STABILITY OF RETICULATED DOMES UNDER MULTIPLE STATIC AND DYNAMIC LOADS

by

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Dissertation submitted to the Graduate Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Civil Engineering

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August, 1976

Blacksburg, Virginia

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(ABSTRACT)

The primary purpose of this dissertation is to investigate the stability of reticulated domes under multiple static and dynamic loads. Two elastic geometrically nonlinear structural models of a reticulated dome with 21 and 39 degrees of freedom are considered.

The nonlinear response of the system to static loads is obtained using nonlinear programming and discrete perturbation techniques. The nonlinear programming technique is used to obtain a starting solution for the discrete perturbation technique and to optimize the choice of the perturbation parameter. Convergence criteria and error estimates to limit errors in a perturbation scheme are developed. A method for selecting a "suitable" perturbation parameter for imperfection sensitive systems is proposed.

The investigation of stability of equilibrium of the system subjected to finite disturbances is based on the concept of "degree of stability" and the associated sufficient stability condition. The stability condition is derived from a theorem on extent of asymptotic stability of Liapunov's direct method of the theory of stability of

motion. Its application requires the determination of the nonlinear fundamental path and the "nearest" unstable post-buckling path. This is obtained via static analysis.

The perturbed motion of the system under a given set of perturbations is obtained by numerically integrating the nonlinear equations of motion. The dynamic stability tests confirm the sufficiency of the dynamic stability condition. However, they also indicate that there is a dynamic disturbance with a specific spatial distribution for which the sufficient condition of stability is also a necessary condition for each equilibrium state tested. Since in practice, the spatial distribution of the disturbances cannot be controlled, the sufficient dynamic stability condition employed is practical for the design of reticulated domes.

The stability boundaries corresponding to two independent loads on the models are presented. Limit points lie on a boundary which is convex towards the region of stability. Bifurcation points lie on a continuous but piecewise differentiable boundary. Each piece of the boundary containing bifurcation points appears to be convex towards the region of stability.

ACKNOWLEDGEMENTS

The work presented in this dissertation was carried out under the direction of Dr. S. M. Holzer and the author wishes to express his deepest gratitude for this guidance.

In addition, grateful appreciation is due to Dr. R. H. Plaut for critically reviewing and proof-reading the entire manuscript and making several valuable comments.

The review of the thesis by Dr. R. D. Walker, Head of Civil Engineering Department, Dr. P. M. Ghare, and Dr. A. E. Somers are gratefully appreciated.

The African-American Institute, whose grant made this work possible, and Ahmadu Bello University (NIGERIA) who nominated the author for the African Graduate (AFGRAD) Fellowship program are to be thanked for their generous support.

The financial support provided by the Department of Civil Engineering and the computer facilities (IBM 360-75) of Virginia Polytechnic Institute and State University are gratefully acknowledged.

This work is dedicated to my parents, Mr. and Mrs. I. O. Abatan, without whose encouragement and love, this work would have remained a dream.

Finally, sincere gratitude is expressed to Frances Carter for her outstanding job in typing the entire manuscript.

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NOTATION

A_i	cross-sectional area of i-th element
C_M	set containing the equilibrium state; a subset of the domain of asymptotic stability of the origin
E_i	modulus of elasticity of i-th element
$E^j(Q_i)$	displacements corresponding to loads Λ^j; E^j are linear in Q_i
h	height of center of dome from supports
L_i	undeformed length of element i
L_i^*	deformed length of element i
ℓ^j	direction cosine from a loading ray in the load space to the load Λ^j
L	Liapunov function
L'	total derivative of L with respect to time
M	energy level at "nearest" unstable equilibrium state
NM	number of members (elements)
NJ	number of joints
O	Landau's error order symbol
P_s	generalized load corresponding to generalized displacement Q_s
P_j	three dimensional force vector at joint j.
P_j^T	transpose of vector P_j
Q_i	generalized displacements
$\dot{Q}_i$	time derivative of Q_i
$q_i'(t)$	generalized velocity perturbation of i-th coordinate

q_i generalized displacement-increment of i-th coordinate

$\dot{q}_i$ first derivative of q_i with respect to ε

R regular matrix

s span of the reticulated dome

T kinetic energy function

t time

t_d duration of exciting force

U_j three-dimensional displacement vector at joint j

V total potential energy function

v total energy; Liapunov function

v_o value of v at time zero

W_L limit point characteristic matrix

W_B bifurcation point characteristic matrix

W_C coincident critical point characteristic matrix

W_S the s-th equilibrium equation

$W_{s,i}$ partial derivatives of the s-th equilibrium equation with respect to the i-th generalized coordinate

$W_{s,ij}$ second order partial derivatives of the s-th equilibrium equation with respect to the i-th and j-th generalized coordinates

det(W) determinant of matrix W

y(t) state vector; resulting motion of system

y_o value of y at time zero; initial state of system

y_1 generalized displacement vector of perturbed motion

y_2	generalized velocity vector of perturbed motion
$\|\|y\|\|$	vector norm; $(y^T y)^{1/2}$
y_1^u	unstable equilibrium states
Λ, λ	load parameters
Λ^j	j-th generalized independent load
Λ_i^c	critical value of Λ_i
Λ_{ok}	critical value of Λ^k when acting singly
ε	perturbation parameter
η	increment in load parameter λ
$\dot{\eta}$	first derivative of η with respect to ε
π	potential energy; strain energy of system
π_i	the strain energy of the i-th element
Ω	potential energy of external forces
δ	limit of errors in a perturbation scheme
γ	unbalanced joint forces; small positive constant
$\in$	is an element of

CHAPTER 1

INTRODUCTION

1.1 Purpose and Scope

The primary purpose of this thesis is to investigate the stability of reticulated domes under multiple static and dynamic loads. An elastic geometrically nonlinear structural model of a reticulated dome is considered. Geometric nonlinearities are due to admissible finite joint displacements. The pre- and post-buckling paths of the imperfection-sensitive system are obtained via nonlinear programming and discrete perturbation techniques. A method for selecting a "suitable" perturbation parameter is proposed.

Stability of equilibrium of the system subjected to instantaneous and transient disturbances is established using the concept of "degree of stability" [45]* and the associated sufficient stability condition. The sufficient stability condition is based on Liapunov's direct method [37].

The perturbed motion of the system for a given class of perturbations is obtained by numerically integrating the nonlinear differential equations of motion. The characteristics of the resulting phase-plane portraits provide two useful guides: 1. a check on the sufficient stability condition, and 2. an indication when the sufficient condition is also necessary for the stability of equilibrium.

Stability boundary curves [59] are presented for the reticulated

*Numbers in brackets refer to the references listed in the bibliography.

dome model to indicate the effect of several independent external loads on the stability of equilibrium.

1.2 "Nearest" Unstable Equilibrium States

In this section, the location of the "nearest" unstable equilibrium states is discussed. The unstable post-buckling path, emanating from an unstable critical point, which is closest to the fundamental path [114] contains the "nearest" unstable equilibrium points.

In this study, the potential energy difference between the stable prebuckling state and the "nearest" unstable equilibrium state is used as a measure of degree of stability of equilibrium [45]. The degree of stability is a quantitative measure of the disturbances an equilibrium state can sustain; it is a safe estimate of the domain of asymptotic stability of the equilibrium state whose stability is in question [45]. Hence the location of the "nearest" unstable equilibrium states with high precision is central to the correct usage of the stability criterion. The concept of degree of stability represents a practical stability criterion.

The nature and type of the critical point in configuration space determines the form of the post-buckling response. Post-buckling paths beyond a limit point, bifurcation point, and a coincident critical point are shown in Fig. 1.1. Definitions of these critical points are given by Thompson [109]. In all three cases shown in Fig. 1.1, point 2 represents the "nearest" unstable equilibrium state with respect to the stable equilibrium point 1. Λ^C, Λ^D represent the critical

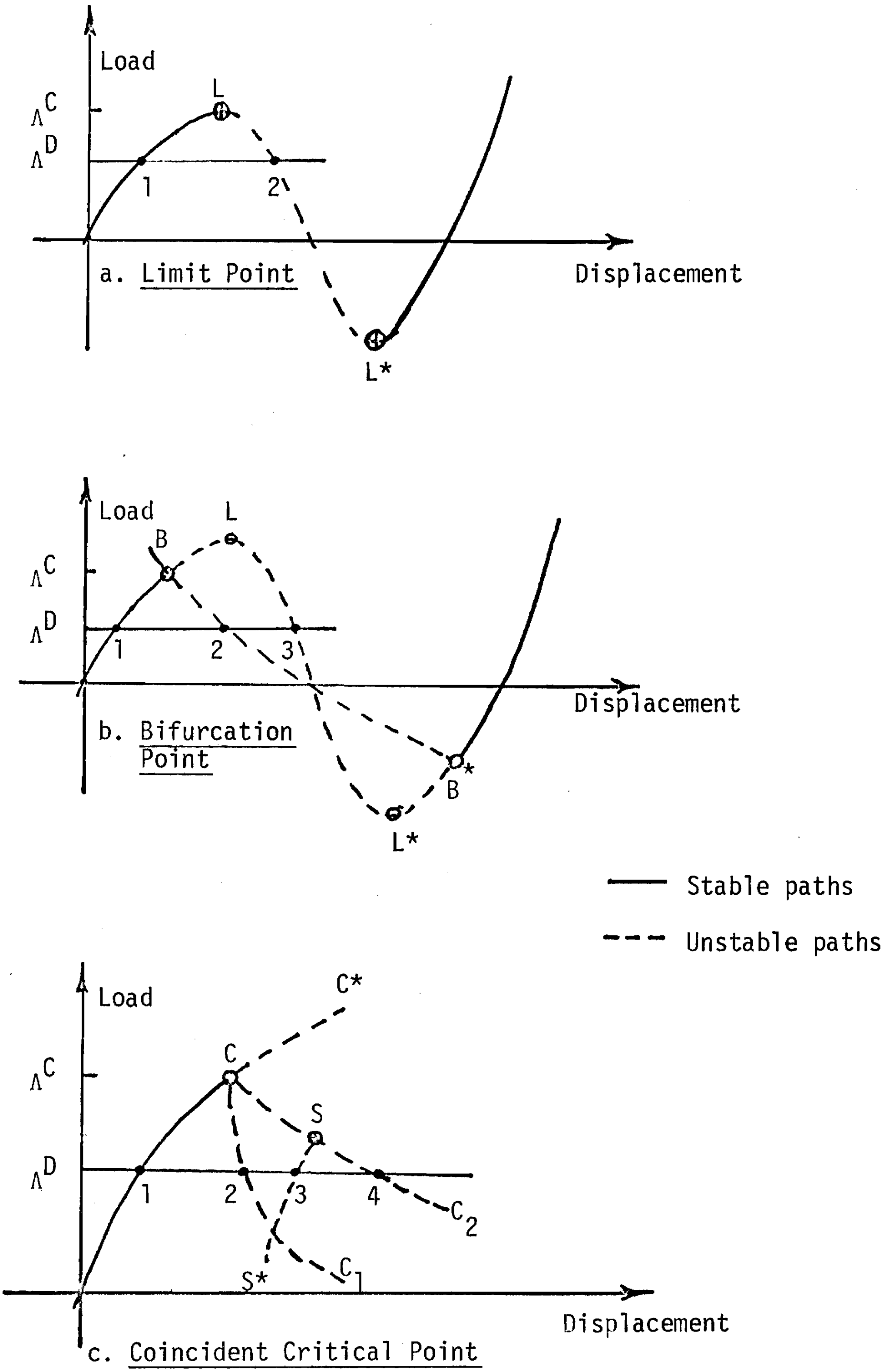


FIG. 1.1 "NEAREST" UNSTABLE EQUILIBRIUM PATHS

load, design load, on the system, respectively. In the case of a limit point, Fig. 1.1a, the unstable post-buckling path, LL*, is unique. If a bifurcation point B (Fig. 1.1b) exists, two unstable paths, BB* and BLL*B*, exist. For some loading conditions, point 2 could be to the right of point 3. In that case, point 3 represents the "nearest" unstable state. The case of a coincident critical point is shown in Fig. 1.1c. For the case of two coincident critical loads, paths CC*, CC_1, CC_2 are feasible. Again, the location of point 2 on CC_1 is required to use the sufficient stability criterion. Secondary bifurcation [7] from an unstable post-buckling path (e.g. path SS*) is not considered.

The static perturbation method [92], a systematic approach, is used to locate all post-buckling paths beyond unstable critical points of the reticulated dome model used.

1.3 Literature Review

There are many factors affecting the stability of structures (Fig. 1.2). Researchers in the field of structural stability have considered several of these factors in selecting mathematical models for several classes of structures. In this section, major works relating to the present study are reviewed and classified on the basis of modelling and solution characteristics. Special attention is given to conservative, geometrically nonlinear structural models. The review contains three sub-sections: stability concepts and theories, solution techniques in stability analysis, and stability of reticulated space structures. The review concludes with a justification and need for research on stability of reticulated domes subjected to multiple

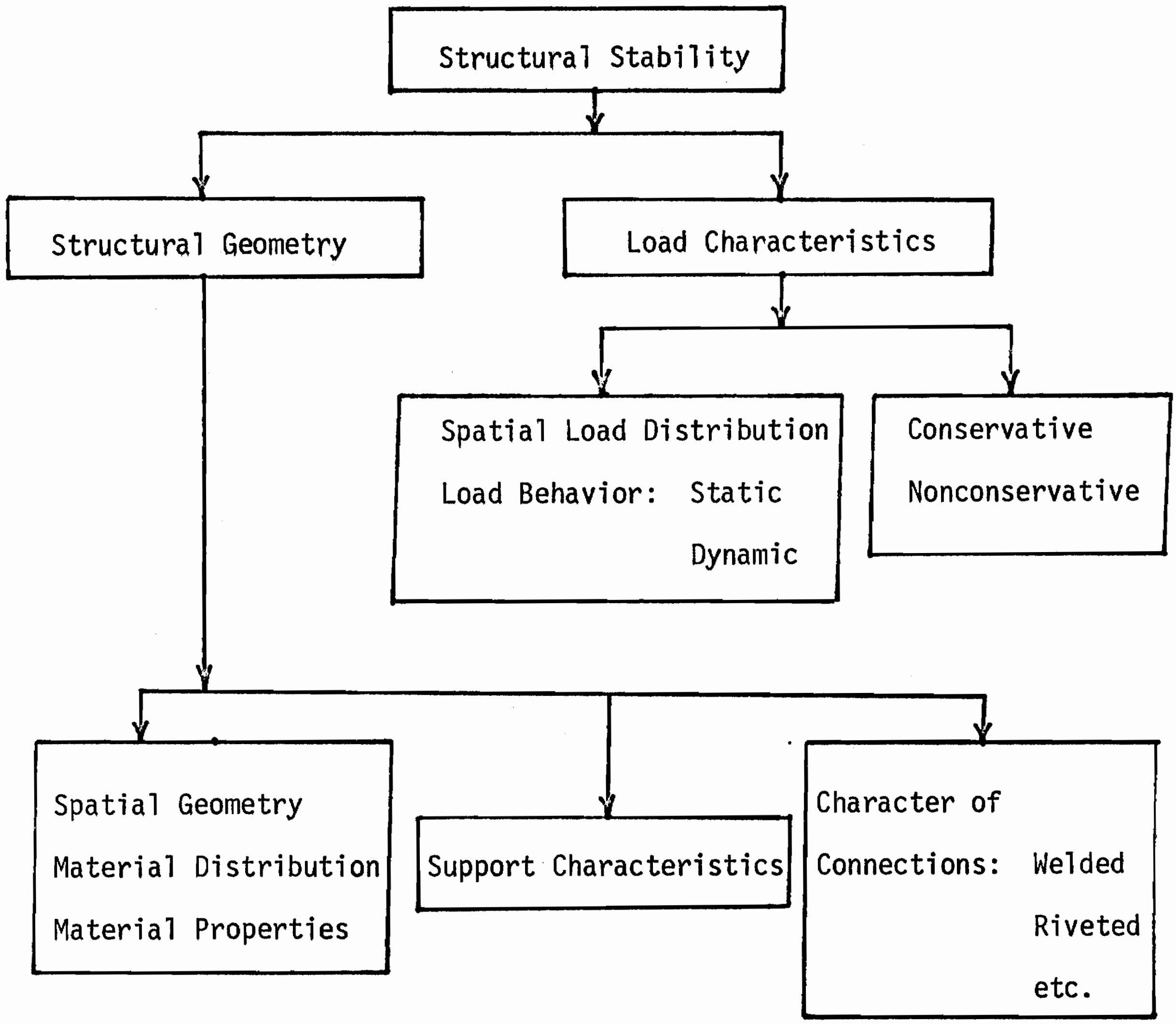


FIG 1.2 FACTORS AFFECTING STRUCTURAL STABILITY

static and dynamic loads.

a. Stability Concepts and Theories

Over two hundred years ago, Euler solved the first problems of elastic instability. He considered the lateral buckling of compressed members [116]. Euler's method permits the question of stability to be replaced by the question of bifurcation of the equilibrium modes [13]. Other eminent scientists such as Lagrange, Dirichlet, Bryan, and Poincaré clarified the connection between the Euler method and the stability concept [13]. Although the Euler method yields accurate results for the stability analysis of conservative elastic systems [116], stability analysis should be based on the theory of stability of motion [12]. The need to utilize dynamic analysis in stability investigations was stressed by Ziegler [122] and later by others [71, 58]. For conservative (divergence) type structures, a static approach to stability is permissible [13, 71] since nontrivial positions of equilibrium always determine the limit of stability. For flutter-prone structures [12, 58], the dynamic approach to stability is essential in order to detect the loss of stability by flutter. Published erroneous results (e.g. Pflüger's static stability analysis of an elastic cantilever rod under a follower load [13]) emphasize the limitations of static analyses in stability investigations.

A rigorous definition of stability of equilibrium (motion) is due to Liapunov [37], whose treatise on the theory of stability of motion (equilibrium) was published in Russia in 1892. Liapunov's direct method

[37] represents a powerful tool for the solution of stability problems and has been successfully used, e.g., [12, 43, 37, 82]. Recently, the method has been used to establish a rigorous basis for the concept of degree of stability of equilibrium of geometrically nonlinear discrete systems subjected to finite aperiodic disturbances [45]. It is the basis of a sufficient stability condition proposed by Hsu [50] and has appeared implicitly in other definitions of dynamic stability [e.g. 18, 54]. A review article [47] places the existing dynamic stability criteria for elastic imperfection-sensitive shells in the framework of Liapunov's theory of stability of motion (equilibrium).

Several other stability theories have been proposed in the literature. Extensive review of some of these theories appears in a paper by Hoff [41] and by several others [e.g. 19, 18, 36, 46]. In the particular case of shallow spherical shells [61, 87, 115], most of these theories are in disagreement with each other and with existing experimental data. Karman and Tsien [61] showed in 1939 that the large discrepancies between experiment and theory for shell structures were due to the highly-unstable post-buckling behavior of these structures. Since then, researchers in shell stability analysis have presented more accurate experimental and theoretical approaches to account for the discrepancies.

In 1945, Koiter, in a classic thesis [65], presented a general theory of stability for elastic systems by means of asymptotic expansion of the potential energy of the system. Koiter's theory received little attention until recently [66] when Thompson [106, 107], and Sewell [92, 94] developed the basic concepts of elastic stability for a discrete system described by N generalized coordinates and a variable loading parameter. These researchers in their theories expanded the potential energy function as a power series and conducted an extensive study relating to the occurrence of limit and bifurcation critical states. This "static perturbation method" [92] became a powerful mathematical tool in a wide class of stability investigations [e.g. 93, 96, 107, 108, 110, 114, 59].

b. Solution Techniques in Stability Analysis

The solution techniques available in structural stability analysis are mostly numerical and/or approximate. This is because the analysis often involves systems of nonlinear equations with non-available exact solutions. Common solution techniques include the Newton-Raphson method [29], the Finite Element Method [68, 79, 80]. Nonlinear Programming [33, 73] and Static Perturbation Methods [92, 109]. The relative advantages and disadvantages of these methods are discussed by Sticklin, et. al. [99] and Oden [80].

The static perturbation method has received considerable attention lately in obtaining post-buckling response and measures of imperfection-sensitivity of structural models. Application of the method is reviewed by Thompson [109]. Walker [117] proposed the technique as a general method of solution for nonlinear simultaneous equations. Various perturbation methods are available in text books [10, 78]. Perturbation approaches have been used in various stability analyses both in the continuum context [2, 77, 85, 98, 108] and in the discrete context [114, 59]. A combined approach using the static perturbation method with other nonlinear analysis techniques appears in recent papers. Techniques used recently in conjunction with the perturbation method include the Newton-Raphson method [117], the Finite Element Method [79], and Nonlinear Programming Techniques [118].

For dynamic stability analysis, Liapunov's direct method [37] represents a consistent approach. This method provides a safe estimate of the domain of asymptotic stability of equilibrium. It can be used to study motion of the system "in the large" [75]. Liapunov's direct method does not require the solution of the differential equations for an investigation of system stability [75]. Its main drawback is the lack of general methods for the construction of Liapunov functions. A Liapunov-type stability analysis provides criteria which are lower bounds on the regions of stability [45, 75, 37]. For conservative systems, the total energy represents a suitable Liapunov function [45].

c. Stability of Reticulated Space Structures

A reticulated space structure is an assemblage of one-dimensional elements interconnected at a finite number of points [46]. The points of connections are structural joints. Because of their three-dimensional rigidity, flexibility and economy of construction, erection, and fabrication, reticulated space structures are widely accepted in the building industry [24]. However, because of their relative lightness in comparison with continuous solid shells, reticulated space structures pose fundamental stability problems under the influence of static and dynamic loads. The stability problems which arise in reticulated space structures are of three types: bar stability (Euler critical load), snap-through buckling, and general buckling [46, 104]. Hence, under increasing external load, the change in the geometry of these structures becomes significant. Therefore, the structure exhibits a nonlinear load-deflection relationship [22].

In reticulated shells, local buckling leads to general buckling because the shell cannot resist shear loads caused by local buckling, and hence the area of buckling increases [24]. An extensive review of the stability of reticulated space structures is given by Holzer and Buchert [46].

In 1963, a large reticulated dome of 93.5 meters span, constructed in Bucharest as the roof structure of the National Economy Pavilion, suffered catastrophic failure after a fall of approximately one meter of fresh snow. This alerted practicing engineers and researchers to the importance of buckling problems of reticulated shell structures. Beles, et. al. [9] and Wright [119] examined the failure of the Bucharest dome and presented criteria for the buckling of reticulated shells using continuous, solid shell analogies. Other related papers appeared in a book on Space Structures [5].

The post-buckling behavior of reticulated shells is often studied by means of Koiter's general post-buckling theory [66]. This theory provides the possibility of determining the sensitivity of a structure to initial imperfections which may cause a great reduction in the buckling load [19]. Application of Koiter's theory to reticulated shells leads to a system of linear equations for the pre-buckling state [104]. At the buckling state, a linear eigenvalue problem is solved. Recent analyses have shown [63] that the assumption of a linear pre-buckling state may lead to inaccurate results. Perturbation techniques have shown excellent results in solving eigenvalue problems with non-linear pre-buckling deformations [117, 118].

In spite of many studies on linear eigenvalue problems of discrete systems [e.g., 104, 90], there are relatively few publications on non-linear eigenvalue analyses [e.g., 6]. The basic difficulty lies in solving nonlinear simultaneous algebraic equations with large numbers of unknowns. Although the 'equivalent' continuous shell analogy for reticulated shell analysis has been developed and utilized in many practical designs [24, 46], one question still remains unanswered: to what extent can these analogies represent the true nature of general reticulated shells of arbitrary composition? Also, it is doubtful if these analogical concepts simulate the behavior of the original reticu-lated shell for localized large deflections common in the buckling of shells [38].

The present study addresses itself to the assessment of degree of stability of equilibrium [45] of reticulated domes under multiple static and dynamic loads. None of the studies cited above included the effect of several independent loads. Multiple load considerations are neces-sary for consideration of symmetrical as well as unsymmetrical load distribution on the dome. Unsymmetrical loads account for wind, snow, and special cases.

In Chapter 2, the basic theorems and definitions used in the sequel are discussed. In Chapter 3, the solution techniques applied are discussed and analyzed in detail with special emphasis on the static perturbation method. Chapter 4 presents the error estimates and convergence criteria in a static perturbation scheme. In Chapter 5, a detailed account of the application of the theory to the stability analysis of geodesic domes is given. Chapter 6 presents the conclusions of the present work.

CHAPTER 2

STABILITY DEFINITIONS, THEOREMS, AND PRINCIPLES

The basic stability definitions, theorems, and principles pertinent to the present study are stated. Their implications and limitations with respect to the model used are discussed where necessary. The discussions on the concept of degree of stability follows those of references 44, 45, and 48. The practical implication of prior knowledge of convexity or concavity of stability boundaries in the case of multiple loads is stated.

2.1 Basic Principles and Theorems

Static and dynamic stability principles and theorems applicable to discrete systems are stated in this section. The discrete system is defined by a total potential function, $V(Q_i, \Lambda^j)$, and kinetic energy function, $T(Q_i, \dot{Q}_i, \Lambda^j)$; $i = 1, 2, \ldots N$, $j = 1, 2, \ldots M$. Q_i, Λ^j are the generalized coordinates (displacements), generalized independent external loads, respectively; N, M are the number of system degrees of freedom, number of generalized independent loads, respectively. At least in the region of interest, V and T are assumed to be single valued and continuous functions of the Q_i; also $\partial V/\partial Q_i$, $\partial T/\partial Q_i$ and $\partial T/\partial \dot{Q}_i$ are assumed continuous in the same region. The system is holonomic and scleronomic, since there are no constraints on the variations of the coordinates, and V and T do not contain time explicity [114].

The energy criterion and the principles of virtual work form the basis of the static stability analysis. In this regard, the following principle is stated [75].

> Virtual Work Principle: A mechanical system is in a state of equilibrium if, and only if, the virtual work of all forces (external and internal) vanishes for any virtual displacement.

This is a necessary and sufficient condition. It implies that V is stationary with respect to every kinematically admissible displacement, Q_i, from a state of equilibrium.

A Theorem, due to Lagrange, relating Liapunov's dynamical definition of stability [37] to the extremum properties of V is now stated.

> Theorem 1: A complete relative minimum of the total potential energy with respect to the generalized coordinates is both necessary and sufficient for stability of equilibrium.

Lagrange's Theorem has not been proved in complete generality for the system under study [114, 59]. By introducing damping with a positive definite energy dissipation, Koiter established a general theorem [59].

> Theorem 2: An equilibrium configuration in which the potential energy has no proper relative minimum is always unstable in the presence of damping with a positive definite energy dissipation.

In view of the presence of damping in actual physical systems, Theorem

2 is adequate for practical purposes [59]. Koiter [65, 66] in his studies on post-buckling behavior provided another theorem.

> Theorem 3: The initial stage of post-critical behavior is completely specified by the stability or instability at the critical point itself.

Theorem 3 is used in determining the sensitivity of a system to imperfections. If the critical point is unstable, the associated system is imperfection-sensitive [47, 25]; i.e., the critical load of the perfect system provides an upper bound to that of the imperfect system.

Two fundamental theorems, due to Thompson [112], concerning the stability of equilibrium beyond bifurcation points are now stated.

> Theorem 4: An initially stable (fundamental) equilibrium path rising monotonically with the loading parameter cannot become (thoroughly) unstable without intersecting a second distinct (post-buckling) equilibrium path.

> Theorem 5: An initially stable (fundamental) equilibrium path rising monotonically with the loading parameter cannot approach an unstable critical equilibrium state (from which the system would snap dynamically) without the approach of a second distinct (post-buckling) equilibrium path at sub-critical values of the

loading parameter.

Topological and analytical proofs of Theorems 4 and 5 for a one degree-of-freedom system are given in [112]. Coincident critical points are excluded in the proofs.

For the discrete system under dynamic loads, Liapunov's definitions of stability, asymptotic stability, and instability of equilibrium are applicable. Let a state vector, $y(t)$, be defined as

$$y(t) = \begin{bmatrix} y_1(t) \\ \hline y_2(t) \end{bmatrix} \tag{2.1}$$

where

$$y_1(t) = [q_i(t)], \ y_2(t) = [q_i'(t)]; \tag{2.2}$$

i.e., $y_1(t)$, $y_2(t)$ are generalized displacement, velocity vectors, respectively, of the perturbed motion. The differential equation of perturbed motion [37] can be expressed in the 2N-dimensional phase-space as

$$y' = f(y) \tag{2.3}$$

Primes in Eqs. 2.2, 2.3 denote differentiation with respect to time. The vector f is assumed to be continuous in a domain D of the 2N-dimensional Euclidean-space, E_{2N}. Let the trivial equilibrium state be an isolated equilibrium point. Consider an initial disturbance y_0 applied at time t = o. The Euclidean norm

$$||y|| = \sqrt{y^T y} = \sqrt{y_1^T y_1 + y_2^T y_2} \tag{2.5}$$

provides a measure of the deviation of the initial state, y_o, and the resulting motion, y(t), from the equilibrium state. The following definitions [37, 75] apply to the resulting motion y(t).

Definition 1: The equilibrium state y = o is STABLE if for any $\varepsilon^* > o$, there exists a $\delta^*(\varepsilon^*) > o$ such that if $||y(o)|| < \delta^*$, then $||y(t)|| < \varepsilon^*$ for all $t \geq o$.

Definition 2: The equilibrium state y = o is ASYMPTOTICALLY STABLE if it is stable and if $\lim_{t\to\infty} ||y(t)|| = o$.

Definition 3: The equilibrium state y = o is UNSTABLE if there exists an $\varepsilon_1^* > o$ such that for arbitrarily small $\delta^* > o$, there is a motion $y(t)$ for which $||y(o)|| < \delta^*$ and $||y(t)|| > \varepsilon_1^*$ at some time t_1.

Definition 1 implies that the motion can be confined to the domain $||y|| < \varepsilon^*$ if the initial disturbance is restricted to the domain $||y|| < \delta^*$. Definition 2 is a stronger stability definition, and it implies that sufficiently small disturbances die out. While the concept of stability refers to all disturbances which are sufficiently small, in the case of instability, Definition 3, it is only necessary that there exist one particular disturbance which leads to an unstable behavior of the system [75].

Of the two distinct methods proposed by Liapunov for investigating stability of equilibrium (motion), Liapunov's direct method [37] is the most powerful. The direct method requires no approximations and the equations of motion need not be solved. However, it involves Liapunov functions, $L(y)$. For conservative systems, the total energy, E, represents a suitable Liapunov function. This follows from Definition 4 and Theorems 6-8 stated below.

Definition 4: In the region D of E_{2N}, the Liapunov function $L(y)$ is said to be

1. Positive Definite if

$$L(y) > o \text{ for all } y \neq o$$

and

$$L(y) = o \text{ at } y = o$$

2. Negative Definite if

$$L(y) < o \text{ for all } y \neq o$$

and

$$L(y) = o \text{ at } y = o$$

3. Positive Semi-definite if

$$L(y) \geq o \text{ for all } y$$

4. Negative Semi-definite if

$L(y) \leq o$ for all y.

The rate of change of L(y) during motion is denoted L'(y); i.e., $L'(y) = dL(y)/dt$. For conservative systems, L = E and $L'(y) = o$. The following theorems, stated without proof, form the basis of Liapunov's direct method [75].

Theorem 6: If there is a function L(y) such that L(y) is positive definite and L'(y) is negative semi-definite, then the equilibrium state y = o is STABLE

Theorem 7: If there is a function L(y) such that L(y) is positive definite and L'(y) is negative definite, then the equilibrium state y = o is ASYMPTOTICALLY STABLE

Theorem 8: If there is a function L(y) such that $L(y_o) > o$ for some y_o with arbitrarily small norm and L'(y) is positive definite, then the equilibrium state y = o is UNSTABLE

To apply these theorems necessitates finding a suitable Liapunov function, L(y). The lack of general methods of constructing functions L(y) and the lack of uniqueness of such functions are the major drawbacks of the direct method. However, the solutions, y(t), are not required. What is needed is L'(y) which comes directly from the equations of motion. Liapunov's theorems provide estimates for the region of asymptotic stability, the domain of attraction of the equilibrium state whose stability is in question. In [45], the theorem on extent of asymptotic stability of Liapunov's direct method was used to derive a subdomain of asymptotic stability for a discrete system. This region is identical to that obtained by Hsu [50]. Geometric interpretations [75, 37] of Theorems 6-8 are given in Fig. 2.1.

Liapunov's theorems permit a study of stability of equilibrium relative to finite disturbances. Hence, if the disturbances which are likely to occur during the life of a system do not cause the perturbed motion to escape the domain of asymptotic stability, equilibrium is stable relative to these disturbances [44, 45]. This is the notion of PRACTICAL STABILITY. This notion is different from that of INFINITESIMAL STABILITY where only stability of equilibrium relative to infinitesimal disturbances is considered. Classical stability analyses [116, 122] ensure only infinitesimal stability.

2.2 Degree of Stability

The discussion of the concept of degree of stability in this section follows that of [44, 45] directly. The degree of stability of equilibrium [45] is a measure of the disturbances an equilibrium state can sustain; i.e., the system continues to perform its intended function. The concept of degree of stability is particularly important in the design of imperfection-sensitive systems, e.g., reticulated domes.

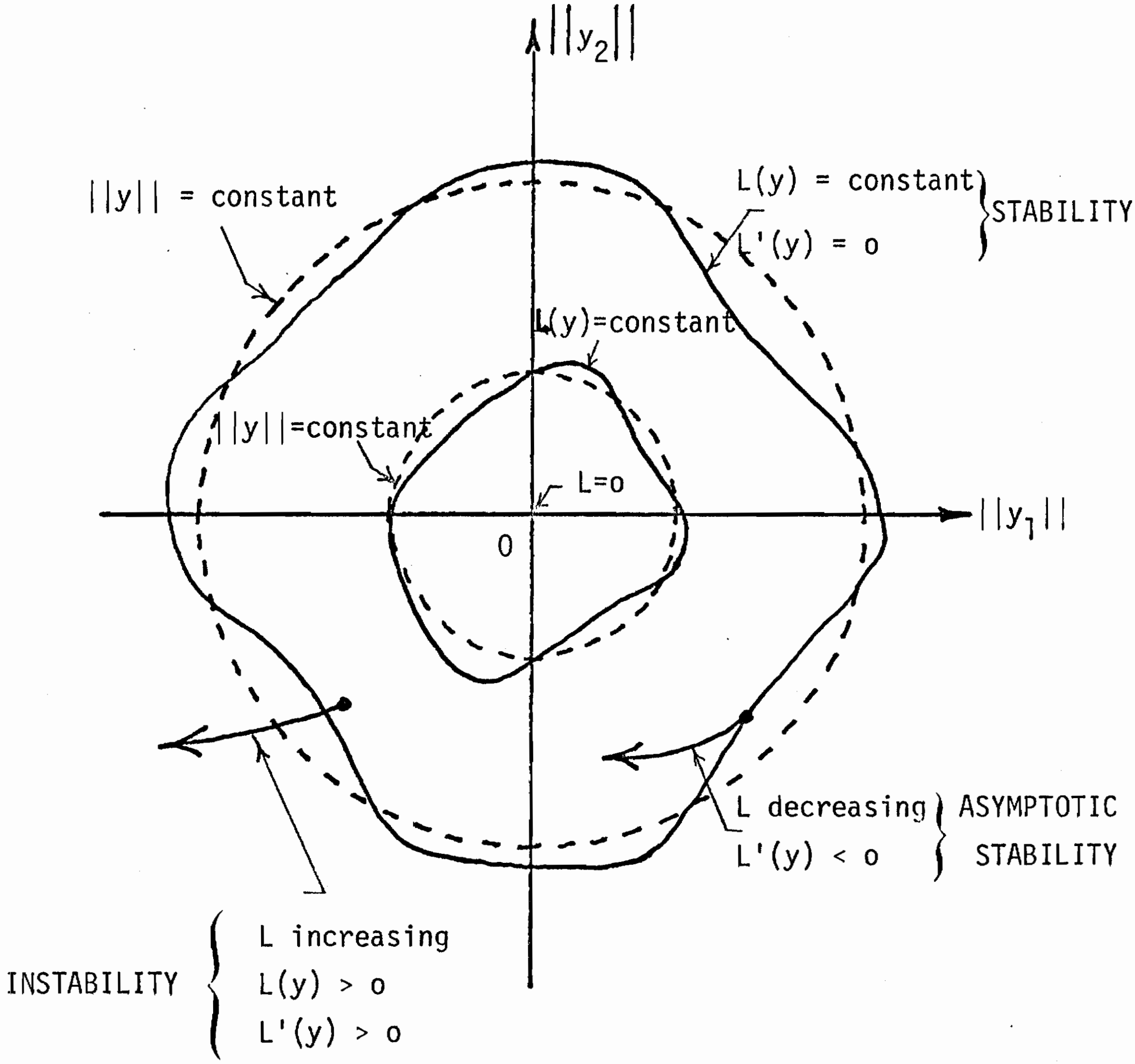


FIG. 2.1 GEOMETRIC INTERPRETATION OF LIAPUNOV'S THEOREMS

This is because imperfection-sensitive systems exhibit multi-equilibrium states below the buckling load, and sufficiently large disturbances can force the system from one equilibrium state to another [45]. It is therefore necessary to know (or at least obtain a safe estimate of) the degree of stability of equilibrium for imperfection-sensitive systems. With such knowledge, imperfection-sensitive systems can be designed for practical stability. In [45], the size of the region of asymptotic stability is identified as a direct measure of degree of stability.

Suppose the origin of the phase-space defined by Eq. 2.3 is asymptotically stable. The finite stability of the origin is in question. Let C_M be a subset of the region of asymptotic stability of the origin [45] having the property

$$C_M = \{y|v(y)<M\}. \tag{2.6}$$

In Eq. 2.6, v(y) is the total energy of the system which is positive definite in C_M; and M is the change in the potential energy between the stable origin and the "nearest" unstable equilibrium state, y_1^u. Mathematically

$$M = v(y_1^u, o). \tag{2.7}$$

Hence, the origin is stable with respect to a specific disturbance if the resulting motion initiates in C_M and the total energy along this motion does not reach the value M. Analytically, the conditions

$$y_o \in C_M \tag{2.8}$$

and

$$v(y) < M; \; o < t \leq t_d \tag{2.9}$$

are sufficient for the stability of the origin. In Eqs. 2.8, 2.9, y_o and y are the initial state and the resulting motion, respectively; t_d denotes the duration of the disturbance. Instantaneous disturbances are defined in terms of initial conditions, and transient disturbances are expressed in terms of exciting forces of finite duration. Eqs. 2.8-2.9 form the basis of a sufficient stability condition proposed by Hsu [50] and it was shown [44] that other definitions of dynamic stability [e.g. 54, 18] are related to it. An application of this sufficient stability condition to the stability of equilibrium of reticulated domes under multiple static and dynamic loads is presented in Chapter 5.

2.3 Stability Boundaries - Multiple Loads

The concept of stability boundaries for discrete systems under several independent external loads is discussed in this section. The practical implications of prior knowledge of the shapes of the stability boundaries for such systems are mentioned.

In classical mechanics of discrete finite systems [114], the external loading on a structural model is represented by a single variable parameter. Thus, the primary target of the stability analysis is the determination of the critical value of this parameter. Such a representation occasionally breaks down [56]. The non-classical studies of Huseyin on problems of multiple loading, which appeared in his recent book [59], emphasize the need to consider several independent loading parameters. The following discussion follows that in reference 59.

Consider the system defined in section 2.1 with the potential function $V(Q_i, \Lambda^j)$. For the system under the independent loads Λ^j, the location of the "stability boundary" corresponds to the location of the critical value of the single variable parameter used in the classical theory. The "stability boundary" is the locus of all critical points associated with an initial loss of stability in the load-space [56]. The load-space is the M-dimensional subspace of the (N+M) dimensional configuration space. The points of the load-space which correspond to states of critical equilibrium constitute the critical surface [107, 93]. The stability boundary refers to portions of the critical surface associated with an initial loss of stability. Hence, the projections of the critical zone (of the configuration space) onto the load space constitute the stability boundary. This is illustrated in Fig. 2.2.

The analysis of a discrete system subjected to independent external loads Λ^j is simplified by the consideration of paths on the equilibrium surface (Fig. 2.2) emerging from an unloaded state and corresponding to the rays [59]

$$\Lambda^j = \ell^j \lambda. \tag{2.10}$$

In Eq. 2.10, ℓ^j are direction cosines and λ is a loading parameter as

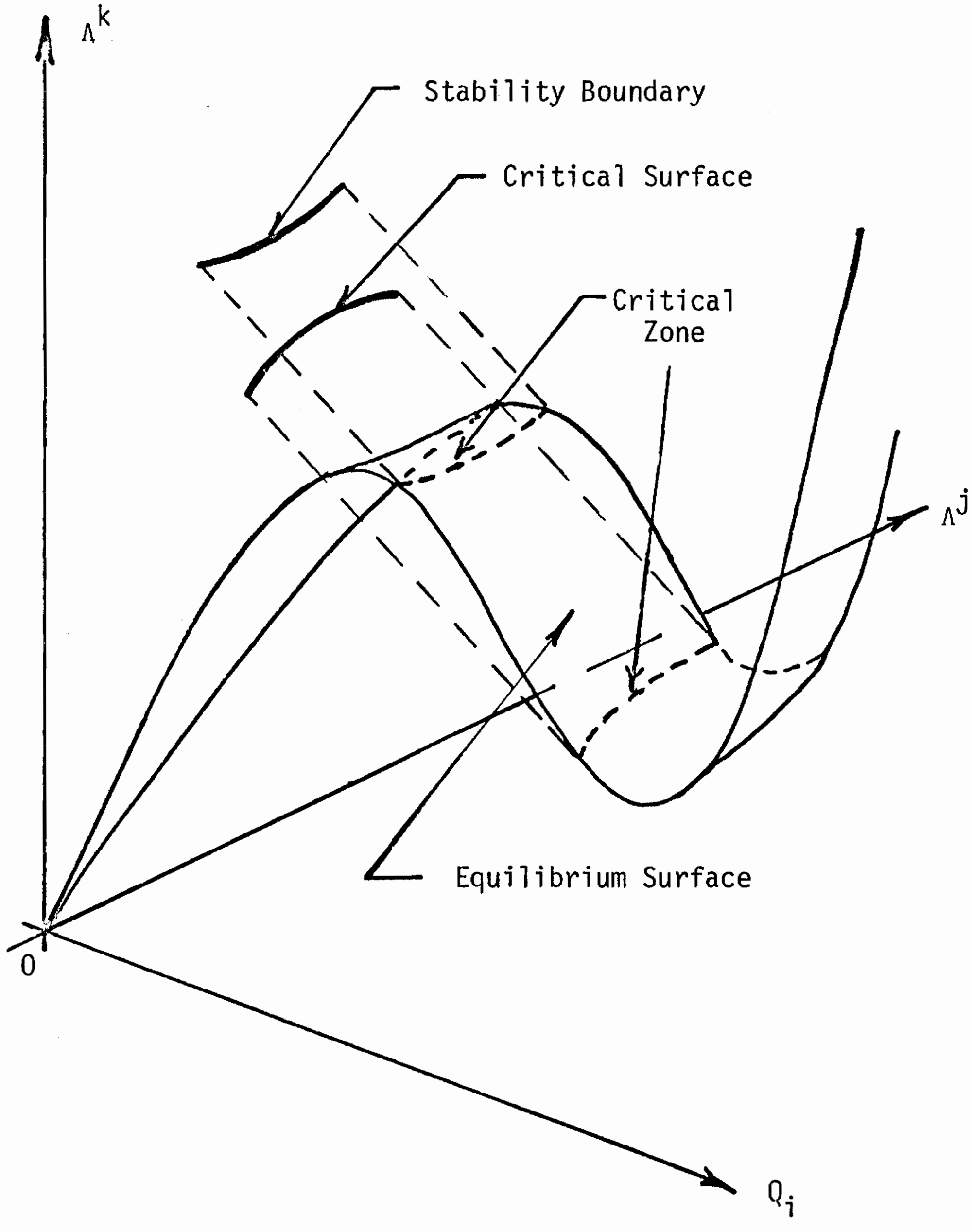


FIG. 2.2 STABILITY BOUNDARY AND EQUILIBRIUM SURFACE

shown in Fig. 2.3.

The concavity or convexity (i.e., the shapes) of the stability boundaries have practical implications. For example, if the stability boundary is concave (as in Fig. 2.4a) for two independent loads, Λ^j and Λ^k, then BA represents a lower bound, while lines BF* and AF* represent upper bounds on the region of stability. BF* and AF* are tangents to the boundary curve BA at B and A, respectively. The critical loads (points A and B) on the axes can be obtained without much computational effort since their location does not involve load interaction. Hence, at least a lower bound can be established immediately if the stability boundary for the system is known a priori to be concave towards the origin, O. Similarly, if the stability boundary is convex towards O, Fig. 2.4b, the line BA represents an upper bound on the region of stability. Again, point F* defining the lower bounds AF* and BF* can be obtained as in Fig. 2.4a. Fig. 2.4c depicts the case of a singular critical point F [59]. Singular points are associated with unstable symmetric points of bifurcation occurring along the loading ray of Fig. 2.3. In this case, the stability boundary takes the form of a cusp (point F, Fig. 2.4c). By joining F to the critical points A and B on the axes, the upper bounds AF and BF on the region of stability are obtained. The line BA represents a lower bound. A safe upper bound is defined by BF* and AF*, where F* connects tangents at B, A to the stability boundary curves BF, AF.

The location of the stability boundary poses serious fundamental difficulties for geometrically nonlinear systems [59]. However, for

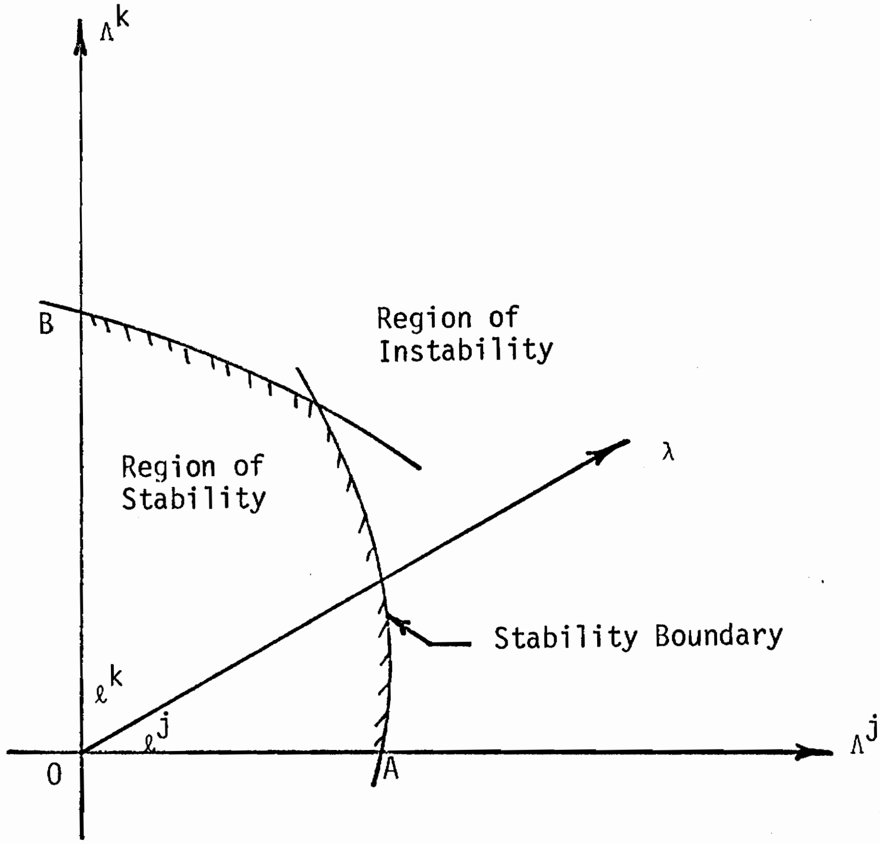


FIG. 2.3 LOADING RAY λ IN LOAD-SPACE

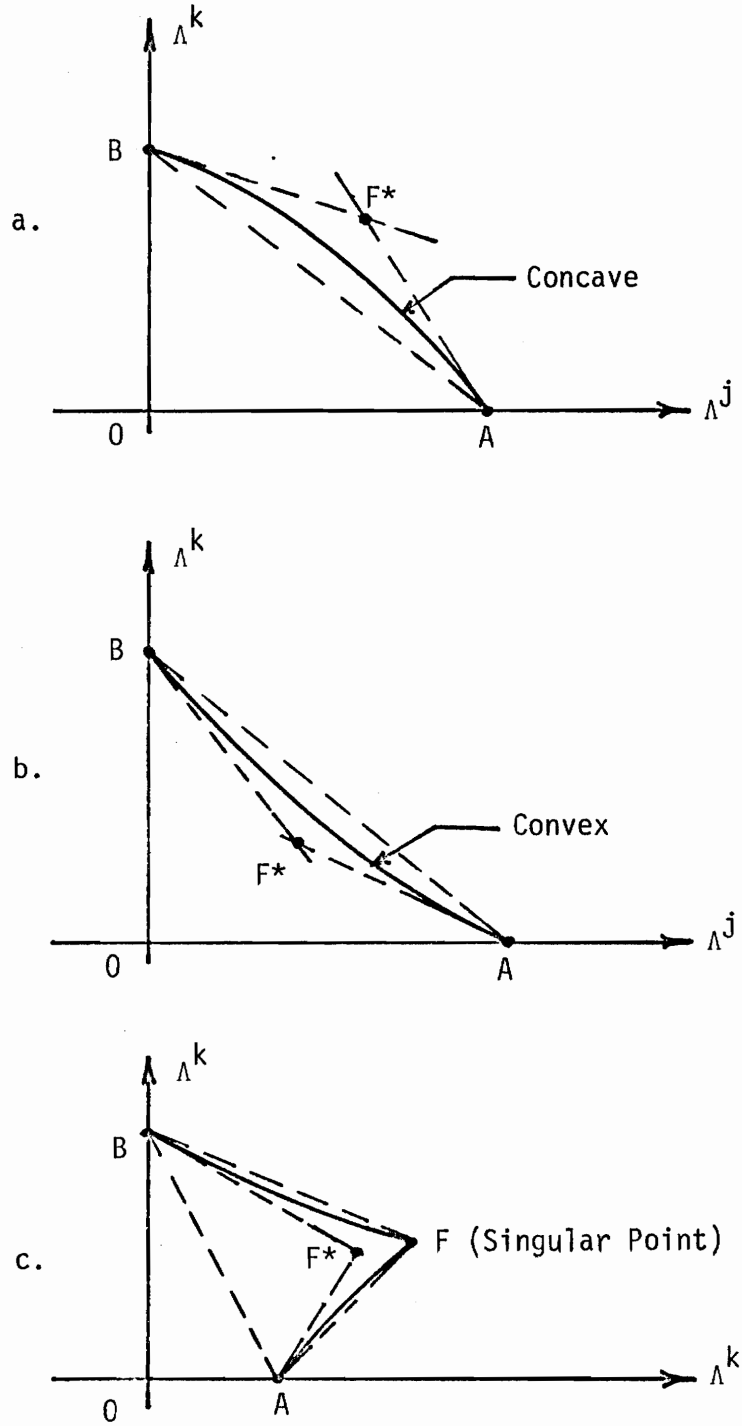


FIG. 2.4 CONVEXITY AND CONCAVITY OF STABILITY BOUNDARIES

some systems, estimates of stability boundaries exist [57], and some theorems have been proposed [59] for these systems.

Stability boundaries for reticulated dome models under two independent external loads are presented in Chapter 5.

CHAPTER 3

SOLUTION TECHNIQUES

The discrete perturbation, nonlinear programming, and dynamic analysis techniques used for obtaining the response of the reticulated dome model to static and dynamic loads are described in this chapter. Special attention is given to the discrete (static) perturbation technique, the key solution process for the determination of the nonlinear response to static loads. A non-diagonalized perturbation scheme [114] is used, in which the generalized displacements are selected as generalized coordinates. This scheme is compared with Thompson's [110, 114], where a set of "sliding" coordinates is introduced as generalized coordinates.

3.1 Discrete Perturbation Method

In this section, the concepts underlying the discrete perturbation technique are introduced. A general analysis is given, and the modifications necessary to obtain response paths beyond critical equilibrium points are described in detail.

a. Introduction

The discrete (static) perturbation method [92] is a parameter perturbation technique. In this technique, the solution is represented by the first few terms of an asymptotic expansion. It is analogous to the continuum perturbation technique [78, 65]. For the elastica problem, it was shown [108] that the contraction of the equilibrium equations in the discrete perturbation method is equivalent to the removal of the secular terms (sources of nonuniformity) in the corresponding continuum analysis. In the discrete perturbation method (Fig. 3.1), the continuum potential function is approximated (discretized) in terms of "appropriate" generalized coordinates. The generalized coordinates reflect response characteristics of the system. Stationarity conditions applied to the approximate total potential function yield a series of nonlinear algebraic equations. A perturbation analysis in terms of a "suitable" parameter is then conducted to obtain the solution to the set of nonlinear algebraic equations. The result is a series of linear, ordered perturbation equations which can be solved recursively. Such an analysis provides the locations of equilibrium points satisfying the nonlinear differential equations of equilibrium. Hence, no information is obtained regarding the stability of the equilibrium states directly. However, the properties of the obtained equilibrium states (e.g., generalized displacements, etc.) can be used in a stability analysis based on higher variations (usually the second variation) of the total potential energy of the system. When the total potential energy assumes a relative minimum, Lagrange's theorem [69] states that the equilibrium state is stable.

b. General Analysis

A general discrete perturbation analysis is presented in this section. The presentation follows the works of Sewell [92, 94], Thompson [114], and Huseyin [56].

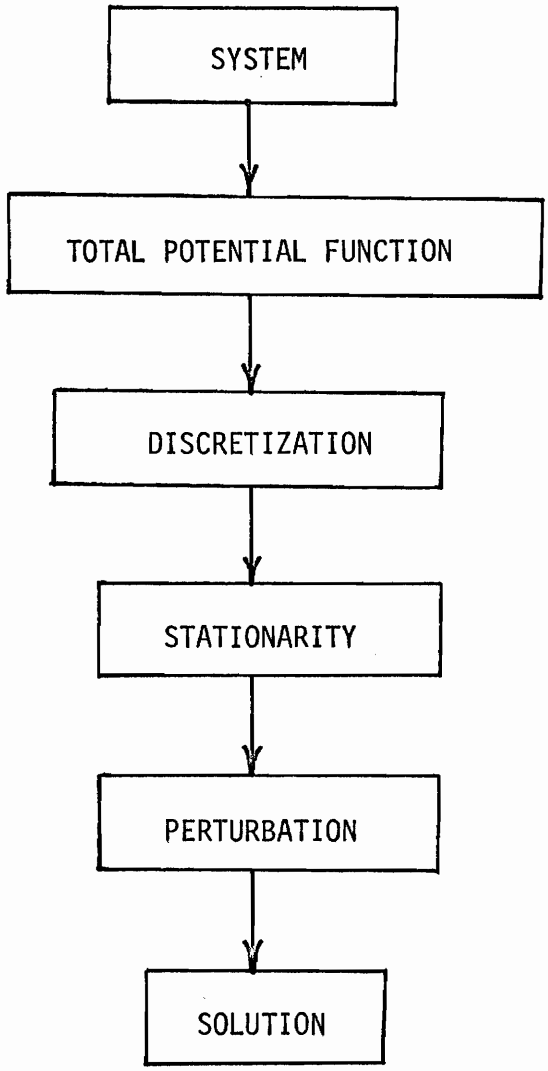


FIG. 3.1 DISCRETE PERTURBATION METHOD

Consider the discrete, conservative system defined in section 2.1. It is assumed that the equilibrium point (Q_i, Λ^j) in configuration space (Fig. 3.2) is known. Using the static perturbation technique, the location of a neighboring equilibrium point $(Q_i+q_i, \Lambda^j+\lambda^j)$ is sought. The search is conducted by considering incremental perturbations q_i in the generalized displacements, Q_i, and λ^j in the independent loads, Λ^j. The equilibrium equations for the known point are

$$W_s(Q_i, \Lambda^j) = o; \quad \left.\begin{array}{l} i = 1, 2, .., N \\ j = 1, 2, .., M \\ s = 1, 2, .., N \end{array}\right\} \tag{3.1}$$

where

$$W_s = \frac{\partial V}{\partial Q_s} \tag{3.2}$$

Correspondingly, at $(Q_i+q_i, \Lambda^j+\lambda^j)$, the equilibrium equations

$$W_s(Q_i+q_i, \Lambda^j+\lambda^j) = o \tag{3.3}$$

are satisfied. In Eqs. 3.1-3.3, N and M are the system degrees of freedom and number of independent loads, respectively, and V is the total potential energy function. V satisfies all properties defined in section 2.1.

The concept of a loading ray (section 2.3) in the M-dimensional load space with η representing the distance along the ray from Λ^j to $\Lambda^j + \lambda^j$ leads to the relation

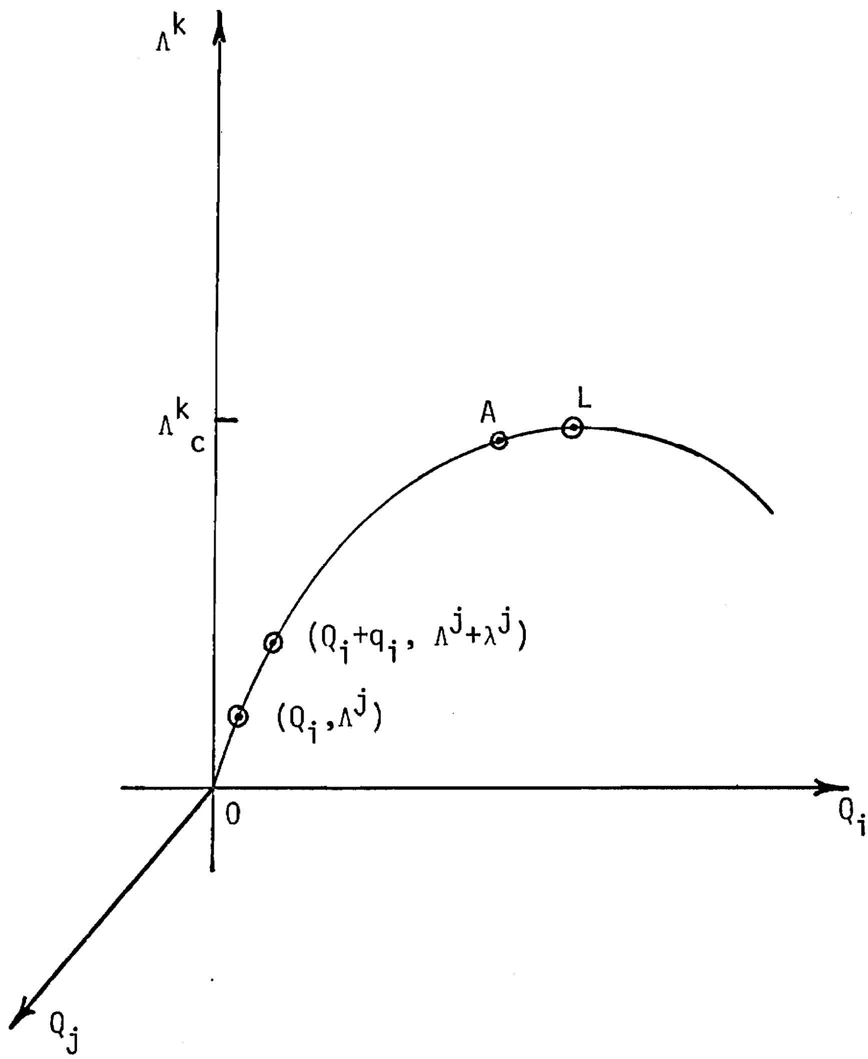


FIG. 3.2 EQUILIBRIUM PATH

$$\lambda^j = \ell^j \eta \tag{3.4a}$$

for a path with proportional loading (Fig. 2.3). From Eqs. 2.10 and 3.4a, one obtains

$$\Lambda^j + \lambda^j = \ell^j (\lambda + \eta) \tag{3.4b}$$

Eq. 3.4b transforms Eqs. 3.3 into the form

$$W_s[Q_i + q_i, \ell^j(\lambda + \eta)] = 0 \tag{3.5}$$

which is the equilibrium equation for the desired equilibrium state defined by parameters q_i and η.

Expanding Eqs. 3.3 about (Q_i, Λ^j) in a Taylor series expansion with increments (q_i, η) gives

$$\begin{aligned}
W_s(Q_i + q_i, \Lambda^j + \lambda^j) &= W_s(Q_i, \Lambda^j) + (W_{s,i}q_i + W_{s,\lambda}\eta) \\
&+ \frac{1}{2!}(W_{s,ij}q_iq_j + 2W_{s,i\lambda}q_i\eta + W_{s,\lambda\lambda}\eta^2) \\
&+ \frac{1}{3!}(W_{s,ijk}q_iq_jq_k + 3W_{s,ij\lambda}q_iq_j\eta \\
&+ 3W_{s,i\lambda\lambda}q_i\eta^2 + W_{s,\lambda\lambda\lambda}\eta^3) \\
&+ O[q_i^4, \eta^4].
\end{aligned} \tag{3.6a}$$

where

$$\left.\begin{aligned} W_{s,i} &= \frac{\partial W_s}{\partial Q_i} \\ W_{s,\lambda} &= \frac{\partial W_s}{\partial \lambda} \\ W_{s,i\lambda} &= \frac{\partial^2 W_s}{\partial Q_i \partial \lambda} \quad ; \; \cdots\cdots \end{aligned}\right\} \qquad (3.6b)$$

All derivatives of W_s are evaluated at $q_i = o$, $\lambda = o$ in Eqs. 3.6 and thereafter; summation convention is utilized. The symbol O in Eqs. 3.6a represents an error term of order q_i^4 or η^4. It is Landau's error symbol [78]. Eqs. 3.1 and 3.3 together with Eqs. 3.6 lead to the following set of nonlinear differential equations

$$\begin{aligned} &(W_{s,i}q_i + W_{s,\lambda}\eta) \\ &+ \frac{1}{2!}(W_{s,ij}q_iq_j + 2W_{s,i\lambda}q_i\eta + W_{s,\lambda\lambda}\eta^2) \\ &+ \frac{1}{3!}(W_{s,ijk}q_iq_jq_k + 3W_{s,ij\lambda}q_iq_j\eta \\ &+ 3W_{s,i\lambda\lambda}q_i\eta^2 + W_{s,\lambda\lambda\lambda}\eta^3) + O[q_i^4, \eta^4] = o. \end{aligned} \qquad (3.7)$$

The simultaneous solution of Eqs. 3.7 can be described in parametric form by the single-valued functions

$$q_i(\varepsilon) = \varepsilon \dot{q}_i + \frac{1}{2!}\, \varepsilon^2 \ddot{q}_i + \frac{1}{3!}\, \varepsilon^3 \dddot{q}_i + O[\varepsilon^4] \tag{3.8}$$

$$\eta(\varepsilon) = \varepsilon\, \dot{\eta} + \frac{1}{2!}\, \varepsilon^2 \ddot{\eta} + \frac{1}{3!}\, \varepsilon^3 \dddot{\eta} + O[\varepsilon^4]. \tag{3.9}$$

In Eqs. 3.8 and 3.9, ε is a "suitable" perturbation parameter, and dots denote differentiation with respect to ε; i.e.

$$\dot{q}_i = \frac{\partial q_i}{\partial \varepsilon},\ \ddot{\eta} = \frac{\partial^2 \eta}{\partial \varepsilon^2},\ \text{etc.} \tag{3.10}$$

The selection of a "suitable" perturbation parameter is discussed in section 4.3. The known state (Q_i, Λ^j) corresponds to $\varepsilon = o$.

There are two fundamental assumptions [92] in the Taylor series representations defined by Eqs. 3.8 and 3.9. First, a non-zero radius of convergence is assumed (for $\varepsilon > o$) to rule out the possibility of converging to the known state (Q_i, Λ^j). Secondly, the parametric representations $q_i = q_i(\varepsilon)$, $\eta = \eta(\varepsilon)$ are assumed to be such that not all $\dot{q}_i$, $\dot{\eta}$ vanish simultaneously at $\varepsilon = o$.

Eqs. 3.8 and 3.9 transform Eqs. 3.7 into

$$\varepsilon\alpha_1 + \varepsilon^2\alpha_2 + \varepsilon^3\alpha_3 + O[\varepsilon^4] = o \tag{3.11}$$

where

$$\alpha_i = \alpha_i(\dot{q}_k, \dot{\eta}, \ldots); \; i = 1, 2, 3. \tag{3.12}$$

These equations must hold for all values of ε. Since sequences of ε are linearly independent, each coefficient $\alpha_i \neq \alpha_i(\varepsilon)$ must vanish independently. Hence, the following linear, ordered perturbation equations are obtained:

$$\varepsilon^1: \quad W_{s,i}\dot{q}_i + W_{s,\lambda}\dot{\eta} = o \tag{3.13}$$

$$\varepsilon^2: \quad W_{s,i}\ddot{q}_i + W_{s,\lambda}\ddot{\eta} + W_{s,ij}\dot{q}_i\dot{q}_j + 2W_{s,i\lambda}\dot{q}_i\dot{\eta} + W_{s,\lambda\lambda}\dot{\eta}^2 = o \tag{3.14}$$

$$\begin{aligned} \varepsilon^3: \quad & W_{s,i}\dddot{q}_i + W_{s,\lambda}\dddot{\eta} + \frac{3}{2} W_{s,ij}(\dot{q}_i\ddot{q}_j + \ddot{q}_i\dot{q}_j) \\ & + 3W_{s,i\lambda}(\dot{q}_i\ddot{\eta} + \ddot{q}_i\dot{\eta}) + 3W_{s,\lambda\lambda}\dot{\eta}\ddot{\eta} \\ & + W_{s,ijk}\dot{q}_i\dot{q}_j\dot{q}_k + 3W_{s,ij\lambda}\dot{q}_i\dot{q}_j\dot{\eta} \\ & + 3W_{s,i\lambda\lambda}\dot{q}_i\dot{\eta}^2 + W_{s,\lambda\lambda\lambda}\dot{\eta}^3 = o \end{aligned} \tag{3.15}$$

where $\varepsilon^k (k = 1, 2, 3)$ is the k-th order perturbation equation.

In many problems of practical interest, the total potential energy is a linear function of the loading parameters [114]*. In such a case,

---

* This is particularly true when Λ^j represent external loads only. If the structural model is acted upon by a nonlinearly elastic loading device, Eq. 3.16 takes the form [93]

$$V = \pi(Q_i) - \int_0^e \bar{\Lambda}^j(e,\Lambda^j)\, de \text{ where } e = e(Q_i) \text{ and } \bar{\Lambda}^j = \bar{\Lambda}^j(\Lambda^j)$$

V can be expressed as [114]

$$V = \pi(Q_i) - \Lambda^j E^j(Q_i). \tag{3.16}$$

In Eq. 3.16, π is the system strain energy and E^j are the deflections corresponding to the loads Λ^j, and E^j are assumed linear in displacements, Q_i. Eqs. 3.13-3.15 are therefore reduced to the following form:

$$\varepsilon^1: \qquad W_{s,i}\dot{q}_i + W_{s,\lambda}\dot{\eta} = o \tag{3.17}$$

$$\varepsilon^2: \qquad W_{s,i}\ddot{q}_i + W_{s,\lambda}\ddot{\eta} + W_{s,ij}\dot{q}_i\dot{q}_j = o \tag{3.18}$$

$$\varepsilon^3: \quad W_{s,i}\dddot{q}_i + W_{s,\lambda}\dddot{\eta} + \frac{3}{2} W_{s,ij}(\dot{q}_i\ddot{q}_j + \ddot{q}_i\dot{q}_j) + W_{s,ijk}\dot{q}_i\dot{q}_j\dot{q}_k = o \tag{3.19}$$

It follows from Eqs. 3.17-3.19 that the key solution is the first order perturbation solution. For example, $\ddot{q}_i$ and $\ddot{\eta}$ of Eqs. 3.18 can be defined completely once $\dot{q}_i$ and $\dot{\eta}$ are obtained from Eqs. 3.17. In general, the solution to the k-th order perturbation equation, ε^k, depends on solutions to the first (k-1) perturbation equations. Hence the new equilibrium state (Q_i+q_i, $\Lambda^j+\lambda^j$) is completely defined by the recursive solutions of Eqs. 3.17-3.19.

Eqs. 3.17 contain (N+1) unknowns, q_i and η. Since only N equations exist, one of the q_i's or η must be specified a priori. Such a specification reduces Eqs. 3.17 to N equations in N unknowns which can be solved simultaneously.

If one of the q_i's is selected as ε, the scheme is called displacement-increment perturbation procedure [92]. This is characterized by the equations

$$\left.\begin{array}{l} q_i = \varepsilon,\ \dot{q}_i = 1,\ \ddot{q}_i = \dddot{q}_i = \ldots = 0 \\[2ex] q_j = \varepsilon \dot{q}_j + \frac{\varepsilon^2}{2!}\ddot{q}_j + \frac{\varepsilon^3}{3!}\dddot{q}_j + O[\varepsilon^4];\ j \neq i \\[2ex] \eta = \varepsilon\dot{\eta} + \frac{\varepsilon^2}{2!}\ddot{\eta} + \frac{\varepsilon^3}{3!}\dddot{\eta} + O[\varepsilon^4]. \end{array}\right\} \qquad (3.20)$$

Similarly if η is selected as ε, the scheme is called load-increment perturbation procedure [92]. This is characterized by the equations

$$\left.\begin{array}{l} \eta = \varepsilon,\ \dot{\eta} = 1,\ \ddot{\eta} = \dddot{\eta} = \ldots = 0 \\[2ex] q_i = \varepsilon\dot{q}_i + \frac{\varepsilon^2}{2!}\ddot{q}_i + \frac{\varepsilon^3}{3!}\dddot{q}_i + O[\varepsilon^4] \end{array}\right\} \qquad (3.21)$$

The above general analysis assumes that only one equilibrium path leaves any equilibrium point. The analysis requires modifications in the neighborhood of critical points. The required perturbation analysis through various critical points is discussed in the next three sub-sections.

c. Perturbation Analysis Through a Limit Point

In this section, the required modification to Eqs. 3.13 to obtain the equilibrium path through a limit point is discussed.

At a limit point, L, the fundamental path [114] OA of Fig. 3.2a reaches a local extremum. The initial post-buckling path associated with the limit point is unstable and falling. Also the coefficient matrix of ε^1 in Eq. 3.13 vanishes [114]; i.e.,

$$\det\,(W_{s,i}) = o \tag{3.22}$$

The coefficient matrix $W_{s,i}$ is referred to as W_L in later discussions since it is associated with the limit point, L.

In addition to Eq. 3.22, the relation (using Eqs. 3.20)

$$\dot{\eta} = o \tag{3.23}$$

holds at the limit point. Hence, Eqs. 3.13 require modification to obtain a defined set of $\dot{q}_i$.

The perturbation analysis for the equlibrium path through a limit point can be established by using the displacement-increment perturbation scheme defined by Eqs. 3.20. The solutions to Eqs. 3.13 are sought by replacing the column corresponding to the "suitable" q_i with that of η. Such a transformation leads to Eqs. 3.24, i.e.,

$$\begin{bmatrix} W_{1,\lambda} & W_{1,2}\cdots W_{1,n} \\ W_{2,\lambda} & W_{2,2}\cdots W_{2,n} \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ W_{n,\lambda} & W_{n,2} \quad W_{n,n} \end{bmatrix} \begin{bmatrix} \dot{\eta} \\ \dot{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{q}_n \end{bmatrix} + \begin{bmatrix} W_{1,1} \\ W_{2,1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ W_{n,1} \end{bmatrix} \dot{q}_1 = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{3.24}$$

if q_1 is assummed to be the "suitable" q_i.

The coefficient matrix of Eqs. 3.24 is denoted W_B for later reference. Since W_B^{-1} exists at equilibrium points excluding a bifurcation point, a unique post-buckling path through the limit point is obtained using Eqs. 3.24. High order approximations are then obtainable from Eqs. 3.14, 3.15.

d. Perturbation Analysis Through a Bifurcation Point

The construction of equilibrium paths through a bifurcation point is presented in this section.

At a point of bifurcation, the fundamental path intersects a second, distinct equilibrium path (Theorems 4 and 5, section 2.1). At bifurcation points, the coeficient matrix of Eqs. 3.24, W_B, becomes singular. Hence, the perturbation scheme of section 3.1c breaks down at bifurcation points and cannot be used for obtaining post-buckling response beyond such points. Consequently, a non-diagonalized perturbation scheme [114] is used, in which the generalized displacements are selected as generalized coordinates. This is presented below.

Let the singularity expressed in Eqs. 3.22 be characterized by the vanishing of the smallest eigenvalue of W_L. This implies the existence of a non-zero minor determinant, det (W_C), where

$$W_C = \begin{bmatrix} W_{2,2} \cdots W_{2,n} \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ \qquad \cdot \\ W_{n,2} \cdots W_{n,n} \end{bmatrix} \tag{3.25}$$

Also, let q_1 be a "suitable" perturbation parameter ε. Hence, Eqs. 3.20 take the form:

$$\left.\begin{aligned} q_1 &= \varepsilon,\ \dot{q}_1 = 1,\ \ddot{q}_1 = \dddot{q}_1 = \ldots = 0 \\ q_i(q_1) &= q_1\dot{q}_i + \frac{1}{2!}q_1^2\,\ddot{q}_i + \frac{1}{3!}q_1^3\,\dddot{q}_i + O[q_1^4];\ i \neq 1 \\ \eta(q_1) &= q_1\dot{\eta} + \frac{1}{2!}q_1^2\,\ddot{\eta} + \frac{1}{3!}q_1^3\,\dddot{\eta} + O[q_1^4] \end{aligned}\right\} \tag{3.26}$$

Inserting Eqs. 3.26 into Eqs. 3.24, one obtains the following two equations:

$$W_{1,\lambda}\dot{\eta} + [W_{1,2}\cdots W_{1,n}]\begin{bmatrix}\dot{q}_2\\ \cdot\\ \cdot\\ \dot{q}_n\end{bmatrix} + W_{1,1} = 0 \tag{3.27}$$

and

$$\begin{bmatrix}W_{2,\lambda}\\ \cdot\\ \cdot\\ \cdot\\ W_{n,\lambda}\end{bmatrix}\dot{\eta} + \begin{bmatrix}W_{2,2}\cdots W_{2,n}\\ \cdot \quad \cdot\\ \cdot \quad \cdot\\ \cdot \quad \cdot\\ W_{n,2}\cdots W_{n,n}\end{bmatrix}\begin{bmatrix}\dot{q}_2\\ \cdot\\ \cdot\\ \cdot\\ \dot{q}_n\end{bmatrix} + \begin{bmatrix}W_{2,1}\\ \cdot\\ \cdot\\ \cdot\\ W_{n,1}\end{bmatrix} = \begin{bmatrix}0\\ \cdot\\ \cdot\\ \cdot\\ 0\end{bmatrix} \tag{3.28}$$

To simplify algebraic operations, the following symbolic representations are introduced:

$$a_{11} = W_{1,\lambda} \tag{3.29}$$

$$a_{12} = [W_{1,2} \cdots W_{1,n}] \tag{3.30}$$

$$a_{21} = \begin{bmatrix} W_{2,\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,\lambda} \end{bmatrix} \tag{3.31}$$

$$b_{11} = W_{1,1} \tag{3.32}$$

$$b_{21} = \begin{bmatrix} W_{2,1} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,1} \end{bmatrix} \tag{3.33}$$

These representations (Eqs. 3.29-3.33) and Eq. 3.25 transform Eqs. 3.28 into

$$\begin{bmatrix} \dot{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{q}_n \end{bmatrix} = - W_c^{-1} (a_{21} \dot{n} + b_{21}) \tag{3.34}$$

and together with Eqs. 3.27 lead to the relation:

$$\alpha\dot{\eta} + \beta = o \tag{3.35}$$

In Eqs. 3.35,

$$\alpha = a_{11} - a_{12}\, W_c^{-1}\, a_{21} \tag{3.36}$$

and

$$\beta = b_{11} - a_{12}\, W_c^{-1}\, b_{21} \tag{3.37}$$

But

$$\det\,(W_B) = o \tag{3.38}$$

at a bifurcation point. Hence, $\dot{\eta}$ is indefinite in both Eqs. 3.24 and 3.35. This indicates that in order for the bifurcation point to satisfy the first order perturbation equations, Eqs. 3.27 and 3.28, the relation

$$\alpha = \beta = o \tag{3.39}$$

must hold in Eqs. 3.35.

The proof of Eq. 3.39 can be established if a regular matrix [34] is introduced as follows:

$$R = \left[\begin{array}{c|c} 1 & -a_{12}\, W_c^{-1} \\ \hline 0 & I \end{array}\right] \tag{3.40}$$

such that

$$\det(R) = 1 \tag{3.41}$$

Pre-multiplying W_B in Eqs. 3.24 by R, one obtains

$$\left[\begin{array}{c|c} 1 & -a_{12} W_c^{-1} \\ \hline o & I \end{array}\right] \left[\begin{array}{c|c} a_{11} & a_{12} \\ \hline a_{21} & W_c \end{array}\right] = \left[\begin{array}{c|c} a_{11} - a_{12} W_c^{-1} a_{21} & o \\ \hline a_{21} & W_C \end{array}\right] \tag{3.42}$$

Taking determinants on both sides of Eq. 3.42 and using conditions expressed in Eqs. 3.38 and 3.41, one finds that

$$(a_{11} - a_{12} W_c^{-1} a_{21}) \cdot \det(W_C) = o \tag{3.43}$$

But

$$\det(W_C) \neq o \tag{3.44}$$

Hence

$$(a_{11} - a_{12} W_c^{-1} a_{21}) = \alpha = o \tag{3.45}$$

Similarly, if W_L is pre-multiplied by R, the resulting equation is

$$\left[\begin{array}{c|c} 1 & -a_{12} W_c^{-1} \\ \hline o & I \end{array}\right] \left[\begin{array}{c|c} b_{11} & a_{12} \\ \hline b_{21} & W_c \end{array}\right] = \left[\begin{array}{c|c} b_{11} - a_{12} W_c^{-1} b_{21} & o \\ \hline b_{21} & W_C \end{array}\right] \tag{3.46}$$

Taking determinants on both sides of Eq. 3.46 leads to

$$(b_{11} - a_{12} W_c^{-1} b_{21}) = \beta = o \tag{3.47}$$

since det (W_L) vanishes at bifurcation points [114].

The conditions expressed in Eqs. 3.45 and 3.47 imply that the first order perturbation equations do not yield a unique solution for $\dot{\eta}$ at a point of bifurcation. In order to obtain definite values for $\dot{\eta}$, the second order perturbation equations, ε^2, must be used. Rewriting ε^2, defined by Eqs. 3.14, in matrix form, one obtains

$$\begin{bmatrix} W_{1,1} & W_{1,2}\cdots W_{1,n} \\ W_{2,1} & W_{2,2}\cdots W_{s,n} \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ W_{n,1} & W_{n,2}\cdots W_{n,n} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \ddot{q}_n \end{bmatrix} + \begin{bmatrix} W_{1,\lambda} \\ W_{2,\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,\lambda} \end{bmatrix} \ddot{\eta}$$

$$+ \begin{bmatrix} W_{1,ij} \\ W_{2,ij} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,ij} \end{bmatrix} \dot{q}_i\dot{q}_j + 2 \begin{bmatrix} W_{1,i\lambda} \\ W_{2,i\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,i\lambda} \end{bmatrix} \dot{q}_i\dot{\eta} + \begin{bmatrix} W_{1,\lambda\lambda} \\ W_{2,\lambda\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,\lambda\lambda} \end{bmatrix} \dot{\eta}^2 = \begin{bmatrix} o \\ o \\ \cdot \\ \cdot \\ \cdot \\ o \end{bmatrix} \tag{3.48}$$

All partial derivatives in Eqs. 3.48 are evaluated at the known equilibrium state $\varepsilon = o$; e.g.

$$W_{i,j} = \left.\frac{\partial W_i}{\partial q_j}\right|_{\varepsilon = o} \quad ; \; W_{i,\lambda} = \left.\frac{\partial W_i}{\partial \eta}\right|_{\varepsilon = o} \tag{3.49}$$

Using Eqs. 3.25 and 3.29-3.31, Eqs. 3.48 can be written as:

$$a_{11}\ddot{\eta} + a_{12}\begin{bmatrix} \ddot{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \ddot{q}_n \end{bmatrix} + W_{1,ij}\dot{q}_i\dot{q}_j$$

$$+ 2W_{1,i\lambda}\dot{q}_i\dot{\eta} + W_{1,\lambda\lambda}\dot{\eta}^2 = o \tag{3.50}$$

and

$$a_{21}\ddot{\eta} + W_C\begin{bmatrix} \ddot{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \ddot{q}_n \end{bmatrix} + \begin{bmatrix} W_{2,ij} \\ \cdot \\ \cdot \\ W_{n,ij} \end{bmatrix}\dot{q}_i\dot{q}_j$$

$$+ 2\begin{bmatrix} W_{2,i\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,i\lambda} \end{bmatrix}\dot{q}_i\dot{\eta} + \begin{bmatrix} W_{2,\lambda\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,\lambda\lambda} \end{bmatrix}\dot{\eta}^2 = \begin{bmatrix} o \\ \cdot \\ \cdot \\ \cdot \\ o \end{bmatrix} \tag{3.51}$$

Eqs. 3.51 imply that

$$\begin{bmatrix} \ddot{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \ddot{q}_n \end{bmatrix} = - W_c^{-1} \left(a_{21} \ddot{\eta} + \begin{bmatrix} W_{2,ij} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,ij} \end{bmatrix} \dot{q}_i \dot{q}_j \right.$$

$$\left. + 2 \begin{bmatrix} W_{2,i\lambda} \\ \cdot \\ \cdot \\ W_{n,i\lambda} \end{bmatrix} \dot{q}_i \dot{\eta} + \begin{bmatrix} W_{2,\lambda\lambda} \\ \cdot \\ \cdot \\ W_{n,\lambda\lambda} \end{bmatrix} \dot{\eta}^2 \right) \tag{3.52}$$

Inserting Eqs. 3.52 into Eqs. 3.50 yields

$$-a_{12} W_c^{-1} \left(\begin{bmatrix} W_{2,ij} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,ij} \end{bmatrix} \dot{q}_i \dot{q}_j + 2 \begin{bmatrix} W_{2,i\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,i\lambda} \end{bmatrix} \dot{q}_i \dot{\eta} \begin{bmatrix} W_{2,\lambda\lambda} \\ \cdot \\ \cdot \\ \cdot \\ W_{n,\lambda\lambda} \end{bmatrix} \dot{\eta}^2 \right)$$

$$+ W_{1,ij} \dot{q}_i \dot{q}_j + 2W_{1,i\lambda} \dot{q}_i \dot{\eta} + W_{1,\lambda\lambda} \dot{\eta}^2$$

$$+ \ddot{\eta} \left(W_{1,\lambda} - a_{12} W_c^{-1} a_{21} \right) = 0 \tag{3.53}$$

The last term of Eqs. 3.53 is equivalent to $\alpha \ddot{\eta}$ which vanishes since α is zero at a bifurcation point. Hence, Eqs. 3.53 in view of Eqs. 3.25 and 3.34 lead to the following quadratic equation in $\dot{\eta}$:

$$a\,\dot{\eta}^2 + b\,\dot{\eta} + c = o \tag{3.54}$$

where

$$\begin{aligned} a = {} & G_{1,k}\, W_{k,ij}\, G_{i,\lambda}\, G_{j,\lambda} + 2W_{1,i\lambda}\, G_{i,\lambda} \\ & + 2G_{1,k}\, W_{k,i\lambda}\, G_{i,\lambda} + G_{i,k}\, W_{k,\lambda\lambda} \\ & + W_{1,\lambda\lambda} + W_{1,ij}\, G_{i,\lambda}\, G_{j,\lambda}; \end{aligned} \tag{3.55}$$

$$\begin{aligned} b = {} & G_{1,k}\, (W_{k,1i} + W_{k,i1})\, G_{i,\lambda} \\ & + G_{1,k}\, W_{k,ij}\, (G_{i,\lambda}\, G_{1,j} + G_{1,j}\, G_{j,\lambda}) \\ & + 2G_{1,k}\, W_{k,i\lambda} + 2G_{1,k}\, W_{k,i\lambda}\, G_{1,i} \\ & + (W_{1,i1} + W_{1,1i})\, G_{i,\lambda} + 2W_{1,i\lambda}\, G_{1,i} \\ & + W_{1,ij}\, (G_{1,i}\, G_{j,\lambda} + G_{i,\lambda}\, G_{1,j}) + 2W_{1,1\lambda}; \end{aligned} \tag{3.56}$$

$$\begin{aligned} c = {} & G_{1,k}\, W_{k,11} + G_{1,k}\, (W_{k,1i} + W_{k,i1})\, G_{1,i} \\ & + G_{1,k}\, W_{k,ij}\, G_{1,i}\, G_{1,j} + G_{1,11} \\ & + (W_{1,i1} + W_{1,1i})\, G_{1,i} + W_{1,ij}\, G_{1,i}\, G_{1,j}; \end{aligned} \tag{3.57}$$

$$i,\ j,\ k = 2,\ 3,\ \ldots\ N.$$

and

$$\begin{bmatrix} G_{1,2} \\ \vdots \\ G_{1,k} \\ \vdots \\ G_{1,n} \end{bmatrix} = - \begin{bmatrix} W_{2,2} & \cdots & W_{2,n} \\ \vdots & & \vdots \\ W_{k,2} & \cdots & W_{k,n} \\ \vdots & & \vdots \\ W_{n,2} & & W_{n,n} \end{bmatrix}^{-1} \cdot \begin{bmatrix} W_{2,1} \\ \vdots \\ W_{k,1} \\ \vdots \\ W_{n,1} \end{bmatrix} \tag{3.58}$$

$$\begin{bmatrix} G_{2,\lambda} \\ \vdots \\ G_{i,\lambda} \\ \vdots \\ G_{n,\lambda} \end{bmatrix} = - \begin{bmatrix} W_{2,2} & \cdots & W_{2,n} \\ \vdots & & \vdots \\ W_{i,2} & \cdots & W_{i,n} \\ \vdots & & \vdots \\ W_{n,2} & \cdots & W_{n,n} \end{bmatrix}^{-1} \begin{bmatrix} W_{2,\lambda} \\ \vdots \\ W_{i,\lambda} \\ \vdots \\ W_{n,\lambda} \end{bmatrix} \tag{3.59}$$

The roots of Eq. 3.54 are

$$\left.\begin{matrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{matrix}\right\} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{3.60}$$

Using $\dot{\eta}_1$, $\dot{\eta}_2$ from Eq. 3.60 in conjunction with Eqs. 3.34, two sets of

$[\dot{q}_2\ \dot{q}_3\ \ldots\ \dot{q}_n]$ are obtained. These together with Eqs. 3.14 and 3.15 yield $\ddot{q}_i\ \ddot{\eta}$, etc. With such information, together with Eqs. 3.26, the construction of the post-buckling paths beyond a bifurcation point becomes possible. For N large, computer programming is required to carry out the analysis described.

If Eqs. 3.17-3.19 instead of Eqs. 3.13-3.15 are used, the resulting quadratic equation in $\dot{\eta}$ is given by

$$a^*\ \dot{\eta}^2 + b^*\ \dot{\eta} + c^* = o \tag{3.61}$$

where

$$a^* = a - (2G_{1,k}\ W_{k,i\lambda}\ G_{i,\lambda} + G_{1,k}\ W_{k,\lambda\lambda} + W_{1,\lambda\lambda}) \tag{3.62}$$

$$b^* = b - (2G_{1,k}\ W_{k,i\lambda} + 2G_{1,k}\ W_{k,i\lambda}\ G_{1,i}$$

$$+ 2W_{1,i\lambda}\ G_{1,i} + 2W_{1,1\lambda}) \tag{3.63}$$

$$c^* = c \tag{3.64}$$

and

$$\left.\begin{matrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{matrix}\right\} = \frac{-b^* \pm \sqrt{b^{*2} - 4a^*\ c^*}}{2a^*} \tag{3.65}$$

Eqs. 3.62 - 3.64 reflect the savings in time and effort in computer analysis when the total potential energy is a linear function of the loading parameters. These equations (i.e., Eqs. 3.62-3.64) form the basis for comparing the non-diagonalized scheme above with that of Thompson [110, 114].

For the purpose of comparison, consider the potential energy function in two generalized coordinates Q_1, Q_2 and a loading parameter λ [114, p. 142]

$$V = \frac{1}{2} Q_1^2 + \frac{1}{2} Q_2^2 - \frac{1}{2} Q_1 Q_2^2 - \lambda Q_1 \tag{3.66}$$

The equilibrium equations defined by $\partial V/\partial Q_i = o$, $i = 1, 2$, yield the fundamental path solutions

$$\begin{aligned} Q_1 &= \lambda \\ Q_2 &= o \end{aligned} \tag{3.67}$$

Thompson [114] defined a set of incremental coordinates X_1, X_2 emerging from the fundamental path as follows

$$\begin{aligned} Q_1 &= \lambda + X_1 \\ Q_2 &= o + X_2 \end{aligned} \tag{3.68}$$

From Eqs. 3.68 and 3.66, one obtains a new function

$$W = \frac{1}{2} X_1^2 + \frac{1}{2} X_2^2 - \frac{1}{2} X_1 X_2^2 - \frac{1}{2} \lambda X_2^2 - \frac{1}{2} \lambda^2 \tag{3.69}$$

It follows from Eqs. 3.66 and 3.69 that the transformation from V to W destroys the linearity in λ when X_1, X_2 are treated as generalized coordinates. Hence Thompson's 'sliding' transformation forces the generalized coordinates to be measured along curvilinear coordinates. This observation was made by Sewell [94]. However, Sewell [94] obtained branching paths beyond bifurcation points by expressing the paths explicitly in terms of orthonormal eigenvectors of an algebraic eigenvalue problem. Such an approach necessitates finding N eigenvalues and eigenvectors of the characteristic stability matrix for paths emanating from the critical point. For N large, this approach is disadvantageous in terms of computing time. It could lead to numerical instability problems for large N [38].

The non-diagonalized perturbation scheme resulting in Eq. 3.61 does not require eigenvalue and eigenvector computations. The generalized displacements used as generalized coordinates prevent any loss of the linearity of V in Λ^j (or V in λ). The conclusion can therefore be drawn that the scheme proposed is computationally advantageous for a large system.

e. Coincident Critical Point

Critical points (limit, bifurcation points) normally lie isolated on the fundamental path. In that case, unique buckling modes are associated with the critical loads. If simultaneous buckling occurs, two or more critical points coincide. At such coincident critical points, buckling modes are mixed [114, 59]. Depending on the number of post-buckling paths leaving a coincident critical point, post-buckling response for systems exhibiting it can be complex and undefined [60, 21].

Coincident critical points have been observed in the analysis of rigid link models with two degrees of freedom [21] and in cylindrical shells [65]. Some general results for N-degree of freedom systems, satisfying certain symmetry and pre-buckling deformation restrictions, have been proposed; e.g., [102, 101, 60, 59].

In the present study of reticulated domes, interest is focused on unstable coincident critical points (if they exist) and the associated unstable post buckling paths. Accordingly, the presence or absence of a coincident critical point is monitored on the basis of the theories of sections 3.1 b-d. For at least two critical points to be coincident, the relations

$$\det (W_L) = \det (W_B) = \det (W_C) = o \tag{3.70}$$

must hold. The relation det $(W_C) = 0$ indicates the presence of at least one zero eigenvalue for the matrix W_C. In this case, the analysis of section 3.1d is extended to include the third order perturbation equation since ε^1 and ε^2 will not yield unique values of $\dot{\eta}$.

In the current investigation, coincident critical points are absent for all loading distributions considered. This fact is established from the non-vanishing of det (W_C) for all cases considered.

3.2 Nonlinear Programming Technique

The basic ideas in nonlinear structural analysis using nonlinear programming technique is discussed in this section. The various nonlinear programming algorithms employed in the nonlinear analysis of geodesic domes (Chapter 5) are referenced. The role of the nonlinear programming technique in obtaining the nonlinear response of reticulated domes to static loads is stated.

The basic law (Theorem 1, section 2.1) underlying the nonlinear programming technique is the law of minimum potential energy. This law states [69] that "a conservative holonomic system is in a configuration of stable equilibrium if, and only if, the value of the total potential energy is a relative minimum."

At the equilibrium point Q_i^*, the equilibrium equations

$$W_s(Q_i, \Lambda^j)\Big|_{Q_i = Q_i^*} = 0; \qquad \begin{array}{l} i = 1, 2, .. N \\ j = 1, 2, .. M \end{array} \tag{3.71}$$

are satisfied. The equilibrium point Q_i^* is stable if

$$V(Q_i^*, \Lambda^j) < V(Q_i, \Lambda^j) \tag{3.72}$$

for all Q_i in some neighborhood of Q_i^*. Eqs. 3.72 imply that the total potential energy surface is convex in the neighborhood of stable equilibrium states. Hence, the minimum potential energy principle provides a useful device for casting the structural analysis problem into a mathematical programming problem [33]. One can prove [80] that such a minimization of a scalar valued function of N variables is mathematically equivalent to solving N simultaneous nonlinear equations (if V is a quadratic form).

For geometrically nonlinear, reticulated dome models, the potential energies of the line elements can be summed to obtain the total potential energy of the assemblage. Hence, Eqs. 3.71 and 3.16 imply that at equilibrium,

$$\sum_{k=1}^{NM} \frac{\partial \pi_k}{\partial Q_s} - P_s = o \tag{3.73}$$

where

$$P_s = P_s(\Lambda^j) \tag{3.74}$$

In Eqs. 3.73 and 3.74, π_k is the strain energy of element k; NM is the total number of elements, and P_s is the generalized load corresponding to the generalized displacement Q_s. A convergence criterion based on Eq. 3.71 or Eq. 3.73 is [73, 80]

$$|W_s(Q_i, \Lambda^j)| < \gamma \tag{3.75}$$

where γ is a small positive constant ($\gamma << 1$). Physically, γ represents the unbalanced joint forces and determines the precision involved in gradient computations.

Three unconstrained nonlinear optimization algorithms are employed in the present study: Davidon-Fletcher-Powell algorithm [31]; Conjugate Gradient technique [32]; Powell's direct search method [83]. The first two techniques are gradient methods. These are efficient for large systems when analytical derivatives of the total potential energy (objective function) are available [39]. The third technique is a direct-search method which is unreliable for large systems because of its slow rate of convergence.

The Conjugate Gradient technique requires less storage space than the Davidon-Fletcher-Powell method. However, for precise location of the minimum, the Davidon-Fletcher-Powell method is more reliable [39]. The main advantage of Powell's direct-search technique is that it requires no derivative expressions. Hence for the direct-search method, the total potential energy need not satisfy regularity and continuity conditions; neither is a proof for the existence of derivatives required [39]. For certain simple functions, Powell's method is quite efficient and requires less function evaluations for convergence compared to gradient methods [39].

In the search for a minimum, all algorithms described above perform a series of one-dimensional searches in specified directions. They are all quadratically convergent algorithms in that they will minimize a quadratic function of N variables in N steps. In highly nonlinear problems, this property is quite desirable since a Taylor series expansion of the total potential energy about stable equilibrium states shows that the quadratic and lower terms dominate in the vicinity of the minimum. Hence, it is sufficient to pass a quadratic polynomial through three points in the neighborhood of the minimum.

Employing the three algorithms stated above provides flexibility since the solution procedure most suitable to a particular test problem can be selected.

The role of the nonlinear programming technique in the present study is to construct the initial portions of the fundamental path [114] at the onset of loading. Such a role provides a starting solution for the static perturbation technique, and optimizes the choice of a "suitable" perturbation parameter. The choice of a "suitable" perturbation parameter is presented in section 4.3.

Detailed descriptions of all nonlinear programming techniques employed are in reference 39.

3.3 Dynamic Analysis

The perturbed motion of the discrete system modeled, under dynamic loads, is obtained using numerical integration. Numerical integration is necessary since the governing differential equations of perturbed motion are nonlinear.

The differential equations of perturbed motion can be expressed as (section 2.1, Eq. 2.3)

$$y' = f(y) \tag{3.76}$$

in the 2N dimensional phase space. The properties of the vector f are given in section 2.1. Consider an initial disturbance y_o applied at time $t = o$. Hamming's modified Predictor-Corrector numerical integration scheme [49, 84] applied to Eqs. 3.76 with the given initial disturbance provides the desired solutions y of Eq. 3.76 during motion.

Phase-plane portraits of perturbed motions provide a valuable check on the sufficient stability criterion [45, 50] used in the present study. Closed trajectories surrounding only the equilibrium point (an isolated singularity) under investigation indicate perturbed motions of stable origin. This corresponds to a stable node (center) in the phase plane. If the perturbed motion encompasses more than one singular (equilibrium) point in the phase plane, the origin is unstable. The perturbed motion surrounding more than one stable node and passing through saddle points is termed a separatrix. A separatrix indicates the domains of attraction, the regions of asymptotic stability, of stable equilibrium states along the real y_1 axis. In general, for conservative systems, closed trajectories enclose odd numbers of singularities (equilibrium points) with the number of centers exceeding the number of saddle points by one [75].

CHAPTER 4

CONVERGENCE CHARACTERISTICS AND ERROR ESTIMATES

In this chapter, the convergence characteristics and error estimates in a perturbation scheme are considered. The role of the right choice of the perturbation parameter in the construction of the equilibrium surface is discussed, and a method for selecting a "suitable" perturbation parameter is proposed.

4.1 Convergence Characteristics of the Static Perturbation Technique

When a series representation for the increments in the generalized displacements, q_i, and the generalized external loads, η, is used in a static perturbation method, the result is a series of successive approximations in the location of a nearby equilibrium state. This section considers the convergent characteristics of such series representations.

Consider the parametric representation of any prospective equilibrium path leaving a known equilibrium point [92] in the form (Chapter 3)

$$\left.\begin{aligned} q_i(\varepsilon) &= \sum_{k=1}^{3} \frac{\varepsilon^k}{k!} \frac{\partial^k q_i}{\partial \varepsilon^k} + O(\varepsilon^4) \\ \eta(\varepsilon) &= \sum_{k=1}^{3} \frac{\varepsilon^k}{k!} \frac{\partial^k \eta}{\partial \varepsilon^k} + O(\varepsilon^4). \end{aligned}\right\} \tag{4.1}$$

In the neighborhood of the equilibrium point (Q_i, Λ^j), information about prospective paths emanating from it is obtained from the following linear, ordered, perturbation equations [92]

$$\left.\begin{aligned} &\frac{d^r}{d\varepsilon^r}\left[W_s\{Q_i(\varepsilon),\ \Lambda^j(\varepsilon)\}\right]\Big|_{\varepsilon=0} = 0; \\ &r = 1, 2, 3, \ldots \\ &s,i = 1, 2, \ldots N \\ &j = 1, 2, \ldots M. \end{aligned}\right\} \tag{4.2}$$

The resulting potential energy, V, for the desired equilibrium state is a series containing terms of homogeneous order,

$$\left.\begin{aligned} &V = V_o + V_1 q_i + V_2 q_i q_j + V_3 q_i q_j q_k + O(q_i q_j q_k q_\ell); \\ &i, j, k, \ell = 1,2, \ldots N. \end{aligned}\right\} \tag{4.3}$$

In Eqs. 4.3, V_o is the potential energy of the known equilibrium state; V_1, V_2, V_3 are the associated coefficients that characterize the first, second, third order perturbation solutions, respectively, of the physical problem. Also

$$V_m \neq V_m(\varepsilon);\ m = 1, 2, 3. \tag{4.4}$$

Eqs. 4.3 can be rewritten as

$$(V-V_o) = V_m \delta_m(\varepsilon) + O(\varepsilon^4) \tag{4.5}$$

where

$$\delta_m(\varepsilon) = O(\varepsilon^m);\ m = 1, 2, 3. \tag{4.6}$$

A 'Ratio Test' [78] applied to the right hand side of Eq. 4.5 establishes the limiting process

$$\lim_{\varepsilon\to 0} \left[\frac{n^{th} \text{ term}}{(n-1)^{th} \text{ term}} \right] = \lim_{\varepsilon\to 0} [O(\varepsilon)] = 0. \qquad (4.7)$$

Eq. 4.7 implies that Eq. 4.5 represents an asymptotic series which is uniformly convergent as $\varepsilon\to 0$. This series belongs to a general class of asymptotic expansions [78] represented as

$$V = \sum_{m=0}^{n-1} a_m \delta_m(\varepsilon) + O[\delta_n(\varepsilon)] \qquad (4.8)$$

where

$$a_m \neq a_m(\varepsilon); \ \varepsilon\to 0 \ . \qquad (4.9)$$

Asymptotic expansions of this type are uniformly valid (in the absence of singularities in the region of interest) and uniformly convergent. For a given asymptotic sequence, $\delta_m(\varepsilon)$, the asymptotic representation of V is unique [78]. The conclusion is that in a static perturbation method, there exists a power series solution in terms of a path parameter, ε, which is uniformly convergent provided $\varepsilon\to 0$.

A proof of convergence of the power series solution, in terms of a parameter, ε, for a system of first order differential equations for "sufficiently" small ε is given by Minorsky [76].

4.2 Error Estimates in a Perturbation Scheme

This section is concerned with the estimation of errors in a static perturbation scheme. An upper bound on the value of the perturbation parameter, ε, for a specified error bound, δ, is given.

Referring to Eqs. 4.1, the pertinent error equations for third order perturbation solutions can be written as follows:

$$\Delta q_i(\varepsilon) = \frac{1}{4!}\,\varepsilon^4 \frac{\partial^4 q_i}{\partial \varepsilon^4} + \frac{1}{5}\,\varepsilon^5 \frac{\partial^5 q_i}{\partial \varepsilon^5} + \ldots;\; i = 1,\, 2,\, ..N \tag{4.10}$$

$$\Delta \eta(\varepsilon) = \frac{1}{4!}\,\varepsilon^4 \frac{\partial^4 \eta}{\partial \varepsilon^4} + \frac{1}{5}\,\varepsilon^5 \frac{\partial^5 \eta}{\partial \varepsilon^5} + \ldots . \tag{4.11}$$

$\Delta q_i(\varepsilon)$, $\Delta\eta(\varepsilon)$, in Eqs. 4.10, 4.11 represent errors in the generalized displacements, errors in the generalized independent loads, respectively. Hence an attempt to limit errors to a value δ can be made by selecting ε to satisfy the inequality

$$\varepsilon \leq \max\left\{\left(\frac{4!\,\delta}{q_i^{(4)}}\right)^{\frac{1}{4}},\ \left(\frac{4!\,\delta}{\eta^{(4)}}\right)^{\frac{1}{4}}\right\} \tag{4.12}$$

where

$$\left.\begin{aligned} q_i^{(4)} &= \frac{\partial^4 q_i}{\partial \varepsilon^4} \\ \eta^{(4)} &= \frac{\partial^4 \eta}{\partial \varepsilon^4} . \end{aligned}\right\} \tag{4.13}$$

Eq. 4.12 represents an upper bound on the value of ε; it also implies that errors in a perturbation scheme can be controlled by limiting the value of the chosen perturbation parameter, ε.

4.3 Choice, and Role of a Perturbation Parameter

Sections 4.1 and 4.2 are developed on the assumption that a "suitable" perturbation parameter, ε, exists. Here, a possible way to choose a "suitable" ε is proposed. Also, the role played by ε in the construction of the equilibrium surface is discussed.

The perturbation parameter, ε, represents an increment in length along the equilibrium path [92]. It is a measure of progress in the response of the system. Geometrically, ε indicates possible expansions or contractions of the equilibrium surface to changing actions. It depicts changes in the physical variables (generalized external loads, generalized displacements) of the system. If the wrong ε is chosen, differentiation with respect to ε is not justified [78]. This leads to nonuniform expansions and, hence, to numerical instability of the solution process. This is illustrated by the response of a 21-degree-of freedom space truss (Chapter 5, Fig. 5.35) for different values of the perturbation parameter.

For some systems exhibiting stable critical states, Fig. 4.1, the load parameter, η, represents a "suitable" choice for ε [114]. As shown in Fig. 4.1, the post-buckling paths of such systems are rising. Hence, at pre-buckling loads, there is a unique equilibrium path. The associated pre-buckling deformations are very small and can be neglected [114,59]. Axially loaded columns, frames, and plates are typical examples in this class [114].

For imperfection-sensitive systems, Fig. 4.2, the critical equilibrium states are unstable. This implies the existence of at least one

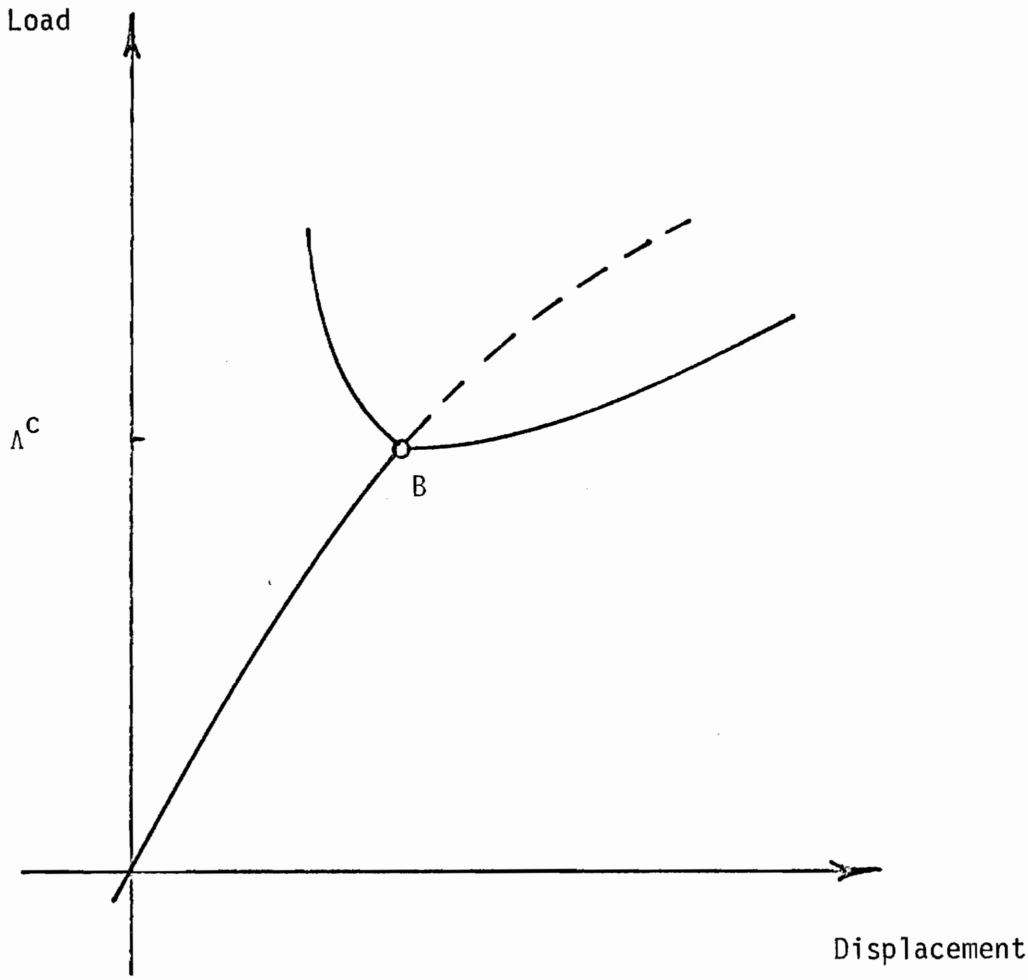


FIG. 4.1 EQUILIBRIUM PATHS THROUGH A STABLE-SYMMETRIC POINT OF BIFURCATION (Imperfection - Insensitive System)

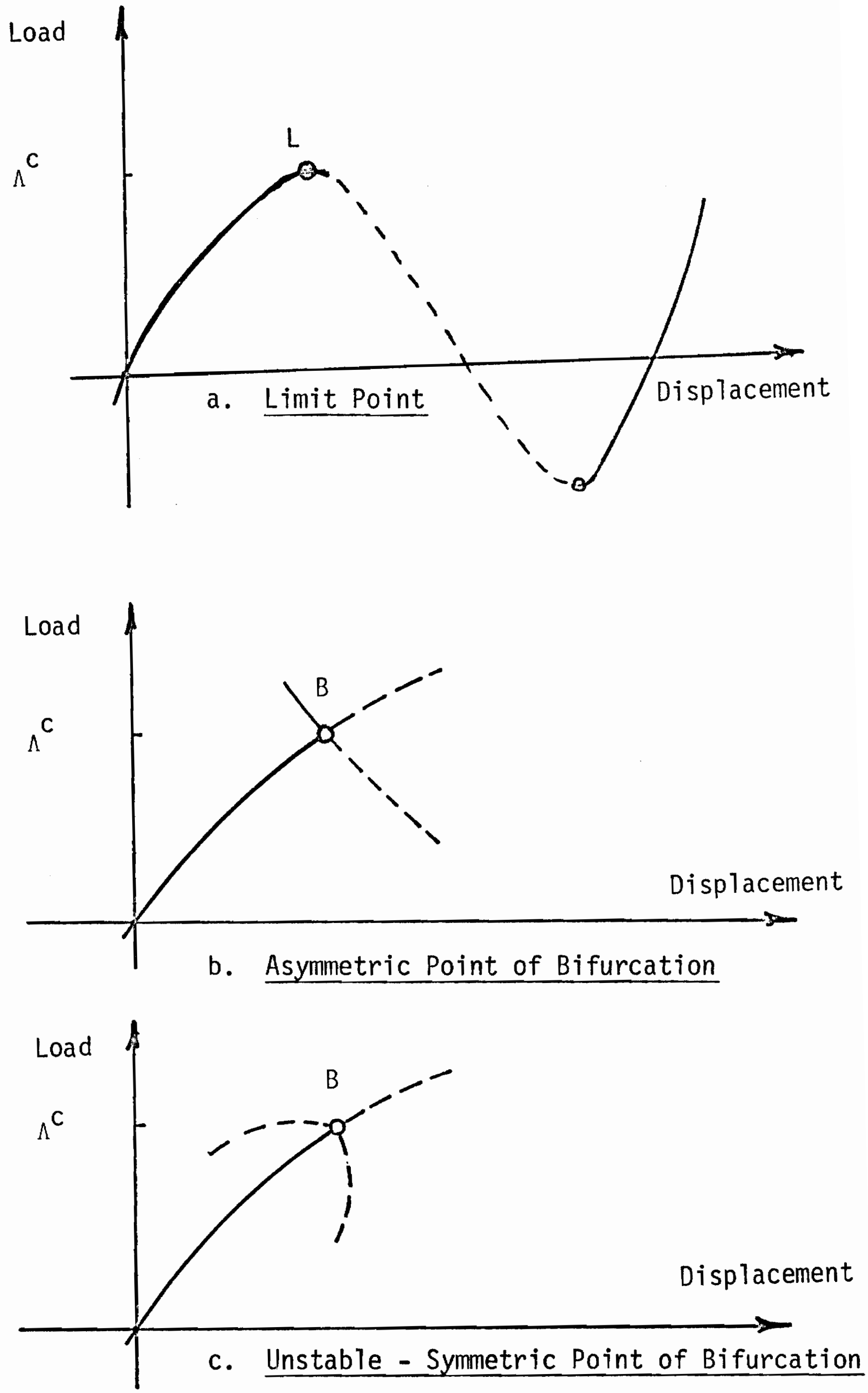


FIG. 4.2 EQUILIBRIUM PATHS THROUGH UNSTABLE CRITICAL POINTS

(Imperfection - Sensitive Systems)

falling post-buckling path. Hence at pre-buckling loads, the system could exhibit multi-equilibrium states [114] as in Fig. 4.2a. Also the pre-buckling deformations are not negligible as they play significant roles in the buckling behavior of the system [57]. In this case, η is an inappropriate choice for post-buckling response, although, estimates of the buckling load can be made. A generalized displacement-increment represents a proper choice for ε. This choice is necessary to obtain complete post-buckling (stable and unstable equilibrium paths) response beyond unstable critical points. Laterally loaded shallow arches, spherical shells, and reticulated domes are typical examples in this class [114].

If the imperfection-sensitive system can be adequately represented by one or two degrees of freedom, (i.e. $N \leq 2$), the choice of ε can be obtained by trial and error between the two generalized displacements. However, for multi-degree of freedom systems with N large, subjected to several independent external loads, the choice of a "suitable" ε is not obvious and cannot be arbitrary. A study in N-space topology can probably provide desirable guidelines [75]. Without topological studies, however, it is essential that the chosen ε ensures the expected shape for the final construction of the equilibrium surface. Also, the condition that all remaining variables are single-valued functions of ε must be satisfied.

The "largest" displacement-increment, q_i, at the initial stage of loading, from the zero-state in configuration-space, is proposed as the "suitable" choice for ε in the case of imperfection-sensitive systems. This choice imposes the necessary control, and restriction, on the generalized displacement that drastically distorts the equilibrium surface if other generalized coordinates are slightly perturbed. Such a control leads to an accurate construction of the equilibrium surface which is desirable in stability investigations [56].

A nonlinear programming technique is used in this study to obtain the generalized displacements corresponding to a prescribed set of external loads at the onset of loading. The associated increments of all the N generalized displacements are stored and compared. This operation is repeated for a few load steps and the "suitable" ε is then chosen as proposed above. A static-perturbation proceedure is then initiated using the chosen ε as the "suitable" perturbation parameter.

CHAPTER 5

DEMONSTRATION PROBLEMS

In this chapter, the theorems, principles, solution techniques, and error measures discussed and developed in earlier chapters are applied to the stability analysis of geodesic domes. Two geodesic dome models (Figs. 5.1, 5.2), with 21 and 39 degrees-of-freedom, are studied in detail. Responses of the dome models to multiple static and dynamic loads are documented. The associated stability boundaries of the models under two independent external loads are obtained. For brevity, the term degree-of-freedom is abbreviated as DOF in subsequent discussions.

The reticulated dome models are formed by an assemblage of one-dimensional elements interconnected at the joints. Depending on the joint coordinates, the domes could be regarded as shallow or deep [87, 88]. Independent loads and finite displacements are admitted at the joints. The resulting geometrically nonlinear discrete system is described by its potential function, $V(Q_i, \Lambda^j)$, where V has all the properties outlined in section 2.1.

The potential energy of the dome model can be expressed as

$$V = \pi + \Omega \tag{5.1}$$

where

$$\pi = \sum_{i=1}^{NM} \frac{A_i E_i}{2L_i} (L_i^* - L_i)^2 \tag{5.2}$$

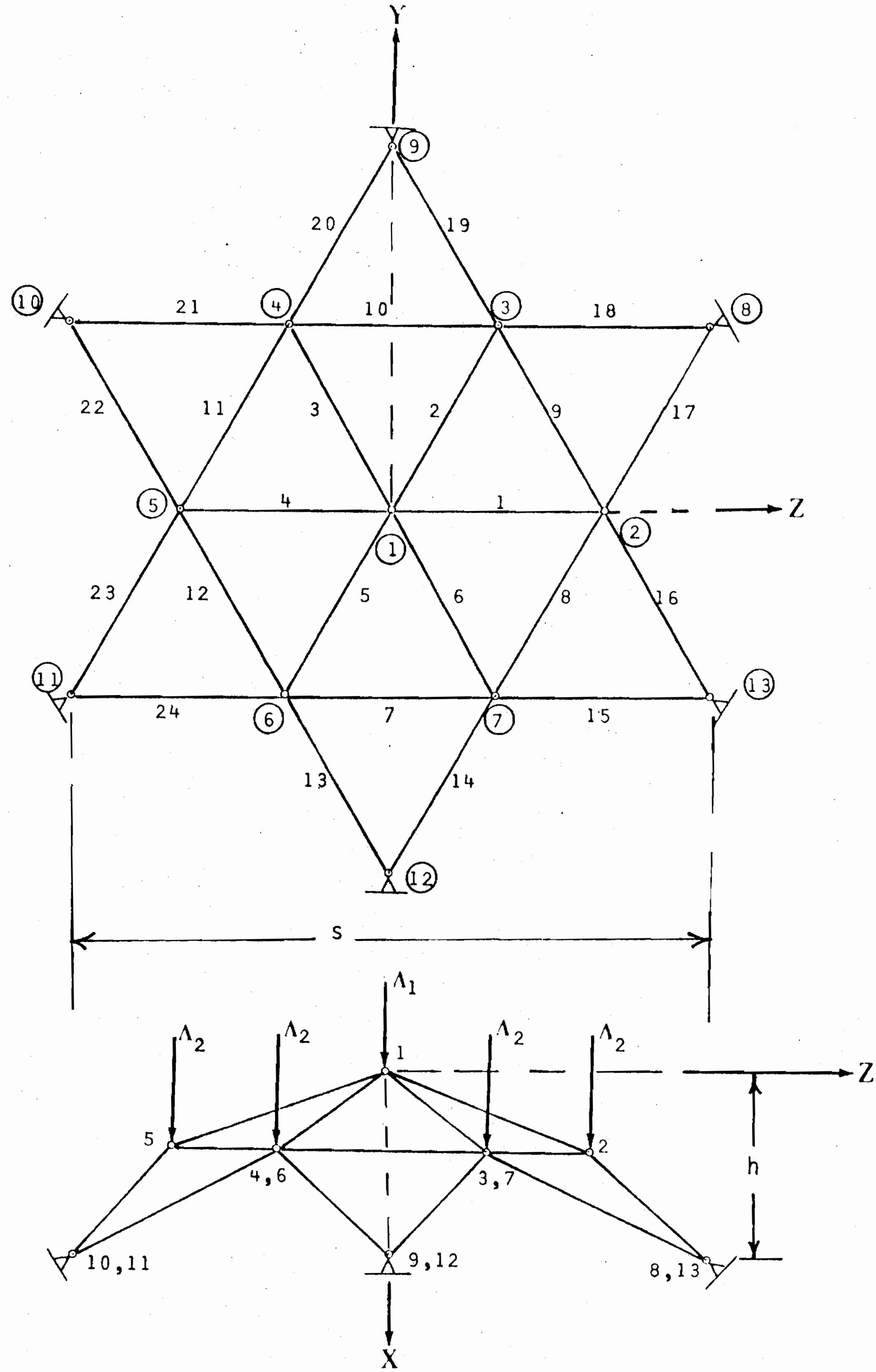


FIG. 5.1 GEODESIC DOME: 21 DOF MODEL

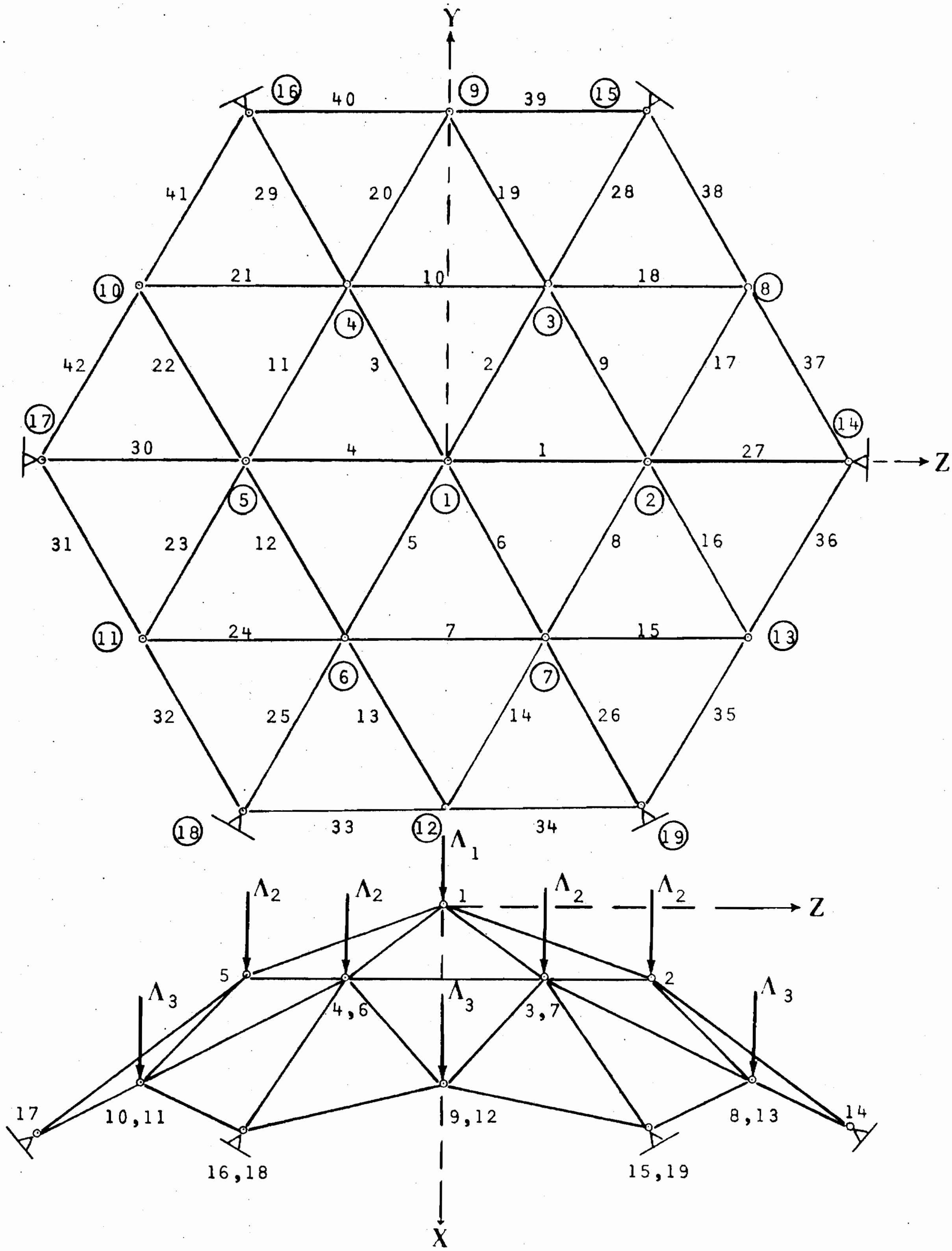


FIG. 5.2 GEODESIC DOME: 39 DOF MODEL

and

$$\Omega = - \sum_{j=1}^{NJ} P_j^T U_j \tag{5.3}$$

In Eqs. 5.2 - 5.3, A_i, E_i, L_i denote the area, the modulus of elasticity, and the initial length of element i, respectively; L_i^* defines the deformed length of the element; NM, NJ are the number of elements, the number of joints, respectively; P_j, U_j are three-dimensional force, displacement vectors at joint j, respectively; the superscript T denotes matrix transposition. The relative axial deflection is the difference between L_i^* and L_i. The initial coordinates and the displacements U_j of the joints completely define L_i and L_i^* [48]; π, Ω are the strain energy, potential energy of external forces, respectively. Eq. 5.1 is identical to Eq. 3.16. Hence, the ordered perturbation equations, Eqs. 3.17 - 3.19, can be used to obtain the nonlinear response of the dome models to static loads.

Eq. 5.1 together with the kinetic energy function $T(Q_i, \dot{Q}_i, \Lambda^j)$ and Rayleigh's dissipation function [75] forms the basis of the dynamic analysis described in section 3.3.

The 21 DOF dome model is subjected to two independent loads Λ_1, Λ_2 while the 39 DOF model is under three independent loads Λ_1, Λ_2 and Λ_3.

5.1 Geodesic Dome: 21 DOF Model

The twenty-one DOF geodesic dome model is shown in Fig. 5.1. This model is analyzed for seven different loading conditions:

1. $\Lambda_1 \neq o$, $\Lambda_2 = o$
2. $\Lambda_1 = \Lambda_2 = \Lambda$
3. $\Lambda_1 > \Lambda_2$
4. $\Lambda_1 < \Lambda_2$
5. Asymmetric Loading; i.e., joints 1,2,3,7 loaded
6. $\Lambda_2 \neq o$, $\Lambda_1 = o$
7. Single concentrated load at joint 2

The variations of Λ_1 and Λ_2 are arbitrary since they are independent loads. At failure, the critical load on the system is denoted Λ^c. If both Λ_1 and Λ_2 are present, Λ^c takes the values of Λ_1 at failure. If only Λ_2 is present, then Λ^c takes the value of Λ_2 at failure. Similarly, if only Λ_1 is present, Λ^c represents the critical value of Λ_1 at failure. This notation is used in all the static response $(\Lambda - Q_i)$ curves.

The effect of shallowness for the dome model is shown in Fig. 5.3. In this illustrative figure, s denotes the span of the dome while h represents the height of joint 1 above the supports. The dome admits only limit points if $s \geq 10h$ while bifurcation states are possible if $s \leq 8h$. For $8 < s/h < 10$, the form of failure depends on the loading configuration; i.e., both limit and bifurcation states are possible in this range. The structure has the freedom to assume any bifurcation configuration. The inequality $s \geq 8h$ satisfies the conditions for shallowness in the classical shallow shell theory [87, 88]. As

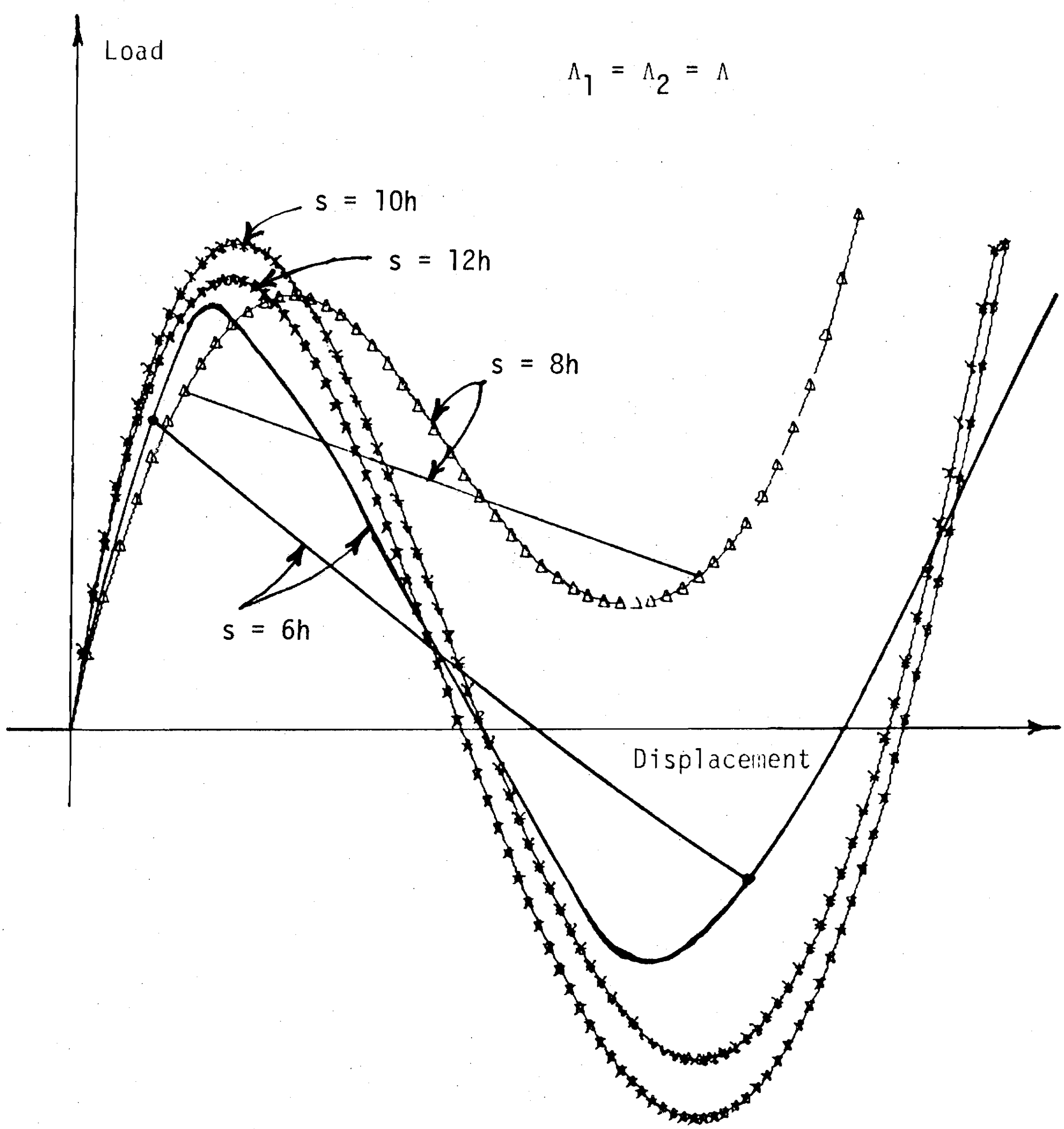


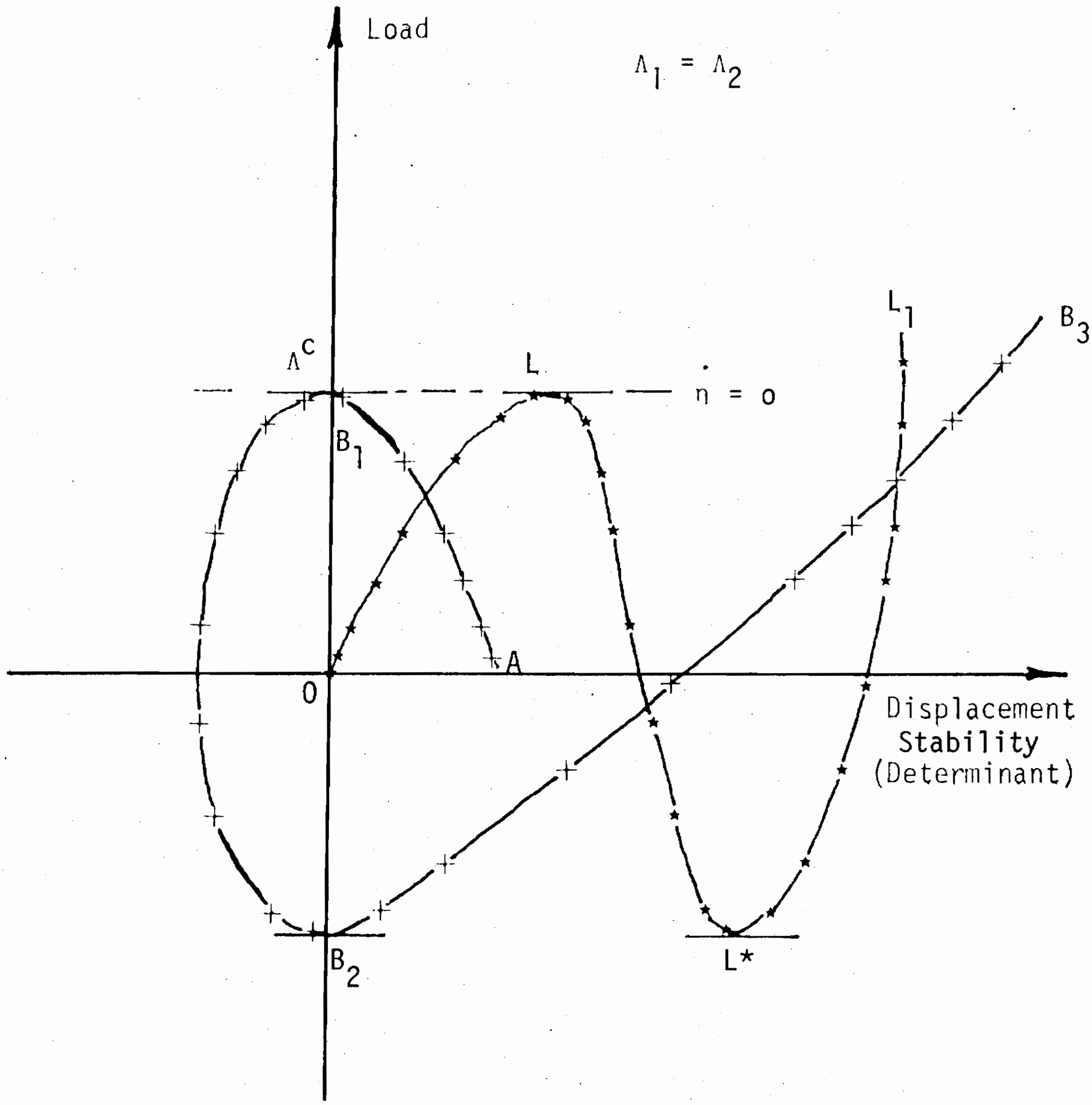
FIG. 5.3 EFFECT OF SHALLOWNESS

shown in Fig. 5.3, a shallow shell admits both limit and bifurcation critical states under some loading conditions. However, if the shell is sufficiently shallow, $s \geq 10h$, only limit points will be admitted as critical states regardless of loading configurations.

As indicated in Fig. 5.3, the reticulated dome exhibits unstable critical (limit and bifurcation) states. Hence, collapse is into an equilibrium configuration which is a finite distance away from the fundamental state. In this case, the buckling load of the structure is sensitive to geometric and/or loading imperfections [114]. At the initial stage of buckling, deformations of the system are symmetric in the case of a limit point. Bifurcation is characterized by initial unsymmetrical deformations in the post-buckling range prior to complete collapse.

Figs. 5.4 and 5.5 depict the process involved in the search for critical states (limit, bifurcation, and coincident critical points). In the case of a limit point (Fig. 5.4), the determinant det (W_L) vanishes and $\dot{\eta} = o$. For bifurcation points (Fig. 5.5), det (W_L) and det (W_B) vanish but $\dot{\eta} \neq o$. The occurrence of a critical state is detected by changes in sign of the stability determinant.

In Figs. 5.6 - 5.8, the effects of various loading conditions on the forms of instability are depicted. When the center joint is loaded at a rate equal to or higher than the ring joints, the model attains a limit point (Figs. 5.6, 5.7). However, when the ring joints take on more loads than the center joint, buckling occurs at a bifurcation



—★— Load vs Displacement

—+— Load vs Stability Determinant

FIG. 5.4 LIMIT POINT LOCATION

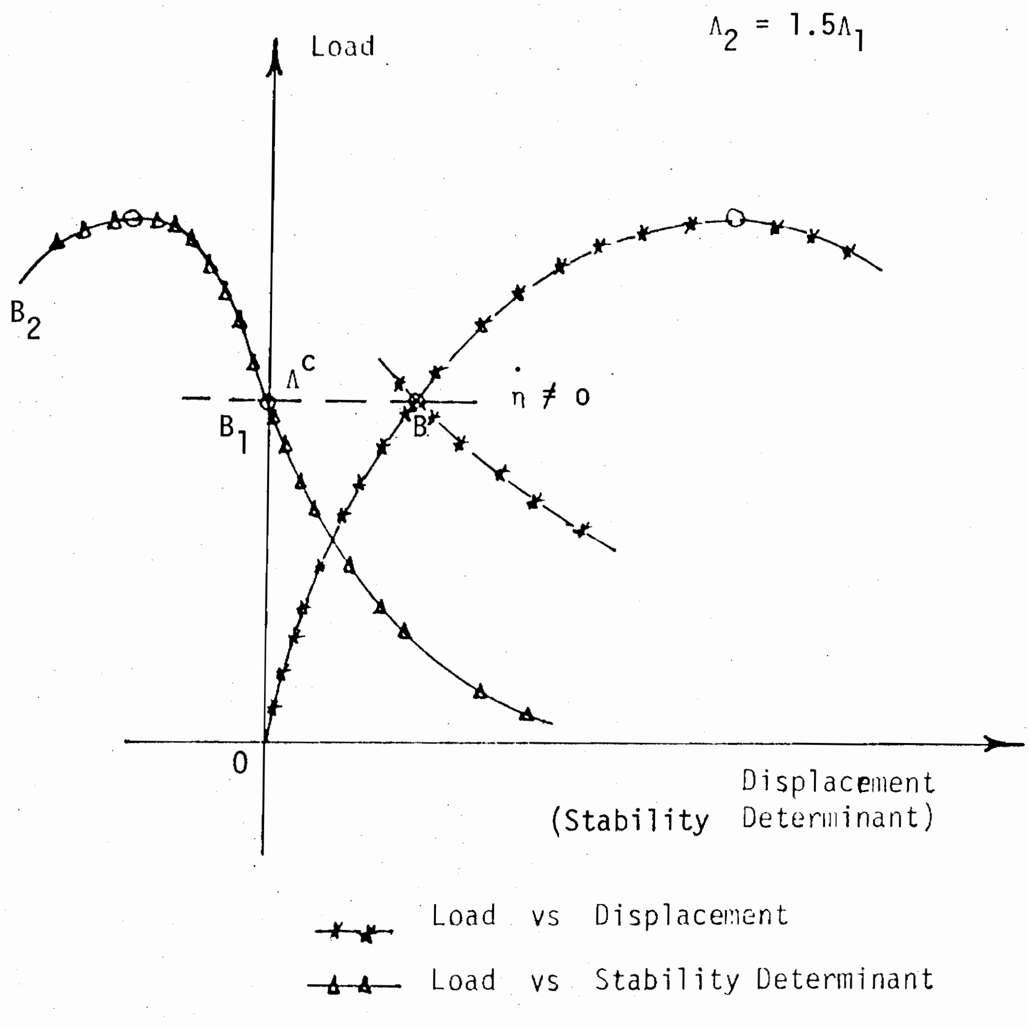


FIG. 5.5 BIFURCATION POINT LOCATION

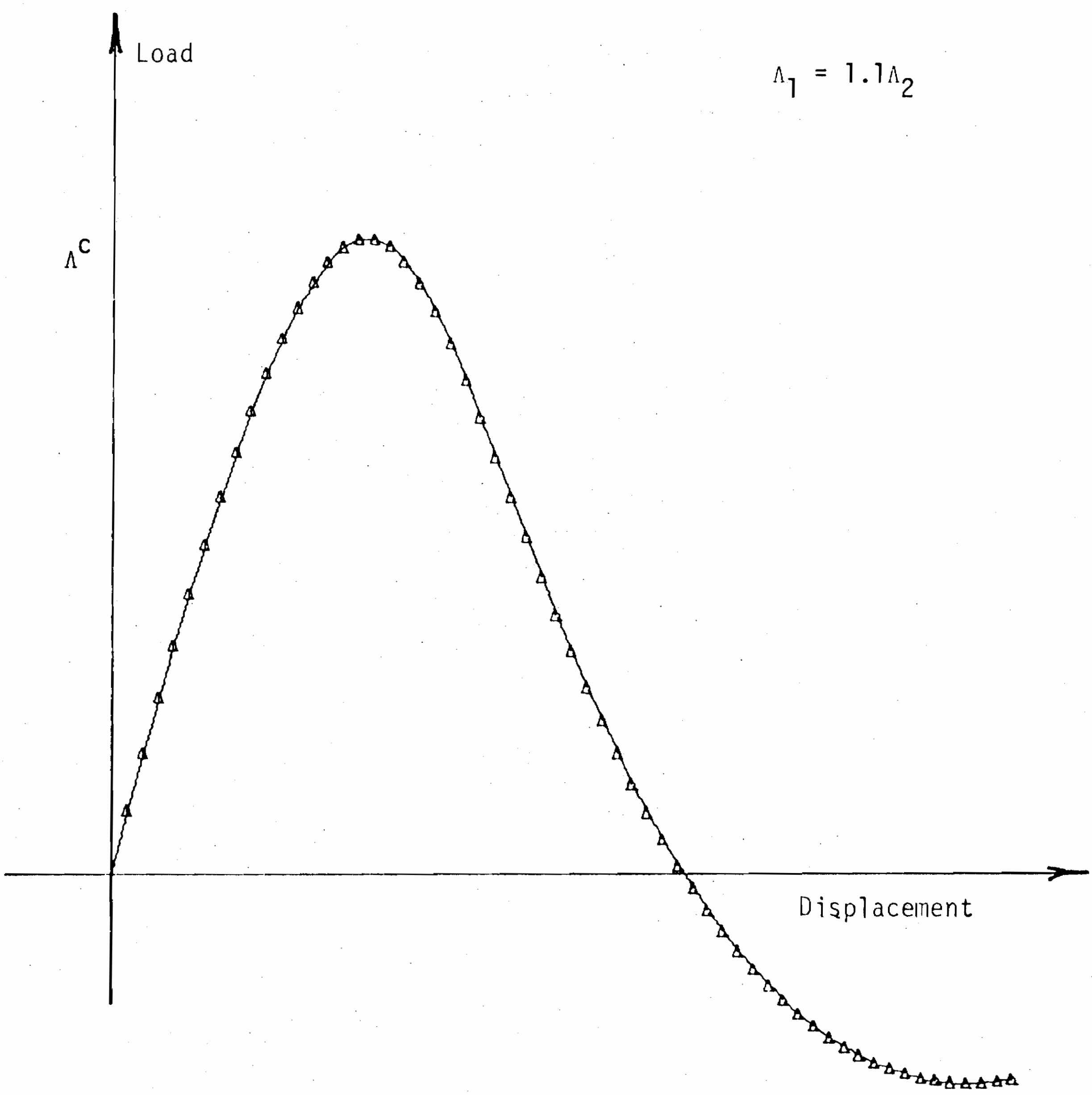


FIG. 5.6 EQUILIBRIUM PATH
(Limit-Type Failure)

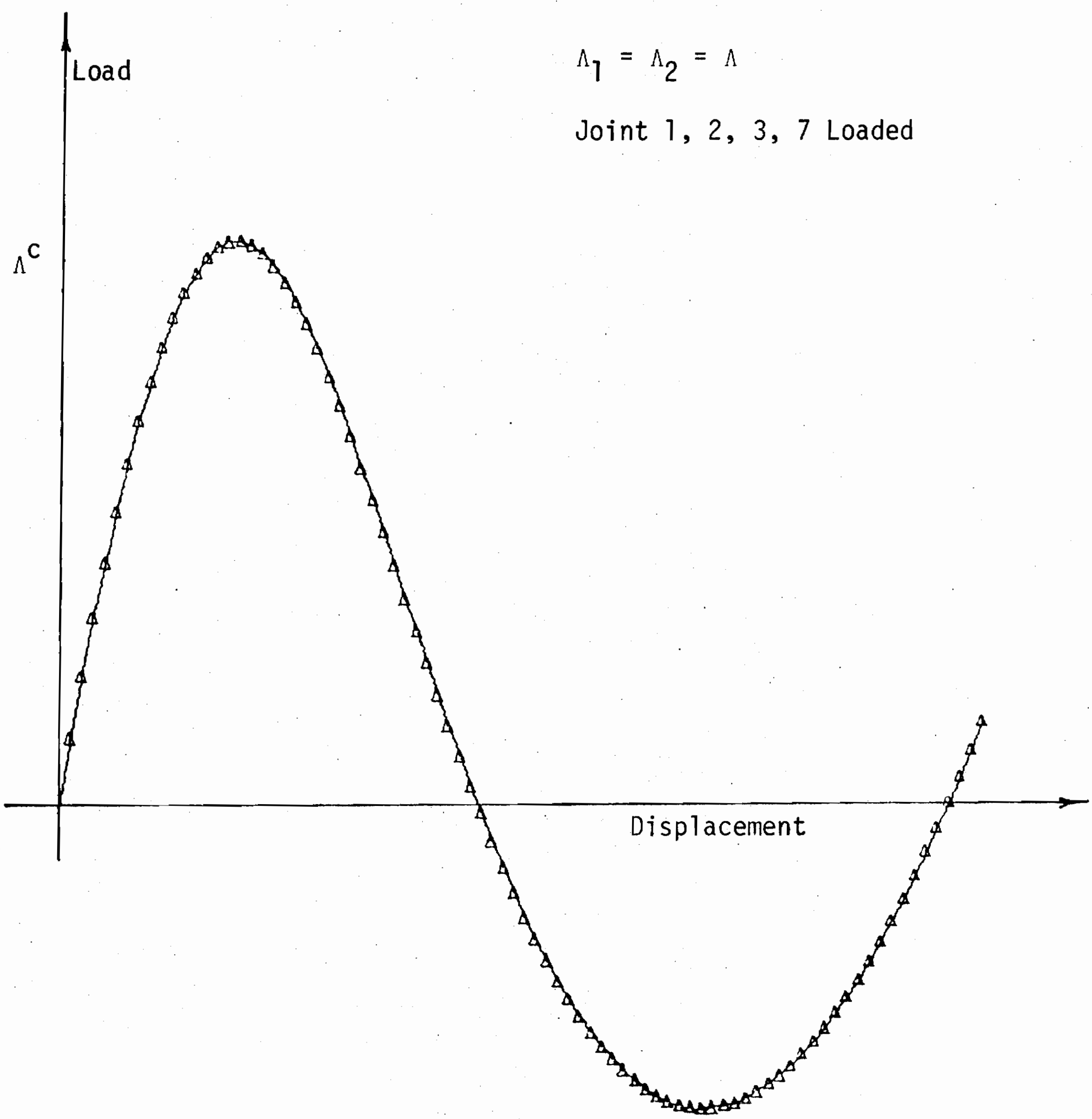


Fig. 5.7 EQUILIBRIUM PATH-ASYMMETRIC LOADING
(Limit-Type Failure)

point and the critical load Λ^c is considerably reduced (Fig. 5.8). In this case, equilibrium on the primary path becomes unstable when the load-level reaches Λ^c and thereafter.

The sensitivity of the reticulated dome model to imperfections is shown in Fig. 5.9. In the presence of loading imperfections, limit points are admitted as critical states. In Fig. 5.9, loading imperfections are introduced at joints 2 and 5, i.e. at these joints, Λ_2/Λ_1 equals 1.8. The perfect system corresponds to the ratio Λ_2/Λ_1 equals 2; i.e. all ring joints loaded at twice the rate at which joint 1 is loaded. The bifurcation load of the perfect system is an upper bound on the limit load of the imperfect system (Fig. 5.9). This figure demonstrates the need to locate all critical states of the perfect system for design. If the bifurcation load were not found, any design based on the limit load of the perfect system would be nonconservative and hence impractical.

Figs. 5.10 and 5.11 illustrate the relation between the load parameter, Λ, and the degree of stability, M. As the load increases, the degree of stability decreases. M approaches zero at the critical load, Λ^c. It appears that equilibrium is more sensitive to small (but finite) disturbances in the neighborhood of bifurcation points (characterized by $\Lambda_2 > \Lambda_1$) than around limit point (characterized by $\Lambda_1 > \Lambda_2$). In both cases, the equilibrium states in the vicinity of the critical point are quite sensitive to disturbances since the slope of the Λ - M curves approaches infinity.

Fig. 5.12 shows the effect of geometric imperfections on the dome

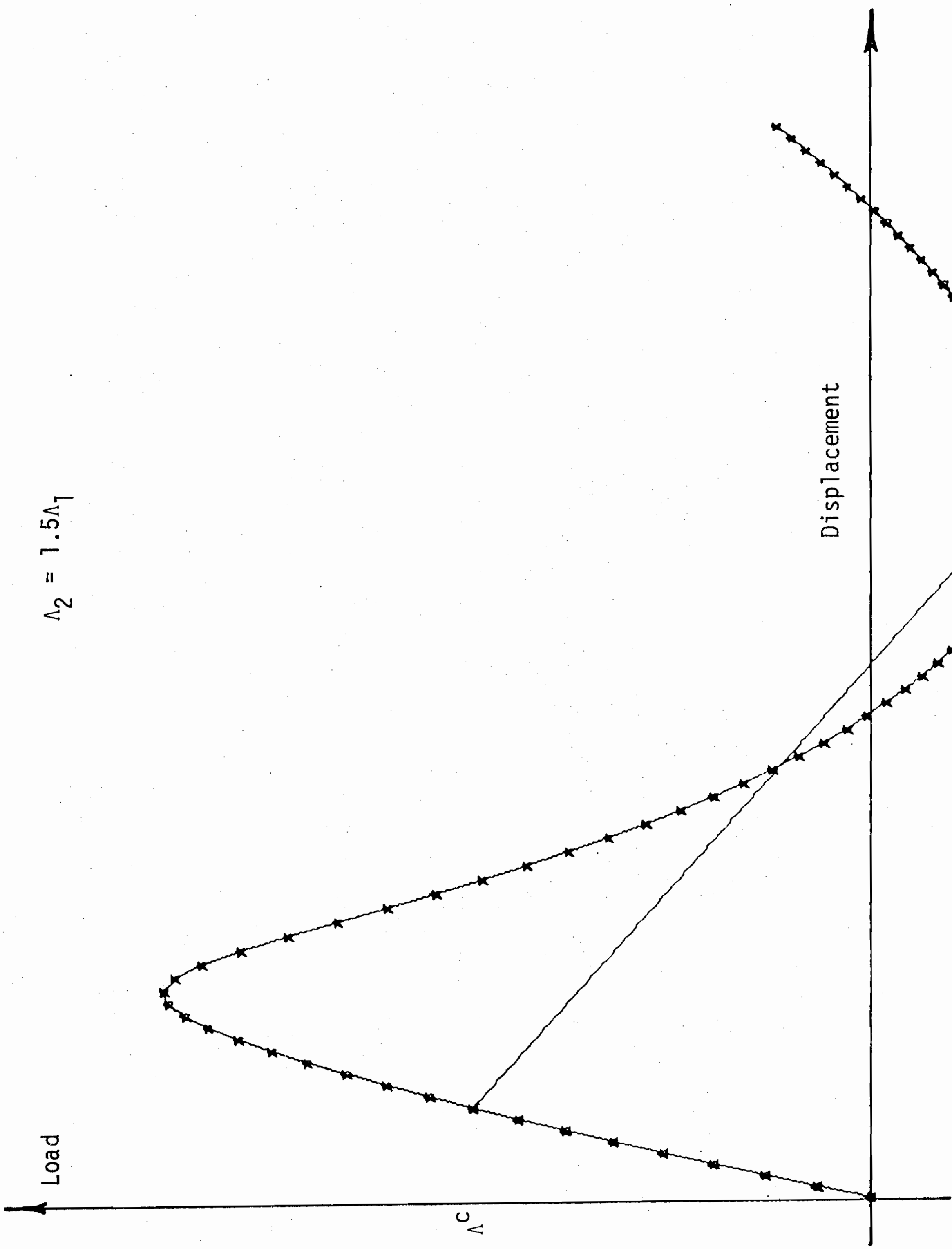


FIG. 5.8 EQUILIBRIUM PATH
(Bifurcation-Type Failure)

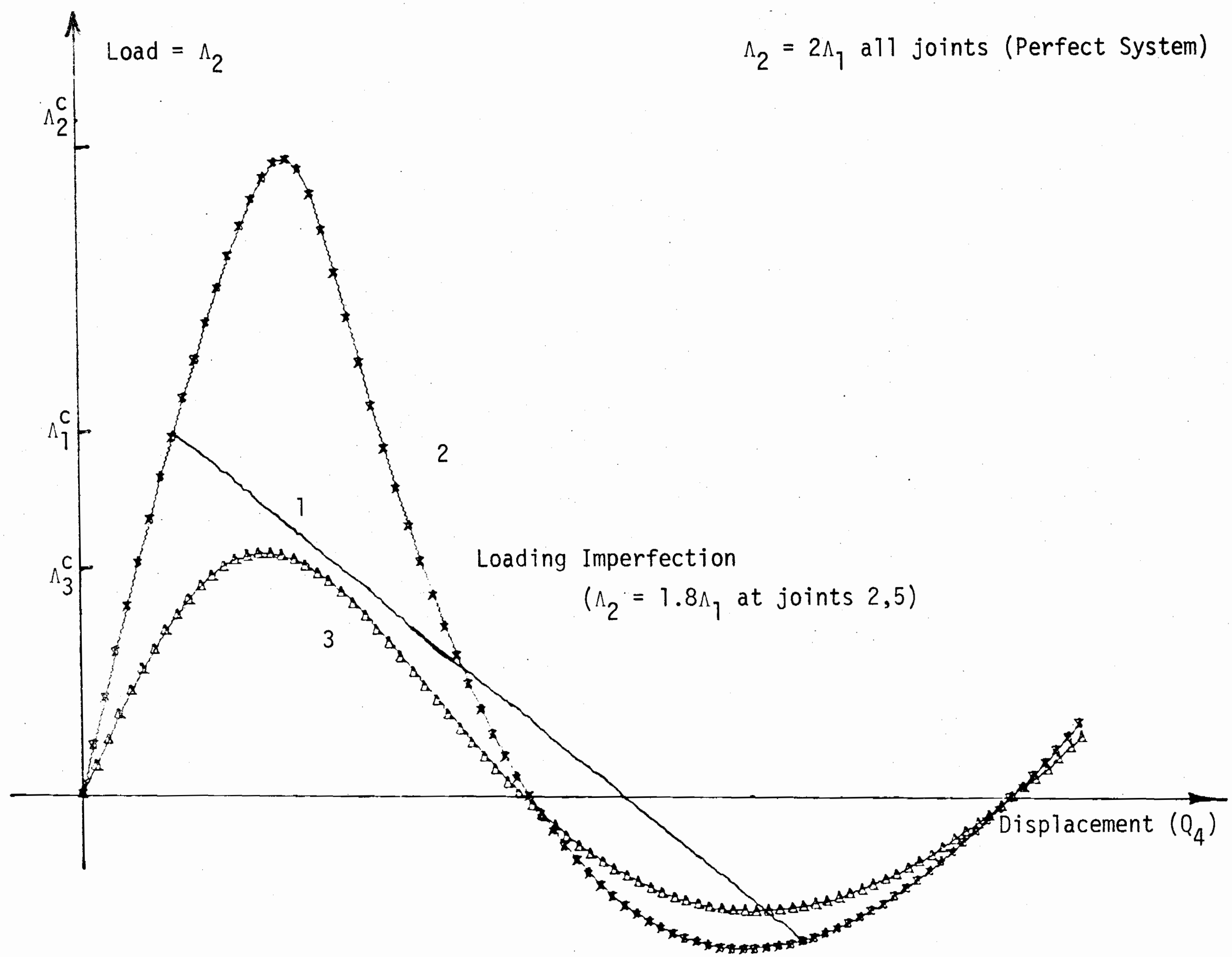


FIG. 5.9 BIFURCATION BUCKLING
(Imperfection-Sensitivity)

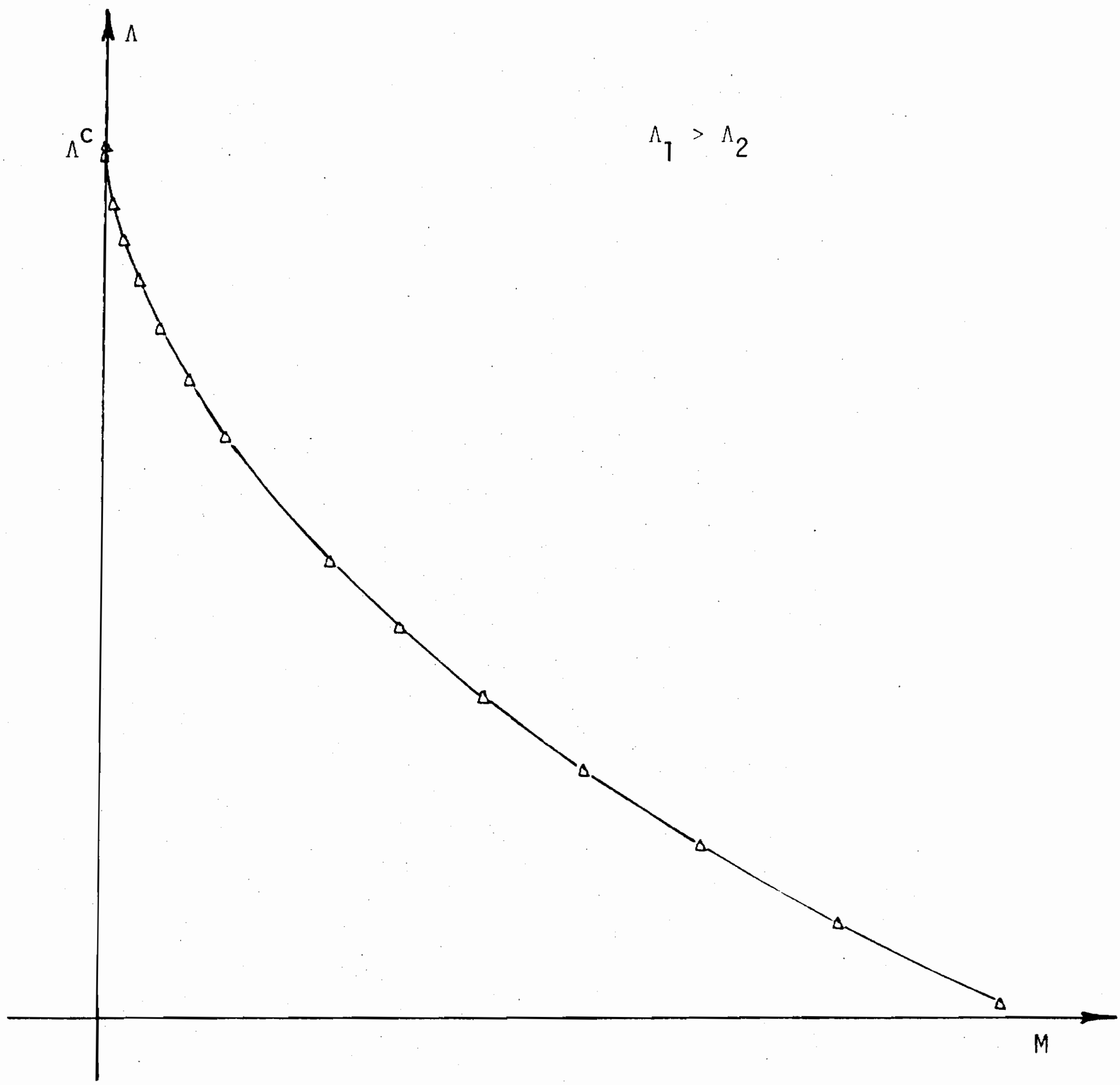


FIG. 5.10 DEGREE OF STABILITY

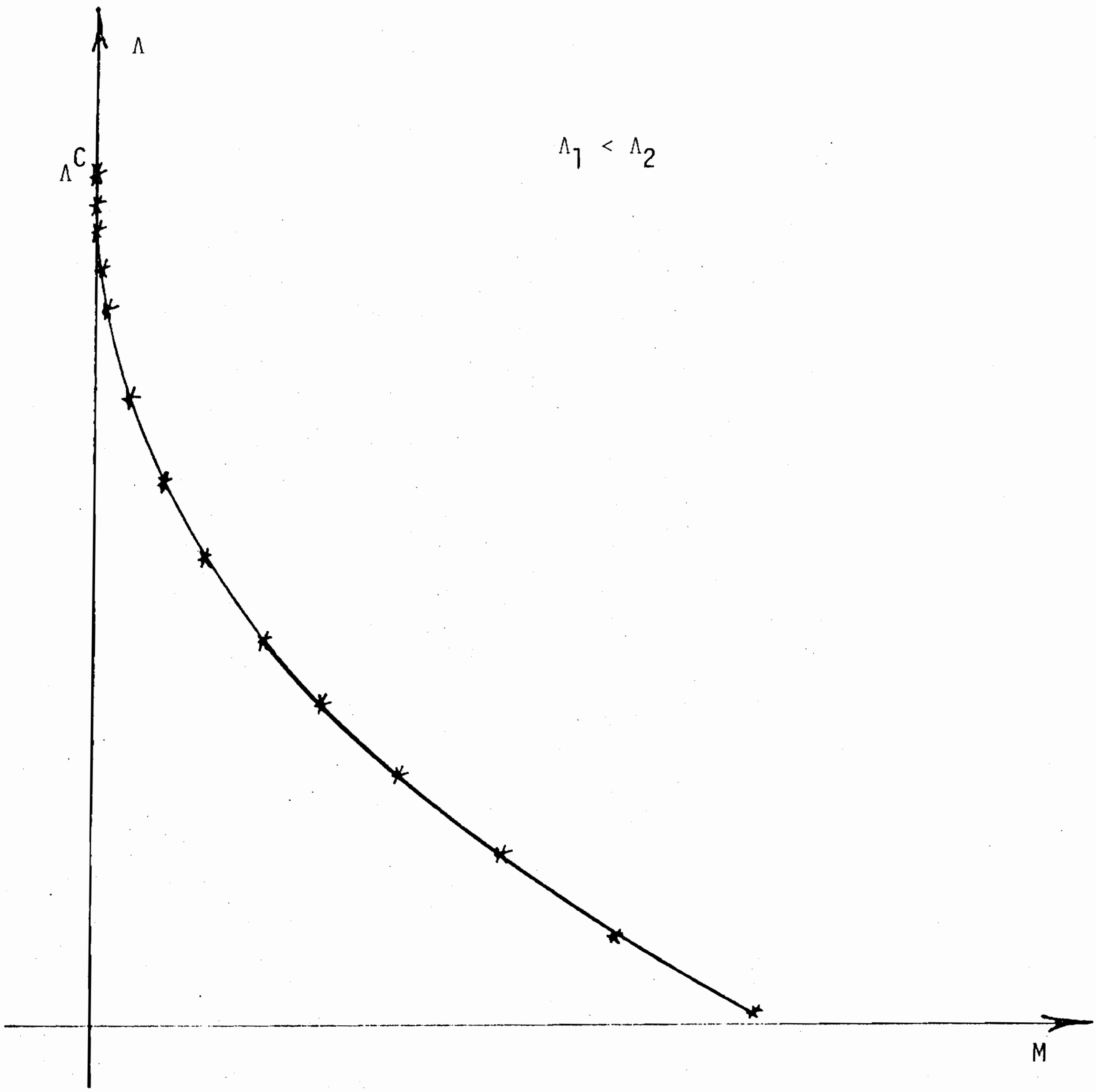


FIG. 5.11 DEGREE OF STABILITY

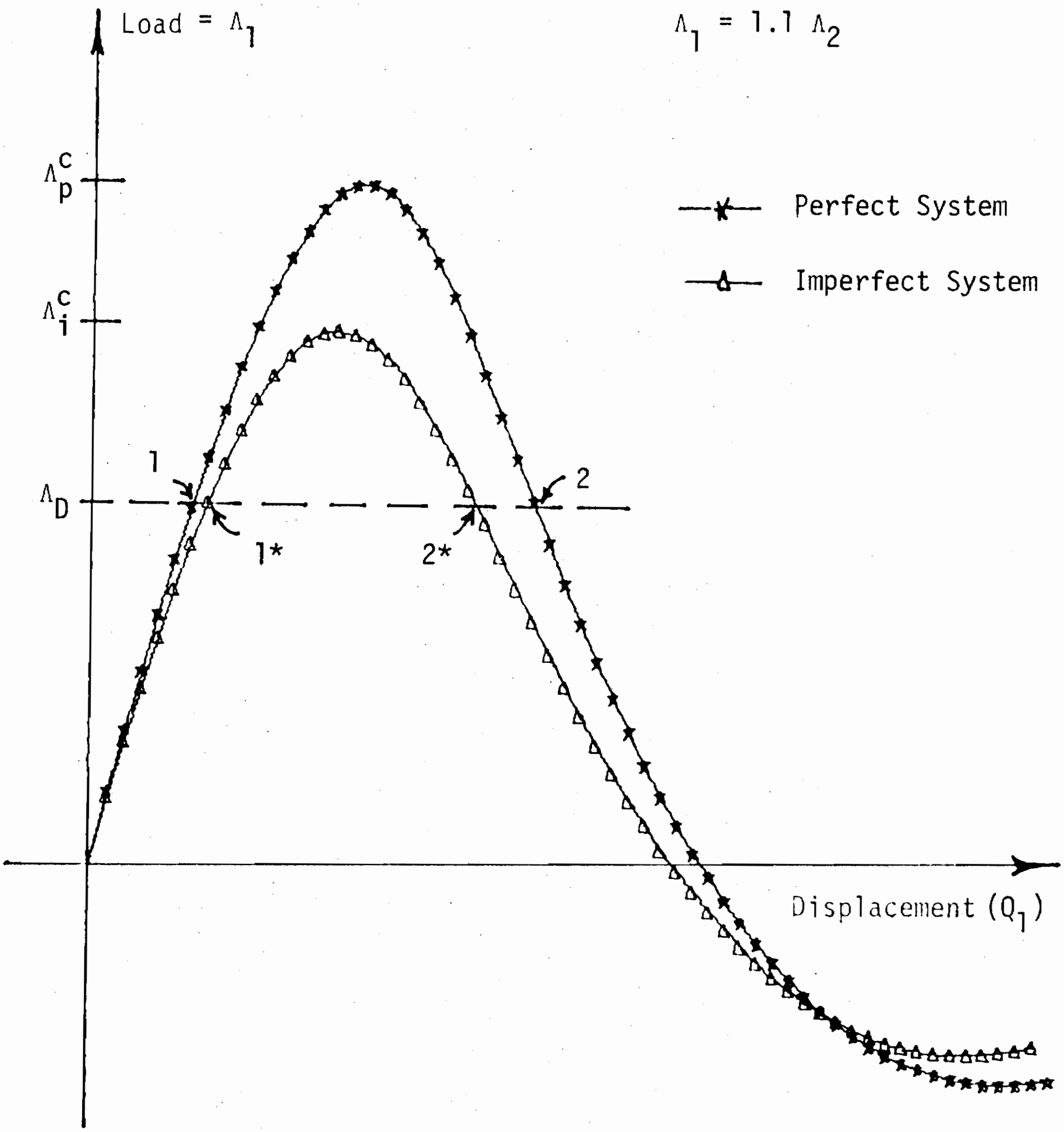


FIG. 5.12 GEOMETRIC IMPERFECTION

model. Geometric imperfections are introduced by reducing by 0.2 the coordinates of joints 2 and 5 in the X-direction (Fig. 5.1). As in the case of loading imperfections (Fig. 5.9), the limit load drops from Λ_p^c for the perfect dome to Λ_i^c for the imperfect dome. Point 2 represents the "nearest" unstable equilibrium state for the perfect dome. Correspondingly, point 2* represents the "nearest" unstable equilibrium state for the imperfect dome for a given design load, Λ_D. As shown in Fig. 5.13, the degree of stability, M_D, for the perfect system is reduced to a value M_D^* for the imperfect system. Hence imperfection sensitivity in reticulated domes are characterized by a reduction of critical load, Λ^c, and a reduction of M at the same load level.

In Figs. 5.14 - 5.16, the selection of the "nearest" unstable equilibrium state is illustrated. In the neighborhood of Λ_B^C, path 1 is "nearest" to the fundamental path. For loads less than Λ_1^*, path 2 is "nearest" to the fundamental path. As shown in Fig. 5.15, for loads Λ less than Λ_1^*, using path 1 as opposed to path 2 to estimate M will be nonconservative. This is further illustrated in Fig. 5.16 which includes the effect of loading imperfection where $\Lambda_2 = 2\Lambda_1$ defines the loading configuration for the perfect system. When the unstable equilibrium states "nearest" to the fundamental path are used, a safe estimate of the degree of stability of equilibrium is obtained. Fig. 5.16 again shows that the Λ - M curve for an imperfect system is a lower bound for the Λ - M curve of the corresponding perfect system.

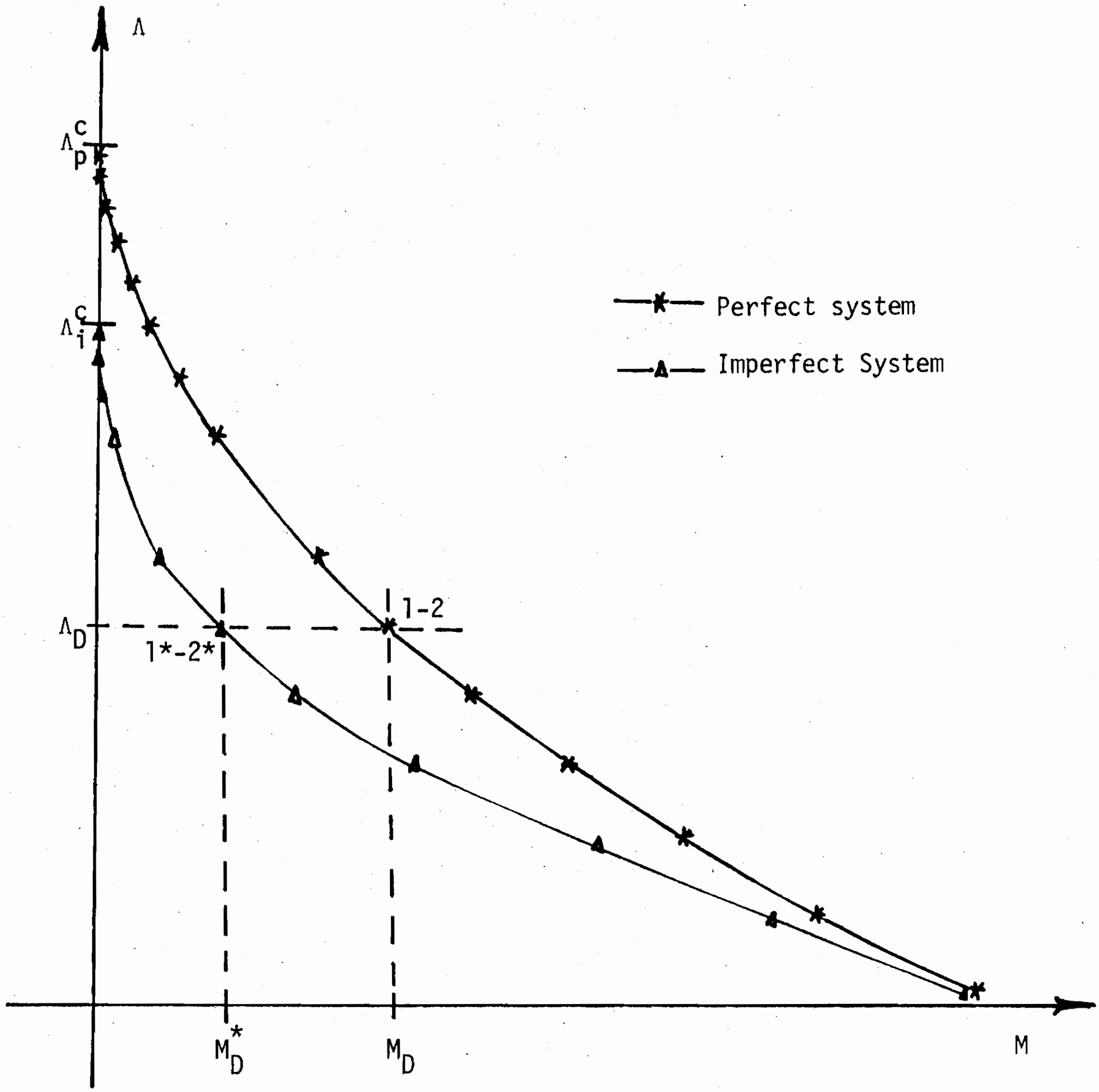


FIG. 5.13 EFFECT OF IMPERFECTIONS ON DEGREE OF STABILITY

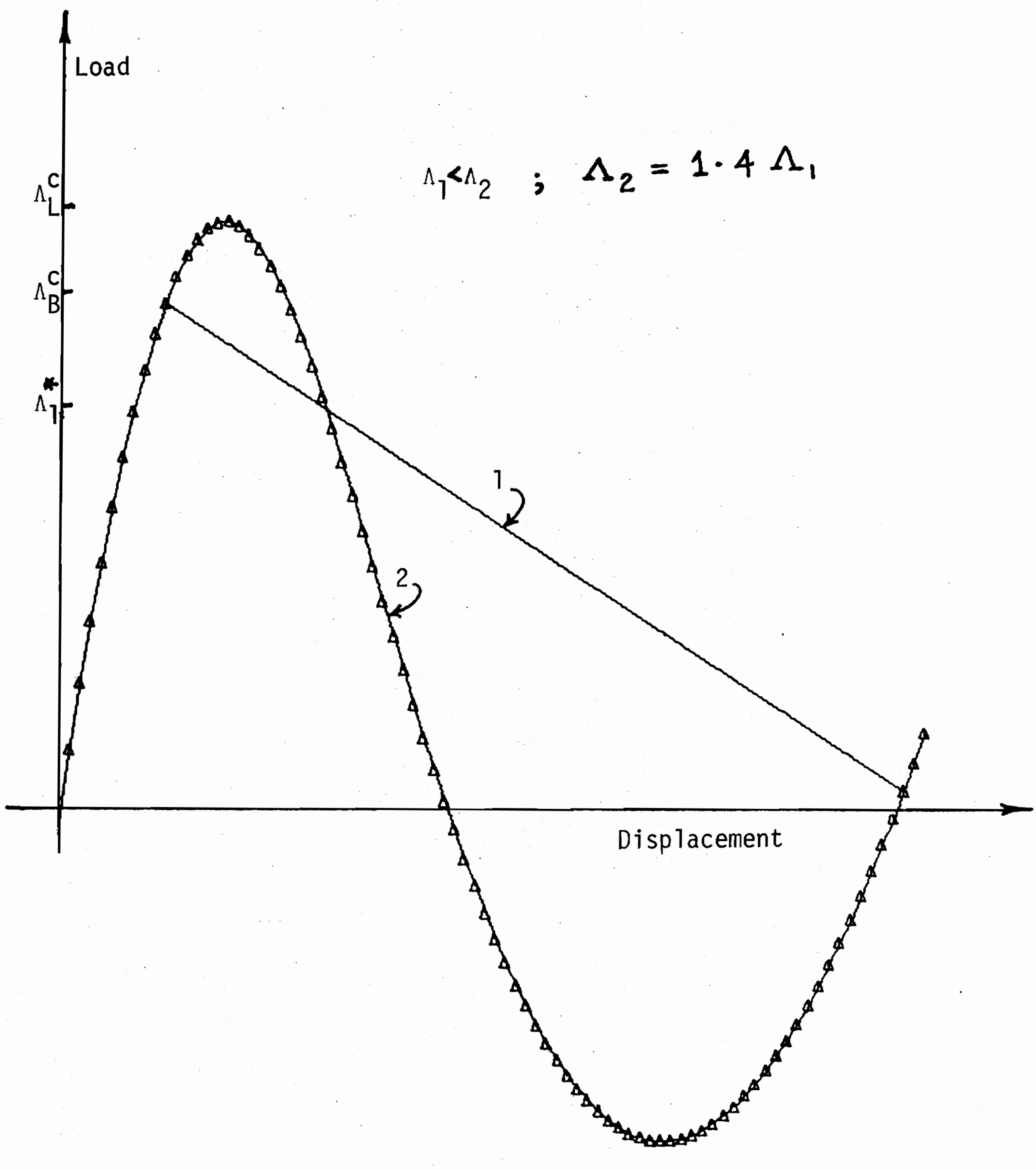


FIG. 5.14 EQUILIBRIUM PATH
("Nearest" Unstable Equilibrium States)

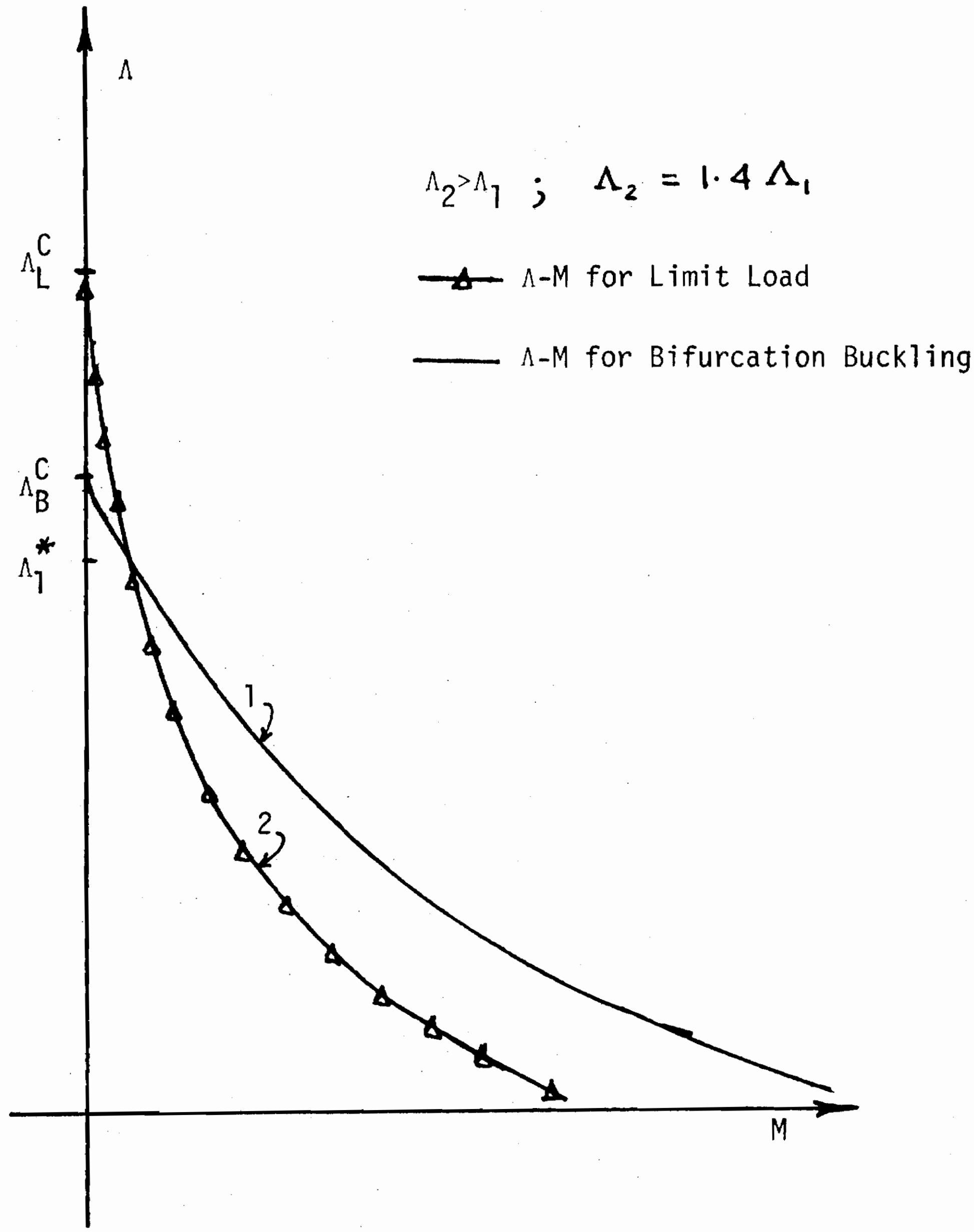


FIG. 5.15 RELATIVE DEGREE OF STABILITY

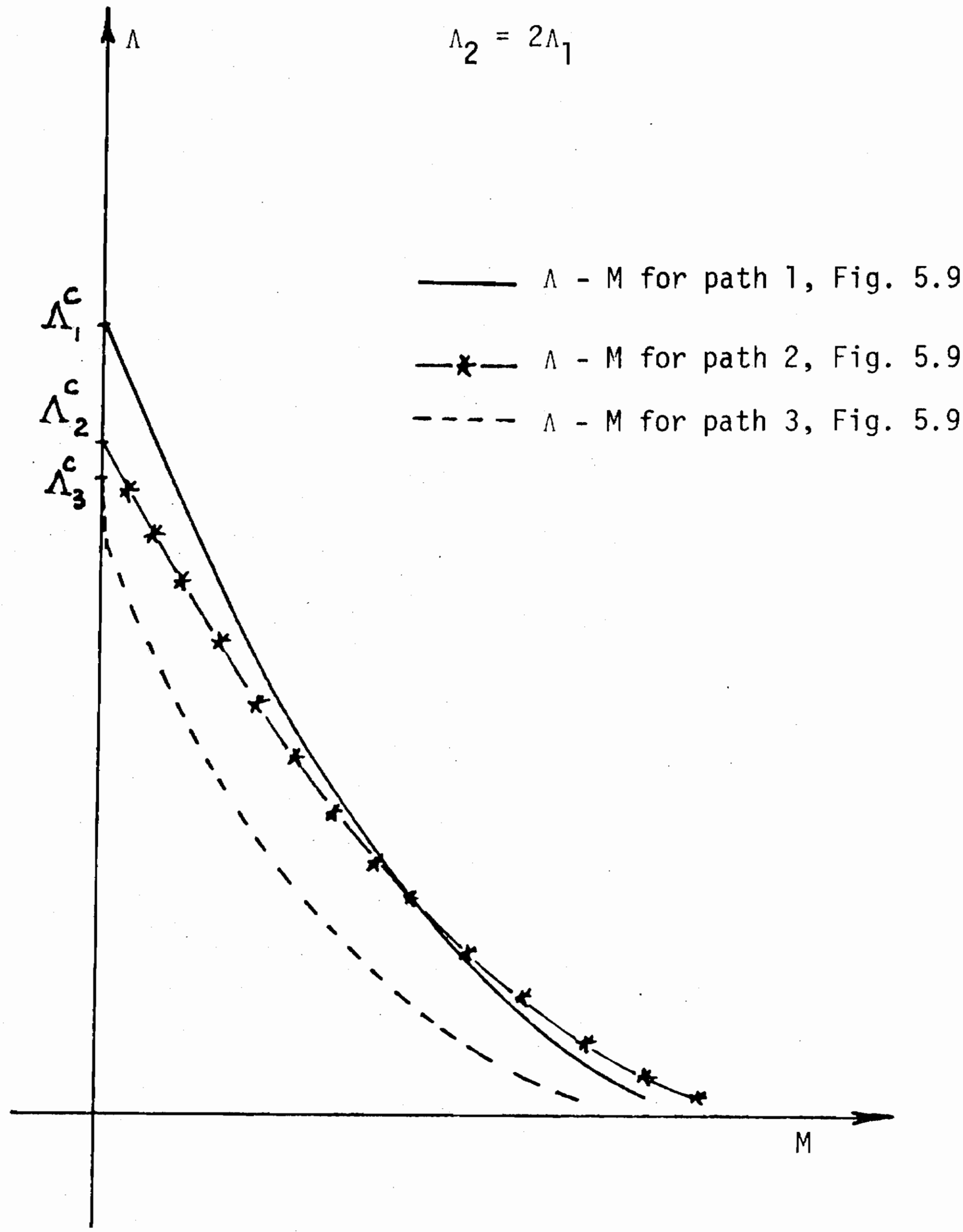


FIG. 5.16 IMPERFECTION SENSITIVITY AND DEGREE OF STABILITY

a. Stability of Equilibrium relative to Instantaneous Disturbances

The stability of equilibrium of the reticulated dome relative to instantaneous disturbances is tested using the sufficient stability condition (section 2.2) and by studying the perturbed motion. Instantaneous disturbances are defined in terms of initial conditions. The equilibrium state of the dome corresponds to a load level that is approximately 0.70 - 0.75 of the critical static load, Λ^c. The sufficient stability criterion defined by Eqs. 2.6 - 2.9 of section 2.2 are used as a basis for selecting initial conditions.

Figs. 5.17 and 5.18 depict the responses of joint 1 and joint 2 to an instantaneous disturbance applied at joint 1. The equilibrium configuration, the origin 0 in the phase plane, corresponds to two independent loads Λ_1, Λ_2 acting at all joint as shown in Fig. 5.1. The loading configuration is such that $\Lambda_1 = 1.1\ \Lambda_2$. The disturbance is defined by an initial state of zero displacements and a single velocity component acting at joint 1 in the X-direction. The initial energy v_o is such that $v_o = 0.95M$. The resulting undamped motions are closed trajectories surrounding the isolated singularity 0. Hence the system (i.e., the origin) is stable with respect to the disturbance associated with an initial energy v_o. This implies that the perturbed motion is confined to the domain of asymptotic stability of the origin. It further implies that the initial disturbance, y_o, belongs to C_M (a subset of the domain of attraction of origin). This is the sufficient stability criterion [45].

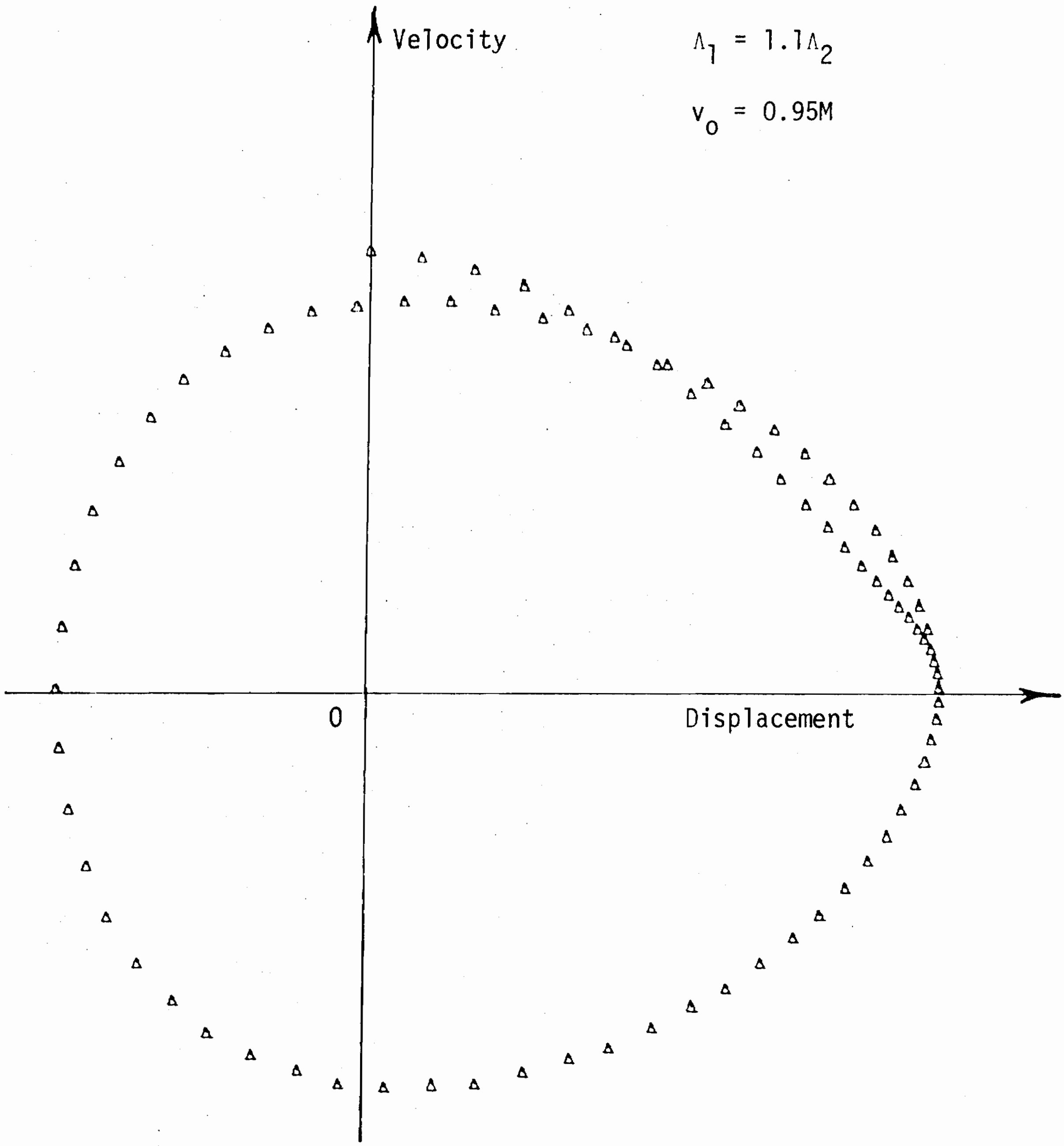


FIG. 5.17 RESPONSE OF JOINT 1
(Joint 1 Loaded with Instantaneous Disturbance)

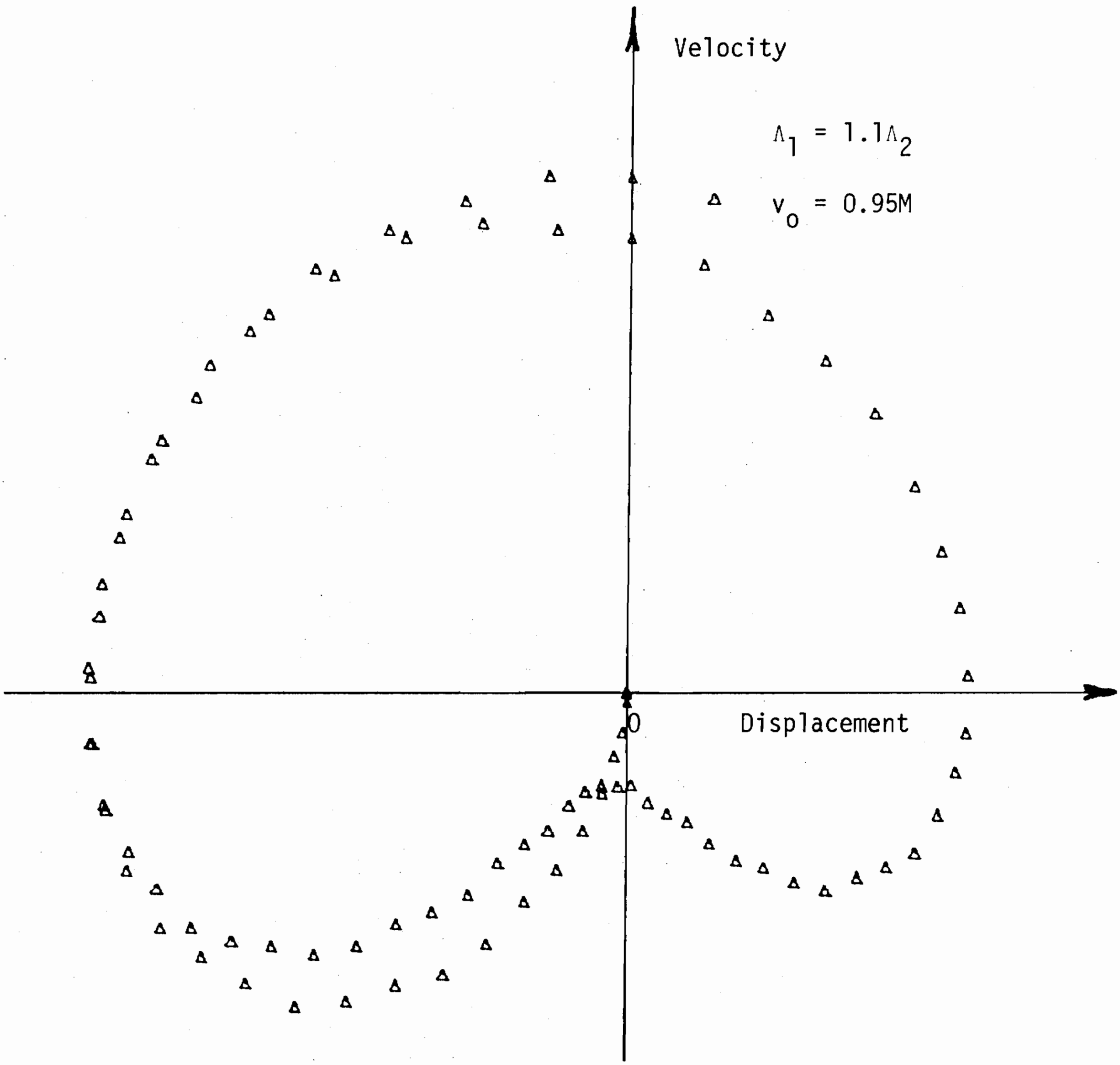


FIG. 5.18 RESPONSE OF JOINT 2
(Joint 1 Loaded with Instantaneous Disturbance)

With damping, the perturbed motion of the system relative to the initial disturbance described above converges to the origin of the phase plane. In this case, equilibrium (origin) is asymptotically stable. This is illustrated in Fig. 5.19.

Fig. 5.20 depicts the same equilibrium configuration subjected to an initial disturbance with an initial energy greater than M. In this case, with v_0 = 1.04M, the motion indicates instability of the origin. The effect of damping is shown in Fig. 5.21. The damped system converges to a new state, the buckled state of the reticulated dome. Since an infinitesimal change in the disturbance causes a finite change in the maximum deviation of the perturbed motion, the sufficient condition is also necessary for stability of equilibrium for this particular case.

With $\Lambda_1 = 1.1\ \Lambda_2$, the responses of joint 2 to initial disturbances at joint 2 are shown in Figs. 5.22 and 5.23. In Fig. 5.22, v_0 = 0.95M and the origin is stable as expected from the sufficient stability condition (section 2.3). However, with v_0 = 1.15M, the response of joint 2 indicates that the origin is a stable equilibrium point. This inference is made since the perturbed motion encloses only the origin in the phase plane. This case confirms that the stability criterion employed is only sufficient and need not be necessary for stability of equilibrium. This is illustrated in Fig. 5.23.

Fig. 5.24 illustrates the responses of joint 2 to instantaneous disturbance applied at joint 2. In this case, $\Lambda_2 = 2\ \Lambda_1$, and the resulting perturbed motions indicate stability, asymptotic stability

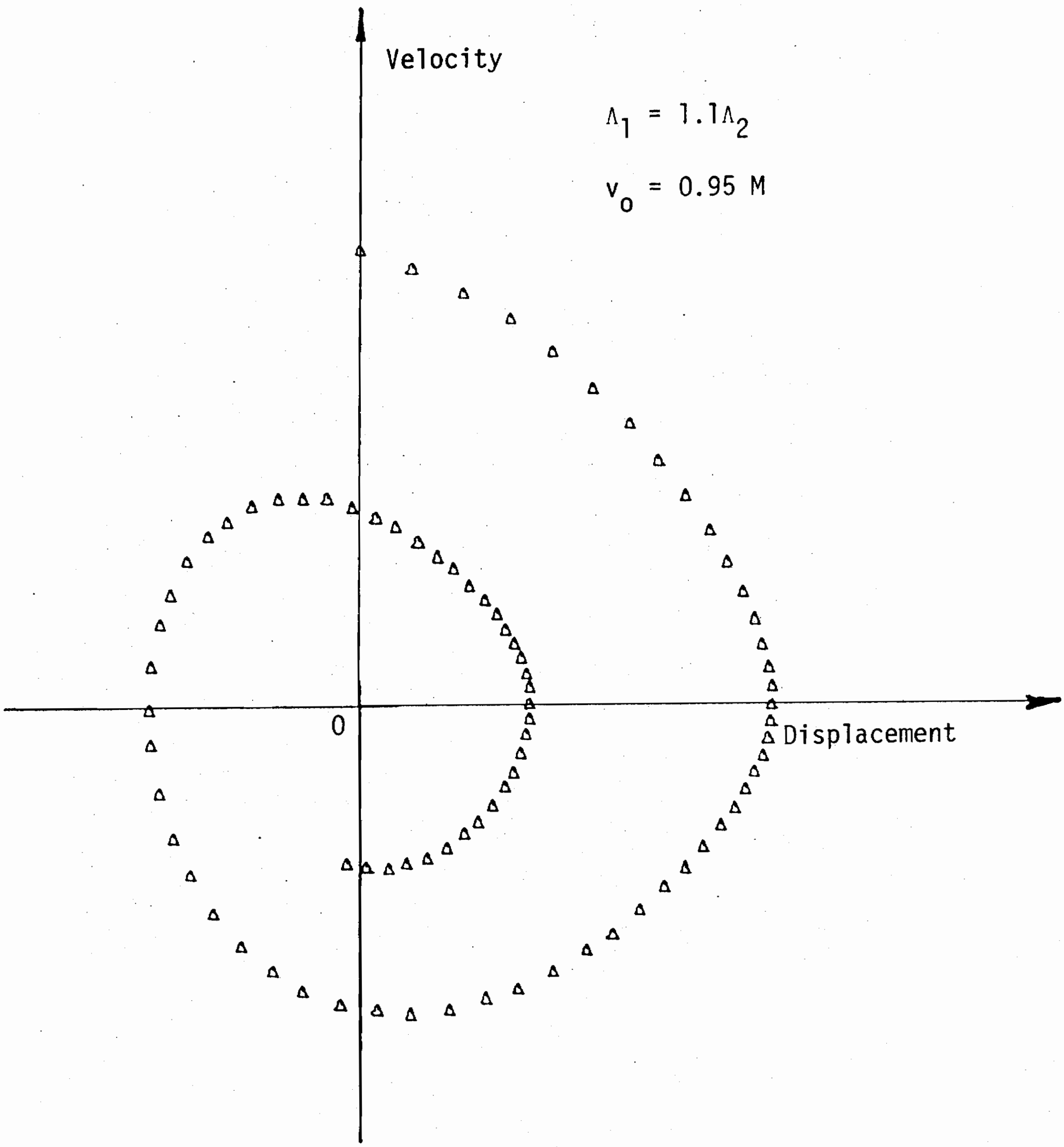


FIG. 5.19 DAMPED RESPONSE OF JOINT 1
(Instantaneous Disturbance at Joint 1)

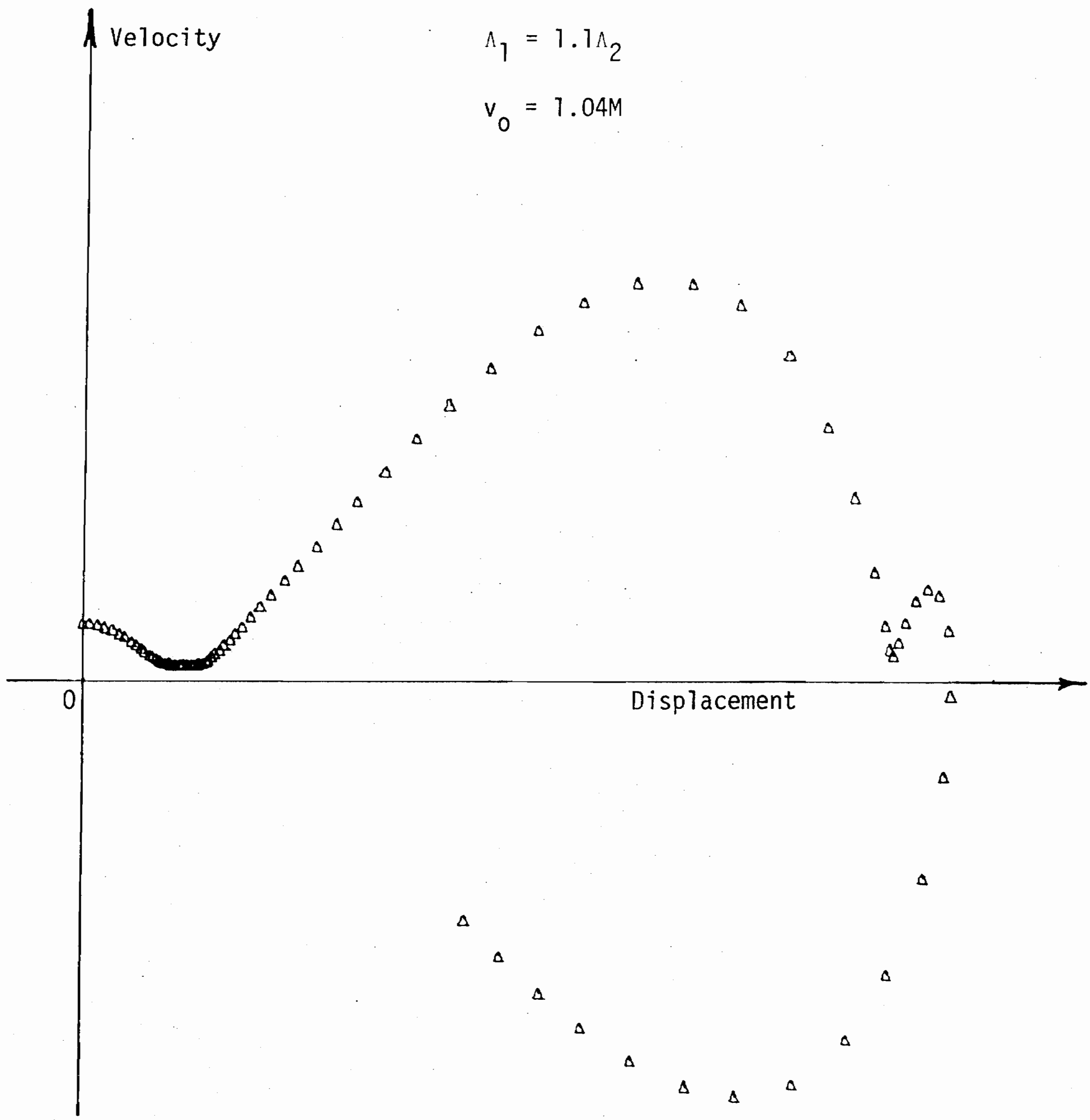


FIG. 5.20 UNDAMPED RESPONSE OF JOINT 1
(Instantaneous Disturbance at Joint 1)

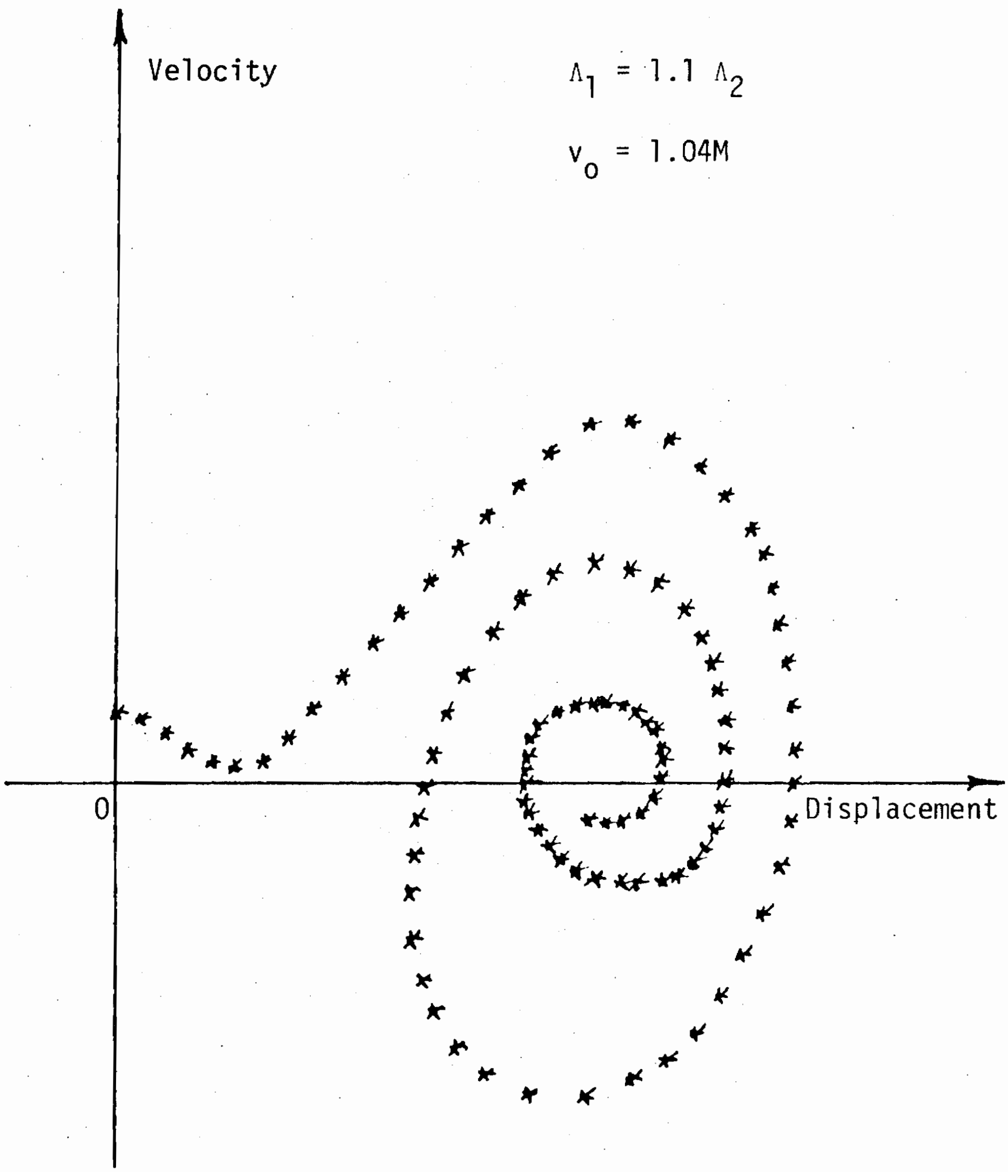


FIG. 5.21 DAMPED RESPONSE OF JOINT 1
(Instantaneous Disturbance at Joint 1)

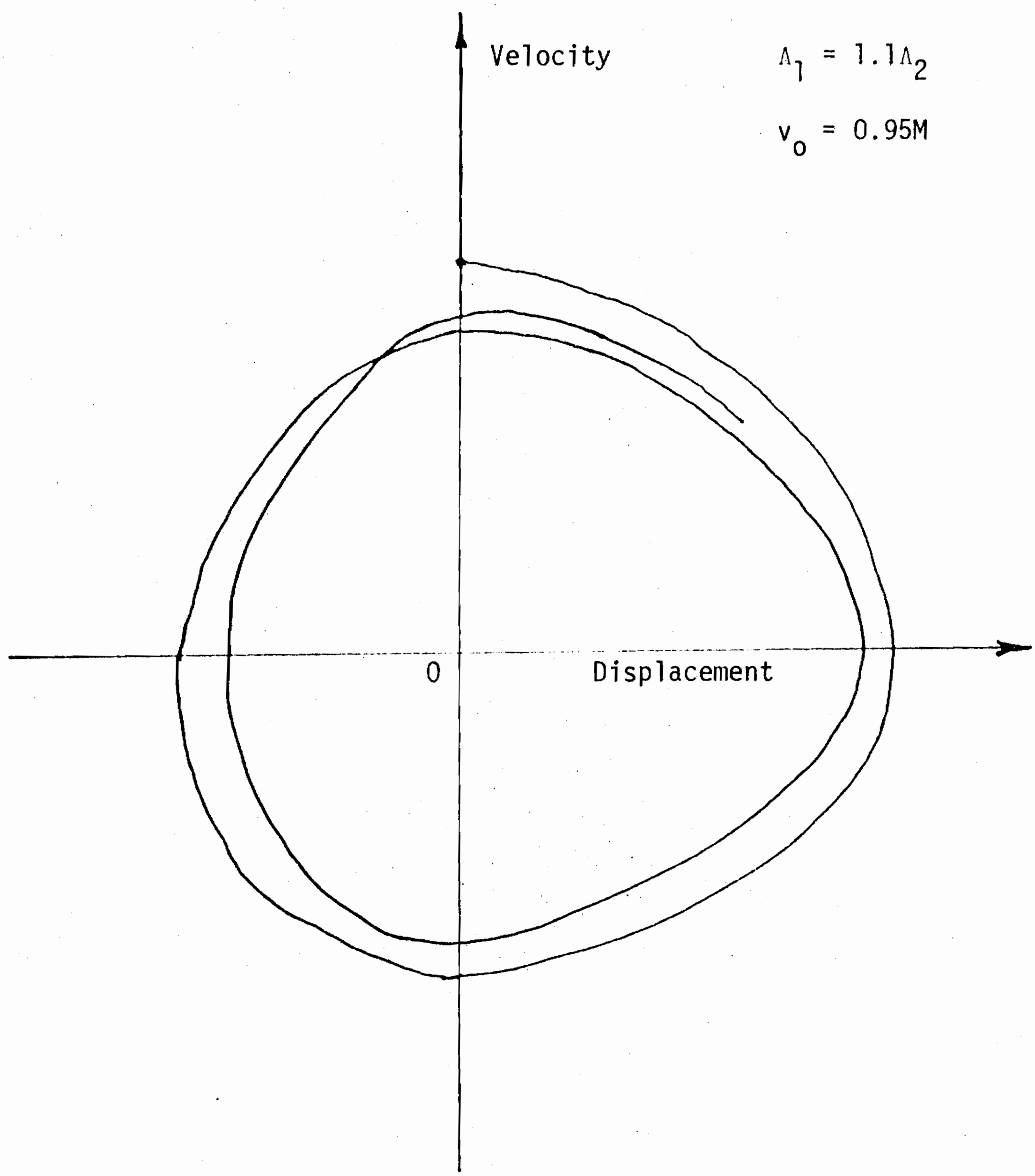


FIG. 5.22 RESPONSE OF JOINT 2
(Instantaneous Disturbance at Joint 2)

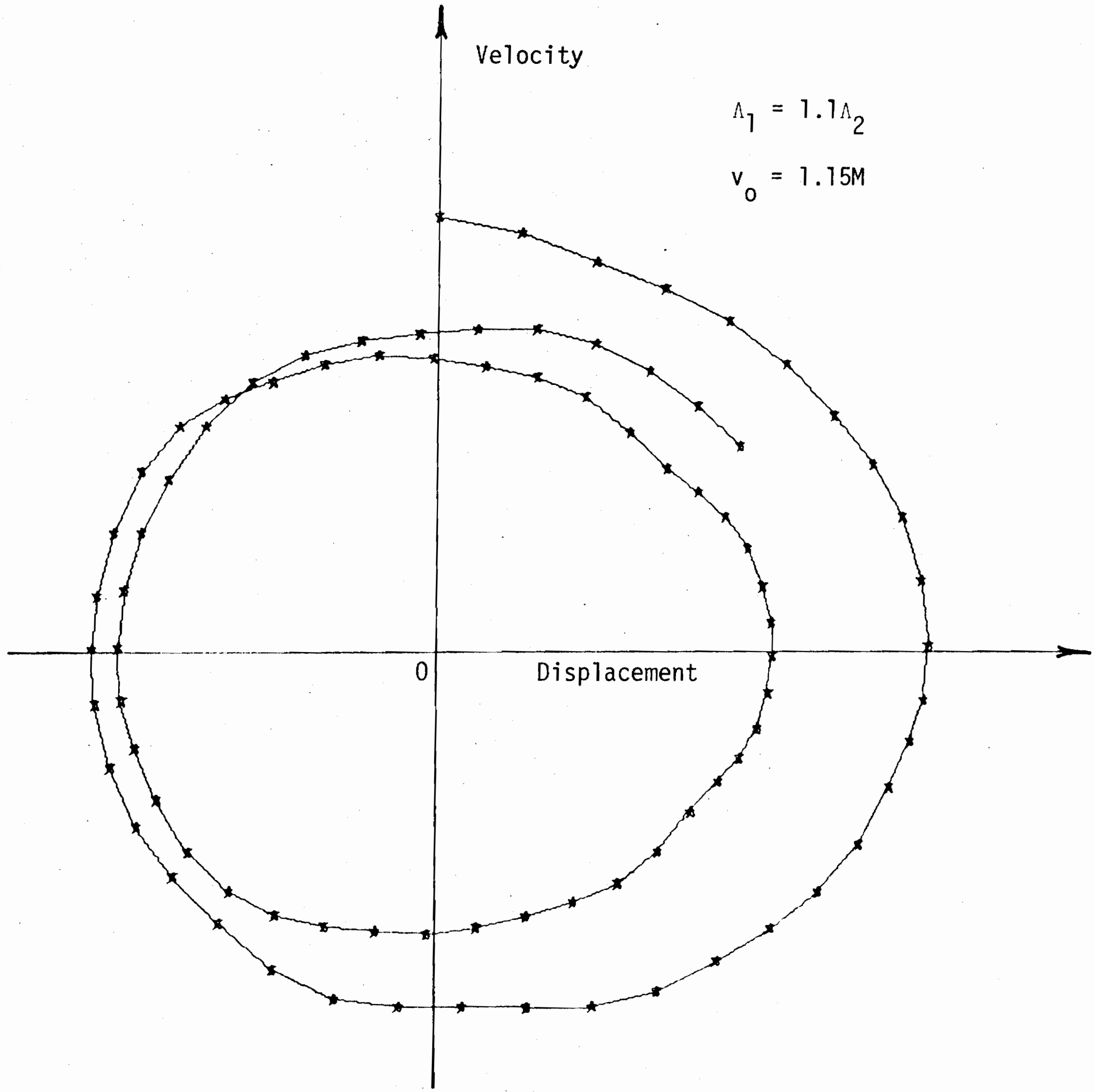


FIG. 5.23 RESPONSE OF JOINT 2
(Instantaneous Disturbance at Joint 2)

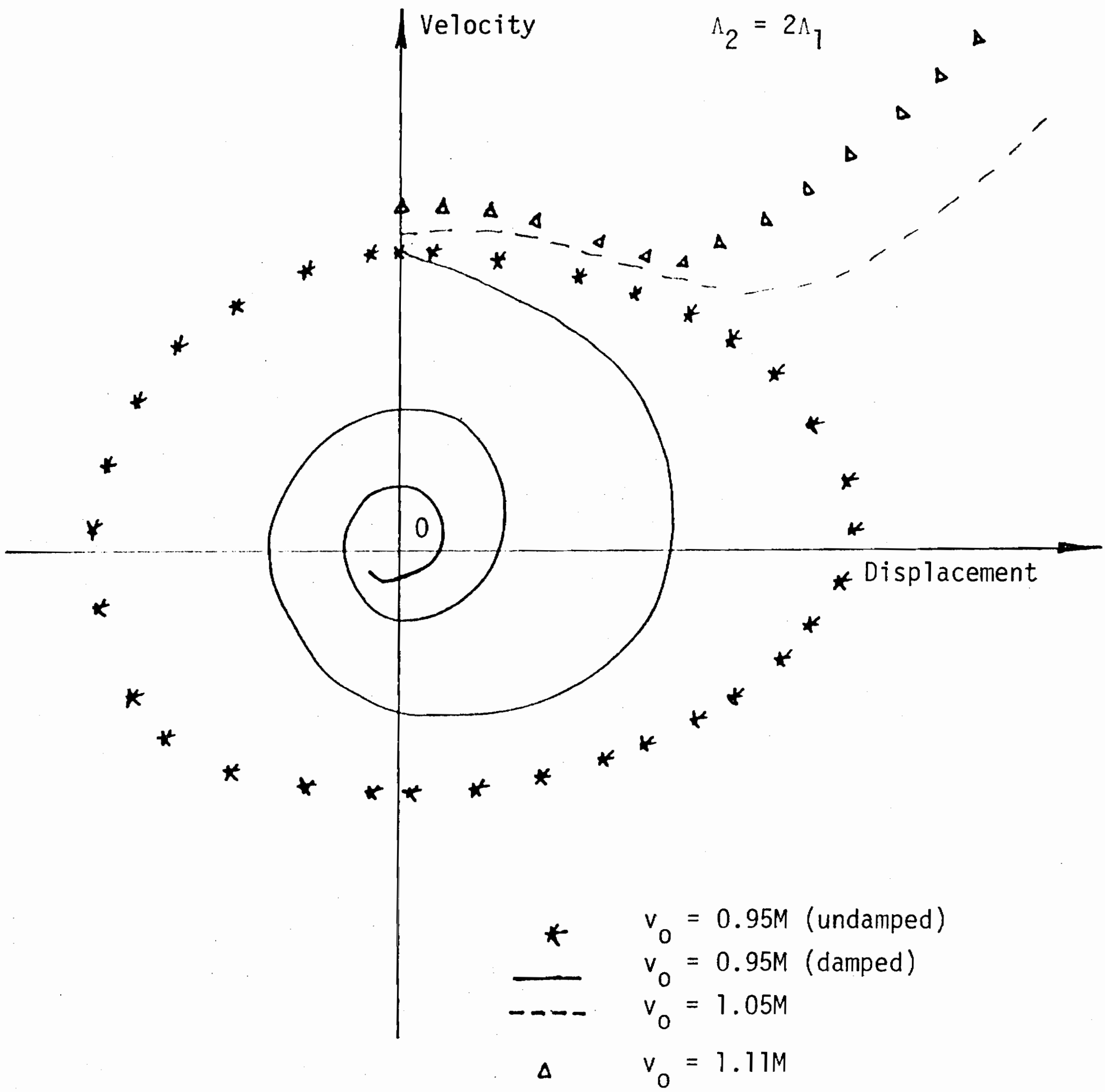


FIG. 5.24 RESPONSE OF JOINT 2
(Instantaneous Disturbance at Joint 2)

and instability of the origin depending on the magnitude of v_0 and the dissipative nature of the system. The sufficient condition is also necessary, in this case, for stability of equilibrium. A qualitatively similar response to that shown in Fig. 5.24 was observed for the case $\Lambda_1 = o$, $\Lambda_2 \neq o$ with joints 2, 5 instantaneously disturbed.

Under the equilibrium configuration corresponding to the load distribution $\Lambda_2 = 2\ \Lambda_1$, the responses of joint 1 to an instantaneous disturbance applied at joint 1 is shown in Fig. 5.25. The perturbed motions indicate that the stability criterion is only sufficient but not necessary for stability of the equilibrium state 0.

With $\Lambda_1 \neq o$ and $\Lambda_2 = o$, the responses of joint 1 to an instantaneous disturbance applied at joint 1 are qualitatively similar to those of Fig. 5.24.

If the dome model is dissipative, the total energy along the perturbed motion might drop below M even when the initial energy, v_0, is greater than M. This is illustrated in Fig. 5.26 for the loading configuration $\Lambda_1 = 1.1\ \Lambda_2$. For such a case, the origin is asymptotically stable and the stability criterion remains only strictly a sufficient stability condition.

The above examples show that the geodesic dome model is more snesitive to disturbances that appear to magnify the equilibrium configuration than to disturbances that tend to excite a different configuration. When the equilibrium configuration is magnified by the dynamic disturbance, the sufficient stability criterion is also necessary for stability of equilibrium. For all cases tested and

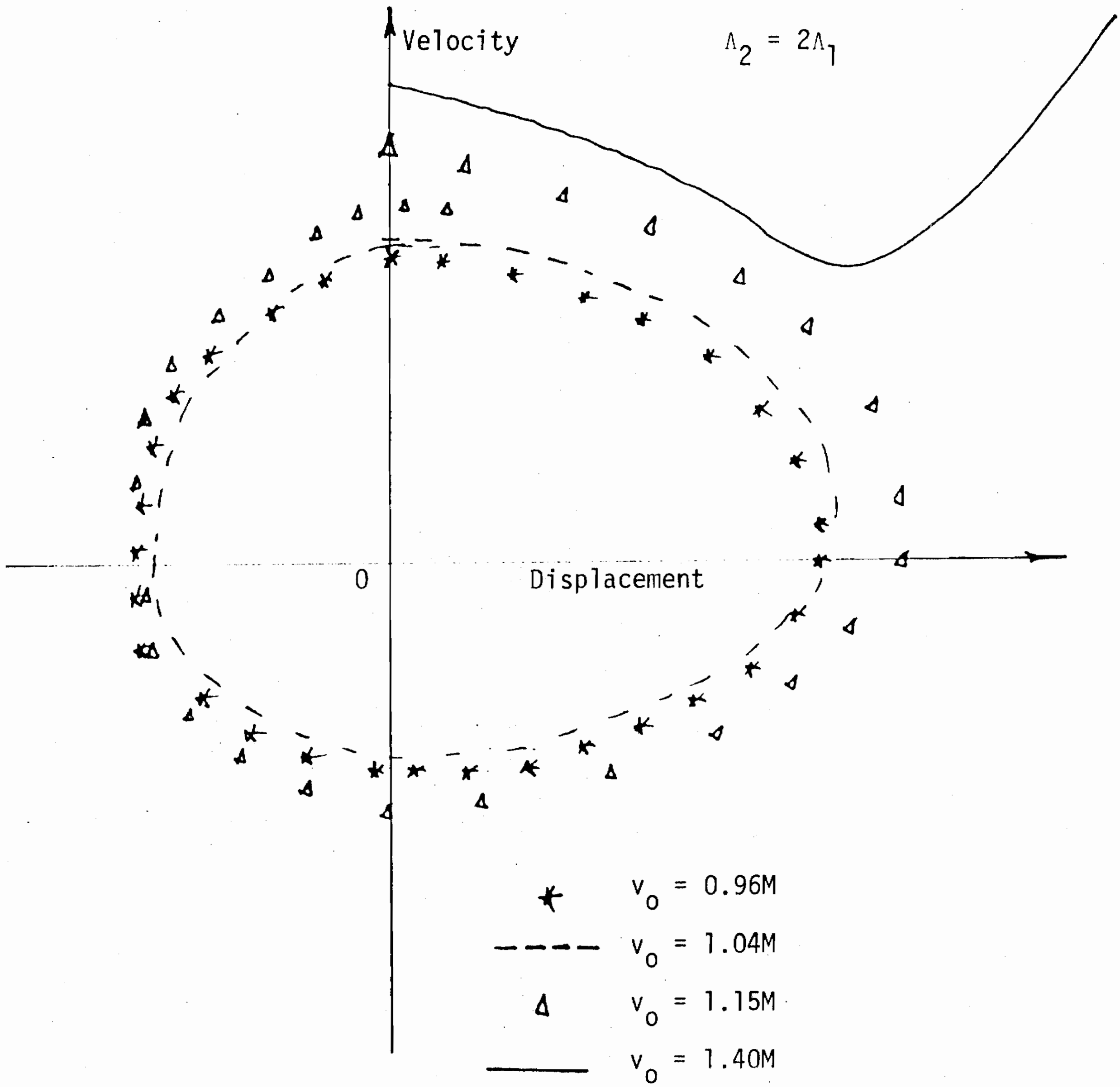


FIG. 5.25 RESPONSE OF JOINT 1
(Instantaneous Disturbance at Joint 1)

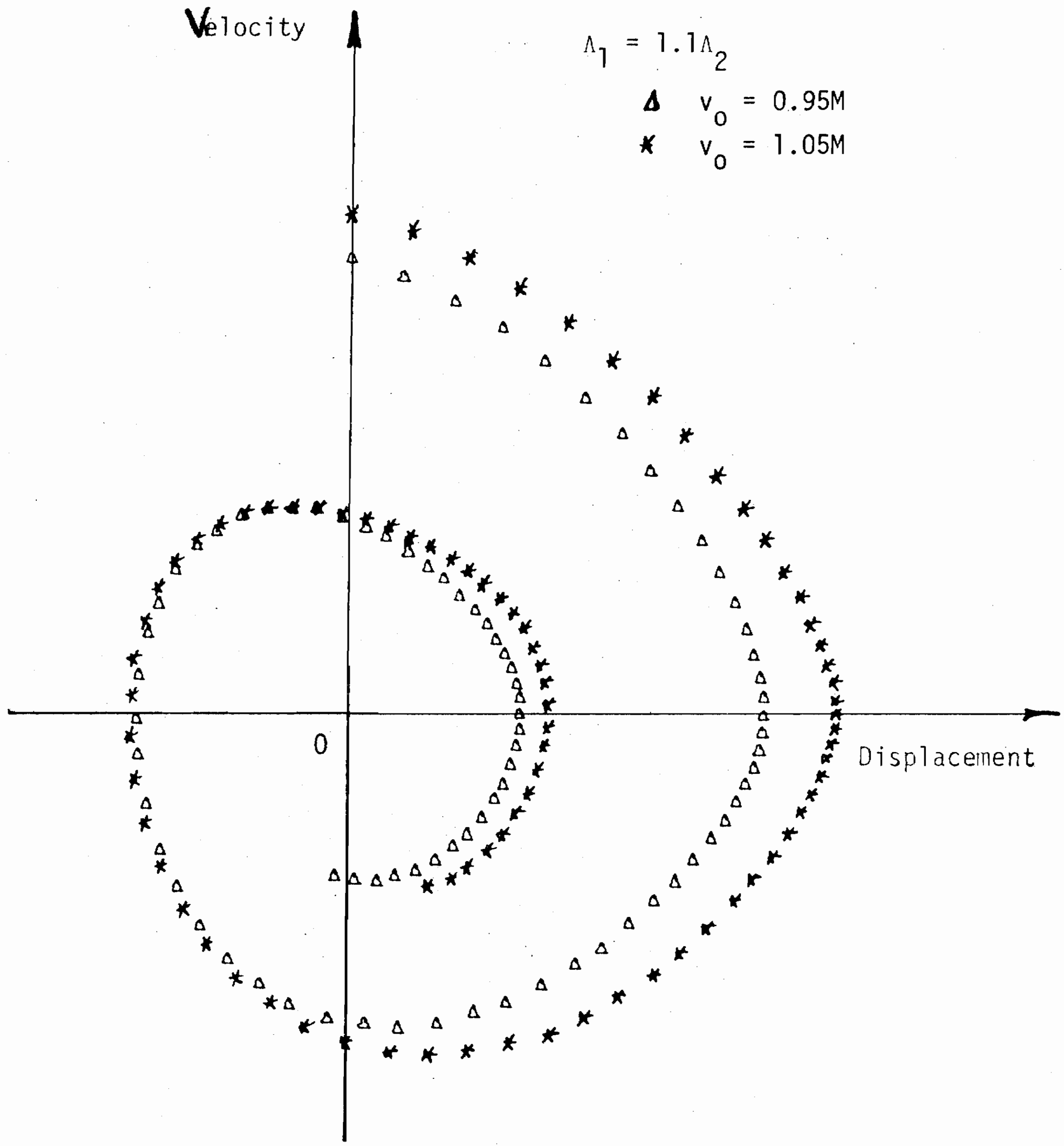


FIG. 5.26 DAMPED RESPONSE OF JOINT 1
(Instantaneous Disturbance at Joint 1)

discussed above, the stability criterion is necessary and sufficient for stability of equilibrium if the following condition is satisfied: the disturbance consisting of a single velocity component is applied in the direction of the generalized displacement whose increment was proposed as a "suitable" choice for the perturbation parameter, ε, in the static analysis. In all cases, the disturbance was applied at a loaded joint. Hence, for each static loading condition, there exists a dynamic disturbance (satisfying condition stated above) with a specific distribution for which the sufficient condition of dynamic stability is also a necessary condition. The sufficient stability condition of section 2.2 is, therefore, practical for the design of reticulated domes.

b. Stability of Equilibrium Relative to Transient Disturbances

Under two independent loads Λ_1, Λ_2 with $\Lambda_1 > \Lambda_2$, Fig. 5.27 shows the response of joint 1 to a transient disturbance applied at joint 1. The equilibrium state corresponds to 0.75 Λ_i^c; where Λ_i^c are the critical values for Λ_1 and Λ_2. A single exciting force with constant amplitude and finite duration (step function) is applied at joint 1. The degree of the disturbance is controlled via the duration of the exciting force. At point 1 along the perturbed motion, the total energy v_1 = 0.94M. According to the stability criterion, equilibrium is stable relative to the prescribed disturbance. This is illustrated by the resulting motion (Fig. 5.27), which initiates at the origin, enters a closed trajectory at the end of the excitation, and remains on this

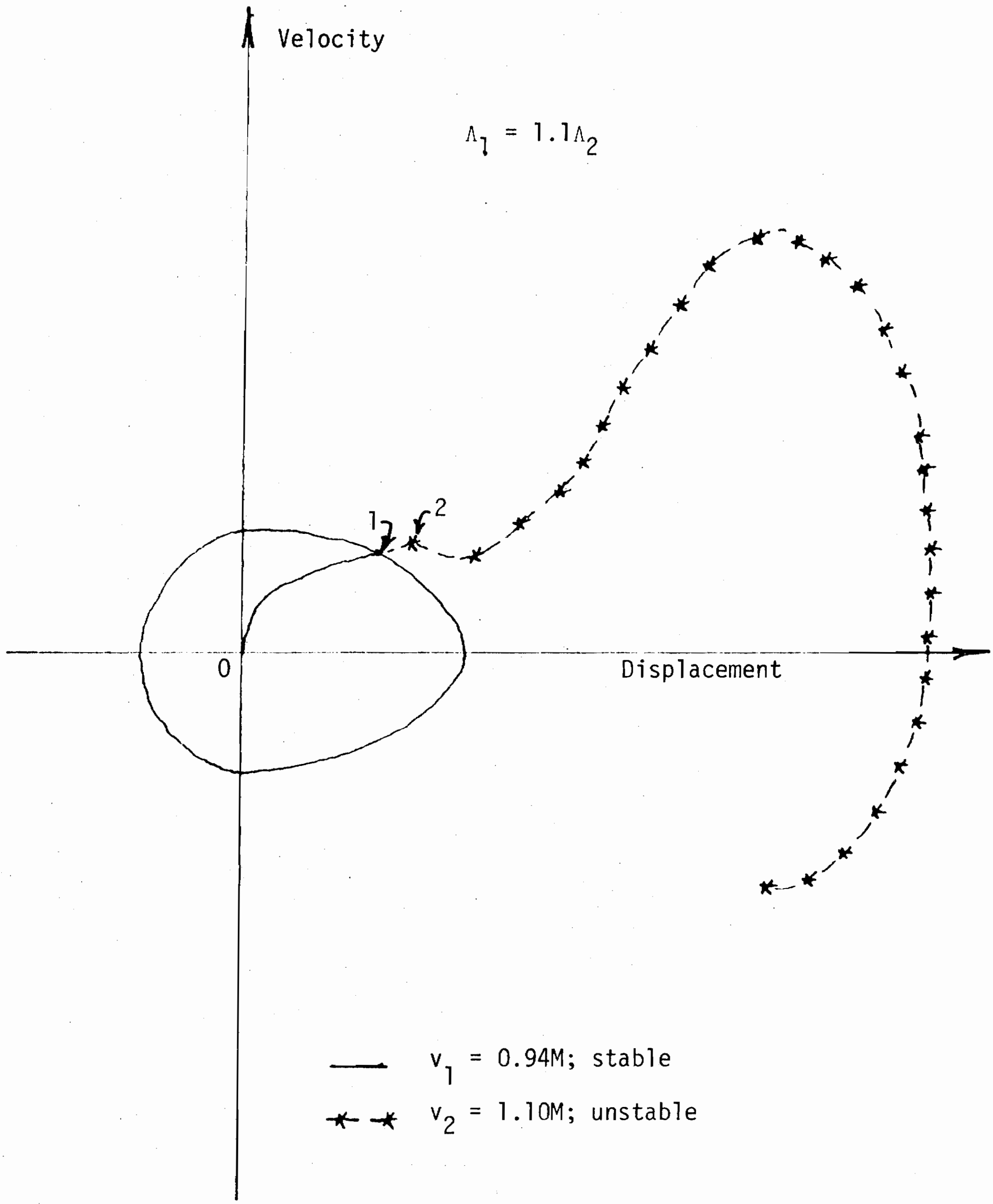


FIG. 5.27 RESPONSE OF JOINT 1
(Transient Disturbance at Join 1)

trajectory. The second excitation causes the motion to escape the domain of attraction of the origin, 0. The maximum energy along the perturbed motion v_2 (point 2, Fig. 5.27) is such that v_2 = 1.1M. In this case, the resulting motion shows instability of the equilibrium state 0.

Fig. 5.28 shows the effect of damping on the responses discussed in relation to Fig. 5.27. At point 1, v_1 = 0.95M and the origin is asymptotically stable. At point 2, v_2 = 1.1M and the perturbed motion converges to the buckled state showing instability of the origin.

5.2 Geodesic Dome: 39 DOF Model

The thirty-nine DOF geodesic dome model is shown in Fig. 5.2. The model is analyzed for different loading conditions:

1. $\Lambda_1 \neq o,\ \Lambda_2 = \Lambda_3 = 0$
2. $\Lambda_1 = \Lambda_2 > \Lambda_3$
3. $\Lambda_1 = 0,\ \Lambda_2 \neq o,\ \Lambda_3 = o$
4. $\Lambda_1 = \Lambda_2,\ \Lambda_3 = o$
5. $\Lambda_1 > \Lambda_2,\ \Lambda_3 = o$
6. $\Lambda_1 < \Lambda_2,\ \Lambda_3 = o$

The 39 DOF geodesic dome model exhibits features that are qualitatively similar to those of the 21 DOF model under comparable loading conditions.

For illustrative purposes, two loading conditions are shown in Figs. 5.29a and 3.29b. Both loading cases display limit points as

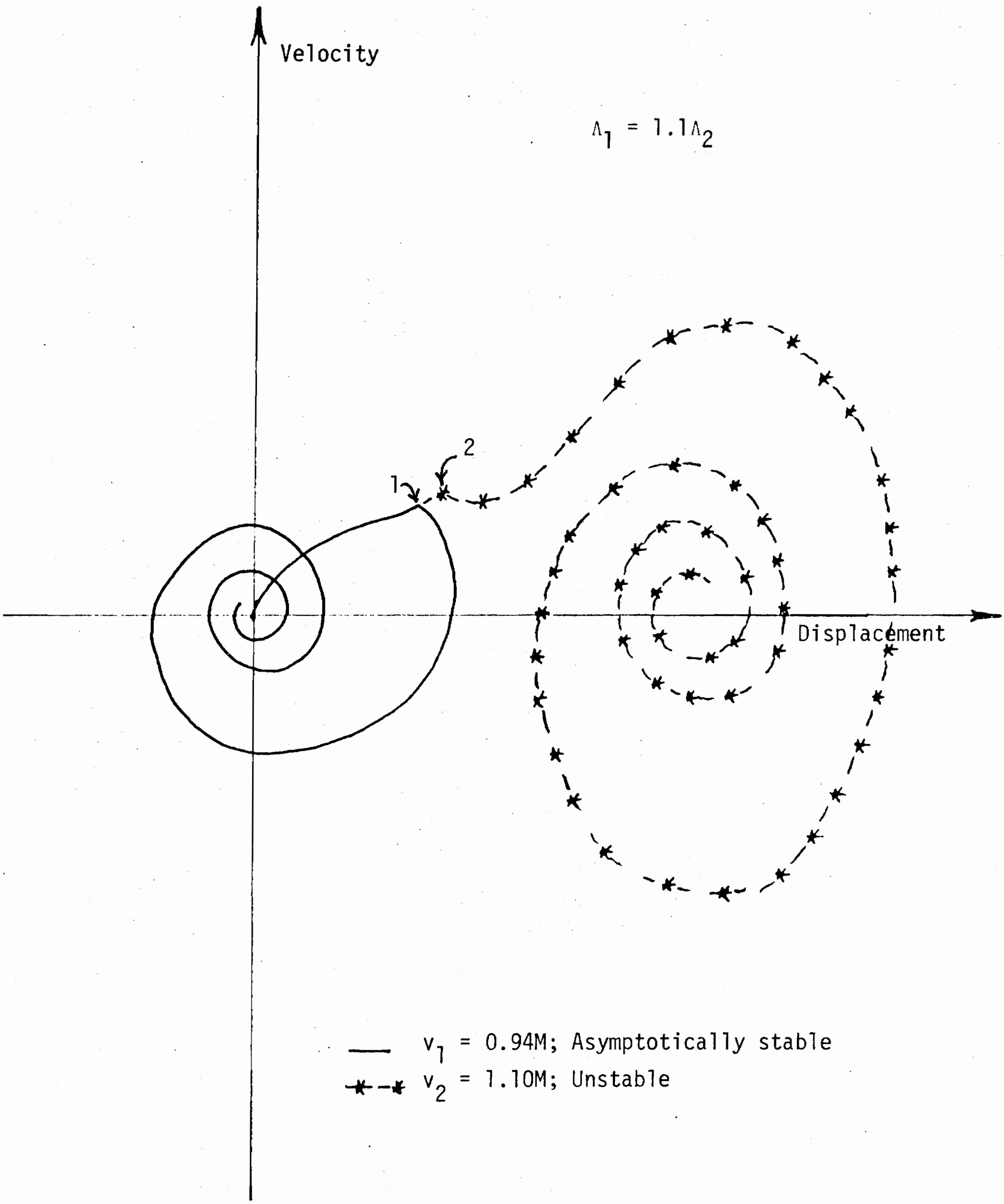


FIG. 5.28 DAMPED RESPONSE OF JOINT 1
(Transient Disturbance at Joint 1)

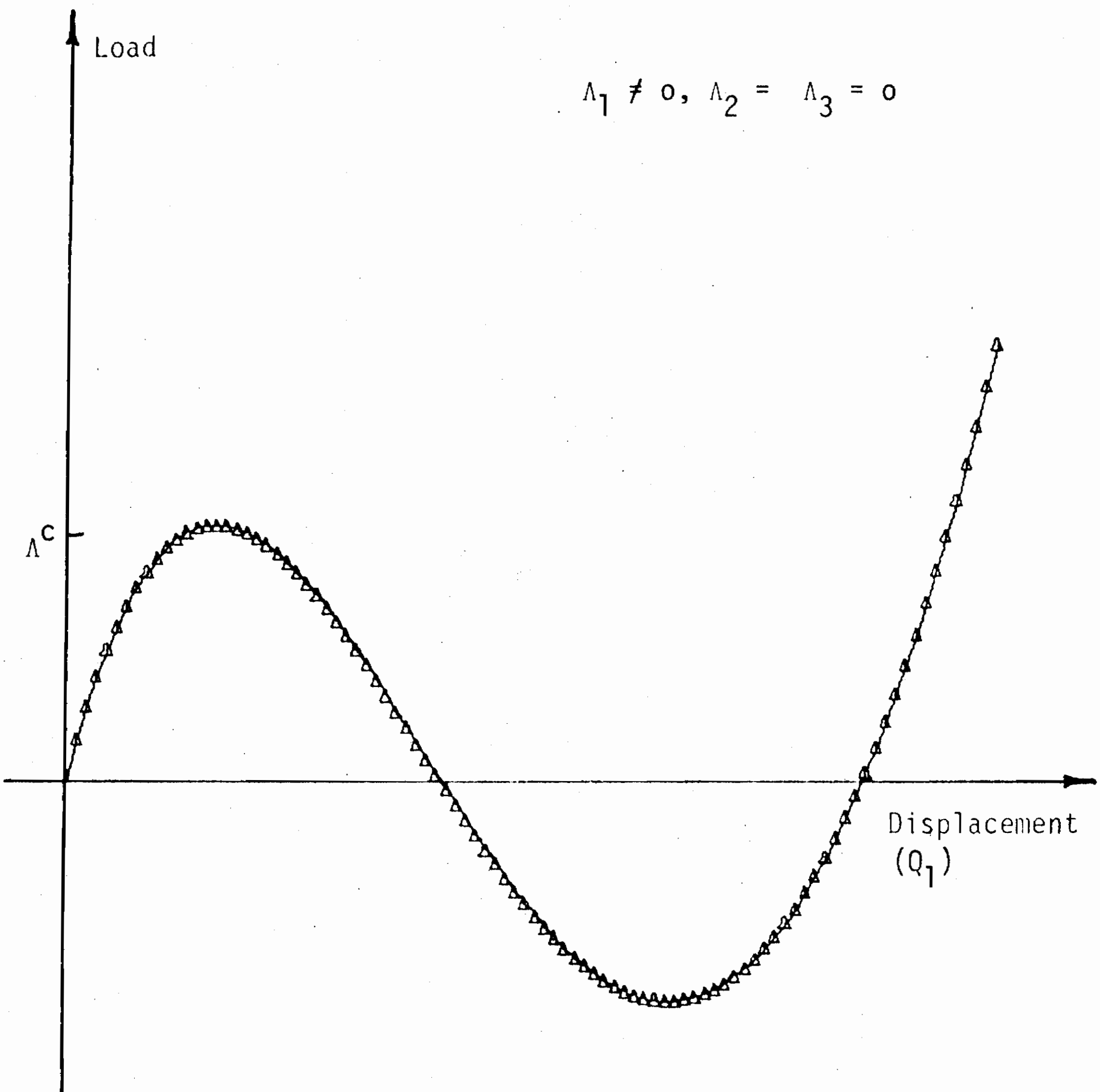


FIG. 5.29a STATIC RESPONSE OF 39 DOF GEODESIC DOME MODEL
(Only Joint 1 Loaded)

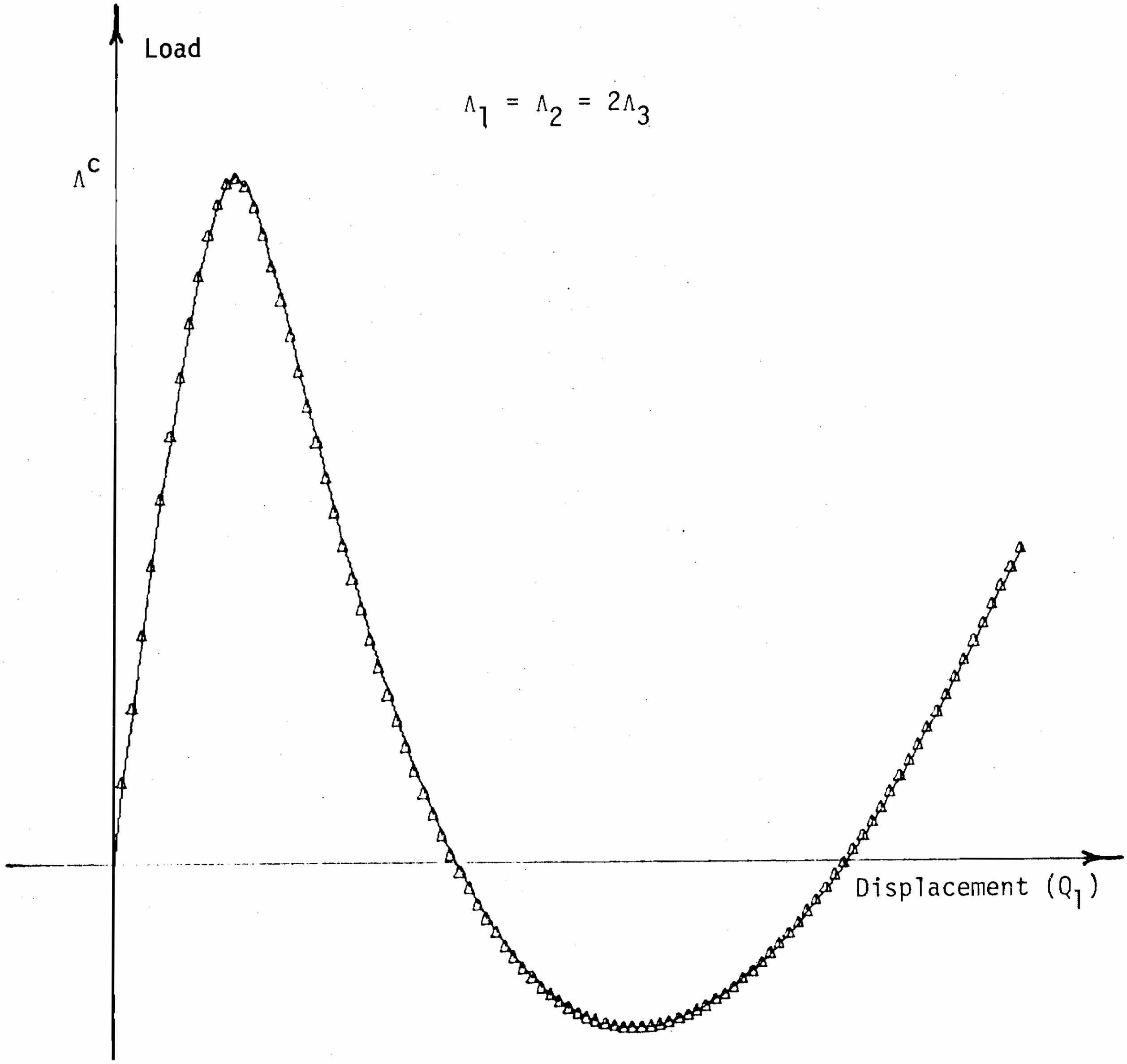


FIG. 5.29b STATIC RESPONSE OF 39 DOF GEODESIC DOME MODEL
(All Joints Loaded)

critical points. Under a single load Λ_1, the dome fails at a much lower value of Λ^c than for the case when all joints are loaded. This is due to the effect of load concentration at joint 1 (Fig. 5.2). This concentration causes local instability of joint 1. However, since the dome cannot resist the shear loads resulting from local instability, general instability cannot be prevented .

The degrees of stability for the loading cases of Figs. 5.29a and 5.29b are illustrated in Fig. 5.30. The dome with all joints loaded is more sensitive to small (but finite) disturbances in the vicinity of the limit load than the dome with the single load. These examples indicate that all static loads that are likely to act on the dome during its lifetime should be included in the model to determine its degree of stability.

With $\Lambda_1 = \Lambda_2 = \Lambda$ and $\Lambda_3 = o$, the response of the dome to an instantaneous disturbance at joint 1 is shown in Fig. 5.31. As with the 21 degree-of-freedom model, the equilibrium state (origin of the phase-plane) is stable or unstable depending on the initial energy. The effect of damping is shown in Fig. 5.32.

When $\Lambda_1 = 2\Lambda_2$, $\Lambda_3 = o$ and joint 2 is instantaneously disturbed, the responses of the ring joints (e.g. joint 2) are shown in Fig. 5.33. When $v_o = 0.96M$, the undamped system is stable while the damped system is asymptotically stable. When $v_o = 1.3M$, the undamped system is still stable. This inference is made from the perturbed motion, a closed trajectory surrounding only the origin. The sufficient dynamic stability condition is not necessary for stability of equilibrium (the

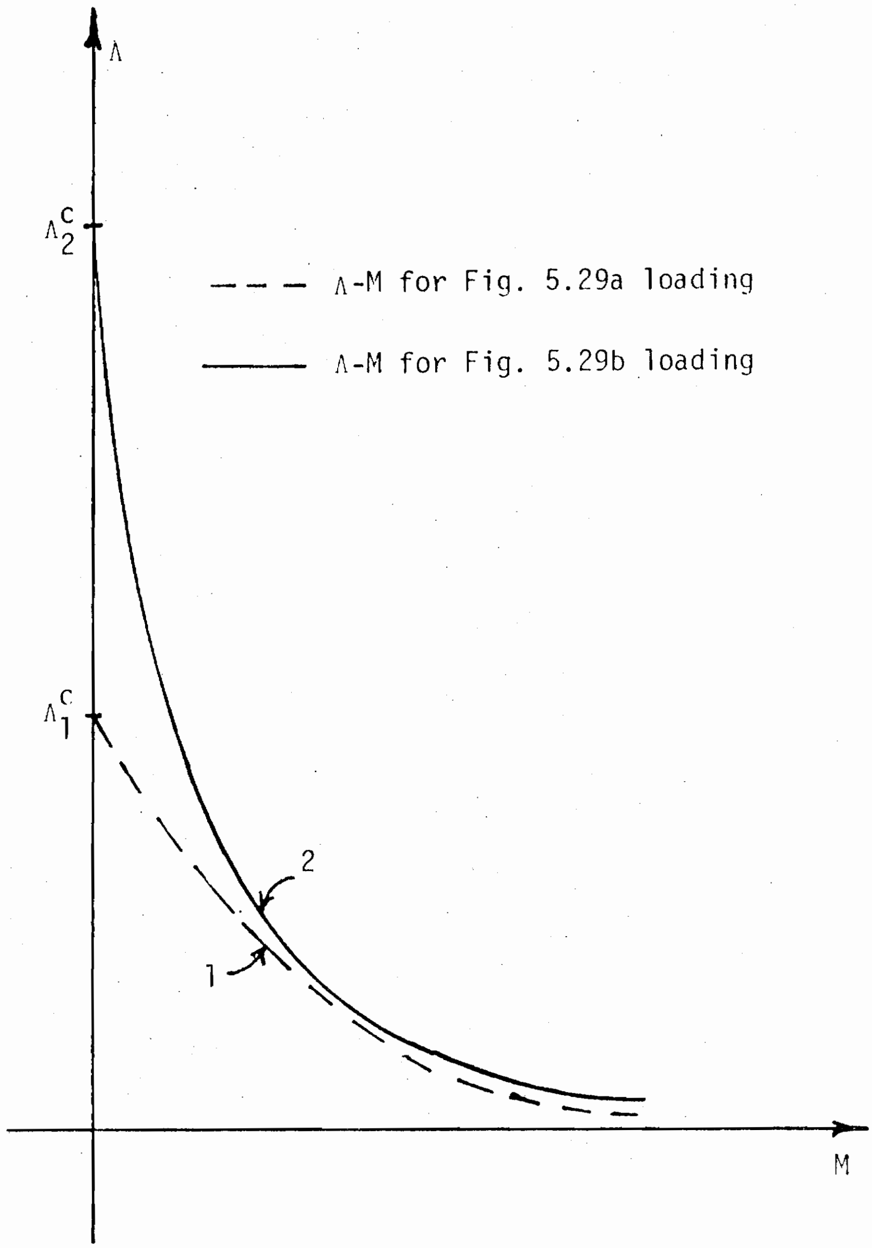
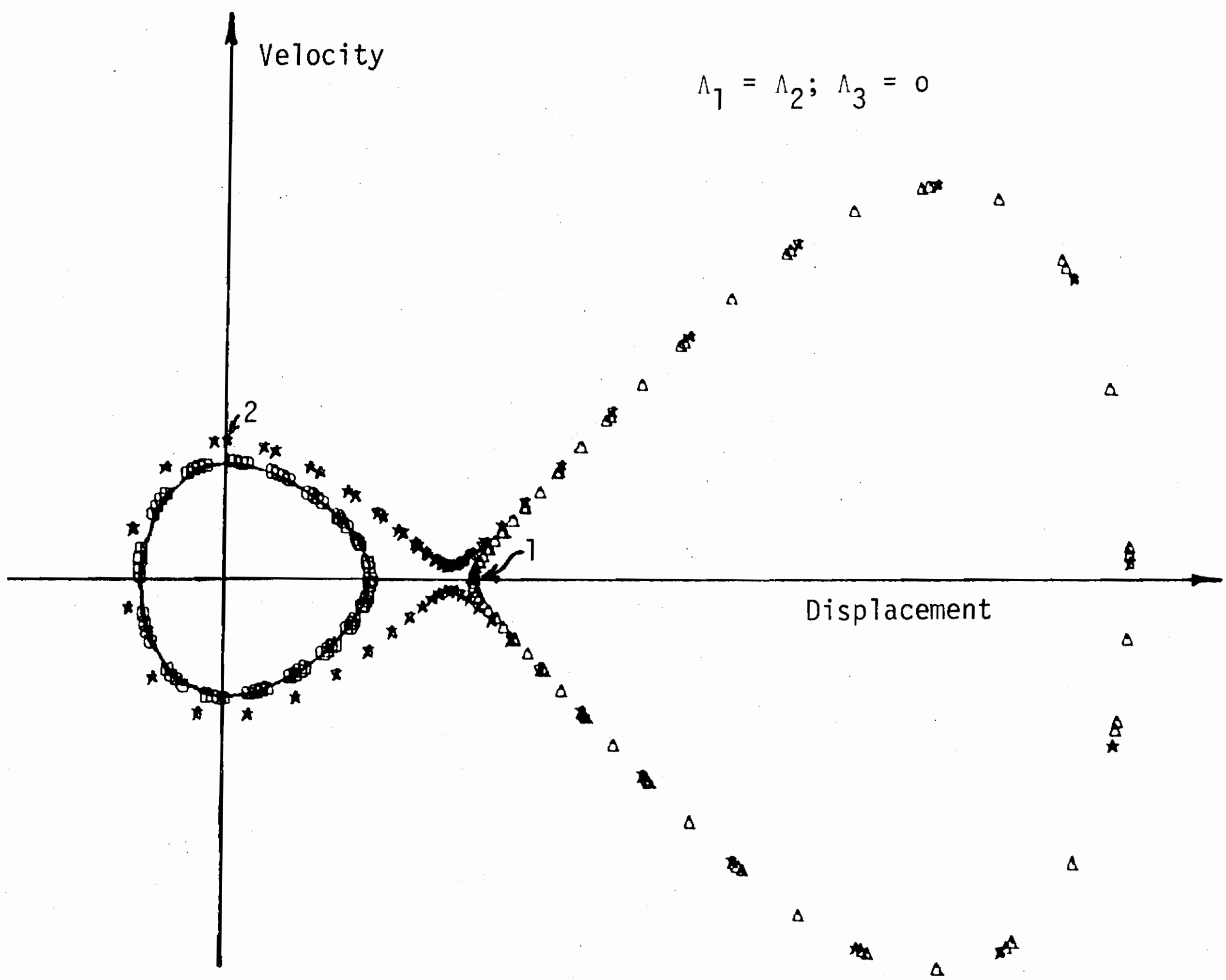


FIG. 5.30 DEGREE OF STABILITY
(Geodesic Dome: 39 DOF Model)



□ v_o = 0.95M; stable

Δ v_1 = 1.08M; unstable

✱ v_2 = 1.05M; unstable

FIG. 5.31 RESPONSE OF JOINT 1
(Instantaneous Disturbance at Joint 1)

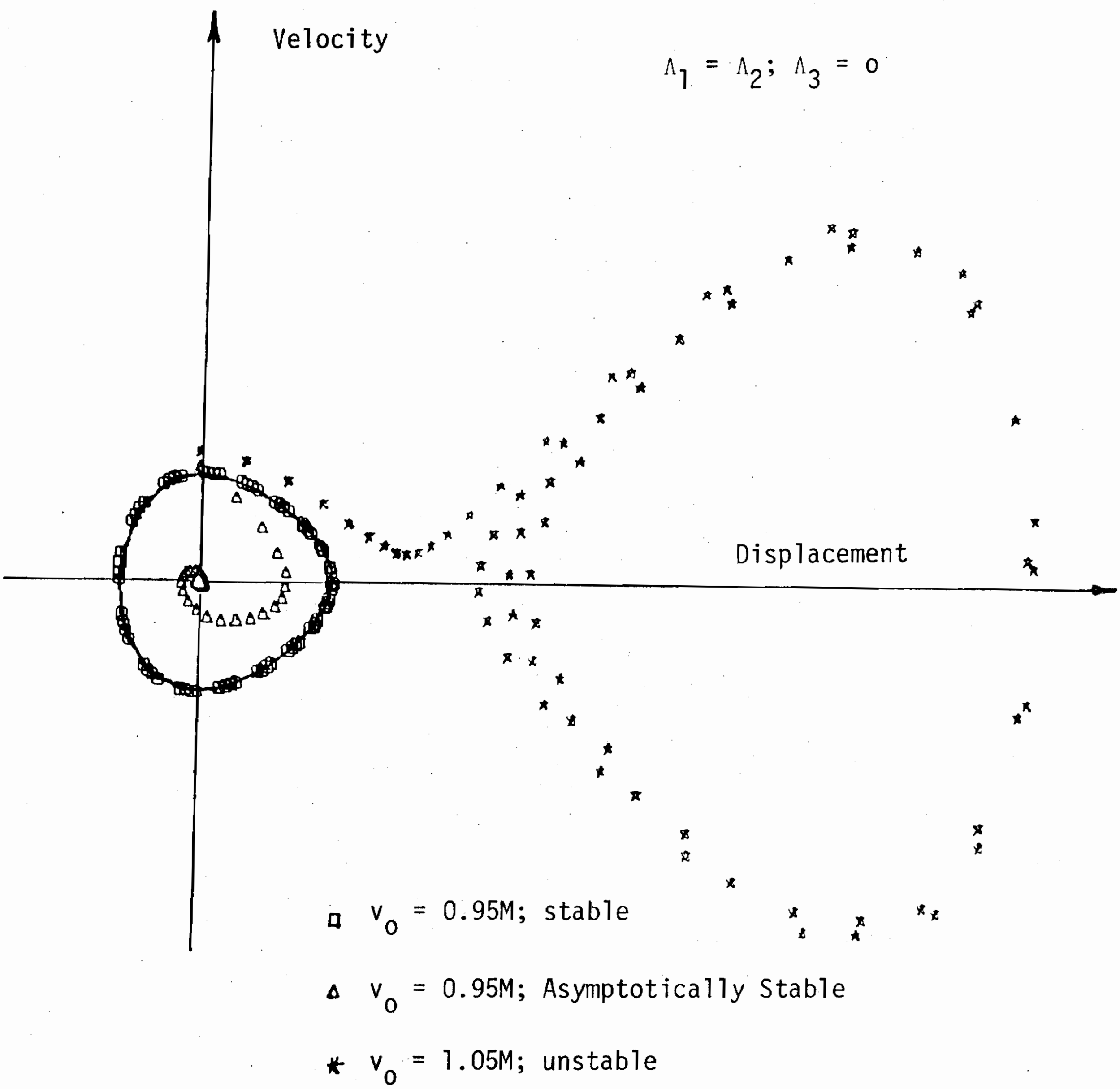


FIG. 5.32 UNDAMPED AND DAMPED RESPONSES OF JOINT 1
(Instantaneous Disturbance at Joint 1)

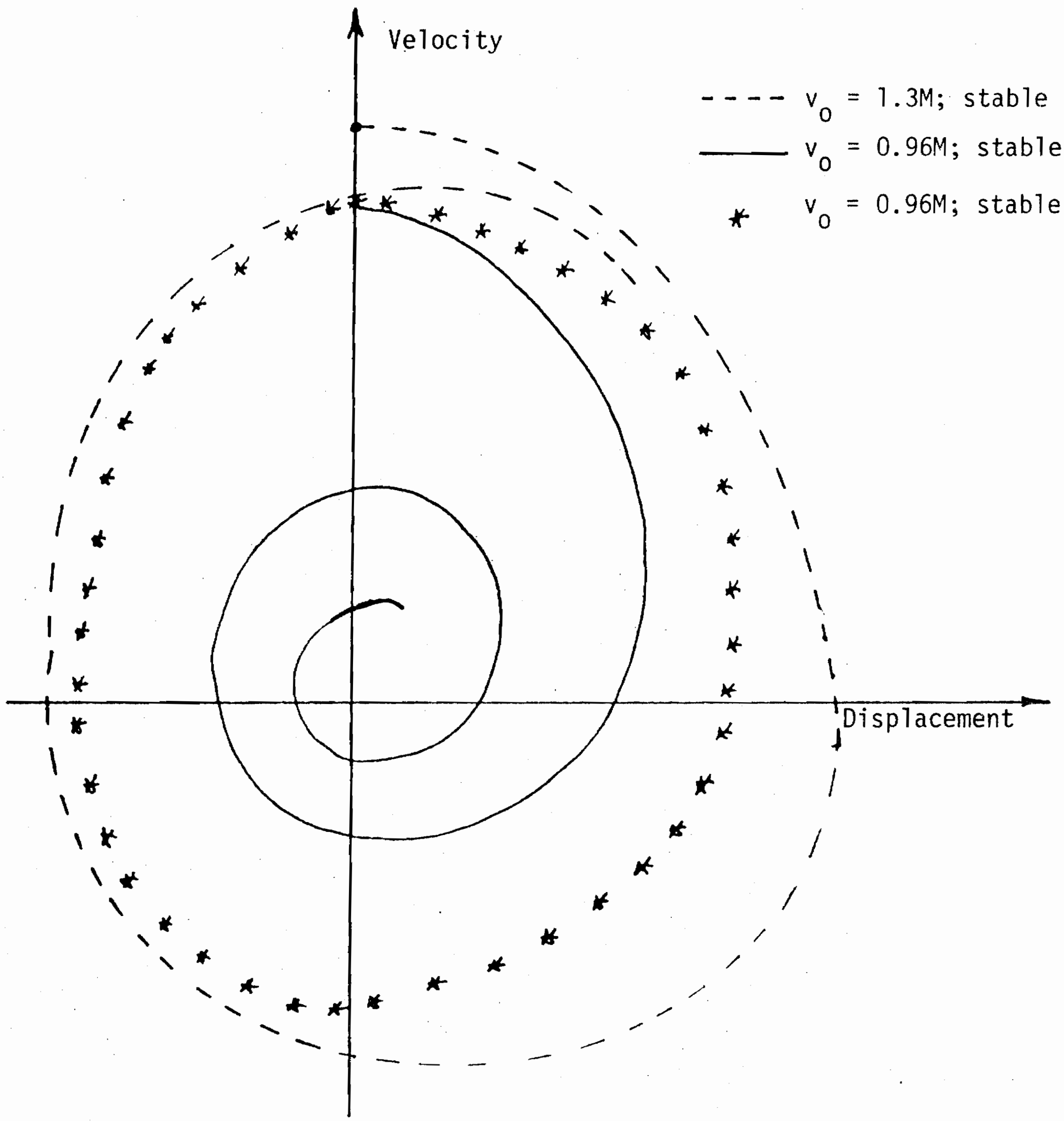


FIG. 5.33 RESPONSE OF RING JOINTS
(Instantaneous Disturbance at Joint 2)

origin in the phase plane). This is in agreement with observations on the 21 DOF model since the disturbance is not applied in the direction corresponding to that of the displacement-increment selected as the "suitable" perturbation parameter, ε.

The response of joint 1 to a transient disturbance at joint 1 is shown in Fig. 5.34. The equilibrium configuration corresponds to a single concentrated load at joint 1. At point 1, the total energy is less than M and the origin is stable. At point 2, the total energy is greater than M and the perturbed motion escapes the domain of attraction of the origin. Later convergence is to the buckled state to which the system is attracted. Hence stability can be decided by integrating the nonlinear equations of perturbed motion for the duration of the disturbance and checking if the total energy at the end of the excitation is less than M.

5.3 Convergence Characteristics

In all figures discussed above, the static analysis was carried out to third order, ε^3, approximation. Also, a proper parameter, ε, was selected for locating equilibrium paths as described in Chapter 4. The convergence characteristics of the perturbation technique are illustrated in Figs. 5.35-5.37.

Fig. 5.35 shows the effect of choosing a poor perturbation parameter. Λ_1^c, Λ_2^c are fictitious critical loads which are results of numerical instability of the perturbation scheme. Point 1 corresponds to choosing q_2 as ε; q_2 is the generalized displacement increment at

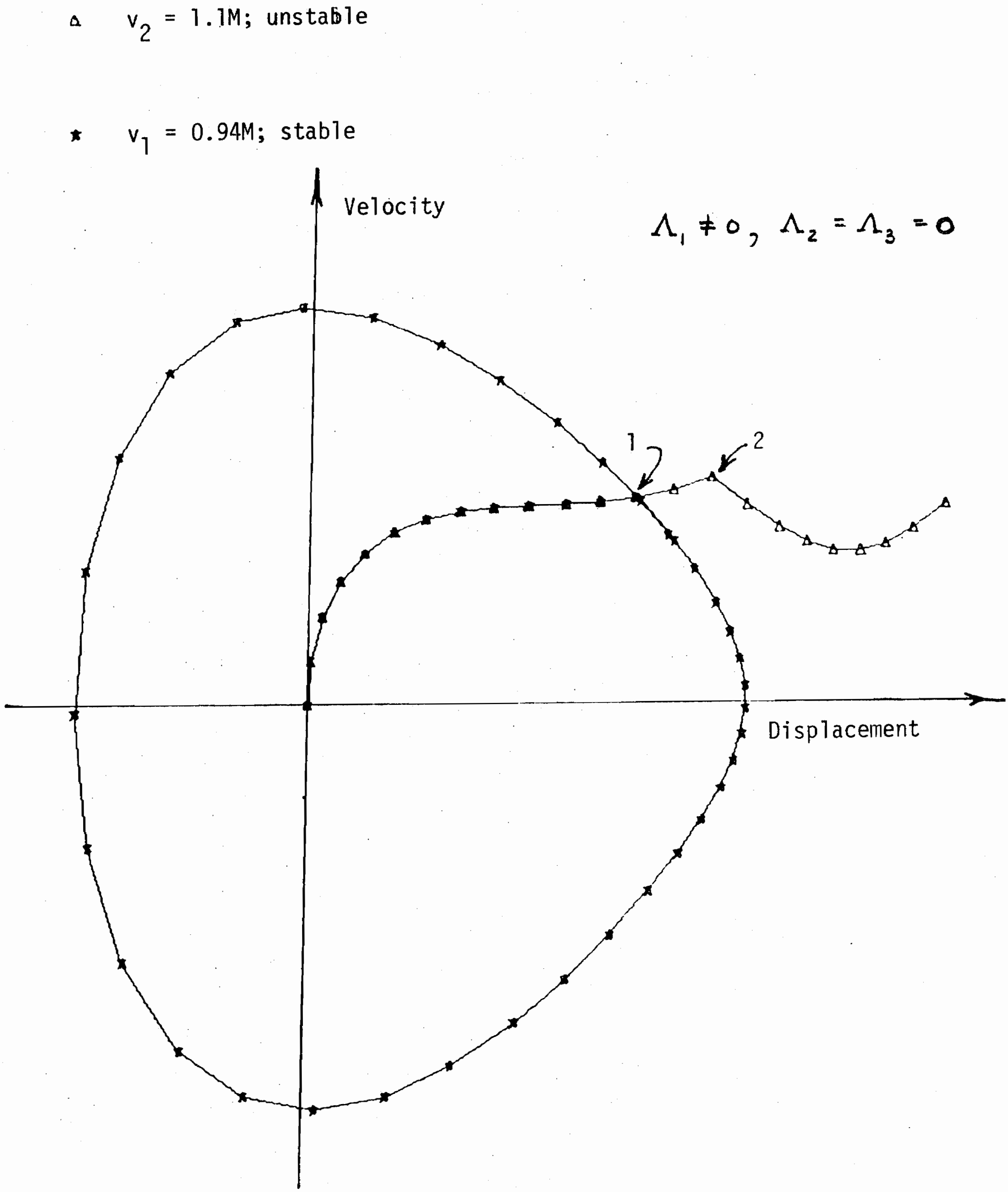


FIG. 5.34 RESPONSE OF JOINT 1 TO TRANSIENT DISTURBANCE

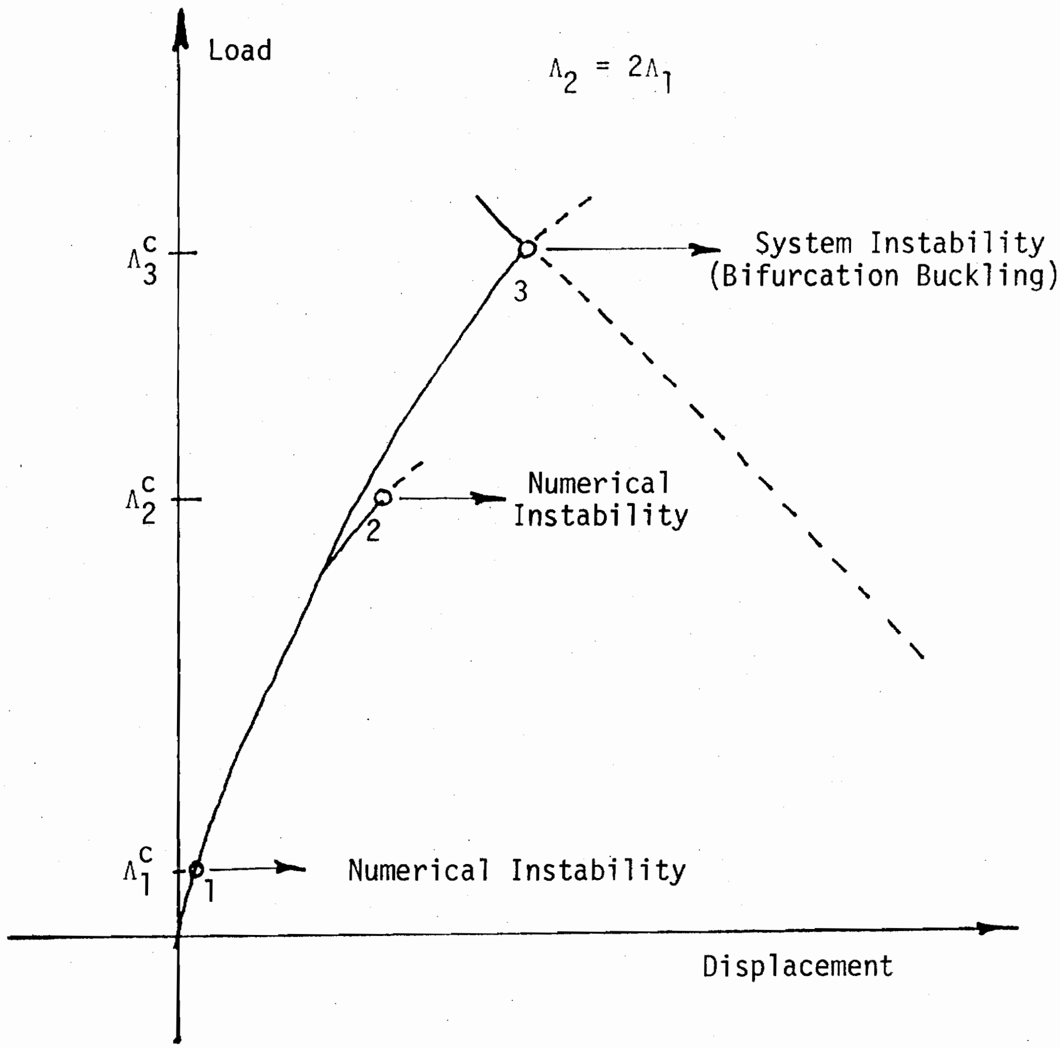


FIG. 5.35 CHOICE OF A PERTURBATION PARAMETER
(Geodesic Dome: 21 DOF Model)

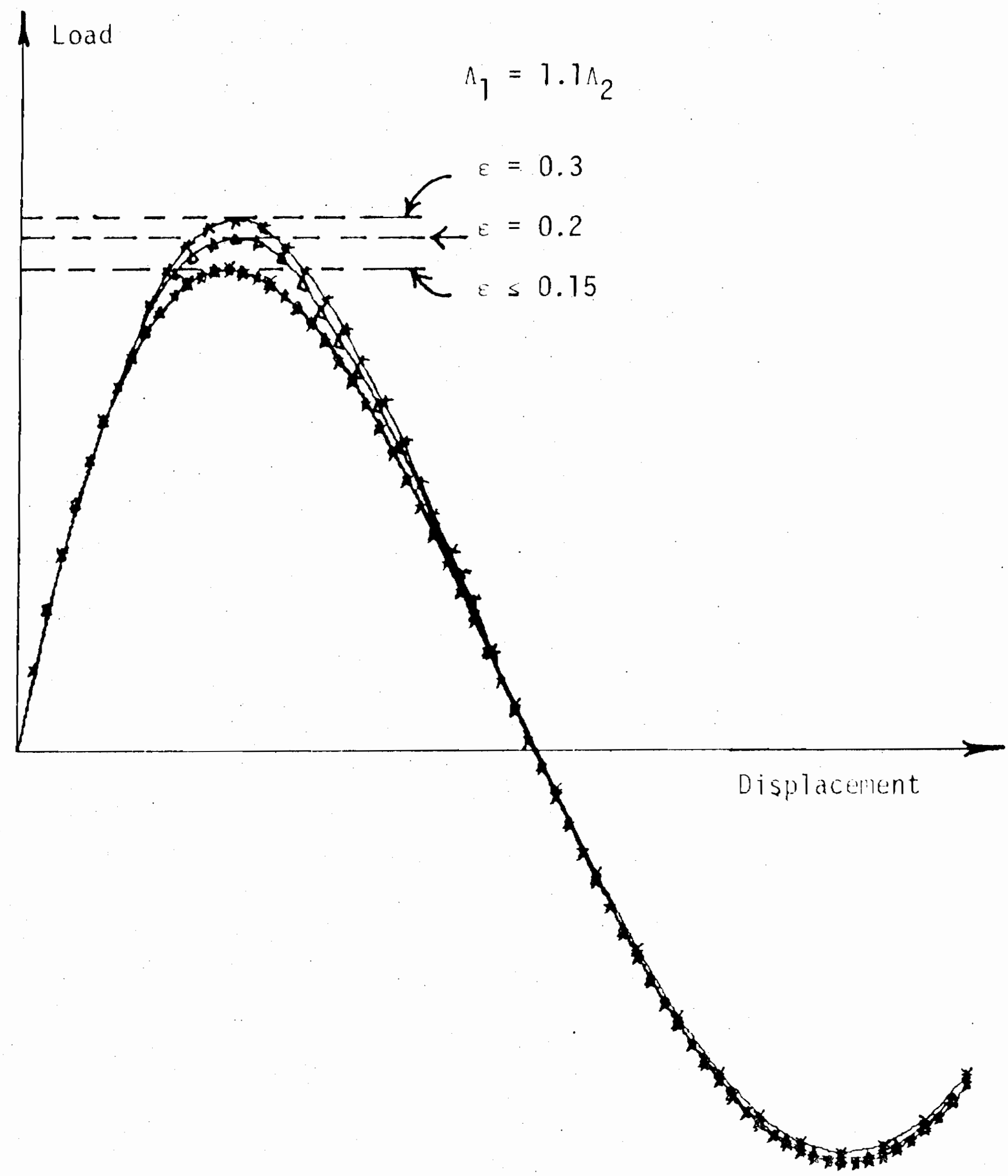


FIG. 5.36 BOUND ON PERTURBATION PARAMETER
(Geodesic Dome: 21 DOF Model)

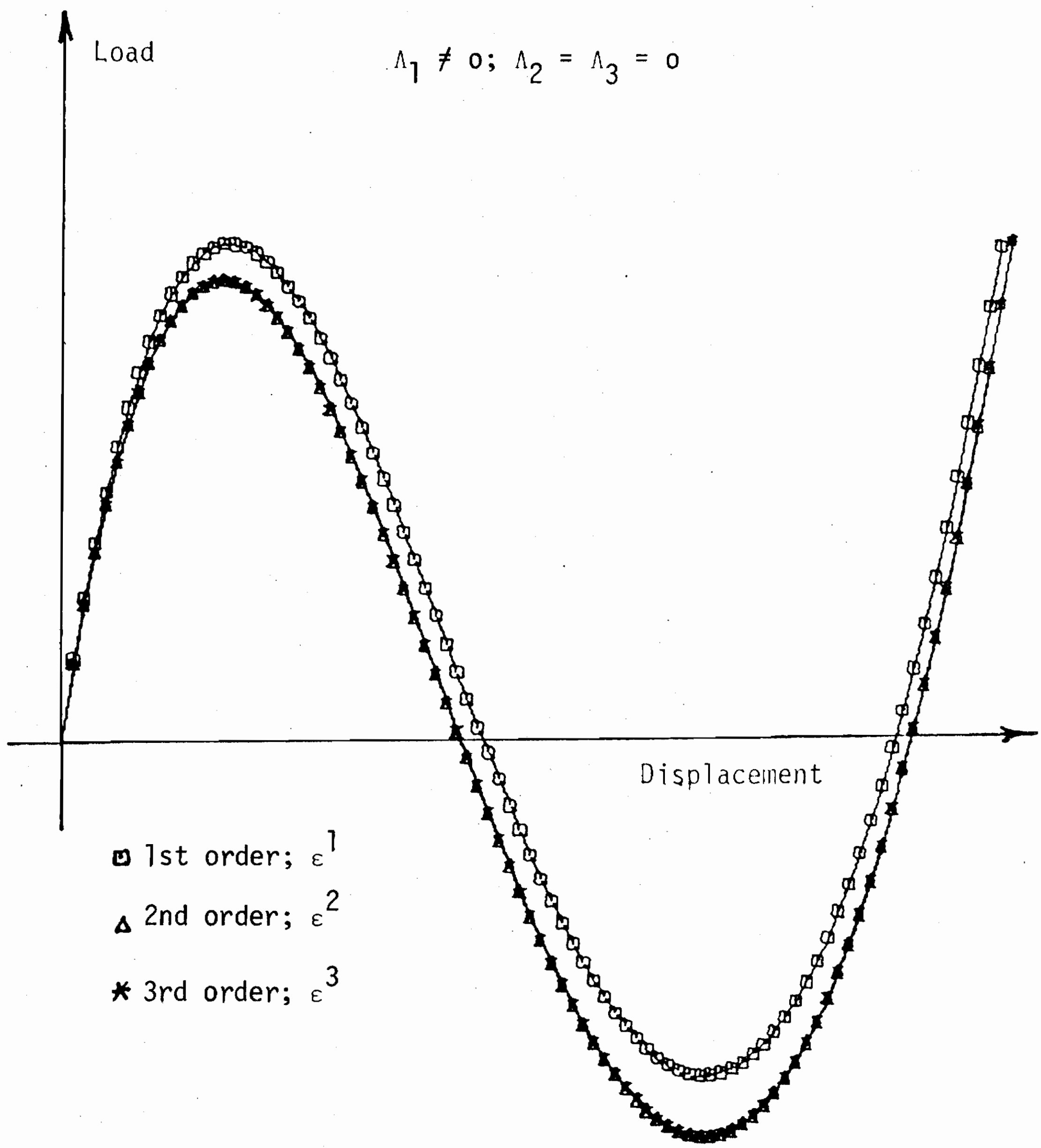


FIG. 5.37 UNIFORM CONVERGENCE OF PERTURBATION SCHEME
(Geodesic Dome: 39 DOF Model)

joint 1 in the Y-direction. Point 2 corresponds to choosing q_1 as ε where q_1 is the displacement increment at joint 1 in the X-direction. The correct choice for ε when $\Lambda_2 > \Lambda_1$ is q_4, the generalized displacement increment at joint 2 in the X-directioh . This is the proposed choice for ε (section 4.3). In this case, the true critical load Λ_3^c is obtained and the post-buckling path beyond the bifurcation point, 3, is obtained without any numerical instability problems.

Fig. 5.36 depicts the characteristics of the nonlinear response to static loads as functions of the value of ε. For an error limit of 10^{-7} and a 0.5 percent allowance for $\overset{....}{q_i}$, the value of ε according to Eq. 4.12 is equal to or lower than 0.15. Convergence of third order solutions for $\varepsilon > 0.15$ show a nonconservative estimate of the critical load Λ. For $\varepsilon \leq 0.15$, a unique critical load is obtained.

Fig. 5.37 shows the uniform convergence of the perturbation solutions for a properly chosen parameter, ε, and a limiting value (based on Eq. 4.12) placed on ε. As in Fig. 5.36, convergence is from above; i.e. the first order perturbation solution provides a nonconservative estimate for the critical load. The solutions for the second and third order perturbation approximations are practically identical (Fig. 5.37).

5.4 Stability Boundaries

The stability boundaries for the two geodesic dome models are depicted in Figs. 5.38-5.40. In both cases, Λ_{01} and Λ_{02} correspond to the critical loads of Λ_1, Λ_2, respectively when acting singly. Hence the locations of the critical states on the axes involve no load

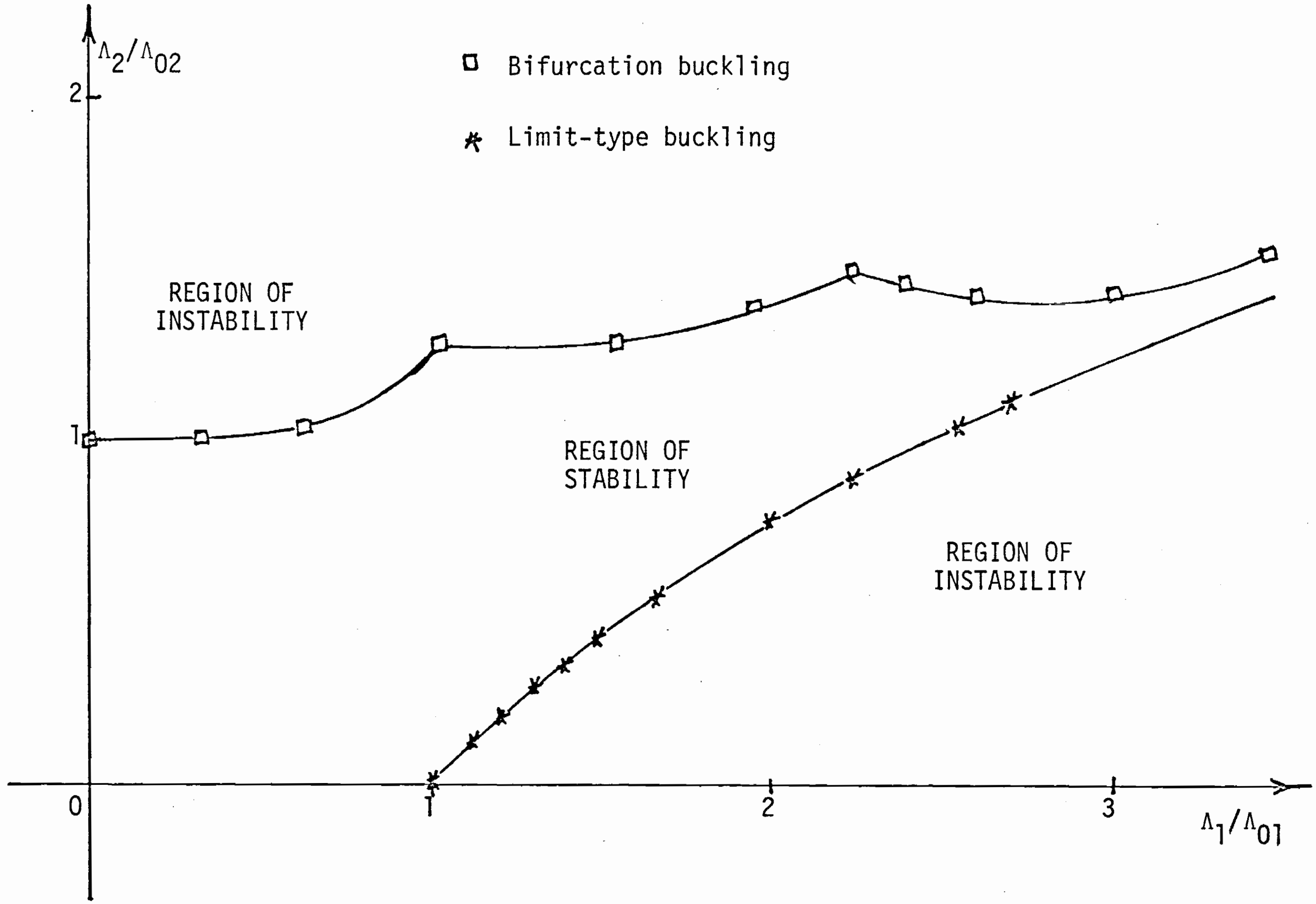


FIG. 5.38 STABILITY BOUNDARIES: 21 DOF MODEL

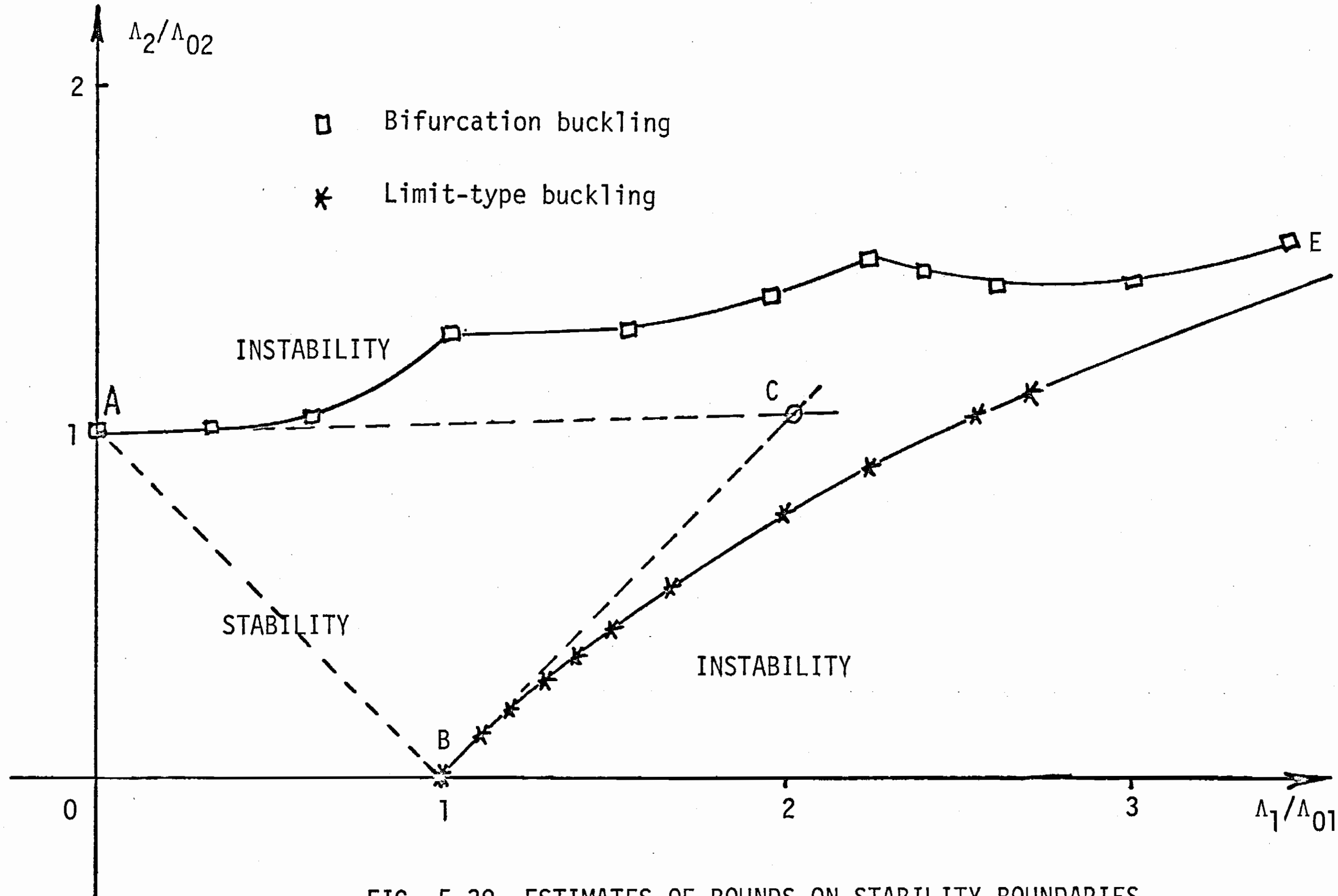


FIG. 5.39 ESTIMATES OF BOUNDS ON STABILITY BOUNDARIES (Geodesic Dome: 21 DOF Model)

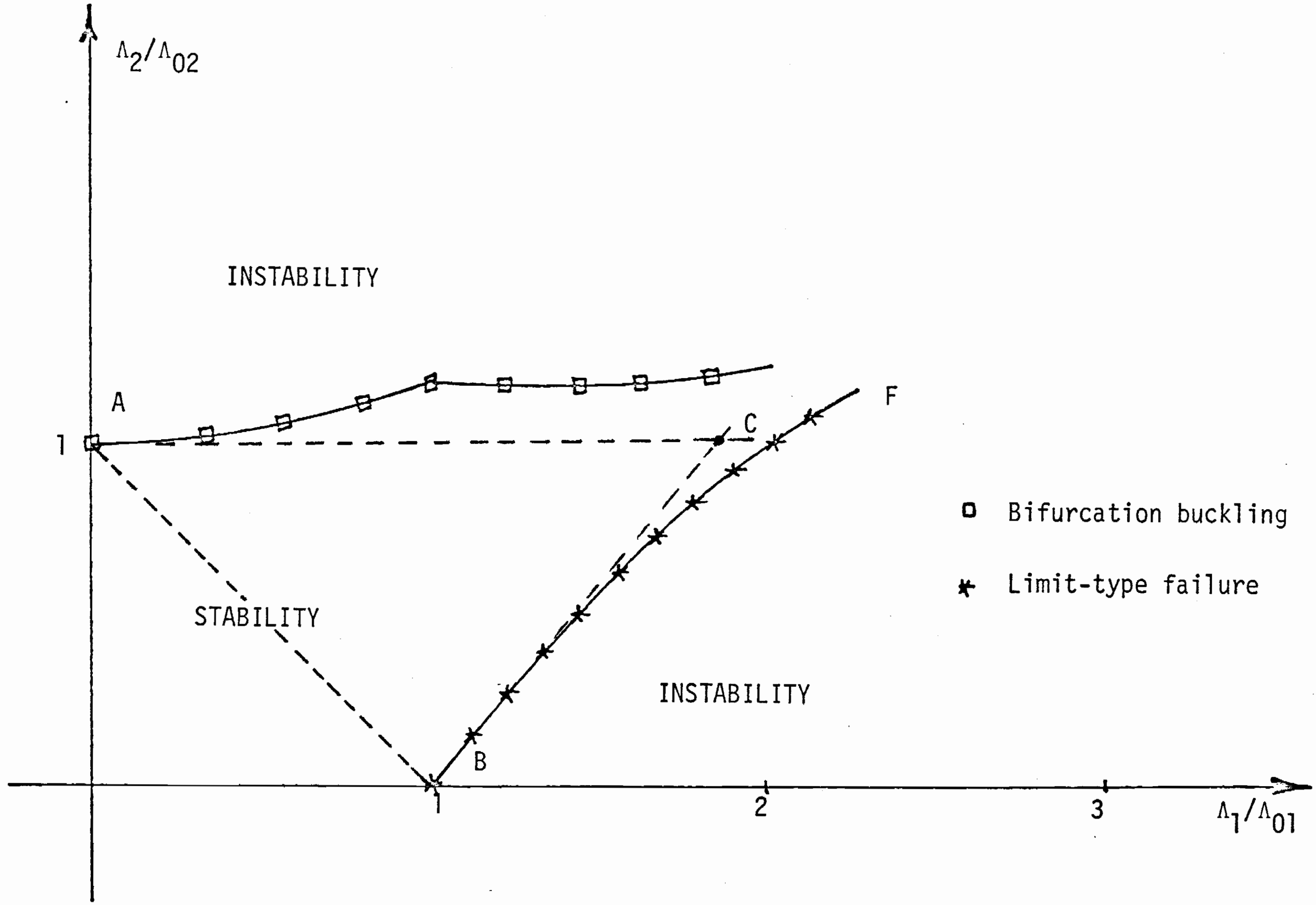


FIG. 5.40 STABILITY BOUNDARIES
(Geodesic Dome: 39 DOF Model)

interaction. The stability boundary BD (Fig. 5.39) for all points corresponding to a limit-type buckling of the 21 DOF model under independent loads Λ_1, Λ_2 is convex towards the region of stability. Bifurcation critical states lie on a stability boundary which is continuous but piecewise differentiable. This boundary (curve AE in Fig. 5.39) appears to be piecewise convex towards the region of stability. All points lying on a particular piece of AE exhibit identical buckling modes at failure.

The line AB represents the least lower bound estimate to the region of stability. The region OACBO represents a safe estimate of the region of stability. AC, BC are safe greatest lower bound estimates of the region of stability obtained from tangents, to AE at A, and to BD at B.

The stability boundaries for the 39 DOF geodesic dome model are shown in Fig. 5.40. Boundary BF is convex toward the region of stability while boundary AF is piecewise convex toward the same region. The two curves approach the singular critical point F associated with an unstable symmetric point of bifurcation. In a way similar to Fig. 5.39, AB represents the least lower bound and AC, BC are safe greatest lower bound estimates on the region of stability. Also, the points lying on each segment of AF exhibit identical buckling modes at failure.

CHAPTER 6

CONCLUSIONS

The stability of reticulated domes under multiple static and dynamic loads is investigated. Two elastic geometrically nonlinear structural models of a reticulated dome are considered .

The nonlinear response of the system to static loads is obtained using nonlinear programming and discrete perturbation techniques. The nonlinear programming technique is used to obtain a starting solution for the discrete perturbation technique and to optimize the choice of the perturbation parameter, ε. Convergence criteria and error estimates to limit errors in a perturbation scheme are developed.

The largest displacement-increment at the initial stage of loading from the zero-state in configuration space is proposed as a "suitable" choice for ε in the case of imperfection-sensitive systems. A poor choice of ε leads to numerical instabilities which appear as 'fictitious' system instabilities.

The investigation of stability of equilibrium of the system subjected to finite disturbances is based on the concept of "degree of stability" and the associated sufficient stability condition, which follow from Liapunov's direct method.

The perturbed motion of the system under a given set of perturbations is obtained by numerically integrating the nonlinear equations of motion . Trajectories in the phase-plane provide valuable checks on the stability condition. These trajectories confirm the sufficiency of the dynamic stability condition. However, they also indicate that there is a dynamic disturbance with a specific spatial distribution for which the sufficient condition of dynamic stability employed is also a necessary condition. This is true for each static loading condition considered. This disturbance corresponds to that which magnifies the static equilibrium configuration . The direction of application of the disturbance corresponds to that of the generalized displacement-increment selected as the "suitable" choice of ε.

The stability boundaries corresponding to two independent loads on the reticulated dome models are presented. Limit points lie on a boundary which is convex towards the region of stability. Bifurcation points lie on a continuous but piecewise differentiable boundary. Each piece of the boundary containing bifurcation points appears to be convex towards the region of stability. Critical points lying on separate pieces of the stability boundary exhibit identical buckling modes. Knowledge of the general properties of such boundaries can form the basis of a rigorous mathematical study to establish the shape of the stability boundaries for reticulated domes exhibiting limit and bifurcation points. This is recommended for future study.

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APPENDIX

COMPUTER PROGRAM

COMPUTER PROGRAM

The digital computer program performs three major tasks in the complete stability analysis of reticulated domes under multiple static and dynamic loads. First, nonlinear programming is used to obtain points on the equilibrium path of the system at the onset of loading, and to optimize the choice of a "suitable" perturbation parameter. Second, a static perturbation scheme is initiated using the chosen perturbation parameter and the known solutions from nonlinear programming technique. The static perturbation scheme is used to obtain the remaining portions of the fundamental path and the entire post-buckling response path. These first two tasks complete the static analysis. Third, the stability of equilibrium (0.70-0.75 of critical static loads) relative to instantaneous and transient disturbances is investigated via the concept of "degree of stability" and the associated sufficient stability condition. A dynamic analysis is performed to check the stability criterion. Hammings modified Predictor-Corrector numerical integration scheme is used to obtain the perturbed motions of the system under a prescribed set of perturbations.

Both the static and dynamic portions of the program generate print and plot outputs. The program is coded in the FORTRAN language and has been used with the IBM 360-75. Only ELASTIC structures are considered. Independent loads and finite displacements are admitted at the joints. All free joints have hinged connections.

The main sequence of operations of the computer program are described within the program using COMMENT cards. Details of coding are omitted . The fortran listing of the basic computer program for two independent joint loads is included for completeness.

PROGRAM LISTING

```
C
C=========    S T A T I C    A N A L Y S I S     ====================
C
C     OBJECTIVE : TO OBTAIN THE NONLINEAR RESPONSE OF RETICULATED
C                 DOMES TO STATIC LOADS
C
C     METHODS : NONLINEAR PROGRAMMING AND
C               STATIC PERTURBATION TECHNIQUES
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*4 P1,P2
      EXTERNAL V
      REAL FS,ACC
      DIMENSION P1(150),P2(150),LABELX(1),LABELY(1)
      DIMENSION U(39),G(39)
      DIMENSION H(294),X(19),Y(19),Z(19)
      DIMENSION ICON(19),LN(39),MW(39)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
C
C     RETICULATED DOMES UNDER TWO INDEPENDENT LOADS
C
      KK=1
      WRITE(6,10)
   10 FORMAT('1',27X,'**********************************************'/
     $28X,'*',44X,'*'/
     $28X,'* A SPACE TRUSS WITH HINGED CONNECTED JOINTS *'/
     $28X,'*   SUBJECTED TO LOADS AT THE JOINTS ONLY    *'/
```

```
      $28X,'*  INDEPENDENT JOINT LOADS ARE CONSIDERED     *'/
      $28X,'*',44X,'*'/
      $28X,'**********************************************'///)
C
C      INPUT DATA FOR MEMBER NUMBERS, NM AND NUMBER OF JOINTS, NJ
C
       READ(5,20) NM,NJ
   20  FORMAT(2I5)
       WRITE(6,30) NM,NJ
   30  FORMAT(//10X,'NUMBER OF ELEMENTS, NM =',I3//10X,'NUMBER OF JOINTS,
      $ NJ =',I3)
C
C      JOINT COORDINATES AND CONSTRAINT OF THE JOINT
C      ICON(J):      MEANS CONSTRAINT AT JOINT 'J'
C      ITS VALUES ARE:      ICON(J)=1 -COMPLETE CONSTRAINED OR FIXED
C                           ICON(J)=0 -COMPLETELY FREE
C
C
C      NF= NUMBER OF UNCONSTRAINED JOINTS
C
       NF=NJ
       WRITE(6,40)
   40  FORMAT(/////20X,'JOINT COORDINATES AND CONSTRAINTS'/////11X,'JOINT
      $ NO',5X,'X-COORD',10X,'Y-COORD',10X,'Z-COORD',8X,'CONSTRAINT'//)
       DO 70 J=1,NJ
       READ(5,50) K,X(K),Y(K),Z(K),ICON(K)
   50  FORMAT(I5,3F15.6,4X,I1)
       WRITE(6,60) K,X(K),Y(K),Z(K),ICON(K)
   60  FORMAT(10X,I5,6X,D12.5,6X,D12.5,5X,D12.5,10X,I1)
       NF=NF-ICON(K)
   70  CONTINUE
C
```

```
C     MEMBER INCIDENCE TABLE
C
      WRITE(6,80)
   80 FORMAT('1',///10X, 'MEMBER INCIDENCE TABLE AND ITS LENGTH'/////5X,
     $'MEMBER NO',10X,'MEMBER INCIDENCE',13X,'MEMBER LENGTH'//)
      DO 110 M=1,NM
      READ(5,90) J,INP(J),INQ(J)
   90 FORMAT(3I5)
      I=INP(J)
      K=INQ(J)
      DX(M)=X(K)-X(I)
      DY(M)=Y(K)-Y(I)
      DZ(M)=Z(K)-Z(I)
      XL(M)= DSQRT(DX(M)**2+DY(M)**2+DZ(M)**2)
      WRITE(6,100) J,INP(J),INQ(J),XL(J)
  100 FORMAT(4X,I6,15X,2I5,15X,D15.5)
  110 CONTINUE
C
C     NUMBER OF DEGREE OF FREEDOM IS EQUAL TO THREE
C     TIMES NUMBER OF UNCONSTRAINED JOINTS, N=3*NF
C
      N=3*NF
      WRITE(6,120) NF,N
  120 FORMAT(//////5X,'NUMBER OF FREE JOINTS, NF =',I3//5X,'NUMBER OF DE
     $GREE OF FREEDOM, N =',I3)
C
C     SOLUTION FOR THE EQUILIBRIUM PATH VIA MINIMIZATION TECHNIQUE
C     USING POWELL-FLECHER METHOD, THIS SOLUTION WILL BE USED AS
C     A STARTING POINT FOR THE PERTURBATION TECHNIQUE TO DESCRIBE
C     THE LOAD VS DISPLACEMENT FOR THE ENTIRE PATH AND ALSO TO SELECT A
C     SUITABLE PERTURBATION PARAMETER , CONJUGATE GRADIENTS AND POWELL'S
C     DIRECT SEARCH OPTIONS ARE AVAILABLE
```

```
C
      WRITE(6,130)
  130 FORMAT('1'///10X,'MINIMIZATION VIA CONJUGATE GRADIENT METHOD'////)
C
C     INITIAL GUESS OF VARIABLES
C
      WRITE(6,140)
  140 FORMAT(//20X,'INITIAL GUESS OF VARIABLES'///11X,'JOINT',8X,'X-DISP
     $.',10X,'Y-DISP.',10X,'Z-DISP.'//)
      DO 160 I=1,NF
      I1=3*I-2
      I2=3*I-1
      I3=3*I
      U(I1)=0.0D0
      U(I2)=0.0D0
      U(I3)=0.0D0
      WRITE(6,150) I,U(I1),U(I2),U(I3)
  150 FORMAT(10X,I5,9X,D12.5,6X,D12.5,6X,D12.5)
  160 CONTINUE
C
C     INITIALIZED ALL JOINT LOADS TO BE ZERO
C
      DO 170 I=1,NF
      PX(I)=0.0D0
      PY(I)=0.0D0
      PZ(I)=0.0D0
  170 CONTINUE
C
C     THE ERROR INDICATOR, IER = 9 IS USED AS THE INITIAL INPUT
C     THE RETURN VALUE ARE SUCH THAT :
C     IER = 0....MEANS CONVERGENCE IS ACHIEVED WITHIN SPECIFIED ACCURACY
C     IER = 1....MEANS NO CONVERGENCE IN THE LIMIT ITERATION
```

```
C     IER =-1....MEANS ERROS IN THE GRADIENT CALCULATION
C     IER = 2.... MEANS IT IS LIKELY THAT THERE EXISTS NO MINIMUM
C
      IER=9
      ACC=1.D-10
      LIMIT=100
      FS=0.0
      KOUNT=0
C
C     INPUT LOADING AT THE JOINT
C
      DO 181 I=1,NF
      READ(5,180) K,PX(K)
  180 FORMAT(I5,F10.6)
  181 CONTINUE
C
C     INPUT THE RATIO OF THE TWO INDEPENDENT LOADS
C     (PROPORTIONAL LOADING)
C
      READ(5,182) FLOAD
  182 FORMAT(F10.5)
  797 CONTINUE
      CALL DFMCG(V,N,U,F,G,FS,ACC,LIMIT,IER,H)
      WRITE(6,186) LIMIT,KOUNT
  186 FORMAT(/////5X,'NUMBER OF ITERATION =',I4//
     $ 5X,I5,5X,'FUNCTION SUBROUTINE CALLED'/////)
      IW=1
      CALL OUTP(U,G,F,N,0,0,IW)
C
C     SEARCH FOR LARGEST DISPLACEMENT-INCREMENT
C
      CALL MAX(N,U,IND)
```

```
C
  284 EPS=0.05
  999 CONTINUE
C
C     INITIALIZE ALL INCREMENTAL SOLUTIONS TO ZERO
C
      PARM1=0.0D0
      PARM2=0.0D0
      PARM3=0.0D0
      DO 280 I=1,N
      Q1(I)=0.0D0
      Q2(I)=0.0D0
      Q3(I)=0.0D0
      Q4(I)=0.0D0
  280 CONTINUE
C
C     USE LARGEST DISPLACEMENT-INCREMENT AS A
C     'SUITABLE' PERTURBATION PARAMETER
C
      Q1(IND)=1.0D0
C
      CALL FIRIN(U,Q1,PARM1,EPS,N,IND,IW)
      CALL SECIN(U,PARM1,PARM2,EPS,N,IND,IW)
      CALL THRIN(U,PARM1,PARM2,PARM3,EPS,N,IND,IW)
      DO 287 I=2,NF
      IF(PX(I).NE.0.0D0) PX(I)=PX(I)+PARM1+PARM2+PARM3
      IF(PY(I).NE.0.0D0) PY(I)=PY(I)+PARM1+PARM2+PARM3
      IF(PZ(I).NE.0.0D0) PZ(I)=PZ(I)+PARM1+PARM2+PARM3
  287 CONTINUE
      IF(PX(1) .NE. 0.0D0) PX(1)=PX(1)+((PARM1+PARM2+PARM3)/FLOAD)
      DO 285 I=1,N
      U(I)=U(I)+EPS*Q1(I)+0.5*EPS**2*Q2(I)+EPS**3*Q3(I)/6.
```

```
  285 CONTINUE                                                          AYO 1930
C                                                                       AYO 1940
C       STORE DISPLACEMENT U(IND) AND THE CORRESPONDING LOAD PX(J)      AYO 1950
C       FOR PLOTTING THE 'EQUILIBRIUM PATH'                             AYO 1960
C                                                                       AYO 1970
        P1(KK)=U(IND)                                                   AYO 1980
        P2(KK)=PX(1)                                                    AYO 1990
C                                                                       AYO 2000
        KK=KK+1                                                         AYO 2010
        KK2=KK-2                                                        AYO 2020
        IF(U(IND).GE.4.5) GO TO 286                                     AYO 2030
        GO TO 284                                                       AYO 2040
  286 CONTINUE                                                          AYO 2050
        DO 410 I=1,KK2                                                  AYO 2060
        WRITE(7,400) P1(I),P2(I)                                        AYO 2070
  400 FORMAT(2E15.4)                                                    AYO 2080
  410 CONTINUE                                                          AYO 2090
C                                                                       AYO 2100
C       PLOT THE 'EQUILIBRIUM PATH'                                     AYO 2110
C                                                                       AYO 2120
        CALL ABPLOT(KK2,P1,P2)                                          AYO 2130
        READ(5,300) LABELX,LABELY                                       AYO 2140
  300 FORMAT(A4,5X,A4)                                                  AYO 2150
        CALL EZPLOT(P1,P2,LABELX,LABELY,1,1,KK2,-11,0)                  AYO 2160
        CALL PLOT(0.0,0.0,-4)                                           AYO 2170
        STOP                                                            AYO 2180
        END                                                             AYO 2190
```

```
      SUBROUTINE V(N,A,VAL,GRAD)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(39),GRAD(39),DE(42)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      KOUNT=KOUNT+1
      WORK=0.0D0
      DO 6000 J=1,NF
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      WORK=WORK+PX(J)*A(J1)+PY(J)*A(J2)+PZ(J)*A(J3)
      DE(J1)=0.0D0
      DE(J2)=0.0D0
      DE(J3)=0.0D0
 6000 CONTINUE
      STENG=0.0D0
      DO 6001 M=1,NM
      J=INP(M)
      K=INQ(M)
      IF(J.GT.NF.AND.K.GT.NF) GO TO 6001
      M1=3*J-2
      M2=3*J-1
      M3=3*J
      M4=3*K-2
      M5=3*K-1
      M6=3*K
      IF(J.GT.NF) GO TO 700
      IF(K.GT.NF) GO TO 750
```

```
      GO TO 760                                                  VEE 0330
  700 DU(M)=A(M4)                                                VEE 0340
      DV(M)=A(M5)                                                VEE 0350
      DW(M)=A(M6)                                                VEE 0360
      SUMX(M)=DX(M)+DU(M)                                        VEE 0370
      SUMY(M)=DY(M)+DV(M)                                        VEE 0380
      SUMZ(M)=DZ(M)+DW(M)                                        VEE 0390
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)             VEE 0400
      DL(M)=XLP(M)-XL(M)                                         VEE 0410
      STENG=STENG+(0.5*DL(M)**2)/XL(M)                           VEE 0420
      DE(M4)=DE(M4)+(DL(M)*SUMX(M))/(XL(M)*XLP(M))               VEE 0430
      DE(M5)=DE(M5)+(DL(M)*SUMY(M))/(XL(M)*XLP(M))               VEE 0440
      DE(M6)=DE(M6)+(DL(M)*SUMZ(M))/(XL(M)*XLP(M))               VEE 0450
      GO TO 6001                                                 VEE 0460
  750 DU(M)=-A(M1)                                               VEE 0470
      DV(M)=-A(M2)                                               VEE 0480
      DW(M)=-A(M3)                                               VEE 0490
      SUMX(M)=DX(M)+DU(M)                                        VEE 0500
      SUMY(M)=DY(M)+DV(M)                                        VEE 0510
      SUMZ(M)=DZ(M)+DW(M)                                        VEE 0520
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)             VEE 0530
      DL(M)=XLP(M)-XL(M)                                         VEE 0540
      STENG=STENG+(0.5*DL(M)**2)/XL(M)                           VEE 0550
      DE(M1)=DE(M1)-(DL(M)*SUMX(M))/(XL(M)*XLP(M))               VEE 0560
      DE(M2)=DE(M2)-(DL(M)*SUMY(M))/(XL(M)*XLP(M))               VEE 0570
      DE(M3)=DE(M3)-(DL(M)*SUMZ(M))/(XL(M)*XLP(M))               VEE 0580
      GO TO 6001                                                 VEE 0590
  760 DU(M)=A(M4)-A(M1)                                          VEE 0600
      DV(M)=A(M5)-A(M2)                                          VEE 0610
      DW(M)=A(M6)-A(M3)                                          VEE 0620
      SUMX(M)=DX(M)+DU(M)                                        VEE 0630
      SUMY(M)=DY(M)+DV(M)                                        VEE 0640
```

```
      SUMZ(M)=DZ(M)+DW(M)                                              VEE 0650
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)                   VEE 0660
      DL(M)=XLP(M)-XL(M)                                               VEE 0670
      STENG=STENG+(0.5*DL(M)**2)/XL(M)                                 VEE 0680
      DE(M1)=DE(M1)-(DL(M)*SUMX(M))/(XL(M)*XLP(M))                     VEE 0690
      DE(M2)=DE(M2)-(DL(M)*SUMY(M))/(XL(M)*XLP(M))                     VEE 0700
      DE(M3)=DE(M3)-(DL(M)*SUMZ(M))/(XL(M)*XLP(M))                     VEE 0710
      DE(M4)=DE(M4)+(DL(M)*SUMX(M))/(XL(M)*XLP(M))                     VEE 0720
      DE(M5)=DE(M5)+(DL(M)*SUMY(M))/(XL(M)*XLP(M))                     VEE 0730
      DE(M6)=DE(M6)+(DL(M)*SUMZ(M))/(XL(M)*XLP(M))                     VEE 0740
 6001 CONTINUE                                                         VEE 0750
      VAL=STENG-WORK                                                   VEE 0760
      DO 6002 J=1,NF                                                   VEE 0770
      J1=3*J-2                                                         VEE 0780
      J2=3*J-1                                                         VEE 0790
      J3=3*J                                                           VEE 0800
      GRAD(J1)=DE(J1)-PX(J)                                            VEE 0810
      GRAD(J2)=DE(J2)-PY(J)                                            VEE 0820
      GRAD(J3)=DE(J3)-PZ(J)                                            VEE 0830
 6002 CONTINUE                                                         VEE 0840
      RETURN                                                           VEE 0850
      END                                                              VEE 0860
```

```
      SUBROUTINE MAX(N,X,JND)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(39)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      XMAX=DABS(X(1))
      JND=1
      IF(N.EQ.1) GO TO 20
      DO 10 I=2,N
      IF(DABS(X(I)).LE.XMAX) GO TO 10
      XMAX=DABS(X(I))
      JND=I
   10 CONTINUE
   20 RETURN
      END
```

```
      SUBROUTINE INVE(A,N,D,L,M)                                        INV 0010
      IMPLICIT REAL*8(A-H,O-Z)                                          INV 0020
      DIMENSION A(1),L(1),M(1)                                          INV 0030
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)                  INV 0040
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)                 INV 0050
      COMMON SUMX(42),SUMY(42),SUMZ(42)                                 INV 0060
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET                     INV 0070
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT                             INV 0080
      D=1.0                                                             INV 0090
      NK=-N                                                             INV 0100
      DO 80 K=1,N                                                       INV 0110
      NK=NK+N                                                           INV 0120
      L(K)=K                                                            INV 0130
      M(K)=K                                                            INV 0140
      KK=NK+K                                                           INV 0150
      BIGA=A(KK)                                                        INV 0160
      DO 20 J=K,N                                                       INV 0170
      IZ=N*(J-1)                                                        INV 0180
      DO 20 I=K,N                                                       INV 0190
      IJ=IZ+I                                                           INV 0200
   10 IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20                               INV 0210
   15 BIGA=A(IJ)                                                        INV 0220
      L(K)=I                                                            INV 0230
      M(K)=J                                                            INV 0240
   20 CONTINUE                                                          INV 0250
      J=L(K)                                                            INV 0260
      IF(J-K) 35,35,25                                                  INV 0270
   25 KI=K-N                                                            INV 0280
      DO 30 I=1,N                                                       INV 0290
      KI=KI+N                                                           INV 0300
      HOLD=-A(KI)                                                       INV 0310
      JI=KI-K+J                                                         INV 0320
```

```
   A(KI)=A(JI)                                                      INV 0330
30 A(JI) =HOLD                                                      INV 0340
35 I=M(K)                                                           INV 0350
   IF(I-K) 45,45,38                                                 INV 0360
38 JP=N*(I-1)                                                       INV 0370
   DO 40 J=1,N                                                      INV 0380
   JK=NK+J                                                          INV 0390
   JI=JP+J                                                          INV 0400
   HOLD=-A(JK)                                                      INV 0410
   A(JK)=A(JI)                                                      INV 0420
40 A(JI) =HOLD                                                      INV 0430
45 IF(BIGA) 48,46,48                                                INV 0440
46 D=0.0                                                            INV 0450
   RETURN                                                           INV 0460
48 DO 55 I=1,N                                                      INV 0470
   IF(I-K) 50,55,50                                                 INV 0480
50 IK=NK+I                                                          INV 0490
   A(IK)=A(IK)/(-BIGA)                                              INV 0500
55 CONTINUE                                                         INV 0510
   DO 65 I=1,N                                                      INV 0520
   IK=NK+I                                                          INV 0530
   HOLD=A(IK)                                                       INV 0540
   IJ=I-N                                                           INV 0550
   DO 65 J=1,N                                                      INV 0560
   IJ=IJ+N                                                          INV 0570
   IF(I-K) 60,65,60                                                 INV 0580
60 IF(J-K) 62,65,62                                                 INV 0590
62 KJ=IJ-I+K                                                        INV 0600
   A(IJ)=HOLD*A(KJ)+A(IJ)                                           INV 0610
65 CONTINUE                                                         INV 0620
   KJ=K-N                                                           INV 0630
   DO 75 J=1,N                                                      INV 0640
```

```
      KJ=KJ+N                                                           INV 0650
      IF(J-K) 70,75,70                                                  INV 0660
   70 A(KJ)=A(KJ)/BIGA                                                  INV 0670
   75 CONTINUE                                                          INV 0680
      IF(DABS(D).LE.1.D-40) D=0.0D0                                     INV 0690
      D=D*BIGA                                                          INV 0700
      A(KK)=1.0/BIGA                                                    INV 0710
   80 CONTINUE                                                          INV 0720
      K=N                                                               INV 0730
  100 K=(K-1)                                                           INV 0740
      IF(K) 150,150,105                                                 INV 0750
  105 I=L(K)                                                            INV 0760
      IF(I-K) 120,120,108                                               INV 0770
  108 JQ=N*(K-1)                                                        INV 0780
      JR=N*(I-1)                                                        INV 0790
      DO 110 J=1,N                                                      INV 0800
      JK=JQ+J                                                           INV 0810
      HOLD=A(JK)                                                        INV 0820
      JI=JR+J                                                           INV 0830
      A(JK)=-A(JI)                                                      INV 0840
  110 A(JI) =HOLD                                                       INV 0850
  120 J=M(K)                                                            INV 0860
      IF(J-K) 100,100,125                                               INV 0870
  125 KI=K-N                                                            INV 0880
      DO 130 I=1,N                                                      INV 0890
      KI=KI+N                                                           INV 0900
      HOLD=A(KI)                                                        INV 0910
      JI=KI-K+J                                                         INV 0920
      A(KI)=-A(JI)                                                      INV 0930
  130 A(JI) =HOLD                                                       INV 0940
      GO TO 100                                                         INV 0950
  150 RETURN                                                            INV 0960
```

```
      SUBROUTINE FIRIN(X,A,PARM,EPS,N,IND,IW)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(N),A(N)
      DIMENSION E(21,21),Q(21),R(21),LN(21),MW(21)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      IORDER=1
       IF(A(IND).EQ.1.0D0) Q(IND)=EPS
       IF(A(IND).NE.1.0D0) INS=2
      IF(INS.EQ.2) GO TO 5000
      INS=1
      CALL FIRST(E,X,N)
      DO 1000 I=1,N
      R(I)=-E(I,IND)
      E(I,IND)=0.0D0
 1000 CONTINUE
      DO 2000 J=1,NF
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      IF(PX(J).NE.0.0D0) E(J1,IND)=-1.0D0
      IF(PY(J).NE.0.0D0) E(J2,IND)=-1.0D0
      IF(PZ(J).NE.0.0D0) E(J3,IND)=-1.0D0
 2000 CONTINUE
      IF(PX(1) .NE. 0.0D0) E(1,IND)=-1.0D0/FLOAD
      CALL INVE(E,N,DET,LN,MW)
      DO 3000 I=1,N
      PARM=PARM+E(IND,I)*R(I)
      DO 3000 J=1,N
```

```
      IF(I.EQ.IND) GO TO 3000
      A(I)=A(I)+E(I,J)*R(J)
 3000 CONTINUE
      DO 4000 I=1,N
      IF(I.EQ.IND) PARM=EPS*PARM
      IF(I.NE.IND) Q(I)=A(I)*EPS
 4000 CONTINUE
      GO TO 9050
 5000 CALL FIRST(E,X,N)
      CALL INVE(E,N,DET,LN,MW)
      DO 6000 I=1,N
      R(I)=0.0D0
 6000 CONTINUE
      DO 7000 J=1,NF
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      IF(PX(J).NE.0.0D0) R(J1)=1.0D0
      IF(PY(J).NE.0.0D0) R(J2)=1.0D0
      IF(PZ(J).NE.0.0D0) R(J3)=1.0D0
 7000 CONTINUE
      DO 8000 I=1,N
      DO 8000 J=1,N
      A(I)=A(I)+E(I,J)*R(J)
 8000 CONTINUE
      PARM=EPS
      DO 9000 I=1,N
      Q(I)=A(I)*EPS
 9000 CONTINUE
 9050 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SECIN(U,PARM1,PARM2,EPS,N,IND,IW)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(N),S(39,4)
      DIMENSION E(21,21),Q(21),R(21),LN(21),MW(21),GRAD(21)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      IORDER=2
      IF(Q1(IND).NE.1.0D0) GO TO 2200
      INS=1
      CALL FIRST(E,U,N)
      DO 2000 I=1,N
      E(I,IND)=0.0D0
 2000 CONTINUE
      DO 2050 J=1,NF
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      IF(PX(J).NE.0.0D0) E(J1,IND)=-1.0D0
      IF(PY(J).NE.0.0D0) E(J2,IND)=-1.0D0
      IF(PZ(J).NE.0.0D0) E(J3,IND)=-1.0D0
 2050 CONTINUE
      IF(PX(1) .NE. 0.0D0) E(1,IND)=-1.0D0/FLOAD
      CALL INVE(E,N,DET,LN,MW)
      CALL SECOND(S,U,N,4)
      DO 2100 I=1,N
      DO 2100 J=1,N
      IF(I.EQ.IND) PARM2=PARM2-E(I,J)*S(J,1)
      IF(I.NE.IND) Q2(I)=Q2(I)-E(I,J)*S(J,1)
 2100 CONTINUE
```

```
      GO TO 2500
 2200 INS=2
      CALL FIRST(E,U,N)
      CALL INVE(E,N,DET,LN,MW)
      CALL SECOND(S,U,N,4)
      PARM2=0.0D0
      DO 2300 I=1,N
      DO 2300 J=1,N
      Q2(I)=Q2(I)-E(I,J)*S(J,1)
 2300 CONTINUE
      PARM=EPS
      DO 2400 I=1,N
      Q(I)=0.5*EPS**2*Q2(I)
 2400 CONTINUE
      GO TO 2800
 2500 Q(IND)=0.0D0
      DO 2700 I=1,N
      IF(I.EQ.IND) GO TO 2600
      Q(I)=0.5*EPS**2*Q2(I)
      GO TO 2700
 2600 PARM2=0.5*EPS**2*PARM2
 2700 CONTINUE
 2800 CONTINUE
      PARM=PARM1+PARM2
      DO 2900 I=1,N
      Q(I)= EPS*Q1(I) + 0.5*EPS**2*Q2(I)
 2900 CONTINUE
      DO 2950 I=2,NF
      IF(PX(I).NE.0.0D0) PX(I)=PX(I)+PARM
      IF(PY(I).NE.0.0D0) PY(I)=PY(I)+PARM
      IF(PZ(I).NE.0.0D0) PZ(I)=PZ(I)+PARM
 2950 CONTINUE
```

```
      IF(PX(1) .NE. 0.0D0) PX(1)=PX(1)+(PARM/FLOAD)                SIN 0650
      DO 3000 I=1,N                                                SIN 0660
      Q(I)=Q(I) + U(I)                                             SIN 0670
 3000 CONTINUE                                                     SIN 0680
      CALL V(N,Q,FUNT,GRAD)                                        SIN 0690
C     CALL OUTP(Q,GRAD,FUNT,N,IORDER,INS,IW)                       SIN 0700
      DO 3050 I=2,NF                                               SIN 0710
      IF(PX(I).NE.0.0D0) PX(I)=PX(I)-PARM                          SIN 0720
      IF(PY(I).NE.0.0D0) PY(I)=PY(I)-PARM                          SIN 0730
      IF(PZ(I).NE.0.0D0) PZ(I)=PZ(I)-PARM                          SIN 0740
 3050 CONTINUE                                                     SIN 0750
      IF(PX(1) .NE. 0.0D0) PX(1)=PX(1)-(PARM/FLOAD)                SIN 0760
      RETURN                                                       SIN 0770
      END                                                          SIN 0780
```

```
      SUBROUTINE THRIN(U,PARM1,PARM2,PARM3,EPS,N,IND,IW)                TRN 0010
      IMPLICIT REAL*8(A-H,O-Z)                                          TRN 0020
      DIMENSION U(N),S(39,4),T(39,3)                                    TRN 0030
      DIMENSION E(21,21),Q(21),R(21),LN(21),MW(21),GRAD(21)             TRN 0040
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)                  TRN 0050
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)                 TRN 0060
      COMMON SUMX(42),SUMY(42),SUMZ(42)                                 TRN 0070
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET                     TRN 0080
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT                             TRN 0090
      IORDER=3                                                          TRN 0100
      IF(Q1(IND).NE.1.0D0) GO TO 3200                                   TRN 0110
      INS=1                                                             TRN 0120
      CALL FIRST(E,U,N)                                                 TRN 0130
      CALL INVE(E,N,DET3,LN,MW)                                         TRN 0140
      SDET=DET3*(10.**25)                                               TRN 0150
      CALL FIRST(E,U,N)                                                 TRN 0160
      DO 3000 I=1,N                                                     TRN 0170
      E(I,IND)=0.0D0                                                    TRN 0180
 3000 CONTINUE                                                          TRN 0190
      DO 3050 J=1,NF                                                    TRN 0200
      J1=3*J-2                                                          TRN 0210
      J2=3*J-1                                                          TRN 0220
      J3=3*J                                                            TRN 0230
      IF(PX(J).NE.0.0D0) E(J1,IND)=-1.0D0                               TRN 0240
      IF(PY(J).NE.0.0D0) E(J2,IND)=-1.0D0                               TRN 0250
      IF(PZ(J).NE.0.0D0) E(J3,IND)=-1.0D0                               TRN 0260
 3050 CONTINUE                                                          TRN 0270
      IF(PX(1) .NE. 0.0D0) E(1,IND)=-1.0D0/FLOAD                        TRN 0280
      CALL INVE(E,N,DET,LN,MW)                                          TRN 0290
      CALL SECOND(S,U,N,4)                                              TRN 0300
      CALL THIRD(T,U,N)                                                 TRN 0310
      DO 3100 I=1,N                                                     TRN 0320
```

```
      DO 3100 J=1,N
      IF(I.EQ.IND) PARM3=PARM3-E(I,J)*(S(J,2)+T(J,1))
      IF(I.NE.IND) Q3(I)=Q3(I)-E(I,J)*(S(J,2)+T(J,1))
3100 CONTINUE
      GO TO 3500
3200 INS=2
      CALL FIRST(E,U,N)
      CALL INVE(E,N,DET,LN,MW)
      CALL SECOND(S,U,N,4)
      CALL THIRD(T,U,N)
      PARM3=0.0D0
      DO 3300 I=1,N
      DO 3300 J=1,N
      Q3(I)=Q3(I)-E(I,J)*(S(J,2)+T(J,1))
3300 CONTINUE
      DO 3400 I=1,N
      Q(I)=EPS**3*Q3(I)/6.
3400 CONTINUE
      GO TO 3800
3500 Q(IND)=0.0D0
      DO 3700 I=1,N
      IF(I.EQ.IND) PARM3=EPS**3*PARM3/6.
      IF(I.NE.IND) Q(I)=EPS**3*Q3(I)/6.
3700 CONTINUE
3800 CALL TAYLOR(Q,U,PARM3,N,INS,IORDER,IW)
      PARM=PARM1+PARM2+PARM3
      DO 3900 I=1,N
      Q(I)=EPS*Q1(I)+0.5*EPS**2*Q2(I)+EPS**3*Q3(I)/6.
3900 CONTINUE
      DO 3950 I=2,NF
      IF(PX(I).NE.0.0D0) PX(I)=PX(I)+PARM
      IF(PY(I).NE.0.0D0) PY(I)=PY(I)+PARM
```

```
      IF(PZ(I).NE.0.0D0) PZ(I)=PZ(I)+PARM
 3950 CONTINUE
      IF(PX(1) .NE. 0.0D0) PX(1)=PX(1)+(PARM/FLOAD)
      DO 4000 I=1,N
      Q(I)=Q(I)+U(I)
 4000 CONTINUE
      CALL V(N,Q,FUNT,GRAD)
      CALL OUTP(Q,GRAD,FUNT,N,IORDER,INS,IW)
      DO 4050 I=2,NF
      IF(PX(I).NE.0.0D0) PX(I)=PX(I)-PARM
      IF(PY(I).NE.0.0D0) PY(I)=PY(I)-PARM
      IF(PZ(I).NE.0.0D0) PZ(I)=PZ(I)-PARM
 4050 CONTINUE
      IF(PX(1) .NE. 0.0D0) PX(1)=PX(1)-(PARM/FLOAD)
      RETURN
      END
```

```
      SUBROUTINE FIRST(X,Q,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(N,N),Q(N)
      DIMENSION DLU(42),DLV(42),DLW(42)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      DO 1000 I=1,N
      DO 1000 J=1,N
      X(I,J)=0.0D0
 1000 CONTINUE
      DO 6000 M=1,NM
      J=INP(M)
      K=INQ(M)
      IF(J.GT.NF.AND.K.GT.NF) GO TO 6000
      IF(J.GT.NF) GO TO 4000
      IF(K.GT.NF) GO TO 5000
      M1=3*J-2
      M2=3*J-1
      M3=3*J
      M4=3*K-2
      M5=3*K-1
      M6=3*K
      DU(M)=Q(M4)-Q(M1)
      DV(M)=Q(M5)-Q(M2)
      DW(M)=Q(M6)-Q(M3)
      SUMX(M)=DX(M)+DU(M)
      SUMY(M)=DY(M)+DV(M)
      SUMZ(M)=DZ(M)+DW(M)
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)
```

```
 DL(M)=XLP(M)-XL(M)                                              1ST 0330
 DLU(M)=-SUMX(M)/XLP(M)                                          1ST 0340
 DLV(M)=-SUMY(M)/XLP(M)                                          1ST 0350
 DLW(M)=-SUMZ(M)/XLP(M)                                          1ST 0360
 X(M1,M1)=X(M1,M1)+(DL(M)*(1.-DLU(M)*DLU(M))/XLP(M)              1ST 0370
$    +  DLU(M)*DLU(M))/XL(M)                                     1ST 0380
 X(M1,M2)=X(M1,M2) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0390
 X(M2,M1)=X(M2,M1) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0400
 X(M1,M3)=X(M1,M3) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0410
 X(M3,M1)=X(M3,M1) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0420
 X(M2,M2)=X(M2,M2) + (DL(M)*(1.-DLV(M)*DLV(M))/XLP(M)            1ST 0430
$   +  DLV(M)*DLV(M))/XL(M)                                      1ST 0440
 X(M2,M3)=X(M2,M3) +(DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)      1ST 0450
 X(M3,M2)=X(M3,M2) +(DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)      1ST 0460
 X(M3,M3)=X(M3,M3)+(DL(M)*(1.-DLW(M)*DLW(M))/XLP(M)              1ST 0470
$  +  DLW(M)*DLW(M))/XL(M)                                       1ST 0480
 X(M4,M4)=X(M4,M4) + (DL(M)*(1.-DLU(M)*DLU(M))/XLP(M)            1ST 0490
$  +  DLU(M)*DLU(M))/XL(M)                                       1ST 0500
 X(M4,M5)=X(M4,M5) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0510
 X(M4,M6)=X(M4,M6) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0520
 X(M5,M4)=X(M5,M4) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0530
 X(M5,M5)=X(M5,M5) +(DL(M)*(1.-DLV(M)*DLV(M))/XLP(M)             1ST 0540
$  +  DLV(M)*DLV(M))/XL(M)                                       1ST 0550
 X(M5,M6)=X(M5,M6) + (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0560
 X(M6,M4)=X(M6,M4) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0570
 X(M6,M5)=X(M6,M5) + (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0580
 X(M6,M6)=X(M6,M6) +  (DL(M)*(1.-DLW(M)*DLW(M))/XLP(M)           1ST 0590
$  +  DLW(M)*DLW(M))/XL(M)                                       1ST 0600
 X(M1,M4)=X(M1,M4) - (DL(M)*(1.-DLU(M)*DLU(M))/XLP(M)            1ST 0610
$  +  DLU(M)*DLU(M))/XL(M)                                       1ST 0620
 X(M1,M5)=X(M1,M5) - (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0630
 X(M1,M6)=X(M1,M6) - (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)     1ST 0640
```

```
      X(M2,M4)=X(M2,M4)  - (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M2,M5)=X(M2,M5)  -(DL(M)*(1.-DLV(M)*DLV(M))/XLP(M)
     $   +   DLV(M)*DLV(M))/XL(M)
      X(M2,M6)=X(M2,M6)  - (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M3,M4)=X(M3,M4)  -  (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M3,M5)=X(M3,M5)  -   (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M3,M6)=X(M3,M6)  - (DL(M)*(1.-DLW(M)*DL(M))/XLP(M)
     $   +   DLW(M)*DLW(M))/XL(M)
      X(M4,M1)=X(M4,M1)  - (DL(M)*(1.-DLU(M)*DLU(M))/XLP(M)
     $   +   DLU(M)*DLU(M))/XL(M)
      X(M5,M1)=X(M5,M1)  - (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M6,M1)=X(M6,M1)  - (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M4,M2)=X(M4,M2)  - (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M5,M2)=X(M5,M2)  -(DL(M)*(1.-DLV(M)*DLV(M))/XLP(M)
     $   +   DLV(M)*DLV(M))/XL(M)
      X(M6,M2)=X(M6,M2)  - (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M4,M3)=X(M4,M3)  -  (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M5,M3)=X(M5,M3)  -   (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M6,M3)=X(M6,M3)  - (DL(M)*(1.-DLW(M)*DL(M))/XLP(M)
     $   +   DLW(M)*DLW(M))/XL(M)
      GO TO 5500
 4000 M4=3*K-2
      M5=3*K-1
      M6=3*K
      DU(M)=Q(M4)
      DV(M)=Q(M5)
      DW(M)=Q(M6)
      SUMX(M)=DX(M)+DU(M)
      SUMY(M)=DY(M)+DV(M)
      SUMZ(M)=DZ(M)+DW(M)
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)
      DL(M)=XLP(M)-XL(M)
```

```
      DLU(M)=-SUMX(M)/XLP(M)                                            1ST 0970
      DLV(M)=-SUMY(M)/XLP(M)                                            1ST 0980
      DLW(M)=-SUMZ(M)/XLP(M)                                            1ST 0990
      X(M4,M4)=X(M4,M4) + (DL(M)*(1.-DLU(M)*DLU(M))/XLP(M)              1ST 1000
     $  +  DLU(M)*DLU(M))/XL(M)                                         1ST 1010
      X(M4,M5)=X(M4,M5) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)       1ST 1020
      X(M5,M4)=X(M5,M4) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)       1ST 1030
      X(M4,M6)=X(M4,M6) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)       1ST 1040
      X(M6,M4)=X(M6,M4) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)       1ST 1050
      X(M5,M5)=X(M5,M5) + (DL(M)*(1.-DLV(M)*DLV(M))/XLP(M)              1ST 1060
     $  +  DLV(M)*DLV(M))/XL(M)                                         1ST 1070
      X(M5,M6)=X(M5,M6) + (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)       1ST 1080
      X(M6,M5)=X(M6,M5) + (DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)       1ST 1090
      X(M6,M6)=X(M6,M6) + (DL(M)*(1.-DLW(M)*DLW(M))/XLP(M)              1ST 1100
     $  +  DLW(M)*DLW(M))/XL(M)                                         1ST 1110
      GO TO 5500                                                        1ST 1120
 5000 M1=3*J-2                                                          1ST 1130
      M2=3*J-1                                                          1ST 1140
      M3=3*J                                                            1ST 1150
      DU(M)=-Q(M1)                                                      1ST 1160
      DV(M)=-Q(M2)                                                      1ST 1170
      DW(M)=-Q(M3)                                                      1ST 1180
      SUMX(M)=DX(M)+DU(M)                                               1ST 1190
      SUMY(M)=DY(M)+DV(M)                                               1ST 1200
      SUMZ(M)=DZ(M)+DW(M)                                               1ST 1210
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)                    1ST 1220
      DL(M)=XLP(M)-XL(M)                                                1ST 1230
      DLU(M)=-SUMX(M)/XLP(M)                                            1ST 1240
      DLV(M)=-SUMY(M)/XLP(M)                                            1ST 1250
      DLW(M)=-SUMZ(M)/XLP(M)                                            1ST 1260
      X(M1,M1)=X(M1,M1)+(DL(M)*(1.-DLU(M)*DLU(M))/XLP(M)                1ST 1270
     $     +  DLU(M)*DLU(M))/XL(M)                                      1ST 1280
```

```
      X(M1,M2)=X(M1,M2) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M2,M1)=X(M2,M1) + (DLU(M)*DLV(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M1,M3)=X(M1,M3) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M3,M1)=X(M3,M1) + (DLU(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M2,M2)=X(M2,M2) + (DL(M)*(1.-DLV(M)*DLV(M))/XLP(M)
     $    +  DLV(M)*DLV(M))/XL(M)
      X(M2,M3)=X(M2,M3) +(DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M3,M2)=X(M3,M2) +(DLV(M)*DLW(M)*(1.-DL(M)/XLP(M)))/XL(M)
      X(M3,M3)=X(M3,M3)+(DL(M)*(1.-DLW(M)*DLW(M))/XLP(M)
     $  +  DLW(M)*DLW(M))/XL(M)
 5500 CONTINUE
 6000 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SECOND(S,Q,N,IORDER)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(3,3),B(3,3),C(3,3)
      DIMENSION RA1(3),RB1(3),RA2(3),RB2(3),RA4(3),RB4(3),RA3(3),RB3(3)
      DIMENSION S(N,IORDER),Q(N)
      DIMENSION DLU(42),DLV(42),DLW(42)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      DO 2000 I=1,N
      DO 2000 J=1,IORDER
      S(I,J)=0.0D0
 2000 CONTINUE
      DO 2080 M=1,NM
      J1=INP(M)
      K1=INQ(M)
      IF(J1.GT.NF.AND.K1.GT.NF) GO TO 2080
      IF(J1.GT.NF) GO TO 2060
      IF(K1.GT.NF) GO TO 2070
      M1=3*J1-2
      M2=3*J1-1
      M3=3*J1
      M4=3*K1-2
      M5=3*K1-1
      M6=3*K1
      RA1(1)=Q1(M1)
      RA1(2)=Q1(M2)
      RA1(3)=Q1(M3)
      RB1(1)=Q1(M4)
      RB1(2)=Q1(M5)
```

```
      RB1(3)=Q1(M6)                                                     2ND 0330
      RA2(1)=Q2(M1)                                                     2ND 0340
      RA2(2)=Q2(M2)                                                     2ND 0350
      RA2(3)=Q2(M3)                                                     2ND 0360
      RB2(1)=Q2(M4)                                                     2ND 0370
      RB2(2)=Q2(M5)                                                     2ND 0380
      RB2(3)=Q2(M6)                                                     2ND 0390
      RA3(1)=Q3(M1)                                                     2ND 0400
      RA3(2)=Q3(M2)                                                     2ND 0410
      RA3(3)=Q3(M3)                                                     2ND 0420
      RB3(1)=Q3(M4)                                                     2ND 0430
      RB3(2)=Q3(M5)                                                     2ND 0440
      RB3(3)=Q3(M6)                                                     2ND 0450
      RA4(1)=Q4(M1)                                                     2ND 0460
      RA4(2)=Q4(M2)                                                     2ND 0470
      RA4(3)=Q4(M3)                                                     2ND 0480
      RB4(1)=Q4(M4)                                                     2ND 0490
      RB4(2)=Q4(M5)                                                     2ND 0500
      RB4(3)=Q4(M6)                                                     2ND 0510
      DU(M)=Q(M4)-Q(M1)                                                 2ND 0520
      DV(M)=Q(M5)-Q(M2)                                                 2ND 0530
      DW(M)=Q(M6)-Q(M3)                                                 2ND 0540
      SUMX(M)=DX(M)+DU(M)                                               2ND 0550
      SUMY(M)=DY(M)+DV(M)                                               2ND 0560
      SUMZ(M)=DZ(M)+DW(M)                                              92ND 0570
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)                    2ND 0580
      DL(M)=XLP(M)-XL(M)                                                2ND 0590
      DLU(M)=-SUMX(M)/XLP(M)                                            2ND 0600
      DLV(M)=-SUMY(M)/XLP(M)                                            2ND 0610
      DLW(M)=-SUMZ(M)/XLP(M)                                            2ND 0620
      D11=(1.-DLU(M)**2)/XLP(M)                                         2ND 0630
      D22=(1.-DLV(M)**2)/XLP(M)                                         2ND 0640
```

```
D33=(1.-DLW(M)**2)/XLP(M)                                          2ND 0650
D12=-DLU(M)*DLV(M)/XLP(M)                                          2ND 0660
D13=-DLU(M)*DLW(M)/XLP(M)                                          2ND 0670
D23=-DLV(M)*DLW(M)/XLP(M)                                          2ND 0680
D111=(-3.*DLU(M)*(1.-DLU(M)**2))/XLP(M)**2                         2ND 0690
D222=(-3.*DLV(M)*(1.-DLV(M)**2))/XLP(M)**2                         2ND 0700
D333=(-3.*DLW(M)*(1.-DLW(M)**2))/XLP(M)**2                         2ND 0710
D112=(-DLV(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                         2ND 0720
D113=(-DLW(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                         2ND 0730
D223=(-DLW(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                         2ND 0740
D122=(-DLU(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                         2ND 0750
D133=(-DLU(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                         2ND 0760
D233=(-DLV(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                         2ND 0770
D123=(3.*DLU(M)*DLV(M)*DLW(M))/XLP(M)**2                           2ND 0780
A(1,1)=(DL(M)*D111+3.*DLU(M)*D11)/XL(M)                            2ND 0790
A(1,2)=(DL(M)*D112+DLV(M)*D11+2.*DLU(M)*D12)/XL(M)                 2ND 0800
A(1,3)=(DL(M)*D113+DLW(M)*D11+2*DLU(M)*D13)/XL(M)                  2ND 0810
A(2,2)=(DL(M)*D122+DLU(M)*D22+2.*DLV(M)*D12)/XL(M)                 2ND 0820
A(2,3)=(DL(M)*D123+DLW(M)*D12+DLU(M)*D23+DLV(M)*D13)/XL(M)         2ND 0830
A(3,3)=(DL(M)*D133+2.*DLW(M)*D13+DLU(M)*D33)/XL(M)                 2ND 0840
A(2,1)=A(1,2)                                                      2ND 0850
A(3,1)=A(1,3)                                                      2ND 0860
A(3,2)=A(2,3)                                                      2ND 0870
B(1,1)=A(1,2)                                                      2ND 0880
B(1,2)=A(2,2)                                                      2ND 0890
B(1,3)=A(3,2)                                                      2ND 0900
B(2,2)=(DL(M)*D222+3.*DLV(M)*D22)/XL(M)                            2ND 0910
B(2,3)=(DL(M)*D223+DLW(M)*D22+2.*DLV(M)*D23)/XL(M)                 2ND 0920
B(3,3)=(DL(M)*D233+2.*DLW(M)*D23+DLV(M)*D33)/XL(M)                 2ND 0930
B(2,1)=B(1,2)                                                      2ND 0940
B(3,1)=B(1,3)                                                      2ND 0950
B(3,2)=B(2,3)                                                      2ND 0960
```

```
      C(1,1)=A(1,3)
      C(1,2)=A(2,3)
      C(1,3)=A(3,3)
      C(2,1)=C(1,2)
      C(2,2)=B(2,3)
      C(2,3)=B(3,3)
      C(3,3)=(DL(M)*D333+3.*DLW(M)*D33)/XL(M)
      C(3,1)=C(1,3)
      C(3,2)=C(2,3)
C
      DO 2050 J=1,3
      DO 2050 K=1,3
      S(M1,1)=S(M1,1) + (RA1(J)*A(J,K)*RA1(K)
     $   -2.*RA1(J)*A(J,K)*RB1(K) + RB1(J)*A(J,K)*RB1(K))
      S(M2,1)=S(M2,1) + (RA1(J)*B(J,K)*RA1(K)
     $   -2.*RA1(J)*B(J,K)*RB1(K) + RB1(J)*B(J,K)*RB1(K))
      S(M3,1)=S(M3,1) + (RA1(J)*C(J,K)*RA1(K)
     $   -2.*RA1(J)*C(J,K)*RB1(K) + RB1(J)*C(J,K)*RB1(K))
      S(M4,1)=S(M4,1) - (RA1(J)*A(J,K)*RA1(K)
     $   -2.*RA1(J)*A(J,K)*RB1(K) + RB1(J)*A(J,K)*RB1(K))
      S(M5,1)=S(M5,1) - (RA1(J)*B(J,K)*RA1(K)
     $   -2.*RA1(J)*B(J,K)*RB1(K) + RB1(J)*B(J,K)*RB1(K))
      S(M6,1)=S(M6,1) - (RA1(J)*C(J,K)*RA1(K)
     $   -2.*RA1(J)*C(J,K)*RB1(K) + RB1(J)*C(J,K)*RB1(K))
      S(M1,2)=S(M1,2) + 3.*(RA1(J)*A(J,K)*RA2(K) - RA1(J)*A(J,K)*RB2(K)
     $   - RB1(J)*A(J,K)*RA2(K) + RB1(J)*A(J,K)*RB2(K))
      S(M2,2)=S(M2,2) + 3.*(RA1(J)*B(J,K)*RA2(K) - RA1(J)*B(J,K)*RB2(K)
     $   - RB1(J)*B(J,K)*RA2(K) + RB1(J)*B(J,K)*RB2(K))
      S(M3,2)=S(M3,2) + 3.*(RA1(J)*C(J,K)*RA2(K) - RA1(J)*C(J,K)*RB2(K)
     $   - RB1(J)*C(J,K)*RA2(K) + RB1(J)*C(J,K)*RB2(K))
      S(M4,2)=S(M4,2) - 3.*(RA1(J)*A(J,K)*RA2(K) - RA1(J)*A(J,K)*RB2(K)
     $   - RB1(J)*A(J,K)*RA2(K) + RB1(J)*A(J,K)*RB2(K))
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 S(M5,2)=S(M5,2) - 3.*(RA1(J)*B(J,K)*RA2(K) - RA1(J)*B(J,K)*RB2(K)  2ND 1290
$  - RB1(J)*B(J,K)*RA2(5) + RB1(J)*B(J,K)*RB2(K))                    2ND 1300
 S(M6,2)=S(M6,2) - 3.*(RA1(J)*C(J,K)*RA2(K) - RA1(J)*C(J,K)*RB2(K)  2ND 1310
$  - RB1(J)*C(J,K)*RA2(K) + RB1(J)*C(J,K)*RB2(K))                    2ND 1320
 S(M1,3)=S(M1,3) + 3.*(RA2(J)*A(J,K)*RA2(K)                          2ND 1330
$  - 2.*RA2(J)*A(J,K)*RB2(K) + RB2(J)*A(J,K)*RB2(K))                 2ND 1340
$ +  4.*(RA1(J)*A(J,K)*RA3(K) - RA1(J)*A(J,K)*RB3(K)                 2ND 1350
$  - RB1(J)*A(J,K)*RA3(K) + RB1(J)*A(J,K)*RB3(K))                    2ND 1360
 S(M2,3)=S(M2,3) + 3.*(RA2(J)*B(J,K)*RA2(K) - 2.*RA2(J)*B(J,K)*RB2(2ND 1370
$K)+ RB2(J)*B(J,K)*RB2(K)) + 4.*(RA1(J)*B(J,K)*RA3(K)                2ND 1380
$  - RA1(J)*B(J,K)*RB3(K) - RB1(J)*B(J,K)*RA3(K)                     2ND 1390
$  + RB1(J)*B(J,K)*RB3(K))                                           2ND 1400
 S(M3,3)=S(M3,3)+3.*(RA2(J)*C(J,K)*RA2(K)-2.*RA2(J)*C(J,K)*RB2(K)    2ND 1410
$ +RB2(J)*C(J,K)*RB2(K))+4.*(RA1(J)*C(J,K)*RA3(K)-RA1(J)*C(J,K)*     2ND 1420
$ RB3(K)-RB1(J)*C(J,K)*RA3(K)+RB1(J)*C(J,K)*RB3(K))                  2ND 1430
 S(M4,3)=S(M4,3) - 3.*(RA2(J)*A(J,K)*RA2(K)                          2ND 1440
$  - 2.*RA2(J)*A(J,K)*RB2(K) + RB2(J)*A(J,K)*RB2(K))                 2ND 1450
$ -  4.*(RA1(J)*A(J,K)*RA3(K) - RA1(J)*A(J,K)*RB3(K)                 2ND 1460
$  - RB1(J)*A(J,K)*RA3(K) + RB1(J)*A(J,K)*RB3(K))                    2ND 1470
 S(M5,3)=S(M5,3) - 3.*(RA2(J)*B(J,K)*RA2(K) - 2.*RA2(J)*B(J,K)*RB2(2ND 1480
$K)+ RB2(J)*B(J,K)*RB2(K)) - 4.*(RA1(J)*B(J,K)*RA3(K)                2ND 1490
$  - RA1(J)*B(J,K)*RB3(K) - RB1(J)*B(J,K)*RA3(K)                     2ND 1500
$  + RB1(J)*B(J,K)*RB3(K))                                           2ND 1510
 S(M6,3)=S(M6,3)-3.*(RA2(J)*C(J,K)*RA2(K)-2.*RA2(J)*C(J,K)*RB2(K)    2ND 1520
$ +RB2(J)*C(J,K)*RB2(K))-4.*(RA1(J)*C(J,K)*RA3(K)-RA1(J)*C(J,K)*     2ND 1530
$ RB3(K)-RB1(J)*C(J,K)*RA3(K)+RB1(J)*C(J,K)*RB3(K))                  2ND 1540
 S(M1,4)=S(M1,4)+5.*(RA1(J)*A(J,K)*RA4(K)-RA1(J)*A(J,K)*RB4(K)       2ND 1550
$ -RB1(J)*A(J,K)*RA4(K)+RB1(J)*A(J,K)*RB4(K))+10.*(RA2(J)*A(J,K)     2ND 1560
$ *RA3(K)-RA2(J)*A(J,K)*RB3(K)-RB2(J)*A(J,K)*RA3(K)+RB2(J)*A(J,K)    2ND 1570
$ *RB3(K))                                                           2ND 1580
 S(M2,4)=S(M2,4) + 5.*(RA1(J)*B(J,K)*RA4(K)                          2ND 1590
$ - RA1(J)*B(J,K)*RB4(K)-RB1(J)*B(J,K)*RA4(K)+RB1(J)*B(J,K)*RB4(K))2ND 1600
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$ + 10.*(RA2(J)*B(J,K)*RA3(K)-RA2(J)*B(J,K)*RB3(K)                 2ND 1610
$ - RB2(J)*B(J,K)*RA3(K) + RB2(J)*B(J,K)*RB3(K))                   2ND 1620
 S(M3,4)=S(M3,4) + 5.*(RA1(J)*C(J,K)*RA4(K)                        2ND 1630
$ - RA1(J)*C(J,K)*RB4(K)-RB1(J)*C(J,K)*RA4(K)+RB1(J)*C(J,K)*RB4(K))2ND 1640
$ + 10.*(RA2(J)*C(J,K)*RA3(K)-RA2(J)*C(J,K)*RB3(K)                 2ND 1650
$ - RB2(J)*C(J,K)*RA3(K) + RB2(J)*C(J,K)*RB3(K))                   2ND 1660
 S(M4,4)=S(M4,4)-5.*(RA1(J)*A(J,K)*RA4(K)-RA1(J)*A(J,K)*RB4(K)     2ND 1670
$ -RB1(J)*A(J,K)*RA4(K)+RB1(J)*A(J,K)*RB4(K))-10.*(RA2(J)*A(J,K)   2ND 1680
$ *RA3(K)-RA2(J)*A(J,K)*RB3(K)-RB2(J)*A(J,K)*RA3(K)+RB2(J)*A(J,K)  2ND 1690
$ *RB3(K))                                                         2ND 1700
 S(M5,4)=S(M5,4) - 5.*(RA1(J)*B(J,K)*RA4(K)                        2ND 1710
$ - RA1(J)*B(J,K)*RB4(K)-RB1(J)*B(J,K)*RA4(K)+RB1(J)*B(J,K)*RB4(K))2ND 1720
$ - 10.*(RA2(J)*B(J,K)*RA3(K)-RA2(J)*B(J,K)*RB3(K)                 2ND 1730
$ - RB2(J)*B(J,K)*RA3(K) + RB2(J)*B(J,K)*RB3(K))                   2ND 1740
 S(M6,4)=S(M6,4) - 5.*(RA1(J)*C(J,K)*RA4(K)                        2ND 1750
$ - RA1(J)*C(J,K)*RB4(K)-RB1(J)*C(J,K)*RA4(K)+RB1(J)*C(J,K)*RB4(K))2ND 1760
$ - 10.*(RA2(J)*C(J,K)*RA3(K)-RA2(J)*C(J,K)*RB3(K)                 2ND 1770
$ - RB2(J)*C(J,K)*RA3(K) + RB2(J)*C(J,K)*RB3(K))                   2ND 1780
2050 CONTINUE                                                        2ND 1790
     GO TO 2080                                                      2ND 1800
2060 M4=3*K1-2                                                       2ND 1810
     M5=3*K1-1                                                       2ND 1820
     M6=3*K1                                                         2ND 1830
     RB1(1)=Q1(M4)                                                   2ND 1840
     RB1(2)=Q1(M5)                                                   2ND 1850
     RB1(3)=Q1(M6)                                                   2ND 1860
     RB2(1)=Q2(M4)                                                   2ND 1870
     RB2(2)=Q2(M5)                                                   2ND 1880
     RB2(3)=Q2(M6)                                                   2ND 1890
     RB3(1)=Q3(M4)                                                   2ND 1900
     RB3(2)=Q3(M5)                                                   2ND 1910
     RB3(3)=Q3(M6)                                                   2ND 1920
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RB4(1)=Q4(M4)                                                    2ND 1930
RB4(2)=Q4(M5)                                                    2ND 1940
RB4(3)=Q4(M6)                                                    2ND 1950
DU(M)=Q(M4)                                                      2ND 1960
DV(M)=Q(M5)                                                      2ND 1970
DW(M)=Q(M6)                                                      2ND 1980
SUMX(M)=DX(M)+DU(M)                                              2ND 1990
SUMY(M)=DY(M)+DV(M)                                              2ND 2000
SUMZ(M)=DZ(M)+DW(M)                                              2ND 2010
XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)                   2ND 2020
DL(M)=XLP(M)-XL(M)                                               2ND 2030
DLU(M)=-SUMX(M)/XLP(M)                                           2ND 2040
DLV(M)=-SUMY(M)/XLP(M)                                           2ND 2050
DLW(M)=-SUMZ(M)/XLP(M)                                           2ND 2060
D11=(1.-DLU(M)**2)/XLP(M)                                        2ND 2070
D22=(1.-DLV(M)**2)/XLP(M)                                        2ND 2080
D33=(1.-DLW(M)**2)/XLP(M)                                        2ND 2090
D12=-DLU(M)*DLV(M)/XLP(M)                                        2ND 2100
D13=-DLU(M)*DLW(M)/XLP(M)                                        2ND 2110
D23=-DLV(M)*DLW(M)/XLP(M)                                        2ND 2120
D111=(-3.*DLU(M)*(1.-DLU(M)**2))/XLP(M)**2                       2ND 2130
D222=(-3.*DLV(M)*(1.-DLV(M)**2))/XLP(M)**2                       2ND 2140
D333=(-3.*DLW(M)*(1.-DLW(M)**2))/XLP(M)**2                       2ND 2150
D112=(-DLV(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                       2ND 2160
D113=(-DLW(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                       2ND 2170
D223=(-DLW(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                       2ND 2180
D122=(-DLU(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                       2ND 2190
D133=(-DLU(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                       2ND 2200
D233=(-DLV(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                       2ND 2210
D123=(3.*DLU(M)*DLV(M)*DLW(M))/XLP(M)**2                         2ND 2220
A(1,1)=(DL(M)*D111+3.*DLU(M)*D11)/XL(M)                          2ND 2230
A(1,2)=(DL(M)*D112+DLV(M)*D11+2.*DLU(M)*D12)/XL(M)               2ND 2240
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      A(1,3)=(DL(M)*D113+DLW(M)*D11+2*DLU(M)*D13)/XL(M)                 2ND 2250
      A(2,2)=(DL(M)*D122+DLU(M)*D22+2.*DLV(M)*D12)/XL(M)                2ND 2260
      A(2,3)=(DL(M)*D123+DLW(M)*D12+DLU(M)*D23+DLV(M)*D13)/XL(M)        2ND 2270
      A(3,3)=(DL(M)*D133+2.*DLW(M)*D13+DLU(M)*D33)/XL(M)                2ND 2280
      A(2,1)=A(1,2)                                                     2ND 2290
      A(3,1)=A(1,3)                                                     2ND 2300
      A(3,2)=A(2,3)                                                     2ND 2310
      B(1,1)=A(1,2)                                                     2ND 2320
      B(1,2)=A(2,2)                                                     2ND 2330
      B(1,3)=A(3,2)                                                     2ND 2340
      B(2,2)=(DL(M)*D222+3.*DLV(M)*D22)/XL(M)                           2ND 2350
      B(2,3)=(DL(M)*D223+DLW(M)*D22+2.*DLV(M)*D23)/XL(M)                2ND 2360
      B(3,3)=(DL(M)*D233+2.*DLW(M)*D23+DLV(M)*D33)/XL(M)                2ND 2370
      B(2,1)=B(1,2)                                                     2ND 2380
      B(3,1)=B(1,3)                                                     2ND 2390
      B(3,2)=B(2,3)                                                     2ND 2400
      C(1,1)=A(1,3)                                                     2ND 2410
      C(1,2)=A(2,3)                                                     2ND 2420
      C(1,3)=A(3,3)                                                     2ND 2430
      C(2,1)=C(1,2)                                                     2ND 2440
      C(2,2)=B(2,3)                                                     2ND 2450
      C(2,3)=B(3,3)                                                     2ND 2460
      C(3,3)=(DL(M)*D333+3.*DLW(M)*D33)/XL(M)                           2ND 2470
      C(3,1)=C(1,3)                                                     2ND 2480
      C(3,2)=C(2,3)                                                     2ND 2490
      DO 2065 J=1,3                                                     2ND 2500
      DO 2065 K=1,3                                                     2ND 2510
      S(M4,1)=S(M4,1) - RB1(J)*A(J,K)*RB1(K)                            2ND 2520
      S(M5,1)=S(M5,1) - RB1(J)*B(J,K)*RB1(K)                            2ND 2530
      S(M6,1)=S(M6,1) - RB1(J)*C(J,K)*RB1(K)                            2ND 2540
      S(M4,2)=S(M4,2) - 3.*RB1(J)*A(J,K)*RB2(K)                         2ND 2550
      S(M5,2)=S(M5,2) - 3.*RB1(J)*B(J,K)*RB2(K)                         2ND 2560
```

```
      S(M6,2)=S(M6,2) - 3.*RB1(J)*C(J,K)*RB2(K)
      S(M4,3)=S(M4,3)-3.*RB2(J)*A(J,K)*RB2(J)-RB1(J)*A(J,K)*RB3(K)*4.
      S(M5,3)=S(M5,3)-3.*RB2(J)*B(J,K)*RB2(J)-RB1(J)*B(J,K)*RB3(K)*4.
      S(M6,3)=S(M6,3)-3.*RB2(J)*C(J,K)*RB2(J)-RB1(J)*C(J,K)*RB3(K)*4.
      S(M4,4)=S(M4,4)-5.*RB1(J)*A(J,K)*RB4(K)-10.*RB2(J)*A(J,K)*RB3(K)
      S(M5,4)=S(M5,4)-5.*RB1(J)*B(J,K)*RB4(K)-10.*RB2(J)*B(J,K)*RB3(K)
      S(M6,4)=S(M6,4)-5.*RB1(J)*C(J,K)*RB4(K)-10.*RB2(J)*C(J,K)*RB3(K)
 2065 CONTINUE
      GO TO 2080
C
 2070 M1=3*J1-2
      M2=3*J1-1
      M3=3*J1
      RA1(1)=Q1(M1)
      RA1(2)=Q1(M2)
      RA1(3)=Q1(M3)
      RA2(1)=Q2(M1)
      RA2(2)=Q2(M2)
      RA2(3)=Q2(M3)
      RA3(1)=Q3(M1)
      RA3(2)=Q3(M2)
      RA3(3)=Q3(M3)
      RA4(1)=Q4(M1)
      RA4(2)=Q4(M2)
      RA4(3)=Q4(M3)
      DU(M)=-Q(M1)                                                     7
      DV(M)=-Q(M2)
      DW(M)=-Q(M3)
      SUMX(M)=DX(M)+DU(M)
      SUMY(M)=DY(M)+DV(M)
      SUMZ(M)=DZ(M)+DW(M)
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)
```

```
DL(M)=XLP(M)-XL(M)
DLU(M)=-SUMX(M)/XLP(M)
DLV(M)=-SUMY(M)/XLP(M)
DLW(M)=-SUMZ(M)/XLP(M)
D11=(1.-DLU(M)**2)/XLP(M)
D22=(1.-DLV(M)**2)/XLP(M)
D33=(1.-DLW(M)**2)/XLP(M)
D12=-DLU(M)*DLV(M)/XLP(M)
D13=-DLU(M)*DLW(M)/XLP(M)
D23=-DLV(M)*DLW(M)/XLP(M)
D111=(-3.*DLU(M)*(1.-DLU(M)**2))/XLP(M)**2
D222=(-3.*DLV(M)*(1.-DLV(M)**2))/XLP(M)**2
D333=(-3.*DLW(M)*(1.-DLW(M)**2))/XLP(M)**2
D112=(-DLV(M)*(1.-3.*DLU(M)**2))/XLP(M)**2
D113=(-DLW(M)*(1.-3.*DLU(M)**2))/XLP(M)**2
D223=(-DLW(M)*(1.-3.*DLV(M)**2))/XLP(M)**2
D122=(-DLU(M)*(1.-3.*DLV(M)**2))/XLP(M)**2
D133=(-DLU(M)*(1.-3.*DLW(M)**2))/XLP(M)**2
D233=(-DLV(M)*(1.-3.*DLW(M)**2))/XLP(M)**2
D123=(3.*DLU(M)*DLV(M)*DLW(M))/XLP(M)**2
A(1,1)=(DL(M)*D111+3.*DLU(M)*D11)/XL(M)
A(1,2)=(DL(M)*D112+DLV(M)*D11+2.*DLU(M)*D12)/XL(M)
A(1,3)=(DL(M)*D113+DLW(M)*D11+2*DLU(M)*D13)/XL(M)
A(2,2)=(DL(M)*D122+DLU(M)*D22+2.*DLV(M)*D12)/XL(M)
A(2,3)=(DL(M)*D123+DLW(M)*D12+DLU(M)*D23+DLV(M)*D13)/XL(M)
A(3,3)=(DL(M)*D133+2.*DLW(M)*D13+DLU(M)*D33)/XL(M)
A(2,1)=A(1,2)
A(3,1)=A(1,3)
A(3,2)=A(2,3)
B(1,1)=A(1,2)
B(1,2)=A(2,2)
B(1,3)=A(3,2)
```

```
      B(2,2)=(DL(M)*D222+3.*DLV(M)*D22)/XL(M)
      B(2,3)=(DL(M)*D223+DLW(M)*D22+2.*DLV(M)*D23)/XL(M)
      B(3,3)=(DL(M)*D233+2.*DLW(M)*D23+DLV(M)*D33)/XL(M)
      B(2,1)=B(1,2)
      B(3,1)=B(1,3)
      B(3,2)=B(2,3)
      C(1,1)=A(1,3)
      C(1,2)=A(2,3)
      C(1,3)=A(3,3)
      C(2,1)=C(1,2)
      C(2,2)=B(2,3)
      C(2,3)=B(3,3)
      C(3,3)=(DL(M)*D333+3.*DLW(M)*D33)/XL(M)
      C(3,1)=C(1,3)
      C(3,2)=C(2,3)
      DO 2075 J=1,3
      DO 2075 K=1,3
      S(M1,1)=S(M1,1) +  RA1(J)*A(J,K)*RA1(K)
      S(M2,1)=S(M2,1) +  RA1(J)*B(J,K)*RA1(K)
      S(M3,1)=S(M3,1) +  RA1(J)*C(J,K)*RA1(K)
      S(M1,2)=S(M1,2) + 3.* RA1(J)*A(J,K)*RA2(K)
      S(M2,2)=S(M2,2) + 3.* RA1(J)*B(J,K)*RA2(K)
      S(M3,2)=S(M3,2) + 3.* RA1(J)*C(J,K)*RA2(K)
      S(M1,3)=S(M1,3)+3.*RA2(J)*A(J,K)*RA2(K)+4.*RA1(J)*A(J,K)*RA3(K)
      S(M2,3)=S(M2,3)+3.*RA2(J)*B(J,K)*RA2(K)+4.*RA1(J)*B(J,K)*RA3(K)
      S(M3,3)=S(M3,3)+3.*RA2(J)*C(J,K)*RA2(K)+4.*RA1(J)*C(J,K)*RA3(K)
      S(M1,4)=S(M1,4)+5.*RA1(J)*A(J,K)*RA4(K)+10.*RA2(J)*A(J,K)*RA3(K)
      S(M2,4)=S(M2,4)+5.*RA1(J)*B(J,K)*RA4(K)+10.*RA2(J)*B(J,K)*RA3(K)
      S(M3,4)=S(M3,4)+5.*RA1(J)*C(J,K)*RA4(K)+10.*RA2(J)*C(J,K)*RA3(K)
 2075 CONTINUE
 2080 CONTINUE
      RETURN
```

```
      SUBROUTINE THIRD(T,Q,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A11(3,3),A12(3,3),A13(3,3),A22(3,3),A23(3,3),A33(3,3)
      DIMENSION RA1(3),RB1(3),RA2(3),RB2(3),RA3(3),RB3(3)
      DIMENSION T(N,3),Q(N)
      DIMENSION DLU(42),DLV(42),DLW(42)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      DO 3000 I=1,N
      DO 3000 J=1,3
      T(I,J)=0.0D0
 3000 CONTINUE
      DO 3300 M=1,NM
      J1=INP(M)
      K1=INQ(M)
      IF(J1.GT.NF.AND.K1.GT.NF) GO TO 3300
      IF(J1.GT.NF) GO TO 3100
      IF(K1.GT.NF) GO TO 3200
      M1=3*J1-2
      M2=3*J1-1
      M3=3*J1
      M4=3*K1-2
      M5=3*K1-1
      M6=3*K1
      RA1(1)=Q1(M1)
      RA1(2)=Q1(M2)
      RA1(3)=Q1(M3)
      RB1(1)=Q1(M4)
      RB1(2)=Q1(M5)
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      RB1(3)=Q1(M6)                                                     3RD 0330
      RA2(1)=Q2(M1)                                                     3RD 0340
      RA2(2)=Q2(M2)                                                     3RD 0350
      RA2(3)=Q2(M3)                                                     3RD 0360
      RB2(1)=Q2(M4)                                                     3RD 0370
      RB2(2)=Q2(M5)                                                     3RD 0380
      RB2(3)=Q2(M6)                                                     3RD 0390
      RA3(1)=Q3(M1)                                                     3RD 0400
      RA3(2)=Q3(M2)                                                     3RD 0410
      RA3(3)=Q3(M3)                                                     3RD 0420
      RB3(1)=Q3(M4)                                                     3RD 0430
      RB3(2)=Q3(M5)                                                     3RD 0440
      RB3(3)=Q3(M6)                                                     3RD 0450
      DU(M)=Q(M4)-Q(M1)                                                 3RD 0460
      DV(M)=Q(M5)-Q(M2)                                                 3RD 0470
      DW(M)=Q(M6)-Q(M3)                                                 3RD 0480
      SUMX(M)=DX(M)+DU(M)                                               3RD 0490
      SUMY(M)=DY(M)+DV(M)                                               3RD 0500
      SUMZ(M)=DZ(M)+DW(M)                                               3RD 0510
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)                    3RD 0520
      DL(M)=XLP(M)-XL(M)                                                3RD 0530
      DLU(M)=-SUMX(M)/XLP(M)                                            3RD 0540
      DLV(M)=-SUMY(M)/XLP(M)                                            3RD 0550
      DLW(M)=-SUMZ(M)/XLP(M)                                            3RD 0560
      D11=(1.-DLU(M)**2)/XLP(M)                                         3RD 0570
      D22=(1.-DLV(M)**2)/XLP(M)                                         3RD 0580
      D33=(1.-DLW(M)**2)/XLP(M)                                         3RD 0590
      D12=-DLU(M)*DLV(M)/XLP(M)                                         3RD 0600
      D13=-DLU(M)*DLW(M)/XLP(M)                                         3RD 0610
      D23=-DLV(M)*DLW(M)/XLP(M)                                         3RD 0620
      D111=(-3.*DLU(M)*(1.-DLU(M)**2))/XLP(M)**2                        3RD 0630
      D222=(-3.*DLV(M)*(1.-DLV(M)**2))/XLP(M)**2                        3RD 0640
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 D333=(-3.*DLW(M)*(1.-DLW(M)**2))/XLP(M)**2                    3RD 0650
 D112=(-DLV(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                    3RD 0660
 D113=(-DLW(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                    3RD 0670
 D223=(-DLW(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                    3RD 0680
 D122=(-DLU(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                    3RD 0690
 D133=(-DLU(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                    3RD 0700
 D233=(-DLV(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                    3RD 0710
 D123=(3.*DLU(M)*DLV(M)*DLW(M))/XLP(M)**2                      3RD 0720
 D1111= (-3.+18.*DLU(M)**2-15.*DLU(M)**4)/XLP(M)**3            3RD 0730
 D2222= (-3.+18.*DLV(M)**2-15.*DLV(M)**4)/XLP(M)**3            3RD 0740
 D3333= (-3.+18.*DLW(M)**2-15.*DLW(M)**4)/XLP(M)**3            3RD 0750
 D1112= (DLU(M)*DLV(M)*(9.-15.*DLU(M)**2))/XLP(M)**3           3RD 0760
 D1113= (DLU(M)*DLW(M)*(9.-15.*DLU(M)**2))/XLP(M)**3           3RD 0770
 D1222= (DLU(M)*DLV(M)*(9.-15.*DLV(M)**2))/XLP(M)**3           3RD 0780
 D2223= (DLW(M)*DLV(M)*(9.-15.*DLV(M)**2))/XLP(M)**3           3RD 0790
 D1333= (DLW(M)*DLU(M)*(9.-15.*DLW(M)**2))/XLP(M)**3           3RD 0800
 D2333= (DLW(M)*DLV(M)*(9.-15.*DLW(M)**2))/XLP(M)**3           3RD 0810
 D1122= (-1.+3.*(DLU(M)**2+DLV(M)**2)                          3RD 0820
$   -15.*DLU(M)**2*DLV(M)**2)/XLP(M)**3                        3RD 0830
 D1133= (-1.+3.*(DLU(M)**2+DLW(M)**2)                          3RD 0840
$   -15.*DLU(M)**2*DLW(M)**2)/XLP(M)**3                        3RD 0850
 D2233= (-1.+3.*(DLV(M)**2+DLW(M)**2)                          3RD 0860
$    -15.*DLV(M)**2*DLW(M)**2)/XLP(M)**3                       3RD 0870
 D1123= DLV(M)*DLW(M)*(3.-15.*DLU(M)**2)/XLP(M)**3             3RD 0880
 D1223= DLU(M)*DLW(M)*(3.-15.*DLV(M)**2)/XLP(M)**3             3RD 0890
 D1233= DLU(M)*DLV(M)*(3.-15.*DLW(M)**2)/XLP(M)**3             3RD 0900
 A11(1,1)= (DL(M)*D1111+4.*DLU(M)*D111+3.*D11**2)/XL(M)        3RD 0910
 A11(1,2)= (DL(M)*D1112+DLV(M)*D111                            3RD 0920
$  +3.*DLU(M)*D112+3.*D11*D12)/XL(M)                           3RD 0930
 A11(1,3)=  (DL(M)*D1113+DLW(M)*D111                           3RD 0940
$   +3.*DLU(M)*D113+3.*D11*D13)/XL(M)                          3RD 0950
 A11(2,2)=  (DL(M)*D1122+2.*DLV(M)*D112+D11*D22                3RD 0960
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$    +2.*DLU(M)*D122+2.*D12**2)/XL(M)                                   3RD 0970
 A11(2,3)=   (DL(M)*D1123+DLW(M)*D112+DLV(M)*D113                       3RD 0980
$   +D23*D11+2.*DLU(M)*D123+2.*D13*D12)/XL(M)                           3RD 0990
 A11(3,3)=   (DL(M)*D1133+2.*DLW(M)*D113+D11*D33                        3RD 1000
$    +2.*DLU(M)*D113+2.*D13**2)/XL(M)                                   3RD 1010
 A11(2,1)=A11(1,2)                                                      3RD 1020
 A11(3,1)=A11(1,3)                                                      3RD 1030
 A11(3,2)=A11(2,3)                                                      3RD 1040
 A12(1,1)=A11(1,2)                                                      3RD 1050
 A12(1,2)=A11(2,2)                                                      3RD 1060
 A12(1,3)=A11(3,2)                                                      3RD 1070
 A12(2,1)=A12(1,2)                                                      3RD 1080
 A12(2,2)=(DL(M)*D1222+DLU(M)*D222+3.*DLV(M)*D122+3.*D22*D12)/XL(M)3RD 1090
 A12(2,3)=  (DL(M)*D1223+DLW(M)*D122+DLU(M)*D223                        3RD 1100
$     +D22*D13+2.*D23*D12+2.*DLV(M)*D123)/XL(M)                         3RD 1110
 A12(3,2)=A12(2,3)                                                      3RD 1120
 A12(3,1)=A12(1,3)                                                      3RD 1130
 A12(3,3)=  (DL(M)*D1233+DLV(M)*D133+DLU(M)*D233+D33*D12                3RD 1140
$    +2.*D23*D13+2.*DLW(M)*D123)/XL(M)                                  3RD 1150
 A13(1,1)=A11(1,3)                                                      3RD 1160
 A13(1,2)=A11(2,3)                                                      3RD 1170
 A13(1,3)=A11(3,3)                                                      3RD 1180
 A13(2,1)=A13(1,2)                                                      3RD 1190
 A13(2,2)=A12(2,3)                                                      3RD 1200
 A13(2,3)=A12(3,3)                                                      3RD 1210
 A13(3,1)=A13(1,3)                                                      3RD 1220
 A13(3,2)=A13(2,3)                                                      3RD 1230
 A13(3,3)=(DL(M)*D1333+DLU(M)*D333+3.*DLW(M)*D133+3.*D22*D13)/XL(M)3RD 1240
 A22(1,1)=A12(1,2)                                                      3RD 1250
 A22(1,2)=A12(2,2)                                                      3RD 1260
 A22(1,3)=A12(3,2)                                                      3RD 1270
 A22(2,1)=A22(1,2)                                                      3RD 1280
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      A22(2,2)= (DL(M)*D2222+4.*DLV(M)*D222+3.*D22**2)/XL(M)            3RD 1290
      A22(2,3)= (DL(M)*D2223+DLW(M)*D222+3.*DLV(M)*D223                 3RD 1300
     $    +3.*D22*D23)/XL(M)                                            3RD 1310
      A22(3,1)=A22(1,3)                                                 3RD 1320
      A22(3,2)=A22(2,3)                                                 3RD 1330
      A22(3,3)= (DL(M)*D2233+2.*DLV(M)*D233+D22*D33                     3RD 1340
     $    +2.*DLW(M)*D223+2.*D23**2)/XL(M)                              3RD 1350
      A23(1,1)=A12(1,3)                                                 3RD 1360
      A23(1,2)=A12(2,3)                                                 3RD 1370
      A23(1,3)=A12(3,3)                                                 3RD 1380
      A23(2,1)=A23(1,2)                                                 3RD 1390
      A23(2,2)=A22(2,3)                                                 3RD 1400
      A23(2,3)=A22(3,3)                                                 3RD 1410
      A23(3,1)=A23(1,3)                                                 3RD 1420
      A23(3,2)=A23(2,3)                                                 3RD 1430
      A23(3,3)=(DL(M)*D2333+DLV(M)*D333+3.*DLW(M)*D233+3.*D33*D23)/XL(M)3RD 1440
      A33(1,1)=A13(1,3)                                                 3RD 1450
      A33(1,2)=A13(2,3)                                                 3RD 1460
      A33(1,3)=A13(3,3)                                                 3RD 1470
      A33(2,1)=A33(1,2)                                                 3RD 1480
      A33(2,2)=A23(2,3)                                                 3RD 1490
      A33(2,3)=A23(3,3)                                                 3RD 1500
      A33(3,1)=A33(1,3)                                                 3RD 1510
      A33(3,2)=A33(2,3)                                                 3RD 1520
      A33(3,3)=(DL(M)*D3333+4.*DLW(M)*D333+3.*D33**2)/XL(M)             3RD 1530
      DO 3050 I=1,3                                                     3RD 1540
      DO 3050 J=1,3                                                     3RD 1550
      T(M1,1)=T(M1,1) + ((Q1(M1)-Q1(M4))*A11(I,J)                       3RD 1560
     $  +(Q1(M2)-Q1(M5))*A12(I,J) + (Q1(M3)-Q1(M6))*A13(I,J))           3RD 1570
     $  *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                 3RD 1580
      T(M2,1)=T(M2,1) + ((Q1(M1)-Q1(M4))*A12(I,J)                       3RD 1590
     $  +(Q1(M2)-Q1(M5))*A22(I,J) + (Q1(M3)-Q1(M6))*A23(I,J))           3RD 1600
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$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1610
 T(M3,1)=T(M3,1) + ((Q1(M1)-Q1(M4))*A13(I,J)                           3RD 1620
$   +(Q1(M2)-Q1(M5))*A23(I,J) + (Q1(M3)-Q1(M6))*A33(I,J))              3RD 1630
$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1640
 T(M4,1)=T(M4,1) - ((Q1(M1)-Q1(M4))*A11(I,J)                           3RD 1650
$   +(Q1(M2)-Q1(M5))*A12(I,J) + (Q1(M3)-Q1(M6))*A13(I,J))              3RD 1660
$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1670
 T(M5,1)=T(M5,1) - ((Q1(M1)-Q1(M4))*A12(I,J)                           3RD 1680
$   +(Q1(M2)-Q1(M5))*A22(I,J) + (Q1(M3)-Q1(M6))*A23(I,J))              3RD 1690
$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1700
 T(M6,1)=T(M6,1) - ((Q1(M1)-Q1(M4))*A13(I,J)                           3RD 1710
$   +(Q1(M2)-Q1(M5))*A23(I,J) + (Q1(M3)-Q1(M6))*A33(I,J))              3RD 1720
$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1730
 T(M1,2)=T(M1,2) + 4.*((Q1(M1)-Q1(M4))*A11(I,J)                        3RD 1740
$    + (Q1(M2)-Q1(M5))*A12(I,J) + (Q1(M3)-Q1(M6))*A13(I,J))            3RD 1750
$ *(RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))           3RD 1760
$ + 2.*((Q2(M1)-Q2(M4))*A11(I,J)+(Q2(M2)-Q2(M5))*A12(I,J)              3RD 1770
$ + (Q2(M3)-Q2(M6))*A13(I,J))                                          3RD 1780
$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1790
 T(M2,2)=T(M2,2) + 4.*((Q1(M1)-Q1(M4))*A12(I,J)                        3RD 1800
$    + (Q1(M2)-Q1(M5))*A22(I,J) + (Q1(M3)-Q1(M6))*A23(I,J))            3RD 1810
$ *(RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))           3RD 1820
$ + 2.*((Q2(M1)-Q2(M4))*A12(I,J)+(Q2(M2)-Q2(M5))*A22(I,J)              3RD 1830
$ + (Q2(M3)-Q2(M6))*A23(I,J))                                          3RD 1840
$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1850
 T(M3,2)=T(M3,2) + 4.*((Q1(M1)-Q1(M4))*A13(I,J)                        3RD 1860
$    + (Q1(M2)-Q1(M5))*A23(I,J) + (Q1(M3)-Q1(M6))*A33(I,J))            3RD 1870
$ *(RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))           3RD 1880
$ + 2.*((Q2(M1)-Q2(M4))*A13(I,J)+(Q2(M2)-Q2(M5))*A23(I,J)              3RD 1890
$ + (Q2(M3)-Q2(M6))*A33(I,J))                                          3RD 1900
$   *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                    3RD 1910
 T(M4,2)=T(M4,2) - 4.*((Q1(M1)-Q1(M4))*A11(I,J)                        3RD 1920
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```
$    + (Q1(M2)-Q1(M5))*A12(I,J)  + (Q1(M3)-Q1(M6))*A13(I,J))               3RD 1930
$ *(RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))                3RD 1940
$ - 2.*((Q2(M1)-Q2(M4))*A11(I,J)+(Q2(M2)-Q2(M5))*A12(I,J)                   3RD 1950
$ + (Q2(M3)-Q2(M6))*A13(I,J))                                               3RD 1960
$  *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                          3RD 1970
 T(M5,2)=T(M5,2) - 4.*((Q1(M1)-Q1(M4))*A12(I,J)                             3RD 1980
$    + (Q1(M2)-Q1(M5))*A22(I,J)  + (Q1(M3)-Q1(M6))*A23(I,J))               3RD 1990
$ *(RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))                3RD 2000
$ - 2.*((Q2(M1)-Q2(M4))*A12(I,J)+(Q2(M2)-Q2(M5))*A22(I,J)                   3RD 2010
$  +  (Q2(M3)-Q2(M6))*A23(I,J))                                             3RD 2020
$  *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                          3RD 2030
 T(M6,2)=T(M6,2) - 4.*((Q1(M1)-Q1(M4))*A13(I,J)                             3RD 2040
$    + (Q1(M2)-Q1(M5))*A23(I,J)  + (Q1(M3)-Q1(M6))*A33(I,J))               3RD 2050
$ *(RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))                3RD 2060
$ - 2.*((Q2(M1)-Q2(M4))*A13(I,J)+(Q2(M2)-Q2(M5))*A23(I,J)                   3RD 2070
$  +  (Q2(M3)-Q2(M6))*A33(I,J))                                             3RD 2080
$  *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                          3RD 2090
 E1=                 20./3.*((Q1(M1)-Q1(M4))*A11(I,J)                       3RD 2100
$ +(Q1(M2)-Q1(M5))*A12(I,J)+(Q1(M3)-Q1(M6))*A13(I,J))*                      3RD 2110
$  (RA1(I)*RA3(J)-RA1(I)*RB3(J)-RB1(I)*RA3(J)+RB1(I)*RB3(J))                3RD 2120
 E2= 10./3.*((Q3(M1)-Q3(M4))*A11(I,J)+(Q3(M2)-Q3(M5))*A12(I,J)              3RD 2130
$ +(Q3(M3)-Q3(M6))*A13(I,J))                                                3RD 2140
$ *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                           3RD 2150
 E3=    10.*((Q2(M1)-Q2(M4))*A11(I,J)                                       3RD 2160
$ +(Q2(M2)-Q2(M5))*A12(I,J)+(Q2(M3)-Q2(M6))*A13(I,J))*                      3RD 2170
$  (RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))                3RD 2180
 E4=    5.*((Q1(M1)-Q1(M4))*A11(I,J)                                        3RD 2190
$ +(Q1(M2)-Q1(M5))*A12(I,J)+(Q1(M3)-Q1(M6))*A13(I,J))                       3RD 2200
$  *(RA2(I)*RA2(J)-2.*RA2(I)*RB2(J)+RB2(I)*RB2(J))                          3RD 2210
 T(M1,3)=T(M1,3) + E1 + E2 +E3 +E4                                          3RD 2220
 F1=                 20./3.*((Q1(M1)-Q1(M4))*A12(I,J)                       3RD 2230
$ +(Q1(M2)-Q1(M5))*A22(I,J)+(Q1(M3)-Q1(M6))*A23(I,J))*                      3RD 2240
```

```
$   (RA1(I)*RA3(J)-RA1(I)*RB3(J)-RB1(I)*RA3(J)+RB1(I)*RB3(J))        3RD 2250
    F2= 10./3.*((Q3(M1)-Q3(M4))*A12(I,J)+(Q3(M2)-Q3(M5))*A22(I,J)     3RD 2260
$  +(Q3(M3)-Q3(M6))*A23(I,J))                                          3RD 2270
$  *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                     3RD 2280
    F3=    10.*((Q2(M1)-Q2(M4))*A12(I,J)                                3RD 2290
$  +(Q2(M2)-Q2(M5))*A22(I,J)+(Q2(M3)-Q2(M6))*A23(I,J))*                3RD 2300
$   (RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))        3RD 2310
    F4=    5.*((Q1(M1)-Q1(M4))*A12(I,J)                                 3RD 2320
$  +(Q1(M2)-Q1(M5))*A22(I,J)+(Q1(M3)-Q1(M6))*A23(I,J))                 3RD 2330
$   *(RA2(I)*RA2(J)-2.*RA2(I)*RB2(J)+RB2(I)*RB2(J))                    3RD 2340
    T(M2,3)=T(M2,3)  + F1  + F2  +F3  + F4                              3RD 2350
    G1=                    20./3.*((Q1(M1)-Q1(M4))*A13(I,J)             3RD 2360
$  +(Q1(M2)-Q1(M5))*A13(I,J)+(Q1(M3)-Q1(M6))*A33(I,J))*                3RD 2370
$   (RA1(I)*RA3(J)-RA1(I)*RB3(J)-RB1(I)*RA3(J)+RB1(I)*RB3(J))        3RD 2380
    G2= 10./3.*((Q3(M1)-Q3(M4))*A13(I,J)+(Q3(M2)-Q3(M5))*A13(I,J)     3RD 2390
$  +(Q3(M3)-Q3(M6))*A33(I,J))                                          3RD 2400
$  *(RA1(I)*RA1(J)-2.*RA1(I)*RB1(J)+RB1(I)*RB1(J))                     3RD 2410
    G3=    10.*((Q2(M1)-Q2(M4))*A13(I,J)                                3RD 2420
$  +(Q2(M2)-Q2(M5))*A13(I,J)+(Q2(M3)-Q2(M6))*A33(I,J))*                3RD 2430
$   (RA1(I)*RA2(J)-RA1(I)*RB2(J)-RB1(I)*RA2(J)+RB1(I)*RB2(J))        3RD 2440
    G4=    5.*((Q1(M1)-Q1(M4))*A13(I,J)                                 3RD 2450
$  +(Q1(M2)-Q1(M5))*A13(I,J)+(Q1(M3)-Q1(M6))*A33(I,J))                 3RD 2460
$   *(RA2(I)*RA2(J)-2.*RA2(I)*RB2(J)+RB2(I)*RB2(J))                    3RD 2470
    T(M3,3)=T(M3,3)  + G1  + G2  + G3  + G4                             3RD 2480
    T(M4,3)=T(M4,3)  - E1  - E2  -E3  -E4                               3RD 2490
    T(M5,3)=T(M5,3)  - F1  - F2  -F3  - F4                              3RD 2500
    T(M6,3)=T(M6,3)  - G1  - G2  - G3  - G4                             3RD 2510
3050 CONTINUE                                                           3RD 2520
    GO TO 3300                                                          3RD 2530
3100 M4=3*K1-2                                                          3RD 2540
    M5=3*K1-1                                                           3RD 2550
    M6=3*K1                                                             3RD 2560
```

```
RB1(1)=Q1(M4)                                                    3RD 2570
RB1(2)=Q1(M5)                                                    3RD 2580
RB1(3)=Q1(M6)                                                    3RD 2590
RB2(1)=Q2(M4)                                                    3RD 2600
RB2(2)=Q2(M5)                                                    3RD 2610
RB2(3)=Q2(M6)                                                    3RD 2620
RB3(1)=Q3(M4)                                                    3RD 2630
RB3(2)=Q3(M5)                                                    3RD 2640
RB3(3)=Q3(M6)                                                    3RD 2650
DU(M)=Q(M4)                                                      3RD 2660
DV(M)=Q(M5)                                                      3RD 2670
DW(M)=Q(M6)                                                      3RD 2680
SUMX(M)=DX(M)+DU(M)                                              3RD 2690
SUMY(M)=DY(M)+DV(M)                                              3RD 2700
SUMZ(M)=DZ(M)+DW(M)                                              3RD 2710
XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)                   3RD 2720
DL(M)=XLP(M)-XL(M)                                               3RD 2730
DLU(M)=-SUMX(M)/XLP(M)                                           3RD 2740
DLV(M)=-SUMY(M)/XLP(M)                                           3RD 2750
DLW(M)=-SUMZ(M)/XLP(M)                                           3RD 2760
D11=(1.-DLU(M)**2)/XLP(M)                                        3RD 2770
D22=(1.-DLV(M)**2)/XLP(M)                                        3RD 2780
D33=(1.-DLW(M)**2)/XLP(M)                                        3RD 2790
D12=-DLU(M)*DLV(M)/XLP(M)                                        3RD 2800
D13=-DLU(M)*DLW(M)/XLP(M)                                        3RD 2810
D23=-DLV(M)*DLW(M)/XLP(M)                                        3RD 2820
D111=(-3.*DLU(M)*(1.-DLU(M)**2))/XLP(M)**2                       3RD 2830
D222=(-3.*DLV(M)*(1.-DLV(M)**2))/XLP(M)**2                       3RD 2840
D333=(-3.*DLW(M)*(1.-DLW(M)**2))/XLP(M)**2                       3RD 2850
D112=(-DLV(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                       3RD 2860
D113=(-DLW(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                       3RD 2870
D223=(-DLW(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                       3RD 2880
```

```
 D122=(-DLU(M)*(1.-3.*DLV(M)**2))/XLP(M)**2
 D133=(-DLU(M)*(1.-3.*DLW(M)**2))/XLP(M)**2
 D233=(-DLV(M)*(1.-3.*DLW(M)**2))/XLP(M)**2
 D123=(3.*DLU(M)*DLV(M)*DLW(M))/XLP(M)**2
 D1111= (-3.+18.*DLU(M)**2-15.*DLU(M)**4)/XLP(M)**3
 D2222= (-3.+18.*DLV(M)**2-15.*DLV(M)**4)/XLP(M)**3
 D3333= (-3.+18.*DLW(M)**2-15.*DLW(M)**4)/XLP(M)**3
 D1112= (DLU(M)*DLV(M)*(9.-15.*DLU(M)**2))/XLP(M)**3
 D1113= (DLU(M)*DLW(M)*(9.-15.*DLU(M)**2))/XLP(M)**3
 D2223= (DLW(M)*DLV(M)*(9.-15.*DLV(M)**2))/XLP(M)**3
 D1333= (DLW(M)*DLU(M)*(9.-15.*DLW(M)**2))/XLP(M)**3
 D2333= (DLW(M)*DLV(M)*(9.-15.*DLW(M)**2))/XLP(M)**3
 D1122= (-1.+3.*(DLU(M)**2+DLV(M)**2)
$    -15.*DLU(M)**2*DLV(M)**2)/XLP(M)**3
 D1133= (-1.+3.*(DLU(M)**2+DLW(M)**2)
$    -15.*DLU(M)**2*DLW(M)**2)/XLP(M)**3
 D2233= (-1.+3.*(DLV(M)**2+DLW(M)**2)
$     -15.*DLV(M)**2*DLW(M)**2)/XLP(M)**3
 D1123= DLV(M)*DLW(M)*(3.-15.*DLU(M)**2)/XLP(M)**3
 D1223= DLU(M)*DLW(M)*(3.-15.*DLV(M)**2)/XLP(M)**3
 D1233= DLU(M)*DLV(M)*(3.-15.*DLW(M)**2)/XLP(M)**3
 A11(1,1)= (DL(M)*D1111+4.*DLU(M)*D111+3.*D11**2)/XL(M)
 A11(1,2)= (DL(M)*D1112+DLV(M)*D111
$   +3.*DLU(M)*D112+3.*D11*D12)/XL(M)
 A11(1,3)=  (DL(M)*D1113+DLW(M)*D111
$    +3.*DLU(M)*D113+3.*D11*D13)/XL(M)
 A11(2,2)=  (DL(M)*D1122+2.*DLV(M)*D112+D11*D22
$    +2.*DLU(M)*D122+2.*D12**2)/XL(M)
 A11(2,3)=  (DL(M)*D1123+DLW(M)*D112+DLV(M)*D113
$   +D23*D11+2.*DLU(M)*D123+2.*D13*D12)/XL(M)
 A11(3,3)=  (DL(M)*D1133+2.*DLW(M)*D113+D11*D33
$    +2.*DLU(M)*D113+2.*D13**2)/XL(M)
```

```
      A11(2,1)=A11(1,2)                                                  3RD 3210
      A11(3,1)=A11(1,3)                                                  3RD 3220
      A11(3,2)=A11(2,3)                                                  3RD 3230
      A12(1,1)=A11(1,2)                                                  3RD 3240
      A12(1,2)=A11(2,2)                                                  3RD 3250
      A12(1,3)=A11(3,2)                                                  3RD 3260
      A12(2,1)=A12(1,2)                                                  3RD 3270
      A12(2,2)=(DL(M)*D1222+DLU(M)*D222+3.*DLV(M)*D122+3.*D22*D12)/XL(M)3RD 3280
      A12(2,3)= (DL(M)*D1223+DLW(M)*D122+DLU(M)*D223                     3RD 3290
     $    +D22*D13+2.*D23*D12+2.*DLV(M)*D123)/XL(M)                      3RD 3300
      A12(3,2)=A12(2,3)                                                  3RD 3310
      A12(3,1)=A12(1,3)                                                  3RD 3320
      A12(3,3)= (DL(M)*D1233+DLV(M)*D133+DLU(M)*D233+D33*D12             3RD 3330
     $   +2.*D23*D13+2.*DLW(M)*D123)/XL(M)                               3RD 3340
      A13(1,1)=A11(1,3)                                                  3RD 3350
      A13(1,2)=A11(2,3)                                                  3RD 3360
      A13(1,3)=A11(3,3)                                                  3RD 3370
      A13(2,1)=A13(1,2)                                                  3RD 3380
      A13(2,2)=A12(2,3)                                                  3RD 3390
      A13(2,3)=A12(3,3)                                                  3RD 3400
      A13(3,1)=A13(1,3)                                                  3RD 3410
      A13(3,2)=A13(2,3)                                                  3RD 3420
      A13(3,3)=(DL(M)*D1333+DLU(M)*D333+3.*DLW(M)*D133+3.*D22*D13)/XL(M)3RD 3430
      A22(1,1)=A12(1,2)                                                  3RD 3440
      A22(1,2)=A12(2,2)                                                  3RD 3450
      A22(1,3)=A12(3,2)                                                  3RD 3460
      A22(2,1)=A22(1,2)                                                  3RD 3470
      A22(2,2)= (DL(M)*D2222+4.*DLV(M)*D222+3.*D22**2)/XL(M)             3RD 3480
      A22(2,3)= (DL(M)*D2223+DLW(M)*D222+3.*DLV(M)*D223                  3RD 3490
     $   +3.*D22*D23)/XL(M)                                              3RD 3500
      A22(3,1)=A22(1,3)                                                  3RD 3510
      A22(3,2)=A22(2,3)                                                  3RD 3520
```

```
      A22(3,3)= (DL(M)*D2233+2.*DLV(M)*D233+D22*D33
     $   +2.*DLW(M)*D223+2.*D23**2)/XL(M)
      A23(1,1)=A12(1,3)
      A23(1,2)=A12(2,3)
      A23(1,3)=A12(3,3)
      A23(2,1)=A23(1,2)
      A23(2,2)=A22(2,3)
      A23(2,3)=A22(3,3)
      A23(3,1)=A23(1,3)
      A23(3,2)=A23(2,3)
      A23(3,3)=(DL(M)*D2333+DLV(M)*D333+3.*DLW(M)*D233+3.*D33*D23)/XL(M)
      A33(1,1)=A13(1,3)
      A33(1,2)=A13(2,3)
      A33(1,3)=A13(3,3)
      A33(2,1)=A33(1,2)
      A33(2,2)=A23(2,3)
      A33(2,3)=A23(3,3)
      A33(3,1)=A33(1,3)
      A33(3,2)=A33(2,3)
      A33(3,3)=(DL(M)*D3333+4.*DLW(M)*D333+3.*D22**2)/XL(M)
      DO 3150 I=1,3
      DO 3150 J=1,3
      T(M4,1)=T(M4,1) + (Q1(M4)*A11(I,J)+Q1(M5)*A12(I,J)
     $ + Q1(M6)*A13(I,J))*RB1(I)*RB1(J)
      T(M5,1)=T(M5,1) + (Q1(M4)*A12(I,J)+Q1(M5)*A22(I,J)
     $ + Q1(M6)*A13(I,J))*RB1(I)*RB1(J)
      T(M6,1)=T(M6,1) + (Q1(M4)*A13(I,J)+Q1(M5)*A23(I,J)
     $ + Q1(M6)*A13(I,J))*RB1(I)*RB1(J)
      T(M4,2)=T(M4,2)+4.*(Q1(M4)*A11(I,J)+Q1(M5)*A12(I,J)
     $  + Q1(M6)*A13(I,J))*RB1(I)*RB2(J)
     $  +2.*(Q2(M4)*A11(I,J)+Q2(M5)*A12(I,J)
     $  + Q2(M6)*A13(I,J))*RB1(I)*RB1(J)
```

```
 T(M5,2)=T(M5,2)+4.*(Q1(M4)*A12(I,J)+Q1(M5)*A22(I,J)
$   + Q1(M6)*A23(I,J))*RB1(I)*RB2(J)
$   +2.*(Q2(M4)*A12(I,J)+Q2(M5)*A22(I,J)
$   + Q2(M6)*A23(I,J))*RB1(I)*RB1(J)
 T(M6,2)=T(M6,2)+4.*(Q1(M4)*A13(I,J)+Q1(M5)*A23(I,J)
$   + Q1(M6)*A33(I,J))*RB1(I)*RB2(J)
$   +2.*(Q2(M4)*A13(I,J)+Q2(M5)*A23(I,J)
$   + Q2(M6)*A33(I,J))*RB1(I)*RB1(J)
 T(M4,3)=T(M4,3) + 20./3.*(Q1(M4)*A11(I,J)+Q1(M5)*A12(I,J)
$    + Q1(M6)*A13(I,J))*RB1(I)*RB3(J)
$ +10./3.*(Q3(M4)*A11(I,J)+Q3(M5)*A12(I,J)
$   + Q3(M6)*A13(I,J))*RB1(I)*RB1(J)
$   +10.*(Q2(M4)*A11(I,J)+Q2(M5)*A12(I,J)
$   +Q2(M6)*A13(I,J))*RB1(I)*RB2(J)
$+5.*(Q1(M4)*A11(I,J)+Q1(M5)*A12(I,J)+Q1(M6)*A13(I,J))
$    *RB2(I)*RB2(J)
 T(M5,3)=T(M5,3) + 20./3.*(Q1(M4)*A12(I,J)+Q1(M5)*A22(I,J)
$    + Q1(M6)*A23(I,J))*RB1(I)*RB3(J)
$ +10./3.*(Q3(M4)*A12(I,J)+Q3(M5)*A22(I,J)
$   + Q3(M6)*A23(I,J))*RB1(I)*RB1(J)
$   +10.*(Q2(M4)*A12(I,J)+Q2(M5)*A22(I,J)
$   +Q2(M6)*A23(I,J))*RB1(I)*RB2(J)
$+5.*(Q1(M4)*A12(I,J)+Q1(M5)*A22(I,J)+Q1(M6)*A23(I,J))
$    *RB2(I)*RB2(J)
 T(M6,3)=T(M6,3) + 20./3.*(Q1(M4)*A13(I,J)+Q1(M5)*A23(I,J)
$    + Q1(M6)*A33(I,J))*RB1(I)*RB3(J)
$ +10./3.*(Q3(M4)*A13(I,J)+Q3(M5)*A23(I,J)
$   + Q3(M6)*A33(I,J))*RB1(I)*RB1(J)
$   +10.*(Q2(M4)*A13(I,J)+Q2(M5)*A23(I,J)
$   +Q2(M6)*A33(I,J))*RB1(I)*RB2(J)
$+5.*(Q1(M4)*A13(I,J)+Q1(M5)*A23(I,J)+Q1(M6)*A33(I,J))
$    *RB2(I)*RB2(J)
```

```
 3150 CONTINUE                                               3RD 4170
      GO TO 3300                                             3RD 4180
 3200 M1=3*J1-2                                              3RD 4190
      M2=3*J1-1                                              3RD 4200
      M3=3*J1                                                3RD 4210
      RA1(1)=Q1(M1)                                          3RD 4220
      RA1(2)=Q1(M2)                                          3RD 4230
      RA1(3)=Q1(M3)                                          3RD 4240
      RA2(1)=Q2(M1)                                          3RD 4250
      RA2(2)=Q2(M2)                                          3RD 4260
      RA2(3)=Q2(M3)                                          3RD 4270
      RA3(1)=Q3(M1)                                          3RD 4280
      RA3(2)=Q3(M2)                                          3RD 4290
      RA3(3)=Q3(M3)                                          3RD 4300
      DU(M)=-Q(M1)                                           3RD 4310
      DV(M)=-Q(M2)                                           3RD 4320
      DW(M)=-Q(M3)                                           3RD 4330
      SUMX(M)=DX(M)+DU(M)                                    3RD 4340
      SUMY(M)=DY(M)+DV(M)                                    3RD 4350
      SUMZ(M)=DZ(M)+DW(M)                                    3RD 4360
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)         3RD 4370
      DL(M)=XLP(M)-XL(M)                                     3RD 4380
      DLU(M)=-SUMX(M)/XLP(M)                                 3RD 4390
      DLV(M)=-SUMY(M)/XLP(M)                                 3RD 4400
      DLW(M)=-SUMZ(M)/XLP(M)                                 3RD 4410
      D11=(1.-DLU(M)**2)/XLP(M)                              3RD 4420
      D22=(1.-DLV(M)**2)/XLP(M)                              3RD 4430
      D33=(1.-DLW(M)**2)/XLP(M)                              3RD 4440
      D12=-DLU(M)*DLV(M)/XLP(M)                              3RD 4450
      D13=-DLU(M)*DLW(M)/XLP(M)                              3RD 4460
      D23=-DLV(M)*DLW(M)/XLP(M)                              3RD 4470
      D111=(-3.*DLU(M)*(1.-DLU(M)**2))/XLP(M)**2             3RD 4480
```

```
      D222=(-3.*DLV(M)*(1.-DLV(M)**2))/XLP(M)**2                        3RD 4490
      D333=(-3.*DLW(M)*(1.-DLW(M)**2))/XLP(M)**2                        3RD 4500
      D112=(-DLV(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                        3RD 4510
      D113=(-DLW(M)*(1.-3.*DLU(M)**2))/XLP(M)**2                        3RD 4520
      D223=(-DLW(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                        3RD 4530
      D122=(-DLU(M)*(1.-3.*DLV(M)**2))/XLP(M)**2                        3RD 4540
      D133=(-DLU(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                        3RD 4550
      D233=(-DLV(M)*(1.-3.*DLW(M)**2))/XLP(M)**2                        3RD 4560
      D123=(3.*DLU(M)*DLV(M)*DLW(M))/XLP(M)**2                          3RD 4570
      D1111=  (-3.+18.*DLU(M)**2-15.*DLU(M)**4)/XLP(M)**3               3RD 4580
      D2222=  (-3.+18.*DLV(M)**2-15.*DLV(M)**4)/XLP(M)**3               3RD 4590
      D3333=  (-3.+18.*DLW(M)**2-15.*DLW(M)**4)/XLP(M)**3               3RD 4600
      D1112=  (DLU(M)*DLV(M)*(9.-15.*DLU(M)**2))/XLP(M)**3              3RD 4610
      D1113=  (DLU(M)*DLW(M)*(9.-15.*DLU(M)**2))/XLP(M)**3              3RD 4620
      D2223=  (DLW(M)*DLV(M)*(9.-15.*DLV(M)**2))/XLP(M)**3              3RD 4630
      D1333=  (DLW(M)*DLU(M)*(9.-15.*DLW(M)**2))/XLP(M)**3              3RD 4640
      D2333=  (DLW(M)*DLV(M)*(9.-15.*DLW(M)**2))/XLP(M)**3              3RD 4650
      D1122=  (-1.+3.*(DLU(M)**2+DLV(M)**2)                             3RD 4660
     $   -15.*DLU(M)**2*DLV(M)**2)/XLP(M)**3                            3RD 4670
      D1133=  (-1.+3.*(DLU(M)**2+DLW(M)**2)                             3RD 4680
     $   -15.*DLU(M)**2*DLW(M)**2)/XLP(M)**3                            3RD 4690
      D2233=  (-1.+3.*(DLV(M)**2+DLW(M)**2)                             3RD 4700
     $    -15.*DLV(M)**2*DLW(M)**2)/XLP(M)**3                           3RD 4710
      D1123=  DLV(M)*DLW(M)*(3.-15.*DLU(M)**2)/XLP(M)**3                3RD 4720
      D1223=  DLU(M)*DLW(M)*(3.-15.*DLV(M)**2)/XLP(M)**3                3RD 4730
      D1233=  DLU(M)*DLV(M)*(3.-15.*DLW(M)**2)/XLP(M)**3                3RD 4740
      A11(1,1)=  (DL(M)*D1111+4.*DLU(M)*D111+3.*D11**2)/XL(M)           3RD 4750
      A11(1,2)=  (DL(M)*D1112+DLV(M)*D111                               3RD 4760
     $  +3.*DLU(M)*D112+3.*D11*D12)/XL(M)                               3RD 4770
      A11(1,3)=   (DL(M)*D1113+DLW(M)*D111                              3RD 4780
     $   +3.*DLU(M)*D113+3.*D11*D13)/XL(M)                              3RD 4790
      A11(2,2)=   (DL(M)*D1122+2.*DLV(M)*D112+D11*D22                   3RD 4800
```

```
$    +2.*DLU(M)*D122+2.*D12**2)/XL(M)
 A11(2,3)=   (DL(M)*D1123+DLW(M)*D112+DLV(M)*D113
$   +D23*D11+2.*DLU(M)*D123+2.*D13*D12)/XL(M)
 A11(3,3)=   (DL(M)*D1133+2.*DLW(M)*D113+D11*D33
$    +2.*DLU(M)*D113+2.*D13**2)/XL(M)
 A11(2,1)=A11(1,2)
 A11(3,1)=A11(1,3)
 A11(3,2)=A11(2,3)
 A12(1,1)=A11(1,2)
 A12(1,2)=A11(2,2)
 A12(1,3)=A11(3,2)
 A12(2,1)=A12(1,2)
 A12(2,2)=(DL(M)*D1222+DLU(M)*D222+3.*DLV(M)*D122+3.*D22*D12)/XL(M)
 A12(2,3)= (DL(M)*D1223+DLW(M)*D122+DLU(M)*D223
$      +D22*D13+2.*D23*D12+2.*DLV(M)*D123)/XL(M)
 A12(3,2)=A12(2,3)
 A12(3,1)=A12(1,3)
 A12(3,3)= (DL(M)*D1233+DLV(M)*D133+DLU(M)*D233+D33*D12
$    +2.*D23*D13+2.*DLW(M)*D123)/XL(M)
 A13(1,1)=A11(1,3)
 A13(1,2)=A11(2,3)
 A13(1,3)=A11(3,3)
 A13(2,1)=A13(1,2)
 A13(2,2)=A12(2,3)
 A13(2,3)=A12(3,3)
 A13(3,1)=A13(1,3)
 A13(3,2)=A13(2,3)
 A13(3,3)=(DL(M)*D1333+DLU(M)*D333+3.*DLW(M)*D133+3.*D22*D13)/XL(M)
 A22(1,1)=A12(1,2)
 A22(1,2)=A12(2,2)
 A22(1,3)=A12(3,2)
 A22(2,1)=A22(1,2)
```

```
 A22(2,2)= (DL(M)*D2222+4.*DLV(M)*D222+3.*D22**2)/XL(M)
 A22(2,3)= (DL(M)*D2223+DLW(M)*D222+3.*DLV(M)*D223
$    +3.*D22*D23)/XL(M)
 A22(3,1)=A22(1,3)
 A22(3,2)=A22(2,3)
 A22(3,3)= (DL(M)*D2233+2.*DLV(M)*D233+D22*D33
$    +2.*DLW(M)*D223+2.*D23**2)/XL(M)
 A23(1,1)=A12(1,3)
 A23(1,2)=A12(2,3)
 A23(1,3)=A12(3,3)
 A23(2,1)=A23(1,2)
 A23(2,2)=A22(2,3)
 A23(2,3)=A22(3,3)
 A23(3,1)=A23(1,3)
 A23(3,2)=A23(2,3)
 A23(3,3)=(DL(M)*D2333+DLV(M)*D333+3.*DLW(M)*D233+3.*D33*D23)/XL(M)
 A33(1,1)=A13(1,3)
 A33(1,2)=A13(2,3)
 A33(1,3)=A13(3,3)
 A33(2,1)=A33(1,2)
 A33(2,2)=A23(2,3)
 A33(2,3)=A23(3,3)
 A33(3,1)=A33(1,3)
 A33(3,2)=A33(2,3)
 A33(3,3)=(DL(M)*D3333+4.*DLW(M)*D333+3.*D22**2)/XL(M)
 DO 3250 I=1,3
 DO 3250 J=1,3
 T(M1,1)=T(M1,1) + (Q1(M1)*A11(I,J)+Q1(M2)*A12(I,J)
$  +  Q1(M3)*A13(I,J))*RA1(I)*RA1(J)
 T(M2,1)=T(M2,1) + (Q1(M1)*A12(I,J)+Q1(M2)*A22(I,J)
$  +  Q1(M3)*A23(I,J))*RA1(I)*RA1(J)
 T(M3,1)=T(M3,1) + (Q1(M1)*A13(I,J)+Q1(M2)*A23(I,J)
```

```
$   +   Q1(M3)*A33(I,J))*RA1(I)*RA1(J)                                  3RD 5450
 T(M1,2)=T(M1,2)  + 4.*(Q1(M1)*A11(I,J)+Q1(M2)*A12(I,J)                 3RD 5460
$   +   Q1(M3)*A13(I,J))*RA1(I)*RA2(J)                                  3RD 5470
$+2.*(Q2(M1)*A11(I,J)+Q2(M2)*A12(I,J)+Q2(M3)*A13(I,J))                  3RD 5480
$    *RA1(I)*RA1(J)                                                     3RD 5490
 T(M2,2)=T(M2,2)  + 4.*(Q1(M1)*A12(I,J)+Q1(M2)*A22(I,J)                 3RD 5500
$   +   Q1(M3)*A23(I,J))*RA1(I)*RA2(J)                                  3RD 5510
$+2.*(Q2(M1)*A12(I,J)+Q2(M2)*A22(I,J)+Q2(M3)*A23(I,J))                  3RD 5520
$    *RA1(I)*RA1(J)                                                     3RD 5530
 T(M3,2)=T(M3,2)  + 4.*(Q1(M1)*A13(I,J)+Q1(M2)*A23(I,J)                 3RD 5540
$   +   Q1(M3)*A33(I,J))*RA1(I)*RA2(J)                                  3RD 5550
$+2.*(Q2(M1)*A13(I,J)+Q2(M2)*A23(I,J)+Q2(M3)*A33(I,J))                  3RD 5560
$    *RA1(I)*RA1(J)                                                     3RD 5570
 T(M1,3)=T(M1,3)  + 20./3.*(Q1(M1)*A11(I,J)+Q1(M2)*A12(I,J)             3RD 5580
$  + Q1(M3)*A13(I,J))*RA1(I)*RA3(J)                                     3RD 5590
$ +10./3.*(Q3(M1)*A11(I,J)+Q3(M2)*A12(I,J)+Q3(M3)*A13(I,J))             3RD 5600
$    *RA1(I)*RA1(J)                                                     3RD 5610
$   +10.*(Q2(M1)*A11(I,J)+Q2(M2)*A12(I,J)+Q2(M3)*A13(I,J))              3RD 5620
$   *RA1(I)*RA2(J)                                                      3RD 5630
$   +5.*(Q1(M1)*A11(I,J)+Q1(M2)*A12(I,J)+Q1(M3)*A13(I,J))               3RD 5640
$    *RA2(I)*RA2(J)                                                     3RD 5650
 T(M2,3)=T(M2,3)  + 20./3.*(Q1(M1)*A12(I,J)+Q1(M2)*A22(I,J)             3RD 5660
$  + Q1(M3)*A23(I,J))*RA1(I)*RA3(J)                                     3RD 5670
$ +10./3.*(Q3(M1)*A12(I,J)+Q3(M2)*A22(I,J)+Q3(M3)*A23(I,J))             3RD 5680
$    *RA1(I)*RA1(J)                                                     3RD 5690
$   +10.*(Q2(M1)*A12(I,J)+Q2(M2)*A22(I,J)+Q2(M3)*A23(I,J))              3RD 5700
$   *RA1(I)*RA2(J)                                                      3RD 5710
$   +5.*(Q1(M1)*A12(I,J)+Q1(M2)*A22(I,J)+Q1(M3)*A23(I,J))               3RD 5720
$    *RA2(I)*RA2(J)                                                     3RD 5730
 T(M3,3)=T(M3,3)  + 20./3.*(Q1(M1)*A13(I,J)+Q1(M2)*A23(I,J)             3RD 5740
$  + Q1(M3)*A33(I,J))*RA1(I)*RA3(J)                                     3RD 5750
$ +10./3.*(Q3(M1)*A13(I,J)+Q3(M2)*A23(I,J)+Q3(M3)*A33(I,J))             3RD 5760
```

```
     $    *RA1(I)*RA1(J)
     $   +10.*(Q2(M1)*A13(I,J)+Q2(M2)*A23(I,J)+Q2(M3)*A23(I,J))
     $   *RA1(I)*RA2(J)
     $   +5.*(Q1(M1)*A13(I,J)+Q1(M2)*A23(I,J)+Q1(M3)*A33(I,J))
     $    *RA2(I)*RA2(J)
3250 CONTINUE
3300 CONTINUE
     RETURN
     END
```

```
      SUBROUTINE TAYLOR(A,B,C,N,INS,IORDER,IW)                          TLR 0010
      IMPLICIT REAL*8(A-H,O-Z)                                          TLR 0020
      DIMENSION A(N),B(N),GRAD(39),X(39)                                TLR 0030
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)                  TLR 0040
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)                 TLR 0050
      COMMON SUMX(42),SUMY(42),SUMZ(42)                                 TLR 0060
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET                     TLR 0070
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT                             TLR 0080
      IF(IW.EQ.0) RETURN                                                TLR 0090
      IF(IORDER.EQ.1) WRITE(6,10)                                       TLR 0100
      IF(IORDER.EQ.2) WRITE(6,15)                                       TLR 0110
   10 FORMAT('1'/////5X,'FIRST ORDER INCREMENTAL SOLUTION FROM PREVIOUS TLR 0120
     $EQUILIBRIUM'/5X,'-------------------------------------------------TLR 0130
     $---------'//)                                                     TLR 0140
   15 FORMAT('1'/////5X,'SECOND ORDER INCREMENTAL SOLUTION FROM PREVIOUSTLR 0150
     $ EQUILIBRIUM'/5X,'------------------------------------------------TLR 0160
     $-------------------'//)                                           TLR 0170
      IF(IORDER.EQ.3) WRITE(6,25)                                       TLR 0180
   25 FORMAT('1'/////5X,'THIRD ORDER INCREMENTAL SOLUTION FROM PREVIOUS TLR 0190
     $EQUILIBRIUM'/5X,'-------------------------------------------------TLR 0200
     $----------'//)                                                    TLR 0210
      IF(INS.EQ.1) WRITE(6,20)                                          TLR 0220
      IF(INS.NE.1) WRITE(6,30)                                          TLR 0230
   20 FORMAT(17X,'BY DISPLACEMENT-INCREMENT APPROACH'/                  TLR 0240
     $17X,'----------------------------------'////)                     TLR 0250
   30 FORMAT(22X,'BY LOAD-INCREMENT APPROACH'/                          TLR 0260
     $22X,'--------------------------'////)                             TLR 0270
      WRITE(6,40)                                                       TLR 0280
   40 FORMAT(1X,'JOINT NO',5X,'X-DISPL',10X,'Y-DISPL',10X,'Z-DISPL',    TLR 0290
     $10X,'LOAD-X',11X,'LOAD-Y',11X,'LOAD-Z'//)                         TLR 0300
      DO 90 I=1,NJ                                                      TLR 0310
      I1=3*I-2                                                          TLR 0320
```

```
      I2=3*I-1
      I3=3*I
      IF(I.GT.NF) GO TO 70
      IF(I.EQ.1) WRITE(6,50) I,A(I1),A(I2),A(I3),C
      IF(I.NE.1) WRITE(6,60) I,A(I1),A(I2),A(I3)
   50 FORMAT(I5,3(6X,D12.5),35X,D12.5)
   60 FORMAT(I5,3(6X,D12.5))
   70 IF(I.LE.NF) GO TO 90
      WRITE(6,80) I
   80 FORMAT(I5)
   90 CONTINUE
C
      WRITE(6,85) SDET
   85 FORMAT(///10X,'STABILITY DETERMINANT =',D12.5)
C
      IF(IORDER.NE.1) RETURN
      DO 100 I=2,NF
      IF(PX(I).NE.0.0D0) PX(I)=PX(I)+C
      IF(PY(I).NE.0.0D0) PY(I)=PY(I)+C
      IF(PZ(I).NE.0.0D0) PZ(I)=PZ(I)+C
  100 CONTINUE
      IF(PX(1) .NE. 0.0D0) PX(1)=PX(1)+(C/FLOAD)
      DO 200 I=1,N
      X(I)=A(I)+B(I)
  200 CONTINUE
      CALL V(N,X,FUNT,GRAD)
      CALL OUTP(X,GRAD,FUNT,N,IORDER,INS,IW)
      DO 300 I=2,NF
      IF(PX(I).NE.0.0D0) PX(I)=PX(I)-C
      IF(PY(I).NE.0.0D0) PY(I)=PY(I)-C
      IF(PZ(I).NE.0.0D0) PZ(I)=PZ(I)-C
  300 CONTINUE
```

```
      IF(PX(1) .NE. 0.0D0) PX(1)=PX(1)-(C/FLOAD)                        TLR 0650
      RETURN                                                            TLR 0660
      END                                                               TLR 0670
```

```
      SUBROUTINE OUTP(Q,GR,FUN,N,INS,INT,IW)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Q(N),GR(N)
      COMMON PX(13),PY(13),PZ(13),DX(42),DY(42),DZ(42)
      COMMON XL(42),XLP(42),DU(42),DV(42),DW(42),DL(42)
      COMMON SUMX(42),SUMY(42),SUMZ(42)
      COMMON Q1(39),Q2(39),Q3(39),Q4(39),FLOAD,SDET
      COMMON INP(42),INQ(42),NM,NJ,NF,KOUNT
      IF(IW.EQ.0) RETURN
      IF(INS.EQ.0.AND.INT.EQ.0) WRITE(6,400)
      IF(INS.EQ.1.AND.INT.EQ.1) WRITE(6,500)
      IF(INS.EQ.1.AND.INT.EQ.2) WRITE(6,600)
      IF(INS.EQ.2.AND.INT.EQ.1) WRITE(6,700)
      IF(INS.EQ.2.AND.INT.EQ.2) WRITE(6,800)
      IF(INS.EQ.3.AND.INT.EQ.1) WRITE(6,900)
      IF(INS.EQ.3.AND.INT.EQ.2) WRITE(6,1000)
  400 FORMAT('1'///10X,'SOLUTION OBTAINED FROM MINIMIZATION VIA CONJUGAT
     $E GRADIENT METHOD'/10X,'--------------------------------------------
     $-----------------------'//)
  500 FORMAT('1'///10X,'FIRST ORDER PERTURBATION SOLUTION VIA DISPLACEME
     $NT INCREMENT'/10X,'---------------------------------------------------
     $-------------'//)
  600 FORMAT('1'///10X,'FIRST ORDER PERTURBATION SOLUTION VIA LOAD INCRE
     $MENT'/10X,'-----------------------------------------------------'//
     $)
  700 FORMAT('1'///10X,'SECOND ORDER PERTURBATION SOLUTION VIA DISPLACEM
     $ENT INCREMENT'/10X,'------------------------------------------------
     $------'//)
  800 FORMAT('1'///10X,'SECOND ORDER PERTURBATION SOLUTION VIA LOAD INCR
     $EMENT'/10X,'-----------------------------------------------------'
     $//)
  900 FORMAT('1'///10X,'THIRD ORDER PERTURBATION SOLUTION VIA DISPLACEME
```

```
     $NT INCREMENT'/10X,'-----------------------------------------------
     $-------------'//)
1000 FORMAT('1'///10X,'THIRD ORDER PERTURBATION SOLUTION VIA LOAD INCRE
     $MENT'/10X,'-----------------------------------------------------'//
     $)
      WRITE(6,10)
   10 FORMAT(//1X,'JOINT NO',5X,'X-DISPL',10X,'Y-DISPL',10X,'Z-DISPL',
     $10X,'LOAD-X',11X,'LOAD-Y',11X,'LOAD-Z'//)
      DO 100 J=1,NJ
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      IF(J.LE.NF) WRITE(6,20) J,Q(J1),Q(J2),Q(J3),PX(J),PY(J),PZ(J)
      IF(J.GT.NF) WRITE(6,30) J
   20 FORMAT(I5,6(6X,D12.5))
   30 FORMAT(I5,7X,'0.0',5(15X,'0.0'))
  100 CONTINUE
      WRITE(6,40)
   40 FORMAT(///1X,'JOINT NO',5X,'GRAD-X',12X,'GRAD-Y',12X,'GRAD-Z'//)
      DO 300 J=1,NJ
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      IF(J.LE.NF) WRITE(6,50) J,GR(J1),GR(J2),GR(J3)
      IF(J.GT.NF) WRITE(6,60) J
   50 FORMAT(I5,3(6X,D12.5))
   60 FORMAT(I5,7X,'0.0',2(15X,'0.0'))
  300 CONTINUE
      WRITE(6,350) FUN
  350 FORMAT(/////10X,'FUNCTIONAL VALUE = ',D12.5/10X,'---------------'
     $)
      RETURN
```

OUT 0650

END

```
C                                                                              INP 0010
C======= DATA FOR UNIFORM LOADING  =============================              INP 0020
C                                                                              INP 0030
   42   19                                                                     INP 0040
    1       0.0          0.0          0.0        0                             INP 0050
    2       2.0          0.0         25.0        0                             INP 0060
    3       2.0         21.65        12.5        0                             INP 0070
    4       2.0         21.65       -12.5        0                             INP 0080
    5       2.0          0.0        -25.0        0                             INP 0090
    6       2.0        -21.65       -12.5        0                             INP 0100
    7       2.0        -21.65        12.5        0                             INP 0110
    8       8.1609      25.0         43.3        1                             INP 0120
    9       8.1609      50.0          0.0        1                             INP 0130
   10       8.1609      25.0        -43.3        1                             INP 0140
   11       8.1609     -25.0        -43.3        1                             INP 0150
   12       8.1609     -50.0          0.0        1                             INP 0160
   13       8.1609     -25.0         43.3        1                             INP 0170
   14      10.87         0.0         50.0        1                             INP 0180
   15      10.87        50.0         25.0        1                             INP 0190
   16      10.87        50.0        -25.0        1                             INP 0200
   17      10.87         0.0        -50.0        1                             INP 0210
   18      10.87       -50.0        -25.0        1                             INP 0220
   19      10.87       -50.0         25.0        1                             INP 0230
    1    2    1                                                                INP 0240
    2    3    1                                                                INP 0250
    3    4    1                                                                INP 0260
    4    5    1                                                                INP 0270
    5    6    1                                                                INP 0280
    6    7    1                                                                INP 0290
    7    6    7                                                                INP 0300
    8    7    2                                                                INP 0310
    9    2    3                                                                INP 0320
```

```
10    3    4                                   INP 0330
11    4    5                                   INP 0340
12    5    6                                   INP 0350
13    6   12                                   INP 0360
14   12    7                                   INP 0370
15    7   13                                   INP 0380
16   13    2                                   INP 0390
17    2    8                                   INP 0400
18    8    3                                   INP 0410
19    3    9                                   INP 0420
20    9    4                                   INP 0430
21    4   10                                   INP 0440
22   10    5                                   INP 0450
23    5   11                                   INP 0460
24   11    6                                   INP 0470
25    6   18                                   INP 0480
26    7   19                                   INP 0490
27    2   14                                   INP 0500
28    3   15                                   INP 0510
29    4   16                                   INP 0520
30    5   17                                   INP 0530
31   17   11                                   INP 0540
32   11   18                                   INP 0550
33   18   12                                   INP 0560
34   12   19                                   INP 0570
35   19   13                                   INP 0580
36   13   14                                   INP 0590
37   14    8                                   INP 0600
38    8   15                                   INP 0610
39   15    9                                   INP 0620
40    9   16                                   INP 0630
41   16   10                                   INP 0640
```

```
  42   10   17                                                        INP 0650
   1  0.00001                                                         INP 0660
   2  0.00001                                                         INP 0670
   3  0.00001                                                         INP 0680
   4  0.00001                                                         INP 0690
   5  0.00001                                                         INP 0700
   6  0.00001                                                         INP 0710
   7  0.00001                                                         INP 0720
  1.0                                                                 INP 0730
DISP      LOAD                                                        INP 0740
```

```
C
C==============  D Y N A M I C    A N A L Y S I S ==================
C
C     OBJECTIVE : TO OBTAIN THE PERTURBED MOTIONS (RESPONSES) OF
C                 RETICULATED DOMES TO MULTIPLE STATIC AND
C                 DYNAMIC LOADS
C
C     METHOD : HAMMING'S MODIFIED PREDICTOR-CRRECTOR NUMERICAL
C              INTEGRATION SCHEME
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      EXTERNAL V
      REAL FS,ACC
      DIMENSION G(21)
      DIMENSION H(294),X(13),Y(13),Z(13)
      DIMENSION ICON(13)
      COMMON U(21),XM(21),C(21)
      COMMON PX(7),PY(7),PZ(7),DX(24),DY(24),DZ(24)
      COMMON XL(24),XLP(24),DU(24),DV(24),DW(24),DL(24)
      COMMON SUMX(24),SUMY(24),SUMZ(24)
      COMMON INP(24),INQ(24),NM,NJ,NF,KOUNT
C
C     A SPACE TRUSS WITH HINGED CONNECTED JOINTS
C
      WRITE(6,10)
   10 FORMAT('1',27X,'**********************************************'/
     $28X,'*',44X,'*'/
     $28X,'* A SPACE TRUSS WITH HINGED CONNECTED JOINTS *'/
     $28X,'*    SUBJECTED TO LOADS AT THE JOINTS ONLY    *'/
     $28X,'*',44X,'*'/
     $28X,'**********************************************'///)
```

```
C
C     INPUT DATA FOR MEMBER NUMBERS, NM AND NUMBER OF JOINTS, NJ
C
      READ(5,20) NM,NJ
   20 FORMAT(2I5)
      WRITE(6,30) NM,NJ
   30 FORMAT(//10X,'NUMBER OF ELEMENTS, NM =',I3//10X,'NUMBER OF JOINTS,
     $ NJ =',I3)
C
C     JOINT COORDINATES AND CONSTRAINT OF THE JOINT
C     ICON(J):     MEANS CONSTRAINT AT JOINT 'J'
C     ITS VALUES ARE:      ICON(J)=1 -COMPLETE CONSTRAINED OR FIXED
C                          ICON(J)=0 -COMPLETELY FREE
C
C
C     NF= NUMBER OF UNCONSTRAINED JOINTS
C
      NF=NJ
      WRITE(6,40)
   40 FORMAT(/////20X,'JOINT COORDINATES AND CONSTRAINTS'/////11X,'JOINT
     $ NO',5X,'X-COORD',10X,'Y-COORD',10X,'Z-COORD',8X,'CONSTRAINT'//)
      DO 70 J=1,NJ
      READ(5,50) K,X(K),Y(K),Z(K),ICON(K)
   50 FORMAT(I5,3F15.6,4X,I1)
      WRITE(6,60) K,X(K),Y(K),Z(K),ICON(K)
   60 FORMAT(10X,I5,6X,D12.5,6X,D12.5,5X,D12.5,10X,I1)
      NF=NF-ICON(K)
   70 CONTINUE
C
C     MEMBER INCIDENCE TABLE
C
      WRITE(6,80)
```

```
   80 FORMAT('1',///10X, 'MEMBER INCIDENCE TABLE AND ITS LENGTH'/////5X,
     $'MEMBER NO',10X,'MEMBER INCIDENCE',13X,'MEMBER LENGTH'//)
      DO 110 M=1,NM
      READ(5,90) J,INP(J),INQ(J)
   90 FORMAT(3I5)
      I=INP(J)
      K=INQ(J)
      DX(M)=X(K)-X(I)
      DY(M)=Y(K)-Y(I)
      DZ(M)=Z(K)-Z(I)
      XL(M)= DSQRT(DX(M)**2+DY(M)**2+DZ(M)**2)
      WRITE(6,100) J,INP(J),INQ(J),XL(J)
  100 FORMAT(4X,I6,15X,2I5,15X,D15.5)
  110 CONTINUE
C
C     NUMBER OF DEGREE OF FREEDOM IS EQUAL TO THREE
C     TIMES NUMBER OF UNCONSTRAINED JOINTS, N=3*NF
C
      N=3*NF
      WRITE(6,120) NF,N
  120 FORMAT(//////5X,'NUMBER OF FREE JOINTS, NF =',I3//5X,'NUMBER OF DE
     $GREE OF FREEDOM, N =',I3)
C
C     SOLUTION FOR THE EQUILIBRIUM PATH VIA MINIMIZATION TECHNIQUE
C     USING POWELL-FLECHER METHOD, THIS SOLUTION WILL BE USED AS
C     A STARTING POINT FOR THE PERTURBATION TECHNIQUE TO DESCRIBE
C     THE LOAD VS DISPLACEMENT FOR THE ENTIRE PATH
C
      WRITE(6,130)
  130 FORMAT('1'///10X,'MINIMIZATION VIA CONJUGATE GRADIENT METHOD'///)
C
C     INITIAL GUESS OF VARIABLES
```

```
C
      WRITE(6,140)
  140 FORMAT(//20X,'INITIAL GUESS OF VARIABLES'///11X,'JOINT',8X,'X-DISP
     $.',10X,'Y-DISP.',10X,'Z-DISP.'//)
      DO 160 I=1,NF
      I1=3*I-2
      I2=3*I-1
      I3=3*I
      U(I1)=0.0D0
      U(I2)=0.0D0
      U(I3)=0.0D0
      WRITE(6,150) I,U(I1),U(I2),U(I3)
  150 FORMAT(10X,I5,9X,D12.5,6X,D12.5,6X,D12.5)
  160 CONTINUE
C
C     INITIALIZED ALL JOINT LOADS TO BE ZERO
C
      DO 170 I=1,NF
      PX(I)=0.0D0
      PY(I)=0.0D0
      PZ(I)=0.0D0
  170 CONTINUE
C
C     THE ERROR INDICATOR, IER = 9 IS USED AS THE INITIAL INPUT
C     THE RETURN VALUE ARE SUCH THAT :
C     IER = 0....MEANS CONVERGENCE IS ACHIEVED WITHIN SPECIFIED ACCURACY
C     IER = 1....MEANS NO CONVERGENCE IN THE LIMIT ITERATION
C     IER =-1....MEANS ERROS IN THE GRADIENT CALCULATION
C     IER = 2.... MEANS IT IS LIKELY THAT THERE EXISTS NO MINIMUM
C
      IER=9
      ACC=1.D-10
```

```
      LIMIT=100
      FS=0.0
      KOUNT=0
C
C     INPUT LOADING AT THE JOINT
C
      DO 305 KQ=1,NF
      READ(5,180) K,PX(K)
  180 FORMAT(I5,E15.5)
  305 CONTINUE
      CALL DFMCG(V,N,U,F,G,FS,ACC,LIMIT,IER,H)
      WRITE(6,186) LIMIT,KOUNT
  186 FORMAT(/////5X,'NUMBER OF ITERATION =',I4//
     $ 5X,I5,5X,'FUNCTION SUBROUTINE CALLED'/////)
      IW=1
      CALL OUTP(U,G,F,N,0,0,IW)
      KOUNT=0
      CALL DYN
      STOP
      END
```

```
      SUBROUTINE V(N,A,VAL,GRAD)
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(21),GRAD(21),DE(21)
      COMMON U(21),XM(21),C(21)
      COMMON PX(7),PY(7),PZ(7),DX(24),DY(24),DZ(24)
      COMMON XL(24),XLP(24),DU(24),DV(24),DW(24),DL(24)
      COMMON SUMX(24),SUMY(24),SUMZ(24)
      COMMON INP(24),INQ(24),NM,NJ,NF,KOUNT
      KOUNT=KOUNT+1
      WORK=0.0D0
      DO 6000 J=1,NF
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      WORK=WORK+PX(J)*A(J1)+PY(J)*A(J2)+PZ(J)*A(J3)
      DE(J1)=0.0D0
      DE(J2)=0.0D0
      DE(J3)=0.0D0
 6000 CONTINUE
      STENG=0.0D0
      DO 6001 M=1,NM
      J=INP(M)
      K=INQ(M)
      IF(J.GT.NF.AND.K.GT.NF) GO TO 6001
      M1=3*J-2
      M2=3*J-1
      M3=3*J
      M4=3*K-2
      M5=3*K-1
      M6=3*K
```

```
      IF(J.GT.NF) GO TO 700
      IF(K.GT.NF) GO TO 750
      GO TO 760
  700 DU(M)=A(M4)
      DV(M)=A(M5)
      DW(M)=A(M6)
      SUMX(M)=DX(M)+DU(M)
      SUMY(M)=DY(M)+DV(M)
      SUMZ(M)=DZ(M)+DW(M)
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)
      DL(M)=XLP(M)-XL(M)
      STENG=STENG+(0.5*DL(M)**2)/XL(M)
      DE(M4)=DE(M4)+(DL(M)*SUMX(M))/(XL(M)*XLP(M))
      DE(M5)=DE(M5)+(DL(M)*SUMY(M))/(XL(M)*XLP(M))
      DE(M6)=DE(M6)+(DL(M)*SUMZ(M))/(XL(M)*XLP(M))
      GO TO 6001
  750 DU(M)=-A(M1)
      DV(M)=-A(M2)
      DW(M)=-A(M3)
      SUMX(M)=DX(M)+DU(M)
      SUMY(M)=DY(M)+DV(M)
      SUMZ(M)=DZ(M)+DW(M)
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)
      DL(M)=XLP(M)-XL(M)
      STENG=STENG+(0.5*DL(M)**2)/XL(M)
      DE(M1)=DE(M1)-(DL(M)*SUMX(M))/(XL(M)*XLP(M))
      DE(M2)=DE(M2)-(DL(M)*SUMY(M))/(XL(M)*XLP(M))
      DE(M3)=DE(M3)-(DL(M)*SUMZ(M))/(XL(M)*XLP(M))
      GO TO 6001
  760 DU(M)=A(M4)-A(M1)
      DV(M)=A(M5)-A(M2)
      DW(M)=A(M6)-A(M3)
```

```
      SUMX(M)=DX(M)+DU(M)                                              VEE 0650
      SUMY(M)=DY(M)+DV(M)                                              VEE 0660
      SUMZ(M)=DZ(M)+DW(M)                                              VEE 0670
      XLP(M)=DSQRT(SUMX(M)**2+SUMY(M)**2+SUMZ(M)**2)                   VEE 0680
      DL(M)=XLP(M)-XL(M)                                               VEE 0690
      STENG=STENG+(0.5*DL(M)**2)/XL(M)                                 VEE 0700
      DE(M1)=DE(M1)-(DL(M)*SUMX(M))/(XL(M)*XLP(M))                     VEE 0710
      DE(M2)=DE(M2)-(DL(M)*SUMY(M))/(XL(M)*XLP(M))                     VEE 0720
      DE(M3)=DE(M3)-(DL(M)*SUMZ(M))/(XL(M)*XLP(M))                     VEE 0730
      DE(M4)=DE(M4)+(DL(M)*SUMX(M))/(XL(M)*XLP(M))                     VEE 0740
      DE(M5)=DE(M5)+(DL(M)*SUMY(M))/(XL(M)*XLP(M))                     VEE 0750
      DE(M6)=DE(M6)+(DL(M)*SUMZ(M))/(XL(M)*XLP(M))                     VEE 0760
 6001 CONTINUE                                                         VEE 0770
      VAL=STENG-WORK                                                   VEE 0780
      DO 6002 J=1,NF                                                   VEE 0790
      J1=3*J-2                                                         VEE 0800
      J2=3*J-1                                                         VEE 0810
      J3=3*J                                                           VEE 0820
      GRAD(J1)=DE(J1)-PX(J)                                            VEE 0830
      GRAD(J2)=DE(J2)-PY(J)                                            VEE 0840
      GRAD(J3)=DE(J3)-PZ(J)                                            VEE 0850
 6002 CONTINUE                                                         VEE 0860
      RETURN                                                           VEE 0870
      END                                                              VEE 0880
```

```
      SUBROUTINE OUTP(Q,GR,FUN,N,INS,INT,IW)
C
C
C     THIS SUBROUTINE GIVES THE OUT-PUT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Q(N),GR(N)
      COMMON U(21),XM(21),C(21)
      COMMON PX(7),PY(7),PZ(7),DX(24),DY(24),DZ(24)
      COMMON XL(24),XLP(24),DU(24),DV(24),DW(24),DL(24)
      COMMON SUMX(24),SUMY(24),SUMZ(24)
      COMMON INP(24),INQ(24),NM,NJ,NF,KOUNT
      IF(IW.EQ.0) RETURN
      IF(INS.EQ.0.AND.INT.EQ.0) WRITE(6,400)
      IF(INS.EQ.1.AND.INT.EQ.1) WRITE(6,500)
      IF(INS.EQ.1.AND.INT.EQ.2) WRITE(6,600)
      IF(INS.EQ.2.AND.INT.EQ.1) WRITE(6,700)
      IF(INS.EQ.2.AND.INT.EQ.2) WRITE(6,800)
      IF(INS.EQ.3.AND.INT.EQ.1) WRITE(6,900)
      IF(INS.EQ.3.AND.INT.EQ.2) WRITE(6,1000)
      IF(INS.EQ.4.AND.INT.EQ.1) WRITE(6,1100)
      IF(INS.EQ.4.AND.INT.EQ.2) WRITE(6,1200)
      IF(INS.EQ.5.AND.INT.EQ.1) WRITE(6,1300)
      IF(INS.EQ.5.AND.INT.EQ.2) WRITE(6,1400)
      IF(INS.EQ.6.AND.INT.EQ.1) WRITE(6,1500)
      IF(INS.EQ.6.AND.INT.EQ.2) WRITE(6,1600)
  400 FORMAT('1'///10X,'SOLUTION OBTAINED FROM MINIMIZATION VIA CONJUGAT
     $E GRADIENT METHOD'/10X,'-----------------------------------------
     $-----------------------'//)
  500 FORMAT('1'///10X,'FIRST ORDER PERTURBATION SOLUTION VIA DISPLACEME
     $NT INCREMENT'/10X,'---------------------------------------------
     $-------------'//)
```

```
600 FORMAT('1'///10X,'FIRST ORDER PERTURBATION SOLUTION VIA LOAD INCREOUTP0330
   $MENT'/10X,'-----------------------------------------------'//OUTP0340
   $)                                                            OUTP0350
700 FORMAT('1'///10X,'SECOND ORDER PERTURBATION SOLUTION VIA DISPLACEMOUTP0360
   $ENT INCREMENT'/10X,'------------------------------------------OUTP0370
   $------'//)                                                   OUTP0380
800 FORMAT('1'///10X,'SECOND ORDER PERTURBATION SOLUTION VIA LOAD INCROUTP0390
   $EMENT'/10X,'-----------------------------------------------'OUTP0400
   $//)                                                          OUTP0410
900 FORMAT('1'///10X,'THIRD ORDER PERTURBATION SOLUTION VIA DISPLACEMEOUTP0420
   $NT INCREMENT'/10X,'-------------------------------------------OUTP0430
   $-------------'//)                                            OUTP0440
1000 FORMAT('1'///10X,'THIRD ORDER PERTURBATION SOLUTION VIA LOAD INCREOUTP0450
   $MENT'/10X,'-----------------------------------------------'//OUTP0460
   $)                                                            OUTP0470
1100 FORMAT('1'///10X,'FOURTH ORDER PERTURBATION SOLUTION VIA DISPLACEMOUTP0480
   $ENT INCREMENT'/10X,'------------------------------------------OUTP0490
   $-----------------'//)                                        OUTP0500
1200 FORMAT('1'///10X,'FOURTH ORDER PERTURBATION SOLUTION VIA LOAD INCROUTP0510
   EMENT'/10X,'-------------------------------------------------'/OUTP0520
   $/)                                                           OUTP0530
1300 FORMAT('1'///10X,'FIFTH ORDER PERTURBATION SOLUTION VIA DISPLACEMEOUTP0540
   $NT INCREMENT'/10X,'-------------------------------------------OUTP0550
   $------------'//)                                             OUTP0560
1400 FORMAT('1'///10X,'FIFTH ORDER PERTURBATION SOLUTION VIA LOAD INCREOUTP0570
   $MENT'/10X,'-----------------------------------------------'//OUTP0580
   $)                                                            OUTP0590
1500 FORMAT('1'///10X,'SIXTH ORDER PERTURBATION SOLUTION VIA DISPLACEMEOUTP0600
   $NT INCREMENT'/10X,'-------------------------------------------OUTP0610
   $------------'//)                                             OUTP0620
1600 FORMAT('1'///10X,'SIXTH ORDER PERTURBATION SOLUTION VIA LOAD INCREOUTP0630
   $MENT'/10X,'-----------------------------------------------'//OUTP0640
```

```
   $)
    WRITE(6,10)
 10 FORMAT(//1X,'JOINT NO',5X,'X-DISPL',10X,'Y-DISPL',10X,'Z-DISPL',
   $10X,'LOAD-X',11X,'LOAD-Y',11X,'LOAD-Z'//)
    DO 100 J=1,NJ
    J1=3*J-2
    J2=3*J-1
    J3=3*J
    IF(J.LE.NF) WRITE(6,20) J,Q(J1),Q(J2),Q(J3),PX(J),PY(J),PZ(J)
    IF(J.GT.NF) WRITE(6,30) J
 20 FORMAT(I5,6(6X,D12.5))
 30 FORMAT(I5,7X,'0.0',5(15X,'0.0'))
100 CONTINUE
    WRITE(6,40)
 40 FORMAT(///1X,'JOINT NO',5X,'GRAD-X',12X,'GRAD-Y',12X,'GRAD-Z'//)
    DO 300 J=1,NJ
    J1=3*J-2
    J2=3*J-1
    J3=3*J
    IF(J.LE.NF) WRITE(6,50) J,GR(J1),GR(J2),GR(J3)
    IF(J.GT.NF) WRITE(6,60) J
 50 FORMAT(I5,3(6X,D12.5))
 60 FORMAT(I5,7X,'0.0',2(15X,'0.0'))
300 CONTINUE
    WRITE(6,350) FUN
350 FORMAT(/////10X,'FUNCTIONAL VALUE = ',D12.5/10X,'----------------'
   $)
    RETURN
    END
```

```
      SUBROUTINE DYN
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      EXTERNAL FCT,OUT
      DIMENSION PRMT(5),X(42),DERY(42),AUX(16,42),XW(13)
      COMMON U(21),XM(21),C(21)
      COMMON PX(7),PY(7),PZ(7),DX(24),DY(24),DZ(24)
      COMMON XL(24),XLP(24),DU(24),DV(24),DW(24),DL(24)
      COMMON SUMX(24),SUMY(24),SUMZ(24)
      COMMON INP(24),INQ(24),NM,NJ,NF,KOUNT
      DO 100 J=1,NJ
      XW(J)=0.0D0
  100 CONTINUE
      DO 200 M=1,NM
      J=INP(M)
      K=INQ(M)
      XW(J)=XW(J)+(0.5*XL(M)*490.)/(32.2*12.**4)
      XW(K)=XW(K)+(0.5*XL(M)*490.)/(32.2*12.**4)
  200 CONTINUE
      DO 400 J=1,NF
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      XM(J1)=XW(J)
      XM(J2)=XW(J)
      XM(J3)=XW(J)
      READ(5,300) C(J1),C(J2),C(J3)
  300 FORMAT(3E20.5)
  400 CONTINUE
      WRITE(6,500)
  500 FORMAT('1'///30X,'INITIAL CONDITIONS FOR DYNAMIC ANALYSIS'/////
```

```
   $ 30X,'DISPLACEMENT',45X,'VELOCITY'/
   $ 30X,'------------',45X,'--------'/
   $ 2X,'JOINTNO',8X,'X-DIRCT',10X,'Y-DIRCT',10X,'Z-DIRCT',
   $ 15X,'X-DIRCT',10X,'Y-DIRCT',10X,'Z-DIRCT'//)
    DO 700 J=1,NF
    J1=6*J-5
    J2=6*J-4
    J3=6*J-3
    J4=6*J-2
    J5=6*J-1
    J6=6*J
    READ(5,600) TQ,X(J1),X(J3),X(J5),X(J2),X(J4),X(J6)
600 FORMAT(F8.5,6D12.5)
    WRITE(6,650) J,X(J1),X(J3),X(J5),X(J2),X(J4),X(J6)
650 FORMAT(I6,7X,6D15.5)
700 CONTINUE
    READ(5,800) (PRMT(I),I=1,5)
    WRITE(6,850) (PRMT(I),I=1,5)
800 FORMAT(5E15.5)
850 FORMAT(/////5X,5D15.5)
    NDIM=6*NF
    XN=NDIM
    DO 900 I=1,NDIM
    DERY(I)=1./XN
900 CONTINUE
    IH=5
    WRITE(6,950)
950 FORMAT('1')
    CALL DHPCG(PRMT,X,DERY,NDIM,IH,FCT,OUT,AUX)
    RETURN
    END
```

```
      SUBROUTINE FCT(T,X,DERY)                                          FCT 0010
C                                                                       FCT 0020
C                                                                       FCT 0030
      IMPLICIT REAL*8(A-H,O-Z)                                          FCT 0040
      DIMENSION X(42),DERY(42),G(21),Q(21)                              FCT 0050
      COMMON U(21),XM(21),C(21)                                         FCT 0060
      COMMON PX(7),PY(7),PZ(7),DX(24),DY(24),DZ(24)                     FCT 0070
      COMMON XL(24),XLP(24),DU(24),DV(24),DW(24),DL(24)                 FCT 0080
      COMMON SUMX(24),SUMY(24),SUMZ(24)                                 FCT 0090
      COMMON INP(24),INQ(24),NM,NJ,NF,KOUNT                             FCT 0100
      N=3*NF                                                            FCT 0110
      DO 100 J=1,NF                                                     FCT 0120
      J1=3*J-2                                                          FCT 0130
      J2=3*J-1                                                          FCT 0140
      J3=3*J                                                            FCT 0150
      J4=6*J-5                                                          FCT 0160
      J5=6*J-3                                                          FCT 0170
      J6=6*J-1                                                          FCT 0180
      Q(J1)=U(J1)+X(J4)                                                 FCT 0190
      Q(J2)=U(J2)+X(J5)                                                 FCT 0200
      Q(J3)=U(J3)+X(J6)                                                 FCT 0210
  100 CONTINUE                                                          FCT 0220
      CALL V(N,Q,VAL,G)                                                 FCT 0230
      DO 200 J=1,N                                                      FCT 0240
      G(J)=-G(J)*3.0D07                                                 FCT 0250
  200 CONTINUE                                                          FCT 0260
      NDIM=2*N                                                          FCT 0270
      DO 300 J=1,N                                                      FCT 0280
      I1=2*J-1                                                          FCT 0290
      I2=2*J                                                            FCT 0300
      DERY(I1)=X(I2)                                                    FCT 0310
      IF(J.EQ.1) DERY(I2)=(G(J)-C(J)*X(I2))/XM(J)                       FCT 0320
```

```
      IF(J.NE.1) DERY(I2)=(G(J)-C(J)*X(I2))/XM(J)                       FCT 0330
  300 CONTINUE                                                          FCT 0340
      RETURN                                                            FCT 0350
      END                                                               FCT 0360
```

```
      SUBROUTINE OUT(T,X,DERY,IH,N,PRMT)                                OUT 0010
C                                                                       OUT 0020
C                                                                       OUT 0030
      IMPLICIT REAL*8(A-H,O-Z)                                          OUT 0040
      DIMENSION Q(21),G(21)                                             OUT 0050
      DIMENSION X(42),DERY(42),PRMT(5)                                  OUT 0060
      COMMON U(21),XM(21),C(21)                                         OUT 0070
      COMMON PX(7),PY(7),PZ(7),DX(24),DY(24),DZ(24)                     OUT 0080
      COMMON XL(24),XLP(24),DU(24),DV(24),DW(24),DL(24)                 OUT 0090
      COMMON SUMX(24),SUMY(24),SUMZ(24)                                 OUT 0100
      COMMON INP(24),INQ(24),NM,NJ,NF,KOUNT                             OUT 0110
      IF(T.EQ.0.0D0) GO TO 40                                           OUT 0120
      JN=INT/100                                                        OUT 0130
      JM=JN*100                                                         OUT 0140
      IF(JM.EQ.INT) GO TO 60                                            OUT 0150
      GO TO 50                                                          OUT 0160
   40 INT=0                                                             OUT 0170
   60 CONTINUE                                                          OUT 0180
      EK=0.0D0                                                          OUT 0190
      DO 15 J=1,NF                                                      OUT 0200
      J1=3*J-2                                                          OUT 0210
      J2=3*J-1                                                          OUT 0220
      J3=3*J                                                            OUT 0230
      J4=6*J-5                                                          OUT 0240
      J5=6*J-3                                                          OUT 0250
      J6=6*J-1                                                          OUT 0260
      K1=6*J-4                                                          OUT 0270
      K2=6*J-2                                                          OUT 0280
      K3=6*J                                                            OUT 0290
      Q(J1)=U(J1)+X(J4)                                                 OUT 0300
      Q(J2)=U(J2)+X(J5)                                                 OUT 0310
      Q(J3)=U(J3)+X(J6)                                                 OUT 0320
```

```
      EK=EK+0.5D0*(XM(J1)*X(K1)**2+XM(J2)*X(K2)**2+XM(J3)*X(K3)**2)
   15 CONTINUE
      CALL  V(21,Q,VAL,G)
      EEK=EK/3.0D07
      TEG=VAL+EEK
      WRITE(6,10) T,VAL,EK,EEK,TEG
   10 FORMAT(/2X,5D20.5//)
      DO 30 J=1,NF
      J1=6*J-5
      J2=6*J-4
      J3=6*J-3
      J4=6*J-2
      J5=6*J-1
      J6=6*J
      WRITE(6,20) J,X(J1),X(J3),X(J5),X(J2),X(J4),X(J6)
   20 FORMAT(16X,I5,6(6X,D12.5))
      WRITE(7,26) T,X(J1),X(J3),X(J5),X(J2),X(J4),X(J6)
   26 FORMAT(F8.5,6D12.5)
   30 CONTINUE
   50 INT=INT+1
      RETURN
      END
```

VITA

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