A New Nonparametric Procedure for the k-sample Problem

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(ABSTRACT)

The k-sample data setting is one of the most common data settings used today. The null hypothesis that is most generally of interest for these methods is that the k-samples have the same location. Currently there are several procedures available for the individual who has data of this type. The most often used method is commonly called the ANOVA F-test. This test assumes that all of the underlying distributions are normal, with equal variances. Thus the only allowable difference in the distributions is a possible shift, under the alternative hypothesis. Under the null hypothesis, it is assumed that all k distributions are identical, not just equally located.

Current nonparametric methods for the k-sample setting require a variety of restrictions on the distribution of the data. The most commonly used method is that due to Kruskal & Wallis (1952). The method, commonly called the Kruskal-Wallis test, does not assume that the data come from normal populations, though they must still be continuous, but maintains the requirement that the populations must be identical under the null, and may differ only by a possible shift under the alternative.

In this work a new procedure is developed which is exactly distribution free when the distributions are equivalent and continuous under the null hypothesis, and simulations are used to study the properties of the test when the distributions are continuous and have the same medians under the null. The power of the statistic under alternatives is also studied. The test bears a resemblance to the two sample sign type tests, which will be pointed out as the development is shown.

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Chapter 1

Introduction

1.1 Motivation

The basic motivation in the area of nonparametric statistics is the removal of unnecessary assumptions. This results in procedures that place as few restrictions as possible on the types of data to which they may be applied. A review of the common methods for comparison of the center of the k underlying distributions in the k-sample setting reveals that even the most relaxed assumptions still require that the observations be gathered independently, that all of the distributions underlying the samples must be identical and continuous under the null, and differ only by shifts under the alternative. This research maintains the assumption of independent data, and continuous underlying distributions. However, there is room for improvement in the area of scale assumptions. This research is intended to begin to address the development of a method that allows the underlying distributions to differ in shape, even under the null hypothesis of equal location. The statistic developed in this work is exactly distribution free when the distributions are identical, and can be adjusted to be valid when the distributions differ under the null hypothesis. A review of the literature will shed light on the history of work in this particular area as a starting point for this work.

1.2 Literature Review

The first method of analysis taught to most beginning statistics students for testing for equal locations in the k-sample setting, with $k \geq 3$, is the ANOVA F-test of the means. This test makes quite restrictive assumptions. First, the distributions underlying the k samples must all come independently from normal distributions. Secondly, they must all be identical under the null hypothesis, and may differ only in their location under the alternative hypothesis. While these assumptions do make the methodology straightforward, and allow for development of a test whose exact distribution can be found, they reduce the usefulness of the method for those data sets that do not meet these assumptions. The method is also not invariant against non-linear transformations of the data.

The attempts to relax the normality and equal shape assumptions have followed two tracks. One approach is that taken by Welch (1951). Welch's improvement was to remove the assumption that the distributions must have the same variance. While this was an improvement, the normality assumption was maintained, and therefore, only a small part of the problem was addressed. Since Welch retained the assumption of normality of all distributions, he also continued to use the sample mean as the estimate of location. Like the F-test, this method is not invariant to non-linear transformations of the data.

The second track is exemplified by Kruskal & Wallis (1952). The authors here develop a test that addresses the need to remove the stringent requirement that the distributions underlying the data be normal, but maintain a requirement that the shapes still be the same, where equal shape implies that the distributions are identical except for location. Thus, under the null, all k distributions must be identical, though not necessarily normal. This was a large improvement, since the distributions underlying the populations need not even be symmetric, but there must still be a belief that the distributions are equally shaped, even if the locations differ. This test is commonly called the Kruskal-Wallis test, and actually tests whether the underlying distributions are the same. Any departure from equally shaped distributions may also cause the test to reject. The Kruskal-Wallis test is in fact much more likely to detect small location shifts than small shape differences, as desired, but can reject for either type of difference between distributions. As an example, the Kruskal-Wallis method might reject in a situation where the underlying distributions are all from the same family, and had equal locations, but varying spreads. Another benefit of the Kruskal-Wallis method is that due to the use of ranks (see section 1.4 for a more detailed discussion of the test) it is also invariant against even non-linear transformations of the data. (A well written discussion of the Kruskal-Wallis test can be found in Daniel (1990, pages 126-131).)

More recently, Rust & Fligner (1984) develop a complicated adjustment to the Kruskal-Wallis test, which seems to do little more than make the use of the method more difficult, and results in assumptions which are identical to the original version. The claim is made that their test is more robust to departures of spread, but the tables of simulation results show their test to be only a slight improvement over the original method.

Fligner (1985) proposes that performing all pairwise Wilcoxon-Mann-Whitney (Wilcoxon 1945, Mann & Whitney 1947) tests is at least as efficient a procedure as the Kruskal-Wallis test. (See section 1.4) Since for a fair comparison the overall Type I error rate for the complete pairwise method must be equal to the Type I error rate chosen for the Kruskal-Wallis method, the pairwise level must be adjusted to maintain the desired overall level. Fligner's paper, however, makes no mention of how to make the necessary adjustment to the pairwise level. (The relationship between the Wilcoxon-Mann-Whitney and Kruskal-Wallis methods will be discussed further in section 1.4.) This method is similar in spirit to the Tukey multiple comparison method. (See Devore (1991, pages 384-385) for a discussion of the significance level for Tukey's method.)

The application of this method to the gap in the literature noted in section 1.1 would be that all pairwise tests could be performed, using either of the two-sample sign type tests that are mentioned in section 2.2. This would allow the samples to come from different underlying distributions. This should result in a test that is distribution free for the problem of interest here. The practical problem with this is the lack of the knowledge necessary to control the pairwise Type I error rate such that the overall error rate could be kept within a chosen bound. The Bonferroni approach is a method which may be used, but often results in a lower than desired Type I error rate, and therefore can substantially affect the power of the test against all alternatives. Since nonparametric tests are already known to be less powerful against equivalent normal theory tests when the normal distribution assumption is valid, giving up any more power could result in a method with very low power.

A search of the literature for further research in this area revealed nothing that was an improvement on this method. Several authors have discussed properties, applications, and extensions of the Kruskal-Wallis test. (Daniel (1990, pages 229-230) gives a good, though dated list.) However, none of these authors has addressed the need for relaxed assumptions.

One procedure that should be discussed, is the extension of the median test that Daniel (1990) introduces. The assumptions that are listed for the extension, are the general assumptions that are desired in the present work, but the test uses the methodology of the two sample median test (pages 84–87) where the assumption of equal shape is made. (See page 84, Assumption D.) It is thus likely that the proposed extension also requires this assumption, and the list of assumptions is incomplete. Daniel does not go into any discussion of the small- or large-sample properties of this test in his text, nor does he give any indication that such work had been done.

1.3 The Topic of This Research

The problems with Daniel's proposed extension have motivated another possible solution that does not exist in the current literature. The method that will be proposed in the next chapter will be to make a k-sample extension of a two-sample method that has the properties desired. The two-sample method used will be distribution free under the null hypothesis in the setting where the two distributions are continuous, regardless of underlying distributions. The Modified Mathisen's Test due to Hettmansperger & McKean

(1998) meets these requirements, and is the method that will be extended in this work. A discussion of this method can be found in section 2.2. A look at the relationship between the Wilcoxon-Mann-Whitney test for two samples, and the Kruskal-Wallis k-sample test will motivate the extension.

1.4 The Kruskal-Wallis as a k-sample Extension of the Wilcoxon-Mann-Whitney, and Wilcoxon Signed-Rank Tests

For one sample, the Wilcoxon Signed Rank statistic subtracts the hypothesized median from the data. The absolute value of the resulting values, which are often called the centered data, are then ranked. Each rank is then given the sign of the centered data value. The test statistic, S_+, is the sum of the ranks associated with positive centered data values. The test assumes that the data are independent and come from a continuous symmetric distribution, such that under the null hypothesis, the centered data should be symmetric about zero. This means that if the two "halves" of the centered distribution are thought of as two separate distributions, the absolute value of the negative centered values and the positive centered values would have the same distribution, though it need not be symmetric.

For two independent samples, the Wilcoxon-Mann-Whitney test groups all of the data together. The data is then ranked, as if all of the observations came from one dataset. Then the Wilcoxon form of the test statistic, W, is the sum of the ranks associated with one of the datasets. The assumptions made by this method are that the data come independently from continuous distributions, and that the distributions are identical, though not necessarily symmetric, under the null hypothesis. (A thorough discussion of the Wilcoxon-Mann-Whitney test can be found in Hettmansperger & McKean (1998, pages 73-93).)

When three or more samples are considered, the Kruskal-Wallis method is the analogous method. The Kruskal-Wallis test also combines all of the data, and then assigns ranks to the observations in the large "dataset." Since there are k samples, $k \geq 3$, simply looking at the sum of the ranks in one of the original datasets is not sufficient. Kruskal and Wallis define R_{ij} to be the rank of the j^{th} observation in the i^{th} dataset. They then look at $R_{i\cdot}$, the sum of the ranks for the i^{th} dataset. Define n_i to be the size of sample i^{th}, and N to be the overall sample size. Then the test statistic for the Krukal-Wallis method is:

$$K = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_{i\cdot}^2}{n_i} - 3(N+1)$$

The assumptions made by this method are the same as those for the Wilcoxon-Mann-Whitney method. Both methods assume that the data come independently from continuous distributions, and that the distributions are identical under the null hypothesis.

Chapter 2

A New Option

2.1 The Sign Test

The new method proposed in this chapter is an extension of the Modified Mathisen's (MM) Test (Hettmansperger & McKean 1998). The MM test is a modification of one of the two common extensions of the Sign test. The Sign test is a one-sample method which assumes only that the data come independently from a continuous distribution. The test statistic is the count of the number of observations above the hypothesized median, which therefore has a binomial distribution under the null hypothesis. Note that the median is used. This enables this method to be invariant against monotone transformations of the data.

2.2 The Modified Mathisen's Test

The most common extension two-sample extension of the Sign test is due to Mood (1950). It combines the data into one large dataset, and then finds the median of that dataset. The method then compares each observation to the median of the combined dataset. The key difference between this method and the other technique, due to Mathisen (1943), is that Mathisen's method compares observations to the median of the other sample.

The original Mathisen's test is intended to test whether two samples come from identical distributions. The Modified Mathisen's (MM) test statistic is a modification intended to test only for location shifts. It is calculated by comparing the observations in one of the samples with the median of the other sample. The statistic is calculated only one way. The median is used to measure the center of the distribution because it is more robust than the mean. Outliers in the sample do not affect the median unless half of the data are outliers. This robust estimation is used since the only assumption made is that the data come independently from continuous distributions. Since the distributions may be different, one of the data sets may be more likely to contain outliers.

Define: $\hat{\theta}_X =$ The median of the first sample.

$Y_j =$ The j^{th} observation in the second sample.

Then the MM test statistic is:

$$S_2 = \sum_{j=1}^{n_2} \operatorname{sgn}(Y_j - \hat{\theta}_X) \tag{2.1}$$

The MM method assumes only that the distributions are continuous, and have the same median under the null hypothesis. These are the general assumptions desired in this work, so the statistic developed in the next section will use this basic form, but extend it to a setting with more than two samples. It should also be noted that the choice of Y and X samples is arbitrary, but not necessarily equivalent. If the two samples come from very different populations, the calculation of the statistic could result in very different values for the two possible choices of naming the distributions. This will affect the logic of the development of the statistic considered in the next section.

2.3 The Paired Sign Statistic

2.3.1 Extending the Modified Mathisen's Test

Now that there are k-samples to consider, with $k \geq 3$, the terminology in section 2.2 must be extended.

$$\text{Define:} \quad \hat{\theta}_l = \text{ The median of the } l^{\text{th}} \text{ sample.}$$

$$X_{ij} = \text{ The } j^{\text{th}} \text{ observation in the } i^{\text{th}} \text{ sample.}$$

The extension can be further simplified if a shortcut notation for the MM statistic is defined.

$$\text{Define: } z_{il} = \sum_{j=1}^{n_i} \text{sgn}(X_{ij} - \hat{\theta}_l) \tag{2.2}$$

Thus z_{il} and z_{li} are the MM statistics for samples i and l. Since the MM statistic lacks symmetry, the statistic proposed here will combine the statistic calculated both ways, i.e. both z_{il} and z_{li} will be calculated. Natural cancelling would happen if there were true separation, since z_{il} and z_{li} would have opposite signs. To avoid this, the absolute value of each z_{il} term will be used. This allows results which should indicate differences, being positive, to compound each other, rather than subtract from each other. The statistic will also be calculated for all possible pairs. For this reason the statistic will be called the Paired Sign (PS) statistic, and the test will be called the Paired Sign test. Combining all of the terms discussed here yields the following statistic.

$$PS = \sum_{l=1}^{k} \sum_{i \neq l} \left| \sum_{j=1}^{n_i} \text{sgn}(X_{ij} - \hat{\theta}_l) \right| = \sum_{l=1}^{k} \sum_{i \neq l} |z_{il}| \tag{2.3}$$

The reader will note that use of the median is continued. This is done because the desired assumptions are only that the distributions from which our data come are continuous, and the data is gathered independently. The use of the median will allow comparison between the samples without possible outliers affecting the analysis. The use of the median also allows the proposed method to be invariant under monotone transformations of the data.

It should also be noted that the calculation of the statistic in this way does not guarantee that the method will be distribution free under the null hypothesis when the distributions differ. This is due to the correlations that will be involved in the calculation of the statistic. Since these covariances, and even the variances, will differ if the distributions differ, the statistic will not meet the ideal assumptions desired in this work. The statistic may however prove to be more flexible than the current methods. The usefulness of the statistic will be evaluated later in this work. This raises questions as to why the statistic may still be expected to be helpful. The construction of the statistic gains information only through the sign of the comparison of each observation to the medians of the other samples. This should result in a method that is more robust than existing methods, but will cause the method to be less powerful. It is known that in general, as methods become more robust, the power will decrease.

If the distributions are equal, the distribution free nature of the statistic is implied by the invariance of the sample median. If any monotone transformation is performed, the ordering of the observations would be maintained, and therefore the value of the PS statistic would be unchanged. The same argument says that the Kruskal-Wallis method is invariant for equal distributions since the ranks are also invariant to monotone transformations.

2.3.2 Practical Considerations

An item of practical importance before the statistic can be applied is that of the definition of the median, $\hat{\theta}_l$, when sample l has an even number of observations. The convention used impacts the distribution of the PS statistic. The most common convention is to declare the average of the two middle values to be the median. However, any value between the two middle values meets the technical definition of the median. Also, if the average of the two middle values is used, the median is no longer invariant to non-linear transformations of the data. Since the invariance feature is desirable, this work will assume that any value from sample i that falls between the two middle values for sample l will be considered to have $\text{sgn}(X_{ij} - \hat{\theta}_l) = 0$. This definition yields a “sign” function that will be:

$$sgn(X_{ij} - \hat{\theta}_l) = \begin{cases} -1 & X_{ij} < X_{l(\frac{n_l}{2})} \\ 1 & X_{ij} > X_{l(\frac{n_l}{2}+1)} \\ 0 & X_{l(\frac{n_l}{2})} < X_{ij} < X_{l(\frac{n_l}{2}+1)} \end{cases}$$

Where $X_{i(j)}$ is the j^{th} order statistic for the i^{th} sample. That is, $X_{i(j)}$ is the j^{th} observation in the i^{th} dataset once it has been reordered from smallest to largest. This convention raises a need to consider how much of the data will be counted as a 0, rather than a ±1.

Proposition 2.1. *For even n_l, if the underlying distributions are identical, the proportion of all possible comparisons, $X_{ij} - \hat{\theta}_l$, that return a zero sign is $\frac{1}{n_l+1}$, where n_l is the number of observations in sample l.*

Proof. Since the underlying distributions are the same, X_{ij} may be thought of as another observation from the distribution of sample l. The event that X_{ij} falls within the median range may be thought of as the event that X_{ij} is the median of the new $n_l + 1$ observation dataset created by adding X_{ij} to sample l. Note that $n_l + 1$ is odd, so the median is uniquely defined as the $\frac{(n_l+1)+1}{2}$ order statistic. By independence, all possible orderings of the $n_l + 1$ observations in the “new” sample are equally likely. Thus X_{ij} is equally likely to be any of the $n_l + 1$ order statistics. This gives X_{ij} a probability of $\frac{1}{n_l+1}$ of being the median, and thus $\frac{1}{n_l+1}$ is the proportion of the comparisons that would result in a zero sign. $\square$

The proportion, $\frac{1}{n+1}$, is just a little less than $\frac{1}{n}$, implying that on average each z_{il} term will have approximately one zero comparison for each calculation where n_l is even. This will reduce the magnitude of the test statistic, even in alternative cases. Once again, while this decision makes the statistic more robust, it will decrease the information gained in alternative settings, thus decreasing the power of the method.

This can be applied to equal or unequal sample sizes. Each sample is compared to the median of all other samples, so that each even sample will have $k - 1$ samples compared to its median range. Odd samples would have no zero comparisons. Thus the overall proportion of zero comparisons could be found by averaging the proportions for each sample size.

Standard Deviations: 1, 1.5, and 2

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
4,4,4	4.84309	24	0.20180	0.20000
6,6,6	5.30468	36	0.14735	0.14286
8,8,8	5.58376	48	0.11633	0.11111
10,10,10	5.76085	60	0.09601	0.09091
12,12,12	5.89402	72	0.08186	0.07692
14,14,14	5.98063	84	0.07120	0.06667
16,16,16	6.07472	96	0.06328	0.05882
18,18,18	6.15094	108	0.05695	0.05263
20,20,20	6.19879	120	0.05166	0.04762
22,22,22	6.22584	132	0.04717	0.04348
24,24,24	6.28302	144	0.04363	0.04000

Table 2.1: Three Normal Distributions

Example 2.1. *If there are three samples with* $n_1 = 5$*,* $n_2 = 6$*, and* $n_3 = 7$*, the expected proportion of zero comparisons would be:*

$$\frac{0 + 12\left(\frac{1}{6+1}\right) + 0}{3(5+6+7)} = \frac{\frac{12}{7}}{3 \cdot 18} = 0.0476$$

Example 2.2. *If there are three samples with* $n_1 = 6$*,* $n_2 = 7$*, and* $n_3 = 8$*, the expected proportion of zero comparisons would be:*

$$\frac{(7+8)\frac{1}{6+1} + 0 + (6+7)\frac{1}{8+1}}{3(6+7+8)} = \frac{\frac{15}{7} + \frac{13}{9}}{3 \cdot 21} = 0.0854$$

Simulations were conducted to study what percentage of all comparisons resulted in a 0 value for the sign when the distributions are not identical under the null hypothesis. Tables 2.1 and 2.2 shows the results for two settings with three normal distributions but differing variances. The average number of zeros is calculated based on counts from 100,000 randomly generated data sets. The "Expected" column represents the proportion that would be expected if the distributions were identical.

The trend for larger samples to result in a larger raw number of zero comparisons, and smaller proportions of zero comparisons, is present in these cases as well, though the numbers were slightly inflated in both

Standard Deviations: 1, 2, and 3

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
4,4,4	4.87044	24	0.20294	0.20000
6,6,6	5.44625	36	0.15128	0.14286
8,8,8	5.83603	48	0.12158	0.11111
10,10,10	6.11881	60	0.10198	0.09091
12,12,12	6.28759	72	0.08733	0.07692
14,14,14	6.45984	84	0.07690	0.06667
16,16,16	6.63583	96	0.06912	0.05882
18,18,18	6.72726	108	0.06229	0.05263
20,20,20	6.82881	120	0.05691	0.04762
22,22,22	6.91730	132	0.05240	0.04348
24,24,24	6.97447	144	0.04843	0.04000

Table 2.2: Three Normal Distributions

of the variance settings for all of the sample sizes. The case with the greater spread of variances showed a higher percentage of zero comparisons than the less drastic case.

2.3.3 Zero Comparisons Under Alternatives

The average number of zero comparisons under alternatives depends on the nature of the distributions considered. If the distributions are symmetric, and unimodal, the number of zero comparisons will decrease on average under alternatives. If the distribution that is shifted is uniform, the average number of zero comparisons will not change unless the shift is drastic enough that the edge of the distribution may be within the median range. If the shifted distribution is not symmetric, the effect of the shift will depend on the direction of the shift. If the distribution is shifted such that values with larger density will fall in the median range, the number of zero comparisons will increase, while a shift that results in the values with lower density being in the median range of the other samples will result in fewer zero comparisons on average.

2.4 Examples to Illustrate the Method

2.4.1 The Phosphorus Content Example

A horticulturist was investigating the phosphorus content of tree leaves from three different varieties of apple trees (1, 2, and 3). Random samples of five leaves from each of the varieties were analyzed for phosphorus content. (Data from Ott (1993).)

Variety 1	Variety 2	Variety 3
.35	.65	.60
.40	.70	.80
.58	.90	.75
.50	.84	.73
.47	.79	.66

Table 2.3: Data for the Phosphorus Content Example

The medians of the three samples are 0.47, 0.79, and 0.73, respectively. Side by side boxplots of the three samples reveal that Variety 1 has a much lower sample median phosphorus content than the other two varieties, which are similarly located (see Figure 2.1). The calculation of the test statistic would be expected to reflect this difference.

The details of the calculation of the statistic are worked out in table 2.4. The -1 in the first cell was found by plugging $i = 1$ and $l = 2$ into the equation $\mathrm{sgn}(X_{i1} - \hat{\theta}_l)$. This yields:

$$\mathrm{sgn}(X_{11} - \hat{\theta}_2) = \mathrm{sgn}(0.35 - 0.79) = \mathrm{sgn}(-0.44) = -1$$

The other values in the table are found in the same way. This results in the value of the test statistic being: $\sum\sum|z_{il}| = 5 + 5 + 5 + 1 + 5 + 3 = 24$. The possible value of the z_{il} terms are: -5, -3, -1, 1, 3, and 5. This implies that the maximum possible value of PS is 30, and the minimum possible is 6, since the absolute value of each z_{il} must be 1, 3, or 5. Our value is much closer to the maximum, thus indicating that at least one of the distributions does not have the same center as the others. This observation will be quantified

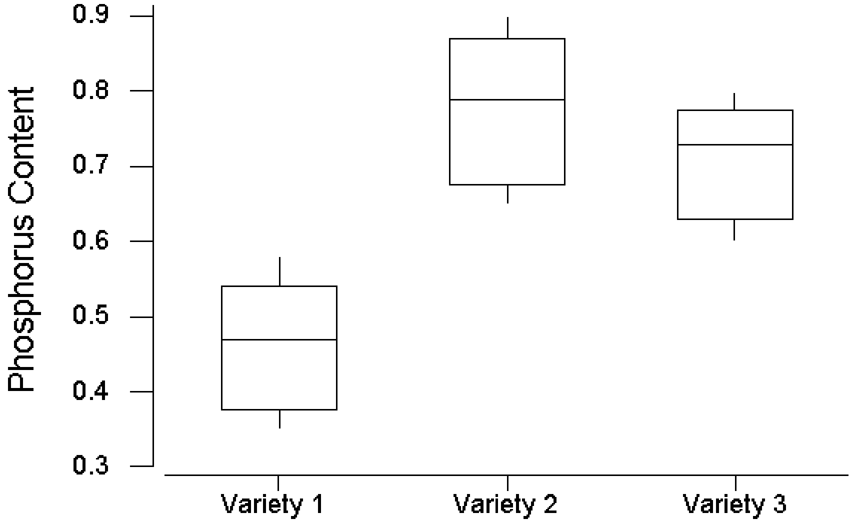

Figure 2.1: Side by Side Boxplots of the Phosphorus Content Data

more directly in section 5.4.1.

Note that comparisons between Variety 1 and either of the other varieties resulted in a positive or negative 5, the strongest evidence possible. The lack of symmetry in the calculation of the statistic can be seen in the comparisons between Varieties 2 and 3. When the data in sample 2 are compared to the median of sample 3, the result was $z_{23} = 1$. When this was reversed, a symmetric method would yield $z_{32} = 1$. However, the calculations above show that $z_{32} = 3$.

2.4.2 The Torque Example

Ryan, Joiner, & Ryan (1985) give measurements for the amount of torque required to turn a log on its axis in order to cut off a thin sheet of wood to be glued with others to produce plywood. The measurements were made at three distinct temperatures: 60° F, 120° F, and 150° F. The data can be found on table 2.5.

Since the sample sizes are all even, each of the samples will have a median range. For the first sample,

	$i = 1,\, l = 2$	$i = 1,\, l = 3$	$i = 2,\, l = 1$	$i = 2,\, l = 3$	$i = 3,\, l = 1$	$i = 3,\, l = 2$
$\text{sgn}(X_{i1} - \hat{\theta}_l)$	-1	-1	1	-1	1	-1
$\text{sgn}(X_{i2} - \hat{\theta}_l)$	-1	-1	1	-1	1	1
$\text{sgn}(X_{i3} - \hat{\theta}_l)$	-1	-1	1	1	1	-1
$\text{sgn}(X_{i4} - \hat{\theta}_l)$	-1	-1	1	1	1	-1
$\text{sgn}(X_{i5} - \hat{\theta}_l)$	-1	-1	1	1	1	-1
z_{il}	-5	-5	5	1	5	-3
$\mid z_{il} \mid$	5	5	5	1	5	3

Table 2.4: Calculation of the PS Statistic for the Phosphorus Content Example

60°	120°	150°
17.30	16.70	15.75
18.05	17.95	16.65
17.40	18.60	15.25
17.40	18.55	15.85
29.55	23.20	22.55
31.50	25.90	22.90
36.75	35.65	28.90
41.20	37.60	35.20

Table 2.5: Data for the Torque Example

the median range is 18.05 to 29.55; for the 120 degree sample, it is 18.60 to 23.20; and for the final sample it was 16.65 to 22.55. A look at side by side boxplots (see Figure 2.2) reveals a slight trend, but certainly not a substantial difference between the three samples. The statistic should indicate some difference, but nothing strong. In section 5.4.2 this is quantified.

The details of the calculation of the statistic are worked out in Table 2.6. The value of the PS statistic here is $0+4+0+4+3+2 = 13$. Since there are median ranges, the absolute value of each z_{il} can take values of 0, 1, 2, 3, 4, 5, 6, 7, or 8. Thus the maximum value of the PS statistic is 48 and the minimum is 0. Our value is closer to the minimum, which suggests only weak evidence of a difference. Once again, a more exact

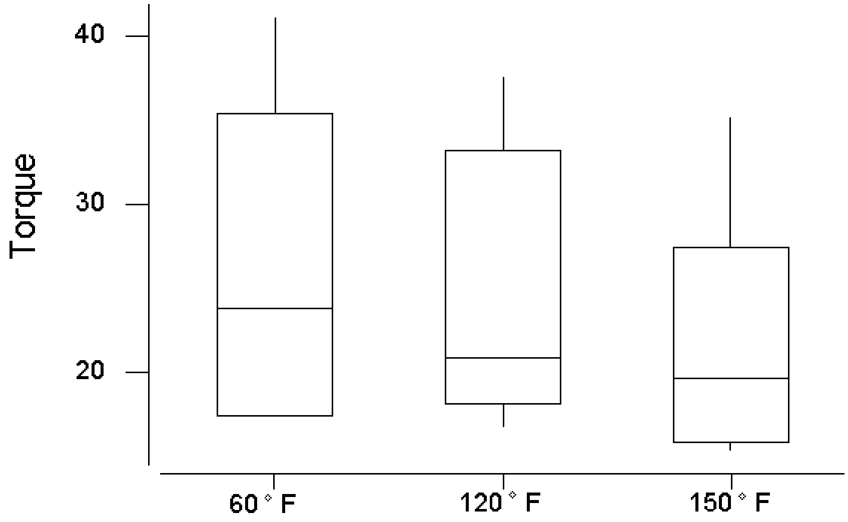

Figure 2.2: Side by Side Boxplots of the Torque Data

conclusion will be made later. A point of interest here is the number of zero comparisons. This example has 17 of the 48 comparisons, or 35.4%, result in a zero. This means that while there are 48 comparisons, only 31 of them result in evidence one way or the other against the null hypothesis.

	$i=1,\ l=2$	$i=1,\ l=3$	$i=2,\ l=1$	$i=2,\ l=3$	$i=3,\ l=1$	$i=3,\ l=2$
$\text{sgn}(X_{i1}-\hat{\theta}_l)$	-1	0	-1	0	-1	-1
$\text{sgn}(X_{i2}-\hat{\theta}_l)$	-1	0	-1	0	-1	-1
$\text{sgn}(X_{i3}-\hat{\theta}_l)$	-1	0	0	0	-1	-1
$\text{sgn}(X_{i4}-\hat{\theta}_l)$	-1	0	0	0	-1	-1
$\text{sgn}(X_{i5}-\hat{\theta}_l)$	1	1	0	1	0	0
$\text{sgn}(X_{i6}-\hat{\theta}_l)$	1	1	0	1	0	0
$\text{sgn}(X_{i7}-\hat{\theta}_l)$	1	1	1	1	0	1
$\text{sgn}(X_{i8}-\hat{\theta}_l)$	1	1	1	1	1	1
z_{il}	0	4	0	4	-3	-2
$\mid z_{il} \mid$	0	4	0	4	3	2

Table 2.6: Calculation of the PS Statistic for the Torque Example

Chapter 3

The Expectation and Variance of the Paired Sign Statistic

3.1 The Practical Usefulness of the Paired Sign Statistic

Nothing discussed to this point allows a statement of significance about the examples in section 2.4. To be useful, the statistic must have some known qualities that reveal whether a particular value of the statistic is to be considered significant at the significance level desired by the user. There are two possible methods by which this may be accomplished. The first method would be the construction of tables covering a variety of sample size combinations and several significance levels such that a user may utilize the tables to look up critical values beyond which the statistic may be considered significant. This is naturally limited, since any table of values is likely to miss sample size combinations of interest to some users.

The second method that can be used to make the statistic useful is a more generally applicable method. If formulae for the mean and variance of the statistic can be developed and made available to the user, then a normal approximation, or some other appropriate approximation, may be possible. The mean and variance may be distributed to the user in the form of tables for common settings of interest, but would be more widely available if a general program were made directly available to any interested user. This would allow the test to be used with any sample size combination for which the normal approximation can be shown to be reasonable. Since this method is more generally applicable, it will be the method of choice if it can be shown to be appropriate. The work in this chapter develops the formulae needed, and a SAS macro is available from the author to perform the calculations for any sample sizes desired for the 3-sample case. A more general macro will soon be available. In Chapter 5 simulations will study whether a normal approximation is appropriate.

3.2 Assumptions of the Theoretical Work

As was mentioned in section 2.3.1, the PS statistic assumes that the data are independent, and come from underlying continuous distributions that differ only in location under the alternatives, and are identical under the null hypothesis.The combinatorial arguments that follow use the assumption that the data must come from distributions that have equal shape under the null hypothesis. The effect of this is that the formulae may give mean and variance values that are not accurate if the statistic is used on data that come from distributions that violate this assumption. This will be studied carefully in the simulation study proposed in Chapter 4.

3.3 The Negative Hypergeometric Distribution

The distribution for the relationship between random choices of balls of two colors, where one is interested only in the number of balls of one color that appear before a certain ball of the other color, is shown in Terrell (1999, section 5.2.3, pages 148–150) to be the negative hypergeometric. The probability mass function of a negative hypergeometric is shown in equation (3.1), where M is the total number of balls for the color being counted, N is the total number of balls for the other color, and n is the number of the ball of the second color where counting will stop. Then X is the random variable defined as the count of the balls of the first color.

$$P(X = x|M, N, n) = f(x) = \frac{\begin{pmatrix} x+n-1 \\ x \end{pmatrix} \begin{pmatrix} M+N-n-x \\ M-x \end{pmatrix}}{\begin{pmatrix} M+N \\ M \end{pmatrix}} \tag{3.1}$$

In terms of the development of the formulae that follow, different "colors" may be considered different samples of interest. This case would be helpful in considering z_{il} by finding the probability that some number of observations in sample i fall below the median of sample l. Since this value is directly associated with z_{il}, this type of distribution will be very helpful in the discussion that follows. However, this is not enough, since only two samples may be considered at a time, and only one median may be considered. Most of the more complicated extensions are developed in Appendix B and referenced in this chapter as needed. Kocherlakota & Kocherlakota (1992, section 6.6) discuss several extensions to the negative hypergeometric, but none that cover the settings needed in this work.

It should be noted here that this distribution does assume that all possible orderings of the balls are equally likely. This is equivalent to assuming that the two underlying distributions from which the data come are equivalent and continuous.

3.4 The Expected Value of the Paired Sign Statistic

The first step will be to establish the expected value of an individual z_{il}, as defined in section 2.3.1. Then it will only be necessary to sum over all possible $\{(i,l) \in \{0,1,\dots k\}$ such that $i \neq l\}$ sets. Considering i and l to be fixed, the densities for the case where n_l is odd and the case where n_l is even can be developed. Since the interest in sample i is in counting observations, and not the location of the median, it is not relevant whether sample i has an odd or even number of observations.

3.4.1 Sample Size Odd

First, consider the case where there are an odd number of observations. The median can be found exactly as the middle observation in the sorted dataset, or $X_{l(\frac{n+1}{2})}$. Define x_i to be the number of observations in sample i that fall below the median of sample l. This situation may be visualized in Figure 3.1, where the median is represented by the dot on the number line that represents possible values of sample l.

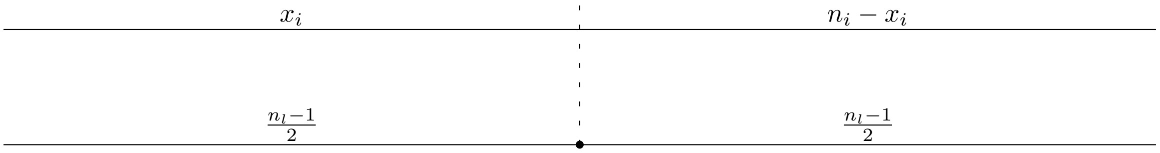

Figure 3.1: Sample l Odd

The relationship between the value of x_i and z_{il}, which is the value of interest, can be summarized as follows.

$$z_{il} = n_i - 2x_i$$

The negative hypergeometric distribution exactly describes this situation, and may be applied in this setting, using the terms defined here, as:

$$f_o(x_i) = \frac{C_1 C_2}{C_T} \tag{3.2}$$

Where:

$$C_1 = \binom{x_i + \frac{n_l - 1}{2}}{x_i}, \; C_2 = \binom{n_i - x_i + \frac{n_l - 1}{2}}{n_i - x_i}, \; C_T = \binom{n_i + n_l}{n_i}$$

Careful inspection reveals that $x_i \in \{0, 1, \ldots, n_i\}$.

3.4.2 Sample Size Even

Finally, consider the case where there are an even number of observations. The median is considered to be the range between $X_{l(\frac{n_l}{2})}$ and $X_{l(\frac{n_l}{2}+1)}$, where $X_{l(j)}$ is the j^{th} order statistic for the l^{th} sample. Define x_i to be the number of observations in sample i that fall below the median range of sample l, and y_i to be the number of observations from sample i that fall within the median range of sample l. The reader may recall that y_i is then the number of observations for which a zero comparison will result. This situation may be visualized in Figure 3.2, where the dots indicate the two middle values.

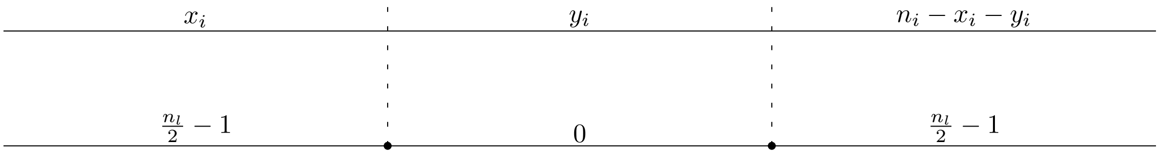

Figure 3.2: Sample l Even

The relationship between the values of x_i and y_i and z_{il} can be summarized as follows.

$$z_{il} = n_i - 2x_i - y_i$$

The negative hypergeometric distribution does not exactly describes this situation. It may be extended and applied in this setting, using the terms defined here, as follows:

$$f_e(x_i, y_i) = \frac{C_1 C_2 C_3}{C_T} \tag{3.3}$$

Where:

$$C_1 = \begin{pmatrix} x_i + \frac{n_l}{2} - 1 \\ \\ x_i \end{pmatrix},\ C_2 = \begin{pmatrix} y_i \\ \\ y_i \end{pmatrix} = 1,\ C_3 = \begin{pmatrix} n_i - x_i - y_i + \frac{n_l}{2} - 1 \\ \\ n_i - x_i - y_i \end{pmatrix}, C_T = \begin{pmatrix} n_i + n_l \\ \\ n_i \end{pmatrix}$$

Careful inspection reveals that in this case $x_i \in \{0, 1, \ldots, n_i\}$ and $y_i \in \{0, 1, \ldots, n_i - x_i\}$.

3.4.3 Combining the Pieces

These two densities may be combined by the following definition.

$$f_{i,l} = \begin{cases} f_o(x_i) & n_l \text{ odd} \\ \\ f_e(x_i, y_i) & n_l \text{ even} \end{cases} \tag{3.4}$$

Where $f_o(x_i)$ and $f_e(x_i, y_i)$ are defined in equations (3.2) and (3.3) respectively. Since the possible values of z_{il} can be written as found in sections 3.4.1 and 3.4.2, the expected value of the absolute value of z_{il} can now be found. The overall expectation, $E(\sum\sum \mid z_{il} \mid) = \sum\sum E(\mid z_{il} \mid)$, can also be found.

3.5 The Variance of the Paired Sign Statistic

This section is concerned with finding the variance of an individual z_{il}. Section 3.6 will be devoted to a discussion of the covariances that are needed to find the overall variance of the PS statistic. According to the definition of variance, the following derivation yields the variance of interest.

$$Var(|z_{il}|) = E(|z_{il}|^2) - E(|z_{il}|)^2 \tag{3.5}$$

In this formula, $f_{i,l}$ would be used, where $f_{i,l}$ is the mass function defined in equation (3.4), for both pieces.

3.6 The Covariances Need for the Variance of the Paired Sign Statistic

The covariances within the formula for z_{il} are zero (see equation (2.2)) since the observations are assumed to be independent. Summing over i and l, however, introduces four distinct types of covariances. The following table lays out the covariances that need to be considered, and the subsection in which each will be considered.

Name	General Form	Subsection
Type I	$Cov(\mid z_{il} \mid, \mid z_{li} \mid)$	3.6.1
Type II	$Cov(\mid z_{il} \mid, \mid z_{il'} \mid)$	3.6.2
Type III	$Cov(\mid z_{il} \mid, \mid z_{i'l} \mid)$	3.6.3
Type IV	$Cov(\mid z_{ih} \mid, \mid z_{hl} \mid)$	3.6.4

Table 3.1: A Table of the Covariances to Be Considered

Each of these cases will require more advanced density functions than the two developed in section 3.4. The derivations follow methods similar to the extension in section 3.4.2. The details can be found in sections B.2 through B.5 in the appendices, and will be referenced in the work that follow.

The only other type of covariance possible is one of the form $Cov(z_{il}, z_{i'l'})$ where the values of i, i', l, and l' are all unique. Since these values are all unique, the independence assumption implies that the covariance would be zero. This implies that moving from three samples to four or more samples will add no further covariance types.

3.6.1 The Type I Covariance

The first extension of interest requires considering the number of observations from sample i that appear before the median of sample l, and the number of observations from sample l that appear before the median of sample i. While this extension involves only two samples, the mass functions developed to this point are not adequate to describe this situation.

In the case where both sample sizes are odd, equation (B.1) defines the needed mass function.

For both sample sizes even, equations (B.2), (B.3), and (B.4) must be combined as follows. (Note that the higher sample for equations (B.2) and (B.4), and the contained sample for equation (B.3) can be called sample i without loss of generality.)

$$f_e^I = \begin{cases} f_e^{Io}(x_i, x_l) & \text{for overlapping median ranges} \\ f_e^{Ic}(x_i, y_i) & \text{for one median range contained within the other} \\ f_e^{Id}(x_i, y_i, x_l, y_l) & \text{for disjoint median ranges} \end{cases}$$

For one sample size odd and the other even, the sample with an odd number of observations will be called sample i, without loss of generality. For this case, equations (B.5), (B.6), and (B.7) will be combined as follows.

$$f_m^I = \begin{cases} f_m^{Ic}(x_i, y_i) & \text{for } \hat{\theta}_i \text{ contained in the median range of sample } l \\ f_m^{Ih}(x_i, y_i, x_l) & \text{for } \hat{\theta}_i \text{ above the median range of sample } l \\ f_m^{Il}(x_i, y_i, x_l) & \text{for } \hat{\theta}_i \text{ below the median range of sample } l \end{cases}$$

Finally, the odd, even, and mixed settings yield the following mass function.

$$f_{i,l}^I = \begin{cases} f_o^I(x_i, x_l) & n_i, n_l \text{ odd} \\ f_e^I & n_i, n_l \text{ even} \\ f_m^I & n_i \text{ odd}, n_l \text{ even} \end{cases} \tag{3.6}$$

Using this equation, and $f_{i,l}(x_i)$ as defined in equation (3.4), $Cov^I(|z_{il}|, |z_{li}|)$ can then be found. By the definition of the covariance:

$$Cov^I(|z_{il}|, |z_{li}|) = E(|z_{il} \cdot z_{li}|) - E(|z_{il}|)E(|z_{li}|) \tag{3.7}$$

The exact values of z_{il} and z_{li} can be found in section B.2.

3.6.2 The Type II Covariance

For this case, interest centers on the number of observations from sample i that appear before the median of sample l, and the number of observations from sample i that appear before the median of sample l'. This case involves three samples, so none of the functions developed to this point are adequate.

In the case where both sample sizes are odd, equation (B.8) defines the needed mass function.

For both sample sizes even, equations (B.9), (B.10), and (B.11) must be combined as follows. (Note that once again the higher sample for equations (B.9) and (B.11), and the contained sample for equation (B.10) can be called sample i without loss of generality.)

$$f_e^{II}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) = \begin{cases} f_e^{IIo}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) & \text{for overlapping median ranges} \\ f_e^{IIc}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) & \text{for one median range contained within the other} \\ f_e^{IId}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) & \text{for disjoint median ranges} \end{cases}$$

For one sample size odd and the other even, the sample with an odd number of observations will be called sample i, without loss of generality. For this case, equations (B.12), (B.13), and (B.14) will be combined as follows.

$$f_m^{II}(x_{Ii}, x_{IIi}, x_{IIIi}) = \begin{cases} f_m^{IIc}(x_{Ii}, x_{IIi}, x_{IIIi}) & \text{for } \hat{\theta}_l \text{ contained in the median range of sample } l' \\ f_m^{IIh}(x_{Ii}, x_{IIi}, x_{IIIi}) & \text{for } \hat{\theta}_l \text{ above the median range of sample } l' \\ f_m^{IIl}(x_{Ii}, x_{IIi}, x_{IIIi}) & \text{for } \hat{\theta}_l \text{ below the median range of sample } l' \end{cases}$$

Combine all of the cases together yields the following general mass function.

$$f^{II}_{i,l,l'} = \begin{cases} f^{II}_o(x_i, y_i) & n_l, n_{l'} \text{ odd} \\ \\ f^{II}_e(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) & n_l, n_{l'} \text{ even} \\ \\ f^{II}_m(x_{Ii}, x_{IIi}, x_{IIIi}) & n_l \text{ odd}, n_{l'} \text{ even} \end{cases} \tag{3.8}$$

Using $f^{II}_{i,l,l'}$ as defined in equation (3.8) and $f_{i,l}(x_i)$ as defined in equation (3.4), $Cov^{II}(|z_{il}|, |z_{il'}|)$ can be found as follows, using the values of z_{il} and $z_{il'}$ as discussed in section B.3.

$$Cov^{II}(|z_{il}|, |z_{il'}|) = E(|z_{il} \cdot z_{il'}|) - E(|z_{il}|)E(|z_{il'}|) \tag{3.9}$$

3.6.3 The Type III Covariance

Here, interest centers on the number of observations from samples i and i' that appear before the median of sample l. While this case, like the previous one, focuses on three samples, the mass functions developed in the previous section cannot be used here.

In this setting, there are only two cases to consider. First, an odd number of observations in sample l. The second case is when there are an even number of observations in sample l. This results, as shown in section B.4, in the mass functions found in equations (B.15) and (B.16) for the odd and even sample size cases, respectively. These cases can then be combined into one general formula.

$$f^{III}_{i,i',l} = \begin{cases} f^{III}_o(x_i, x_{i'}) & n_l \text{ odd} \\ \\ f^{III}_e(x_i, y_i, x_{i'}, y_{i'}) & n_l \text{ even} \end{cases} \tag{3.10}$$

Using $f^{III}_{i,i',l}$ and $f_{i,l}(x_i)$ as defined in equation (3.4), $Cov^{III}(|z_{il}|, |z_{i'l}|)$ can be found as follows, using the values of z_{il} and $z_{i'l}$ as discussed in section B.4.

$$Cov^{III}(|z_{il}|, |z_{i'l}|) = E(|z_{il} \cdot z_{i'l}|) - E(|z_{il}|)E(|z_{i'l}|) \tag{3.11}$$

3.6.4 The Type IV Covariance

In this final case a function must be developed for assigning probabilities when counting the number of observations of sample i that appear before the median of sample h, and the number of observations of sample h that appear before the median of sample l are of interest. Since the interest for sample h is both on the median and counting the observations, none of the previous functions are sufficient to describe this situation.

The first case to be considered is when both samples h and l have an odd number of observations. Equations (B.17) and (B.18) must be combined in the following manner.

$$f_o^{IV}(x_i, y_i, x_h) = \begin{cases} f_o^{IVh}(x_i, y_i, x_h) & \hat{\theta}_h > \hat{\theta}_l \\ f_o^{IVl}(x_i, y_i, x_h) & \hat{\theta}_h < \hat{\theta}_l \end{cases}$$

For both sample sizes even, equations (B.19), (B.20), (B.21), (B.22), (B.23), and (B.24) must be combined as follows.

$$f_e^{IV} = \begin{cases} f_e^{IVho}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h) & \text{for overlapping median ranges with the range for } h \text{ higher} \\ f_e^{IVlo}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h) & \text{for overlapping median ranges with the range for } l \text{ higher} \\ f_e^{IVhc}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h, y_h) & \text{for the median range of } h \text{ contained within the range for } l \\ f_e^{IVlc}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) & \text{for the median range of } l \text{ contained within the range for } h \\ f_e^{IVhd}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h, y_h) & \text{for the median range of } h \text{ entirely above the range for } l \\ f_e^{IVld}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h, y_h) & \text{for the median range of } l \text{ entirely above the range for } h \end{cases}$$

For n_h odd and n_l even, equations (B.25), (B.26), and (B.27) will be combined as follows.

$$f_{oe}^{IV}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h, y_h) = \begin{cases} f_{oe}^{IVc}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h, y_h) & \hat{\theta}_h \text{ contained within the median range of sample } l \\ f_{oe}^{IVh}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h, y_h) & \hat{\theta}_h \text{ above the median range of sample } l \\ f_{oe}^{IVl}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h, y_h) & \hat{\theta}_h \text{ below the median range of sample } l \end{cases}$$

For n_h odd and n_l even, equations (B.28), (B.29), and (B.30) will be combined as follows.

$$f_{eo}^{IV} = \begin{cases} f_{eo}^{IVc}(x_{Ii}, x_{IIi}, x_{IIIi}) & \hat{\theta}_l \text{ contained within the median range of sample } h \\ f_{eo}^{IVl}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h) & \hat{\theta}_l \text{ above the median range of sample } h \\ f_{eo}^{IVh}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h) & \hat{\theta}_l \text{ below the median range of sample } h \end{cases}$$

Combine all of the cases together yields the following general mass function.

$$f_{i,h,l}^{IV} = \begin{cases} f_o^{IV} & \text{both } n_h \text{ and } n_l \text{ odd} \\ f_e^{IV} & \text{both } n_h \text{ and } n_l \text{ even} \\ f_{oe}^{IV} & n_h \text{ odd and } n_l \text{ even} \\ f_{eo}^{IV} & n_h \text{ even and } n_l \text{ odd} \end{cases} \tag{3.12}$$

Following the procedures from the previous subsections in this section, and using $f_{i,h,l}^{IV}$ and $f_{i,l}(x_i)$ as defined in equation (3.4), as well as the exact values for z_{ih} and z_{hl}, which can be found in section B.5, the Type IV covariance is:

$$Cov^{IV}(|z_{hl}|, |z_{ih}|) = E(|z_{hl} \cdot z_{ih}|) - E(|z_{hl}|)E(|z_{ih}|) \tag{3.13}$$

3.7 The Overall Variance of the Paired Sign Statistic

Putting all of these variance and covariance equations together allows the calculation of the overall variance of the test statistic. The $k = 3$ case will be considered as an example, since no additional covariances are introduced for $k \geq 4$.

3.7.1 A Special Case

When $k = 3$ the PS statistic can be rewritten as follows.

$$PS = \sum_{l=1}^{3} \sum_{i \neq l} |z_{il}| \tag{3.14}$$

The following formula gives the overall variance for the 3-sample case.

$$\begin{aligned} Var\left(\sum_{l=1}^{3} \sum_{i \neq l} |z_{il}|\right) =& Var(|z_{12}|) + Var(|z_{13}|) + Var(|z_{21}|) \\ & + Var(|z_{23}|) + Var(|z_{31}|) + Var(|z_{32}|) \\ & + 2 \cdot Cov^{I}(|z_{12}|, |z_{21}|) + 2 \cdot Cov^{I}(|z_{13}|, |z_{31}|) + 2 \cdot Cov^{I}(|z_{23}|, |z_{32}|) \\ & + 2 \cdot Cov^{II}(|z_{12}|, |z_{13}|) + 2 \cdot Cov^{II}(|z_{21}|, |z_{23}|) + 2 \cdot Cov^{II}(|z_{31}|, |z_{32}|) \\ & + 2 \cdot Cov^{III}(|z_{12}|, |z_{32}|) + 2 \cdot Cov^{III}(|z_{13}|, |z_{23}|) + 2 \cdot Cov^{III}(|z_{21}|, |z_{31}|) \\ & + 2 \cdot Cov^{IV}(|z_{12}|, |z_{23}|) + 2 \cdot Cov^{IV}(|z_{13}|, |z_{32}|) + 2 \cdot Cov^{IV}(|z_{21}|, |z_{13}|) \\ & + 2 \cdot Cov^{IV}(|z_{23}|, |z_{31}|) + 2 \cdot Cov^{IV}(|z_{31}|, |z_{12}|) + 2 \cdot Cov^{IV}(|z_{32}|, |z_{21}|) \end{aligned} \tag{3.15}$$

3.8 The Expected Value and Variance for Several Equal Sample Size Settings

The following table gives the expected value and variance for a range of settings for three samples of equal size.

Sample Sizes			Expectation	Variance
3	3	3	10.8000	9.18857
4	4	4	10.6286	23.8200
5	5	5	14.2857	22.9274
6	6	6	14.0649	39.4492
7	7	7	17.1329	38.9526
8	8	8	16.8988	56.5917
9	9	9	19.5919	56.3517
10	10	10	19.3572	74.7371
11	11	11	21.7852	74.6751
12	12	12	21.5545	93.6014
13	13	13	23.7824	93.6688
14	14	14	23.5575	113.010
15	15	15	25.6276	113.175
16	16	16	25.4090	132.846
17	17	17	27.3506	133.087
18	18	18	27.1383	153.030
19	19	19	28.9726	153.332
20	20	20	28.7665	173.504
21	21	21	30.5094	173.855
22	22	22	30.3092	194.223
23	23	23	31.9731	194.615
24	24	24	31.7784	215.153
25	25	25	33.3732	215.579

Table 3.2: Values of the Expectation and Variance

Chapter 4

The Design of the Simulation Study

The work done in the previous chapter has established the expectation and variance under the null hypothesis if the distributions are identical. This does not establish that the null distribution can be approximated by a normal distribution. Since the Paired Sign statistic is by definition discrete, it is unlikely that the normal distribution will accurately approximate the distribution everywhere. Since the interest for testing is in critical values and P-values, in this work interest is in the tails of the distribution. The first topic to be considered through simulations, then, is whether the normal distribution can accurately approximate tail probabilities. It is also of interest to study how the power of the PS test compares to the other existing methods. The following sections lay out the simulation study that will be carried out to investigate the behavior of the PS method in a variety of settings.

4.1 Tests

The proposed method will be compared through simulation studies to the two methods that are most commonly used in practice. When the assumption of normality is valid, and often when it is not, the F-test is the method used. Since this method is used with data that is clearly non-normal, the behavior of this method when the data come from such distributions is still of interest. Knowledgeable users who fear their data may not be normal are likely to use the Kruskal-Wallis method discussed in some detail in section 1.4. When the data appear to come from different underlying distributions, the Kruskal-Wallis test is often used, since the literature offers no alternatives that are appropriate. These two methods will be compared to the Paired Sign method across the settings discussed in this chapter. For the F-test, the exact F distribution will be used to determine whether the test should reject. For the Kruskal-Wallis method, since most tables only contain values up to three samples of size five, the Chi-squared approximation will be used for decision making. For the PS method, cutoff values will be discussed in section 5.1.

4.2 Distributions

4.2.1 Setting 1

The first distributional setting will be chosen to meet the assumptions of the F-test, so that the performance of the other methods may be compared to the F-test, when the F-test is known to be the optimal method. This will allow comparison between the Kruskal-Wallis method and the Paired Sign test with regard to how much power these methods give up. Three standard normals will be used under the null, and only shifts will be added under the alternatives discussed in section 4.5. The density can be seen in Figure 4.1.

4.2.2 Setting 2

In this setting the normality assumption will not be violated, but the variances will be different for each of the datasets. Two cases will be considered. They are outlined in the following table.

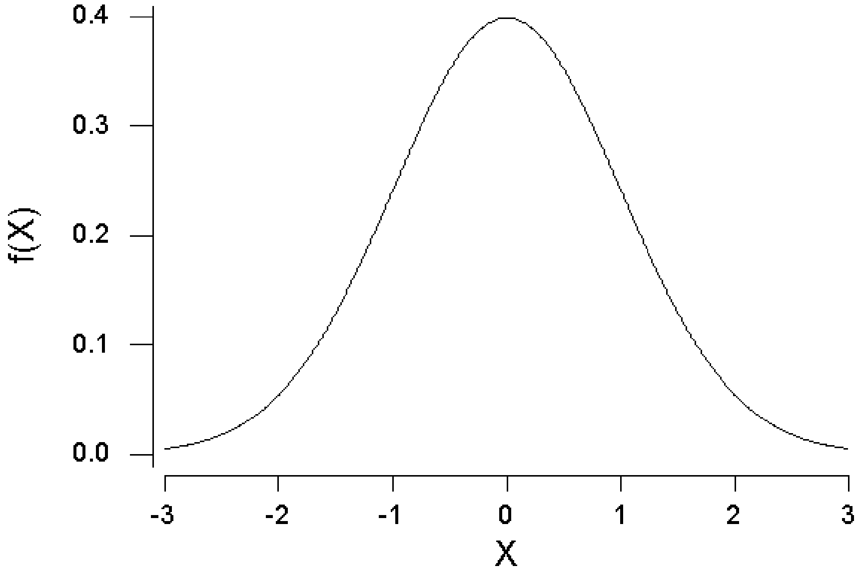

Figure 4.1: Density Function for a Standard Normal

	Standard Deviations
Setting 2a	$\sigma_1 = 1$, $\sigma_2 = 1.5$, and $\sigma_3 = 2$
Setting 2b	$\sigma_1 = 1$, $\sigma_2 = 2$, and $\sigma_3 = 3$

This will allow comparison between different degrees of unequal variances. While the distributions are normal and symmetric, the F-test's assumption of equal variances, and the Kruskal-Wallis and Paired Sign methods' requirement of equal shape, are violated. The results will give an indication of the robustness of the methods to this type of violations of their assumptions.

4.2.3 Setting 3

The third setting will return to identical distributions under the null hypothesis, but will no longer utilize normal distributions. The underlying distributions will now be identical Laplace distributions. This should cause noticeable difficulties with the F-test, and allow direct comparison of the Kruskal-Wallis and Paired Sign methods when both are valid. The F-test should overestimate the within sample variation, since the Laplace distribution has heavier tails than the normal distribution, which will make it harder to detect between sample variation. This implies that the F-test will be overly conservative. Figure 4.2 shows the density function used.

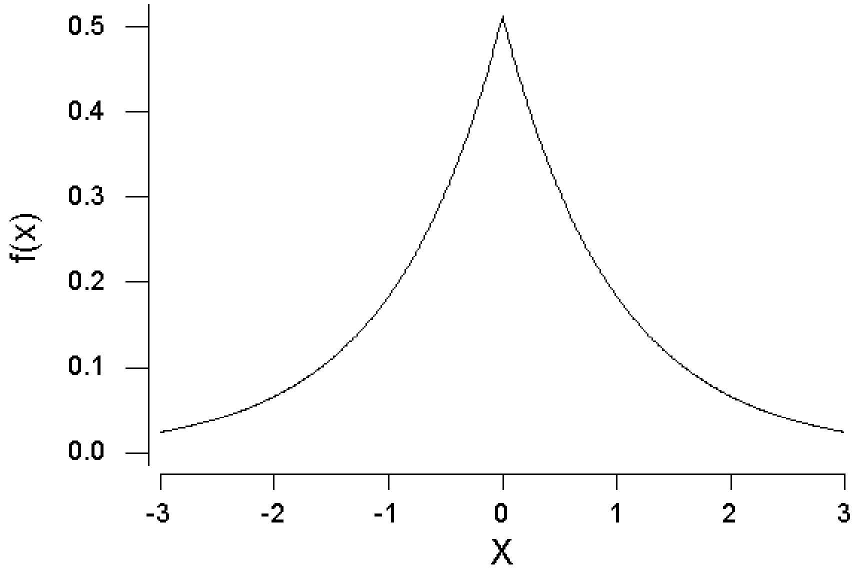

Figure 4.2: Density Function for the Laplace

4.2.4 Setting 4

The final setting will represent the type of data for which the proposed method was developed, though the true assumptions of the paired sign method, and both of the other methods, are violated in this setting. The first sample will be from a Laplace distribution, with the spread controlled by using a scale parameter of 0.973085, which yields an interquartile range (IQR) of 1.34898. This matches the IQR of a standard normal distribution. That is for X_{1j} the density function is:

$$f(x) = \frac{1}{2}(0.973085)e^{-(0.973085)|x|} \tag{4.1}$$

This is the density function shown in Figure 4.2.

The second distribution will be a special case of the generalized Lambda distribution due to Ramberg & Schmeiser (1974) that is developed by Randles, Fligner, Policello & Wolfe (1980). This distribution was chosen because it severely right-skewed, which will violate the assumptions of both the F-test and the Kruskal-Wallis test. Data can be generated using random uniform variates, through the following formula.

$$F^{-1}(u) = \lambda_1 + \frac{(u^{\lambda_3} - (1-u)^{\lambda_4})}{\lambda_2} \tag{4.2}$$

The particular distribution of interest is what Randles et al. (1980) call setting 14. Table 4.1 gives the original values that were used for the four parameters, and the adjusted values that are used to control the center, measured by the median, and spread, using the IQR again, of the distribution. The values used in the simulations yield a median of 0, and an IQR of 1.34898. The adjusted values lead to the density function shown in Figure 4.3.

	λ_1	λ_2	λ_3	λ_4
Original	0	-1.0	-0.0001	-0.17
Adjusted	-0.781515	-0.15993	-0.0001	-0.17

Table 4.1: Original and Adjusted Values for the Generalized Lambda Distribution

The final distribution is a left-skewed normal mixture distribution due to Marron & Wand (1992). Once again, the distribution is adjusted to have median 0 and IQR 1.34898. Table 4.2 gives the weights and exact distributions used. The resulting density is given in Figure 4.4.

To aid in the comparison of the distributions, Figure 4.5 shows an overlay of the cumulative distribution functions for the three distributions in this setting. Note that the Generalized Lambda distribution appears stochastically larger than the other two distributions. It is not actually stochastically larger, as the cdf of

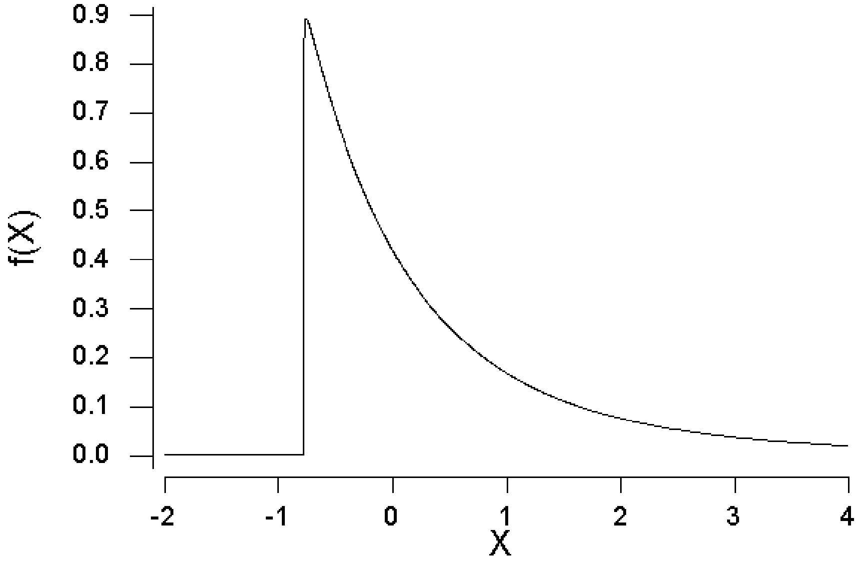

Figure 4.3: Density Function for the Generalized Lambda

the Laplace distribution actually does cross under that of the Generalized Lambda briefly when X is just below 0. These distributions should still result in higher power when the second sample is moved than when the other samples are shifted. The proportion of zero counts for even sample sizes in this setting has been studied through simulations. The results, found on Table A.3, indicate that the proportion of zeros is significantly lower in this setting than would be expected if the distributions were identical.

4.3 Levels

Three common significance levels, 0.10, 0.05, and 0.01, will be considered. The three levels were chosen because they are commonly used in practice, and provide data for a reasonable range of values which will reveal the effect of the various factors on each test across the significance levels.

Weight	Mean	Standard Deviation
0.2	-0.839838	1.46155
0.2	-0.339838	0.97437
0.6	0.243495	0.81197

Table 4.2: The Distributions in the Normal Mixture

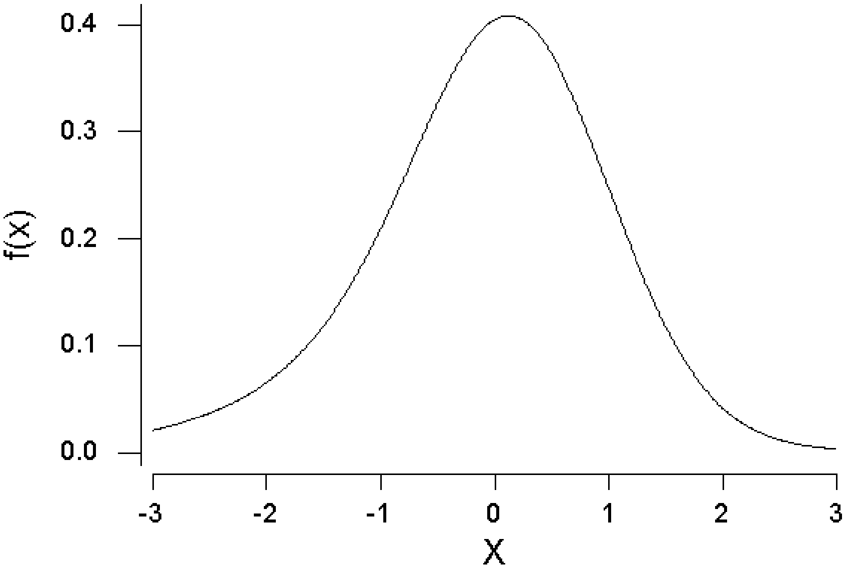

Figure 4.4: Density Function for the Normal Mixture

4.4 Sample Sizes

Since the central limit theorem indicates that for large samples normal theory methods are less affected by violations of their assumptions, interest here centers on the behavior of the methods of interest for a range of small sample sizes. Two types of settings will be considered: equal and unequal sample sizes.

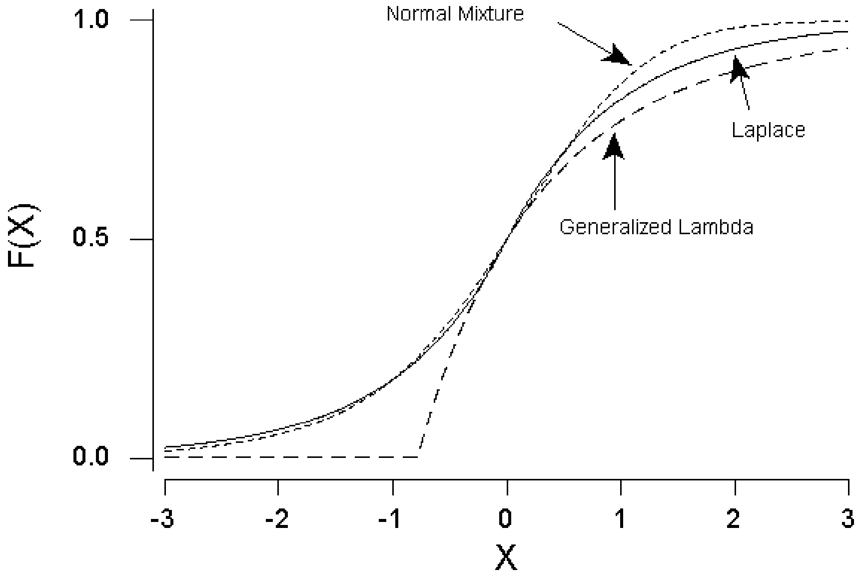

Figure 4.5: Overlay of the Cumulative Distribution Functions

4.4.1 Equal Sample Size Settings

Since small sample sizes are of primary interest, therefore equal sample sizes from 5 through 10 will be considered. The simulations will enable a discussion of the properties of the statistic for both very small samples and moderate sample sizes.

4.4.2 Unequal Sample Size Settings

It is not always possible, or practical, for users to have equal sample sizes. Also, studies designed to have equal sample sizes may have dropouts or other difficulties that result in unequal sample sizes despite the plans. For this reason several unequal sample size settings were chosen. Two were chosen such that there were two odd samples and one even, and two that have one odd sample and two even samples. This will allow the effect of mixtures of odd and even sample sizes to be investigated. Also, research may or may not result in unequal sample sizes that are similar. For each of the odd and even mixtures both similar and dissimilar settings were selected. The following four choices will be used in the studies:

- 5, 6, and 7
- 6, 7, and 8
- 5, 8, and 11
- 6, 9, and 12

For cases where the three distributions are not equal, all possible arrangements will be used. The results will then be given for each possible arrangement, and averaged over all cases, since it is unlikely that the practitioner would know which sample would have each type of distribution.

Also of interest is the count of zero comparisons in each case. For equal underlying distributions, this has been discussed in section 2.3.2. For unequal underlying distributions, the results in section A.2 reveal results similar to those for equal sample sizes discussed earlier. The case with three normals with different variances has elevated zero counts, when averaged across all of the permutations of sample sizes within each combination. Closer inspection reveals that for the minimum number of zero comparisons, the even sample or samples should get the smallest variance, or variances. If there are two even samples, the smallest variance should go to the smaller of the even samples, and the next smallest variance should get the larger of the even sample sizes. The largest proportion of zero counts reverses these trends. For one even sample, the smallest variance should be the largest of the samples with an odd number of observations, and the largest variance should be for the distribution underlying the even sample. For two even samples, the odd sample should come from the smallest variance population, and the smallest even sample should come from the distribution with the largest variance.

The three different distributions case also retains the features of the equal sample size settings, namely the proportion remains below the expected number of zero counts, when averaged over the different arrangements. For the smallest proportion, when there is one even sample, the smallest odd sample should come from the Laplace distribution, the larger odd sample from the generalized Lambda, and the even sample from the normal mixture. For the highest proportion, the larger odd sample should come from the normal mixture, and the even sample should be generalized Lambda data. For the cases with two even distributions, the minimum proportion of zero comparisons is obtained when the smaller even sample comes from the normal mixture, and the other even sample comes from the generalized Lambda. This implies that the odd sample must come from the Laplace distribution. For the maximum proportion, the smaller even sample should come from the Laplace distribution, and the odd sample should come the normal mixture. Thus the larger even sample should remain generalized Lambda.

4.4.3 Expected Value and Variance for the Settings of Interest

The following table gives the expected value and variance for the sample size settings that will be considered in the simulation studies. These will be used in the calculation of the approximations discussed in Chapter 5.

Sample Sizes			Expectation	Variance
5	5	5	14.2857	22.9274
6	6	6	14.0649	39.4492
7	7	7	17.1329	38.9526
8	8	8	16.8988	56.5917
9	9	9	19.5919	56.3517
10	10	10	19.3572	74.7371
5	6	7	15.1997	36.1434
6	7	8	15.9408	43.2770
5	8	11	18.1953	55.0646
6	9	12	18.7038	61.0470

Table 4.3: Values of the Expectation and Variance for the Simulations

4.5 True Locations

In addition to the null case where all the medians will be the same, several alternatives will be considered. The cases considered will enable the consideration of the performance of the statistics to be compared against a range of true differences in location. The alternatives considered are given in table 4.4. Comparing results for the (0,0,1) case and the (0,0,2) case will give a sense of how the statistics improve as the difference in medians becomes greater. The (0,1,2) case will reveal how well the statistics can detect differences when all three distributions have different locations. For settings where the underlying distributions differ, that is in Settings 2a and 2b and in Setting 4, all possible arrangements of the medians will be considered. Table 4.5 outlines the possible alternatives, and shows the cases in which each will be considered.

	Medians		
Null	0	0	0
Alternative 1	0	0	1
Alternative 2	0	0	2
Alternative 3	0	1	2

Table 4.4: The True Locations Considered

Medians	Setting 1	Setting 2 (both)	Setting 3	Setting 4
0,0,1	√	√	√	√
0,1,0		√		√
1,0,0		√		√
0,0,2	√	√	√	√
0,2,0		√		√
2,0,0		√		√
0,1,2	√	√	√	√
0,2,1		√		√
1,0,2		√		√
1,2,0		√		√
2,0,1		√		√
2,1,0		√		√

Table 4.5: Alternatives Considered in Each Setting

Chapter 5

The Null Distribution

5.1 Two Approximations

The most commonly used approximation is the normal approximation. This approach would utilize the mean and variance of the statistic to adjust selected critical values from the normal distribution. Since differences in location result in larger values of the PS statistic, upper tailed critical values would be used. For instance, the point from a standard normal distribution that would have 5% above it is 1.6449. This point would be adjusted for the setting of interest by multiplying by the standard deviation and adding the mean. The resulting value would be the cutoff for significant results. Values of the PS statistic above this point would lead to the conclusion that the samples come from distributions that differ in location.

Another approach, which would be more conservative, is to use what might be called a “half-normal” approximation, though it does not make use of the half-normal distribution. Since the construction of the statistic involves the absolute value, it may be conjectured that the distribution that results from the summing over all samples of the z_{il} pieces results in a distribution whose upper tail would be considerably heavier than that estimated by the normal distribution. For this reason, it seemed reasonable to consider whether two-tailed critical values from a normal distribution might give better estimates of the true cutoff points, despite the fact that interest for the PS statistic is in only the upper tail. Since interest centers on levels of 0.01, 0.05, and 0.10, upper tail normal critical values for 0.005, 0.025, and 0.05, respectively, were used to estimate the needed critical values.

The areas of interest for application of this method are primarily small sample size settings. For larger sample sizes, random sampling ensures independence between observations, and allows for the central limit theorem (CLT) to be invoked, suggesting the that normal theory methods becomes approximate even for nonnormal data. This leads to the general application of the F-test for larger samples. Also, while assymptotic results are desirable, the correlation of the terms made central limit theorem type arguments for normality, or half-normality, intractable. For this reason, the effectiveness of the two approximations will be evaluated empirically. 100,000 random data sets from each of the distributional settings discussed in section 4.2 were generated under the null hypothesis that the medians were equal. For each data set, the value of the statistic was calculated. This allows for the construction of empirical critical values which may be compared to those found by the two approximations discussed above. The approximations will first be compared to the two cases where the distributions meet the additional assumption that was necessary for the mean and variance calculations, namely that the distributions are identical. Tables 5.1 through 5.3 show the approximate and empirical cutoff values, enabling an evaluation of the effectiveness of the approximations.

5.1.1 Calculating the Approximate Critical Values

The approximation values were found by calculating the value based on the approximation, and then recording the next larger possible value. For instance, for three samples of size five, the 0.10 critical value for a standard normal distribution was found to be 1.2816. This was multiplied by the appropriate standard deviation, 4.788 (see Table 3.2), and then added to the mean, 14.2857 (again, see Table 3.2). This resulted in an approximate cutoff of 20.4223. Since this implies that the null hypothesis should be rejected if the PS statistic takes a value larger than 20.4223, and the first possible value larger than 20.4223 is 22 in this case, 22 was recorded in the table.

5.1.2 Calculating the Empirical Critical Values

In each of the settings, 100,000 values of the test statistic were generated, with the null hypothesis true. That is, the underlying distributions were controlled such that the medians were equal. The empirical value was then calculated by looking at output from Proc Freq in SAS. Since large values of the statistic indicate differences in location, cutoff values were chosen such that the proportion of values of the test statistic that were at or above the cutoff value was below the specified significance level. For instance, the 0.10 level cutoff value for three samples of size five each in Setting 1 was found to be 22, since the cumulative proportion 22 or greater was 0.0899. The cumulative proportion for 20, the next lower possible value, was 0.1747, which is too large.

5.1.3 Comparison of the Critical Values

It is apparent from the values in these tables, that there is not one method of approximation which is always closer to the empirical cuttoff value than the other. Also, neither approximation does a good job of estimating the empirical cutoff values, even for the larger sample sizes. These tables indicate that the approximations suggested are not effective ways of determining critical values, and suggest that the approximations would also not be useful for estimating P-values.

Due to the distribution free nature of the PS method when the distributions are equal under the null hypothesis (see the final paragraph in section 2.3.1) the cutoff values calculated using the normal distribution may be referred to as identical case cutoff values. These values may be used whenever the

	Approximation		Empirical
	Normal	"Half-Normal"	Identical
5,5,5	22 (0.0899)	24 (0.0377)	22 (0.0899)
6,6,6	23 (0.0949)	25 (0.0492)	23 (0.0949)
7,7,7	26 (0.1168)	28 (0.0669)	28 (0.0669)
8,8,8	27 (0.1060)	30 (0.0507)	28 (0.0855)
9,9,9	30 (0.1182)	32 (0.0758)	32 (0.0758)
10,10,10	31 (0.1065)	34 (0.0568)	32 (0.0878)
5,6,7	23 (0.1114)	26 (0.0452)	24 (0.0900)
6,7,8	25 (0.1139)	27 (0.0658)	26 (0.0894)
5,8,11	28 (0.1183)	31 (0.0513)	29 (0.0880)
6,9,12	29 (0.1226)	32 (0.0652)	31 (0.0807)

Table 5.1: Approximate and Empirical Cutoff Values at the 0.10 Level

	Approximation		Empirical
	Normal	"Half-Normal"	Identical
5,5,5	24 (0.0377)	24 (0.0377)	24 (0.0377)
6,6,6	25 (0.0492)	27 (0.0219)	25 (0.0492)
7,7,7	28 (0.0669)	30 (0.0340)	30 (0.0340)
8,8,8	30 (0.0507)	32 (0.0277)	31 (0.0376)
9,9,9	32 (0.0758)	36 (0.0252)	34 (0.0455)
10,10,10	34 (0.0568)	37 (0.0262)	35 (0.0438)
5,6,7	26 (0.0452)	27 (0.0259)	26 (0.0452)
6,7,8	27 (0.0658)	29 (0.0344)	28 (0.0500)
5,8,11	31 (0.0513)	33 (0.0282)	32 (0.0423)
6,9,12	32 (0.0652)	35 (0.0289)	34 (0.0393)

Table 5.2: Approximate and Empirical Cutoff Values at the 0.05 Level

	Approximation		Empirical
	Normal	"Half-Normal"	Identical
5,5,5	26 (0.0121)	28 (0.0026)	28 (0.0026)
6,6,6	29 (0.0078)	31 (0.0023)	29 (0.0078)
7,7,7	32 (0.0150)	34 (0.0056)	34 (0.0056)
8,8,8	35 (0.0089)	37 (0.0034)	35 (0.0089)
9,9,9	38 (0.0128)	40 (0.0059)	40 (0.0059)
10,10,10	40 (0.0109)	42 (0.0056)	41 (0.0078)
5,6,7	30 (0.0066)	31 (0.0026)	29 (0.0094)
6,7,8	32 (0.0107)	33 (0.0063)	33 (0.0063)
5,8,11	36 (0.0107)	38 (0.0045)	37 (0.0057)
6,9,12	37 (0.0152)	39 (0.0073)	39 (0.0073)

Table 5.3: Approximate and Empirical Cutoff Values at the 0.01 Level

underlying distributions are identical under the null hypothesis, regardless of what the common distribution may be. The behavior of the statistic in other distributional settings is yet to be considered.

5.2 Critical Values Across Settings

As previously mentioned in section 5.1.2, for each of these settings, 100,000 values of the test statistic were generated, with the null hypothesis true. That is, the underlying distributions were controlled such that the medians were equal. The empirical value was then calculated by looking at output from Proc Freq in SAS. It should also be noted in the cases that follow that this is a theoretical exercise, since users of the procedure would not know, in practice, the exact distributions underlying their data. The study of the cutoff values is intended to gain some perspective on how different the empirical cutoff values would be if the truth were known to the user and correct values were calculated.

Tables 5.4 through 5.9 will allow for comparison of the empirical cutoffs from the identical distribution

	Empirical			
	Identical	Setting 2a	Setting 2b	Setting 4
5,5,5	22 (0.0899)	24 (0.0490)	24 (0.0630)	26 (0.0529)
6,6,6	23 (0.0949)	24 (0.0917)	25 (0.0851)	28 (0.0777)
7,7,7	28 (0.0669)	28 (0.0879)	30 (0.0666)	32 (0.0990)
8,8,8	28 (0.0855)	29 (0.0880)	30 (0.0946)	35 (0.0840)
9,9,9	32 (0.0758)	34 (0.0654)	34 (0.0936)	40 (0.0764)
10,10,10	32 (0.0878)	33 (0.0991)	35 (0.0927)	42 (0.0847)

Table 5.4: Empirical Cutoff Values at the 0.10 Level

settings, Setting 1 and Setting 3, to the other settings, Setting 2a, Setting 2b, and Setting 4.

Several interesting trends can be noted. First, when comparing Settings 2a and 2b to the identical underlying distribution case shows an increase in the cutoff value from the identical case to Setting 2a, and from Setting 2a to setting 2b. This indicates that the levels seen in simulations should be higher in Settings 2a and 2b than for the identical distributions cases, with Setting 2b being the highest. This does not mean that the level will be higher than stated for all sample sizes. The few sample sizes where the cutoffs are the same will have higher levels than in the identical distributions case, but the levels will still be at or below the stated level.

In Setting 4, the cutoff values differ from the identical case values even more dramatically than the values in Settings 2a and 2b. This is especially true for the larger sample sizes. The most notable example is that for three samples of size five, the 0.01 level empirical cutoff value for Setting 4 is 30, the maximum possible value of the statistic. Thus, the null hypothesis would only be rejected if the PS statistic took on the most extreme value, if a true 0.01 level test is constructed. This indicates that use of the identical cases to determine cutoff values will result in a severely inflated level when the distributions are as different as the distributions in Setting 4. This will be verified in the null distribution simulations. All of the simulations that follow use the identical case cutoff values, unless otherwise indicated.

	Empirical			
	Identical	Setting 2a	Setting 2b	Setting 4
5,6,7	24 (0.0900)	24 (0.0918)	25 (0.0707)	29 (0.0757)
5,7,6	–	24 (0.0953)	25 (0.0792)	29 (0.0630)
6,5,7	–	25 (0.0704)	25 (0.0916)	29 (0.0682)
6,7,5	–	25 (0.0865)	26 (0.0949)	27 (0.0971)
7,5,6	–	25 (0.0863)	26 (0.0951)	28 (0.0994)
7,6,5	–	25 (0.0945)	27 (0.0696)	27 (0.0946)
6,7,8	26 (0.0894)	26 (0.0984)	27 (0.0911)	32 (0.0944)
6,8,7	–	27 (0.0789)	27 (0.1000)	32 (0.0784)
7,6,8	–	27 (0.0771)	27 (0.0981)	32 (0.0969)
7,8,6	–	27 (0.0898)	28 (0.0928)	31 (0.0880)
8,6,7	–	27 (0.0992)	29 (0.0806)	31 (0.0948)
8,7,6	–	28 (0.0816)	29 (0.0875)	31 (0.0811)
5,8,11	29 (0.0880)	28 (0.0886)	28 (0.0957)	38 (0.0953)
5,11,8	–	29 (0.0791)	29 (0.0884)	37 (0.0726)
8,5,11	–	29 (0.0954)	31 (0.0755)	37 (0.0928)
8,11,5	–	31 (0.0890)	33 (0.0757)	33 (0.0967)
11,5,8	–	31 (0.0957)	33 (0.0948)	35 (0.0970)
11,8,5	–	32 (0.0998)	34 (0.0968)	33 (0.0927)
6,9,12	31 (0.0807)	30 (0.0870)	30 (0.0989)	41 (0.0857)
6,12,9	–	31 (0.0840)	32 (0.0844)	39 (0.0910)
9,6,12	–	31 (0.0875)	32 (0.0943)	41 (0.0784)
9,12,6	–	33 (0.0819)	34 (0.0931)	37 (0.0850)
12,6,9	–	33 (0.0947)	35 (0.0994)	39 (0.0756)
12,9,6	–	34 (0.0890)	36 (0.0979)	37 (0.0771)

Table 5.5: Empirical Cutoff Values at the 0.10 Level

	Empirical			
	Identical	Setting 2a	Setting 2b	Setting 4
5,5,5	24 (0.0377)	24 (0.0490)	26 (0.0237)	28 (0.0141)
6,6,6	25 (0.0492)	26 (0.0483)	27 (0.0437)	29 (0.0497)
7,7,7	30 (0.0340)	30 (0.0477)	32 (0.0344)	34 (0.0491)
8,8,8	31 (0.0376)	32 (0.0411)	33 (0.0433)	37 (0.0445)
9,9,9	34 (0.0455)	36 (0.0390)	38 (0.0349)	42 (0.0415)
10,10,10	35 (0.0438)	37 (0.0419)	39 (0.0405)	45 (0.0362)

Table 5.6: Empirical Cutoff Values at the 0.05 Level

5.2.1 Critical Values for Other Sample Sizes

If critical values are desired for other sample size combinations not given in this work, a SAS macro is available from the author that will generate empirical cutoff values, under the assumption that the data come from identical distributions.

5.3 Comparison of Methods in the Null Case

5.3.1 Setting 1: Three Standard Normal Distributions

Based on the results found in Table C.2 (page 119), it can be seen that in this setting the F-test is consistently around the correct level, which is to be expected since the distribution of the F-test is exact for normal data with equal variances. The Kruskal-Wallis method, and the PS method are both conservative. The PS statistic is quite conservative in some settings, though in several cases the method has a higher level than the Kruskal-Wallis method. For example, in the $\alpha = 0.01$ level case where there are samples of sizes 5, 6, and 7, the PS statistic has a level of 0.0094 while the level of the Kruskal-Wallis method is only 0.0041 in this setting. The conservative nature of the PS test is a result of the discrete nature of the statistic. In every

	Empirical			
	Identical	Setting 2a	Setting 2b	Setting 4
5,6,7	26 (0.0452)	26 (0.0468)	27 (0.0332)	31 (0.0299)
5,7,6	–	26 (0.0496)	27 (0.0394)	31 (0.0244)
6,5,7	–	27 (0.0338)	27 (0.0455)	31 (0.0254)
6,7,5	–	27 (0.0425)	28 (0.0465)	29 (0.0445)
7,5,6	–	27 (0.0420)	29 (0.0285)	30 (0.0429)
7,6,5	–	27 (0.0472)	29 (0.0322)	29 (0.0420)
6,7,8	28 (0.0500)	29 (0.0393)	30 (0.0391)	34 (0.0479)
6,8,7	–	29 (0.0427)	30 (0.0439)	34 (0.0385)
7,6,8	–	29 (0.0430)	30 (0.0435)	34 (0.0469)
7,8,6	–	29 (0.0499)	31 (0.0383)	33 (0.0453)
8,6,7	–	30 (0.0429)	31 (0.0450)	33 (0.0472)
8,7,6	–	30 (0.0452)	31 (0.0495)	33 (0.0390)
5,8,11	32 (0.0423)	31 (0.0366)	31 (0.0404)	41 (0.0298)
5,11,8	–	31 (0.0461)	32 (0.0439)	39 (0.0412)
8,5,11	–	32 (0.0481)	33 (0.0440)	39 (0.0462)
8,11,5	–	34 (0.0433)	35 (0.0429)	36 (0.0447)
11,5,8	–	34 (0.0484)	36 (0.0473)	37 (0.0497)
11,8,5	–	35 (0.0413)	37 (0.0396)	36 (0.0410)
6,9,12	34 (0.0393)	33 (0.0411)	33 (0.0488)	43 (0.0496)
6,12,9	–	34 (0.0418)	35 (0.0387)	42 (0.0428)
9,6,12	–	34 (0.0438)	35 (0.0467)	43 (0.0403)
9,12,6	–	36 (0.0405)	37 (0.0461)	40 (0.0408)
12,6,9	–	36 (0.0496)	39 (0.0381)	41 (0.0401)
12,9,6	–	37 (0.0448)	39 (0.0481)	39 (0.0429)

Table 5.7: Empirical Cutoff Values at the 0.05 Level

	Empirical			
	Identical	Setting 2a	Setting 2b	Setting 4
5,5,5	28 (0.0026)	28 (0.0043)	28 (0.0064)	30 (0.0018)
6,6,6	29 (0.0078)	30 (0.0078)	31 (0.0070)	33 (0.0045)
7,7,7	34 (0.0056)	34 (0.0097)	36 (0.0063)	38 (0.0062)
8,8,8	35 (0.0089)	37 (0.0067)	38 (0.0082)	41 (0.0074)
9,9,9	40 (0.0059)	42 (0.0054)	44 (0.0051)	46 (0.0084)
10,10,10	41 (0.0078)	43 (0.0076)	45 (0.0071)	49 (0.0088)

Table 5.8: Empirical Cutoff Values at the 0.01 Level

case, if a lower cutoff value were used, the statistic would not maintain the stated level. It is of interest to note that the level is always lower in cases where three odd samples are considered than in adjacent cases where three even samples were obtained. The effect of sample size is not clear in this setting, since the choice to have even or odd sample sizes affects the level dramatically. For instance, at the $\alpha = 0.01$ level, the true level for three samples of size eight each (0.0093) is higher than that for three samples of size nine (0.0060). This is once again a factor of the discrete nature of the statistic.

5.3.2 Settings 2a and 2b: Three Normal Distributions with Different Variances

Tables D.2 and D.3, for Setting 2a, and D.29 and D.30, for Setting 2b, reveal how differing variances affect the statistics. (See pages 126 and 127, and 153 and 154, respectively.) The F-test almost always exceeds the stated level. The exceptions are in the unequal sample size cases where the sample with the largest number of observations comes from the distribution with the largest variance. The Kruskal-Wallis method is conservative for small sample sizes and at the 0.01 level, but for larger sample sizes and higher levels can demonstrate a level that is quite elevated.

In Setting 2a, the Paired Sign test remains slightly conservative only in equal sample size situations where the three samples are either of size five or seven, and the lower two levels are considered. In general,

	Empirical			
	Identical	Setting 2a	Setting 2b	Setting 4
5,6,7	29 (0.0094)	30 (0.0077)	30 (0.0099)	33 (0.0080)
5,7,6	–	30 (0.0086)	31 (0.0053)	33 (0.0060)
6,5,7	–	30 (0.0098)	31 (0.0073)	33 (0.0061)
6,7,5	–	31 (0.0058)	31 (0.0089)	33 (0.0039)
7,5,6	–	31 (0.0063)	32 (0.0084)	33 (0.0046)
7,6,5	–	31 (0.0069)	32 (0.0083)	32 (0.0094)
6,7,8	33 (0.0063)	33 (0.0079)	34 (0.0080)	38 (0.0064)
6,8,7	–	33 (0.0085)	34 (0.0097)	37 (0.0082)
7,6,8	–	33 (0.0095)	35 (0.0059)	37 (0.0085)
7,8,6	–	34 (0.0075)	35 (0.0071)	37 (0.0056)
8,6,7	–	34 (0.0089)	35 (0.0089)	37 (0.0054)
8,7,6	–	34 (0.0098)	36 (0.0070)	36 (0.0099)
5,8,11	37 (0.0057)	35 (0.0087)	35 (0.0098)	44 (0.0088)
5,11,8	–	36 (0.0094)	37 (0.0064)	43 (0.0072)
8,5,11	–	37 (0.0076)	38 (0.0095)	43 (0.0062)
8,11,5	–	39 (0.0057)	39 (0.0096)	41 (0.0051)
11,5,8	–	39 (0.0073)	41 (0.0068)	41 (0.0088)
11,8,5	–	40 (0.0075)	41 (0.0083)	40 (0.0073)
6,9,12	39 (0.0073)	38 (0.0094)	39 (0.0077)	48 (0.0082)
6,12,9	–	39 (0.0081)	40 (0.0084)	47 (0.0068)
9,6,12	–	39 (0.0098)	41 (0.0076)	47 (0.0063)
9,12,6	–	41 (0.0083)	42 (0.0099)	45 (0.0063)
12,6,9	–	42 (0.0083)	44 (0.0081)	45 (0.0076)
12,9,6	–	43 (0.0067)	45 (0.0064)	43 (0.0092)

Table 5.9: Empirical Cutoff Values at the 0.01 Level

the levels are elevated beyond the desired goals. The performance of the method is no better, in general, than the performance of the F-test. In some cases the F-test is much closer to the desired level, while in others the PS method is clearly preferable. When the sample sizes are allowed to differ, some more interesting trends emerge. For the two smaller sample size choices, the PS method generally has level higher than the other two methods. For the larger sample sizes, the level of the PS method is between that of the F-test and the Kruskal-Wallis method for the smaller two levels, but is higher again at the 0.10 level.

In Setting 2b the method clearly violates its level in all of the cases except when the three samples were all of size five, and the lowest level was used. For unequal sample sizes, the PS method has a true average level between the other two methods in only two cases, when the samples are of sizes six, seven, and eight, and when the sizes of the samples are five, eight, and eleven. Within the permutations of the differing sample sizes, the PS method varies widely. In some arrangements, the PS is much better than the F-test, while a similar setting shows that the F-test is to be preferred. In this more extreme setting, the PS method consistently has the highest levels of the three methods, which indicates that the PS test should not be conducted with the identical case cutoff values in this type of setting.

Generally, the Kruskal-Wallis test is the best method for Setting 2a. In Setting 2b, the Kruskal-Wallis method emerges even more clearly as the best of the three methods. In the cases where the difference in variances is more extreme, the PS method tends to have a higher level than either of the other tests.

While there are some violations of the stated level in this setting, no current nonparametric method is designed for this setting. Ideally Welch's ANOVA would be used for this type of data, however, the F-test and Kruskal-Wallis method are more often used, despite the lack of validity for both of these methods. For this reason, the simulations considering alternatives to the null will continue the use of the cutoff values determined from the identical distribution case.

5.3.3 Setting 3: Three Laplace Distributions

The behavior of the PS test and the Kruskal-Wallis method in this setting, as seen in Table E.2 (page 180), is identical, except for slight differences due to different simulated data sets, to that in Setting 1 (three standard normal distributions). The F-test, however, is consistently more conservative than it was in Setting 1. This is caused by the fact that the normal distribution has lighter tails than the Laplace. This causes the F-test trouble because there is more within sample variability in this data than would be expected for a sample from a normal population, making it harder to detect differences in location. It should be noted that at the 0.01 level, the F-test still is in general the closest method to the desired level, though the PS method is closest for even sample size settings and the smallest unequal sample size setting. At the 0.05 level, the F-test and the Kruskal-Wallis procedure have essentially the same level, though the Kruskal-Wallis is usually slightly higher. When the 0.10 level is considered, the Kruskal-Wallis method is consistently closer to the stated level than the other methods. The PS method is usually most conservative method for the higher two levels, due to the discrete nature of the statistic.

5.3.4 Setting 4: Three Different Distributions

In this setting, the behavior of all three methods is very poor. None of the three methods are able to maintain any of the desired levels. Since none of the methods is able to maintain the stated level, none are valid. (See Tables F.2 and F.3, pages 187 and 188) It is quite clear that none of the methods should be used in this setting. Interestingly, for the larger levels, the Kruskal-Wallis method has a higher level than either of the other methods. While the method maintains its level well in Settings 2a and 2b, it fails to maintain its level here. The PS test should not be used with the identical case cutoff values either. However, in practice, it is rarely known whether the underlying populations differ so severely, and the usual methods are often used anyway. Thus the simulations of alternative cases will still be run. Interestingly, the F-test consistently has the lowest true level, especially at the higher levels. It is still of interest to consider whether the new method, which maintains the stated level better than the Kruskal-Wallis method, has competitive power, since for data of this type, nonparametric methods are more likely to be applied. When the user fears the data do not meet the assumptions of the F-test, the Kruskal-Wallis method is often applied, regardless of the true underlying distributions. The simulations of alternative cases will reveal whether the PS test may be preferable to the Kruskal-Wallis method in this setting.

To see if the method may be more properly applied in this setting, the simulations were also rerun using the cutoffs that were determined for this setting. For equal cases, the cutoffs can be found on Tables 5.4, 5.6, and 5.8. For unequal sample sizes, the only useful option is to use the average of the cutoff values for the various arrangements. This was done by averaging the six values in each case, and then rounding up to the next possible value of the statistic. Table 5.10 summarizes the results.

Sample Size	0.01 level	0.05 level	0.10 level
5,5,5	30	28	26
6,6,6	33	29	28
7,7,7	38	34	32
8,8,8	41	37	35
9,9,9	46	42	40
10,10,10	49	45	42
5,6,7	33	30	29
6,7,8	37	34	32
5,8,11	42	38	36
6,9,12	46	42	39

Table 5.10: Empirical Cutoffs for Setting 4

These cutoffs will be used to study the power of the statistic in this case. The PS test will not be compared to the existing methods here since they are not valid in this setting. The purpose of the study is to determine whether the PS method gives reasonable power in this setting when the correct cutoff values are used. Tables F.29 and F.30 (pages 214 and 215) display results for the true level when the null hypothesis is true, and the corrected critical values are used. As in the Setting 1, the test is consistently conservative. In one equal sample size setting the true level seems to be elevated. This is in the case of three samples of size seven each. In the case where the level was the highest, the level was 0.1014 which exceeded the desired 0.1000 level. The earlier distributional studies using SAS' Proc Freq had indicated that the level should have been about 0.0990, which would be below the desired level.

How conservative the test is in other cases depends on the setting, and how close to the true level the empirical proportion was able to come. In the case of the 0.01 level testing with equal sample sizes, the true level ranged from 0.0016, when the three samples were of size five each, to 0.0085, when the three samples were each of size nine. At the 0.05 level with equal sample sizes, the true level was highest when the three samples were all of size six. In that case the level was 0.0500. In the case where the three samples were of sizes five, the lowest power was obtained, 0.0136. When the sample sizes were unequal, the highest true level for any arrangement, when 0.05 was the target value, was the 0.0937 obtained when the samples were of sizes five, eight, and eleven, in that order. The average for the three samples of sizes five, eight, and eleven exceeded the desired level at both the 0.01 and 0.05 levels.

When the 0.10 level was claimed, the power ranged from 0.0518 to the 0.1014 discussed earlier. The minimum was obtained when the three sample were all of size five, while the maximum came from the case where the three samples were each of size seven. The true level exceeded the claim in several of the permutations of the two largest unequal sample size settings. The averages for each of these cases was below the stated level, but for two of the permutations of the largest setting, and three of the arrangements for the five, eight, and eleven case, the true level was higher than desired. This should be expected, since the cutoff value was chosen to be the average of the cutoff values for each combination, thus occasionally some of the permutations of the sample sizes will have a true level higher than desired, while the average should be at or below the desired level.

5.4 The Examples Revisited

5.4.1 The Phosphorus Content Example

Recall that in section 2.4.1 the value of the PS statistic in this example was found to be 24. This is now known to be equal to the critical value for a 0.05 level test. Thus if a 0.05 or greater test is desired, the conclusion would be to reject the null hypothesis that the three varieties of apples all have the same phosphorus content. For comparison, when the F-test was conducted, the P-value given by Minitab was 0.000 and the Kruskal-Wallis method gives a P-value 0.008. The empirical P-value looked up in the frequency information given from Proc Freq in SAS for the PS method was actually 0.0377, which is higher than the other methods.

5.4.2 The Torque Example

In section 5.4.2 the PS statistic was calculated to be 13. This value is well below the 0.10 level cutoff value, which for three samples of size eight each was 28. This indicates clearly that the three temperature settings do not make a significant difference in the amount of torque required to turn the log. In this situation, the F-test gives a P-value of 0.576, while the P-value for the Kruskal-Wallis test on this data is 0.281. The empirical Paired Sign P-value is 0.7026.

Chapter 6

The Alternative Cases

The discussion in this chapter will be about the power of the methods in the various cases when the alternatives listed in Table 4.5 were considered.

6.1 Three Normal Distributions

6.1.1 Setting 1: Three Standard Normal Distributions

In situations with common samples sizes, the fact that the actual level was not strictly increasing in the sample size carries over to the power of the Paired Sign method. (See Tables C.3 through C.8, pages 120 - 123.) In section 5.3.1 it was noticed that the true level of the Kruskal-Wallis test was lower than that of the PS test in several cases. This did not always result in the PS method having higher power in these settings for the alternatives considered. For the case where two of the medians were zero, and the third was 1, the PS method had higher power at the 0.01 level for three samples of size six (0.0827 compared to 0.0701) and a higher average power for the smallest of the unequal sample size settings (0.0927 compared to 0.0705). The power was also higher in this unequal sample size setting regardless of which of the sample sizes was taken from the shifted population. The power of the PS method never exceeded that of the Kruskal-Wallis test in the case where one population was shifted further from the other two. When all of the medians differed, the PS method had higher power than the Kruskal-Wallis method in the two cases mentioned for the milder shift.

The F test has the highest power against all of the alternatives considered, but in each case the Kruskal-Wallis method was closer to the power of the F-test as the level of the tests increased. In all of the cases, the power of the PS method was less than that of the F-test, but was reasonable. An interesting trend revealed itself in the case where all three medians were different. For all three of the methods, the highest power was obtained when the smallest sample was the one in the middle. This meant that the larger samples were further apart, and therefore easier for all of the methods to detect. Conversely, when the median of the largest sample is between the other medians, the power is the least for all of the tests.

6.1.2 Setting 2a: Three Normals with Mildly Different Variances

The discussion in this section is based on trends found in Tables D.4 through D.17, pages 128 through 141. For the two situations where the sample with the largest variance was moved, the F-test consistently had higher power than the other methods. This should be expected, since the F-test had a higher level than the other methods. The Kruskal-Wallis method has higher power than the PS method even in most of the cases where the PS test had the level that was closest to the desired level without exceeding it. The only case where the level of the PS method was preferable and the PS method maintains a higher power is at the 0.01 level, and the three samples are each of size seven. Even here, the PS method has higher power only when the second or third population is shifted.

When the sample with the smallest variance was moved the power of the Kruskal-Wallis and PS methods improve dramatically. The power of the Kruskal-Wallis test is higher in most of the settings for the more extreme case than that of the F-test. This is especially true as the sample size and level increase. The F-test also exhibits an increase in power for larger sample sizes. The increase in power when the sample with the smallest variance is shifted is most noticeable in the rank based Kruskal-Wallis method, because moving the sample with the smallest population variance makes it more likely that the sample of interest will have ranks grouped higher than the other samples. This scenario also increases the probability that the value of z_{il} will be extreme when the first sample is compared to the other samples.

For the cases where all of the medians are different, the Paired Sign method rarely has higher power than the Kruskal-Wallis method, except in the case where the medians are 1,0,2 or 1,2,0, but the F-test maintains higher power than either of the other methods across all of the cases considered. In these cases, and each of the others in this distributional setting, the PS method has reasonable power.

6.1.3 Setting 2b: Three Normals with More Extreme Variances

For the case where the population with the largest variance had a different median than the other populations, the power of the three methods at the 0.05 and 0.01 levels are quite similar, especially for larger sample sizes. (See Tables D.31 through D.44, pages 155 - 168.) For example, when the level was 0.10 and the three samples had sizes five, eight, and eleven, in some order, the average powers were 0.2269, 0.2002, and 0.2347 for the F-test, Kruskal-Wallis and PS methods, respectively. This example is also a case in which the PS statistic has higher power than the other methods. The power of the PS statistic looks good, but this is due largely to the elevated level, not any great feature of the method. In most of the cases considered, the PS method had the highest level, so it is not surprising that the method also has the highest power in many of the cases.

When the sample from the distribution with the smallest variance is moved instead, the Kruskal-Wallis method is once again more powerful than the F-test in most of the cases considered. As an example of the increase in power, consider Kruskal-Wallis tests conducted at the 0.10 level in the equal sample size cases.

When the population with the largest variance is shifted to have median 2, the power reaches a maximum of 0.5307 in the case of samples of size ten. When the smallest variance population is moved, a power of 0.5318 is reached when the samples are only of size six each. (In the case where the population with the largest variance is shifted, the power for three samples of size six was only 0.3689.)

When all of the populations have different medians, the Kruskal-Wallis method was close to the F-test and outperformed the F-test when the level was 0.10. In most of the cases considered, the PS statistic maintains a power that is reasonably close to the other methods, and is often the method with the highest power. For example, when the level is 0.10, and the three samples are of size six each with medians zero, one, and two, the PS method has power 0.3616, while the F-test and Kruskal-Wallis methods have powers 0.3576 and 0.3593, respectively. As mentioned previously, this should not be considered a strong endorsement of the PS method since the level was elevated.

6.2 Setting 3: Three Laplace Distributions

While the Kruskal-Wallis and PS methods have identical levels in Settings 1 and 3, the power of both methods drop significantly from Setting 1 to Setting 3. (See Tables E.3 through E.8, pages 181 - 184.) In settings where only one sample is shifted to have a different median, the Kruskal-Wallis method has higher power than the other methods in most of the cases considered. The power of the Paired Sign method is generally close to that of the other two methods when the shift is mild, but can be quite different when the shift is larger. The power of the PS test is especially close to that of the F-test in most cases, even exceeding the power of the F-test in several of the cases considered. For the case where the three samples are all of size ten, and the shift is mild, the power for the two methods is summarized in Table 6.1. Note that for two of the three levels, the PS test has higher power than the F-test.

When the shift is larger, the power of the PS method can be much less in some settings. For example,

Method	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
F-test	0.1590	0.3632	0.4922
PS	0.1402	0.3802	0.5220

Table 6.1: Power Values for Selected Cases

consider samples of sizes six, seven, and eight. The power of all three methods for $\alpha = 0.01$ is displayed in the Table 6.2. Regardless of the ordering of the samples, the power of the PS method is less than two thirds of the power of the Kruskal-Wallis method, while the F-test has the highest power. For other levels, the power of the PS test is closer to that of the Kruskal-Wallis and F-tests, but is not as close as in the case of a milder shift.

Sample Sizes	F	KW	PS
6,7,8	0.5375	0.4775	0.3076
6,8,7	0.5065	0.4325	0.2555
7,8,6	0.4652	0.3826	0.2218
Average	0.5031	0.4309	0.2616

Table 6.2: Power for the Tests at the 0.01 Level

When all three populations have different medians, the PS test has power more similar to the other methods than in the large shift cases. Once again the Kruskal-Wallis method generally has the highest power. The PS method actually has the highest power in four equal sample size cases and the highest average power in two of the unequal sample size cases. It also has higher power than the F-test in several additional cases where the Kruskal-Wallis method had the highest power.

6.3 Setting 4: Three Different Distributions

6.3.1 Using Identical Case Cutoff Values

The power of the PS test is competitive with the F-test, especially for the higher levels, across the range of alternatives. (See Tables F.4 through F.27 on pages 189 through 212) When one of the distributions was shifted to have median one, the choice of which population is shifted makes a significant difference. All three of the methods perform best when the third population is shifted, and worst when the first was moved. An interesting feature is that the difference is least visible with the Paired Sign method. In fact, in the case where the first population is shifted, the PS is almost always the most powerful of the methods.

When the shifted distribution is moved to have median two, instead of one, the trends change. The highest power for all of the methods is in the case where the second distribution is moved. Shifting the third distribution still results in the lowest power for the F and Kruskal-Wallis methods, but the lowest power for the PS method is when the first distribution is shifted. The power of the Paired Sign test is often lower than the other methods, but is usually quite close. In the case where the first population is shifted, the PS method usually has the highest power, as it did in the case of the smaller shift.

Finally, the case where the populations are shifted such that all of the medians differ must be considered. The highest power was generally attained when the third sample was the lowest, and the second sample is the highest. The PS test sometimes has much lower power than the other methods, but does have reasonable power in this setting. The rest of the cases, listed in order of decreasing power, were: 0,2,1; 0,1,2; 1,0,2; 2,0,1,; 2,1,0. The Kruskal-Wallis method generally has the highest power across the various settings, but this is largely attributable to the elevated level that the method had in all of the settings. The power of the F-test and PS method are often similar, with each having the advantage in some settings. Only in the setting where the power is the lowest is the PS method clearly preferable to the F-test. In most of the other cases the F-test has generally higher power, while it also maintained a true level closer to that desired.

6.3.2 Using Corrected Cutoff Values

In section 5.3.4 it was discussed that the identical case cutoff values were very different from the empirical cutoff values for this setting. For this reason it was proposed to conduct simulations using the empirical values for this setting, or the corrected cutoff values, to see if the method would work well if some adjustment were made. These simulations are discussed in this section.

As the study of the level would indicate, the cases involving three samples of size five exhibit power values significantly lower than the other cases considered, since the effect of the discrete nature of the statistic was most evident in this small sample setting. (See Tables F.31 through F.54 on pages 216 through 239 for the values discussed in this section.) When only one population is shifted, the general trends discussed for the power in the previous section, i.e. when the power is highest in each of the alternative settings, hold true here as well. When all of the populations have different medians, it may be of interest which ordering gives the highest power. This may be of use to a researcher who suspect that if there is a difference in location, it should be in a certain direction. It might be of interest to the individual to know whether he or she should expect to have good power. The ordering discussed in the previous section (i.e. when the identical case cutoff values were used) no longer describes the decreasing power values. With corrected cutoff values, the alternatives, sorted from highest to lowest power, are: 0,1,2; 0,2,1; 1,0,2; 1,2,0; 2,0,1; 2,1,0 (where listing “0,1,2” first means that the highest power values were generally seen when the Laplace distribution had median zero, the generalized lambda distribution had median one, and the normal mixture had median 2). While this ordering is true in general, there are isolated cases where this ordering does not hold. However, these exceptions exhibit no pattern. Overall, the power values of the Paired Sign test are not overwhelming, but the power does increase under each alternative, and increases when the shift is increased from one to two. The power achieves a high of 0.8784 at the 0.10 level when the three samples are of size 10 each, and the second population is shifted two units higher.

The conclusion is that with adjusted cutoff values, the PS method gives a test that is valid and has reasonable power. This cannot be claimed by the other methods considered. Naturally, some method must be used to decide which cutoff values will be used, since it is rare that a researcher would know that their distributions exhibit the unique behavior that is seen in this setting. Here, an exploratory data analysis type approach is suggested. The user is encouraged to construct boxplots of the data, and look for differences in the shapes for the three samples. If the shapes look quite different, the corrected, or adjusted, cutoff values would be suggested, while the identical case critical values should be used if the samples appear similar.

6.4 The Effect of Sample Size

This is the area of concern where the discreteness of the statistic most hinders the study of the behavior of the statistic. Due to the discrete nature of the statistic, it is not possible to state whether using larger sample sizes enables the method to more closely attain the desired level. Due to the increased number of possible values of the statistic as the sample sizes increase, it may be conjectured that for much larger samples this discreteness may have a less noticeable effect. This should cause larger sample sizes to more closely attain the desired level. It may also be conjectured that due to the fact that the zero comparisons yield more possible values for even sample sizes than for similar odd sample sizes, the even sample size settings may be able to closely meet the desired level for smaller sample sizes than for odd cases. The sample sizes studied in this research, however, appear to have been too small to observe these trends.

While trends did not appear when the null hypothesis cases were considered, it was apparent from the study that power did increase as sample size increased. Even in the cases where the shift was mildest the power was almost always increasing in the sample sizes. The only exceptions were in cases where the actual level was quite low for the some sample size combination. When one of the distributions was shifted more drastically, the trend was quite clear. There did not seem to be a difference in the power due only to whether the sample sizes were odd or even.

Total Sample Size	Equal Setting	Unequal Setting
18	6,6,6	5,6,7
21	7,7,7	6,7,8
24	8,8,8	5,8,11
27	9,9,9	6,9,12

Table 6.3: Equivalent Equal and Unequal Sample Size Settings

	Significance Level		
Total Sample Size	0.01	0.05	0.10
18	varied	equal	equal
21	varied	varied	varied
24	equal	varied	varied
27	varied	equal	varied

Table 6.4: Sample Size Choice resulting in Higher True Level

A final issue of practical importance is to compare the cases where the sample sizes varied, with an equal sample size case where there are the same number of total observations. Table 6.3 shows the cases with equal numbers of total observations. These may be compared to discuss whether there may be some advantage to using unequal sample sizes in practice.

In the null distribution cases, there is a clear pattern. The pattern holds, except in Setting 4. Table 6.4 shows whether equal or varied sample sizes had higher level for Settings 1, 2a, 2b, and 3. Tables 6.5 and 6.6 show which had the higher level for Settings 4 without corrected cutoff values and with corrected cutoff values, respectively. The is likely due to the discreteness of the statistic, since the trend carries across most of the settings. This is also indicated by the fact that changing the cutoff values used has more effect on the pattern than changing the underlying distributions.

	Significance Level		
Total Sample Size	0.01	0.05	0.10
18	varied	equal	equal
21	equal	varied	varied
24	equal	varied	equal
27	varied	equal	equal

Table 6.5: Sample Size Choice resulting in Higher True Level for Setting 4

	Significance Level		
Total Sample Size	0.01	0.05	0.10
18	varied	equal	equal
21	varied	equal	equal
24	varied	varied	varied
27	varied	equal	varied

Table 6.6: Sample Size Choice resulting in Higher True Level for Setting 4 with Corrected Cutoff Values

As the alternative cases are considered, the problems of discreteness appear to be less important than the significance level chosen. As all of the alternative cases are studied, it becomes clear that the choice of whether to vary the sample sizes depends largely on the level at which the user wishes to conduct the test. At the 0.01 level the unequal sample size settings tend to have higher power than the equal settings. For the 0.05 and 0.10 levels, the trend reverses, and the equal sample size settings generally have greater power than the equivalent variable sample size cases.

6.5 General Observations

The statistic introduced in this work does not achieve the same power as the other methods, but is generally close. The main benefit to the method seems to be in Setting 4. When the distributions differ significantly, as in this setting, none of the methods are able to hold their level. Of the three standard methods, the PS test with the identical case critical values is the most conservative, though it violates its level in most cases. The power of the PS method, however, is quite competitive across the alternatives considered. This indicates that the PS method should be preferred, even in this form, to the existing methods in situations where the populations are thought to differ significantly in shape. The PS test also has the advantage of a natural adjustment to restore the ability of the method to maintain the desired level. There is no known adjustment suggested for the F-test or Kruskal-Wallis method to make them valid in this setting. The proposed method does not address this situation perfectly, but does offer an improvement on the existing methods, as desired.

Chapter 7

Summary of Results and Further Research

7.1 Summary of the Results

Each of the methods considered had areas where it would be preferred to the other two methods. In this section, these areas of strength will be restated for clarity. More details will be given about the statistic proposed in this research, since the other two methods have been studied in depth previously.

7.1.1 Areas of Strength for the F-test

The F-test proved to be the best method only when all of the samples were normal, with the same variance, the distribution of the F-statistic is exact. Thus the level of the method was correct in all cases, and the test was most powerful against all alternatives. The test also performed reasonably well in Setting 3, despite the non-normal nature of the data. The test performed conservatively in this setting due to the symmetric, heavy-tailed nature of the Laplace distribution. Had another distribution been chosen for the three populations, the F-test may have performed quite differently.

7.1.2 Areas of Strength for the Kruskal-Wallis Method

The Kruskal-Wallis method performed best in Setting 3, as expected. Setting 3 met the assumptions of this method exactly, and the method proved to be the best. The Kruskal-Wallis test was also the best method in Settings 2a and 2b. This was surprising due to the difference in the shapes of the distributions. The fact that the F-test was affected by the differing variances was not surprising. It was interesting to note that the PS test was more affected by the variance changes than the Kruskal-Wallis method. Perhaps the preservation of the general shape was a key to why the method worked well in Settings 2a and 2b, but performed very poorly in Setting 4, where the shapes were not related.

7.1.3 Areas of Strength for the Paired Sign Statistic

While the PS method had lower power than the other methods in many of the settings, and failed to maintain the desired level in the settings where the distributions were not the same, the performance of the statistic in Setting 4 was promising. The PS method was consistently closer to the stated level than was the Kruskal-Wallis method, and had power which rivaled that of the F-test. If a user were interested in applying a method to data from underlying distributions that differ significantly, without wanting to make an adjustment to the method, the PS method would be a reasonable choice, based on the findings from the simulations conducted in this research.

The PS method also has another advantage, in that the user is not required to use the method with identical case cutoff values. In this work, cutoff values for the extreme case were found, and shown to make the method valid, while maintaining reasonable power values. The power values in the corrected cutoff case were not as high as those in the identical case cutoffs, but should be considered superior due to the invalid nature of the test when identical case cutoff values were used. The power is also not comparable to the power of the identical distribution cases. This is to be expected, since the differences between the underlying distributions is quite extreme. The critical values were pushed farther out toward the limits of the possible values of the statistic, thus causing less possible values for which the test would reject. Even when the alternatives are true, a shift of one of the distributions from median zero to median one becomes more difficult to detect. This loss of power would also be noticed for the other methods, if some method of controlling the level of the test were to be proposed and studied.

The only remaining question is how to choose the adjustment of the cutoff values. Some adjustment would be appropriate for data of the type in Settings 2a and 2b, but clearly not as drastic an adjustment as was necessary for data from Setting 4. Currently it is left to the user to decide, likely based on boxplots or nonparametric density estimates, the extent of adjustment necessary. The variety of correct cutoffs listed on Tables 5.4 through 5.9 on pages 51 through 56 may give the user a range of values to guide the selection process.

7.1.4 Choosing the Method to Use

The results summarized above may be joined into a logical process for making the decision of which method to use in practice. If the practitioner is comfortable, based on knowledge of the subject area, that the data may come from identical normal distributions, the F-test should be conducted. If the subject matter specialist does not believe that normality is likely, but does believe that the distributions should be the same under the null hypothesis assumption of equal locations, then the Kruskal-Wallis method is suggested. If the researcher believes that there may be differences between the distributions, even under the null hypothesis, then the Paired Sign method should be chosen, due to the stability it displayed across the settings considered, when compared to the F-test and the Kruskal-Wallis method.

7.2 Further Research

The first issue for future research would be to make the decision as to which critical values to use less subjective. Some method which would test whether the distributions are equal, and, if they differ, quantify how different, could be used to decide which values were appropriate for the particular data of interest. The behavior of the statistic when sampling from distributions common in certain applications that may be of interest should be considered as well. An example would be the Weibull distributions used regularly in reliability applications.

It would also be of interest to further investigate the large sample properties of the proposed method. While interest in the method is primarily for small sample settings, it would be of interest to know how the statistic would behave in larger settings, and to consider the possibility of an approximation for larger sample sizes. An adjustment for ties may also be considered. While the assumption of continuity assures that there are no real ties, rounding in the recording of values may sometimes result in ties. Ties are not as serious a problem with the proposed statistic as they are with the Kruskal-Wallis method, since comparisons are only to the median of each sample, whereas the Kruskal-Wallis statistic requires the ranking of each observation. However, since ties may happen in practice, the effect of ties on the statistic, and how they should be handled, should be considered.

Also, it would be valuable to consider alternatives that lie between those considered in this work. Approximate power curves may be drawn if a fine enough mesh of alternatives is considered. In this work only two sets of medians, (0,0,1) and (0,0,2), were considered to see how a more severe shift of one of the populations affects the power. Several values between zero and one, and between one and two, as well as some beyond two, could be chosen to more accurately study how quickly the power increases.

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Appendix A

Full Results for the Effect of the Median Range

A.1 Equal Sample Sizes

Variances: 1, 1.5, and 2

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
4,4,4	4.84309	24	0.20180	0.20000
6,6,6	5.30468	36	0.14735	0.14286
8,8,8	5.58376	48	0.11633	0.11111
10,10,10	5.76085	60	0.09601	0.09091
12,12,12	5.89402	72	0.08186	0.07692
14,14,14	5.98063	84	0.07120	0.06667
16,16,16	6.07472	96	0.06328	0.05882
18,18,18	6.15094	108	0.05695	0.05263
20,20,20	6.19879	120	0.05166	0.04762
22,22,22	6.22584	132	0.04717	0.04348
24,24,24	6.28302	144	0.04363	0.04000

Table A.1: Three Normal Distributions

Variances: 1, 2, and 3

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
4,4,4	4.87044	24	0.20294	0.20000
6,6,6	5.44625	36	0.15128	0.14286
8,8,8	5.83603	48	0.12158	0.11111
10,10,10	6.11881	60	0.10198	0.09091
12,12,12	6.28759	72	0.08733	0.07692
14,14,14	6.45984	84	0.07690	0.06667
16,16,16	6.63583	96	0.06912	0.05882
18,18,18	6.72726	108	0.06229	0.05263
20,20,20	6.82881	120	0.05691	0.04762
22,22,22	6.91730	132	0.05240	0.04348
24,24,24	6.97447	144	0.04843	0.04000

Table A.2: Three Normal Distributions

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
4,4,4	4.27782	24	0.17824	0.20000
6,6,6	4.67223	36	0.12978	0.14286
8,8,8	4.86277	48	0.10131	0.11111
10,10,10	4.97375	60	0.08290	0.09091
12,12,12	5.07898	72	0.07054	0.07692
14,14,14	5.16337	84	0.06147	0.06667
16,16,16	5.21363	96	0.05431	0.05882
18,18,18	5.29709	108	0.04905	0.05263
20,20,20	5.34851	120	0.04457	0.04762
22,22,22	5.39569	132	0.04088	0.04348
24,24,24	5.43946	144	0.03777	0.04000

Table A.3: Three Different Distributions

A.2 Unequal Sample Sizes

Variances: 1, 1.5, and 2

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
5,6,7	1.73821	36	0.04828	0.04762
5,7,6	2.34044	36	0.06501	0.04762
6,5,7	1.05063	36	0.02918	0.04762
6,7,5	1.09752	36	0.03049	0.04762
7,5,6	2.45788	36	0.06827	0.04762
7,6,5	1.89734	36	0.05270	0.04762
Average	**1.7637**	**36**	**0.04899**	**0.04762**
6,7,8	3.39816	42	0.08091	0.08466
6,8,7	2.87099	42	0.06836	0.08466
7,6,8	4.34186	42	0.10338	0.08466
7,8,6	4.58410	42	0.10915	0.08466
8,6,7	3.19435	42	0.07606	0.08466
8,7,6	3.93238	42	0.09363	0.08466
Average	**3.7203**	**42**	**0.08858**	**0.08466**
5,8,11	1.71347	48	0.03570	0.03704
5,11,8	2.44588	48	0.05096	0.03704
8,5,11	1.04252	48	0.02172	0.03704
8,11,5	1.15843	48	0.02413	0.03704
11,5,8	2.70318	48	0.05632	0.03704
11,8,5	2.10334	48	0.04382	0.03704
Average	**1.8611**	**48**	**0.03878**	**0.03704**
6,9,12	3.51515	54	0.06510	0.07326
6,12,9	3.10131	54	0.05743	0.07326
9,6,12	4.83718	54	0.08958	0.07326
9,12,6	5.44946	54	0.10092	0.07326
12,6,9	3.97831	54	0.07367	0.07326
12,9,6	4.97213	54	0.09208	0.07326
Average	**4.3089**	**54**	**0.07980**	**0.07326**

Table A.4: Three Normal Distributions

Variances: 1, 2, and 3

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
5,6,7	1.83487	36	0.05097	0.04762
5,7,6	2.66744	36	0.07410	0.04762
6,5,7	0.75826	36	0.02106	0.04762
6,7,5	0.80462	36	0.02235	0.04762
7,5,6	2.78141	36	0.07726	0.04762
7,6,5	2.06862	36	0.05746	0.04762
Average	**1.8192**	**36**	**0.05053**	**0.04762**
6,7,8	3.36435	42	0.08010	0.08466
6,8,7	2.61719	42	0.06231	0.08466
7,6,8	4.79619	42	0.11420	0.08466
7,8,6	5.12546	42	0.12203	0.08466
8,6,7	3.13804	42	0.07472	0.08466
8,7,6	4.07743	42	0.09708	0.08466
Average	**3.8531**	**42**	**0.09174**	**0.08466**
5,8,11	1.76046	48	0.03668	0.03704
5,11,8	2.75851	48	0.05747	0.03704
8,5,11	0.73962	48	0.01541	0.03704
8,11,5	0.85300	48	0.01777	0.03704
11,5,8	3.16008	48	0.06584	0.03704
11,8,5	2.37963	48	0.04958	0.03704
Average	**1.9419**	**48**	**0.04046**	**0.03704**
6,9,12	3.30030	54	0.06112	0.07326
6,12,9	2.68342	54	0.04969	0.07326
9,6,12	5.35099	54	0.09909	0.07326
9,12,6	6.15297	54	0.11394	0.07326
12,6,9	4.08508	54	0.07565	0.07326
12,9,6	5.38702	54	0.09976	0.07326
Average	**4.4933**	**54**	**0.08321**	**0.07326**

Table A.5: Three Normal Distributions

Sample Sizes	Average Number of Zeros	Comparisons	Proportion	Expected
5,6,7	1.86694	36	0.05186	0.04762
5,7,6	0.95761	36	0.02660	0.04762
6,5,7	1.82688	36	0.05075	0.04762
6,7,5	1.85470	36	0.05152	0.04762
7,5,6	0.98003	36	0.02722	0.04762
7,6,5	1.80092	36	0.05003	0.04762
Average	**1.5478**	**36**	**0.04300**	**0.04762**
6,7,8	3.10536	42	0.07394	0.08466
6,8,7	3.87591	42	0.09228	0.08466
7,6,8	3.13332	42	0.07460	0.08466
7,8,6	2.74149	42	0.06527	0.08466
8,6,7	3.82462	42	0.09106	0.08466
8,7,6	2.75971	42	0.06571	0.08466
Average	**3.2401**	**42**	**0.07714**	**0.08466**
5,8,11	1.97784	48	0.04121	0.03704
5,11,8	0.98620	48	0.02055	0.03704
8,5,11	1.86982	48	0.03895	0.03704
8,11,5	1.93326	48	0.04028	0.03704
11,5,8	1.04417	48	0.02175	0.03704
11,8,5	1.85011	48	0.03854	0.03704
Average	**1.6102**	**48**	**0.03355**	**0.03704**
6,9,12	3.84342	54	0.07117	0.07326
6,12,9	4.55394	54	0.08433	0.07326
9,6,12	3.92731	54	0.07273	0.07326
9,12,6	2.91830	54	0.05404	0.07326
12,6,9	4.42231	54	0.08189	0.07326
12,9,6	2.97490	54	0.05509	0.07326
Average	**3.7734**	**54**	**0.06988**	**0.07326**

Table A.6: Three Different Distributions

Appendix B

The Theoretical Distributional Work

B.1 An Explanation of What Follows

The figures that follow help to justify the formulae that are developed for the needed mass functions.

B.1.1 The Figures

In the figures, there is a number line for each sample of interest. For those samples where the value of the median is of interest in the calculation of z_{il}, the median is indicated by a dot on the number line for odd samples, or two dots at the middle values for even sample sizes. This splits the number lines up into several regions. Depending on the setting, there will be between two and five regions. Each of these regions has a certain number of observations in it for each sample.

The regions along the line are labelled corresponding to the number of observations in the region. An asterisk on an unknown value indicates that the number is a nuisance value in that it has no effect on the value of an of the z_{il}'s under consideration. The density is then a function of the remaining unknowns.

B.1.2 The Mass Functions

Once the number of observations in each of the regions has been written out, the mass functions can then developed. The assumption that the underlying densities are equally shaped implies that within each region, all possible arrangements are equally likely. Thus, all that is needed is what will be called C_r, the count of the number of unique possible arrangements of the observations in region r. The total number of arrangements will be called C_T. Thus once all of the C_i's and the value of C_T are known for a particular case, the mass function can be developed. This is what is done in each of the following cases.

B.2 The Distributions for the Type I Covariances

B.2.1 Both Sample Sizes Odd

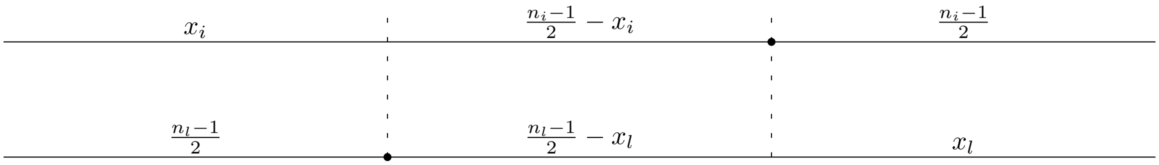

Figure B.1: Both Sample Sizes Odd

$$z_{il} = n_i - 2x_i \qquad z_{li} = 2x_l - n_l$$

$$f_o^I(x_i, x_l) = \frac{C_1 C_2 C_3}{C_T} \tag{B.1}$$

Where:

$$C_1 = \begin{pmatrix} x_i + \frac{n_l-1}{2} \\ x_i \end{pmatrix},\ C_2 = \begin{pmatrix} \frac{n_i-1}{2} - x_i + \frac{n_l-1}{2} - x_l \\ \frac{n_i-1}{2} - x_i \end{pmatrix},\ C_3 = \begin{pmatrix} \frac{n_i-1}{2} + x_l \\ \frac{n_i-1}{2} \end{pmatrix},\ \text{and}\ \ C_T = \begin{pmatrix} n_i + n_l \\ n_i \end{pmatrix}$$

$$x_i \in \left\{0, 1, \ldots, \frac{n_i - 1}{2}\right\} \quad x_l \in \left\{0, 1, \ldots, \frac{n_l - 1}{2}\right\}$$

B.2.2 Both Sample Sizes Even

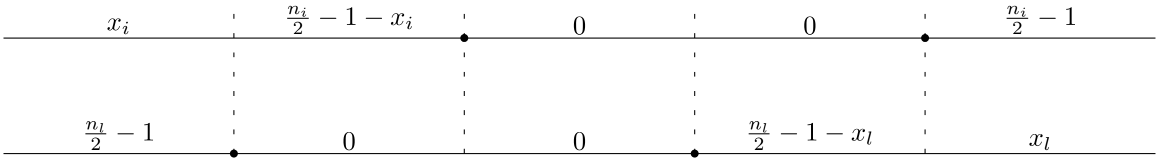

Figure B.2: Both Sample Sizes Even, Overlapping Median Ranges

$$z_{il} = \frac{n_i}{2} - x_i \qquad z_{li} = x_l - \frac{n_l}{2}$$

$$f_e^{Io}(x_i, x_l) = \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.2}$$

Where:

$$C_1 = \begin{pmatrix} x_i + \frac{n_l}{2} - 1 \\ x_i \end{pmatrix},\ C_2 = \begin{pmatrix} \frac{n_i}{2} - 1 - x_i \\ \frac{n_i}{2} - 1 - x_i \end{pmatrix} = 1,\ C_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1,$$

$$C_4 = \begin{pmatrix} \frac{n_l}{2} - 1 - x_l \\ \frac{n_l}{2} - 1 - x_l \end{pmatrix} = 1,\ C_5 = \begin{pmatrix} \frac{n_i}{2} - 1 + x_l \\ \frac{n_i}{2} - 1 \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l \\ n_i \end{pmatrix}$$

$$x_i \in \left\{0, 1, \ldots, \frac{n_i}{2} - 1\right\} \quad x_l \in \left\{0, 1, \ldots, \frac{n_l}{2} - 1\right\}$$

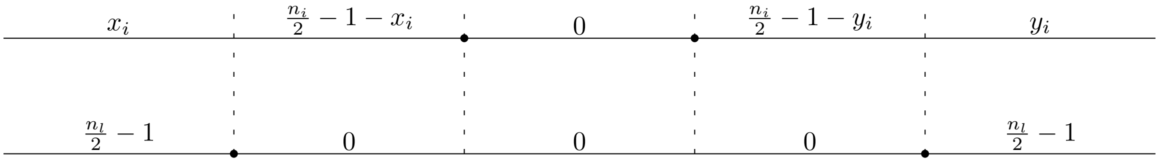

Figure B.3: Both Sample Sizes Even, Contained Median Ranges

$$z_{il} = y_i - x_i \quad z_{li} = 0$$

$$f_e^{Ic}(x_i, y_i) = \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.3}$$

Where:

$$C_1 = \begin{pmatrix} x_i + \frac{n_l}{2} - 1 \\ x_i \end{pmatrix},\ C_2 = \begin{pmatrix} \frac{n_i}{2} - 1 - x_i \\ \frac{n_i}{2} - 1 - x_i \end{pmatrix} = 1,\ C_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1,$$

$$C_4 = \begin{pmatrix} \frac{n_i}{2} - 1 - y_i \\ \frac{n_i}{2} - 1 - y_i \end{pmatrix} = 1,\ C_5 = \begin{pmatrix} y_i + \frac{n_l}{2} - 1 \\ y_i \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l \\ n_i \end{pmatrix}$$

$$x_i \in \left\{0, 1, \ldots, \frac{n_i}{2} - 1\right\} \quad y_i \in \left\{0, 1, \ldots, \frac{n_i}{2} - 1\right\}$$

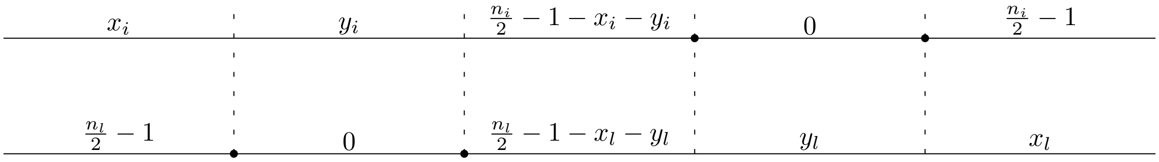

Figure B.4: Both Sample Sizes Even, Disjoint Median Ranges

$$z_{il} = n_i - 2x_i - y_i \quad z_{li} = 2x_l + y_l - n_l$$

$$f_e^{Id}(x_i, y_i, x_l, y_l) = \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.4}$$

Where:

$$C_1 = \binom{x_i + \frac{n_l}{2} - 1}{x_i},\ C_2 = \binom{y_i}{y_i} = 1,\ C_3 = \binom{\frac{n_i}{2} - 1 - x_i - y_i + \frac{n_l}{2} - 1 - x_l - y_l}{\frac{n_i}{2} - 1 - x_i - y_i},$$

$$C_4 = \binom{y_l}{y_l} = 1,\ C_5 = \binom{\frac{n_i}{2} - 1 + x_l}{\frac{n_i}{2} - 1},\ C_T = \binom{n_i + n_l}{n_i}$$

$$x_i \in \left\{0, 1, \ldots, \frac{n_i}{2} - 1\right\} \quad y_i \in \left\{0, 1, \ldots, \frac{n_i}{2} - 1 - x_i\right\} \quad x_l \in \left\{0, 1, \ldots, \frac{n_l}{2} - 1\right\} \quad y_l \in \left\{0, 1, \ldots, \frac{n_l}{2} - 1 - x_l\right\}$$

B.2.3 One Sample Size Odd/One Sample Size Even

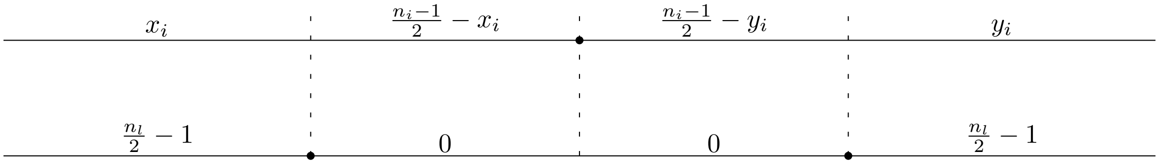

Figure B.5: Sample i Odd, Sample l Even, i contained in the median range of l

$$z_{il} = y_i - x_i \quad z_{li} = 0$$

$$f_m^{Ic}(x_i, y_i) = \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.5}$$

Where:

$$C_1 = \begin{pmatrix} x_i + \frac{n_l}{2} - 1 \\ x_i \end{pmatrix},\ C_2 = \begin{pmatrix} \frac{n_i-1}{2} - x_i \\ \frac{n_i-1}{2} - x_i \end{pmatrix} = 1,\ C_3 = \begin{pmatrix} \frac{n_i-1}{2} - y_i \\ \frac{n_i-1}{2} - y_i \end{pmatrix} = 1,$$

$$C_4 = \begin{pmatrix} y_i + \frac{n_l}{2} - 1 \\ y_i \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l \\ n_i \end{pmatrix}$$

$$x_i \in \left\{0, 1, \ldots, \frac{n_i - 1}{2}\right\} \quad y_i \in \left\{0, 1, \ldots, \frac{n_i - 1}{2}\right\}$$

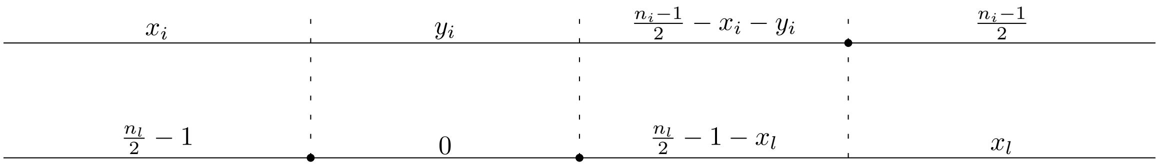

Figure B.6: Sample i Odd, Sample l Even, i not in the median range of l, i "high"

$$z_{il} = n_i - 2x_i - y_i \quad z_{li} = 2x_l - n_l$$

$$f_m^{Ih}(x_i, y_i, x_l) = \frac{C_1 C_3 C_4}{C_T} \tag{B.6}$$

Where:

$$C_1 = \begin{pmatrix} x_i + \frac{n_l}{2} - 1 \\ x_i \end{pmatrix},\ C_2 = \begin{pmatrix} y_i \\ y_i \end{pmatrix} = 1,\ C_3 = \begin{pmatrix} \frac{n_i-1}{2} - x_i - y_i + \frac{n_l}{2} - 1 - x_l \\ \frac{n_i-1}{2} - x_i - y_i \end{pmatrix},$$

$$C_4 = \begin{pmatrix} \frac{n_i-1}{2} + x_l \\ \frac{n_i-1}{2} \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l \\ n_i \end{pmatrix}$$

$$x_i \in \left\{0, 1, \ldots, \frac{n_i - 1}{2}\right\} \quad y_i \in \left\{0, 1, \ldots, \frac{n_i - 1}{2} - x_i\right\} \quad x_l \in \left\{0, 1, \ldots, \frac{n_l}{2} - 1\right\}$$

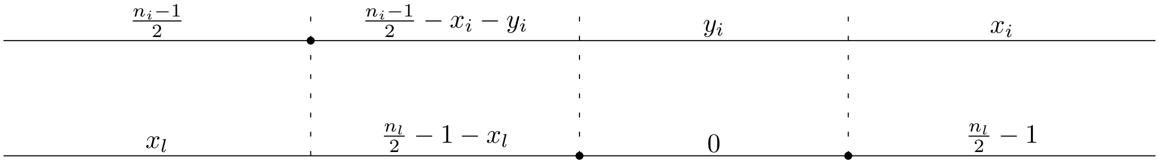

Figure B.7: Sample i Odd, Sample l Even, i not in the median range of l, i "low"

$$z_{il} = 2x_i + y_i - n_i \quad z_{li} = n_l - 2x_l$$

$$f_m^{Il}(x_i, y_i, x_l) = \frac{C_1 C_2 C_4}{C_T} \tag{B.7}$$

Where:

$$C_1 = \binom{\frac{n_i-1}{2}+x_l}{\frac{n_i-1}{2}},\ C_2 = \binom{\frac{n_i-1}{2}-x_i-y_i+\frac{n_l}{2}-1-x_l}{\frac{n_i-1}{2}-x_i-y_i},\ C_3 = \binom{y_i}{y_i} = 1,$$
$$C_4 = \binom{x_i+\frac{n_l}{2}-1}{x_i},\ C_T = \binom{n_i+n_l}{n_i}$$

$$x_i \in \left\{0, 1, \ldots, \frac{n_i-1}{2}\right\} \quad y_i \in \left\{0, 1, \ldots, \frac{n_i-1}{2} - x_i\right\} \quad x_l \in \left\{0, 1, \ldots, \frac{n_l}{2} - 1\right\}$$

B.3 The Distributions for the Type II Covariances

B.3.1 Odd Sample Sizes

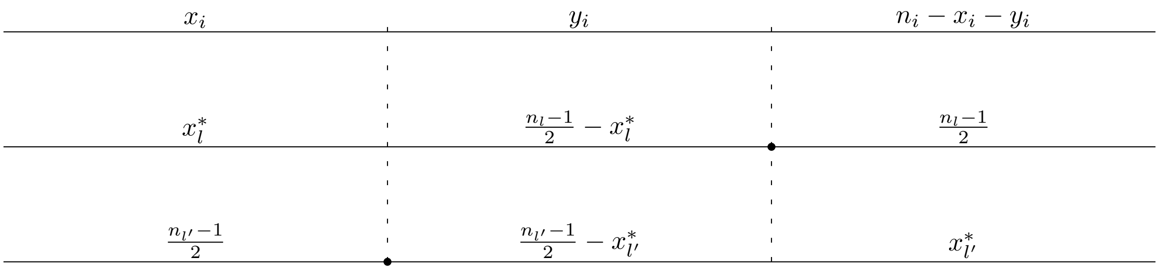

Figure B.8: Both Sample Sizes Odd

$$z_{il} = n_i - 2x_i - 2y_i \quad z_{il'} = n_i - 2x_i$$

$$f_o^{II}(x_i, y_i) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \sum_{x_{l'}^*=0}^{\frac{n_{l'}-1}{2}} \frac{C_1 C_2 C_3}{C_T} \tag{B.8}$$

Where:

$$C_1 = \begin{pmatrix} x_i + x_l^* + \frac{n_{l'}-1}{2} \\ \\ x_i \quad x_l^* \quad \frac{n_{l'}-1}{2} \end{pmatrix}, \ C_2 = \begin{pmatrix} y_i + \frac{n_l-1}{2} - x_l^* + \frac{n_{l'}-1}{2} - x_{l'}^* \\ \\ y_i \quad \frac{n_l-1}{2} - x_l^* \quad \frac{n_{l'}-1}{2} - x_{l'}^* \end{pmatrix}, \ C_3 = \begin{pmatrix} n_i - x_i - y_i + \frac{n_l-1}{2} + x_{l'}^* \\ \\ n_i - x_i - y_i \quad \frac{n_l-1}{2} \quad x_{l'}^* \end{pmatrix},$$

$$C_T = \begin{pmatrix} n_i + n_l + n_{l'} \\ \\ n_i \quad n_l \quad n_{l'} \end{pmatrix}$$

$$x_i \in \{0, 1, \ldots, n_i\} \quad y_i \in \{0, 1, \ldots, n_i - x_i\}$$

B.3.2 Even Sample Sizes

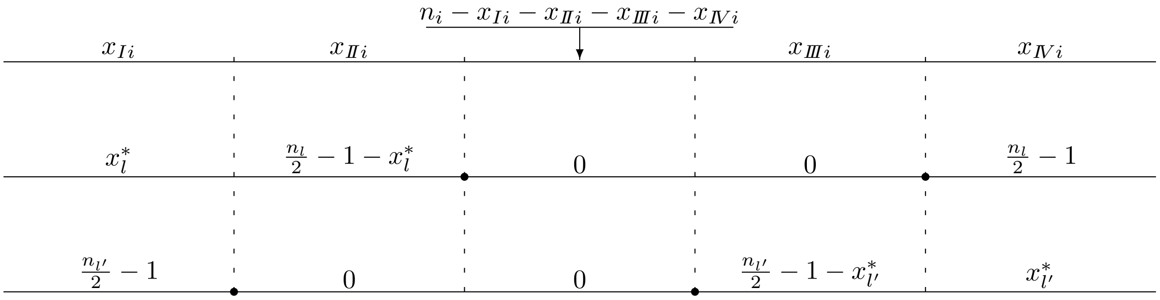

Figure B.9: Both Sample Sizes Even, Overlapping median ranges

$$z_{il} = x_{IVi} - x_{Ii} - x_{IIi} \quad z_{il'} = x_{IIIi} + x_{IVi} - x_{Ii}$$

$$f_e^{IIo}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \sum_{x_{l'}^*=0}^{\frac{n_{l'}}{2}-1} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.9}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + x_l^* + \frac{n_{l'}}{2} - 1 \\ x_{Ii} \quad x_l^* \quad \frac{n_{l'}}{2} - 1 \end{pmatrix},\ C_2 = \begin{pmatrix} x_{IIi} + \frac{n_l}{2} - 1 - x_l^* \\ x_{IIi} \end{pmatrix},\ C_3 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \end{pmatrix} = 1,$$

$$C_4 = \begin{pmatrix} x_{IIIi} + \frac{n_{l'}}{2} - 1 - x_{l'}^* \\ x_{IIIi} \end{pmatrix},\ C_5 = \begin{pmatrix} x_{IVi} + \frac{n_l}{2} - 1 + x_{l'}^* \\ x_{IVi} \quad \frac{n_l}{2} - 1 \quad x_{l'}^* \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l + n_{l'} \\ n_i \quad n_l \quad n_{l'} \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\}$$

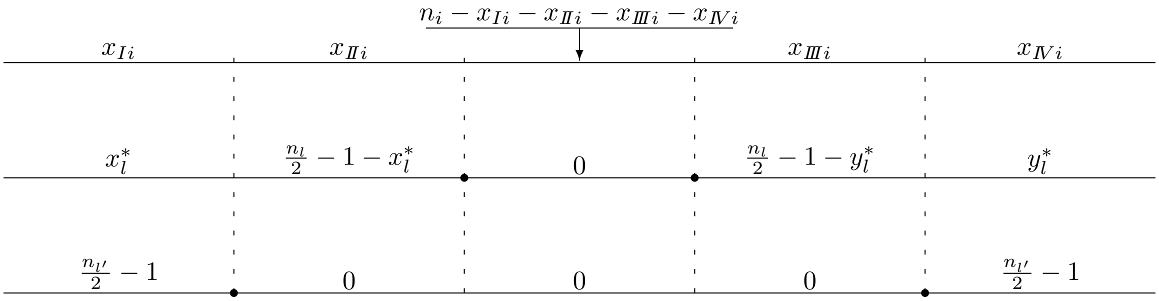

Figure B.10: Both Sample Sizes Even, Contained median ranges

$$z_{il} = x_{IIIi} + x_{IVi} - x_{Ii} - x_{IIi} \quad z_{il'} = x_{IVi} - x_{Ii}$$

$$f_e^{IIc}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \sum_{y_l^*=0}^{\frac{n_l}{2}-1} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.10}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + x_l^* + \frac{n_{l'}}{2} - 1 \\ \\ x_{Ii} \quad x_l^* \quad \frac{n_{l'}}{2} - 1 \end{pmatrix}, \; C_2 = \begin{pmatrix} x_{IIi} + \frac{n_l}{2} - 1 - x_l^* \\ \\ x_{IIi} \end{pmatrix}, \; C_3 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \\ \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \end{pmatrix} = 1,$$

$$C_4 = \begin{pmatrix} x_{IIIi} + \frac{n_l}{2} - 1 - y_l^* \\ \\ x_{IIIi} \end{pmatrix}, \; C_5 = \begin{pmatrix} x_{IVi} + y_l^* + \frac{n_{l'}}{2} - 1 \\ \\ x_{IVi} \quad y_l^* \quad \frac{n_{l'}}{2} - 1 \end{pmatrix}, \; C_T = \begin{pmatrix} n_i + n_l + n_{l'} \\ \\ n_i \quad n_l \quad n_{l'} \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\}$$

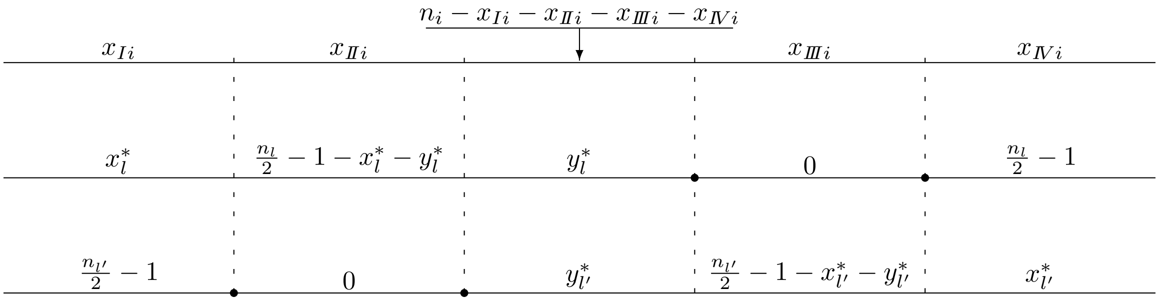

Figure B.11: Both Sample Sizes Even, Disjoint median ranges

$$z_{il} = x_{IIIi} + 2x_{IVi} - n_i \quad z_{il'} = n_i - 2x_{Ii} - x_{IIi}$$

$$f_e^{IId}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \sum_{y_l^*=0}^{\frac{n_l}{2}-1-x_l^*} \sum_{x_{l'}^*=0}^{\frac{n_{l'}}{2}-1} \sum_{y_{l'}^*=0}^{\frac{n_{l'}}{2}-1-x_{l'}^*} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.11}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + x_l^* + \frac{n_{l'}}{2} - 1 \\ x_{Ii} \quad x_l^* \quad \frac{n_{l'}}{2} - 1 \end{pmatrix},\ C_2 = \begin{pmatrix} x_{IIi} + \frac{n_l}{2} - 1 - x_l^* - y_l^* \\ x_{IIi} \end{pmatrix},$$

$$C_3 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} + y_l^* + y_{l'}^* \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \quad y_l^* \quad y_{l'}^* \end{pmatrix},\ C_4 = \begin{pmatrix} x_{IIIi} + \frac{n_{l'}}{2} - 1 - x_{l'}^* - y_{l'}^* \\ x_{IIIi} \end{pmatrix},$$

$$C_5 = \begin{pmatrix} x_{IVi} + x_{l'}^* + \frac{n_{l'}}{2} - 1 \\ x_{IVi} \quad x_{l'}^* \quad \frac{n_{l'}}{2} - 1 \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l + n_{l'} \\ n_i \quad n_l \quad n_{l'} \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\}$$

B.3.3 One Sample Size Odd/One Sample Size Even

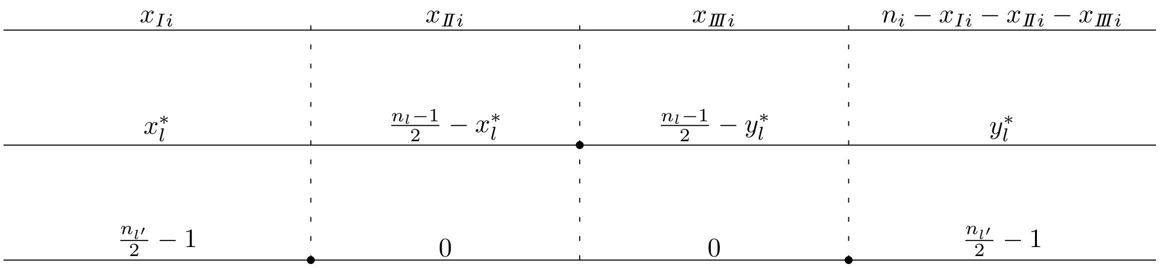

Figure B.12: Sample l Odd, Sample l' Even, l in the median range of l'

$$z_{il} = n_i - 2x_{Ii} - 2x_{IIi} \quad z_{il'} = n_i - 2x_{Ii} - x_{IIi} - x_{IIIi}$$

$$f_m^{IIc}(x_{Ii}, x_{IIi}, x_{IIIi}) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \sum_{y_l^*=0}^{\frac{n_l-1}{2}} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.12}$$

Where:

$$C_1 = \binom{x_{Ii} + x_l^* + \frac{n_{l'}}{2} - 1}{x_{Ii} \quad x_l^* \quad \frac{n_{l'}}{2} - 1},\ C_2 = \binom{x_{IIi} + \frac{n_l-1}{2} - x_l^*}{x_{IIi}},\ C_3 = \binom{x_{IIIi} + \frac{n_l-1}{2} - y_l^*}{x_{IIIi}},$$

$$C_4 = \binom{n_i - x_{Ii} - x_{IIi} - x_{IIIi} + y_l^* + \frac{n_{l'}}{2} - 1}{n_i - x_{Ii} - x_{IIi} - x_{IIIi} \quad y_l^* \quad \frac{n_{l'}}{2} - 1},\ C_T = \binom{n_i + n_l + n_{l'}}{n_i \quad n_l \quad n_{l'}}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

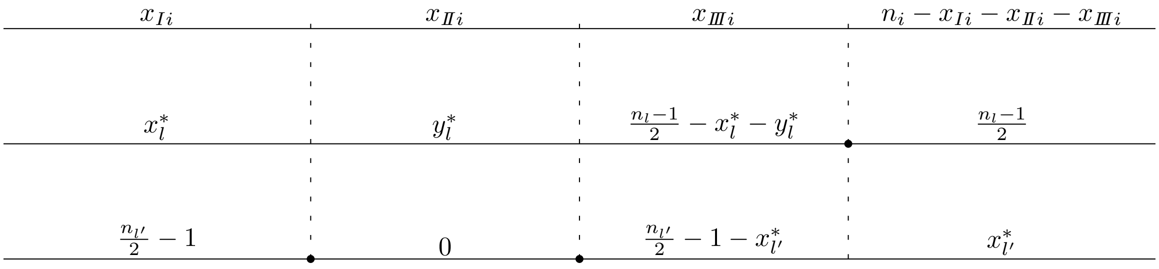

Figure B.13: Sample l Odd, Sample l' Even, l not in the median range of l', l "high"

$$z_{il} = n_i - 2x_{Ii} - 2x_{\mathit{II}i} - 2x_{\mathit{III}i} \quad z_{il'} = n_i - 2x_{Ii} - x_{\mathit{II}i}$$

$$f_m^{\mathit{II}h}(x_{Ii}, x_{\mathit{II}i}, x_{\mathit{III}i}) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \sum_{y_l^*=0}^{\frac{n_l-1}{2}-x_l^*} \sum_{x_{l'}^*=0}^{\frac{n_{l'}}{2}-1} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.13}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + x_l^* + \frac{n_{l'}}{2} - 1 \\ x_{Ii} \quad x_l^* \quad \frac{n_{l'}}{2} - 1 \end{pmatrix},\ C_2 = \begin{pmatrix} x_{\mathit{II}i} + y_l^* \\ x_{\mathit{II}i} \end{pmatrix},\ C_3 = \begin{pmatrix} x_{\mathit{III}i} + \frac{n_l-1}{2} - x_l^* - y_l^* + \frac{n_{l'}}{2} - 1 - x_{l'}^* \\ x_{\mathit{III}i} \quad \frac{n_l-1}{2} - x_l^* - y_l^* \quad \frac{n_{l'}}{2} - 1 - x_{l'}^* \end{pmatrix},$$

$$C_4 = \begin{pmatrix} n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} + \frac{n_l-1}{2} + x_{l'}^* \\ n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} \quad \frac{n_l-1}{2} \quad x_{l'}^* \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l + n_{l'} \\ n_i \quad n_l \quad n_{l'} \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{\mathit{II}i} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{\mathit{III}i} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{\mathit{II}i}\}$$

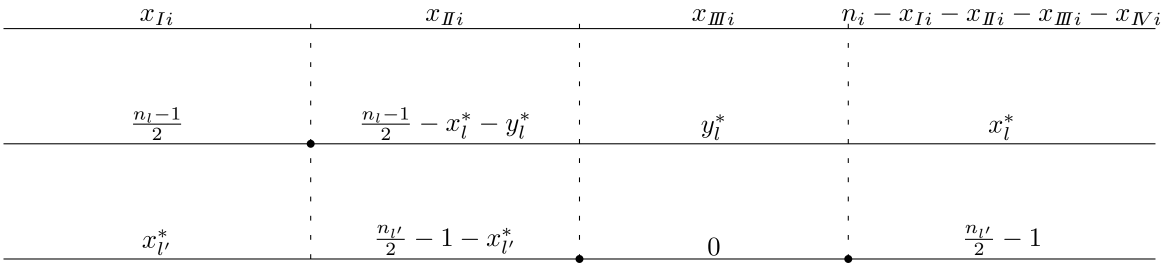

Figure B.14: Sample l Odd, Sample l' Even, l not in the median range of l', l “low”

$$z_{il} = n_i - 2x_{Ii} \quad z_{il'} = n_i - 2x_{Ii} - 2x_{IIi} - x_{IIIi}$$

$$f_m^{IIl}(x_{Ii}, x_{IIi}, x_{IIIi}) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \sum_{y_l^*=0}^{\frac{n_l-1}{2}-x_l^*} \sum_{x_{l'}^*=0}^{\frac{n_{l'}}{2}-1} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.14}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + \frac{n_l-1}{2} + x_{l'}^* \\ x_{Ii} \quad \frac{n_l-1}{2} \quad x_{l'}^* \end{pmatrix},\ C_2 = \begin{pmatrix} x_{IIi} + \frac{n_l-1}{2} - x_l^* - y_l^* + \frac{n_{l'}}{2} - 1 - x_{l'}^* \\ x_{IIi} \quad \frac{n_l-1}{2} - x_l^* - y_l^* \quad \frac{n_{l'}}{2} - 1 - x_{l'}^* \end{pmatrix},\ C_3 = \begin{pmatrix} x_{IIIi} + y_l^* \\ x_{IIIi} \end{pmatrix},$$

$$C_4 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} + x_l^* + \frac{n_{l'}}{2} - 1 \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} \quad x_l^* \quad \frac{n_{l'}}{2} - 1 \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_l + n_{l'} \\ n_i \quad n_l \quad n_{l'} \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

B.4 The Distributions for the Type III Covariances

B.4.1 Sample Size Odd

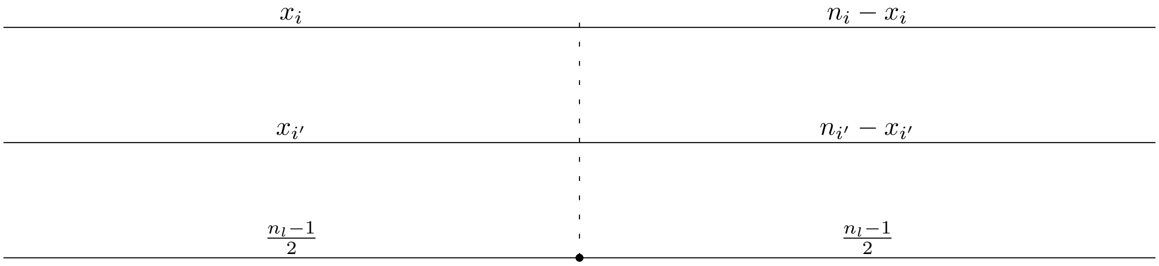

Figure B.15: Sample l Odd

$$z_{il} = n_i - 2x_i \quad z_{i'l} = n_{i'} - 2x_{i'}$$

$$f_o^{\mathit{III}}(x_i, x_{i'}) = \frac{C_1 C_2}{C_T} \tag{B.15}$$

Where:

$$C_1 = \begin{pmatrix} x_i + x_{i'} + \frac{n_l-1}{2} \\ \\ x_i \quad x_{i'} \quad \frac{n_l-1}{2} \end{pmatrix},\ C_2 = \begin{pmatrix} n_i - x_i + n_{i'} - x_{i'} + \frac{n_l-1}{2} \\ \\ n_i - x_i \quad n_{i'} - x_{i'} \quad \frac{n_l-1}{2} \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_{i'} + n_l \\ \\ n_i \quad n_{i'} \quad n_l \end{pmatrix}$$

$$x_i \in \{0, 1, \dots, n_i\} \quad x_{i'} \in \{0, 1, \dots, n_{i'}\}$$

B.4.2 Sample Size Even

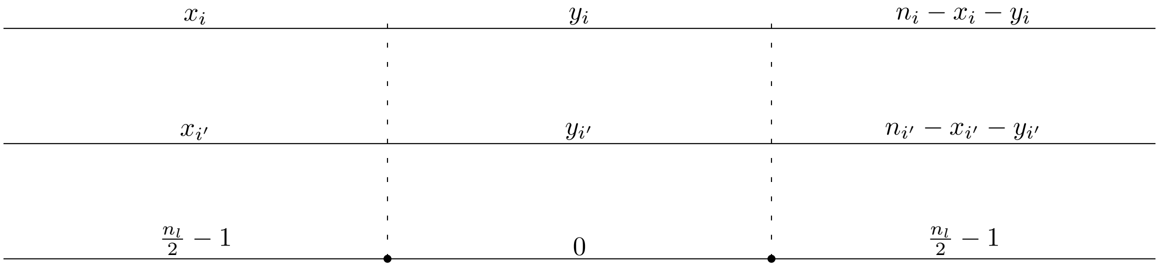

Figure B.16: Sample l Even

$$z_{il} = n_i - 2x_i - y_i \quad z_{li} = n_{i'} - 2x_{i'} - y_{i'}$$

$$f_e^{\mathit{III}}(x_i, y_i, x_{i'}, y_{i'}) = \frac{C_1 C_2 C_3}{C_T} \tag{B.16}$$

Where:

$$C_1 = \begin{pmatrix} x_i + x_{i'} + \frac{n_l}{2} - 1 \\ x_i \quad x_{i'} \quad \frac{n_l}{2} - 1 \end{pmatrix},\ C_2 = \begin{pmatrix} y_i + y_{i'} \\ y_i \end{pmatrix},\ C_3 = \begin{pmatrix} n_i - x_i - y_i + n_{i'} - x_{i'} - y_{i'} + \frac{n_l}{2} - 1 \\ n_i - x_i - y_i \quad n_{i'} - x_{i'} - y_{i'} \quad \frac{n_l}{2} - 1 \end{pmatrix},$$

$$C_T = \begin{pmatrix} n_i + n_{i'} + n_l \\ n_i \quad n_{i'} \quad n_l \end{pmatrix}$$

$$x_i \in \{0, 1, \ldots, n_i\} \quad y_i \in \{0, 1, \ldots, n_i - x_i\} \quad x_{i'} \in \{0, 1, \ldots, n_{i'}\} \quad y_{i'} \in \{0, 1, \ldots, n_{i'} - x_{i'}\}$$

B.5 The Distributions for the Type IV Covariances

B.5.1 Both Sample Sizes Odd

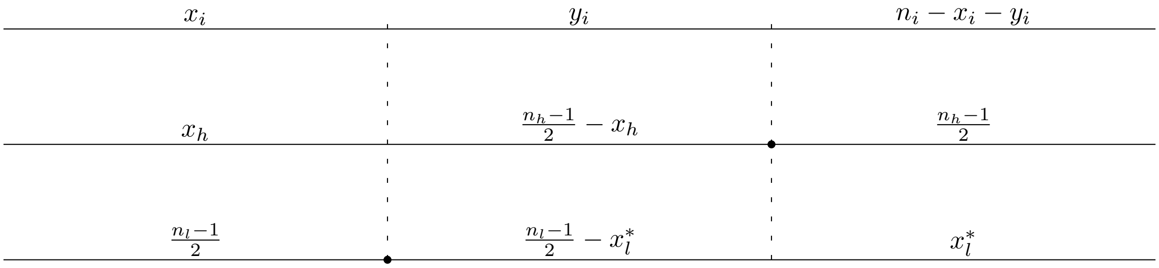

Figure B.17: Type IV Cov - $Cov(z_{ih}, z_{hl})$, Both Sample Sizes Odd, h “high”

$$z_{ih} = n_i - 2x_i - 2y_i \quad z_{hl} = n_h - 2x_h$$

$$f_o^{IVh}(x_i, y_i, x_h) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \frac{C_1 C_2 C_3}{C_T} \tag{B.17}$$

Where:

$$C_1 = \begin{pmatrix} x_i + x_h + \frac{n_l-1}{2} \\ \\ x_i \quad x_h \quad \frac{n_l-1}{2} \end{pmatrix},\ C_2 = \begin{pmatrix} y_i + \frac{n_h-1}{2} - x_h + \frac{n_l-1}{2} - x_l^* \\ \\ y_i \quad \frac{n_h-1}{2} - x_h \quad \frac{n_l-1}{2} - x_l^* \end{pmatrix},$$

$$C_3 = \begin{pmatrix} n_i - x_i - y_i + \frac{n_h-1}{2} + x_l^* \\ \\ n_i - x_i - y_i \quad \frac{n_h-1}{2} \quad x_l^* \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_i \in \{0, 1, \ldots, n_i\} \quad y_i \in \{0, 1, \ldots, n_i - x_i\} \quad x_h \in \left\{0, 1, \ldots, \frac{n_h - 1}{2}\right\}$$

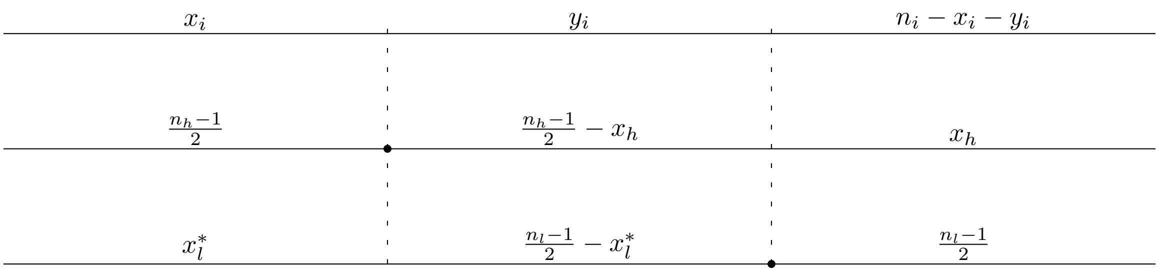

Figure B.18: Both Sample Sizes Odd, h "low"

$$z_{ih} = n_i - 2x_i \quad z_{hl} = 2x_h - n_h$$

$$f_o^{IVl}(x_i, y_i, x_h) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \frac{C_1 C_2 C_3}{C_T} \tag{B.18}$$

Where:

$$C_1 = \begin{pmatrix} x_i + \frac{n_h-1}{2} + x_l^* \\ \\ x_i \quad \frac{n_h-1}{2} \quad x_l^* \end{pmatrix},\ C_2 = \begin{pmatrix} y_i + \frac{n_h-1}{2} - x_h + \frac{n_l-1}{2} - x_l^* \\ \\ y_i \quad \frac{n_h-1}{2} - x_h \quad \frac{n_l-1}{2} - x_l^* \end{pmatrix},$$

$$C_3 = \begin{pmatrix} n_i - x_i - y_i + x_h + \frac{n_l-1}{2} \\ \\ n_i - x_i - y_i \quad x_h \quad \frac{n_l-1}{2} \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_i \in \{0, 1, \ldots, n_i\} \quad y_i \in \{0, 1, \ldots, n_i - x_i\} \quad x_h \in \left\{0, 1, \ldots, \frac{n_h - 1}{2}\right\}$$

B.5.2 Both Sample Sizes Even

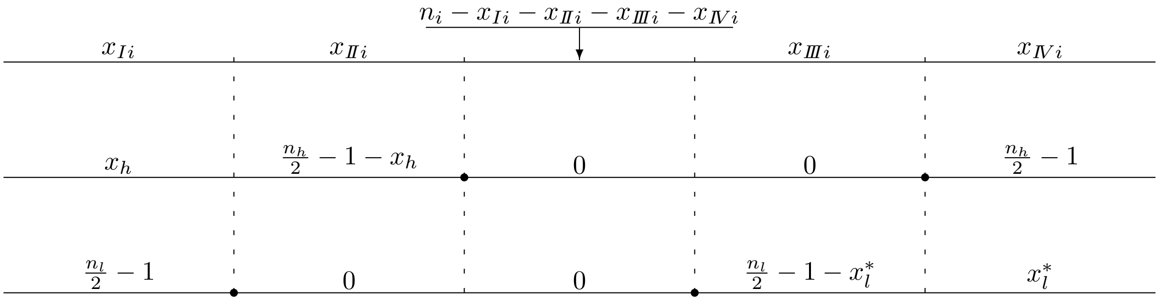

Figure B.19: Both Sample Sizes Even, Overlapping median ranges, h “high”

$$z_{ih} = x_{IVi} - x_{Ii} - x_{IIi} \quad z_{hl} = \frac{n_h}{2} - x_h$$

$$f_e^{IVho}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.19}$$

Where:

$$C_1 = \left(\begin{array}{c} x_{Ii} + x_h + \frac{n_l}{2} - 1 \\ x_{Ii} \quad x_h \quad \frac{n_l}{2} - 1 \end{array}\right),\ C_2 = \left(\begin{array}{c} x_{IIi} + \frac{n_h}{2} - 1 - x_h \\ x_{IIi} \end{array}\right),\ C_3 = \left(\begin{array}{c} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \end{array}\right) = 1,$$

$$C_4 = \left(\begin{array}{c} x_{IIIi} + \frac{n_l}{2} - 1 - x_l^* \\ x_{IIIi} \end{array}\right),\ C_5 = \left(\begin{array}{c} x_{IVi} + \frac{n_h}{2} - 1 + x_l^* \\ x_{IVi} \quad \frac{n_h}{2} - 1 \quad x_l^* \end{array}\right),\ C_T = \left(\begin{array}{c} n_i + n_h + n_l \\ n_i \quad n_h \quad n_l \end{array}\right)$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\} \quad x_h \in \left\{0, 1, \ldots, \tfrac{n_h}{2} - 1\right\}$$

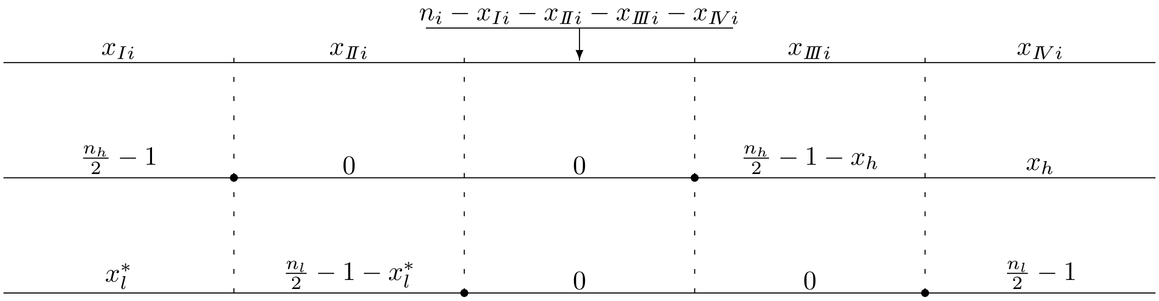

Figure B.20: Both Sample Sizes Even, Overlapping median ranges, h "low"

$$z_{ih} = x_{IIIi} + x_{IVi} - x_{Ii} \quad z_{hl} = x_h - \frac{n_h}{2}$$

$$f_e^{IVlo}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.20}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + \frac{n_h}{2} - 1 + x_l^* \\ x_{Ii} \quad \frac{n_h}{2} - 1 \quad x_l^* \end{pmatrix},\ C_2 = \begin{pmatrix} x_{IIi} + \frac{n_l}{2} - 1 - x_l^* \\ x_{IIi} \end{pmatrix},\ C_3 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \end{pmatrix} = 1,$$

$$C_4 = \begin{pmatrix} x_{IIIi} + \frac{n_h}{2} - 1 - x_h \\ x_{IIIi} \end{pmatrix},\ C_5 = \begin{pmatrix} x_{IVi} + x_h + \frac{n_l}{2} - 1 \\ x_{IVi} \quad x_h \quad \frac{n_l}{2} - 1 \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\} \quad x_h \in \left\{0, 1, \ldots, \tfrac{n_h}{2} - 1\right\}$$

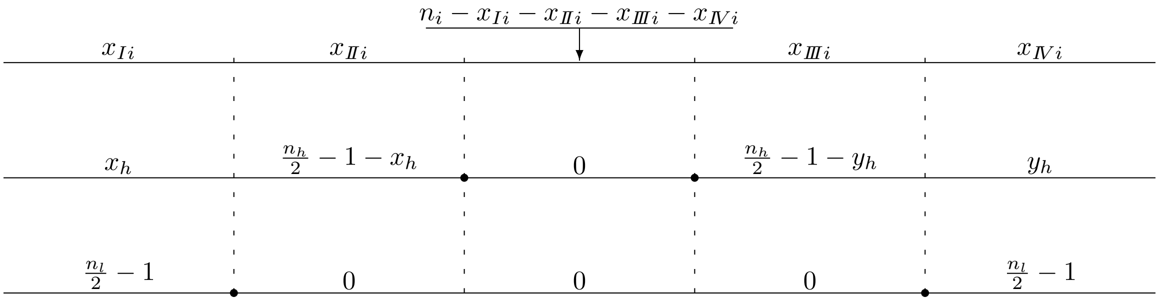

Figure B.21: Both Sample Sizes Even, Contained median ranges, h “inside”

$$z_{ih} = x_{\mathit{III}i} + x_{\mathit{IV}i} - x_{\mathit{I}i} - x_{\mathit{II}i} \quad z_{hl} = y_h - x_h$$

$$f_e^{\mathit{IV}hc}(x_{\mathit{I}i}, x_{\mathit{II}i}, x_{\mathit{III}i}, x_{\mathit{IV}i}, x_h y_h) = \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.21}$$

Where:

$$C_1 = \binom{x_{\mathit{I}i} + x_h + \frac{n_l}{2} - 1}{x_{\mathit{I}i} \quad x_h \quad \frac{n_l}{2} - 1},\ C_2 = \binom{x_{\mathit{II}i} + \frac{n_h}{2} - 1 - x_h}{x_{\mathit{II}i}},\ C_3 = \binom{n_i - x_{\mathit{I}i} - x_{\mathit{II}i} - x_{\mathit{III}i} - x_{\mathit{IV}i}}{n_i - x_{\mathit{I}i} - x_{\mathit{II}i} - x_{\mathit{III}i} - x_{\mathit{IV}i}} = 1,$$

$$C_4 = \binom{x_{\mathit{III}i} + \frac{n_h}{2} - 1 - y_h}{x_{\mathit{III}i}},\ C_5 = \binom{x_{\mathit{IV}i} + y_h + \frac{n_l}{2} - 1}{x_{\mathit{IV}i} \quad y_h \quad \frac{n_l}{2} - 1},\ C_T = \binom{n_i + n_h + n_l}{n_i \quad n_h \quad n_l}$$

$$x_{\mathit{I}i} \in \{0, 1, \ldots, n_i\} \quad x_{\mathit{II}i} \in \{0, 1, \ldots, n_i - x_{\mathit{I}i}\} \quad x_{\mathit{III}i} \in \{0, 1, \ldots, n_i - x_{\mathit{I}i} - x_{\mathit{II}i}\}$$

$$x_{\mathit{IV}i} \in \{0, 1, \ldots, n_i - x_{\mathit{I}i} - x_{\mathit{II}i} - x_{\mathit{III}i}\} \quad x_h \in \left\{0, 1, \ldots, \tfrac{n_h}{2} - 1\right\} \quad y_h \in \left\{0, 1, \ldots, \tfrac{n_h}{2} - 1\right\}$$

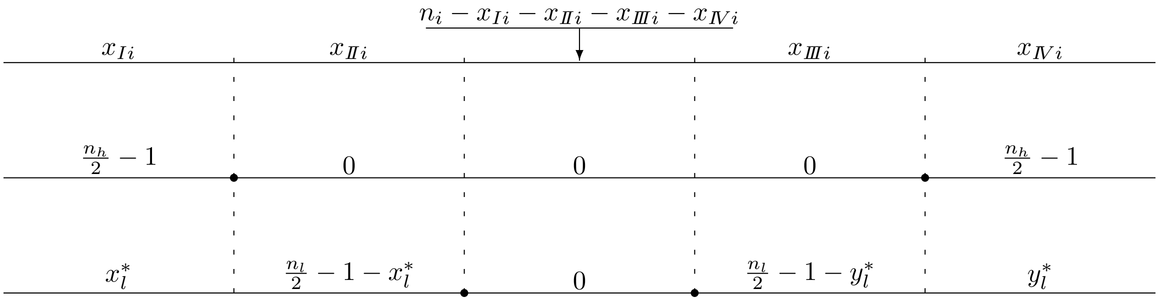

Figure B.22: Both Sample Sizes Even, Contained median ranges, h "outside"

$$z_{ih} = x_{IVi} - x_{Ii} \quad z_{hl} = 0$$

$$f_e^{IVlc}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \sum_{y_l^*=0}^{\frac{n_l}{2}-1} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.22}$$

Where:

$$C_1 = \left(\begin{array}{c} x_{Ii} + \frac{n_h}{2} - 1 + x_l^* \\ x_{Ii} \quad \frac{n_h}{2} - 1 \quad x_l^* \end{array}\right), \; C_2 = \left(\begin{array}{c} x_{IIi} + \frac{n_l}{2} - 1 - x_l^* \\ x_{IIi} \end{array}\right), \; C_3 = \left(\begin{array}{c} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \end{array}\right) = 1,$$

$$C_4 = \left(\begin{array}{c} x_{IIIi} + \frac{n_l}{2} - 1 - y_l^* \\ x_{IIIi} \end{array}\right), \; C_5 = \left(\begin{array}{c} x_{IVi} + \frac{n_h}{2} - 1 + y_l^* \\ x_{IVi} \quad \frac{n_h}{2} - 1 \quad y_l^* \end{array}\right), \; C_T = \left(\begin{array}{c} n_i + n_h + n_l \\ n_i \quad n_h \quad n_l \end{array}\right)$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\}$$

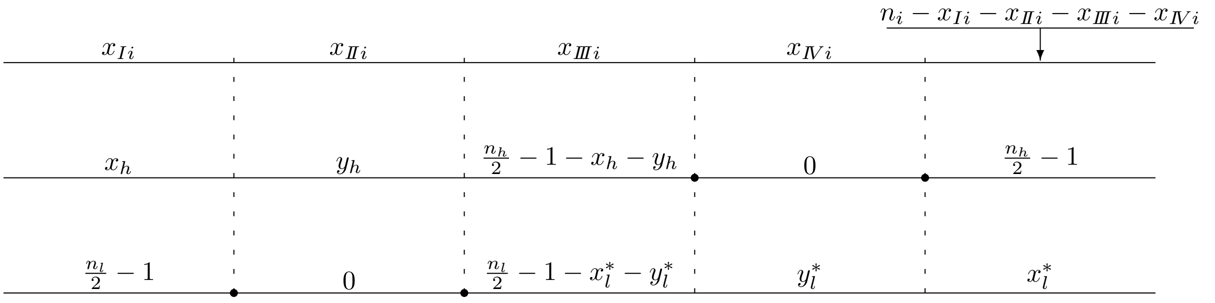

Figure B.23: Both Sample Sizes Even, Disjoint ranges, h "high"

$$z_{ih} = n_i - 2x_{Ii} - 2x_{IIi} - 2x_{IIIi} - x_{IVi} \quad z_{hl} = n_h - 2x_h - y_h$$

$$f_e^{IVhd}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h, y_h) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \sum_{y_l^*=0}^{\frac{n_l}{2}-1-x_l^*} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.23}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + x_h + \frac{n_l}{2} - 1 \\ x_{Ii} \quad x_h \quad \frac{n_l}{2} - 1 \end{pmatrix}, \; C_2 = \begin{pmatrix} x_{IIi} + y_h \\ x_{IIi} \end{pmatrix}, \; C_3 = \begin{pmatrix} x_{IIIi} + \frac{n_h}{2} - 1 - x_h - y_h + \frac{n_l}{2} - 1 - x_l^* - y_l^* \\ x_{IIIi} \quad \frac{n_h}{2} - 1 - x_h - y_h \quad \frac{n_l}{2} - 1 - x_l^* - y_l^* \end{pmatrix},$$

$$C_4 = \begin{pmatrix} x_{IVi} + y_l^* \\ x_{IVi} \end{pmatrix}, \; C_5 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} + \frac{n_h}{2} - 1 + x_l^* \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \quad \frac{n_h}{2} - 1 \quad x_l^* \end{pmatrix}, \; C_T = \begin{pmatrix} n_i + n_h + n_l \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\} x_h \in \left\{0, 1, \ldots, \frac{n_h}{2} - 1\right\} \quad y_h \in \left\{0, 1, \ldots, \frac{n_h}{2} - 1 - x_h\right\}$$

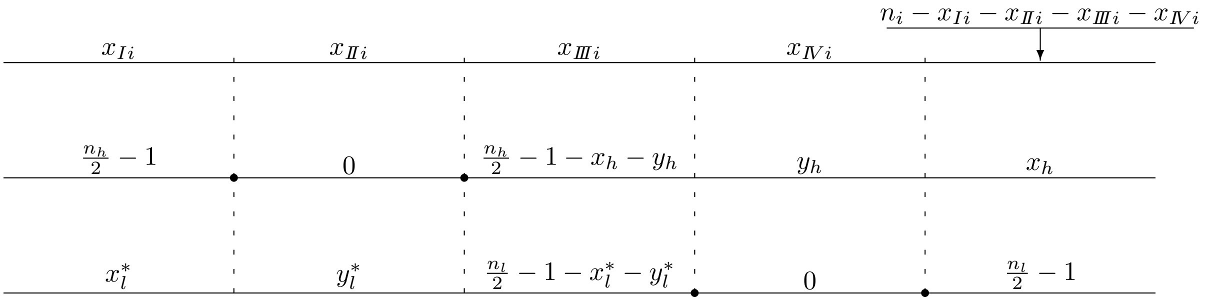

Figure B.24: Both Sample Sizes Even, Disjoint ranges, h "low"

$$z_{ih} = n_i - 2x_{Ii} - x_{IIi} \quad z_{hl} = 2x_h + y_h - n_h$$

$$f_e^{IVld}(x_{Ii}, x_{IIi}, x_{IIIi}, x_{IVi}, x_h, y_h) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \sum_{y_l^*=0}^{\frac{n_l}{2}-1-x_l^*} \frac{C_1 C_2 C_3 C_4 C_5}{C_T} \tag{B.24}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + \frac{n_h}{2} - 1 + x_l^* \\ x_{Ii} \quad \frac{n_h}{2} - 1 \quad x_l^* \end{pmatrix},\ C_2 = \begin{pmatrix} x_{IIi} + y_l^* \\ x_{IIi} \end{pmatrix},\ C_3 = \begin{pmatrix} x_{IIIi} + \frac{n_h}{2} - 1 - x_h - y_h + \frac{n_l}{2} - 1 - x_l^* - y_l^* \\ x_{IIIi} \quad \frac{n_h}{2} - 1 - x_h - y_h \quad \frac{n_l}{2} - 1 - x_l^* - y_l^* \end{pmatrix},$$

$$C_4 = \begin{pmatrix} x_{IVi} + y_h \\ x_{IVi} \end{pmatrix},\ C_5 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} + x_h + \frac{n_l}{2} - 1 \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} - x_{IVi} \quad x_h \quad \frac{n_l}{2} - 1 \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\}$$

$$x_{IVi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi} - x_{IIIi}\} x_h \in \left\{0, 1, \ldots, \tfrac{n_h}{2} - 1\right\} \quad y_h \in \left\{0, 1, \ldots, \tfrac{n_h}{2} - 1 - x_h\right\}$$

B.5.3 Sample Size h Odd/Sample Size l Even

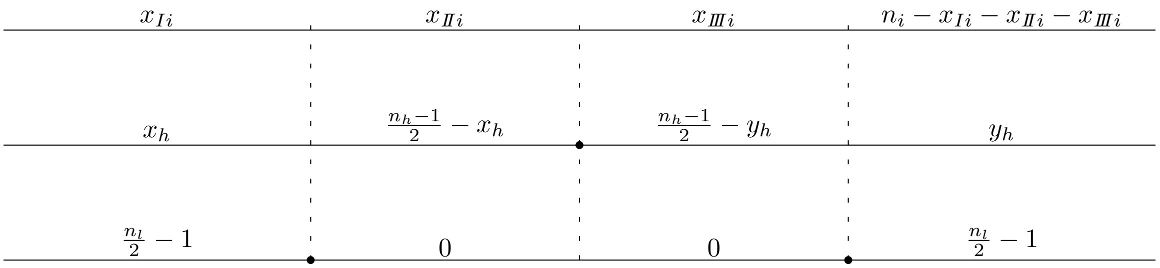

Figure B.25: Sample h Odd, Sample l Even, h in the median range of l

$$z_{ih} = n_i - 2x_{Ii} - 2x_{\mathit{II}i} \quad z_{hl} = y_h - x_h$$

$$f_{oe}^{\mathit{IV}c}(x_{Ii}, x_{\mathit{II}i}, x_{\mathit{III}i}, x_h, y_h) = \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.25}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + x_h + \frac{n_l}{2} - 1 \\ \\ x_{Ii} \quad x_h \quad \frac{n_l}{2} - 1 \end{pmatrix},\ C_2 = \begin{pmatrix} x_{\mathit{II}i} + \frac{n_h-1}{2} - x_h \\ \\ x_{\mathit{II}i} \end{pmatrix},\ C_3 = \begin{pmatrix} x_{\mathit{III}i} + \frac{n_h-1}{2} - y_h \\ \\ x_{\mathit{III}i} \end{pmatrix},$$

$$C_4 = \begin{pmatrix} n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} + y_h + \frac{n_l}{2} - 1 \\ \\ n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} \quad y_h \quad \frac{n_l}{2} - 1 \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{\mathit{II}i} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{\mathit{III}i} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{\mathit{II}i}\}$$

$$x_h \in \left\{0, 1, \ldots, \tfrac{n_h-1}{2}\right\} \quad y_h \in \left\{0, 1, \ldots, \tfrac{n_h-1}{2}\right\}$$

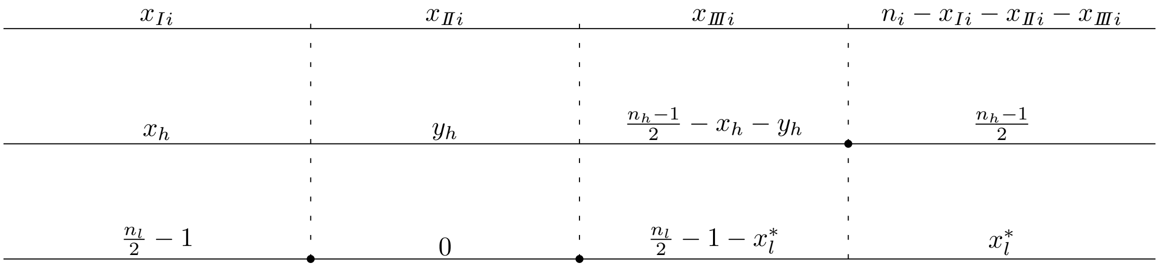

Figure B.26: Sample h Odd, Sample l Even, h not in the median range of l, h "high"

$$z_{ih} = n_i - 2x_{Ii} - 2x_{\mathit{II}i} - 2x_{\mathit{III}i} \quad z_{hl} = n_h - 2x_h - y_h$$

$$f_{oe}^{\mathit{IV}h}(x_{Ii}, x_{\mathit{II}i}, x_{\mathit{III}i}, x_h, y_h) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.26}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + x_h + \frac{n_l}{2} - 1 \\ x_{Ii} \quad x_h \quad \frac{n_l}{2} - 1 \end{pmatrix},\ C_2 = \begin{pmatrix} x_{\mathit{II}i} + y_h \\ x_{\mathit{II}i} \end{pmatrix},\ C_3 = \begin{pmatrix} x_{\mathit{III}i} + \frac{n_h-1}{2} - x_h - y_h + \frac{n_l}{2} - 1 - x_l^* \\ x_{\mathit{III}i} \quad \frac{n_h-1}{2} - x_h - y_h \quad \frac{n_l}{2} - 1 - x_l^* \end{pmatrix},$$

$$C_4 = \begin{pmatrix} n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} + \frac{n_h-1}{2} + x_l^* \\ n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} \quad \frac{n_h-1}{2} \quad x_l^* \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{\mathit{II}i} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{\mathit{III}i} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{\mathit{II}i}\}$$

$$x_h \in \left\{0, 1, \ldots, \tfrac{n_h-1}{2}\right\} \quad y_h \in \left\{0, 1, \ldots, \tfrac{n_h-1}{2} - x_h\right\}$$

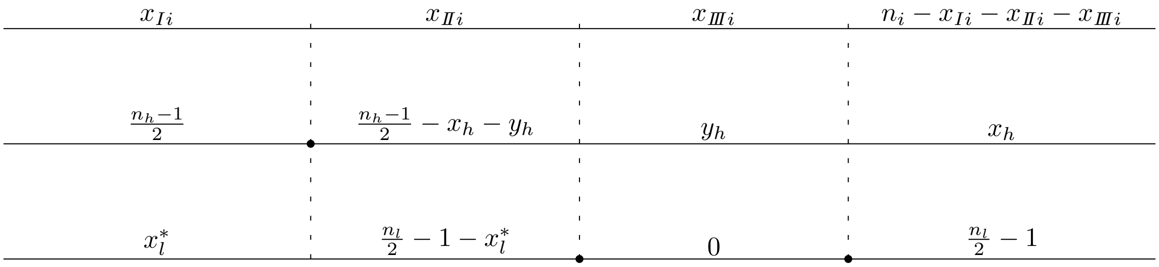

Figure B.27: Sample h Odd, Sample l Even, h not in the median range of l, h "low"

$$z_{ih} = n_i - 2x_{\mathit{I}i} \quad z_{hl} = 2x_h + y_h - n_h$$

$$f_{oe}^{\mathit{IV}l}(x_{\mathit{I}i}, x_{\mathit{II}i}, x_{\mathit{III}i}, x_h, y_h) = \sum_{x_l^*=0}^{\frac{n_l}{2}-1} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.27}$$

Where:

$$C_1 = \left(\begin{array}{c} x_{\mathit{I}i} + \frac{n_h-1}{2} + x_l^* \\ \\ x_{\mathit{I}i} \quad \frac{n_h-1}{2} \quad x_l^* \end{array} \right), \; C_2 = \left(\begin{array}{c} x_{\mathit{II}i} + \frac{n_h-1}{2} - x_h - y_h + \frac{n_l}{2} - 1 - x_l^* \\ \\ x_{\mathit{II}i} \quad \frac{n_h-1}{2} - x_h - y_h \quad \frac{n_l}{2} - 1 - x_l^* \end{array} \right), \; C_3 = \left(\begin{array}{c} x_{\mathit{III}i} + y_h \\ \\ x_{\mathit{III}i} \end{array} \right),$$

$$C_4 = \left(\begin{array}{c} n_i - x_{\mathit{I}i} - x_{\mathit{II}i} - x_{\mathit{III}i} + x_h + \frac{n_l}{2} - 1 \\ \\ n_i - x_{\mathit{I}i} - x_{\mathit{II}i} - x_{\mathit{III}i} \quad x_h \quad \frac{n_l}{2} - 1 \end{array} \right), \; C_T = \left(\begin{array}{c} n_i + n_h + n_l \\ \\ n_i \quad n_h \quad n_l \end{array} \right)$$

$$x_{\mathit{I}i} \in \{0, 1, \ldots, n_i\} \quad x_{\mathit{II}i} \in \{0, 1, \ldots, n_i - x_{\mathit{I}i}\} \quad x_{\mathit{III}i} \in \{0, 1, \ldots, n_i - x_{\mathit{I}i} - x_{\mathit{II}i}\}$$

$$x_h \in \left\{0, 1, \ldots, \tfrac{n_h-1}{2}\right\} \quad y_h \in \left\{0, 1, \ldots, \tfrac{n_h-1}{2} - x_h\right\}$$

B.5.4 Sample Size h Even/Sample Size l Odd

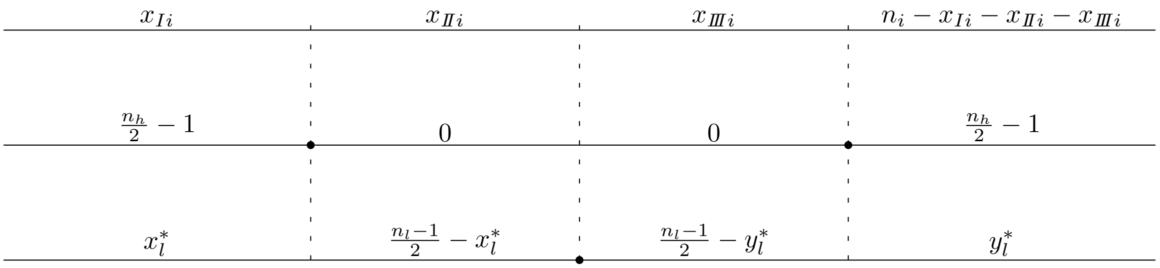

Figure B.28: Sample h Even, Sample l Odd, l in the median range of h

$$z_{ih} = n_i - 2x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} \quad z_{hl} = 0$$

$$f_{eo}^{\mathit{IV}c}(x_{Ii}, x_{\mathit{II}i}, x_{\mathit{III}i}) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \sum_{y_l^*=0}^{\frac{n_l-1}{2}} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.28}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + \frac{n_h}{2} - 1 + x_l^* \\ \\ x_{Ii} \quad \frac{n_h}{2} - 1 \quad x_l^* \end{pmatrix},\ C_2 = \begin{pmatrix} x_{\mathit{II}i} + \frac{n_l-1}{2} - x_l^* \\ \\ x_{\mathit{II}i} \end{pmatrix},$$

$$C_3 = \begin{pmatrix} x_{\mathit{III}i} + \frac{n_l-1}{2} - y_l^* \\ \\ x_{\mathit{III}i} \end{pmatrix},\ C_4 = \begin{pmatrix} n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} + \frac{n_h}{2} - 1 + y_l^* \\ \\ n_i - x_{Ii} - x_{\mathit{II}i} - x_{\mathit{III}i} \quad \frac{n_h}{2} - 1 \quad y_l^* \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{\mathit{II}i} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{\mathit{III}i} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{\mathit{II}i}\}$$

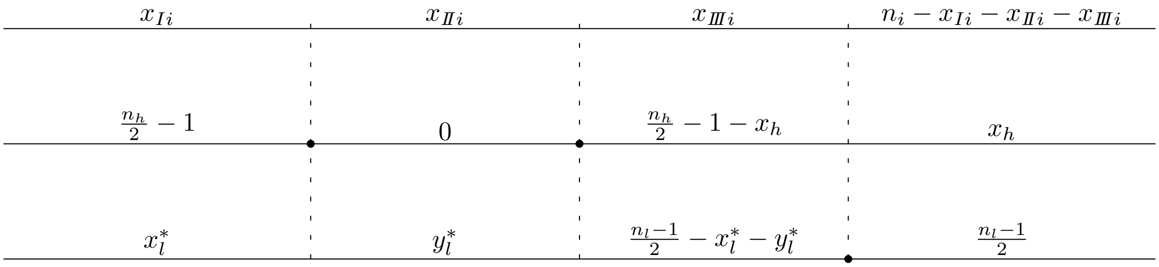

Figure B.29: Sample h Even, Sample l Odd, l not in the median range of h, l "high"

$$z_{ih} = n_i - 2x_{Ii} - x_{IIi} \quad z_{hl} = 2x_h - n_h$$

$$f_{eo}^{IVl}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \sum_{y_l^*=0}^{\frac{n_l-1}{2}-x_l^*} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.29}$$

Where:

$$C_1 = \begin{pmatrix} x_{Ii} + \frac{n_h}{2} - 1 + x_l^* \\ \\ x_{Ii} \quad \frac{n_h}{2} - 1 \quad x_l^* \end{pmatrix},\ C_2 = \begin{pmatrix} x_{IIi} + y_l^* \\ \\ x_{IIi} \end{pmatrix},\ C_3 = \begin{pmatrix} x_{IIIi} + \frac{n_h}{2} - 1 - x_h + \frac{n_l-1}{2} - x_l^* - y_l^* \\ \\ x_{IIIi} \quad \frac{n_h}{2} - 1 - x_h \quad \frac{n_l-1}{2} - x_l^* - y_l^* \end{pmatrix},$$

$$C_4 = \begin{pmatrix} n_i - x_{Ii} - x_{IIi} - x_{IIIi} + x_h + \frac{n_l-1}{2} \\ \\ n_i - x_{Ii} - x_{IIi} - x_{IIIi} \quad x_h \quad \frac{n_l-1}{2} \end{pmatrix},\ C_T = \begin{pmatrix} n_i + n_h + n_l \\ \\ n_i \quad n_h \quad n_l \end{pmatrix}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\} \quad x_h \in \left\{0, 1, \ldots, \frac{n_h}{2} - 1\right\}$$

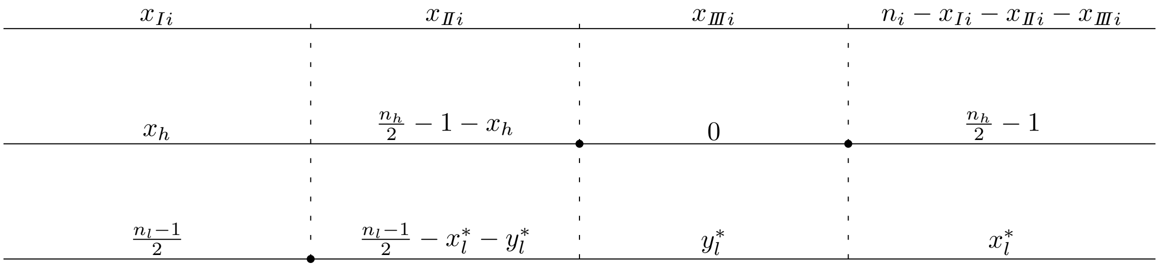

Figure B.30: Sample h Even, Sample l Odd, l not in the median range of h, l "low"

$$z_{ih} = n_i - 2x_{Ii} - 2x_{IIi} - x_{IIIi} \quad z_{hl} = n_h - 2x_h$$

$$f_{eo}^{IVh}(x_{Ii}, x_{IIi}, x_{IIIi}, x_h) = \sum_{x_l^*=0}^{\frac{n_l-1}{2}} \sum_{y_l^*=0}^{\frac{n_l-1}{2}-x_l^*} \frac{C_1 C_2 C_3 C_4}{C_T} \tag{B.30}$$

Where:

$$C_1 = \binom{x_{Ii} + x_h + \frac{n_l-1}{2}}{x_{Ii} \quad x_h \quad \frac{n_l-1}{2}},\ C_2 = \binom{x_{IIi} + \frac{n_h}{2} - 1 - x_h + \frac{n_l-1}{2} - x_l^* - y_l^*}{x_{IIi} \quad \frac{n_h}{2} - 1 - x_h \quad \frac{n_l-1}{2} - x_l^* - y_l^*},\ C_3 = \binom{x_{IIIi} + y_l^*}{x_{IIIi}},$$

$$C_4 = \binom{n_i - x_{Ii} - x_{IIi} - x_{IIIi} + \frac{n_h}{2} - 1 + x_l^*}{n_i - x_{Ii} - x_{IIi} - x_{IIIi} \quad \frac{n_h}{2} - 1 \quad x_l^*},\ C_T = \binom{n_i + n_h + n_l}{n_i \quad n_h \quad n_l}$$

$$x_{Ii} \in \{0, 1, \ldots, n_i\} \quad x_{IIi} \in \{0, 1, \ldots, n_i - x_{Ii}\} \quad x_{IIIi} \in \{0, 1, \ldots, n_i - x_{Ii} - x_{IIi}\} \quad x_h \in \left\{0, 1, \ldots, \frac{n_h}{2} - 1\right\}$$

Appendix C

Setting 1: Three Standard Normal Distributions

This appendix contains tables of the results for Setting 1 of the simulation studies. In each case 100,000 Monte Carlo data sets were generated and for each the three tests were conducted. Whether each test would reject at each of the specified levels was monitored.

Medians	Sample Sizes	Table	Page
0,0,0	All	C.2	119
0,0,1	Equal	C.3	120
0,0,1	Unequal	C.4	120
0,0,2	Equal	C.5	121
0,0,2	Unequal	C.6	121
0,1,2	Equal	C.7	122
0,1,2	Unequal	C.8	123

Table C.1: List of Tables for Setting 1

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0096	0.0032	0.0026	0.0492	0.0436	0.0369	0.1004	0.0922	0.0903
6,6,6	0.0101	0.0045	0.0079	0.0498	0.0420	0.0496	0.0985	0.0977	0.0947
7,7,7	0.0101	0.0054	0.0061	0.0491	0.0442	0.0329	0.0996	0.0977	0.0655
8,8,8	0.0103	0.0064	0.0093	0.0504	0.0456	0.0373	0.1009	0.0977	0.0857
9,9,9	0.0100	0.0064	0.0060	0.0507	0.0461	0.0452	0.1004	0.0988	0.0756
10,10,10	0.0104	0.0070	0.0075	0.0497	0.0460	0.0447	0.1012	0.0990	0.0883
5,6,7	0.0096	0.0041	0.0094	0.0492	0.0417	0.0450	0.1000	0.0956	0.0898
6,7,8	0.0102	0.0056	0.0063	0.0499	0.0442	0.0497	0.0993	0.0974	0.0895
5,8,11	0.0098	0.0056	0.0061	0.0498	0.0452	0.0421	0.0995	0.0978	0.0878
6,9,12	0.0103	0.0064	0.0076	0.0505	0.0459	0.0399	0.1006	0.0989	0.0812

Table C.2: Setting 1: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0998	0.0387	0.0250	0.2822	0.2472	0.1832	0.4183	0.3833	0.3415
6,6,6	0.1366	0.0701	0.0827	0.3476	0.2990	0.2761	0.4869	0.4660	0.4053
7,7,7	0.1787	0.1124	0.0835	0.4107	0.3704	0.2697	0.5511	0.5254	0.3871
8,8,8	0.2246	0.1595	0.1491	0.4724	0.4308	0.3335	0.6109	0.5837	0.4947
9,9,9	0.2723	0.2056	0.1381	0.5292	0.4894	0.4144	0.6640	0.6366	0.5135
10,10,10	0.3170	0.2512	0.1930	0.5800	0.5400	0.4553	0.7070	0.6823	0.5913

Table C.3: Setting 1: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1493	0.0876	0.1069	0.3660	0.3240	0.2790	0.5072	0.4785	0.4082
5,7,6	0.1354	0.0709	0.0906	0.3447	0.3013	0.2693	0.4857	0.4599	0.4018
6,7,5	0.1220	0.0529	0.0807	0.3182	0.2677	0.2331	0.4542	0.4190	0.3543
Average	0.1356	0.0705	0.0927	0.3430	0.2977	0.2605	0.4824	0.4525	0.3881
6,7,8	0.1935	0.1327	0.1022	0.4325	0.3936	0.3518	0.5743	0.5490	0.4683
6,8,7	0.1795	0.1142	0.0901	0.4111	0.3706	0.3239	0.5534	0.5280	0.4382
7,8,6	0.1600	0.0973	0.0776	0.3826	0.3352	0.3036	0.5222	0.4922	0.4202
Average	0.1777	0.1147	0.0900	0.4087	0.3665	0.3264	0.5500	0.5231	0.4422
5,8,11	0.2572	0.1946	0.1335	0.5148	0.4779	0.3862	0.6517	0.6283	0.5294
5,11,8	0.2225	0.1574	0.0995	0.4710	0.4323	0.3383	0.6093	0.5844	0.4839
8,11,5	0.1512	0.0801	0.0639	0.3621	0.3136	0.2466	0.4977	0.4650	0.3837
Average	0.2103	0.1440	0.0990	0.4493	0.4079	0.3237	0.5862	0.5592	0.4657
6,9,12	0.3085	0.2432	0.1939	0.5730	0.5360	0.4276	0.7040	0.6815	0.5653
6,12,9	0.2714	0.2065	0.1481	0.5295	0.4891	0.3674	0.6639	0.6377	0.5106
9,12,6	0.1941	0.1220	0.1059	0.4252	0.3783	0.2969	0.5640	0.5323	0.4387
Average	0.2580	0.1906	0.1493	0.5092	0.4678	0.3640	0.6440	0.6172	0.5049

Table C.4: Setting 1: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.5497	0.2875	0.1143	0.8271	0.7727	0.5399	0.9111	0.8799	0.8133
6,6,6	0.7021	0.5012	0.3481	0.9072	0.8636	0.7302	0.9575	0.9439	0.8791
7,7,7	0.8162	0.6843	0.3752	0.9530	0.9305	0.8045	0.9802	0.9722	0.9015
8,8,8	0.8897	0.8070	0.5804	0.9769	0.9644	0.8756	0.9916	0.9868	0.9589
9,9,9	0.9393	0.8891	0.6018	0.9891	0.9823	0.9470	0.9963	0.9940	0.9727
10,10,10	0.9666	0.9369	0.7494	0.9951	0.9915	0.9673	0.9985	0.9973	0.9888

Table C.5: Setting 1: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.7383	0.5860	0.4438	0.9261	0.8945	0.7561	0.9678	0.9550	0.8905
5,7,6	0.7014	0.4989	0.3512	0.9067	0.8643	0.7781	0.9571	0.9425	0.8926
6,7,5	0.6433	0.3843	0.2991	0.8756	0.8137	0.6227	0.9394	0.9121	0.7792
Average	0.6943	0.4897	0.3647	0.9028	0.8575	0.7190	0.9548	0.9365	0.8541
6,7,8	0.8406	0.7415	0.4653	0.9624	0.9461	0.8801	0.9856	0.9793	0.9405
6,8,7	0.8125	0.6816	0.3849	0.9529	0.9307	0.8350	0.9805	0.9718	0.9179
7,8,6	0.7731	0.6139	0.3272	0.9364	0.9014	0.7761	0.9719	0.9583	0.8963
Average	0.8087	0.6790	0.3925	0.9506	0.9261	0.8304	0.9793	0.9698	0.9182
5,8,11	0.9261	0.8795	0.6377	0.9867	0.9801	0.9310	0.9954	0.9932	0.9756
5,11,8	0.8892	0.8052	0.4373	0.9759	0.9636	0.8890	0.9910	0.9862	0.9567
8,11,5	0.7489	0.5242	0.2531	0.9223	0.8734	0.6556	0.9646	0.9442	0.8518
Average	0.8547	0.7363	0.4427	0.9616	0.9390	0.8252	0.9837	0.9745	0.9280
6,9,12	0.9614	0.9327	0.7993	0.9944	0.9911	0.9612	0.9984	0.9975	0.9860
6,12,9	0.9379	0.8880	0.5964	0.9897	0.9820	0.9127	0.9966	0.9941	0.9681
9,12,6	0.8464	0.7049	0.4134	0.9615	0.9357	0.7902	0.9837	0.9737	0.9209
Average	0.9152	0.8419	0.6030	0.9819	0.9696	0.8880	0.9929	0.9884	0.9583

Table C.6: Setting 1: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.4002	0.1988	0.1538	0.7032	0.6423	0.5240	0.8219	0.7832	0.7233
6,6,6	0.5408	0.3653	0.4011	0.8082	0.7537	0.6999	0.8956	0.8736	0.8144
7,7,7	0.6571	0.5181	0.4381	0.8774	0.8428	0.7289	0.939	0.9231	0.8310
8,8,8	0.7545	0.6515	0.6217	0.9256	0.8988	0.8228	0.9663	0.9547	0.9128
9,9,9	0.8302	0.7499	0.6363	0.9544	0.9361	0.8891	0.9798	0.9728	0.9312
10,10,10	0.8850	0.8263	0.7592	0.9730	0.9607	0.9270	0.9891	0.9847	0.9636

Table C.7: Setting 1: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.5222	0.3510	0.4094	0.7965	0.7429	0.6595	0.8890	0.8609	0.7806
5,7,6	0.4831	0.3034	0.3855	0.7630	0.7023	0.6501	0.8642	0.8344	0.7702
6,5,7	0.5790	0.4164	0.4234	0.8348	0.7865	0.7080	0.9141	0.8937	0.8277
6,7,5	0.4847	0.3047	0.3878	0.7662	0.7044	0.6517	0.8670	0.8380	0.7749
7,5,6	0.5791	0.4155	0.4249	0.8363	0.7869	0.7101	0.9146	0.8937	0.8282
7,6,5	0.5227	0.3529	0.4108	0.7977	0.7427	0.6615	0.8877	0.8605	0.7793
Average	0.5285	0.3573	0.4070	0.7991	0.7443	0.6735	0.8894	0.8635	0.7935
6,7,8	0.6469	0.5115	0.4539	0.8716	0.8324	0.7792	0.9360	0.9162	0.8615
6,8,7	0.6125	0.4713	0.4389	0.8506	0.8070	0.7563	0.9223	0.9030	0.8418
7,6,8	0.6089	0.4689	0.4371	0.8471	0.8041	0.7527	0.9186	0.8994	0.8386
7,8,6	0.6110	0.4712	0.4396	0.8487	0.8051	0.7541	0.9205	0.9011	0.8395
8,6,7	0.6949	0.5665	0.4739	0.8977	0.8674	0.8004	0.9507	0.9370	0.8791
8,7,6	0.6465	0.5121	0.4561	0.8735	0.8325	0.7805	0.9354	0.9171	0.8622
Average	0.6368	0.5003	0.4499	0.8649	0.8248	0.7705	0.9306	0.9123	0.8538
5,8,11	0.7009	0.5712	0.5026	0.8976	0.8623	0.7830	0.9497	0.9340	0.8786
5,11,8	0.6200	0.4819	0.4491	0.8506	0.8078	0.7366	0.9203	0.8993	0.8392
8,5,11	0.8328	0.7469	0.5505	0.9564	0.9407	0.8630	0.9816	0.9758	0.9346
8,11,5	0.6189	0.4829	0.4487	0.8492	0.8064	0.7353	0.9196	0.8984	0.8394
11,5,8	0.8317	0.7449	0.5505	0.9574	0.9403	0.8622	0.9819	0.9762	0.9349
11,8,5	0.7000	0.5705	0.5041	0.8978	0.8633	0.7834	0.9501	0.9347	0.8768
Average	0.7174	0.5997	0.5009	0.9015	0.8701	0.7939	0.9505	0.9364	0.8839
6,9,12	0.7917	0.6933	0.6430	0.9389	0.9162	0.8518	0.9719	0.9619	0.9215
6,12,9	0.7207	0.6130	0.5882	0.9047	0.8739	0.8046	0.9531	0.9382	0.8864
9,6,12	0.8885	0.8288	0.6971	0.9746	0.9634	0.8994	0.9901	0.9856	0.9510
9,12,6	0.7212	0.6114	0.5897	0.9075	0.8755	0.8072	0.9545	0.9403	0.8891
12,6,9	0.8883	0.8274	0.6960	0.9760	0.9645	0.8983	0.9902	0.9863	0.9514
12,9,6	0.7943	0.6942	0.6443	0.9387	0.9154	0.8520	0.9718	0.9621	0.9214
Average	0.8008	0.7114	0.6431	0.9401	0.9182	0.8522	0.9719	0.9624	0.9201

Table C.8: Setting 1: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

Appendix D

Setting 2: Three Normal Distributions, with Differing Variances

This appendix contains tables of the results for Settings 2a and 2b of the simulation studies. In each case 100,000 Monte Carlo data sets were generated and for each the three tests were conducted and proportion of tests rejecting is displayed.

D.1 Setting 2a: Standard Deviations 1, 1.5, and 2

Medians	Sample Sizes	Table	Page
0,0,0	Equal	D.2	126
0,0,0	Unequal	D.3	127
0,0,1	Equal	D.4	128
0,0,1	Unequal	D.5	129
0,1,0	Equal	D.6	130
0,1,0	Unequal	D.7	131
1,0,0	Equal	D.8	132
1,0,0	Unequal	D.9	133
0,0,2	Equal	D.10	134
0,0,2	Unequal	D.11	135
0,2,0	Equal	D.12	136
0,2,0	Unequal	D.13	137
2,0,0	Equal	D.14	138
2,0,0	Unequal	D.15	139
0,1,2	Equal	D.16	140
0,1,2	Unequal	D.17	141
0,2,1	Equal	D.18	142
0,2,1	Unequal	D.19	143
1,0,2	Equal	D.20	144
1,0,2	Unequal	D.21	145
1,2,0	Equal	D.22	146
1,2,0	Unequal	D.23	147
2,0,1	Equal	D.24	148
2,0,1	Unequal	D.25	149
2,1,0	Equal	D.26	150
2,1,0	Unequal	D.27	151

Table D.1: List of Tables for Setting 2a

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0143	0.0052	0.0046	0.0575	0.0488	0.0481	0.1069	0.0961	0.1091
6,6,6	0.0147	0.0063	0.0129	0.0611	0.0487	0.0668	0.1116	0.1091	0.1219
7,7,7	0.0144	0.0074	0.0099	0.0587	0.0506	0.0488	0.1084	0.1061	0.0883
8,8,8	0.0144	0.0086	0.0152	0.0572	0.0506	0.0534	0.1057	0.1019	0.1129
9,9,9	0.0143	0.0083	0.0110	0.0576	0.0519	0.0660	0.1067	0.1048	0.1033
10,10,10	0.0142	0.0091	0.0142	0.0565	0.0509	0.0654	0.1060	0.1043	0.1186

Table D.2: Setting 2a: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0096	0.0049	0.0107	0.0449	0.0403	0.0467	0.0873	0.0893	0.0911
5,7,6	0.0123	0.0054	0.0115	0.0518	0.0425	0.0511	0.0981	0.0946	0.0967
6,5,7	0.0115	0.0057	0.0138	0.0506	0.0450	0.0594	0.0963	0.0963	0.1126
6,7,5	0.0171	0.0069	0.0169	0.0667	0.0509	0.0697	0.1206	0.1053	0.1290
7,5,6	0.0165	0.0073	0.0183	0.0650	0.0522	0.0685	0.1190	0.1098	0.1238
7,6,5	0.0203	0.0082	0.0207	0.0759	0.0569	0.0763	0.1330	0.1144	0.1367
Average	0.0146	0.0064	0.0153	0.0592	0.0480	0.0620	0.1091	0.1016	0.1150
6,7,8	0.0107	0.0061	0.0082	0.0465	0.0438	0.0561	0.0894	0.0936	0.0990
6,8,7	0.0129	0.0069	0.0090	0.0527	0.0461	0.0623	0.0990	0.0987	0.1082
7,6,8	0.0176	0.0089	0.0114	0.0663	0.0518	0.0693	0.1197	0.1081	0.1169
7,8,6	0.0174	0.0085	0.0112	0.0651	0.0520	0.0693	0.1180	0.1079	0.1158
8,6,7	0.0162	0.0083	0.0132	0.0630	0.0529	0.0778	0.1145	0.1101	0.1284
8,7,6	0.0194	0.0098	0.0144	0.0729	0.0568	0.0821	0.1290	0.1151	0.1338
Average	0.0157	0.0081	0.0112	0.0611	0.0506	0.0695	0.1116	0.1056	0.1170
5,8,11	0.0065	0.0045	0.0038	0.0319	0.0335	0.0294	0.0654	0.0764	0.0641
5,11,8	0.0105	0.0059	0.0056	0.0461	0.0413	0.0373	0.0884	0.0894	0.0805
8,5,11	0.0082	0.0055	0.0071	0.0390	0.0405	0.0470	0.0783	0.0906	0.0936
8,11,5	0.0230	0.0097	0.0143	0.0808	0.0600	0.0775	0.1411	0.1183	0.1403
11,5,8	0.0203	0.0104	0.0162	0.0740	0.0628	0.0795	0.1328	0.1237	0.1459
11,8,5	0.0313	0.0128	0.0206	0.1013	0.0714	0.0977	0.1692	0.1351	0.1747
Average	0.0166	0.0081	0.0113	0.0622	0.0516	0.0614	0.1125	0.1056	0.1165
6,9,12	0.0070	0.0051	0.0057	0.0328	0.0357	0.0319	0.0676	0.0797	0.0682
6,12,9	0.0109	0.0065	0.0081	0.0457	0.0411	0.0406	0.0881	0.0881	0.0835
9,6,12	0.0090	0.0064	0.0099	0.0417	0.0436	0.0451	0.0820	0.0939	0.0884
9,12,6	0.0221	0.0107	0.0166	0.0789	0.0584	0.0664	0.1358	0.1172	0.1248
12,6,9	0.0186	0.0103	0.0203	0.0722	0.0611	0.0783	0.1287	0.1214	0.1415
12,9,6	0.0290	0.0138	0.0259	0.0948	0.0680	0.0893	0.1605	0.1311	0.1554
Average	0.0161	0.0088	0.0144	0.0610	0.0513	0.0586	0.1105	0.1052	0.1103

Table D.3: Setting 2a: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0515	0.0198	0.0142	0.1536	0.1247	0.1121	0.2404	0.2054	0.2139
6,6,6	0.0627	0.0300	0.0441	0.1744	0.1361	0.1578	0.2665	0.2410	0.2439
7,7,7	0.0762	0.0408	0.0413	0.2011	0.1634	0.1424	0.2967	0.2674	0.2186
8,8,8	0.0909	0.0535	0.0678	0.2235	0.1839	0.1713	0.3247	0.2902	0.2830
9,9,9	0.1040	0.0639	0.0605	0.2474	0.2037	0.2143	0.3515	0.3144	0.2861
10,10,10	0.1206	0.0776	0.0831	0.2735	0.2272	0.2325	0.3812	0.3408	0.3351

Table D.4: Setting 2a: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0532	0.0300	0.0428	0.1565	0.1327	0.1325	0.2456	0.2314	0.2146
5,7,6	0.0585	0.0280	0.0417	0.1623	0.1290	0.1360	0.2508	0.2260	0.2172
6,5,7	0.0585	0.0322	0.0529	0.1687	0.1422	0.1555	0.2614	0.2428	0.2486
6,7,5	0.0670	0.0273	0.0477	0.1785	0.1319	0.1500	0.2701	0.2228	0.2380
7,5,6	0.0716	0.0339	0.0575	0.1904	0.1467	0.1686	0.2852	0.2518	0.2566
7,6,5	0.0748	0.0303	0.0549	0.1928	0.1412	0.1546	0.2864	0.2341	0.2436
Average	0.0639	0.0303	0.0496	0.1749	0.1373	0.1495	0.2666	0.2348	0.2364
6,7,8	0.0671	0.0409	0.0407	0.1841	0.1585	0.1727	0.2795	0.2620	0.2520
6,8,7	0.0722	0.0400	0.0397	0.1933	0.1571	0.1692	0.2869	0.2595	0.2493
7,6,8	0.0814	0.0414	0.0419	0.2064	0.1563	0.1731	0.3012	0.2581	0.2530
7,8,6	0.0811	0.0406	0.0408	0.2060	0.1555	0.1727	0.3017	0.2589	0.2517
8,6,7	0.0826	0.0449	0.0503	0.2143	0.1722	0.1950	0.3119	0.2776	0.2805
8,7,6	0.0863	0.0437	0.0487	0.2137	0.1599	0.1912	0.3089	0.2634	0.2739
Average	0.0785	0.0419	0.0437	0.2030	0.1599	0.1790	0.2984	0.2633	0.2601
5,8,11	0.0620	0.0463	0.0327	0.1807	0.1708	0.1382	0.2781	0.2787	0.2306
5,11,8	0.0758	0.0447	0.0289	0.1968	0.1636	0.1361	0.2920	0.2664	0.2265
8,5,11	0.0753	0.0554	0.0518	0.2053	0.1946	0.1830	0.3077	0.3088	0.2841
8,11,5	0.0926	0.0379	0.0386	0.2190	0.1554	0.1632	0.3125	0.2505	0.2631
11,5,8	0.1101	0.0622	0.0674	0.2587	0.2076	0.2166	0.3622	0.3190	0.3262
11,8,5	0.1145	0.0477	0.0564	0.2539	0.1747	0.1926	0.3514	0.2742	0.3024
Average	0.0884	0.0490	0.0460	0.2191	0.1778	0.1716	0.3173	0.2829	0.2722
6,9,12	0.0761	0.0572	0.0531	0.2081	0.1966	0.1635	0.3127	0.3120	0.2579
6,12,9	0.0890	0.0554	0.0494	0.2214	0.1850	0.1571	0.3205	0.2917	0.2506
9,6,12	0.0905	0.0685	0.0698	0.2365	0.2204	0.1916	0.3460	0.3395	0.2933
9,12,6	0.1062	0.0511	0.0564	0.2407	0.1757	0.1709	0.3376	0.2776	0.2693
12,6,9	0.1241	0.0757	0.0868	0.2802	0.2264	0.2260	0.3875	0.3424	0.3328
12,9,6	0.1254	0.0599	0.0795	0.2693	0.1916	0.2120	0.3688	0.2974	0.3131
Average	0.1019	0.0613	0.0658	0.2427	0.1993	0.1869	0.3455	0.3101	0.2862

Table D.5: Setting 2a: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0459	0.0193	0.0176	0.1472	0.1329	0.1220	0.2397	0.2266	0.2315
6,6,6	0.0555	0.0291	0.0498	0.1701	0.1525	0.1719	0.2688	0.2738	0.2649
7,7,7	0.0670	0.0427	0.0499	0.1963	0.1867	0.1587	0.3035	0.3070	0.2422
8,8,8	0.0791	0.0574	0.0791	0.2199	0.2141	0.1905	0.3319	0.3400	0.3119
9,9,9	0.0933	0.0721	0.0723	0.2455	0.2438	0.2409	0.3651	0.3748	0.3217
10,10,10	0.1030	0.0853	0.0937	0.2697	0.2689	0.2601	0.3924	0.4055	0.3747

Table D.6: Setting 2a: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0442	0.0243	0.0426	0.1445	0.1357	0.1388	0.2371	0.2474	0.2297
5,7,6	0.0537	0.0312	0.0507	0.1670	0.1527	0.1495	0.2672	0.2689	0.2376
6,5,7	0.0439	0.0215	0.0475	0.1440	0.1326	0.1492	0.2358	0.2392	0.2414
6,7,5	0.0676	0.0381	0.0706	0.1967	0.1738	0.1973	0.3036	0.2928	0.3012
7,5,6	0.0573	0.0269	0.0576	0.1738	0.1500	0.1616	0.2750	0.2643	0.2523
7,6,5	0.0721	0.0366	0.0732	0.2042	0.1761	0.2024	0.3111	0.2970	0.3077
Average	0.0565	0.0298	0.0570	0.1717	0.1535	0.1665	0.2716	0.2683	0.2617
6,7,8	0.0554	0.0370	0.0400	0.1711	0.1685	0.1725	0.2716	0.2849	0.2557
6,8,7	0.0663	0.0453	0.0500	0.1928	0.1855	0.2046	0.2988	0.3073	0.2941
7,6,8	0.0790	0.0514	0.0588	0.2201	0.2037	0.2110	0.3333	0.3318	0.2984
7,8,6	0.0803	0.0519	0.0605	0.2220	0.2063	0.2151	0.3347	0.3339	0.3031
8,6,7	0.0675	0.0412	0.0542	0.1953	0.1791	0.2133	0.3000	0.3001	0.3055
8,7,6	0.0818	0.0513	0.0637	0.2251	0.2059	0.2236	0.3382	0.3320	0.3157
Average	0.0717	0.0464	0.0545	0.2044	0.1915	0.2067	0.3128	0.3150	0.2954
5,8,11	0.0484	0.0375	0.0251	0.1552	0.1643	0.1268	0.2521	0.2800	0.2194
5,11,8	0.0744	0.0553	0.0426	0.2090	0.2040	0.1671	0.3186	0.3316	0.2722
8,5,11	0.0422	0.0254	0.0310	0.1412	0.1439	0.1369	0.2333	0.2533	0.2296
8,11,5	0.1143	0.0792	0.0878	0.2877	0.2660	0.2643	0.4138	0.4039	0.3834
11,5,8	0.0750	0.0415	0.0573	0.2073	0.1855	0.1924	0.3143	0.3050	0.3032
11,8,5	0.1315	0.0836	0.1006	0.3091	0.2688	0.2853	0.4338	0.4015	0.4088
Average	0.0810	0.0538	0.0574	0.2183	0.2054	0.1955	0.3277	0.3292	0.3028
6,9,12	0.0595	0.0489	0.0426	0.1820	0.1927	0.1477	0.2872	0.3155	0.2447
6,12,9	0.0868	0.0690	0.0696	0.2361	0.2355	0.2042	0.3519	0.3688	0.3131
9,6,12	0.0537	0.0391	0.0468	0.1678	0.1740	0.1519	0.2677	0.2917	0.2503
9,12,6	0.1310	0.0982	0.1108	0.3143	0.2992	0.2604	0.4435	0.4410	0.3747
12,6,9	0.0858	0.0559	0.0843	0.2309	0.2153	0.2271	0.3444	0.3392	0.3394
12,9,6	0.1417	0.1002	0.1218	0.3283	0.2940	0.2788	0.4552	0.4297	0.3935
Average	0.0931	0.0686	0.0793	0.2432	0.2351	0.2117	0.3583	0.3643	0.3193

Table D.7: Setting 2a: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0410	0.0163	0.0111	0.1429	0.1376	0.1084	0.2421	0.2400	0.2255
6,6,6	0.0503	0.0263	0.0401	0.1696	0.1610	0.1666	0.2754	0.2940	0.2696
7,7,7	0.0600	0.0407	0.0364	0.1904	0.1950	0.1456	0.3044	0.3276	0.2397
8,8,8	0.0705	0.0555	0.0646	0.2183	0.2287	0.1852	0.3389	0.3635	0.3229
9,9,9	0.0812	0.0712	0.0558	0.2428	0.2600	0.2399	0.3706	0.4002	0.3332
10,10,10	0.0945	0.0891	0.0821	0.2712	0.2923	0.2675	0.4057	0.4422	0.3962

Table D.8: Setting 2a: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0333	0.0180	0.0313	0.1267	0.1284	0.1205	0.2205	0.2441	0.2136
5,7,6	0.0389	0.0196	0.0365	0.1415	0.1377	0.1290	0.2404	0.2568	0.2195
6,5,7	0.0436	0.0238	0.0384	0.1508	0.1501	0.1477	0.2522	0.2733	0.2555
6,7,5	0.0591	0.0306	0.0545	0.1876	0.1744	0.1819	0.2988	0.3040	0.2971
7,5,6	0.0614	0.0344	0.0520	0.1914	0.1770	0.1691	0.3024	0.3041	0.2752
7,6,5	0.0718	0.0383	0.0621	0.2119	0.1901	0.1958	0.3285	0.3203	0.3100
Average	0.0514	0.0275	0.0458	0.1683	0.1596	0.1573	0.2738	0.2838	0.2618
6,7,8	0.0414	0.0289	0.0279	0.1508	0.1629	0.1595	0.2565	0.2905	0.2514
6,8,7	0.0488	0.0325	0.0342	0.1673	0.1734	0.1813	0.2757	0.3031	0.2772
7,6,8	0.0680	0.0455	0.0442	0.2090	0.2056	0.2002	0.3270	0.3407	0.2949
7,8,6	0.0667	0.0448	0.0442	0.2094	0.2057	0.1975	0.3282	0.3417	0.2930
8,6,7	0.0723	0.0497	0.0461	0.2169	0.2135	0.2253	0.3358	0.3450	0.3294
8,7,6	0.0797	0.0544	0.0516	0.2334	0.2232	0.2273	0.3552	0.3594	0.3303
Average	0.0628	0.0426	0.0414	0.1978	0.1974	0.1985	0.3131	0.3301	0.2960
5,8,11	0.0242	0.0186	0.0164	0.1065	0.1301	0.0991	0.1961	0.2488	0.1890
5,11,8	0.0355	0.0240	0.0242	0.1374	0.1517	0.1215	0.2399	0.2755	0.2197
8,5,11	0.0508	0.0436	0.0273	0.1777	0.1992	0.1589	0.2898	0.3338	0.2775
8,11,5	0.0967	0.0692	0.0657	0.2691	0.2619	0.2439	0.4006	0.4062	0.3733
11,5,8	0.1092	0.0760	0.0625	0.2794	0.2633	0.2452	0.4047	0.4006	0.3786
11,8,5	0.1441	0.0967	0.0911	0.3385	0.2993	0.2994	0.4688	0.4393	0.4349
Average	0.0768	0.0547	0.0479	0.2181	0.2176	0.1947	0.3333	0.3507	0.3122
6,9,12	0.0331	0.0294	0.0293	0.1338	0.1650	0.1244	0.2367	0.2950	0.2251
6,12,9	0.0448	0.0370	0.0420	0.1639	0.1869	0.1554	0.2792	0.3228	0.2648
9,6,12	0.0624	0.0576	0.0449	0.2051	0.2313	0.1725	0.3254	0.3700	0.2903
9,12,6	0.1087	0.0875	0.0831	0.2903	0.2897	0.2347	0.4237	0.4340	0.3593
12,6,9	0.1212	0.0945	0.0939	0.3054	0.2930	0.2780	0.4355	0.4348	0.4077
12,9,6	0.1537	0.1130	0.1192	0.3570	0.3297	0.3017	0.4920	0.4750	0.4292
Average	0.0873	0.0698	0.0687	0.2426	0.2493	0.2111	0.3654	0.3886	0.3294

Table D.9: Setting 2a: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.2083	0.0893	0.0500	0.4316	0.3551	0.2855	0.5615	0.4830	0.4735
6,6,6	0.2770	0.1513	0.1552	0.5181	0.4196	0.4060	0.6429	0.5816	0.5443
7,7,7	0.3492	0.2156	0.1607	0.5937	0.5050	0.4180	0.7104	0.6472	0.5424
8,8,8	0.4187	0.2808	0.2614	0.6599	0.5719	0.4957	0.7670	0.7033	0.6583
9,9,9	0.4876	0.3479	0.2579	0.7196	0.6342	0.5962	0.8156	0.7543	0.6894
10,10,10	0.5476	0.4117	0.3413	0.7679	0.6853	0.6445	0.8519	0.7970	0.7604

Table D.10: Setting 2a: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.2614	0.1652	0.1715	0.5082	0.4416	0.3795	0.6383	0.5915	0.5238
5,7,6	0.2626	0.1424	0.1420	0.5004	0.4087	0.3893	0.6277	0.5693	0.5232
6,5,7	0.2763	0.1747	0.1947	0.5245	0.4548	0.4205	0.6538	0.6050	0.5675
6,7,5	0.2669	0.1226	0.1434	0.4948	0.3795	0.3605	0.6168	0.5222	0.4945
7,5,6	0.2954	0.1635	0.1835	0.5360	0.4334	0.4357	0.6609	0.5926	0.5670
7,6,5	0.2863	0.1334	0.1596	0.5185	0.3963	0.3581	0.6385	0.5368	0.4970
Average	0.2748	0.1503	0.1658	0.5137	0.4191	0.3906	0.6393	0.5696	0.5288
6,7,8	0.3342	0.2283	0.1785	0.5885	0.5205	0.4982	0.7100	0.6619	0.6105
6,8,7	0.3325	0.2053	0.1507	0.5772	0.4894	0.4577	0.6946	0.6354	0.5773
7,6,8	0.3374	0.1940	0.1438	0.5742	0.4599	0.4361	0.6890	0.6080	0.5615
7,8,6	0.3365	0.1942	0.1433	0.5722	0.4575	0.4345	0.6877	0.6070	0.5606
8,6,7	0.3645	0.2269	0.1830	0.6091	0.5158	0.4889	0.7222	0.6549	0.6082
8,7,6	0.3545	0.2048	0.1651	0.5880	0.4693	0.4689	0.7013	0.6167	0.5940
Average	0.3433	0.2089	0.1607	0.5849	0.4854	0.4641	0.7008	0.6307	0.5854
5,8,11	0.3746	0.3048	0.1959	0.6394	0.6044	0.4935	0.7589	0.7387	0.6420
5,11,8	0.3736	0.2509	0.1391	0.6199	0.5407	0.4397	0.7346	0.6778	0.5958
8,5,11	0.4171	0.3383	0.2531	0.6756	0.6384	0.5586	0.7896	0.7658	0.6963
8,11,5	0.3539	0.1718	0.1190	0.5748	0.4344	0.3873	0.6826	0.5734	0.5471
11,5,8	0.4633	0.3076	0.2363	0.6948	0.5979	0.5558	0.7946	0.7241	0.6953
11,8,5	0.3952	0.1938	0.1553	0.6149	0.4581	0.4181	0.7154	0.5947	0.5844
Average	0.3963	0.2612	0.1831	0.6366	0.5457	0.4755	0.7460	0.6791	0.6268
6,9,12	0.4547	0.3735	0.2928	0.7088	0.6699	0.5720	0.8135	0.7918	0.7048
6,12,9	0.4462	0.3215	0.2242	0.6868	0.6073	0.4992	0.7884	0.7362	0.6462
9,6,12	0.4934	0.4070	0.3269	0.7418	0.6992	0.5990	0.8379	0.8129	0.7308
9,12,6	0.4202	0.2363	0.1882	0.6409	0.5099	0.4473	0.7413	0.6447	0.5985
12,6,9	0.5255	0.3757	0.3075	0.7463	0.6535	0.5780	0.8350	0.7700	0.7158
12,9,6	0.4600	0.2565	0.2361	0.6754	0.5312	0.5024	0.7681	0.6654	0.6468
Average	0.4667	0.3284	0.2626	0.7000	0.6118	0.5330	0.7974	0.7368	0.6738

Table D.11: Setting 2a: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.2004	0.0924	0.0609	0.4471	0.4191	0.3310	0.5947	0.5807	0.5366
6,6,6	0.2715	0.1633	0.1857	0.5397	0.5103	0.4566	0.6802	0.6863	0.6054
7,7,7	0.3479	0.2588	0.1985	0.6248	0.6139	0.4827	0.7548	0.7614	0.6282
8,8,8	0.4230	0.3516	0.3117	0.6980	0.6945	0.5671	0.8129	0.8198	0.7358
9,9,9	0.5006	0.4456	0.3159	0.7627	0.7620	0.6846	0.8615	0.8676	0.7766
10,10,10	0.5684	0.5289	0.4133	0.8126	0.8137	0.7303	0.8973	0.9033	0.8407

Table D.12: Setting 2a: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.2327	0.1404	0.1604	0.4921	0.4735	0.4247	0.6382	0.6518	0.5855
5,7,6	0.2756	0.1874	0.2047	0.5490	0.5302	0.4343	0.6913	0.6957	0.5830
6,5,7	0.2190	0.1104	0.1564	0.4718	0.4380	0.3953	0.6166	0.6140	0.5464
6,7,5	0.3195	0.2187	0.2548	0.5968	0.5694	0.5226	0.7326	0.7281	0.6707
7,5,6	0.2589	0.1331	0.1846	0.5208	0.4790	0.3955	0.6634	0.6466	0.5416
7,6,5	0.3195	0.1959	0.2424	0.5925	0.5536	0.5245	0.7267	0.7160	0.6720
Average	0.2709	0.1643	0.2006	0.5372	0.5073	0.4495	0.6781	0.6754	0.5999
6,7,8	0.3092	0.2317	0.1716	0.5861	0.5839	0.5046	0.7234	0.7397	0.6380
6,8,7	0.3560	0.2817	0.2248	0.6358	0.6291	0.5870	0.7647	0.7736	0.7074
7,6,8	0.3997	0.3161	0.2460	0.6757	0.6627	0.5813	0.7977	0.7988	0.7010
7,8,6	0.3969	0.3131	0.2415	0.6743	0.6609	0.5809	0.7981	0.8002	0.7000
8,6,7	0.3315	0.2331	0.2024	0.6059	0.5775	0.5497	0.7366	0.7342	0.6793
8,7,6	0.3881	0.2899	0.2326	0.6636	0.6412	0.5617	0.7873	0.7829	0.6872
Average	0.3636	0.2776	0.2198	0.6402	0.6259	0.5609	0.7680	0.7716	0.6855
5,8,11	0.3214	0.2691	0.1376	0.6023	0.6168	0.4750	0.7385	0.7635	0.6427
5,11,8	0.4317	0.3769	0.2558	0.7072	0.7111	0.5833	0.8213	0.8341	0.7332
8,5,11	0.2444	0.1492	0.1146	0.5054	0.4955	0.3913	0.6479	0.6608	0.5585
8,11,5	0.5524	0.4839	0.3710	0.8044	0.7945	0.7060	0.8909	0.8900	0.8266
11,5,8	0.3430	0.2096	0.1852	0.6081	0.5671	0.4622	0.7363	0.7170	0.6326
11,8,5	0.5558	0.4440	0.3410	0.7994	0.7643	0.6963	0.8846	0.8678	0.8196
Average	0.4081	0.3221	0.2342	0.6711	0.6582	0.5524	0.7866	0.7889	0.7022
6,9,12	0.4033	0.3645	0.2361	0.6818	0.6957	0.5380	0.8025	0.8230	0.6937
6,12,9	0.5128	0.4698	0.3875	0.7730	0.7813	0.6804	0.8702	0.8842	0.8028
9,6,12	0.3235	0.2452	0.1932	0.5977	0.5958	0.4633	0.7302	0.7423	0.6310
9,12,6	0.6229	0.5725	0.4471	0.8486	0.8453	0.7197	0.9204	0.9221	0.8337
12,6,9	0.4186	0.3105	0.2846	0.6838	0.6523	0.5796	0.7994	0.7876	0.7278
12,9,6	0.6163	0.5311	0.4043	0.8411	0.8144	0.6872	0.9131	0.9015	0.8142
Average	0.4829	0.4156	0.3255	0.7377	0.7308	0.6114	0.8393	0.8435	0.7505

Table D.13: Setting 2a: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1938	0.0946	0.0525	0.4644	0.4632	0.3235	0.6233	0.6329	0.5510
6,6,6	0.2698	0.1787	0.1736	0.5613	0.5616	0.4782	0.7143	0.7414	0.6455
7,7,7	0.3530	0.2925	0.1844	0.6576	0.6774	0.5001	0.7934	0.8183	0.6705
8,8,8	0.4349	0.4007	0.3202	0.7339	0.7588	0.6104	0.8503	0.8703	0.7925
9,9,9	0.5157	0.5033	0.3133	0.7949	0.8193	0.7371	0.8931	0.9096	0.8310
10,10,10	0.5958	0.6023	0.4378	0.8502	0.8708	0.7923	0.9272	0.9396	0.8932

Table D.14: Setting 2a: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1915	0.1123	0.1384	0.4698	0.4816	0.3778	0.6366	0.6740	0.5477
5,7,6	0.2144	0.1251	0.1549	0.4996	0.5060	0.3806	0.6613	0.6918	0.5415
6,5,7	0.2450	0.1624	0.1649	0.5321	0.5418	0.4613	0.6906	0.7206	0.6478
6,7,5	0.2936	0.1957	0.2053	0.5954	0.5910	0.5158	0.7424	0.7603	0.6951
7,5,6	0.3180	0.2279	0.2236	0.6114	0.6002	0.4979	0.7521	0.7578	0.6562
7,6,5	0.3448	0.2462	0.2468	0.6400	0.6243	0.5478	0.7748	0.7766	0.7030
Average	0.2679	0.1783	0.1890	0.5581	0.5575	0.4635	0.7096	0.7302	0.6319
6,7,8	0.2706	0.2223	0.1472	0.5765	0.6070	0.4995	0.7308	0.7724	0.6499
6,8,7	0.2936	0.2375	0.1658	0.6001	0.6246	0.5363	0.7501	0.7863	0.6828
7,6,8	0.3784	0.3121	0.2073	0.6822	0.6944	0.5766	0.8108	0.8302	0.7176
7,8,6	0.3776	0.3140	0.2081	0.6813	0.6943	0.5800	0.8112	0.8310	0.7192
8,6,7	0.3970	0.3352	0.2372	0.6951	0.6993	0.6426	0.8180	0.8281	0.7695
8,7,6	0.4289	0.3596	0.2491	0.7177	0.7148	0.6364	0.8355	0.8409	0.7639
Average	0.3577	0.2968	0.2025	0.6588	0.6724	0.5786	0.7927	0.8148	0.7172
5,8,11	0.1959	0.1476	0.0983	0.4870	0.5485	0.3675	0.6590	0.7307	0.5524
5,11,8	0.2429	0.1778	0.1270	0.5492	0.5935	0.4057	0.7143	0.7677	0.5922
8,5,11	0.3661	0.3411	0.1709	0.6721	0.7080	0.5691	0.8036	0.8380	0.7445
8,11,5	0.5138	0.4624	0.2778	0.7952	0.8039	0.6860	0.8917	0.9021	0.8314
11,5,8	0.5402	0.4658	0.3320	0.7969	0.7799	0.7115	0.8862	0.8793	0.8382
11,8,5	0.6217	0.5404	0.4005	0.8508	0.8272	0.7704	0.9220	0.9116	0.8781
Average	0.4134	0.3559	0.2344	0.6919	0.7102	0.5850	0.8128	0.8382	0.7395
6,9,12	0.2792	0.2617	0.1737	0.5975	0.6678	0.4708	0.7545	0.8174	0.6596
6,12,9	0.3351	0.3049	0.2174	0.6571	0.7099	0.5323	0.7998	0.8467	0.7113
9,6,12	0.4547	0.4530	0.2806	0.7501	0.7825	0.6224	0.8627	0.8871	0.7816
9,12,6	0.5872	0.5628	0.3686	0.8450	0.8547	0.6946	0.9231	0.9309	0.8318
12,6,9	0.6195	0.5659	0.4825	0.8509	0.8423	0.7915	0.9222	0.9194	0.8860
12,9,6	0.6852	0.6276	0.5141	0.8895	0.8755	0.8003	0.9454	0.9397	0.8898
Average	0.4935	0.4627	0.3395	0.7650	0.7888	0.6520	0.8680	0.8902	0.7934

Table D.15: Setting 2a: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1435	0.0618	0.0446	0.3464	0.3126	0.2515	0.4840	0.4521	0.4249
6,6,6	0.1925	0.1127	0.1423	0.4229	0.3794	0.3728	0.5627	0.5501	0.5090
7,7,7	0.2439	0.1696	0.1491	0.4943	0.4623	0.3731	0.6327	0.6176	0.5055
8,8,8	0.2992	0.2301	0.2474	0.5583	0.5306	0.4616	0.6924	0.6791	0.6241
9,9,9	0.3547	0.2885	0.2422	0.6184	0.5926	0.5568	0.7443	0.7325	0.6564
10,10,10	0.4145	0.3537	0.3327	0.6759	0.6524	0.6146	0.7895	0.7780	0.7341

Table D.16: Setting 2a: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1529	0.0940	0.1318	0.3724	0.3493	0.3141	0.5170	0.5128	0.4502
5,7,6	0.1542	0.0846	0.1292	0.3693	0.3328	0.3154	0.5081	0.4970	0.4438
6,5,7	0.1884	0.1226	0.1478	0.4273	0.3969	0.3686	0.5717	0.5651	0.5204
6,7,5	0.1909	0.1000	0.1534	0.4139	0.3620	0.3610	0.5517	0.5207	0.5038
7,5,6	0.2257	0.1385	0.1645	0.4661	0.4126	0.3897	0.6028	0.5747	0.5280
7,6,5	0.2216	0.1251	0.1711	0.4572	0.3948	0.3777	0.5918	0.5453	0.5126
Average	0.1890	0.1108	0.1496	0.4177	0.3747	0.3544	0.5572	0.5359	0.4931
6,7,8	0.2095	0.1519	0.1411	0.4520	0.4351	0.4165	0.5965	0.5967	0.5371
6,8,7	0.2045	0.1416	0.1412	0.4410	0.4144	0.4107	0.5846	0.5785	0.5301
7,6,8	0.2406	0.1600	0.1558	0.4840	0.4397	0.4246	0.6213	0.6004	0.5427
7,8,6	0.2414	0.1601	0.1571	0.4833	0.4391	0.4263	0.6185	0.5989	0.5429
8,6,7	0.2823	0.1983	0.1753	0.5369	0.4953	0.4784	0.6703	0.6460	0.6010
8,7,6	0.2767	0.1868	0.1765	0.5256	0.4700	0.4697	0.6572	0.6225	0.5883
Average	0.2425	0.1665	0.1578	0.4871	0.4489	0.4377	0.6247	0.6072	0.5570
5,8,11	0.1827	0.1471	0.1241	0.4243	0.4377	0.3568	0.5746	0.6034	0.5022
5,11,8	0.1891	0.1318	0.1296	0.4189	0.4038	0.3502	0.5607	0.5631	0.4908
8,5,11	0.3062	0.2641	0.1648	0.5803	0.5810	0.4749	0.7163	0.7267	0.6352
8,11,5	0.2712	0.1807	0.1801	0.5157	0.4633	0.4414	0.6487	0.6147	0.5817
11,5,8	0.4098	0.3055	0.2295	0.6634	0.6035	0.5523	0.7766	0.7363	0.6961
11,8,5	0.3714	0.2440	0.2361	0.6152	0.5298	0.5165	0.7302	0.6703	0.6568
Average	0.2884	0.2122	0.1774	0.5363	0.5032	0.4487	0.6679	0.6524	0.5938
6,9,12	0.2452	0.2102	0.1994	0.5099	0.5217	0.4362	0.6548	0.6814	0.5812
6,12,9	0.2420	0.1890	0.2036	0.4927	0.4814	0.4277	0.6325	0.6376	0.5645
9,6,12	0.3712	0.3329	0.2501	0.6465	0.6447	0.5181	0.7728	0.7771	0.6670
9,12,6	0.3280	0.2387	0.2519	0.5793	0.5326	0.4798	0.7043	0.6763	0.6140
12,6,9	0.4631	0.3670	0.3242	0.7125	0.6606	0.6012	0.8148	0.7829	0.7309
12,9,6	0.4216	0.2970	0.3181	0.6645	0.5907	0.5695	0.7734	0.7246	0.6938
Average	0.3452	0.2725	0.2579	0.6009	0.5720	0.5054	0.7254	0.7133	0.6419

Table D.17: Setting 2a: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1312	0.0535	0.0322	0.3574	0.3515	0.2493	0.5131	0.5243	0.4500
6,6,6	0.1809	0.1046	0.1195	0.4396	0.4373	0.3752	0.5994	0.6316	0.5346
7,7,7	0.2385	0.1819	0.1265	0.5225	0.5462	0.3858	0.6792	0.7125	0.5461
8,8,8	0.2981	0.2659	0.2287	0.5961	0.6280	0.4849	0.7433	0.7767	0.6772
9,9,9	0.3610	0.3457	0.2235	0.6624	0.6971	0.6028	0.7964	0.8268	0.7158
10,10,10	0.4274	0.4279	0.3241	0.7245	0.7596	0.6630	0.8418	0.8697	0.7936

Table D.18: Setting 2a: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1273	0.0690	0.0965	0.3562	0.3654	0.3054	0.5172	0.5592	0.4666
5,7,6	0.1596	0.0965	0.1198	0.4114	0.4179	0.3205	0.5730	0.6087	0.4714
6,5,7	0.1425	0.0738	0.1069	0.3755	0.3782	0.3297	0.5335	0.5668	0.4954
6,7,5	0.2228	0.1432	0.1590	0.5016	0.4977	0.4323	0.6577	0.6788	0.6047
7,5,6	0.1953	0.1082	0.1367	0.4540	0.4416	0.3568	0.6098	0.6217	0.5085
7,6,5	0.2450	0.1536	0.1709	0.5247	0.5071	0.4479	0.6734	0.6826	0.6148
Average	0.1821	0.1074	0.1316	0.4372	0.4347	0.3654	0.5941	0.6196	0.5269
6,7,8	0.1793	0.1363	0.1045	0.4421	0.4743	0.3980	0.6036	0.6567	0.5391
6,8,7	0.2178	0.1709	0.1285	0.4921	0.5202	0.4628	0.6533	0.6949	0.6034
7,6,8	0.2827	0.2253	0.1531	0.5776	0.5938	0.4861	0.7264	0.7544	0.6269
7,8,6	0.2847	0.2257	0.1531	0.5771	0.5934	0.4847	0.7240	0.7523	0.6246
8,6,7	0.2503	0.1880	0.1475	0.5326	0.5375	0.4996	0.6844	0.7041	0.6402
8,7,6	0.3032	0.2345	0.1633	0.5930	0.5971	0.5071	0.7368	0.7513	0.6457
Average	0.2530	0.1968	0.1417	0.5358	0.5527	0.4731	0.6881	0.7190	0.6133
5,8,11	0.1366	0.1102	0.0757	0.3770	0.4303	0.3141	0.5418	0.6195	0.4792
5,11,8	0.2159	0.1764	0.1240	0.4996	0.5423	0.3940	0.6627	0.7165	0.5654
8,5,11	0.1674	0.1298	0.0962	0.4194	0.4590	0.3646	0.5811	0.6357	0.5339
8,11,5	0.4399	0.3923	0.2368	0.7333	0.7440	0.6187	0.8483	0.8618	0.7742
11,5,8	0.3079	0.2257	0.1803	0.5894	0.5765	0.4902	0.7272	0.7269	0.6555
11,8,5	0.4964	0.4131	0.2772	0.7655	0.7410	0.6610	0.8647	0.8555	0.8013
Average	0.2940	0.2413	0.1650	0.5640	0.5822	0.4738	0.7043	0.7360	0.6349
6,9,12	0.1898	0.1795	0.1412	0.4660	0.5315	0.3886	0.6320	0.7061	0.5576
6,12,9	0.2813	0.2620	0.2092	0.5830	0.6361	0.4995	0.7362	0.7898	0.6635
9,6,12	0.2261	0.2052	0.1683	0.5066	0.5529	0.4268	0.6647	0.7163	0.5957
9,12,6	0.5014	0.4746	0.3093	0.7824	0.7973	0.6217	0.8822	0.8947	0.7702
12,6,9	0.3720	0.3091	0.2820	0.6565	0.6528	0.5873	0.7826	0.7884	0.7319
12,9,6	0.5509	0.4900	0.3587	0.8067	0.7938	0.6640	0.8942	0.8898	0.7978
Average	0.3536	0.3201	0.2448	0.6335	0.6607	0.5313	0.7653	0.7975	0.6861

Table D.19: Setting 2a: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1512	0.0763	0.0718	0.3459	0.2938	0.2915	0.4723	0.4224	0.4386
6,6,6	0.1976	0.1141	0.1816	0.4103	0.3495	0.3897	0.5382	0.5019	0.5006
7,7,7	0.2517	0.1651	0.1996	0.4804	0.4233	0.4012	0.6062	0.5661	0.5049
8,8,8	0.3045	0.2175	0.2904	0.5407	0.4835	0.4695	0.6613	0.6195	0.6029
9,9,9	0.3555	0.2682	0.2938	0.5942	0.5371	0.5496	0.7092	0.6691	0.6280
10,10,10	0.4122	0.3220	0.3732	0.6484	0.5911	0.5984	0.7553	0.7178	0.6986

Table D.20: Setting 2a: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1824	0.1163	0.1715	0.3991	0.3543	0.3601	0.5321	0.5072	0.4824
5,7,6	0.2024	0.1242	0.1804	0.4206	0.3621	0.3661	0.5510	0.5137	0.4821
6,5,7	0.1762	0.1064	0.1876	0.3852	0.3383	0.3711	0.5147	0.4798	0.4863
6,7,5	0.2141	0.1225	0.2077	0.4313	0.3592	0.3992	0.5550	0.5007	0.5132
7,5,6	0.1924	0.1062	0.1968	0.4024	0.3359	0.3716	0.5277	0.4793	0.4798
7,6,5	0.2112	0.1147	0.2110	0.4248	0.3458	0.3921	0.5494	0.4890	0.5049
Average	0.1965	0.1151	0.1925	0.4106	0.3493	0.3767	0.5383	0.4950	0.4915
6,7,8	0.2410	0.1685	0.1912	0.4727	0.4302	0.4473	0.6025	0.5752	0.5466
6,8,7	0.2565	0.1742	0.2018	0.4885	0.4349	0.4618	0.6160	0.5793	0.5626
7,6,8	0.2635	0.1723	0.2063	0.4918	0.4223	0.4511	0.6152	0.5653	0.5479
7,8,6	0.2639	0.1710	0.2081	0.4932	0.4251	0.4500	0.6176	0.5679	0.5480
8,6,7	0.2449	0.1582	0.2129	0.4664	0.4019	0.4579	0.5917	0.5471	0.5553
8,7,6	0.2600	0.1653	0.2171	0.4841	0.4113	0.4551	0.6072	0.5543	0.5510
Average	0.2550	0.1683	0.2062	0.4828	0.4210	0.4539	0.6084	0.5649	0.5519
5,8,11	0.2754	0.2189	0.1713	0.5254	0.4993	0.4256	0.6562	0.6451	0.5616
5,11,8	0.3213	0.2370	0.1964	0.5646	0.5137	0.4506	0.6853	0.6532	0.5838
8,5,11	0.2272	0.1671	0.1999	0.4581	0.4277	0.4325	0.5893	0.5702	0.5502
8,11,5	0.3232	0.2112	0.2515	0.5530	0.4761	0.4937	0.6703	0.6166	0.6088
11,5,8	0.2682	0.1705	0.2333	0.4926	0.4189	0.4476	0.6121	0.5560	0.5651
11,8,5	0.3231	0.1983	0.2637	0.5446	0.4510	0.4877	0.6570	0.5869	0.6033
Average	0.2897	0.2005	0.2194	0.5231	0.4645	0.4563	0.6450	0.6047	0.5788
6,9,12	0.3351	0.2771	0.2621	0.5879	0.5585	0.4891	0.7096	0.6938	0.6109
6,12,9	0.3772	0.2916	0.2916	0.6236	0.5692	0.5204	0.7375	0.7045	0.6399
9,6,12	0.2854	0.2217	0.2697	0.5256	0.4913	0.4749	0.6508	0.6315	0.5923
9,12,6	0.3750	0.2618	0.3176	0.6083	0.5347	0.5194	0.7207	0.6702	0.6301
12,6,9	0.3185	0.2203	0.3084	0.5466	0.4750	0.5036	0.6626	0.6097	0.6130
12,9,6	0.3692	0.2468	0.3280	0.5957	0.5072	0.5198	0.7061	0.6385	0.6276
Average	0.3434	0.2532	0.2962	0.5813	0.5227	0.5045	0.6979	0.6580	0.6190

Table D.21: Setting 2a: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1490	0.0726	0.0687	0.3459	0.2946	0.2912	0.4715	0.4210	0.4390
6,6,6	0.2006	0.1178	0.1868	0.4155	0.3524	0.3928	0.5426	0.5045	0.5043
7,7,7	0.2528	0.1657	0.1988	0.4796	0.4219	0.4005	0.6036	0.5653	0.5050
8,8,8	0.3018	0.2161	0.2924	0.5399	0.4816	0.4697	0.6610	0.6200	0.6016
9,9,9	0.3597	0.2694	0.2957	0.5976	0.5402	0.5492	0.7115	0.6716	0.6274
10,10,10	0.4112	0.3193	0.3704	0.6475	0.5910	0.5972	0.7559	0.7178	0.6974

Table D.22: Setting 2a: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1860	0.1179	0.1739	0.3993	0.3552	0.3611	0.5311	0.5047	0.4802
5,7,6	0.2037	0.1247	0.1810	0.4267	0.3663	0.3718	0.5536	0.5183	0.4862
6,5,7	0.1765	0.1063	0.1892	0.3871	0.3376	0.3704	0.5142	0.4795	0.4858
6,7,5	0.2130	0.1232	0.2079	0.4303	0.3582	0.3982	0.5552	0.4999	0.5122
7,5,6	0.1929	0.1070	0.1977	0.4040	0.3350	0.3735	0.5293	0.4814	0.4821
7,6,5	0.2111	0.1155	0.2093	0.4245	0.3474	0.3927	0.5511	0.4911	0.5072
Average	0.1972	0.1158	0.1932	0.4120	0.3500	0.3780	0.5391	0.4958	0.4923
6,7,8	0.2380	0.1656	0.1899	0.4712	0.4243	0.4422	0.5993	0.5691	0.5412
6,8,7	0.2565	0.1735	0.1999	0.4880	0.4336	0.4589	0.6156	0.5790	0.5597
7,6,8	0.2656	0.1744	0.2096	0.4925	0.4232	0.4491	0.6147	0.5648	0.5467
7,8,6	0.2640	0.1730	0.2081	0.4944	0.4247	0.4516	0.6180	0.5688	0.5477
8,6,7	0.2429	0.1561	0.2124	0.4676	0.4041	0.4568	0.5916	0.5472	0.5557
8,7,6	0.2612	0.1655	0.2184	0.4856	0.4127	0.4569	0.6071	0.5568	0.5537
Average	0.2547	0.1680	0.2064	0.4832	0.4204	0.4526	0.6077	0.5643	0.5508
5,8,11	0.2749	0.2186	0.1706	0.5234	0.4976	0.4234	0.6546	0.6442	0.5592
5,11,8	0.3221	0.2367	0.1956	0.5651	0.5117	0.4502	0.6869	0.6534	0.5824
8,5,11	0.2243	0.1655	0.1970	0.4559	0.4245	0.4278	0.5870	0.5686	0.5476
8,11,5	0.3261	0.2142	0.2518	0.5550	0.4753	0.4930	0.6707	0.6150	0.6106
11,5,8	0.2705	0.1711	0.2336	0.4930	0.4199	0.4499	0.6120	0.5556	0.5662
11,8,5	0.3210	0.1973	0.2631	0.5439	0.4508	0.4882	0.6588	0.5864	0.6044
Average	0.2898	0.2006	0.2186	0.5227	0.4633	0.4554	0.6450	0.6039	0.5784
6,9,12	0.3347	0.2762	0.2640	0.5864	0.5566	0.4878	0.7099	0.6929	0.6128
6,12,9	0.3801	0.2938	0.2933	0.6224	0.5714	0.5215	0.7356	0.7041	0.6399
9,6,12	0.2855	0.2195	0.2693	0.5271	0.4912	0.4742	0.6537	0.6315	0.5915
9,12,6	0.3712	0.2596	0.3139	0.6058	0.5320	0.5177	0.7143	0.6656	0.6285
12,6,9	0.3159	0.2177	0.3058	0.5461	0.4749	0.5021	0.6637	0.6091	0.6124
12,9,6	0.3695	0.2443	0.3286	0.5940	0.5059	0.5195	0.7032	0.6387	0.6260
Average	0.3428	0.2519	0.2958	0.5803	0.5220	0.5038	0.6967	0.6570	0.6185

Table D.23: Setting 2a: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1334	0.0532	0.0324	0.3598	0.3558	0.2502	0.5174	0.5275	0.4546
6,6,6	0.1801	0.1040	0.1182	0.4372	0.4377	0.3759	0.5971	0.6311	0.5344
7,7,7	0.2375	0.1813	0.1240	0.5230	0.5433	0.3825	0.6782	0.7128	0.5457
8,8,8	0.3000	0.2664	0.2286	0.5969	0.6277	0.4826	0.7429	0.7754	0.6748
9,9,9	0.3613	0.3466	0.2241	0.6644	0.6985	0.6028	0.7980	0.8287	0.7166
10,10,10	0.4303	0.4308	0.3241	0.7274	0.7616	0.6640	0.8450	0.8721	0.7960

Table D.24: Setting 2a: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1293	0.0688	0.0962	0.3552	0.3652	0.3056	0.5146	0.5591	0.4653
5,7,6	0.1612	0.0955	0.1189	0.4125	0.4223	0.3221	0.5751	0.6080	0.4734
6,5,7	0.1423	0.0746	0.1073	0.3768	0.3809	0.3309	0.5348	0.5686	0.4973
6,7,5	0.2227	0.1421	0.1593	0.5010	0.4967	0.4299	0.6567	0.6799	0.6032
7,5,6	0.1952	0.1107	0.1387	0.4535	0.4432	0.3561	0.6083	0.6201	0.5079
7,6,5	0.2440	0.1521	0.1713	0.5244	0.5083	0.4475	0.6753	0.6826	0.6125
Average	0.1825	0.1073	0.1320	0.4372	0.4361	0.3654	0.5941	0.6197	0.5266
6,7,8	0.1792	0.1365	0.1032	0.4427	0.4743	0.3990	0.6051	0.6575	0.5396
6,8,7	0.2141	0.1684	0.1259	0.4931	0.5207	0.4613	0.6559	0.6988	0.6064
7,6,8	0.2835	0.2254	0.1520	0.5802	0.5959	0.4858	0.7290	0.7543	0.6266
7,8,6	0.2809	0.2238	0.1495	0.5747	0.5913	0.4818	0.7234	0.7504	0.6221
8,6,7	0.2519	0.1904	0.1493	0.5329	0.5368	0.4999	0.6839	0.7050	0.6395
8,7,6	0.3015	0.2332	0.1625	0.5952	0.5991	0.5066	0.7393	0.7554	0.6457
Average	0.2519	0.1963	0.1404	0.5365	0.5530	0.4724	0.6894	0.7202	0.6133
5,8,11	0.1365	0.1107	0.0754	0.3781	0.4327	0.3148	0.5432	0.6184	0.4801
5,11,8	0.2176	0.1776	0.1217	0.4986	0.5397	0.3937	0.6592	0.7145	0.5647
8,5,11	0.1684	0.1307	0.0978	0.4255	0.4625	0.3655	0.5844	0.6380	0.5363
8,11,5	0.4391	0.3915	0.2351	0.7307	0.7409	0.6151	0.8457	0.8597	0.7723
11,5,8	0.3102	0.2257	0.1803	0.5898	0.5769	0.4901	0.7287	0.7277	0.6547
11,8,5	0.4996	0.4159	0.2795	0.7662	0.7440	0.6647	0.8669	0.8557	0.8013
Average	0.2952	0.2420	0.1650	0.5648	0.5828	0.4740	0.7047	0.7357	0.6349
6,9,12	0.1884	0.1785	0.1419	0.4629	0.5304	0.3880	0.6303	0.7072	0.5558
6,12,9	0.2809	0.2616	0.2109	0.5848	0.6375	0.5012	0.7384	0.7925	0.6637
9,6,12	0.2253	0.2066	0.1667	0.5063	0.5521	0.4274	0.6658	0.7156	0.5957
9,12,6	0.5005	0.4772	0.3107	0.7802	0.7952	0.6213	0.8811	0.8937	0.7695
12,6,9	0.3750	0.3141	0.2859	0.6598	0.6567	0.5913	0.7860	0.7917	0.7347
12,9,6	0.5522	0.4923	0.3595	0.8061	0.7950	0.6659	0.8940	0.8903	0.7978
Average	0.3537	0.3217	0.2459	0.6334	0.6612	0.5325	0.7659	0.7985	0.6862

Table D.25: Setting 2a: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1430	0.0633	0.0459	0.3493	0.3149	0.2551	0.4864	0.4560	0.4286
6,6,6	0.1925	0.1132	0.1418	0.4229	0.3809	0.3753	0.5646	0.5522	0.5107
7,7,7	0.2444	0.1683	0.1494	0.4932	0.4623	0.3732	0.6319	0.6197	0.5045
8,8,8	0.2998	0.2298	0.2482	0.5602	0.5324	0.4635	0.6940	0.6808	0.6292
9,9,9	0.3586	0.2910	0.2444	0.6249	0.5965	0.5599	0.7482	0.7344	0.6582
10,10,10	0.4167	0.3570	0.3336	0.6762	0.6531	0.6154	0.7919	0.7800	0.7345

Table D.26: Setting 2a: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1548	0.0955	0.1325	0.3742	0.3518	0.3157	0.5174	0.5165	0.4504
5,7,6	0.1550	0.0855	0.1296	0.3715	0.3340	0.3171	0.5108	0.4975	0.4468
6,5,7	0.1889	0.1223	0.1463	0.4226	0.3949	0.3624	0.5648	0.5640	0.5182
6,7,5	0.1896	0.0989	0.1515	0.4153	0.3632	0.3603	0.5545	0.5207	0.5024
7,5,6	0.2267	0.1389	0.1678	0.4678	0.4147	0.3899	0.6055	0.5796	0.5297
7,6,5	0.2251	0.1269	0.1734	0.4616	0.4005	0.3799	0.5976	0.5524	0.5159
Average	0.1900	0.1113	0.1502	0.4188	0.3765	0.3542	0.5584	0.5385	0.4939
6,7,8	0.2071	0.1503	0.1388	0.4494	0.4323	0.4163	0.5944	0.5942	0.5357
6,8,7	0.2074	0.1429	0.1424	0.4435	0.4175	0.4120	0.5852	0.5791	0.5311
7,6,8	0.2409	0.1619	0.1580	0.4832	0.4387	0.4233	0.6205	0.5981	0.5410
7,8,6	0.2411	0.1600	0.1569	0.4845	0.4400	0.4268	0.6196	0.5995	0.5438
8,6,7	0.2816	0.1978	0.1741	0.5367	0.4951	0.4802	0.6687	0.6473	0.6003
8,7,6	0.2756	0.1877	0.1771	0.5251	0.4700	0.4709	0.6570	0.6236	0.5902
Average	0.2423	0.1668	0.1579	0.4871	0.4489	0.4383	0.6242	0.6070	0.5570
5,8,11	0.1825	0.1467	0.1243	0.4262	0.4395	0.3568	0.5771	0.6069	0.5033
5,11,8	0.1890	0.1316	0.1287	0.4157	0.4006	0.3484	0.5590	0.5608	0.4897
8,5,11	0.3084	0.2644	0.1642	0.5806	0.5820	0.4770	0.7165	0.7269	0.6370
8,11,5	0.2721	0.1810	0.1816	0.5164	0.4651	0.4424	0.6480	0.6148	0.5853
11,5,8	0.4074	0.3031	0.2289	0.6607	0.6006	0.5485	0.7738	0.7361	0.6943
11,8,5	0.3673	0.2407	0.2339	0.6125	0.5264	0.5124	0.7289	0.6680	0.6531
Average	0.2878	0.2113	0.1769	0.5354	0.5024	0.4476	0.6672	0.6523	0.5938
6,9,12	0.2400	0.2073	0.1955	0.5034	0.5194	0.4316	0.6519	0.6763	0.5781
6,12,9	0.2429	0.1883	0.2043	0.4904	0.4804	0.4262	0.6310	0.6352	0.5631
9,6,12	0.3699	0.3313	0.2492	0.6468	0.6453	0.5201	0.7727	0.7773	0.6673
9,12,6	0.3257	0.2389	0.2505	0.5775	0.5303	0.4788	0.7038	0.6755	0.6117
12,6,9	0.4639	0.3672	0.3241	0.7116	0.6588	0.5982	0.8130	0.7818	0.7294
12,9,6	0.4230	0.3017	0.3210	0.6677	0.5917	0.5687	0.7751	0.7282	0.6942
Average	0.3442	0.2725	0.2574	0.5996	0.5710	0.5039	0.7246	0.7124	0.6406

Table D.27: Setting 2a: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

D.2 Setting 2b: Standard Deviations 1, 2, and 3

Medians	Sample Sizes	Table	Page
0,0,0	Equal	D.29	153
0,0,0	Unequal	D.30	154
0,0,1	Equal	D.31	155
0,0,1	Unequal	D.32	156
0,1,0	Equal	D.33	157
0,1,0	Unequal	D.34	158
1,0,0	Equal	D.35	159
1,0,0	Unequal	D.36	160
0,0,2	Equal	D.37	161
0,0,2	Unequal	D.38	162
0,2,0	Equal	D.39	163
0,2,0	Unequal	D.40	164
2,0,0	Equal	D.41	165
2,0,0	Unequal	D.42	166
0,1,2	Equal	D.43	167
0,1,2	Unequal	D.44	168
0,2,1	Equal	D.45	169
0,2,1	Unequal	D.46	170
1,0,2	Equal	D.47	171
1,0,2	Unequal	D.48	172
1,2,0	Equal	D.49	173
1,2,0	Unequal	D.50	174
2,0,1	Equal	D.51	175
2,0,1	Unequal	D.52	176
2,1,0	Equal	D.53	177
2,1,0	Unequal	D.54	178

Table D.28: List of Tables for Setting 2b

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0193	0.0079	0.0069	0.0671	0.0571	0.0638	0.1176	0.1043	0.1365
6,6,6	0.0195	0.0090	0.0197	0.0673	0.0532	0.0868	0.1176	0.1146	0.1477
7,7,7	0.0184	0.0096	0.0156	0.0642	0.0557	0.0663	0.1129	0.1110	0.1150
8,8,8	0.0192	0.0108	0.0250	0.0649	0.0574	0.0775	0.1136	0.1126	0.1476
9,9,9	0.0184	0.0110	0.0188	0.0641	0.0572	0.0930	0.1133	0.1118	0.1400
10,10,10	0.0179	0.0113	0.0244	0.0634	0.0570	0.0946	0.1115	0.1123	0.1599

Table D.29: Setting 2b: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0122	0.0062	0.0139	0.0468	0.0422	0.0563	0.0873	0.0925	0.1064
5,7,6	0.0160	0.0075	0.0158	0.0571	0.0460	0.0613	0.1027	0.0978	0.1123
6,5,7	0.0139	0.0074	0.0210	0.0534	0.0484	0.0778	0.0970	0.1026	0.1420
6,7,5	0.0251	0.0096	0.0258	0.0813	0.0581	0.0957	0.1378	0.1149	0.1675
7,5,6	0.0221	0.0105	0.0272	0.0748	0.0592	0.0946	0.1289	0.1222	0.1623
7,6,5	0.0302	0.0120	0.0326	0.0926	0.0663	0.1047	0.1524	0.1267	0.1784
Average	0.0199	0.0089	0.0227	0.0677	0.0534	0.0817	0.1177	0.1095	0.1448
6,7,8	0.0130	0.0076	0.0119	0.0489	0.0464	0.0711	0.0911	0.0947	0.1189
6,8,7	0.0163	0.0086	0.0127	0.0585	0.0497	0.0796	0.1043	0.1023	0.1322
7,6,8	0.0234	0.0111	0.0174	0.0780	0.0583	0.0914	0.1320	0.1180	0.1452
7,8,6	0.0242	0.0115	0.0172	0.0781	0.0592	0.0924	0.1327	0.1194	0.1478
8,6,7	0.0222	0.0115	0.0218	0.0743	0.0631	0.1111	0.1279	0.1245	0.1749
8,7,6	0.0269	0.0132	0.0244	0.0850	0.0632	0.1139	0.1426	0.1274	0.1776
Average	0.0210	0.0106	0.0176	0.0705	0.0567	0.0933	0.1218	0.1144	0.1494
5,8,11	0.0061	0.0046	0.0043	0.0294	0.0322	0.0322	0.0574	0.0704	0.0682
5,11,8	0.0130	0.0071	0.0066	0.0498	0.0415	0.0439	0.0903	0.0870	0.0896
8,5,11	0.0090	0.0069	0.0117	0.0381	0.0443	0.0662	0.0751	0.0955	0.1221
8,11,5	0.0363	0.0144	0.0218	0.1022	0.0689	0.1084	0.1642	0.1284	0.1865
11,5,8	0.0277	0.0148	0.0316	0.0871	0.0748	0.1236	0.1455	0.1410	0.2078
11,8,5	0.0514	0.0209	0.0403	0.1339	0.0873	0.1504	0.2065	0.1564	0.2489
Average	0.0239	0.0115	0.0194	0.0734	0.0582	0.0875	0.1232	0.1131	0.1539
6,9,12	0.0064	0.0055	0.0076	0.0316	0.0349	0.0380	0.0628	0.0763	0.0774
6,12,9	0.0137	0.0079	0.0106	0.0513	0.0430	0.0506	0.0934	0.0897	0.1000
9,6,12	0.0094	0.0072	0.0149	0.0389	0.0454	0.0602	0.0763	0.0967	0.1126
9,12,6	0.0333	0.0137	0.0257	0.0971	0.0673	0.0941	0.1564	0.1287	0.1636
12,6,9	0.0259	0.0152	0.0386	0.0836	0.0735	0.1240	0.1419	0.1393	0.2061
12,9,6	0.0462	0.0202	0.0484	0.1231	0.0841	0.1425	0.1935	0.1544	0.2259
Average	0.0225	0.0116	0.0243	0.0709	0.0580	0.0849	0.1207	0.1142	0.1476

Table D.30: Setting 2b: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0386	0.0160	0.0123	0.1164	0.0936	0.0980	0.1849	0.1560	0.1891
6,6,6	0.0462	0.0217	0.0372	0.1285	0.0971	0.1337	0.1990	0.1817	0.2134
7,7,7	0.0520	0.0276	0.0357	0.1393	0.1125	0.1180	0.2136	0.1928	0.1869
8,8,8	0.0570	0.0318	0.0541	0.1484	0.1212	0.1392	0.2250	0.2015	0.2364
9,9,9	0.0631	0.0380	0.0489	0.1616	0.1302	0.1729	0.2400	0.2143	0.2373
10,10,10	0.0700	0.0438	0.0639	0.1748	0.1446	0.1873	0.2554	0.2320	0.2780

Table D.31: Setting 2b: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0324	0.0180	0.0315	0.1025	0.0879	0.0998	0.1669	0.1617	0.1692
5,7,6	0.0394	0.0191	0.0313	0.1135	0.0878	0.1062	0.1800	0.1621	0.1755
6,5,7	0.0373	0.0211	0.0426	0.1110	0.0959	0.1325	0.1809	0.1742	0.2145
6,7,5	0.0531	0.0216	0.0431	0.1417	0.1018	0.1393	0.2147	0.1724	0.2248
7,5,6	0.0514	0.0251	0.0518	0.1389	0.1054	0.1507	0.2153	0.1927	0.2323
7,6,5	0.0603	0.0246	0.0508	0.1546	0.1078	0.1454	0.2313	0.1841	0.2321
Average	0.0457	0.0216	0.0419	0.1270	0.0978	0.1290	0.1982	0.1745	0.2081
6,7,8	0.0394	0.0236	0.0298	0.1169	0.1024	0.1354	0.1865	0.1788	0.2022
6,8,7	0.0444	0.0241	0.0278	0.1261	0.1018	0.1339	0.1971	0.1789	0.2043
7,6,8	0.0575	0.0281	0.0340	0.1485	0.1079	0.1474	0.2244	0.1911	0.2183
7,8,6	0.0579	0.0282	0.0356	0.1514	0.1092	0.1478	0.2253	0.1917	0.2174
8,6,7	0.0582	0.0315	0.0456	0.1513	0.1232	0.1743	0.2273	0.2065	0.2532
8,7,6	0.0667	0.0327	0.0464	0.1630	0.1184	0.1798	0.2404	0.2041	0.2585
Average	0.0540	0.0280	0.0365	0.1429	0.1105	0.1531	0.2168	0.1919	0.2257
5,8,11	0.0286	0.0213	0.0174	0.0956	0.0946	0.0847	0.1585	0.1688	0.1505
5,11,8	0.0449	0.0250	0.0180	0.1246	0.1000	0.0924	0.1928	0.1744	0.1648
8,5,11	0.0376	0.0299	0.0376	0.1164	0.1186	0.1422	0.1887	0.2037	0.2249
8,11,5	0.0757	0.0295	0.0359	0.1752	0.1150	0.1550	0.2541	0.1925	0.2491
11,5,8	0.0774	0.0404	0.0645	0.1851	0.1502	0.2044	0.2679	0.2392	0.3075
11,8,5	0.0990	0.0397	0.0613	0.2150	0.1371	0.2010	0.2992	0.2228	0.3114
Average	0.0605	0.0310	0.0391	0.1520	0.1193	0.1466	0.2269	0.2002	0.2347
6,9,12	0.0364	0.0280	0.0313	0.1115	0.1080	0.1066	0.1798	0.1889	0.1777
6,12,9	0.0503	0.0300	0.0312	0.1379	0.1112	0.1113	0.2090	0.1888	0.1865
9,6,12	0.0436	0.0345	0.0490	0.1293	0.1286	0.1393	0.2049	0.2176	0.2215
9,12,6	0.0817	0.0353	0.0493	0.1828	0.1252	0.1506	0.2622	0.2097	0.2380
12,6,9	0.0801	0.0474	0.0807	0.1903	0.1535	0.2069	0.2762	0.2461	0.3088
12,9,6	0.1031	0.0450	0.0830	0.2208	0.1491	0.2126	0.3059	0.2391	0.3116
Average	0.0659	0.0367	0.0541	0.1621	0.1293	0.1546	0.2397	0.2150	0.2407

Table D.32: Setting 2b: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0346	0.0158	0.0156	0.1102	0.1014	0.1098	0.1826	0.1766	0.2098
6,6,6	0.0393	0.0216	0.0473	0.1223	0.1109	0.1533	0.1986	0.2106	0.2364
7,7,7	0.0428	0.0274	0.0443	0.1301	0.1290	0.1389	0.2104	0.2283	0.2146
8,8,8	0.0485	0.0343	0.0697	0.1419	0.1464	0.1651	0.2259	0.2472	0.2719
9,9,9	0.0518	0.0415	0.0633	0.1492	0.1584	0.2069	0.2375	0.2644	0.2788
10,10,10	0.0566	0.0483	0.0806	0.1629	0.1755	0.2172	0.2553	0.2867	0.3197

Table D.33: Setting 2b: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0276	0.0160	0.0340	0.0912	0.0900	0.1121	0.1545	0.1742	0.1882
5,7,6	0.0343	0.0199	0.0412	0.1102	0.1015	0.1222	0.1837	0.1912	0.1959
6,5,7	0.0292	0.0160	0.0422	0.0973	0.0955	0.1343	0.1670	0.1809	0.2218
6,7,5	0.0501	0.0270	0.0658	0.1459	0.1273	0.1806	0.2296	0.2236	0.2791
7,5,6	0.0427	0.0223	0.0569	0.1287	0.1145	0.1533	0.2072	0.2107	0.2381
7,6,5	0.0564	0.0296	0.0729	0.1595	0.1353	0.1926	0.2460	0.2349	0.2906
Average	0.0401	0.0218	0.0522	0.1221	0.1107	0.1492	0.1980	0.2026	0.2356
6,7,8	0.0321	0.0219	0.0326	0.1038	0.1083	0.1416	0.1754	0.2008	0.2133
6,8,7	0.0396	0.0258	0.0410	0.1230	0.1220	0.1701	0.2012	0.2184	0.2496
7,6,8	0.0526	0.0340	0.0524	0.1531	0.1438	0.1843	0.2394	0.2467	0.2611
7,8,6	0.0547	0.0344	0.0534	0.1537	0.1431	0.1829	0.2410	0.2484	0.2615
8,6,7	0.0475	0.0302	0.0545	0.1390	0.1342	0.2027	0.2212	0.2337	0.2912
8,7,6	0.0590	0.0354	0.0615	0.1611	0.1484	0.2070	0.2505	0.2536	0.2897
Average	0.0476	0.0303	0.0492	0.1390	0.1333	0.1814	0.2215	0.2336	0.2611
5,8,11	0.0221	0.0179	0.0161	0.0808	0.0941	0.0855	0.1431	0.1757	0.1544
5,11,8	0.0404	0.0285	0.0296	0.1219	0.1225	0.1176	0.1987	0.2159	0.1985
8,5,11	0.0218	0.0160	0.0268	0.0809	0.0967	0.1197	0.1444	0.1822	0.2005
8,11,5	0.0816	0.0518	0.0808	0.2085	0.1863	0.2417	0.3095	0.3016	0.3505
11,5,8	0.0549	0.0327	0.0639	0.1540	0.1471	0.2006	0.2406	0.2482	0.3064
11,8,5	0.1051	0.0616	0.1062	0.2466	0.2066	0.2897	0.3546	0.3228	0.4108
Average	0.0543	0.0348	0.0539	0.1488	0.1422	0.1758	0.2318	0.2411	0.2702
6,9,12	0.0271	0.0245	0.0295	0.0949	0.1104	0.1033	0.1628	0.2007	0.1760
6,12,9	0.0458	0.0353	0.0506	0.1353	0.1398	0.1515	0.2155	0.2432	0.2407
9,6,12	0.0272	0.0232	0.0402	0.0941	0.1136	0.1258	0.1630	0.2064	0.2098
9,12,6	0.0865	0.0596	0.0936	0.2141	0.2025	0.2197	0.3175	0.3227	0.3197
12,6,9	0.0568	0.0406	0.0874	0.1622	0.1614	0.2271	0.2523	0.2670	0.3371
12,9,6	0.1024	0.0664	0.1224	0.2445	0.2167	0.2681	0.3509	0.3356	0.3754
Average	0.0576	0.0416	0.0706	0.1575	0.1574	0.1826	0.2437	0.2626	0.2765

Table D.34: Setting 2b: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0311	0.0132	0.0104	0.1081	0.1033	0.0958	0.1830	0.1836	0.2021
6,6,6	0.0352	0.0186	0.0330	0.1172	0.1127	0.1411	0.1957	0.2179	0.2323
7,7,7	0.0387	0.0266	0.0309	0.1259	0.1340	0.1219	0.2124	0.2410	0.2055
8,8,8	0.0401	0.0312	0.0493	0.1345	0.1500	0.1475	0.2246	0.2606	0.2691
9,9,9	0.0442	0.0391	0.0419	0.1454	0.1662	0.1947	0.2401	0.2824	0.2797
10,10,10	0.0487	0.0479	0.0598	0.1559	0.1860	0.2126	0.2557	0.3091	0.3305

Table D.35: Setting 2b: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0211	0.0117	0.0238	0.0808	0.0852	0.0936	0.1456	0.1741	0.1732
5,7,6	0.0273	0.0143	0.0284	0.0963	0.0934	0.1053	0.1693	0.1880	0.1820
6,5,7	0.0272	0.0156	0.0307	0.0985	0.1024	0.1232	0.1715	0.2004	0.2208
6,7,5	0.0447	0.0222	0.0488	0.1381	0.1258	0.1637	0.2240	0.2269	0.2697
7,5,6	0.0450	0.0237	0.0455	0.1386	0.1283	0.1520	0.2257	0.2337	0.2495
7,6,5	0.0558	0.0280	0.0562	0.1613	0.1419	0.1798	0.2548	0.2475	0.2884
Average	0.0369	0.0193	0.0389	0.1189	0.1128	0.1363	0.1985	0.2118	0.2306
6,7,8	0.0238	0.0170	0.0201	0.0914	0.1056	0.1259	0.1657	0.2045	0.2038
6,8,7	0.0299	0.0203	0.0259	0.1058	0.1149	0.1459	0.1842	0.2165	0.2317
7,6,8	0.0468	0.0298	0.0365	0.1447	0.1417	0.1679	0.2351	0.2528	0.2532
7,8,6	0.0446	0.0289	0.0362	0.1429	0.1416	0.1660	0.2353	0.2532	0.2513
8,6,7	0.0475	0.0320	0.0386	0.1462	0.1493	0.1959	0.2377	0.2571	0.2955
8,7,6	0.0560	0.0358	0.0468	0.1647	0.1565	0.2021	0.2599	0.2697	0.2951
Average	0.0414	0.0273	0.0340	0.1326	0.1349	0.1673	0.2197	0.2423	0.2551
5,8,11	0.0115	0.0097	0.0083	0.0544	0.0737	0.0622	0.1085	0.1572	0.1306
5,11,8	0.0226	0.0155	0.0162	0.0859	0.0966	0.0911	0.1543	0.1900	0.1705
8,5,11	0.0241	0.0221	0.0193	0.0937	0.1219	0.1193	0.1675	0.2228	0.2168
8,11,5	0.0684	0.0436	0.0582	0.1922	0.1832	0.2185	0.2966	0.3012	0.3369
11,5,8	0.0687	0.0462	0.0521	0.1875	0.1827	0.2175	0.2883	0.2950	0.3433
11,8,5	0.1103	0.0667	0.0900	0.2632	0.2216	0.2894	0.3747	0.3442	0.4227
Average	0.0509	0.0340	0.0407	0.1462	0.1466	0.1663	0.2317	0.2517	0.2701
6,9,12	0.0145	0.0142	0.0158	0.0653	0.0932	0.0780	0.1278	0.1854	0.1528
6,12,9	0.0251	0.0199	0.0284	0.0958	0.1145	0.1142	0.1715	0.2164	0.2025
9,6,12	0.0287	0.0296	0.0299	0.1063	0.1406	0.1250	0.1883	0.2486	0.2245
9,12,6	0.0722	0.0526	0.0713	0.2003	0.1991	0.2007	0.3079	0.3234	0.3098
12,6,9	0.0731	0.0558	0.0797	0.1983	0.2004	0.2479	0.3020	0.3202	0.3748
12,9,6	0.1078	0.0726	0.1087	0.2606	0.2343	0.2771	0.3762	0.3623	0.3978
Average	0.0536	0.0408	0.0556	0.1544	0.1637	0.1738	0.2456	0.2761	0.2770

Table D.36: Setting 2b: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1136	0.0484	0.0316	0.2625	0.2097	0.1949	0.3675	0.3042	0.3355
6,6,6	0.1433	0.0738	0.0937	0.3087	0.2340	0.2702	0.4170	0.3689	0.3828
7,7,7	0.1750	0.0984	0.0963	0.3560	0.2855	0.2686	0.4683	0.4109	0.3730
8,8,8	0.2103	0.1249	0.1537	0.4019	0.3277	0.3211	0.5153	0.4509	0.4665
9,9,9	0.2440	0.1563	0.1481	0.4425	0.3636	0.3985	0.5540	0.4904	0.4892
10,10,10	0.2778	0.1829	0.1957	0.4844	0.4026	0.4332	0.5965	0.5307	0.5578

Table D.37: Setting 2b: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1190	0.0711	0.0925	0.2816	0.2378	0.2323	0.3930	0.3614	0.3429
5,7,6	0.1290	0.0660	0.0827	0.2887	0.2210	0.2381	0.3984	0.3487	0.3478
6,5,7	0.1310	0.0788	0.1173	0.3012	0.2534	0.2838	0.4132	0.3805	0.4085
6,7,5	0.1518	0.0642	0.0962	0.3140	0.2248	0.2605	0.4193	0.3328	0.3771
7,5,6	0.1586	0.0844	0.1232	0.3294	0.2492	0.3020	0.4404	0.3835	0.4139
7,6,5	0.1651	0.0713	0.1113	0.3326	0.2359	0.2606	0.4391	0.3459	0.3766
Average	0.1424	0.0726	0.1039	0.3079	0.2370	0.2629	0.4172	0.3588	0.3778
6,7,8	0.1520	0.0953	0.0940	0.3329	0.2855	0.3165	0.4488	0.4112	0.4180
6,8,7	0.1640	0.0923	0.0847	0.3403	0.2742	0.2955	0.4530	0.3994	0.3998
7,6,8	0.1812	0.0942	0.0882	0.3559	0.2612	0.2946	0.4647	0.3903	0.3974
7,8,6	0.1823	0.0955	0.0897	0.3574	0.2617	0.2931	0.4632	0.3902	0.3971
8,6,7	0.1891	0.1063	0.1172	0.3728	0.2996	0.3416	0.4849	0.4252	0.4466
8,7,6	0.1956	0.1018	0.1113	0.3733	0.2721	0.3414	0.4840	0.4043	0.4481
Average	0.1774	0.0976	0.0975	0.3554	0.2757	0.3138	0.4664	0.4034	0.4178
5,8,11	0.1431	0.1125	0.0763	0.3309	0.3130	0.2583	0.4554	0.4468	0.3877
5,11,8	0.1751	0.1014	0.0611	0.3558	0.2907	0.2454	0.4678	0.4147	0.3735
8,5,11	0.1741	0.1383	0.1308	0.3764	0.3576	0.3582	0.5017	0.4942	0.4854
8,11,5	0.2098	0.0898	0.0802	0.3813	0.2554	0.2827	0.4821	0.3693	0.4122
11,5,8	0.2494	0.1466	0.1656	0.4481	0.3594	0.4078	0.5609	0.4855	0.5386
11,8,5	0.2554	0.1153	0.1267	0.4305	0.2861	0.3343	0.5306	0.4010	0.4760
Average	0.2012	0.1173	0.1068	0.3872	0.3104	0.3145	0.4998	0.4353	0.4456
6,9,12	0.1845	0.1453	0.1308	0.3887	0.3624	0.3226	0.5135	0.4994	0.4499
6,12,9	0.2097	0.1319	0.1090	0.4037	0.3319	0.2924	0.5189	0.4593	0.4223
9,6,12	0.2123	0.1695	0.1700	0.4262	0.3987	0.3698	0.5523	0.5366	0.5025
9,12,6	0.2431	0.1176	0.1212	0.4217	0.2989	0.3048	0.5220	0.4221	0.4329
12,6,9	0.2789	0.1780	0.2051	0.4838	0.3900	0.4161	0.5957	0.5196	0.5479
12,9,6	0.2817	0.1339	0.1770	0.4629	0.3250	0.3850	0.5627	0.4500	0.5107
Average	0.2350	0.1460	0.1522	0.4312	0.3512	0.3485	0.5442	0.4812	0.4777

Table D.38: Setting 2b: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1007	0.0497	0.0439	0.2612	0.2593	0.2452	0.3850	0.3981	0.4069
6,6,6	0.1287	0.0771	0.1272	0.3110	0.3128	0.3308	0.4420	0.4829	0.4571
7,7,7	0.1582	0.1201	0.1367	0.3620	0.3857	0.3403	0.4993	0.5478	0.4673
8,8,8	0.1912	0.1668	0.2094	0.4144	0.4515	0.3994	0.5568	0.6107	0.5651
9,9,9	0.2270	0.2134	0.2070	0.4639	0.5093	0.5007	0.6051	0.6610	0.6060
10,10,10	0.2670	0.2669	0.2677	0.5164	0.5699	0.5397	0.6576	0.7159	0.6749

Table D.39: Setting 2b: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0993	0.0622	0.1026	0.2606	0.2706	0.2808	0.3840	0.4323	0.4160
5,7,6	0.1242	0.0820	0.1283	0.3061	0.3139	0.2913	0.4375	0.4745	0.4115
6,5,7	0.0958	0.0521	0.1080	0.2552	0.2594	0.2934	0.3779	0.4153	0.4256
6,7,5	0.1619	0.1075	0.1831	0.3663	0.3628	0.4020	0.5020	0.5275	0.5407
7,5,6	0.1279	0.0691	0.1387	0.3096	0.3022	0.3032	0.4382	0.4592	0.4244
7,6,5	0.1685	0.1021	0.1864	0.3733	0.3599	0.4092	0.5103	0.5269	0.5498
Average	0.1296	0.0792	0.1412	0.3119	0.3115	0.3300	0.4417	0.4726	0.4613
6,7,8	0.1269	0.0994	0.1058	0.3141	0.3450	0.3381	0.4479	0.5078	0.4531
6,8,7	0.1570	0.1255	0.1415	0.3606	0.3872	0.4183	0.4983	0.5486	0.5352
7,6,8	0.1938	0.1536	0.1632	0.4152	0.4322	0.4152	0.5552	0.5931	0.5285
7,8,6	0.1918	0.1505	0.1622	0.4134	0.4316	0.4122	0.5535	0.5915	0.5272
8,6,7	0.1569	0.1148	0.1504	0.3596	0.3702	0.4334	0.4951	0.5314	0.5541
8,7,6	0.2002	0.1496	0.1716	0.4219	0.4313	0.4294	0.5598	0.5900	0.5443
Average	0.1711	0.1322	0.1491	0.3808	0.3996	0.4078	0.5183	0.5604	0.5237
5,8,11	0.1123	0.0990	0.0663	0.2927	0.3374	0.2669	0.4244	0.4992	0.4095
5,11,8	0.1792	0.1554	0.1278	0.3969	0.4276	0.3546	0.5365	0.5943	0.5028
8,5,11	0.0926	0.0643	0.0780	0.2530	0.2909	0.2793	0.3770	0.4450	0.4149
8,11,5	0.2933	0.2539	0.2597	0.5498	0.5668	0.5495	0.6863	0.7166	0.6865
11,5,8	0.1722	0.1128	0.1531	0.3760	0.3785	0.3793	0.5083	0.5298	0.5264
11,8,5	0.3284	0.2562	0.2835	0.5791	0.5569	0.5856	0.7062	0.6989	0.7179
Average	0.1963	0.1569	0.1614	0.4079	0.4264	0.4025	0.5398	0.5806	0.5430
6,9,12	0.1454	0.1431	0.1196	0.3498	0.4046	0.3182	0.4904	0.5687	0.4632
6,12,9	0.2154	0.2012	0.2098	0.4517	0.4983	0.4478	0.5940	0.6597	0.5876
9,6,12	0.1224	0.1052	0.1240	0.3093	0.3568	0.3136	0.4449	0.5150	0.4583
9,12,6	0.3322	0.3068	0.2919	0.5929	0.6203	0.5221	0.7249	0.7608	0.6603
12,6,9	0.2080	0.1648	0.2354	0.4305	0.4444	0.4825	0.5677	0.5969	0.6239
12,9,6	0.3552	0.3038	0.3107	0.6098	0.6035	0.5483	0.7339	0.7370	0.6812
Average	0.2298	0.2042	0.2152	0.4573	0.4880	0.4388	0.5926	0.6397	0.5791

Table D.40: Setting 2b: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0932	0.0459	0.0288	0.2644	0.2837	0.2132	0.3988	0.4318	0.3965
6,6,6	0.1180	0.0805	0.1003	0.3186	0.3442	0.3285	0.4655	0.5318	0.4787
7,7,7	0.1512	0.1309	0.1017	0.3755	0.4310	0.3243	0.5316	0.6043	0.4801
8,8,8	0.1820	0.1825	0.1810	0.4313	0.5018	0.4133	0.5924	0.6669	0.6106
9,9,9	0.2178	0.2385	0.1701	0.4864	0.5673	0.5304	0.6458	0.7230	0.6577
10,10,10	0.2597	0.3024	0.2514	0.5442	0.6299	0.5874	0.6998	0.7737	0.7404

Table D.41: Setting 2b: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0731	0.0473	0.0728	0.2317	0.2690	0.2387	0.3696	0.4501	0.3864
5,7,6	0.0875	0.0559	0.0866	0.2630	0.2926	0.2480	0.4076	0.4756	0.3854
6,5,7	0.0993	0.0685	0.0886	0.2807	0.3139	0.2948	0.4232	0.4936	0.4670
6,7,5	0.1428	0.0943	0.1339	0.3599	0.3779	0.3737	0.5097	0.5578	0.5448
7,5,6	0.1506	0.1034	0.1291	0.3629	0.3753	0.3400	0.5084	0.5436	0.4887
7,6,5	0.1734	0.1179	0.1536	0.4038	0.4047	0.4030	0.5507	0.5731	0.5637
Average	0.1211	0.0812	0.1108	0.3170	0.3389	0.3164	0.4615	0.5156	0.4727
6,7,8	0.0997	0.0885	0.0742	0.2914	0.3579	0.3245	0.4450	0.5417	0.4626
6,8,7	0.1150	0.1014	0.0912	0.3229	0.3836	0.3732	0.4787	0.5688	0.5187
7,6,8	0.1722	0.1485	0.1238	0.4110	0.4576	0.4080	0.5665	0.6294	0.5439
7,8,6	0.1717	0.1473	0.1222	0.4132	0.4560	0.4069	0.5685	0.6300	0.5441
8,6,7	0.1823	0.1555	0.1303	0.4171	0.4522	0.4678	0.5688	0.6159	0.6097
8,7,6	0.2075	0.1761	0.1474	0.4538	0.4824	0.4645	0.6033	0.6450	0.6005
Average	0.1581	0.1362	0.1149	0.3849	0.4316	0.4075	0.5385	0.6051	0.5466
5,8,11	0.0513	0.0483	0.0399	0.1998	0.2897	0.2031	0.3425	0.4776	0.3469
5,11,8	0.0812	0.0696	0.0640	0.2680	0.3485	0.2530	0.4230	0.5402	0.4099
8,5,11	0.1258	0.1325	0.0744	0.3440	0.4271	0.3459	0.4995	0.5969	0.5285
8,11,5	0.2607	0.2409	0.1866	0.5376	0.5780	0.5198	0.6885	0.7322	0.6896
11,5,8	0.2757	0.2284	0.1883	0.5257	0.5345	0.5261	0.6624	0.6786	0.6852
11,8,5	0.3721	0.3088	0.2760	0.6371	0.6166	0.6336	0.7609	0.7517	0.7714
Average	0.1945	0.1714	0.1382	0.4187	0.4657	0.4136	0.5628	0.6295	0.5719
6,9,12	0.0757	0.0895	0.0771	0.2623	0.3812	0.2693	0.4247	0.5716	0.4380
6,12,9	0.1118	0.1196	0.1172	0.3345	0.4400	0.3462	0.5021	0.6272	0.5168
9,6,12	0.1619	0.1875	0.1323	0.4051	0.4975	0.3926	0.5673	0.6629	0.5709
9,12,6	0.2992	0.3017	0.2375	0.5861	0.6367	0.5128	0.7314	0.7785	0.6724
12,6,9	0.3076	0.2843	0.2890	0.5757	0.5937	0.6109	0.7121	0.7350	0.7493
12,9,6	0.4020	0.3614	0.3427	0.6687	0.6666	0.6309	0.7891	0.7936	0.7578
Average	0.2264	0.2240	0.1993	0.4721	0.5360	0.4605	0.6211	0.6948	0.6175

Table D.42: Setting 2b: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0778	0.0333	0.0234	0.2107	0.1902	0.1661	0.3179	0.2960	0.3059
6,6,6	0.0963	0.0545	0.0740	0.2436	0.2165	0.2444	0.3576	0.3593	0.3616
7,7,7	0.1164	0.0761	0.0750	0.2809	0.2667	0.2319	0.4007	0.4036	0.3459
8,8,8	0.1398	0.1012	0.1295	0.3194	0.3106	0.2923	0.4444	0.4479	0.4483
9,9,9	0.1649	0.1281	0.1231	0.3580	0.3535	0.3749	0.4873	0.4946	0.4760
10,10,10	0.1915	0.1576	0.1746	0.3947	0.3936	0.4171	0.5237	0.5367	0.5522

Table D.43: Setting 2b: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0700	0.0432	0.0637	0.1978	0.1902	0.1850	0.3033	0.3200	0.2957
5,7,6	0.0788	0.0417	0.0661	0.2089	0.1867	0.1942	0.3162	0.3173	0.3014
6,5,7	0.0879	0.0575	0.0789	0.2343	0.2240	0.2356	0.3470	0.3634	0.3681
6,7,5	0.1062	0.0515	0.0879	0.2560	0.2174	0.2512	0.3700	0.3441	0.3785
7,5,6	0.1207	0.0706	0.0968	0.2838	0.2465	0.2676	0.3988	0.3867	0.3918
7,6,5	0.1293	0.0665	0.1031	0.2943	0.2450	0.2683	0.4110	0.3714	0.3956
Average	0.0988	0.0552	0.0828	0.2459	0.2183	0.2337	0.3577	0.3505	0.3552
6,7,8	0.0902	0.0642	0.0645	0.2381	0.2382	0.2584	0.3553	0.3768	0.3655
6,8,7	0.0983	0.0629	0.0683	0.2483	0.2374	0.2635	0.3636	0.3738	0.3733
7,6,8	0.1252	0.0778	0.0831	0.2903	0.2601	0.2847	0.4096	0.4018	0.3895
7,8,6	0.1251	0.0772	0.0816	0.2909	0.2594	0.2839	0.4095	0.4011	0.3908
8,6,7	0.1409	0.0934	0.0928	0.3162	0.2928	0.3298	0.4397	0.4318	0.4498
8,7,6	0.1496	0.0944	0.1011	0.3267	0.2844	0.3351	0.4474	0.4257	0.4485
Average	0.1216	0.0783	0.0819	0.2851	0.2621	0.2926	0.4042	0.4018	0.4029
5,8,11	0.0628	0.0529	0.0436	0.1933	0.2167	0.1816	0.3040	0.3556	0.2966
5,11,8	0.0825	0.0540	0.0508	0.2169	0.2136	0.1952	0.3268	0.3459	0.3118
8,5,11	0.1181	0.1064	0.0746	0.2974	0.3242	0.2874	0.4275	0.4713	0.4337
8,11,5	0.1578	0.0928	0.1045	0.3333	0.2871	0.3186	0.4540	0.4224	0.4527
11,5,8	0.2139	0.1452	0.1342	0.4184	0.3738	0.4021	0.5397	0.5116	0.5510
11,8,5	0.2300	0.1353	0.1538	0.4267	0.3439	0.4001	0.5430	0.4784	0.5436
Average	0.1442	0.0978	0.0936	0.3143	0.2932	0.2975	0.4325	0.4309	0.4316
6,9,12	0.0844	0.0753	0.0780	0.2361	0.2673	0.2334	0.3585	0.4131	0.3605
6,12,9	0.1045	0.0759	0.0903	0.2599	0.2603	0.2501	0.3765	0.3998	0.3720
9,6,12	0.1436	0.1344	0.1114	0.3397	0.3656	0.3093	0.4756	0.5150	0.4557
9,12,6	0.1760	0.1146	0.1412	0.3653	0.3263	0.3266	0.4882	0.4698	0.4558
12,6,9	0.2352	0.1740	0.1882	0.4485	0.4081	0.4304	0.5734	0.5469	0.5725
12,9,6	0.2468	0.1565	0.2063	0.4519	0.3824	0.4303	0.5702	0.5237	0.5593
Average	0.1651	0.1218	0.1359	0.3502	0.3350	0.3300	0.4737	0.4781	0.4626

Table D.44: Setting 2b: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0660	0.0302	0.0205	0.2065	0.2226	0.1779	0.3274	0.3635	0.3423
6,6,6	0.0834	0.0507	0.0744	0.2472	0.2733	0.2655	0.3805	0.4494	0.4022
7,7,7	0.1008	0.0838	0.0726	0.2848	0.3387	0.2572	0.4286	0.5078	0.3972
8,8,8	0.1217	0.1209	0.1281	0.3320	0.4022	0.3222	0.4839	0.5737	0.5100
9,9,9	0.1452	0.1615	0.1194	0.3751	0.4647	0.4314	0.5355	0.6301	0.5570
10,10,10	0.1700	0.2050	0.1729	0.4179	0.5159	0.4712	0.5781	0.6787	0.6288

Table D.45: Setting 2b: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0509	0.0298	0.0519	0.1771	0.2086	0.1962	0.2959	0.3707	0.3296
5,7,6	0.0684	0.0436	0.0669	0.2158	0.2452	0.2063	0.3462	0.4130	0.3314
6,5,7	0.0609	0.0340	0.0607	0.1982	0.2242	0.2255	0.3184	0.3855	0.3736
6,7,5	0.1095	0.0710	0.1111	0.3002	0.3202	0.3308	0.4447	0.4967	0.4883
7,5,6	0.0953	0.0552	0.0889	0.2656	0.2814	0.2599	0.4011	0.4442	0.3934
7,6,5	0.1299	0.0811	0.1257	0.3291	0.3363	0.3526	0.4714	0.5062	0.5067
Average	0.0858	0.0525	0.0842	0.2477	0.2693	0.2619	0.3796	0.4361	0.4038
6,7,8	0.0653	0.0555	0.0491	0.2188	0.2764	0.2538	0.3535	0.4515	0.3788
6,8,7	0.0872	0.0742	0.0714	0.2640	0.3205	0.3237	0.4075	0.4969	0.4602
7,6,8	0.1275	0.1079	0.0925	0.3345	0.3814	0.3383	0.4847	0.5559	0.4681
7,8,6	0.1290	0.1093	0.0925	0.3402	0.3869	0.3444	0.4891	0.5582	0.4726
8,6,7	0.1138	0.0910	0.0916	0.3078	0.3472	0.3794	0.4499	0.5114	0.5176
8,7,6	0.1491	0.1225	0.1109	0.3675	0.4001	0.3815	0.5149	0.5671	0.5134
Average	0.1120	0.0934	0.0847	0.3055	0.3521	0.3369	0.4499	0.5235	0.4685
5,8,11	0.0394	0.0371	0.0251	0.1529	0.2204	0.1582	0.2674	0.3868	0.2855
5,11,8	0.0775	0.0690	0.0543	0.2399	0.3110	0.2282	0.3848	0.4932	0.3783
8,5,11	0.0563	0.0494	0.0386	0.1957	0.2642	0.2241	0.3219	0.4258	0.3743
8,11,5	0.2206	0.2051	0.1655	0.4822	0.5256	0.4783	0.6367	0.6878	0.6433
11,5,8	0.1519	0.1142	0.1129	0.3623	0.3863	0.3734	0.5035	0.5402	0.5366
11,8,5	0.2901	0.2384	0.2209	0.5488	0.5418	0.5574	0.6843	0.6906	0.7080
Average	0.1393	0.1189	0.1029	0.3303	0.3749	0.3366	0.4664	0.5374	0.4877
6,9,12	0.0526	0.0608	0.0496	0.1962	0.2907	0.2022	0.3272	0.4655	0.3460
6,12,9	0.0974	0.1042	0.0987	0.2897	0.3832	0.3108	0.4440	0.5689	0.4700
9,6,12	0.0737	0.0807	0.0715	0.2415	0.3279	0.2578	0.3816	0.4937	0.4154
9,12,6	0.2464	0.2536	0.1974	0.5177	0.5772	0.4489	0.6718	0.7322	0.6098
12,6,9	0.1726	0.1572	0.1857	0.4029	0.4429	0.4675	0.5497	0.6013	0.6198
12,9,6	0.3105	0.2815	0.2569	0.5779	0.5884	0.5270	0.7113	0.7288	0.6714
Average	0.1589	0.1563	0.1433	0.3710	0.4351	0.3690	0.5143	0.5984	0.5221

Table D.46: Setting 2b: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0856	0.0432	0.0429	0.2155	0.1821	0.2060	0.3131	0.2789	0.3294
6,6,6	0.1074	0.0604	0.1181	0.2480	0.2082	0.2776	0.3519	0.3331	0.3732
7,7,7	0.1296	0.0817	0.1244	0.2853	0.2499	0.2771	0.3909	0.3703	0.3663
8,8,8	0.1518	0.1021	0.1820	0.3195	0.2838	0.3245	0.4300	0.4073	0.4427
9,9,9	0.1773	0.1271	0.1840	0.3566	0.3170	0.3917	0.4674	0.4439	0.4673
10,10,10	0.1999	0.1498	0.2310	0.3873	0.3489	0.4209	0.5021	0.4780	0.5216

Table D.47: Setting 2b: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0867	0.0540	0.1008	0.2220	0.1993	0.2353	0.3222	0.3160	0.3355
5,7,6	0.1029	0.0596	0.1058	0.2469	0.2073	0.2441	0.3484	0.3271	0.3382
6,5,7	0.0876	0.0524	0.1184	0.2201	0.1967	0.2639	0.3197	0.3080	0.3677
6,7,5	0.1237	0.0672	0.1396	0.2735	0.2205	0.2965	0.3768	0.3337	0.3987
7,5,6	0.1098	0.0603	0.1351	0.2512	0.2067	0.2749	0.3514	0.3245	0.3695
7,6,5	0.1289	0.0670	0.1475	0.2787	0.2202	0.2972	0.3823	0.3355	0.4027
Average	0.1066	0.0601	0.1245	0.2487	0.2085	0.2687	0.3501	0.3241	0.3687
6,7,8	0.1122	0.0758	0.1079	0.2645	0.2425	0.2995	0.3707	0.3626	0.3866
6,8,7	0.1278	0.0819	0.1191	0.2864	0.2505	0.3174	0.3938	0.3725	0.4100
7,6,8	0.1446	0.0873	0.1300	0.3057	0.2544	0.3158	0.4112	0.3772	0.4024
7,8,6	0.1449	0.0874	0.1296	0.3044	0.2545	0.3170	0.4137	0.3791	0.4044
8,6,7	0.1318	0.0812	0.1398	0.2869	0.2464	0.3420	0.3927	0.3659	0.4330
8,7,6	0.1483	0.0867	0.1454	0.3091	0.2546	0.3399	0.4152	0.3793	0.4272
Average	0.1349	0.0834	0.1286	0.2928	0.2505	0.3219	0.3996	0.3728	0.4106
5,8,11	0.1054	0.0857	0.0785	0.2620	0.2590	0.2416	0.3751	0.3850	0.3506
5,11,8	0.1463	0.0997	0.0961	0.3163	0.2807	0.2721	0.4284	0.4057	0.3833
8,5,11	0.0960	0.0743	0.1141	0.2413	0.2388	0.2939	0.3473	0.3579	0.3969
8,11,5	0.1869	0.1089	0.1633	0.3592	0.2911	0.3661	0.4690	0.4176	0.4747
11,5,8	0.1498	0.0904	0.1634	0.3099	0.2636	0.3466	0.4160	0.3822	0.4555
11,8,5	0.2062	0.1152	0.1998	0.3814	0.2972	0.3969	0.4884	0.4201	0.5125
Average	0.1484	0.0957	0.1359	0.3117	0.2717	0.3195	0.4207	0.3948	0.4289
6,9,12	0.1310	0.1091	0.1305	0.3046	0.2975	0.2937	0.4213	0.4277	0.3995
6,12,9	0.1743	0.1250	0.1571	0.3546	0.3189	0.3326	0.4690	0.4504	0.4430
9,6,12	0.1205	0.0959	0.1553	0.2818	0.2750	0.3103	0.3918	0.3998	0.4149
9,12,6	0.2116	0.1321	0.2049	0.3932	0.3287	0.3665	0.5028	0.4581	0.4714
12,6,9	0.1712	0.1129	0.2181	0.3418	0.2971	0.3869	0.4513	0.4186	0.4943
12,9,6	0.2252	0.1336	0.2349	0.4051	0.3269	0.4014	0.5143	0.4527	0.5076
Average	0.1723	0.1181	0.1835	0.3469	0.3074	0.3486	0.4584	0.4346	0.4551

Table D.48: Setting 2b: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0854	0.0430	0.0439	0.2132	0.1811	0.2058	0.3098	0.2764	0.3296
6,6,6	0.1076	0.0601	0.1171	0.2484	0.2069	0.2765	0.3518	0.3320	0.3704
7,7,7	0.1290	0.0813	0.1238	0.2833	0.2499	0.2780	0.3901	0.3691	0.3684
8,8,8	0.1538	0.1046	0.1847	0.3204	0.2843	0.3246	0.4319	0.4070	0.4443
9,9,9	0.1741	0.1245	0.1804	0.3519	0.3145	0.3897	0.4637	0.4404	0.4654
10,10,10	0.1999	0.1485	0.2292	0.3861	0.3499	0.4204	0.5019	0.4787	0.5214

Table D.49: Setting 2b: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0864	0.0537	0.0987	0.2208	0.1973	0.2336	0.3212	0.3153	0.3316
5,7,6	0.1015	0.0601	0.1058	0.2452	0.2061	0.2428	0.3480	0.3264	0.3378
6,5,7	0.0882	0.0530	0.1195	0.2208	0.1972	0.2663	0.3213	0.3088	0.3683
6,7,5	0.1220	0.0667	0.1384	0.2705	0.2174	0.2945	0.3747	0.3310	0.3980
7,5,6	0.1080	0.0593	0.1353	0.2505	0.2064	0.2762	0.3544	0.3260	0.3734
7,6,5	0.1296	0.0675	0.1480	0.2799	0.2208	0.2979	0.3818	0.3357	0.4006
Average	0.1060	0.0601	0.1243	0.2480	0.2075	0.2686	0.3502	0.3239	0.3683
6,7,8	0.1116	0.0754	0.1077	0.2619	0.2388	0.2974	0.3678	0.3589	0.3818
6,8,7	0.1272	0.0805	0.1179	0.2828	0.2477	0.3179	0.3907	0.3696	0.4092
7,6,8	0.1425	0.0871	0.1287	0.3025	0.2525	0.3161	0.4099	0.3756	0.4030
7,8,6	0.1417	0.0853	0.1285	0.3037	0.2533	0.3192	0.4126	0.3770	0.4049
8,6,7	0.1293	0.0792	0.1397	0.2852	0.2454	0.3408	0.3914	0.3654	0.4313
8,7,6	0.1500	0.0882	0.1474	0.3117	0.2574	0.3417	0.4169	0.3818	0.4314
Average	0.1337	0.0826	0.1283	0.2913	0.2492	0.3222	0.3982	0.3714	0.4103
5,8,11	0.1056	0.0850	0.0784	0.2614	0.2586	0.2416	0.3748	0.3852	0.3513
5,11,8	0.1484	0.1017	0.0976	0.3190	0.2830	0.2750	0.4305	0.4089	0.3875
8,5,11	0.0945	0.0728	0.1129	0.2377	0.2335	0.2894	0.3434	0.3541	0.3921
8,11,5	0.1900	0.1127	0.1642	0.3627	0.2944	0.3691	0.4722	0.4222	0.4789
11,5,8	0.1500	0.0909	0.1652	0.3103	0.2629	0.3464	0.4153	0.3805	0.4564
11,8,5	0.2062	0.1134	0.1973	0.3833	0.2960	0.3961	0.4926	0.4209	0.5147
Average	0.1491	0.0961	0.1359	0.3124	0.2714	0.3196	0.4215	0.3953	0.4302
6,9,12	0.1322	0.1102	0.1301	0.3053	0.2966	0.2932	0.4231	0.4270	0.3998
6,12,9	0.1756	0.1258	0.1580	0.3575	0.3196	0.3318	0.4720	0.4495	0.4422
9,6,12	0.1199	0.0953	0.1539	0.2818	0.2753	0.3090	0.3944	0.4004	0.4163
9,12,6	0.2102	0.1311	0.2047	0.3914	0.3292	0.3680	0.5022	0.4568	0.4717
12,6,9	0.1718	0.1143	0.2163	0.3399	0.2941	0.3846	0.4495	0.4160	0.4918
12,9,6	0.2270	0.1366	0.2396	0.4072	0.3291	0.4056	0.5169	0.4569	0.5101
Average	0.1728	0.1189	0.1838	0.3472	0.3073	0.3487	0.4597	0.4344	0.4553

Table D.50: Setting 2b: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0668	0.0300	0.0214	0.2061	0.2235	0.1782	0.3276	0.3641	0.3433
6,6,6	0.0838	0.0500	0.0734	0.2485	0.2742	0.2659	0.3830	0.4496	0.4027
7,7,7	0.1018	0.0841	0.0715	0.2878	0.3419	0.2580	0.4319	0.5127	0.3970
8,8,8	0.1236	0.1230	0.1283	0.3306	0.4036	0.3209	0.4825	0.5711	0.5079
9,9,9	0.1439	0.1610	0.1211	0.3746	0.4615	0.4296	0.5336	0.6281	0.5554
10,10,10	0.1718	0.2067	0.1740	0.4206	0.5214	0.4713	0.5835	0.6818	0.6323

Table D.51: Setting 2b: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0503	0.0289	0.0512	0.1765	0.2072	0.1957	0.2940	0.3698	0.3296
5,7,6	0.0686	0.0434	0.0681	0.2177	0.2463	0.2068	0.3483	0.4137	0.3308
6,5,7	0.0603	0.0330	0.0592	0.1961	0.2230	0.2266	0.3204	0.3900	0.3773
6,7,5	0.1116	0.0730	0.1121	0.2996	0.3193	0.3265	0.4418	0.4932	0.4866
7,5,6	0.0950	0.0547	0.0890	0.2656	0.2825	0.2564	0.3986	0.4441	0.3917
7,6,5	0.1274	0.0792	0.1247	0.3257	0.3340	0.3524	0.4686	0.5030	0.5071
Average	0.0855	0.0520	0.0841	0.2469	0.2687	0.2607	0.3786	0.4356	0.4039
6,7,8	0.0667	0.0561	0.0504	0.2180	0.2778	0.2547	0.3513	0.4502	0.3796
6,8,7	0.0864	0.0753	0.0709	0.2624	0.3191	0.3223	0.4053	0.4943	0.4583
7,6,8	0.1287	0.1085	0.0939	0.3395	0.3863	0.3427	0.4898	0.5591	0.4707
7,8,6	0.1296	0.1106	0.0960	0.3415	0.3894	0.3425	0.4916	0.5595	0.4733
8,6,7	0.1147	0.0930	0.0922	0.3074	0.3455	0.3799	0.4519	0.5123	0.5196
8,7,6	0.1495	0.1210	0.1080	0.3657	0.3979	0.3779	0.5130	0.5659	0.5101
Average	0.1126	0.0941	0.0852	0.3058	0.3527	0.3367	0.4505	0.5236	0.4686
5,8,11	0.0389	0.0368	0.0236	0.1517	0.2201	0.1573	0.2689	0.3893	0.2873
5,11,8	0.0771	0.0696	0.0544	0.2427	0.3120	0.2278	0.3852	0.4914	0.3753
8,5,11	0.0573	0.0496	0.0404	0.1970	0.2659	0.2254	0.3254	0.4296	0.3748
8,11,5	0.2220	0.2074	0.1674	0.4842	0.5289	0.4797	0.6379	0.6902	0.6436
11,5,8	0.1525	0.1142	0.1121	0.3628	0.3862	0.3741	0.5048	0.5415	0.5370
11,8,5	0.2902	0.2393	0.2195	0.5502	0.5431	0.5603	0.6874	0.6909	0.7099
Average	0.1397	0.1195	0.1029	0.3314	0.3760	0.3374	0.4683	0.5388	0.4880
6,9,12	0.0533	0.0624	0.0499	0.1968	0.2917	0.2035	0.3300	0.4670	0.3476
6,12,9	0.0982	0.1046	0.1006	0.2941	0.3884	0.3138	0.4481	0.5733	0.4714
9,6,12	0.0734	0.0800	0.0706	0.2409	0.3268	0.2574	0.3806	0.4944	0.4154
9,12,6	0.2468	0.2548	0.1972	0.5204	0.5776	0.4510	0.6721	0.7329	0.6123
12,6,9	0.1732	0.1581	0.1843	0.4050	0.4432	0.4666	0.5495	0.6018	0.6207
12,9,6	0.3068	0.2788	0.2540	0.5763	0.5882	0.5276	0.7104	0.7297	0.6717
Average	0.1586	0.1565	0.1428	0.3723	0.4360	0.3700	0.5151	0.5999	0.5232

Table D.52: Setting 2b: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0782	0.0333	0.0231	0.2111	0.1915	0.1644	0.3158	0.2954	0.3051
6,6,6	0.0965	0.0540	0.0746	0.2444	0.2165	0.2435	0.3581	0.3605	0.3606
7,7,7	0.1183	0.0776	0.0757	0.2821	0.2696	0.2336	0.4022	0.4066	0.3453
8,8,8	0.1396	0.1006	0.1297	0.3188	0.3089	0.2947	0.4440	0.4479	0.4489
9,9,9	0.1639	0.1283	0.1229	0.3586	0.3513	0.3721	0.4885	0.4955	0.4751
10,10,10	0.1892	0.1583	0.1747	0.3968	0.3913	0.4166	0.5260	0.5389	0.5501

Table D.53: Setting 2b: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0707	0.0435	0.0642	0.1994	0.1939	0.1882	0.3083	0.3242	0.2997
5,7,6	0.0766	0.0398	0.0640	0.2106	0.1865	0.1946	0.3163	0.3175	0.3030
6,5,7	0.0882	0.0566	0.0776	0.2337	0.2247	0.2358	0.3479	0.3644	0.3688
6,7,5	0.1061	0.0510	0.0882	0.2586	0.2197	0.2542	0.3722	0.3490	0.3815
7,5,6	0.1210	0.0709	0.0970	0.2831	0.2465	0.2685	0.3993	0.3892	0.3907
7,6,5	0.1289	0.0666	0.1020	0.2920	0.2442	0.2663	0.4069	0.3699	0.3927
Average	0.0986	0.0547	0.0822	0.2462	0.2193	0.2346	0.3585	0.3524	0.3561
6,7,8	0.0897	0.0641	0.0645	0.2385	0.2373	0.2586	0.3539	0.3753	0.3637
6,8,7	0.0979	0.0638	0.0681	0.2471	0.2371	0.2658	0.3633	0.3764	0.3748
7,6,8	0.1262	0.0785	0.0824	0.2912	0.2592	0.2839	0.4093	0.4012	0.3891
7,8,6	0.1268	0.0777	0.0814	0.2916	0.2603	0.2855	0.4117	0.4026	0.3924
8,6,7	0.1404	0.0928	0.0919	0.3147	0.2922	0.3312	0.4391	0.4333	0.4495
8,7,6	0.1485	0.0936	0.0997	0.3241	0.2828	0.3327	0.4445	0.4233	0.4470
Average	0.1216	0.0784	0.0813	0.2845	0.2615	0.2930	0.4036	0.4020	0.4028
5,8,11	0.0639	0.0535	0.0447	0.1912	0.2165	0.1803	0.3020	0.3539	0.2955
5,11,8	0.0829	0.0538	0.0524	0.2191	0.2139	0.1970	0.3272	0.3441	0.3120
8,5,11	0.1185	0.1060	0.0743	0.2981	0.3236	0.2874	0.4301	0.4715	0.4352
8,11,5	0.1572	0.0927	0.1041	0.3329	0.2893	0.3174	0.4537	0.4240	0.4523
11,5,8	0.2135	0.1452	0.1358	0.4161	0.3723	0.3982	0.5390	0.5091	0.5469
11,8,5	0.2297	0.1340	0.1524	0.4264	0.3426	0.3990	0.5449	0.4786	0.5432
Average	0.1443	0.0975	0.0940	0.3140	0.2930	0.2966	0.4328	0.4302	0.4309
6,9,12	0.0855	0.0760	0.0776	0.2373	0.2685	0.2340	0.3592	0.4150	0.3595
6,12,9	0.1048	0.0761	0.0891	0.2574	0.2584	0.2468	0.3753	0.3975	0.3707
9,6,12	0.1442	0.1348	0.1131	0.3412	0.3660	0.3100	0.4778	0.5169	0.4571
9,12,6	0.1752	0.1142	0.1414	0.3635	0.3248	0.3259	0.4866	0.4687	0.4538
12,6,9	0.2333	0.1723	0.1887	0.4494	0.4071	0.4302	0.5736	0.5503	0.5728
12,9,6	0.2485	0.1573	0.2063	0.4561	0.3839	0.4284	0.5760	0.5244	0.5566
Average	0.1653	0.1218	0.1360	0.3508	0.3348	0.3292	0.4748	0.4788	0.4618

Table D.54: Setting 2b: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

Appendix E

Setting 3: Three Laplace Distributions

This appendix contains tables of the results for Settings 3 of the simulation studies. In each case 100,000 Monte Carlo data sets were generated and for each the three tests were conducted and proportion of tests rejecting is displayed.

Table E.1: List of Tables for Setting 3

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0064	0.0034	0.0027	0.0432	0.0447	0.0372	0.0926	0.0919	0.0906
6,6,6	0.0070	0.0041	0.0079	0.0425	0.0415	0.0485	0.0932	0.0987	0.0943
7,7,7	0.0070	0.0052	0.0057	0.0447	0.0454	0.0336	0.0957	0.0996	0.0663
8,8,8	0.0074	0.0064	0.0086	0.0441	0.0446	0.0365	0.0940	0.0971	0.0855
9,9,9	0.0073	0.0063	0.0059	0.0455	0.0458	0.0457	0.0956	0.0986	0.0754
10,10,10	0.0074	0.0063	0.0076	0.0460	0.0453	0.0444	0.0959	0.0978	0.0871
5,6,7	0.0070	0.0043	0.0097	0.0423	0.0410	0.0454	0.0910	0.0948	0.0895
6,7,8	0.0072	0.0055	0.0063	0.0445	0.0441	0.0505	0.0948	0.0985	0.0892
5,8,11	0.0080	0.0054	0.0060	0.0470	0.0462	0.0431	0.0984	0.0995	0.0886
6,9,12	0.0081	0.0062	0.0076	0.0473	0.0463	0.0395	0.0998	0.0993	0.0819

Table E.2: Setting 3: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0570	0.0314	0.0181	0.1907	0.2010	0.1495	0.3025	0.3138	0.2857
6,6,6	0.0744	0.0543	0.0633	0.2253	0.2353	0.2282	0.3462	0.3792	0.3459
7,7,7	0.0945	0.0833	0.0610	0.2631	0.2889	0.2108	0.3861	0.4293	0.3192
8,8,8	0.1155	0.1111	0.1069	0.2973	0.3327	0.2686	0.4217	0.4754	0.4260
9,9,9	0.1362	0.1420	0.0962	0.3288	0.3765	0.3343	0.4583	0.5191	0.4375
10,10,10	0.1590	0.1737	0.1402	0.3632	0.4198	0.3802	0.4922	0.5625	0.5220

Table E.3: Setting 3: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0806	0.0668	0.0807	0.2388	0.2555	0.2293	0.3597	0.3934	0.3481
5,7,6	0.0742	0.0539	0.0695	0.2288	0.2386	0.2234	0.3480	0.3787	0.3493
6,7,5	0.0672	0.0417	0.0600	0.2100	0.2164	0.1884	0.3254	0.3460	0.3003
Average	0.0740	0.0541	0.0701	0.2259	0.2368	0.2137	0.3444	0.3727	0.3326
6,7,8	0.1020	0.0942	0.0731	0.2739	0.3010	0.2879	0.3979	0.4427	0.3993
6,8,7	0.0963	0.0857	0.0651	0.2650	0.2901	0.2646	0.3871	0.4313	0.3749
7,8,6	0.0862	0.0730	0.0575	0.2449	0.2623	0.2505	0.3642	0.4023	0.3611
Average	0.0948	0.0843	0.0652	0.2613	0.2845	0.2677	0.3831	0.4254	0.3784
5,8,11	0.1358	0.1364	0.0969	0.3261	0.3671	0.3162	0.4516	0.5116	0.4574
5,11,8	0.1163	0.1128	0.0730	0.2980	0.3341	0.2762	0.4236	0.4783	0.4142
8,11,5	0.0789	0.0565	0.0436	0.2268	0.2476	0.1935	0.3412	0.3807	0.3192
Average	0.1103	0.1019	0.0712	0.2836	0.3163	0.2620	0.4055	0.4569	0.3969
6,9,12	0.1586	0.1697	0.1396	0.3659	0.4170	0.3539	0.4934	0.5620	0.4947
6,12,9	0.1362	0.1418	0.1054	0.3303	0.3769	0.2969	0.4584	0.5212	0.4375
9,12,6	0.0986	0.0881	0.0753	0.2635	0.2940	0.2373	0.3840	0.4333	0.3716
Average	0.1311	0.1332	0.1068	0.3199	0.3626	0.2960	0.4453	0.5055	0.4346

Table E.4: Setting 3: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.3160	0.1806	0.0725	0.5851	0.5687	0.3942	0.7125	0.7009	0.6359
6,6,6	0.4197	0.3051	0.2359	0.6782	0.6605	0.5728	0.7876	0.8030	0.7362
7,7,7	0.5057	0.4305	0.2417	0.7470	0.7559	0.5927	0.8409	0.8620	0.7414
8,8,8	0.5870	0.5408	0.4034	0.8057	0.8224	0.6981	0.8829	0.9042	0.8556
9,9,9	0.6606	0.6414	0.3998	0.8525	0.8766	0.8095	0.9140	0.9375	0.8860
10,10,10	0.7265	0.7262	0.5372	0.8897	0.9141	0.8604	0.9387	0.9591	0.9376

Table E.5: Setting 3: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.4449	0.3538	0.2939	0.7031	0.6991	0.5895	0.8081	0.8243	0.7440
5,7,6	0.4167	0.3020	0.2392	0.6767	0.6614	0.5898	0.7865	0.7999	0.7496
6,7,5	0.3705	0.2327	0.2012	0.6364	0.6112	0.4723	0.7550	0.7489	0.6377
Average	0.4107	0.2962	0.2448	0.6721	0.6572	0.5505	0.7832	0.7910	0.7104
6,7,8	0.5375	0.4775	0.3076	0.7731	0.7861	0.7185	0.8591	0.8824	0.8250
6,8,7	0.5065	0.4325	0.2555	0.7492	0.7555	0.6600	0.8401	0.8611	0.7877
7,8,6	0.4652	0.3826	0.2218	0.7167	0.7065	0.6187	0.8151	0.8294	0.7564
Average	0.5031	0.4309	0.2616	0.7463	0.7494	0.6657	0.8381	0.8576	0.7897
5,8,11	0.6453	0.6259	0.4207	0.8457	0.8719	0.7886	0.9094	0.9361	0.8955
5,11,8	0.5878	0.5395	0.2895	0.8049	0.8245	0.7181	0.8814	0.9057	0.8457
8,11,5	0.4395	0.3217	0.1653	0.6904	0.6820	0.4996	0.7957	0.7996	0.6972
Average	0.5575	0.4957	0.2918	0.7803	0.7928	0.6688	0.8622	0.8805	0.8128
6,9,12	0.7143	0.7152	0.5635	0.8846	0.9126	0.8451	0.9353	0.9594	0.9263
6,12,9	0.6628	0.6445	0.4117	0.8548	0.8775	0.7520	0.9155	0.9378	0.8756
9,12,6	0.5334	0.4616	0.2795	0.7672	0.7669	0.6251	0.8538	0.8674	0.7938
Average	0.6368	0.6071	0.4182	0.8355	0.8523	0.7407	0.9015	0.9215	0.8652

Table E.6: Setting 3: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.2251	0.1392	0.1019	0.4740	0.4766	0.4053	0.6114	0.6173	0.5936
6,6,6	0.3007	0.2335	0.2877	0.5619	0.5669	0.5760	0.6875	0.7206	0.7049
7,7,7	0.3729	0.3342	0.3082	0.6318	0.6619	0.5890	0.7487	0.7891	0.7118
8,8,8	0.4436	0.4308	0.4747	0.6953	0.7342	0.7033	0.8000	0.8432	0.8299
9,9,9	0.5138	0.5219	0.4828	0.7514	0.7975	0.7920	0.8408	0.8854	0.8580
10,10,10	0.5753	0.6010	0.6161	0.7950	0.8445	0.8488	0.8744	0.9163	0.9145

Table E.7: Setting 3: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.2922	0.2268	0.2907	0.5533	0.5620	0.5371	0.6818	0.7075	0.6698
5,7,6	0.2684	0.1985	0.2764	0.5256	0.5318	0.5351	0.6573	0.6843	0.6687
6,5,7	0.3295	0.2649	0.3041	0.5914	0.5970	0.5777	0.7146	0.7432	0.7122
6,7,5	0.2674	0.1977	0.2751	0.5226	0.5305	0.5355	0.6568	0.6831	0.6686
7,5,6	0.3306	0.2652	0.3049	0.5941	0.6010	0.5790	0.7169	0.7469	0.7147
7,6,5	0.2924	0.2274	0.2902	0.5526	0.5631	0.5399	0.6821	0.7089	0.6731
Average	0.2968	0.2301	0.2902	0.5566	0.5642	0.5507	0.6849	0.7123	0.6845
6,7,8	0.3677	0.3320	0.3248	0.6266	0.6521	0.6648	0.7431	0.7826	0.7665
6,8,7	0.3449	0.3085	0.3133	0.6013	0.6297	0.6415	0.7231	0.7648	0.7465
7,6,8	0.3420	0.3056	0.3119	0.6014	0.6281	0.6424	0.7232	0.7647	0.7469
7,8,6	0.3395	0.3052	0.3115	0.6000	0.6263	0.6435	0.7222	0.7630	0.7461
8,6,7	0.4029	0.3651	0.3327	0.6587	0.6858	0.6727	0.7699	0.8085	0.7753
8,7,6	0.3655	0.3305	0.3237	0.6258	0.6519	0.6637	0.7441	0.7818	0.7650
Average	0.3604	0.3245	0.3197	0.6190	0.6457	0.6548	0.7376	0.7776	0.7577
5,8,11	0.4012	0.3752	0.3558	0.6567	0.6945	0.6697	0.7681	0.8129	0.7905
5,11,8	0.3411	0.3177	0.3220	0.5974	0.6407	0.6350	0.7182	0.7690	0.7584
8,5,11	0.5208	0.5036	0.3919	0.7564	0.7928	0.7355	0.8447	0.8844	0.8456
8,11,5	0.3407	0.3180	0.3175	0.5963	0.6401	0.6325	0.7176	0.7664	0.7547
11,5,8	0.5191	0.5021	0.3897	0.7554	0.7905	0.7322	0.8436	0.8838	0.8451
11,8,5	0.3980	0.3720	0.3545	0.6527	0.6922	0.6673	0.7647	0.8106	0.7899
Average	0.4202	0.3981	0.3552	0.6692	0.7085	0.6787	0.7762	0.8212	0.7974
6,9,12	0.4762	0.4737	0.4958	0.7182	0.7638	0.7496	0.8164	0.8641	0.8474
6,12,9	0.4166	0.4139	0.4591	0.6683	0.7166	0.7112	0.7773	0.8283	0.8143
9,6,12	0.5846	0.5911	0.5245	0.7995	0.8419	0.7844	0.8778	0.9156	0.8798
9,12,6	0.4158	0.4152	0.4593	0.6658	0.7170	0.7101	0.7766	0.8276	0.8135
12,6,9	0.5836	0.5913	0.5249	0.8007	0.8422	0.7844	0.8787	0.9161	0.8808
12,9,6	0.4753	0.4708	0.4954	0.7194	0.7638	0.7486	0.8175	0.8645	0.8473
Average	0.4920	0.4927	0.4932	0.7287	0.7742	0.7481	0.8241	0.8694	0.8472

Table E.8: Setting 3: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

Appendix F

Setting 4: Three Different Distributions

This appendix contains tables of the results for Setting 4, with identical case and corrected cutoff values, of the simulation studies. In each case 100,000 Monte Carlo data sets were generated and for each the three tests were conducted and proportion of tests rejecting is displayed.

F.1 Identical Case Critical Values

Table F.1: List of Tables for Setting 4: Identical Case Cutoff Values

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0401	0.0187	0.0143	0.1385	0.1539	0.1325	0.2327	0.2641	0.2591
6,6,6	0.0479	0.0316	0.0504	0.1587	0.1805	0.1889	0.2577	0.3183	0.2981
7,7,7	0.0559	0.0491	0.0504	0.1769	0.2229	0.1782	0.2812	0.3606	0.2814
8,8,8	0.0652	0.0678	0.0845	0.1944	0.2610	0.2205	0.3031	0.4031	0.3663
9,9,9	0.0736	0.0875	0.0769	0.2136	0.2966	0.2886	0.3237	0.4408	0.3856
10,10,10	0.0827	0.1079	0.1068	0.2300	0.3296	0.3141	0.3475	0.4792	0.4474

Table F.2: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0611	0.0414	0.0750	0.1838	0.2032	0.2199	0.2861	0.3331	0.3300
5,7,6	0.0516	0.0308	0.0645	0.1596	0.1802	0.2002	0.2524	0.3101	0.3101
6,5,7	0.0606	0.0440	0.0684	0.1875	0.2099	0.2072	0.2970	0.3440	0.3254
6,7,5	0.0368	0.0225	0.0443	0.1304	0.1507	0.1459	0.2188	0.2748	0.2445
7,5,6	0.0454	0.0333	0.0539	0.1583	0.1850	0.1892	0.2609	0.3199	0.3088
7,6,5	0.0340	0.0226	0.0419	0.1286	0.1523	0.1485	0.2207	0.2776	0.2476
Average	0.0483	0.0324	0.0580	0.1580	0.1802	0.1852	0.2560	0.3099	0.2944
6,7,8	0.0696	0.0610	0.0630	0.1986	0.2383	0.2568	0.3040	0.3761	0.3651
6,8,7	0.0588	0.0506	0.0541	0.1754	0.2173	0.2189	0.2729	0.3532	0.3196
7,6,8	0.0439	0.0372	0.0431	0.1487	0.1890	0.1998	0.2436	0.3241	0.2976
7,8,6	0.0428	0.0361	0.0423	0.1447	0.1877	0.1990	0.2424	0.3244	0.2973
8,6,7	0.0561	0.0518	0.0470	0.1801	0.2272	0.2207	0.2889	0.3654	0.3233
8,7,6	0.0423	0.0381	0.0393	0.1493	0.1918	0.1938	0.2471	0.3270	0.2937
Average	0.0523	0.0458	0.0481	0.1661	0.2086	0.2148	0.2665	0.3450	0.3161
5,8,11	0.1072	0.0960	0.1087	0.2541	0.3024	0.3160	0.3623	0.4394	0.4434
5,11,8	0.0691	0.0630	0.0741	0.1810	0.2433	0.2396	0.2721	0.3818	0.3584
8,5,11	0.1130	0.1071	0.0941	0.2871	0.3241	0.3184	0.4112	0.4672	0.4612
8,11,5	0.0305	0.0261	0.0288	0.1169	0.1605	0.1318	0.2036	0.2884	0.2334
11,5,8	0.0645	0.0694	0.0496	0.2076	0.2690	0.2298	0.3266	0.4149	0.3702
11,8,5	0.0296	0.0285	0.0252	0.1209	0.1677	0.1334	0.2145	0.2977	0.2339
Average	0.0690	0.0650	0.0634	0.1946	0.2445	0.2282	0.2984	0.3816	0.3501
6,9,12	0.1140	0.1178	0.1390	0.2675	0.3360	0.3368	0.3780	0.4762	0.4664
6,12,9	0.0746	0.0812	0.0915	0.1958	0.2811	0.2448	0.2926	0.4242	0.3711
9,6,12	0.1212	0.1290	0.1351	0.3000	0.3551	0.3410	0.4258	0.5028	0.4698
9,12,6	0.0384	0.0420	0.0516	0.1362	0.2004	0.1657	0.2301	0.3390	0.2736
12,6,9	0.0769	0.0931	0.0754	0.2309	0.3087	0.2452	0.3546	0.4606	0.3778
12,9,6	0.0381	0.0439	0.0436	0.1443	0.2088	0.1635	0.2486	0.3511	0.2770
Average	0.0772	0.0845	0.0894	0.2125	0.2817	0.2495	0.3216	0.4257	0.3726

Table F.3: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1874	0.0812	0.0449	0.3955	0.3825	0.2905	0.5256	0.5404	0.4937
6,6,6	0.2423	0.1357	0.1532	0.4655	0.4668	0.4253	0.5937	0.6481	0.5773
7,7,7	0.2927	0.2222	0.1624	0.5246	0.5671	0.4389	0.6513	0.7218	0.5912
8,8,8	0.3425	0.3056	0.2695	0.5787	0.6466	0.5276	0.6984	0.7830	0.7028
9,9,9	0.3930	0.3882	0.2645	0.6292	0.7157	0.6469	0.7400	0.8340	0.7385
10,10,10	0.4407	0.4697	0.3674	0.6734	0.7726	0.6929	0.7783	0.8753	0.8054

Table F.4: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.2704	0.1718	0.2061	0.4948	0.5033	0.4545	0.6191	0.6658	0.5972
5,7,6	0.2401	0.1302	0.1720	0.4508	0.4579	0.4346	0.5745	0.6346	0.5904
6,5,7	0.2746	0.1795	0.1942	0.5119	0.5152	0.4539	0.6415	0.6790	0.6137
6,7,5	0.2049	0.0995	0.1362	0.4115	0.4023	0.3441	0.5395	0.5853	0.4992
7,5,6	0.2432	0.1403	0.1553	0.4774	0.4790	0.4405	0.6118	0.6561	0.6031
7,6,5	0.2060	0.1035	0.1337	0.4213	0.4143	0.3542	0.5553	0.5955	0.5024
Average	0.2399	0.1375	0.1663	0.4613	0.4620	0.4136	0.5903	0.6361	0.5677
6,7,8	0.3232	0.2584	0.1989	0.5528	0.5942	0.5588	0.6706	0.7415	0.6837
6,8,7	0.2856	0.2141	0.1690	0.5083	0.5554	0.4963	0.6305	0.7139	0.6319
7,6,8	0.2552	0.1791	0.1465	0.4752	0.5081	0.4596	0.6036	0.6802	0.5930
7,8,6	0.2523	0.1773	0.1455	0.4756	0.5095	0.4611	0.6019	0.6786	0.5925
8,6,7	0.3024	0.2330	0.1644	0.5447	0.5809	0.5058	0.6711	0.7337	0.6403
8,7,6	0.2563	0.1830	0.1402	0.4882	0.5191	0.4669	0.6211	0.6889	0.6052
Average	0.2792	0.2075	0.1608	0.5075	0.5445	0.4914	0.6331	0.7061	0.6244
5,8,11	0.4054	0.3658	0.2919	0.6206	0.6831	0.6195	0.7232	0.8058	0.7444
5,11,8	0.3196	0.2770	0.2037	0.5256	0.6143	0.5301	0.6361	0.7600	0.6742
8,5,11	0.4496	0.4062	0.2862	0.6891	0.7192	0.6458	0.7921	0.8324	0.7703
8,11,5	0.2128	0.1333	0.1088	0.4229	0.4652	0.3569	0.5511	0.6437	0.5302
11,5,8	0.3697	0.3287	0.1793	0.6300	0.6757	0.5733	0.7539	0.8081	0.7186
11,8,5	0.2238	0.1456	0.1003	0.4559	0.4863	0.3633	0.5952	0.6623	0.5396
Average	0.3302	0.2761	0.1950	0.5574	0.6073	0.5148	0.6753	0.7521	0.6629
6,9,12	0.4504	0.4455	0.3947	0.6649	0.7467	0.6806	0.7618	0.8517	0.7879
6,12,9	0.3615	0.3607	0.2782	0.5752	0.6884	0.5679	0.6843	0.8132	0.7134
9,6,12	0.5004	0.4858	0.4041	0.7296	0.7773	0.6863	0.8236	0.8742	0.7925
9,12,6	0.2625	0.2148	0.1793	0.4871	0.5666	0.4432	0.6167	0.7275	0.6066
12,6,9	0.4276	0.4158	0.2757	0.6815	0.7375	0.5962	0.7933	0.8509	0.7418
12,9,6	0.2811	0.2326	0.1764	0.5245	0.5872	0.4606	0.6608	0.7438	0.6312
Average	0.3806	0.3592	0.2847	0.6105	0.6840	0.5725	0.7234	0.8102	0.7122

Table F.5: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1066	0.0737	0.0513	0.3191	0.3201	0.2749	0.4645	0.4524	0.4464
6,6,6	0.1557	0.1313	0.1511	0.3953	0.3837	0.3731	0.5419	0.5507	0.5039
7,7,7	0.2106	0.1885	0.1652	0.4685	0.4728	0.3891	0.6078	0.6167	0.5023
8,8,8	0.2664	0.2500	0.2484	0.5298	0.5365	0.4491	0.6632	0.6735	0.6051
9,9,9	0.3177	0.3093	0.2466	0.5861	0.5957	0.5419	0.7131	0.7270	0.6299
10,10,10	0.3729	0.3716	0.3190	0.6411	0.6524	0.5866	0.7579	0.7694	0.7012

Table F.6: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1581	0.1155	0.1659	0.3969	0.3615	0.3525	0.5414	0.5094	0.4786
5,7,6	0.1522	0.1349	0.1711	0.3885	0.3879	0.3678	0.5354	0.5315	0.4936
6,5,7	0.1638	0.1164	0.1599	0.4016	0.3578	0.3612	0.5414	0.5177	0.4802
6,7,5	0.1498	0.1493	0.1599	0.3908	0.4116	0.3898	0.5395	0.5739	0.5172
7,5,6	0.1557	0.1268	0.1718	0.3954	0.3881	0.3619	0.5380	0.5400	0.4905
7,6,5	0.1503	0.1425	0.1651	0.3911	0.4133	0.3700	0.5377	0.5706	0.5130
Average	0.1550	0.1309	0.1656	0.3941	0.3867	0.3672	0.5389	0.5405	0.4955
6,7,8	0.2117	0.1785	0.1664	0.4636	0.4384	0.4233	0.6036	0.5896	0.5338
6,8,7	0.2050	0.1926	0.1654	0.4598	0.4592	0.4396	0.6009	0.6083	0.5558
7,6,8	0.2035	0.2047	0.1731	0.4640	0.4917	0.4587	0.6073	0.6348	0.5717
7,8,6	0.2039	0.2039	0.1723	0.4633	0.4928	0.4592	0.6069	0.6367	0.5718
8,6,7	0.2075	0.1820	0.1634	0.4631	0.4630	0.4375	0.6043	0.6121	0.5479
8,7,6	0.2035	0.1981	0.1690	0.4619	0.4897	0.4503	0.6060	0.6373	0.5600
Average	0.2059	0.1933	0.1683	0.4626	0.4725	0.4448	0.6048	0.6198	0.5568
5,8,11	0.2509	0.1848	0.1739	0.5094	0.4440	0.4022	0.6421	0.5863	0.5429
5,11,8	0.2234	0.2206	0.1847	0.4795	0.4867	0.4380	0.6236	0.6276	0.5830
8,5,11	0.2705	0.1892	0.1785	0.5168	0.4580	0.4142	0.6405	0.6001	0.5406
8,11,5	0.2460	0.2959	0.1880	0.5186	0.5899	0.5035	0.6601	0.7220	0.6508
11,5,8	0.2381	0.2102	0.1864	0.4924	0.4980	0.4251	0.6272	0.6501	0.5571
11,8,5	0.2473	0.2847	0.1763	0.5238	0.5957	0.4704	0.6661	0.7348	0.6271
Average	0.2460	0.2309	0.1813	0.5068	0.5121	0.4422	0.6433	0.6535	0.5836
6,9,12	0.3064	0.2391	0.2300	0.5659	0.5105	0.4467	0.6914	0.6460	0.5739
6,12,9	0.2774	0.2738	0.2450	0.5439	0.5537	0.4854	0.6778	0.6862	0.6184
9,6,12	0.3223	0.2496	0.2457	0.5731	0.5260	0.4599	0.6908	0.6638	0.5846
9,12,6	0.3022	0.3563	0.2782	0.5801	0.6434	0.5440	0.7135	0.7650	0.6815
12,6,9	0.2966	0.2757	0.2499	0.5568	0.5719	0.4803	0.6809	0.7120	0.6125
12,9,6	0.3070	0.3526	0.2640	0.5868	0.6542	0.5220	0.7176	0.7789	0.6572
Average	0.3020	0.2912	0.2521	0.5678	0.5766	0.4897	0.6953	0.7087	0.6214

Table F.7: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.0605	0.0331	0.0246	0.1541	0.1554	0.1479	0.2264	0.2416	0.2644
6,6,6	0.0655	0.0452	0.0690	0.1615	0.1696	0.2084	0.2339	0.2805	0.3086
7,7,7	0.0744	0.0617	0.0715	0.1733	0.2053	0.2005	0.2467	0.3142	0.2905
8,8,8	0.0811	0.0777	0.1127	0.1822	0.2327	0.2470	0.2576	0.3453	0.3782
9,9,9	0.0856	0.0929	0.1051	0.1882	0.2566	0.3047	0.2623	0.3740	0.3896
10,10,10	0.0921	0.1121	0.1436	0.1976	0.2847	0.3385	0.2713	0.4053	0.4568

Table F.8: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.0752	0.0473	0.0820	0.1774	0.1772	0.2160	0.2533	0.2873	0.3164
5,7,6	0.0751	0.0487	0.0827	0.1715	0.1801	0.2113	0.2425	0.2864	0.3138
6,5,7	0.0660	0.0431	0.0826	0.1680	0.1737	0.2037	0.2481	0.2769	0.3045
6,7,5	0.0659	0.0460	0.0699	0.1563	0.1716	0.1906	0.2251	0.2759	0.2912
7,5,6	0.0572	0.0420	0.0719	0.1493	0.1654	0.1932	0.2252	0.2653	0.2940
7,6,5	0.0588	0.0441	0.0634	0.1471	0.1640	0.1899	0.2172	0.2704	0.2891
Average	0.0664	0.0452	0.0754	0.1616	0.1720	0.2008	0.2352	0.2770	0.3015
6,7,8	0.0829	0.0641	0.0839	0.1875	0.2121	0.2605	0.2656	0.3246	0.3548
6,8,7	0.0812	0.0641	0.0752	0.1800	0.2105	0.2498	0.2520	0.3215	0.3428
7,6,8	0.0743	0.0635	0.0709	0.1698	0.2045	0.2436	0.2390	0.3151	0.3363
7,8,6	0.0734	0.0624	0.0679	0.1654	0.1997	0.2402	0.2343	0.3104	0.3326
8,6,7	0.0674	0.0600	0.0682	0.1638	0.1926	0.2360	0.2397	0.3023	0.3273
8,7,6	0.0670	0.0594	0.0661	0.1608	0.1982	0.2375	0.2335	0.3072	0.3312
Average	0.0744	0.0623	0.0720	0.1712	0.2029	0.2446	0.2440	0.3135	0.3375
5,8,11	0.1060	0.0837	0.1021	0.2190	0.2496	0.2818	0.2991	0.3657	0.3928
5,11,8	0.0904	0.0795	0.0892	0.1867	0.2406	0.2595	0.2556	0.3590	0.3734
8,5,11	0.0772	0.0638	0.0861	0.1930	0.2150	0.2454	0.2814	0.3212	0.3628
8,11,5	0.0732	0.0736	0.0616	0.1639	0.2261	0.2240	0.2308	0.3426	0.3416
11,5,8	0.0538	0.0549	0.0672	0.1496	0.1965	0.2177	0.2302	0.3014	0.3312
11,8,5	0.0611	0.0675	0.0542	0.1516	0.2153	0.2136	0.2197	0.3257	0.3290
Average	0.0770	0.0705	0.0767	0.1773	0.2239	0.2403	0.2528	0.3359	0.3551
6,9,12	0.1091	0.1015	0.1394	0.2203	0.2715	0.3019	0.2995	0.3900	0.4153
6,12,9	0.0956	0.0973	0.1212	0.1945	0.2708	0.2844	0.2646	0.3974	0.4003
9,6,12	0.0856	0.0818	0.1171	0.2008	0.2384	0.2708	0.2867	0.3510	0.3828
9,12,6	0.0790	0.0907	0.0956	0.1720	0.2546	0.2583	0.2408	0.3775	0.3769
12,6,9	0.0628	0.0717	0.0955	0.1637	0.2227	0.2454	0.2444	0.3349	0.3576
12,9,6	0.0689	0.0848	0.0893	0.1611	0.2402	0.2452	0.2336	0.3557	0.3646
Average	0.0835	0.0880	0.1097	0.1854	0.2497	0.2677	0.2616	0.3678	0.3829

Table F.9: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.3802	0.1759	0.0720	0.6160	0.5370	0.3989	0.7328	0.6896	0.6317
6,6,6	0.4722	0.2453	0.2302	0.7017	0.6365	0.5508	0.8013	0.7863	0.7032
7,7,7	0.5583	0.3921	0.2476	0.7711	0.7398	0.5932	0.8540	0.8537	0.7285
8,8,8	0.6325	0.5002	0.3957	0.8229	0.8131	0.6743	0.8922	0.8990	0.8234
9,9,9	0.6978	0.6048	0.3967	0.8633	0.8654	0.7823	0.9194	0.9319	0.8460
10,10,10	0.7529	0.6969	0.5227	0.8954	0.9058	0.8219	0.9406	0.9554	0.8998

Table F.10: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.4948	0.2994	0.2954	0.7192	0.6698	0.5738	0.8134	0.8085	0.7173
5,7,6	0.4622	0.2360	0.2447	0.6856	0.6275	0.5760	0.7832	0.7795	0.7182
6,5,7	0.5071	0.3105	0.2845	0.7401	0.6818	0.5812	0.8329	0.8162	0.7364
6,7,5	0.4283	0.2023	0.2094	0.6545	0.5679	0.4671	0.7585	0.7342	0.6309
7,5,6	0.4830	0.2512	0.2313	0.7198	0.6489	0.5930	0.8191	0.7953	0.7334
7,6,5	0.4403	0.2124	0.2053	0.6714	0.5789	0.4784	0.7777	0.7402	0.6306
Average	0.4693	0.2520	0.2451	0.6984	0.6291	0.5449	0.7975	0.7790	0.6945
6,7,8	0.5756	0.4300	0.2899	0.7820	0.7612	0.6895	0.8610	0.8675	0.7948
6,8,7	0.5466	0.3837	0.2577	0.7536	0.7330	0.6405	0.8386	0.8497	0.7658
7,6,8	0.5161	0.3233	0.2218	0.7313	0.6839	0.5992	0.8221	0.8160	0.7252
7,8,6	0.5135	0.3220	0.2235	0.7279	0.6820	0.5978	0.8202	0.8161	0.7233
8,6,7	0.5732	0.4013	0.2510	0.7860	0.7464	0.6492	0.8675	0.8575	0.7724
8,7,6	0.5294	0.3313	0.2194	0.7462	0.6909	0.6078	0.8363	0.8195	0.7372
Average	0.5424	0.3653	0.2439	0.7545	0.7162	0.6307	0.8410	0.8377	0.7531
5,8,11	0.6601	0.5703	0.4058	0.8327	0.8441	0.7379	0.8941	0.9187	0.8384
5,11,8	0.5892	0.4719	0.2868	0.7749	0.7941	0.6816	0.8484	0.8875	0.7929
8,5,11	0.7179	0.6101	0.4033	0.8871	0.8684	0.7641	0.9379	0.9347	0.8576
8,11,5	0.4899	0.2704	0.1797	0.6994	0.6455	0.5015	0.7950	0.7868	0.6901
11,5,8	0.6767	0.5272	0.2727	0.8637	0.8283	0.7200	0.9249	0.9103	0.8226
11,8,5	0.5173	0.2881	0.1720	0.7380	0.6628	0.5151	0.8306	0.8003	0.7008
Average	0.6085	0.4563	0.2867	0.7993	0.7739	0.6534	0.8718	0.8731	0.7837
6,9,12	0.7214	0.6657	0.5306	0.8722	0.8916	0.7945	0.9206	0.9477	0.8736
6,12,9	0.6521	0.5771	0.3986	0.8219	0.8500	0.7158	0.8853	0.9240	0.8363
9,6,12	0.7752	0.6990	0.5407	0.9131	0.9077	0.7975	0.9538	0.9571	0.8754
9,12,6	0.5680	0.3930	0.2798	0.7666	0.7425	0.6018	0.8476	0.8564	0.7542
12,6,9	0.7415	0.6311	0.4087	0.8999	0.8768	0.7411	0.9471	0.9388	0.8545
12,9,6	0.6009	0.4124	0.2837	0.8035	0.7565	0.6238	0.8799	0.8661	0.7742
Average	0.6765	0.5631	0.4070	0.8462	0.8375	0.7124	0.9057	0.9150	0.8280

Table F.11: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.5124	0.4207	0.3321	0.7952	0.8325	0.7131	0.8883	0.9230	0.8759
6,6,6	0.6518	0.6142	0.6105	0.8743	0.9160	0.8533	0.9368	0.9706	0.9345
7,7,7	0.7520	0.7777	0.6597	0.9228	0.9642	0.8887	0.9642	0.9884	0.9537
8,8,8	0.8321	0.8830	0.8048	0.9548	0.9852	0.9443	0.9811	0.9952	0.9856
9,9,9	0.8882	0.9413	0.8189	0.9733	0.9936	0.9809	0.9895	0.9981	0.9924
10,10,10	0.9256	0.9716	0.9047	0.9851	0.9977	0.9908	0.9945	0.9994	0.9977

Table F.12: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.6632	0.6187	0.6390	0.8837	0.9183	0.8712	0.9426	0.9713	0.9498
5,7,6	0.6659	0.6828	0.6742	0.8836	0.9302	0.8744	0.9435	0.9745	0.9444
6,5,7	0.6262	0.5244	0.5853	0.8585	0.8860	0.7887	0.9266	0.9578	0.8831
6,7,5	0.6565	0.6707	0.6501	0.8771	0.9274	0.8627	0.9381	0.9722	0.9369
7,5,6	0.6102	0.5233	0.5728	0.8503	0.8871	0.7869	0.9201	0.9563	0.8797
7,6,5	0.6362	0.6069	0.6044	0.8676	0.9154	0.8599	0.9323	0.9696	0.9444
Average	0.6430	0.6045	0.6210	0.8701	0.9107	0.8406	0.9339	0.9670	0.9231
6,7,8	0.7645	0.7819	0.6759	0.9289	0.9638	0.9154	0.9673	0.9880	0.9617
6,8,7	0.7679	0.8179	0.7116	0.9315	0.9700	0.9429	0.9695	0.9901	0.9754
7,6,8	0.7623	0.8152	0.7098	0.9279	0.9699	0.9449	0.9676	0.9898	0.9791
7,8,6	0.7614	0.8137	0.7093	0.9279	0.9685	0.9441	0.9669	0.9891	0.9784
8,6,7	0.7265	0.7263	0.6337	0.9107	0.9507	0.8931	0.9568	0.9841	0.9497
8,7,6	0.7465	0.7800	0.6548	0.9214	0.9643	0.9136	0.9639	0.9888	0.9625
Average	0.7549	0.7892	0.6825	0.9247	0.9645	0.9257	0.9653	0.9883	0.9678
5,8,11	0.8513	0.8849	0.7223	0.9636	0.9851	0.9533	0.9853	0.9952	0.9872
5,11,8	0.8361	0.9135	0.8239	0.9573	0.9883	0.9772	0.9833	0.9964	0.9938
8,5,11	0.7645	0.6958	0.6253	0.9177	0.9514	0.8486	0.9572	0.9848	0.9284
8,11,5	0.8343	0.9106	0.8101	0.9560	0.9865	0.9743	0.9819	0.9953	0.9920
11,5,8	0.7265	0.6956	0.5974	0.9018	0.9506	0.8362	0.9493	0.9847	0.9192
11,8,5	0.8161	0.8756	0.6758	0.9485	0.9829	0.9410	0.9771	0.9948	0.9834
Average	0.8048	0.8293	0.7091	0.9408	0.9741	0.9218	0.9724	0.9919	0.9673
6,9,12	0.8990	0.9425	0.8297	0.9783	0.9936	0.9626	0.9915	0.9985	0.9865
6,12,9	0.8913	0.9583	0.9206	0.9758	0.9953	0.9861	0.9913	0.9987	0.9935
9,6,12	0.8365	0.8420	0.7403	0.9503	0.9804	0.9085	0.9765	0.9945	0.9669
9,12,6	0.8898	0.9542	0.9092	0.9737	0.9944	0.9898	0.9902	0.9984	0.9972
12,6,9	0.8167	0.8444	0.7379	0.9441	0.9804	0.9118	0.9737	0.9948	0.9673
12,9,6	0.8792	0.9394	0.8040	0.9710	0.9926	0.9607	0.9884	0.9980	0.9882
Average	0.8688	0.9135	0.8236	0.9655	0.9895	0.9533	0.9853	0.9972	0.9833

Table F.13: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.2824	0.1701	0.1549	0.4695	0.4571	0.4634	0.5678	0.5896	0.6283
6,6,6	0.3359	0.2389	0.3480	0.5223	0.5362	0.6030	0.6170	0.6823	0.7130
7,7,7	0.3842	0.3272	0.3834	0.5655	0.6206	0.6252	0.6568	0.7462	0.7297
8,8,8	0.4311	0.4145	0.5261	0.6110	0.6957	0.7167	0.6973	0.8044	0.8292
9,9,9	0.4707	0.4900	0.5384	0.6435	0.7500	0.7964	0.7278	0.8458	0.8560
10,10,10	0.5110	0.5636	0.6442	0.6802	0.8008	0.8447	0.7596	0.8820	0.9063

Table F.14: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.3358	0.2182	0.3544	0.5188	0.5147	0.5627	0.6096	0.6530	0.6743
5,7,6	0.3249	0.2098	0.3407	0.5005	0.5103	0.5620	0.5926	0.6472	0.6791
6,5,7	0.3487	0.2498	0.3888	0.5395	0.5451	0.6197	0.6364	0.6835	0.7334
6,7,5	0.3243	0.2303	0.3251	0.5062	0.5288	0.5752	0.5986	0.6722	0.7022
7,5,6	0.3429	0.2675	0.3948	0.5382	0.5643	0.6225	0.6364	0.6953	0.7339
7,6,5	0.3296	0.2531	0.3459	0.5184	0.5497	0.5798	0.6153	0.6891	0.6977
Average	0.3344	0.2381	0.3583	0.5203	0.5355	0.5870	0.6148	0.6734	0.7034
6,7,8	0.3832	0.3103	0.3995	0.5612	0.5979	0.6758	0.6512	0.7273	0.7653
6,8,7	0.3695	0.3021	0.3641	0.5435	0.5894	0.6525	0.6334	0.7188	0.7463
7,6,8	0.3729	0.3210	0.3672	0.5533	0.6159	0.6682	0.6436	0.7442	0.7621
7,8,6	0.3740	0.3198	0.3660	0.5528	0.6152	0.6684	0.6421	0.7429	0.7616
8,6,7	0.3951	0.3536	0.4109	0.5855	0.6422	0.6989	0.6782	0.7660	0.7838
8,7,6	0.3834	0.3423	0.3965	0.5692	0.6343	0.6998	0.6619	0.7600	0.7897
Average	0.3797	0.3249	0.3840	0.5609	0.6158	0.6773	0.6517	0.7432	0.7681
5,8,11	0.4029	0.3168	0.4149	0.5718	0.6130	0.6566	0.6585	0.7304	0.7637
5,11,8	0.3603	0.2971	0.3634	0.5237	0.5986	0.6270	0.6102	0.7238	0.7385
8,5,11	0.4698	0.4249	0.4877	0.6621	0.7037	0.7448	0.7499	0.8105	0.8362
8,11,5	0.3971	0.3863	0.3457	0.5713	0.6749	0.6699	0.6574	0.7918	0.7860
11,5,8	0.4468	0.4580	0.5011	0.6495	0.7359	0.7613	0.7453	0.8370	0.8528
11,8,5	0.4181	0.4315	0.4123	0.6074	0.7129	0.7154	0.6984	0.8229	0.8250
Average	0.4158	0.3858	0.4209	0.5976	0.6732	0.6958	0.6866	0.7861	0.8004
6,9,12	0.4453	0.4085	0.5285	0.6135	0.6808	0.7261	0.6971	0.7869	0.8179
6,12,9	0.4032	0.3904	0.4789	0.5690	0.6722	0.7008	0.6535	0.7857	0.8039
9,6,12	0.5116	0.5026	0.5766	0.6932	0.7578	0.7723	0.7749	0.8504	0.8551
9,12,6	0.4392	0.4701	0.4874	0.6128	0.7406	0.7353	0.6965	0.8409	0.8352
12,6,9	0.4958	0.5303	0.5958	0.6876	0.7885	0.7956	0.7731	0.8765	0.8721
12,9,6	0.4686	0.5168	0.5613	0.6500	0.7749	0.7905	0.7348	0.8696	0.8741
Average	0.4606	0.4698	0.5381	0.6377	0.7358	0.7534	0.7217	0.8350	0.8431

Table F.15: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.3271	0.2046	0.1937	0.5817	0.5443	0.5679	0.7104	0.6830	0.7202
6,6,6	0.4260	0.3045	0.4551	0.6795	0.6453	0.6971	0.7907	0.7808	0.7866
7,7,7	0.5130	0.4230	0.5075	0.7518	0.7328	0.7389	0.8478	0.8396	0.8145
8,8,8	0.5935	0.5293	0.6583	0.8123	0.8007	0.8091	0.8911	0.8856	0.8843
9,9,9	0.6653	0.6169	0.6835	0.8585	0.8523	0.8719	0.9225	0.9207	0.9101
10,10,10	0.7293	0.6977	0.7763	0.8948	0.8933	0.9066	0.9442	0.9456	0.9439

Table F.16: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.4037	0.2692	0.4508	0.6556	0.6158	0.6827	0.7724	0.7474	0.7773
5,7,6	0.3846	0.2705	0.4670	0.6338	0.6059	0.6850	0.7529	0.7435	0.7703
6,5,7	0.4476	0.3098	0.4349	0.7068	0.6519	0.6803	0.8141	0.7868	0.7767
6,7,5	0.3943	0.3061	0.4735	0.6460	0.6290	0.7041	0.7629	0.7655	0.7943
7,5,6	0.4558	0.3340	0.4548	0.7149	0.6702	0.6900	0.8216	0.7984	0.7841
7,6,5	0.4216	0.3344	0.4828	0.6798	0.6564	0.7129	0.7941	0.7868	0.8090
Average	0.4179	0.3040	0.4606	0.6728	0.6382	0.6925	0.7863	0.7714	0.7853
6,7,8	0.4994	0.3977	0.4904	0.7380	0.7055	0.7586	0.8347	0.8212	0.8293
6,8,7	0.4721	0.3879	0.5065	0.7128	0.6983	0.7721	0.8160	0.8132	0.8416
7,6,8	0.4827	0.4193	0.5325	0.7274	0.7240	0.7794	0.8285	0.8342	0.8402
7,8,6	0.4790	0.4127	0.5270	0.7240	0.7194	0.7710	0.8253	0.8309	0.8358
8,6,7	0.5484	0.4528	0.5106	0.7837	0.7586	0.7906	0.8713	0.8577	0.8559
8,7,6	0.5173	0.4473	0.5225	0.7600	0.7463	0.7763	0.8543	0.8513	0.8366
Average	0.4998	0.4196	0.5149	0.7410	0.7254	0.7747	0.8384	0.8348	0.8399
5,8,11	0.5186	0.4001	0.5234	0.7482	0.7150	0.7647	0.8400	0.8261	0.8390
5,11,8	0.4584	0.3899	0.5593	0.6898	0.6930	0.7757	0.7931	0.8039	0.8451
8,5,11	0.6742	0.5505	0.5071	0.8681	0.8234	0.7801	0.9274	0.9046	0.8625
8,11,5	0.4997	0.4909	0.5949	0.7399	0.7670	0.8231	0.8365	0.8612	0.8955
11,5,8	0.6838	0.5908	0.5442	0.8755	0.8452	0.8026	0.9333	0.9179	0.8727
11,8,5	0.5776	0.5473	0.5996	0.8097	0.8165	0.8382	0.8906	0.8992	0.9039
Average	0.5687	0.4949	0.5548	0.7885	0.7767	0.7974	0.8702	0.8688	0.8698
6,9,12	0.5990	0.5116	0.6396	0.8075	0.7861	0.8154	0.8832	0.8756	0.8830
6,12,9	0.5413	0.4939	0.6702	0.7606	0.7668	0.8346	0.8478	0.8587	0.8979
9,6,12	0.7375	0.6462	0.6491	0.9008	0.8721	0.8294	0.9477	0.9332	0.8938
9,12,6	0.5824	0.5826	0.7071	0.8041	0.8267	0.8461	0.8841	0.9015	0.8958
12,6,9	0.7496	0.6790	0.6874	0.9122	0.8930	0.8658	0.9554	0.9468	0.9205
12,9,6	0.6630	0.6394	0.7092	0.8630	0.8690	0.8535	0.9253	0.9316	0.9017
Average	0.6455	0.5921	0.6771	0.8414	0.8356	0.8408	0.9073	0.9079	0.8988

Table F.17: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.4343	0.2445	0.1273	0.7293	0.6804	0.4957	0.8392	0.7889	0.7246
6,6,6	0.5662	0.4387	0.3285	0.8184	0.7633	0.6588	0.8981	0.8817	0.8006
7,7,7	0.6763	0.5706	0.3539	0.8775	0.8551	0.6902	0.9342	0.9258	0.8011
8,8,8	0.7603	0.6821	0.5145	0.9181	0.9052	0.7786	0.9582	0.9538	0.9021
9,9,9	0.8271	0.7798	0.5210	0.9464	0.9396	0.8609	0.9735	0.9734	0.9145
10,10,10	0.8762	0.8475	0.6478	0.9651	0.9643	0.9024	0.9842	0.9851	0.9594

Table F.18: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.5310	0.3431	0.3103	0.7946	0.7167	0.5740	0.8804	0.8253	0.7209
5,7,6	0.5427	0.4000	0.3175	0.8018	0.7437	0.5915	0.8875	0.8443	0.7352
6,5,7	0.5523	0.3914	0.3355	0.8066	0.7351	0.6602	0.8886	0.8567	0.7879
6,7,5	0.5784	0.4916	0.3478	0.8254	0.7900	0.7105	0.9028	0.8995	0.8365
7,5,6	0.5736	0.4519	0.3799	0.8236	0.7860	0.6421	0.8981	0.8889	0.7778
7,6,5	0.5909	0.4989	0.3867	0.8352	0.8172	0.6786	0.9084	0.9101	0.8224
Average	0.5615	0.4295	0.3463	0.8145	0.7648	0.6428	0.8943	0.8708	0.7801
6,7,8	0.6475	0.5272	0.3345	0.8597	0.8025	0.7002	0.9219	0.9009	0.8182
6,8,7	0.6541	0.5577	0.3422	0.8639	0.8223	0.7297	0.9268	0.9131	0.8459
7,6,8	0.6832	0.5999	0.3702	0.8828	0.8735	0.7564	0.9388	0.9357	0.8560
7,8,6	0.6831	0.6029	0.3730	0.8810	0.8726	0.7563	0.9377	0.9345	0.8533
8,6,7	0.6833	0.5828	0.3956	0.8819	0.8659	0.7758	0.9370	0.9337	0.8669
8,7,6	0.6985	0.6202	0.4078	0.8914	0.8867	0.7819	0.9432	0.9441	0.8680
Average	0.6750	0.5818	0.3706	0.8768	0.8539	0.7501	0.9342	0.9270	0.8514
5,8,11	0.6607	0.4893	0.2939	0.8646	0.7762	0.6083	0.9238	0.8672	0.7729
5,11,8	0.6696	0.5754	0.3174	0.8694	0.8127	0.6615	0.9296	0.8985	0.8238
8,5,11	0.7177	0.5735	0.3726	0.8915	0.8516	0.7257	0.9393	0.9227	0.8382
8,11,5	0.7785	0.7524	0.4271	0.9271	0.9322	0.8392	0.9643	0.9695	0.9287
11,5,8	0.7485	0.6871	0.4812	0.9137	0.9166	0.7889	0.9549	0.9638	0.8871
11,8,5	0.8006	0.8009	0.5107	0.9408	0.9565	0.8508	0.9722	0.9829	0.9384
Average	0.7293	0.6464	0.4005	0.9012	0.8743	0.7457	0.9474	0.9341	0.8649
6,9,12	0.7495	0.6084	0.4152	0.9096	0.8524	0.7144	0.9520	0.9244	0.8423
6,12,9	0.7553	0.6626	0.4450	0.9146	0.8832	0.7590	0.9559	0.9414	0.8818
9,6,12	0.7939	0.6911	0.4934	0.9279	0.9043	0.7658	0.9615	0.9544	0.8716
9,12,6	0.8400	0.8379	0.5671	0.9528	0.9577	0.8566	0.9784	0.9816	0.9394
12,6,9	0.8213	0.7852	0.6194	0.9450	0.9493	0.8619	0.9726	0.9795	0.9330
12,9,6	0.8621	0.8688	0.6494	0.9635	0.9745	0.8896	0.9841	0.9905	0.9512
Average	0.8037	0.7423	0.5316	0.9356	0.9202	0.8079	0.9674	0.9620	0.9032

Table F.19: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.2608	0.1476	0.1115	0.4650	0.4310	0.4207	0.5803	0.5731	0.5902
6,6,6	0.3248	0.2016	0.3006	0.5353	0.5142	0.5674	0.6477	0.6683	0.6805
7,7,7	0.3814	0.2931	0.3301	0.5951	0.6028	0.5877	0.6986	0.7362	0.6938
8,8,8	0.4384	0.3766	0.4797	0.6465	0.6723	0.6806	0.7425	0.7927	0.7968
9,9,9	0.4897	0.4548	0.4933	0.6884	0.7338	0.7577	0.7765	0.8354	0.8180
10,10,10	0.5391	0.5299	0.6038	0.7287	0.7862	0.8135	0.8089	0.8747	0.8819

Table F.20: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.3496	0.2259	0.3064	0.5668	0.5399	0.5557	0.6734	0.6924	0.6771
5,7,6	0.3404	0.2133	0.2976	0.5492	0.5263	0.5563	0.6561	0.6807	0.6759
6,5,7	0.3216	0.2097	0.3132	0.5428	0.5207	0.5613	0.6571	0.6660	0.6805
6,7,5	0.3115	0.1975	0.3023	0.5133	0.4900	0.5436	0.6211	0.6406	0.6666
7,5,6	0.2986	0.1911	0.3039	0.5156	0.4979	0.5578	0.6335	0.6429	0.6717
7,6,5	0.2914	0.1885	0.3002	0.4969	0.4733	0.5384	0.6102	0.6255	0.6570
Average	0.3189	0.2043	0.3039	0.5308	0.5080	0.5522	0.6419	0.6580	0.6715
6,7,8	0.4079	0.3219	0.3450	0.6210	0.6274	0.6561	0.7196	0.7569	0.7469
6,8,7	0.4010	0.3071	0.3353	0.6084	0.6200	0.6523	0.7075	0.7501	0.7474
7,6,8	0.3654	0.2765	0.3308	0.5725	0.5791	0.6342	0.6750	0.7177	0.7244
7,8,6	0.3708	0.2802	0.3351	0.5764	0.5815	0.6369	0.6793	0.7207	0.7282
8,6,7	0.3623	0.2868	0.3404	0.5799	0.5838	0.6493	0.6886	0.7198	0.7421
8,7,6	0.3511	0.2684	0.3393	0.5614	0.5673	0.6400	0.6672	0.7076	0.7288
Average	0.3764	0.2902	0.3377	0.5866	0.5932	0.6448	0.6895	0.7288	0.7363
5,8,11	0.5025	0.4331	0.3703	0.7005	0.7236	0.6738	0.7831	0.8308	0.7808
5,11,8	0.4781	0.4028	0.3564	0.6673	0.7096	0.6679	0.7525	0.8234	0.7680
8,5,11	0.4425	0.3768	0.3896	0.6683	0.6718	0.6806	0.7681	0.7884	0.7734
8,11,5	0.3648	0.2863	0.3534	0.5608	0.5918	0.6464	0.6607	0.7300	0.7558
11,5,8	0.3816	0.3293	0.3805	0.6096	0.6237	0.6681	0.7201	0.7479	0.7638
11,8,5	0.3360	0.2768	0.3636	0.5428	0.5655	0.6530	0.6499	0.7023	0.7626
Average	0.4176	0.3509	0.3690	0.6249	0.6477	0.6650	0.7224	0.7705	0.7674
6,9,12	0.5529	0.5131	0.5080	0.7383	0.7792	0.7341	0.8151	0.8715	0.8214
6,12,9	0.5241	0.4902	0.4898	0.7101	0.7686	0.7304	0.7878	0.8640	0.8267
9,6,12	0.5061	0.4601	0.5082	0.7209	0.7382	0.7308	0.8088	0.8399	0.8208
9,12,6	0.4201	0.3709	0.4789	0.6152	0.6718	0.7036	0.7098	0.7914	0.7960
12,6,9	0.4445	0.4098	0.5006	0.6664	0.6910	0.7241	0.7656	0.7999	0.8148
12,9,6	0.3905	0.3498	0.4889	0.5998	0.6430	0.7071	0.7029	0.7658	0.7929
Average	0.4730	0.4323	0.4957	0.6751	0.7153	0.7217	0.7650	0.8221	0.8121

Table F.21: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1915	0.1259	0.0723	0.5132	0.6116	0.4083	0.6959	0.7839	0.6634
6,6,6	0.2893	0.2677	0.2386	0.6289	0.7371	0.5916	0.7867	0.8826	0.7620
7,7,7	0.3888	0.4433	0.2608	0.7251	0.8443	0.6308	0.8555	0.9333	0.7948
8,8,8	0.4915	0.6072	0.4384	0.8029	0.9080	0.7490	0.9035	0.9637	0.8965
9,9,9	0.5833	0.7278	0.4472	0.8605	0.9468	0.8675	0.9385	0.9806	0.9305
10,10,10	0.6654	0.8201	0.5984	0.9026	0.9695	0.9065	0.9611	0.9894	0.9621

Table F.22: Setting 4: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.3448	0.3209	0.2823	0.6946	0.7868	0.6511	0.8380	0.9073	0.8225
5,7,6	0.3341	0.3799	0.3307	0.6850	0.8017	0.6670	0.8335	0.9118	0.8198
6,5,7	0.2873	0.1964	0.2142	0.6224	0.6927	0.4925	0.7777	0.8528	0.6623
6,7,5	0.2779	0.3222	0.2986	0.6165	0.7564	0.6061	0.7770	0.8819	0.7593
7,5,6	0.2404	0.1597	0.1912	0.5597	0.6430	0.4755	0.7248	0.8218	0.6512
7,6,5	0.2410	0.2249	0.2313	0.5659	0.6919	0.5687	0.7333	0.8498	0.7548
Average	0.2876	0.2673	0.2581	0.6240	0.7288	0.5768	0.7807	0.8709	0.7450
6,7,8	0.4499	0.5016	0.2973	0.7784	0.8743	0.7240	0.8918	0.9491	0.8436
6,8,7	0.4369	0.5463	0.3426	0.7703	0.8823	0.7705	0.8861	0.9507	0.8733
7,6,8	0.3792	0.4893	0.3249	0.7177	0.8509	0.7626	0.8522	0.9346	0.8716
7,8,6	0.3793	0.4888	0.3251	0.7142	0.8484	0.7625	0.8475	0.9338	0.8702
8,6,7	0.3357	0.3424	0.2190	0.6671	0.7835	0.6244	0.8103	0.9063	0.7690
8,7,6	0.3400	0.3974	0.2512	0.6751	0.8115	0.6663	0.8196	0.9158	0.8003
Average	0.3868	0.4610	0.2934	0.7205	0.8418	0.7184	0.8513	0.9317	0.8380
5,8,11	0.6569	0.7286	0.3841	0.9044	0.9507	0.8420	0.9624	0.9829	0.9423
5,11,8	0.5977	0.7691	0.5329	0.8736	0.9530	0.8956	0.9480	0.9824	0.9583
8,5,11	0.4362	0.3268	0.1990	0.7383	0.8077	0.5439	0.8536	0.9222	0.7332
8,11,5	0.4321	0.6177	0.4531	0.7537	0.8953	0.8178	0.8736	0.9545	0.9097
11,5,8	0.3077	0.2328	0.1563	0.6115	0.7225	0.4787	0.7534	0.8716	0.6684
11,8,5	0.3502	0.4588	0.2589	0.6761	0.8328	0.6812	0.8151	0.9253	0.8393
Average	0.4635	0.5223	0.3307	0.7596	0.8603	0.7099	0.8677	0.9398	0.8419
6,9,12	0.7284	0.8258	0.5230	0.9342	0.9729	0.8532	0.9767	0.9911	0.9377
6,12,9	0.6799	0.8482	0.6774	0.9137	0.9747	0.9108	0.9678	0.9914	0.9598
9,6,12	0.5319	0.5242	0.3277	0.8144	0.8941	0.6769	0.9055	0.9613	0.8398
9,12,6	0.5306	0.7297	0.6221	0.8268	0.9393	0.8875	0.9196	0.9761	0.9467
12,6,9	0.4039	0.4140	0.2674	0.7097	0.8381	0.6010	0.8344	0.9337	0.7797
12,9,6	0.4467	0.6018	0.3910	0.7603	0.8962	0.7391	0.8728	0.9575	0.8664
Average	0.5536	0.6573	0.4681	0.8265	0.9192	0.7781	0.9128	0.9685	0.8884

Table F.23: Setting 4: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.2418	0.1127	0.0681	0.4367	0.4033	0.3224	0.5386	0.5217	0.5153
6,6,6	0.2955	0.1961	0.1908	0.4882	0.4610	0.4556	0.5845	0.6174	0.5979
7,7,7	0.3437	0.2631	0.2039	0.5335	0.5526	0.4684	0.6244	0.6866	0.5968
8,8,8	0.3838	0.3294	0.3134	0.5689	0.6170	0.5535	0.6581	0.7370	0.7164
9,9,9	0.4261	0.4041	0.3146	0.6075	0.6779	0.6519	0.6905	0.7880	0.7414
10,10,10	0.4624	0.4727	0.4129	0.6375	0.7316	0.7086	0.7167	0.8304	0.8195

Table F.24: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.2967	0.1894	0.2087	0.4912	0.4584	0.4658	0.5875	0.6102	0.6045
5,7,6	0.3069	0.2096	0.2297	0.4932	0.4851	0.4562	0.5874	0.6320	0.5990
6,5,7	0.2777	0.1614	0.1916	0.4777	0.4418	0.4020	0.5783	0.5708	0.5385
6,7,5	0.3037	0.2153	0.2175	0.4894	0.4884	0.4515	0.5865	0.6359	0.5967
7,5,6	0.2718	0.1676	0.1841	0.4727	0.4461	0.4029	0.5756	0.5755	0.5433
7,6,5	0.2919	0.1999	0.1866	0.4850	0.4661	0.4585	0.5827	0.6219	0.6024
Average	0.2915	0.1905	0.2030	0.4849	0.4643	0.4395	0.5830	0.6077	0.5807
6,7,8	0.3442	0.2601	0.2198	0.5316	0.5447	0.5293	0.6231	0.6789	0.6429
6,8,7	0.3533	0.2752	0.2305	0.5365	0.5676	0.5578	0.6271	0.6993	0.6736
7,6,8	0.3523	0.2806	0.2257	0.5355	0.5741	0.5590	0.6255	0.7031	0.6749
7,8,6	0.3526	0.2820	0.2257	0.5376	0.5752	0.5609	0.6279	0.7044	0.6759
8,6,7	0.3221	0.2454	0.1884	0.5186	0.5122	0.5075	0.6159	0.6548	0.6357
8,7,6	0.3371	0.2656	0.2022	0.5293	0.5548	0.5261	0.6223	0.6879	0.6471
Average	0.3436	0.2682	0.2154	0.5315	0.5548	0.5401	0.6236	0.6881	0.6584
5,8,11	0.3892	0.3149	0.2497	0.5692	0.6044	0.5631	0.6550	0.7258	0.6927
5,11,8	0.3938	0.3618	0.2948	0.5662	0.6523	0.5993	0.6517	0.7716	0.7279
8,5,11	0.3277	0.2291	0.1867	0.5324	0.5098	0.4484	0.6302	0.6308	0.6071
8,11,5	0.3982	0.3823	0.2648	0.5767	0.6700	0.5987	0.6629	0.7873	0.7434
11,5,8	0.3115	0.2322	0.1699	0.5168	0.5161	0.4426	0.6189	0.6385	0.6066
11,8,5	0.3801	0.3413	0.2084	0.5701	0.6312	0.5625	0.6609	0.7518	0.7130
Average	0.3668	0.3103	0.2291	0.5552	0.5973	0.5358	0.6466	0.7176	0.6818
6,9,12	0.4248	0.3872	0.3365	0.6004	0.6598	0.5975	0.6816	0.7731	0.7239
6,12,9	0.4309	0.4338	0.3893	0.6026	0.7140	0.6596	0.6841	0.8206	0.7804
9,6,12	0.3776	0.3068	0.2670	0.5718	0.5719	0.5330	0.6626	0.6999	0.6756
9,12,6	0.4362	0.4528	0.3731	0.6110	0.7263	0.6606	0.6926	0.8319	0.7857
12,6,9	0.3668	0.3135	0.2529	0.5655	0.5809	0.5322	0.6600	0.7111	0.6844
12,9,6	0.4219	0.4152	0.3077	0.6061	0.6870	0.6031	0.6926	0.7970	0.7450
Average	0.4097	0.3849	0.3211	0.5929	0.6567	0.5977	0.6789	0.7723	0.7325

Table F.25: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,5,5	0.1227	0.0587	0.0398	0.2921	0.3142	0.2465	0.4130	0.4489	0.4183
6,6,6	0.1557	0.1093	0.1255	0.3403	0.3699	0.3530	0.4669	0.5367	0.4930
7,7,7	0.1858	0.1670	0.1367	0.3870	0.4494	0.3533	0.5199	0.6060	0.4838
8,8,8	0.2183	0.2243	0.2172	0.4329	0.5188	0.4324	0.5669	0.6636	0.6042
9,9,9	0.2517	0.2827	0.2099	0.4784	0.5820	0.5244	0.6138	0.7162	0.6293
10,10,10	0.2850	0.3448	0.2852	0.5203	0.6362	0.5797	0.6553	0.7639	0.7152

Table F.26: Setting 4: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
Sample Sizes	F	KW	PS	F	KW	PS	F	KW	PS
5,6,7	0.1722	0.1044	0.1327	0.3683	0.3815	0.3266	0.4978	0.5271	0.4606
5,7,6	0.1434	0.0852	0.1222	0.3194	0.3484	0.3064	0.4421	0.4983	0.4445
6,5,7	0.1886	0.1340	0.1430	0.3953	0.4055	0.3797	0.5296	0.5649	0.5273
6,7,5	0.1278	0.0883	0.1380	0.2893	0.3344	0.3350	0.4064	0.4922	0.4692
7,5,6	0.1648	0.1313	0.1473	0.3645	0.4028	0.3618	0.4967	0.5659	0.5135
7,6,5	0.1345	0.1077	0.1503	0.3074	0.3613	0.3398	0.4316	0.5199	0.4713
Average	0.1552	0.1085	0.1389	0.3407	0.3723	0.3416	0.4674	0.5281	0.4811
6,7,8	0.2065	0.1713	0.1337	0.4134	0.4437	0.4169	0.5478	0.5967	0.5445
6,8,7	0.1723	0.1483	0.1327	0.3631	0.4149	0.3919	0.4897	0.5696	0.5112
7,6,8	0.1572	0.1463	0.1329	0.3363	0.4116	0.3927	0.4602	0.5708	0.5173
7,8,6	0.1584	0.1448	0.1334	0.3371	0.4135	0.3922	0.4608	0.5730	0.5164
8,6,7	0.1986	0.1886	0.1530	0.4101	0.4806	0.4452	0.5452	0.6322	0.5675
8,7,6	0.1657	0.1616	0.1437	0.3536	0.4381	0.4305	0.4833	0.5952	0.5561
Average	0.1765	0.1602	0.1382	0.3689	0.4337	0.4116	0.4978	0.5896	0.5355
5,8,11	0.2559	0.1942	0.1306	0.4871	0.4831	0.3808	0.6202	0.6195	0.5396
5,11,8	0.1696	0.1391	0.1160	0.3526	0.4220	0.3315	0.4843	0.5644	0.4790
8,5,11	0.3374	0.2968	0.1767	0.5831	0.5939	0.5111	0.7062	0.7271	0.6622
8,11,5	0.1345	0.1434	0.1516	0.2887	0.4092	0.3827	0.4025	0.5605	0.5164
11,5,8	0.2567	0.2852	0.1860	0.4995	0.5950	0.5037	0.6360	0.7362	0.6643
11,8,5	0.1488	0.1777	0.1768	0.3282	0.4638	0.4284	0.4540	0.6220	0.5713
Average	0.2172	0.2061	0.1563	0.4232	0.4945	0.4230	0.5505	0.6383	0.5721
6,9,12	0.2906	0.2592	0.2024	0.5316	0.5464	0.4541	0.6627	0.6837	0.6038
6,12,9	0.1978	0.2012	0.1804	0.3999	0.4823	0.3934	0.5350	0.6261	0.5323
9,6,12	0.3700	0.3570	0.2503	0.6181	0.6510	0.5349	0.7370	0.7716	0.6854
9,12,6	0.1592	0.1984	0.2003	0.3336	0.4797	0.4206	0.4572	0.6297	0.5652
12,6,9	0.2935	0.3459	0.2669	0.5407	0.6524	0.5413	0.6735	0.7818	0.6876
12,9,6	0.1828	0.2398	0.2434	0.3798	0.5401	0.4976	0.5114	0.6888	0.6410
Average	0.2490	0.2669	0.2240	0.4673	0.5587	0.4737	0.5961	0.6970	0.6192

Table F.27: Setting 4: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

F.2 Corrected Critical Values

Medians	Sample Sizes	Table	Page
0,0,0	Equal	F.29	214
0,0,0	Unequal	F.30	215
0,0,1	Equal	F.31	216
0,0,1	Unequal	F.32	217
0,1,0	Equal	F.33	218
0,1,0	Unequal	F.34	219
1,0,0	Equal	F.35	220
1,0,0	Unequal	F.36	221
0,0,2	Equal	F.37	222
0,0,2	Unequal	F.38	223
0,2,0	Equal	F.39	224
0,2,0	Unequal	F.40	225
2,0,0	Equal	F.41	226
2,0,0	Unequal	F.42	227
0,1,2	Equal	F.43	228
0,1,2	Unequal	F.44	229
0,2,1	Equal	F.45	230
0,2,1	Unequal	F.46	231
1,0,2	Equal	F.47	232
1,0,2	Unequal	F.48	233
1,2,0	Equal	F.49	234
1,2,0	Unequal	F.50	235
2,0,1	Equal	F.51	236
2,0,1	Unequal	F.52	237
2,1,0	Equal	F.53	238
2,1,0	Unequal	F.54	239

Table F.28: List of Tables for Setting 4: Corrected Cutoff Values

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0016	0.0136	0.0518
6,6,6	0.0040	0.0500	0.0781
7,7,7	0.0063	0.0497	0.1014
8,8,8	0.0079	0.0440	0.0824
9,9,9	0.0085	0.0422	0.0761
10,10,10	0.0084	0.0353	0.0842

Table F.29: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0075	0.0596	0.0745
5,7,6	0.0060	0.0515	0.0630
6,5,7	0.0060	0.0530	0.0689
6,7,5	0.0037	0.0332	0.0435
7,5,6	0.0042	0.0433	0.0534
7,6,5	0.0033	0.0321	0.0425
Average	0.0051	0.0455	0.0576
6,7,8	0.0101	0.0478	0.0968
6,8,7	0.0077	0.0381	0.0775
7,6,8	0.0058	0.0314	0.0675
7,8,6	0.0057	0.0314	0.0669
8,6,7	0.0058	0.0335	0.0724
8,7,6	0.0049	0.0271	0.0606
Average	0.0067	0.0349	0.0736
5,8,11	0.0245	0.0937	0.1530
5,11,8	0.0158	0.0649	0.1066
8,5,11	0.0144	0.0754	0.1375
8,11,5	0.0039	0.0234	0.0459
11,5,8	0.0062	0.0416	0.0837
11,8,5	0.0024	0.0186	0.0401
Average	0.0112	0.0529	0.0945
6,9,12	0.0193	0.0681	0.1372
6,12,9	0.0120	0.0442	0.0920
9,6,12	0.0125	0.0622	0.1354
9,12,6	0.0048	0.0229	0.0521
12,6,9	0.0046	0.0291	0.0756
12,9,6	0.0025	0.0158	0.0427
Average	0.0093	0.0404	0.0892

Table F.30: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0074	0.0446	0.1413
6,6,6	0.0190	0.1525	0.2181
7,7,7	0.0294	0.1625	0.2841
8,8,8	0.0398	0.1659	0.2714
9,9,9	0.0463	0.1660	0.2672
10,10,10	0.0512	0.1661	0.3156

Table F.31: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0307	0.1729	0.2060
5,7,6	0.0256	0.1523	0.1707
6,5,7	0.0231	0.1569	0.1965
6,7,5	0.0164	0.1060	0.1379
7,5,6	0.0159	0.1373	0.1555
7,6,5	0.0150	0.1045	0.1322
Average	0.0211	0.1383	0.1665
6,7,8	0.0423	0.1576	0.2734
6,8,7	0.0352	0.1299	0.2296
7,6,8	0.0283	0.1171	0.2096
7,8,6	0.0279	0.1157	0.2054
8,6,7	0.0275	0.1219	0.2288
8,7,6	0.0226	0.1061	0.1984
Average	0.0306	0.1247	0.2242
5,8,11	0.0843	0.2649	0.3840
5,11,8	0.0660	0.1875	0.2818
8,5,11	0.0572	0.2497	0.3880
8,11,5	0.0209	0.0894	0.1530
11,5,8	0.0311	0.1591	0.2764
11,8,5	0.0149	0.0834	0.1518
Average	0.0457	0.1723	0.2725
6,9,12	0.0750	0.2274	0.3952
6,12,9	0.0511	0.1530	0.2787
9,6,12	0.0615	0.2315	0.4022
9,12,6	0.0265	0.0938	0.1787
12,6,9	0.0294	0.1355	0.2798
12,9,6	0.0182	0.0822	0.1758
Average	0.0436	0.1539	0.2851

Table F.32: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0099	0.0527	0.1429
6,6,6	0.0247	0.1516	0.2039
7,7,7	0.0396	0.1648	0.2651
8,8,8	0.0504	0.1661	0.2495
9,9,9	0.0598	0.1705	0.2497
10,10,10	0.0662	0.1663	0.2763

Table F.33: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0307	0.1347	0.1680
5,7,6	0.0300	0.1397	0.1705
6,5,7	0.0303	0.1422	0.1615
6,7,5	0.0234	0.1429	0.1595
7,5,6	0.0292	0.1403	0.1700
7,6,5	0.0242	0.1327	0.1645
Average	0.0280	0.1388	0.1657
6,7,8	0.0455	0.1331	0.2175
6,8,7	0.0417	0.1333	0.2186
7,6,8	0.0383	0.1339	0.2266
7,8,6	0.0388	0.1343	0.2265
8,6,7	0.0406	0.1301	0.2167
8,7,6	0.0400	0.1361	0.2283
Average	0.0408	0.1335	0.2224
5,8,11	0.0520	0.1473	0.2180
5,11,8	0.0514	0.1599	0.2388
8,5,11	0.0622	0.1633	0.2342
8,11,5	0.0473	0.1713	0.2680
11,5,8	0.0518	0.1650	0.2406
11,8,5	0.0363	0.1493	0.2404
Average	0.0502	0.1594	0.2400
6,9,12	0.0560	0.1419	0.2325
6,12,9	0.0534	0.1461	0.2464
9,6,12	0.0588	0.1482	0.2461
9,12,6	0.0456	0.1575	0.2819
12,6,9	0.0495	0.1447	0.2501
12,9,6	0.0405	0.1450	0.2621
Average	0.0506	0.1472	0.2532

Table F.34: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0050	0.0248	0.0704
6,6,6	0.0098	0.0710	0.1002
7,7,7	0.0143	0.0699	0.1239
8,8,8	0.0173	0.0674	0.1118
9,9,9	0.0193	0.0651	0.1053
10,10,10	0.0199	0.0620	0.1203

Table F.35: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0132	0.0713	0.0822
5,7,6	0.0125	0.0669	0.0828
6,5,7	0.0133	0.0645	0.0817
6,7,5	0.0091	0.0546	0.0687
7,5,6	0.0109	0.0578	0.0732
7,6,5	0.0087	0.0569	0.0651
Average	0.0113	0.0620	0.0756
6,7,8	0.0183	0.0633	0.1126
6,8,7	0.0163	0.0590	0.1064
7,6,8	0.0132	0.0527	0.0974
7,8,6	0.0130	0.0526	0.0980
8,6,7	0.0141	0.0528	0.0952
8,7,6	0.0123	0.0490	0.0924
Average	0.0145	0.0549	0.1003
5,8,11	0.0286	0.0916	0.1429
5,11,8	0.0188	0.0755	0.1242
8,5,11	0.0211	0.0736	0.1177
8,11,5	0.0097	0.0495	0.0896
11,5,8	0.0132	0.0571	0.0968
11,8,5	0.0105	0.0493	0.0873
Average	0.0170	0.0661	0.1098
6,9,12	0.0251	0.0776	0.1402
6,12,9	0.0179	0.0627	0.1214
9,6,12	0.0211	0.0638	0.1162
9,12,6	0.0103	0.0460	0.0988
12,6,9	0.0143	0.0497	0.0978
12,9,6	0.0087	0.0403	0.0892
Average	0.0162	0.0567	0.1106

Table F.36: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0123	0.0725	0.2070
6,6,6	0.0351	0.2318	0.3183
7,7,7	0.0518	0.2438	0.3992
8,8,8	0.0731	0.2627	0.3983
9,9,9	0.0833	0.2589	0.3942
10,10,10	0.0962	0.2694	0.4625

Table F.37: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0515	0.2526	0.2970
5,7,6	0.0431	0.2316	0.2461
6,5,7	0.0408	0.2320	0.2823
6,7,5	0.0310	0.1644	0.2095
7,5,6	0.0295	0.2150	0.2301
7,6,5	0.0263	0.1689	0.2082
Average	0.0370	0.2108	0.2455
6,7,8	0.0718	0.2397	0.3921
6,8,7	0.0633	0.2020	0.3330
7,6,8	0.0521	0.1879	0.3102
7,8,6	0.0526	0.1876	0.3119
8,6,7	0.0507	0.1955	0.3360
8,7,6	0.0443	0.1762	0.3054
Average	0.0558	0.1982	0.3314
5,8,11	0.1308	0.3703	0.5007
5,11,8	0.1100	0.2770	0.4018
8,5,11	0.0942	0.3550	0.5115
8,11,5	0.0389	0.1486	0.2432
11,5,8	0.0640	0.2606	0.4154
11,8,5	0.0297	0.1442	0.2439
Average	0.0779	0.2593	0.3861
6,9,12	0.1217	0.3339	0.5284
6,12,9	0.0858	0.2338	0.3986
9,6,12	0.1081	0.3455	0.5406
9,12,6	0.0523	0.1591	0.2779
12,6,9	0.0580	0.2243	0.4112
12,9,6	0.0380	0.1490	0.2852
Average	0.0773	0.2409	0.4070

Table F.38: Setting 4: $\theta_1 = 0$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.1358	0.3291	0.5356
6,6,6	0.2719	0.6100	0.6876
7,7,7	0.3713	0.6614	0.7797
8,8,8	0.4683	0.7122	0.8067
9,9,9	0.5226	0.7343	0.8204
10,10,10	0.5926	0.7723	0.8784

Table F.39: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.3064	0.6030	0.6373
5,7,6	0.3283	0.6350	0.6743
6,5,7	0.2504	0.5322	0.5848
6,7,5	0.2898	0.6157	0.6562
7,5,6	0.2341	0.5267	0.5701
7,6,5	0.2471	0.5687	0.6065
Average	0.2760	0.5802	0.6215
6,7,8	0.4042	0.6247	0.7371
6,8,7	0.4296	0.6629	0.7817
7,6,8	0.4110	0.6614	0.7808
7,8,6	0.4095	0.6602	0.7766
8,6,7	0.3395	0.5788	0.7025
8,7,6	0.3585	0.6017	0.7263
Average	0.3921	0.6316	0.7508
5,8,11	0.4944	0.6994	0.7950
5,11,8	0.5677	0.8051	0.8891
8,5,11	0.3594	0.5882	0.6847
8,11,5	0.4986	0.7754	0.8750
11,5,8	0.3120	0.5634	0.6702
11,8,5	0.3693	0.6480	0.7685
Average	0.4336	0.6799	0.7804
6,9,12	0.5128	0.7084	0.8260
6,12,9	0.6012	0.8103	0.9190
9,6,12	0.4350	0.6302	0.7438
9,12,6	0.5549	0.7889	0.9091
12,6,9	0.3750	0.6083	0.7405
12,9,6	0.4194	0.6701	0.8011
Average	0.4831	0.7027	0.8233

Table F.40: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0517	0.1562	0.3040
6,6,6	0.1095	0.3509	0.4234
7,7,7	0.1550	0.3860	0.5071
8,8,8	0.2022	0.4165	0.5240
9,9,9	0.2409	0.4424	0.5408
10,10,10	0.2808	0.4703	0.6067

Table F.41: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.1127	0.3198	0.3564
5,7,6	0.0995	0.3007	0.3394
6,5,7	0.1278	0.3543	0.3866
6,7,5	0.0875	0.2971	0.3268
7,5,6	0.1304	0.3561	0.3954
7,6,5	0.0964	0.3110	0.3452
Average	0.1091	0.3232	0.3583
6,7,8	0.1718	0.3565	0.4687
6,8,7	0.1429	0.3188	0.4339
7,6,8	0.1377	0.3168	0.4389
7,8,6	0.1394	0.3193	0.4407
8,6,7	0.1732	0.3582	0.4773
8,7,6	0.1573	0.3502	0.4763
Average	0.1537	0.3366	0.4560
5,8,11	0.1972	0.3858	0.4785
5,11,8	0.1398	0.3333	0.4352
8,5,11	0.2638	0.4645	0.5613
8,11,5	0.1249	0.3217	0.4385
11,5,8	0.2385	0.4672	0.5757
11,8,5	0.1606	0.3790	0.5009
Average	0.1875	0.3919	0.4984
6,9,12	0.2323	0.4107	0.5257
6,12,9	0.1673	0.3451	0.4791
9,6,12	0.2690	0.4533	0.5803
9,12,6	0.1446	0.3368	0.4866
12,6,9	0.2517	0.4523	0.5952
12,9,6	0.1919	0.4065	0.5612
Average	0.2095	0.4008	0.5380

Table F.42: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0535	0.1934	0.3838
6,6,6	0.1356	0.4522	0.5344
7,7,7	0.2059	0.5100	0.6378
8,8,8	0.2854	0.5529	0.6568
9,9,9	0.3438	0.5919	0.6866
10,10,10	0.4062	0.6271	0.7498

Table F.43: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.1321	0.4171	0.4548
5,7,6	0.1390	0.4328	0.4631
6,5,7	0.1206	0.3947	0.4329
6,7,5	0.1398	0.4268	0.4717
7,5,6	0.1260	0.4307	0.4574
7,6,5	0.1452	0.4397	0.4845
Average	0.1338	0.4236	0.4607
6,7,8	0.2039	0.4446	0.5725
6,8,7	0.2156	0.4602	0.5890
7,6,8	0.2356	0.4877	0.6068
7,8,6	0.2358	0.4869	0.6051
8,6,7	0.2078	0.4549	0.5901
8,7,6	0.2191	0.4704	0.5976
Average	0.2196	0.4675	0.5935
5,8,11	0.2396	0.4981	0.6039
5,11,8	0.2778	0.5347	0.6344
8,5,11	0.2152	0.4765	0.5926
8,11,5	0.2966	0.5694	0.6779
11,5,8	0.2479	0.5250	0.6360
11,8,5	0.2758	0.5658	0.6791
Average	0.2588	0.5283	0.6373
6,9,12	0.2763	0.5033	0.6390
6,12,9	0.3072	0.5408	0.6712
9,6,12	0.2702	0.5168	0.6504
9,12,6	0.3501	0.5912	0.7058
12,6,9	0.2812	0.5399	0.6879
12,9,6	0.3302	0.5809	0.7084
Average	0.3025	0.5455	0.6771

Table F.44: Setting 4: $\theta_1 = 0$, $\theta_2 = 1$, and $\theta_3 = 2$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0331	0.1254	0.2900
6,6,6	0.0744	0.3288	0.4188
7,7,7	0.1146	0.3538	0.5151
8,8,8	0.1506	0.3800	0.5164
9,9,9	0.1752	0.3879	0.5202
10,10,10	0.2022	0.4069	0.5891

Table F.45: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0748	0.2579	0.3081
5,7,6	0.0749	0.2642	0.3156
6,5,7	0.0846	0.3081	0.3354
6,7,5	0.0754	0.3203	0.3445
7,5,6	0.0966	0.3275	0.3784
7,6,5	0.0893	0.3289	0.3837
Average	0.0826	0.3012	0.3443
6,7,8	0.1110	0.2827	0.4216
6,8,7	0.1102	0.2885	0.4336
7,6,8	0.1187	0.3072	0.4530
7,8,6	0.1196	0.3072	0.4548
8,6,7	0.1357	0.3396	0.4924
8,7,6	0.1329	0.3453	0.5038
Average	0.1214	0.3118	0.4599
5,8,11	0.1020	0.2584	0.3624
5,11,8	0.1021	0.2735	0.3888
8,5,11	0.1572	0.3559	0.4786
8,11,5	0.1576	0.4046	0.5608
11,5,8	0.1892	0.4469	0.5713
11,8,5	0.1774	0.4678	0.6154
Average	0.1476	0.3679	0.4962
6,9,12	0.1188	0.2709	0.4127
6,12,9	0.1265	0.2956	0.4506
9,6,12	0.1565	0.3310	0.4924
9,12,6	0.1550	0.3721	0.5646
12,6,9	0.1975	0.4315	0.6198
12,9,6	0.1932	0.4547	0.6496
Average	0.1579	0.3593	0.5316

Table F.46: Setting 4: $\theta_1 = 0$, $\theta_2 = 2$, and $\theta_3 = 1$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0276	0.1125	0.2514
6,6,6	0.0685	0.3011	0.3802
7,7,7	0.1023	0.3343	0.4661
8,8,8	0.1464	0.3663	0.4811
9,9,9	0.1740	0.3827	0.4908
10,10,10	0.2103	0.4123	0.5660

Table F.47: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0669	0.2719	0.3059
5,7,6	0.0652	0.2786	0.2998
6,5,7	0.0706	0.2817	0.3145
6,7,5	0.0682	0.2649	0.3005
7,5,6	0.0682	0.2846	0.3073
7,6,5	0.0677	0.2640	0.3006
Average	0.0678	0.2743	0.3048
6,7,8	0.1086	0.3002	0.4299
6,8,7	0.1076	0.2875	0.4137
7,6,8	0.1066	0.2876	0.4109
7,8,6	0.1081	0.2908	0.4130
8,6,7	0.1092	0.2969	0.4223
8,7,6	0.1095	0.2976	0.4224
Average	0.1083	0.2934	0.4187
5,8,11	0.1283	0.3443	0.4626
5,11,8	0.1306	0.3407	0.4567
8,5,11	0.1397	0.3679	0.4850
8,11,5	0.1204	0.3252	0.4388
11,5,8	0.1400	0.3566	0.4717
11,8,5	0.1222	0.3281	0.4421
Average	0.1302	0.3438	0.4595
6,9,12	0.1541	0.3610	0.5099
6,12,9	0.1410	0.3396	0.4917
9,6,12	0.1559	0.3596	0.5061
9,12,6	0.1435	0.3341	0.4777
12,6,9	0.1560	0.3577	0.5016
12,9,6	0.1532	0.3510	0.4902
Average	0.1506	0.3505	0.4962

Table F.48: Setting 4: $\theta_1 = 1$, $\theta_2 = 0$, and $\theta_3 = 2$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0136	0.0708	0.2030
6,6,6	0.0332	0.2371	0.3265
7,7,7	0.0540	0.2612	0.4323
8,8,8	0.0763	0.2846	0.4352
9,9,9	0.0938	0.2965	0.4471
10,10,10	0.1155	0.3209	0.5315

Table F.49: Setting 4: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0434	0.2420	0.2832
5,7,6	0.0546	0.2788	0.3276
6,5,7	0.0279	0.1643	0.2140
6,7,5	0.0454	0.2401	0.2957
7,5,6	0.0243	0.1553	0.1925
7,6,5	0.0317	0.1923	0.2308
Average	0.0379	0.2121	0.2573
6,7,8	0.0642	0.2271	0.3802
6,8,7	0.0793	0.2716	0.4445
7,6,8	0.0729	0.2592	0.4283
7,8,6	0.0715	0.2591	0.4301
8,6,7	0.0420	0.1663	0.3005
8,7,6	0.0514	0.1929	0.3342
Average	0.0636	0.2294	0.3863
5,8,11	0.1093	0.3467	0.5152
5,11,8	0.1729	0.4965	0.6625
8,5,11	0.0369	0.1636	0.2718
8,11,5	0.1200	0.4039	0.5656
11,5,8	0.0283	0.1297	0.2228
11,8,5	0.0548	0.2233	0.3597
Average	0.0870	0.2940	0.4329
6,9,12	0.1000	0.3127	0.5217
6,12,9	0.1611	0.4486	0.6799
9,6,12	0.0476	0.1742	0.3286
9,12,6	0.1326	0.3937	0.6210
12,6,9	0.0335	0.1341	0.2706
12,9,6	0.0602	0.2139	0.3893
Average	0.0892	0.2795	0.4685

Table F.50: Setting 4: $\theta_1 = 1$, $\theta_2 = 2$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0167	0.0684	0.1718
6,6,6	0.0347	0.1891	0.2525
7,7,7	0.0517	0.2019	0.3200
8,8,8	0.0681	0.2120	0.3152
9,9,9	0.0785	0.2146	0.3131
10,10,10	0.0916	0.2229	0.3645

Table F.51: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0433	0.1868	0.2080
5,7,6	0.0462	0.1927	0.2304
6,5,7	0.0385	0.1547	0.1906
6,7,5	0.0399	0.1815	0.2188
7,5,6	0.0336	0.1473	0.1799
7,6,5	0.0319	0.1679	0.1874
Average	0.0389	0.1718	0.2025
6,7,8	0.0629	0.1757	0.2775
6,8,7	0.0630	0.1890	0.3007
7,6,8	0.0571	0.1857	0.3001
7,8,6	0.0578	0.1845	0.2980
8,6,7	0.0466	0.1513	0.2521
8,7,6	0.0507	0.1611	0.2660
Average	0.0564	0.1746	0.2824
5,8,11	0.0880	0.2322	0.3330
5,11,8	0.0928	0.2668	0.3719
8,5,11	0.0518	0.1584	0.2388
8,11,5	0.0664	0.2319	0.3479
11,5,8	0.0414	0.1440	0.2255
11,8,5	0.0537	0.1902	0.2980
Average	0.0657	0.2039	0.3025
6,9,12	0.0863	0.2107	0.3385
6,12,9	0.0922	0.2412	0.3897
9,6,12	0.0588	0.1587	0.2653
9,12,6	0.0710	0.2221	0.3763
12,6,9	0.0475	0.1437	0.2543
12,9,6	0.0527	0.1733	0.3101
Average	0.0681	0.1916	0.3224

Table F.52: Setting 4: $\theta_1 = 2$, $\theta_2 = 0$, and $\theta_3 = 1$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,5,5	0.0066	0.0397	0.1184
6,6,6	0.0171	0.1272	0.1779
7,7,7	0.0263	0.1352	0.2308
8,8,8	0.0341	0.1341	0.2169
9,9,9	0.0414	0.1385	0.2134
10,10,10	0.0478	0.1350	0.2442

Table F.53: Setting 4: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Sample Sizes	PS	PS	PS
5,6,7	0.0166	0.1039	0.1325
5,7,6	0.0158	0.0969	0.1215
6,5,7	0.0177	0.1234	0.1435
6,7,5	0.0209	0.1196	0.1386
7,5,6	0.0192	0.1212	0.1480
7,6,5	0.0226	0.1215	0.1500
Average	0.0188	0.1144	0.1390
6,7,8	0.0249	0.1023	0.1856
6,8,7	0.0287	0.1074	0.1845
7,6,8	0.0299	0.1027	0.1768
7,8,6	0.0292	0.1025	0.1788
8,6,7	0.0303	0.1203	0.2090
8,7,6	0.0301	0.1122	0.1954
Average	0.0289	0.1079	0.1884
5,8,11	0.0219	0.1052	0.1786
5,11,8	0.0249	0.0967	0.1574
8,5,11	0.0349	0.1565	0.2587
8,11,5	0.0435	0.1363	0.2038
11,5,8	0.0374	0.1594	0.2601
11,8,5	0.0448	0.1522	0.2324
Average	0.0346	0.1344	0.2152
6,9,12	0.0268	0.1041	0.2020
6,12,9	0.0322	0.1032	0.1828
9,6,12	0.0300	0.1267	0.2515
9,12,6	0.0371	0.1112	0.2010
12,6,9	0.0404	0.1444	0.2689
12,9,6	0.0426	0.1329	0.2394
Average	0.0349	0.1204	0.2243

Table F.54: Setting 4: $\theta_1 = 2$, $\theta_2 = 1$, and $\theta_3 = 0$

Vita

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Professional Academic Experience

Lecturer 2001–2002
Messiah College, Department of Mathematics, Grantham, PA

- Undergraduate teaching experience –
  - Applied Mathematics for Management
  - Calculus I
  - Calculus II (planned)
  - Introductory Statistics

Instructor 2000–2001
Virginia Polytechnic Institute & State University, Department of Statistics, Blacksburg, VA

- Undergraduate teaching experience –
  - Statistical Methods I & II
  - Probability and Statistics for Electrical Engineers
  - Statistics, a Liberal Arts Approach
- Taught various course-loads with as many as 280 students and three sections per semester.
- Provided out-of-class tutoring and assistance.
- Responsible for course, exam, homework, and syllabus design.
- Supervised Statistical Methods II students in the collection and analysis of data for individual projects of their own choosing.
- Supervised graduate assistants employed as graders.

Teaching Assistant 1997–2000
Virginia Polytechnic Institute & State University, Department of Statistics, Blacksburg, VA

- Undergraduate teaching experience –
  - Statistical Methods I
  - Biological Statistics I & II
- Supervised graduate assistants employed as graders.
- Supervised Biological Statistics II students in the collection and analysis of data for individual projects of their own choosing.
- Led fifty-student recitation sections –
  - Introduction to Statistics
- Graded and reported student homeworks –
  - Introduction to Statistics
  - Statistical Methods for the Social Sciences
  - Probability and Statistics for Electrical Engineers

Statistical Consultant 1997–2000
Virginia Polytechnic Institute & State University, Department of Statistics Consulting Center, Blacksburg, VA

- Lead and assisted consultant teams to gather information about projects and determine goals and appropriate methods of analysis.
- Interacted with clients with varying levels of statistical knowledge.
- Participated in projects from a variety of departments such as Biology, Mechanical Engineering, Food Science, Fisheries & Wildlife, Education, Wood Products, Geology, Chemistry, and Psychology.

Academic Degrees

Ph.D., Statistics 2001
Virginia Polytechnic Institute & State University, College of Arts & Sciences, Blacksburg, VA

- *Dissertation Title:* A New Nonparametric Procedure for the k-sample Problem
- *Dissertation Summary:* Research focused on the development of a new statistical method for determining whether data come from equally located distributions
- *Dissertation Committee Chair:* Dr. George Terrell
- Chapter Vice-President, Mu Sigma Rho, 1998-1999

M.S., Statistics 1997
Virginia Polytechnic Institute & State University, College of Arts & Sciences, Blacksburg, VA

- Member, Mu Sigma Rho, National Statistical Honor Society

B.A., Mathematics, Statistics Minor, Cum Laude 1996
Messiah College, Grantham, PA

- Member, Sigma Zeta, National Honorary Science Society

Paper Submitted for Publication

"Prevalence of Cognitively Intact Individuals Residing in Extended Care Nursing Facilities",
Co-author, submitted to *The Gerontologist.*

Additional Skills

- SAS: General Procs, Macro Programming, International Matrix Language, and Data Management
- Minitab: Graphics, Data Summary, Data Manipulation and Analysis
- LaTeX: Equations, Tables, General Chapters, Indexes, Bibliographies, Type-Setting
- Working knowledge of UNIX and Windows environments
- Working knowledge of Microsoft Office and Correl Suite word processing and spreadsheet software