Designing Order Picking Systems for Distribution Centers

Pratik J. Parikh

Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Industrial and Systems Engineering

Russell D. Meller, Chair
Deborah F. Cook
Kimberly P. Ellis
Michael R. Taaffe
G. Don Taylor

September 1, 2006
Blacksburg, Virginia

Keywords: Distribution Center, Order Picking, Blocking, Storage System, Picking Strategy

Designing Order Picking Systems for Distribution Centers

Pratik J. Parikh

(ABSTRACT)

This research addresses decisions involved in the design of an order picking system in a distribution center. A distribution center (DC) in a logistics system is responsible for obtaining materials from different suppliers and assembling (or sorting) them to fulfill a number of different customer orders. Order picking, which is a key activity in a DC, refers to the operation through which items are retrieved from storage locations to fulfill customer orders.

Several decisions are involved when designing an order picking system (OPS). Some of these decisions include the identification of the picking-area layout, configuration of the storage system, and determination of the storage policy, picking method, picking strategy, material handling system, pick-assist technology, etc. For a given set of these parameters, the best design depends on the objective function (e.g., maximizing throughout, minimizing cost, etc.) being optimized. The overall goal of this research is to develop a set of analytical models for OPS design. The idea is to help an OPS designer to identify the best performing alternatives out of a large number of possible alternatives. Such models will complement experienced-based or simulation-based approaches, with the goal of improving the efficiency and efficacy of the design process.

In this dissertation we focus on the following two key OPS design issues: configuration of the storage system and selection between batch and zone order picking strategies. Several factors that affect these decisions are identified in this dissertation; a common factor amongst these being picker blocking. We first develop models to estimate picker blocking (Contribution 1) and use the picker blocking estimates in addressing the two OPS design issues, presented as Contributions 2 and 3.

In Contribution 1 we develop analytical models using discrete-time Markov chains to

estimate pick-face blocking in wide-aisle OPSs. Pick-face blocking refers to the blocking experienced by a picker at a pick-face when another picker is already picking at that pick-face. We observe that for the case when pickers may pick only one item at a pick-face, similar to in-the-aisle blocking, pick-face blocking first increases with an increase in pick-density and then decreases. Moreover, pick-face blocking increases with an increase in the number of pickers and pick to walk time ratio, while it decreases with an increase in the number of pick-faces. For the case when pickers may pick multiple items at a pick-face, pick-face blocking increases monotonically with an increase in the pick-density. These blocking estimates are used in addressing the two OPS design issues, which are presented as Contributions 2 and 3.

In Contribution 2 we address the issue of configuring the storage system for order picking. A storage system, typically comprised of racks, is used to store pallet-loads of various stock keeping units (SKU) — a SKU is a unique identifier of products or items that are stored in a DC. The design question we address is related to identifying the optimal height (i.e., number of storage levels), and thus length, of a one-pallet-deep storage system. We develop a cost-based optimization model in which the number of storage levels is the decision variable and satisfying system throughput is the constraint. The objective of the model is to minimize the system cost, which is comprised of the cost of labor and space. To estimate the cost of labor we first develop a travel-time model for a person-aboard storage/retrieval (S/R) machine performing Tchebyshev travel as it travels in the aisle. Then, using this travel-time model we estimate the throughput of each picker, which helps us estimate the number of pickers required to satisfy the system throughput for a given number of storage levels. An estimation of the cost of space is also modeled to complete the total cost model. Results from an experimental study suggest that a low (in height) and long (in length) storage system tends to be optimal for situations where there is a relatively low number of storage locations and a relatively high throughput requirement; this is in contrast with common industry perception of the higher the better. The primary reason for this contrast is because the industry does not consider picker blocking and vertical travel of the S/R machine. On the other hand, results from the same optimization model suggest that a manual OPS should, in almost all situations, employ a high (in height) and short (in length) storage system; a result that is consistent with industry practice. This consistency is expected as picker blocking and vertical travel, ignored in industry, are not a factor in a manual OPS.

In Contribution 3 we address the issue of selecting between batch and zone picking strategies. A picking strategy defines the manner in which the pickers navigate the picking aisles of a storage area to pick the required items. Our aim is to help the designer in identifying the least expensive picking strategy to be employed that meets the system throughput requirements. Consequently, we develop a cost model to estimate the system cost of a picking system that employs either a batch or a zone picking strategy. System cost includes the cost of pickers, equipment, imbalance, sorting system, and packers. Although all elements are modeled, we highlight the development of models to estimate the cost of imbalance and sorting system. Imbalance cost refers to the cost of fulfilling the left-over items (in customer orders) due to workload-imbalance amongst pickers. To estimate the imbalance cost we develop order batching models, the solving of which helps in identifying the number of items unfulfilled. We also develop a comprehensive cost model to estimate the cost of an automated sorting system. To demonstrate the use of our models we present an illustrative example that compares a sort-while-pick batch picking system with a simultaneous zone picking system.

To summarize, the overall goal of our research is to develop a set of analytical models to help the designer in designing order picking systems in a distribution center. In this research we focused on two key design issues and addressed them through analytical approaches. Our future research will focus on addressing other design issues and incorporating them in a decision support system.

This work received support from the Center for High Performance Manufacturing at Virginia Tech and Volvo Trucks North America, Inc.

Dedicated to
my parents
and
wife, Priti

Acknowledgements

Life's journey never ends. It is full roadblocks and one needs support and guidance from the near and dear ones to overcome these roadblocks. I, therefore, wish to express my gratitude to all those who have supported me during my stay at Virginia Tech.

Firstly, I would like to express my deepest gratitude to my advisor, Russell D. Meller, Ph.D., for his support, guidance, and encouragement throughout my doctoral studies and research. He has not only helped me to master the research process, but has also challenged me at times with a view to prepare me for life's journey. The best part of my association with him was for me to observe and imbibe the thought process of a true researcher and achiever like him. He has certainly developed a researcher in me.

Secondly, I also thank my committee members, Deborah F. Cook, Ph.D., Kimberly P. Ellis, Ph.D., Michael R. Taaffe, Ph.D., and G. Don Taylor, Ph.D., for carefully reading my dissertation and providing constructive feedbacks that has improved the overall quality of this research immensely. Also many thanks to Kevin D.Creehan, Ph.D. for his willingness to support my education during the last year of my stay at Virginia Tech. Moreover, thanks to Heather Nachtmann, Ph.D. at the Department of Industrial Engineering, University of Arkansas for her insightful comments regarding order picking system cost.

My warmest regards to my friends with whom I have not just shared academic experiences, but also experiences of life: Sivasankkar Selvanathan, Natalie Cherbaka, Qi Liu, Weiping Chen, Hungda Wan, Radu Babiceanu, Leonardo Rivera, Rami Musa, Walid Nasr, Javier Rueda, Lixin Wang, Liming Yao, Hitesh Attri, and many others. I also appreciate the support of the following ISE (Virginia Tech) and IE (University of Arkansas) staff members: Lovedia Cole, Kim Ooms, Dot Cupp, Nicole Lafon, Margie Zelinski, Lynette Kessinger, Will West, Brian Burgess, Steve Shirley, Karen Standley, Karen Hendrix, and Bonnie Swayze.

My parents, Jitesh P. Parikh, Ph.D. and Smita J. Parikh, deserve a special thanks for nurturing me through the early part of my life. It is because of their love and support during my academic endeavors that I have reached this stage. And thanks to my younger brother, Dr. Pranav J. Parikh, who has always been there supporting my decisions.

Not to forget my in-laws who were always there for me during the past 3 years: Dr. Kishore Shastri, Dr. Ranjan Shastri, Ashok Shastri, Hansa Shastri, Kunal Shastri, Pinky Kapadia, Nishith Kapadia, the lovely Deeya Kapadia, and all relatives.

Last, but no least, I would like to thank my lovely wife Priti for her understanding and love during the past few years. Her unwavering belief in GOD and her constant support not just helped this dissertation see through its end, but also helped me become a better person.

The journey of life continues ...

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Logistics can be defined as a process of effectively and efficiently managing the flow of goods and services between manufacturers and customers. As mentioned in the *15th Annual State of Logistics Report* (considered by many as a premier benchmark for U.S. Logistics Activity and excerpts of which appear in Cooke (2004)), total U.S. logistics costs represents approximately 8.5% of the U.S. Gross Domestic Product in 2003. A logistics system combines the material management, material flow, and physical distribution systems into an overall flow system (Tompkins et al., 2003). Though the material management and the material flow systems handle the flow of materials within and outside the manufacturing facility (i.e., from manufacturer to warehouse), it is the physical distribution system that forms the basis of fulfilling customer orders.

The physical location where the physical distribution system exists is referred to as *a distribution center* (*DC*) (Tompkins et al., 2003) or *a distribution warehouse* (van den Berg and Zijm, 1999). A DC, as shown in Figure 1.1, is responsible for obtaining materials from different suppliers and assembling (or sorting) them to fulfill a number of different customer orders. Various activities in a DC include order receipt, material receipt, labeling, put-away (the act of placing items into storage), replenishment, inventory control, assembly, order picking, sorting and packing, staging, shipping, returns processing, etc.

Amongst these activities, *order picking* has been identified as the highest-priority activity in a DC for productivity improvements. A few reasons for an increased focus on order picking are as follows (Tompkins et al., 2003): an estimated 50% of the total DC operating costs have been attributed to order picking, new operating programs such as Just-in-Time (JIT), cycle time reduction, quick response, etc., have been introduced that require frequent delivery of smaller orders and inclusion of more stock keeping units (SKUs) in the order picking system (OPS), and customer service has been improved due to renewed emphasis on minimizing

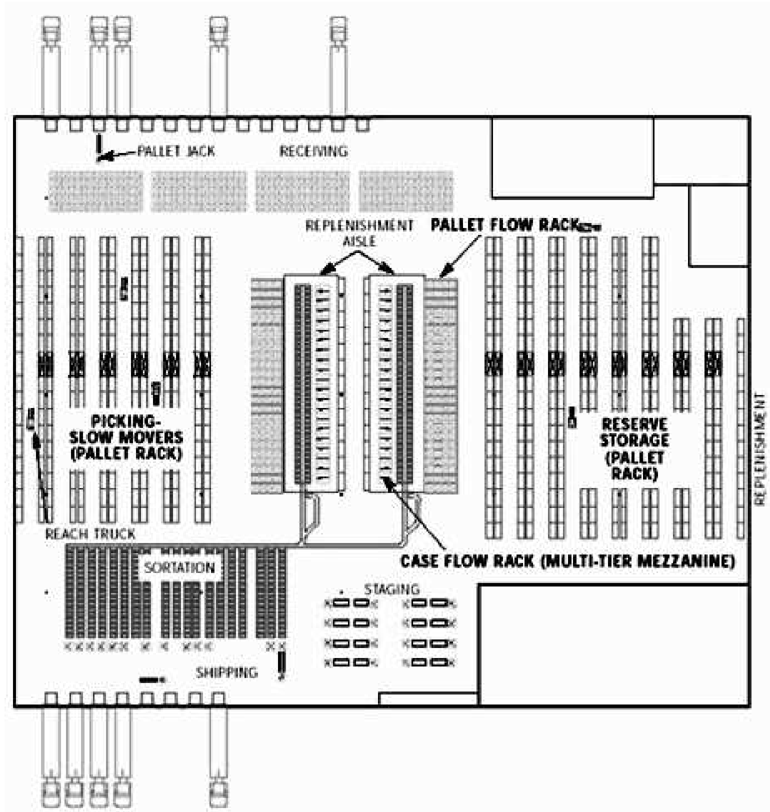


Figure 1.1: Schematic of a Distribution Center (Design Plans and Ideas, 2002).

product damage, transaction times, and picking errors. (A SKU is defined as a unique identifier of each product or item that is stocked in a DC.) This makes research in areas related to improving order picking important.

The specific problem addressed in this research is related to the design of an OPS in a DC, a problem that we detail in Chapter 3.

1.1. Motivation

An order-picking-system designer must consider the following question: *which OPS best meets a given set of objectives?* The design of an OPS depends on various system-design parameters; e.g., the configuration of the picking area and the storage system, as well as the determination of the storage policy, picking method, picking strategy, material handling system, pick-assist technology, etc. For a given set of these parameters, the best design depends on the objective being optimized. Some commonly used objectives include maximizing system throughput, or minimizing area, cost, response time, or picking error-rate. In making decisions regarding these parameters, the decision maker considers models of these systems, which helps him/her make decisions based on model-approximations or stochastic estimates of performance measures.

Owing to the complexity involved in designing an OPS, designers typically rely on simulation-based estimates of a stochastic model. However, simulation models are difficult to build and modify. Moreover, a number of what-if type analyses are required before an "optimal" design is reached. A considerable amount of the resources required for simulation models can be minimized if analytical models are available to assist in making design decisions.

This research is motivated by the fact that there is a lack of analytical approaches in certain areas of the OPS design literature. We intend to not only identify these critical areas, but also to model the dynamics involved in these areas through analytical approaches.

1.2. Background on Order Picking

As mentioned before, order picking is one of the main activities performed in a DC. Order picking refers to an operation through which the items are retrieved from their storage locations to fulfill customer orders. Customer orders are generally received by a DC throughout the day. These orders are recorded and converted into a format that can be used by the OPS to pick items from the storage area. Some of the order picking work elements include traveling to the item, searching for the item, reaching and extracting the item from storage, documenting the pick, sorting items, etc. Depending on the type of OPS employed, one or more of the above activities may be unnecessary (e.g., sorting items while picking is not relevant when employing a zone picking strategy).

An order picking system is a complex mix of the aforementioned design parameters to ensure that a specific objective (e.g., throughput, space, cost, etc.) is optimized. Below is a list of some of the major strategies and policies that a designer can choose from while designing an OPS.

Reserve and Forward Area: Most DCs are generally split into two separate areas; namely, the bulk-storage (or reserve) and the picking (or forward) areas. In the bulk-storage area are stored SKUs in large quantities to increase space utilization. The picking area is used for fast picking of frequently requested SKUs in the form of cases or pieces and is designed to increase picking efficiency. The picking area is replenished with SKUs from the reserve area.

Storage System Configuration in the Picking Area: For a given volume of SKUs to be stored in a picking area, the key decisions required in configuring a storage system include:

- *number of storage levels*; i.e., number of storage locations at a pick-face, where a pick-face is defined as a two-dimensional section of the storage area from where the pickers extract items to be picked. A pick-face may represent a column of pallet rack, a bay of flow rack, or a section of bin shelving;

- *lane-depth*; i.e., depth of the pick-face in terms of the number of unit-loads stored; and

- *number and width of aisles*; in terms of width, aisles can be considered wide (10-12 ft) or narrow (5-10 ft), and this determines which picking method can be used and if the pickers may pass one another in the aisle.

Storage Policy: There are various policies that are employed for storing the SKUs in a storage area of an OPS:

- *randomized storage*, in which each item of a SKU is stored randomly in the storage area;

- *volume-based dedicated storage*, in which the SKUs are assigned storage locations, with highest expected volume (or turn-over) SKU placed closest to the pick-up/drop-off (p/d) point;

- *class-based storage*, in which the SKUs are divided into various classes according to either frequency of requests or similarity of SKUs, and the storage area is divided into various regions, with SKUs in the same class stored randomly in one of the storage regions;

- *correlated storage or family grouping*, in which the SKUs are stored in nearby positions if they are often ordered together by customers;

- *cube-per-order index (COI) storage*, in which SKUs are assigned to storage locations on the basis of the ratio of the required space to the order frequency (cube-per-order index or COI), with the lowest COI SKUs assigned closest to the p/d point; and

- *shared storage*, in which items of different SKUs are successively stored in the same storage location depending on the duration of stay of those SKUs.

Picking Strategy: An order picking strategy defines the manner in which the pickers navigate the order picking aisles of a storage area to pick the required items. There are three basic order picking strategies:

- *discrete picking*, in which a picker is responsible for picking all the items in a single order during a pick-tour;

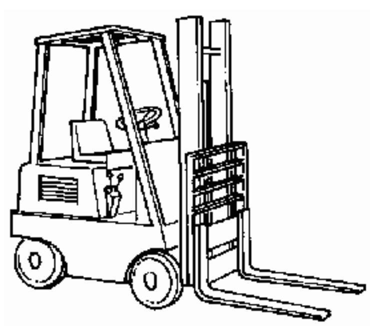
- *batch picking*, in which several orders are batched (or grouped) together and a picker picks all the items in a given batch; and

- *zone picking*, in which each picker is assigned to a specific region of the storage area and is responsible for picking the items (for single or batch of orders) in that region only.

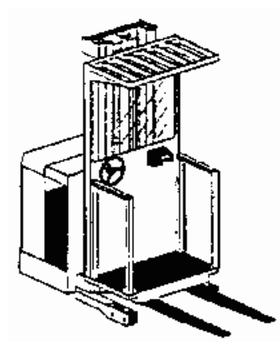
For OPSs configured in a manner similar to a flow-line in a production system, *bucket brigade picking* — a control policy for discrete picking — may be employed. According to this strategy, each picker follows the rule (Bartholdi et al., 2001): "Pick forward until someone takes over your work; then go back for more." That is, as soon as the most downstream picker completes an order, he walks back to take over the order the picker immediately upstream of him is currently picking. The latter, in turn, takes over the order of his predecessor, and so on until the most upstream picker begins a new order. Moreover, in batch or zone picking, if orders are required to be picked in a predefined time-window (known as a wave), then it is referred to as *wave picking*.

Picking Method: Depending on whether or not humans are involved in the system, there are three methods in which order picking can be accomplished; namely, *manual*, *semi-automated*, and *automated*. A manual (or picker-to-product) OPS is one in which the order pickers travel to the point where the item to be picked is located (e.g., pick-to-tote/cart/truck). A semi-automated (or product-to-picker) OPS is one in which the items to be picked are brought to a stationary picker through mechanical means (e.g., carousel, vertical lift module, etc.). An automated OPS has the potential of picking orders without any human intervention (e.g., A-frame).

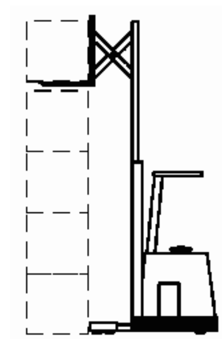
Material Handling Equipment: In an OPS, material handling equipment can be used either to assist in manual, semi-automated, or automated picking of items. A variety of material handling equipment are employed in a modern DC to increase the productivity of an OPS. These include totes, carts, conveyors, trucks (counter-balanced (CB) lift, order-picker, reach, etc.), horizontal and vertical carousels, vertical lift modules (VLMs), A-frame dispenser, end-of-aisle mini-load automated storage and retrieval systems (AS/RS), etc. Some of these equipments are illustrated in Figure 1.2.



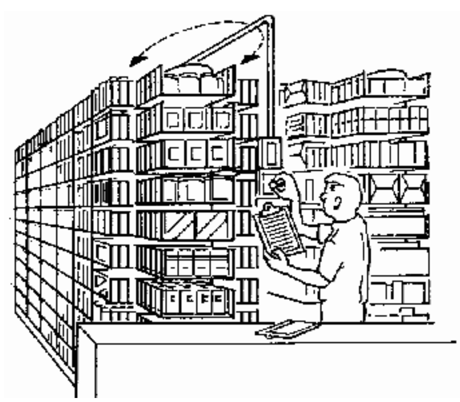
(a) CB-Lift Truck



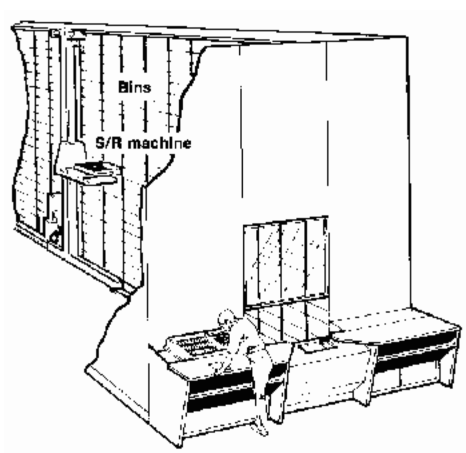
(b) Order Picker Truck



(c) Narrow-Aisle Reach Truck



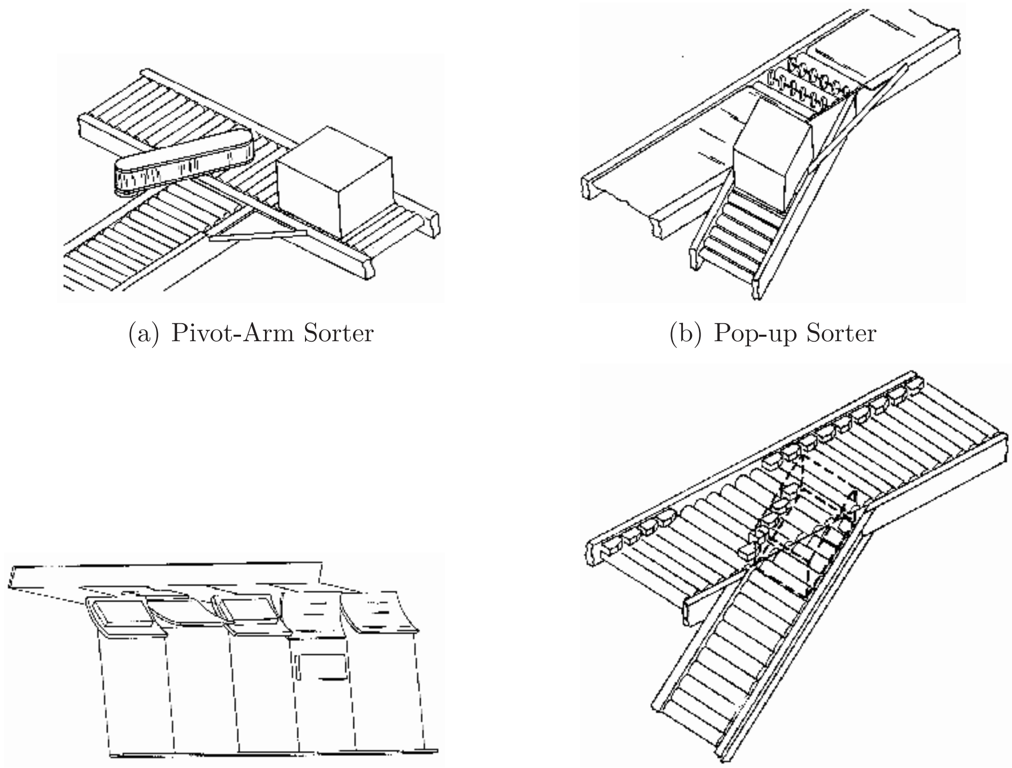
(d) Horizontal Carousel



(e) End-of-Aisle Mini-Load AS/RS

Figure 1.2: Material Handling Equipment Employed within an OPS in a DC (MHIA, 2006).

Furthermore, some OPSs may require that the items, which were picked from the storage area, be sorted (or consolidated) according to customer orders. For this purpose, a manual or an automated sorting system (e.g., tilt-tray, sliding-shoe, cross-belt, etc.) may be employed. Figure 1.3 illustrates a few automated sortation systems typically used in a DC.



(a) Pivot-Arm Sorter (b) Pop-up Sorter

(c) Tilt-Tray Sorter (d) Sliding-Shoe Sorter

Figure 1.3: Examples of Automated Sorting Systems (MHIA, 2006).

Pick-Assist Technology: For each item to be picked, a pick-assist technology helps a picker in locating and picking the right quantity of an item. Pick-assist technologies can take the form of either a *paper-based* or *paper-less* picking system. A paper-based picking system is one in which every picker receives a pick-list that contains the items to be picked, their location, and quantity. A paper-less picking system uses either a *light-directed*, a *voice-directed*, a *light-and-voice-directed*, or a *radio-frequency-directed* (or *RF-directed*) picking system. A light-directed picking (or pick-to-light) system employs picking-lights (in the form of LEDs) along with a small display at each storage location to assist pickers in locating and picking the correct quantity of an item. In a voice-directed picking system, picking requirements are communicated to the pickers wearing head-sets through voice-commands. A light-and-voice-directed picking system employs both picking-lights at storage locations and voice-commands through head-sets to assist pickers in locating and picking an item. In an RF-directed picking system, pickers carry a hand-held RF terminal that displays picking instructions to the pickers on the terminal screen.

1.3. Research Objectives

As mentioned earlier, a basic OPS design problem is the selection of the best combination of design parameters (i.e., storage area configuration, storage policy, picking method, picking strategy, material handling system, pick-assist technology, etc.) in order to optimize a given set of objectives. Solving this problem is a complex task. The designer typically follows the standard engineering-design process (Tompkins et al., 2003), which involves the following phases: Phase 1: define the problem; Phase 2: analyze the problem, generate alternatives, evaluate the alternatives, and select the preferred design; and Phase 3: implement the design.

So for a given problem definition (i.e., Phase 1), the design process requires the designer to perform a search through a number of scenarios (generated through combinations of various design parameters) to find the best one (i.e., Phase 2). The designer typically constructs simulation models for various scenarios and evaluates them against one or more objectives. In so doing, the designer invests significant person-hours, thus lengthening the time-frame of the design process. A procedure based on analytical models to evaluate and optimize one or more of the design parameters can reduce this time investment. This is precisely what this research aims to do.

We develop a set of analytical models to assist a designer in designing an OPS for a medium-to-high-throughput DC. Since we use the term, "medium-to-high throughput DC" throughout the remainder of this dissertation, we believe it is necessary to provide some characterization of such a DC. What follows is one way to do so based on first principles.

Note that a picking area in a DC is designed to increase the efficiency of order picking. The two major time-components of an order picking operation are the expected time required to pick an item at a pick-face (t_p) and the expected time required to walk past a pick-face (t_w). An efficient medium-to-high throughput DC could be characterized as a system in which the total time to pick (T_p) is at least as much as the total time to walk (T_w). The worst-case scenario would be the one in which T_p:T_w is 1:1. Therefore, in any efficient medium-to-high throughput DC, it would be natural to expect that the percentage of total time spent in picking would be more than 50%.

If in an efficient DC the pick:walk time ratio (t_p:t_w) and the total pick:walk time ratio

(T_p:T_w) are both 1:1, then this would correspond to a system where the pickers stop at every pick face (to pick exactly one item) as they walk through the picking area. Such a system will have a pick-density, the probability that a picker will pick at a pick-face (Gue et al., 2006), value of 1.0. For example, consider a picking area with 100 storage locations with floor storage (i.e., no stacking). If each storage location is a foot in length, depth, and height, then the storage area is 100 ft. long. Let the walking speed of the pickers be 1 ft/s. If $t_p = t_w = 1$ s (such that the pick:walk time ratio is 1:1) and T_p:T_w = 1:1, then $T_p = T_w = 100$ s. This means that the pickers pick at every pick-face, hence the pick-density equals 1.0.

Based on our experience, the pick:walk time ratio typically ranges between 5:1 and 10:1. Continuing the above example, if the pick:walk time ratio is 10:1 and $T_p = T_w = 100$ s, then each picker picks an item at exactly 10 out of the 100 storage locations. This corresponds to a pick-density value of 0.1. This means that an efficient medium-to-high throughput DC with pick:walk time ratio of 10:1 will correspond to the pick-density of at least 0.10. The fact that the pickers may pick multiple items at a pick-face in a realistic DC further increases the pick-density value. Therefore, we assume that the value of 0.10 serves as a lower bound for the pick-density value for a medium-to-high throughput DC with a pick:walk time ratio of 10:1. In fact, most DCs that we visited as part of this research, and would characterize as medium-to-high throughput DCs (e.g., J.Crew, eToys, etc.), had pick:walk time ratios of approximately 10:1, as well as pick-densities greater than 0.25. Therefore, the percentage of total time to pick (T_p) in such DCs was greater than 50%.

So if the designer knows which stock keeping units (SKUs) are to be stored in the picking area and their quantities, then Phase 1 may involve the following problem definitions in an order picking environment:

- What type of storage system should be used?
- Which order picking strategy (i.e., discrete, batch, zone, or bucket brigade) should be used?
- Which picking system (i.e., manual, semi-automated, or automated) should be used?
- Which pick-assist technology (i.e., paper-based or paper-less) should be used?

In this research Phase 1 represents the first two design problem definitions; i.e., configuring a storage system and selecting an appropriate picking strategy. As part of Phase 2 we identify several factors that affect these two design problems and develop analytical models for the same. A common factor in both these design problems is picker blocking. We first develop analytical models to estimate picker blocking and then use these estimates to address the two design problems. That is, we address three critical issues in OPS design, which are listed below in the form of research contributions. These research contributions are explained in more detail in Chapter 3.

Contribution 1: As mentioned earlier, a crucial factor in configuring a storage system and selecting an appropriate picking strategy in an OPS is *picker blocking*. Blocking can reduce picker productivity, which can increase the number of pickers to satisfy system throughput. This additional increase in the number of pickers increases the cost of labor, which needs to be accounted for in the two design decisions. Therefore, we have developed analytical models to estimate picker blocking in DCs. We specifically develop models to estimate pick-face blocking, which is defined as the blocking experienced by pickers requiring to pick at the same pick-face at the same time instant. We then use these blocking estimates in addressing the two OPS design problems (presented as Contributions 2 and 3).

Contribution 2: While designing the best configuration of a storage system, a number of factors need to be considered, which include the available area, number and quantity of SKUs to be stored, type of storage system used, etc. However, there are two other factors that have not been given due consideration in the OPS literature and in the industry; these are picker blocking and travel-time. For example, a high (in height) and short (in length) storage system can lead to high blocking as pickers will be concentrated in a small area. Moreover, if a high (in height) and short (in length) storage system is used in a semi-automated OPS employing a person-aboard S/R machine, then such a configuration may increase total travel-time due to excessive vertical travel involved in high systems. (Note that the vertical travel speed of a S/R machine is significantly lower than the horizontal travel speed.) An increase in both the picker blocking and the travel-time may reduce picker productivity, which means more pickers will be required to satisfy system throughput; this increases the labor (and equipment) cost. In contrast, a high (in height) and short (in length) storage system means less storage space required, which decreases space cost. Therefore, in terms of system cost (sum of labor, equipment, and space costs) there exists a trade-off between these cost components. To address this trade-off we have developed analytical models to identify the best configuration of the storage system in the order picking area.

Contribution 3: To decide when to employ a batch or a zone picking strategy is a critical design question. There exists a trade-off in terms of throughput and cost in deciding when to batch-pick or zone-pick. For example, batch picking may be less expensive as it is relatively simple to employ and requires less equipment, but may lead to higher picker blocking (which reduces throughput), while zone picking will eliminate picker blocking, but may experience high workload-imbalance and may require downstream sorting (which increase system cost). The objective is to analyze this trade-off in selecting an appropriate picking strategy (i.e., batch or zone) for a given application by developing an analytical model.

Our main research aim is to deliver a set of analytical models to a designer to aid in the design of an OPS in a DC. Through the above three contributions, we intend to address key design questions in certain areas of OPS design that have either not been addressed or only partly addressed in the OPS literature. By combining our analytical models with those available in the OPS literature, we intend to take a step closer towards building a complete set of analytical models for OPS design. Such models will complement the experience-based and simulation-based approaches, with the goal of improving the efficiency and efficacy of the design process.

1.4. Dissertation Outline

The remainder of this dissertation is organized as follows. Chapter 2 presents a comprehensive literature review on various aspects of DC design models, with an emphasis on order picking. Chapter 3 revisits the basic question of designing an OPS, but this time in further detail. Various trade-offs that exist during the design process are highlighted as the basis for our research. The three contributions of this research are explained in detail in this chapter. Chapter 4 is a paper that addresses Contribution 1 of our research; i.e., estimating picker blocking in wide-aisle order picking systems. Chapter 5 is a paper that addresses Contribution 2 of this research; i.e., configuring the storage system for order picking in a distribution center. Chapter 6 is a paper that addresses Contribution 3 of this research; i.e., selecting between batch and zone order picking strategies in a distribution center. Finally, Chapter 7 summarizes the conclusions we draw from this research, and also presents avenues for future research.

Chapter 2

Literature Review

This chapter reviews literature in the area of distribution center (DC) design, with a focus on the OPS within the DC. Section 2.1 focuses on general models for designing a DC. Models and procedures that have been proposed for the design or selection of an OPS are covered in Section 2.2. Section 2.3 briefly reviews literature in specific areas of order picking, which include the forward-reserve problem, storage area configuration, storage policy, picking strategy, simultaneous design of two or more interacting parameters, picking method, and material handling system. Finally, in Section 2.4 important questions yet to be addressed in the area of OPS design are identified, which serve as objectives of our research.

From our literature search, we observe that there are a large number of contributions in the areas of DC and OPS design; so we concentrate on a few key contributions in each of these areas and refer the reader to the four survey papers by Cormier and Gunn (1992), Gu et al. (2005), Rouwenhorst et al. (2000), and van den Berg (1999) for other contributions in these areas.

2.1. Design Models for a Distribution Center

As mentioned in Chapter 1, designing a DC (or a warehouse) is a complex task involving a number of design parameters. A designer is faced with the challenge of how best to address all the design issues without committing too large an amount of resources. To this end, several models have been presented in the literature to design a DC. Below we present a brief review of some of these models; details on several other models are presented in the four survey papers (Cormier and Gunn (1992), Gu et al. (2005), Rouwenhorst et al. (2000), and van den Berg (1999)).

Ashayeri and Gelders (1985) review various analytical- and simulation-based approaches proposed for warehouse design problems. They conclude that, in general, neither a pure analytical approach nor a pure simulation approach will lead to a practical design method. As a potential solution, they suggest a two-step procedure for warehouse design. As a first step of this procedure, analytical models could be used to quickly compare different design alternatives and to select only a few feasible alternatives, thus, reducing the search space. As a second step, an elaborate simulation model can be built to include dynamics that could not be modeled using analytical approaches. We also advocate such a two-step procedure and concentrate our efforts on the first step; i.e., the development of analytical models to aid in the design of a DC.

Gray et al. (1992) propose a multi-stage hierarchical decision approach to model the composite design and operating problems for a typical order-consolidation warehouse. As an objective function, they consider minimization of annualized incremental initial costs plus the warehouse operating costs. The various decisions to be made include warehouse layout, equipment and technology selection, item location, zoning, picker routing, pick list generation, and order batching. Their hierarchical approach utilizes a sequence of coordinated mathematical models to evaluate the major economic tradeoffs and to prune the decision space to a few superior alternatives. These alternatives can then be evaluated and compared using detailed simulation models. The main elements of this hierarchical approach include facility design and technology selection, item allocation, and operating policy (in the form of order batch size and number of zones in the forward area). They also illustrate the application of their method via an automotive spare-parts distribution center example.

van den Berg (1999) presents a literature survey on methods and techniques for the planning and control of warehousing systems. Planning-related decisions include inventory management and storage location assignment, while control-related decisions include picker-routing, sequencing, scheduling, and order batching. Based on the survey, he makes two summarizing remarks related to: (1) a need for completely different approaches to solve most of the planning- and control-related problems, instead of using dedicated heuristic approaches; and (2) a need for approaches that consider trade-offs between productivity (in terms of throughput) and urgency (in terms of order completion deadlines).

Rouwenhorst et al. (2000) present a reference framework and a classification of warehouse design and control problems. They define a warehouse design as a “structured approach of decision making at a strategic, tactical, and operational level in an attempt to meet a number of well-defined performance criteria." The strategic level includes decisions concerning the design of process flow and level of automation. Decisions at the tactical level include resource requirements (e.g., storage system, number of employees, etc.), determination of a layout, and organizational issues. The main decisions at the operational level are concerned with assignment and control problems of people and equipment. Through a comprehensive literature review, the authors identify a strong need for design-oriented approaches as opposed to analysis-oriented approaches for warehouse design problems.

From our review, we observe that apart from the work by Gray et al. (1992), all other design approaches are qualitative in nature. Such qualitative approaches, although helpful in making higher-level decisions, do not aid in quantitatively analyzing the performance of various OPSs in terms of throughput, space, response time, etc. Gray et al. (1992) present such an analytical approach; however, their approach overlooks critical design questions, such as which picking strategy is appropriate, what is the effect of storage area configuration on pick-density, how to develop models to compare various picking methods (and associated material handling equipment) in terms of throughput, etc. In this research, we address these critical design questions through analytical approaches and present a more comprehensive analytical modeling framework for OPS design.

2.2. Order Picking System Design Models

Order picking has been identified as a major activity in a DC and the prime contributor to overall DC operating expenses. Although there have apparently been several research contributions that propose models for the design and selection of an appropriate OPS for a given application, most of these articles are not a part of the published OPS literature. We review two contributions that are a part of the published literature; the other unpublished contributions are reviewed in survey papers by Rouwenhorst et al. (2000) and van den Berg (1999).

Brynzer et al. (1994) present a *zero-based analysis* methodology for the evaluation of an OPS as a basis for system design and managerial decisions. The basic idea of a zero-based analysis is to divide the resource consumption in an OPS into three parts. The first part is the necessary work, which refers to the time consumed in reaching, grabbing, and transferring an item to an appropriate position. The second part refers to time losses incurred due to reading and identification, traveling, and waiting. The third part refers to the cost involved with picking preparation and finishing, quality assurance, and other administrative activities. By measuring all three parts of resource consumption in appropriate units, various OPS designs can be compared and the best design can be selected. This approach is limited by the fact that it is not clear as to what constitutes necessary work. That is, is necessary work considered in reference to the current number of items being picked per unit time or the maximum number of items that can be picked in that time? Also, this approach is limited to a picker-to-product OPS; other OPSs, such as product-to-picker and automated systems may exhibit different resource consumption schemes, for which the zero-based analysis is not applicable.

Yoon and Sharp (1996) present a structured procedure for the analysis and design of order picking systems that considers interdependent relationships between different functional areas (e.g., receiving, picking, sorting, etc.) of an OPS. This procedure is divided into three stages.

In Stage 1 (or input stage) the data requirements are identified. These requirements include eliciting the managerial considerations in terms of objectives and constraints, collecting transactional data in terms of products and orders, developing an order profile spreadsheet, and identifying overall department and flow structures. In Stage 2 (or selection stage) storage capacities are calculated and appropriate operating strategies, equipment, information transformation requirements, etc., are specified. To assist in the selection stage, the authors develop simple analytical expressions to determine the number of aisles and number of workers required for various OPSs employing bin shelving, power cart, horizontal carousel, etc. In Stage 3 (or evaluation stage) the final OPS design is compared against various constraints (e.g., cost, space, throughput, etc.).

The authors provide an example of their approach by designing an OPS for a hypothetical application. The main limitation of this approach is that it is deterministic in nature, while OPSs consists of many stochastic elements. Further, the authors do not present models to decide between various picking strategies, configuring a storage area, or throughput models for all types of involved OPSs. We propose to address some of these issues by developing models that consider the various stochastic elements in the operation of an OPS.

2.3. Specific Parameters of Order Picking System Design

The approaches presented above consider the higher level design of a DC or an OPS, and there were only a few to review. In contrast, there has been considerable literature that addresses specific parameters of OPS design: e.g., the forward-reserve problem, storage area configuration, storage policy, picking strategy, picking method and material handling equipment, etc. What follows is a brief review of a few noteworthy contributions related to each of these parameters; other contributions are covered in the survey papers by Gu et al. (2005), Rouwenhorst et al. (2000), and van den Berg (1999).

2.3.1 The Forward-Reserve Problem

As mentioned in Chapter 1, most DCs store SKUs in two different storage areas: the forward and reserve areas. The forward area is used for efficient order picking, while the reserve area is used for efficiently storing the large quantities of items used to replenish the forward area. The forward-reserve problem considers the joint determination of the sizes of the forward and reserve areas and the products and their quantities that should be stored in the forward area. The objective is to minimize the expected amount of work involved in order picking and replenishing the forward area. For example, the amount of work involved in order picking may be reduced by designing a small forward area. However, a small forward area can only hold relatively few quantities of SKUs, which increases the work involved in replenishing the forward area with these SKUs. By increasing the number of SKUs that can be stored in the forward area, the work involved in replenishment decreases. However, a large forward area results in an increase in the work involved in order picking.

Hackman and Rosenblatt (1990) address the problem of deciding which SKUs to assign to the forward area, and how to allocate space amongst the assigned SKUs, for a fixed-capacity forward area. The objective is to minimize the total material handling costs of order picking and replenishment, assuming either none or all items of a SKU are assigned to the forward area. The authors assume that one trip is required to replenish a product, irrespective of the allocated quantity. They present a knapsack-based heuristic that assigns the optimal product quantities to the forward area in decreasing-cost-savings order until it is full. Using real-world data, they compare their heuristic with a ranking-based approach and demonstrate considerable cost savings.

van den Berg et al. (1998) consider a warehouse with busy and idle periods where reserve-picking is possible. They assume that the size of the forward area is given as well as the number of unit-loads of each product that need to be stored in the warehouse, either in the forward area or in the reserve area. They allow for the possibility of replenishing a unit-load during a replenishment trip during idle periods in order to reduce the replenishment activity during subsequent busy periods. The objective is to decide which quantities of which products should be stored in the forward area to minimize the expected total labor-time for order picking and replenishment during a busy period. The authors model the problem as a binary programming problem and present a knapsack-based heuristic that provides tight performance guarantees. Experiments with random data demonstrate savings of up to 30% may be possible when compared to procedures used in practice.

Recently, Heragu et al. (2005) consider the problem of joint determination of product allocation and functional area size determination in the design of a warehouse. The three functional areas they identified include the forward, reserve, and cross-docking area. They developed a mixed integer program (MIP) and a heuristic algorithm to solve the two problems jointly so that annual handing and storage costs can be minimized. Through experiments on various problem sizes, the authors demonstrate that a branch-and-bound algorithm failed to obtain a solution for large-sized problems. On the other hand, their heuristic is able to achieve good solutions for such problems in a reasonable time (when compared to a lower bound obtained through an LP relaxation of the MIP model). Further, their heuristic achieves the same (good) quality solution for large-sized problems as that obtained through a simulated annealing algorithm, but requires significantly less CPU time.

2.3.2 Storage Area Configuration

The problem of configuring a storage area requires the identification of storage requirements, the selection of storage equipment, the determination of a layout (i.e., length, width, and number of aisles and cross-aisles), etc. Though approaches to configure the bulk storage area are readily available in the OPS literature (see survey papers by Rouwenhorst et al. (2000) and van den Berg (1999) for details), there has been limited work related to the problem of configuring a storage system in the picking area. In this section, we review contributions that come close to addressing this problem.

Yoon and Sharp (1995) present a cognitive design procedure for an OPS. Similar to Yoon and Sharp (1996) discussed in Section 2.2, they divide the procedure into three stages; i.e., input, selection, and evaluation stages. In this work, the authors present details on various analytical models that a designer can use to identify the best OPS design. With respect to storage area configuration, the authors focus on identifying storage requirements, aisle layout (i.e., length, width, and number of aisles), and the number of workers for various OPS configurations, which include OPSs for case-picking (e.g., person-aboard S/R machine) and item-picking (e.g., horizontal carousel and bin-shelving). Through a numerical case study, they illustrate the use of their analytical models.

Malmborg (1996) formulates an integrated cost-based evaluation model that links three major policy issues associated with distribution systems. These issues include inventory management, space allocation between reserve and retrieval (or forward) storage areas, and storage area layout. Specific to storage area layout, the author develops analytical models to estimate the number of storage locations and the number of aisles required to store a given product volume. In so doing, the author considers two types of storage policies: randomized storage and volume-based dedicated storage. The author embeds these cost-based models into a computational evaluation model to assist a designer in analyzing the costs associated with decisions regarding the three aforementioned policy issues.

However, both of the above contributions do not explicitly provide models or approaches to configure a storage system, which is the decision regarding the length, height, and depth of the racks for storing a given volume of SKUs. There are several factors that affect the decision of configuring the storage system. The two key factors, which are neither addressed in the OPS literature nor in industry, are picker blocking and travel-time. Different combinations of length, height, and depth may yield the same storage volume, but for each of these combinations the amount of picker blocking and travel-time may be different. Both these factors largely govern the required number of pickers in the system, which in turn affects the cost of labor. Therefore, due consideration needs to be given to these two factors when configuring the storage system for the picking area.

2.3.3 Storage Policy

A storage policy defines the manner in which SKUs are allocated to storage locations in the storage area. The objective is to determine the best policy to allocate SKUs to the storage locations such that the picker travel time is minimized. As mentioned in Chapter 1, the various storage policies that can be employed in a forward area of an OPS include randomized, volume-based dedicated, class-based, cube-per-order-index-based (or COI-based), family-based, and shared storage policies.

A randomized storage policy is the simplest policy to employ, but may not turn out to be optimal in terms of SKU allocation, replenishment, and picking costs. To this end, Hausman et al. (1976) analyze the class-based storage policy and compare it with both randomized and volume-based dedicated storage policies in terms of crane travel times in automated warehousing systems. They mention that the latter storage policies are extreme cases of the class-based storage policy; i.e., randomized storage considers a single class, while volume-based dedicated storage considers one class for each product. It turns out that, in terms of throughput, volume-based dedicated storage is the most efficient and randomized storage is the least efficient policy. However, due to the difficulty in knowing *a priori*, and keeping track from then on, the turnover of every pallet to be stored in the system, a class-based storage policy is recommended to mask a lack of knowledge concerning pallet turnover. Through experiments, the authors suggest that a two-class and a three-class storage policy can yield 70% and 85% of the benefits (in terms of the reduction in crane travel times) gained through a volume-based dedicated storage policy, respectively.

Lee (1992) addresses the storage assignment policy in a man-on-board AS/RS. The problem examined considers allocating items to storage locations so that the total travel time required to pick all the given orders per period is minimized. This problem is formulated as a generalized assignment problem and a heuristic procedure is developed to solve it based on a group technology concept (which considers both order structure and order frequency). The procedure first generates an initial storage layout (i.e., an assignment of items to storage locations) and then systematically improves the layout through pairwise exchanges of item locations until no further improvement can be achieved. The initial layout is obtained through the COI storage policy and then the items are assigned, one-by-one, to storage locations following a spacefilling curve. Based on a travel-time matrix, which contains the distances between the input-output point and the storage locations, the total tour length across all orders and time periods is obtained. Lee demonstrates through experiments that there can be a reduction of about 2.2% to 11.2% in the S/R machine travel time through the use of a storage layout generated by this heuristic as compared to a layout obtained solely through the COI-based storage policy.

Frazelle and Sharp (1989) introduce the concept of a correlated or family-based storage policy. According to this policy, products that are frequently ordered together form a family and are stored together. The proximity of products frequently ordered together can reduce travel times for order picking. The authors present a simple rule for identifying correlated products from a given order set. They report reductions of 30-40% in the number of retrieval trips with the use of this policy in an end-of-aisle mini-load AS/RS OPS.

Goetschalckx and Ratliff (1990) introduce the duration-of-stay-based shared storage policy for a unit load warehouse. This storage policy, unlike a volume-based dedicated storage policy, considers successively storing units of different products in the same storage location depending on the duration of stay of those products. They develop an optimal storage policy with respect to travel time and storage space for a perfectly balanced system, in which the number of arriving units is equal to the number of departing units for every time period. For unbalanced systems, they develop two heuristics: a greedy heuristic and an adaptive heuristic (based on the duration of stay of units). Between the greedy and adaptive heuristics, the greedy approach performed on average 4% better (in terms of reduction in travel time) than the adaptive approach when the products are fewer and the system imbalance is larger. For a relatively balanced system with large products, the adaptive approach performed as much as 14% better than the greedy approach. The authors also compare the duration-of-stay-based shared storage policy with randomized, dedicated, and class-based storage policies. Through their simulation experiments, the authors show that the duration-of-stay-based shared policy resulted in a lower average travel time over all the other policies. Specifically, the shared policy demonstrated a savings of 25% over the optimal dedicated storage policy. The authors claim that the shared storage policy has a potential of reducing storage space requirements in an unbalanced system. However, no results were presented to justify their claim.

Jane (2000) considers a sequential zone picking system, which he refers to as a *relay* picking system. He addresses the problem of assigning n products into m storage zones (one picker per zone) with the objective of minimizing the differences that might exist between each picker's total number of picks. Using historical data, the author proposes calculating the total time all products are demanded by all customer orders (Q), the total time item i is requested by all orders (Q_i), and the workload of picker j (Q^j) estimated by summing all items requested from zone j ($j = 1, 2, ..., m$). The heuristic then attempts to make all Q^j approach Q/m, or make the load-rate, ρ ($= \frac{Q^j}{Q/m}$), equal to 1. Using industry data, the author demonstrates that the load rate of the existing assignment of $n = 152$ products to $m = 5$ storage zones varied from 0.49 to 1.35. However, when the products were reassigned using the proposed heuristic, the load rate was close to one; i.e., all pickers pick nearly the same number of items.

Jane and Laih (2005) propose a clustering algorithm for item assignment in a simultaneous zone picking system. The authors propose a similarity measure between any two items for measuring the co-appearance of both items in the same order. Accordingly, items frequently ordered together are located in different zones to minimize the idle time in the simultaneous zone system. To make these assignments, they develop a natural cluster model and solve it through a heuristic approach. With empirical data from a DC they evaluate their approach by observing that the utilization of the OPS, on average, could be improved by at least 28% if the items were reassigned to the zones using their model. Furthermore, the authors employ their natural cluster model to analyze the zone size of the order picking system. They indicate that the average utilization of the system increases as the number of zones decreases. However, as the number of zones decreases, the workload of the pickers increases too. Therefore, a proper zone size should be decided by trading-off the system utilization and the picker workload.

Several other storage policies have been proposed in the literature, which are reviewed in survey papers by Rouwenhorst et al. (2000) and van den Berg (1999).

All the afore-mentioned contributions do not address the issue of determining a detailed product layout in the storage locations; e.g., Product A should be placed in Aisle 2, in Rack 5, within Location 3, and oriented with the width of the unit facing front. Determining such a detailed product layout is referred to as *slotting*. The main objective of slotting is to minimize the costs associated with order picking, replenishing, and rewarehousing (reassignment of products to the storage locations) in the storage area, and "to achieve ergonomic efficiency by putting popular and/or heavy items at waist level (the *golden zone*, from which it is easiest to pick)" (Bartholdi and Hackman, 2006).

To this end, Sadiq et al. (1996) describe an approach, a conceptual framework, and a heuristic algorithm to address the problem of determining a detailed product layout (i.e., slotting). Specifically, the authors explore the feasibility of periodically reassigning products to storage locations for a dynamic product mix. The product mix is assumed to include two or more product families with different life cycles, correlated demand within families, and commonality of demand across product families. The authors propose an improvement-type algorithm, the dynamic stock location assignment algorithm (SLAA), that attempts to minimize the mean order-processing time (OPT), which is the sum of order picking time and order picking system rewarehousing time. If the mean OPT is minimized, then the authors assume that the total costs will be minimized. Note that OPT does not include the replenishment time. The SLAA employs a two-phase procedure.

In the first phase of the SLAA, decisions regarding which items should be in the system are made. In the second phase, clusters of products are formed and they are allotted specific storage locations according to COI index. To improve assignments by trading off the rewarehousing time and the order picking time, the SLAA performs a series of pairwise exchanges between same-sized clusters and adjacent clusters. When no further improvement is possible, the heavier and faster-moving items are assigned to ergonomically preferred locations within respective clusters to improve the ergonomic efficiency. The authors compare the SLAA with the COI algorithm (that minimizes order picking time) and the Global algorithm (that minimizes rewarehousing time). With the COI algorithm as the standard for comparison, the authors demonstrate that the Global algorithm results in a 37% higher mean OPT while the SLAA results in a 21% lower mean OPT as compared to the mean OPT obtained through the COI algorithm.

2.3.4 Picking Strategy

A picking strategy defines the manner in which pickers navigate the order picking area to pick items from storage locations. The primary objective of a picking strategy is to maximize throughput or minimize cost or response time. In Chapter 1 we defined three basic picking strategies: discrete, batch, and zone. Another picking strategy, called bucket brigade picking, was proposed by Bartholdi et al. (2001). Bucket brigade, a concept that originated in assembly lines, was developed as a way of coordinating workers who progressively assemble a product along a line. The authors apply the bucket brigade concept to an order picking system in a DC. According to this strategy, each picker follows the rule (Bartholdi et al., 2001): "Pick forward until someone takes over your work; then go back for more." That is, as soon as the most downstream picker completes an order, he walks back to take over the order the picker immediately upstream of him is currently picking. The latter, in turn, takes over the order of his predecessor, and so on until the most upstream picker begins a new order.

Characterizing bucket brigade picking as a self-balancing strategy, the authors develop stochastic models to analyze the performance of this strategy in high-volume OPSs. Through simulation experiments comparing bucket brigade picking to sequential zone picking, they suggest that the production rate efficiency (which is a ratio of realized production rate and maximum possible rate) of a simulated bucket brigade is similar to that of zone picking for the case when the OPS had identical pickers in terms of their walk and pick times. With an increasing difference in the walk and pick times of individual pickers, bucket brigade picking is more productive. Through a real-world study, the authors mention that bucket brigade picking improved pick-rates by 34% as compared to a sequential zone picking strategy. The only limitation this strategy faces is that it can be effective only in cases when the picking aisle is structured as a flow line, and when the time to hand-off an order is low.

Petersen (2000) states that the choice of a picking strategy can have a tremendous effect on the efficiency and cost of an OPS in mail order companies. To this end, he evaluates five order picking strategies — discrete (or strict), batch, sequential (or pick and pass) zone, simultaneous zone (which he calls batch zone), and simultaneous zone-wave — using a simulation model developed in C (a common computer language). The author considers labor requirements, total processing time, and customer service (in terms of mean percentage of orders not fulfilled in an 8-hr. shift) as performance measures across which these strategies are compared. Using an experimental design approach, the author considers three key factors that include picking strategy, daily order volume, and demand skewness with several levels for each factor. Based on the results, Petersen concludes that simultaneous zone-wave picking and batch picking are superior and that their performance is not adversely affected by changes in demand skewness patterns or daily order volume. On the other hand, the performance of sequential zone and batch zone picking deteriorates as order volume increases.

Gue et al. (2006) address the effect of pick density on order picking areas in narrow-aisle DCs. Since a batch picking strategy may induce picker congestion, which can reduce system throughput, they develop analytical models to estimate *in-the-aisle* picker blocking in a narrow-aisle DC. Using discrete time Markov chains, the authors estimate bounds on the percentage of time each picker is blocked. The critical assumptions in their work include: the picking area may be represented as a circle, the ratio of pick to walk time is 1:1 or ∞:1, and pickers can make only one pick at a pick-face. They perform simulation studies to estimate blocking for cases when pick to walk time ratios are 5:1, 10:1, and 20:1. They derive analytical expressions that suggest that congestion is at a maximum in a batch picking system when the pick-density is between 0.33 and 0.37. Through their results, the authors suggest that blocking increases with an increase in the number of pickers in narrow-aisle DCs, but decreases with an increase in the picking area (for the same number of pickers). They also indicate a progression of picking strategies with an increase in the pick-density: batch picking is preferred for very low pick-densities, zone picking is preferred for intermediate pick-densities, and batch picking is preferred for relatively high pick-densities.

Though the contribution by Gue et al. (2006) can be considered noteworthy in terms of analyzing the trade-offs (in terms of reduction in throughput due to blocking) between batch and zone picking systems, their work is limited to narrow-aisle DCs only. There are a large number of DCs that have aisles wide enough to allow pickers to pass, for which analytical approaches to select between a batch and a zone picking strategy are needed. Since blocking is one of the keys factors between these two picking systems, models to estimate blocking in wide-aisle DCs need to be developed. Moreover, as assumed in Gue et al. (2006), the pickers may pick only one item at a pick-face. This is not true in general as pickers tend to make multiple picks at a pick-face.

In the area of picking strategy, there are two additional design issues: determining an *order batching policy* and a *routing policy* for the OPS. A number of contributions are available in the OPS literature in the area of finding the optimal order batching policy and routing policy for a given picking strategy. Below, we review a few contributions in each of these areas; several other contributions are reviewed in the survey papers by Rouwenhorst et al. (2000) and van den Berg (1999).

Order Batching: If individual customer orders consist of only a relatively few number of items, then picking efficiency can be increased by grouping orders into batches. This problem of simultaneously assigning orders to a batch to be picked together in one picking tour is referred to as the *order batching problem*. The objective of the order batching problem is to reduce travel time (and hence, increase throughput) by determining the size of a batch and the orders to be included in that batch. This problem is critical when employing a batch or a simultaneous zone picking strategy.

de Koster et al. (1999) evaluate two types of order batching algorithms: seed and time savings algorithms. Seed algorithms begin by selecting a seed (or initial) order from those orders not yet added to a route, and then add (to this seed order) not yet selected orders until the order picking device is filled to capacity. They consider two routing policies, S-shape and largest gap, in their evaluation. They also consider the maximum capacity of the pick-device, which they vary from 12 to 150 items in their experiments. The heuristics are compared in terms of travel time, number of batches formed, robustness, simplicity, and CPU time. The authors demonstrate that seed algorithms are best when used in conjunction with the S-shape routing policy and a large pick-device capacity. Time savings algorithms perform best in conjunction with the largest gap routing policy and a small pick-device capacity.

Recently, Gademann and van de Velde (2005) address the order batching problem in a parallel-aisle warehouse to minimize the total travel time needed to pick all items. They mention that though a number of heuristics are proposed in the literature, a fundamental analysis of this problem is still lacking. They prove that this problem is NP-hard in the strong sense, but that it is solvable in polynomial time if no batch contains more than two orders. Furthermore, the authors design a branch-and-price algorithm that solves modestly-sized batching problems to optimality. For larger-sized batching problems, they present an approximation algorithm based on an iterated descent local search method. This algorithm shows excellent computational performance, both in terms of solution quality and computation time.

Routing Policy: A *routing policy* defines the route that each picker will follow to retrieve items from storage locations in the forward area. The goal of a routing policy is to reduce the travel distance (or time) by determining the sequence of picks and the route to be followed. Finding the optimal routing policy is a core problem when employing a discrete or batch picking strategy.

Ratliff and Rosenthal (1983) present an optimal routing policy for a rectangular warehouse that contains crossovers only at the ends of aisles. The routing problem is modeled as a traveling salesman problem (TSP) and an approach based on graph theory and dynamic programming is used to obtain an optimal solution. The authors indicate that the computation time of this algorithm is linear in the number of aisles and that a 50-aisle routing problem can be solved optimally in about one minute.

Hall (1993) evaluates and compares various routing strategies employed in a warehouse. Depending on the type of warehouse, Hall divides the routing strategies into three categories: narrow-aisle, wide-aisle, and zoned warehouses. Routing strategies for narrow-aisle warehouses include: (1) traversal (or S-shape), (2) mid-point return, (3) largest-gap return, and (4) optimal. For wide-aisles, the author considers the routing strategy proposed by Goetschalckx and Ratliff (1988). The author derives expected route-lengths for all these strategies and compares them across a variety of warehouse configurations. Several rules-of-thumb are developed to select the best strategy for a given application (e.g., traversal strategy is preferred for four or more picks per aisle, etc.).

de Koster and van der Poort (1998) study the order routing problem for centralized and decentralized warehouses. The authors define a centralized warehouse as one in which pickers are issued pick-lists and deposit items at a central depot. Similarly, they define a decentralized warehouse as one in which pickers are issued pick-lists and deposit items at the end of every aisle (later transferred to a sorting area via a delivery conveyor). The authors extend the well-known polynomial algorithm of Ratliff and Rosenthal (1983) (that considered a centralized warehouse) to address the routing policy of a decentralized warehouses. A comparison of the performance of this new algorithm and the S-shape heuristic is provided for three order picking systems: (1) a narrow-aisle pallet storage area employing a zone picking strategy with decentralized depositing of picked items (2) a shelf storage area employing a batch picking strategy with decentralized depositing of picked items; and (3) a wide-aisle pallet storage area employing a discrete picking strategy with centralized depositing of picked items. The authors demonstrate that their algorithm yields a reduction in travel time per route of between 7% and 34% as compared to the S-shape heuristic.

The survey papers by Rouwenhorst et al. (2000) and van den Berg (1999) review several other approaches for order batching and routing policy problems proposed in the OPS literature.

2.3.5 Simultaneous Design of Two or More Interacting Design Parameters

From the above discussion, we note that there exists significant literature dealing with various problems in the design of an OPS (e.g., storage policy, picking strategy, order batching, routing policy, etc.). However, we may not be able to completely isolate these problems from one another. This is true since design decisions made related to one problem can affect design decisions (to be made) related to other problems. For example, a traversal strategy may not be an effective routing policy when using a volume-based dedicated storage policy as compared to a randomized storage policy. This section reviews a few recent contributions that analyze this inter-dependency between decisions at various stages of OPS design; other contributions are covered in review papers by Gu et al. (2005), Rouwenhorst et al. (2000), and van den Berg (1999).

Ruben and Jacobs (1999), through simulation, explore the relationship between five order-batching heuristics and three storage policies (randomized, class-based, and family-based). They compare various combinations of order-batching heuristics and storage policies in terms of total work-force-hours, total distance, and total idleness. To include the effect of congestion that occurs when multiple pickers occupy an aisle, they partition each aisle into 20 sections of equal area (each having about 17.6 ft wide rack space) and restricting access to each area to a single picker. They mention that the use of such large partitioned sections enables them to model the delays en route to storage locations that would occur in crowded conditions. They measure congestion in terms of an increase in work-force-hours. They suggest that the productivity of the work-force is significantly affected by the chosen storage policy, the level of order-structure, and the batch construction heuristic used. Through their simulation experiments, they conclude that an increase in work-force and batch-size (or number of items in a batch) to meet higher demands leads to higher congestion. When employing a class-based policy, pickers are more likely to pick simultaneously in those sections of the forward area that store SKUs with highest turnover, thus increasing congestion. As a result, the amount of congestion is more pronounced when using a class-based policy as compared to a randomized or a family-based policy.

Petersen and Aase (2004) evaluate several picking, routing, and storage policies through simulation. The objective is to determine which policy or combination of policies will provide the greatest reduction in total picking time compared to the current policies of an example DC. The example DC currently employs a policy that entails picking one order at a time, storing product in a random fashion, and using a traversal policy for routing pickers. The effects of order size, warehouse shape, location of pick-up/drop-off point, and demand distribution are analyzed. Their results indicate that the batching of orders yields the greatest savings, particularly when smaller order sizes are common. Furthermore, the impact of switching from traversal to optimal routing on the reduction in picker travel is significantly less than changing picking or storage policies.

Jewkes et al. (2004) address the problem of jointly determining product location, picker home base location, and allocating products to each picker following a discrete order picking strategy. Picker home base location refers to a specific location where a picker will begin and end his tour within his zone. The main objective is to maximize the expected number of orders that can be completed per unit time, or, equivalently, to minimize the expected order cycle time. For this problem the authors show that a greedy approach is optimal for grouping products into bins and that their results apply for several alternate “out and back” picking strategies. Furthermore, for the case when product location is fixed, they develop a dynamic programming (DP) algorithm to determine the optimal picker home base location and product allocation. Though the authors demonstrate the effectiveness of the greedy and DP algorithms for small-sized problems, these algorithms are not suitable for large-sized problems.

2.3.6 Picking Method and Material Handling Equipment

The selection of appropriate material handling equipment to use in an OPS depends on the picking method being employed. For example, if employing a manual picking method, then a designer may select either a pick-cart, a tote, or one of the many varieties of order picking trucks. If employing a semi-automated picking method, then the choices are carousels, VLMs, and end-of-aisle mini-load AS/RSs, etc. An end-of-aisle mini-load AS/RS is used to store unit loads and to pick items from the retrieved loads (i.e., split-case picking). Furthermore, a sorting system may be required when employing certain picking strategies.

A major objective in the selection of a picking method and the associated material handling equipment is the throughput of the resulting system. Analytical models to estimate the throughput of some of these picking method and material handling equipment combinations are reviewed below.

Bozer and White (1996) develop an algorithm to design and evaluate the performance of an end-of-aisle mini-load AS/RS. The algorithm is based on an approximate analytical model to estimate expected picker and S/R machine utilization. This algorithm can be used for a variety of configurations including multiple pick positions per aisle and multiple aisles per picker. The authors also investigate the possibility of improving picker utilization by sequencing container retrievals within each order.

Meller and Klote (2004) develop stochastic models to estimate the throughput of pods of carousels and VLMs with human order-pickers picking batches of orders. A pod represents multiple carousels/VLMs that are grouped together to more efficiently utilize the order-picker (i.e., the order-picker rarely has to wait on the carousel/VLM, while it rotates, to retrieve the next item). The authors use a cyclic queueing model to estimate picker utilization in the case of carousel/VLM pods.

de Koster (1994) presents an approximation method for early-stage evaluation of design alternatives of a pick-to-conveyor OPS. The author considers a pick and pass zone picking strategy, in which orders (in bins) travel on a conveyor and get diverted only to the zones in which there are items to be picked. The author employs an approach based on Jackson network modeling and analysis to develop this approximate method. This method can be used to evaluate the effects of changing the layout of the system, the number of picking stations, the number of pickers, the conveyor speed, the number of bins to be processed per day, the number of order-lines per bin, etc. Note that a pick-to-conveyor OPS employing a pick and pass zone picking strategy is not as common as the one that employs a simultaneous zone picking strategy. In the latter, pickers pick items from the storage locations simultaneously and induct them on to the conveyor directly. This type of OPS is a standard configuration for case picking operations, throughput models for which need to be developed.

Sorting System: Sorting systems are employed to sort or distribute items that are picked from the forward storage area into customer orders. Such systems are integral to an OPS when employing a batch picking system that does not allow for sorting items while picking or a simultaneous zone picking strategy. Several researchers have addressed various issues related to a sorting system; see survey papers by Rouwenhorst et al. (2000) and van den Berg (1999) for details. Below, we review two of these contributions.

Johnson and Meller (2002) develop models to estimate the performance of split-case sorting systems. Split-case sorting systems are typically used when items must be distributed (or sorted) in less-than-case quantities. The authors develop stochastic models to estimate throughput for different system configurations (e.g., side-by-side, split, etc.) by considering potential blocking during induction. Through their models, they develop several insights related to the placement of inductors, the relative performance of various system configurations, and the effect of presorting in split configurations.

Russell and Meller (2003) address the basic problem of whether or not to automate the sorting system by developing cost and throughput models for manual and automated sorting systems. Their research is aimed at providing a design aid deciding whether or not to automate the sorting process. They develop a descriptive model for the sorting system design decision, based on demand levels, labor rates, order sizes, and other factors, and incorporate the descriptive model into a cost-based optimization model to recommend a solution. Embedded in this model are the analytical models for automated split-case sorting systems developed in Johnson and Meller (2002). In the case when the descriptive model suggested a manual sorting system, the authors developed a prescriptive analytical model to determine an optimal batching level.

2.4. Summary

In this chapter we briefly reviewed literature in the areas of: (1) overall DC (or warehouse) design, (2) design of an OPS, and (3) specific problems related to OPS design. From our review we observe that there are several other critical design issues in OPS design that have not been completely addressed in the literature, a few of which are as follows:

1. estimating the amount of blocking in wide-aisle DCs;
2. determining the best configuration of the storage system for storing the SKUs; and
3. selecting a batch or a zone picking strategy for medium-to-high throughput DCs.

By addressing the above issues in this dissertation we intend to contribute to the set of analytical models available for the design of an OPS for the forward area of a DC. Through these analytical models, a designer would be able to compare various OPS design alternatives quickly. How we propose to address the above critical issues is discussed next in detail (as contributions of this research) in Chapter 3.

Chapter 3

Problem Statement

As mentioned in Chapter 1, the design of an OPS is a complex task. The complexity arises due to the numerous combinations of design parameters (e.g., storage configuration, storage policy, picking method, picking strategy, material handling system, pick-assist technology, etc.). The design decisions related to these design parameters are highlighted in Figure 3.1. The problems that have not yet been addressed completely in the OPS literature are italicized.

During the design process, a designer aims at optimizing one or more objectives, which include (but are not limited to):

- throughput — how to maximize the average number of orders fulfilled per hour considering current and future customer order patterns?
- cost — how to minimize the system cost (which includes the costs related to the storage area, equipment, labor, etc.)?
- space — how to minimize the space required to store the SKUs?
- response time — how to minimize the time required to fulfill an order once received?
- picking error-rate — how to minimize the proportion of customer orders not fulfilled correctly?

Some of these objectives are conflicting in nature. For example, while optimizing for throughput, a designer may select a highly specialized material handling system, which may not minimize system cost. However, a less expensive system may result in a high error-rate in fulfilling customer orders. And so on with the other combinations of objectives. Furthermore, the objective to be optimized itself varies between various time-periods for a given

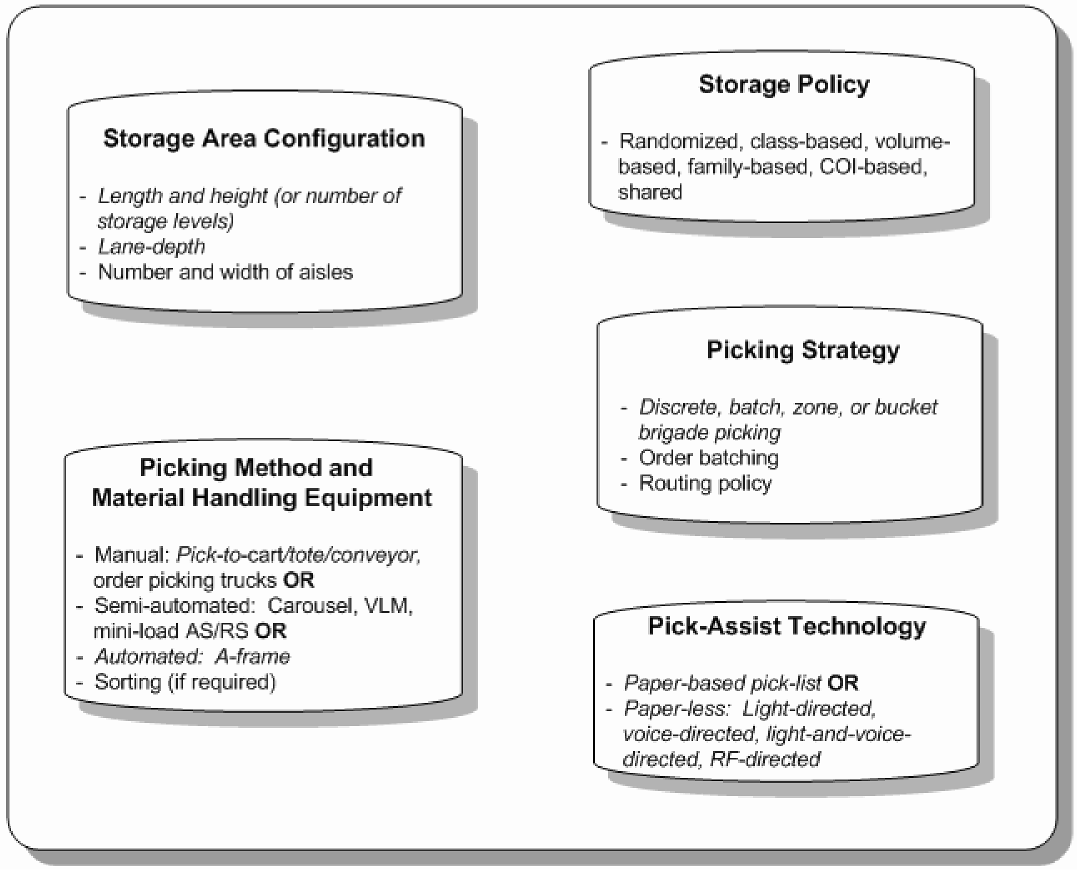


Figure 3.1: Design Decisions for an OPS Design in a DC.

application or between various applications. For example, as business within a DC grows, the designer, who initially designed the system to minimize cost, may now consider re-designing (or modifying) the system to maximize throughput.

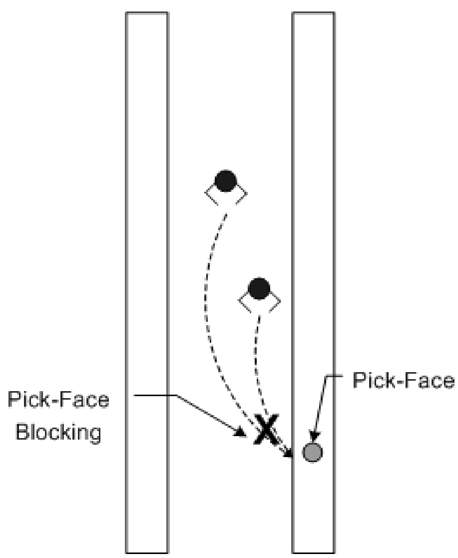
In this research we assume that the designer is tasked with making design decisions for an OPS in a DC. As discussed previously, this is a complex task, with no comprehensive set of design models to assist the designer. Thus, our overall research goal is to develop a set of analytical models for designing an OPS for the picking area in a DC. As can be seen in Figure 3.1, there are a number of areas of modeling that must first be addressed to achieve this research objective.

In Section 3.1 we discuss our first research contribution on estimating picker blocking in in wide-aisle DCs. This will aid in addressing the issues of storage system configuration and picking strategy selection. In Section 3.2 we discuss our second research contribution related to configuring the storage system for order picking, an issue under "Storage Area Configuration" in Figure 3.1. In Section 3.3 we address the "batch versus zone problem"

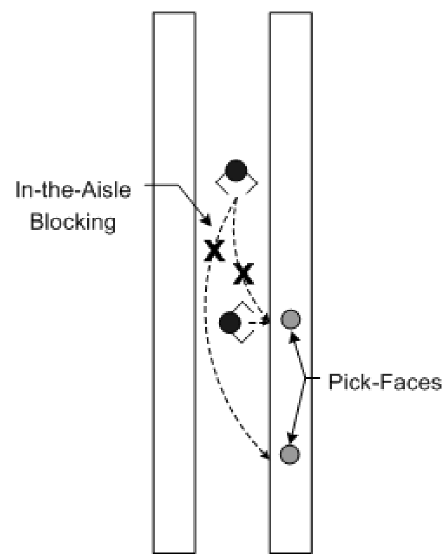
— a problem related to "Discrete, batch, zone, or bucket brigade picking" under "Picking Strategy" in Figure 3.1. — by developing an analytical model.

3.1. Contribution 1: Picker Blocking Estimation

In Chapter 2 we highlighted that the aspect of picker blocking has not been completely addressed in the literature. Picker blocking can take two forms: (1) *pick-face blocking*, in which pickers get blocked at a pick-face and (2) *in-the-aisle blocking*, in which pickers get blocked not only at a pick-face, but also within an aisle due to their inability to pass other pickers in a narrow-aisle DC. Both these forms of blocking are illustrated in Figure 3.2.



(a) Wide-Aisle DC



(b) Narrow-Aisle DC

Figure 3.2: Forms of Picker Blocking: (a) Pick-Face Blocking in a Wide-Aisle DC and (b) In-the-Aisle Blocking in a Narrow Aisle DC.

Picker blocking is a crucial factor in configuring a storage system and in deciding between a batch and a zone picking strategy. When configuring a storage system, if the storage system is high (in height) and short (in length), then the pickers may block each other when picking in such a small picking area. When deciding between a batch and a zone picking strategy, pickers may be blocked in a batch picking system as they are free to move in the storage area. Note that blocking can potentially reduce picker productivity, which may increase the number of pickers required to satisfy system throughput; this will increase the cost of labor.

To estimate in-the-aisle picker blocking in a narrow-aisle DC, Gue et al. (2006) developed analytical models based on a Markov chain modeling approach. A narrow-aisle DC is one in which the aisles are too narrow for pickers to pass each other within the aisle. However, there are a large number of DCs that have aisles wide enough to allow pickers to pass each other in the aisle. For such systems, pickers may experience blocking at a pick-face, which we refer to as *pick-face blocking*. Models to estimate pick-face blocking have yet to be developed.

As the *first* contribution of this research, we have built analytical models to estimate pick-face blocking in a wide-aisle DC. Also, in so doing, instead of restricting pickers to pick only one item at a pick-face or assuming that the pick-time at each stop is constant (as assumed in Gue et al. (2006)), we include the possibility of pickers picking multiple items at a pick-face; i.e., we assume that the pick-time at each stop is variable. A discrete time Markov chain modeling approach is used to estimate the amount of blocking in a system with two pickers for cases when pickers make single and multiple picks at a pick-face. The analytical models we develop are for the cases when the pick to walk time ratios are 1:1 and ∞:1. For other ratios (e.g., 5:1, 10:1, and 20:1), we develop simulation models to estimate pick-face blocking. Using these models and the models developed by Gue et al. (2006), a designer will be able to include an estimate of blocking as a factor in configuring the storage system and selecting an appropriate picking strategy.

In Chapter 4 we present a paper based on Contribution 1. This paper provides details on the development of our models. In this paper we compare our results with those presented in Gue et al. (2006) and provide general observations on the blocking phenomena in narrow- and wide-aisle DCs.

3.2. Contribution 2: Storage System Configuration

Configuring the storage system for order picking requires an estimation of the length and height (for a given depth) of the storage equipment (e.g., shelves, racks, etc.). We define the height of a storage area in terms of the number of storage levels at a pick-face. Figure 3.3 illustrates the relationship between a storage location, storage level, and pick-face.

In this research we focus on the problem of identifying the best configuration of the storage system in terms of its height and length. For example, consider a hypothetical scenario where the total storage volume required to store the SKUs is 12 cubic feet. Assuming that each storage location is a unit foot in length, height, and depth, 12 storage locations will be required to store these SKUs. For this scenario, three possibilities of length and height of

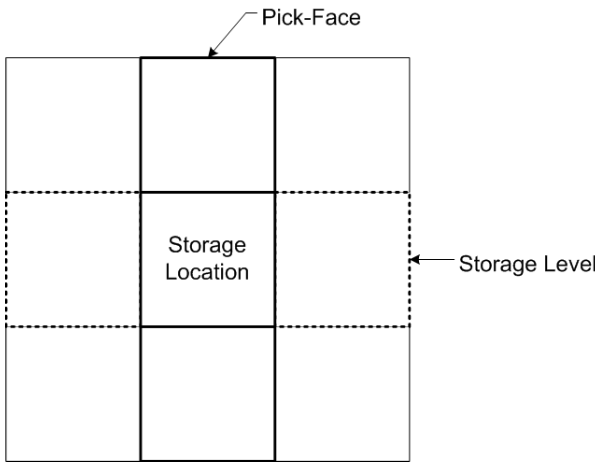


Figure 3.3: Storage Area Schematic.

the storage area are illustrated in Figure 3.4. The decision we wish to help a designer make is the following: *what is the optimal height, and thereby, length, of the storage system in the picking area that minimizes system cost?* System cost is the combined cost of pickers, equipment, and space. For the hypothetical example illustrated in Figure 3.4, the designer will have to identify the optimal storage system amongst 1-level, 2-level, or 3-level storage systems.

Factors that affect the storage system configuration problem include available area, number and quantity of SKUs required to be stored, type of storage system used, etc. Two other factors that are crucial for this problem — but are overlooked in the OPS literature and in industry — are picker blocking and travel-time.

We explained earlier how picker blocking affects the storage system configuration problem. We also need to consider the effect of travel-time on this problem. For example, in a semi-automated OPS employing a person-aboard storage/retrieval (S/R) machine the vertical speed of the S/R machine is significantly lower than its horizonal speed. As a result, as the storage system height increases (which decreases system length), it is possible that an increase in the vertical travel-time may offset a decrease in the horizontal travel-time of the S/R machine. Consequently, the total travel-time, which is the sum of horizontal and vertical travel-times, may increase. An increase in the total travel-time results in a decrease in the throughput of a picker, which means more pickers will be required to satisfy the system throughput. That is, picker blocking and increase in travel-time tend to increase the cost of labor and equipment. In contrast, as the storage system height increases the amount of

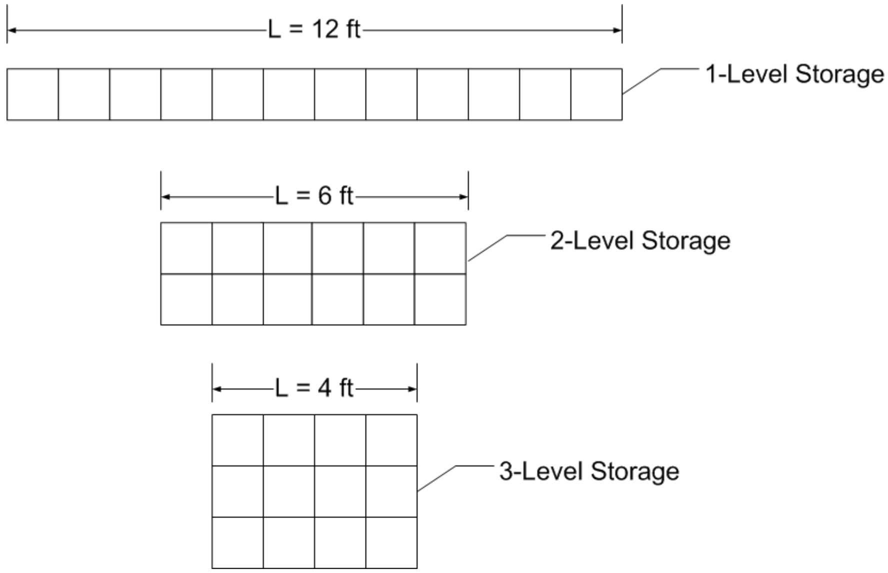


Figure 3.4: Relationship Between Length and Height (or Number of Storage Levels) of the Storage Area.

space required decreases, which decreases the cost of space.

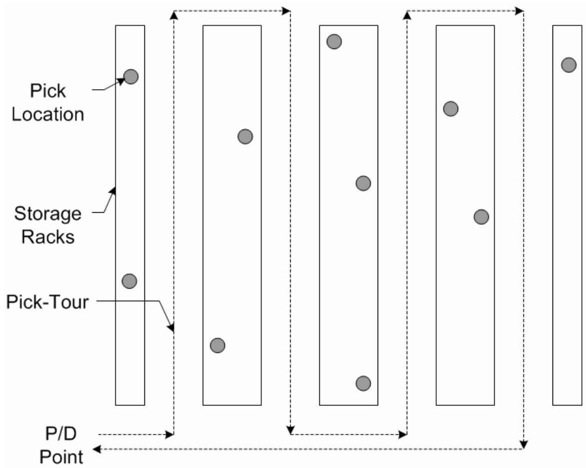
Therefore, there exists a trade-off between the three cost components, labor, equipment, and space, in configuring the storage system. We pursue an analytical approach based on system cost to address this trade-off, details of which are presented in Chapter 5.

3.3. Contribution 3: Picking Strategy Selection

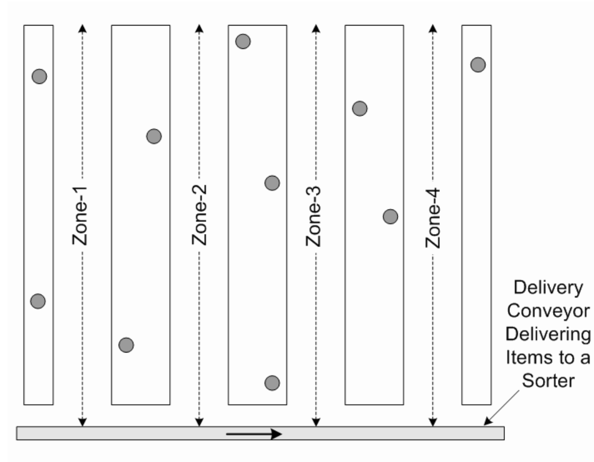
Discrete order picking, though simple to implement, can be labor-intensive for medium-to-high throughput DCs. Moreover, bucket brigade picking is limited to applications where handing-off the items to a downstream picker is easy. It is our observation that most DCs typically do not employ these two strategies; instead they prefer to consider batch or zone picking strategies.

Selecting between batch and zone picking strategies, which we refer to as the *batch versus zone problem*, depends on their relative advantages and disadvantages. Batch picking is simple to implement and may not require a sorting system. However, as pickers move freely in the storage area the probability of blocking increases (see Figure 6.1(a)), which decreases picker productivity.

On the other hand, a zone picking strategy is not affected by blocking as pickers are assigned to specific regions of the storage area (see Figure 6.1(b)). However, in zone picking



(a) Batch Picking Strategy



(b) Zone Picking Strategy

Figure 3.5: Schematic of Batch and Zone Picking Strategies.

systems there are issues related to workload-imbalance (with the pick-and-pass method) and the need for downstream sorting (with the simultaneous method). A pick-and-pass or sequential zone picking strategy is one in which picking occurs in one zone at a time, after which the order is passed to the next zone. A simultaneous or synchronized zone picking strategy is one in which all applicable zones are picked simultaneously and then orders are consolidated through a sorting system (Tompkins et al., 2003). Thus, there exists a trade-off between the two strategies. No models exist in the literature that a designer can utilize for the batch versus zone problem.

In our research we propose to address this problem through an analytical approach. Specifically, we develop an analytical model that will assist a designer in deciding between batch and zone picking strategies based on throughput requirements of a DC. Some of the parameters considered in the model include customer orders (in terms of orders/day and distribution of items in orders), workload-imbalance, number of waves, requirement of a sorting system, amount of blocking, etc. We include further details on this contribution in Chapter 6.

3.4. Summary

The design of an OPS is a complex task that requires a designer to consider some sort of a trade-off while making many of the design decisions. Analytical models that address issues in certain areas of OPS design exist in the OPS literature. However, there are number of

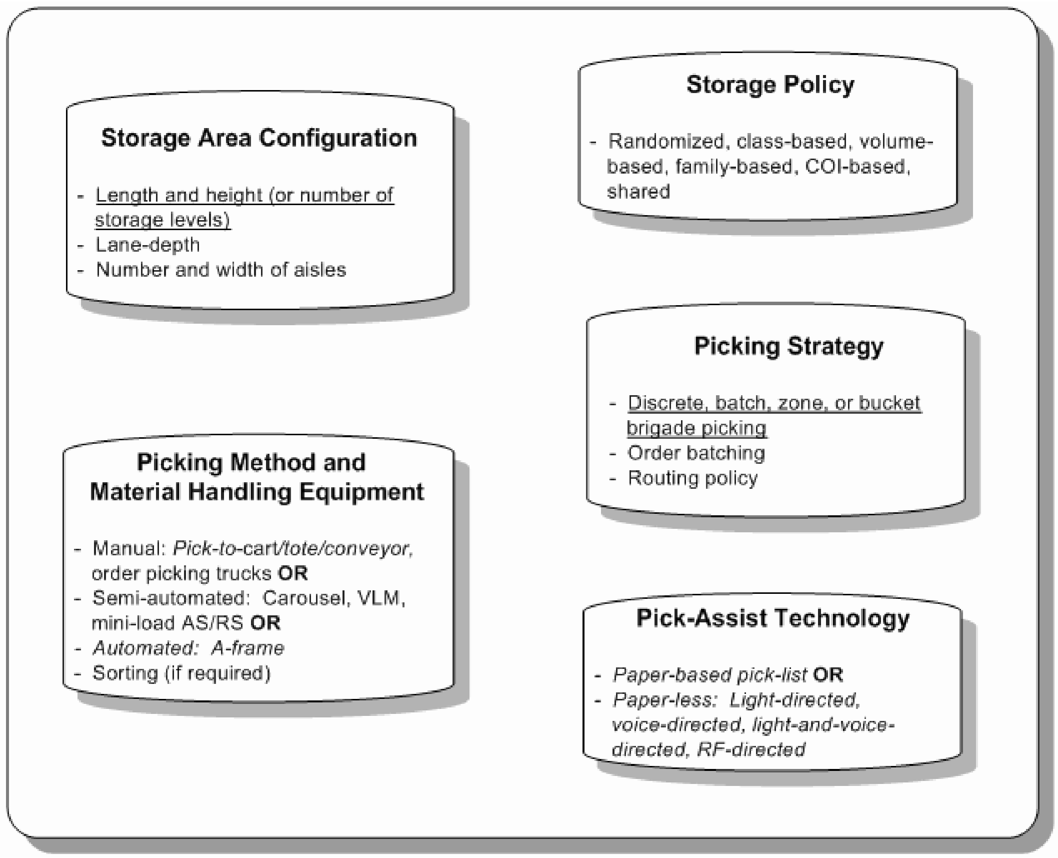


Figure 3.6: Design Decisions Addressed in this Research.

other issues that have not been completely addressed. Our research aims at addressing these issues through analytical approaches. In this dissertation we focus on the following three contributions:

Contribution 1. Estimating pick-face blocking in wide-aisle order picking systems;

Contribution 2. Configuring the storage system for order picking in a distribution center;

Contribution 3. Selecting between batch and zone picking strategies in a distribution center.

These three contributions are underlined in Figure 3.6. Note that Contribution 1 is included with Contributions 2 and 3 within “Length and height (or number of storage levels)” under “Storage Area Configuration” and within “Discrete, batch, zone, or bucket brigade picking” under “Picking Strategy,” respectively.

Contributions 1, 2, and 3 have been addressed, the details of which are presented in Chapters 4, 5, and 6, respectively. We expect that our research will help an OPS designer to identify the best OPS configuration out of a large number of possible configurations.

Chapter 4

Estimating Picker Blocking in Wide-Aisle Order Picking Systems

Abstract

Several factors influence the design of an order picking system (OPS) in a distribution center (DC); e.g., storage area configuration, storage policy, picking strategy, etc. Amongst these, selection of a picking strategy is critical when the objective is to maximize throughput. Picking strategy — discrete, batch, or zone — refers to the act of assigning and routing pickers in the forward area to fulfill customer orders. For medium-to-high throughput OPSs, batch picking (in which pickers circumnavigate a forward area) and zone picking (in which pickers are assigned to specific sections of the forward area) strategies are common. A crucial factor in deciding between a batch and a zone picking strategy is *picker blocking*. In this chapter we develop analytical models to estimate blocking in an OPS that has picking aisles wide enough to allow pickers to pass other pickers in the aisle. In such OPSs, pickers can experience blocking at a pick-face when two or more pickers need to pick at the same pick-face. Our results suggest that when pickers may pick only one item at a pick-face, blocking is less in a wide-aisle OPS compared to that in a narrow-aisle OPS. However, when pickers may pick multiple items at a pick-face, blocking increases monotonically with an increase in the number of items to be picked.

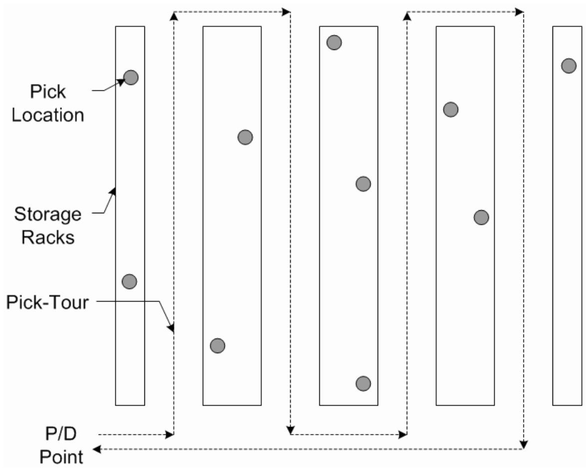
4.1. Introduction

A distribution center (DC) is an integral element of a logistics system. A DC is responsible for obtaining materials from different suppliers, performing value-added activities, and assembling (or sorting) them to fulfill a number of different customer orders. Some of the major activities within a DC include the receipt of items and customer orders, storing items, order picking, shipping, customer service and reclamation, and control. Amongst these activities, *order picking* has been identified as the highest priority activity in a DC for productivity improvements due to its relatively high (about 50%) contribution to the total DC operating cost (Tompkins et al., 2003). Order picking refers to an operation through which the items are retrieved from a storage location to fulfill customer orders. Some of the order picking work elements include traveling to the item, searching for the item, reaching and extracting the item from the storage, documenting the pick, sorting items, etc.

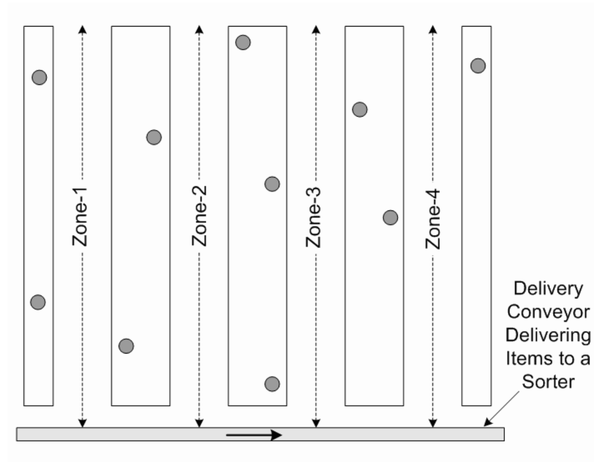
While designing an OPS, a designer is always posed with the question: *which OPS best meets a given set of objectives?* Some of the objectives the designer is required to optimize include maximizing throughput or minimizing cost, space, response time, and error-rate. Optimization of one or more of these objectives requires a complex task of designing (or selecting) several parameters; e.g., forward-reserve area allocation, forward picking area configuration, storage policy (randomized, class-based, volume-based, family-based, COI-based, etc.), picking strategy (discrete, batch, or zone), picking method (manual, semi-automated, or automated), and material handling equipment (order picker trucks, carousels, VLMs, etc.).

Amongst these design parameters, the problem of which picking strategy to employ is crucial when the objective is to maximize system throughput. A picking strategy defines the manner in which the pickers navigate in the order picking aisles of a picking area to pick required items. There are three basic order picking strategies: (1) *discrete picking*, in which a picker picks all the items in a single order during a pick-tour; (2) *batch picking*, in which several orders are batched (or grouped) together and a picker picks all items in a given batch; and (3) *zone picking*, in which each picker is assigned to a specific region (consisting of one or more aisles) of the picking area and picks items (for a single order or batch of orders) in that region only.

Discrete picking, though simple to implement, can be labor-intensive for medium-to-high throughput OPSs. Such OPSs typically employ a batch or a zone picking strategy. Selecting between these two strategies, which we refer to as the *batch versus zone problem*, depends on their relative advantages and disadvantages. Batch picking results in less worker travel per item picked, which increases picker productivity; however, as pickers move freely in the picking area there is an increased chance of blocking (see Figure 4.1(a)). On the contrary, blocking does not exist when employing a zone picking strategy as pickers are assigned to specific regions of the picking area (see Figure 4.1(b)). However, there are issues related to zone imbalance or the need for downstream sortation. Thus, there exists a trade-off between these two strategies.



(a) Batch Picking Strategy

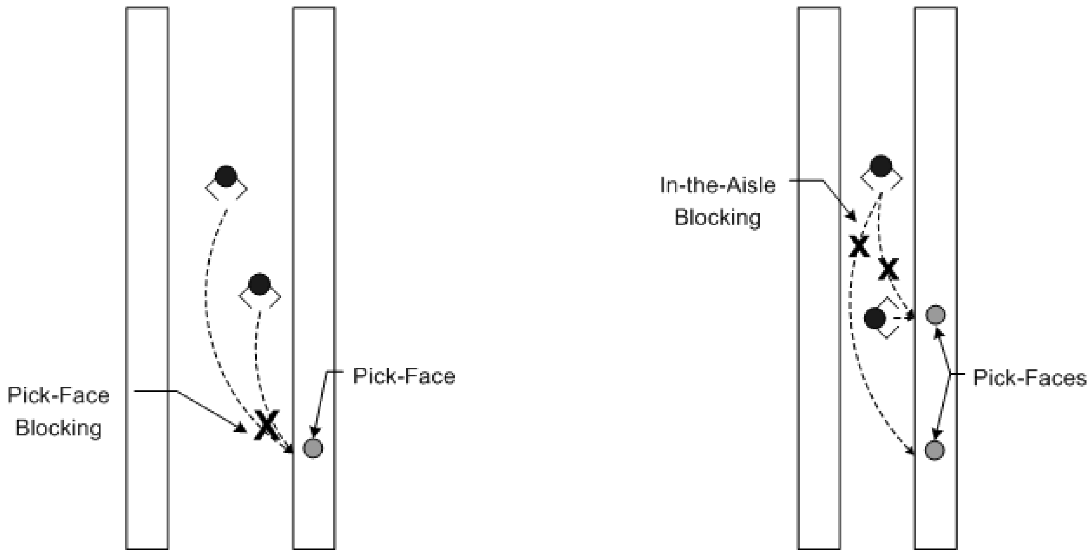


(b) Zone Picking Strategy

Figure 4.1: Schematic of Batch and Zone Picking Strategies.

In this chapter we analyze this trade-off by estimating blocking in OPSs. Blocking can be prominent in a high throughput system that requires a large number of pickers in the picking area. Blocking can lead to increased picker idle time and reduce their productivity, thus, affecting system throughput. We develop analytical models to estimate blocking in wide-aisle OPSs. A wide-aisle OPS is one in which the aisles are wide enough to allow pickers to pass each other in the aisle. In such OPSs pickers can experience blocking at a pick-face when two or more pickers need to pick at the same pick-face — we refer to this form of blocking as *pick-face blocking*. On the other hand, a narrow-aisle OPS is one in which aisles are too narrow to allow pickers to pass each other in the aisle. For such OPSs, pickers not only experience pick-face blocking, but also experience blocking due to their inability to pass other pickers in the aisle. This form of blocking is referred to as *in-the-aisle* blocking, models to estimate which have been developed by Gue et al. (2006). Figure 5.4 illustrates pick-face blocking in wide-aisle OPSs and in-the-aisle blocking in narrow-aisle OPSs.

The remainder of this chapter is organized as follows. In Section 4.2 we briefly review literature in the area of design of DCs and OPSs. In Section 4.3 we present the assumptions that we make in developing the blocking models. In Section 4.4 we develop analytical models to estimate pick-face blocking, considering two extreme values of pick-to-walk-time ratios,



(a) Wide-Aisle OPS (b) Narrow-Aisle OPS

Figure 4.2: Forms of Picker Blocking: (a) Pick-Face Blocking in a Wide-Aisle OPS and (b) In-the-Aisle Blocking in a Narrow Aisle OPS.

for the case when pickers may pick *only one* item at a pick-face. For deterministic pick-times, the possibility of picking only one item leads to *no variance* in the time to pick items at a pick-face for each picker. Similarly, in Section 4.5 we develop analytical models to estimate pick-face blocking, again considering two extreme values of pick-to-walk-time ratios, for the case when pickers may pick *multiple* items at a pick-face. The possibility of picking multiple items leads to *high variance* in the time to pick items at a pick-face for each picker. In Section 4.6 we summarize our understanding of the blocking phenomenon and its importance in the batch versus zone problem.

4.2. Literature Review

In the area of overall DC (or a warehouse) design, Gray et al. (1992) propose a multi-stage hierarchical decision approach to model the composite design and operating problems for a typical order-consolidation warehouse. Their hierarchical approach utilizes a sequence of coordinated mathematical models to evaluate the major economic tradeoffs and to prune the decision space to a few superior alternatives. Rouwenhorst et al. (2000) present a reference framework and a classification of warehouse design and control problems. They define a warehouse design problem as a "structured approach of decision making at a strategic, tactical, and operational level in an attempt to meet a number of well-defined performance criteria."

Specific to the design of an OPS in a DC, Brynzer et al. (1994) present a zero-based analysis methodology for the evaluation of OPSs as a base for system design and managerial decisions. Yoon and Sharp (1996) propose a structured procedure for the analysis and design of an OPS that considers the inter-dependent relationships between different functional areas (e.g., receiving, picking, sorting, etc.). The survey papers by van den Berg (1999), Rouwenhorst et al. (2000), and Gu et al. (2005) provide a good review of various contributions in the area of DC and OPS design.

The issue of which picking strategy is best for a given application is a crucial design problem. Petersen (2000) demonstrated through simulation studies that batch and zone picking systems are superior to any other strategy. Ruben and Jacobs (1999) explore the relationship between five order-batching heuristics and three storage policies (randomized, class-based, and family-based). Through simulation studies, they conclude that blocking plays a vital role in the simultaneous selection of order-batching heuristics and storage policies. Rouwenhorst et al. (2000) hint at, but do not address, the batch versus zone selection problem.

To understand the effect of pick-density (the probability that a picker will pick at a pick-face) on blocking in narrow-aisle OPSs, Gue et al. (2006) develop analytical models to estimate *in-the-aisle* picker blocking in such systems. They use discrete time Markov chains to estimate bounds on the percentage of time each picker is blocked. The critical assumptions in their work include: the order picking aisle may be represented as a circle, the ratio of walk to pick time is 1:1 or ∞:1, and pickers can make only one pick at a pick-face. They perform simulation studies to estimate blocking for cases when walk to pick time ratios are 5:1, 10:1, and 20:1. Through their results, the authors suggest that blocking increases with an increase in the number of pickers in narrow-aisle OPSs, but decreases with an increase in the picking area (for the same number of pickers). Skufca (2005) derives an analytical expression to estimate blocking in a circular warehouse with k workers walking at infinite speed for the case when the aisle is too narrow to allow passing and pickers pick at most one item at a pick-face.

From our review of the OPS literature, we conclude that the only two contributions that develop analytical models to estimate blocking in an OPS are those by Gue et al. (2006) and Skufca (2005). However, both these contributions consider only a narrow-aisle OPS, in which pickers are unable to pass within the aisle. Many OPSs in the real-world have aisles wide enough to allow pickers to pass. Models to estimate blocking in such OPSs will certainly help a designer to not only decide between batch versus zone picking strategies, but also to quickly analyze the trade-off between storage space utilization (for a wide- and narrow-aisle OPS) and blocking. Further, pickers may pick multiple items at a pick-face — a situation not considered in Gue et al. (2006) and Skufca (2005).

With this perspective, we next develop blocking models to estimate pick-face blocking in wide-aisle OPSs in which pickers may pick only one item (i.e., there is no variance in the time to pick items) or multiple items (i.e., there is high variance in the time to pick items) at a pick-face. For ease of presentation, a picker will be referred to as *he* as we develop our models.

4.3. Blocking Models for Wide-Aisle Order Picking Systems

In developing the blocking models, we assume that:

- The order picking area consists of n pick-faces. A pick-face might represent a column of pallet rack, a bay of flow rack, or a section of bin shelving. The place at which a picker stops is considered the pick-face (however, in reality each pick-face might consist of several storage locations).

- Pickers follow a traversal or S-shape routing policy, which means that they visit each aisle and travel through that aisle in only one direction. Consequently, we assume the layout of the picking area may be represented as a circle.

- When a picker stops at a pick-face, he can access only one side of the aisle to pick an item. In practice, pick-faces are located on both sides of an aisle. However, for ease in modeling, we assume that the pick-faces on both sides are merged together. That is, if each side of the aisle has $n/2$ pick-faces, then after merging there are n pick-faces on one side of the aisle and 0 on the other side.

- If a picker picks I ($I < n$) items on average, then the probability with which a picker stops at a pick-face, p, can be calculated as $p = \frac{I}{n}$.

- We assume average times to pick (t_p) and walk past a pick-face (t_w). The time to pick (t_p) is averaged across the number of stock keeping units (SKUs) picked from a storage location at a pick-face; i.e., t_p includes the time spent in picking multiple items of the same SKU. (A SKU is defined as a unique identifier of each product or item that is stocked in a DC.)

- There are only two pickers in the order picking aisle at a given time with identical values of t_p, t_w, and p.

- A picker is either picking, walking, or idle due to blocking.

Let $\lambda_{batch}(k)$ and $\lambda_{zone}(k)$ represent the throughput of a batch or a zone picking system with k pickers. The expressions for these throughput values can be obtained from Gue et al. (2006) as

$$\lambda_{batch}(k) = k\left[\frac{I}{It_p + nt_w}\right](1 - b(k)) \qquad \text{and} \qquad \lambda_{zone}(k) = k\left[\frac{I}{It_p + nt_w}\right]. \tag{4.1}$$

In these expressions it is clear that the throughput of batch and zone picking systems differ only in the term $(1 - b(k))$, where $b(k)$ represents the fraction of time each of the k pickers is blocked ($0 \leq b(k) \leq 1$). Since in Gue et al. (2006) narrow-aisle OPSs with no passing are considered, $b(k)$ represents in-the-aisle blocking. Here, we assume wide-aisle OPSs, in which pickers can pass, and therefore, $b(k)$ represents pick-face blocking.

We now develop models to estimate pick-face blocking ($b(k)$) for the cases when the pickers may pick only one item (Section 4.4), and when the pickers may pick multiple items (Section 4.5).

4.4. Blocking Models when Pickers May Pick Only One Item at a Pick-Face

Consider a circular aisle with n pick-faces (Figure 4.3). In a given epoch (i.e., a specific time period), a picker would either pick at a pick-face with probability p or walk to the next pick-face with probability q $(= 1 - p)$. It is assumed that if a picker picks in a given epoch, he would walk to the next pick-face in the next epoch with a probability equal to 1. In this way, we develop analytical models for the case when pickers may pick only one item at a pick-face; i.e., there is no variance in the time to pick items at a pick-face. In so doing, we consider two situations; *first*, in which the pick:walk time ratio is 1:1, and *second*, in which the pick:walk time ratio is ∞:1. For other ratios (e.g., 5:1, 10:1, and 20:1), we develop simulation models to estimate pick-face blocking.

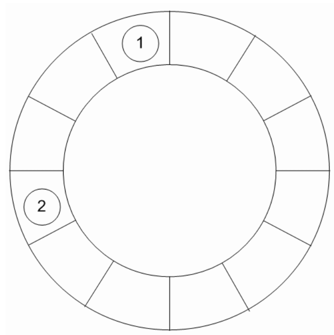


Figure 4.3: A Circular Order Picking Aisle.

4.4.1 Analytical Model for a Pick:Walk Time Ratio of 1:1

We consider the situation where the pick and walk times are identical and equal to one unit. That is, in one time unit (or epoch), a picker would either pick an item or he would walk to the next pick-face.

Let D_t be the distance between Picker 1 and Picker 2 at epoch t with picker numbers being assigned arbitrarily, *a priori*, and never change. For a circular aisle, D_t = (Picker 1 position) - (Picker 2 position) mod n such that $0 \leq D_t \leq n-1$. That is, if $n = 6$, then the pickers could be at the same pick-face (i.e., $D_t = 0$) or as far away as faces 1 and 6 (i.e., $D_t = 5$).

We model this system using a discrete time Markov chain approach. For this, we need to identify the states such that they form a Markov chain. The sequence $\{D_t\}$ is not a Markov chain. This is because if a picker picked during the last epoch, then he must move during this epoch (with probability 1). Hence, D_{t+1} depends on D_t and on what the pickers performed (i.e., picked or walked) in the previous step. To establish the Markov property, we must consider what happened in the previous epoch. The sequence r_{xy} would be a Markov chain, where r ($0 \leq r \leq n-1$) is the distance between the two pickers and x and y represent the previous action of Pickers 1 and 2, respectively. For example, State 2_{pw} indicates that pickers are 2 pick-faces apart with Picker 1 just picked and Picker 2 just walked. Then, the State 0_{pp} would represent that pickers are at the same pick-face and both just picked, which is not possible. This state would be referred to as the *blocked* state. If this happens (i.e., both pickers need to pick at the same pick-face), then it is assumed that either Picker 1 or Picker 2 picks in that epoch while the other waits (i.e., gets blocked). Hence, State 0_{pp} can transit to State 1_{wp} or $(n-1)_{pw}$ with probability equal to 0.5. All other states are achievable. The states can be ordered as $\{0_{pp}$ (or *blocked*$), 0_{pw}, 0_{wp}, 0_{ww}, 1_{pp}, \cdots, (n-1)_{wp}, (n-1)_{ww}\}$. The transition-probability diagram for this case is illustrated in Figure 4.4.

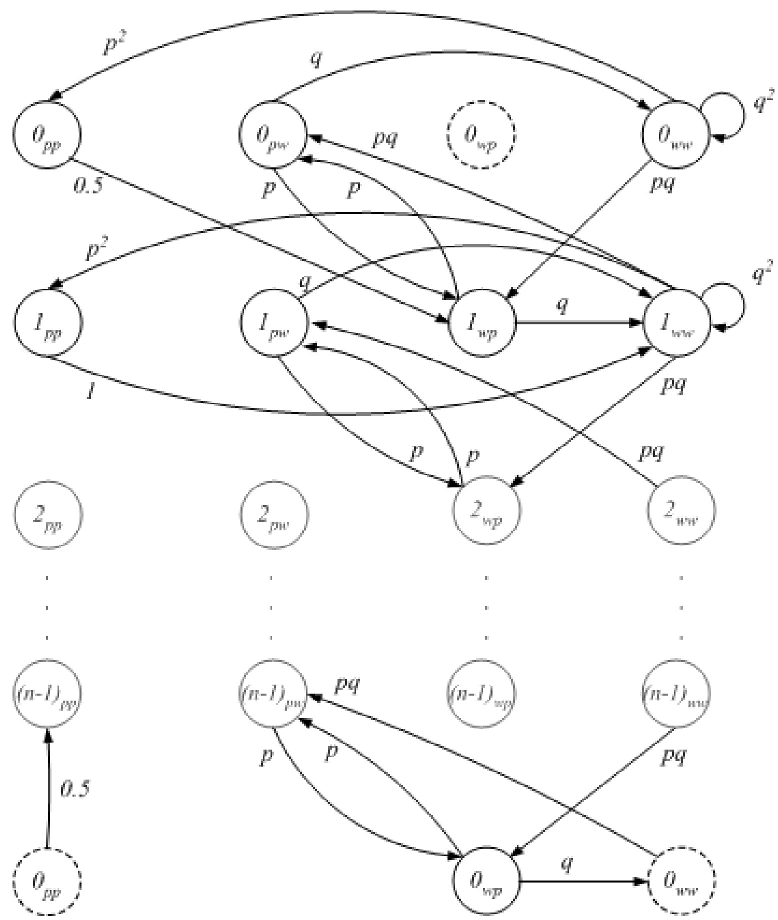


Figure 4.4: Transition-Probability Diagram for the 1:1 Ratio in a Circular Aisle when Pickers May Pick Only One Item.

Note that States 0_{pp}, 0_{wp}, and 0_{ww} are duplicated in this diagram (i.e., shown with both full and dotted lines). Such a representation helps in clearly indicating their relationships with each other and states starting with 0, 1, and $n-1$. The resulting transition matrix is

$$A_L = \begin{bmatrix} D_0 & U_0 & 0 & \cdots & Y \\ L & D & U & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & L & D & U \\ X & \cdots & 0 & L & D_{n-1} \end{bmatrix},$$

where

$$D_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q \\ 0 & 0 & 0 & q \\ p^2 & 0 & 0 & q^2 \end{bmatrix}, \ U_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & pq & 0 \end{bmatrix}, \ Y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & pq & 0 & 0 \end{bmatrix};$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & pq & 0 & 0 \end{bmatrix}, \ D = D_{n-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & q \\ 0 & 0 & 0 & q \\ p^2 & 0 & 0 & q^2 \end{bmatrix}, \text{ and } U = X = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & pq & 0 \end{bmatrix}.$$

Matrix X indicates the possible transitions from the states starting with $n-1$ to the states starting with 0, while matrix Y indicates the possible transitions from 0 to $n-1$.

The procedure to solve A_L is presented in Appendix A. For $\mathbf{z}A_L = \mathbf{z}$, we can verify that

$$\mathbf{z} = \left[\overbrace{\frac{2p^2(1-p)}{2-p}, p, p, \frac{2(1-p)}{2-p}}^{x=0}, \overbrace{p^2, p, p, 1}^{x=1}, \overbrace{p^2, p, p, 1}^{x=2}, \ldots, \overbrace{p^2, p, p, 1}^{x=n-2}, \overbrace{p^2, p, p, 1}^{x=n-1} \right].$$

Note that x refers to the states starting with $0, 1, 2, ..., n-1$ (e.g., $x = 0$ refers to the states 0_{pp}, 0_{pw}, 0_{wp}, and 0_{ww}). The stationary density for the Markov process is determined by scaling by $\|\mathbf{z}\|$. Since the blocked state is State 0_{pp}, we have

$$b_1(2) = \frac{z_1/2}{\sum_j z_j} = \frac{2p^2(1-p)/(2(2-p))}{(n(2-p)(1+p)^2 - p^3 - p)/(2-p)} = \frac{p^2(1-p)}{n(2-p)(1+p)^2 - p^3 - p}, \quad (4.2)$$

where $b_1(2)$ is the average amount of time that each of the two pickers is blocked for 1:1 ratio. Observe that $b_1(2)$ is a decreasing function of n.

To obtain the value of p that maximizes or minimizes (4.2), we take the first derivative and equate it to zero, which yields

$$\frac{4np - p^2 - 3np^2 + 2p^3 - 6np^3 + p^4 + np^4}{(n(2-p)(1+p)^2 - p^3 - p)^2} = 0. \quad (4.3)$$

The procedure of obtaining roots of the above equation is fairly complicated, so we resort to a commercially available software (i.e., MATLAB (2004)). By substituting $n = 2$ in (4.3), we obtain -1.085, 0, 0.653, and 3.765 as roots of the above equation. However, as p is a variable representing the probability of picking, we constrain $0 \leq p \leq 1$. Therefore, we arrive at $p = 0.653$. This value of p corresponds to the extremum of (4.2). To check if this extremum is a maximum or a minimum, we take the second derivative of (4.2) and evaluate it at $p = 0.653$. As the evaluated value is less than 0, we conclude that $p = 0.653$ maximizes blocking obtained through (4.2) for $n = 2$.

By repeating the above steps for a very large value of n (i.e., $n \to \infty$), we observe that the value of p that maximizes blocking is 0.628. The two extreme values of n (i.e., $n = 2$ and $n \to \infty$) define a range of p for which blocking is maximum; we refer to this range as the *critical range* of p for the 1:1 ratio with passing allowed. Figure 4.5 illustrates the percentage of time each picker is blocked for several values of n. As mentioned before, blocking decreases as n increases, with maximum blocking for various values of n occurring when $0.628 \leq p \leq 0.653$. It is interesting to observe that this range is quite narrow. Although we do not have an explanation for why this is the case, Gue et al. (2006) also observed a similar narrow range in the probability of picking (that maximizes blocking) for narrow aisle systems. From Figure 4.5 we observe that that percentage of time each picker is blocked in

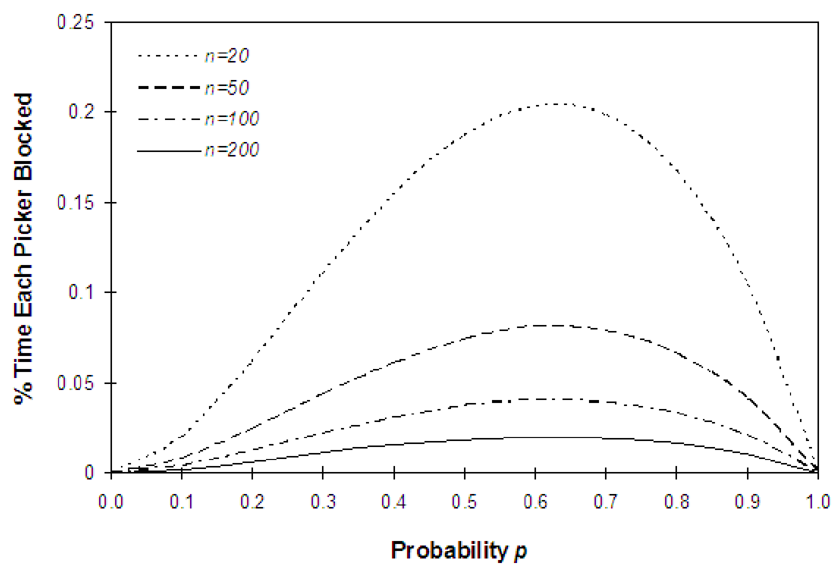


Figure 4.5: Percentage of Time Each Picker is Blocked for Various Values of n in a Circular Aisle, Pickers May Pick Only One Item, and Pick:Walk Time Ratio is 1:1.

a circular aisle with two pickers is very small. Hence, blocking does not seem to be a critical issue for such systems where pickers may pick only one item and the pick:walk time ratio is 1:1.

Table 4.1 compares the average blocking values experienced by the two pickers, as obtained through analytical and simulation models, in a circular aisle with $n = 20$ for the 1:1 ratio. As observed, for a very low value of p (i.e., $p = 0.05$), the estimate of blocking obtained through the analytical model is different from that of the simulation model (although the discrepancy is small in an absolute sense). This discrepancy is due to the fact that the probability of making at least one round of the aisle without picking an item is high for low values of p — an aspect not considered in the analytical model, but captured by the simulation model. This affects the dynamics in the system and hence, blocking. Note that such a low value of p may result if $I = 1$ and $n = 20$ or $I = 5$ and $n = 100$. Very low values of p do not correspond to medium-to-high throughput OPSs; for such OPSs, p is relatively high. For $p \geq 0.1$, the analytical and simulation estimates of pick-face blocking follow closely. Further, our experiments indicate that the %-difference decreases as n increases. Therefore, for the systems of interest to us in our research; i.e., medium to large systems with a value of $p \geq 0.25$, our analytical model performs very well.

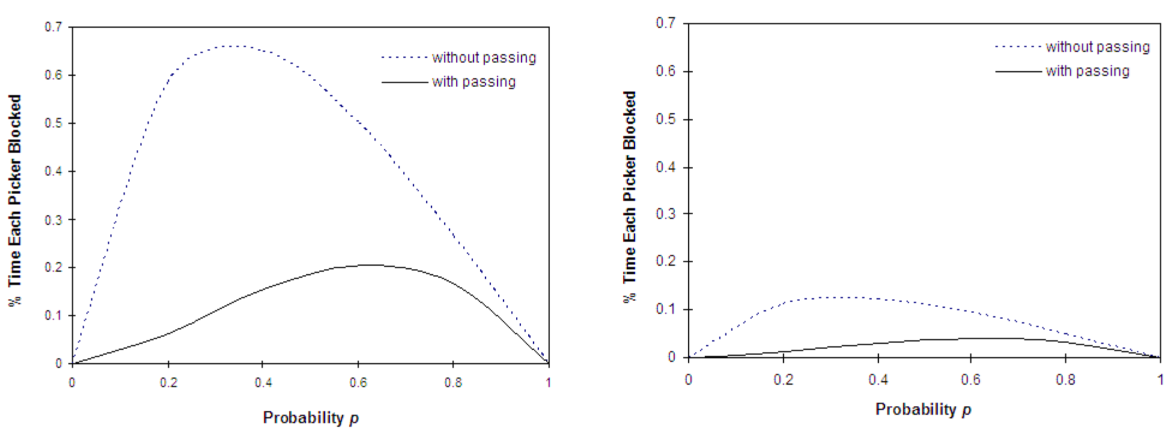
Table 4.1: Comparison of Analytical and Simulation Results of Percentage of Time Each Picker is Blocked in a Circular Aisle with $n = 20$, Pickers May Pick only One Item, and Pick:Walk Time Ratio is 1:1.

Probability p	**Analytical**	**Simulation**	**Std. Error**	**% Difference**
0	0.000	0.000	0.0000	0.00
0.05	0.005	0.006	0.0002	-2.86
0.1	0.196	0.195	0.0003	0.21
0.2	0.062	0.062	0.0005	0.00
0.3	0.110	0.111	0.0009	-0.15
0.4	0.154	0.153	0.0012	0.38
0.5	0.186	0.187	0.0013	-0.06
0.6	0.203	0.204	0.0015	-0.39
0.7	0.198	0.199	0.0014	-0.58
0.8	0.167	0.168	0.0017	-0.15
0.9	0.104	0.103	0.0018	0.44
1.0	0.000	0.000	0.0000	0.00

For the cases where $n = 20$ and $n = 100$, Figure 4.6 compares pick-face blocking (when pickers can pass) and in-the-aisle blocking (when pickers cannot pass — as modeled in Gue et al. (2006)). It is clear that the amount of pick-face blocking is much less as compared to in-the-aisle blocking. This result is not surprising as in-the-aisle blocking accounts not only for pick-face blocking, but also blocking due to the inability of the pickers to pass each other.

With respect to Figure 4.6, when passing is not allowed, Gue et al. (2006) explain the change in in-the-aisle blocking with an increase in the probability of picking as follows. They suggest that for extremely low values of p, blocking is low as pickers make almost no picks. As the value of p increases, blocking increases as pickers make sufficient stops to cause interaction. For relatively high values of p, pickers tend not to block one another because they spend more time picking and less time traveling (and blocking only occurs when a picker is traveling). The critical range of p they derived for which in-the-aisle blocking is maximized is between 0.33 and 0.37.

For our case (i.e., when passing is allowed), a possible explanation for the change in pick-face blocking with an increase in the probability of picking can be given as follows. Similar to Gue et al. (2006), for low values of p, blocking is low as pickers rarely stop to make picks. As p increases, the number of stops the pickers make increases and blocking increases, but only slightly. This is due to the fact that for blocking to occur in our case (as opposed to in the case modeled in Gue et al. (2006)), both pickers must stop at the same location, an event that is relatively rare until the level of picking increases substantially; i.e., p increases to a relatively high value (around 0.6). If p increases further, then the pickers tend to stop at virtually every pick-face to make picks, resulting in virtually no interaction between them.



(a) $n = 20$ (b) $n = 100$

Figure 4.6: Comparison of Percentage of Time Pickers are Blocked When Pickers are Allowed and Not Allowed to Pass for the 1:1 Ratio in a Circular Aisle with $n = 20$ and $n = 100$.

4.4.2 Analytical Model for a Pick:Walk Time Ratio of ∞:1

We next consider the situation where pickers require 0 time units to walk past a pick-face (alternatively, walking speed is ∞), while their pick-time equals one time unit. This situation corresponds to a pick:walk time ratio of ∞:1.

Similar to the 1:1 case, we can use the sequence r_{xy} to represent a Markov chain. Since both pickers travel at an infinite speed, the time between the two pickers stopping is equal to zero. Thus, both pickers, regardless of pick locations, start and end picking at one of the n pick-faces at the same time each epoch, except in the case where both attempt to pick at the same location.) As a result, x and y (in r_{xy}) would always be "p" for picked or "p" for attempted to pick when $r = 0$. Therefore, the subscripts x and y can be dropped. Hence, to model this system using a Markov chain, we use states that are represented by r (instead of r_{xy}).

We use a Markov chain approach to model this system. Accordingly, denote the desired number of locations moved by Picker 1 and Picker 2 in epoch t by X_t^1 and X_t^2. For a circular aisle, both of these random variables follow a distribution given by

$$f(x) = q^{x-1}p \qquad\qquad \text{for } x = 1, 2, ... \text{ and } x \neq n, 2n, 3n, ...$$

Note that if x were to equal $0, n, 2n, ...$, then that would mean that the picker picks multiple items at the same pick-face during the next epoch; a case not considered here and treated in later sections. However, also note that if x were to equal $n + 1, n + 2, ...$, then that would mean that the picker visits all n pick-faces at least once before making a pick. Such a situation does not make sense in a real system, however it may arise mathematically for very low values of p and n. However, for medium-to-high throughput OPSs, the values of p and n would lead to a very low probability of pickers moving beyond $x > n - 1$.

In the Markov model, let $Y_t = X_t^1 - X_t^2$ be the change in distance between the two pickers. We notice that when Pickers 1 and 2 are r distance units apart (i.e., the Markov chain is in State r), to affect a change in distance of $Y_t = y$ units there can be two scenarios; i.e.,

Scenario 1, the pickers did not pass each other in the aisle; or

Scenario 2, the pickers passed each other in the aisle.

To illustrate the two scenarios, consider an example of a circular aisle with n pick-faces. Let Picker 1 and Picker 2 be one distance unit apart (i.e., $r = 1$) at the end of an epoch.

This is illustrated in Figure 4.7 with direction of travel and measurement of distance being clockwise.

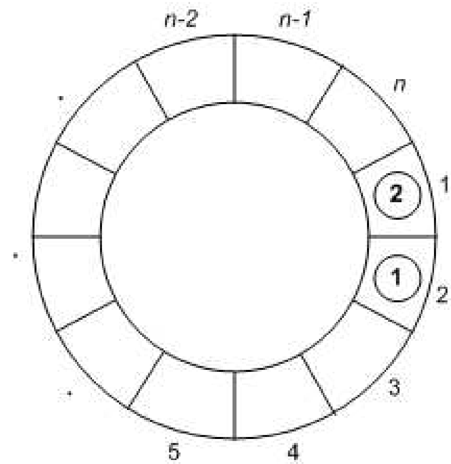


Figure 4.7: A Circular Aisle with n Pick-Faces and Two Pickers.

At the end of the next epoch, if Pickers 1 and 2 are three distance units apart (i.e., $r = 3$), then the change in distance y equals 2. For this situation to occur, examples of the two scenarios could be

> *Scenario 1*, Picker 1 moves three distance units (e.g.., $X_t^1 = 3$) and Picker 2 moves one distance unit (e.g., $X_t^2 = 1$); i.e., Picker 2 *did not pass* Picker 1; and
>
> *Scenario 2*, Picker 1 moves one distance unit (e.g., $X_t^1 = 1$) and Picker 2 moves $n - 1$ distance units (e.g., $X_t^2 = n - 1$); i.e., Picker 2 *passed* Picker 1.

This means, Y_t has two distributions, one for each scenario. We derive these two distributions and present them in Appendix B. These two distributions are used to develop the transition probability matrix in the Markov model. Let a_{ij} represent the transition probability from State i to State j. Using the distributions given by (4.7) and (4.8) and arranging the states in the order $0, 1, 2, ..., n-1$, the state transition probability matrix is given by

$$A_U = \begin{bmatrix} g_0(0) & g_0(1) & g_0(2) & \cdots & g_0(n-3) & g_0(n-2) & g_0(n-1) \\ g_1(-1) & g_1(0) & g_1(1) & \cdots & g_1(n-4) & g_1(n-3) & g_1(n-2) \\ g_2(-2) & g_2(-1) & \ddots & \ddots & \ddots & g_2(n-4) & g_2(n-3) \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ g_{n-3}(-n+3) & g_{n-3}(-n+4) & \ddots & \ddots & \ddots & g_{n-3}(1) & g_{n-3}(2) \\ g_{n-2}(-n+2) & g_{n-2}(-n+3) & g_{n-2}(-n+4) & \cdots & g_{n-2}(-1) & g_{n-2}(0) & g_{n-2}(1) \\ g_{n-1}(-n+1) & g_{n-1}(-n+2) & g_{n-1}(-n+3) & \cdots & g_{n-1}(-2) & g_{n-1}(-1) & g_{n-1}(0) \end{bmatrix}.$$

For example, $g_0(0) = 0$, $g_0(1) = g_0(n-1) = \dfrac{p + pq^{n-2}}{2 \cdot \sum_{y \in \Gamma} h_0(y)}$, and $g_1(-1) = \dfrac{pq + pq^{n-1}}{(1+q) \cdot \sum_{y \in \Gamma} h_i(y)}$.

Due to difficulties in obtaining a closed-form expression for the blocking probability for each picker, we solve the transition matrix A_U for given values of n and p using a commercially available software (i.e., MATLAB (2004)). We accomplish this by obtaining the limiting probabilities of being in State i $(0 \leq i \leq n-1)$. Notice that State 0 refers to the blocking state. However, the limiting value of the blocking probability actually refers to the sum of blocking experienced by the two pickers in the system. We divide this probability value by 2 to obtain the probability of each picker being blocked.

Table 4.2 compares the analytical and simulation results for the percentage of time each picker is blocked for various values of p in a circular aisle with $n = 20$. Similar to the results for a ratio of 1:1, for a very low value of p (i.e., $p = 0.05$), the %-difference observed in the analytical and simulation estimates of blocking is relatively high. For all other values of p, this difference is within 1%. Hence, we conclude that the analytical model developed above performs well in estimating the pick-face blocking for a pick:walk time ratio of ∞:1.

Table 4.2: Comparison of Analytical and Simulation Results of Percentage of Time Each Picker is Blocked in a Circular Aisle with $n = 20$, Pickers May Pick only One Item, and Pick:Walk Time Ratio of ∞:1.

Probability p	**Analytical**	**Simulation**	**Std. Error**	**% Difference**
0	0.000	0.000	0.0000	0.00
0.05	2.391	2.440	0.0032	-2.01
0.1	2.358	2.372	0.0038	-0.58
0.2	2.229	2.237	0.0027	-0.37
0.3	2.078	2.074	0.0037	0.16
0.4	1.898	1.902	0.0031	-0.19
0.5	1.695	1.697	0.0037	-0.15
0.6	1.461	1.457	0.0034	0.31
0.7	1.188	1.187	0.0033	0.11
0.8	0.863	0.865	0.0033	-0.20
0.9	0.474	0.470	0.0033	0.82
1.0	0.000	0.000	0.0000	0.00

4.4.3 Simulation Results for Various Pick:Walk Time Ratios and Number of Pickers

Having obtained analytical expressions that estimate pick-face blocking for two extreme pick:walk time ratios, we now present simulation results of pick-face blocking for realistic pick:walk time ratios; namely, 5:1, 10:1, and 20:1. Figure 4.8 illustrates the blocking probabilities obtained for various pick:walk time ratios for a circular aisle with $n = 20$. The smooth curves represent the estimates obtained through analytical models, while all the remaining curves are generated through simulation models.

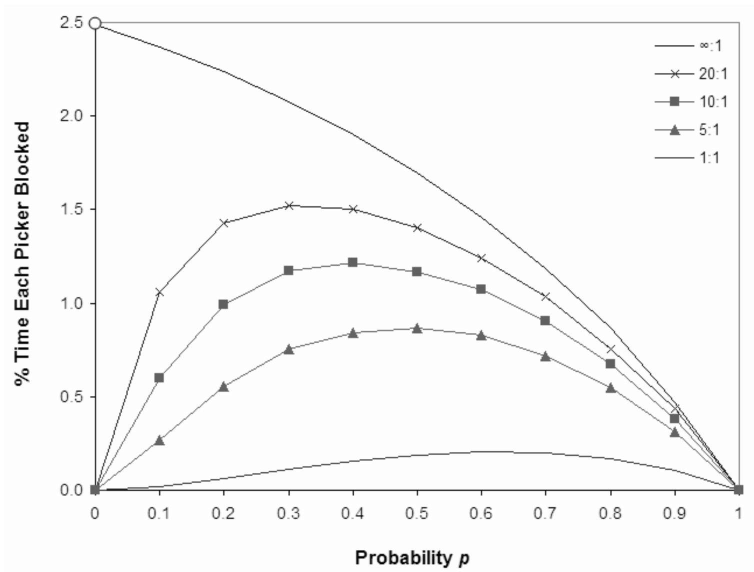
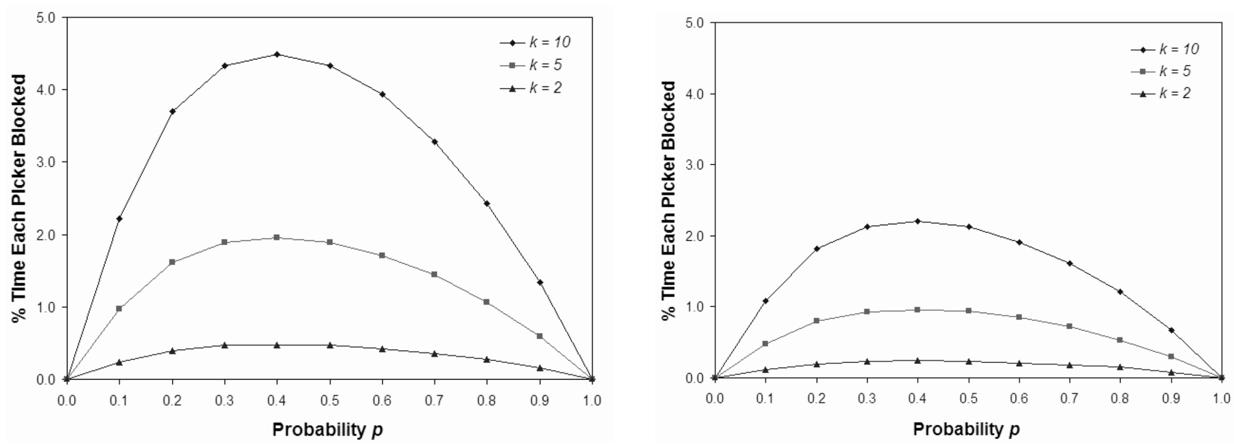


Figure 4.8: Blocking Experienced by Each Picker in a Circular Aisle with Two Pickers, $n = 20$, Pickers May Pick Only One Item, and Standard Pick:Walk Time Ratios.

From Figure 4.8 we observe that the blocking values for the ∞:1 ratio are higher as compared to the blocking values for the 1:1 ratio. This is explained by the observation that in a system with ∞:1 ratio the pickers spend all of their time in picking and no time in walking. Hence, unlike the 1:1 case where the pickers spend equal time in picking and walking, there is a lot more interaction between the pickers in the ∞:1 case; thus leading to increased blocking. For $p = 0$, the amount of blocking is zero for any pick:walk time ratio; this is indicated by a hole in the Figure 4.8 at $p = 0$ for the ∞:1 ratio.

It is not easy to extend our analytical results for 1:1 and ∞:1 ratios for picking areas with more than two pickers. For such cases, we rely on our simulation models to estimate pick-face blocking for systems with more than two pickers. Figure 4.9 illustrates the estimates of blocking in a circular aisle with $k \geq 2$ pickers, a pick:walk ratio of 10:1, and two different picking areas. It is clear that with an increase in the number of pickers, pick-face blocking increases. On the other hand, for the same the number of pickers, blocking decreases as the size of the picking area increases.



(a) $n = 50$ (b) $n = 100$

Figure 4.9: Blocking Experienced by Each of the k Pickers in a Circular Aisle with $n = 50$ and $n = 100$, Pickers May Pick Only One Item, and Pick:Walk Time Ratio of 10:1.

4.5. Blocking Models when Pickers May Pick Multiple Items at a Pick-Face

Now consider the case when the pickers may pick multiple items of a different SKU located at some other storage location at the same pick-face. This case is different from the previous case where the pickers may pick only one item; i.e., this case involves high variance in the time to pick items at a pick-face. We assume that at a pick-face there can be at most m storage locations, each containing a distinct SKU (and no two storage locations at a pick-face have identical SKUs). If the picker picks multiple items at a pick-face, then the SKUs picked may be identical (if picked from the same storage location at that pick-face) or distinct (if picked from separate storage locations). As mentioned before, the time required to pick identical SKUs at a storage location can be handled through the average pick time (t_p).

We assume here that the pickers only pick distinct SKUs. (That is, this case deals with the modeling dynamics involved with picking distinct SKUs from the storage locations at a pick-face.) Thus, if there are I distinct SKUs to be picked in a pick-tour and a picker picks on average a ($a \geq 1$) items in a SKU at a pick-face, then the total items the picker picks (say, I') equals $a \cdot I$. Note that for this case the pick-time equals $a \cdot t_p$. Let u represent the probability of picking one or more items at a pick-face (and $v = 1 - u$).

4.5.1 Analytical Model for a Pick:Walk Time Ratio of 1:1

For the case when the pickers may pick multiple at a pick-face, starting in State 0_{pp} (i.e., blocked state), this state can be achieved again in the next epoch with probability u. This means that if Picker 1 picks at a pick-face in the current epoch while Picker 2 waits, then Picker 1 may pick multiple items in the next epoch with probability u forcing Picker 2 to wait in that epoch too. However, in the next epoch if Picker 1 decides to walk, then Picker 2 would pick at that pick-face with probability 1 (as he wanted to pick at that pick-face in the previous epoch). A similar situation may arise when Picker 2 picks at a pick-face and Picker 1 waits. Hence, state 0_{pp} can transit to either state 1_{wp} or state $(n-1)_{pw}$ with probability $v/2$. With all the other arguments presented in the case when pickers pick only one item at a pick-face holding true for this case also, the transition-probability diagram for this case is illustrated in Figure 4.10. The resulting transition matrix is

$$A_L^R = \begin{bmatrix} D_0 & U_0 & 0 & \cdots & Y \\ L & D & U & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & L & D & U \\ X & \cdots & 0 & L & D_{n-1} \end{bmatrix},$$

where

$$D_0 = \begin{bmatrix} u & 0 & 0 & 0 \\ u^2 & 0 & 0 & v^2 \\ u^2 & 0 & 0 & v^2 \\ u^2 & 0 & 0 & v^2 \end{bmatrix}, \quad U_0 = \begin{bmatrix} 0 & 0 & v/2 & 0 \\ 0 & 0 & uv & 0 \\ 0 & 0 & uv & 0 \\ 0 & 0 & uv & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & v/2 & 0 & 0 \\ 0 & uv & 0 & 0 \\ 0 & uv & 0 & 0 \\ 0 & uv & 0 & 0 \end{bmatrix};$$

$$L = \begin{bmatrix} 0 & uv & 0 & 0 \\ 0 & uv & 0 & 0 \\ 0 & uv & 0 & 0 \\ 0 & uv & 0 & 0 \end{bmatrix}, \quad D = D_{n-1} = \begin{bmatrix} u^2 & 0 & 0 & v^2 \\ u^2 & 0 & 0 & v^2 \\ u^2 & 0 & 0 & v^2 \\ u^2 & 0 & 0 & v^2 \end{bmatrix}, \quad \text{and } U = X = \begin{bmatrix} 0 & 0 & uv & 0 \\ 0 & 0 & uv & 0 \\ 0 & 0 & uv & 0 \\ 0 & 0 & uv & 0 \end{bmatrix}.$$

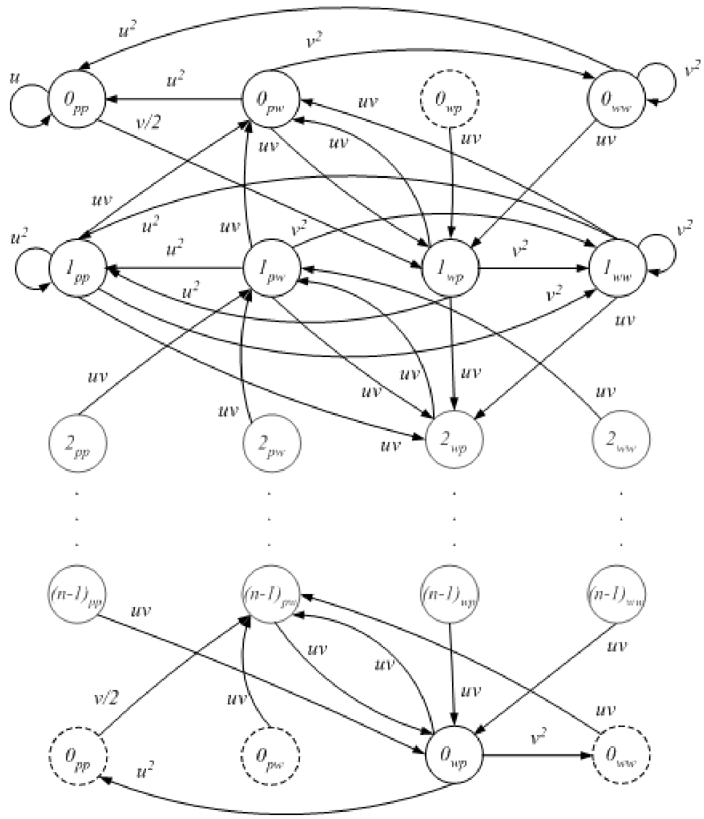


Figure 4.10: Transition-Probability Diagram for the 1:1 Ratio in a Circular Aisle when Pickers May Pick Multiple Items.

We use a procedure similar to that presented in Appendix A to solve A_L^R. For $\mathbf{z}A_L^R = \mathbf{z}$, we can verify that

$$\mathbf{z} = \Big[\overbrace{\frac{2u^2}{(1-u)^2(2-u)}, \frac{u}{1-u}, \frac{u}{1-u}, \frac{2(1-u)}{2-u}}^{x=0}, \overbrace{\frac{u^2}{(1-u)^2}, \frac{u}{1-u}, \frac{u}{1-u}, 1}^{x=1},$$
$$\overbrace{\frac{u^2}{(1-u)^2}, \frac{u}{1-u}, \frac{u}{1-u}, 1}^{x=2}, \ldots, \overbrace{\frac{u^2}{(1-u)^2}, \frac{u}{1-u}, \frac{u}{1-u}, 1}^{x=n-2}, \overbrace{\frac{u^2}{(1-u)^2}, \frac{u}{1-u}, \frac{u}{1-u}, 1}^{x=n-1}\Big]$$

The stationary density for the Markov process is determined by scaling by $\|\mathbf{z}\|$. Since the blocked state is the State 0_{pp}, we have

$$b_1^r(2) = \frac{z_1/2}{\sum_j z_j} = \frac{2u^2/(2(1-u)^2(2-u))}{(n(2-u)-u+2u^2)/((1-u)^2(2-u))} = \frac{u^2}{n(2-u)-u+2u^2}, \qquad (4.4)$$

where $b_1^r(2)$ is the average amount of time that each of the two pickers is blocked for 1:1 ratio. We use a non-linear solver in Microsoft Excel 2003 to obtain the relationship between $b_1^r(2)$ and u for various values of n. The results are illustrated in Figure 4.11. For $u = 1.0$, the amount of blocking is zero for any value of n; this is indicated by the holes in the Figure 4.11 at $u = 1.0$.

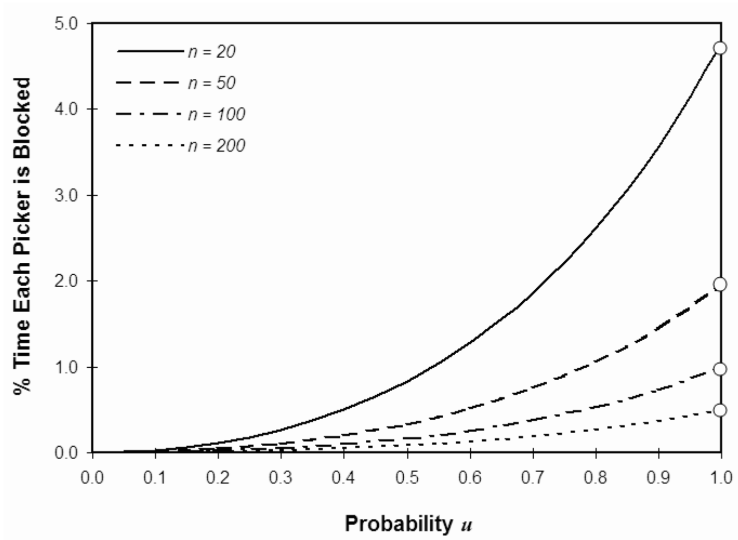


Figure 4.11: Percentage of Time Each Picker is Blocked for Various Values of n in a Circular Aisle, Pickers May Pick Multiple Items, and Pick:Walk Time Ratio of 1:1.

Table 4.3 compares the blocking probabilities obtained through analytical and simulation models. The %-difference values across all values of u are within 1%; hence, we deem our analytical model satisfactory in estimating the pick-face blocking for a pick:walk ratio of 1:1 for such OPSs.

4.5.2 Analytical Model for a Pick:Walk Time Ratio of ∞:1

We next consider the instance when the pick:walk time ratio is ∞:1. Using the assumptions presented for the ∞:1 ratio in the case when pickers may pick only one item, we develop a Markov chain model to obtain the blocking probabilities.

Let the desired number of locations moved by Picker 1 and Picker 2 in epoch t be represented by X_t^1 and X_t^2. Let $\beta = \{i|i = 0, n, 2n, ...\}$ and $\gamma = \{i|i > 0, i \notin \beta\}$. For a circular aisle, if a picker picks multiple items at a pick-face, then we have

$$f(x) = v^x u \qquad\qquad \text{for } x \in \beta.$$

Table 4.3: Comparison of Analytical and Simulation Results of Percentage of Time Each Picker is Blocked in a Circular Aisle with $n = 20$, Pickers May Pick Multiple Items, and Pick:Walk Time Ratio of 1:1.

Probability u	**Analytical**	**Simulation**	**Std. Error**	**% Difference**
0	0.000	0.000	0.0000	0.00
0.05	0.006	0.006	0.0002	-0.34
0.1	0.027	0.026	0.0004	0.13
0.2	0.111	0.112	0.0001	-0.42
0.3	0.266	0.265	0.0018	0.13
0.4	0.501	0.502	0.0023	-0.08
0.5	0.833	0.834	0.0033	-0.04
0.6	1.280	1.278	0.0070	0.16
0.7	1.865	1.866	0.0091	-0.09
0.8	2.614	2.613	0.0137	0.07
0.9	3.565	3.572	0.0231	-0.19
1.0	0.000	0.000	0.0000	0.00

However, if a picker picks only one item at a pick-face (with a probability $v = 1 - u$), then we have

$$f(x) = (1-u)(v^{x-1}u) = v^x u \qquad \text{for } x \in \gamma.$$

Combining the above two cases, we get the distribution of X_t for each picker as

$$f(x) = v^x u I_{\{x\in\beta\}} + v^x u I_{\{x\in\gamma\}} \qquad \text{for } x \geq 0. \tag{4.5}$$

In this expression, $I_{\{x\in\beta\}}$ is an indicator function that is equal to 1 when $x \in \beta$, and 0 when $x \in \gamma$. Similarly, $I_{\{x\in\gamma\}}$ is an indicator function that equals 1 when $x \in \gamma$, and 0 when $x \in \beta$.

Similar to the ∞:1 ratio for case when pickers may pick only one item, we consider two scenarios; *Scenario 1*, the pickers did not pass each other in the aisle, and *Scenario 2*, the pickers passed each other in the aisle. The two corresponding distributions are derived and presented in Appendix C. These two distributions are used to develop the transition probability matrix in the Markov model. Let a_{ij} represent the transition probability from State i to State j. Using the distributions given by (4.11) and (4.18) and arranging the states in the order $0, 1, 2, ..., n-1$, the state transition probability matrix is given by

$$A_U^R = \begin{bmatrix} k_0(0) & k_0(1) & k_0(2) & \cdots & k_0(n-3) & k_0(n-2) & k_0(n-1) \\ k_1(-1) & k_1(0) & k_1(1) & \cdots & k_1(n-4) & k_1(n-3) & k_1(n-2) \\ k_2(-2) & k_2(-1) & \ddots & \ddots & \ddots & k_2(n-4) & k_2(n-3) \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ k_{n-3}(-n+3) & k_{n-3}(-n+4) & \ddots & \ddots & \ddots & k_{n-3}(1) & k_{n-3}(2) \\ k_{n-2}(-n+2) & k_{n-2}(-n+3) & k_{n-2}(-n+4) & \cdots & k_{n-2}(-1) & k_{n-2}(0) & k_{n-2}(1) \\ k_{n-1}(-n+1) & k_{n-1}(-n+2) & k_{n-1}(-n+3) & \cdots & k_{n-1}(-2) & k_{n-1}(-1) & k_{n-1}(0) \end{bmatrix}.$$

For example, $k_0(0) = \dfrac{u}{\sum_{y\in\Gamma} l_0(y)}$, $k_0(1) = \dfrac{uv + uv^{n-1}}{\sum_{y\in\Gamma} l_0(y)}$, and $k_1(-1) = \dfrac{uv + uv^{n-1}}{(1+v)\cdot\sum_{y\in\Gamma} l_i(y)}$.

As before, we use MATLAB (2004) to obtain the total blocking probability and divide this value by 2 to obtain the probability of each picker being blocked. Table 4.4 presents a comparison of analytical and simulation results for the percentage of time each picker is blocked for various values of u in a circular aisle with 20 pick-faces. We observe that for cases where $u \leq 0.1$, the blocking estimates obtained through analytical and simulation models are not within 1% (but they are within 2%) error. This error can be attributed to the approximation of $1 - v^n \approx 1$ we used in deriving $l_i^1(y)$ and $l_i^2(y)$ (considering $n \geq 20$ and $0 \leq v \leq 0.85$). The %-difference values for $u > 0.1$ are less than 1%, which indicates that this analytical model gives very good estimates of pick-face blocking for a pick:walk time ratio of ∞:1.

4.5.3 Simulation Results for Various Pick:Walk Time Ratios and Number of Pickers

Similar to the case when pickers may pick only one item at a pick-face, we now present simulation results to estimate pick-face blocking for pick:walk time ratios of 5:1, 10:1, and 20:1. The percentage of time each picker is blocked is illustrated in Figure 4.12. The smooth curves represent estimates obtained through analytical models, while all the remaining curves are generated through simulation models. As before, the amount of blocking is zero for $u = 1.0$ for all pick:walk time ratios. From Figure 4.12, we notice that

Theorem 1 *In a wide-aisle OPS, when pickers may pick multiple items at a pick-face and* t_p:t_w=∞:1, *all blocking curves converge to a single blocking value as* $u \rightarrow 1$.

Table 4.4: Comparison of Analytical and Simulation Results of Percentage of Time Each Picker is Blocked in a Circular Aisle with $n = 20$, Pickers May Pick Multiple Items, and Pick:Walk Time Ratio of ∞:1.

Probability u	**Analytical**	**Simulation**	**Std. Error**	**% Difference**
0	0.000	0.000	0.0000	0.00
0.05	2.587	2.556	0.0036	1.20
0.1	2.652	2.624	0.0036	1.09
0.2	2.754	2.760	0.0040	-0.22
0.3	2.928	2.915	0.0049	0.45
0.4	3.086	3.087	0.0053	-0.04
0.5	3.275	3.279	0.0072	-0.14
0.6	3.504	3.496	0.0083	0.25
0.7	3.760	3.741	0.0126	0.51
0.8	4.040	4.025	0.0166	0.37
0.9	4.367	4.365	0.0260	0.05
1.0	0.000	0.000	0.0000	0.00

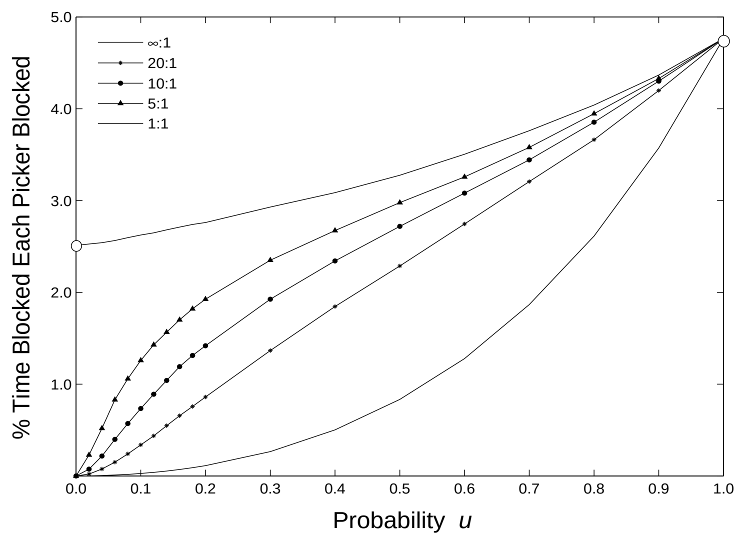
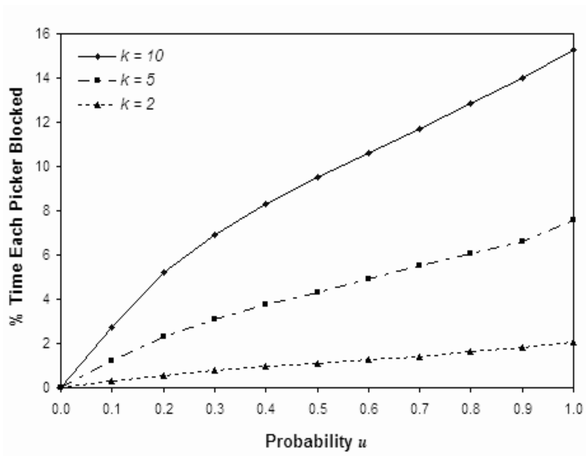


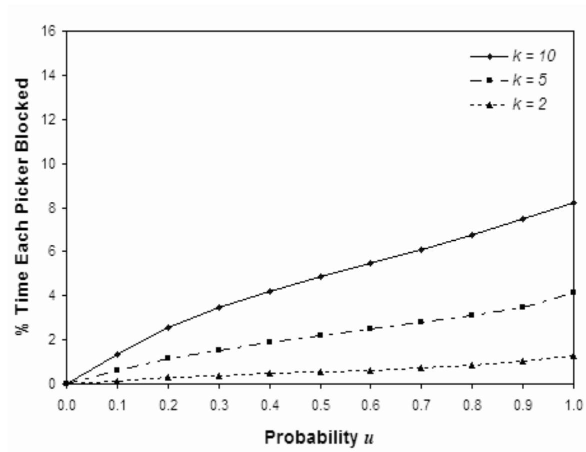
Figure 4.12: Blocking Experienced by Each Picker in a Circular Aisle with Two Pickers, $n = 20$, Pickers May Pick Multiple Items, and Standard Pick:Walk Time Ratios.

Proof: Consider each pick:walk time ratio as a system by itself. Consider first a 1:1 system. With very high values of u, this suggest that the pickers spend almost all of their time picking at the pick-faces and almost no time in walking. Therefore, the ratio of the total time to pick to the total time to walk approaches infinity. Now consider systems with a higher pick:walk time ratio. For such systems, the ratio of the total time to pick to the total time to walk approaches infinity even faster. Hence, the systems represented by different pick:walk time ratios become equivalent as $u \to 1$. Consequently, the blocking value as $u \to 1$ in each of these systems is identical, and can be obtained via the 1:1 ratio results presented in (4.4). This value is equal to $\dfrac{1}{n+1}$. $\square$

As before, it is not easy to extend our analytical results for 1:1 and ∞:1 ratios for more than two pickers. Instead we rely on simulation models to estimate pick-face blocking for systems with $k > 2$. Figure 4.13 illustrates the estimates of blocking in a circular aisle with $k \geq 2$ pickers, a pick:walk ratio of 10:1, and two different picking areas. As observed, pick-face blocking increases with an increase in the number of pickers; while it decreases with an increase in the size of the picking area (for the same number of pickers).



(a) $n = 50$



(b) $n = 100$

Figure 4.13: Blocking Experienced by Each of the k Pickers in a Circular Aisle with $n = 50$ and $n = 100$, Pickers May Pick Multiple Items, and Pick:Walk Time Ratio of 10:1.

4.6. Conclusions

In this chapter we identified the problem of selecting between a batch or a zone picking strategy — a problem we define as the batch versus zone problem — for designing an OPS in a DC. Blocking plays a crucial role in deciding which strategy to employ for a specific application. To this end, we developed analytical models to estimate pick-face blocking in a wide-aisle OPS. We considered a circular aisle with two pickers and two cases: (i) when pickers may pick only one item at a pick-face leading to no variance in the time to pick items at a pick-face, and (ii) when pickers may pick multiple items at a pick-face leading to high variance in the time to pick items at a pick-face. Based on the results from both the analytical and simulation models, we conclude that:

1. For the case when pickers may pick only one item at a pick-face, blocking is less of a problem in wide-aisle OPSs compared to that in narrow-aisle OPSs (as illustrated in Figure 4.6).

2. Comparing the cases when pickers may either pick only one item or pick multiple items at a pick-face in wide-aisle OPSs, blocking is more pronounced in the latter case (as illustrated by comparing Figures 4.8 and 4.12).

3. Blocking increases with an increase in the number of pickers and decreases with an increase in the size of the picking area for same number of pickers (as illustrated in Figures 4.9 and 4.13).

4. Similar to blocking estimates in the narrow-aisle OPS, the amount of blocking changes with pick-density for the case when pickers may pick only one item at a pick-face (as illustrated in Figure 4.8). As a result, we observe a possible progression of picking strategies with an increase in the pick-density: batch picking for very low pick-densities, zone picking for intermediate pick-densities, and batch picking for relatively high pick-densities.

5. In contrast to Conclusion 4 (i.e., when pickers may pick only one item), in a system where pickers may pick multiple items, blocking increases monotonically with an increase in pick-density (as illustrated in Figure 4.12). By identifying a tolerable blocking value for a given system (i.e., defined by the number of pickers, a pick:walk time ratio, the number of pick-faces, etc.), one can obtain the associated pick-density (say, p'). A possible progression of picking strategies for this case will be to employ batch picking until p' and employ zone picking for pick-density values higher than p'.

In practice, as pickers tend to make multiple picks at a pick-face, we believe that Conclusion 5 is stronger than Conclusion 4. This is true because if pickers may pick multiple items at a pick-face, then the expected number of picks at a pick-face increases. This means that the mean time spent at a pick-face increases, and the variance also increases (as pickers may now pick either 1, 2, ..., or g items at a pick-face). If we were to use the blocking models for Case I (i.e., when pickers may pick only one item) to account for multiple picks at a pick-face, then we will have to inflate the value of t_p to reflect the increase in the mean time to pick at a pick-face. In so doing, the expected number of items picked in Case I and Case II (i.e., when pickers may pick multiple items) will be equal. However, as Case I considers deterministic values of t_p, it cannot account for any variance in the time to pick items at a pick-face. Ignoring this variance may under-estimate pick-face blocking. For example, consider an OPS with 50 pick-faces, 10 pickers, pick to walk time ratio of 10:1, and each picker required to pick 40 items. As estimated by the simulation models for Cases I and II, the average blocking experienced by the pickers is 2.76% and 8.92%, respectively. Such under-estimation may affect design decisions in which these blocking estimates are used: e.g., storage area configuration, picking strategy selection, etc.

We believe that the results presented in this chapter along with those presented in Gue et al. (2006) will enhance our understanding of the selection of an appropriate picking strategy for an OPS in a DC. As future research in the area of OPS design, we intend to employ these blocking estimates in the analytical models we are developing to (i) configure the storage system for order picking and (ii) to select an appropriate picking strategy.

Appendix A

This appendix briefly describes the procedure to obtain a closed-form expression for $b_1(2)$ for the case when pickers may pick only one item at a pick-face and the pick:walk ratio is 1:1.

It is clear from the transition-probability diagram, presented in Section 4.1.1, that there exists a symmetry in the some of the state transitions; i.e., states starting with $2, 3, 4, ..., n-2$ are symmetric, while states starting with $0, 1$, and $n-1$ are not symmetric. As a result, we only need to solve for states $0, 1, 2$, and $n-1$ such that it satisfies $\mathbf{z}A_L$=$\mathbf{z}$. Table 4.5 summarizes the reduced transition-probability matrix.

From Table 4.5, we have the following set of 16 simultaneous equations:

$$
\begin{aligned}
z_1 &= p^2 \cdot z_4 \\
z_2 &= p \cdot z_7 + pq \cdot z_8 \\
z_3 &= p \cdot z_{14} + pq \cdot z_{16} \\
z_4 &= q \cdot z_2 + q \cdot z_3 + q^2 \cdot z_4 \\
z_5 &= p^2 \cdot z_8 \\
z_6 &= p \cdot z_{11} + pq \cdot z_{12} \\
z_7 &= \frac{z_1}{2} + p \cdot z_2 + pq \cdot z_4 \\
z_8 &= z_5 + q \cdot z_6 + q \cdot z_7 + q^2 \cdot z_8 \\
z_9 &= p^2 \cdot z_{12} \\
z_{10} &= p \cdot z_{15} + pq \cdot z_{16} \\
z_{11} &= p \cdot z_6 + pq \cdot z_8 \\
z_{12} &= z_9 + q \cdot z_{10} + q \cdot z_{11} + q^2 \cdot z_{12} \\
z_{13} &= p^2 \cdot z_{16} \\
z_{14} &= \frac{z_1}{2} + p \cdot z_3 + pq \cdot z_4 \\
z_{15} &= p \cdot z_{10} + pq \cdot z_{12} \\
z_{16} &= z_{13} + q \cdot z_{14} + q \cdot z_{15} + q^2 \cdot z_{16}.
\end{aligned}
$$

As one of these equations is redundant, we have 16 unknowns and 15 equations. To solve for the unknowns, we assume $z_{16} = 1$. Solving for the remaining 15 unknowns yields

Table 4.5: Reduced Transition-Probability Matrix for the Case When Pickers May Pick Only One Item and Pick:Walk Time Ratio is 1:1.

		0_{pp}	0_{pw}	0_{wp}	0_{ww}	1_{pp}	1_{pw}	1_{wp}	1_{ww}	2_{pp}	2_{pw}	2_{wp}	2_{ww}	$(n-1)_{pp}$	$(n-1)_{pw}$	$(n-1)_{wp}$	$(n-1)_{ww}$
		z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}	z_{16}
0_{pp}	z_1							0.5							0.5		
0_{pw}	z_2				q			p									
0_{wp}	z_3				q										p		
0_{ww}	z_4	p^2			q^2			pq							pq		
1_{pp}	z_5								1								
1_{pw}	z_6								q			p					
1_{wp}	z_7		p						q								
1_{ww}	z_8		pq			p^2			q^2			pq					
2_{pp}	z_9												1				
2_{pw}	z_{10}												q			p	
2_{wp}	z_{11}						p						q				
2_{ww}	z_{12}						pq			p^2			q^2			pq	
$(n-1)_{pp}$	z_{13}																1
$(n-1)_{pw}$	z_{14}			p													q
$(n-1)_{wp}$	z_{15}										p						q
$(n-1)_{ww}$	z_{16}			pq							pq			p^2			q^2

$$\mathbf{z} = \left[\overbrace{\frac{2p^2(1-p)}{2-p}, p, p, \frac{2(1-p)}{2-p}}^{x=0}, \overbrace{p^2, p, p, 1}^{x=1}, \overbrace{p^2, p, p, 1}^{x=2}, \ldots, \overbrace{p^2, p, p, 1}^{x=n-2}, \overbrace{p^2, p, p, 1}^{x=n-1} \right]$$

Note that x refers to the states starting with $0, 1, 2, ..., n-1$; e.g., $x = 0$ refers to the states 0_{pp}, 0_{pw}, 0_{wp}, and 0_{ww}.

Appendix B

This appendix presents the derivation of the distributions corresponding to the two scenarios used to develop the transition probability matrix in the Markov model for the case when pickers may pick only one item and pick:walk time ratio is ∞:1.

Scenario 1: When no passing is involved, we use the distribution derived in Gue et al. (2006). This is given by

$$h_i^1(y) = \frac{pq^{|y|}}{1+q} \qquad \text{for } -\infty \leq y \leq \infty,$$

where $h_i^1(y)$ represents the distribution to estimate the probability of change in distance of y (for Scenario 1) starting in State i, where $1 \leq i \leq n-1$. For State $i = 0$, this distribution is derived later.

Scenario 2: When pickers pass, we have $Y_t = X_t^1 - X_t^2 + n$. (To understand this, consider the case when Picker 1 is ahead of Picker 2. According to this scenario, if Picker 2 passes Picker 1 in the next epoch, then we have $X_t^2 > X_t^1$. As a result, $Y_t \leq 0$. But when Picker 2 passes Picker 1, the actual change in distance is $Y_t = X_t^1 - X_t^2 + n$, as measured in the clockwise direction. For the example presented Section 4.2, $Y_t = X_t^1 - X_t^2 + n = 1 - (n-1) + n = 2$.) Moreover, we have $1 \leq X_t^1 \leq \infty$ and $1 \leq X_t^2 \leq \infty$.

Let $h_i^2(y)$ represent the distribution to estimate the probability of change in distance of y (for Scenario 2) starting in State i, where $1 \leq i \leq n-1$. Since X_t^1 and X_t^2 are independent and identically distributed,

$$\begin{aligned} h_i^2(y) &= P(Y_t = y) \\ &= \sum_{x=1}^{\infty} P(X_t^1 = x) P(X_t^2 = x + n - y) \\ &= \sum_{x=1}^{\infty} f(x) f(x + n - y). \end{aligned}$$

In the case of a circular aisle, Y_t is symmetric. So we consider only non-negative values for

y to obtain an expression for $h_i^2(y)$. That is,

$$\begin{aligned} h_i^2(y) &= \sum_{x=1}^{\infty} f(x)f(x+n-y) \\ &= \sum_{x=1}^{\infty} q^{x-1}p \cdot q^{x+n-y-1}p \\ &= p^2q^{n-y}\sum_{x=1}^{n-1}\left(q^2\right)^{x-1} = \frac{p^2q^{n-y}}{1-q^2} \\ &= \frac{pq^{n-y}}{1+q} \qquad \text{for } 1 \leq y \leq \infty. \end{aligned}$$

For y equal to 0, the above term does not have to be evaluated. This is true since for Scenario 2 the only way y equals 0 and pickers pass each other in the aisle is when Picker 2 passes Picker 1 and visits all the remaining pick-faces at least once (i.e., makes at least one round of the aisle) before making a pick. Though such a situation is possible (mathematically, but not practically) for very low values of p (e.g., $p \leq 0.05$), it is very unlikely for relatively high values of p. Note that such low values of p may result due to high values of n. Therefore, for this scenario we have

$$h_i^2(y) = \frac{pq^{n-|y|}}{1+q} \qquad \text{for } -\infty \leq y \leq \infty, y \neq 0.$$

Adding $h_i^1(y)$ and $h_i^2(y)$, we get

$$h_i(y) = \begin{cases} \dfrac{pq^{|y|}}{1+q} & \text{for } y = 0 \\ \dfrac{pq^{|y|} + pq^{n-|y|}}{1+q} & \text{for } -\infty \leq y \leq \infty, y \neq 0. \end{cases} \tag{4.6}$$

The above expression indicates that $Y_t = y$ has an infinite state space. However, for a circular aisle with finite number of pick-faces (n in our case), the range of y is limited. If Γ represents the set containing all possible values of y, where $\Gamma = \{i | -n+1 \leq i \leq n-1\}$, then we have $\sum_{y\in\Gamma} h_i(y) \neq 1$. This means that $h_i(y)$ is not a valid probability mass function. To obtain a valid probability mass function (say, $g_i(y)$), we divide $h_i(y)$ by $\sum_{y\in\Gamma} h_i(y)$. That is,

$$g_i(y) = \frac{h_i(y)}{\sum\limits_{y\in\Gamma} h_i(y)} \qquad \text{for } -n+1 \leq y \leq n-1. \tag{4.7}$$

This means that if r represents the distance between the two pickers at the end of the previous epoch, then in the next epoch, the probability of events with blocking or no blocking can be estimated using $g_i(y)$.

Note that $y = -r$ suggests the event with blocking (i.e., transition into State 0). That is, if the change in distance between the pickers at the end of next epoch equals the distance between them at the end of the previous epoch, then this leads to blocking. As the pickers may pick only one item, once in the blocked state (i.e., State 0), the probability of being in that same state in the next epoch is 0. State 0 can only transition to the other non-blocking states, the probabilities for which are obtained next.

Transition Probabilities of Exiting State 0: We notice here that whenever State 0 is reached at the end of an epoch, there are two instances in which State 0 is exited at the end of the next epoch:

Instance 1, Picker 1 picks and Picker 2 waits; or

Instance 2, Picker 2 picks and Picker 1 waits.

Instance 1 suggests that the change in distance y between the pickers at the end of the next epoch is solely due to Picker 1 (as Picker 2 does not move). Similarly, Instance 2 suggests that the change in distance y occurs solely due to Picker 2 (as Picker 1 does not move). However, each of these instances can occur with a probability equal to 0.5. Next we obtain the transition probability distribution for each of these instances.

Instance 1: According to this instance, if both pickers need to pick at a pick-face in a given epoch, then Picker 1 picks first at the blocked pick-face while Picker 2 waits during the epoch. In the next epoch, Picker 1 leaves that pick-face to pick at some other pick-face in the aisle, while Picker 2 picks at the blocked pick-face with probability 1. Given that Picker 1 picks first, let A represent the event that Picker 1 stops at a pick-face y distance units away from the blocked pick-face and B represent the event that Picker 2 is blocked and picks at the same pick-face during the next epoch. Note that both of these events are independent. Let $h_0^a(y)$ be the distribution that provides the probability of exiting State 0 through Instance 1. Then,

$$\begin{aligned} h_0^1(y) = P[A] \cdot P[B] = pq^{y-1} \cdot 1 &= pq^{y-1} \qquad \text{for } 1 \le y \le \infty \\ &= pq^{|y|-1} \qquad \text{for } -\infty \le y \le \infty, y \ne 0. \end{aligned}$$

Instance 2: According to this instance, Picker 2 picks first at the blocked pick-face while Picker 1 waits during an epoch. In the next epoch, Picker 2 leaves that pick-face to pick at some other pick-face in the aisle, while Picker 1 picks at the blocked pick-face with probability 1. Given that Picker 2 picked first, let C represent the event that Picker 1 picks at the blocked pick-face and D represent the event that Picker 2 stops at a pick-face $n-y$ distance units (in clockwise direction) away from the blocked pick-face during the next epoch. Note that when Picker 2 moves a distance equal to $n-y$, the actual change in distance between Pickers 1 and 2 (in clockwise direction) becomes $n-(n-y)=y$. If $h_0^2(y)$ is the distribution that provides the probability of exiting State 0 through Instance 2, then

$$\begin{aligned} h_0^2(y) = P[C]\cdot P[D] = 1\cdot pq^{n-y-1} &= pq^{n-y-1} \qquad \text{for } 1 \le y \le \infty \\ &= pq^{n-|y|-1} \qquad \text{for } -\infty \le y \le \infty, y \neq 0. \end{aligned}$$

Adding $h_0^1(y)$ and $h_0^2(y)$ yields

$$\begin{aligned} h_0(y) &= 0.5(h_0^1(y)) + 0.5(h_0^2(y)) = 0.5(pq^{|y|-1}) + 0.5(pq^{n-|y|-1}) \\ &= \frac{pq^{|y|-1} + pq^{n-|y|-1}}{2} \qquad \text{for } -\infty \le y \le \infty, y \neq 0. \end{aligned}$$

As mentioned earlier, the probability of starting and ending in State 0 (such that $y=0$) is equal to 0; i.e., $h_0(y) = h_0(0) = 0$.

As before, the range of y is restricted by the finite number of pick-faces in the aisle; specifically, $y \in \Gamma$. Therefore, $h_0(y)$ is not a valid probability mass function because $\sum_{y\in\Gamma} h_0(y) \neq 1$. To obtain a valid probability mass function (say, $g_0(y)$), we divide $h_0(y)$ by $\sum_{y\in\Gamma} h_0(y)$. That is,

$$g_0(y) = \frac{h_0(y)}{\sum\limits_{y\in\Gamma} h_0(y)} \qquad \text{for } -n+1 \le y \le n-1. \tag{4.8}$$

Appendix C

This appendix presents the derivation of the distributions corresponding to the two scenarios used to develop the transition probability matrix in the Markov model for the case when pickers may pick multiple items and pick:walk time ratio is ∞:1.

Scenario 1: When no passing is involved, we have $Y_t = X_t^1 - X_t^2$. Let $l_i^1(y)$ represent the distribution to estimate the probability of change in distance of y (for Scenario 1) starting in State i, where $1 \leq i \leq n-1$. (As before, the distribution for State $i = 0$ is derived later). Since X_t^1 and X_t^2 are independent and identically distributed,

$$\begin{aligned} l_i^1(y) &= P(Y_t = y) \\ &= \sum_{x=0}^{\infty} P(X_t^1 = x + y) P(X_t^2 = x) \\ &= \sum_{x=0}^{\infty} f(x+y) f(x). \end{aligned} \tag{4.9}$$

From (4.5) and (4.9), we have

$$\begin{aligned} l_i^1(y) &= \sum_{x=0}^{\infty} \Big(v^{x+y} u I_{\{x \in \beta\}} + v^{x+y} u I_{\{x \in \gamma\}} \Big) \Big(v^x u I_{\{x \in \beta\}} + u v^x I_{\{x \in \gamma\}} \Big) \\ &= \sum_{x=0}^{\infty} \Big(v^{2x+y} u^2 I_{\{x \in \beta\}} + v^{2x+y} u^2 I_{\{x \in \gamma\}} \Big) \\ &\qquad + \sum_{x=0}^{\infty} \Big(v^{x+y} u I_{\{x \in \beta\}} \cdot v^x u I_{\{x \in \gamma\}} + v^x u I_{\{x \in \beta\}} \cdot u v^{x+y} I_{\{x \in \gamma\}} \Big). \end{aligned}$$

Note, the second summation equals 0 because when $x \in \beta$, $I_{\{x \in \gamma\}} = 0$ and when $x \in \gamma$, $I_{\{x \in \beta\}} = 0$. Therefore, we have

$$\begin{aligned} l_i^1(y) &= \sum_{x=0}^{\infty} \Big(v^{2x+y} u^2 I_{\{x \in \beta\}} + v^{2x+y} u^2 I_{\{x \in \gamma\}} \Big) \\ &= v^y u^2 \sum_{x=0}^{\infty} v^{2x} I_{\{x \in \beta\}} + v^{y+2} u^2 \sum_{x=0}^{\infty} (v^2)^{x-1} I_{\{x \in \gamma\}} \\ &= v^y u^2 (1 + v^{2n} + v^{3n} + v^{4n} + \ldots) + v^{y+2} u^2 (1 + v^2 + v^4 + v^6 + \ldots) \\ &= v^y u^2 \sum_{i=0}^{\infty} v^{in} + v^{y+2} u^2 \sum_{i=0, i \neq n}^{\infty} v^{2i} \\ &= v^y u^2 \left(\frac{1}{1 - v^n} \right) + v^{y+2} u^2 \left(\frac{1}{1 - v^2} \right) \qquad \left(\text{where } \sum_{i=0, i \neq n}^{\infty} v^{2i} \approx \left(\frac{1}{1 - v^2} \right) \right) \\ &= v^y u^2 + \frac{v^{y+2} u}{1 + v} \qquad (\text{where } 1 - v^n \approx 1 \text{ for } n \geq 20 \text{ and } 0 \leq v \leq 0.85) \\ &= v^y \left(u^2 + \frac{v^2 u}{1 + v} \right) = v^y \left((1 - v)^2 + \frac{v^2 (1 - v)}{1 + v} \right) \\ &= \frac{u v^y}{1 + v} \qquad \text{for } y \geq 0 \text{ or} \\ &= \frac{u v^{|y|}}{1 + v} \qquad \text{for } -\infty < y < \infty. \end{aligned}$$

Scenario 2: When pickers pass, we have $Y_t = X_t^1 - X_t^2 + n$. Let $l_i^2(y)$ represent the distribution to estimate the probability of change in distance of y (for Scenario 2) starting in State i, where $1 \leq i \leq n-1$. Since X_t^1 and X_t^2 are independent and identically distributed,

$$
\begin{aligned}
l_i^1(y) &= P(Y_t = y) \\
&= \sum_{x=0}^{\infty} P(X_t^1 = x) P(X_t^2 = x + n - y) \\
&= \sum_{x=0}^{\infty} f(x) f(x + n - y) \\
&= \sum_{x=0}^{\infty} \left(v^x u I_{\{x\in\beta\}} + v^x u I_{\{x\in\gamma\}} \right) \left(v^{x+n-y} u I_{\{x\in\beta\}} + v^{x+n-y} u I_{\{x\in\gamma\}} \right) \\
&= \sum_{x=0}^{\infty} \left(v^{2x+n-y} u^2 I_{\{x\in\beta\}} + v^{2x+n-y} u^2 I_{\{x\in\gamma\}} \right) \\
&\qquad + \sum_{x=0}^{\infty} \left(v^x u I_{\{x\in\beta\}} \cdot v^{x+n-y} u I_{\{x\in\gamma\}} + v^{x+n-y} u I_{\{x\in\beta\}} \cdot v^x u I_{\{x\in\gamma\}} \right) \\
&= \sum_{x=0}^{\infty} \left(u^2 v^{n-y} I_{\{x\in\beta\}} + u^2 v^{2x+n-y} I_{\{x\in\gamma\}} \right) \\
&= v^{n-y} u^2 \sum_{x=0}^{\infty} v^{2x} I_{\{x\in\beta\}} + v^{n-y+2} u^2 \sum_{x=0}^{\infty} (v^2)^{x-1} I_{\{x\in\gamma\}} \\
&= v^{n-y} u^2 (1 + v^{2n} + v^{3n} + v^{4n} + \ldots) + v^{y+2} u^2 (1 + v^2 + v^4 + v^6 + \ldots) \\
&= v^{n-y} u^2 \sum_{i=0, i\neq n}^{\infty} v^{in} + v^{y+2} u^2 \sum_{i=0}^{\infty} v^{2i} \\
&= v^{n-y} u^2 \left(\frac{1}{1 - v^n} \right) + v^{y+2} u^2 \left(\frac{1}{1 - v^2} \right) \qquad \left(\text{where } \sum_{i=0, i\neq n}^{\infty} v^{2i} \approx \left(\frac{1}{1 - v^2} \right) \right) \\
&= v^{n-y} u^2 + \frac{v^{y+2} u}{1 + v} \qquad \left(\text{where } 1 - v^n \approx 1 \text{ for } n \geq 20 \text{ and } 0 \leq v \leq 0.85 \right) \\
&= v^{n-y} \left(u^2 + \frac{u v^2}{1 + v} \right) \\
&= \frac{u v^{n-y}}{1 + v} \qquad \text{for } y \geq 1 \text{ or} \\
&= \frac{u v^{n-|y|}}{1 + v} \qquad \text{for } -\infty < y < \infty, y \neq 0.
\end{aligned}
$$

Adding $l_i^1(y)$ and $l_i^2(y)$, we get

$$l_i(y) = \begin{cases} \dfrac{uv^{|y|}}{1+v} & \text{for } y = 0 \\ \dfrac{uv^{|y|} + uv^{n-|y|}}{1+v} & \text{for } -\infty \leq y \leq \infty, y \neq 0. \end{cases} \tag{4.10}$$

It is interesting to note that $l_i(y)$ from above is identical to $h_i(y)$ in (4.6). That is, the probabilities of exiting State i are the same whether or not pickers pick multiple items at a pick-face. As before, $l_i(y)$ for $y \in \Gamma$ is not a valid probability mass function because $\sum_{y\in\Gamma} l_i(y) \neq 1$. To obtain a valid probability mass function (say, $k_i(y)$), we divide $l_i(y)$ by $\sum_{y\in\Gamma} l_i(y)$. That is,

$$k_i(y) = \frac{l_i(y)}{\displaystyle\sum_{y\in\Gamma} l_i(y)} \qquad \text{for } -n+1 \leq y \leq n-1. \tag{4.11}$$

Note that $y = -r$ suggests the event with blocking (i.e., transition into State 0). As pickers may pick multiple items, once in blocked state (i.e., State 0), this state can transit to any other state, including itself.

Transition Probabilities of Exiting State 0: To obtain the probabilities of either staying in the blocked state (i.e., State 0) or exiting it, similar to the case when pickers may pick only one item, we consider two instances; *Instance 1*, Picker 1 picks and Picker 2 waits, and *Instance 2*, Picker 2 picks and Picker 1 waits.

Instance 1: According to this instance, Picker 1 picks first from one of the m storage locations at the blocked pick-face while Picker 2 waits during an epoch. If, in the next epoch, Picker 1 needs to pick multiple items at the same pick-face (but at a different storage location), then Picker 2 is blocked again for one epoch. Therefore, *starting in State 0*, the probability of staying in the blocked state is given by

$$\begin{aligned} P(\text{staying in the blocked state}) &= P(\text{Picker 1 picks again}) \cdot P(\text{Picker 2 wants to pick}) \\ &= u \cdot 1 = u \qquad \text{for } y = 0. \end{aligned} \tag{4.12}$$

If Picker 1 does not need to pick again at the same pick-face, then he leaves that pick-face and walks (with infinite speed) to a different pick-face. Given that Picker 1 picks first, let E represent the event that Picker 1 stops at a pick-face y distance units away from the blocked pick-face and F represent the event that Picker 2 picks at the blocked pick-face during the next epoch. Let $l_0^1(y)$ be the distribution that provides the probability of exiting State 0 through Instance 1. The probability of this event happening gives us the probability of exiting State 0, and is given by

$$\begin{aligned} l_0^1(y) &= P(\text{Picker 1 does not pick again}) \cdot P[E] \cdot P[F] \\ &= (1-u) \cdot uv^{y-1} \cdot 1 \\ &= uv^y \qquad \text{for } 0 \le y \le \infty, y \ne 0 \\ &= uv^{|y|} \qquad \text{for } -\infty \le y \le \infty, y \ne 0. \end{aligned} \tag{4.13}$$

Instance 2: If both Pickers 1 and 2 need to pick at a pick-face, then according to this instance, Picker 2 picks first at that pick-face while Picker 1 waits during an epoch. Therefore, the probability of staying in the blocked state is given by

$$\begin{aligned} P(\text{staying in the blocked state}) &= P(\text{Picker 2 picks again}) \cdot P(\text{Picker 1 wants to pick}) \\ &= u \cdot 1 = u \qquad \text{for } y = 0. \end{aligned} \tag{4.14}$$

If Picker 2 does not need to pick at the same pick-face, then he leaves that pick-face and walks (with infinite speed) to a different pick-face. Given that Picker 2 picked first, let G represent the event that Picker 1 picks at the blocked pick-face and H represent the event that Picker 2 stops at a pick-face $n-y$ distance units (in clockwise direction) away from the blocked pick-face during the next epoch. If $l_0^2(y)$ is the distribution that provides the probability of exiting State 0 through Instance 2, then

$$\begin{aligned} l_0^2(y) &= P(\text{Picker 2 does not pick again}) \cdot P[G] \cdot P[H] = (1-u) \cdot 1 \cdot uv^{n-y-1} \\ &= uv^{n-y} \qquad \text{for } 1 \le y \le \infty \\ &= uv^{n-|y|} \qquad \text{for } -\infty \le y \le \infty, y \ne 0. \end{aligned} \tag{4.15}$$

Adding (4.12) and (4.14) yields

$$l_0(y) = 0.5(u) + 0.5(u) = u \qquad \text{for } y = 0. \tag{4.16}$$

Adding (4.13) and (4.15) yields

$$\begin{aligned} l_0(y) &= 0.5(l_0^1(y)) + 0.5(l_0^2(y)) \\ &= 0.5(uv^{|y|}) + 0.5(uv^{n-|y|}) \\ &= \frac{uv^{|y|} + uv^{n-|y|}}{2} \qquad \text{for } -\infty \le y \le \infty, y \ne 0. \end{aligned} \tag{4.17}$$

Once again, $l_0(y)$ for $y \in \Gamma$ is not a valid probability mass function because $\sum_{y\in\Gamma} l_0(y) \ne 1$. To obtain a valid probability mass function (say, $k_0(y)$), we divide $l_0(y)$ by $\sum_{y\in\Gamma} l_0(y)$. That is,

$$k_0(y) = \frac{l_0(y)}{\sum_{y\in\Gamma} l_0(y)} \qquad \text{for } -n+1 \le y \le n-1. \tag{4.18}$$

Chapter 5

Configuring the Storage System for Order Picking in a Distribution Center

Abstract

The design of an order picking system in a distribution center depends on several factors; e.g., forward-reserve allocation, storage system configuration and the chosen storage policy and picking strategy. In this chapter we address the problem of configuring the storage system for order picking in a distribution center; i.e., deciding the length and height of the storage system for a given volume of items to be stored. A common perception is to configure the storage system as high as possible, which reduces the length for a given number of storage locations. Such a configuration may reduce the space needed, but it may increase the cost of pickers and equipment since such a configuration may increase total travel-time (which includes travel in the vertical direction) and order picker blocking (which increases as the height increases). We present an analytical model to determine the optimal storage system configuration for a semi-automated order picking system. Our results suggest that as the required throughput of such a system increases, the optimum height of the storage racks tends to be less than the maximum allowable height. However, in contrast, for a manual system (where the vertical travel-time is negligible) the optimum height is always equal to the maximum allowable height.

5.1. Introduction

To be competitive amidst increasing competition from other companies and pressure from customers requiring swift and seamless service and low delivery times, companies need effectively-designed and efficiently-managed supply chains. This means companies must make every effort possible to reduce costs incurred due to various non-value added activities: e.g., material handling, storage, sorting, etc.

One key component in a supply chain is a distribution center (DC), which plays the vital role of obtaining materials from different suppliers, performing value-added activities, and assembling (or sorting) them to fulfill customer orders. The activities within a DC include receipt of items and customer orders, storing items, order picking, shipping, customer service and reclamation, and control. Though the DCs are designed to deliver some level of service to the manufacturers and customers, they do so at the cost of extensive material handling and storage. This increases the non-value added cost of products. Therefore, designing efficient DCs is necessary to provide a better flow of material through it to reduce these non-value added costs.

Specifically, we focus on the order picking activity, as it has been identified as the highest priority activity in a DC for productivity improvement due to its relatively high (about 50%) contribution to the total DC operating cost (Tompkins et al., 2003). Order picking refers to the operation of retrieving items from storage locations to fulfill customer orders. The design of the order picking system (OPS) directly relates to the service a DC is able to provide to its customers. While designing an order picking system, a designer must consider the following question: *which OPS best meets a given set of objectives?* Some of the objectives the designer is required to optimize include maximizing throughput or minimizing cost, space, response time, and/or error-rate. If the designer knows what stock keeping units (a unique identifier of each product or item that is stocked in a DC), and their quantities, are to be stored in the picking area, then the decisions that he is required to make in meeting one or more of these objectives include:

- What type of storage system should be used?
- Which order picking strategy (i.e., discrete, batch, zone, or bucket brigade) should be used?
- Which picking system (i.e., manual, semi-automated, or automated) should be used?

- Which pick-assist technology (i.e., paper-based or paper-less) should be used?

In this chapter our focus is on the first decision in the above list; i.e., (optimally) configuring the storage system for order picking in a DC. We focus on this decision because it forms the basis for all other decisions in the above list.

We characterize an order picking area as consisting of a given number of storage (flow- or pallet-) racks, with each storage rack having one of more storage levels (in the vertical direction) to store a specific set of SKUs. The decision we wish to help a designer make is the following: *what is the optimal height, and thereby, length, of the storage system in the picking area that minimizes system cost?* System cost is the combined cost of pickers, equipment, and space.

In this chapter we consider semi-automated OPSs. Such OPSs employ semi-automated equipment to assist manual pickers in picking items from storage racks. The use of a semi-automated equipment improves the productivity of the pickers as it results in increased speed of operation and reduced fatigue (Tompkins et al., 2003). Moreover, these equipment facilitate the design of high storage systems to utilize the vertical space. However, these equipment are expensive and employing them requires a cost justification in terms of the relative increase in system throughput and the decrease in floorspace. Commonly used semi-automated OPSs include carousels, vertical-lift modules, and person-aboard storage/retrieval (S/R) machines (such as an order picker truck, counter-balanced lift truck, reach truck, and turret truck).

Our focus will be on person-aboard S/R machines or trucks that are typically used in narrow-aisle OPSs. (A narrow-aisle OPS is one in which there is only one-way traffic possible.) We select this type of semi-automated OPS for two reasons: (i) these systems are commonly employed in DCs; and (ii) no analytical models exist for designing such systems. An examples of a person-aboard S/R machine (in which a picker, onboard an S/R machine, physically picks items or cases from storage locations) is an order picker truck (see Figure 5.1).

It may seem at first that the decision regarding the length and height of a storage system is relatively simple. A seemingly attractive solution is to configure the storage system as high as possible to reduce length, and hence, space. A common perception in industry is that such a solution is optimal. However, this perception may not be entirely accurate because travel-time and picker blocking increase as the storage system gets higher, which leads to a tradeoff between capital and operational expenses.

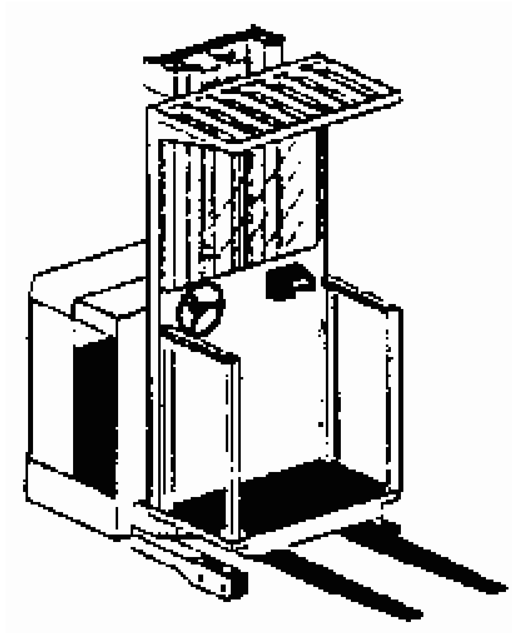


Figure 5.1: Schematic of an Order Picker Truck (MHIA, 2006)

To illustrate this point, consider a scenario where 1260 pallet-loads need to be stored in the picking area with narrow aisles, and each pallet-rack opening is 5 ft each in length, width, and height. Let the racks be one-pallet-deep, which results in a total floorspace requirement of 31,500 ft^2. Assume that the racks can extend no more than 35 ft in height. Hence, we get an option of 10 ft high racks (i.e., 2-level storage), 15 ft high racks (i.e., 3-level storage), 20 ft high racks (i.e., 4-level storage), and so on, up to 35 ft high racks (i.e., 7-level storage). This is illustrated in Figure 5.2. Also assume that the pickers pick 30 cases per pick-tour, pick-time is 0.167 min per case, and the horizontal and vertical speeds of the person-aboard S/R machine are 300 fpm and 50 fpm, respectively.

If the common perception of designing the storage systems as high as possible is used, the selected configuration of the storage system will have 35 ft high racks and a 180 ft long aisle (assuming one-long aisle for ease of explanation) with storage racks on either side. The reasoning behind this perception is that a high (in height) and short (in length) storage system reduces space, which reduces the cost of space. Moreover, a high (in height) and short (in length) will also minimize the pick-tour length (compared to a configuration that is less high in height, say 30 ft high and 210 ft long). Minimal pick-tour lengths decrease horizontal travel-time, which increases the throughput of pickers, thus requiring less pickers (and associated equipment) to meet a given throughput. This reduces the cost

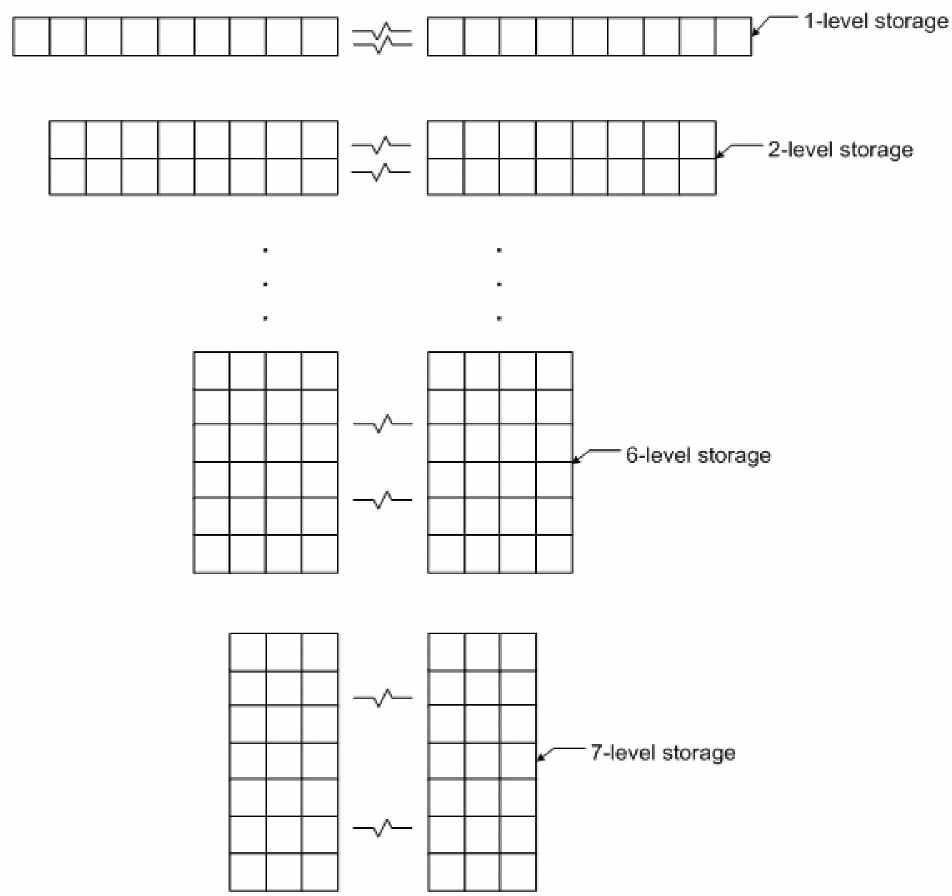


Figure 5.2: Schematic of Various Storage Configurations to Store 1260 Pallet-Loads.

of pickers and equipment. Although this is all correct, the common industry perception does not consider is an association between an increase in storage height and an increase in the vertical travel-time of person-aboard S/R machine. Vertical travel-time increases because these equipment are at least 6 times slower when traveling vertically compared to traveling horizontally (Raymond Corporation, 2005). This increase in vertical travel-time may offset the decrease in horizontal travel-time, which may increase the total travel-time and, therefore, decrease the throughput of a picker. Due to such factors, in our example the throughput per picker for the 35 ft high system is 142.56 cases/hr, while the throughput for the 30 ft high system (which is approximately 17% longer in length) is actually higher at 152.38 cases/hr).

Moreover, using a simulation model similar to that employed by Gue et al. (2006) and which is discussed later, in the system with 35 ft high racks, picker blocking is relatively high compared to that in the system with 30 ft high racks. Blocking results in waiting times, and hence, decreases picker productivity. Taller systems increase the pick-density of each stop, which can result in more picker blocking, which also decreases the throughput of a picker.

This combined effect of increases in travel-time and picker blocking causes the productivity of pickers to drop substantially in the system with 35 ft racks. For our example, if the given system throughput is 1000 cases/hr, then the systems with 30 ft and 35 ft high racks will require 8 and 9 pickers, respectively. So though the cost of space for the taller storage system is low, the cost of an extra picker (and equipment) can be significant enough that the system with 35 ft racks is more expensive than the system with 30 ft racks. This means, from a system cost perspective, the system with 30 ft high racks is preferred.

The above example suggests that a high (in height) and short (in length) system may not be optimal in all cases. That is, the problem of configuring the storage system is not as easy as it first seems. In this chapter we address the general problem of configuring the storage system in an OPS. Specifically, we develop an analytical model to identify the optimal storage height in an OPS employing a person aboard S/R machine system, such that the total cost of pickers, equipment, and space is minimized.

The remainder of this chapter is organized as follows. In Section 5.2 we review literature in the area of DC and OPS design along with past research in storage system configuration. In Section 5.3 we present a travel-time model for a semi-automated OPS employing a person-aboard S/R machine. In Section 5.4 we present an approach to estimate the blocking experienced by pickers in narrow-aisle OPSs. In Section 5.5 we develop a cost-based optimization model to identify the optimal level of storage racks. In Section 5.6, using this optimization model, we examine two semi-automated system configurations, and analyze the effect of space cost on the optimal storage height. In Section 5.7 we briefly discuss the storage configuration problem for a manual OPS. Finally, we present our conclusions in Section 5.8.

5.2. Related Research

In the area of overall DC (or warehouse) design, Gray et al. (1992) propose a multi-stage hierarchical decision approach to model the composite design and operating problems for a typical order-consolidation warehouse. Their hierarchical approach utilizes a sequence of coordinated mathematical models to evaluate the major economic tradeoffs and to prune the decision space to a few superior alternatives. Rouwenhorst et al. (2000) present a reference framework and a classification of warehouse design and control problems. They define a warehouse design problem as a "structured approach of decision making at a strategic, tactical, and operational level in an attempt to meet a number of well-defined performance criteria."

Specific to the design of an OPS in a DC, Brynzer et al. (1994) present a zero-based analysis methodology for the evaluation of OPSs as a base for system design and managerial decisions. Yoon and Sharp (1996) propose a structured procedure for the analysis and design of an OPS that considers the inter-dependent relationships between different functional areas (e.g., receiving, picking, sorting, etc.). The survey papers by van den Berg (1999), Rouwenhorst et al. (2000), and Gu et al. (2005) provide a complete review of various contributions in the area of DC and OPS design.

The problem of configuring the order picking area is critical as other decisions related to storage policy, picking strategy selection, etc., depend on it. A poorly configured order picking area can significantly affect the throughput. Yoon and Sharp (1995) present a cognitive design procedure for an OPS. They divide the procedure into three stages: input, selection, and evaluation stages. In their work, the authors present details on various analytical models that a designer can use to identify the best OPS design. With respect to storage area configuration, the authors focus on identifying storage requirements, aisle layout (i.e., length, width, and number of aisles), and the number of workers for various OPS configurations. These OPS configurations include picking systems for case-picking (e.g., person-aboard S/R machine) and item-picking (e.g., horizontal carousel and bin-shelving). Through a numerical case study, they illustrate the use of their analytical models.

Malmborg (1996) formulates an integrated cost-based evaluation model that links three major policy issues associated with distribution systems. These issues include inventory management, space allocation between reserve and retrieval (or forward) storage areas, and storage area layout. Specific to storage area layout, the author develops analytical models to estimate the number of storage locations and the number of aisles required to store a given product volume. In so doing, the author considers two types of storage policies: randomized storage and volume-based dedicated storage. The author embeds these cost-based models into a computational evaluation model to assist a designer in analyzing decisions regarding the three aforementioned policy issues.

Both of the Yoon and Sharp (1995) and Malmborg (1996) contributions overlook one key issue, which is the decision regarding the length, height, and depth of a storage area for storing a given volume of SKUs. Different combinations of length, height, and depth may yield the same storage volume, but for each of these combinations the pick-density may be different. Pick-density is defined as the probability that a picker will pick at any given pick-face during his tour. (A pick-face might represent a column of pallet rack, a bay of flow rack, or a section of bin shelving; see Figure 5.3.) For a given storage volume, the shorter (in length) and higher (in height) the storage area, the higher the pick-density. An increase in pick-density may result in an increase in blocking, as pointed out by Parikh and Meller (2006b) in their work related to estimating blocking in picking systems.

As mentioned before, an increase in travel-time and blocking in a short and high system may reduce system throughput, and increase system cost. Similarly, a long and low storage system may not reduce system cost either, because of an increase in pick-tour lengths (which decreases throughput). It is therefore essential to identify the system configuration that minimizes system cost. Before we examine the trade-off between the cost of pickers and equipment, and cost of space, we first present a model to estimate the travel-time of a pick-tour in a semi-automated OPS.

5.3. Estimating the Travel-Time of a Pick-Tour

In Section 5.1 we provided an example where we compared a 30 ft high storage system with a 35 ft high storage system. One component of the example was the vertical travel-time. In this section we develop an analytical model to estimate the travel-time of a person-aboard S/R machine. In developing this model we make the following assumptions:

- Pickers may pick multiple items at a pick-face.
- S/R machines can travel in both the horizontal and vertical directions simultaneously; i.e., we assume a Tchebychev travel metric.
- We model a multi-aisle picking area as an area with one-long aisle to estimate travel-time. Later, we add time to travel cross-aisles and travel to and from the depot. (A depot is the physical location in the picking area from where the pickers pick the pick-lists and deposit items. We assume that the depot is located in the middle of the front-aisle.)
- For ease in modeling, we merge the pick-faces on both sides of an aisle. That is, if each side of the aisle has $n/2$ pick-faces, then after merging there are n pick-faces on one

side of the aisle and 0 on the other side. Consequently, for a given item distribution, we merge the locations of the items to be picked so that they all fall on the same side as the pick-faces. Therefore, pickers only pick from one side of an aisle.

- The person on-board S/R machines do not backtrack while retrieving items.

The notation used in our model, along with their description, are presented below.

I = average number of items to be picked during a pick-tour

s = total number of storage locations required to store all the SKUs

g = number of storage levels at a pick-face

g^{min} = minimum possible storage locations at a pick-face

g^{max} = maximum possible storage locations at a pick-face

n = total number of pick-faces = $\left\lceil \dfrac{s}{2g} \right\rceil$

a = number of aisles

a_w = width of an aisle (ft)

a_l = length of an aisle (ft)

a_l^{max} = maximum allowable length of an aisle (ft)

c_w = width of a cross-aisle (ft)

c_l = length of a cross-aisle between successive aisles (ft)

p_d = depth of a pick-face (ft)

p_h = height of a pick-face (ft)

p_w = width of a pick-face (ft)

v_h = horizontal speed of the S/R machine (fpm)

v_v = vertical speed of the S/R machine (fpm); we assume that the upward and downward travel speeds are identical

t_p = time required by each picker to pick an item from the storage location (min)

t_w = time required to travel past a pick-face (min) = $\frac{p_w}{v_h}$

$E[T_{depot}]$ = expected time to travel to and from the depot (min)

$E[T_{cross}]$ = expected total time to travel the cross-aisles (min)

$E[T_{aisles}]$ = expected total time to travel the aisles (min)

$E[T_{travel}]$ = expected total travel-time (min)

$E[T_{pick}]$ = expected total time to pick I items (min) = It_p

$E[T]$ = expected total time required to pick all the items (min) = $E[T_{travel}] + E[T_{pick}]$

λ = theoretical pick-rate of a picker (items/hr) = $\frac{60I}{E[T]}$

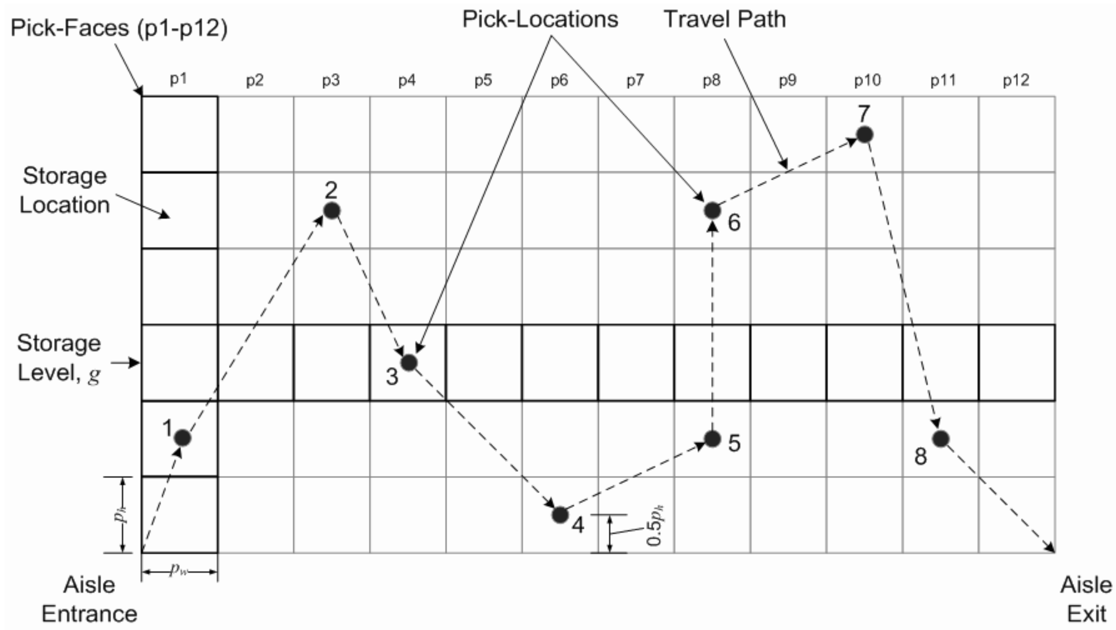


Figure 5.3: Schematic of Travel-Path Followed by a Person-Aboard S/R Machine.

Λ_{req} = throughput requirement of the system (items/hr)
K = actual number of pickers required to achieve required throughput (Λ_{req})
S = total space corresponding to a specific storage level g (ft^2)

Figure 5.3 illustrates a schematic of the travel-path followed by a person-aboard S/R machine within an aisle. The travel-path includes the following three components: (i) entering the aisle, (ii) picking items from the storage locations at various pick-faces, and (iii) exiting the aisle. Once we estimate the three travel-time components, we may then add the time to travel the cross-aisles and time to travel to and from the depot (in a multi-aisle system). This way we can obtain the total expected time to pick all items in the system. Using this logic, we have

$$E[T_{travel}] = E[T_{aisles}] + E[T_{cross}] + E[T_{depot}]. \tag{5.1}$$

We can easily estimate $E[T_{cross}]$ and $E[T_{depot}]$ as we know the the number of aisles (a), the width of aisles (a_w), the length of cross-aisles (c_l), the depth of a pick-face (p_d), and the horizonal speed of the equipment (v_h). That is,

$$E[T_{cross}] = \frac{(a-1)c_l}{v_h}, \qquad \text{and}$$
$$E[T_{depot}] = \frac{a_w(a-1) + p_d(2a-2)}{v_h}. \tag{5.2}$$

Next we develop a model to estimate $E[T_{aisles}]$.

Estimating $E[T_{aisles}]$: The estimation of $E[T_{aisles}]$ depends on the distribution of the items within the storage locations. We can decompose $E[T_{aisles}]$ into four travel-time components; i.e.,

$$E[T_{aisles}] = E[T_1] + E[T_2] + E[T_3] + E[T_4], \tag{5.3}$$

where

$E[T_1]$ = expected travel-time to reach the first pick (min),

$E[T_2]$ = expected total vertical travel-time at pick-faces with at least one pick (min),

$E[T_3]$ = expected total travel-time between pick-faces (min), and

$E[T_4]$ = expected travel-time to reach the end of the aisle from the last pick (min).

We next develop models to estimate these four components of $E[T_{aisles}]$.

To estimate $E[T_1]$, similar to Roodbergen and Vis (2006), we assume that all items are uniformly distributed in the picking area. Accordingly, we can estimate the expected number of pick-faces with at least one pick ($E[P]$) as

$$E[P] = n\left[1 - \left(1 - \frac{1}{n}\right)^I\right],$$

where $\left[1 - \left(1 - \frac{1}{n}\right)^I\right]$ is the probability of a pick-face with at least one pick. Using $E[P]$, we can estimate expected number of items to be picked at a pick-face ($E[I_P]$) as

$$E[I_P] = \sum_{i=0}^{I} i\left[\binom{I}{i}\left(\frac{1}{E[P]}\right)^i\left(1 - \frac{1}{E[P]}\right)^{I-i}\right],$$

where $\left[\binom{I}{i}\left(\frac{1}{E[P]}^i\right)\left(1 - \frac{1}{E[P]}\right)^{I-i}\right]$ is the (binomial) probability density function for picking i ($1 \leq i \leq I$) items at one of the $E[P]$ pick-faces.

Next we estimate the expected nearest stop in the system. Using the approach presented in Roodbergen and Vis (2006), we get the probability that i is the nearest stop (out of n pick-faces) as $\left(\frac{n-i+1}{n}\right)^I - \left(\frac{n-i}{n}\right)^I$, which is the probability that all picks fall in pick-faces $1, ..., n-i+1$ minus the probability that all picks fall in pick-faces $1, ..., n-i$.

Therefore, the expected nearest stop ($E[P_{near}]$) is given by

$$E[P_{near}] = \sum_{i=1}^{n} i \left[\left(\frac{n-i+1}{n} \right)^I - \left(\frac{n-i}{n} \right)^I \right],$$

and the expected time to reach the nearest stop ($E[T(P_{near})]$) is

$$E[T(P_{near})] = \frac{\frac{p_w}{2} + (E[P_{near}] - 1)p_w}{v_h},$$

where the numerator represents distance from the beginning of the aisle to the nearest stop.

Similarly, we get the probability that i is the farthest stop (out of n pick-faces) as $\left(\frac{i}{n}\right)^I - \left(\frac{i-1}{n}\right)^I$, which is the probability that all picks fall in pick-faces $1, ..., i$ minus the probability that all picks fall in pick-faces $1, ..., i-1$. Therefore, the expected farthest stop in the picking area ($E[P_{far}]$) is given by

$$E[P_{far}] = \sum_{i=1}^{n} i \left[\left(\frac{i}{n} \right)^I - \left(\frac{i-1}{n} \right)^I \right],$$

and the expected time to reach the farthest stop ($E[T(P_{far})]$) is

$$E[T(P_{far})] = \frac{\frac{p_w}{2} + (E[P_{far}] - 1)p_w}{v_h}.$$

Next we estimate the expected time to reach the bottom-most pick at a stop having at least one pick. We use a continuous approximation of the discrete storage locations at a pick-face. The probability distribution of picking i ($0 < i < I$) items at a stop follows a binomial distribution with parameters I and $\frac{1}{n}$. Let l_i ($i = 1, 2, ..., I$) be the location of the ith item to be picked. If we order l_i such that $l_1 \leq l_2 \leq l_3 \leq \ldots \leq l_I$, then l_i is called an ith order statistic. As a result, we can use standard expressions from the order statistics' literature for I uniformly distributed items in the continuous storage locations at a stop.

Treating the location of the first pick (i.e., l_1) as an independent random variable with a uniform distribution, we get the expected location of the first pick as $g\left(\frac{1}{i+1}\right)$. The product of i, its probability of occurrence, and the corresponding expected value of the location of the first pick among i picks gives the location of the bottom-most pick at a stop. By taking the sum of the above product over $1 \leq i \leq I$, we get the expected location of the bottom-most (or first) pick ($E[P_{bottom}]$) which is given by

$$\begin{aligned}
E[P_{bottom}] &= 0.5 + \sum_{i=1}^{I} g\left(\frac{1}{i+1}\right) \mathrm{P}(X = i | i > 0) \\
&= 0.5 + \sum_{i=1}^{I} g\left(\frac{1}{i+1}\right) \left[\frac{\binom{I}{i}\left(\frac{1}{n}\right)^{i}\left(1-\frac{1}{n}\right)^{I-i}}{1-\left(1-\frac{1}{n}\right)^{I}} \right],
\end{aligned}$$

where $\binom{I}{i}\left(\frac{1}{n}\right)^{i}\left(1-\frac{1}{n}\right)^{I-i}$ represents the binomial probability distribution of picking i items at one of the n pick-faces, and $1-\left(1-\frac{1}{n}\right)^{I}$ represents $P(i > 0)$, or one minus the probability of having no picks at a stop. The constant 0.5 in the above expression accounts for the fact that we assume the picks are located at the center of a storage location, and that the first storage location is at least 0.5 distance units above the floor (see Figure 5.3). The expected time to reach the bottom-most pick at a stop ($E[T(P_{bottom})]$) can be calculated as

$$E[T(P_{bottom})] = \frac{\frac{p_h}{2} + (E[P_{bottom}] - 1)p_h}{v_v}.$$

Again using the results from order statistics, the expected location of the last pick (i.e., l_I) is given by $g\left(\frac{i}{i+1}\right)$. Using the above arguments, the expected top-most (or last) pick at a stop with at least one pick ($E[P_{top}]$)is given by

$$\begin{aligned}
E[P_{top}] &= 0.5 + \sum_{i=1}^{I} g\left(\frac{i}{i+1}\right) \mathrm{P}(X = i | i > 0) \\
&= 0.5 + \sum_{i=1}^{I} g\left(\frac{i}{i+1}\right) \left[\frac{\binom{I}{i}\left(\frac{1}{n}\right)^{i}\left(1-\frac{1}{n}\right)^{I-i}}{1-\left(1-\frac{1}{n}\right)^{I}} \right].
\end{aligned}$$

Therefore, the expected time to reach the top-most pick ($E[T(P_{top})]$) can be calculated as

$$E[T(P_{top})] = \frac{\frac{p_h}{2} + (E[P_{top}] - 1)p_h}{v_v}.$$

We note that the S/R machine performs Tchebychev travel. This means that the time to reach the first pick ($E[T_1]$) is the maximum of the horizontal and vertical times to reach that pick. That is, the expected time to reach the first pick ($E[T_1]$) is given by

$$E[T_1] = \max\left\{E[T(P_{near})], E[T(P_{bottom})]\right\}. \tag{5.4}$$

Having reached the first pick, the S/R machine then travels vertically at that stop, picking all the required items from the two pick-faces. Therefore, the expected vertical travel time at a stop can be estimated by taking the difference of the expected time between the first and last picks at that stop. Therefore, the expected total travel time at all stops ($E[T_2]$) is given by

$$E[T_2] = E[P]\big(E[T(P_{top})] - E[T(P_{bottom})]\big). \tag{5.5}$$

We now need to estimate the expected total time to travel-between pick-faces. This time is the sum of expected times to travel between pick-faces at successive pick-faces with at least one pick. Because the S/R machine performs Tchebychev travel, we can calculate the individual travel-between-times as the maximum of horizontal and vertical travel-times to reach a pick location at the succeeding stop from the current stop. For example, consider that the picker is at Pick Location 1 at a pick-face (in Figure 5.3), and moves to Pick Location 2 at some other pick-face for the next pick. If d_h^{12} and d_v^{12} are the horizontal and vertical distances between pick locations 1 and 2, respectively, and t_3^{12} represents the time required for this travel, then considering Tchebychev travel, $t_3^{12} = \max\left\{\frac{d_h^{12}}{v_h}, \frac{d_v^{12}}{v_v}\right\}$. By estimating the travel-time for all such travel-between pick-faces, we can obtain the total travel-time between pick-faces (T_3) for a given distribution of pick locations.

We estimate the expected travel-time between pick-faces ($E[T_3]$) as the product of the expected travel-time between a pair of successive pick-faces with at least one pick and the total number of pairs of pick-faces with at least one pick ($E[P]-1$). For example, in Figure 5.3 the pairs of successive pick-faces with at least one pick are (p1, p3), (p3, p4), (p4, p6), (p6, p8), (p8, p10), and (p10, p11). The expected travel-time between a pair of successive pick-faces with at least one pick can be estimated as follows. If $E[P_d]$ be the expected distance between the first and last pick-faces between which all the I items are distributed, then $E[P_d] = E[P_{far}] - E[P_{near}]$. Similar to before, for a given pick-face with at least one pick, we get the probability that i is the next successive pick-face with at least one pick (p_i) as $\left(\frac{E[P_d]-i+1}{E[P_d]}\right)^{I-1} - \left(\frac{E[P_d]-i}{E[P_d]}\right)^{I-1}$, which is the probability that $I-1$ picks fall in the range of pick-faces 1, ..., $E[P_d]-i+1$ minus the probability that $I-1$ picks fall in the range of pick-faces $1, ..., E[P_d]-i$. Now let q_j be the probability that the top-most pick at a given pick-face is at location j, and let q_k be the probability that next closest pick at the next successive pick-face, i, is at location k. Similar to before, we can estimate q_j and q_k as

$$q_j = \left(\frac{j}{g}\right)^{E[I_P]} - \left(\frac{j-1}{g}\right)^{E[I_P]} \qquad \text{and} \qquad q_k = \left(\frac{k}{g}\right)^{E[I_P]} - \left(\frac{k-1}{g}\right)^{E[I_P]}.$$

The corresponding horizontal and vertical travel-times can be estimated as $\frac{ip_w}{v_h}$ and $\frac{\mid j-k \mid p_h}{v_v}$, respectively. We can, therefore, estimate the total travel-time between pick-faces ($E[T_3]$) as

$$E[T_3] = (E[P]-1) \sum_{i=1}^{E[P_d]-1} \sum_{j=1}^{g} \sum_{k=1}^{g} p_i q_j q_k \max\left\{\frac{ip_w}{v_h}, \frac{\mid j-k \mid p_h}{v_v}\right\}. \tag{5.6}$$

Finally, we estimate the expected time to reach the end of the aisle from the last pick ($E[T_4]$). Note that we can estimate this time in the same manner as we estimated $E[T_1]$, albeit, measuring distances from the end of the aisle (rather than the beginning of the aisle). As the distribution of the nearest point from each end of the aisle is the same, we have

$$E[T_4] = E[T_1]. \tag{5.7}$$

Substituting (5.4), (5.5), (5.6), and (5.7) in (5.3) we get the expected total travel-time in the aisles ($E[T_{aisles}]$). From (5.1), the expected total travel-time ($E[T]$) is the sum of $E[T_{aisles}]$, $E[T_{cross}]$, and $E[T_{depot}]$.

To evaluate the quality of our analytical travel-time model we developed a simulation model of a narrow-aisle OPS using a person-aboard S/R machine. We simulated various cases, recorded the simulated travel-times for each of them, and then compared the simulated travel-times with the travel-time estimates obtained through our analytical model. Table 5.1 provides a comparison of results for some of these cases.

From the above table, we observe that the travel-time model developed in this section performs reasonably well, with most estimates within 3% of the simulated results. However, there are a few cases with very low pick-density for which the %-Diff values are relatively high; e.g., when $s = 500$, $g = 3$, and $I = 10$, %-Diff = -4.26%. A possible reason for such large %-Diff value is the high degree of stochasticity (in the distribution of items within pick-faces) at low pick-densities (i.e., a low number of total items to be picked). The current model is unable to capture this, and hence, the high %-Diff values for such cases. However, medium to high throughput OPSs, the focus of this chapter, have a significantly large number of items to be picked (i.e., such OPSs have high pick-density). For such cases, our model performs

Table 5.1: Comparison of Analytical and Simulation Results of Total Travel-Time ($E[T]$ in min) for Various Semi-Automated OPS Configurations.

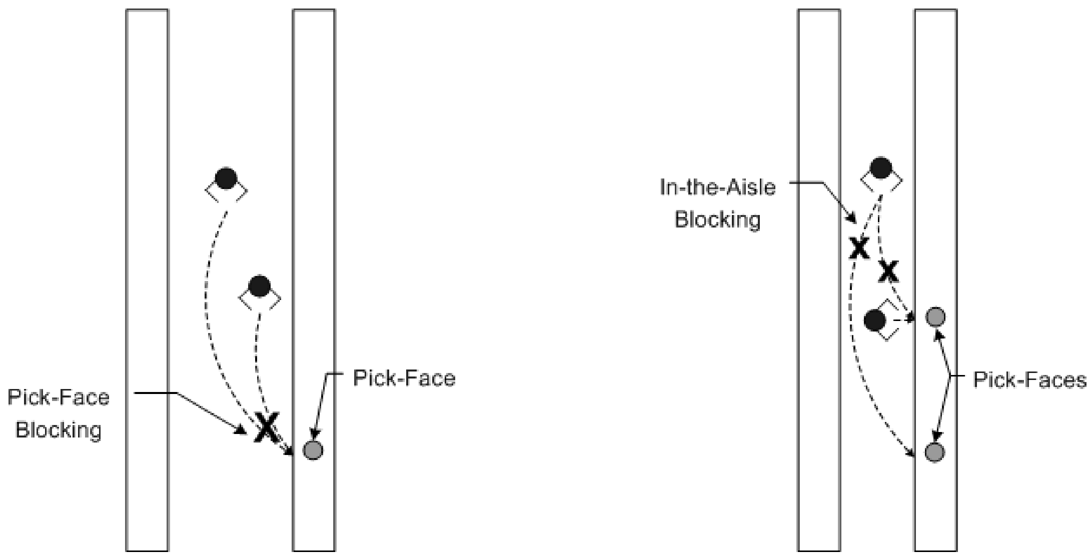
s	g	I	**Analytical**	**Simulation**	**Std. Err.**	**%-Diff**
500	3	10	1.67	1.74	0.0005	-4.26
	5	10	2.08	2.11	0.0012	-1.06
	7	10	2.76	2.77	0.0017	-0.30
	3	20	2.50	2.49	0.0020	0.15
	5	20	3.58	3.54	0.0075	1.04
	7	20	4.82	4.76	0.0208	1.23
	3	30	3.32	3.26	0.0058	2.04
	5	30	5.00	4.92	0.0237	1.55
	7	30	6.84	6.67	0.0270	2.66
1000	3	10	2.73	2.96	0.0005	-7.76
	5	10	2.44	2.54	0.0007	-3.70
	7	10	2.95	3.00	0.0020	-1.42
	3	20	3.37	3.45	0.0002	-2.21
	5	20	3.92	3.94	0.0078	-0.33
	7	20	5.11	5.08	0.0182	0.48
	3	30	4.12	4.13	0.0068	-0.33
	5	30	5.40	5.36	0.0228	0.73
	7	30	7.20	7.13	0.0517	1.06

fairly well (%-Diff values $< 3\%$) in estimating the total travel-time of a person-aboard S/R machine. Hence, we will use our travel-time model for further analysis.

We now provide an approach to estimate blocking; the second factor that may affect throughput as the number of storage levels increases.

5.4. Estimating Picker Blocking

Picker blocking can be prominent in a high throughput system that requires a large number of pickers in the picking area, and can lead to increased picker idle time and reduction in their productivity; thus, affecting system throughput. Blocking can take two forms depending on the aisle-widths. Aisles in an OPS can be either wide or narrow. A wide-aisle OPS is one in which the aisles are wide enough to allow pickers to pass each other in the aisle. Parikh and Meller (2006b) observe that in such OPSs, though pickers may pass, they may still experience blocking when two or more pickers need to pick at the same pick-face. They referred to this form of blocking as *pick-face blocking.* In contrast, a narrow-aisle OPS is one in which aisles are too narrow to allow pickers to pass each other in the aisle. For such OPSs, pickers may not only experience pick-face blocking, but may also experience blocking due to their inability to pass other pickers in the aisle. Gue et al. (2006) refer to this cumulative blocking effect as *in-the-aisle blocking*. Figure 5.4 illustrates these two forms of blocking.



(a) Wide-Aisle OPS (b) Narrow-Aisle OPS

Figure 5.4: Forms of blocking: (a) Pick-Face Blocking in a Wide-Aisle OPS and (b) In-the-Aisle Blocking in a Narrow-Aisle OPS.

Next we summarize the results developed by Gue et al. (2006) and Parikh and Meller (2006b) for narrow- and wide-aisle systems, respectively, based on a discrete-time Markov chain modeling approach. They assume that pickers follow a traversal or S-shape routing policy, which means they visit each aisle and travel through that aisle in only one direction. Consequently, they assume that the layout of the picking area may be represented as a circle. Moreover, the pick to walk time ratios are either 1:1 or ∞:1. For all other pick to walk time ratios: e.g., 5:1, 10:1, and 20:1, they resort to simulation.

Gue et al. (2006) model the narrow-aisle picking system using a discrete-time Markov chain, with states r_{xy}, where x and y represent the previous action of Pickers 1 and 2, respectively. For example, State 2_{pw} indicates that pickers are 2 pick-faces apart with Picker 1 having just picked and Picker 2 having just walked. Then, the State 1_{wp} would be referred to as the *blocked* state. They find the limiting probability of visiting the blocked state and obtain closed-form expressions to estimate average in-the-aisle blocking experienced by pickers. They consider that there are only two pickers in the system and the ratios of time to pick an item and time to walk past a pick-face are 1:1 and ∞:1. Furthermore, they assume the time per stop at a pick-face is constant, which results in a low variability in the length of time spent when stopped at a pick-face.

Since in most OPSs there are multiple items located at each pick-face, there exists the possibility that once stopped, a picker will pick multiple items. In such a situation, the blocking phenomenon may be quite different, since once stopped, the picker will stay stopped for longer (and with higher variance), which increases the potential for blocking other pickers. The variability in the length of time spent at a pick-face when stopped is relatively high for such OPSs. Parikh and Meller (2006b) represent the probability of picking at a location in this case as p. They reformulate the Markov chain model presented by Gue et al. (2006) for wide-aisle OPSs for the cases when there are two pickers in the system, and when (i) pickers may pick one item when stopped (i.e., the time per stop is constant), and (ii) pickers may pick multiple items when stopped (i.e., the time per stop is of variable length). For each of these cases, they obtain the average blocking for each picker in the system.

We present a summary of the results obtained by Gue et al. (2006) and Parikh and Meller (2006b) in Table 5.2. In the table we use the abbreviation "AENCF" for "analytical expression is not closed-form" and "TBD" for still needs "to be developed."

Table 5.2: Summary of Analytical Results for Wide- and Narrow-Aisle Order Picking Systems.

	t_p:t_w	**Wide** (Parikh and Meller, 2006b)	**Narrow** (Gue et al., 2006)
May Pick One Item	1:1	$\frac{p^2(1-p)}{n(2-p)(1+p)^2-p^3-p}$	$\frac{pq}{(n-1)(p+1)^2-2p^2}$
	∞:1	AENCF	$\frac{1-p}{2(1-p)+(n-1)p}$
May Pick Multiple Items	1:1	$\frac{p^2}{n(2-p)-p+2p^2}$	TBD
	∞:1	AENCF	TBD

Both Gue et al. (2006) and Parikh and Meller (2006b) suggest that blocking depends on four factors: number of pick-faces (n), pick-density (p), number of pickers in the system (K), and their pick to walk time ratio (t_p:t_w). However, analytical models are only available for t_p:t_w of 1:1 and ∞:1, while actual ratios are likely to fall between these two extreme points. That is, if we assume that the time to pick an item (t_p) is 0.167 min and the speed at which a S/R machine passes a pick-face 5 ft wide (v_h) is 300 fpm (therefore, $t_w = 0.017$ min), then t_p:t_w is 10:1. As there are no analytical models available to help estimate blocking for this ratio (as well as for cases with more than two pickers), a simulation or some other estimation method would need to be used. However, since simulation is relatively accurate but time-consuming, we will provide a regression model based on a set of simulation runs. In this way, the simulation runs we provide can be used as a foundation for an analytical statistical regression model that will fill the gaps in currently available analytical blocking models.

Note that semi-automated OPSs are typically employed in a narrow-aisle OPS. Though rare, a semi-automated OPS may be employed in a wide-aisle OPS. However, the blocking experienced by pickers in wide-aisle OPSs, referred to as pick-face blocking, tends to be significantly lower than in-the-aisle blocking for a given system configuration and number of pickers (Parikh and Meller, 2006b). So, though blocking is one of the factors (apart from travel-time) that may affect the problem of configuring the storage system, the effect of low values of blocking is negligible. Consequently, we limit our discussion in this chapter to narrow-aisle OPSs only.

To develop an analytical statistical regression model to estimate in-the-aisle blocking, we first developed a simulation model for a narrow-multi-aisle semi-automated OPS. We model the picking area as one circular aisle with pickers moving along the circle picking items from storage locations. The key parameters we used in the simulation model include number of pick-faces (n), pick-density (p), number of pickers (K), and pick to walk time ratio (t_p:t_w). We generated 315 data points using this simulation model, which we then used to develop a regression model.

We first limited our model development to a simple and intuitive regression model with the three key parameters, n, p, and K, and one or more interaction terms. The idea was that such an intuitive model would provide very good insight into the blocking phenomenon for a given narrow-aisle OPS. This intuitive model concurred with our general understanding that (i) when n, p, and K are zero, blocking, $B(\cdot)$, is zero, (ii) as K increases (i.e., the picking area gets more crowded), $B(\cdot)$ increases, and (iii) as p (i.e., the picking frequency at a pick-face increases) and/or n increases (i.e., the picking area becomes larger), $B(\cdot)$ decreases. The R^2 value for this model was high (0.90), however, the sum square error value was high too (2,021.37), which suggests that this model would not provide very accurate estimates of blocking. Therefore, we shifted our focus towards developing a relatively accurate model, even if that meant sacrificing the intuition within the model.

We used the 315 data points from our simulation model and considered the following 13 parameters to identify the top-performing regression models: n, p, K, np, pk, nK, npK, n^2, p^2, K^2, $\sqrt{n}$, $\sqrt{p}$, and $\sqrt{K}$. All of the top-performing models had an R^2 value in excess of 0.98 and a sum square error less than 1,000. We then generated a test set of 50 data points from the same simulation model. This test set was used to identify the most robust regression model, which we defined as the model that yielded the least sum square error on the test set. It turned out that the following model was the most robust of all the models we considered.

$$B(n, p, K, t_p{:}t_w) = -0.168n + 2.89K + 0.000336n^2 - 1.29pK - 0.00716nK + 1.26\sqrt{n}. \quad (5.8)$$

Note that for the original set of 315 data points, the above model had a coefficient of determination (R^2) of 0.99, a sum square error of 388.49, and an estimated standard deviation of 1.12. Further details of the above regression model are presented in the Appendix.

Note that the above model uses the pick density (p) as an input parameter. In the next section we clarify how this parameter can be specified for a picking system.

5.4.1 Estimating Pick-Density

Pick-density (p) must model both the probability of stopping at a pick-face and picking (repeatedly) at that pick-face. We let $q = 1 - p$ be the probability of not picking at a stop. Given the number of items to be picked during a pick-tour (I), and the number of possible pick-faces in the picking area (n), the average number of items picked by a picker at each possible pick-face (I_{avg}) can be determined as $I_{avg} = \frac{I}{n}$. The value of I_{avg} also corresponds to the number of items a picker picks at a stop from a total of $2g$ storage locations. (Note that each pick-face has g storage locations, and there are two pick-faces at a stop in a narrow-aisle system.)

The value of I_{avg} corresponds to $\sum_{i=0}^{2g} i f(i)$, where $f(i)$ represents the probability mass function (pmf) that defines the probability of picking i items at a stop before exiting that stop, and is given as

$$f(i) = \begin{cases} q & \text{if } i = 0, \\ pq & \text{if } i = 1, \\ \vdots & \\ p^{2g-1}q & \text{if } i = 2g-1, \\ p^{2g} & \text{if } i = 2g, \end{cases}$$

where $q = 1 - p$. Hence, we have

$$\begin{aligned}
\frac{I}{n} = I_{avg} &= \sum_{i=0}^{2g} if(i) \\
&= 0q + 1pq + 2p^2q + 3p^3q + s + (2g-1)p^{2g-1}q + 2gp^{2g} \\
&= q\sum_{i=0}^{2g-1} ip^i + 2gp^{2g} \\
&= q\left(\frac{p - 2gp^{2g} + (2g-1)p^{2g+1}}{(p-1)^2}\right) + 2gp^{2g} \\
&= \left(\frac{p - 2gp^{2g} + (2g-1)p^{2g+1}}{1-p}\right) + 2gp^{2g}. \qquad (5.9)
\end{aligned}$$

Solving (5.9) in terms of p allows us to specify p in (5.8) so that we can estimate the blocking in the system.

5.5. Storage System Configuration Model

With models developed to estimate the labor requirement of a picking system, we now address a cost-based optimization model for storage system configuration. The idea behind the cost-based optimization model is to identify the optimal storage level (g), such that the total system cost is minimized. We define system cost as the sum of cost of pickers, equipment, and space. Let

C_K = yearly cost of a picker (\$),
C_Q = yearly cost of a person-aboard S/R machine (\$),
C_S = yearly cost of storage space (\$/ft^2), and
C = yearly system cost (\$),

where $C = K(C_K + C_Q) + SC_S$.

If we know the hourly rate of the pickers, then it is relatively straightforward to estimate the yearly loaded cost of pickers (C_K), which includes all the benefits that pickers receive from their employer. To estimate the cost of equipment (C_Q), we need to know the one-time purchase cost and yearly maintenance cost of the equipment. For a given cost of capital, we can then estimate the yearly cost of equipment. For example, the cost of purchasing a person-aboard S/R machine (e.g., an order picker truck) is about \$30,000, and yearly maintenance cost is about \$1,500. If the useful life of the S/R machine is 5 years, and the cost of capital is 10%, then the yearly cost of equipment will be about \$10,000. We assume that every picker will require one S/R machine. Cost of space (C_S) depends on whether the land is owned or leased, and whether the facility is to be constructed or already constructed. We assume that C_S does not vary with the number of storage levels (g).

Our objective is to identify the optimal value of g that corresponds to the minimum system cost (C). We refer to this as the *master* problem. This means that for each value of g ($g^{min} \leq g \leq g^{max}$), we need to identify the number of pickers (K) that satisfies the system throughput (Λ_{req}) such that the cost of a system with g storage levels is minimized. Note that for each value of g the space (S), and hence, the space cost (SC_S), is fixed. Therefore, the minimum system cost for a given value of g will correspond to the system with the lowest number of pickers (K) that meets system throughput.

Because for each value of g ($g^{min} \leq g \leq g^{max}$) the minimum system cost has to be estimated, we can decompose the master problem into $g^{max} - g^{min} + 1$ number of *subproblems*. For each of these subproblems the objective function is the minimization of the cost of the system with g storage levels ($C(g)$). Out of the $g^{max} - g^{min} + 1$ cost values, the optimal storage level (g^*) will correspond to $\underset{g}{\operatorname{argmin}}\{C(g)\}$. Below we present the optimization model for one of these subproblems.

The decision variable in the model is the number of pickers (K). In (5.10) and (5.11) we calculate the number of pick-faces (n) and the required number of aisle (a) for a given value of g. The total space required (S) corresponding to a system with g storage levels, which is the sum of the space requirements for aisles, racks, and cross-aisles, is calculated in (5.12). In (5.13) we ensure that the total throughput of the system (Λ_{req}), which depends on the number of pickers (K), the throughput of the individual pickers (λ), and the corresponding blocking estimate ($B(n, p, K, t_p : t_w)$), is satisfied. Note that the pick-density (p), blocking ($B(n, p, K, t_p{:}t_w)$), and travel-time ($E[T]$) can be estimated from the models presented in previous sections. The throughput of individual pickers can then be estimated as, $\lambda = \dfrac{60I}{E[T]}$. The objective in the above optimization model is to minimize system cost ($C(g)$), which in this case will be achieved by picking the minimum value of K that satisfies the throughput constraint, (5.13).

$$
\begin{aligned}
\text{minimize} \quad & C(g) \\
\text{subject to} \quad & n = \left\lceil \frac{s}{2g} \right\rceil && (5.10) \\
& a = \left\lceil \frac{n p_w}{a_l^{max}} \right\rceil && (5.11) \\
& S = a a_l a_w + 2 a a_l p_d + 2(a a_w c_w + 2 a p_d c_w) && (5.12) \\
& K\lambda \left[1 - B(n, p, K, t_p{:}t_w)\right] \geq \Lambda_{req} && (5.13) \\
& K \in I^+
\end{aligned}
$$

The procedure we follow to solve each subproblem is as follows. We first set the number of storage levels (g) as $g = g^{min}$. Then

Step 1. Calculate the number of pick-faces (n) as $n = \left\lceil \frac{s}{2g} \right\rceil$, and the number of aisles (a) as $a = \left\lceil \frac{n p_w}{a_l^{max}} \right\rceil$.

Step 2. Estimate the travel-time of the S/R machine using the models presented in Section 5.3.

Step 3. Estimate the pick-density (p) using (5.9).

Step 4. Calculate the lower bound on number of pickers required, $K^{LB} = \left\lceil \frac{\Lambda_{req}}{\lambda} \right\rceil$. Set $K = K^{LB}$.

Step 5. Estimate the blocking ($B(n, p, K, t_p{:}tw)$) in the system with K pickers using (5.8).

Step 6. Check if $K\lambda B(n, p, K, t_p{:}tw) \geq \Lambda_{req}$. If this is true, then estimate the cost of pickers and equipment ($K(C_K + C_Q)$) with the value of K and combine with the cost of space (SC_S) based on S. Calculate $C(g)$, the system cost corresponding to storage level g, and exit. Else, set $K{=}K + 1$ and go to Step 6.

Now set $g = g + 1$ and repeat the above procedure to solve the remaining subproblems while $g \leq g^{max}$. We now have the $C(g)$ values for all of the $g^{max} - g^{min} + 1$ subproblems. The optimal storage level (g^*) can now be calculated as $g^* = \underset{g}{\operatorname{argmin}}\{C(g)\}$.

5.6. Examining Semi-Automated Order Picking System Configurations

To illustrate the use of the optimization model presented in Section 5.5, we applied it to determine the optimal storage level (g^*) for two narrow-aisle semi-automated system configurations: $s = 500$ and $s = 1000$ with $g^{min} = 2$ and $g^{max} = 7$, and three values of required system throughput: $\Lambda_{req} = 250$, 500, and 1000 cases/hr. Notice that we do not set $g^{min} = 1$. Our reasoning behind this is addressed later in this section.

Table 5.3 presents the results for an example scenario where $s = 1000$ and $\Lambda_{req} = 1000$ cases/hr. The highlighted row illustrates the optimal system configuration corresponding to minimum system cost (C).

Table 5.3: Results for $s = 1000$, $\Lambda_{req} = 1000$ cases/hr, $C_K = \$35,000$, $C_Q = \$10,000$, $C_S = \$25/\text{ft}^2$, $v_h = 300$ fpm, $v_v = 50$ fpm, and Pickers Pick 30 cases per tour with $t_p = 0.167$ min; therefore, t_p:t_w = 10:1. (Note: Cost values are in thousands.)

g	n	S	$E[T]$	λ	p	K	$B(\cdot)$	C_K	C_Q	C_S	C
2	250	23,296	10.48	171.32	0.1082	6	4.11	\$210	\$60	\$582.4	\$852.4
3	167	14,976	9.86	182.30	0.1527	6	6.98	\$210	\$60	\$374.4	\$644.4
4	125	11,232	10.15	177.05	0.1938	7	10.23	\$245	\$70	\$280.8	\$595.8
5	100	9,312	10.86	165.53	0.2309	7	11.90	\$245	\$70	\$232.8	\$547.8
6	**84**	**7,488**	**11.47**	**156.97**	**0.2632**	**8**	**14.81**	**\$280**	**\$80**	**\$187.2**	**\$547.2**
7	72	6,528	12.32	146.07	0.2942	9	17.49	\$315	\$90	\$163.2	\$568.2

For the case presented in Table 5.3, we see that the optimal storage level (g^*), which corresponds to minimum system cost (C), is equal to 6. Notice that the space required (S) decreases in g. For example, by moving up one level from $g = 2$ to $g = 3$, we save about 8,320 ft^2, which accounts for savings of about \$208,000 in the cost of space. Furthermore, notice that travel-time ($E[T]$) first decreases in g, and then increases in g. This is because, for very low values of g, by moving one level up the reduction in horizontal travel-time is large as compared to the increase in vertical travel-time; e.g., compare $E[T]$ for $g = 2$ and $g = 3$. However, after a certain value of g, the increase in vertical travel-time dominates the decrease in horizontal travel-time, which causes $E[T]$ to increase. For the case presented in Table 5.3, throughput per picker (λ) is at its maximum when $g = 3$. After this point, as g increases, λ decreases. Hence, more pickers are required in the system. Moreover, as g increases, pick-density (g) increases. An increase in the number of pickers (K) and the pick-density (p) increases the probability of blocking ($B(\cdot)$), which further increases K; e.g., as the value of g increases from 3 to 7 observe how the value of K increases from 6 to 9. The value of g^* corresponds to the value of g that achieves the minimum system cost, but does not achieve the minimum cost of the any of the three cost components, C_K, C_Q, or C_S, individually.

Here we would like to mention the possible impact of a difference in the cost of space (C_S), from location to location, on g^*. Assume that the cost of pickers and equipment are consistent within the U.S. For regions where space cost is relatively low (e.g., rural areas of the U.S.), as g increases, an increase in picker (and equipment) cost will dominate the corresponding reduction in space cost. Hence, for a DC in such a region, a relatively low (in height) and long (in length) storage system will be optimal. In contrast, for regions that have a relatively high space cost (e.g., just outside a major metropolitan area in the U.S.), as g increases, a reduction in space cost will dominate a corresponding increase in picker and equipment cost. Hence, for a DC in such a region, a high (in height) and short (in length) storage system will be optimal. This is clearly evident from Table 5.4, where we provide g^* for two values of space cost (C_S) and two values of number of storage locations (s), with all other parameter values being identical as before.

Table 5.4: The Impact of Space Cost, C_S, on Optimal Storage Level, g^*.

(a) $C_s = \$25/\text{ft}^2$

$g*$	Λ_{req} (cases/hr)		
	250	500	1000
$s = 500$	7	5	4
$s = 1000$	7	7	6

(b) $C_S = \$50/\text{ft}^2$

$g*$	Λ_{req} (cases/hr)		
	250	500	1000
$s = 500$	7	7	6
$s = 1000$	7	7	7

Also note from Table 5.4 that as the required system throughput (Λ_{req}) increases, the optimal storage level g^* may not always be equal to g_{max}. This is in contrast to the common perception that under any condition, $g^* = g_{max}$; i.e., under any condition a high (in height) and short (in length) system represents an optimal storage configuration.

We mentioned earlier that we did not consider $g = 1$ in the optimization model. This is because our cost-based optimization model may not be directly applicable as the cost components are not the same. A system with $g = 1$, typical in low throughput OPSs, will employ floor-storage (not requiring racks) and pallet-trucks (instead of a person-aboard

S/R machines). However, a designer can still use our cost-based optimization model if the appropriate cost components are included (i.e., the cost of the racks) or modified (i.e., the cost of the equipment).

Note that a configuration with $g = 1$ can be cost effective for low throughput OPSs because of two reasons: (i) floor-storage eliminates the need of racks; and (ii) a pallet truck is less expensive than an order picker truck. These two reasons can lead a 1-level storage system to be optimal for low throughput OPSs. For example, consider a low throughput semi-automated OPS using floor-storage, employing a pallet truck, and $\Lambda_{req} = 100$ items/hr. The other system parameters are $s = 250$, t_p:t_w = 10:1, and cost of space (C_S) equals \$1.50/ft^2 (possible if the facility is already constructed on leased land). Let us evaluate two storage system configurations, $g = 1$ and $g = 2$, using our cost-based optimization model presented in Section 5.5 after appropriately accounting for the cost of racks and equipment. Using the travel-time, pick-density, and blocking models presented in previous sections we find that, for the system with $g = 1$, only one picker is required to meet the throughput; however, for the system with $g = 2$, two pickers are required. Let the annual cost of a pallet truck and an order picker truck be \$1,000 and \$10,000, respectively. Also, let the cost of racks required to store 250 unit-loads be approximately \$10,000. (Note that no racks are required for $g = 1$ due to floor-storage.) Therefore, the system costs (which now includes the cost of the racks), estimated by the optimization model, for systems with $g = 1$ and $g = 2$ are \$56,232 and \$63,832, respectively. This means that for this case the system with $g = 1$ is less expensive than the system with $g = 2$. We also evaluated other systems with $3 \leq g \leq 7$, but the system cost for all of these configurations were higher than \$56,232. That is, a 1-level storage system can be optimal for low throughput OPSs.

The point we wish to make here is that the optimal storage level depends on the number of storage locations and the throughput level of an order picking system. For a low throughput system, a 1-level storage system may be optimal. Additionally, for a medium-to-high throughput system, the optimal storage level may be less than the maximum allowable storage level (but may not be equal to it).

This general understanding of configuring a storage system, however, is based on a model developed specifically for a semi-automated OPS and may not be applicable for a manual OPS. For example, in a manual OPS the issues of an increase in the vertical travel-time and picker blocking may not be significant. We next analyze a manual OPS to identify key points in configuring a storage system for such OPSs.

5.7. Analyzing a Special Case: Manual Order Picking System

Manual OPSs use either a pick cart, pallet jack, or pallet truck to pick items or cases from storage racks. With these equipment, pickers can only pick items stored up to a height of 7 ft, which is a typical height most pickers can reach without any assistance. There is no vertical travel involved in a manual OPS, unlike a semi-automated OPS; hence, we treat a manual OPS as a special case of a semi-automated OPS. Note that we can still use the optimization model presented in Section 5.5, but with different values of various parameters such as pick-time, walk-speed, aisle-lengths, etc. Moreover, a storage level for manual OPSs is a shelf and not a pallet-location; accordingly, we need to change the scale, in terms of the number of storage levels, of the problem.

Again, the common industry perception for medium-to-high throughput manual OPSs is to utilize the full 7 ft. height at a pick-face for picking purposes. The idea is to reduce the storage space, and hence, cost. To test this perception, we used the optimization model (presented in Section 5.5) for a manual OPS. We estimated the pick-density and blocking in a manner similar to the semi-automated system. We assumed that the pickers follow a traversal strategy to estimate the travel-time. We then obtained the optimal number of storage levels for several manual OPS configurations obtained by varying s, Λ_{req}, C_S, and t_p:t_w.

Interestingly, these optimal values coincided with the common industry perception; i.e., the optimal number of storage level was 7 in all cases examined. A possible reason being that in a manual OPS the optimum value of storage level depends only on blocking. This is in contrast to the semi-automated system, where it depends on increase in blocking and travel-time (due to the increase in vertical travel as the number of storage levels increases). Moreover, the blocking experienced by the pickers in a manual OPS is negligible for very low values of pick to walk time ratio (t_p:t_w). (Note that t_p:$t_w = 1.6$:1 in the case of a manual OPS with parameters $t_p = 0.067$ min and $t_w = \frac{p_w}{v_h} = \frac{5}{120} = 0.042$ min, where v_h is the walk-speed of pickers and equals 120 fpm.) The relatively small level of blocking has only a minimal effect on the number of pickers required. As a result, the system cost is almost nearly a decreasing function in the number of storage levels since the cost of space and travel-time are both decreasing in the number of storage levels.

However, consider the case when the walk-speed of pickers is 300 fpm and the pick-time is 0.27 min (therefore, t_p:t_w=16:1), with all other cost parameters same as before. For such a case, the optimal number of storage level is 6. Such a scenario is highly unlikely as pickers would have to run to achieve a speed of 300 fpm, and the pick-time would need to be four times longer than usual. Therefore, such a scenario can serve as a limiting case; i.e., any further decrease in the value of t_p:t_w from 16:1 will result in an optimal number of storage level value equal to 7.

5.8. Conclusions

Designing an order picking system for a distribution center is a complex task. A designer is required to make several decisions: e.g., forward-reserve allocation, storage area configuration, storage policy, picking strategy and system selection, identification of labor requirements, routing policy selection, etc. Out of these, we focused on configuring the storage system for order picking, as it forms the basis of a number of the above decisions.

Specifically, we developed a cost-based optimization model for a semi-automated order picking system to identify the optimal level (or height), and thus, length of a storage system to store a given volume of SKUs in one-pallet-deep storage racks. The objective of the model was to minimize system cost. The system cost is comprised of the cost of pickers, equipment, and space. To estimate the cost of pickers and equipment, we had to estimate the throughput of pickers. For this, we developed analytical models to (i) estimate the travel-time of a person-aboard storage/retrieval machine; (ii) estimate the pick-density for picking in the storage area; and (iii) estimate blocking experienced by the pickers in narrow-aisle storage areas.

Our results suggest that the optimal storage level is influenced by travel-time and blocking. In general, low (in height) and long (in length) storage systems tend to be optimal for situations when there is a relatively low number of storage locations and a relatively high throughput requirement. This is in contrast to the common perception in industry that high (in height) and short (in length) storage systems are optimal for all situations. On the other hand, results from the same optimization model suggests that a manual OPS should, in almost all situations, employ a high (in height) and short (in length) storage system; a result that is consistent with with industry practice.

An obvious question is why the model's results would be consistent with industry practice in one case and not the other. We believe the answer to this question lies in the consideration, or rather lack thereof, of vertical travel-time by the S/R machine in the semi-automated case. That is, when vertical travel is not considered (as is what we believe is the practice in industry), one would tend to overestimate the height of the system, which is what we found from comparing our results (that do consider vertical travel) with what industry believes is optimal. However, when considering the manual model, in which vertical travel is minimal and not an issue, industry practice more closely matches our model, and the two give consistent results. Thus, although there may be other factors at work, we believe the fact that the model's results are consistent with industry practice in one case and not the other adds a degree of validity to our model.

Future research in this area may include extending our model to include the decision of optimal storage depth, referred to as lane depth, along with storage height in a semi-automated OPS. Evaluating lane depths greater than one is needed for certain applications where space costs are relatively high. For such applications, it may be appropriate to sacrifice the throughput of a picker, due to an increase in pick-time at a pick-face, in order to save space by employing a multi-deep storage system. From the standpoint of minimizing the system cost, it then becomes important to simultaneously identify the optimal storage height and lane depth of storage systems for such applications. Another interesting aspect is the impact of various storage policies and product-slotting techniques on the storage system configuration problem.

Other potential research topics in the area of OPS design include developing analytical models to (i) select an appropriate order picking strategy (discrete, batch, zone, or bucket brigade) and (ii) select an appropriate picking system (manual, semi-automated, or automated).

Appendix

This appendix provides details on the regression model developed for estimating blocking in a narrow-aisle semi-automated OPS having a pick to walk time ratio of 10:1.

The regression model given by (5.8) was developed using simulation estimates of $B(n, p, K, t_p{:}t_w)$ for various combinations of n (50, 75, 100, 125, 150, 200, 300), p (0.1, 0.2, 0.3, 0.4, 0.5), and K (2, 3, 4, 5, 6, 7, 8, 9, 10). Note that the ratio $t_p{:}t_w$ is fixed and equals 10:1; hence, it is not included in the regression models.

Table 5.5 illustrates the data used to build the regression model, along with it's prediction ability (which includes squared deviation).

Table 5.5: Details of the Regression Model Developed to Estimate $B(n, p, K, t_p{:}t_w)$.

n	p	K	Actual	Predicted	Sq. Dev.
50	0.1	2	4.87	6.16	1.66
		3	8.92	8.56	0.13
		4	12.58	10.96	2.62
		5	15.47	13.36	4.43
		6	18.51	15.77	7.52
		7	20.89	18.17	7.40
		8	23.18	20.57	6.79
		9	25.37	22.98	5.73
		10	27.30	25.38	3.69
50	0.2	2	4.47	5.90	2.04
		3	8.66	8.17	0.24
		4	12.14	10.45	2.87
		5	15.04	12.72	5.38
		6	17.90	14.99	8.45
		7	20.38	17.27	9.69
		8	22.45	19.54	8.46
		9	24.82	21.82	9.03
		10	26.70	24.09	6.81
50	0.3	2	3.92	5.64	2.96
		3	7.75	7.78	0.00
		4	10.67	9.93	0.55
		5	13.72	12.07	2.71
		6	16.32	14.22	4.41
		7	18.88	16.36	6.33
		8	20.91	18.51	5.76
		9	23.03	20.65	5.64
		10	24.52	22.80	2.96
50	0.4	2	3.35	5.38	4.13
		3	6.92	7.40	0.23
		4	9.81	9.41	0.16
		5	12.18	11.43	0.56
		6	14.60	13.45	1.33
		7	17.06	15.46	2.56
		8	19.10	17.48	2.63
		9	21.03	19.49	2.36
		10	22.80	21.51	1.67
50	0.5	2	3.13	5.12	3.97
		3	5.61	7.01	1.96
		4	8.94	8.90	0.00
		5	11.09	10.78	0.09
		6	13.40	12.67	0.53
		7	15.56	14.56	1.00
		8	17.69	16.45	1.55
		9	19.19	18.33	0.74
		10	21.02	20.22	0.64

continued ...

n	p	K	Actual	Predicted	Sq. Dev.
75	0.1	2	3.45	4.65	1.44
		3	6.41	6.87	0.22
		4	9.24	9.10	0.02
		5	11.37	11.32	0.00
		6	13.74	13.55	0.04
		7	15.87	15.77	0.01
		8	17.69	17.99	0.09
		9	19.68	20.22	0.29
		10	21.28	22.44	1.35
75	0.2	2	2.98	4.39	1.99
		3	6.35	6.49	0.02
		4	8.51	8.58	0.01
		5	11.03	10.68	0.12
		6	13.12	12.77	0.12
		7	15.22	14.87	0.12
		8	17.18	16.96	0.05
		9	18.88	19.06	0.03
		10	20.65	21.15	0.25
75	0.3	2	2.90	4.13	1.52
		3	5.27	6.10	0.69
		4	7.59	8.07	0.23
		5	9.88	10.03	0.02
		6	11.83	12.00	0.03
		7	13.79	13.96	0.03
		8	15.44	15.93	0.24
		9	16.98	17.90	0.84
		10	18.74	19.86	1.26
75	0.4	2	2.43	3.88	2.09
		3	4.62	5.71	1.19
		4	6.79	7.55	0.58
		5	8.83	9.39	0.31
		6	10.55	11.22	0.45
		7	12.03	13.06	1.06
		8	14.02	14.90	0.77
		9	15.54	16.73	1.43
		10	16.92	18.57	2.73
75	0.5	2	2.39	3.62	1.51
		3	4.24	5.33	1.18
		4	6.04	7.03	0.99
		5	7.81	8.74	0.87
		6	9.33	10.45	1.25
		7	11.10	12.16	1.12
		8	12.64	13.87	1.50
		9	14.46	15.57	1.24
		10	15.64	17.28	2.70
100	0.1	2	2.65	3.25	0.36
		3	5.04	5.30	0.07
		4	7.11	7.34	0.05

continued ...

n	p	K	Actual	Predicted	Sq. Dev.
		5	9.12	9.39	0.07
		6	11.10	11.43	0.11
		7	12.71	13.48	0.59
		8	14.51	15.52	1.02
		9	15.97	17.57	2.54
		10	17.29	19.61	5.38
100	0.2	2	2.31	2.99	0.47
		3	4.65	4.91	0.07
		4	6.72	6.82	0.01
		5	8.55	8.74	0.04
		6	10.45	10.66	0.04
		7	12.15	12.57	0.18
		8	13.97	14.49	0.27
		9	15.24	16.40	1.35
		10	16.57	18.32	3.06
100	0.3	2	2.07	2.73	0.44
		3	4.42	4.52	0.01
		4	5.85	6.31	0.21
		5	7.61	8.10	0.24
		6	9.38	9.88	0.25
		7	10.72	11.67	0.90
		8	12.69	13.46	0.59
		9	13.74	15.24	2.26
		10	15.06	17.03	3.88
100	0.4	2	1.75	2.48	0.53
		3	3.39	4.13	0.55
		4	4.99	5.79	0.64
		5	6.72	7.45	0.53
		6	8.03	9.11	1.16
		7	9.79	10.77	0.95
		8	11.08	12.42	1.81
		9	12.00	14.08	4.33
		10	13.69	15.74	4.20
100	0.5	2	1.66	2.22	0.31
		3	3.12	3.75	0.39
		4	4.73	5.28	0.30
		5	6.22	6.81	0.34
		6	7.31	8.33	1.05
		7	8.71	9.86	1.33
		8	9.56	11.39	3.36
		9	11.08	12.92	3.39
		10	12.31	14.45	4.58
125	0.1	2	2.14	2.07	0.01
		3	4.15	3.94	0.05
		4	5.91	5.80	0.01
		5	7.29	7.67	0.14
		6	9.22	9.53	0.10
		7	10.49	11.40	0.83
					continued ...

n	p	K	Actual	Predicted	Sq. Dev.
		8	12.25	13.27	1.03
		9	13.49	15.13	2.69
		10	14.85	17.00	4.61
125	0.2	2	2.07	1.81	0.07
		3	4.42	3.55	0.76
		4	5.85	5.29	0.32
		5	7.61	7.02	0.35
		6	9.38	8.76	0.39
		7	10.72	10.50	0.05
		8	12.69	12.23	0.21
		9	13.74	13.97	0.05
		10	15.06	15.71	0.42
125	0.3	2	1.86	1.55	0.09
		3	3.44	3.16	0.08
		4	4.67	4.77	0.01
		5	6.27	6.38	0.01
		6	7.50	7.99	0.24
		7	8.99	9.59	0.36
		8	10.28	11.20	0.85
		9	11.63	12.81	1.39
		10	12.70	14.42	2.95
125	0.4	2	1.57	1.30	0.08
		3	2.64	2.77	0.02
		4	4.26	4.25	0.00
		5	5.68	5.73	0.00
		6	6.58	7.21	0.40
		7	7.89	8.69	0.64
		8	8.82	10.17	1.82
		9	10.36	11.65	1.66
		10	11.22	13.13	3.64
125	0.5	2	1.54	1.04	0.25
		3	2.44	2.39	0.00
		4	3.74	3.74	0.00
		5	4.69	5.09	0.16
		6	6.05	6.44	0.15
		7	7.01	7.79	0.60
		8	8.25	9.14	0.79
		9	9.27	10.49	1.48
		10	10.32	11.84	2.30
150	0.1	2	1.74	1.17	0.33
		3	3.67	2.85	0.67
		4	5.04	4.54	0.25
		5	6.57	6.23	0.12
		6	7.84	7.91	0.01
		7	9.04	9.60	0.31
		8	10.38	11.29	0.82
		9	11.59	12.97	1.92
		10	12.88	14.66	3.17
					continued ...

n	p	K	Actual	Predicted	Sq. Dev.
150	0.2	2	1.53	0.91	0.39
		3	3.16	2.47	0.48
		4	4.85	4.02	0.68
		5	6.08	5.58	0.25
		6	7.61	7.14	0.22
		7	8.85	8.70	0.02
		8	10.12	10.26	0.02
		9	11.21	11.81	0.36
		10	12.28	13.37	1.19
150	0.3	2	1.35	0.65	0.49
		3	2.70	2.08	0.39
		4	3.99	3.51	0.23
		5	5.22	4.94	0.08
		6	6.42	6.37	0.00
		7	7.94	7.79	0.02
		8	8.73	9.22	0.24
		9	9.79	10.65	0.74
		10	10.83	12.08	1.57
150	0.4	2	1.15	0.39	0.57
		3	2.40	1.69	0.50
		4	3.70	2.99	0.50
		5	4.65	4.29	0.13
		6	5.65	5.59	0.00
		7	6.87	6.89	0.00
		8	7.83	8.19	0.13
		9	8.79	9.49	0.49
		10	9.85	10.79	0.89
150	0.5	2	1.12	0.13	0.97
		3	2.17	1.30	0.75
		4	2.94	2.48	0.22
		5	4.10	3.65	0.21
		6	5.38	4.82	0.32
		7	5.97	5.99	0.00
		8	6.92	7.16	0.06
		9	7.93	8.33	0.16
		10	8.97	9.50	0.28
200	0.1	2	1.34	0.32	1.05
		3	2.56	1.65	0.84
		4	3.94	2.98	0.93
		5	4.76	4.30	0.21
		6	6.14	5.63	0.26
		7	7.13	6.96	0.03
		8	8.20	8.29	0.01
		9	9.38	9.62	0.06
		10	10.10	10.95	0.72
200	0.2	2	1.29	0.06	1.52
		3	2.18	1.26	0.85
		4	3.65	2.46	1.42
					continued ...

n	p	K	Actual	Predicted	Sq. Dev.
		5	4.54	3.66	0.78
		6	5.57	4.86	0.51
		7	6.79	6.06	0.53
		8	8.03	7.26	0.59
		9	8.53	8.46	0.01
		10	9.54	9.66	0.01
200	0.3	2	1.17	0.20	1.87
		3	2.08	0.87	1.46
		4	3.13	1.94	1.41
		5	4.17	3.01	1.34
		6	4.94	4.09	0.73
		7	5.94	5.16	0.61
		8	6.93	6.23	0.49
		9	7.78	7.30	0.23
		10	8.46	8.37	0.01
200	0.4	2	0.91	0.46	1.87
		3	1.58	0.49	1.20
		4	2.92	1.43	2.23
		5	3.57	2.37	1.44
		6	4.20	3.31	0.79
		7	5.07	4.25	0.67
		8	6.01	5.20	0.66
		9	6.85	6.14	0.51
		10	7.54	7.08	0.21
200	0.5	2	0.75	0.71	2.15
		3	1.56	0.10	2.14
		4	2.31	0.91	1.96
		5	3.18	1.72	2.12
		6	3.96	2.54	2.02
		7	4.55	3.35	1.44
		8	5.56	4.16	1.95
		9	6.33	4.98	1.83
		10	6.88	5.79	1.19
300	0.1	2	0.95	2.89	3.76
		3	1.68	3.50	3.32
		4	2.59	4.12	2.33
		5	3.37	4.73	1.85
		6	4.21	5.34	1.28
		7	5.14	5.95	0.66
		8	5.74	6.57	0.69
		9	6.67	7.18	0.26
		10	7.08	7.79	0.51
300	0.2	2	0.72	2.63	3.66
		3	1.64	3.12	2.18
		4	2.32	3.60	1.64
		5	3.08	4.08	1.01
		6	4.05	4.57	0.27
		7	4.63	5.05	0.18
					continued ...

n	p	K	Actual	Predicted	Sq. Dev.
		8	5.50	5.54	0.00
		9	5.98	6.02	0.00
		10	6.61	6.50	0.01
300	0.3	2	0.65	2.37	2.97
		3	1.59	2.73	1.30
		4	2.27	3.08	0.66
		5	2.73	3.44	0.50
		6	3.47	3.79	0.10
		7	3.90	4.15	0.06
		8	4.82	4.50	0.10
		9	5.20	4.86	0.12
		10	5.68	5.21	0.22
300	0.4	2	0.50	2.12	2.61
		3	1.60	2.34	0.55
		4	1.85	2.57	0.52
		5	2.65	2.79	0.02
		6	3.01	3.02	0.00
		7	3.67	3.25	0.18
		8	4.00	3.47	0.28
		9	4.91	3.70	1.47
		10	5.37	3.92	2.09
300	0.5	2	0.34	1.86	2.30
		3	1.35	1.95	0.37
		4	1.75	2.05	0.09
		5	2.45	2.15	0.09
		6	3.00	2.25	0.57
		7	3.13	2.34	0.62
		8	3.94	2.44	2.25
		9	4.14	2.54	2.57
		10	5.01	2.63	5.65

Chapter 6

Selecting Between Batch and Zone Order Picking Strategies in a Distribution Center

Abstract

An order picking strategy defines the manner in which pickers navigate the picking area in a distribution center (DC) to pick items from storage locations. The primary objective of a picking strategy is to maximize throughput, or minimize cost or response time, or a combination thereof. Order picking strategies include discrete, batch, zone, and bucket brigade picking. Since most DCs employ some form of a batch or zone picking strategy, we focus on the problem of selecting between a batch picking and a zone picking strategy; we refer to this as the batch versus zone problem. For this problem, we propose a cost model to estimate the cost of each type of picking strategy in an order picking system with the best picking strategy defined as the one that results in least picking system cost. In our cost model we consider the effects of pick-rate, picker blocking, workload-imbalance, and the sorting system requirement. Models for estimating the cost associated with workload-imbalance and the sorting system are presented. Through an example problem, we show how parameters such as system throughput, order sizes, item distribution in orders, and wavelength affect the batch versus zone decision problem.

6.1. Introduction

Order picking refers to the operation of retrieving items from storage locations to fulfill customer orders. It has been identified as the highest priority activity in a distribution center

(DC) for productivity improvement due to its relatively high (about 50%) contribution to the total DC operating cost (Tompkins et al., 2003). While designing an order picking system, a designer must consider the following question: *which picking system best meets a given set of objectives?* Some of the objectives a designer is required to optimize include maximizing throughput or minimizing either cost, space, response time, or error-rate, or a combination thereof. To answer this question, the designer typically follows the standard engineering design process (Tompkins et al., 2003), which involves the following phases: Phase 1: define the problem; Phase 2: analyze the problem, generate alternatives, evaluate the alternatives, and select the preferred design; and Phase 3: implement the design.

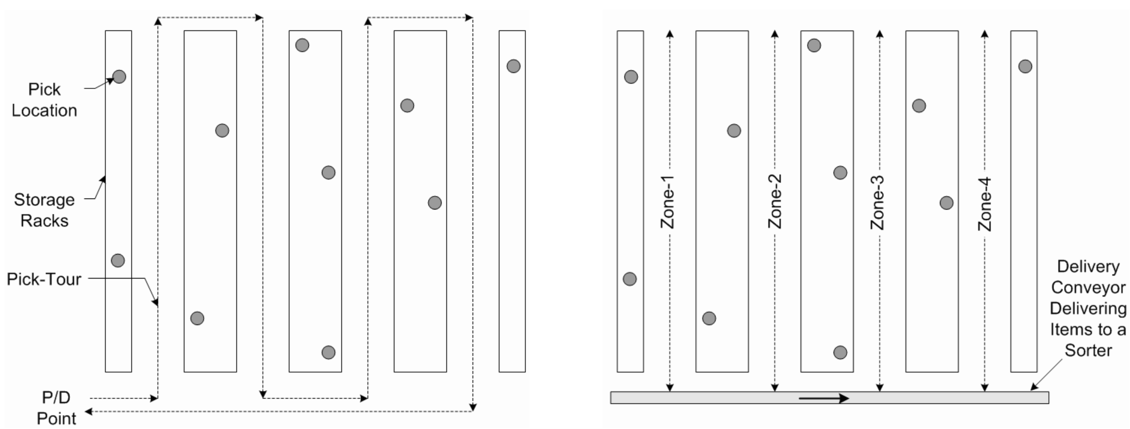
If the designer knows which stock keeping units (a unique identifier of each type of product or item that is stocked in a DC) are to be stored in the picking area and their quantities, then Phase 1 may involve the following problem definitions in an order picking environment:

- What type of storage system should be used?
- Which order picking strategy (i.e., discrete, batch, zone, or bucket brigade) should be used?
- Which picking system (i.e., manual, semi-automated, or automated) should be used?
- Which pick-assist technology (i.e., paper-based or paper-less) should be used?

The first problem has been addressed in Parikh and Meller (2006a). In this chapter we focus on the second problem, selecting an appropriate order picking strategy within a DC. To analyze this problem, according to Phase 2 of the engineering design process, the designer first has to identify possible alternatives for this problem. After that, the designer has to devise a proper methodology to identify the best performing alternative(s) before moving to Phase 3. The focus of this chapter is on identifying various picking strategies and developing a proper methodology to select the best strategy for a picking system (i.e., Phase 2).

There are four order picking strategies: discrete, batch, zone, and bucket brigade. In *discrete picking* a picker is responsible for picking all the items in a single order during a pick-tour. In *batch picking* several orders are batched (or grouped) together and a picker picks all the items in a given batch. *Zone picking* requires that each picker is assigned to a specific region of the storage area and is responsible for picking the items in that region only. *Bucket brigade picking*, which is actually a control policy for executing discrete order picking, requires that as soon as the most downstream picker completes an order, he/she walks back to take over the order the picker immediately upstream of him/her is currently picking. The latter, in turn, takes over the order of his predecessor, and so on until the most upstream picker begins a new order (Bartholdi et al., 2001). Note that, in batch or zone picking, if orders are required to be picked in a predefined time-window (known as a wave), then it is referred to as *wave picking*.

Discrete order picking, though simple to implement, can be labor-intensive for medium-to-high throughput DCs. Moreover, bucket brigade picking is limited to applications where handing-off the items to a downstream picker is easy. It is our observation that most DCs typically do not employ these two strategies; instead they prefer to consider batch or zone picking strategies. Therefore, we consider these two strategies, as illustrated in Figure 6.1, in our modeling.



(a) Batch Picking Strategy (b) Zone Picking Strategy

Figure 6.1: Schematic of Batch and Zone Picking Strategies.

Two types of batch picking strategies exist: pick-and-sort and sort-while pick (Tompkins et al., 2003). In *pick-and-sort* batch picking the pickers *do not* sort (while picking) the items into customer orders. Such a situation can occur if the cart capacity is too small for the pickers to sort the picked items in the cart. The picked items are, therefore, consolidated downstream through a manual or automated sorting system. This means that pick-and-sort batch picking maintains a high pick-rate (items picked per unit time) for the pickers (as no sorting is involved during picking), but requires a downstream sorter. In contrast, in *sort-while-pick* batch picking the pickers simultaneously pick and sort the items into customer orders. Such a situation can occur if the pick-cart capacity is large enough for the pickers to sort the picked items in available compartments in the pick-cart. This means that sort-while-pick batch picking reduces the pick-rate of pickers (due to sorting while picking), but eliminates the need for a downstream sorter. Therefore, there exists a trade-off between pick-rate and the requirement of a sorting system with the two types of batch picking systems.

Note that in both of these types of batch picking systems, pickers travel the entire picking area, which increases their total travel-time. They may also block each other while picking. We elaborate on both of these issues later in this section.

As in batch picking, two types of zone picking strategies exist: sequential and simultaneous (Tompkins et al., 2003). In *sequential* (also known as progressive or pick-and-pass) zone picking, picking occurs one order at a time, and in one zone at a time, after which the order is passed to the next zone. Because only one order is handled at a time, sequential zone picking reduces the pick-rate of pickers, but eliminates the requirement of a downstream sorter. In contrast, in *simultaneous* (also known as synchronized) zone picking, all items corresponding to batched orders are picked simultaneously from all the zones, and then orders are consolidated through a sorting system. Therefore, simultaneous zone picking increases the pick-rate of pickers as the entire batch is picked at a time, but requires a downstream sorter. As a result, with zone picking there exists a trade-off between pick-rate and the requirement of a sorting system. The sequential and simultaneous zone picking strategies are illustrated in Figures 6.2 and 6.3, respectively.

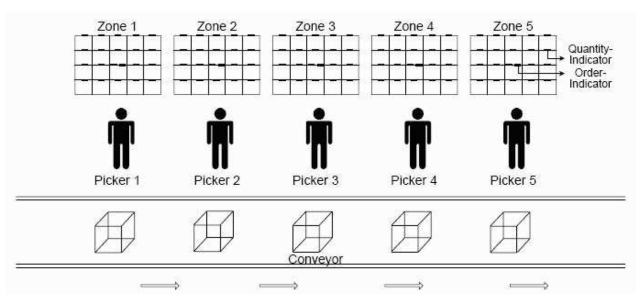


Figure 6.2: A Sequential Zone Picking Strategy (Jane, 2000).

From the above description, we can see that, at a minimum, *pick-rate* and *the requirement of a sorting system* affect the selection of an appropriate picking strategy. In addition, there are two other factors that must be considered: *blocking* and *workload-imbalance*. Blocking may be significant in batch picking systems where pickers move freely in traveling to picks. Blocking results in waiting-times, which reduces the productivity of pickers. Additionally,

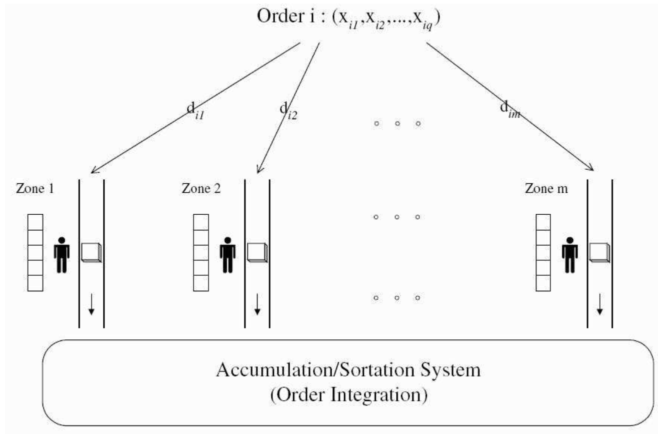


Figure 6.3: A Simultaneous Zone Picking Strategy (Jane and Laih, 2005).

workload-imbalance results when the orders are batched such that unequal workloads are assigned to pickers. Unequal workload may result in orders not being fulfilled in the scheduled hours of operation during the day. If these left over orders are not to affect the next shift's operations, then they will have to be fulfilled through additional labor; e.g., by incurring overtime. Based on our discussions with industry personnel workload-imbalance is an issue more closely related to zone picking systems. Table 6.1 provides a summary of these factors with respect to the advantages and disadvantages of batch and zone picking types.

In deciding whether to employ a batch or zone picking strategy a designer has to account for each of the four factors just discussed: *pick-rate*, *blocking*, *workload-imbalance*, and *sorting*. We refer to this decision problem as the *batch versus zone problem*. One way to address this problem is to estimate the effect of these four factors and incorporate them into a model to estimate batch or zone picking system cost. The components of the system cost may include *labor*, *equipment*, *imbalance*, and *sorting* costs. Labor cost includes the total annualized cost of labor (i.e., pickers, along with pick-carts, and packers) in the system. Equipment cost includes cost of racks, conveyors, etc. Imbalance cost is the cost incurred to fulfill items that were not fulfilled during the scheduled hours of operation in a day; this cost component is explained in more detail later. Sorting cost is the annualized cost of a sorting system, a model for which we develop later.

To illustrate how the four factors mentioned earlier affect the four cost components, assume that a designer is required to select between a sort-while-pick batch picking system and a simultaneous zone picking system. Because sorting is part of the picking process in sort-while-pick batch picking, the pick-rate of the pickers is relatively low. Therefore, the labor

Table 6.1: Summary of Advantages and Disadvantages of Various Types of Batch and Zone Picking Strategies.

Strategy	**Type**	**Advantages**	**Disadvantages**
Batch Picking	Pick-and-Sort	• Increases pick-rate of pickers as sorting not part of picking. • Reduces chances of workload-imbalance.	• Requires a sorting system. • Increases probability of blocking. • Decreases pick-rate of pickers as pick-tours are long.
	Sort-while-Pick	• Reduces chances of workload-imbalance. • Does not require a sorting system.	• Decreases pick-rate as sorting part of picking process. • Decreases pick-rate of pickers as pick-tours are long. • Increases probability of blocking.
Zone Picking	Progressive	• Increases pick-rate of pickers as pick-tours are short. • Does not require a sorting system. • Eliminates blocking.	• Increases chances of workload-imbalance.
	Synchronized	• Increases pick-rate of pickers as pick-tours are short. • Eliminates blocking.	• Requires a sorting system. • Increases chances of workload-imbalance.

cost in sort-while-pick batch picking is high relative to simultaneous zone picking. The equipment cost in simultaneous zone picking may be high if take-away conveyors and additional technology (e.g., pick-to-light) is employed. Excessive workload-imbalance in simultaneous zone picking, compared to sort-while-pick batch picking, will increase the number of orders not fulfilled during the day, which increases the imbalance cost. Furthermore, simultaneous zone picking requires a sorter to sort items into orders, which results in the cost of a sorter.

(Sort-while-pick batch picking does not require a sorter.) Therefore, the designer will have to estimate all four cost components (labor, equipment, imbalance, and sorting) before deciding which one of the two picking strategies should be employed. Our aim in this chapter is to address the batch versus zone problem by developing an appropriate cost model for these two picking systems and to discuss how other combinations of batch and zone picking systems can be compared using the cost model we develop.

The remainder of this chapter is organized as follows. In Section 6.2 we briefly review literature in the area of picking system design. In Section 6.3 we present a cost model to estimate the cost of batch picking and zone picking systems. As mentioned earlier, since our cost model requires estimating costs due to workload-imbalance and sorting system, in Sections 6.4 and 6.5, we present methods for doing so. In Section 6.6 we revisit our cost model for the batch versus zone problem through an illustrative example. Finally, in Section 6.7 we summarize our understanding of the batch versus zone problem.

6.2. Related Research

In the area of overall DC (or warehouse) design, Gray et al. (1992) propose a multi-stage hierarchical decision approach to model the composite design and operating problems for a typical order-consolidation warehouse. Their hierarchical approach utilizes a sequence of coordinated mathematical models to evaluate the major economic tradeoffs and to prune the decision space to a few superior alternatives. Rouwenhorst et al. (2000) present a reference framework and a classification of warehouse design and control problems. They define a warehouse design problem as a "structured approach of decision making at a strategic, tactical, and operational level in an attempt to meet a number of well-defined performance criteria."

In the area of design of a picking system in a DC, Brynzer et al. (1994) present a zero-based analysis methodology for the evaluation of OPSs as a base for system design and managerial decisions. Yoon and Sharp (1996) propose a structured procedure for the analysis and design of a picking system that considers the inter-dependent relationships between different functional areas (e.g., receiving, picking, sorting, etc.). The survey papers by van den Berg (1999), Rouwenhorst et al. (2000), and Gu et al. (2005) provide a complete review of various contributions in the area of DC and picking system design.

The problem of selecting an order picking strategy is critical as it affects the design of aisles and storage systems, picker-routing, etc., all of which affect the throughput of the system. The primary objective of a picking strategy is to maximize throughput, or minimize cost (or response time).

Petersen (2000) states that the choice of a picking strategy can have a tremendous effect on the efficiency and cost of a picking system in mail order companies. To this end, he evaluates five order picking strategies — discrete (or strict), batch, sequential (or pick and pass) zone, simultaneous zone (which he calls batch-zone), and simultaneous zone-wave — using a simulation model. Based on the results, Petersen concludes that simultaneous zone-wave picking and batch picking are superior, and that their performance is not adversely affected by changes in demand skewness patterns or daily order volume. On the other hand, the author notes that the performance of sequential zone and batch-zone picking deteriorates as order volume increases.

Characterizing bucket brigade picking as a self-balancing strategy (and a control policy for discrete order picking), Bartholdi et al. (2001) develop stochastic models to analyze the performance of this strategy in high-volume OPSs. Through simulation experiments comparing bucket brigade picking to sequential zone picking, they suggest that the production rate efficiency (a ratio of realized production rate to maximum possible rate) of a simulated bucket brigade is similar to that of zone picking for the case when the picking system had identical pickers (in terms of their walk and pick times). As the difference in the walk and pick times of individual pickers increases, bucket brigade picking is more productive. The limitation of this strategy is that it is most effective when the picking aisle is structured as a flow line, and when the time to hand-off an order is low.

Jane (2000) considers a sequential zone picking system, which he refers to as a *relay* picking system. He addresses the problem of assigning n products into m storage zones (one picker per zone) with the objective of minimizing the differences that might exist between each picker's total number of picks. Jane and Laih (2005) propose a clustering algorithm for item assignment in a simultaneous zone picking system. The authors propose a similarity measure between any two items for measuring the co-appearance of both items in the same order. Accordingly, items frequently ordered together are located in different zones to minimize the idle time in the simultaneous zone system.

Recently, there has been some work related to the batch versus zone problem. For example, Russell and Meller (2003) develop cost and throughput models for manual and automated sorting systems. Sorting systems are required in pick-and-sort batch picking and simultaneous zone picking systems, and as described earlier, play a major role in the batch versus zone problem. Their research is aimed at providing a design aid deciding whether or not to automate the sorting process. They develop a descriptive model for the sorting system design decision, based on demand levels, labor rates, order sizes, and other factors, and incorporate the descriptive model into a cost-based optimization model to recommend a solution.

Gue et al. (2006) address the effect of pick density on order picking areas in narrow-aisle DCs. Since a batch picking strategy may induce picker-blocking, which can reduce system throughput, they develop analytical models to estimate *in-the-aisle* picker-blocking in a narrow-aisle DC. Using discrete time Markov chains, the authors estimate bounds on the percentage of time each picker is blocked for the case when pickers pick only one item from a pick-face when stopped (e.g., when there may be multiple picks at a stop). Parikh and Meller (2006b) suggest that in many DCs, the aisles are wide enough to permit passing. In such DCs the pickers may still experience blocking whenever two or more pickers need to pick at a pick-face at the same time. They refer this form of blocking as *pick-face* blocking. Similar to Gue et al. (2006), they employ a discrete time Markov chain approach to estimate average picker-blocking in wide-aisle DCs. Parikh and Meller (2006b) also consider the case when there is a high variance in the time spent picking once stopped (which is likely to occur when multiple picks may be possible at a single stop). As mentioned earlier, picker-blocking can play a significant role in the batch versus zone problem.

To our knowledge there has been no research that addresses the batch versus zone problem. We next present a possible approach to address this problem with a cost-model-based approach.

6.3. A Cost Model for the Batch versus Zone Problem

We model the batch versus zone problem on the basis of cost. Accordingly, we develop a cost model to estimate the cost of picking systems employing batch and zone picking strategies. The cost model includes the expected costs of pickers, equipment, imbalance, sorting, and packers. (Note that we represent the expected cost of labor as the expected cost of pickers and packers and throughout the chapter, for simplicity, we use x instead of $E[x]$ when representing the expected value of x.) Let C^b and C^z represent batch and zone picking system costs, respectively, which can be estimated as follows:

$$C^b = P^b c_p + c_e^b + U^b c_u + c_s(\alpha^b I) + K^b c_k \qquad \text{and}$$
$$C^z = P^z c_p + c_e^z + U^z c_u + c_s(\alpha^z I) + K^z c_k, \tag{6.1}$$

where

- I = expected number of items to be picked,
- P^b (P^z) = expected number of pickers required in batch (zone) picking,
- c_p = annualized loaded cost of a picker ($); including the cost of picking vehicle,
- c_e^b (c_e^z) = annualized equipment cost in batch (zone) picking; including the cost of racks, conveyors, etc. ($),
- U^b (U^z) = expected number of items in orders unfulfilled due to workload-imbalance in a day in batch (zone) picking,
- c_u = yearly unit imbalance cost to fulfil an item ($/unit/year); including the costs associated with overtime of pickers, packers, sorter, shipping personnel, etc.,
- K^b (K^z) = expected number of packers required in batch (zone) picking to pack the picked (and sorted) items into shipping cartons before sending these cartons to shipping,
- c_k = annualized loaded cost of a packer ($),
- α^b (α^z) = binary parameter for batch (zone) picking; 1, if sorter required, and 0, otherwise; $\alpha^b = 1$ for pick-and-sort batch picking, while $\alpha^z = 1$ for simultaneous zone picking, and
- $c_s(I)$ = annualized cost of a sorting system to sort I ($I > 0$) items ($); includes the cost of a sorting conveyor, sort-lanes (with diverting mechanisms), induction stations, and inductors.

Some of the cost components in 6.1 are relatively straightforward to estimate, while others are more difficult to estimate. For example, estimating the costs associated with pickers, equipment, and packers is not difficult. However, estimating the imbalance and sorting system costs is difficult. Therefore, in the next two sections we develop models to estimate these two cost components. Once all the cost components are estimated, the designer can use (6.1) to estimate the cost of employing either a batch picking or a zone picking strategy. We define the best picking strategy as the one that corresponds to the minimum of the two system costs, C^b and C^z; i.e., if C represents the cost of the least expensive picking system, then the batch versus zone problem can be defined as

$$\begin{aligned} \text{minimize} \quad & C \\ \text{subject to} \quad & C = \min\{C^b, C^z\}. \end{aligned}$$

6.4. Estimating the Imbalance Cost in Picking Systems

In picking systems it is important that the pickers are assigned a workload that is nearly the same for the pickers within and across all waves. A balanced workload will help all the pickers finish their workload at the exact same time. The advantage of doing so, in addition to minimizing the labor cost, is the efficient scheduling of sorting and shipping operations. Moreover, a balanced workload ensures that all customer orders are fulfilled in the scheduled hours of operation during the day so that there is no overtime, and hence, eliminating the imbalance cost.

Workload-imbalance occurs when the workload assigned amongst pickers is not equal. To illustrate this point further, consider an example of a picking area that consists of 2 aisles in which 2 pickers perform the picking operation. The system operates 2 waves, and each picker can pick a maximum of 10 items in a wave. Assume that 10 orders (with a total of 40 items) have to be fulfilled in a day. The distribution of items within these 10 orders over 2 aisles is illustrated in Table 6.2.

For this example, if a batch picking system is employed (where both pickers travel the 2 aisles), then we need to only focus on the last column in Table 6.2. A possible way in which the 10 orders can be batched to reduce workload-imbalance between the two pickers is illustrated in Table 6.3. In the table we observe that all 10 orders have been assigned to one of the two pickers, and both the pickers have been assigned a balanced workload of 10 items in each wave. That is, all the orders have been assigned, and the workload amongst the pickers is balanced.

Note that we have used the total number of items picked as a surrogate for workload (as

Table 6.2: Distribution of Items in the 10-Order Example.

Order #	**Aisle 1**	**Aisle 2**	**Total Items**
1	1	2	3
2	5	0	5
3	2	3	5
4	2	0	2
5	0	4	4
6	1	4	5
7	3	2	5
8	0	4	4
9	2	2	4
10	1	2	3
Total	17	23	40

opposed to the total time for a picker directly). Our reasoning for this is as follows: The total time for a picker is the sum of total travel-time, which depends on the routing of pickers, and total pick-time. Since we are mostly concerned with medium-to-high throughput DCs, we assume here that the pickers pass by each pick location once during a pick-tour (Gue et al., 2006); therefore, the total travel-time is fixed, and we can eliminate it from our model. This leaves us with pick-time, which depends on the number of items picked. Furthermore, similar to Gue et al. (2006), we assume that pick-time is linear in the number of items picked.

Table 6.3: A Possible Order Batching Solution for the Batch Picking System for the 10-Order Example.

		Orders Assigned	**Total Items**
Wave 1	Picker 1	2, 3	10
	Picker 2	6, 7	10
Wave 2	Picker 1	1, 9, 10	10
	Picker 2	4, 5, 8	10
Total			40

Now consider a zone picking system, where each aisle is a zone and one picker is assigned to each zone. We therefore now need to focus on the middle two columns in Table 6.2. A possible way by which orders can be batched to reduce workload-imbalance between the two pickers is illustrated in Table 6.4. Notice that a total of 9 out of 10 orders have been assigned during the two waves. Order #8 is not assigned because it could not be accommodated during either of the waves. That is, if this order were included in one of the waves, then it would violate the capacity of Zone 2 (which is 10 items). As a result, Order #8 will have to be picked during overtime. That is, not all orders have been assigned, and the workload amongst zones is not balanced.

Table 6.4: A Possible Order Batching Solution for a Zone Picking System.

		Orders Assigned	**Total Items**
Wave 1	Zone 1	1, 2, 5, 9, 10	9
	Zone 2	1, 2, 5, 9, 10	10
Wave 2	Zone 1	3, 4, 6, 7	8
	Zone 2	3, 4, 6, 7	9
Total			36

In general the following two factors affect the process of generating a balanced workload for pickers: (i) item to zone assignment; and (ii) order batching. Considering (i), if the items are inappropriately assigned to zones, then it implies that a few zones are assigned a larger pool of frequently requested items than other zones. As a result, when the orders are batched, the pickers in these few zones may be required to pick a larger number of items than the pickers in other zones. This results in workload-imbalance. Considering (ii), if the orders are inappropriately batched, then more workload may be assigned to some pickers than others. This may be because of an inappropriate policy used for batching orders. This results in workload-imbalance. Workload-imbalance, due to either (i) and/or (ii), may result in customer orders not being fulfilled during the scheduled hours of operation in a day. We, therefore, use the number of items (in customer orders) unfulfilled as a measure of workload-imbalance and refer to the cost of fulfilling these unfulfilled items as the *imbalance cost*.

In this chapter we currently assume that the items are appropriately assigned to zones. Therefore, we eliminate reason (i) listed above, and focus on (ii), which relates to the order batching policy used. We also assume that the order pool is static: i.e., the order pool contains only those orders received before a certain time in the day.

To quantify the effects of workload-imbalance for batch and zone picking strategies on equal footing, we formulate mathematical models to batch orders. With these models, we can identify the orders that were not fulfilled during the scheduled hours of operation (which determines the imbalance cost). We use a dual bin-packing problem analogy to discuss our model formulations.

6.4.1 Order Batching Models

The classical bin-packing problem (BPP) is defined as follows: *given a number of items of integer sizes, what is the minimum number of bins, each having the same integer capacity, necessary to assign all items?* We can draw an analogy between our problem and the BPP by considering that (i) a bin represents a picker, (ii) the capacity of the bin represents the predefined capacity of the picker in terms of the number of items that can be picked, and (iii) the assignment of items to bins represents the assignment of orders to pickers. Therefore, we can define the order batching problem (based on the BPP) as follows: *given a number of orders consisting of items (integers), what is the minimum number of pickers, each having the same integer capacity, necessary to fulfill all orders?* This analogy suggests that the number of pickers is a decision variable. That is, if the number of orders to be fulfilled changes daily, then the number of pickers may also change daily. As the hiring and firing of pickers on daily basis is generally not feasible, most DCs fix the number of pickers for a certain time-period based on the expected demand for that time-period. Hence, the BPP analogy does not hold completely for the order batching problem.

Instead, formulating the order batching problem analogous to the dual bin packing problem (dual BPP) seems appropriate. The dual BPP is defined as follows: *given a fixed number of capacitated bins, what is the maximum number of items that can be packed?* The analogy between items and orders, and bins and pickers still holds; however, the definition of the order batching problem (based on the dual BPP) changes and is as follows: *given a fixed number of pickers, what is the maximum number of items that can be fulfilled?* In the order batching considered here we are given a fixed number of pickers picking in a given number of waves. Each picker can pick no more than a certain fixed number of items during each wave. The objective of the order batching problem is to balance the workload amongst pickers such that the number of items fulfilled during the scheduled hours of operation is maximized. That is, the order batching problem we consider follows very closely to the dual BPP. We assume that any unfulfilled items (in orders) during the scheduled hours of operation will need to be fulfilled by incurring overtime, which will increase the imbalance cost.

First consider the order batching problem for batch picking, which we will use to identify the optimal assignment of orders to pickers in a batch picking system. To facilitate model formulation we use the number of items as a surrogate for the workload of each picker. Let

- i = index for orders; $i = 1, ..., D$,
- j = index for pickers; $j = 1, ..., P$,
- k = index for waves; $k = 1, ..., W$,
- d_i = parameter for the number of items in order i,
- M^b = parameter for the maximum number of items that can be picked by a picker in a wave in a batch picking system, and
- $x_{ijk} = \begin{cases} 1, & \text{if order } i \text{ is assigned to picker } j \text{ in wave } k \\ 0, & \text{otherwise.} \end{cases}$

Note that the parameter M^b depends on the expected pick-rate of the pickers; i.e., number of items picked in unit time. The expected value of pick-rate depends on the following three factors: (i) pick-time, (ii) travel-time, and (iii) pick-cart capacity. The expected value of pick-time can be obtained through time studies. The expected value of travel-time depends on the picker-routing policy (e.g., traversal, largest gap, return, etc.) and can be estimated through models proposed in the order picking literature (e.g., Hall (1993), Ratliff and Rosenthal (1983), or Roodbergen and Vis (2006)). The pick-cart capacity is assumed to be a fixed constant depending on the type of the pick-cart used and the sizes of items picked. For simplicity, we assume that the expected value of pick-rate of the pickers has already been established through time-studies after accounting for factors (i), (ii), and (iii).

The mathematical model for the order batching problem for batch picking (labeled OBP-BP) is as follows:

$$\begin{aligned}
\text{OBP-BP : maximize} \quad & \sum_{i=1}^{D}\sum_{j=1}^{Z}\sum_{k=1}^{W} d_i x_{ijk} && && (6.2)\\
\text{subject to} \quad & \sum_{i=1}^{D} d_i x_{ijk} = b_{jk} && \forall j,k && (6.3)\\
& b_{jk} \le M^b && \forall j,k && (6.4)\\
& \sum_{j=1}^{P}\sum_{k=1}^{W} x_{ijk} \le 1 && \forall i && (6.5)\\
& x_{ijk} \in \{0,1\} && \forall i,j,k &&
\end{aligned}$$

In the above formulation the decision variable is x_{ijk}; i.e., the assignment of order i to picker j in wave k. We calculate the current number of items assigned to each picker in a wave (i.e., b_{jk}) through (6.3). In (6.4) we ensure that no b_{jk} value exceeds the capacity of a picker (M^b). We use (6.5) to ensure that no order is assigned to more than one picker in one of the W waves. That is, an order i, if assigned to a wave k, may be assigned to no more than one picker j. The objective, represented by (6.2), is to maximize the number of items fulfilled during the scheduled hours of operation.

By solving the model OBP-BP we can obtain the values of b_{jk}, which provide the current workload of each picker during a wave. The orders with their x_{ijk} value equal to zero could not be included in the batches formed, and hence will not be fulfilled during the scheduled hours of operation. The total number of items corresponding to these unfulfilled orders (say U^b) can be calculated as the difference between the total items to be picked and the objective function value. That is,

$$U^b = \sum_{i=1}^{D} d_i - \sum_{i=1}^{D}\sum_{j=1}^{Z}\sum_{k=1}^{W} d_i x_{ijk}.$$

Next we consider the order batching problem for zone picking in which we assume that only one picker is assigned to each zone, and modify the OBP-BP model, to identify the optimal assignment of orders to pickers in a zone picking system. There are two key differences between the batch and zone picking models: (i) along with the number of items in an order, we need to consider the distribution of items in that order, and (ii) we only need to consider the assignment of orders to waves, because when an order is assigned to a wave, it is automatically assigned to all the applicable zones from where the items within that order are to be picked. Accordingly, let

- d_{ij} = parameter for the number of items in order i to be picked from zone j,
- M^z = parameter for the maximum number of items that can be picked by a picker in a wave in a batch picking system, and
- $x_{ik} = \begin{cases} 1, & \text{if order } i \text{ is assigned to wave } k \\ 0, & \text{otherwise.} \end{cases}$

Note that the values of M^b and M^z may be different as the pick-rates of pickers in a batch picking and a zone picking system are likely to be different (as illustrated later).

A mathematical model for the order batching problem for zone picking (labeled OBP-ZP) is as follows:

$$
\begin{aligned}
\text{OBP-ZP : maximize} \quad & \sum_{i=1}^{D}\sum_{j=1}^{Z}\sum_{k=1}^{W} d_{ij}x_{ik} && && (6.6)\\
\text{subject to} \quad & \sum_{i=1}^{D} d_{ij}x_{ik} = b_{jk} && \forall j,k && (6.7)\\
& b_{jk} \leq M^z && \forall j,k && (6.8)\\
& \sum_{k=1}^{W} x_{ik} \leq 1 && \forall i && (6.9)\\
& x_{ik} \in \{0,1\} && \forall i,k &&
\end{aligned}
$$

In this formulation the decision variable is x_{ik}; i.e., the assignment of order i to wave k. The current number of items assigned to each zone in a wave (b_{jk}) is specified using (6.7). In (6.8) we ensure that no b_{jk} value exceeds the capacity of a zone (M^z). We use (6.9) to ensure that no order is assigned to more than one wave. The objective, represented by (6.6), is to maximize the total number of items fulfilled.

Similar to the OBP-BP model, by solving the OBP-ZP model we can obtain the values of b_{jk}, which provide the current workload of each picker during a wave. The total number of items corresponding to the unfulfilled orders (say U^z) can be calculated as the difference between the total items to be picked and the objective function value. That is,

$$U^z = \sum_{i=1}^{D}\sum_{j=1}^{Z} d_{ij} - \sum_{i=1}^{D}\sum_{j=1}^{Z}\sum_{k=1}^{W} d_{ij}x_{ik}.$$

Note that the number of binary decision variables in the OBP-ZP model (i.e., x_{ik}) is smaller than the number of binary decision variables in the OBP-BP model (i.e., x_{ijk}). This is because in zone picking an order is assigned directly to wave k, while in batch picking an order is assigned to a specific picker j in a wave k. Consequently, the OBP-BP model may be considerably harder to solve as compared to the OBP-ZP model, as we illustrate in Section 6.6.

We next, through a small experimental study, illustrate the use of our order batching models to estimate workload-imbalance (in terms of the number of items unfulfilled).

6.4.2 Experimental Study

In our experimental study we consider an example problem where the picking area consists of 16 aisles, where each aisle is 100 ft long. An 8-hr shift with twelve 40-min waves is considered. Two picking systems are considered, sort-while-pick batch picking and simultaneous zone picking. The pick-tour in a batch picking system covers all 16 aisles; however, the pick-tour in a zone picking system will cover only the number of aisles corresponding to the zone-size (i.e., the number of aisles in a zone). (In this example the zone-size in a zone picking system may be 1, 2, 4, or 8 aisles.) Smaller zone-sizes in a zone picking system, compared to that in a batch picking system, decrease the travel-time of pickers, which increases their pick-rates. Moreover, the pick-time in a simultaneous zone picking system is relatively small as pickers are not required to sort items while picking, unlike in a sort-while-pick batching picking system. In this example we assume that the the expected value of pick-rate of pickers in a sort-while-pick batch picking system is 150 items/hr. The zone-sizes in a simultaneous zone picking system, and the corresponding expected value of pick-rates of pickers, are as follows: (a) for a zone-size equal to 8 aisles (2 pickers, each assigned 8 aisles), the expected pick-rate is 375 items/hr; (b) for a zone-size equal to 4 aisles (4 pickers, each assigned 4 aisles), the expected pick-rate is 469 items/hr; and (c) for a zone-size equal to 2 aisles (8 pickers, each assigned 2 aisles), the expected pick-rate is 564 items/hr.

The following parameters (and their values) were included in estimating these pick-rates: (i) the walk-speed is 100 fpm, (ii) the pick-time in a sort-while-pick batch picking system is 8 s, while it is 6 s in a simultaneous zone picking system, (iii) the cross-aisle width is 10 ft, (iv) the time to drop-off the picked items at the depot at the end of each tour in a sort-while-pick batch picking system is 20 s, while the time to drop-off picked items at the end of each aisle in a simultaneous zone picking system is 11 s, (v) the cart capacity is 75 items, and (vi) the wavelength is 60 min. So, for example, if the pickers follow a traversal routing policy in a sort-while-pick batch picking system, then they will have to tour the picking area 2 times in a 60-min wave. Accordingly, the total travel-time of pickers in such a system will be 39.12 min, and the total drop-off time is 0.67 min. Consequently, the time available for picking is 20.21 min, which at 8 s per pick gives 150 items/hr. However, in a simultaneous zone picking system, with a zone-size equal to 8 aisles, the total travel-time of pickers (touring the picking area twice in a wave) is 19.56 min and the total drop-off time is 2.93 min. Therefore, the total time available for picking is 37.51 min, which at 6 s per pick gives 375 items/hr. All other pick-rates were estimated in a similar manner.

Note that the order batching problem for batch picking requires only the number of items in an order. In contrast, the order batching problem for zone picking requires both the number and distribution of items in orders over the zones. For example, an order may consist of 20 items. There exist numerous possibilities by which these 20 items may be distributed over the 16 aisles (e.g., 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, and 2, or 5, 4, 0, 0, 0, 3, 0, 3, 0, 3, 0, 2, 0, 0, 0, and 0, etc.). That is, the items may be *nearly* uniformly distributed across the aisles or they may be *skewed* (due to seasonal fluctuations). Notice that the distribution of items in an order will not affect the order batching problem for batch picking, however, it may affect the order batching problem for zone picking.

In addition to solving the order batching problem using the OBP-BP and OBP-ZP models, we also include the following four order batching policies:

1. a first-come-first-serve (FCFS) order batching policy, in which orders are batched in the order they are received, with the batching initiated by first considering the lowest indexed picker (or wave);

2. a look-ahead FCFS policy (FCFS-50), in which the FCFS order batching policy is used and whenever an order cannot be allocated to a picker (or wave) because of the picker-capacity (M^b or M^z) constraint, the subsequent 50 orders are checked for inclusion in the current batch;

3. a look-ahead FCFS policy (FCFS-200), in which, instead of checking the subsequent 50 orders, the subsequent 200 orders are checked for inclusion in the current batch; and

4. a rule similar to first-fit-decreasing (FFD) policy typically employed for solving BPPs and dual BPPs, in which the orders are first sorted in decreasing order in terms of number of the items, and then each successive order is allocated to a picker (or wave) starting from the lowest indexed picker (or wave).

With this setup, we designed a small experiment to illustrate the effects of order sizes, item distribution in orders, and number of waves on the order batching problem and identified the resulting number of items unfulfilled. To analyze the effect of order sizes, we consider two customer order sizes: small and large. Small-sized orders (typical of direct-to-consumer DCs) contain on average 3 items with a range of 1–5 items, while large-sized (typical of retail DCs) orders contain on average 20 items with a range of 10–30 items. For each order size, we consider three picking system throughputs: low, medium, and high. In our experiments, for small-sized orders, low, medium, and high system throughputs correspond to 300 (6,000), 750 (15,000), and 1,800 (36,000) orders (items) per day, respectively. For large-sized orders, these values correspond to 2,000 (6,000), 5,000 (15,000), and 12,000 (36,000) orders (items) per day, respectively. To analyze the effect of item distribution, we consider two distributions, D1 (a *uniform* distribution) and D2 (a *right-skewed* distribution). We generate one set of order for each of these possibilities; a total of 12 sets of orders.

To analyze the effect of number of waves, we consider 6 possibilities: one 8-hr wave, two 4-hr waves, four 2-hr waves, eight 1-hr waves, twelve 40-min waves, and sixteen 30-min waves. The selection of an appropriate number of waves is critical for any order picking system as its selection has cost implications, most notably in terms of the size of the sorting system. That is, the size of the sorting system is much larger in terms of the number of sort-lanes (and thus, expensive) in a system with four 1-hr waves due to the relatively large number of orders to be sorted compared to a system with sixteen 30-min waves.

A lower bound (LB) was obtained for each of the problems by solving a linear relaxation of the OBP-BP and OBP-ZP models. The experimental outcomes are included in various tables and figures in Appendices A and B. The following observations summarize the information in these appendices:

Observation 1: In terms of the number of items unfulfilled in the order batching problem for batch and zone picking, the FFD policy performs at least as well as, and in most cases out-performs, the other three policies, FCFS, FCFS-50, and FCFS-200, for the two order sizes and the two item distributions. Moreover, amongst the FCFS policies the FCFS policy performs the worst, while the FCFS-200 policy does not perform worst than the FCFS-50 policy.

Theorem 2 *The number of items fulfilled (i.e., the objective function value in the OBP-BP and OBP-ZP models, as presented in (6.2) and (6.6)) using the FCFS-200 policy is never* *less than the number of items fulfilled using the FCFS-50 policy.*

Proof: By contradiction. Accordingly, we assume that the number of items fulfilled (i.e., the objective function value in the OBP-BP and OBP-ZP models, as presented in (6.2) and (6.6)) using the FCFS-50 and FCFS-200 policies follow the relationship

$$N_{200} < N_{50}, \tag{6.10}$$

where N_{50} and N_{200} are the objective function values obtained using the FCFS-50 and FCFS-200 policies, respectively.

Note that both the FCFS-50 and FCFS-200 batching policies start assigning orders to the current batch in the same way as the FCFS policy. Let N represent the objective function value of the current batch when using the FCFS-50 and FCFS-200 policies just before identifying that the next order, say d_v, cannot be assigned to the current batch as it will violate the batch-capacity.

When the order d_v is encountered, the subsequent 50 and 200 orders for FCFS-50 and FCFS-200, respectively, are scanned. Let n_{50} and n_{200} represent the number of items in orders found in the subsequent 50 and 200 orders, respectively, starting from order number d_v, that fit in the current batch. Then, $N_{50} = N + n_{50}$ and $N_{200} = N + n_{200}$. Therefore, from (6.10),

$$n_{200} < n_{50}. \tag{6.11}$$

As the next 50 or 200 orders (depending on the policy used) are scanned only once before the current batch is considered *completely assigned* (after which it is removed from further consideration for the batch), the set of orders $\{1, 2, \ldots, 50\}$ is actually a *subset* of the set of orders $\{1, 2, \ldots, 50, \ldots, 200\}$. Therefore, $n_{200} \geq n_{50}$, which contradicts (6.11). □

The FFD policy consistently achieves the optimal (or near-optimal) solution to the order batching problem for both the batch and zone picking systems when the order sizes are small. However, as the order size increases the solution gap between the FFD policy and the optimal (or best) solution obtained by solving the OBP-BP and OBP-ZP models increases to a point where the gap may be a motivation to use the OBP optimization models. (The remaining observations are therefore based on the solutions obtained from the FFD policy and/or the OBP optimization models.)

Observation 2: The number of items unfulfilled due to workload-imbalance is higher in a zone picking system than in a batch picking system for various order sizes, item distributions, and number of waves. For the small-sized order problems we solved, there were never any items remaining unfulfilled in the batch picking system; however, the number of unfulfilled items were high in the zone picking system depending on the distribution of items in orders. For large-sized orders, workload-imbalance affects both the batch picking and zone picking systems; however, this effect is always greater in the latter.

Observation 3: The item distribution in orders has a major effect on the number of items unfulfilled in zone picking systems. In our study we observed that the number of items unfulfilled in orders generated using Distribution D2 (skewed) is higher than that using Distribution D1 (uniform). As noted earlier, item distribution in the orders does not impact the order batching problem for batch picking systems.

Observation 4: For small-sized orders, the number of items unfulfilled is unaffected by changes in the number of waves. However, for large-sized orders, as the number of waves increases, the number of items unfulfilled increases (nearly-linearly).

Observation 5: In terms of solution time, all four policies (FCFS, FCFS-50, FCFS-200, and FFD) take less than 20 seconds on a standard personal computer to solve the order batching problems during the study. While all instances of the OBP-ZP model for small-sized orders were solved optimally in a reasonable time (less than 305 seconds), a few instances for large-sized problems could not be solved optimally even after 24-hrs of processing. In contrast, except for one instance no other OBP-BP model for small-sized or large-sized orders could be solved optimally after 24-hrs of processing. As was highlighted earlier, this is likely due to the number of binary variables in the OBP-BP model, which is larger than in the OBP-ZP model. Although a large number of binary decision variables in the OBP-BP model provides flexibility in assigning orders to pickers, and thus, decreases the number of unfulfilled items, solving such a problem is difficult. In contrast, the number of binary variables in the OBP-ZP model is relatively small, which decreases the flexibility of the model to assign orders to pickers. As a result, the number of items unfulfilled in a zone picking system is relatively high.

Though we observed that the FFD policy performed very well when assigning items to pickers, the solution obtained does not guarantee that all pickers will be assigned a nearly-equal number of orders. That is, in the operational sense, a few pickers will be assigned a relatively small number of orders that each contain a large number of items, while the others will be assigned a relatively large number of orders that each contain a small number of items. It is possible to devise a simple algorithm that would accept the order-to-picker assignment obtained through the FFD policy and then perform a local search on the assignment of orders to pickers without changing the objective function value (i.e., the assignment of items to pickers). In so doing, our objective will be to obtain a batching that has a nearly-equal number of items, and a nearly-equal number of orders, assigned to each picker.

As stated at the outset, the workload-imbalance cost issue is only one issue that must be considered in the batch versus zone problem. Now that we have this issue of the problem modeled, the other issue needing further study is the cost of a sorting system.

6.5. Estimating the Sorting System Cost

We are most interested in modeling the cost of an automated sorting system, which is comprised of a sorting conveyor, sort-lanes (with diverting mechanisms), one or more inductors at induction stations, and hardware and software controls. A sorting conveyor is required to transfer the items to the sort-lanes. The sort-lanes are indexed according to customer order(s) and typically employ gravity chutes or powered roller conveyors to carry the sorted item(s) to packers. The packers pack these orders into shipping cartons and then send them to shipping. A diverting mechanism at the sort-lane helps to divert items on the sorting conveyor into the sort-lane. This can be achieved either through a pivot arm, pop-up wheels, a tilt-tray, a bomb-bay tray, a cross-belt conveyor, or sliding shoes. An induction station consists of one or more inductors (manual or automatic) that place items onto the sorting conveyor. The hardware and software controls associate the incoming item with the sort-lane, execute the sorting operation, and ensure proper system operation.

We estimate the cost of a sorting system as the sum of fixed and variable costs. The fixed cost of a sorter is the cost of the sorting conveyor and associated hardware and software controls. The variable cost depends on the number of induction stations, inductors per induction station, and sort-lanes (with diverting mechanisms). Therefore, the cost of a sorting system ($c_s(I)$) can be estimated as follows:

$$c_s(I) = c_f + n_l c_l + n_s c_s + n_i c_i, \tag{6.12}$$

where

- c_f = fixed cost of the sorter,
- n_l = number of sort-lanes required,
- c_l = annualized cost of a sort-lane ($),
- n_s = number of induction stations required,
- c_s = annualized cost of an induction station ($),
- n_i = number of inductors required, and
- c_i = annualized loaded cost of labor ($).

Note that our model is similar to the one presented in Russell and Meller (2003). However, our contribution is two-fold:

1. In our model we consider the cost of sort-lanes. This is important since the number of sort-lanes impacts the system cost and to not consider the sort-lanes would lead to an inaccurate model.

2. We develop our model from the point-of-view of the number of orders that needs to be sorted and not by the components in the sorter. That is, by evaluating the sorter cost over the entire range or orders that a sorter (with a given maximum conveyor speed) is capable of sorting, we can accurately reflect the economies-of-scale and diminishing returns present in a sorter. By focusing on what the sorter does (sort orders) instead of how it does this (with components), we ensure that the costs can be accurately represented to specify sorter cost *a priori* of making a decision on the size of the sorter to be used. This aspect is explained through a parametric cost function later in this section.

Note that in a sorter the number of sort-lanes depends on the total number of orders to be sorted and the number of orders assigned to each sort-lane. That is, if D^w represents the total number of orders to be sorted in a wave and d_l represents the number of orders that can be assigned to a sort-lane, then the number of sort-lanes required (n_l) can be estimated as, $n_l = \left\lceil \frac{D^w}{d_l} \right\rceil$.

To estimate the number of inductions stations (n_s) and inductors (n_i) required for the sorting system, we adopt the approach presented in Russell and Meller (2003). We summarize key expressions from their work here. Let y represent the total induction rate in items/min; $y = (D^w I_{avg})/60$, where D^w and I_{avg} represent the number of orders in a wave and the average number of items in an order, respectively. Let s represent the speed of the sorting conveyor (in items/min) and λ represent the nominal induction rate of the inductors (in items/min). Therefore, the total number of induction stations (n_s) can be estimated as

$$n_s = \begin{cases} 1, & \text{if } y \leq s \\ \left\lceil \dfrac{y}{2s - y} \right\rceil, & \text{otherwise.} \end{cases}$$

The total number of inductors (n_i) can be estimated using the model developed by Johnson and Meller (2002). Using their geometric model, Russell and Meller (2003) constructed a table (Table A1 in the Appendix of their article) to estimate the induction rate of each inductor at an induction station. They assume that at most 4 identical induction stations may be used with no more than 4 inductors at each induction station. Using the appropriate row in their table, the effective rate of each inductor as a percentage of the conveyor speed can be determined. This percentage, when multiplied by the conveyor speed, determines the actual induction rate (in items/min) of each inductor at each induction station. The number of inductors at each station can then be estimated by adding the number of items for the first inductor, then the second inductor, and so on, until the required number of items to be inducted at the station is reached (i.e., y/n_s). The total number of inductors (n_i) is the sum of inductors at all n_s stations.

For example, let $y = 50$ items/min, $s = 100$ items/min, and $\lambda = 20$ items/min. As $y < s$, $n_s = 1$; i.e., the number of induction stations required is equal to 1. With a nominal induction rate of 20 items/min per inductor, the nominal rate as a percentage of conveyor speed is 20% (20/100). From the Table A1 presented in Russell and Meller (2003), we observe that the induction rates (in %) for the 4 possible inductors at the induction station are 20.00%, 19.05%, 17.73%, and 15.84%, respectively; the corresponding induction rates (in items/min) are 20.00, 19.05, 17.73, and 15.84, respectively. Consequently, to satisfy the required total induction rate ($y = 50$ items/min), 3 inductors are required ($20.00 + 19.05 + 17.73 = 56.78 > 50$).

Thus, we can estimate the required sorting system parameter values, and consequently, the cost of the sorting system using (6.12). Figure 6.4 illustrates a parametric cost function to estimate the cost of a sorting system, where l represents the minimum increments of sort-lanes to consider; an actual cost function is illustrated in Section 6.6.

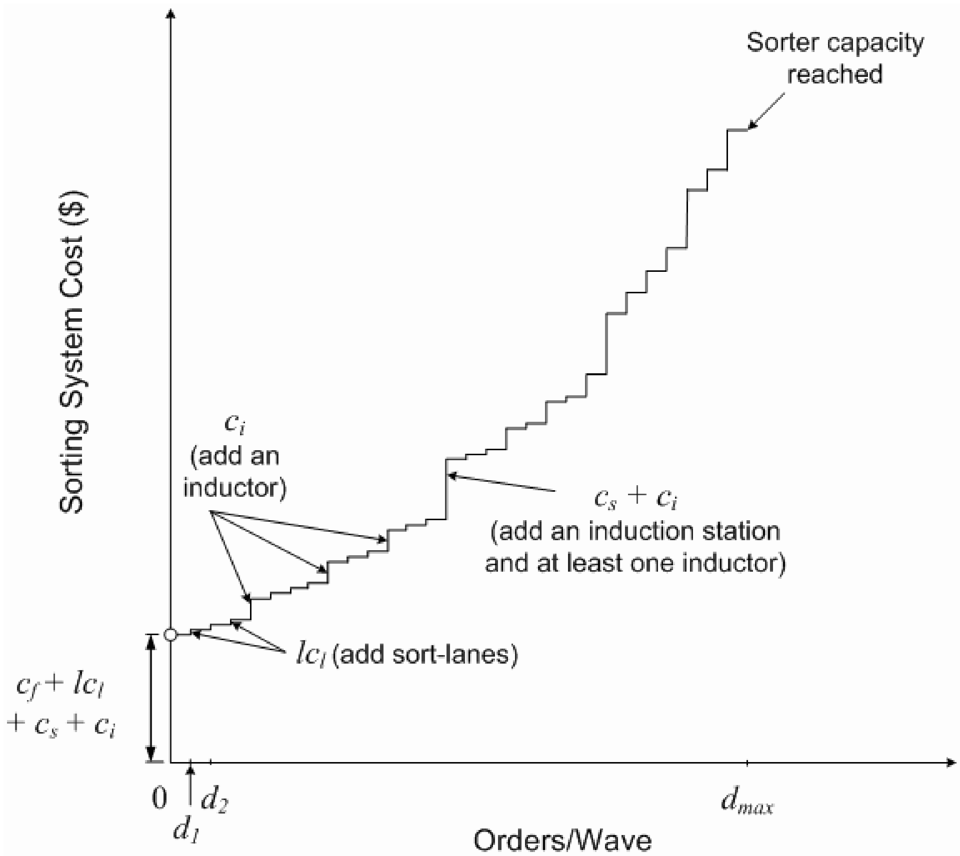


Figure 6.4: A Parametric Cost Function to Estimate the Cost of a Sorting System.

In Figure 6.4 the term $c_f + lc_l + c_s + c_i$ indicates the initial cost of the sorter installed with l sort-lanes and one induction station with one inductor. Depending on the nominal induction rate of inductors and the conveyor speed such a configuration may be suitable for sorting a certain number of orders (say, d_1). If more orders were to be sorted (say, d_2), then additional sort-lanes will be required, the cost of which may be estimated as lc_l. If the orders to be sorted increases further, it is possible that one inductor at the induction station may not be able to meet the total induction rate of the system (y). This means that more inductors will be required at that induction station; this is indicated by the cost component c_i in Figure 6.4. If the sum of the induction rates of all inductors at an induction station is greater than s but less than the total induction rate (y), then a second induction station will need to be employed; this is indicated by the term $c_s + c_i$ in Figure 6.4. Note that at least one additional inductor will be required and that reallocation of the inductors across all induction stations will occur. The process of adding sort-lanes, inductors, and induction stations will continue until the total induction rate (y), corresponding to orders to be sorted in a wave, is met.

Note that as with any other system, the law of diminishing returns applies to the sorting system as well. Accordingly, the additional number of orders that can be sorted by adding the second induction station is less than the number of orders that can be sorted with the first station. The primary reason for this relationship, as shown by Johnson and Meller (2002), is the blocking experienced by inductors as the number of inductors or induction stations increases. Moreover, space constraints may prohibit the installation of more induction stations. These constraints require that a sorter will only be able to sort a certain number of orders in a wave; this is denoted by d_{max} in Figure 6.4.

Note that the relationship between the cost of the sorting system and the number of orders to be sorted in a wave is not linear; it is non-linear, in fact, strictly convex (for $I > 0$). The non-linearity is because of the diminishing returns in the induction rates due to increased levels of induction blocking in larger sorting systems. Therefore, the cost to increase the number of orders sorted increases rapidly as d_{max} is approached.

A detailed example of utilizing this sorter cost model is provided in the next section where we revisit the batch versus zone problem cost model presented in Section 6.3.

6.6. Cost Model Revisited Through an Example

We now present an illustrative example to indicate how our models can be used to estimate the system cost of batch picking and zone picking systems. We use the identical picking area and system parameters mentioned in Section 6.4.2 and summarize these values (along with other example parameter values) in Table 6.5. Note that the pack-rate of pickers is typically higher in batch picking compared to zone picking as in the latter the pickers may (i) have to sort items at the end of the sort-lane into individual customer orders if more than one order is assigned to each sort-lane, and (ii) have to walk from one sort-lane to another to perform the packing operation. The cost components related to the sorter are representative values for a tilt-tray sorter. All cost values have been annualized (e.g., if the initial cost of an induction station is \$120,000, then its annualized cost for a 5 year service and at 15% interest rate is \$17,798).

For the system parameters mentioned in Table 6.5 we compare a sort-while-pick batch picking (SWPBP) to a simultaneous zone picking (SZP) system based on their system cost. In our comparison we consider three picking system throughput values (Λ_{sys}), low, medium,

Table 6.5: Notation Used in the Illustrative Example.

Parameters	**Symbol**	**Value**
Number of waves	w	12 per shift
Wavelength	l_w	40 minutes
Avg. # of items in an order	I_{avg}	
- small-sized orders		3 items
- large-sized orders		20 items
Total number of (narrow) aisles		16
Total number of pick-faces	n	800 (i.e., 50 pick-faces/aisle)
Pick-rate (batch picking)	r_p^b	150 items/hr
Pick-rate (zone picking)	r_p^z	
- 2 pickers (zone size = 8 aisles)		375 items/hr
- 4 pickers (zone size = 4 aisles)		469 items/hr
- 8 pickers (zone size = 2 aisles)		564 items/hr
Speed of conveyor	s	100 items/min
Nominal induction rate	λ	20 items/min
# of orders per sort-lane	d_l	
- small-sized orders		5 orders/sort-lane
- large-sized orders		1 order/sort-lane
Pack-rate (batch picking)	r_k^b	180 orders/hr
Pack-rate (zone picking)	r_k^z	100 orders/hr
Imbalance cost	c_u	\$75 per item/year
Years of service		5 years
Interest rate		15%
Annualized cost of equipment	c_e	\$20,000 (batch) & \$40,000 (zone)
Annualized fixed cost of a sorter	c_f	\$96,405
Annualized cost of an induction station	c_s	\$17,798
Annualized cost of a sort-lane	c_l	\$371
Annual cost of a picker, inductor, or packer	c_p, c_i, or c_k	\$40,000

and high, and two order sizes, small and large. Based on the system throughput and order sizes, the five cost components (pickers, equipment, imbalance, sorting, and packers) are then estimated. Note that in a SWPBP system pickers sort the items into customer orders while they are picking; i.e., pickers carry along a compartmentalized pick-cart with enough compartments for the customer orders to be assigned to them. Table 6.6 summarizes the results of the SWPBP system.

For the SZP system we need to explicitly incorporate the sorter cost model. Figure 6.5 shows the actual sorting system cost function, generated using the expressions developed in Section 6.5, for sorting 1000 small-sized orders in a 40-min wave in a SZP. The tuple (a,b) in Figure 6.5 indicates the induction station number (#a) and the number of inductors (b) at that induction station, respectively. Therefore, (1,2) indicates that there is only one induction station, induction station #1, and it has 2 inductors assigned to it. Similarly, (1,3) & (2,2) indicates that there are two induction stations; induction station #1 has 3 inductors assigned to it and induction station #2 has 2 inductors assigned to it. This actual sorting system cost function is incorporated into the cost model for a SZP system. See Table 6.6 for a summary of the cost calculations.

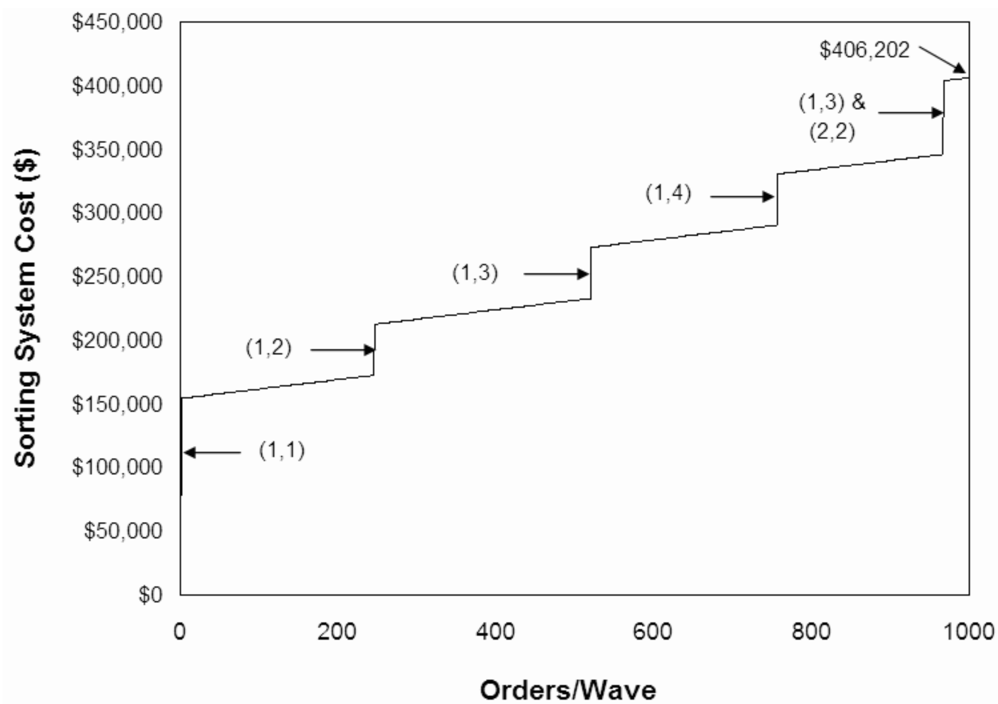


Figure 6.5: An Actual Sorting System Cost Function for a Sorter Required to Sort 1000 Small-Sized Orders in a 40-min Wave in a Simultaneous Zone Picking System.

For our illustrative example, we observe that a SWPBP system saves $116,079 annually over a SZP system when the required order picking system throughput (Λ_{sys}) is low. Though a majority of these savings comes from the SWPBP system not requiring a sorting system, around 10% of these savings are derived from the imbalance cost. However, as the system throughput increases, the savings derived by not requiring a sorting system in the SWPBP system is offset by the increase in the number of pickers required (from 5 to 13). As a result, for medium system throughput, in terms of system cost, the SWPBP and SZP systems are essentially equally expensive. For high system throughput, the SWPBP system requires 30

Table 6.6: Comparison of Sort-while-Pick Batch Picking (SWPBP) and Simultaneous Zone Picking (SZP) Systems for a Given System Throughput, Order Sizes, 12 Waves/Shift, and $I_{avg} = 3$ and 20 for Small-Sized and Large-Sized Orders, Respectively.

	Low Λ_{sys}		**Medium** Λ_{sys}		**High** Λ_{sys}	
	SWPBP	SZP	SWPBP	SZP	SWPBP	SZP
Avg. daily orders, D						
- small-sized orders	2,000	2,000	5,000	5,000	12,000	12,000
- large-sized orders	300	300	750	750	1,800	1,800
Orders/wave, D^w $(=\lceil D/w \rceil)$						
- small-sized orders	167	167	417	417	1,000	1,000
- large-sized orders	25	25	63	63	150	150
Items/wave, I $(= D^w I_{avg})$	500	500	1,250	1,250	3,000	3,000
Pick-rate/wave, r_p^w $(= (r_p l_w)/60)$	100	250	100	313	100	376
# of pickers, P $(= \lceil I/r_p \rceil)$	5	2	13	4	30	8
Picker-blocking, $b(P)$	0%	-	2.2%	-	7.4%	-
(1) Cost of pickers, $\left(= \left\lceil \frac{P}{1-b(P)/100} \right\rceil c_p\right)$	$200,000	$80,000	$560,000	$160,000	$1,320,000	$320,000
(2) Equipment cost, c_e	$20,000	$40,000	$20,000	$40,000	$20,000	$40,000
Daily avg. # of items unfulfilled, U						
- small-sized orders	0	131	0	379	0	814
- large-sized orders	15	181	156	617	767	1,371
(3) Imbalance cost, Uc_u						
- small-sized orders	$0	$9,825	$0	$28,425	$0	$61,050
- large-sized orders	$975	$13,575	$11,700	$46,275	$0	$102,825
# of sort-lanes, n_l $(= \lceil D^w/d_l \rceil)$						
- small-sized orders (d_l=5)	-	34	-	84	-	200
- large-sized orders (d_l=1)	-	25	-	63	-	150
# of induction stations, n_s	-	1	-	1	-	2
# of inductors, n_i	-	1	-	2	-	5
(4) Sorting cost, $c_s(I)$ $(= c_f + n_l c_l + n_s c_s + n_i c_i)$						
- small-sized orders	-	$166,818	-	$225,368	-	$406,202
- large-sized orders	-	$163,479	-	$217,577	-	$387,652
Pack-rate/wave, r_k^w $(= (r_k l_w)/60)$ (orders/wave)	120	67	120	67	120	67
# of packers, K $(= \lceil D^w/r_k^w \rceil)$	2	3	4	7	9	15
(5) Cost of packers, Kc_k	$80,000	$120,000	$160,000	$280,000	$360,000	$600,000
System cost, C (= (1)+(2)+(3)+(4)+(5))						
- small-sized orders	$300,000	$416,643	$740,000	$733,793	$1,700,000	$1,427,252
- large-sized orders	$300,975	$417,054	$751,700	$743,852	$1,757,525	$1,450,477

pickers in the picking area, while the SZP system requires only 8 pickers. This savings in labor cost offsets the increase in the imbalance cost and the sorting system cost. In fact, the SZP system is now $272,748 less expensive annually than the SWPBP system.

In general, we see a progression of the most cost effective picking strategy; the SWPBP system is cost effective for low values of Λ_{sys}, both the SWPBP and SZP systems are equally cost effective for medium values of Λ_{sys}, while the SZP system is cost effective for high values of Λ_{sys}. This progression is dependent on the number of waves employed during the shift. As the number of waves decreases (i.e., the wavelength increases) the size of the sorting system, and its cost, will potentially increase. This means that as the number of waves decreases the SZP system may turn out to be more expensive even for medium values of Λ_{sys}.

Note that for the illustrative example we considered only one type of batch picking strategy along with one type of zone picking strategy (i.e., SWPBP and SZP). Our models are equally applicable for the other types of batch and zone picking strategies (i.e., pick-and-sort batch and progressive zone) with the appropriate changes in model parameters (e.g., pick-rate, pack-rate, and requirement of a sorter). For example, if we were to compare the SWPBP strategy with the pick-and-pass zone picking strategy, then we notice that neither of these strategies require a sorter. Consequently, the imbalance cost (apart from the labor cost) is expected to play a major role in differentiating the costs of the two systems, and hence, identifying the least expensive picking strategy.

6.7. Conclusions

Selecting an appropriate picking strategy is critical to the overall operation of a distribution center as it largely governs the cost of the picking system for a given throughput requirement. To address this problem a designer would typically follow these three phases of the engineering design process: Phase 1: define the problem; Phase 2: analyze the problem, generate alternatives, evaluate the alternatives, and select the preferred design; and Phase 3: implement the design.

For an order picking environment, out of several potential problem definitions, we focused on the order picking strategy selection problem and considered it as Phase 1. Consequently, Phase 2 helped us identify the four potential order picking strategies: discrete, batch, zone, and bucket brigade picking. Our focus was on batch and zone picking strategies as they are frequently employed. We referred to this selection problem as the batch versus zone problem.

Various factors, such as pick-rate, picker-blocking, workload-imbalance, and sorting system requirement, that affect the batch versus zone problem were considered.

To address the batch versus zone problem we developed a cost model to estimate the cost of a batch picking and a zone picking system. The cost model comprised of the cost of pickers, equipment, imbalance, sorting system, and packers. Imbalance cost is a direct result of workload-imbalance in a picking system, and depends on the assignment of items to the aisles and the order batching policy employed. Considering a given assignment of items to the aisles, we developed mathematical models, analogous to the dual bin packing problem, to estimate the effect of order batching on workload-imbalance. Based on our experiments, we found that workload-imbalance is greater in zone picking systems as compared to batch picking systems. Moreover, our experimental study showed that workload-imbalance is more prominent when (i) the order sizes increase, (ii) the item distribution is more non-uniform, and (iii) the number of waves increases.

We then developed a sorting system cost model, which in-part is based on the approach presented in (Russell and Meller, 2003) to estimate cost of sorting system. In our sorting system cost model, we explicitly considered the cost of sort-lanes. We also indicated through a parametric cost function that the relationship between the sorting system cost and the number of orders is strictly convex. Finally, we demonstrated the use of our models through an illustrative example in which we compared sort-while-pick batch picking (SWPBP) and simultaneous zone picking (SZP) systems. For this example, a gradual progression of the most cost effective picking strategy based system throughput was observed: SWPBP for low system throughput, SWPBP or SZP for medium system throughput, and SZP for large system throughput.

An avenue for future research in this problem area would be to conduct a comprehensive experimental study focusing on the workload-imbalance issue by considering a larger set of parameter values as well as considering multiple sets of customers orders. Moreover, considering the item-to-aisle assignment problem in zone picking in conjunction with the order batching problem may provide a broader perspective on the workload-imbalance issue in picking systems. It would also be helpful to develop a similar cost model for other order picking strategies (e.g., bucket brigade picking). Yet another area of future research would be to develop analytical models to select an appropriate picking system (manual, semi-automated, or automated).

Appendix A

This appendix summarizes the effect of order sizes and item distribution in orders on workload-imbalance for the example problem presented in Section 6.4.2. Note that we generated orders according to distributions D1 and D2. In our experiments the Distribution D1 is uniform, while Distribution D2 is skewed and is as illustrated in Table 6.7. Note that the number of items in each order generated according to distributions D1 and D2 is identical.

Table 6.7: The Percentage of Items in Customer Orders Generated According to Distribution D2 to be Fulfilled from the 16 Picking Aisles.

Aisle	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
% of Items	8	8	7.5	7.5	7	7	6.5	6.5	6	6	5.5	5.5	5	5	4.5	4.5

Because there are two order sizes (small and large), experiments were first conducted on small-sized orders (generated according to distributions D1 and D2) using the four order batching policies and the OBP-BP and OBP-ZP models presented earlier. These experiments were then repeated for large-sized orders. The order batching solutions obtained in terms of the total number of items included in the order batches by employing the four policies and the OBP-BP and OBP-ZP models are then compared to each other. The number of items unfulfilled is the difference of the total number of items to be fulfilled and the total number of items included in the batches.

Tables 6.8, 6.9, and 6.10 summarize the results obtained for small-sized orders, while Tables 6.11, 6.12, and 6.13 summarize the results for large-sized orders. ("NA" in Table 6.8 means CPLEX was unable to solve the problem on a computer with a 3.2GHz processor and 2GB RAM; the number of binary decision variables for this problem was 4.32 million.)

Table 6.8: Results for Batch Picking: Small-Sized Orders and Distributions D1 and D2.

Orders (Items)		**FCFS**	**FCFS-50**	**FCFS-200**	**FFD**	**OBP-BP**		**LB**
							Time	
2,000 (6000)	Obj.	5,940	6,000	6,000	6,000	6,000	1,436.62s	6,000
	Gap	1.00%	0.00%	0.00%	0.00%	0.00%		
	Unful.	60	0	0	0	0		0
5,000 (15,000)	Obj.	14,846	15,000	15,000	15,000	14,949	86,400s	15,000
	Gap	1.03%	0.00%	0.00%	0.00%	0.36%		
	Unful.	154	0	0	0	51		0
12,000 (36,000)	Obj.	35,580	36,000	36,000	36,000	NA	-	NA
	Gap	1.17%	0.00%	0.00%	0.00%	-		
	Unful.	420	0	0	0	-		-

Table 6.9: Results for Zone Picking: Small-Sized Orders and Distribution D1.

Orders (Items)		**FCFS**	**FCFS-50**	**FCFS-200**	**FFD**	**OBP-ZP**		**LB**
							Time	
2,000 (6,000)	Obj.	5,793	5,905	5,960	5,987	5,987	46.51s	5,987
	Gap	3.24%	1.37%	0.45%	0.00%	0.00%		
	Unful.	207	95	40	13	13		13
5,000 (15,000)	Obj.	14,284	14,519	14,854	14,905	14,905	95.16s	14,905
	Gap	4.17%	2.59%	0.34%	0.00%	0.00%		
	Unful.	716	481	146	95	95		95
12,000 (36,000)	Obj.	33,796	34,489	35,322	35,878	35,878	1,974.93s	35,878
	Gap	5.80%	3.87%	1.55%	0.00%	0.00%		
	Unful.	2,204	1,511	678	122	122		122

Table 6.10: Results for Zone Picking: Small-Sized Orders and Distribution D2.

Orders (Items)		**FCFS**	**FCFS-50**	**FCFS-200**	**FFD**	**OBP-ZP**		**LB**
							Time	
2,000 (6000)	Obj.	5,470	5,503	5,562	5,689	5,751	10.05s	5,751
	Gap	4.69%	4.14%	3.16%	1.04%	0.00%		
	Unful.	530	497	438	311	249		249
5,000 (15,000)	Obj.	13,083	13,170	13,323	14,154	14,338	35.24s	14,338
	Gap	8.42%	7.84%	6.81%	1.23%	0.00%		
	Unful.	1,917	1,830	1,677	846	662		662
12,000 (36,000)	Obj.	30,882	31,475	31,740	34,108	34,494	150.18s	34,494
	Gap	10.07%	8.41%	7.68%	1.08%	0.00%		
	Unful.	5,118	4,525	4,260	1,892	1,506		1,506

Table 6.11: Results for Batch Picking: Large-Sized Orders and Distribution D1 and D2.

Orders (Items)		**FCFS**	**FCFS-50**	**FCFS-200**	**FFD**	**OBP-BP**		**LB**
							Time	
300 (6,000)	Obj.	5,401	5,778	5,805	5,837	5,987	86,400s	6,000
	Gap	9.98%	3.70%	3.25%	2.72%	0.22%		
	Unful.	599	222	195	163	13		0
750 (15,000)	Obj.	13,521	14,428	14,507	14,717	14,844	86,400s	15,000
	Gap	9.86%	3.81%	3.29%	1.89%	1.04%		
	Unful.	1,479	572	493	283	156		0
1,800 (36,000)	Obj.	32,237	34,588	34,822	35,231	35,233	86,400s	36,000
	Gap	10.45%	3.92%	3.27%	2.14%	2.18%		
	Unful.	3,763	1,412	1,178	769	767		0

Table 6.12: Results for Zone Picking: Large-Sized Orders and Distribution D1.

Orders (Items)		**FCFS**	**FCFS-50**	**FCFS-200**	**FFD**	**OBP-ZP**		**LB**
							Time	
300 (6,000)	Obj.	5,654	5,795	5,851	5,867	5,981	10.68s	5,981
	Gap	5.46%	3.11%	2.17%	1.90%	0.00%		
	Unful.	346	205	149	133	19		19
750 (15,000)	Obj.	13,980	14,109	14,172	14,380	14,883	86,400s	14,886
	Gap	6.08%	5.21%	4.79%	3.39%	0.02%		
	Unful.	1,020	891	828	620	117		114
1,800 (36,000)	Obj.	33,432	34,223	34,452	34,997	35,709	86,400s	35,748
	Gap	6.46%	4.25%	3.61%	2.09%	0.11%		
	Unful.	2,568	1,777	1,548	1,003	291		252

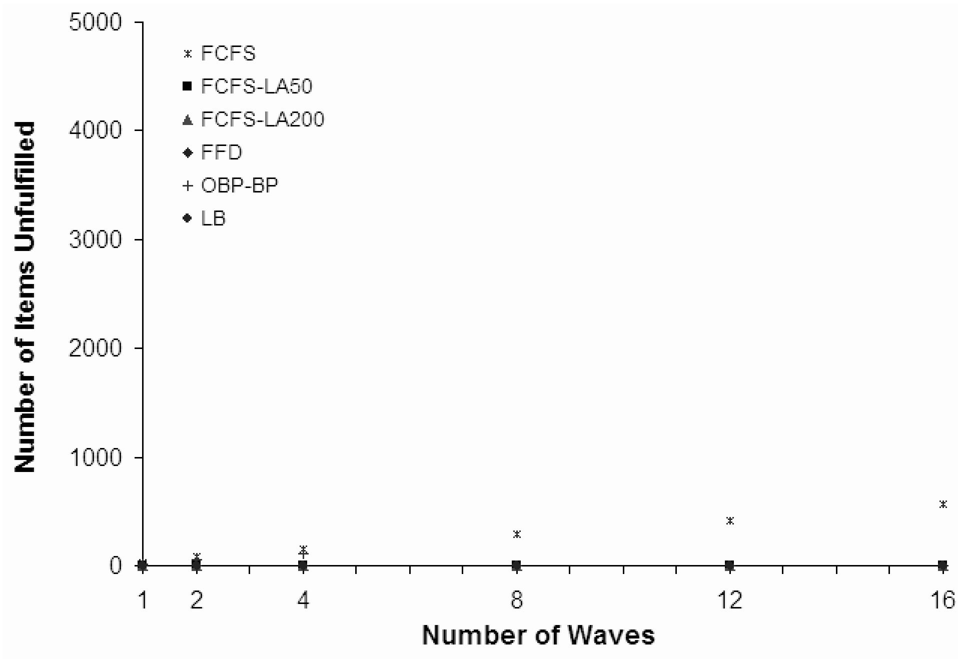
Table 6.13: Results for Zone Picking: Large-Sized Orders and Distribution D2.

Orders (Items)		**FCFS**	**FCFS-50**	**FCFS-200**	**FFD**	**OBP-ZP**		**LB**
							Time	
300 (6,000)	Obj.	5,415	5,509	5,509	5,521	5,658	25.19s	5,658
	Gap	4.06%	2.49%	2.49%	2.29%	0.00%		
	Unful.	585	491	491	479	342		342
750 (15,000)	Obj.	13,025	13,193	13,195	13,206	13,944	117.47s	13,944
	Gap	6.17%	5.04%	5.03%	4.95%	0.00%		
	Unful.	1,975	1,807	1,805	1,794	1,056		1,056
1,800 (36,000)	Obj.	30,566	31,024	31,214	31,661	33,549	785.14s	33,549
	Gap	8.31%	7.04%	6.51%	5.26%	0.00%		
	Unful.	5,434	4,976	4,786	4,339	2,451		2,451

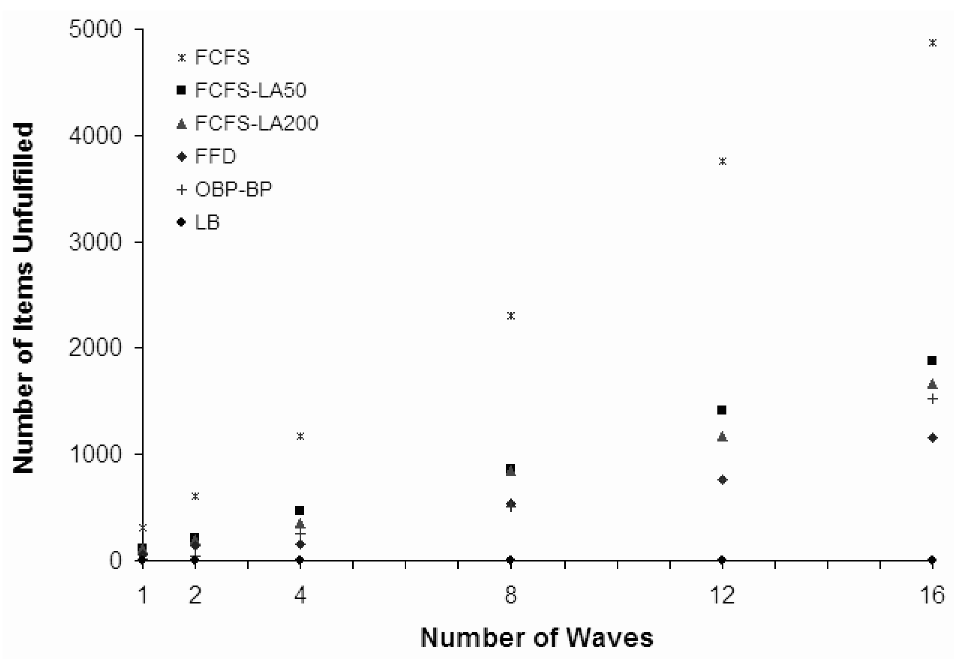
Appendix B

This appendix summarizes the effect of the number of waves on workload-imbalance for the example problem presented in Section 6.4.2.

For our experimentation, both the batch and zone picking systems are considered. There are 12,000 small-sized orders and 1,800 large-sized orders generated as per the Distribution D1 with a total of 36,000 items in each of them. Figure 6.6 illustrates the number of items unfulfilled in small-sized and large-sized orders in a batch picking system. Figure 6.7 illustrates the number of items unfulfilled in small- and large-sized orders in a zone picking system.

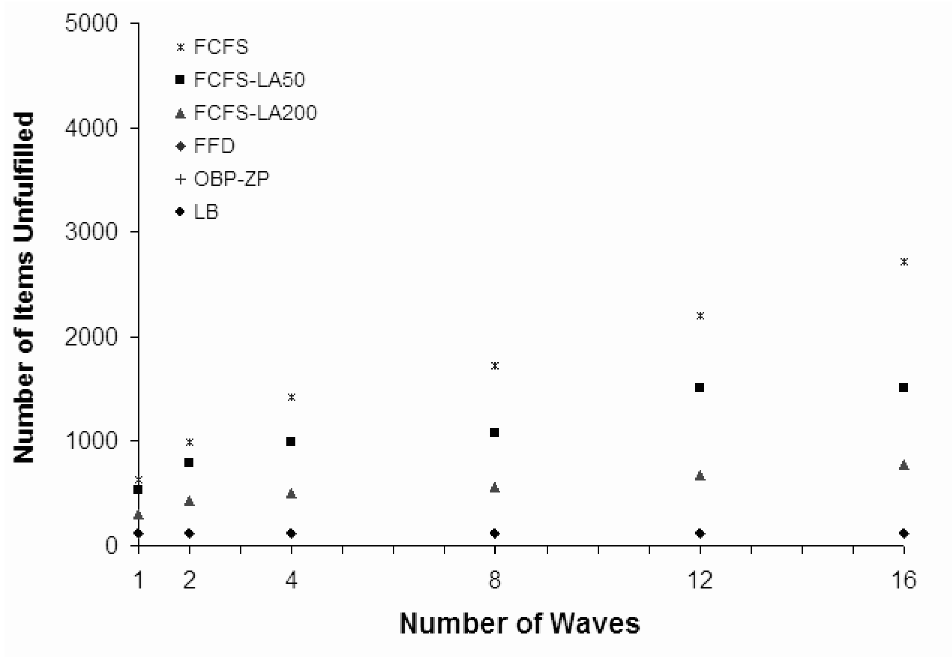


(a) Small-Sized Orders.

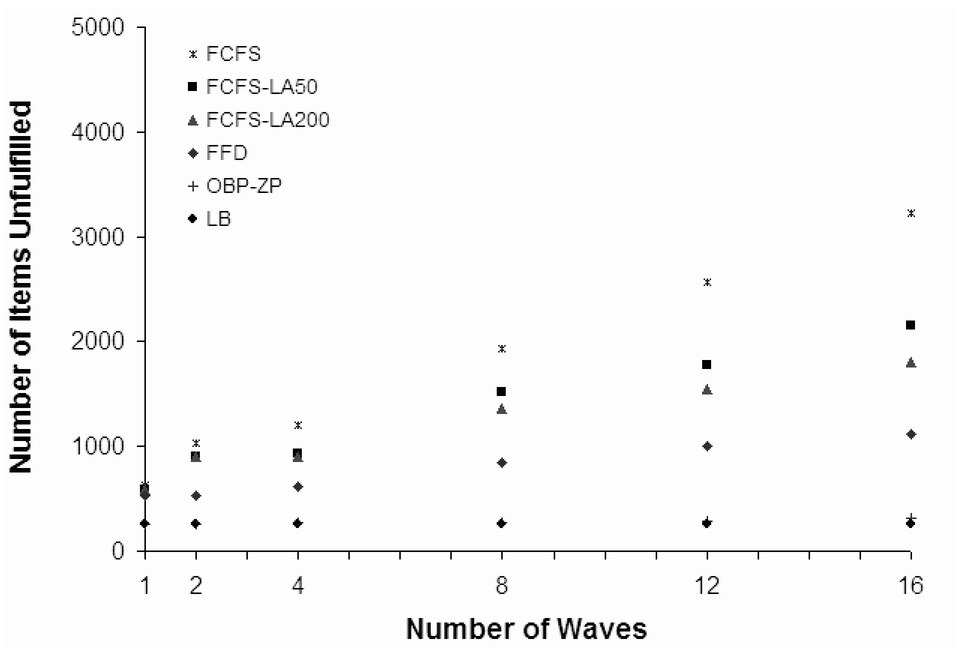


(b) Large-Sized Orders.

Figure 6.6: Effect of Wavelength on the Number of Items Unfulfilled in a Batch Picking System.



(a) Small-Sized Orders.



(b) Large-Sized Orders.

Figure 6.7: Effect of Wavelength on the Number of Items Unfulfilled in a Zone Picking System.

Chapter 7

Conclusions and Future Research

A distribution center (DC) in a logistics system is responsible for obtaining materials from different suppliers and assembling (or sorting) them to fulfill a number of different customer orders. Various activities in a DC include order receipt, material receipt, labeling, put-away, replenishment, inventory control, order picking, sorting and packing, staging, shipping, and returns processing. Amongst these activities, *order picking* has been identified in the literature as the highest priority activity in a DC for productivity improvements because an estimated 50% of the total DC operating costs have been attributed to order picking. Order picking refers to an operation through which items are retrieved from storage locations to fulfill customer orders.

The focus of this research is on developing analytical models to design an order picking system (OPS) in a DC. The design of an OPS depends on various parameters; e.g., identification of the picking-area layout, configuration of the storage system, and determination of the storage policy, picking method, picking strategy, material handling system, pick-assist technology, etc. For a given set of these parameters, the best design would depend on the objective (e.g., maximizing throughout, minimizing cost, etc.) to be optimized. The main question we wish to address in this research is as follows: *which OPS best meets a given set of objectives?*

This research is motivated because of two reasons: (i) the prevalent use of experienced-based decision making or simulation models to design an OPS, which are sub-optimal and/or time-consuming; and (ii) the lack of analytical approaches in certain areas of the OPS design literature. Our overall goal is to develop a set of analytical models so that an OPS designer can use this set when entrusted with the task of designing an OPS.

The two key OPS design issues addressed in this dissertation are the configuration of a storage system and the selection between batch and zone order picking strategies. We have identified several factors that affect both these decisions, with a common factor amongst these being picker blocking. We first developed models to estimate picker blocking (Contribution 1) and then use this factor, along with other factors, to address the two key OPS design issues (Contributions 2 and 3).

Contribution 1 - Estimating Picker Blocking: As previously mentioned, picker blocking is an important factor when configuring a storage system and deciding which order picking strategy to use. Picker blocking can take two forms, *pick-face blocking* and *in-the-aisle blocking*. Pick-face blocking occurs when two or more pickers need to pick at the same pick-face. In-the-aisle blocking occurs when the aisles are too narrow for pickers to pass each other in the aisle. We developed analytical models using discrete time Markov chains to estimate pick-face blocking in wide-aisle DCs considering that pickers may pick one item or may pick multiple items at a pick-face. We considered systems where there are two pickers in the system and the pick to walk time ratio is either 1:1 or ∞:1. We then compared pick-face blocking with in-the-aisle blocking (as modeled in Gue et al. (2006)) for the case when pickers may pick only one item at a pick-face. For identical OPS configurations, our results show that (i) the curves representing both these forms of blocking had a similar pattern (i.e., concave) in increasing pick-density, (ii) the maximum values of pick-face and in-the-aisle blocking occurred at different values of pick-density, and (iii) pick-face blocking was lower than in-the-aisle blocking. For the case when pickers may pick multiple items at a pick-face, we found instead that blocking increases monotonically with an increase in pick-density. For situations when there are more than two pickers and/or the pick to walk time ratio is between 1:1 and ∞:1 we developed simulation models to estimate the pick-face blocking. The models we developed to estimate blocking were used in Contributions 2 and 3 of this research.

Contribution 2 - Configuring the Storage System: A storage system is required to store SKUs that are required to fulfill customer orders. For a given rack type, configuring the storage system refers to the identification of the storage system length, height, and depth. A sub-optimal storage system configuration may lead to increased pick-times and/or travel-times, which can reduce picker throughput. To identify an optimal storage system configuration we proposed a cost-based optimization model. The cost model was used to identify the optimal storage level (or height), and thus, length of a storage for a given volume of SKUs. Our model is more comprehensive than a view of this problem from industry's perspective for two reasons. First, we include the impact of picker blocking in our cost model, using models developed as part of Contribution 1. Furthermore, we take a more comprehensive view of travel-time than is typically employed in that we consider vertical travel in addition to horizontal travel. Analytical models to estimate the travel-time of a type of semi-automated OPS, a person-aboard storage/retrieval machine were developed to do so. Experimental results show that a low (in height) and long (in length) storage systems tends to be optimal for situations where there is a relatively low number of storage locations and a relatively high throughput requirement; this is in contrast with common industry perception of the higher the better. Such a contrast between our model and industry practice was expected since picker blocking and vertical travel, which are ignored in practice, are significant factors when configuring the storage system for a semi-automated OPS. On the other hand, results from the same optimization model suggests that a manual OPS should, in almost all situations, employ a high (in height) and short (in length) storage system; a result that is consistent with industry practice. The consistency between our model and industry practice for a manual OPS was expected since vertical travel, which is ignored in practice, is not a factor in a manual OPS.

Contribution 3 - Selecting Between Batch and Zone Order Picking Strategies: An order picking strategy defines the manner in which the pickers navigate the picking aisles of a storage area to pick the required items. Amongst the various picking strategies; i.e., discrete, batch, zone, and bucket brigade, we focused our attention on deciding between batch and zone picking strategies as they are typically employed for medium-to-high throughput DCs. We referred to this decision problem as the batch versus zone problem. Our objective was to analyze the trade-off in terms of pick-rate, travel-time, workload-imbalance, and sorting system requirement in selecting an appropriate picking strategy (i.e., batch or zone) for a given application. To model this problem we pursued a cost-based approach, whereby we developed a cost model to estimate the cost of operating a picking system employing a batch or a zone picking strategy. The cost components in the cost model included the annualized cost of pickers, equipment, imbalance, sorting system, and packers. We specifically focused on estimating the cost of imbalance and a sorting system. Imbalance cost was defined as the cost of fulfilling the left-over items (in customer orders) due to workload-imbalance. To estimate the imbalance cost we developed order batching models (using a dual bin-packing model analogy) and solved them using several batching policies and the optimal approach. Through a small experimental study we demonstrated that workload-imbalance (i) depends on order-sizes, item distribution in orders, and the number of waves employed, and (ii) is typically greater in zone picking as compared to batch picking. We also developed a comprehensive sorting system cost model to estimate the cost of an automated sorting system. Through an illustrative example we demonstrated the use of our models.

In summary, we initiated our research with a goal of developing a set of analytical models for designing order picking systems in a DC. The intention was to aid the designer in selecting a few viable alternatives from a large set of alternative OPS system designs. We subsequently focused on two key design problems of OPS design in this dissertation: storage system configuration and picking strategy selection. We identified various factors that affect these problems, developed appropriate analytical models (including picker blocking models), and illustrated the use of our models through examples. However, the research presented in this dissertation does not completely address our overall research goal. That is, several research questions emerged during our research. We discuss these research questions in the context of future research next.

Future Research: We first present the possible research questions that have emerged out of this dissertation research.

In Contribution 1 we developed analytical models to estimate picker blocking in wide-aisle DCs. We considered the case where pickers pick one item at a pick-face as well as the case where multiple items may be picked at a pick-face. However, analytical models for estimating in-the-aisle blocking in narrow-aisle DCs considering the case where pickers may pick multiple items at a pick-face have yet to be developed. Note that narrow-aisles are space efficient compared to wide-aisles, but they tend to increase picker blocking as compared to wide-aisles. It would be interesting to develop analytical models for the multiple item case in narrow aisles as well, which will complete the set of blocking models for both wide- and narrow-aisle DCs. This set of models will not only aid in comparing the blocking phenomenon in DCs with either narrow or wide aisle-widths, but also help in possibly designing DCs that have a few aisles or sections of aisles that are narrow and others wide; we call such an arrangement of aisles *offset aisles*.

In Contribution 2 we developed a cost-based optimization model to determine the optimal storage height. In doing so we assumed that the storage depth (lane depth) was equal to one. We made this assumption because lane depths greater than one affect the pick-time at a pick-face as pickers may have to remove SKUs at the front of the lane to reach SKUs at the back. This handling will result in a reduction in the throughput of a picker. However, higher lane depths result in a shorter (in length) storage system, which reduces the space cost. Applications where space costs are relatively high may benefit from using a multi-deep storage system. However, the corresponding reduction in picker-throughput may increase the number of pickers to satisfy system throughput, thus increasing labor cost. Thus, from the standpoint of minimizing the system cost, it would be an interesting research endeavor to simultaneously identify the optimal storage height and lane depth of storage systems for such applications. Furthermore, understanding the impact of various storage policies and product-slotting techniques on the storage system configuration problem seems to be an interesting area for future research.

In Contribution 3 one of the factors we identified in the batch versus zone problem was workload-imbalance. We also provided a means for quantifying workload-imbalance by developing new formulations for order batching. However, our analysis of the workload-imbalance issue was dependent on a small experimental study. It is worth investigating in the workload-imbalance issue further by considering multiple sets of customers orders and other batching policies. Moreover, considering the item-to-aisle assignment problem in zone picking in conjunction with the order batching problem may provide a broader perspective on the workload-imbalance issue in picking systems. Furthermore, a cost model for other picking strategies (e.g., bucket brigade picking) may prove helpful when evaluating all possible order picking strategies.

Note that all of the analytical models developed in this dissertation were based on an assumption that a randomized storage policy is used. However, there are several non-randomized storage policies that have been employed in DCs. Modifying our models or developing new models that consider these storage policies would be an interesting extension of our research.

We now present several other research questions that, although were not addressed in this dissertation research, form a part of our overall research goal.

Case- and piece-picking OPSs typically employ a pick-to-conveyor OPS, where pickers pick items from pick-faces and immediately induct them onto a take-away conveyor installed in the order picking aisle. Throughput models for carousels, VLMs, and end-of-aisle mini-load AS/RSs are available in the OPS literature. However, a throughput model for a pick-to-conveyor OPS have yet to be developed. If such a model were developed, then a designer would benefit from comparing a pick-to-conveyor OPS with other OPSs (e.g., a carousel, VLM, end-of-aisle mini-load AS/RS, etc.) in terms of throughput, and thus select an appropriate OPS for a given application.

Another research question is related to the amount of automation to be utilized in an OPS. A manual (or picker-to-product) OPS is one in which the order pickers travel to the point where the item to be picked is located (e.g., pick-to-tote/cart/truck). A semi-automated (or product-to-picker) OPS is one in which the items to be picked are brought to a stationary picker through mechanical means (e.g., a carousel, vertical lift module, etc.). An automated OPS has the potential of picking orders without any human intervention (e.g., an A-frame). A high level of automation in an OPS will significantly improve picking productivity, but it comes at a cost. So for a specific application, it would be helpful if a model could be developed that would help the designer in making this decision during the initial stages of OPS design.

Similarly, there are several pick-assist technologies, as highlighted in Chapter 1, that can be used to execute the order picking operation. These technologies are either paper-based (e.g., pick-list) or paper-less (e.g., voice, light, and radio-frequency). There exists a trade-off between productivity and cost when determining an appropriate pick-assist technology. An analytical model that can suggest an appropriate pick-assist technology considering this trade-off will certainly be helpful to a designer.

Note that all the models that we have developed in this research and that would form basis of future research address isolated problems in the OPS design domain. An OPS designer will certainly benefit from these models; however, if a decision support system was created that included all of these analytical models (appropriately linked with each other), then such a system would be a considerable aid to OPS system designers. They can use this system to quickly evaluate OPS design alternatives instead of relying on potentially inefficient experience-based approaches or resource-heavy simulation approaches as is standard in industry.

Bibliography

Ashayeri, J. and Gelders, L. F. (1985). Warehouse design optimization. *European Journal of Operation Research*, 21(3):285–294.

Bartholdi, J. J., Eisenstein, D. D., and Foley, R. D. (2001). Performance of bucket brigades when work is stochastic. *Operations Research*, 49(5):710–719.

Bartholdi, J. J. and Hackman, S. T. (2006). *Warehouse and Distribution Science.* Working book. Available at http://www.isye.gatech.edu/people/faculty/John_Bartholdi/wh/book/wh-sci-0.55.pdf.

Bozer, Y. A. and White, J. A. (1996). A generalized design and performance analysis model for end-of-aisle order picking systems. *IIE Transactions*, 28(4):271–280.

Brynzer, H., Johansson, M., and Medbo, L. (1994). A methodology for evaluation of order picking systems as a base for system design and managerial decisions. *International Journal of Operations and Production Management*, 14(3):126–139.

Cooke, J. A. (2004). The two faces of globalization: Globalization may be keeping consumer prices low, but it's creating new challenges for logistics managers. *Logistics Management*, July 1.

Cormier, G. and Gunn, E. A. (1992). A review of warehouse models. *European Journal of Operational Research*, 58(1):3–13.

de Koster, R. (1994). Performance approximation of pick-to-belt order picking systems. *European Journal of Operational Research*, 72(3):558–573.

de Koster, R. and van der Poort, E. (1998). Routing order pickers in a warehouse: a comparison between optimal and heuristic solutions. *IIE Transactions*, 30(5):469–480.

de Koster, R., van der Poort, E., and Wolters, M. (1999). Efficient order batching methods in warehouse. *International Journal of Production Research*, 37(7):1479–1504.

Design Plans and Ideas (2002). *Dry Grocery Goods DC.* Reed Business Information, Waltham, MA. Available at http://www.contentconvergence.com/july2002/industrialtrucks.html.

Frazelle, E. A. and Sharp, G. P. (1989). Correlated assignment strategy can improve order-picking operation. *Industrial Engineering*, 21(4):33–37.

Gademann, N. and van de Velde, S. (2005). Order batching to minimize total travel time in a parallel-aisle warehouse. *IIE Transactions*, 37(1):63–75.

Goetschalckx, M. and Ratliff, H. D. (1988). An efficient algorithm to cluster order picking items in a wide aisle. *Engineering Costs and Production Economics*, 13(4):263–271.

Goetschalckx, M. and Ratliff, H. D. (1990). Shared storage policies based on the duration stay of unit loads. *Management Science*, 36(9):1120–1132.

Gray, A. E., Karmarkar, U. S., and Seidmann, A. (1992). Design and operation of an order-consolidation warehouse — models and application. *European Journal of Operational Research*, 58(1):14–36.

Gu, J., Goetschalckx, M., and McGinnis, L. F. (2005). Research on warehouse operation: A comprehensive review. Working paper, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA.

Gue, K. R., Meller, R. D., and Skufca, J. D. (2006). The effects of pick density on order picking areas with narrow aisles. *IIE Transactions*, 38(10):859–868.

Hackman, S. T. and Rosenblatt, M. J. (1990). Allocating items to an automated storage and retrieval systems. *IIE Transactions*, 22(1):7–14.

Hall, R. W. (1993). Distance approximations for routing manual pickers in a warehouse. *IIE Transactions*, 25(4):76–87.

Hausman, W. H., Schwarz, L. B., and Graves, S. C. (1976). Optimal storage assignment in automatic warehousing systems. *Management Science*, 22(6):629–638.

Heragu, S. S., Du, L., Mantel, R. J., and Schuur, P. C. (2005). Mathematical model for warehouse design and product allocation. *International Journal of Production Research*, 43(2):327–338.

Jane, C.-C. (2000). Storage location assignment in a distribution center. *International Journal of Physical Distribution and Logistics Management*, 30(1):55–71.

Jane, C.-C. and Laih, Y.-W. (2005). A clustering algorithm for item assignment in a synchronized zone order picking system. *European Journal of Operational Research*, 166(2):489–496.

Jewkes, E., Lee, C., and Vickson, R. (2004). Product location, allocation, and server home base location for an order picking line with multiple servers. *Computers and Operations Research*, 31(4):623–636.

Johnson, M. E. and Meller, R. D. (2002). Performance analysis of split-case sorting systems. *Manufacturing & Service Operations Management*, 4(4):258–274.

Lee, M. K. (1992). A storage assignment policy in a man-on-board automated storage-retrieval system. *International Journal of Production Research*, 30(10):2281–2292.

Malmborg, C. J. (1996). An integrated storage system evaluation model. *Applied Mathematical Modeling*, 20(5):359–370.

MATLAB (2004). *Help Manual (MATLAB 7.0)*. The MathWorks, Inc., Natwick, MA.

Meller, R. D. and Klote, J. F. (2004). A throughput model for carousel/vlm pods. *IIE Transactions*, 36(8):725–741.

MHIA (2006). *Material Handling Taxonomy - Order Picker*. Material Handling Industry of America, Charlotte, NC. Available at http://www.mhia.org/et/mhe_tax.htm.

Parikh, P. J. and Meller, R. D. (2006a). Configuring the storage system for order picking in a distribution center. Working paper, Grado Department of Industrial and Systems Engineering, Virginia Tech, Blacksburg, VA.

Parikh, P. J. and Meller, R. D. (2006b). Estimating picker blocking in wide-aisle order picking systems. Working paper, Grado Department of Industrial and Systems Engineering, Virginia Tech, Blacksburg, VA.

Petersen, C. G. (2000). An evaluation of order picking policies for mail order companies. *Production and Operations Management*, 9(4):319–335.

Petersen, C. G. and Aase, G. (2004). A comparison of picking, storage, and routing policies in manual order picking. *International Journal of Production Economics*, 92(1):11–19.

Ratliff, H. D. and Rosenthal, A. S. (1983). Order-picking in a rectangular warehouse: a solvable case of the traveling salesman problem. *Operations Research*, 31:507–521.

Raymond Corporation (2005). *Order Picker Truck.* Greene, NY. Available at http://www.raymondcorp.com.

Roodbergen, K. J. and Vis, I. F. A. (2006). A model for warehouse layout. *IIE Transactions*, 38(10):799–812.

Rouwenhorst, B., Reuter, B., Stockrahm, V., van Houtum, G. J., Mantel, R. J., and Zijm, W. H. M. (2000). Warehouse design and control: Framework and literature review. *European Journal of Operational Research*, 122(3):515–533.

Ruben, A. R. and Jacobs, F. R. (1999). Batch construction heuristics and storage assignment for walk/ride and pick systems. *Management Science*, 45(4):575–596.

Russell, M. L. and Meller, R. D. (2003). Cost and throughput modeling of manual and automated order fulfillment systems. *IIE Transactions*, 35(7):589–603.

Sadiq, M., Landers, T. L., and Taylor, G. D. (1996). An assignment algorithm for dynamic picking systems. *IIE Transactions*, 28(8):607–616.

Skufca, J. D. (2005). k workers in a circular warehouse: A random walk on a circle, without passing. *SIAM Review*, 47(2):301–314.

Tompkins, J. A., White, J. A., Bozer, Y. A., and Tanchoco, J. M. A. (2003). *Facilities Planning.* John Wiley & Sons, New York, NY, 3rd edition.

van den Berg, J. P. (1999). A literature survey on planning and control of warehousing systems. *IIE Transactions*, 31(8):751–762.

van den Berg, J. P., Sharp, G. P., Gademann, A. J. R. M., and Pochet, Y. (1998). Forward-reserve allocation in a warehouse with unit load replenishment. *European Journal of Operational Research*, 111(1):98–113.

van den Berg, J. P. and Zijm, W. H. M. (1999). Models for warehouse management: Classification and examples. *International Journal of Production Economics*, 59(3):519–528.

Yoon, C. S. and Sharp, G. P. (1995). Example application of the cognitive design procedure for an order pick system: Case study. *European Journal of Operational Research*, 87(2):223–246.

Yoon, C. S. and Sharp, G. P. (1996). A structured procedure for analysis and design of order pick systems. *IIE Transactions*, 28(5):379–389.

Vita

Pratik J. Parikh was born in the city of Ahmedabad in the state of Gujarat, India. He was raised in the city of Vadodara where he obtained his Bachelor of Engineering degree in Mechanical Engineering from The Maharaja Sayajirao University of Baroda. In 2001 he was awarded the prestigious Dr. Bhagwan Gajwani Fellowship for pursuing graduate studies at Binghamton University (S.U.N.Y.), Binghamton, NY. He finished his M.S. degree in Systems Science in Fall 2002. After briefly attending the University of Texas at Austin, he joined the Grado Department of Industrial and Systems Engineering at Virginia Polytechnic Institute and State University (Virginia Tech) as a Ph.D. student in Fall 2003. Under the tutelage of Dr. Russell D. Meller, he initiated his research in the area of design of order picking systems for distribution centers and completed this research in Fall 2006.

During his stay at Virginia Tech, he served as a teaching assistant for the department and a research assistant for the Center for High Performance Manufacturing (CHPM). As a research assistant, he worked on projects with CHPM member companies in the areas of material flow optimization, facility layout, and radio frequency identification. He was awarded the Material Handling Education Foundation Fellowship twice (2004-05 and 2005-06) and the Dover Fellowship once (2004-05). He was also nominated to attend the 2006 IIE Doctoral Colloquium held in conjunction with the 2006 Industrial Engineering Research Conference in Orlando, FL.

Pratik's research either has been published in or is currently under review with journals such as *IEEE Transactions on Robotics* and *IIE Transactions*. He has also served as a referee for several international journals and conferences. He is currently a student member of IIE and INFORMS, and will be listed in the 2007 edition of Marquis Who's Who in Science and Engineering.

Pratik will join the Science Group at Manhattan Associates in Atlanta, GA as a Scientist in the Warehouse Management Systems domain.