CHAPTER 2.

GEOMETRICAL MODELING OF WOOD TRANSVERSE THERMAL CONDUCTIVITY

Summary

This chapter discusses development of geometric models for predicting thermal conductivity in radial and tangential directions for softwood and hardwood species. The models were generated from wood anatomical structure observation and measurements. Models were separated for the radial and tangential directions in both softwood and hardwood species due the different structure in the two directions. Structure observation was made under the Scanning Electron Microscope and Environmental Scanning Electron Microscope and measurements were preformed with the help of image analysis. Modeling the effective thermal conductivity in radial and tangential direction is helpful to understand the heat transfer mechanism in the two directions and predict the values in a wide range when the practical experiments for obtaining those values are hard to make.

Model estimations provide insights for changes of the heat transfer parameters -- thermal conductivity, with the independent variables used in the model. Validation tests for the model-estimated thermal conductivities were performed on three species -- southern yellow pine, Scots pine and soft maple. Radial and tangential thermal conductivity were measured for all of these three species to compare with model output. Discrepancies between model and testing results were analyzed. Theoretical model outputs were compared with the testing results and empirical model results to evaluate the model predictions. A sensitivity study provided insights for the relative importance of input parameters in the model estimations.

2.1 Introduction

Wood and wood-based materials have many applications in areas that require good insulating properties. Their low thermal conductivity and good strength make them of special interest for building construction, refrigerators, cars and beer barrels, etc. (Kollmann & Cote 1968, Ward 1960). The poor heat conductance of wood is due to the paucity of free electrons which are media for energy transmission and due to the porosity of wood. Wood is a typical porous material. Its structure is complicated, which makes it a strongly anisotropic material in the area of drying shrinkage and mechanical applications.

The complicated structure of wood consists of many cell types which have a cell wall surrounding a cell lumen in the center. For example, these types of cells are thick-walled latewood tracheids, thin-walled earlywood tracheids in the softwood species, and thick-walled fibers, thin-walled vessels in the hardwood species. Wood cells are also arranged in two main directions. Most of the cells are in the longitudinal direction as the tree's grain direction. Some of wood cells, such as ray cells, are arranged in the radial-transverse direction, perpendicular to the grain direction. It is the structure of wood that makes its physical and mechanical properties direction dependent.

The thermal conductivity of wood varies with the direction of heat flow with respect to the grain, with specific gravity, with defects, with extractives, and also with the moisture content in wood and temperature (MacLean 1941). Thermal conductivity of wood is the combination of the thermal conductivity values of the substances in wood. The theoretical models for examining the relationship of wood structure and the directional dependent thermal conductivity have been studied for years (Kollmann & Malmquist 1956, Siau 1968, Couturier 1996). But no theoretical values for wood thermal conductivity in three directions ---- longitudinal, radial and tangential ---- have been given and compared with experiments. And, no theoretical prediction for the thermal conductivity change with structure change and moisture content change is available.

Thermal conductivity of wood is usually measured by the steady-state method, which usually requires some time for wood samples to reach equilibrium. If the wood samples contain high moisture content, it will take a fairly long time for the moisture distribution in wood to reach the equilibrium state. It is not quite realistic to do the test. So with the help of theoretical understanding of the wood thermal conductivity, it will be possible to predict the change of this property with the extended range of moisture content.

Theoretical models can be set up on dry wood samples. Then experimental results on these dry samples can be used as a comparison to evaluate the model. After the model is validated, it can be used for prediction in an extended range.

Thermal conductivity, referred to as a transport property, provides an indication of the rate at which energy is transferred by the diffusion process. It depends on the physical structure of matter, atomic and molecular, which is related to the state of the matter (Incropera & DeWitt 1981). A knowledge of thermal conductivity of wood in a large range is important in kiln-drying operations, preservation impregnation, hot pressing of wood based composites, wood thermal degradation and other process in which wood is subjected to a temperature change.

2.2 Background

2.2.1 Wood anatomical structure

The thermal conductivity of wood is a structure dependent property. It varies in the three main directions of wood as they are usually referred to in the wood lumber industry ---- longitudinal direction (parallel to the grain, along the length of a tree), radial direction (perpendicular to the grain, along the radius of the round tree on the cross section), and tangential direction (perpendicular to the grain, tangent to each growth ring). The anatomical structure of wood in these three directions is different. And, most of the anisotropic properties of wood are due to this structure difference.

The majority component of softwood species is long slender cells called longitudinal tracheids, which occupy about 90-95% of the total wood volume (Haygreen & Bowyer 1982). On the cross section of a tree, they are close to rectangular shape with different cell wall thickness. Tracheids that are formed early in a growing season are thin-walled cells with greater diameters, while those formed later in the year are thick-walled cells with smaller diameters. Tracheids give softwood the mechanical strength required (especially the thick-walled latewood tracheids) and provide for heat and mass transport. The heat transfer in wood is mainly by conduction through the cell walls, and partly by the convection of air in the cell lumens. The mass (moisture) transfer in wood from one tracheid to another takes place through the bordered pits. There are numerous pits on the tracheid's radial cell walls, and only a few on the tangential walls. Gronli (1996) reported that there are about 200 pits on each earlywood tracheid and only 10-50 rather small bordered pits on each latewood tracheid. Another important type of cells in softwood are ray cells, which include ray tracheids and ray parenchyma arranging in radial strands perpendicular to the grain direction. Softwood rays are from one to many cells in height but are usually only one cell wide. Ray cells are small compared to the longitudinal tracheids. Resin canals are another characteristic for the softwood species, especially in pine. One typical softwood genus is pine. Southern yellow pine is a popular species group for construction application in America, and Scots pine is a popular species for the European construction industry. Figure 2.1 and Figure 2.2 show the general structure of these two species (Hoadley 1980).

Comparing to the softwood structure, hardwood structure is much more complex. Hardwoods are composed of more diverse types of cells, more varied arrangement of cells and varied sizes of rays. One major type of cells is the specialized conducting cells called vessel

Figure 2. 1 Southern yellow pine (*Pinus* spp.) cross section structure 20× (Hoadley 1980).

Figure 2. 2 Red pine (*P.resinosa*) cross section structure 20× (Hoadley 1980).

elements. They are very large in diameter with very thin walls. Fiber is another major type of cells with very thick walls in hardwoods. All the vessels and fibers tend to be rounded in the cross section as compared to the nearly rectangular shape of softwood tracheids. Hardwoods are characterized by very large rays, which have a significant effect on some of the physical properties of wood. In general, comparing the softwood with hardwood structure, the softwood is more homogenous while the hardwood is relatively heterogeneous. As an example of hardwood species, the microscope structure of red maple is shown in Figure 2.3 (Hoadley 1980).

Figure 2. 3 Red maple (*Acer rubrum*) cross section structure 20× (Hoadley 1980).

Differential shrinkage and swelling in the three main directions of wood lumber and different mechanical and physical properties in the three directions have been shown to be all related to the orientation of structural units of the cell wall (Wangaard 1939, 1942). The orientation of fibrils in the cell walls of wood has shown to accounte for the greater part of the variation in thermal conductivity values predicted on the basis of specific gravity and moisture content. Cell wall structure is characterized with 3 layers. The outer and inner layer are very thin compared to the middle layer of the secondary wall (S2). The S2 layer takes about 90-95% of the whole cell wall. So it plays the major roll in material properties. The microfibrils in the S2 layer are oriented from 10° to 30° from the cell axis, and are nearly parallel to the long axis of the cell when observed under the microscope. This orientation is responsible for the significant difference in a lot of physical properties between the longitudinal and transverse direction, such as thermal conductivity. Wangaard (1942) showed that the transverse thermal conductivity deviations in Douglas-fir are greatly dependent on the variation of microfibril slope.

Cell wall substance plays an important roll in material properties. Wood anisotropic material properties and shrinkage has been demonstrated to be related to the cell wall amount and structure in the wood (Skaar 1988 and Pentony 1952). Boyd (1974) found that variations in shape of cell cross-section and wall thickness has an influence on anisotropy of shrinkage. Trenard & Guenean (1977) pointed out that differential shrinkage is more closely related to certain aspects of cellular structure, e.g. lumen dimensions and cell wall thickness. Wood density has a close relation with the cell wall percentage in the wood samples. Nicolls' (1984) results showed that density was influenced more by the radial than by the tangential component of both tracheid width and cell wall thickness. Excellent relationships were found between density and proportion of cell wall on tracheid cross-section data, and he also proved that the maximum density was mainly controlled by the cell wall thickness. Quirk (1984) gave his testing results to show that wood basic density was highly correlated with cell-wall thickness. Cell wall thickness in a single cell and between earlywood and latewood cells is also different. Nobuchi et al. (1993) used a pinning method to investigate the differentiating zones in the cell wall structure. The cell wall structure which suffered direct effects of pinning injuries in stems of Cryptomeria japonica was observed under an ordinary light microscope and a polarizing microscope. They found that the tangential wall thickness of tracheids was thicker than that of the radial wall in the earlywood area, but thinner than the radial wall thickness for the latewood tracheids. The different cell wall thickness and structure in the radial and tangential directions can be correlated with the different properties in the two directions.

In terms of the influence of the gross wood structure upon the thermal conductivity, Wangaard (1940) did a study examining the effect of the type, size and disposition of the longitudinal cells on this physical property. His results failed to show the significant effect from these gross structures.

2.2.2 Thermal conductivity of wood

Wood is a hygroscopic, porous material. The unique structure of wood causes the anisotropic nature of wood in its mechanical and physical properties. Thermal conductivity has been shown by many scientists to have a very close relationship with the wood structure and moisture. The ability of a material to conduct heat as a result of transmitting molecular vibrations from one atom or molecule to another varies greatly depending upon the chemical nature of the material and its gross structure or texture. Thermal conductivity, k, is expressed in terms of quantity of heat, Q, that flows across unit thickness, x, of a material with a unit cross section, A, under unit temperature difference between the two faces, T, in unit time, t:

$$k = \frac{Q * x}{A * T * t}$$

Equa. (2. 1)

The heat conductivity of wood is dependent on a number of factors with varying degree of importance. The more significant variables affecting the rate of heat flow in wood are: 1).density of wood; 2).moisture content of wood; 3).direction of heat flow with respect to the grain; 4).relative density of latewood and earlywood and proportion of latewood and earlywood in the lumber; 5).extractives or chemical substances in the wood, and defects, etc.. From numerous and varied factors affecting thermal conductivity of wood, Van Dusen (1920) first found that there was nearly a linear relationship between conductivity and density. So did Rowley (1933). In his preliminary investigation in 1929 over 100 species, he found an indication of a linear relation between the conductivity and density, and between the conductivity and moisture content. There is also a questionable conclusion in his findings ---- somewhat greater conductivity may be expected tangentially to grain than radially with strong marked annual rings. No data or explanation was shown for this conclusion. In the test, he also found that the inherent variability of wood makes a considerable number of tests of each species necessary to arrive at a fairly representative value for each species. MacLean (1940) did a five-year systematic study on 32 wood species, comprising both softwoods and hardwoods. He tested the effect of different density and different moisture content in wood on the heat conductivity. The experiments were made on a number of species with the moisture content ranging from 0 to that of wood in the green condition. Also he did several runs on Douglas-fir to compare the conductivity in the radial and tangential directions. He found from the results based on the oven-dry samples that the conductivity was, in general, directly proportional to the specific gravity. The empirical equation from 84 tests results was concluded as *Equation 2.2*.

$$k = 1.39 * S + 0.165 \qquad [Btu \cdot in / ft^2 \cdot hr \cdot ^\circ F]$$

Equa. (2. 2)

Where, *S*----specific gravity;

Urakami and Fukuyama (1981) later used an advanced technique to examine the influence of density (ρ) on the thermal conductivity perpendicular to the grain at room temperature, and gave the relationship as:

$$k = 0.0174 + 1.86 * 10^{-4} * \rho \quad [W/m \cdot K]$$

Equa. (2. 3)

The moisture content effect tests gave MacLean (1941) enough data to modify his equation (*Equation 2.2*) by adding up the moisture effect. *Equation 2.4* is for moisture values from 0 to 40%, while *Equation 2.5* is for moisture from 40% and higher.

$$k = S * (1.39 + 0.028 * MC) + 0.165 \qquad for\ MC < 40\%$$

Equa. (2. 4)

$$k = S * (1.39 + 0.038 * MC) + 0.165 \qquad for\ MC > 40\%$$

Equa. (2. 5)

(* Note: Unit for the thermal conductivity *k* is Btu.in/ft^2.hr.°F)

Moisture content's significant influence on wood thermal conductivity is easy to understand. Since wood is a porous material, its insulating nature results from the dead air in cell lumens. According to Jay (1957), water vapor in these spaces has little effect, but free water in the spaces increases the conductivity considerably. There are some deviations of the experimental values from the computed values based on the above equations obtained from MacLean's tests. First he assumed the deviation was influenced by the effect of extractives or resins in the wood. But the tests on a number of runs on resinous southern yellow pine showed that resin's effect on the conductivity was not much different from that of the wood cell wall substances. MacLean also mentioned that, although there appeared to be a tendency for higher conductivity in the tangential direction, a small number of tests made on Douglas-fir to compare the radial and tangential thermal conductivity values, showed no pronounced differences. From his tests, he also found that small knots and checks and a thin film of glue in the Douglas-fir plywood had no influence on the thermal conductivity values. In the end he did a few runs to investigate the effect of differences of the average testing temperature on the thermal conductivity of the wood. He found no significant changes of thermal conductivity by the average temperature change. This is questionable and challenged by some researchers later.

Kollmann (1951) derived an empirical relationship (*Equation 2.6*), which measures the temperature effect on the thermal conductivity. It can be applied within the range of temperature of -50°<θ<+100°C. However, only five scattered points were used for getting this equation.

$$\lambda_2 - \lambda_1 * [1 - (1.1 - 0.98\rho_0) * (\theta_1 - \theta_2)/100]$$

Equa. (2. 6)

where, λ_2, λ_1----thermal conductivity at two temperatures;

θ_1, θ_2----two different temperatures;

Ward (1960) found the effect of temperature to be somewhat less than that which Kollmann (1951) predicts at lower densities. Ward implemented a dynamic method for simultaneously determining the thermal conductivity and specific heat of wood and wood-based materials as a function of temperature. In his result, he found the thermal conductivity and specific heat each appeared to be related linearly to temperature for both virgin wood samples and particleboard. But due to a small number of tests, he couldn't draw any quantitative conclusions regarding the relationships. Lewis' (1940) study on the temperature effect on the thermal conductivity of 3 wood species in the range of -50°C~+40°C proved Ward's conclusion ---- linear relationship, but not as great as that predicted by Kollmann (1951). In Lewis conclusions, he pointed out that the relationship between thermal conductivity and temperature, and between thermal conductivity and moisture content are linear, but he found the thermal conductivity increases with density in a non-linear manner. He also found that thermal conductivity increases in a linear fashion with an increase in moisture content. He explained this as a result of greater molecular mobility. As moisture in wood increases, the space between the molecules and the amount of moisture molecules both increase, which gives the greater molecular mobility, then more energy transportation results. Later in 1998, Harada et al. used a laser flash method to measure the thermal constants of wood during the heating process on 13 Japanese wood species. They analyzed the relations between the three parameters--thermal conductivity, density and temperature--using the multiple regression analysis and obtained the equations as:

$$k = 0.00249 + 0.000145 * \rho + 0.000184 * T \qquad [W/m \cdot K]$$

Equa. (2. 7)

Suleiman et al. (1999) did a test with a recently developed Transient Plane Source (TPS) technique on six oven-dry samples of birch, and found the thermal conductivity slightly increases in both longitudinal and transverse direction as the temperature increases from 20°C to 100°C.

The wood structure influence, such as the grain orientation, on thermal conductivity has been proved by several scientists (Griffths and Kaye 1923, Wangaard 1940, MacLean 1941, and Hendricks 1940). The conductivity in wood longitudinal direction is found to be about 2.25 to 2.75 times the conductivity in transverse direction. This ratio has been proved by all the studies. For example, Wangaard (1940) found the conductivity in longitudinal direction was about 2.5 times that in the transverse direction. Kollmann (1951) found that longitudinal conductivity for most common woods was somewhat less than 3 times as great as transverse conductivity. Hendricks' (1940) experiments on basswood showed the ratio of longitudinal over transverse is about 2.8. But for the difference of conductivity across the grain, there were some conflicting discoveries. Griffiths and Kaye (1923) found the thermal conductivity of wood in the radial direction to be about 5% to 10% greater than in the tangential direction. Rowley (1933) found that in species with strongly marked springwood and summerwood, tangential conductivity was somewhat greater than radial conductivity, but no appreciable differences exists between conductivities in radial and tangential direction in woods which possess uniformity of structure throughout the annual ring. However, Wangaard (1940) found that in hardwoods radial conductivity exceeds tangential conductivity by a comparatively small but significant amount. He attributed this difference to the influence of the significant amount of wood rays in hardwoods. The ray volume in hardwoods is about 17 to 20%, while in softwoods the ray volume is only about 7%. This is why he found there was no significant difference between radial and tangential conductivity in softwoods. MacLean's (1941) small number of tests made on Douglas-fir specimens showed no pronounced differences for the thermal conductivity in the two directions, although in practically, there appeared to be a tendency for higher conductivity in the tangential direction. In Suleiman's (1999) study, he pointed out that radial conductivity may be higher than tangential conductivity and this may be attributed to the influence of the radially oriented wood rays. According to Steinhagen's (1977) review, it appears that the ratio of the tangential to radial conductivity is primarily determined by the volume of the ray cell in hardwoods and by the latewood volume in softwood.

Heat flow in wood is governed by the laws of diffusion. There are two conditions for measuring the thermal conductivity -- steady-state and unsteady-state (Wangaard 1969). Fick's first law of diffusion (*Equation 2.8*) can be applied to the steady-state thermal conductivity measurement:

$$Q = k * A * t * [(T_1 - T_2) / x]$$

Equa. (2. 8)

Where, Q ---- total heat transferred;

k ---- thermal conductivity;

A ---- cross area of the heat flow;

t ---- time;

x ---- distance of the heat flow;

(T_1-T_2) ---- temperature difference between the two faces;

The unsteady-state heat flow follows Fick's second law:

$$\frac{du}{dt} = h^2 \frac{d^2u}{dx^2}$$

Equa. (2. 9)

where, du/dt ---- temperature change with time;

h^2 ---- thermal diffusivity, $h^2 = k/c \cdot \rho$;

k ---- thermal conductivity;

c ---- specific heat;

ρ ---- density;

du/dx ---- temperature change with distance from surface;

The guarded hot plate is the most widely used apparatus for determining thermal conductivity of materials. It has been standardized by the American Society for Testing Materials (1). The hot plate apparatus consists of a guarded hot plate for which power input can be controlled and measured, two identical testing specimens, and two cooling plates or heat sinks. Most of the early research results (Grober 1910, Poensgen 1912, Van Dusen 1920, Rowley 1933, Kollmann 1934, Wangaard 1940, and MacLean 1941) were obtained by this technique. Kamke and Zylkowski (1989) used a guarded heat-flux apparatus (R-MATIC, Dynatech Corporation) to determine the thermal conductivity values for several types of wood-based composites. This steady-state method proved to be precise and valuable for determining the wood thermal conductivity. But the limitation is the requirement for relatively large size testing specimens, plus a long time is involved for each test, especially when the specimen contains moisture in it. In early 1950s, a variable-state method, involving a heat source and heat sink of known thermal capacity, which permits the simultaneous determination of thermal conductivity k and specific heat c was introduced (Clarke and Kingston 1950). Ward (1960) implemented this method in his test to determine the specific heat and thermal conductivity of wood. He improved the apparatus by making it possible to determine thermal conductivity and specific heat simultaneously and automatically throughout a wide range of temperature. Hendrick (1962) used the same technique to examine the temperature effect on the thermal conductivity in the range of -40°C to +50°C. All the above studies were performed below 100°C. Both steady-state and unsteady-state methods are designed for solid wood and composites. Avramidis and Lau (1992) introduced a new technique for measuring thermal conductivity -- Transient Heat Flow, which was first developed by Van der Held and Van Drunen (1975). The principle of the THF method was founded on the basis of theoretical considerations of the temperature response of an infinite homogenous medium to an imbedded line heat source of constant strength. Avramidis and Lau modified the apparatus to improve its accuracy and handling. They used this method to test different sizes of wood flakes. They found that thermal conductivity of wood particles was a strong positive function of moisture content, bulk density and temperature. Suleiman et al. (1999) used this method on oven-dry hardwood (birch) samples to investigate the influence of temperature, density, porosity and anisotropy on heat conduction. They found that the thermal conductivity was higher at higher temperatures with a higher slope for the axial sample and the conductivity decreases as porosity increases. According to their analysis, it seems that the dominant mechanism of heat transfer across the cell lumen in these types of wood is the heat conduction through the voids. Harada et al. (1998) introduced the laser flash method, which enables us to measure the thermal constants at high temperature and under vacuum conditions. He measured the thermal conductivity of several

Japanese wood species up to 240°C and under the vacuum condition. With this method, he extended the relationship between the temperature and thermal conductivity to a larger range.

2.2.3 Theoretical modeling of thermal conductivity

Investigations on wood thermal conductivity for the past 80 years have provided us with the empirical models for predicting thermal conductivity from its influences, such as density or specific gravity, moisture content, and temperature, etc. Based on understanding of the heat transfer mechanism in wood and testing results, some theoretical models were proposed.

After analyzing the influence of fiber orientation on heat transmission in wood and wood-based materials, Kollmann and Malmquist (1956) proposed a model with two extreme cases for heat transfer -- Maximum heat transfer factor (λ_{max}) with parallel arrangement of the fibers in the direction of heat flow, and minimum heat transfer factor (λ_{min}) with parallel arrangement of the fibers perpendicular to the direction of heat flow, shown in Figure 2. 4.

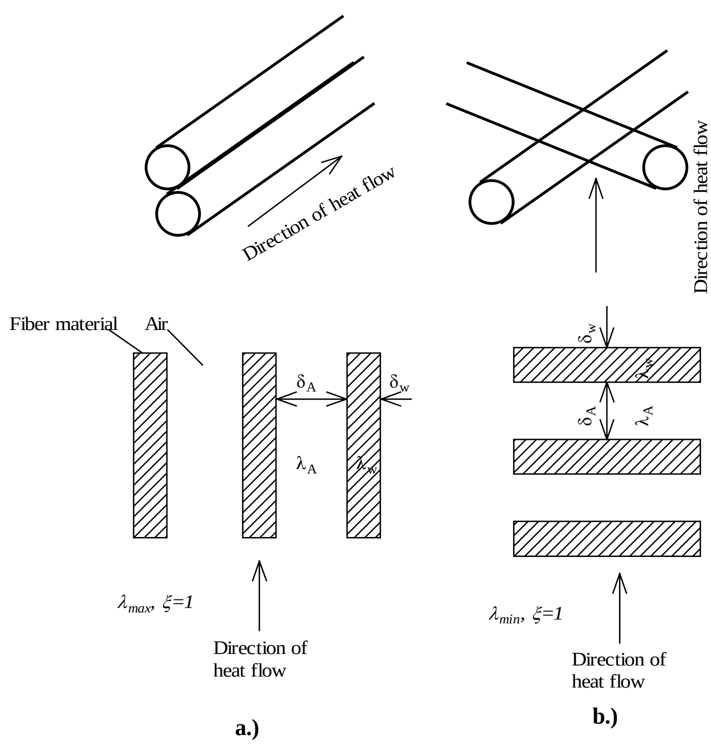

Figure 2. 4 Kollmann's model for the thermal conductivity (Kollmann & Malmquist 1956).

For a cellular structure material like wood, the mixed arrangement of these two cases is usually found. So the resultant heat transfer in wood can be represented by the combination of these two cases with a "bridge-factor" ξ (*Equation 2.10*).

$$\lambda = \xi\lambda_{\max} + (1-\xi)\lambda_{\min}$$

Equa. (2. 10)

For the two extreme cases described in Figure 2.4, they gave the thermal conductivity as in *Equation 2.11*.

$$\lambda_{\min} = \frac{\lambda_a * \lambda_w * (\delta_a + \delta_w)}{\lambda_a * \delta_w + \lambda_w * \delta_a}$$
$$\lambda_{\max} = \frac{\lambda_a * \delta_a + \lambda_w \delta_w}{\delta_w + \delta_a}$$

Equa. (2. 11)

where, λ_a ---- conductivity of the air;

λ_w ---- conductivity of the poreless wood substance;

δ_a ---- thickness of the air layer;

δ_w ---- thickness of the wood substance layer;

By employing the definition of density of wood, as follows

$$\rho = \frac{\delta_w}{\delta_a + \delta_w}\rho_w$$

Equa. (2. 12)

and introducing *Equation 2.11* and *Equation 2.12* into *Equation 2.10*, the effective thermal conductivity can be obtained by:

$$\lambda = \xi[\lambda_a + \frac{\rho}{\rho_w}(\lambda_w - \lambda_a)] + \frac{(1-\xi)\lambda_a \cdot \lambda_w}{\lambda_w - \frac{\rho}{\rho_w}(\lambda_w - \lambda_a)}$$

Equa. (2. 13)

They used the bridge-factor ξ=1 for heat conduction along the grain in solid wood, and ξ=0.58 for heat conduction perpendicular to the grain. They also gave the bridge factor for fiberboards and particleboards as 0.14 and 0.44, respectively. The model-predicted and test results from many resources are shown in Figure 2.5.

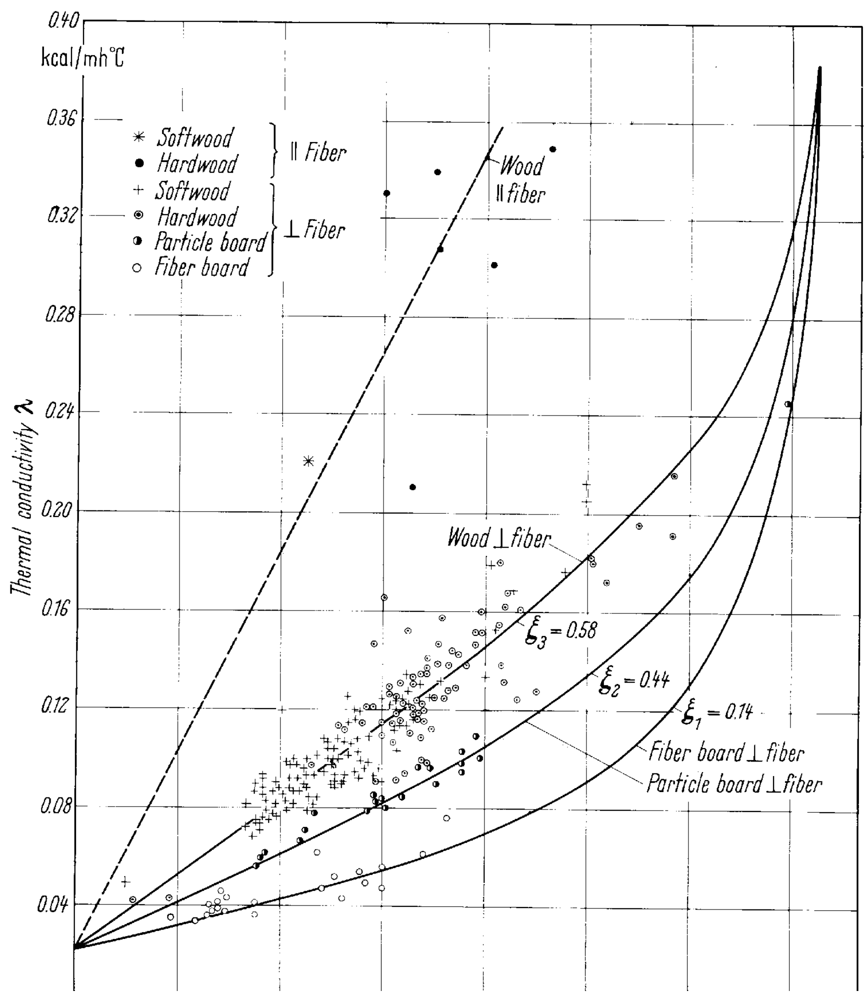

Figure 2. 5 Kollmann's plot for the relation between thermal conductivity and specific gravity of wood, particle boards and fiber boards (Kollmann & Malmquist 1956).

According to Siau (1968,1995), most heat and mass transport occurs mainly through the cell wall substance and sometimes through the air in the lumens. So he derived a model for transverse thermal conductivity on a geometrical model for a single wood cell shown in Figure 2.6. He idealized this geometrical cell model by assuming equal thickness for all the walls, the ratio of cell-wall thickness to the width of the cell is assumed to be the same no matter what type of cell it is, and neglecting the ends of the cells, pit openings on the radial walls, and transverse-oriented cells in wood. The geometrical model was transferred to an equivalent configuration (Figure 2.7 a) for transverse thermal conductivity model. By adopting the analogous electrical circuit (Figure 2.7 b), the effective conductivity can be derived as:

$$\frac{1}{k_{qT}} = \frac{(1-a)}{k_{qT}^{w}} + \frac{a}{(1-a)k_{qT}^{w} + ak_{qa}};$$

Equa. (2. 14)

Where, k_{qT} ---- transverse thermal conductivity of wood;

K_{qa} ---- thermal conductivity of dead air;

K_{qT}^{w} ---- transverse thermal conductivity of cell wall substance;

a ---- square root of porosity;

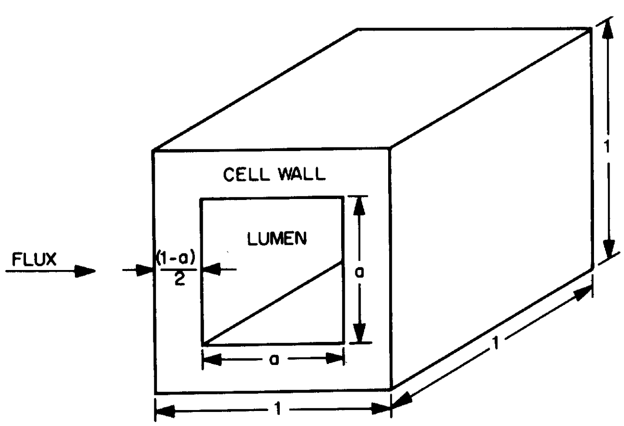

Figure 2. 6 Siau's Geometrical model for a single cell. (Siau 1984).

In order to modify his model to represent the nonuniformity of the heat flux in the sidewalls, he introduced a factor Z to account for the decreased conductivity effectiveness of the cross walls.

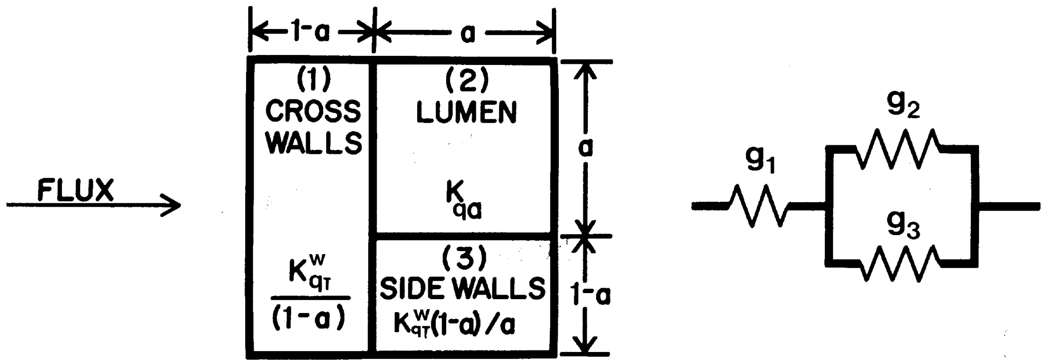

Figure 2. 7 a). (left) Siau's transverse thermal conductivity model ; b). (right) analogous electrical circuits.

The modified model is shown in Figure 2. 8 . The corresponding equation for the modified model was written as:

$$\frac{1}{k_{qT}} = \frac{(1-a)}{k_{qT}^{w} Z} + \frac{a}{(1-a)k_{qT}^{w} + ak_{qa}};$$

Equa. (2. 15)

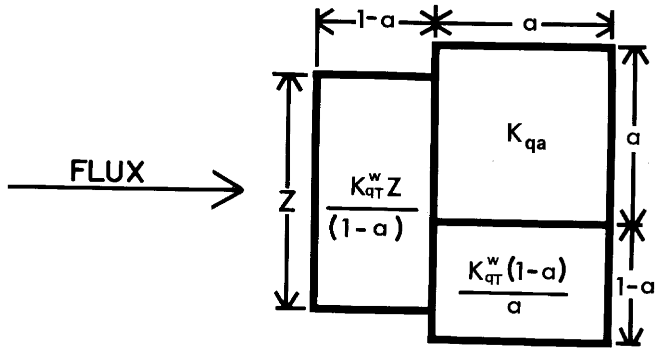

Figure 2. 8 Siau's transverse model modified for reduced conductivity efficiency of the cross walls.

Siau (1968) tested this model on wood-polymer composites and obtained good agreement between the experimental and theoretical values. Based on Siau's single cell geometrical model for wood-polymer composites, Couturier et al. (1996) extended the model by taking into account the swelling of the cell wall and the possibility that air bubbles do not extend the full length of the cell lumens. He proposed models for both conduction in longitudinal direction and transverse direction. Figure 2.9 and Figure 2.10 show their model transformed configuration and thermal circuit for the two directions. The overall thermal conductivity in longitudinal and transverse direction is the appropriate combination of the individual thermal conductivities (*Equation 2.16* and *Equation 2.17*). The testing results on the untreated red maple and some wood-polymer composites showed a promising validation of the model he used.

$$k_{WPC,L} = (1-\frac{\varepsilon_W}{\beta})k^*_{CW,L} + (\frac{\varepsilon_W}{\beta} - \frac{\varepsilon_{WPC}}{\lambda})k_p + \frac{\varepsilon_{WPC}}{\lambda}(\frac{\lambda}{k_a} + \frac{(1-\lambda)}{k_p})^{-1}$$

Equa. (2. 16)

$$k_{WPC,T} = k^*_{CW,T}(1-\sqrt{\frac{\varepsilon_W}{\beta}}) + (1-\sqrt{\lambda\beta\frac{\varepsilon_{WPC}}{\varepsilon_W}})\{\frac{\sqrt{\beta/\varepsilon_W}-1}{k^*_{CW,T}} + \frac{1}{k_p}\}^{-1}$$
$$+\sqrt{\lambda\beta\frac{\varepsilon_{WPC}}{\varepsilon_W}}\{\frac{\sqrt{\beta/\varepsilon_W}-1}{k^*_{CW,T}} + \frac{1-\sqrt{\beta\varepsilon_{WPC}/\lambda\varepsilon_W}}{k_p} + \frac{\sqrt{\beta\varepsilon_{WPC}/\lambda\varepsilon_W}}{k_a}\}^{-1}$$

Equa. (2. 17)

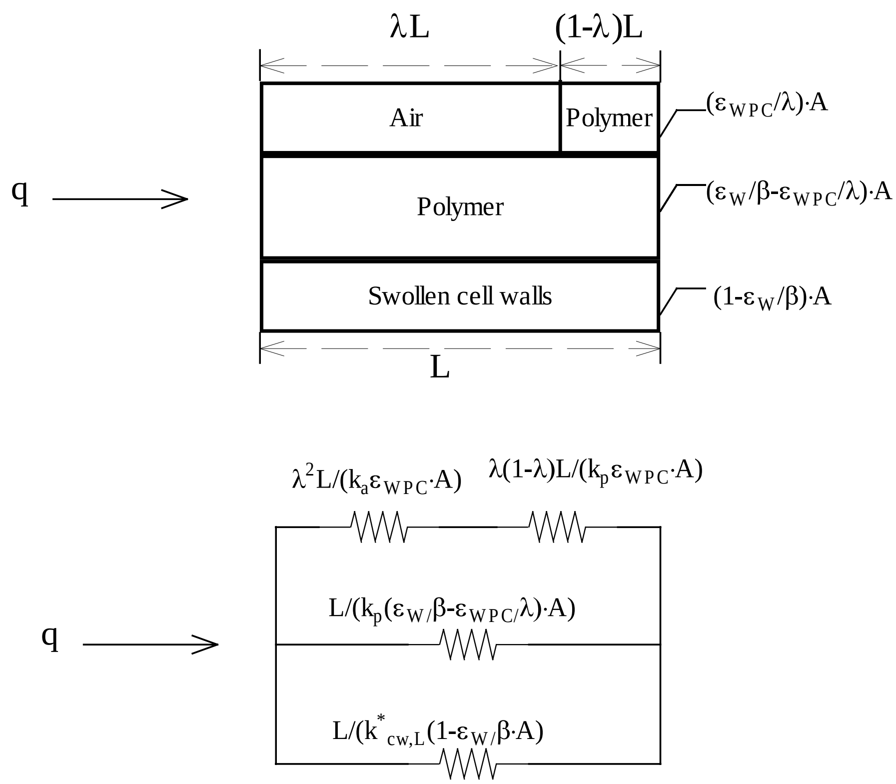

Figure 2. 9 Equivalent configuration and thermal circuit for the longitudinal conductivity model by Couturier et al. (1996).

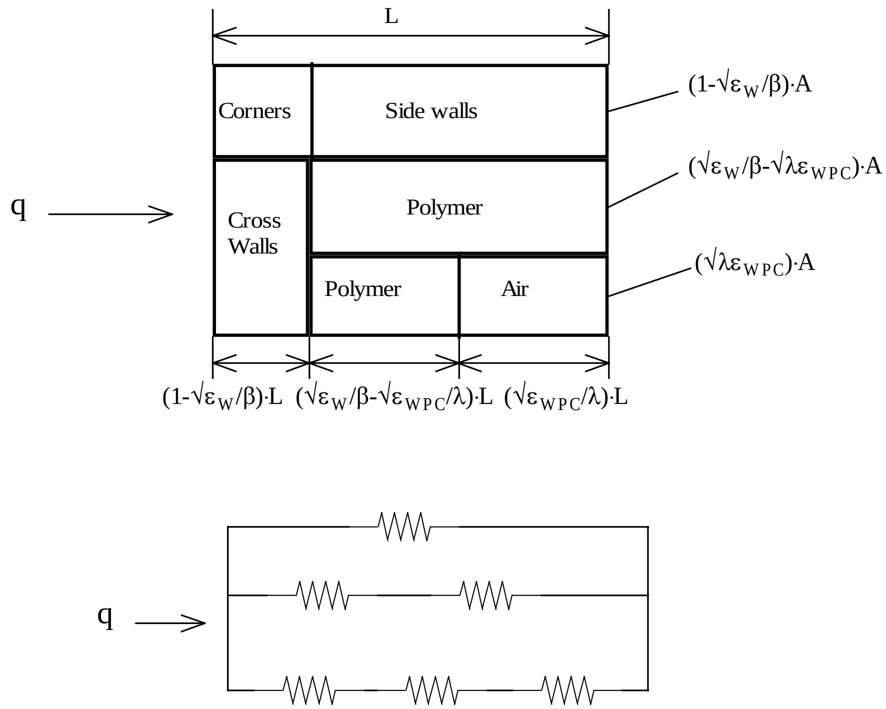

Figure 2. 10 Equivalent configuration and thermal circuit for the transverse conductivity model by Couturier et al. (1996).

The physico-chemical property of a multiphase mixture was a classical problem. It was a big challenge to the scientists and engineers of all areas before the 1960s (Meredith and Tobias 1962). It has been the interesting study in a wide engineering area for a long time. Scientists were trying to solve this problem by all kinds of analytical methods. The relation of thermal or electrical conductance of such mixtures to the conductance of the pure phases represents a complex problem. Maxwell (1881) studied the simplest kind of two-phase dispersion consisting of spherical particles of conductivity k_d imbedded in a medium of conductivity k_c. Using the technique of solving a Laplace equation and the principle of continuity, he obtained the expression for estimating the effective thermal conductivity of the mixture:

$$K_m = \frac{K_d + 2 - 2f(1 - K_d)}{K_d + 2 + f(1 - K_d)}$$

Equa. (2. 18)

Where, $K_m = k_m/k_c$;

$K_d = k_d/k_c$;

Maxwell's model was for the dispersion of spheres, and practically it was found that the estimation was only accurate when the volume fraction of the disperse phase is up to about 0.1. Rayleigh (1892) replaced the spheres in the Maxwell's derivation by infinitely long parallel cylinders. For parallel cylinders, in which the field is in a direction normal to the axis of the cylinders, Rayleigh gave the estimate of the mixture's effective thermal conductivity as:

$$K_m = \frac{1 + K_d - f(1 - K_d)}{1 + K_d + f(1 - K_d)}$$

Equa. (2. 19)

If wood is considered as a mixture of cell wall substances and air, Maxwell’s and Rayleigh’s model can be applied to estimate the thermal conductivity of wood. But the k_m value calculated from Rayleigh’s model for wood should be the longitudinal thermal conductivity of wood.