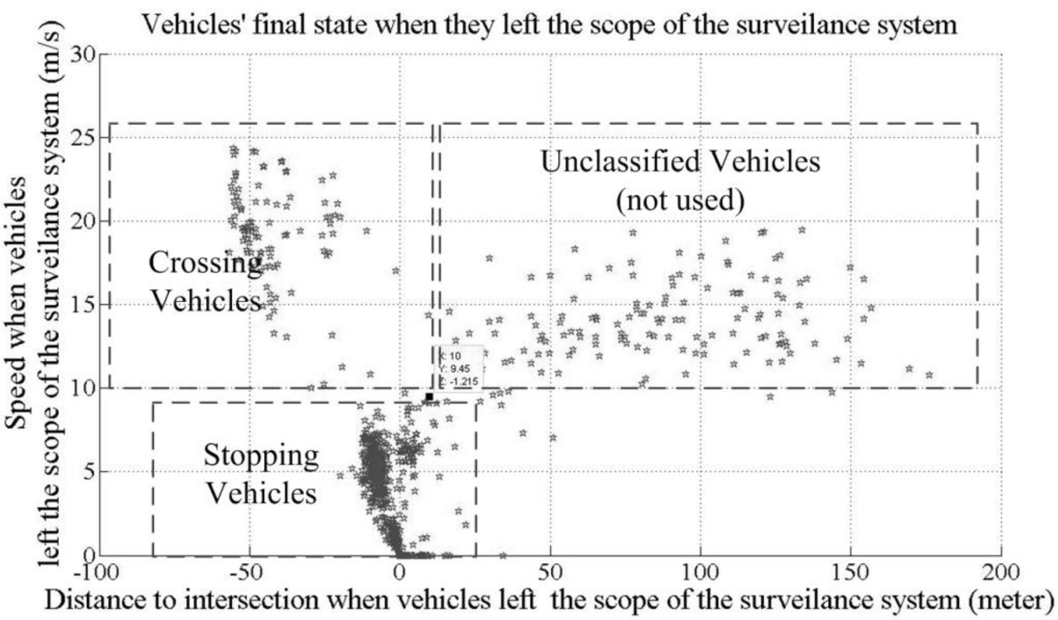


Figure 3-3B Vehicles states when they left DAS

3.3.1 Stopping Probability Model

There are many determinant factors that can have influence on a drivers' stopping probability and we selected one crucial factor from the perspective of traffic signal control, the vehicles' time to intersection at the yellow onset. TTI can reflect the vehicles' instantaneous speeds and distances. By collecting the observed stopping vehicles and their TTIs at the yellow onset, the cumulative stopping curve was plotted. The author used cumulative normal curve (the probit model) to fit the observed stopping vehicle at the yellow onset and get the stopping probability curve with an R^2 value of 0.95 and the other models, such as logistic regression model, can definitely be applied when other factors are included.

$$P_{stop}(TTI) = \Phi\left(\frac{TTI - \mu}{\sigma}\right), \text{ where } \mu=3.75 \text{ and } \sigma= 1.35 \qquad (3\text{-}13)$$

We found that the observed Type II dilemma zone boundaries range from approximately 2 seconds to 5 seconds TTI, which is close to the ITE recommendation.

3.3.2 Observed Acceleration/deceleration at the Yellow Onset

The observed data shows the TTI-dependent feature of a vehicles' acceleration/deceleration at the yellow onset. As in the previous studies [74, 75], a family of the normal distributions was used to represent the acceleration/deceleration randomness and they are functions of TTI: As illustrated in Fig. 3-4, each dot represents the average acceleration/deceleration of the vehicles with certain TTI at the yellow onset and all vehicles in the crossing group either nearly kept constant speeds or slightly accelerated to cross the intersection. On the other hand, vehicles in the stopping group braked more heavily when they were closer to the intersection. We used one single aggregated variance to reflect the vehicles randomness, and the corresponding distributions were used to generate acceleration/deceleration rates for each individual vehicle in the simulation. From a variation perspective, it seems that the stopping group had more difficulty selecting deceleration rates since their values were more scattered. Aggregated variances were used to reflect this phenomenon.

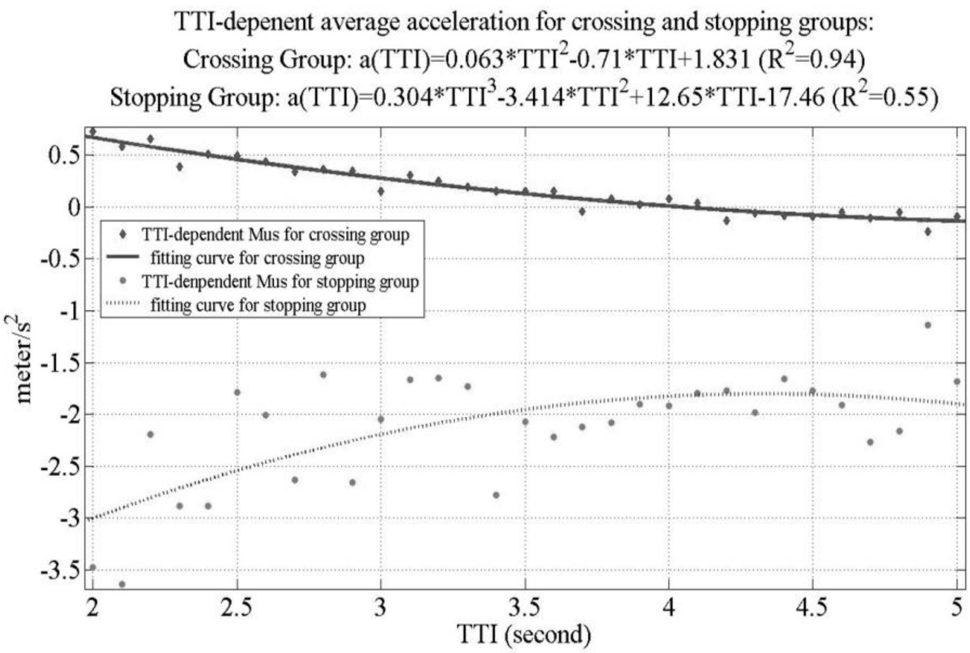


Figure 3-4 Vehicles' TTI-dependent acceleration/deceleration at the yellow onset

3.3.3 Monte Carlo Simulation

It is hard to derive a close-form analytical model of the DZ hazard. Therefore, we developed a Monte Carlo simulation framework to calculate the TTI-dependent dilemma hazard. The Monte Carlo simulation technique is widely applied to poorly modeled or model-less problems. The Monte Carlo simulation technique produces samples generated from given probability distributions. The average of the output variable (the dilemma hazard in this paper) can be used as an estimator of the system evaluation.

The simulation environment was designed in such a way that it had exactly the same road geometry and controller settings as the intersection where the data was collected. The only variable input was the TTI. In other words, the simulation was conducted according to various TTI values (Fig. 3-5).

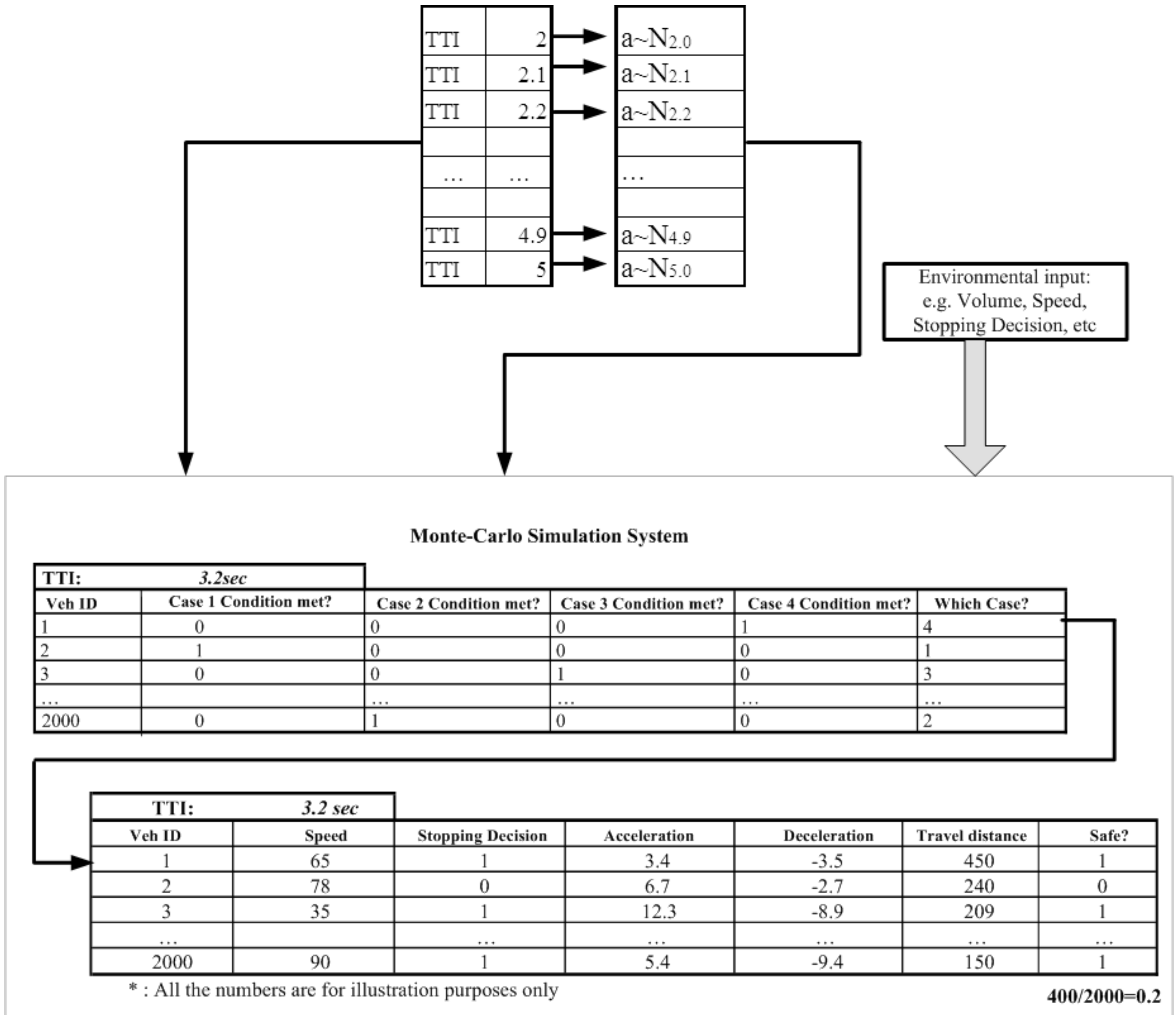


Figure 3-5 The Monte Carlo simulation framework

For each TTI value, a total of 2,000 independent vehicles were simulated. Each vehicle was randomly and independently assigned a speed, a stop-or-cross decision, an acceleration/deceleration rate, and headway from its lead vehicles according to the following rules:

Speed: the ITE's recommendation about the speed distribution of the approaching vehicles were used to generate individual vehicle's speed.

Stopping probability: The probit model, in which TTI is the only determinant variable (Eq. (3-13)) was used.

Acceleration/deceleration rate: as previously discussed, the random acceleration/deceleration rates are functions of TTI (Fig. 3-4) and the aggregated values are fit with polynomial functions as follows:

Stopping Group:

$$\mu(TTI) = 0.304TTI^3 - 3.414TTI^2 + 12.65TTI - 17.46 \qquad (R^2 = 0.55) \tag{3-14}$$

Crossing Group:

$$\mu(TTI) = 0.063TTI^2 - 0.71TTI + 1.831 \qquad (R^2 = 0.94) \tag{3-15}$$

With Eq. (3-14, 3-15) and the TTI value, the Monte Carlo simulation framework generates acceleration/deceleration rates for each vehicle according to the calibrated distribution. In this chapter, the ITE-recommended values were used, i.e., -10 feet/s^2 for the maximum deceleration and 15 feet/s^2 for the maximum acceleration. As mentioned above, the headway in light or moderate traffic is considered exponentially distributed.

Perception-Reaction time: the ITE recommendation (1.0 second P-R time) was used.

In the Monte Carlo simulation, we designed a decision tree to identify which case an individual vehicle belongs to and whether it is safe or not, as illustrated in Fig. 3-6. Each generated vehicle will fall into one of the four modeling scenarios, previously described in the multiple-vehicle condition. After we assign a vehicle with the speed, stopping decision, acceleration/deceleration rate, headway from its lead vehicle and the lead vehicle's stopping decision (if the lead vehicle is also in the DZ), we can calculate the dilemma hazard values under each case scenario and decide which case this vehicle belongs to. For each TTI value, the ratio of the unsafe vehicles to the overall 2000 vehicles was used as the estimator of the dilemma hazard for that TTI.

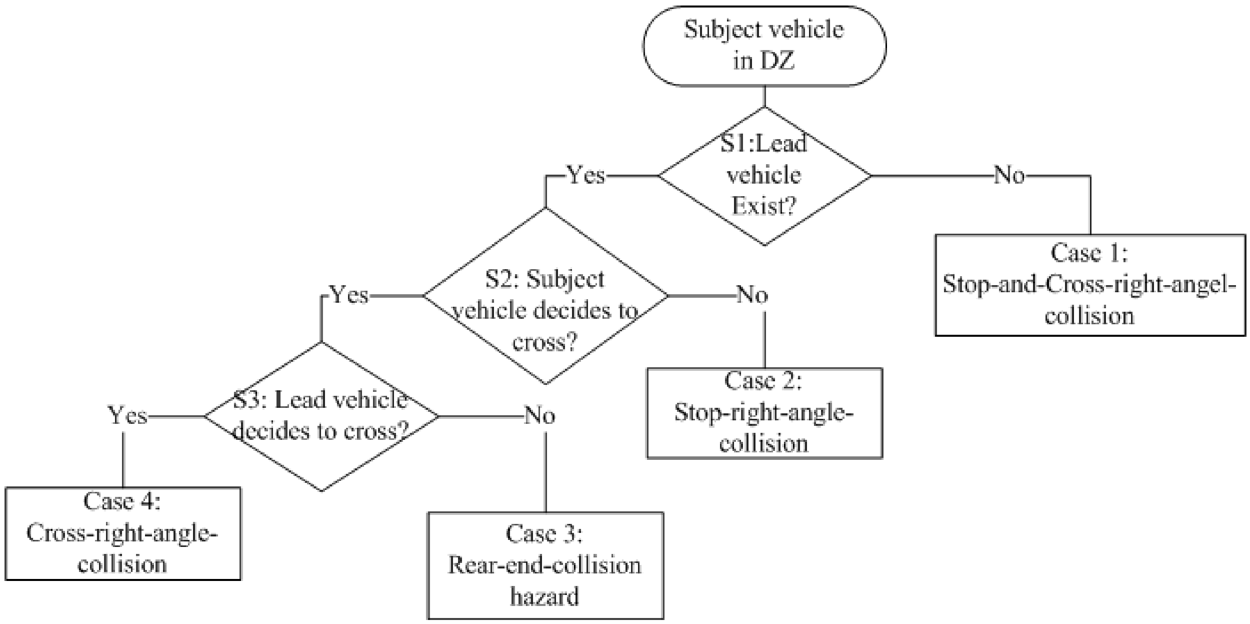


Figure 3-6 The subject vehicle's dilemma hazard decision tree

3.3.4 Dilemma Hazard Values from Monte Carlo simulation

We plotted the simulated dilemma hazard values with respect to TTIs. We also fit those data with polynomial fitting curves as shown in Fig. 3-7. It should be noted that the vehicles' dilemma hazard is higher in the middle than on the edges. The derived dilemma hazard function is:

$$H = -0.202 * TTI^2 + 1.565 * TTI - 2.218 \quad (R^2 = 0.88) \qquad (3\text{-}16)$$

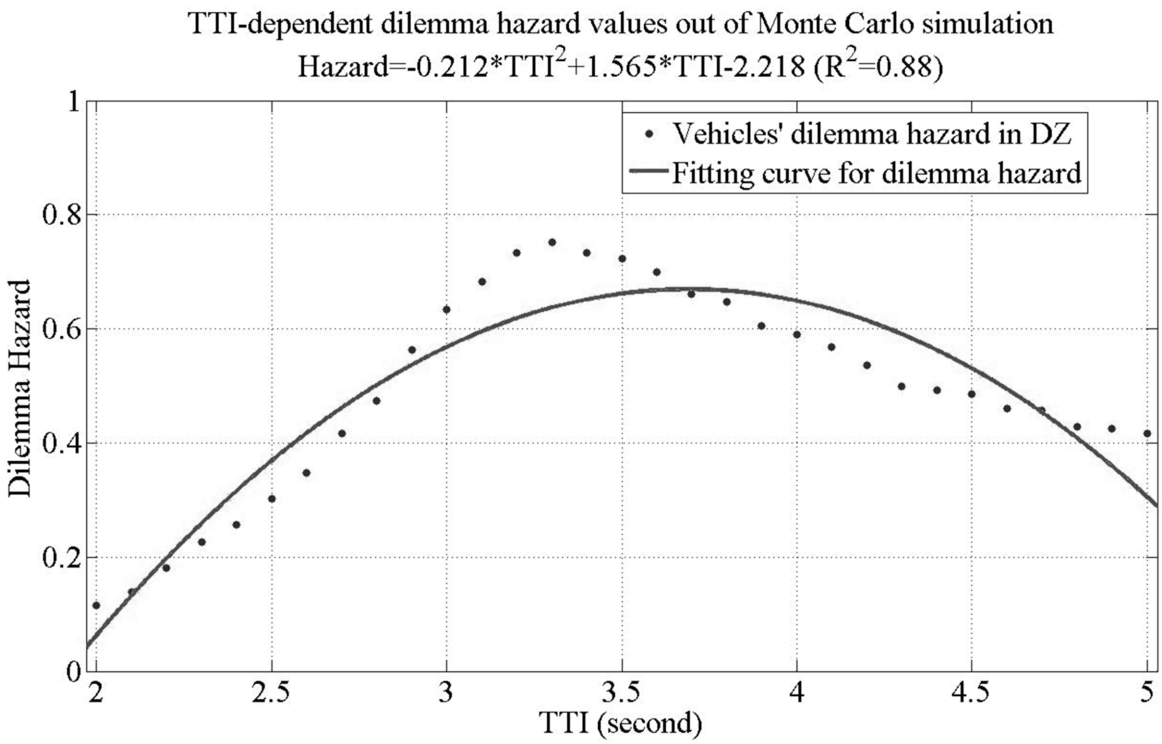


Figure 3-7 Dilemma hazard values and fitting at the study site

3.3.4 Controller Yellow and All-red Settings Impact on the Dilemma Hazard

We first compared the cumulative dilemma hazard in the controller settings being used in the field with the ITE recommendation (Fig. 3-8). From Fig. 3-8, it seems that the yellow and all-red clearance settings have a significant impact on a vehicles' overall dilemma hazard (the area under the curve). We further investigated several yellow and all-red settings around the ITE recommended values and those used at the data-collection site. Fig. 3-9 implies the overall dilemma hazard values for different yellow and all-red clearance intervals. This figure illustrates that the overall dilemma hazard is sensitive to the yellow and all-red clearance settings and therefore can be used to optimize the yellow and all-red settings at intersections. For this data collection site, the optimal yellow and all-red values were found to be 6.0. How to split the yellow and all-red to minimize the overall dilemma hazard is out of scope of this research.

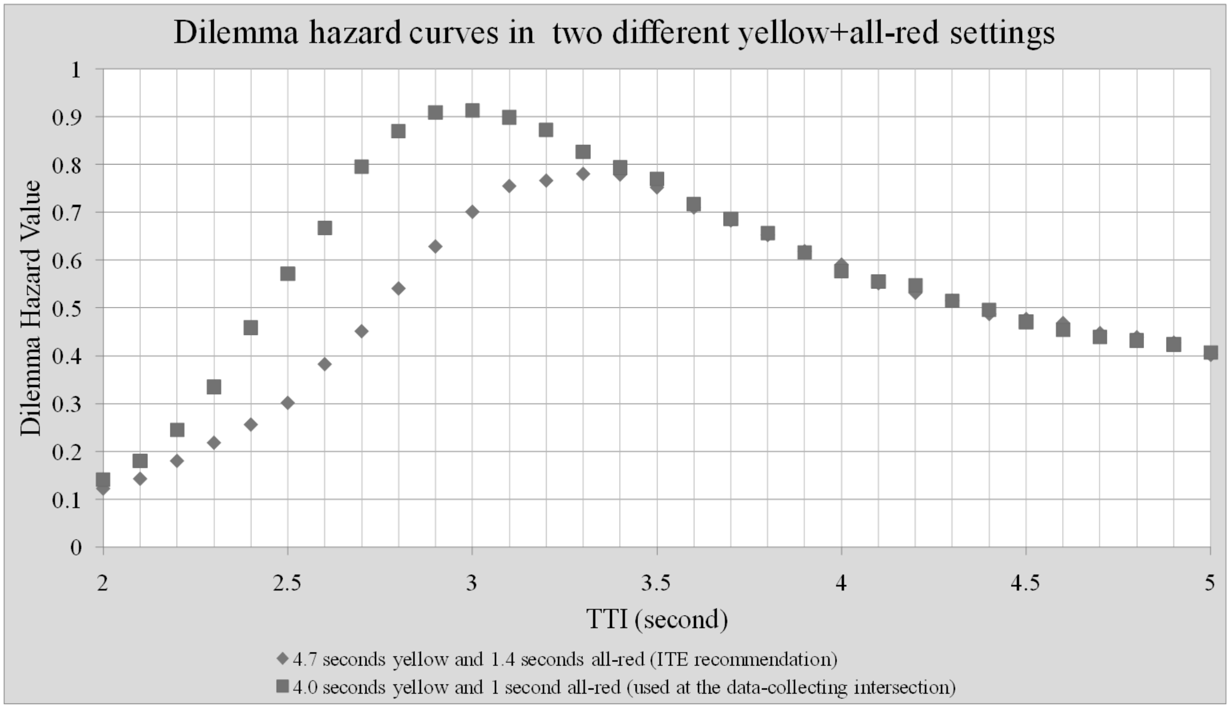


Figure 3-8 Dilemma hazard curves for different yellow and all-red clearance intervals

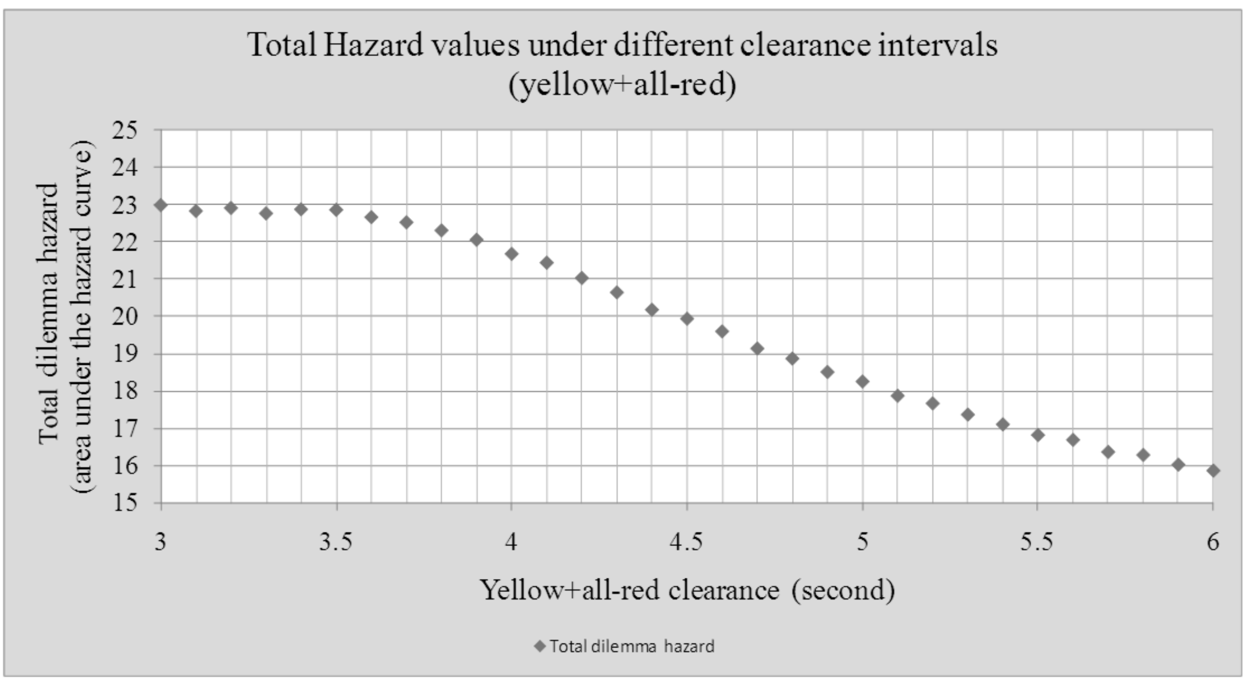


Figure 3-9 Total dilemma hazard values for different yellow and all-red settings

3.4 Conclusion of this chapter

A new traffic conflict potential measure, namely the dilemma hazard, was defined in this chapter. The dilemma hazard model uses a vehicle kinematics method as in GHM model (Type I DZ) to analyze the crash potential for vehicles in a Type II DZ. Monte Carlo simulation is used to mimic the driving behaviors in the DZ and calculate the dilemma hazard, which is a function of time-to-intersection.

There are many potential applications for the dilemma hazard model. For example, it can be used as a new safety measure for DZ protection systems as well as for the determination of the optimal clearance interval. The optimal clearance interval for the study site was found to be 6.0 seconds. While this value is specific to the study site, the proposed framework can be easily expanded to a wide range of scenarios to improve the intersection safety.

Chapter 4 Optimal Advance Detectors Design for the Multi-detector Green Extension Systems at High-Speed Signalized Intersections

4.1 Introduction

The multi-detector green extension system (GES) is a traditional DZ protection system and its mechanism is to extend the green via advance detectors until no vehicles exist in the dilemma zone (gap-out). When gap-out occurs, zero vehicles will be caught in the dilemma zone whereas when max-out occurs, GES provides no protection for the vehicles in the DZ. As a result, GES is in essence an all-or-nothing system. Depending on the local traffic conditions and controller/detector settings, GES will have different max-out ratios [15]. The crucial part of in the GES design is to configure the advance detectors properly to keep a low max-out ratio.

4.2 Significance of the Research

Previous studies proved that GES could effectively reduce the number of DZ-associated accidents [13, 14, 15]. It was also reported that there is no one optimum design for all the design speeds [78]. Meanwhile the prevailing GES designs are partially based on engineering judgments and therefore they cannot guarantee the optimal configurations. Another issue is that all the traditional GES designs consider safety only, rather than both safety and efficiency. Therefore, safety might be achieved at the cost of efficiency if the efficiency is not taken into account in the GES design.

This chapter addressed these issues by answering the following two questions:

- How to determine the operational efficiency under different GES designs?

- How to optimize a GES design?

The GES design can be recast as an optimizing problem (Eq. (4-1)).

objective function:
$\mathrm{Min}(U_1,U_2)=F(n,t_1,t_2,\ldots t_n,x_1,x_2,\ldots x_n)$
where,
U_i: DZ cost and delay cost;
n: the number of detectors;
t_i: the detector i's extension time(sec) and;
x_i: the detector i's location(feet); (4-1)

The following sections describe in detail each component of the optimization problem.

4.3 Simulation-based Objective Function

Obviously, there is no close-form objective function for this problem. Therefore, the author developed a simulation system in C++ to evaluate various GES configurations. The simulation system was developed for a high-speed major-minor signalized intersection and the major approaches were equipped with GES. As illustrated in Fig. 4-1, it consists of three modules: “Vehicle Generator” (to generate vehicles in our simulation engine), “Phase Terminator” (to decide when to end the green according to the Vehicle Generator Module) and “Cost Calculator” (to calculate the dilemma hazard after the green ends).

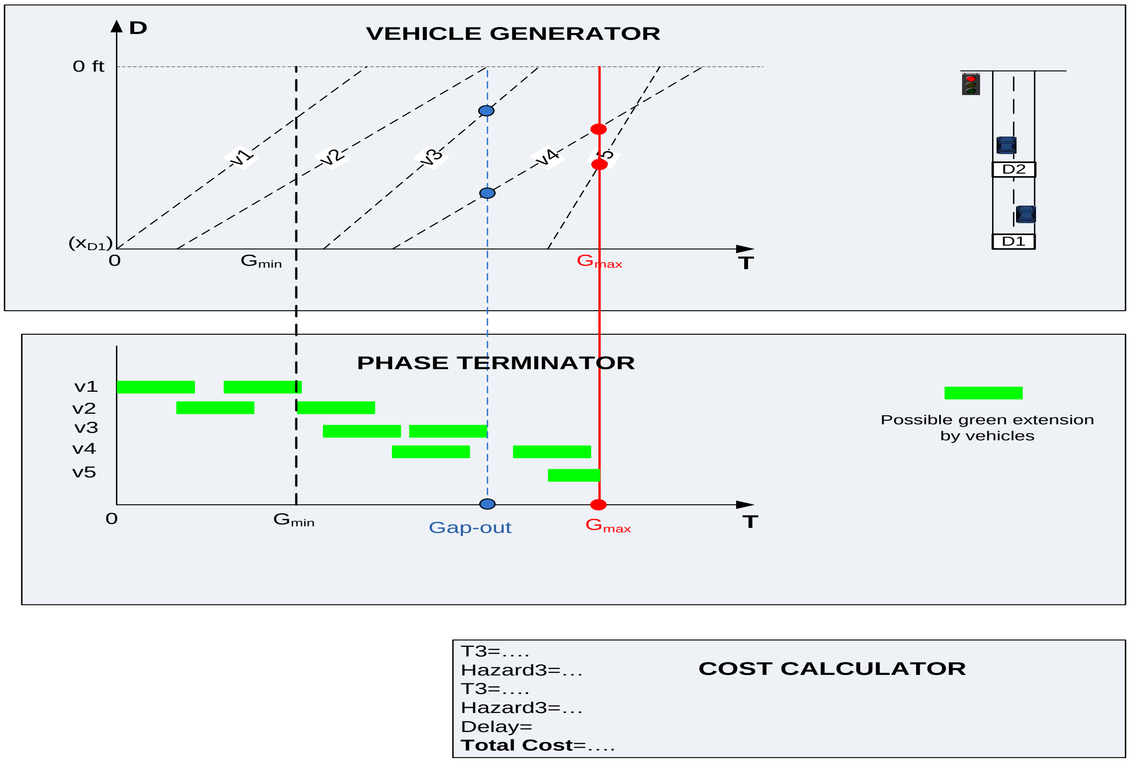


Figure 4-1 GES simulation system

The approaching vehicles are assumed to be moving at their desired speeds in GES, which has been validated in light and moderate traffic with multiple lanes according to the previous field observations. Following this assumption, the author designed the Vehicle Module in such a way that each generated vehicle moves at a constant speed. The vehicles are sequentially generated according to the local speed limit (for the speed generation) and traffic volume (for the headway generation). For instance, given the generated (*i-1*) vehicles' headways, h_i, the ith vehicle will enter the system at $\sum_{i=1}^{n} h_i$. Each vehicle's speed is generated independently.

The Phase Terminator made decision whether to end the green between 15 seconds (the minimum green) and 55 seconds (the maximum green). In the time horizon, each detector will extend the green whenever it detects a vehicle. When all the detector extensions expire at a particular point in time, the green phase will gap out. If the green keeps being extended until the maximum green, the phase will max out. The other minor phases are set as fixed 15-seconds duration.

Previous studies concluded that vehicles in light traffic have normally distributed speeds and exponentially distributed headways [73, 76, 79] . The "Vehicle Generator" module generates vehicles following these conclusions. When a vehicle is generated, it will be assigned a speed and headway from its front vehicle. The entering time of each vehicle is determined by adding all its previous vehicles' headways. Meanwhile, the generated vehicles in light traffic are assumed to move at desired speeds and can change lanes easily to keep their desired speeds when they meet slow lead vehicles. The Vehicle Module calculates the vehicle trajectories according to these assumptions (Fig. 4-1).

The green extension logic is activated between the minimum green and the maximum green. If only one vehicle were in the system, the logic would extend the green when the vehicle passes any advance detector. If this vehicle moves fast enough to reach the next detector before the extension ends, the extension timer will be reset. Otherwise the green will gap out. In reality, vehicles in multiple lanes will move simultaneously and the extend-green timer will be reset whenever a vehicle hits any of the advance detectors. Gap-out will occur only when the extend-green timer counts down to zero. If the timer keeps being reset, the green will max out (Fig. 4-1).

When either gap-out or max-out occurs, the location of each vehicle in the system will be checked to see whether the vehicle is caught in the DZ or not. If a vehicle is in the DZ, its dilemma hazard will be calculated with the dilemma hazard function (Eq. (3- 16)). After all the vehicles have been checked, the individual vehicle dilemma hazards are summed up as the total dilemma hazard incurred in this signal cycle. Meanwhile, given that the developed simulation system only simulates the driving behaviors before the stop line and there is little consideration of interactions between vehicles, the control delay in this cycle is calculated with the associated HCM 2000 control delay models [80]. This calculation repeats in each cycle until one simulation hour is reached. Then the dilemma hazards and control delays generated in all the signal cycles are summed up as the hourly dilemma hazard and control delay under that detector configuration. Multiple replications with common random numbers were deployed to simulate/evaluate each detector configuration. Multiple replications can reduce the randomness, but the common random numbers ensured the difference between different detector configurations was only caused by the various detector configurations.

4.4 Evaluation of Green Extension System

Safety Evaluation- the hourly dilemma hazard: the author used the dilemma hazard model to evaluate the performance of different GES designs. Readers can refer to Chapter 3 to understand how the dilemma hazard model was derived.

Operation Evaluation-the hourly control delay: Unlike the traditional GES designs, the new design presented in this chapter takes into account both safety and efficiency. HCM 2000 provides control delay formulas for signalized intersections [80]. Although these formulas are for fixed timing strategies, the control delay can still be approximated by calculating equivalent green for a certain observing period. In HCM 2000, the control delay is composed of two parts if there is no initial queue delay: uniform control delay and random control delay.

$$d=d_1(PF)+d_2\ ;$$

$$d_1=\frac{0.5C(1-{}^{g}\!/\!_{C})^2}{1-[\min(1,X)\,{}^{g}\!/\!_{C}]}\ ;$$

$$d_2=900T\left[(X-1)+\sqrt{(X-1)^2+\frac{8kIX}{cT}}\right] \tag{4-2}$$

Where:

- *d*: control delay per vehicle;
- d_1:uniform control delay;
- d_2: random delay due to random arrivals;
- *PF*: progression factor, which is 1 for isolated intersections and random arrivals;
- *X*: lane group degree of saturation;
- *T*: duration of analysis period (h);
- *I*: lane group capacity (veh/h);
- *k*: incremental delay factor;
- *g*: green duration;
- *C*: cycle length;

Safety and efficiency can be converted to monetary values. Previous studies have concluded that the probability that a traffic conflict becomes a real accident is about 0.0001 [23], and the average cost for each real accident is $56,706 [81]. Therefore the unit cost for a dilemma hazard is approximately $5.67. A reported delay cost per vehicle-hour is about $17.02/hour/vehicle [82]. These values may be different in different regions and they were used here to illustrate the concept. The optimization was deployed with the following two objectives:

$$\begin{aligned} &U_1=5.67*DilemmaHazard(safety) \\ &U_2=\frac{17.02}{3600}*\sum d_i*v_i(delay) \end{aligned} \tag{4-3}$$

Where:

- U_1,U_2: safety and delay cost in terms of dollars;

- d_i: control delay per each vehicle on phase i; and
- v_i: traffic volume on phase i (veh/h)

4.5 Optimizing algorithm: the Genetic Algorithm

Abbas et al. developed a multiple-objective optimizing program using the Genetic Algorithm [53]. The chromosome is composed of two parts, the detectors' location and the detectors' extensions. In the process of optimization, crossover only occurs between locations or between extensions. Locations and extensions do not cross over. In this way, the optimization program can simultaneously optimize the detectors' locations and extensions. Specifically, initial detector configurations (locations and extensions) in a binary form are randomly generated. Each solution is then evaluated in terms of safety and efficiency. Then the best solutions are selected to cross and generate a new set of detector configurations. This process continues until the pre-determined number of iterations is reached. The number of detectors also needs to be selected carefully. In practice, it can be 1, 2 or 3. To be general, three-detector GES designs were used to represent all feasible configurations. If the optimal design has two or even three detectors to be placed at exactly the same place, it implies that the optimal design needs only two detectors or one detector.

Other parameters governing the optimizing behaviors are selected with commonly used values. They are: crossover probability (0.8), mutation probability (0.02), population size (100) and iteration time (50). Obviously, these values will affect the converging rate of the optimization, but how to select these parameters appropriately is out of scope of this research. To prevent the optimizing process from stopping at certain local optimums, large numbers of population size and iterations are necessary. After some preliminary runs, 100 population size and 50 generations were proved to be enough.

Another issue in the optimizing process is to prevent the optimizer from converging to some unpractical solutions. The objective function will be unaware of the generated vehicles until they reach the first detector. Consequently, the first advance detector has to be placed upstream enough to ensure all the vehicles caught in the DZ will be detected. According to a previous study [83], 97.5th percentile vehicle speed is approximately 10 MPH higher than the design speed. With this conclusion, if a local speed limit is 45 MPH and the first detector is placed where vehicles at 55 MPH speed enter the dilemma zone, then 97.5% of vehicles can be detected when they enter the dilemma zone.

Based on this information, the first detector's allowed range is between 404 feet (5 seconds TTI for 55MPH vehicles) and 588 feet (5 seconds TTI for 80 MPH vehicles). This constraint was set to ensure at least 97.5% vehicles could be detected when they entered the dilemma zone at 45MPH intersections. The allowable locations for the second and third detectors ranged between the stop line and the first detector's location. As a result, this scheme enables the GA optimizer to reduce the number of detectors by coinciding a detector's location with another one if it is not needed.

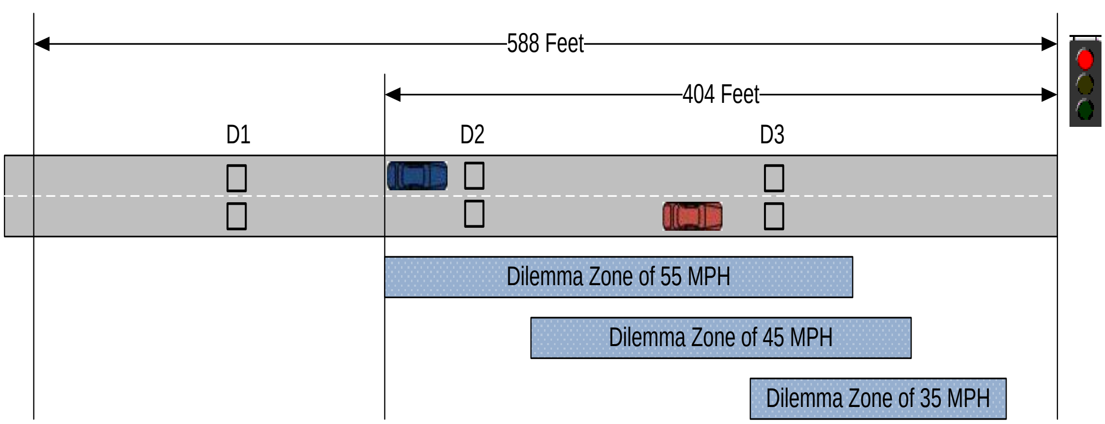


Figure 4-2 Detectors' allowable locations

The multi-objective GA optimizer was deployed, and. U_1 and U_2 defined in Eq. (4-3) were used as the objective functions. The optimizer finally recommended the top 7 solutions. As shown in Figure 4-3, the solutions converged fast, implying the selected population size was enough.

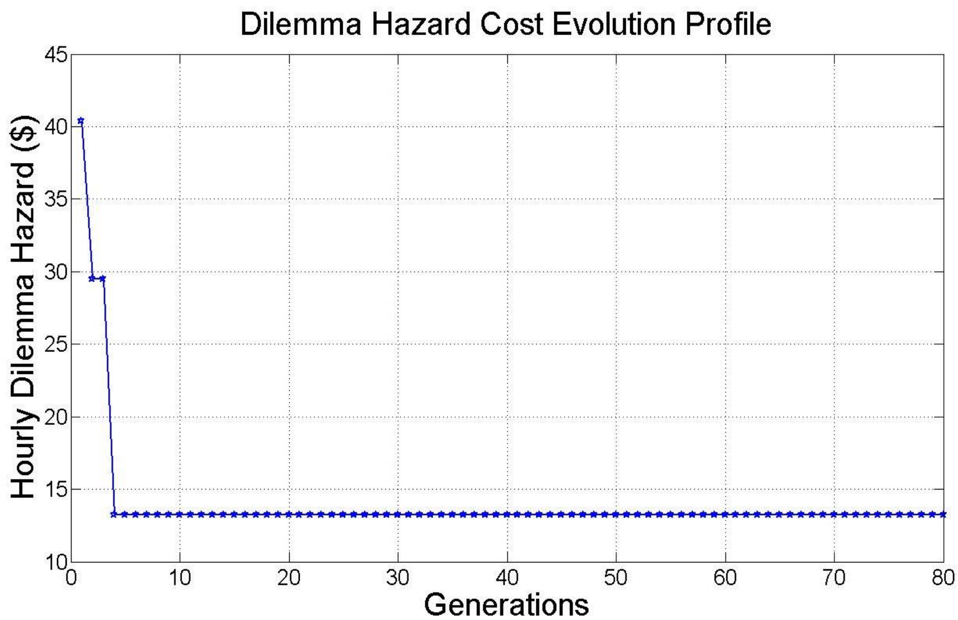


Figure 4-3 The evolution of optimal detector configurations

4.5 Results Analysis

Figure 4-4 shows that the difference among the top seven designs is primarily on the safety side. This phenomenon implies that the benefit of the optimal design will be primarily to reduce the dilemma hazard. The author compared the new optimal design with the traditional GES designs. This comparison among different designs was conducted with multiple replications and Figure 4-5 shows the safety improvement is more considerable than the efficiency improvement with the optimal GES design.

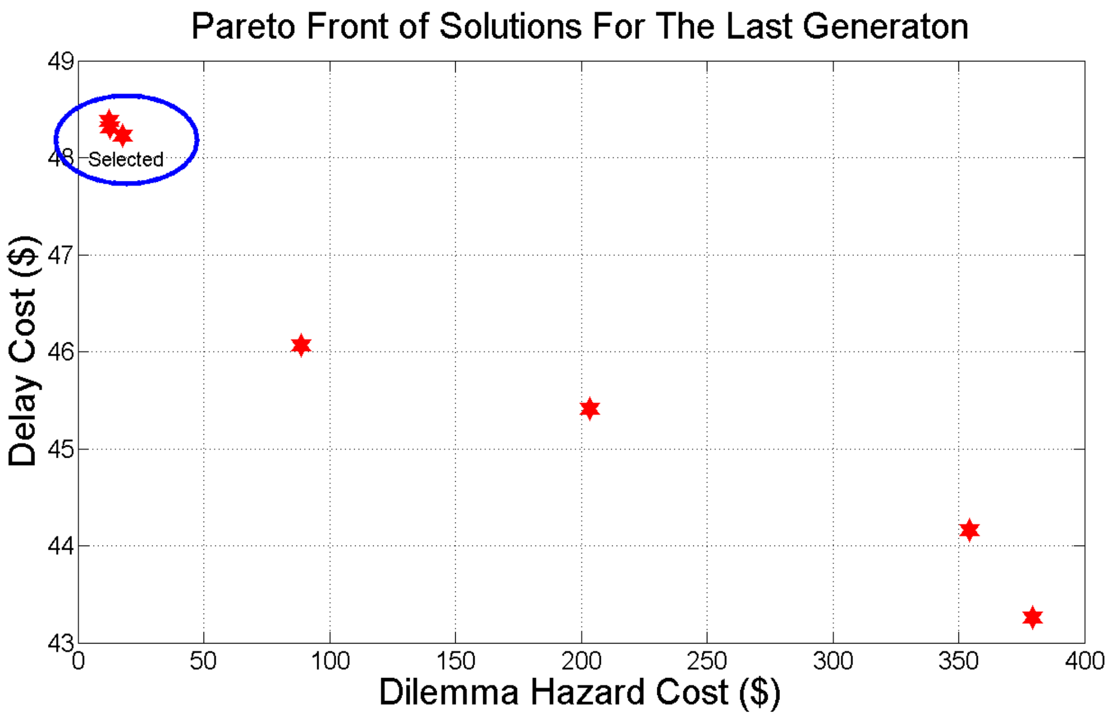


Figure 4-4 Pareto front of the top seven designs

Figure 4-5 shows that the difference among the candidate solutions is primarily on the safety side. This phenomenon implies that the benefit from the optimal design will be primarily from the reduction of the dilemma hazard. The authors compared the new optimal design with the traditional GES designs. This comparison among various GES was deployed using multiple replications with common random numbers and the result supported the speculation. Figure 4-6 shows the safety improvement is more considerable than the efficiency improvement with the optimal GES design.

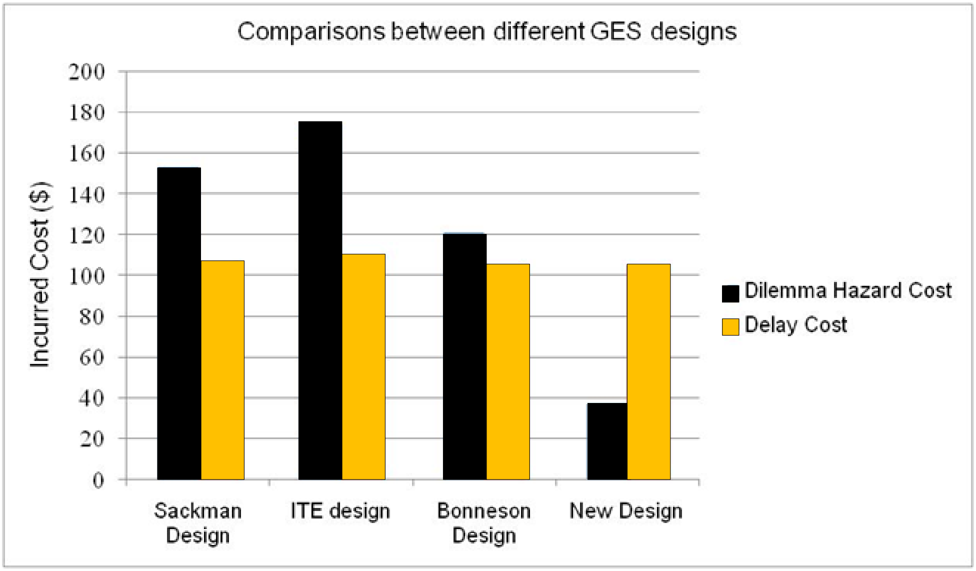


Figure 4-5 Dilemma hazard and control delay costs under different GES designs

4.6 Conclusion for this Chapter

This chapter recasts the design of the multi-detector green extension system (GES) as an optimization problem. Using the multiple-objective genetic algorithm, the author optimized detector configurations to lower both the dilemma hazard and the control delay. The new optimal design was also compared with other traditional designs under the same traffic conditions and the results showed that the new GES could substantially increase the safety but has less significant improvement on the efficiency.

Chapter 5 A New Dilemma Zone Protection Algorithm Based On the Prediction of Vehicles Trajectories and Markov Process

5.1 Introduction

Markov process (MP) is a stochastic process whose future-state probabilities are determined only by its most recent state [84]. The Markov process has been proved capable of simulating a wide range of systems. However, in the field of the traffic signal control, few MP-based signal control applications have been reported. In this paper, we will consider the number of vehicles in the DZ during the green as a Markov process. In light of this idea, the predicted number of vehicles in the DZ, namely state, is calculated with the current state and the state transition matrix. Specifically, using the transition matrix, the new algorithm first predicts the number of vehicles in the DZ for all the time steps from now until the maximum green. Then it compares the predicted number with the current number of vehicles in the DZ to decide whether to end the green immediately or extend one more step. This process is repeated until either the algorithm considers the current time step is the best time (fewest vehicles in the DZ) to end the green or the green phase reaches the maximum.

In order to explain the new algorithm in detail, we structured this chapter into three parts:

The first part includes the literature review on the MP applications in the transportation field and discussion how to apply the Markov Process to the dilemma zone protection issues.

The second part explains how the proposed MP-based algorithm works and describes how a VISSIM-based simulation environment was designed in order to test and evaluate the new algorithm;

The last part is to apply the new algorithm to a high-speed signalized intersection in Christiansburg, VA. The geometry of that intersection and its dynamic traffic patterns were modeled into the simulation environment to obtain a close-to-reality traffic network. The calculation of the current number of vehicles in the DZ (current state) is crucial in the new algorithm and we proposed two methods to calculate the current state: the detector-based method and the vehicle-infrastructure-integration-based (VII) method. The detector-based method is to apply the car-following models with data from advance detectors and predict vehicle trajectories and the number of vehicles in the DZ; the VII-based method can detect the exact number of vehicles in the DZ. Obviously, the VII-based method has higher fidelity, but it is not commercially-off-the-shelf yet. As a result, we put emphasis on the detector-based method and only used the VII-based method to do cross comparison.

5.2 Significance of the Research

Although previous studies have proved that the Markov process could be applied to many traffic problems, its applications to the dilemma zone protection issues have been limited. Adam et al. designed a dilemma zone protection policy using reinforcement learning [85]. This policy compares the current number of vehicles in the DZ and the predicted number of vehicles in the DZ to make decisions (i.e., end the green vs. extend the green). The state transition matrix used in Adam's paper is stationary and therefore cannot respond to changing traffic conditions. Meanwhile, Adam's method needs more comprehensive evaluation.

This chapter addresses the above issues as follows:

1. Presents a new algorithm for updating the state transition matrix periodically using both historical data and the new incoming data.

2. Evaluates the new MP-based dilemma zone protection algorithm under dynamic traffic conditions. If a new signal control algorithm involves complex computing, it will have difficulty in embedding into most commercial traffic simulation packages. As a result, many previous studies evaluated their signal-control algorithms in simplified simulation environments, which is likely to cause bias since many driving behaviors are ignored. Unlike the traditional methods, the new algorithm was directly deployed and evaluated in VISSIM® via a middleware, namely *VTDatex*.

5.4 Model Description

5.4.1 Calculate the Number of Vehicles in the DZ with Advance detectors

The advance detectors are typically placed 700~1,000 feet upstream of the intersection on each of its major road approaches and used to identify approaching vehicle speeds and types. According to the locations of the advance detectors and vehicle speeds, each vehicle's trajectory can be projected in the time-space diagram. If a fast vehicle catches its front vehicle in the DZ, the fast vehicle decelerates to maintain a safe headway from its front vehicle. 1.5 seconds was used as the safe headway in this research, but other values can also apply. After vehicle trajectories are projected, we can calculate the number of vehicles in the DZ. In Fig. 5-1, the fast vehicle 2 slows down to maintain a safe headway from vehicle 1 and its dilemma zone is changed accordingly. The vehicles in the DZ can be counted at any time point.

Obviously, this assumption is rather conservative due to the fact that vehicles may change lanes to keep their desired speeds. Nonetheless, some previous research concluded that this assumption holds well in moderate or high traffic volumes [18, 19, 33].

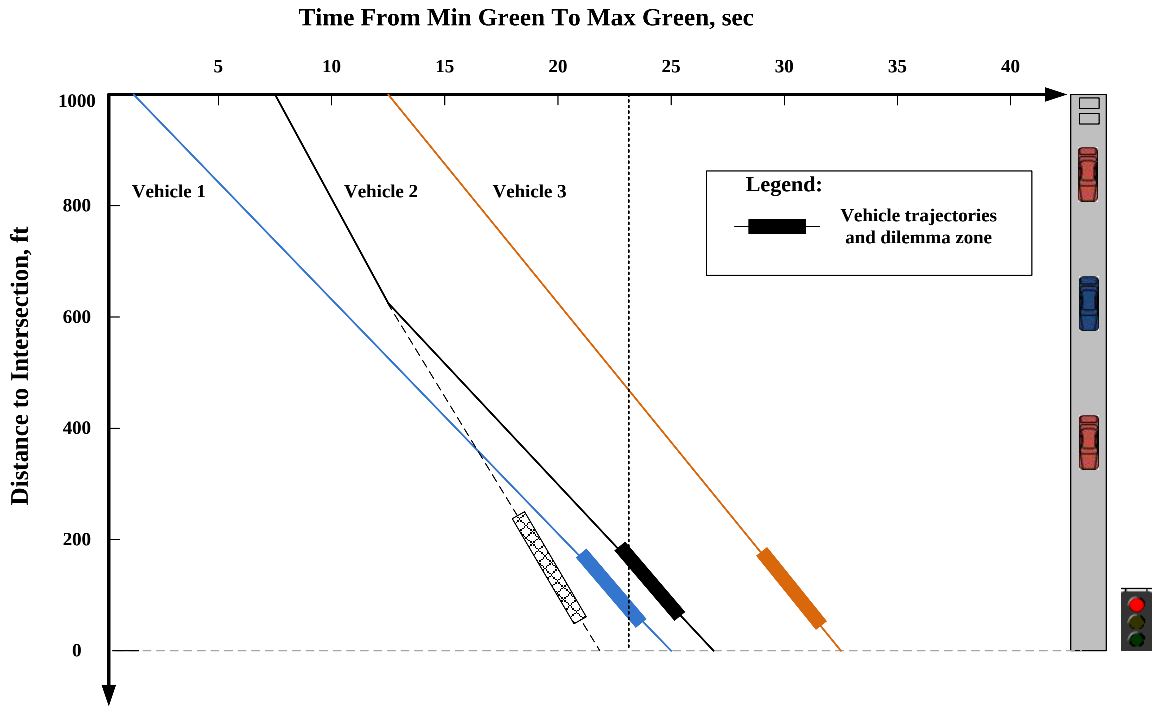


Figure 5-1 Count the vehicles in DZ with advance detectors

5.4.2 Markov-process-based Dilemma Zone Protection Algorithm

The goal of the new algorithm is to determine the best time to end the green so as to minimize the number of vehicles in the DZ. Specifically, when the green is between the minimum green and the maximum green and there are neither trucks in the DZ nor queues, the new algorithm will make the decision whether to end or extend the green at each time step. The decision is made according to the observed vehicles in the DZ and the predicted numbers of vehicles in the DZ in future using the Markov matrix.

Let N_{t0} denote the current number of vehicles in the DZ at time t_0 and $P(t_0) = \left[p_{ij}(t_0) \right]$ denote the state transition matrix. Then the state transition matrix at time t_n after n time steps is:

$$P(t_n) = \left[p_{ij}(t_0) \right]^n = \left[p_{ij}(t_n) \right] \tag{5-1}$$

And the probabilities in which the state transits from N_{t0} to other states after n time steps are:

$$\Pi^{t_n} = (p_{N_{t_0},0}(t_n), p_{N_{t_0},1}(t_n), p_{N_{t_0},2}(t_n), \ldots) \quad (\sum_j p_{N_{t_0},j}(t_n) = 1) \tag{5-2}$$

Where $p_{N_{t_0},j}(t_n)$ is the corresponding element in the state transition matrix $P(t_n)$.

With Eq. (1) and (2), the predicted number of vehicles at time t_n is:

$$N_t = 0 \times \pi_0^{t_n} + 1 \times \pi_1^{t_n} + 2 \times \pi_2^{t_n} + \ldots. \qquad \text{Where } \pi_j^{t_n} = p_{N_{t_0},j}(t_n) \tag{5-3}$$

If at a particular point in time, the observed number of vehicles in the DZ is less than any predicted number at all future time steps, the green will end. Otherwise, the algorithm extends the green one time step. In the case where the observed number and certain predicted number are equally minimal, the green will be extended. The rationale is to lengthen the cycle length and reduce the hourly number of vehicles in the DZ.

5.4.3 Update the State Transition Matrix

The state transition matrix $\left[p_{ij}(t) \right]$ is crucial for the prediction. In order to respond to the latest traffic, the matrix needs updating periodically according to the new incoming observations. We applied the *rolling horizon* concept to the matrix updating. The rolling horizon concept is used by operations research analysts in production-inventory control and was introduced into the signal control field by Gartner in his adaptive signal control algorithm, OPAC [48]. OPAC divides time into stages and, in each stage, uses the observation during the "head" time to predict the traffic during the "tail" time. Then OPAC decides the timing plans. The time is then shifted ahead a head time long and this process is repeated. In light of the rolling horizon concept, the new algorithm collects state transitions during the head time of each stage, updates the matrix according to the new data then applies the new matrix during the tail time (Fig. 5-2).

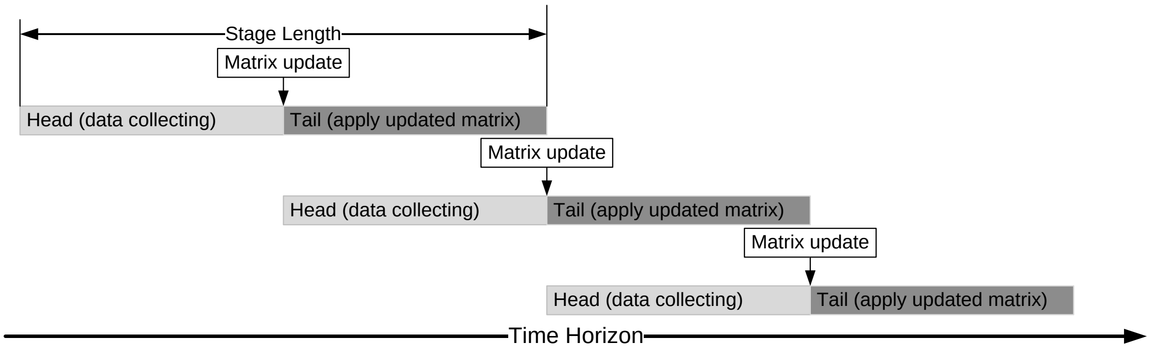


Figure 5-2 Updating Markov matrix using rolling horizon technique

Let Ω denote the finite state space and $X_1, X_{2,}...$ be the transitions between states. Then X is a Markov chain on Ω with the transition matrix $\left[p_{ij}(t)\right]$. The empirical state transition matrix can be derived using the new observations with entries:

$$\hat{p}_n(i,j) = \frac{\sum_{k=1}^{n} 1(X_k = i, X_{k+1} = j)}{\sum_{k=1}^{n} 1(X_k = i)} (i, j \in \Omega) \tag{5-4}$$

We can reasonably assume that the denominator in Eq. (5-4) is positive for all possible states after long observation (i.e., the head time is long enough). While deriving the transition matrix with the observations, one issue is that the observed transitions within one head time may be biased due to the system's inherent randomness. To mitigate this bias, the algorithm does not derive the transition matrix merely with new incoming observations. Rather, it estimates the number of observations using both the historical observations and the new observations. The underlying rationale is that although the traffic is highly dynamic, the traffic pattern is repeated day by day. As a result, the traffic patterns at the same time of different days are similar and associated.

As in Fig. 5-3, the historical observations were stored in a series of time-dependent matrixes and each cell of those matrixes stands for the transition between states during certain time. Whenever new observations come in, the algorithm estimates the transition for each cell using the new incoming transitions, the historical transition at the same time of a day, the historical transition head-time units before that time of a day, the historical transition two head-time units before that time of a day, the historical transition one head-time unit after that time of a day and the historical transition two head-time units after that time of a day. This means the incoming observations will be combined with the archived data within the same hour of a day.

Least-square estimation is used and its mathematical expression is as follows:

Let $N_{i,j}(t)$ (variable) denote the estimation of the transition between *i* and *j* at time t; $N_{i,j}^{old}(t)$ denote the historical transition between *i* and *j* at time t; $N_{i,j}^{new}(t)$ denote the new incoming transition between *i* and *j*. Then the estimation using the least-square estimation can be formulized as:

$$\begin{aligned} Min\ & \left(N_{i,j}(t)-N_{i,j}^{new}(t)\right)^2+\left(N_{i,j}(t)-N_{i,j}^{old}(t-1)\right)^2+\left(N_{i,j}(t)-N_{i,j}^{old}(t-2)\right)^2 \\ & +\left(N_{i,j}(t)-N_{i,j}^{old}(t)\right)^2+\left(N_{i,j}(t)-N_{i,j}^{old}(t+1)\right)^2+\left(N_{i,j}(t)-N_{i,j}^{old}(t+2)\right)^2 \end{aligned} \tag{5-5}$$

After all $N_{i,j}(t)$s are estimated, they are first used to derive the new transition matrix, and then they replace the corresponding data in the historical data matrixes and the approximating function for each cell is updated as well. This method will not only mitigate the possible bias generated during the matrix updating but also prevent the historical data from getting obsolete.

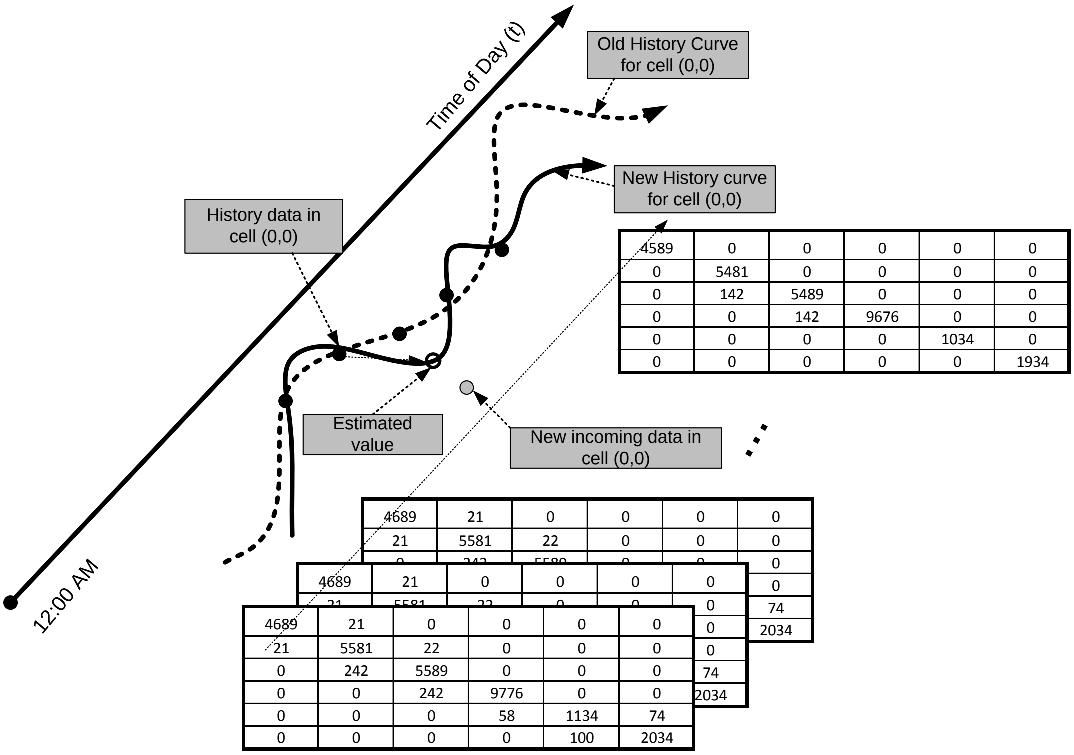


Figure 5-3 Illustrations how to update the transition matrix using both historical data and new incoming data

In summary, the MP-based dilemma zone protection algorithm can be described as in Fig. 5-4:

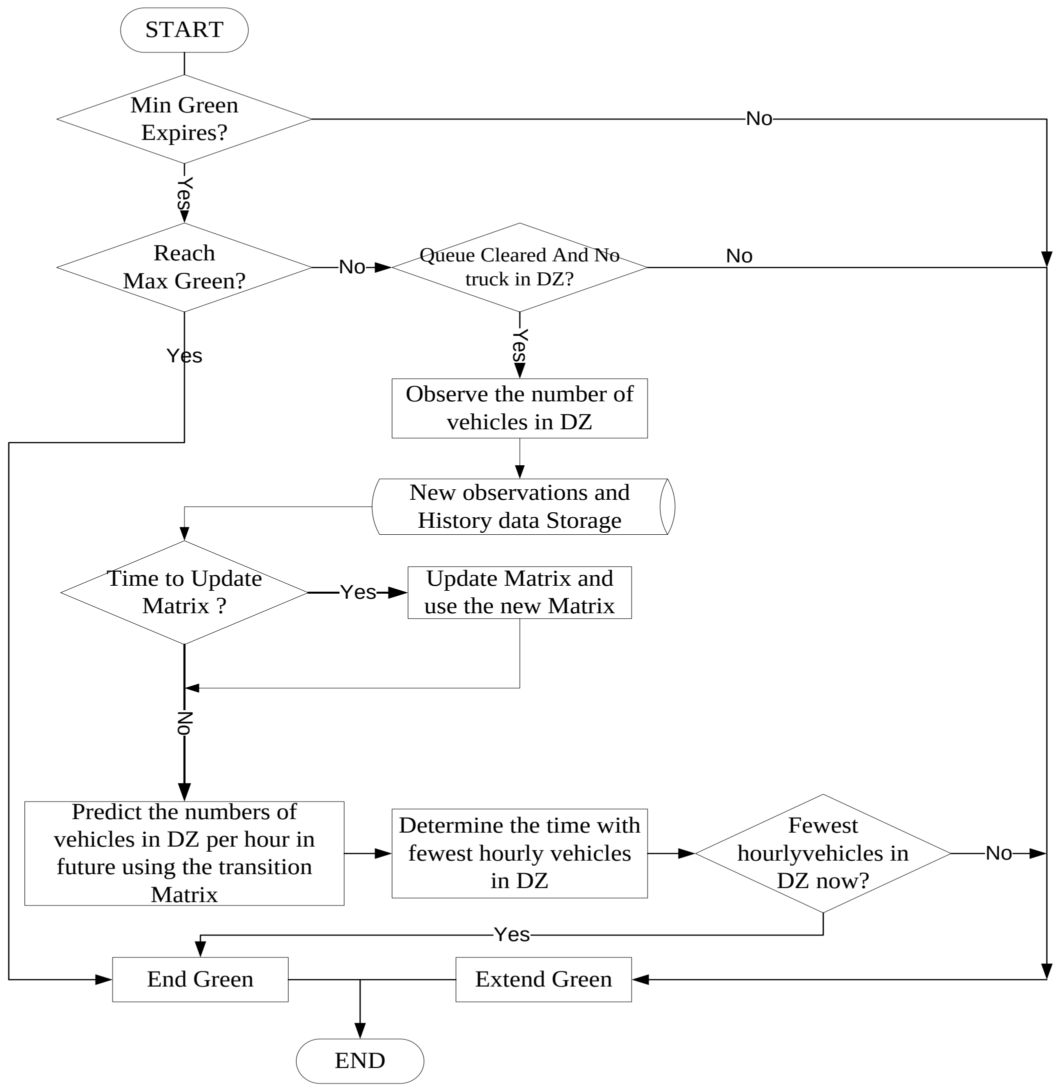


Figure 5-4 Flow chart of the new MP-based dilemma zone protection algorithm

5.5 Algorithm Deployment in VISSIM

The new algorithm was deployed and evaluated in a prevailing microscopic traffic simulation environment, VISSIM [86]. The advantages of VISSIM over other simulation packages include:

1. VISSIM provides the largest flexibility for users to calibrate driving behaviors and traffic conditions;

2. VISSIM was developed under .NET framework, which brings flexibility for add-on program development;

3. VISSIM provides the best tools for the development of signal control strategies, such as the NEMA controller emulator, Vehicle Actuated Programming (VAP) language, signal control Application Programming Interfaces (SCAPI), etc.

VISSIM SCAPI method was used to develop the MP-based signal control emulator in this research. SCAPIs were written in C++ language and the original version of SCAPI controller requires signal control algorithms be embedded into a single dynamic link library (DLL) file. To facilitate the development, we developed a middleware namely "*VTDatex*", which can synchronously collect all the real-time detectors/phases states from the VISSIM network to the controller emulator then return the new desired phase states back to the VISSIM network.

The advance detectors are used to collect vehicle speeds/types and predict vehicle trajectories and the number of vehicles in the DZ. Meanwhile, the actual number of vehicles in the DZ is also collected through VISSIM Common Object Module (VISSIM COM) to evaluate the new algorithm's performance.

At each time step, the controller runs the algorithm, makes decisions according to the current state and the Markov state-transit matrix then returns the new desired phase states to the VISSIM network. The concept of this simulation environment is illustrated as in Fig. 5-5.

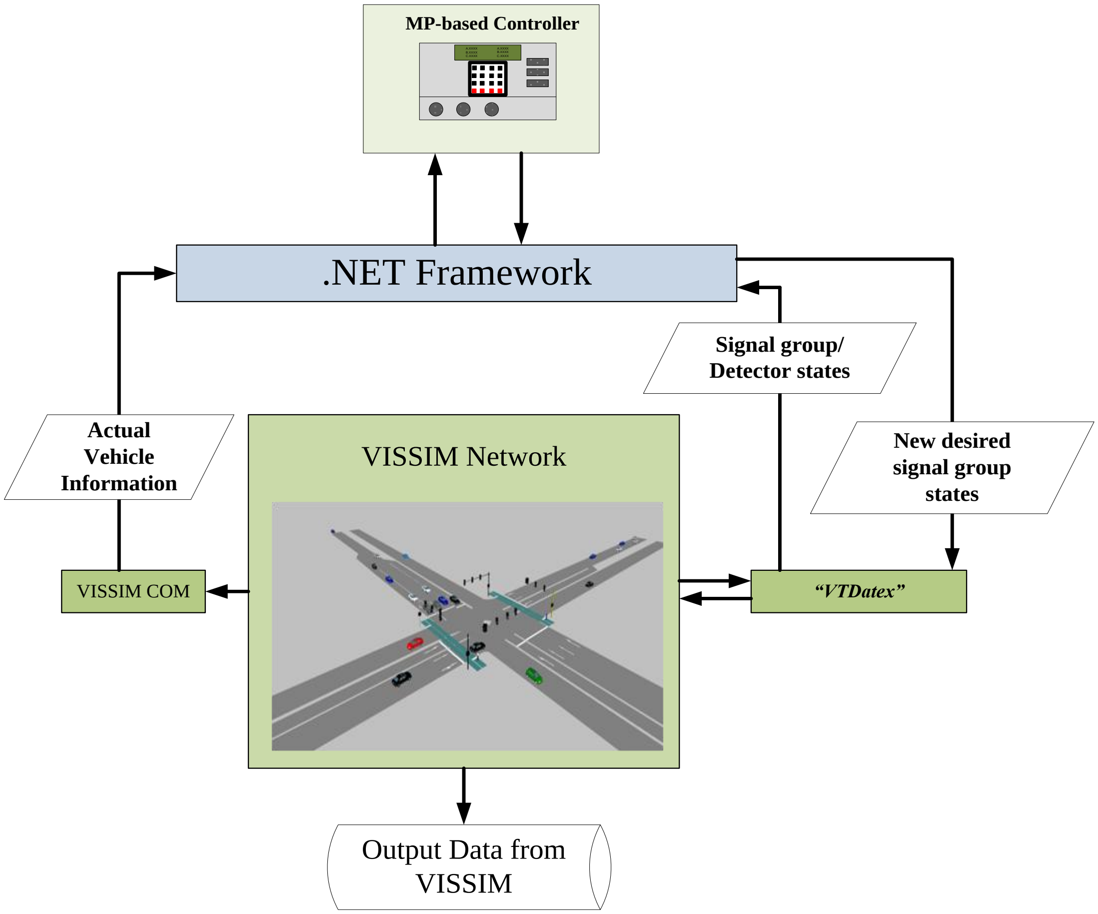


Figure 5-5 Illustration of VISSIM Simulation Environment

5.6 Study Intersection

The study intersection is located on the Peppers Ferry Road and the North Franklin Street at Christiansburg, VA. Its geometry is illustrated in Fig. 5-6. It has high-speed (45 MPH) lanes dedicated to the through traffic on the north-bound and south-bound approaches. Those lanes are our study lanes.

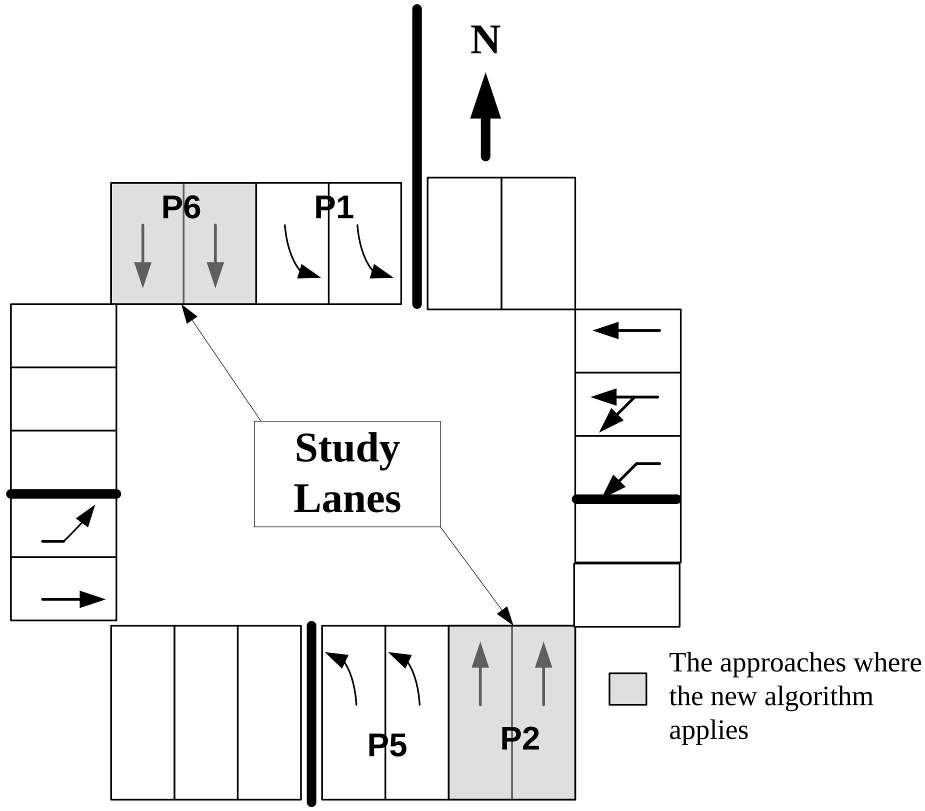


Figure 5-6 Geometry of the study intersection

Traffic volumes: the purpose of this experiment is to make an informal evaluation of the new algorithm under a close-to-reality traffic condition. We counted the traffic volume on the study lanes every 15 minutes with a data acquisition system on the high-speed approaches and the whole counting lasted 9 hours. The through-traffic volumes were plotted as in Fig. 5-7. These volumes were modeled into the VISSIM network to provide a close-to-reality traffic volume profile. The volumes on other approaches are listed in Table 5-1.

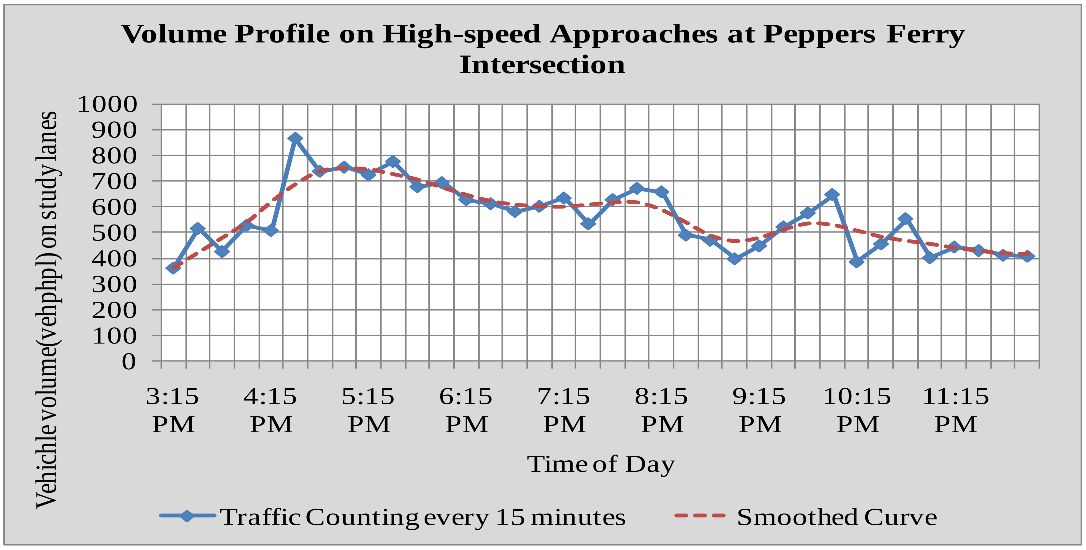
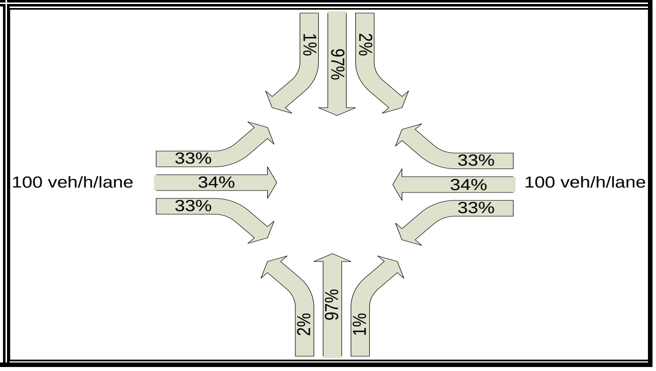


Figure 5-7 Traffic volume profiles on the study lanes

Other VISSIM networks inputs: Table 5-1 lists all the other necessary inputs for the network modeling, except the signal control settings. They were selected to reflect a prevailing traffic condition. Specifically, the Wiedemann 99 model was selected because the vehicles under that model will maintain their desired speeds unless they have to slow down and yield to a slower front vehicle to keep safe headways. This assumption has been widely accepted and is also consistent with the assumption used to predict the vehicle trajectories. Given the purpose of this study is to test the performance of the new algorithm in a general traffic condition, the VISSIM network was not specifically calibrated for the local conditions.

Table 5-1 Network Inputs for the VISSIM network

Driver Behavior:	Model Type:	*Wiedemann 99*
	Reaction to amber Signal:	*One Decision Model*
	Other Settings:	*Default Values*
Traffic O-D distribution		

Signal control emulator phasing: The NEMA phasing sequence is shown in Fig. 5-8.

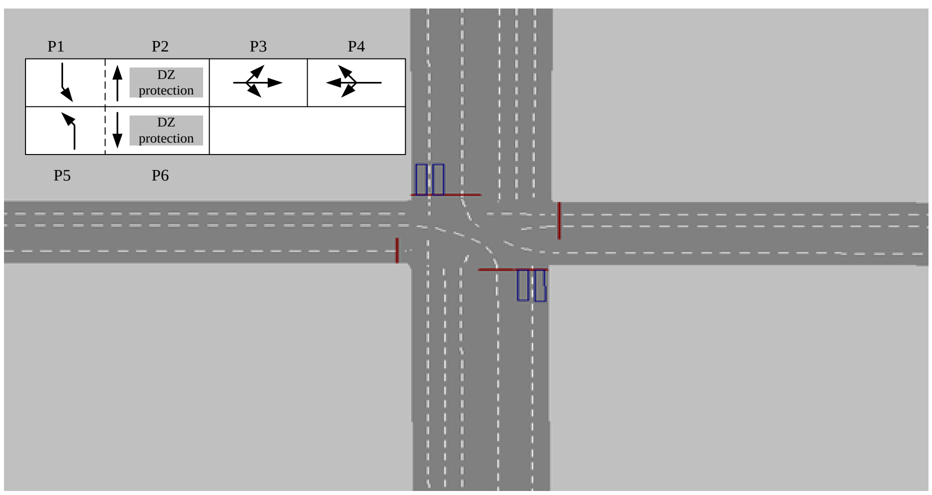


Figure 5-8 NEMA Ring Structure of the control emulators

Phase 2 and 6 were equipped with the dilemma zone protection system and therefore their possible durations range from the minimum green to the maximum green. Meanwhile, phase 2 and 6 were also equipped with the stop line detectors to ensure the queues were cleared before the green ended. The other phases were set as fixed.

5.7 Results Analysis

We compared the new MP-based algorithm with the detection-control system (D-CS). D-CS is a DZ protection system developed by Texas Transportation Institute. It also uses advance detectors to predict the number of vehicles in the DZ and to determine the best time to end the green accordingly. All but the control logic were set exactly the same for D-CS and the MP-based algorithm. Simulation was conducted with multiple replications to reduce randomness and the results are as in Fig. 5-9. From Fig. 5-9, the MP-based algorithm caught fewer vehicles in the DZ than D-CS did in both high and moderate traffic volumes.

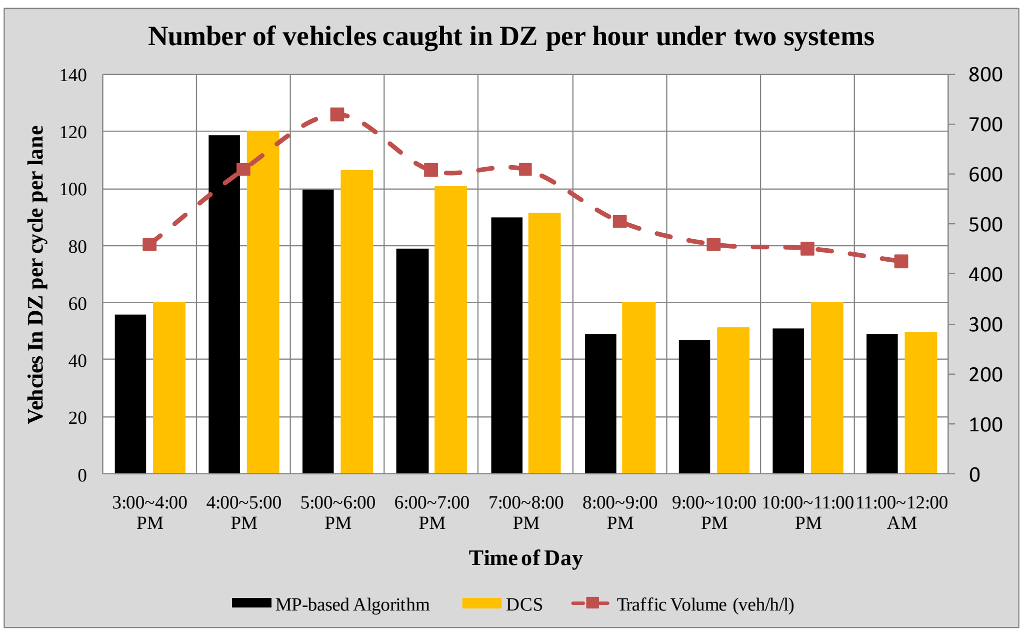


Figure 5-9 Comparison between the MP-based algorithm and D-CS

The detector-based prediction of the vehicle trajectories is of low fidelity and therefore may have a negative impact on the algorithm's performance. To address this issue, we directly input the algorithm with the actual number of vehicles rather than the predicted number from the detectors. In practice, this can be achieved with the newly emerged vehicle-infrastructure-integration (VII) technique. Via VII technique, the controller will know the exact position of each approaching vehicle and therefore the state (number of vehicles in the dilemma zone) can be precisely calculated rather roughly predicted. To examine the potential benefits brought by VII, the advance detectors were replaced with the VII technique in the new algorithm. The results show that the performance can be significantly improved if the VII technique applies (Fig. 5-10). In other words, the new MP-based algorithm will have a better performance if the VII techniques become mature in future.

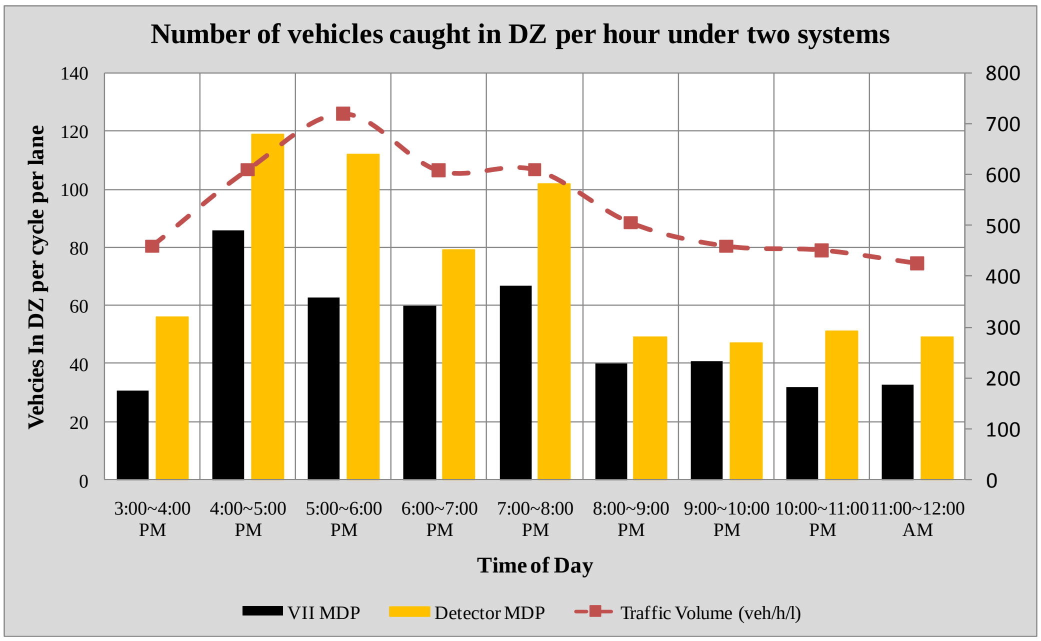


Figure 5-10 Comparison between detector-based data collection and VII-based data collection

5.8 Conclusion for this Chapter

The dilemma zone problem is a leading cause for crashes at intersections and therefore determining how to protect vehicles caught in the dilemma zone has been a widely concerning issue. The authors of this paper addressed this topic by applying the Markov process concept to the design of new dilemma zone protection algorithm. The new algorithm predicts the future states with the Markov state-transit matrix and compares them with the current state. Then it determines the best time to end the green. The Markov state-transit matrix is crucial in the algorithm. In order to respond to the latest traffic, the state transition matrix is periodically updated using the rolling-horizon approach.

The authors also compared the new algorithm with another dilemma zone protection system, the detection-control system. The simulation was conducted with multiple replications and results show that the new algorithm has a better performance than D-CS in dynamic traffic conditions.

How to calculate the number of vehicles in the DZ is another important issue in the new algorithm. We adopted a practical method based on advance detectors and car-following models to address this issue. However, the detector-based method is of low fidelity and so it may have a negative impact on the overall performance. In order to investigate the impact, we input the algorithm with the actual number of vehicles in the DZ and results showed the algorithm's performance considerably improved if the number of vehicles in the DZ can be precisely calculated. This means that the new algorithm will be more beneficial when the VII techniques are mature in future.

Chapter 6 Simulation-based Stochastic Optimization of traffic signal systems under Retrospective Approximation Framework

6.1 Introduction

Traffic signal control systems have a significant impact on the performance of transportation networks. According to a study by Texas Transportation Institute, the unit cost caused by the control delay is as high as $17.02 per vehicle-hour [82]. As a result, many research efforts have been dedicated to optimizing the traffic signal systems to reduce delays and travel times.

The traditional model-based optimization for traffic systems is to first set up analytical models to abstract the key properties of the problems, and then seek the optimums according to these models. Although this traditional methodology is useful, it is questionable due to the fact that it typically considers traffic systems as deterministic and therefore it cannot deal with the inherent randomness of the traffic systems. As a result, the suggested solution may be biased.

A newly emerged methodology using simulation-based stochastic optimization can better approximate traffic signal systems due to its capability of modeling the random driving behaviors and the highly complex feedback mechanisms between signal control parameters and traffic movement. Although the suggested solution is more valid with the simulation-based optimization, there are two key issues that have not been well addressed yet: (1) simulation-based optimization usually takes excessive long time, and (2) the estimators of the performance measures may be biased due to insufficient simulation replications, eventually resulting in a biased optimal solution. To address these two issues, it is necessary to ensure fast convergence of the simulation-based optimization algorithm. On the other hand, to ensure the unbiased estimate for the performance measures, it is necessary to apply sufficient simulation replications. There is a dilemma while addressing these two issues together: sufficient simulation replications can achieve a relatively unbiased estimate for the performance measures, but excessive replications will also make the optimization algorithms converge rather slowly.

To address these issues, the author investigated a recent development of simulation-based optimization and introduced the retrospective approximation (RA) framework into the optimization of traffic signal systems. We structured this paper into four parts: in the first part, we reviewed the development of simulation-based optimization and the retrospective approximation framework; in the second part, we analyzed the major issues in the latest practice of traffic signal optimization; in the third part, we designed a variant of the Markov Monotonic Search Algorithm that can converge quickly as well as accommodate the requirements by the RA framework. We developed the RA-based optimization engine using VISSIM and VC.NET. We applied the new RA-based optimization engine to the search for optimal maximum green parameters for the multi-detector green extension systems.

Retrospective Approximation (RA) is a variant of SAA and it was first proposed by Chen and Schmeiser [87]. Later on, Pasupathy and Schmeiser extended the RA technique [88] to multiple dimensions and Pasupathy investigated how to make the RA algorithm converge faster [89]. RA reflects the latest theoretical development of simulation-based optimization. Instead of generating a single approximate function *S*, a sequence of approximate functions $\{S_i\}$ are generated with increasing sample sizes $\{m_i\} \to \infty$, and solved to decreasing tolerances $\{\varepsilon_i\} \to 0$. The rationale is that in the early iterations, the approximate objective functions $\{s_i\}$ are not close to the true objective function G due to their small sample sizes. Therefore, the optimization in early iterations is primarily to better understand the problem. For instance, with certain heuristic searching algorithms, the optimum in an early iteration may be used as the initial guess for the next iteration. This measure, consequently, can increase the converging rate in the next iteration. The small sample size and large error tolerance can ensure the early iterations will not expend excessive efforts. With the sample size increasing, it is necessary to lower the error tolerance so that the sample path of the optimal solutions in all iterations can gradually approach the true optimum with probability one (Fig. 6-1). Please note the curves in Fig. 6-1 are for illustration only and in practice they may be in other shapes.

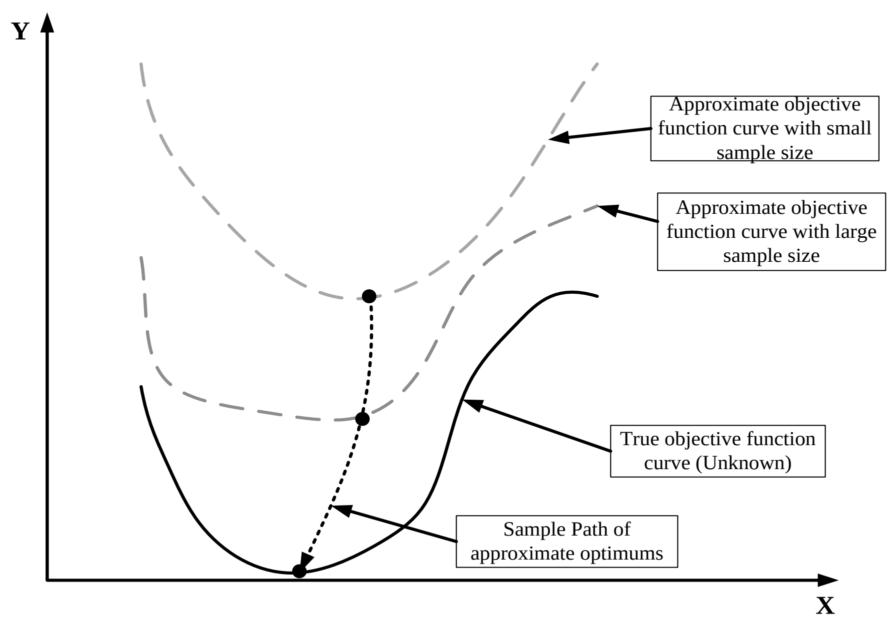


Figure 6-1 Conceptual Illustration of the RA Technique

The following work is necessary when the RA framework is applied:

1. Provide an initial sample size m and a rule for successively increasing m_i, $m_i \geq 2$

2. Provide a rule for computing an error-tolerance sequence $\{\varepsilon_i\} \to 0$

3. Provide a rule for stopping the RA algorithm.

4. Provide a mechanism of sampling the simulation system in order to generate the approximate problem S_i. The sampling is conducted with m_i independent random seeds then averaging to obtain $\overline{Y}_m(x)$, defined before;

5. Select a numerical optimization algorithm (e.g., the Genetic Algorithm, the Simulated Annealing Algorithm, or some other search algorithms) to search the optimal solution x_i^* to the approximate problem S_i to the error tolerance ε_i .

The iterations continue until certain stopping criteria are satisfied. It is also worth noting that the RA framework is very generic and therefore we have the flexibility of selecting the best numerical optimization algorithms according to the problem.

6.2 Significance of the Research

The problem with traditional analytical models is that they ignore the driving behaviors and the randomness of traffic systems. Recent research efforts, however, can directly connect the commercial microscopic simulation software with optimization algorithms [66, 70]. In this way, the driving behaviors can be modeled in detail. Nevertheless, the converging rate of the stochastic optimization has been a major issue in practice. For example, the commonly used genetic algorithm requires a significant amount of time to obtain an “acceptable” solution. When dealing with large-scale traffic networks, the required simulation time may be too long to afford. The state-of-practice methods addressed this issue either by using small sample sizes to approximate the objective function or by reducing the searching efforts (e.g., reduce the population size in the genetic algorithm). Although these measures can help maintain an acceptable computing time, the globality and unbiasness of the solutions are almost surely compromised. As a result, it is necessary to design better numerical optimization algorithms that can converge fast and ensure the solutions' globality as well.

Another issue is how to determine the number of simulation replications, which reflects the approximate efforts to the true problems. The more replications there are, the better the approximation is. However, a large number of replications also means longer simulation time. Meanwhile, a specific sample size may be insufficient for one traffic system but excessive for another system. As a result, using one single sample size to approximate all systems will not always be acceptable.

We address these issues by applying the RA framework to the optimization of traffic signal systems. Unlike SAA, the RA framework does not need to determine the sample size beforehand. Rather, it gradually increases the sample size in the process of the optimization until the sufficient approximation is achieved. From the output difference between the RA iterations, we can tell whether extra approximate efforts are needed. Meanwhile, to increase the optimization efficiency, we designed a variant of a stochastic global optimization algorithm, the *Markov Monotonic Search Algorithm*, which can meet the requirements of the RA framework as well as converge fast.

6.3 Algorithm Description

Under the RA framework, the numerical optimization algorithm can be selected according to the problems. The proposed optimization algorithm in this section is designed for one-dimension optimization problems. These results can either expand to multiple dimensions or to other optimization algorithms following the same procedure.

Initialization: the sample size and error tolerance for the first iteration are set as $m_1 = 1$ and

$$\varepsilon_1 = \frac{1}{\sqrt{m_1}}.$$

Rules for successive increasing sample size and decreasing error tolerance:

$m_i = RoundUp\left[(1+10\%)m_{i-1}\right](i \geq 2)$ and $\varepsilon_i = \frac{1}{\sqrt{m_i}}$.

The increasing rate of sample size is changeable: a large increasing rate may cause a rapid increase of simulation efforts and therefore considerably increase the simulation time. 10% is used here, but the increasing rate is obviously changeable according to the problem.

Inheritable Markov Monotonic Search Algorithm: the *Markov Monotonic Search Algorithm* (MMSA) is an optimization algorithm [90]. MMSA is based on the Markov random search in the optimization space, which means that the random search in each step makes use of the information of last observation. It is similar to the simulated annealing algorithm except that MMSA uses greedy search (i.e., MMSA always search the minimum in each step). If the feasible space of a problem is smooth enough, which is typical of traffic systems, MMSA is likely to obtain a relatively global optimum? The author selected MMSA to obtain the global optimums in the sequence of the RA iterations. In a particular step of MMSA, the current solution is replaced by the best solution randomly selected according to a particular state probability *P*. *P* determines how to assign the search efforts in the optimization space and may be updated while the optimization is progressing.

Let $\Re$ denote the stopping criterion for MMSA. We can formulate the MMSA as follows:

```
x=x0                                    //Choose initial solution x0
WHILE (TRUE)
{
  η =NewSolutionGenerate();//generate new solution according to  the state probability P.
  IF  f(η) ≤ f(x0)          // the new solution is better than the current solution
  {
      x= η ;          //current solution is replaced with the new (better) solution
  }
  ELSE
  {
      x keeps unchanged
  }
  IF( ℜ )                  // stopping criterion is satisfied
  {
      RETURN X;     //return the final optimum
  }
}
```

Clearly, the state probability P is crucial to generate the next solution properly and ensure fast converging. In this paper, we designed a variant of MMSA, namely Inheritable MMSA (IMMSA), which can lead to a visible improvement in efficiency as well as accommodate the requirements by the RA framework. Rather than the “pure” random search, IMMSA will focus the search effort based on the updated learn state probability P since the last RA iteration. Then IMMSA will distribute the search efforts according to P in the new RA iteration. The rationale is: the sequence of approximate problems (S_i) with increasing sample sizes will gradually approach the true problem (G). These approximate problems should have similar properties, such as convexity or monotonicity, which implies that their optimums should be close, too. Consequently the new RA iteration can use (or, “inherit”) the search experience from last RA iteration and assign search efforts according to such experience. In other words, the new RA iteration should put more search efforts where the better solutions lie in last RA iteration. This rationale is validated by our preliminary experiments (Fig. 6-7).

To update the state probability *P*, all the attempted solutions in a particular RA iteration are ranked in an ascending order according to their corresponding objective function values. Those solutions with less than 80^{th} percentile objective function values are selected as "promising" solutions and the nearby areas around them are more likely to have the global optimum. We can ascertain where to put more searching efforts in the next iteration according to the empirical distribution of these "better" solutions. In other words, we should put emphasis on searching where more "promising" solutions are. Please note that 80^{th} percentile can be replaced with other values. The higher the percentile value is, the better the IMMSA can learn from the last RA iteration. However, high percentile values may also result in very small samples to derive the empirical distributions. This idea is illustrated in Fig. 6-2.

All recorded solutions in this iteration

x	F(x)
49.42322	5.86713
60.08746	6.714719
27.69129	3.562483
17.4188	1.55525
44.51757	5.433007
43.55765	5.343936
55.3045	6.348583
43.02819	5.294166
29.86977	3.856134
49.80472	5.899552
51.20713	6.017236
55.44726	6.359816
26.16863	3.34195
45.58811	5.530652
21.93831	2.634067
25.52128	3.243652
42.41558	5.235989
62.72627	6.908421
28.06735	3.614879
27.96367	3.600509
59.33359	6.658347
21.29205	2.508396
30.58444	3.947713
51.99897	6.082678
56.11994	6.412483
56.2324	6.421246

Sort F(x)

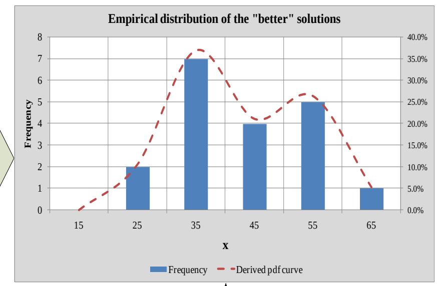
x	F(x)
17.4188	1.55525
21.29205	2.508396
21.93831	2.634067
25.52128	3.243652
26.16863	3.34195
27.69129	3.562483
27.96367	3.600509
28.06735	3.614879
29.86977	3.856134
30.58444	3.947713
42.41558	5.235989
43.02819	5.294166
43.55765	5.343936
44.51757	5.433007
45.58811	5.530652
49.42322	5.86713
49.80472	5.899552
51.20713	6.017236
51.99897	6.082678
55.3045	6.348583
55.44726	6.359816
56.11994	6.412483
59.33359	6.658347
60.08746	6.714719
62.72627	6.908421
56.2324	6.421246

F(x) in ascending order

"promising" solutions with objective function value less than 80^{th} percentile value

80^{th} percentile objective function value

Infer distribution



The empirical distribution is used to decide where more search efforts should be devoted

*** numbers are only for illustration**

Figure 6-2 Illustration of How to decide the search efforts in the next RA iteration based on the recorded solutions in current RA iteration

Now we are ready to formulate the RA-based stochastic optimizaiton method, which is shown in Fig. 6-3.

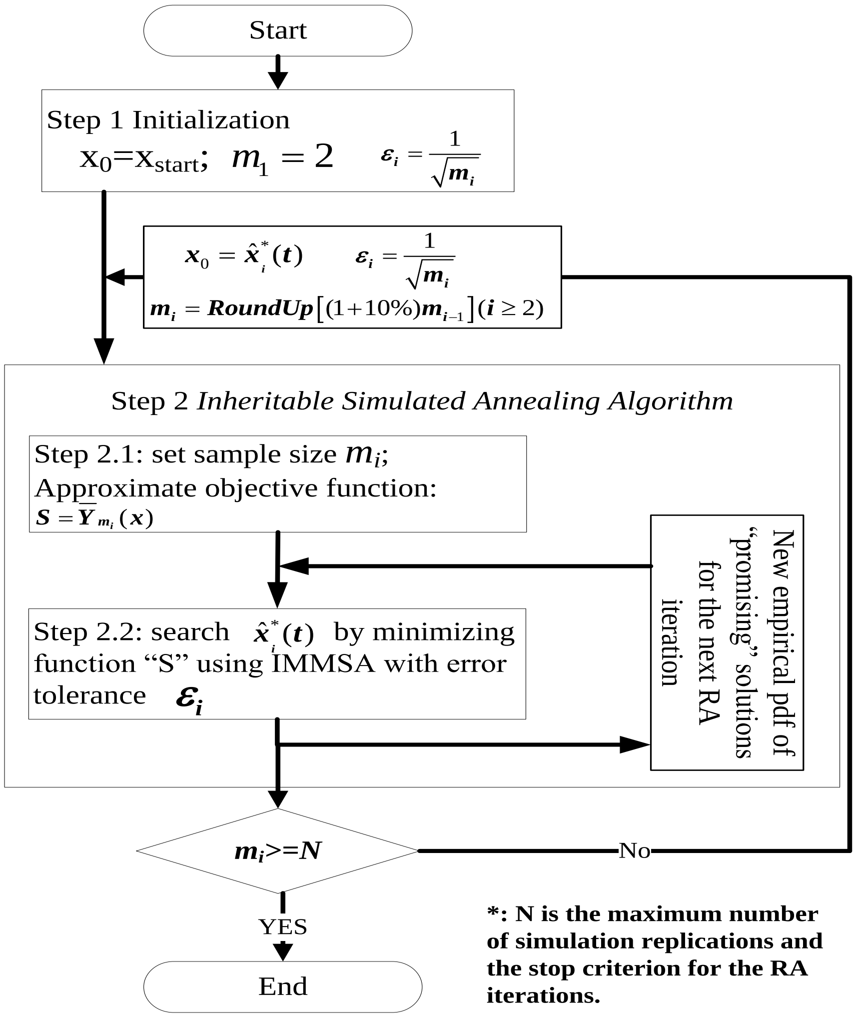


Figure 6-3 Flowchart of the RA-based stochastic optimization

6.4 Simulation-based Optimization Using VISSIM

The new algorithm was coupled with a commercial microscopic traffic simulation environment, VISSIM [86]. The advantages of VISSIM over other simulation packages include:

1. VISSIM provides the largest flexibility for users to calibrate the driving behaviors and traffic conditions;
2. VISSIM was developed under .NET framework, which brings flexibility for integrating the optimizing algorithm with the simulation engine;

The optimization algorithm sends each candidate solution into VISSIM and then drives the simulation via VISSIM COM. After each simulation run, the output is sent back to the optimization algorithm and IMMSA determines the optimum. Fig. 6-4 illustrates the architecture of VISSIM-based stochastic optimization system.

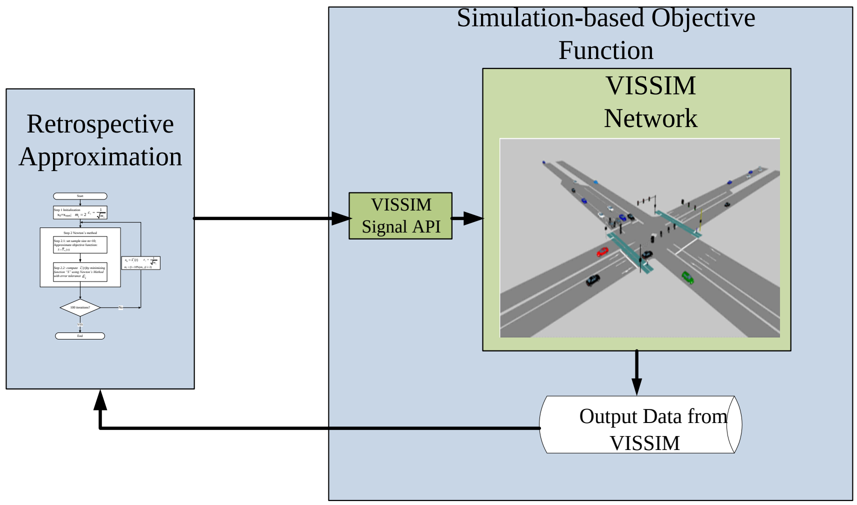


Figure 6-4 Illustration of the VISSIM-based optimization

6.5 Optimal Maximum Green Setting for the Multi-Detector Green Extension System

In this section, we conducted the optimization for the dilemma zone protection issues to illustrate the potentials of the RA-based stochastic optimization.

Problem statement: Dilemma zone (DZ) is an area at high-speed signalized intersections, where drivers can neither cross safely nor stop comfortably at the yellow onset. The dilemma zone problem is a leading cause for crashes at intersections and therefore how to protect vehicles in the DZ is a widely concerning issue. The multi-detector green extension system (GES) is a widely used dilemma zone protection system. Its mechanism is to hold the green using multiple advance detectors until there are no vehicles caught in the DZ. Meanwhile, to avoid having excessively long queues on the conflicting approaches, the GES-equipped phases have maximum greens. When the maximum green is reached, the green will end regardless of the number of vehicles in the DZ (i.e., max-out). Obviously, a max-out is hazardous and should be avoided. The longer the maximum green is, the less likely a max-out will occur. As a result, the maximum green setting has a substantial impact on the performance of GES. In this section, we conducted an illustrative study using the RA-based optimization algorithm to provide an optimal maximum green setting for GES.

Significance of the problem: It is nearly impossible to set up analytical models for GES and therefore the maximum green for GES is usually calculated indirectly. A typical method is to calculate the phase splits under pre-timed signal control and then derive the maximum green for the GES-equipped phases accordingly. The HCM model is commonly used to calculate the phase splits of fixed timing plans [80]:

$$G_0 = \frac{v * C}{S * N * (\frac{v}{c})} \tag{6-4}$$

Where:

- G_0: Maximum green (sec);
- $\frac{v}{c}$: Raito of volume to capacity, the design objective and 0.95 is used in this paper;
- C: Cycle Length (sec);
- S: Saturation flow (1,600 vehicles per hour per lane in this paper);
- N: Number of lanes;

The suggested maximum green needs to be longer than G_0 to ensure a low max-out probability. Taken as an example, the prevailing GES design by Bonneson [91] suggests that the maximum green should be at least 1.5 times longer than G_0. In practice, many GES designs follow this suggestion. However, this method is partially derived from engineering judgments and thus it may not guarantee the optimum. Meanwhile, since we cannot set up an analytical objective function for this problem, we can only approximate the objective function through simulation sampling and deploy simulation-based optimization.

Traffic scenario: the GES design needs to reflect local traffic scenarios. Fig. 6-5 shows the geometry of a local intersection at Christiansburg, VA and the traffic volumes.

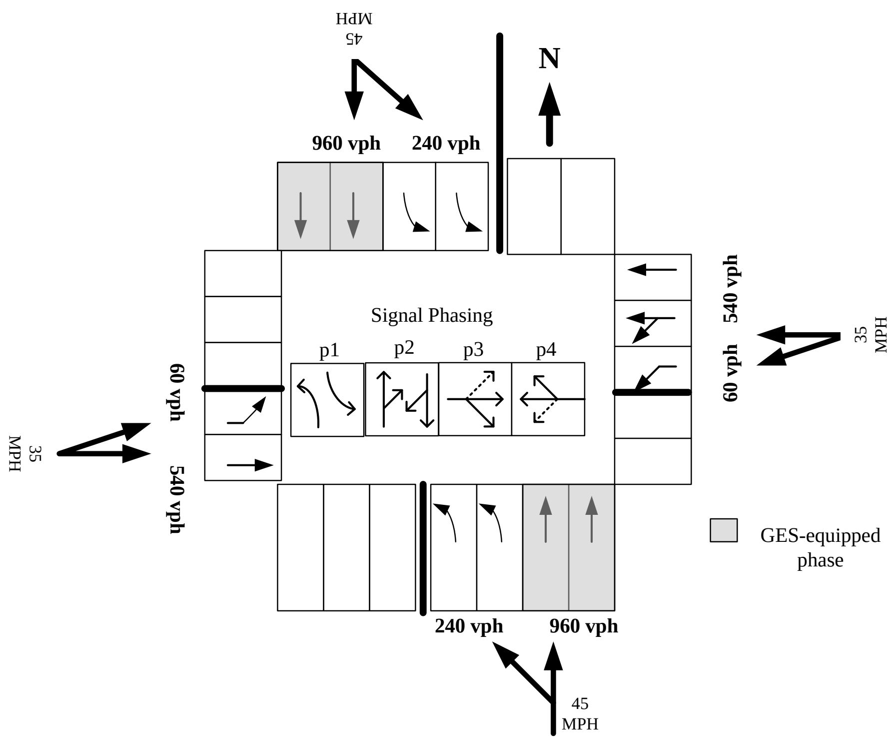


Figure 6-5 traffic scenario used in optimal maximum green design

Signal phasing: phase 2 is equipped with GES and the other phases are pre-timed. We selected 120 seconds as the cycle length to calculate the green length for phase 1, 3 and 4. The yellow interval and all-red clearance are set as 3 seconds and 2 seconds respectively. According to Eq.4 and Fig.5, the green lengths for the fixed phase 1, 3 and 4 are G_1=10 seconds, G_3=31 seconds and G_4=21 seconds. We increased G_1 to 15 seconds to meet the minimum green requirement. The possible maximum green of the GES-equipped phase can in theory range from 15 seconds (minimum green) to infinitive. However, in practice, an excessively long cycle length may make drivers on the conflict approaches disrespect the signal. Therefore we allowed the maximum green of phase 2 to be 100 seconds.

Advance detector configurations: we used the "Constant-Speed" detector configuration suggested by Bonneson et al. [91]. The locations and extensions of the advance detectors are listed in Table 6-1:

Table 6-1 Bonneson's Constant-speed advance detectors configuration (45 MPH speed)

Detector ID	Position (feet)	Extension (seconds)
1	445	0.9
2	364	1.2
3	283	3.1

Composite cost from the stochastic objective function: the traffic volumes, the controller settings and advance detector configuration are all modeled into VISSIM. The VISSIM simulation engine is used to sample and approximate GES. There are two variables in the VISSIM network: the maximum green for phase 2 and the random seed. For one particular maximum-green value, the VISSIM network is sampled with independent random seeds and then the composite cost (the performance measure) is calculated and then averaged. The composite cost covers both delay cost and safety cost caused by the dilemma zone. The unit delay cost is about \$17.02/hour/vehicle [82]; the safety is evaluated with the hazard caused by the dilemma zone. The number of vehicles in the dilemma zone is traditionally used to measure the safety, but this measure implies the vehicles in the dilemma zone are equally unsafe regardless of their speeds and locations. As a result, we used a traffic-conflict-based dilemma hazard model to measure each vehicle's unsafe level at the yellow onset. In light of the dilemma hazard model, the unsafe level, namely dilemma hazard, of the vehicle in the dilemma zone primarily depends on its time to intersection (TTI) at the yellow onset [92]. Each caught vehicle's TTI-dependent dilemma hazard can be calculated with Eq. 5 [92].

$$H = -0.202 * TTI^2 + 1.565 * TTI - 2.218 \tag{6-5}$$

Previous studies also concluded that the probability that a traffic conflict becomes a real accident is about 0.0001 [23], and the average cost for each real accident is $56,706 [81]. Therefore the unit cost for a dilemma hazard is approximately $5.67.

When converted to monetary values, the composite outputs from VISSIM can be calculated as:

$$\mathrm{Y} = 5.67 * \mathrm{DilemmaHazard(safety)} + \frac{17.02}{3600} * \mathrm{delay} \tag{6-6}$$

Stochastic Optimization Design: The proposed algorithm was customized to solve this one-dimension optimization problem as follows:

- Initialization for the 1st RA iteration:
    - Initial maximum green (initial guess) x_0: 57 seconds (calculated with the Bonneson's method);
    - The initial state probability $P=P_0$: uniformly distributed between 15 seconds and 100 seconds;
    - Initial sample size for VISSIM: 1;
- Rules for the RA progressing:
    - From the 2nd RA iteration, the sample size $\boldsymbol{m_i}$ is increased as

      $\boldsymbol{m_i} = \boldsymbol{RoundUp}\left[(1+10\%)\boldsymbol{m_{i-1}}\right](\boldsymbol{i} \geq 2)$;
    - Decreasing error tolerance: $\boldsymbol{\varepsilon_i} = \dfrac{1}{\sqrt{\boldsymbol{m_i}}}$;
    - The Common Random Numbers (CRN) rule applies;

- When m_i reaches 5, the whole RA iterating ends;

- Inheritable Markov Monotonic Search Algorithm in each RA iteration:

  - Fig. 6-6 illustrates the optimizing process of IMMSA;

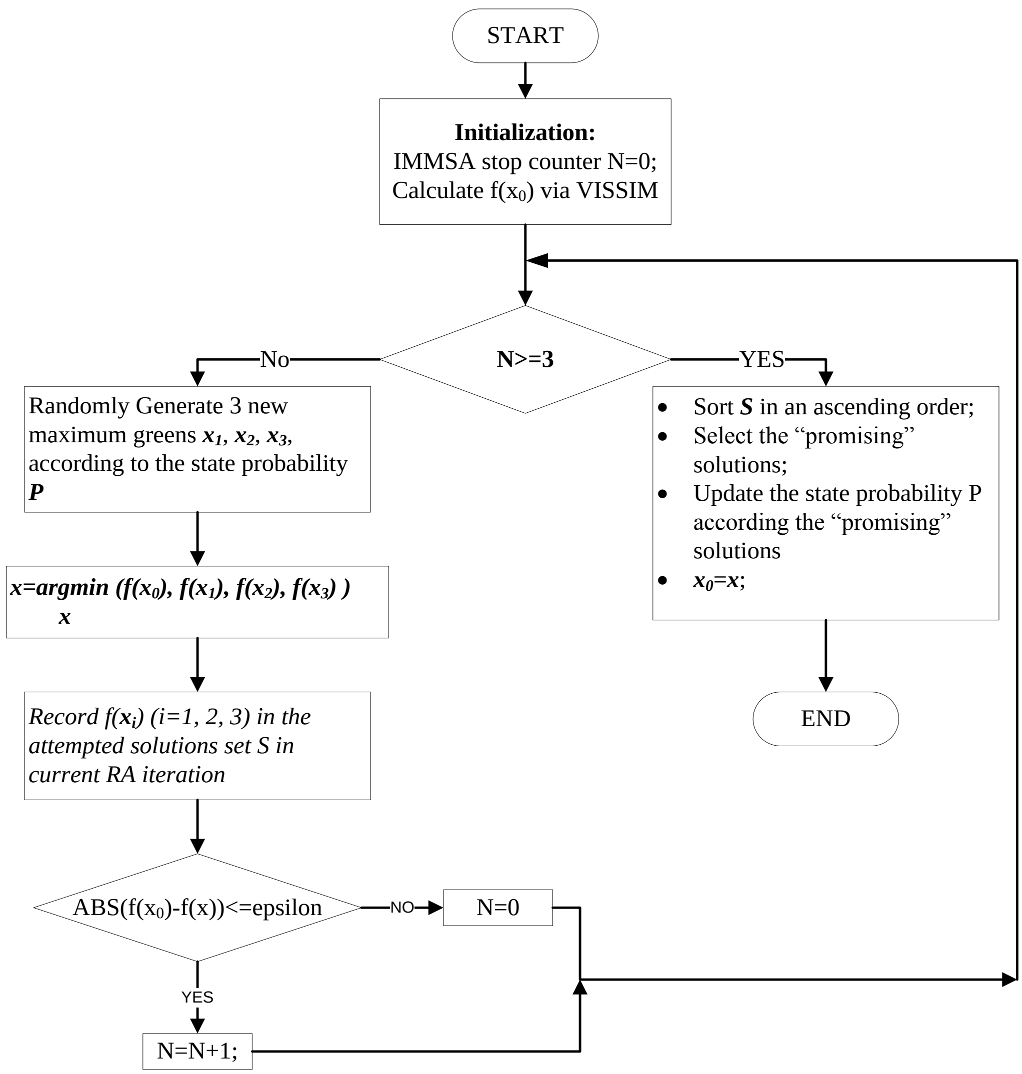


Figure 6-6 Flow chart of Inheritable Markov Monotonic Search Algorithm

Preliminary Study of the problem properties: Before the optimization was deployed, we used small simulation replications to investigate the composite costs under various maximum greens. Maximum greens were selected from 15 seconds to 100 seconds with half-second increments

(Fig. 6-7). Although such selection was of low fidelity due to its large increments, it helped us better understand the problem properties as well as validate the results from the optimization. Meanwhile, it is clear that the two approximate curves in Fig. 6-7 show very similar patterns and their optimum should be very close. This finding is supportive to the speculation in the proceeding section.

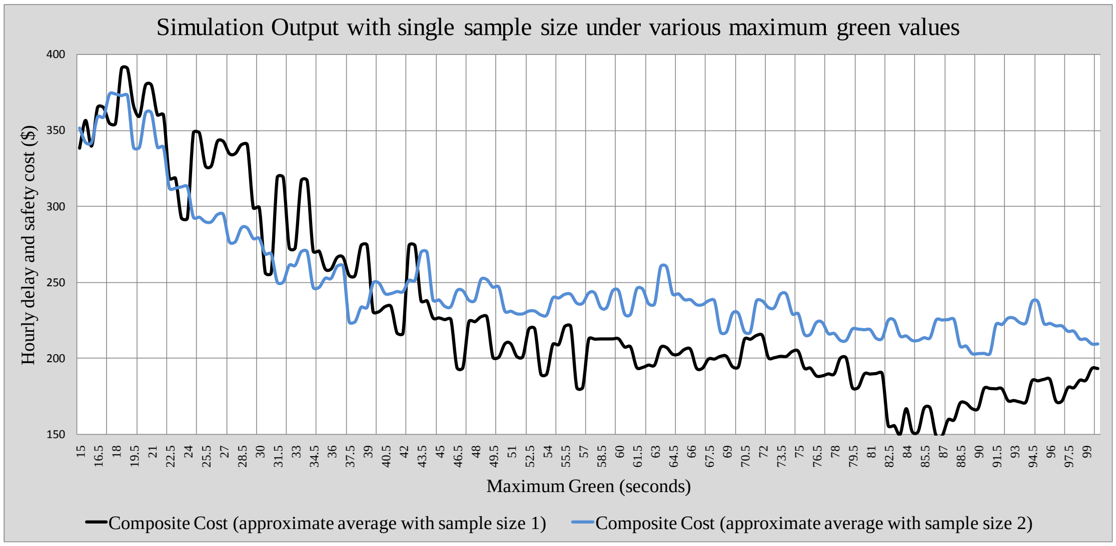


Figure 6-7 Composite cost curves with small number of simulation replications

Optimization results: Fig. 6-8 shows how the optimal solution evolved while the RA-based stochastic optimization was progressing, and Fig. 6-9 shows how the generated composite cost by VISSIM decreased. From Fig. 6-8, we can conclude that increasing the sample size (i.e., increase the approximate efforts) can help reduce the necessary optimization efforts because a larger sample size lowers the random noise residing in the simulation-based objective functions. For instance, in Fig. 6-8, the 1st RA iteration needs 13 random searches to reach the global optimum, whereas the search efforts considerably decrease when the sample size increases to 2~5.

Fig. 6-8 and 6-9 also show that the generated composite cost under the same maximum green value may change due to an increase in the sample size in VISSIM. However, when the sample size becomes large enough, such difference will be negligible. From Fig.9, the optimal maximum green value (solution) from the previous RA iteration always generates a higher composite cost when the sample size of VISSIM is increased in the next RA iteration. However, such increase is nearly negligible when the sample size increases from 3 to 4 or from 4 to 5. As a result, Fig. 6-9 can answer the question how many approximate efforts are necessary but not excessive when simulation-based optimization is deployed. For the problem in this paper, a sample size of 4 is sufficient since the optimum in the 4th RA iteration increases the composite output less than 1% and the optimum stays unchanged when the sample size increase from 4 to 5 (Fig. 6-9).

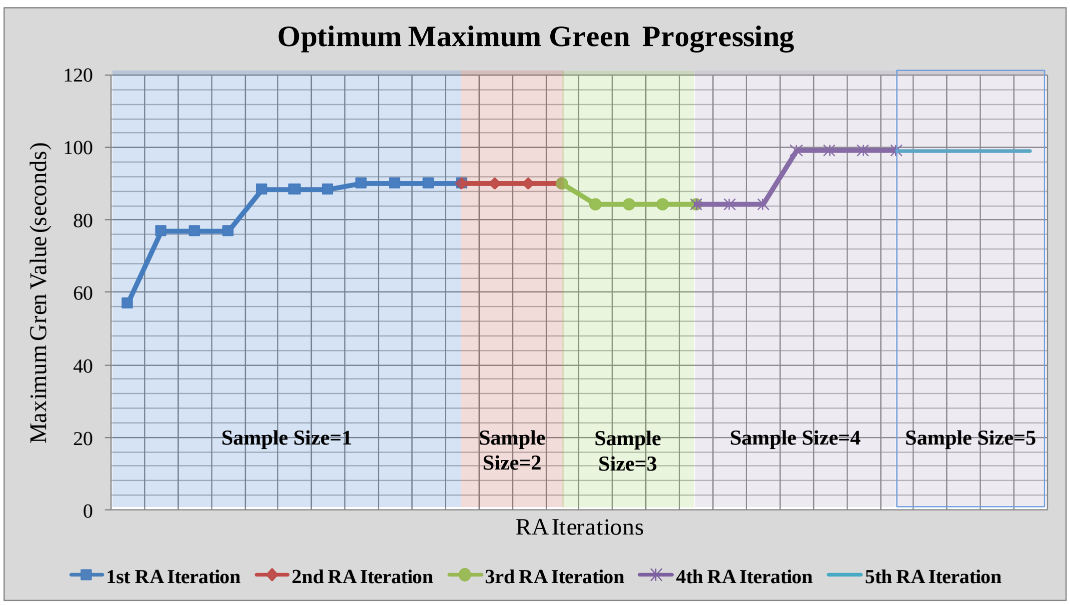


Figure 6-8 Optimum Maximum green progressing

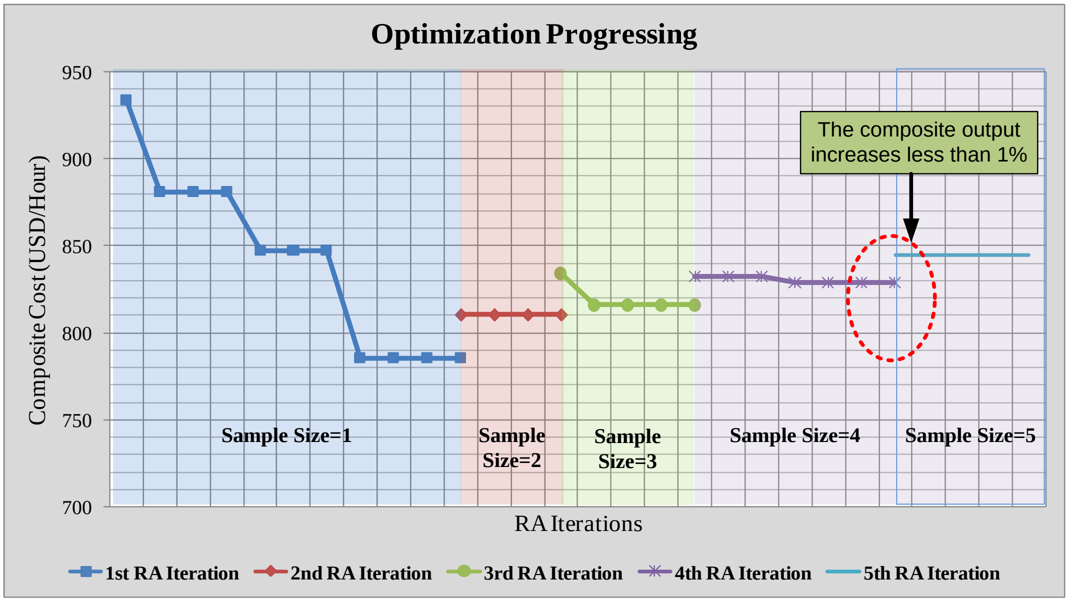


Figure 6-9 Minimal composite cost progressing

Improvement by the optimal maximum green for GES: Since we concluded that a sample size of 4 or higher is sufficient (and thus valid) for the approximate efforts, we only compared the optimal maximum green (99 seconds) obtained in the 4th and 5th RA iterations with the maximum green value (57 seconds) calculated with the Bonneson's method. The comparison is listed in Table 6-2 and it shows that the new optimal maximum green can significantly lower the hourly composite cost compared with the traditional maximum green.

Table 6-2 Improvements by the optimal maximum green

Sample Size	Hourly composite cost under the optimal maximum green (USD)	Hourly composite cost under the Bonneson's maximum green (USD)	Improvements
4	828	968	14.46%
5	844	944	10.59%

6.6 Conclusion for This Chapter and Future Work

Most traffic signal systems have to be optimized through traffic simulation samplings. The state-of-practice method is to approximate the true (and unknown) problem with a single approximate problem, and then optimize the approximate function. The authors in this paper introduce the

Retrospective Approximation (RA) concept into the optimization of traffic signal systems. RA reflects the latest theoretical development of the simulation-based optimization and is able to synchronize approximate efforts and optimization efforts when the simulation-based optimization is deployed. When the approximate efforts are low, RA will lower the optimization efforts and put emphasis on a better understanding of the system; when the approximate efforts are gradually increased, RA will also increase the optimization efforts to obtain the global optimum.

Based on the RA concept, we developed a stochastic optimization engine using VISSIM, and applied it to the maximum green setting issue for the multi-detector green extension systems. The result shows that a sample size of 4 is sufficient for VISSIM to approximate a typical multi-detector green extension system. The optimal maximum green for the GES-equipped phase at the intersection of Christiansburg, VA was determined to be 99 seconds. This new design can significantly decrease the hourly composite cost (safety and delay cost) compared with the traditional value.

In the future, we are planning to extend the IMMSA algorithm to multiple dimensions and further improve the converging rate. We are also planning to integrate other prevailing traffic simulation engines with the optimization engine developed in this paper and investigate how adaptive those simulation engines in the market will be when the simulation-based optimization is deployed.

Conclusions

The author addressed several issues surrounding dilemma zone protection using stochastic methods. First, the author modeled the decision process and behaviors of drivers when they encounter a yellow onset. Using the Monte Carlo simulation method, the author found that the crash-potential, namely the dilemma hazard, changes while vehicles are approaching intersections and will reach a maximum when they are in the middle of the dilemma zone. Using the field data, the author derived the dilemma hazard function, which is dependent on a vehicles' time to intersection at the yellow onset.

Based on the dilemma hazard function, the author optimized a traditional dilemma zone protection system, the multi-detector green extension system, using the genetic algorithm. By using this new configuration of advance detectors, the safety of an intersection can be significantly improved.

Although many dilemma zone protection systems implicitly include the random properties of traffic systems, they are mainly based on deterministic models. The author addressed this issue by introducing the Markov process into the dilemma zone protection issue. The new algorithm considers the number of vehicles in the DZ as the Markov process, and therefore it is only determined by the most recent state. Based on this assumption, the author designed a new Markov-Process-based dilemma zone protection algorithm and found that the new algorithm could outperform the prevailing D-CS system in dynamic traffic conditions and the effectiveness of the new algorithm may be further improved if a better data collection technique is available (e.g., Vehicle-Infrastructure-Integration techniques).

At last, the author addressed an important issue on the simulation-based simulation. How to simulate traffic systems sufficiently, but not excessively during the optimization is a widely concerning issue. The state-of-practice method cannot answer this question well. The author applied the recent theoretical development on the simulation-based simulation, the retrospective approximation framework (RA), to optimizing the maximum green of the green extension system. The author also designed a new algorithm to accommodate the requirement by RA. The optimization results showed that the new maximum green value could considerably improve the safety and four replications in VISSIM simulation are sufficient to simulate an isolated intersection.

References

1. Messer, C.J., S. R. Sunkari, Hassan A. Charara, and R.T. Parker, *Development of Advance Warning Systems for End-of-green Phase at High-Speed Traffic Signals*, in *FHWA/TX-04/0-4260-4*. 2003.
2. Gazis, D., H. Robert, and M. A., *The problem of the ambler signal Light in Traffic flow*. Operations Research, 1960. **8**(1): p. 112-132.
3. Institute of Transportation Engineers, *Recommended Practice: Determining Vehicle Change Intervals*, in *Technical Committee 4A-16*. 1985: Washington D.C.
4. Liu, C. and R. Herman, *A Review of The Yellow Interval Dilemma.* Transportation Research: Part A, 1996. **30**(5): p. 333-348.
5. Crawfoard, A. and D.C. Taylor, *Critical amber periods.* Traffic Engineering and Control, 1961. **3,**: p. 473~478.
6. Olson, P.L. and R.W. Rothery, *Driver Response to the Amber Phase of Traffic Signals.* Operations Research, 1961. **9**(5): p. 650-663.
7. Williams, W.L., *Driver Behavior During The Yellow Interval.* Transportation Research Record, 1977. **644**: p. 75-78.
8. Chang, M.S., C. J. Messer, and A.J. Santiago, *Timing Traffic Signal Change Intervals Based On Driver Behavior*. Transportation Research Record, 1985. **1027**: p. 20-30.
9. Lin, F.B., *Timing design of signal change intervals.* Transportation Research Record, 1986. **1069**: p. 45-51.
10. May , A.D., *Clearance Interval At Traffic Signals.* Journal of Highway Research Board, 1968. **221**: p. 41-71.

11. Liu, Y., G. Chang, R. Tao, T. Hicks, and E. Tabacek, *Empirical Observations of Dynamic Dilemma Zones at Signalized Intersections.* Transportation Research Record, 2007. **1948**: p. 20p.

12. Easa, S.M., *Reliability-based design of intergreen interval at traffic signals.* Journal of Transportation Engineering, 1993. **119**(2): p. 255-271.

13. Parsonson, P.S., R.W. Roseveare, and J.M. Thomas(Southern Section ITE Technical Council Committee 18), *Small-area Detection at Intersection approaches.* Journal of Transportation Engineering, 1974. **44**(5): p. 8-17.

14. Zegeer, C.V., *Effectiveness of Green-Extension Systems at High-speed Intersections.* . 1977, Department of Transportation: Lexington, Kentucky.

15. Bonneson, J.A., P.T. McCoy, and Brian A. Moen, *Traffic Detector Design and Evaluation Guidelines*. 1994, Department of Civil Engineering, University of Nebraska: Lincoln, Nebraska. p. 79.

16. Papaioannou, P., *Driver Behaviour, Dilemma Zone and Safety Effects at Urban Signalised Intersections in Greece.* Journal of Accident Analysis and Prevention, 2007. **39**(1): p. 147-158.

17. Peterson, A., *LHOVRA a new traffic signal control strategy for isolated junctions.* Traffic engineering and control, 1986. **July/August**.

18. Kronborg, P., *SOS–Self Optimizing Signal Control: Development and Field Trials of the SOS Algorithm for Self-Optimizing Signal Control at Isolated Intersections*, in *TFK (Transport Research Institute) Report 1997:2E, Stockholm, Sweden*. 1997.

19. Bonneson, J., D. Middleton, K. Zimmerman, H. Charara, and M. Abbas, *Intelligent Detection-control System For Rural Signalized Intersections*. 2002.

20. Chaudhary, N.A., M. Abbas, and H. A. Charara, *Development and field testing of platoon identification and accommodation system.* Transportation Research Record, 2006. **1978**: p. 141-148.

21. Perkins, S.R. and J.I. Harris, *Traffic Conflict Characteristics-Accident Potential at Intersections.* Journal of Highway Research Record, 1968. **225**: p. 35-43.

22. Cooper, D.F. and N. Feguson, *Traffic Studies at T-Junctions: 2. A conflict simulation model.* Traffic Engineering and Control 1976: p. 306~309.

23. Baker, W.T., *An Evaluation of the Traffic Conflict Technique.* Highway Research Record, 1972: p. 1-8.

24. Glauz, W.D. and D.J. Migletz, *Application of Traffic conflict analysis at intersections*, in *NCHRP report 219*. 1980: Washington, D.C.

25. Paddock, R.D., *An accident prediction method*. 1974: Ohio Department of Transportation.

26. Glauz, W.D., K. M. Bauer, and D.J. Migletz, *Expected Traffic Conflict Rates and Their Use in predicting Accidents.* Transportation Research Record, 1985(1026): p. 1-12.

27. Gettman, D. and L. Head, *Surrogate Safety Measures from Traffic Simulation Models.* . 2003, FHWA: Washington D.C.

28. M.R.Crabtree, R.H., *MOVA Traffic Control Manual*. 2006.

29. Peterson Alf *LHOVRA a new traffic signal control strategy for isolated junctions.* Traffic engineering and control, 1986. **July/August**.

30. Zimmerman K.H, *Providing Additional Dilemma Zone Protection for Trucks at High-Speed Signalized Intersections*, in *Transportation Research Board 86th Annual Meeting*. 2007, Transportation Research Board. p. 16p.

31. McCoy P.T, *Dilemma Zone Protection on High-Speed Signalized Intersection Approaches: Advance Detection Versus Advance Warning Flashers and Advance Detection*, in *Transportation Research Board 83th Annual conference CD-ROM*. 2003: Washington D.C.

32. Sunkari Srinivasa R, Messer Carroll J, and Charara Hassan, *Performance of Advance Warning for End of Green System for High-Speed Signalized Intersections*. 2005: Transportation Research Board. pp 176-184.

33. Chaudhary, N.A., M. M. Abbas, and H. Charara, *Platoon Identification And Accommodation System For Isolated Traffic Signals On Arterials*. 2003, Texas Transportation Institute: College Station, TX.

34. Yosef, S. and H. Mahmassani, *Model of Drive Behavior At High-speed Signalized Intersections.* Transportation Science, 1981. **15**(1): p. 50-61.

35. Gates, T.J., D. A. Noyce, and L. Laracuente, *Analysis of Dilemma Zone Driver Behavior at Signalized Intersections*. Transportation Research Record. Vol. 2030. 2007: Transportation Research Board (CD-ROM).

36. Sackman, H.B., M. Monahan, P. Parsonson, and A.F. Trvino, *Vehicle Detector Placement for High-speed isolated, Traffic-Actuated Intersection Control*. 1977.

37. Southern Section ITE, *Large-area Detection At Intersection Approaches.* Journal of Traffic Engineering, 1976. **46**(June): p. 28-37.

38. Botma, H., *Queuing Model to Determine Variability of Recurrent Congestion at Freeway Links over a year.* Transportation Research Record, 1999. **1678**: p. 83-89.

39. Hoogendoorn, S.P., V.L. Knoop, and H.J. Van Zuylen, *Robust control of traffic networks under uncertain conditions.* Journal of Advanced Transportation, 2008. **42**(3): p. 357-377.

40. Alfa, A.S. and M.F. Neuts, *Modelling vehicular traffic using the discrete time Markovian arrival process.* Transportation Science, 1995. **29**(2): p. 109-117.

41. Viti, F. and H.J. Van Zuylen, *Modeling queues at signalized intersections.* Transportation Research Record, 2004(1883): p. 68-77.

42. Geroliminis, N. and A. Skabardonis, *Prediction of arrival profiles and queue lengths along signalized arterials by using a Markov decision process.* Transportation Research Record, 2005(1934): p. 116-124.

43. Yu, X.H. and W.W. Recker, *Stochastic adaptive control model for traffic signal systems.* Transportation Research Part C: Emerging Technologies, 2006. **14**(4): p. 263-282.

44. Kim, S., M.E. Lewis, and C.C. White, *State space reduction for nonstationary stochastic shortest path problems with real-time traffic information.* IEEE Transactions on Intelligent Transportation Systems, 2005. **6**(3): p. 273-284.

45. Robertson, D.I., *TRANSYT: A Traffic Network Study Tool*, in *Transport and Road Research Laboratory*. 1969: United Kingdom.

46. Hunt, P.B., D. I. Robertson, R. D. Bretherton, and R. I. Winton, *SCOOT - A Traffic Responsive Method of Coordinating Signals*, in *TRRL Laboratory Report*. 1981, Transport and Road Research Laboratory: Great Britain. p. 41.

47. Sims, A.G. and K.W. Dobinson, *The Sydney Coordinated Adaptive Traffic (SCAT) System philosophy and benefits.* IEEE Transactions on Vehicular Technology, 1980. **t-29**(2): p. 130-7.

48. Gartner, N.H., *OPAC: a demand-responsive strategy for traffic signal control.* Transportation Research Record, 1983. **906**: p. 75-81.

49. Head, L.K., P.B. Mirchandani, and D. Sheppard, *Hierarchical Real-Time Traffic Control.* Transportation Research Record, 1992. **1360**: p. 82-88.

50. Abbas, M. and A. Sharma, *Multi-Objective Plan Selection Optimization for Traffic Responsive Control.* Journal of transportation engineering, 2006. **132**(5): p. 376-284.

51. Nakatsuji, T. and T. Kaku, *Development of A Self-organizing Traffic Control System Using Neural Network Models.* Transportation Research Record, 1991(1324): p. 137-145.

52. Park, B., C.J. Messer, and T. Urbanik II, *Initial Evaluations of New TRANSYT-7F Ver 8.1 Program.* Transportation Research Board, 1999(1683): p. 127-132.

53. Abbas, M., N.A.Chaudhary, G.Pesti, and A.Sharm., *Guidelines for Determination of Optimal Traffic Responsive Plan Selection Control Parameters*. 2005: College Station, TX.

54. Lin, F.-B., *Use of binary choice decision process for adaptive signal control.* Journal of Transportation Engineering, 1990. **115**(3): p. 270-282.

55. Lin, F.-B. and S. Vijayakumar, *Adaptive Signal Control at Isolated Intersections.* Journal of Transportation Engineering, 1988. **114**(5): p. 555-573.

56. Zuylen, V., Henk J, and F. Viti. *Delay at controlled intersections: The old theory revised*. 2006. Toronto, ON, Canada: Institute of Electrical and Electronics Engineers Inc., Piscataway, NJ 08855-1331, United States.

57. Adam, Z., M.M. Abbas, and P. Li. *Modelling the Complexity of Driving Behavior During Signal Yellow Interval Using Reinforcement Learning* in *88th Annual Meeting of Transportation Research Board*. 2009. Washing, D.C.

58. Adam, Z., M.M. Abbas., and P. Li. *Evaluating Green-Extension Policies Using Reinforcement Learning and Markovian Traffic State Estimation*. in *88th Annual Meeting of Transportation Research Board*. 2009. Washington, D.C.

59. Sun, S., G. Yu, and C. Zhang. *Short-term traffic flow forecasting using sampling Markov Chain method with incomplete data*. in *Intelligent Vehicles Symposium, IEEE*. 2004. Parma, Italy: Institute of Electrical and Electronics Engineers Inc., Piscataway, NJ 08855-1331, United States.

60. Sherlaw-Johnson, C., S. Gallivan, and J. Burridge, *Estimating a Markov transition matrix from observational data.* Journal of the Operational Research Society, 1995. **46**(3): p. 405-410.

61. Hazelton, M.L. and D.P. Watling, *Computation of equilibrium distributions of Markov traffic-assignment models.* Transportation Science, 2004. **38**(3): p. 331-342.

62. Robbins, H. and S. Monro, *A stochastic Approximation Method.* Annals of Mathematical Statistics, 1951. **22**: p. 400-407.

63. Healy, K. and L.W. Schruben. *Retrospective Simulation Response Optimization*. in *Proceedings of the 1991 Winter Simulation Conference*. 1991. Piscataway, New Jersey: IEEE.

64. Rubinstein, R.Y. and A. Shapiro, *Discrete Event Systems: Sensitivity Analysis and Stochastic Optimization by the Score Function Method*. 1993, New York: John Wiley & Sons.

65. Shapiro, A. and T. Homem-de-Mello, *A Simulation-based approach to two-stage stochastic programming with recourse.* . Mathematical Programming, 1997. **81**: p. 301-325.

66. Park, B., C.J. Messer, and T. Urbanik, II, *Traffic signal optimization program for oversaturated conditions: genetic algorithm approach.* Transportation Research Record, 1999(1683): p. 133-142.

67. Park, B., C.J. Messer, and T.I. Urbanik, *Enhanced Genetic Algorithm for Signal-timings Optimization of Oversaturated Intersections.* Transportation Research Record, 2000. **1727**: p. 32-41.

68. Park, B. and J.D. Schneeberger, *Evaluation of Traffic Signal Timing Optimization Methods Using a Stochastic and Microscopic Simulation Program*. 2003, University of Virginia: Charlottesville, VA.

69. Park, B. and I. Yun, *Stochastic Optimization Method for Coordinated Actuated Signal Systems*. 2005, University of Virginia: Charlottesville, VA.

70. Stevanovic, J., A. Stevanovic, P.T. Martin, and T. Bauer, *Stochastic optimization of traffic control and transit priority settings in VISSIM.* Transportation Research Part C: Emerging Technologies, 2008. **16**(3): p. 332-349.

71. National Safety Council, *Injury Facts.* 2007. **2006-2007**.

72. AASHTO, *A Policy on Geometric Design of Highways and Streets*. 2004, American Association of State Highway and Transportation Officials: Washington, D.C.

73. Institute of Transportation Engineers, *Transportation and Traffic Engineering Handbook*. 1985, Prentice Hall.

74. Toledo, T., H. N. Koutsopoulos, and M. E. Ben-Akiva, *Modeling Integrated Lane-changing Behavior.* Transportation Research Record, 2003(1857): p. p. 30-38.

75. Ahmed, K.I., *Modeling Drivers' Acceleration and Lane Changing Behavior*, in *Department of Civil Engineering*. 1999, M.I.T: Cambridge.

76. May, A.D., *Traffic Flow Fundamentals*. 1990, Englewood Cliffs, N.J.: Prentice Hall.

77. Doerzaph, Z.R., *Intersection stopping behavior as influenced by driver state: Implications for Intersection Decision Support systems*. 2004, Virginia Polytechnic Institute and State University: Blacksburg,VA.

78. Si J., T. Urbanik II, and L.D. Han, *Effectiveness of Alternative Detector Configurations for Option Zone Protection on High-Speed Approaches to Traffic Signals.* Journal of the Transportation Research Board, 2007. **2035**: p. 107-113.

79. Baker, M., *Fuel Consumption and Emission Models for Evaluating Traffic Control and Route Guidance Strategies*. 1994, Queen's University: Kingston, Ontario, Canada.

80. National Research Council, *Highway Capacity Manual (4th edition)*, in *Transportation Research Board*. 2000: Washington, D.C.

81. Sharma, A., D. M. Bullock, and S. Peeta, *Recasting Dilemma Zone Design as a Marginal Cost-Benefits Problem.* Transportation Research Record, 2007(2035): p. 88-96.

82. Schrank, D. and T. Lomax, *The 2007 Urban Mobility Report*. 2007, Texas Transportation Institute: College Station, TX.

83. Institution of Transportation Engineers, *Transportation and Traffic Engineering Handbook*. 1985, Prentice Hall.

84. Ross, M.S., *Introduction to Probability Models, 9th edition*. 2006, New York: Academic Press, ELSEVIER.

85. Adam, Z., M.M. Abbas, and P. Li, *Evaluating Green-Extension Policies Using Reinforcement Learning and Markovian Traffic State Estimation*. Transportation Research Board 88th Annual Meeting. 2009: Transportation Research Board. 19p.

86. PTV-AG Inc., *VISSIM 5.10 User Manual*. 2008. 438.

87. Chen, H. and B.W. Schmeiser, *Stochastic Root Finding via Retrospective Approximation.* Institute of Industrial Engineers Transactions, 2001. **33**: p. 259-275.

88. Pasupathy, R. and B.W. Schmeiser. *Retrospective Approximation Algorithms for the Multidimensional Stochastic Root-finding Problem*. in *Proceedings of the 2004 Winter Simulation Conference*. 2004. Washington, DC, USA.

89. Pasupathy, R. *On Choosing Parameters In Retrospective-Approximation Algorithms for Simulation-optimization*. in *Proceedings of the 2006 Winter Simulation Conference*. 2006. Monterey, CA.

90. Emile, A. and K. Jan, *Simulated Annealing and Boltzmann Machines: A Stochastic Approach to Combinatorial Optimization and Neural Computing*. 1991.

91. Bonneson, J.A. and P.T. McCoy, *Traffic Detector Designs for Isolated Intersections.* ITE Journal, 1996(August): p. 42-47.

92. Pengfei Li and M.M. Abbas. ***Optimal Advance Detectors Design for the Multi-detector Green Extension System at High-Speed Signalized Intersections*** in *88th Annual Meeting of Transportation Research Board*. 2009. Washington D.C., US: Transportation Research Board of National Academies.