

Intra-Household Decision Making

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ABSTRACT

This dissertation consists of three essays. In the first one (Chapter three), “*Gains and Losses from Household Formation*,” I introduce a general equilibrium model, wherein a household may consist of more than one member, each with their own preferences and endowments. In these models at first, individuals form households. Then, collective decisions (or bargaining) within the household specifies the consumption plans of household members. Finally, competition across households determines a feasible allocation of resources. I consider a model with two types of individuals and pure group externalities. I investigate the competitive equilibrium allocation and stability of the equilibrium in that setting. Specifically, I show that under a certain set of assumptions a competitive equilibrium with free exit is also a competitive equilibrium with free household formation. Similar results are obtained for a special case of consumption externality. Illustrative examples, where prices may change as household structures change, are used to show how general equilibrium model with variable household structure works and some interesting results are discussed at the end of the first essay.

In the second essay (Chapter four), “*Effects of the Price System on Household Labor Supply*,” I introduce leisure and labor into the two-type economy framework that was constructed in the first essay. The main objective of this essay is to investigate the effects of exogenous prices on the labor supply decisions, and completely analyze the partial equilibrium model outcomes in a two-type economy setting. I assume a wage gap and explore the effect of that gap on labor supply. The main content of the second essay is the analysis of the effect of change in wages, price of the private good, power of each individual in the household, relative importance of private consumption compared to leisure, and the level of altruism on individual’s decisions about how much private good or leisure he/she wants to consume. The effect of a relative price change on labor supply, private consumption and utility level is also investigated. Moreover, one of the variations of Spence’s signaling model is borrowed to explain why higher education of women in Iran does not necessarily translate into higher female labor force participation. Finally, fixed point theorem is used to calculate the power (or alternatively labor supply) of individuals in the household endogenously for the two-type economy with labor at the end of this essay.

In the third essay (Chapter five), “*Dynamics of Poverty in Iran: What Are the Determinants of the Probability of Being Poor?*,” I explore the characteristics of the households who fall below the poverty line and stay there as well as those who climb up later. I decompose poverty in Iran into chronic and transient poverty, and investigate the relation of each component of poverty with certain characteristics of households. I also study mobility and the main characteristics of growth in expenditure of households. One of the main issues in economic policy making nowadays is the evaluation of effectiveness of anti-poverty programs. In order to achieve this goal one should be able to track down a household for a period of time. In this essay, I am going to investigate the dynamics of

poverty in Iran during 1992-95. I am especially interested in finding the characteristics of the households that fall below the poverty line and stay there in addition to those that climb up later. Obviously, if policy-makers want to have efficient policies to reduce poverty, they should target the former group. I decompose poverty in Iran into chronic and transient poverty, and investigate the relation of each component of poverty with certain characteristics of households. I also study mobility in this period with an emphasis on mobility in and out of poverty and review the main characteristics of the growth in expenditure of households.

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1 Introduction

“In recent years, economists have used economic theory more boldly to explain behavior outside the monetary market sector, and increasing numbers of noneconomists have been following their examples. ... Yet, one type of behavior has been almost completely ignored by economists, although scarce resources are used and it has been followed in some form by practically all adults in every recorded society. I refer to marriage.”
(Becker, 1973)

In his seminal paper and his 1978 and 1991 monographs, Gary Becker pioneers the economic analysis of marriage broadly defined. He is the first to investigate the reasons why people marry from an economic perspective, examining the gains from marriage, which is also a major topic of my dissertation. He also studies the socioeconomic patterns of marriage, developing the concept of assortive mating. Assortive mating is one of the channels by which education not only affects a person’s job market opportunities, but also the person’s marriage market opportunities. More generally, in the words of Gersbach and Haller (2001):

“The allocation of resources among consumers and the ensuing welfare properties are obviously affected by the specifics of a pre-existing partition of the population into households. Conversely, the formation of households can – partly or fully – be driven by economic considerations, by the anticipated effects of the emerging household structure on the allocation of economic resources.”

This dissertation deals with household decisions and household formation in the spirit of Becker and of recent work by Gersbach and Haller. Chapter 2 introduces the theoretical framework and notation. Chapters 3 and 4 are theoretical and cast within the general equilibrium framework of Gersbach and Haller (2001, 2005, 2008). Chapter 5 is devoted to an empirical investigation of

poverty in Iran, which relies on a decomposition of poverty into chronic and transient.

1.1 Review of “Household” Literature

Abstract models of marriage and matching predate Becker’s work. A combinatorial lemma by Hall (1935) and Maak (1936) became later known as the “marriage theorem”. The seminal contribution to two-sided matching is Gale and Shapley (1962) whose setting transcends the marriage market. The main results are surveyed in Roth and Sotomayor (1990). The matching (and assignment game) literature (see Roth and Sotomayor, 1990) is primarily focused on group formation and group stability, and not on competitive exchange of commodities. There are no active commodity markets, since there is at most one commodity. Both matching and the subsequent “household” literature deal with formation and stability of groups or households. In that sense they are similar. Matching models refrain from investigating the relation between household formation and competitive market allocation of private goods while the household models do. Despite this major difference, Gersbach and Haller (2003, 2005, 2008), adopt and adapt concepts from the matching literature.

There are two types of models dealing with group formation and competitive market allocation of commodities, comprising the “household” literature and “club” literature. Since it provides the formal framework for my work, I am going to elaborate on the “household” literature first.

1.1.1 Household Models

This dissertation builds upon the contributions of Haller (2000) and Gersbach and Haller (1999, 2001, 2003). Like the previous work, it adopts Chiappori’s (1988, 1992) collective rationality condition for multi-member households in a general equilibrium context. Chiappori’s collective rationality of multi-member households leads to an efficient consumption plan within the household’s budget set. Haller (2000) initiated this line of research. He introduces a model of a pure exchange economy with given multi-member households and derives a version of the first welfare theorem for the case of positive externalities, among others.

Gersbach and Haller (1999) show the existence of a competitive equilibrium. Gersbach and Haller (2001, 2003) allow for a variable and, consequently, endogenous household structure. I develop a variety of examples akin to earlier ones in Haller (2000) and Gersbach and Haller (2000a,b).

Haller (2000) considers a model of a pure exchange economy with given multi-member households. He shows that when externalities are absent, the competitive equilibria among households yield allocations that can be individually decentralized. When there are certain, for instance, positive externalities and each household is able to internalize the intra-household externalities, global efficiency is achieved in equilibrium. If negative externalities are present or individual preferences are satiated, sub-optimality can occur. Gersbach and Haller (2001) focus on the efficiency of the outcomes with a variable household structure. They conclude that if there is no externality, or there are specific ones, household formation and collective decisions within households will not affect Pareto efficiency, but will change the allocation of goods among individuals within the household. In general, neither market nor efficient choices within the household will necessarily lead to a fully optimal allocation. Moreover, if externalities are not too positive, every fully Pareto optimal allocation can be decentralized. Gersbach and Haller (2000a) introduce several concepts of power within households. Gersbach and Haller (2000b) investigate the impact of an exogenous shift of bargaining power within households on the equilibrium allocation and equilibrium welfare.

As already mentioned, the work of Gersbach and Haller as well as mine incorporates the collective rationality model of Chiappori into general equilibrium models. I will refer to these models as “household models”. Household models can be viewed as a special combination of general equilibrium theory and cooperative game theory. The latter refers to the collective rationality assumption. In some household models, an individual can decide to join or leave households. Therefore, either the household structure is exogenously given and fixed or it can be variable and, therefore, endogenous. The competition of all individuals in the market determines a feasible allocation of resources while within households collective decisions are made about consumption. Household decisions are subject

to a household budget constraint. In equilibrium, commodity market clears and the equilibrium is “stable” if nobody wants to leave the household (and, for example, become single) at the current market prices.

Chiappori (1988) models the household as a two-member collectivity, taking Pareto efficient decisions. He constructs a three-good model, in which total consumption and each member’s labor supply are the only observables. He concludes that under an egoistic assumption, one may derive falsifiable conditions upon household labor supplies from both parametric and nonparametric viewpoints. Whereas, under an altruistic assumption, restrictions obtain only in the nonparametric context. Chiappori (1992) develops a general “collective” model of household labor supply, which characterizes agents based on their own preferences and assumes that all household decisions are Pareto efficient. An alternative interpretation is that there are two stages in the internal decision process: First, agents share non-labor income, according to some given sharing rule; second, each one optimally chooses his or her own labor supply and consumption. Then, he shows that this setting generates testable restrictions on labor supplies. Also, the observation of labor supply behavior is sufficient for recovering individual preferences and the sharing rule. His new setting adapts the traditional tools of welfare analysis. For empirical tests and empirical implementation of models with intra-household allocations refer to Browning, Bourguignon, Chiappori, and Lechene (1994) and Browning, Chiappori (1998).

There is another line of research which is based on the risk sharing property of households (see Chiappori (1999) and Mazzocco (2004a, b, c, and d)). These models are only indirectly relevant to mine. They, too, consider individuals rather than households as the building block of the economy. However, their approach to the problem and their methodology are different from that of Gersbach and Haller.

1.1.2 Club Theory

Club Theory is a related, yet different literature. Some club may only provide social benefits to their members, or pure group externalities in the terminology of Gersbach and Haller. Many clubs provide local public good or public projects and

club memberships offer access to those goods, services and projects. In contrast, each consumer purchases on his or her own his or her private consumption goods. Traditional club models used to have at most one private good. For club models with multiple private goods, see Cole and Prescott (1997), Ellickson (1979), Ellickson, Grodal, Scotchmer, and Zame (1999 and 2001), Gilles and Diamantaras (1998), Gilles and Scotchmer (1997), and Wooders (1988, 1989, and 1997). In club models and household models alike, we have the allocation of individuals into groups and allocation of consumption goods to individuals. Also, individuals engage in competition in a market and are affected by market conditions. In other words, like household models, club models permit for endogenous group (or household) formation and competitive market allocation of private goods. This begs the question how club and household models differ.

For a detailed discussion on why club theory and households model are different, I refer to Gersbach and Haller (2001 and 2008). In club models, individuals “compete” for club membership and private consumptions. This competition yields optimal choices subject to individual budget constraints, and at equilibrium prices markets for both club memberships and private goods clear. Gersbach and Haller (2008) show that in the absence of consumption externalities, their household model and the club model of Gilles and Scotchmer (1997) are equivalent in a certain sense: Equivalence means that the respective equilibrium concepts amount to the same allocations. This equivalence holds despite different descriptive aims: In household models collective decisions are made about private consumption - that is the household’s total consumption is subject to the household’s budget constraint – whereas in club models, each individual is subject to an individual budget constraint. With certain consumption externalities, the equivalence breaks down, and the allocative implications of the two models differ. Moreover, other kinds of club models allow for multiple club memberships, in contrast to the current household model where each individual belongs to exactly one household. However, Gersbach and Haller suggest the possibility of a future meta-model that encompasses the features of both models.

1.2 My Contributions

As part of the “household” literature, the first essay of this dissertation (Chapter 3) considers the individual, rather than the household, as the smallest unit of the economy. It uses a general equilibrium model of a pure exchange economy to investigate the gains (or losses) of individuals from forming a household, rather than remaining single. Thus, it focuses on the incentives for household formation and potential gains of both spouses. Consequently, it considers the extent to which the set of opportunities for a household is “bigger” than for single persons. Previous studies have shown that when there are no externalities, there is no gain from household formation. However, externalities are almost always present in real situations and thus there is a gain (or loss) associated with household formation, which deserves serious investigation.

I consider a model with two types of individuals, who may form two-person households (with one member of each type) to benefit from specific forms of group or consumption externalities. The analysis of this model in Chapter 3 leads to five main results. First, when utility functions for individuals are concave, strictly monotone, and continuously differentiable I show the existence of equilibrium for the two-type economy. It is important to notice that moving from “standard” models with households as the building blocks of an economy, to models that treat individuals as the smallest units, necessitates a re-examination of the existence of equilibrium. In other words, I cannot assume that the previous existence theorems in the standard models are necessarily valid in the new setting. Gersbach and Haller (1999) show existence of competitive equilibria among households, if the household structure is exogenously given, in a general equilibrium setting. The novelty here is a variable instead of a fixed household structure. I get existence in a special case with a variable household structure where as many individuals as possible are matched in heterosexual pairs as one possible household structure or individuals remain single as another possible structure. In other words, individuals can exit the household and chose to become single as an outside option. The household structure is stable in the weak sense that at the current prices, no matched individual has an incentive to leave its

household and go single. What I show in Chapter 3, by proposition 1 (and later by proposition 2 and 3 for a slightly different set-up) is that under a certain set of assumptions a competitive equilibrium with free exit is also a competitive equilibrium with free household formation. These equilibrium concepts are introduced in Gersbach and Haller (2003). I assume each utility function concave, strictly monotone and continuously differentiable on \mathbb{R}_{++}^{ℓ} so that the first order approach applies; each endowment strictly positive; all males of the same type with strict preference for marriage; all females of the same type with strict preference for marriage; an equal number of males and females.¹

Second, with equal numbers of males and females, the above household structure is stable in the strong sense that no group of individuals can benefit from forming a new household. This result is of interest, since Gersbach and Haller (2003) have shown by example that in a bilateral matching model with both group and consumption externalities and two goods, there need not exist a competitive equilibrium with a household structure that is stable in the strong sense. My results show that there is at least one special case with group externality where we do have competitive equilibrium with a household structure that is stable in the above sense. Gersbach and Haller (2003) present sufficient conditions for existence which are different from mine.

Third, with equal numbers of males and females and after introducing the extended core for a heterosexual two-person household, I show that the extended core for the economy (obtained by replicating the extended core for households) consists of all possible competitive equilibrium allocations. The allocations belonging to equilibria, where the household structure is stable in a weak or strong sense, form proper subsets of the extended core of the economy. Thus, I not only show the existence of equilibrium, but also can specify or restrict the set of possible equilibria.

Fourth, I show that among all possible equilibria there exists one “trivial” or default equilibrium. Indeed, existence of equilibria is a direct result of the existence of such a default equilibrium. Thereafter, by assuming continuity, I

¹ I assume positive pure group externality to enforce that each type has a strict preference for marriage

construct the set of all possible equilibria around that trivial equilibrium. Finally, I show that my results, which are derived for a particular pure group externality, can be extended to another special case of an additive positive consumption externality.

In another section of this essay, I study the gains and losses from household formation in a different setting. I consider examples with variable household structures and perform comparative statics across household structures to study the effects of household formation. To be precise, I compare individual welfare across equilibria with different pre-set household structures, although the approach also applies to different endogenous household structures. This approach is essentially different from the one that is used in the first section of this essay, where the price system remains unchanged while I compare the options prior to and after an individual exits a household. The effect of change in the structure of households potentially affects the outcome in two different ways: (i) the effects of the presence of group externality, and (ii) the effect of a price change, which can be viewed as a “feedback” effect. As an interesting result, I find that in the presence of certain externalities, for some values of the model parameters, some individuals may find it more attractive to live in a society consisting of singles, which means that there are losses from household formation. The reason is effect (ii) that the equilibrium terms of trade (relative prices) are sensitive to the prevailing household structure.

In my second essay (Chapter 4), I add labor supply (and consumption of leisure) to the two-type economy. The main findings in this chapter (see results 1-11 in section 4.4) are as follows. First, a higher wage rate leads to an increase in consumption. Second, a higher wage rate leads to an increase in spouse’s consumption. Third, a higher wage rate leads to a decrease in leisure. Fourth, a higher wage rate leads to an increase in spouse’s leisure. Fifth, a higher price of private good leads to a decrease in consumption. Sixth, higher leisure leads to an increase in own consumption. Seventh, a higher power of an individual in the household leads to a higher private consumption. Eighth, a higher power of an individual in the household leads to a higher leisure. Ninth, a higher weight of consumption in one’s utility function leads to a decrease in leisure. Tenth, the

more altruistic an individual is, the less is his/her own consumption and the more is the spouse's consumption.

Then, I investigate the effect of change in relative prices (same amount multiplied by or added to all prices) on consumption, leisure and utility level of each individual. I also show how power of individuals in the household can be calculated endogenously in the two-type economy model. Finally, I observe that in Iran education of girls has increased a lot while their labor force participation rate did not. Referring to Spence's signaling model, I find one way (among other possible ways) to show that it is possible that high-productivity women have to spend more on their education to convince the employer that they belong to the high productivity group compared to their male counterparts. This means that female workers need to signal "quality" through education.

In Chapter 5, I decompose poverty in Iran into chronic and transient poverty. I find the determinants of chronic and transient poverty and especially look at the role of education, gender, and employment of the head of household, region (especially rural/urban), and size of household in determining the extent of chronic and transient poverty. Quintile and poverty transition matrices both suggest a high mobility in Iran which indicates the importance of this research in targeting "actual" poor for policy-makers. I also investigate the determinants of growth (or in general, change) in household expenditures.

The current work continues with the outline of a general equilibrium model in Chapter 2, introduction of a simple pure exchange two-type economy, and comparative statics by means of some general equilibrium examples in Chapter 3. In Chapter 4, I will introduce labor to the two-type model and do partial equilibrium analysis and refer to Spence's signaling model (1974) to explain why, despite a highly educated female population, the Iranian female labor participation rate is one of the lowest in the region. The investigation of poverty dynamics in Iran in Chapter 5 constitutes the empirical part of this dissertation. The review of literature on poverty, inequality, and income mobility is also presented in Chapter 5. A summary of the results as well as possible directions for improvement of the model conclude the current work in Chapter 6.

2 General Equilibrium Framework

In this section, I introduce the general equilibrium model with households as the building blocks of the economy.² Consider a finite pure exchange economy and a finite number of consumers, represented by the set $I = \{1, \dots, n\}$. A generic consumer is denoted by i or j . The population I is partitioned into households. P denotes a partition of I into non-empty subsets, and is called a **household structure in I** . In the next section, I consider two special household structures. First, P^0 which is the partition of I into singletons $\{i\}$, $i = 1, 2, \dots, I$. Second, P^1 which is a partition of I into pairs of individuals when $|I| = 2k, k = 1, 2, \dots$. Specifically, when I talk about a two-type economy with equal numbers of individuals of each type, I use the special case of P^1 with two-member households of heterogeneous type, P^2 . \mathcal{P} is the set of all household structures in I . If P consists of H households, then $h = 1, 2, \dots, H$ is used to label them. At times in particular in the next section, I consider a fixed, possibly exogenously given household structure. At other times, I treat the household structure as an object of endogenous choice and hence consider variable household structures.³ h (and sometimes g) serves as a symbol for a “household” throughout this dissertation.

2.1 Commodity and Consumer Allocations

There exist a finite number $\ell \geq 1$ of commodities. Thus the commodity space is \mathbb{R}^ℓ . Each commodity is formally treated as a private good, possibly with externalities in consumption. Each consumer $i \in I$ has a consumption set $X_i = \mathbb{R}_+^\ell$ so that the **commodity allocation space** is $\mathcal{X} \equiv \prod_{j \in I} X_j$. Let $\mathbf{x} = (x_i)$, $\mathbf{y} = (y_i)$ denote generic elements of \mathcal{X} .

² I adopt the basic framework of Haller (2000) and Gersbach and Haller (2001) and their notation.

³ see section (3.1)

I distinguish between a fixed and a variable household structure. The general presumption is that the consumer population is divided into households. Therefore the **consumer allocation space** is \mathcal{P} .

2.2 Household Structures

A fixed household structure means that there is a given household structure $P \in \mathcal{P}$, partitioning the consumer population I into households. A variable household structure means that households are endogenously formed so that some household structure $P \in \mathcal{P}$ is ultimately realized. Relative to P , I use the following terminology regarding $i \in I$ and $h \subseteq I$, $h \neq \emptyset$:

$$\begin{cases} h \in P : & \text{"household } h \text{ exists" or "household } h \text{ is formed"}; \\ i \in h : & \text{"}i \text{ belongs to } h \text{" or "individual } i \text{ is a member of household } h \text{"}. \end{cases}$$

2.3 Feasible Allocation of Commodities and Consumers

An **allocation** is a pair $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ specifying the consumption bundle and household membership of each consumer. After the specification of individual preferences, by means of utility representations, an allocation determines the welfare of each and every member of society. In particular, the set of feasible allocations determines the set of feasible utility allocations.

- The allocation of commodities has the form $\mathbf{x} = (x_i) = (x_i)_{i \in I}$, meaning that consumption bundle $x_i \in X_i$ is assigned to individual i .
- The ALLOCATION OF CONSUMERS assumes the form $P = \{1, \dots, H\}$, meaning that consumers are grouped into households $h \in P$.
- HOUSEHOLD CONSUMPTION. For a potential household $h \subseteq I$, $h \neq \emptyset$, set $\mathcal{X}_h = \prod_{i \in h} X_i$, the consumption set for household h . \mathcal{X}_h has generic elements $\mathbf{x}_h = (x_i)_{i \in h}$. If $\mathbf{x} \in \mathcal{X}$ is a commodity allocation, then consumption for household h is the restriction of $\mathbf{x} = (x_i)_{i \in I}$ to h , $\mathbf{x}_h = (x_i)_{i \in h}$. If $(\mathbf{x}; P)$ is an allocation, then a household $h \in P$ attains the household consumption $\mathbf{x}_h \in \mathcal{X}_h$.
- FEASIBILITY. The economic units endowed with resources are households rather than individuals. Notice, however, that in an environment

with endogenous household formation, each singleton $\{i\}$ is a potential one-person household with its own endowment. For a potential household $h \subseteq I, h \neq \emptyset$, its **endowment** is a commodity bundle $\omega_h \in \mathbb{R}^\ell, \omega_h \geq \mathbf{0}$. A special case is

- **(IPR) Individual Property Rights:** $\omega_h = \sum_{i \in h} \omega_{\{i\}}$ for each household h .

In general, the social endowment with resources depends on the household structure. Namely, if the household structure $P \in \mathcal{P}$ is in place, then the **social endowment** is

$$\omega_P \equiv \sum_{h \in P} \omega_h.$$

A different household structure can yield a different social endowment. Allowing the endowment of a household to differ from the sum of endowments of the potential one-person households formed by its members can be interpreted as resource costs of setting up households or, in the opposite direction, as economies of scale enjoyed by larger households.

I call an allocation $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ **feasible**, if

$$(1) \quad \sum_{i \in I} x_i = \omega_P.$$

A state of the economy is defined as a triple $(p, \mathbf{x}; P)$ such that $p \in \mathbb{R}^\ell$ is a price system and $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ is an allocation.

2.4 Consumer Preferences

In principle, a consumer might have preferences on the allocation space $\mathcal{X} \times \mathcal{P}$ and care about each and every detail of an allocation. For individual $i \in I$, I assume that i has preferences on $\mathcal{X} \times \mathcal{P}$ represented by a

- utility function $U_i : \mathcal{X} \times \mathcal{P} \rightarrow \mathbb{R}$

It is reasonable to assume that an individual does not care about the features of an allocation beyond the boundaries of his own household. Condition HSP is a formal expression of this assumption.

- **(HSP) Household-Specific Preferences:**

$$U_i(\mathbf{x}; P) = U_i(\mathbf{x}_h; h) \text{ for } i \in h, h \in P, (\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}, U_i : \mathcal{X}_h \times P \rightarrow \mathbb{R}.$$

The notation $\mathbb{U}_i(\mathbf{x}_h; h)$ indicates that the individual's welfare depends only on the arguments \mathbf{x}_h and h . If a fixed household structure P is given, then the arguments P or h of the utility functions may also be omitted and HSP reduces to the condition of **intra-household externalities** employed in Haller (2000).

HSP allows for pure group externalities which solely depend on the persons belonging to a household. It also permits various kinds of consumption externalities. Consumption externalities can be anonymous. An individual cares only about its own consumption and aggregate consumption in the household, not the composition of the household or who consumes exactly what among fellow household members. Consumption externalities can also be personal and therefore the extent of the externalities depends not only on the level of consumption, but also on the specific persons who consume in the household. To formulate these externalities, I need more notations. For $i \in I$, define $\mathcal{H}_i \equiv \{h \subseteq I \mid i \in h\}$. \mathcal{H}_i is the set of potential households of which i would be a member. If $h \in \mathcal{H}_i$ and $\mathbf{x}_h \in \mathcal{X}_h$, then I can write $\mathbf{x}_h = (x_i, x_{h \setminus i})$ where $h \setminus i$ serves as shorthand for $h \setminus \{i\}$ and

$$x_{h \setminus i} \in X_{h \setminus i} = \prod_{j \in h \setminus i} X_j$$

describes the consumption of household members j other than i . Now I am prepared to formulate externalities as well as separability and monotonicity properties. I commence with the latter.

- **(MON) Monotonicity:** $\mathbb{U}_i(x_i, x_{h \setminus i}; h)$ is increasing in x_i for all $i \in I, h \in \mathcal{H}_i$.
Intra-household consumption externalities exists if $U_i(\mathbf{x}) = \mathbb{U}_i(\mathbf{x}_h)$, for $i \in h, \mathbf{x} \in \mathcal{X}$.
- **(NNE) Non-Negative Externalities:** $\mathbb{U}_i(x_i, x_{h \setminus i}; h)$ is non-decreasing in $x_{h \setminus i}$ for all $i \in I, h \in \mathcal{H}_i$.
- **(NPE) Non-Positive Externalities:** $\mathbb{U}_i(x_i, x_{h \setminus i}; h)$ is non-increasing in $x_{h \setminus i}$ for all $i \in I, h \in \mathcal{H}_i$.
- **(SEP) Separable Externalities** $\mathbb{U}_i(x_i, x_{h \setminus i}; h) = V_i(x_i) + \sum_j v_{i,j}(x_j)$ for $i \in h, j \in h, i \neq j, V_i : X_i \rightarrow \mathbb{R}$, and $v_{i,j} : X_j \rightarrow \mathbb{R}$.

- **(PGE) Pure Group Externalities** For each consumer i , there exist functions $U_i^c : X_i \rightarrow \mathbb{R}$ and $U_i^g : \mathcal{H}_i \rightarrow \mathbb{R}$ such that

$$\mathbb{U}_i(\mathbf{x}_h; h) = U_i^c(x_i) + U_i^g(h) \text{ for } \mathbf{x}_h \in \mathcal{X}_h, h \in \mathcal{H}_i.$$

PGE assumes that one can additively separate the pure consumption effect $U_i^c(x_i)$ from the pure group effect $U_i^g(h)$. A special case of PGE is **group size externality** where $U_i^g(h) = U_i^g(|h|)$. A very special case of PGE is the absence of externalities, corresponding to $U_i^g \equiv 0$. Since I will refer to it repeatedly, let us distinguish this case by its own acronym. In the next section I will especially look at the case of additively separable fixed group externalities, i.e. $\mathbb{U}_i(\mathbf{x}_h; h) = U_i^c(x_i) + U_i^g(h)$ where $i \in I$ is a member of household h with only two persons of different type. Furthermore,

$$U_i^g(h) = \begin{cases} B_i & \text{for } |h| = 2, \\ 0 & \text{otherwise,} \end{cases}$$

where B_i is the benefit from formation of household.⁴

- **(ABS) Absence of Externalities:** $U_i(\mathbf{x}; P) = V_i(x_i)$ for $i \in I$, $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ and $V_i : X_i \rightarrow \mathbb{R}$.

2.5 Pareto Optimal Allocations

Which allocations qualify as “optimal” or “efficient” depends on how much freedom a social planner is granted to allocate resources and people. In this section, I consider two cases. In the first case, the social planner is constrained by a fixed household structure and can only allocate the available resources. In the second case, the planner can allocate both people and resources.

Constrained optimality refers to commodity allocations that are optimal relative to a fixed household structure. Suppose then a fixed household structure P . A commodity allocation $\mathbf{x} \in \mathcal{X}$ is P -**feasible**, if the allocation $(\mathbf{x}; P)$ is feasible. Denote by $\mathcal{X}(P)$ the set of P -feasible allocations. A commodity

⁴ One can also think about the cost of leaving the household (in the case of divorce). Taking into the account both the cost of leaving a household and benefit of forming a household, may become very crucial in the case that one is interested in a household formation game, or a dynamic model. In our static model benefit would be enough for most applications.

allocation $\mathbf{x} \in \mathcal{X}$ is **constrained Pareto optimal with respect to P** or **P -optimal**, if $\mathbf{x} \in \mathcal{X}(P)$ and there is no $\mathbf{y} \in \mathcal{X}(P)$ with

$$(U_i(\mathbf{y}; P))_{i \in I} > (U_i(\mathbf{x}; P))_{i \in I}.$$

Next suppose that a social planner can allocate both commodities and consumers. An allocation $(\mathbf{x}; P)$ is called **(fully) Pareto optimal** or an **optimum optimum**, if “there is no better one”, i.e. if $(\mathbf{x}; P)$ is feasible and there is no feasible allocation $(\mathbf{x}'; P')$ satisfying

$$(U_i(\mathbf{x}'; P'))_{i \in I} > (U_i(\mathbf{x}; P))_{i \in I}.$$

Denote by \mathcal{M}^* the set of Pareto optimal allocations. Gersbach and Haller (2001) show the existence of a fully Pareto optimal allocation in the case when the utility functions $U_i(\cdot; h), i \in I, h \in \mathcal{H}_i$ are continuous.

2.6 Fixed Household Structure

For a fixed household structure, I define the concept of a competitive equilibrium among households. In an equilibrium among households, a household chooses an efficient consumption schedule for its members, subject to the household budget constraint. Throughout this section, I take a household structure $P \in \mathcal{P}$ as given. First, I consider a household $h \in P$ and a price system $p \in \mathbb{R}^\ell$. For $\mathbf{x}_h = (x_i)_{i \in h} \in \mathcal{X}_h$, denote

$$p * \mathbf{x}_h = p \cdot \bar{\mathbf{x}}_h = p \cdot \left(\sum_{i \in h} x_i \right).$$

Then h 's **budget set** is defined as

$$B_h(p) = \{\mathbf{x}_h \in \mathcal{X}_h : p * \mathbf{x}_h \leq p \cdot \omega_h\}.$$

A demand correspondence for household h is defined as $D_h : \mathbb{R}^\ell \Rightarrow \mathcal{X}_h$ with $D_h(p) \subseteq B_h(p)$ for all $p \in \mathbb{R}^\ell$.

I define next the **efficient budget set** $EB_h(p)$ by: $\mathbf{x}_h = (x_i)_{i \in h} \in EB_h(p)$ if and only if $\mathbf{x}_h \in B_h(p)$ and there is no $\mathbf{y}_h \in B_h(p)$ such that

- $U_i(\mathbf{y}_h; h) \geq U_i(\mathbf{x}_h; h)$ for all $i \in h$;
- $U_i(\mathbf{y}_h; h) > U_i(\mathbf{x}_h; h)$ for some $i \in h$.

Finally, a **competitive equilibrium among households (given household structure P)** or a **P -equilibrium** for short is defined as a price system p together with an allocation $\mathbf{x} = (x_i)$ satisfying (1) and

$$\mathbf{x}_h \in EB_h(p) \text{ for all } h \in P.$$

Thus in a competitive equilibrium, each household makes an efficient choice under its budget constraint and markets clear. Efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household. Following Haller (2000), I explore a budget exhaustion property for welfare conclusions:

- **(BE) Budget Exhaustion:** The budget exhaustion property holds for the economy with household structure P , if

$$(2) \mathbf{x}_h \in EB_h(p) \Rightarrow p * \mathbf{x}_h = p \cdot \omega_h$$

holds for each household $h \in P$, any household consumption profile $\mathbf{x}_h \in \mathcal{X}_h$, and any price system $p \in \mathbb{R}^\ell$.

The budget exhaustion property holds for the economy with variable household structure, if condition (2) is satisfied for all h , $\mathbf{x}_h \in \mathcal{X}_h$, and $p \in \mathbb{R}^\ell$.

Monotonicity (MON) together with non-negative externalities (NNE) implies BE for any household structure.

Existence of P -equilibria for a given household structure P under Budget Exhaustion is shown in Gersbach and Haller (1999). Constrained optimality of P -equilibria is addressed by Haller (2000) who obtains an abstract version of the first welfare theorem for a fixed households structure, suggesting that the interaction of efficient collective household decisions and markets produces efficient outcomes.

2.7 Variable Household Structure

Suppose I allow a social planner to rearrange households and thus choose an arbitrary household structure, while the resource allocation is left to the market. With variable household structure, there is the option to leave a household. Also, household membership is an endogenous outcome. This allows definitions of

normative concepts such as Pareto optimality and inquiry into the interaction of collective decisions and markets with flexible boundaries.

Let $D = (D_h)_{h \in \mathcal{H}}$ be a profile of demand correspondences for households and $(p, \mathbf{x}; P)$ be a state of the economy.

The state $(p, \mathbf{x}; P)$ is a **competitive D-equilibrium** if the allocation $(\mathbf{x}; P)$ is feasible and $\mathbf{x}_h \in D_h(p)$ for $h \in P$.

The state $(p, \mathbf{x}; P)$ is a **competitive equilibrium with free exit** (CEFE) if the allocation $(\mathbf{x}; P)$ is feasible and:

- $\mathbf{x}_h \in EB_h(p)$ for all $h \in P$;
- There is no $h \in P$, $i \in h$ and $y_i \in B_{\{i\}}(p)$ such that $U_i(y_i; \{i\}) > U_i(\mathbf{x}_h; h)$.

The state $(p, \mathbf{x}; P)$ is a **competitive equilibrium with free household formation** (CEFH) if the allocation $(\mathbf{x}; P)$ is feasible and:

- $\mathbf{x}_h \in EB_h(p)$ for all $h \in P$;
- There is no $h \in P$, $i \in h$ and $y_i \in B_{\{i\}}(p)$ such that $U_i(y_i; \{i\}) > U_i(\mathbf{x}_h; h)$;
- There is no h and $g \in P$, $i \in h$ and $\mathbf{y}_{g \cup \{i\}} \in B_{g \cup \{i\}}(p)$ such that $U_j(\mathbf{y}_{g \cup \{i\}}; g \cup \{i\}) > U_j(\mathbf{x}_g; g)$ for all $j \in g$:
 $U_i(\mathbf{y}_{g \cup \{i\}}; g \cup \{i\}) > U_i(\mathbf{x}_h; h)$.

For a more detailed discussion on related equilibrium concepts such as competitive equilibrium with free exit (CEFE) or competitive equilibrium with free household formation (CEFH) refer to Gersbach and Haller (2003). With endogenous household formation, we need to add the stability requirements like “at the current prices, no individual should benefit from exit; no individual should benefit from joining another household; no group of individuals should benefit from forming a new household” to our equilibrium concept. In a competitive equilibrium each household makes a collective choice given its budget set and markets clear, and the “stability” in the above sense exists. In Chapter 3, what I use as the equilibrium concept is a combination of (CEFE) and (CEFH). Since the only plausible household structures in the two-type economy with equal

number of each type are P^0 and P^2 , using the notion of stability, as defined in that chapter, simplifies the discussions.

The notion of an “optimal” household structure can be formalized. Namely, set $\mathcal{P}^* \equiv \{P \in \mathcal{P} \mid \exists \mathbf{x} \in \mathcal{X} : (\mathbf{x}; P) \in \mathcal{M}^*\}$.

Then a household structure P will be called **optimal**, if $P \in \mathcal{P}^*$, that is if it is part of an optimum optimorum (full Pareto optimum).

3 Gains and Losses in Household Formation

In this chapter a pure exchange economy with a specific group or consumption externality is investigated.

3.1 Two-Type Economy

Let us start with a very simple model, where there are only two types of individuals. There are $\ell \geq 1$ commodities. There are two types of consumers, females and males, of finite numbers $n_F \geq 1$ and $n_M \geq 1$, respectively. Let $I = F \cup M$, $F \cap M = \emptyset$, be the set of the individuals in the economy, where F is the set of females and M is the set of males. Individual $i \in I$ has an endowment bundle $\omega_i \in \mathbb{R}_{++}^\ell$. Assume i has utility $U_i(x_i)$ when single and consuming $x_i \in \mathbb{R}_+^\ell$. I assume well behaved preferences, which guarantees the existence of equilibrium. To be precise, I assume each U_i to be concave, strictly monotone, and twice continuously differentiable on \mathbb{R}_{++}^ℓ , such that the first order approach applies. This type economy is characterized by the existence of $U_F : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$, $U_M : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$, such that $U_i = U_F, \forall i \in F$ and $U_j = U_M, \forall j \in M$.

3.2 Two-Type Economy with Group Externalities

In this section we consider specific pure group externalities in two person households. The group externality takes the simple form of constant additive utility:

$$U_i^h(x_i) = U_i(x_i) + B_i, i \in F$$

$$U_j^h(x_j) = U_j(x_j) + B_j, j \in M$$

where B_i and B_j represent the benefit of getting together and forming a household for individuals of type F and M , respectively. Otherwise, each individual who remains single enjoys his or her own utility $U_i(x_i)$, $i \in I$. This means that only if a female and a male form a household they will enjoy the

group externality. Otherwise there is no externality. Further, let us assume there exists a joint utility for the household of the form

$$\begin{aligned} U_{i+j}^h(x_i, x_j) &= a_i [U_i(x_i) + B_i] + a_j [U_j(x_j) + B_j] \\ &= a_i U_i^h(x_i) + a_j U_j^h(x_j), \quad i \in F, j \in M, h = \{i, j\}, \end{aligned}$$

U_{i+j}^h being the utilitarian social welfare function for the household h that consists of two individuals $i \in F$ and $j \in M$.

Without loss of generality, I assume $n_M \geq n_F$. Let us start with the simplest possible case. Suppose $n_F = n_M = 1$, and some a_i and a_j . Let $F = \{i\}$, $M = \{j\}$. Also, suppose that (x^*, p^*) is the P^0 -equilibrium of the economy, i.e. if there is no multi-member household formation. Each agent maximizes her or his own utility subject to her or his budget set, i.e. x_k^* is the solution to the following optimization problem:

$$\max_{x_k} U_k(x_k) \text{ s.t. } p^* x_k \leq p^* \omega_k, \quad k = i, j.$$

I know that there exist numbers μ_k , such that

$$\begin{aligned} \text{grad} U_k(x_k^*) &= \mu_k p^*, \quad k = i, j \\ \sum_k x_k^* &\leq \sum_k \omega_k, \end{aligned}$$

These are the tangency and social feasibility conditions. Also,

$$U_k(x_k) \geq U_k(\omega_k), \quad k = i, j.$$

In other words, the agent will voluntarily trade and end up with an allocation that provides both of them with at least as much utility as they can get from their initial endowments.

Suppose that the household structure is P^2 , and one takes the Pareto optimal allocation, $x^* = (x_i^*, x_j^*) \gg 0$, without group externalities. In this case, x^* solves

$$\begin{aligned} \max_{x_i} U_i(x_i) \\ \text{s.t. } x_i + x_j &\leq \omega_i + \omega_j, \\ U_j(x_j) &\geq U_j(x_j^*) \end{aligned}$$

which gives

$$(3) \quad \text{grad} U_i(x_i^*) = \beta \text{grad} U_j(x_j^*).$$

Note that (3) is the necessary and sufficient condition for the solution of the above problem, disregarding boundary solutions.⁵ On the other hand, market equilibrium satisfies the first order condition:

$$(4) \quad \text{grad}U_k(x_k^*) = \mu_k p^*, \quad k = i, j.$$

These two sets of conditions (3) and (4) are identical for $\beta = \frac{\mu_i}{\mu_j}$. Therefore, the first order conditions for a Pareto efficient allocation are the same as the first order conditions for maximizing a weighted sum of utilities, that is:

$$\max_{x_i, x_j} a_i U_i^h + a_j U_j^h \quad \text{s.t.} \quad p^*(x_i + x_j) \leq p^*(\omega_i + \omega_j).$$

The first order condition for this problem is:

$$\begin{aligned} a_k \text{grad}U_k(x_k^*) &= \lambda p^*, \quad k = i, j, \text{ or} \\ \text{grad}U_i(x_i^*) &= \frac{a_j}{a_i} \text{grad}U_j(x_j^*), \end{aligned}$$

which is the same as the problem above for $\beta = \frac{a_j}{a_i}$. Note that if I fix the parameters $a_k \quad k = i, j$, then I can reach any particular point on the contract curve.⁶ For example, if $a_i = 0$,⁷ then the allocation in which the type M individual consumes all of his endowments is a Pareto optimal outcome.⁸ The question here is which outcomes are stable.

Given any household structure, **an outcome (allocation) is stable** if there is no possible outside option that in the current situation (price system) would make anyone strictly better off. In this definition, the word “possible” plays a very important role. Notice that I use the word stable in a “positive” sense. If

⁵ If I switch the role of females and males the resulting optimization problem would retain the same form of solutions.

⁶ One should notice that $a_k, k = i, j$, in the Utilitarian Social Welfare function can be viewed as the power that each type has in the household.

⁷ One should note that F.O.C. holds only for interior solutions, but this particular result is true without using F.O.C.

⁸ We may have seen such a phenomena, or at least similar cases, in some traditional marriage norms in the past.

one considers a case where the household structure is fixed— for example, the cost of divorce is very high — then any interior point on the contract curve is a stable outcome for a particular choice of a_i and a_j , $i \in F, j \in M$.

Definition 1. A feasible allocation $x = (x_i, x_j)$ belongs to the **household extended core of household (i, j) with group externality** (briefly, extended core) if it belongs to the contract curve of the economy consisting of household members i and j . That is $grad(U_i)$ and $grad(U_j)$ are proportional, $U_i(x_i) + B_i \geq U_i(\omega_i)$, and $U_j(x_j) + B_j \geq U_j(\omega_j)$.

Assumption (ETP) Equal Treatment Property in Equilibrium: If there are several individuals with the same types, utilities, and initial conditions (endowments, etc.) in the equilibrium, they would all end up with the same allocation.

Definition 2 The **extended core of the replica economy** (briefly, extended core of the economy) is the replica of the extended core for the household.

I use (ETP) in the proof for proposition 1, 2 and 3. The idea behind this assumption is simple and appealing: Given the same type, utility and initial condition, in equilibrium there should not be any discrimination among players based on their “labels” or “names”.⁹

The extended core of the economy is the largest set that contains all of the stable Pareto optimal solutions for the case of an equal number of individuals of each type, where the individuals inside the household cannot trade after separation (as an outside option), and should consume their endowments after “divorce” (see the proposition below). One can think about it this way: The individual only has the opportunity to trade before and during her/his marriage, but is not allowed to trade after divorce. If this is the case, and I have an equal number of individuals of each type, the extended core of the replica economy represents the set of all stable P -optimal outcomes where P is the household structure with $|h| \leq 2$ and there is one member of each type in each household of size two. In other words, I have household structure P^2 . Furthermore, I allow

⁹ Many game theory and network models use the axiom of anonymity in equilibrium, which is a related concept: a mutations of players’ name will not change the outcome.

single individuals to be present. Notice that if I allow for trade after the separation, I should reconsider the stability issue.¹⁰ If $n_F = n_M = 1$, and if I allow for trade after separation, then the P^0 -equilibrium allocation will be stable, and will be a point in the core, and thus a point in the extended core. Pareto optimality implies that if one divorces, the best she or he can maintain is the utility of P^0 -equilibrium¹¹. In this case, there is not another individual to make collusion with, after the trade. If this is the case, then she or he will not enjoy the benefit associated with household formation. Thus individuals prefer to stay in the household after it is formed.

It is clear that the P^0 -equilibrium is a stable P^2 -equilibrium. However, this leads us to the question of uniqueness. Based on the assumption of continuity, I can conclude that a subset of the extended core in a neighborhood of the P^0 -equilibrium is the set of all stable P^2 -optimal outcomes.¹² This will change if I change a_i and a_j , $i \in F, j \in M$. Now, suppose $n_F = n_M = n$. I can look at one out of n formed households, and investigate the outside option for each individual in that household if I allow for trade after divorce. First, let us state the following propositions.

Proposition 1 *If $n_F = n_M = n$, and if the individual cannot trade after separation, then the extended core of the economy is the set of all stable P^2 -optimal allocations. These are supportable with some market price, given a suitably chosen set of weights in the utilitarian social welfare function for the household.*

Proof. Using the above discussion and the Equal Treatment Property (ETP) in a Replica Economy, I can conclude that each individual will match with an individual of the other type in equilibrium. I allow for trades inside the household in the extended core, but after formation of a household, there is no trade outside the household: Everybody has already achieved her or his highest attainable

¹⁰ Obviously the way it affects the outcomes is different, too.

¹¹ Assuming P^0 -prices are the going prices.

¹² Remember that if one offers a point on the contract curve outside the extended core, an individual can do better if she consumes her endowments. Thus, arbitrary trade should occur in the extended core.

utility, and there is no Pareto-improving movement left. Further, there will be no gain for an individual from leaving the household. \downarrow

Proposition 2 *If $n_F = n_M = n$ and individuals are allowed to trade after separation, then a proper subset of the extended core of economy is the set of all stable P^2 -optimal allocations that are supportable with some market price, given a suitably chosen set of weights in the utilitarian social welfare function for the households. The subset is non empty. It contains the P^0 -equilibrium allocation and a neighborhood around it.*

Proof. I use the above discussion for $n_F = n_M = 1$. Using the assumption of the Equal Treatment Property in a Replica Economy, if no new household formation is possible, then the P^0 -equilibrium is a stable P^2 -equilibrium. By the continuity assumption, a proper subset of the extended core of economy in a neighborhood of the P^0 -equilibrium contains the stable P^2 -optimal outcomes.

Moreover, for any household h , if $x_h^* = (x_i^*, x_j^*)$ solves

$$\max_{x_i, x_j} a_i U_i^h + a_j U_j^h \text{ s.t. } p^*(x_i + x_j) \leq p^*(\omega_i + \omega_j), i \in F, j \in M,$$

then $x_h^* = (x_i^*, x_j^*) \in EB_h(p^*)$.

Now, let us allow for the formation of new households. That is, allow two different individuals of different types in two households to end their relations with their partners and form a new household together (divorce and remarriage). Let us distinguish between two cases here. In one case, all household utilitarian social welfare weights are equal; in the other, they are different. First, suppose $a_i, i \in F, M$ are equal for all households. P^0 -equilibrium (x^*, p^*) is P^2 -equilibrium and $x_h^* = (x_i^*, x_j^*) \in EB_h(p^*)$ for all $h \in P^2$. It is obvious that, by the definition of an efficient budget set, there is no other allocation that makes at least one person better off without making others worse off. Therefore, the P^2 -equilibrium is stable. By continuity, there is a neighborhood around the P^0 -equilibrium allocation that is stable. Now suppose that different households have different parameters $a_i, i \in F, M$. Take any female i and any male j belonging to households g and h , respectively, and suppose that they form a new household k , where they get $U_i^k(y_i^*) > U_i^g(x_i^*), U_j^k(y_j^*) > U_j^h(x_j^*)$ (a more relaxed

situation is at least one of them is better off). This means $(x_i^*, x_j^*) \notin EB_h(p^*)$, which is a contradiction. So, P^0 -equilibrium is stable and, by the assumption of continuity, a proper subset of the extended core in a neighborhood of the P^0 -equilibrium is the stable P^2 -optimal outcomes. †

Now, consider the case where $n_F \neq n_M$. Assuming well behaved utility functions for individuals, and using the properties of a Replica Economy (mainly I assume (ETP)), I know that in order to maximize the sum of utilities I must have the maximum possible household formation. Without loss of generality, I may assume $n_M > n_F$. Thus, $n_M - n_F$ individuals remain single. Notice that remaining single (i.e. structure P^0) is not a stable structure for households: Any two individuals with different types can get together and be better off. Now the question is what is the optimal outcome for this case where $n_M \neq n_F$. One may ask whether there is any equilibrium. If yes, is it unique? Also, under what conditions are the households that already exist “stable”? This dissertation does not answer the above questions.

3.3 Two-Type Economy with Consumption Externality

Let us consider the two-type economy introduced before and (SEP) and (MON). Suppose that utility takes the form that is introduced in (SEP). Also suppose that $v_{i;j}(x_j) = \alpha_{i;j}U_j(x_j)$. Notice that in the two-type economy, defined earlier, we have considered the household structure P^2 . If there is a positive intra-household consumption externality ($\alpha_{i;j} \geq 0$), then for a typical household h :

$$\begin{aligned} U_h &= \sum_{i \in h, j \in h, j \neq i} a_i [U_i(x_i) + v_{i;j}(x_j)] = \sum_{i \in h, j \in h, j \neq i} a_i [U_i(x_i) + \alpha_{i;j}U_j(x_j)] \\ &= \sum_{i \in h, j \in h, j \neq i} (a_i + a_j \alpha_{j;i}) U_i(x_i) = \sum_{i \in h} \beta_i U_i(x_i), \end{aligned}$$

where $\beta_i = (a_i + a_j \alpha_{j;i})$, $i \in h, j \in h, j \neq i$. If I assume $a_i, \alpha_{i;j} > 0, \forall i, j \in h, i \neq j$, then $\beta_i > 0$. Hence, the household exhausts its budget. If I have a single-person household, it is clear that she or he will exhaust her or his budget as well. So a competitive equilibrium allocation among households, i.e. the P^2 -equilibrium, is P^2 -optimal, and can be individually decentralized. The next question is whether

or not this equilibrium is stable. Let us first look at the system of equations that relates parameters β_i and a_i :

$$\begin{aligned} a_i + a_j \alpha_{j;i} &= \beta_i, \\ a_j + a_i \alpha_{i;j} &= \beta_j, \end{aligned}$$

or in matrix form

$$\begin{pmatrix} 1 & \alpha_{j;i} \\ \alpha_{i;j} & 1 \end{pmatrix} \begin{pmatrix} a_i \\ a_j \end{pmatrix} = \begin{pmatrix} \beta_i \\ \beta_j \end{pmatrix}.$$

If $\det \begin{pmatrix} 1 & \alpha_{j;i} \\ \alpha_{i;j} & 1 \end{pmatrix} = 1 - \alpha_{i;j} \alpha_{j;i} \neq 0$, the matrix has an inverse and

$$\begin{aligned} \begin{pmatrix} a_i \\ a_j \end{pmatrix} &= \frac{1}{1 - \alpha_{i;j} \alpha_{j;i}} \begin{pmatrix} 1 & -\alpha_{j;i} \\ -\alpha_{i;j} & 1 \end{pmatrix} \begin{pmatrix} \beta_i \\ \beta_j \end{pmatrix}, \text{ or} \\ \begin{pmatrix} a_i \\ a_j \end{pmatrix} &= \frac{1}{1 - \alpha_{i;j} \alpha_{j;i}} \begin{pmatrix} \beta_i - \alpha_{j;i} \beta_j \\ \beta_j - \alpha_{i;j} \beta_i \end{pmatrix}. \end{aligned}$$

For reasonable results, I should have $a_i, a_j > 0$. So, for $a_i > 0$ I get

$$\beta_i - \alpha_{j;i} \beta_j > 0, \text{ and } 1 - \alpha_{i;j} \alpha_{j;i} > 0, \text{ or } \beta_i - \alpha_{j;i} \beta_j < 0, \text{ and } 1 - \alpha_{i;j} \alpha_{j;i} < 0,$$

and for $a_j > 0$,

$$\beta_j - \alpha_{i;j} \beta_i > 0, \text{ and } 1 - \alpha_{i;j} \alpha_{j;i} > 0, \text{ or } \beta_j - \alpha_{i;j} \beta_i < 0, \text{ and } 1 - \alpha_{i;j} \alpha_{j;i} < 0.$$

Under these conditions, I have a one-to-one relationship between β_i 's and a_i 's. In addition, I get a positive a_i for any positive β_i . If these conditions hold, then I can state the following proposition:

Proposition 3. *If $n_F = n_M = n$, individuals are allowed to trade after separation, and the above conditions on β_i 's and a_i 's hold, then a proper neighborhood around P^0 -equilibrium on the contract curve is the set of all stable P^2 -optimal allocations that are supportable with some market price given a suitably chosen set of weights in a utilitarian social welfare function for the households.*

Proof. First, because of positive externality, as many households as possible will form. It is clear that the P^0 -equilibrium is a stable P^2 -equilibrium. Therefore, the subset is not empty. By continuity, a proper neighborhood around P^0 -equilibrium on the contract curve is also stable P^2 -optimal. \dagger

3.4 Two-Person Households: Illustrative Examples

I use several examples to illustrate some of the simple facts and to shed light on the more general cases. In this section I go beyond the two-type economy model and use the general setting from Gersbach and Haller (2001). A distinction should be made between stability as defined in section 3.2 and the free exit option discussed in Gersbach and Haller (2001) and Haller (2000). In the previous section, I define a household to be stable if under given price system, the individuals in the household do not have any incentive to leave the household. This means that under the current prices the level of their individual utility will become lower by leaving the household compared to staying in the household. In other words, I am conducting the stability analysis under fixed equilibrium prices. The free exit equilibrium in Gersbach and Haller's sense is more or less the same. In this section, however, I will compare two different household structures, and therefore conduct a comparative static analysis. Thus the prices will no longer stay the same. In other words, I am contrasting two different utility levels for each and every individual under two different price systems (which is a result of change in household structure).

In the sequel, $\ell \geq 1$ stands for the number of commodities, and I stands for the finite set of consumers or individuals. I begin with a simple example of a three-person pure exchange economy without any externalities. One can think of these three persons as different types of individuals. This will make the example more general. The example illustrates that if there are no externalities, then there is basically no incentive for the individuals to form two-member households. The market outcomes will not change, regardless of whether individuals stay alone or join together. Note that in the examples, superscripts denote the commodity and subscripts denote the individual or household.

3.4.1 Example 1

Let $\ell = 2$ and $I = \{1, 2, 3\}$. $U_i(x_h; h) = u_i(x_i) = u_i(x_i^1, x_i^2)$ represents preferences, where x_i^k denotes the quantity of good k , $k = 1, \dots, \ell$, consumed by individual i . Specifically, let us assume:

$$\begin{aligned} u_1(x_1^1, x_1^2) &= x_1^1, \\ u_2(x_2^1, x_2^2) &= x_2^2, \\ u_3(x_3^1, x_3^2) &= x_3^1 x_3^2, \end{aligned}$$

and assume the individuals endowments $\omega_1 = (x, x)$, $\omega_2 = (x, x)$, $\omega_3 = (x, x)$.

I normalize commodity prices so that $p_1 = 1$ (i.e. $p = (1, p_2)$). Thus, I have three persons with the same endowments. One of them prefers to use both commodities, while the other two only want to use one of the goods. Suppose each individual acts separately in the market, i.e., the household structure is $P^0 = \{\{1\}, \{2\}, \{3\}\}$. If so, there exists a unique market equilibrium calculated by using the demand for commodities as follows:

$$x_1^1 = \frac{x(1+p_2)}{1}, x_1^2 = 0, x_2^1 = 0, x_2^2 = \frac{x(1+p_2)}{p_2}, x_3^1 = \frac{x(1+p_2)}{2}, x_3^2 = \frac{x(1+p_2)}{2p_2}.$$

In equilibrium the excess demand for good 1 is zero so that:

$$x_1^1 + x_2^1 + x_3^1 = \frac{x(1+p_2)}{1} + \frac{x(1+p_2)}{2} = 3x.$$

The solution is $p_2 = 1$, and the market equilibrium is given by:

$$\begin{aligned} p &= (1, 1), \\ x_1 &= (2x, 0), U_1 = 2x, x_2 = (0, 2x), U_2 = 2x, x_3 = (x, x), U_3 = x^2. \end{aligned}$$

The first two individuals will trade, and hence will be better off. The third one will not trade. Now consider the household structure

$$P^1 = \{g, h\} = \{\{1, 2\}, \{3\}\}.$$

Further assume that household g maximizes a Nash product of the form $W = (U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)} = (x_1^1)^\alpha (x_2^2)^{(1-\alpha)} = (x_g^1)^\alpha (x_g^2)^{(1-\alpha)}$, $0 < \alpha < 1$. One can think of α as the power of an individual in the household. The household endowments will be: $\omega_g = (2x, 2x)$, $\omega_h = (x, x)$. Again, assume $p_1 = 1$. Therefore, there exists a unique market equilibrium. Namely, the demand for commodities is as follows:

$$x_g^1 = \frac{2\alpha x(1 + p_2)}{1}, x_g^2 = \frac{2(1 - \alpha)x(1 + p_2)}{p_2}, x_h^1 = \frac{x(1 + p_2)}{2}, x_h^2 = \frac{x(1 + p_2)}{2p_2}.$$

In equilibrium, the excess demand for good 1 is zero, i.e.

$$x_g^1 + x_h^1 = 3x.$$

The solution is:

$$p_2 = \frac{5 - 4\alpha}{4\alpha + 1},$$

$$x_g^1 = 12\alpha \frac{x}{4\alpha + 1}, x_g^2 = 12x \frac{1 - \alpha}{5 - 4\alpha},$$

$$x_h^1 = \frac{3x}{4\alpha + 1}, x_h^2 = \frac{3x}{5 - 4\alpha}.$$

Hence, the market equilibrium is given by:

$$P = (1, \frac{5 - 4\alpha}{4\alpha + 1}),$$

$$x_g = (12\alpha \frac{x}{4\alpha + 1}, 12x \frac{1 - \alpha}{5 - 4\alpha}), U_g = 12\alpha^\alpha x (4\alpha + 1)^{-\alpha} (1 - \alpha)^{1-\alpha} (5 - 4\alpha)^{-(1-\alpha)},$$

$$x_h = (\frac{3x}{4\alpha + 1}, \frac{3x}{5 - 4\alpha}), U_h = 9 \frac{x^2}{(4\alpha + 1)(5 - 4\alpha)}.$$

Note that if $\alpha = \frac{1}{2}$, I will arrive at the competitive equilibrium that I calculated in the first part. If I assume that a member of the household will leave the household if her or his utility decreases as a result of household formation, then this is the only equilibrium under the assumption of free exit. In other words, there is no gain from household formation in the absence of externalities. This result is an instance of Gersbach and Haller's (2000a) No Power Theorem: If there are no externalities, individuals in multi-member households remain powerless.

In the next example, I introduce externalities. One may predict that positive externalities make it more attractive for individuals to form a household. Here I consider only group externalities; which means that if individuals form households their utilities change.

3.4.2 Example 2

This example introduces a group externality associated with household formation. When individuals 1 and 2 form a household, their preferences regarding private consumption will change. The group externality is of the form:

$$u_1(x_1^1, x_1^2) = \begin{cases} x_1^1 x_1^2, & \text{if 1 and 2 form a household} \\ x_1^1, & \text{otherwise} \end{cases}$$

$$u_2(x_2^1, x_2^2) = \begin{cases} x_2^1 x_2^2, & \text{if 1 and 2 form a household} \\ x_2^2, & \text{otherwise} \end{cases}$$

$$u_3(x_3^1, x_3^2) = x_3^1 x_3^2.$$

Consider this particular form of externality as follows: If 1 and 2 remain single, one goes to the movies, and the other drinks. If they form a household, they will enjoy both activities together. If individuals 1 and 2 do not form a household, I get the same results as in the first part of example 1; i.e., the market equilibrium is:

$$p = (1,1),$$

$$x_1 = (2x, 0), U_1 = 2x, x_2 = (0, 2x), U_2 = 2x, x_3 = (x, x), U_3 = x^2.$$

But when 1 and 2 form the household $g = \{1,2\}$, the market equilibrium outcome changes. Assume household g maximizes a Nash product of the form:

$$W = (U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)} = [(x_1^1)^\alpha (x_2^1)^{(1-\alpha)}] [(x_1^2)^\alpha (x_2^2)^{(1-\alpha)}]$$

and use Lemma 1 from Gersbach and Haller (2000b). Let $x_g^1 = x_1^1 + x_2^1$ and $x_g^2 = x_1^2 + x_2^2$ denote the total amounts of commodities 1 and 2, respectively, purchased by household g . According to the lemma, maximization of the Nash product $(U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)}$ requires

$$x_1^1 = \frac{\alpha}{\alpha + (1-\alpha)} x_g^1 = \alpha x_g^1, x_2^1 = \frac{1-\alpha}{\alpha + (1-\alpha)} x_g^1 = (1-\alpha) x_g^1,$$

$$x_1^2 = \frac{\alpha}{\alpha + (1-\alpha)} x_g^2 = \alpha x_g^2, x_2^2 = \frac{1-\alpha}{\alpha + (1-\alpha)} x_g^2 = (1-\alpha) x_g^2.$$

Substituting these values in W , I get:

$$W = (U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)} = [(x_1^1)^\alpha (x_2^1)^{(1-\alpha)}] [(x_1^2)^\alpha (x_2^2)^{(1-\alpha)}]$$

$$= [\alpha^{2\alpha} (1-\alpha)^{2\alpha}] x_g^1 x_g^2.$$

Thus, it is as if I am dealing with a simple Cobb-Douglas household again: $W_g = Ax_g^1 x_g^2$ with $A = [\alpha^{2\alpha}(1-\alpha)^{2\alpha}]$. The household endowments will be $\omega_g = (2x, 2x)$, $\omega_h = (x, x)$. Again, let us assume $p_1 = 1$. Now, I can solve for the equilibria as follows:

$$x_g^1 = x(1 + p_2), x_g^2 = x \frac{1 + p_2}{p_2},$$

$$x_h^1 = \frac{x(1 + p_2)}{2}, x_h^2 = \frac{x(1 + p_2)}{2p_2}.$$

In equilibrium the excess demand for good 1 is zero, so:

$$x_g^1 + x_h^1 = \frac{3}{2}x(1 + p_2) = 3x,$$

The solution is $p_2 = 1$, and

$$x_g^1 = 2x, x_g^2 = 2x, U_g = [\alpha^{2\alpha}(1-\alpha)^{2\alpha}]4x^2,$$

$$x_h^1 = x, x_h^2 = x, U_h = x^2.$$

This is a no-trade equilibrium. Inside the household, the endowments will be distributed according to the individual power:

$$x_1^1 = 2\alpha x, x_2^1 = 2(1-\alpha)x,$$

$$x_1^2 = 2\alpha x, x_2^2 = 2(1-\alpha)x,$$

$$U_1 = 4\alpha^2 x^2, U_2 = 4(1-\alpha)^2 x^2.$$

Suppose that $\alpha = \frac{1}{2}$.¹³ Then formation of household is beneficial for individual 1 if and only if $x^2 \geq 2x$, or $x \geq 2$. This means that incentives for household formation depend on the size of the endowments. Notice that these kinds of preferences exist when single individuals enjoy different activities when they are alone but enjoy both activities, e.g. going to the movies and drinking together, when they join.

3.4.3 Example 3

The following example is a more general case of the last example. I am going to allow for different individual preferences after household formation. Consider the last example with the following change:

¹³ This is actually the competitive equilibrium with free exit. For a definition of this refer to Gersbach and Haller (2000a)

$$\begin{aligned}
u_1(x_1^1, x_1^2) &= \begin{cases} (x_1^1)^\beta (x_1^2)^{1-\beta}, & \text{if 1 and 2 form a household} \\ x_1^1, & \text{otherwise} \end{cases} \\
u_2(x_2^1, x_2^2) &= \begin{cases} (x_2^1)^\gamma (x_2^2)^{1-\gamma}, & \text{if 1 and 2 form a household} \\ x_2^2, & \text{otherwise} \end{cases} \\
u_3(x_3^1, x_3^2) &= x_3^1 x_3^2.
\end{aligned}$$

If individuals 1 and 2 do not form a household together, then the same results as in the first part of example 1 hold; i.e., the market equilibrium will be:

$$\begin{aligned}
p &= (1,1), \\
x_1 &= (2x, 0), U_1 = 2x, x_2 = (0, 2x), U_2 = 2x, x_3 = (x, x), U_3 = x^2.
\end{aligned}$$

If individuals 1 and 2 form the household $g = \{1, 2\}$, the market equilibrium changes. Assume that household g maximizes a Nash product of the form

$$W = [(x_1^1)^{\beta\alpha} (x_2^1)^{\gamma(1-\alpha)}] [(x_1^2)^{(1-\beta)\alpha} (x_2^2)^{(1-\gamma)(1-\alpha)}]$$

Using *lemma 1* from Gersbach and Haller (2000b), let $x_g^1 = x_1^1 + x_2^1$ and $x_g^2 = x_1^2 + x_2^2$ denote the total amounts purchased by household h . Maximization of the Nash product $(U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)}$ requires that:

$$x_1^1 = \frac{\sigma}{\sigma + \tau} x_g^1, x_2^1 = \frac{\tau}{\sigma + \tau} x_g^1, x_1^2 = \frac{\sigma^*}{\sigma^* + \tau^*} x_g^2, x_2^2 = \frac{\tau^*}{\sigma^* + \tau^*} x_g^2,$$

where

$$\sigma = \alpha\beta, \tau = (1 - \alpha)\gamma, \sigma^* = \alpha(1 - \beta), \tau^* = (1 - \alpha)(1 - \gamma).$$

Substituting these values in W , I get:

$$W = (U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)} = A \left[(x_g^1)^{[\alpha\beta + \gamma(1-\alpha)]} (x_g^2)^{[\alpha(1-\beta) + (1-\alpha)(1-\gamma)]} \right].$$

So I am dealing with a simple Cobb-Douglas household again:

$$W_g = A (x_g^1)^\delta (x_g^2)^{(1-\delta)},$$

where

$$\begin{aligned}
A &= \left[\left(\frac{\sigma}{\sigma + \tau} \right)^{\alpha\beta} \left(\frac{\tau}{\sigma + \tau} \right)^{\gamma(1-\alpha)} \left(\frac{\sigma^*}{\sigma^* + \tau^*} \right)^{\alpha(1-\beta)} \left(\frac{\tau^*}{\sigma^* + \tau^*} \right)^{(1-\alpha)(1-\gamma)} \right], \\
\delta &= [\alpha\beta + \gamma(1-\alpha)] = \alpha\beta + \gamma - \gamma\alpha, \\
1 &= [\alpha\beta + \gamma(1-\alpha)] + [\alpha(1-\beta) + (1-\alpha)(1-\gamma)].
\end{aligned}$$

The household endowments will be: $\omega_g = (2x, 2x)$, $\omega_h = (x, x)$. For $p_1 = 1$, I can solve for the equilibria as follows:

$$x_g^1 = 2x\delta(1 + p_2), x_g^2 = 2x(1 - \delta)\frac{1 + p_2}{p_2}, x_h^1 = \frac{x(1 + p_2)}{2}, x_h^2 = \frac{x(1 + p_2)}{2p_2}.$$

In equilibrium, the excess demand for good 1 is zero, so:

$$x_g^1 + x_h^1 = 2x\delta(1 + p_2) + \frac{x(1 + p_2)}{2} = 2x\delta + 2x\delta p_2 + \frac{1}{2}x + \frac{1}{2}xp_2 = 3x,$$

The solution is:

$$\begin{aligned} p_2 &= \frac{5 - 4\delta}{4\delta + 1}, \\ x_g^1 &= 12x\frac{\delta}{4\delta + 1}, x_g^2 = 12(1 - \delta)\frac{x}{5 - 4\delta}, \\ x_h^1 &= 3\frac{x}{4\delta + 1}, x_h^2 = 3\frac{x}{5 - 4\delta}. \end{aligned}$$

The equilibrium price depends only on δ , which is in a sense the household relative evaluation of good 1 compared to good 2.¹⁴ Note that individual 3 values both goods equally. Also, notice that δ is itself a weighted average of individual evaluations of the relative importance of goods with respect to the power in the household. Now, inside the household I have:

$$\begin{aligned} x_1^1 &= \frac{\sigma}{\sigma + \tau}x^1 = \frac{12\alpha\beta x}{(4\delta + 1)}, x_2^1 = \frac{12(1 - \alpha)\gamma x}{(4\delta + 1)}, \\ x_1^2 &= \frac{12\alpha(1 - \beta)x}{(5 - 4\delta)}, x_2^2 = \frac{12(1 - \alpha)(1 - \gamma)x}{(5 - 4\delta)}. \end{aligned}$$

For allocations inside the household the power of individuals in the household and their personal relative evaluations of goods are directly related to the optimal choice, which is exactly what one would expect.

Notice that β and γ show the relative tendency of individuals 1 and 2 toward good 1, respectively (and so do $1 - \beta$ and $1 - \gamma$ for good 2).¹⁵ α and $1 - \alpha$ represent the power of individuals 1 and 2 in the household. Hence, it makes sense that the equilibrium allocation of good 1 for person 1 is directly related to α and β , and for person 2, it is directly related to $1 - \alpha$ and γ . Moreover, the equilibrium allocation of good 2 for person 1 relates directly to α

¹⁴ which is equal to $\frac{d}{d + (1 - d)}$.

¹⁵ To be precise, they are the share of expenditures on good 1 in the total expenditures (=income).

and $1 - \beta$, and for person 2 relates directly to $1 - \alpha$ and $1 - \gamma$. Again, notice that if $\alpha = \beta = \gamma = \frac{1}{2}$, then $\delta = \frac{1}{2}$, and I get back to the no trade equilibrium.

3.4.4 Example 4

Suppose that there is another type of group externality associated with the formation of households, i.e. individuals 1 and 2 care about the same good if they are alone, but if they form a household, their preference will change. Thus, I have a group externality of the form:

$$\begin{aligned} u_1(x_1^1, x_1^2) &= \begin{cases} x_1^1 x_1^2, & \text{if 1 and 2 form a household} \\ x_1^1, & \text{otherwise} \end{cases} \\ u_2(x_2^1, x_2^2) &= \begin{cases} x_2^1 x_2^2, & \text{if 1 and 2 form a household} \\ x_2^1, & \text{otherwise} \end{cases} \\ u_3(x_3^1, x_3^2) &= x_3^1 x_3^2. \end{aligned}$$

One can think about this problem as follows: Suppose good 2 is beverage, and both 1 and 2 are social drinkers. They will not drink if they are alone, but once they get together, they will enjoy drinking.¹⁶ If individuals 1 and 2 do not form a household together, then:

$$x_1^1 = \frac{x(1 + p_2)}{1}, x_1^2 = 0, x_2^1 = \frac{x(1 + p_2)}{1}, x_2^2 = 0, x_3^1 = \frac{x(1 + p_2)}{2}, x_3^2 = \frac{x(1 + p_2)}{2p_2}.$$

In equilibrium, the excess demand for good 1 is zero, so:

$$x_1^1 + x_2^1 + x_3^1 = \frac{x(1 + p_2)}{1} + \frac{x(1 + p_2)}{1} + \frac{x(1 + p_2)}{2} = \frac{5}{2}x(1 + p_2) = 3x.$$

The solution is $p_2 = \frac{1}{5}$, and the equilibrium is:

$$\begin{aligned} p &= (1, \frac{1}{5}), x_1 = (\frac{6}{5}x, 0), U_1 = \frac{6}{5}x, x_2 = (\frac{6}{5}x, 0), \\ U_2 &= \frac{6}{5}x, x_3 = (\frac{3}{5}x, 3x), U_3 = \frac{9}{5}x^2. \end{aligned}$$

If 1 and 2 form a household, the outcome of market equilibrium changes. Assume that household g maximizes a Nash product of the form:

$$W = (U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)} = [(x_1^1)^\alpha (x_2^1)^{(1-\alpha)}] [(x_1^2)^\alpha (x_2^2)^{(1-\alpha)}]$$

¹⁶ Note that they will still enjoy the other good (good 1), even if they do not form a household.

Using lemma 1 from Gersbach and Haller (2000b), let $x_g^1 = x_1^1 + x_2^1$ and $x_g^2 = x_1^2 + x_2^2$ denote the total amount purchased by household h . Further, maximization of the Nash product $(U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)}$ requires

$$x_1^1 = \alpha x_g^1, x_2^1 = (1 - \alpha)x_g^1, x_1^2 = \alpha x_g^2, x_2^2 = (1 - \alpha)x_g^2.$$

From substituting these values in W , I get:

$$W = (U_1(x_1))^\alpha (U_2(x_2))^{(1-\alpha)} = [\alpha^{2\alpha} (1 - \alpha)^{2\alpha}] x_g^1 x_g^2.$$

Consequently, I am dealing with a simple Cobb-Douglas household again: $W_g = Ax_g^1 x_g^2$, where $A = [\alpha^{2\alpha} (1 - \alpha)^{2\alpha}]$. The household endowments are $\omega_g = (2x, 2x)$, $\omega_h = (x, x)$. Assume $p_1 = 1$. Now, I can solve for the equilibrium as follows:

$$x_g^1 = x(1 + p_2), x_g^2 = x \frac{1 + p_2}{p_2}, x_h^1 = \frac{x(1 + p_2)}{2}, x_h^2 = \frac{x(1 + p_2)}{2p_2}.$$

In equilibrium:

$$x_g^1 + x_h^1 = \frac{3}{2}x(1 + p_2) = 3x.$$

The solution is $p_2 = 1$, so:

$$x_g^1 = x(1 + p_2) = 2x, x_g^2 = x \frac{1 + p_2}{p_2} = 2x, \\ x_h^1 = \frac{x(1 + p_2)}{2} = x, x_h^2 = \frac{x(1 + p_2)}{2p_2} = x.$$

Now, in the household,

$$x_1^1 = 2\alpha x, x_2^1 = 2(1 - \alpha)x, \\ x_1^2 = 2\alpha x, x_2^2 = 2(1 - \alpha)x,$$

If $\alpha = \frac{1}{2}$, then $U_1 = x^2$ after the household formation. Compared to $U_1 = \frac{6}{5}x$ in the case of remaining single, household formation is beneficial if $x^2 \geq 1.2x$, or $x \geq 1.2$. Again this shows that benefit from household formation depends on how large the endowment is.

In Chapter 6, I state the insights from the simple model and examples I introduced so far. Further, I try to relate it to real situations. I also suggest some ways for extending the model.

4 Effects of the Price System on Household Labor Supply

4.1 Introduction

In Chapter 3, I have discussed the outcomes of a pure exchange economy when individuals rather than households are the building blocks of the economy. In this chapter, I consider the effects of the price system on the household labor supply if labor is added to my previous model. Specifically, the choice of labor vs. leisure for different genders in a two-type economy is explored. An individual decides about his/her choice of labor (more labor means lower leisure) and private consumption given the price of private good and wage rates. In this model, there is a wage difference that could have been caused by an inherent difference between each gender's productivity. Of course, genders' different productivities are not the only explanation for wage differences. The dynamic behind this gender difference is not the main concern of this chapter, and I assume it is given. This is not an unusual assumption. For example, Becker (1991) specifies the sources of difference in productivity between genders. He refers to "biological" differences that lead to the assumption that an hour of household or market time of each spouse is not a perfect substitute for the other spouse's time. He also mentions that women have a "comparative advantage" over men in the household sector. He later suggests that since "household activities are much effort intensive than leisure-oriented activities and may be more or less effort intensive than market activities" married women allocate less energy to each hour of work than married men who spend equal time in the labor force, and this can be an explanation of why they are paid less per hour of work. My motivation behind this chapter comes from my personal experience with labor force participation in Iran. During the years after the revolution, the investment in human capital in general and especially in women increased tremendously. Parents spent a lot of resources on educating their daughters. Moreover, girls increased their share of college education, to the point where more than sixty percent of college students were female. Despite this fact, the market seemed to

still discriminate between the genders by lower wages offered to women. My goal in this chapter is to see how a wage differential for female workers affects labor force participation and also to see if there is an economic model that supports my observation about female labor force participation in Iran.

There are six main findings in this chapter (see results 1-11 in section 4.4). First, a higher wage rate leads to an increase in self and spouse's consumption, a decrease in own leisure, and increase in spouse's leisure, *ceteris paribus*. Second, a higher price of private good leads to a decrease in consumption, *ceteris paribus*. Third, higher leisure leads to an increase in own consumption, *ceteris paribus*. Fourth, a higher power of an individual in the household leads to a higher private consumption and higher leisure. Fifth, a higher weight of consumption in one's utility function leads to a decrease in leisure, *ceteris paribus*. And finally, the more altruistic an individual is, the less is his/her own consumption and the more is the spouse's consumption. Section 4.6 revisits Spence's explanation of sustainable wage discrimination. As a result, female labor participation remained low.

4.2 The Model

Let us start with one of the simplest models that one can think of, where there are only two types of individuals (two-type economy). This simplistic model will give us some insight as to whether the interaction of different types results in a 'better' (in a normative sense) outcome for a household in the presence of externalities. Suppose that there are two types of consumers, female and male, and the numbers of the two types are n_F and n_M , respectively.¹⁷

Let $I = F \cup M = \{1, 2, \dots, n_m + n_F\}$, be the set of the individuals in the economy, where F is the set of female and M is the set of male individuals. When appropriate, we use $i \in F$ and $j \in M$ to represent a female and a male, respectively. Let $U_i(x_i, L_i)$ be the utility associated with an individual i , $i \in F, M$, and $x_i \in \mathbb{R}_+$, the commodity consumed (one can consider it as a

¹⁷ Here we are interested in the household and the effect of marriage on the well-being of an individual, but nothing can stop us from thinking about partnerships, etc.

composite good) and $L_i \in \mathbb{R}_+$ is the amount of leisure consumed. Without loss of generality, let us assume that the total amount of time available is 1 unit. That means $1 - l_i = L_i$, where l_i is the fraction of the available time that individual i spends on working (i.e., labor). We assume well behaved preferences that guarantee the existence of equilibrium. To be precise, we assume each U_i is concave, strictly monotone, and twice continuously differentiable on \mathbb{R}_{++}^2 , such that the first order approach applies. Additionally, p, w_i are the price of the private good and individual i 's wage rate, respectively.

If we consider individual i alone he or she solves the following problem

$$\max_{x_i} U_i(x_i, L_i) \quad \text{s.t. } px_i \leq w_i(1 - L_i), \quad i \in I.$$

Equating the marginal rate of substitution to the price ratio, one gets

$$MRS_{x_i, L_i} = \frac{\partial U_i / \partial x_i}{\partial U_i / \partial L_i} = \frac{p}{w_i}, \quad i \in F, M.$$

Now, suppose two individuals i, j form a household. Suppose further that after marriage they maximize a joint utilitarian social welfare function subject to the joint budget set, as follows.¹⁸

$$\begin{aligned} & \max_{x, L} \alpha U_i(x_i, L_i) + (1 - \alpha) U_j(x_j, L_j) \\ & \text{s.t. } p(x_i + x_j) \leq w_i(1 - L_i) + w_j(1 - L_j), \quad i \in F, j \in M. \end{aligned}$$

Solving for the first order conditions resulting from Lagrange leads to

$$MRS_{x_i, L_i} = \frac{\partial U_i / \partial x_i}{\partial U_i / \partial L_i} = \frac{p}{w_i}, \quad MRS_{x_j, L_j} = \frac{\partial U_j / \partial x_j}{\partial U_j / \partial L_j} = \frac{p}{w_j}.$$

Further, note that

$$\frac{\partial U_i / \partial x_i}{\partial U_j / \partial L_j} = \frac{1 - \alpha}{\alpha}.$$

To learn more about this result I look at the following two examples.

¹⁸ In this set up α and $1 - \alpha$ can be viewed as the power of each individual in the household.

4.3 Illustrative Examples

4.3.1 Example 1:

Suppose there are two individuals in the household with equal weights, $\alpha = \frac{1}{2}$, and Cobb-Douglas utility functions of the form:

$$U_1 = x_1 L_1, U_2 = x_2 L_2.$$

Then,

$$\frac{\partial U_1 / \partial x_1}{\partial U_2 / \partial L_2} = \frac{L_1}{L_2} = \frac{1 - \alpha}{\alpha} = 1 \rightarrow L_1 = L_2 \rightarrow l_1 = l_2.$$

Moreover,

$$MRS_{x_i, L_i} = \frac{\partial U_i / \partial x_i}{\partial U_i / \partial L_i} = \frac{L_i}{x_i} = \frac{1 - l_i}{x_i} = \frac{p}{w_i} \rightarrow x_i = (1 - l_i) \frac{w_i}{p}, i = 1, 2.$$

Now, using the budget equation, and the fact that $l_1 = l_2$, yields

$$p(x_1 + x_2) = (w_1 + w_2)l_1.$$

Substituting x_1 and x_2 from the above, with $l_1 = l_2$ in previous equation, gives

$$p\left((1 - l_1) \frac{w_1}{p} + (1 - l_1) \frac{w_2}{p}\right) = (w_1 + w_2)l_1 \rightarrow (1 - l_1)(w_1 + w_2) = (w_1 + w_2)l_1$$

$$\rightarrow \boxed{l_1 = l_2 = \frac{1}{2}}$$

Which in turn leads to: $\boxed{x_1 = \frac{w_1}{2p}}, \boxed{x_2 = \frac{w_2}{2p}}$ and $U_1 = \frac{w_1}{4p}, U_2 = \frac{w_2}{4p}$.

This makes sense: If the power in the household is equal, each spouse works the same amount, and their consumption of private good is directly related to their wage rate and inversely related to the price of private good. It is interesting to note that even under the assumption of equal power inside the household when there is a difference in treatment of each gender in the society (here, wage difference) that difference leads to a difference in the private consumption within households and therefore in the utility level of each individual. Thus unequal treatment of women in the workforce can induce their unequal treatment in household. For the case where $w_1 = w_2$, the outcome is identical for both genders; there is no difference in utilities and private consumption levels.

4.3.2 Example 2:

Suppose there are two individuals in the households with given powers α and $1 - \alpha$, and Cobb-Douglas utility functions of the form: $U_1 = x_1 L_1, U_2 = x_2 L_2$, respectively. Then,

$$\frac{L_1}{L_2} = \frac{1 - \alpha}{\alpha} \rightarrow L_1 = \frac{1 - \alpha}{\alpha} L_2.$$

Also,

$$x_1 = L_1 \frac{w_1}{p} = (1 - l_1) \frac{w_1}{p} \text{ and } x_2 = L_2 \frac{w_2}{p} = (1 - l_2) \frac{w_2}{p}.$$

Now, substituting x_1 and x_2 , and $L_1 = \frac{1 - \alpha}{\alpha} L_2$ in the budget line

$$p(x_1 + x_2) = w_1(1 - L_1) + w_2(1 - L_2)$$

gives

$$\begin{aligned} p\left(L_1 \frac{w_1}{p} + L_2 \frac{w_2}{p}\right) &= w_1\left(1 - \frac{1 - \alpha}{\alpha} L_2\right) + w_2(1 - L_2) \\ w_1 \frac{1 - \alpha}{\alpha} L_2 + L_2 w_2 &= w_1 - w_1 \frac{1 - \alpha}{\alpha} L_2 + w_2 - w_2 L_2, \end{aligned}$$

which in turn leads to:

$$\begin{aligned} L_2 &= \frac{\alpha}{2} \frac{w_1 + w_2}{w_1(1 - \alpha) + \alpha w_2}, & L_1 &= \frac{1 - \alpha}{2} \frac{w_1 + w_2}{w_1(1 - \alpha) + \alpha w_2} \\ x_1 &= \frac{(1 - \alpha)w_1}{2p} \frac{w_1 + w_2}{w_1(1 - \alpha) + \alpha w_2}, & x_2 &= \frac{\alpha w_2}{2p} \frac{w_1 + w_2}{w_1(1 - \alpha) + \alpha w_2} \end{aligned}$$

It is interesting to look at the special case when $w_1 = w_2$. In this case,

$$L_1 = 1 - \alpha, L_2 = \alpha, x_1 = \frac{(1 - \alpha)w_1}{p}, x_2 = \frac{\alpha w_1}{p}, U_1 = \frac{(1 - \alpha)^2 w_1}{p}, U_2 = \frac{\alpha^2 w_1}{p}.$$

Comparing this result with the outcome of previous example where power was equal, and assuming that \bar{U}_1 and \bar{U}_2 are the maximum utilities that each individual can gain if they remain single, leads to the conclusion that this household forms only if

$$U_1 = \frac{(1 - \alpha)^2 w_1}{p} \geq \bar{U}_1 = \frac{w_1}{4p} \rightarrow \frac{(1 - \alpha)^2 w_1}{p} \geq \frac{w_1}{4p} \rightarrow (1 - \alpha)^2 \geq \frac{1}{4} \rightarrow \alpha \leq \frac{1}{2}$$

and

$$U_2 = \frac{\alpha^2 w_1}{p} \geq \overline{U_2} = \frac{w_1}{4p} \rightarrow \frac{\alpha^2 w_1}{p} \geq \frac{w_1}{4p} \rightarrow \alpha^2 \geq \frac{1}{4} \rightarrow \alpha \geq \frac{1}{2}$$

Both of these conditions hold simultaneously only if $\alpha = \frac{1}{2}$. This makes sense: If the wage rates are the same, households form only if the power of each sex in the household is equal. Otherwise, the spouse with lower power will work less (and have more leisure time) and consume more private good and at the same time earns higher utility level! Needless to say this has to do with the specific utility function that we are using here, Cobb-Douglas.

4.3.3 Example 3:

Now, suppose there are two individuals in the households with given powers α and $1 - \alpha$, and Cobb-Douglas utility functions of the form:

$$U_1 = x_1^\beta L_1^{1-\beta}, U_2 = x_2^\beta L_2^{1-\beta}.$$

Again, we will have, $L_1 = \frac{1-\alpha}{\alpha} L_2$. Also,

$$MRS_{x_i, L_i} = \frac{\partial U_i / \partial x_i}{\partial U_i / \partial L_i} = \frac{\beta L_i}{(1-\beta)x_i} = \frac{p}{w_i} \rightarrow x_i = \frac{\beta}{(1-\beta)} \frac{w_i}{p} L_i, i = 1, 2.$$

Substituting x_1 and x_2 , and $L_1 = \frac{1-\alpha}{\alpha} L_2$ in the budget equation yields

$$p \left(\frac{\beta}{(1-\beta)} \frac{w_1}{p} \frac{1-\alpha}{\alpha} L_2 + \frac{\beta}{(1-\beta)} \frac{w_2}{p} L_2 \right) = w_1 \left(1 - \frac{1-\alpha}{\alpha} L_2 \right) + w_2 (1 - L_2)$$

$$\frac{\beta}{1-\beta} \left(w_1 \frac{1-\alpha}{\alpha} L_2 + w_2 L_2 \right) = w_1 - w_1 \frac{1-\alpha}{\alpha} L_2 + w_2 - w_2 L_2,$$

which in turn leads to:

$L_1 = (1-\alpha)(1-\beta) \frac{w_1 + w_2}{w_1(1-\alpha) + \alpha w_2}$	$L_2 = \alpha(1-\beta) \frac{w_1 + w_2}{w_1(1-\alpha) + \alpha w_2}$
$x_1 = (1-\alpha)\beta \frac{w_1}{p} \frac{w_1 + w_2}{w_1(1-\alpha) + \alpha w_2}$	$x_2 = \alpha\beta \frac{w_2}{p} \frac{w_1 + w_2}{w_1(1-\alpha) + \alpha w_2}$

Again, it is interesting to look at the special case when $w_1 = w_2$. In this case,

$$L_1 = 2(1-\alpha)(1-\beta), L_2 = 2\alpha(1-\beta), x_1 = 2(1-\alpha)\beta \frac{w_1}{p}, x_2 = 2\beta\alpha \frac{w_1}{p},$$

$$U_1 = \frac{4(1-\alpha)^2 \beta(1-\beta)w_1}{p}, U_2 = \frac{4\alpha^2 \beta(1-\beta)w_1}{p}.$$

Comparing this result with the outcome of previous examples leads to the conclusion that this household forms only if

$$U_1 = \frac{4(1-\alpha)^2\beta(1-\beta)w_1}{p} \geq \bar{U}_1 = \beta(1-\beta)\frac{w_1}{p} \rightarrow$$

$$\frac{4(1-\alpha)^2\beta(1-\beta)w_1}{p} \geq \beta(1-\beta)\frac{w_1}{p} \rightarrow (1-\alpha)^2 \geq \frac{1}{4} \rightarrow \alpha \leq \frac{1}{2}$$

and

$$U_2 = \frac{4\alpha^2\beta(1-\beta)w_2}{p} \geq \bar{U}_2 = \beta(1-\beta)\frac{w_2}{p} \rightarrow$$

$$\frac{4\alpha^2\beta(1-\beta)w_2}{p} \geq \beta(1-\beta)\frac{w_2}{p} \rightarrow \alpha^2 \geq \frac{1}{4} \rightarrow \alpha \geq \frac{1}{2}$$

Hence, $\frac{1}{2} \leq \alpha \leq \frac{1}{2}$. And of course, this inequality is only valid if $\alpha = \frac{1}{2}$. These three examples are in line with the No Power Theorem which was stated in previous chapter. In these examples, too, in the absence of externalities there is no gain from household formation even though they are not in the context of a pure exchange economy.

4.4 Positive Additive Externality

Let us assume that the individual utility function of each gender takes the form of $U_i(x_i, L_i) = \beta \ln x_i + (1-\beta) \ln L_i, i \in F, M$. Note that the choice of the same utility functions for both types of individuals is intentional because the central idea in this chapter is to investigate the effect of exogenous price (wage rate) differences on labor supply. Further, suppose after forming a household each individual cares about the well being of his/her spouse, and so their utilities can be expressed as

$$U_1^2(x_1, L_1, x_2, L_2) = U_1(x_1, L_1) + \xi_{12}U_2(x_2, L_2),$$

$$U_2^1(x_1, L_1, x_2, L_2) = U_2(x_2, L_2) + \xi_{21}U_1(x_1, L_1),$$

with $0 \leq \xi_{ij} \leq 1, i, j = 1, 2$. ξ_{ij} represents how much an individual cares about the well being of his/her spouse, and therefore represents altruism within the household. The household utilitarian social welfare function is of the form $\alpha U_1^2(\cdot) + (1-\alpha)U_2^1(\cdot)$ where α and $1-\alpha$ are the power of each individual in the household. A typical household wants to solve the following problem:

$$\max_{x,L} \alpha U_1^2(x_1, L_1, x_2, L_2) + (1 - \alpha) U_2^1(x_1, L_1, x_2, L_2),$$

subject to:

$$p(x_1 + x_2) \leq w_1(1 - L_1) + w_2(1 - L_2),$$

$$U_1^2(x_1, L_1, x_2, L_2) \geq \beta \ln\left(\beta \frac{w_1}{p}\right) + (1 - \beta) \ln(1 - \beta),$$

$$U_2^1(x_1, L_1, x_2, L_2) \geq \beta \ln\left(\beta \frac{w_2}{p}\right) + (1 - \beta) \ln(1 - \beta),$$

$$x_1 \geq 0, x_2 \geq 0, 0 \leq L_1 \leq 1, 0 \leq L_2 \leq 1.$$

Note that the sum of labor and leisure is one.¹⁹ To ensure that given the endowment, the utility level is positive we also assume:

$$\left(\beta \frac{\min(w_1, w_2)}{p}\right)^\beta (1 - \beta)^{(1-\beta)} > 1.$$

The first order conditions for the above problem lead to:

$$x_1 = \frac{\beta}{1 - \beta} \frac{w_1}{p} L_1, \quad x_2 = \frac{\beta}{1 - \beta} \frac{w_2}{p} L_2$$

The solution to the above optimization problem is:

$$x_1 = \beta \frac{w_1 + w_2}{p} \frac{1}{1 + \gamma_1}, \quad x_2 = \beta \frac{w_1 + w_2}{p} \frac{\gamma_1}{1 + \gamma_1},$$

$$L_1 = (1 - \beta) \frac{w_1 + w_2}{w_1} \frac{1}{1 + \gamma_1}, \quad L_2 = (1 - \beta) \frac{w_1 + w_2}{w_2} \frac{\gamma_1}{1 + \gamma_1},$$

where $\gamma_1 = \frac{\alpha \xi_{12} + (1 - \alpha)}{\alpha + (1 - \alpha) \xi_{21}}$.

There are three important properties for γ_1 ,

$$\begin{aligned} \frac{d\gamma_1}{d\alpha} &= \frac{d}{d\alpha} \left(\frac{1 - \alpha(1 - \xi_{12})}{\alpha(1 - \xi_{21}) + \xi_{21}} \right) = \frac{-(1 - \xi_{12})[\alpha(1 - \xi_{21}) + \xi_{21}] - (1 - \xi_{21})[1 - \alpha(1 - \xi_{12})]}{(\alpha(1 - \xi_{21}) + \xi_{21})^2} \\ &= - \frac{\alpha(1 - \xi_{21})(1 - \xi_{12}) + \xi_{21}(1 - \xi_{12}) + (1 - \xi_{21}) - \alpha(1 - \xi_{12})(1 - \xi_{21})}{(\alpha(1 - \xi_{21}) + \xi_{21})^2} \\ &= - \frac{\xi_{21}(1 - \xi_{12}) + (1 - \xi_{21})}{(\alpha(1 - \xi_{21}) + \xi_{21})^2} < 0, \end{aligned}$$

¹⁹ That is $l_i + L_i = 1$, assuming that the total time endowment is normalized to one. Later I relax this assumption to $l_i + L_i = \varepsilon$ and use homotheticity.

$$\frac{d\gamma_1}{d\xi_{12}} = \frac{d}{d\xi_{12}} \left(\frac{\alpha\xi_{12} + (1-\alpha)}{\alpha + (1-\alpha)\xi_{21}} \right) = \frac{\alpha(\alpha + (1-\alpha)\xi_{21}) - 0}{(\alpha + (1-\alpha)\xi_{21})^2} = \frac{\alpha}{\alpha + (1-\alpha)\xi_{21}} > 0,$$

$$\frac{d\gamma_1}{d\xi_{21}} = \frac{d}{d\xi_{21}} \left(\frac{\alpha\xi_{12} + (1-\alpha)}{\alpha + (1-\alpha)\xi_{21}} \right) = \frac{0 - (1-\alpha)(\alpha\xi_{12} + (1-\alpha))}{(\alpha + (1-\alpha)\xi_{21})^2} = -\frac{(1-\alpha)(\alpha\xi_{12} + (1-\alpha))}{(\alpha + (1-\alpha)\xi_{21})^2} < 0.$$

If $l_i + L_i = \varepsilon$, then using homotheticity, gives $x_{i, new}^* = \varepsilon x_{i, old}^*$ and $L_{i, new}^* = \varepsilon L_{i, old}^*$, and I have to assume

$$\varepsilon > \frac{1}{(\beta \frac{\min(w_1, w_2)}{p})^\beta (1-\beta)^{(1-\beta)}}$$

to ensure that $u^* > 0$. Hence, without loss of generality from now on I assume $\varepsilon = 1$. Let us define household wage as $w_h = w_1 + w_2$. The following observations are especially interesting:

1. $\frac{\partial x_i}{\partial w_i} > 0$, higher wage rate leads to an increase in consumption, ceteris paribus.
2. $\frac{\partial x_i}{\partial w_j} > 0$, higher wage rate for the spouse leads to an increase in own consumption, ceteris paribus. This is the direct result of externality.
3. $\frac{\partial x_i}{\partial w_h} > 0$, higher household wage rate leads to an increase in consumption, ceteris paribus.
4. $\frac{\partial x_i}{\partial p} < 0$, higher price for private consumption leads to a decrease in consumption, ceteris paribus.
5. $\frac{\partial x_i}{\partial L_i} > 0$, higher leisure leads to an increase in own consumption, ceteris paribus. At first glance, this result might look counter-intuitive, however, this is a result of using specific utility function (remember that for Cobb-Douglas utility function the share of expenditure on each “good” is fixed).
6. $\frac{\partial x_i}{\partial \alpha} = \frac{\partial x_i}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \alpha} > 0$, because $\frac{\partial x_i}{\partial \gamma_1} < 0$ and $\frac{\partial \gamma_1}{\partial \alpha} < 0$. The higher the power of an individual in the household, the higher the private consumption.

7. $\frac{\partial x_1}{\partial \xi_{12}} = \frac{\partial x_1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \xi_{12}} < 0$, because $\frac{\partial x_1}{\partial \gamma_1} < 0$ and $\frac{\partial \gamma_1}{\partial \xi_{12}} > 0$.
8. $\frac{\partial x_1}{\partial \xi_{21}} = \frac{\partial x_1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \xi_{21}} > 0$, because $\frac{\partial x_1}{\partial \gamma_1} < 0$ and $\frac{\partial \gamma_1}{\partial \xi_{21}} < 0$.
9. $\frac{dL_i}{dw_i} < 0$, higher wage rate leads to a decrease in leisure, ceteris paribus.
10. $\frac{\partial L_i}{\partial w_j} > 0$, higher wage rate for the spouse leads to an increase in own leisure, ceteris paribus. This is happening as a result of externality.
11. $\frac{\partial L_i}{\partial \alpha} = \frac{\partial L_i}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \alpha} > 0$, because $\frac{\partial L_i}{\partial \gamma_1} < 0$ and $\frac{\partial \gamma_1}{\partial \alpha} < 0$. The higher the power of individual in the household, the higher the amount of leisure.
12. $\frac{\partial L_i}{\partial \beta} < 0$ or $\frac{\partial L_i}{\partial (1-\beta)} > 0$, higher weight of consumption in utility function leads to a decrease in leisure, ceteris paribus.

Next, I will derive the effect of change in relative prices. First, what happens if all prices change by the same percentage, that is, if $w_1' = \delta w_1$, $w_2' = \delta w_2$, $p' = \delta p$, then using

$$x_1 = \beta \frac{w_1 + w_2}{p} \frac{1}{1 + \gamma_1}, x_2 = \beta \frac{w_1 + w_2}{p} \frac{\gamma_1}{1 + \gamma_1},$$

$$L_1 = (1 - \beta) \frac{w_1 + w_2}{w_1} \frac{1}{1 + \gamma_1}, L_2 = (1 - \beta) \frac{w_1 + w_2}{w_2} \frac{\gamma_1}{1 + \gamma_1},$$

yields $x_1' = x_1$, $x_2' = x_2$, $L_1' = L_1$, $L_2' = L_2$. This means the above change will not have any effect on the utility level of individuals. Second, if all prices change by the same amount, that is $w_1' = w_1 + \delta$, $w_2' = w_2 + \delta$, $p' = p + \delta$, I use the following lemmas to investigate what happens to consumption, leisure and utility level of each individual.

Lemma 4.1:

If $A > B$, then $\frac{A}{B} > \frac{A + \delta}{B + \delta}$.

Proof:

$$A > B \Leftrightarrow A\delta > B\delta \Leftrightarrow AB + A\delta > AB + B\delta \Leftrightarrow A(B + \delta) > B(A + \delta) \Leftrightarrow \frac{A}{B} > \frac{A + \delta}{B + \delta}.$$

Lemma 4.2:

$$\text{If } A > 2B, \text{ then } \frac{A}{B} > \frac{A + 2\delta}{B + \delta}.$$

Proof:

$$A > 2B \Leftrightarrow A\delta > 2B\delta \Leftrightarrow AB + A\delta > AB + 2B\delta \Leftrightarrow A(B + \delta) > B(A + 2\delta) \Leftrightarrow \frac{A}{B} > \frac{A + 2\delta}{B + \delta}.$$

Lemma 4.3:

If $w_2 > w_1 > p$ then $x'_1 < x_1, x'_2 < x_2, L'_1 < L_1, L'_2 > L_2$ and $u'_1 < u_1$ and one cannot tell what happens to u_2 .

Proof:

$w_2 > w_1 > p$ so, $w_2 + w_1 > 2p$, $w_2 + w_1 > 2w_1$ and $w_2 + w_1 < 2w_2$. Thus,

$$w_2 + w_1 > 2p \Rightarrow \frac{w_2 + w_1}{p} > \frac{w_2 + w_1 + 2\delta}{p + \delta} \Rightarrow x'_1 < x_1, x'_2 < x_2,$$

$$w_2 + w_1 > 2w_1 \Rightarrow \frac{w_2 + w_1}{w_1} > \frac{w_2 + w_1 + 2\delta}{w_1 + \delta} \Rightarrow L'_1 < L_1, \text{ and } u'_1 < u_1,$$

$$w_2 + w_1 < 2w_2 \Rightarrow \frac{w_2 + w_1}{w_2} > \frac{w_2 + w_1 + 2\delta}{w_2 + \delta} \Rightarrow L'_2 > L_2, \text{ and one cannot tell what happens to } u_2.$$

Lemma 4.4:

If $w_2 < w_1 < p$ then $x'_1 > x_1, x'_2 > x_2, L'_1 > L_1, L'_2 < L_2$ and $u'_1 > u_1$ and one cannot tell what happens to u_2 .

Proof: Similar to previous proof. Just repeat the above proof, changing the direction of all inequalities.

Lemma 4.5:

If $w_1 > w_2 > p$ then $x'_1 < x_1, x'_2 < x_2, L'_1 > L_1, L'_2 < L_2$ and $u'_2 < u_2$ and one cannot tell what happens to u_1 .

Proof:

$w_1 > w_2 > p$ so, $w_2 + w_1 > 2p$, $w_2 + w_1 > 2w_2$ and $w_2 + w_1 < 2w_1$. Thus,

$$w_2 + w_1 > 2p \Rightarrow \frac{w_2 + w_1}{p} > \frac{w_2 + w_1 + 2\delta}{p + \delta} \Rightarrow x_1' < x_1, x_2' < x_2,$$

$$w_2 + w_1 < 2w_1 \Rightarrow \frac{w_2 + w_1}{w_1} < \frac{w_2 + w_1 + 2\delta}{w_1 + \delta} \Rightarrow L_1' > L_1, \text{ and one cannot tell what}$$

happens to u_1 , and

$$w_2 + w_1 > 2w_2 \Rightarrow \frac{w_2 + w_1}{w_2} > \frac{w_2 + w_1 + 2\delta}{w_2 + \delta} \Rightarrow L_2' < L_2, u_2' < u_2.$$

Lemma 4.6:

If $w_1 < w_2 < p$ then $x_1' > x_1, x_2' > x_2, L_1' < L_1, L_2' > L_2$ and $u_2' > u_2$ and one cannot tell what happens to u_1 .

Proof: Similar to previous proof. Just repeat the above proof where changing the direction of all inequalities.

4.5 Calculation of a Fixed Point for the Model

In the household economics literature, child labor supply has been investigated in various papers. An interesting survey of the literature on child labor is provided in Basu (1999). In that survey, he introduces a model in which each member of household's labor supply depends on his/her powers in the household and power of that household member depends on the labor supply. He proposes using the Fixed Point Theorem to calculate the powers endogenously in that model. In this section, I use Basu's proposed method and apply it to my model.

Let us assume that the power of each individual in the household is exactly equal to his or her earning share which is defined as following

$$\alpha = \frac{w_1(1 - L_1)}{w_1(1 - L_1) + w_2(1 - L_2)}, \quad 1 - \alpha = \frac{w_2(1 - L_2)}{w_1(1 - L_1) + w_2(1 - L_2)}.$$

Using Brouwer's Fixed Point Theorem, one can show the existence of an endogenous solution to the model in section 4.4. Leisure (or labor), which is a function of power, can be endogenously calculated if we assume power is a function of leisure (or labor income) as mentioned above. I can also endogenously calculate the powers (which are functions of labor) but I only show the former one. Here is the statement of Brouwer's Fixed Point Theorem in Mas-Colell,

Whinston and Green (1995): “Suppose that $A \subset \mathbb{R}^N$ is a nonempty, compact, convex set and that $f : A \rightarrow A$ is a continuous function from A into itself. Then $f(\cdot)$ has a fixed point; that is, there is an $x \in A$ such that $x = f(x)$.” In my model, leisure is a function of power and power is a function of leisure (or labor), so we can use the method proposed by Basu (1999).

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} f_1(L_1, L_2) \\ f_2(L_1, L_2) \end{bmatrix} = f(L_1, L_2), f : \mathbb{R}^2 \rightarrow \mathbb{R}^2.$$

By substituting $A = (1 - \beta)(w_1 + w_2)$ in the abovementioned formulas for L_1 and L_2 we get:

$$L_1 = (1 - \beta) \frac{w_1 + w_2}{w_1} \frac{1}{1 + \gamma_1} = \frac{A}{w_1} \frac{1}{1 + \gamma_1}$$

$$L_2 = (1 - \beta) \frac{w_1 + w_2}{w_2} \frac{\gamma_1}{1 + \gamma_1} = \frac{A}{w_2} \frac{\gamma_1}{1 + \gamma_1}$$

Now using $\alpha = \frac{w_1(1 - L_1)}{w_1(1 - L_1) + w_2(1 - L_2)}$ we need to calculate $\frac{1}{1 + \gamma_1}$ and $\frac{\gamma_1}{1 + \gamma_1}$, which can be done as follows.

$$\begin{aligned} \gamma_1 &= \frac{\alpha \xi_{12} + (1 - \alpha)}{\alpha + (1 - \alpha) \xi_{21}} = \frac{\xi_{12} \left(\frac{w_1(1 - L_1)}{w_1(1 - L_1) + w_2(1 - L_2)} \right) + \left(\frac{w_2(1 - L_2)}{w_1(1 - L_1) + w_2(1 - L_2)} \right)}{\left(\frac{w_1(1 - L_1)}{w_1(1 - L_1) + w_2(1 - L_2)} \right) + \left(\frac{w_2(1 - L_2)}{w_1(1 - L_1) + w_2(1 - L_2)} \right) \xi_{21}} \\ &= \frac{\xi_{12} w_1(1 - L_1) + w_2(1 - L_2)}{w_1(1 - L_1) + \xi_{21} w_2(1 - L_2)}, \\ 1 + \gamma_1 &= \frac{w_1(1 - L_1) + \xi_{21} w_2(1 - L_2) + \xi_{12} w_1(1 - L_1) + w_2(1 - L_2)}{w_1(1 - L_1) + \xi_{21} w_2(1 - L_2)} \\ &= \frac{w_1(1 - L_1)(1 + \xi_{12}) + w_2(1 - L_2)(1 + \xi_{21})}{w_1(1 - L_1) + \xi_{21} w_2(1 - L_2)}, \\ \frac{1}{1 + \gamma_1} &= \frac{w_1(1 - L_1) + \xi_{21} w_2(1 - L_2)}{w_1(1 - L_1)(1 + \xi_{12}) + w_2(1 - L_2)(1 + \xi_{21})} \end{aligned}$$

And finally,

$$\frac{\gamma_1}{1 + \gamma_1} = \gamma_1 \times \frac{1}{1 + \gamma_1} = \frac{\xi_{12} w_1 (1 - L_1) + w_2 (1 - L_2)}{w_1 (1 - L_1) + \xi_{21} w_2 (1 - L_2)} \times \frac{w_1 (1 - L_1) + \xi_{21} w_2 (1 - L_2)}{w_1 (1 - L_1) (1 + \xi_{12}) + w_2 (1 - L_2) (1 + \xi_{21})},$$

$$\frac{\gamma_1}{1 + \gamma_1} = \frac{\xi_{12} w_1 (1 - L_1) + w_2 (1 - L_2)}{w_1 (1 - L_1) (1 + \xi_{12}) + w_2 (1 - L_2) (1 + \xi_{21})}.$$

Using the above calculation for $1/(1 + \gamma_1)$ and $\gamma_1/(1 + \gamma_1)$, L_1 and L_2 are derived as functions of wage, leisure, and known parameters. More importantly they are independent of α .

$$L_1 = \frac{A}{w_1} \frac{1}{1 + \gamma_1} = \frac{A}{w_1} \frac{w_1 (1 - L_1) + \xi_{21} w_2 (1 - L_2)}{w_1 (1 - L_1) (1 + \xi_{12}) + w_2 (1 - L_2) (1 + \xi_{21})} = f_1(L_1, L_2), \quad (*)$$

$$L_2 = \frac{A}{w_2} \frac{\gamma_1}{1 + \gamma_1} = \frac{A}{w_2} \frac{\xi_{12} w_1 (1 - L_1) + w_2 (1 - L_2)}{w_1 (1 - L_1) (1 + \xi_{12}) + w_2 (1 - L_2) (1 + \xi_{21})} = f_2(L_1, L_2). \quad (**)$$

Note that $f(L_1, L_2) = [f_1(L_1, L_2) \ f_2(L_1, L_2)]^T$, as described above, is defined on a non-empty set (for example, if $\beta = \frac{1}{2}$, $w_1 = w_2$, $\xi_{12} = \xi_{21} = 0$, then $L_1 = L_2 = \frac{1}{2}$ is a solution) and f is a continuous function from \mathbb{R}^N into itself. Moreover, by design powers of individuals in the household, i.e. α and $1 - \alpha$, which are defined to be the share of individual's labor income are bounded. Both $1/(1 + \gamma_1)$ and $\gamma_1/(1 + \gamma_1)$ are less than one and therefore are bounded. Also, if we assume $w_1 = \eta w_2$ (or $w_2 = (1/\eta) w_1$), then using $(1 - \beta) \leq 1$ we have:

$$\frac{A}{w_1} = \frac{(1 - \beta)(w_1 + w_2)}{w_1} = (1 - \beta) \left(1 + \frac{1}{\eta}\right) \leq \left(1 + \frac{1}{\eta}\right),$$

and

$$\frac{A}{w_2} = \frac{(1 - \beta)(w_1 + w_2)}{w_2} = (1 - \beta)(\eta + 1) \leq (\eta + 1).$$

This means $f_1(L_1, L_2) < (1 + \frac{1}{\eta})$ and $f_2(L_1, L_2) < (1 + \eta)$. In other words, f is bounded. Also, note that η is close to one since it is the relative wage gap.

Hence, by using Brouwer's Fixed Point Theorem there exists a solution to $L = [L_1 L_2]^T = f(L_1, L_2)$.

Now divide L_1 by L_2 ,

$$\begin{aligned} \frac{L_1}{L_2} &= \frac{\frac{A}{w_1} \frac{w_1(1-L_1) + \xi_{21}w_2(1-L_2)}{w_1(1-L_1)(1+\xi_{12}) + w_2(1-L_2)(1+\xi_{21})}}{\frac{A}{w_2} \frac{\xi_{12}w_1(1-L_1) + w_2(1-L_2)}{w_2(1-L_1)(1+\xi_{12}) + w_2(1-L_2)(1+\xi_{21})}} \\ &= \frac{w_2}{w_1} \times \frac{w_1(1-L_1) + \xi_{21}w_2(1-L_2)}{\xi_{12}w_1(1-L_1) + w_2(1-L_2)} \end{aligned}$$

to get,

$$\begin{aligned} \xi_{12}(w_1)^2 L_1(1-L_1) + w_1 w_2 L_1(1-L_2) &= w_1 w_2 L_2(1-L_1) + \xi_{21}(w_2)^2 L_2(1-L_2), \text{ or} \\ \xi_{12}(w_1)^2 L_1 - \xi_{12}(w_1)^2 (L_1)^2 + w_1 w_2 L_1 - w_1 w_2 L_1 L_2 \\ &= w_1 w_2 L_2 - w_1 w_2 L_1 L_2 + \xi_{21}(w_2)^2 L_2 - \xi_{21}(w_2)^2 (L_2)^2. \end{aligned}$$

Rewrite this equation to get the following quadratic equation:

$$\left[-\xi_{12}(w_1)^2 \right] (L_1)^2 + \left[\xi_{12}(w_1)^2 + w_1 w_2 \right] (L_1) + \left[\xi_{21}(w_2)^2 (L_2)^2 - \xi_{21}(w_2)^2 L_2 - w_1 w_2 L_2 \right] = 0$$

$$\text{If } \left[\xi_{12}(w_1)^2 + w_1 w_2 \right]^2 - 4 \left[-\xi_{12}(w_1)^2 \right] \left[\xi_{21}(w_2)^2 (L_2)^2 - \xi_{21}(w_2)^2 L_2 - w_1 w_2 L_2 \right] \geq 0,$$

the above quadratic equation has two solutions.

Note that in the coefficients of this quadratic equation are known parameters or expressions of L_2 . So, the solution would be of the form $L_1 = f(L_2)$.

$$\begin{aligned} L_1 &= \frac{2 \left[\xi_{12}(w_1)^2 + w_1 w_2 \right]}{2 \left[-\xi_{12}(w_1)^2 \right]} \pm \\ &\quad \frac{\sqrt{\left[\xi_{12}(w_1)^2 + w_1 w_2 \right]^2 - 4 \left[-\xi_{12}(w_1)^2 \right] \left[\xi_{21}(w_2)^2 (L_2)^2 - \xi_{21}(w_2)^2 L_2 - w_1 w_2 L_2 \right]}}{2 \left[-\xi_{12}(w_1)^2 \right]} \end{aligned}$$

This is exactly what is needed to solve for the fixed point. Plugging this value of L_1 in (**) leads to an equation with one variable, L_2 , and that gives L_2 in terms of known parameters. Of course, with known L_2 , L_1 is easily calculated using the solution to the above quadratic equation.

4.6 Labor Participation Rate for Females in Iran

My original interest in the labor force participation stems from an observation I made, while working on a project that dealt with comparing rural women in Iran with those in South Korea, Indonesia and Bangladesh (Mousavi and Moheemkar-Kheirandish, 1997). Later, a World Bank report on MENA (Middle East and North Africa) countries confirmed my earlier findings (MENA Development Report, 2004). This Report suggests:

“Women’s education has had large payoffs in lower fertility, in better family health status, and in more education for children. Despite the benefit in those crucial aspects of well being, investments in girls’ education have not achieved full payoffs in terms of the economic well-being of women, their families, and the economy as a whole (...) the rate [of participation of women in the labor force] remains low in MENA compared to that in other regions.”

In Iran, while female education has improved considerably since 1960’s, the female labor force participation rate has not improved proportionally. For example, the literacy rate among women more than tripled between 1960 and 2000 while it increased by a growth factor of 1.84 for men during the same period. During the same period, the gap between male and female average years of schooling (defined as male average years of schooling divided by female average years of schooling times 100) decreased from 220% to 135%. The same trend was observed for primary and secondary school gross enrollment ratios. The success rate of female students in the national college and universities entrance exams became so high (at some point, more than 60% of students entered the universities were female) that the government even started to consider affirmative actions in favor of male students! At the same time, the labor force participation rate stayed more or less about 20% until 1990 and only increased to 30% (that is, a 52% increase between 1960 and 2000 which is close to average for MENA) while some other countries experienced increases of more than 100%

(and in some cases even more than 500%) over the same period of time. Also, the unemployment rate among the women with education above middle school (middle school, high school, or higher education) was considerably higher than those with low education (illiterate, some primary, or primary). In the year 2000, the ratio of women's wages to men's wages was 0.82, the percentage of the wage gap unexplained by productive characteristics was 56% and the percentage increase in women's wages if discrimination were eliminated was 12%. Average years of education for female wage earners was 9.1 years, for female labor force participants was 6.9 years, and for female population was 5.8 years. The corresponding numbers for men were 7.0, 5.9, and 6.8 years, respectively. These statistics were my primary motivation for thinking that maybe female workers need to signal "quality" through education and that might be one explanation for why the education of girls has increased a lot while their labor force participation rate has not.

Is there any theoretical model that can back up this observation and reasoning? My original presumption was that this model has to be a general equilibrium model. The reason I was thinking this way was simple: If we get stuck in a "bad" equilibrium, there has to be a feedback that keeps us there and make it possible to justify staying there. What I do in the rest of this section is use Spence's market signaling model (see Spence, 1973, 1974, 1976 and 2002) to explain that the Iranian women's behavior could be consistent and explainable with his signaling model. In one of many variations of his model, Spence assumes that there are two groups, men and women. There are two productivity levels, high and low, and the cost of education for the group with lower productivity is higher. Education serves as "observable, alterable" characteristic while gender is the "observable, unalterable" characteristic. He assumes that the distribution of productive capabilities and the incidence of signaling costs are the same within each group, though later he relaxes this assumption. He poses the following question: "*How could sex have an informational impact on the market?*" The way the model is set up, the unconditional probability and conditional probability that a person drawn at random from population has a high productivity given that he is a man (or she is a woman) are the same. In other words, gender and

productivity are uncorrelated in the population. Hence, employers cannot use gender as a predictor for productivity. Further, men and women of equal productivity have the same signaling (education) costs. Spence argues that even though the signaling cost is the same for both genders, gender still may have informational impact since the opportunity sets of men and women of comparable productivity need not be the same. He reasons that if the employer's beliefs distributions are conditional on both gender and education, then only other individuals of the same gender feel the external impact of a gender's signaling decision. It follows that if at some point of time investment in education is not the same for the two genders, then returns to education will be different the next round. He says:

“...There are externalities implicit in the fact that an individual is treated as average member of the group of people who look the same and that, as a result, and in spite of an apparent sameness the opportunity sets facing two or more groups that are visibly distinguishable may in fact be different. The employer now has two potential signals to consider: Education and sex. At the start he does not know whether either education or sex will be correlated with productivity. Uninformative potential signals or indices are discarded in the course of reaching an equilibrium...there is at least the logical possibility that men and women will settle into different stable signaling equilibria in the market and stay there.”

This means it is possible that high-productivity women have to spend more on education (and have less to spend on other goods) to convince the employer that they belong to high productivity group compared to their male counterparts. My conjecture is that this is indeed the case and Spence's model provides a plausible explanation of the situation in Iran.

I close this chapter here and move on to the empirical part of this dissertation where households are dealing with poverty and mobility. The decision making process in the following chapter has to do with how government

should “target” troubled households. Thus next chapter investigates a completely different aspect of household economics.

5 Dynamics of Poverty in Iran: What Are the Determinants of the Probability of Being Poor?

5.1 Introduction

After The Iranian revolution in 1979, which was based on egalitarian ideas and rooted in Islamic beliefs of the population, one of the goals of the government (or at least one of its claimed goals) has always been to reduce poverty. In this chapter I am trying to look at a specific period of time in Iran, i.e. four years after the end of the war between Iran and Iraq, when foreign debt reached its maximum and the government started to pay back its debts. To have some idea about the distribution of income in an economy, one should know about three different concepts: Poverty, inequality and income mobility. Altogether, they give the researcher a better picture of income distribution and its dynamics, and as a result a better picture about how the wealth is distributed in the economy. This chapter will focus on poverty, which is one of the basic concerns of every government, especially in developing countries like Iran. Moreover, I will explain the trend of poverty through time. I will also talk about income mobility in Iran. In particular, I focus on the characteristics of the households that slip below the poverty line and stay there as well as those that climb up later.

In the literature, there are lots of theoretical and empirical papers on identifying the poor, in order to target them for policy purposes. Many authors focus on Bernoulli-type regression models (e.g. logit and probit) to explain the heterogeneities in the probability of being poor, based on the socio-economic characteristics of households. The results from these models suggest the proper policies for targeting poor families and assigning governmental aids to these households. In the current chapter, I use regression models to specify the main characteristics of chronic and transient poverty. In other words, my goal is to associate transient and permanent poverty with household socio-economic characteristics. If there is a difference in the set of explanatory variables of

transitory and permanent poverty, policy-makers can use that difference for targeting purposes.

After constructing a well-defined model one can be more confident in suggesting that a policy that affects the permanent poor but not the transitory poor will be a proper strategy for reducing poverty in the long-run.

Some of the early contributions to the literature on poverty dynamics were made by those who studied income mobility. Mobility in and out of the lowest income quintiles can be viewed as a measure of the rate of slipping into and skipping out of poverty. There has been a wide variety of studies of mobility, including some studies by authors who use a transition matrix as a tool to measure mobility. I use transition matrices to study the movements between different categories, which can be either a relative measure like percentile and quintile, or an absolute one like different income groups. Lillard and Willis (1978) and Shorocks (1978) are among those taking such an approach.

Masoumi (1990) and Masoumi and Zandvakil (1992) introduce an entropy measure for mobility, based on an axiomatic approach, and use it to analyze “within” and “without” mobility for different demographic groups, especially for African-Americans and women in the U.S..

Fields and Ok (1996, 1998) present a measure of mobility that satisfies several axioms and show that this measure is unique.²⁰ They apply their measure to U.S. data to calculate mobility and its trends in the 1990s. Fields et al (2000) examine income mobility in four different countries: Indonesia, South Africa, Spain and Venezuela. Based on univariate and multivariate analysis, they conclude that there is unconditional convergence: Low-income households gained ground and high-income households lost ground. They found that except for base year income, employment has the greatest effect on household income change. Household composition accounts for a smaller but yet noticeable effect, while human capital characteristics are either not correlated with changes in per-capita household income, or have a small effect.

²⁰ They argue, using some illustrative examples, that their axioms should be satisfied by any “good” mobility measure.

The other important branch of research on income distribution is the inequality literature. Atkinson (1970) started a new line of research into inequality, and Sen (1973), in response to Atkinson's paper, started an axiomatic approach, in which he showed that the Gini coefficient does not satisfy all the desirable axioms. Theil introduced an information-based index, named after him, with lots of applications, which is not the focus of my research. There are also other indices that have been proposed as improvements on the above indexes. However, this line of research is not the primary focus of this chapter.

Recently, poverty, which is the other main research branch in income distribution, became one of the mainstream subjects of interest, especially for international agencies and governmental policy-makers. As collecting longitudinal data has become more popular and access to this kind of data has become easier, there have been a lot of studies of the dynamics of poverty.

Jalan and Ravallion (2000) suggest a systematic way of distinguishing between chronic and transitory poverty. They propose a measure that allows poverty to be decomposed into its chronic and transitory components. Using data from rural China, they identify the determinants of each of these two categories and test if they are different. Jalan and Ravallion define a person to be chronically poor if the person's average expenditure over a period falls short of the poverty line. They measure the person's chronic poverty by using his or her average expenditure in place of expenditure (or income) in the squared gap measure of poverty (also known as Foster-Greer-Thorbecke, or FGT measure of poverty). Transient poverty is then defined as the average of differences between total poverty and chronic poverty. To see this more clearly, let $(y_{i1}, y_{i2}, \dots, y_{iT})$ be household i 's expenditure over time, and $P(y_{i1}, y_{i2}, \dots, y_{iT})$ the inter-temporal squared poverty gap measure for household i . Then if $m(y_i)$ is the average of expenditure over time, chronic poverty is defined as

$$C_i = P(m(y_i), m(y_i), \dots, m(y_i)),$$

where instead of expenditures in each year, the average of expenditures over time is substituted in the function $P(\cdot)$ and Transient poverty is defined as residual poverty, or actual poverty minus chronic poverty

$$T_i = P(y_{i1}, y_{i2}, \dots, y_{iT}) - C_i.$$

To compute $P(\cdot)$ they use the squared poverty gap measure for individual households at each time period, which is:

$$p(y_{it}) = \begin{cases} \left(1 - \frac{y_{it}}{PL}\right)^2 & \text{if } \frac{y_{it}}{PL} < 1 \\ 0 & \text{otherwise} \end{cases}$$

where y_{it} is real household expenditure and PL is the poverty line in the base year. They measure total poverty during the period by averaging $p(y_{it})$ over T time periods, $\frac{1}{T} \sum_t p(y_{it})$, which insures additivity. This, in turn, allows them to decompose total poverty into chronic and transient poverty. They employed censored conditional quintile estimators to show that average household wealth is an important determinant of both types of poverty, but household demographics, education and health affect only chronic poverty.

Baulch and McCulloch (1997, 2000) investigate poverty dynamics in rural Pakistan. They find that most of the poor are temporarily poor and only three percent of them remained poor in all five years of the available panel data. They use logit regression models to identify the determinants of being poor. They show that the probability of being poor increases with household size, and dependency ratio, but decreases with secondary education, land, and the value of assets owned. It also depends on district of residence. They do not find any meaningful relationship between poverty status and age, sex and basic education of household head. Based on the partial likelihood proportional hazards model, they found that the larger the household size, the greater the probability of entering poverty and the lower the probability of exiting poverty. Moreover, they discover that the higher the level of education, the greater the probability of exiting poverty. Finally, the greater the value of livestock owned the lower the probability of entering poverty for relatively low poverty lines. Since a big portion of people under the poverty line are temporarily poor, it would be more effective for government to choose the policies that increase the exit probability and lower the entry probability.

Baulch and McCulloch (1999) investigate the possibility of improving targeting accuracy by distinguishing between chronically and transitorily poor, based on household characteristics. They use panel data, which consists of 686 households in rural areas in Pakistan, and utilize ordered and multinomial logit analysis in their paper. They mention that most of the poor in their sample are transitorily poor (70-74 percent). In other words, the change in income for poor households in each year is high. Based on logit models they discover that poor households are more likely to be larger, less educated, have fewer livestock and land and live in certain locations. They find that the dependency ratio can be used as a tool to distinguish between permanent and temporary poor, and so they suggest that it be used by policy-makers for targeting purposes. Then they compare and contrast the poverty impacts of “growth” and “smoothing” policies. They demonstrate that income smoothing policies like micro-credit for consumption, seasonal public works, crop insurance or price stabilization schemes tend to decrease transitory poverty and will be effective in the short run. On the other hand, to reduce chronic poverty, policy-makers need a long-run plan for sustained growth in the income of households.

Carter and May (1999) investigate the KwaZulu-Natal income dynamics data. They show that poverty rates among non-white households have increased. They observe that two third of poor households in South Africa remained poor after five years and more households fell below poverty line than climbed up. Using non-parametric methods, they explore the extent to which initial endowments of social and human capital predict the growth in future well being.

Okrasa (1999) uses four-year Polish panel data to identify transiently and permanently poor households. He finds that human capital, fertility level, unemployment among household members, age and education of the head of the household, and family assets, as well as location (rural or urban) have some effects on staying permanently poor. He also reports a tendency toward long-term poverty between 1993 and 1996 in Poland. Based on his findings, he suggests the proper policies for policy-makers.

Speder (1998) focuses on descriptive statistics based on Hungarian panel data and concludes that living in villages, having low education and more children, being unemployed and not having a spouse are positively related to being permanently poor. Macroeconomic changes and events in the life cycle of individual job careers may explain transitory poverty.

In Iran, Tabibian et al (1998), prepare a yearly report on inequality and poverty for the Plan and Budget Organization, based on the Iranian (cross section) expenditure survey, to give specific policy recommendations to the government. Salehi-Isfahani (2008) provides a more recent poverty analysis in Iran. Cross section data have the drawback of not allowing researchers to follow up a household through time. That is one of the main reasons for using the Iranian panel data in this chapter to distinguish between different kinds of poverty.

5.2 Description of Data

The data I am using in this chapter is panel data on household social and economic characteristics collected for four years; 1992-95. I use the raw data, and a dictionary file to transfer the data into STATA. This panel data is one of the only two available panel data sets in Iran gathered by the Statistic Center of Iran (SCI).²¹

This panel shares some characteristics with the expenditure surveys, also collected by SCI. Compared to expenditure surveys, this panel data is more detailed in demographic and socio-economic characteristics but less detailed in reporting on expenditure and income. The panel is a nationally representative, clustered sample of about 5000 households over four years. About 3300 households are present in all four years (I will call it the balanced sample or balanced panel hereafter).

Despite the fact that the panel data is smaller in size and is not as up-to-date (not collected after 1995) as the expenditure surveys, it helps us understand some aspects of poverty that expenditure surveys cannot reveal. The most important

²¹ The other one was collected by SCI in 12 rounds between 1987 and 1989.

aspect is the extent to which poverty is a transitory phenomenon. I am able to see whether those who are classified as poor in 1992 remain poor one, two and three years later. I can decompose poverty into its transitory and chronic components, using the method proposed by Jalan and Ravallin, and see whether different types of individuals are more susceptible to one or the other type of poverty.

5.3 Attrition in the Panel Data

The sample begins with 5090 households in 1992, of which 3364 or 66 percent appear in all four years that constitute the balanced sample for this analysis. Most of the attrition takes place in the first year, 17.5 percent, but attrition continues at about 11 percent or less per year in the remaining two years. The number of households was 4255, 3982 and 3662 in 1993, 1994 and 1995, respectively. So the panel data is unbalanced with the attrition rate of 28% in four years. Yearly attrition was 17.5, 10.3 and 11 percent between 1992 and 1994, respectively. Note that if I only choose the households that stay in the panel for four years, I will have a selected sample, because the characteristics of those who left the panel and those who stayed may be different.

The use of the balanced sample raises an important concern regarding selection. If families drop out of the panel for reasons related to the characteristics I am analyzing here, the balanced sample will suffer from a selection problem and our conclusions will be subject to selection bias. Along certain dimensions there are clearly selection biases. For example, those who leave are less educated than those who stay in the panel. Fortunately, there is less of a selection problem along the poverty dimension. As table 5.1 shows, roughly similar proportions of the poor and the non-poor dropped out in the first and during the entire sample period. This tells us that at least as far as measuring poverty is concerned, the panel years are comparable. This is not true for a comparison of education levels because the attrition rates are not the same for all education levels. The same applies to some other socio-economic characteristics.

Status in 1992	Percent dropping out	
	1992-93	1992-95
Poor	16.0	29.0
Non-poor	17.8	30.5

Table 5.1 Sample selectivity: The poor are only slightly less likely to drop out

5.4 Expenditures Distribution and Rural vs. Urban Comparison

In the literature on poverty and inequality, there has been a debate over the choice of income or expenditures as a measure of household wealth. I have chosen expenditure over income, since other studies in Iran showed that expenditures that are reported in the surveys are more reliable.

The distribution of household expenditure for each of the four years of panel data is graphed in Fig. 5.1. As might be expected, there is clear skewness in the distribution of expenditure. The right side of Fig. 5.1 depicts the logarithm of expenditures, which has a distribution that is close to a normal distribution. In fact, the logarithmic transformation changes the distribution to one that is not exactly normal but at least symmetric.

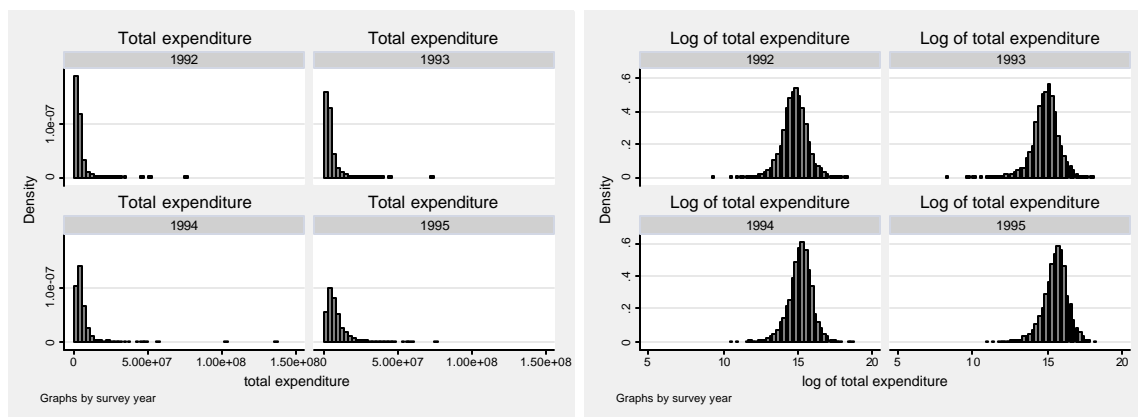


Fig 5.1: Total expenditure and Logarithm of total expenditure

Rural and urban areas have similarities as well as differences. The differences between rural and urban areas play such an important role that throughout this chapter I will run separate regressions and tables for each area.

Fig 5.2 shows the Rural and Urban Logarithm of per capita expenditure distributions. The solid vertical lines are the corresponding poverty lines (which I will explain in more detail in the next section).

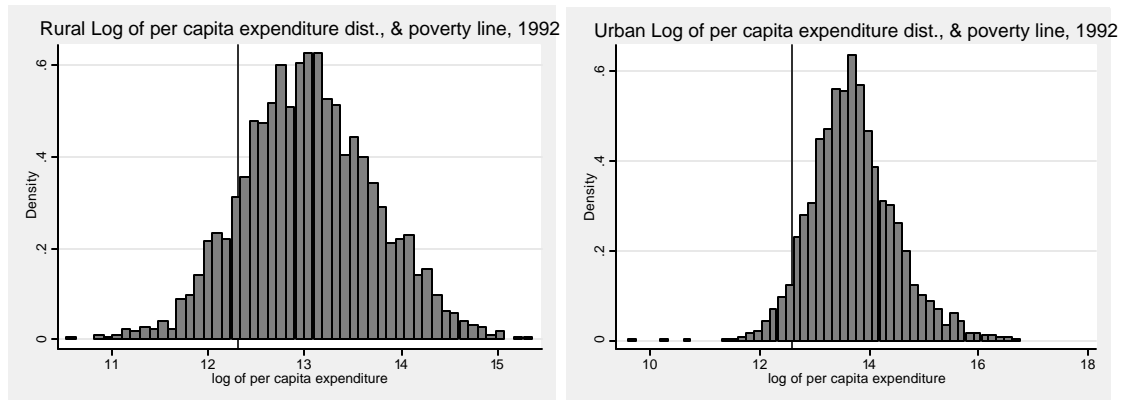


Fig 5.2: Rural and Urban Logarithm of per capita expenditure and poverty line (solid vertical line represents Poverty Line)

5.5 Comparison with Expenditure Surveys

As I mentioned earlier, the panel data I am using in this study is one of the two available panel data sets in Iran and the only one during the period of particular interest to me, a few years after the Iran-Iraq war. Therefore, there is no alternative choice of data for us in this study. However, one should always be aware of the shortcomings of the available data. For this reason, I use Table 5.2 to make a direct comparison of poverty for 1994 measured by the budget survey and by the panel data. Average expenditures in the two samples are somewhat different. The average urban (rural) family expenditures according to the expenditure survey exceed the panel average by 13 percent (12 percent) in 1994. Although the differences in poverty rates are smaller, they do not accord with the differences in average expenditures. The rural poverty rate is actually smaller in the panel survey than the expenditure survey, 25.9 percent compared to 27.7 percent, whereas the urban poverty rate is slightly higher (17.9 compared to 17.0 percent). In the table below, I use the household size as a weighting factor, to get individual-level data, and compare the resulting numbers to household-level

data.²² In the rest of this chapter, I always use household-level data. For more information on individual-level analysis, please see Salehi-Isfahani and Mohemkar-Kheirandish (2002).

Year 1994	Panel data		Expenditure survey
	Household level	Individual level	Individual level
a) Per capita expenditure			
<i>Rural</i>	689871	638001	772,907
<i>Urban</i>	1357494	1196716	1,534,468
b) Headcount ratio			
<i>Rural</i>	23.07	25.85	27.69
<i>Urban</i>	13.86	17.87	16.97

Table 5.2 Comparing panel data and expenditure survey

5.6 The Choice of a Poverty Line

I use separate poverty lines for households in rural areas, urban areas, and the province of Tehran, calculated by Jamshid Pajouyan. Another measurement of poverty line is provided by Statistical Center of Iran (SCI). The table below shows the value of the poverty lines I use in this study and also those provided by SCI. The fact that the province of Tehran is treated separately is mostly due to its socio-economic differences with other provinces in Iran.

Year	Pajouyan ^a				SCI ^b	
			Tehran			
	Rural	Urban	Rural	Urban	Rural	Urban
1992	207873.9	293678.9	229237.9	407463.2	170574	281798
1993	255159.6	356531.0	281383.3	494666.9	209552	346193
1994	366556.0	481994.0	404228.3	668739.9	283305	468036
1995	563103.3	720002.6	620975.5	998963.7	423195	699144

Table 5.3 Calorie poverty line (Per capita Iranian Rials) calculated by Pajouyan and SCI

- a. Poverty Line Calculated by Pajouyan (1994)
- b. Poverty Line Calculated by Statistical Center of Iran

²² Source: Salehi-Isfahani and Mohemkar-Kheirandish (2002)

For each region (rural, urban, and Tehran), Pajouyan considers the consumption basket, of which the food component provides a “minimum” necessary level of Calories. The Rial value of this basket is the poverty line for that group of households.

5.7 Poverty

In this section I discuss poverty and its dynamics in Iran in detail. As mentioned earlier, households are considered the units in this study. This contrasts with Salehi-Isfahani and Moheemkar-Kheirandish (2002) where the same data set was used but each household was weighted by the household size. I will compare and contrast their result with my findings henceforth.

5.7.1 Changes in Poverty During the Panel Years

As with many other data sets for developing countries that have been reported in the literature, expenditure is greater than income in Iranian panel data. Table 5.4 shows nominal income and expenditure. This discrepancy is one of the reasons I think expenditure is a more reliable measure of household wealth. The panel years cover an interesting period in Iran’s reform program. The first two years of the survey correspond to relatively good economic conditions while the last two years are considered difficult years, as the reform program fell apart and the government imposed harsh import restrictions. Imports were cut from a level above \$25 billion in 1992-93 to under \$15 billion for 1994-95. Evidence from the panel data indicates that the defeat of the reform program and the ensuing import compression had serious implications for household welfare and poverty. In 1988, Iran and Iraq accepted UN resolution 598 and its cease-fire, and immediately after that Iranian government began a series of large scale projects that took a few years to complete. Some of those projects were finished during this panel data period.

Survey Year	Total		Rural		Urban	
	Mean of Total Expenditure	Mean of Total Income	Mean of Total Expenditure	Mean of Total Income	Mean of Total Expenditure	Mean of Total Income
1992	3748834	3167641	2646016	2068718	4608915	4024684
1993	4121023	3602317	2757644	2466319	5184315	4488276
1994	5051272	5404680	3648809	4069227	6145044	6446191
1995	7810144	6980217	6325299	5507869	8968165	8128492

Table 5.4 Nominal total expenditure and income in current Rials

Compared to 1992, real per capita expenditure was lower in both rural and urban areas in 1995 (Table 5.5). Urban expenditure fell continuously and was lower by 19 percent in 1995. Rural expenditure recovered partially in 1995 but was still down by 11 percent.

Year	Total	Rural	Urban
1992	1313281	866024	1662095
1993	1172686	748039	1503865
1994	1064963	689871	1357495
1995	1068352	757640	1310674

Table 5.5 Average real per capita expenditures during the panel years in 1994 prices

Poverty rates responded similarly, rising in urban areas from 13.2 percent in 1992 to 15.3 percent in 1995, and in rural areas from 17.8 percent to 21.9 percent (Table 5.6). Individual level data show higher figures but the same direction of change, as reported in Salehi-Isfahani and Mohemkar-Kheirandish (2002).

Year	Head Count Ratio		
	Total	Rural	Urban
1992	15.2	17.8	13.2
1993	16.6	22.8	11.8
1994	17.9	23.1	13.9
1995	18.2	21.9	15.3

Table 5.6 Poverty rates 1992-1995

Note: Poverty lines are measured using Pajouyan's calculations for 1994 and adjusted for other years using the CPI.

5.7.2 Duration of Poverty: Years under Poverty Line

As Table 5.7 shows, less than three percent of the rural and urban population were poor in all four years. Of those who were poor in at least one year of the panel, around half stayed poor only one year. The average number of years in poverty for those who were poor at least once was 1.82 years in rural areas and 1.77 years for urban areas, or 1.8 years for the total population. In urban areas close to 70 percent of households were never under the poverty line, while the corresponding number for rural areas is around 53 percent. This difference is additional evidence for the necessity of treating rural and urban areas separately.

Years Poor	Total		Rural		Urban	
	Percent	Cum.	Percent	Cum.	Percent	Cum.
0	62.3	62.3	53.1	53.1	69.5	69.5
1	18.7	81.0	22.2	75.3	15.9	85.5
2	10.5	91.5	13.6	88.9	8.0	93.5
3	5.8	97.3	8.1	97.1	4.0	97.5
4	2.7	100.0	2.9	100.0	2.5	100.0
Total	100.00		100.00		100.00	

Table 5.7 Length of poverty (years in poverty)

Education of the household head had a lot to do with how many years the household experienced poverty during the panel, supporting the findings in many other studies in developing countries, which identified education as an important correlate of poverty status. In urban areas, those experiencing poverty in all four years were 58 percent illiterate, compared to 23 percent for those who were not poor in any panel year (Table 5.8). In rural areas there was much less of a difference: 62 percent of those poor all four years and 49 percent of the “never-poor” were illiterate. Higher education can be viewed as a factor that reduces the risk of spells under the poverty line. But the low percentage of household heads with higher education in the data set prevents any definite conclusion.

The relationship of the gender of the head with the number of years in poverty shows an interesting pattern (Table 5.9). Consistent with other findings

regarding gender, those living in a female headed household do not appear very different in terms of frequency of poverty spells from those in male headed households. Roughly 7 percent of those poor any number of years, live in female headed households, which is the same as their 7.5 percent share in total population. But the story is different for those who are always poor. In rural areas the share of female headed households in the “always poor” category is much larger (17.5 percent) than their share in population (7 percent), indicating their higher vulnerability to poverty spells. This vulnerability is less obvious in Salehi-Isfahani and Mohemkar-Kheirandish (2002). For urban areas the share is 7.81 which is less than the female headed household share in population namely 8.5 percent. However, the difference is not significant.

	Education Categories							
	Years poor	Illiterate	Some Primary	Primary School	Middle School	High School	Higher Education	Total
Total	0	32.76	19.63	20.96	9.24	10.36	7.05	100
	1	45.84	23.06	18.36	8.52	3.58	0.64	100
	2	48.48	22.82	17.65	7.02	3.61	0.43	100
	3	57.25	22.21	15.02	5.01	0.51	0.00	100
	4	59.78	26.45	9.64	3.03	1.10	0.00	100
	Total	39.00	20.94	19.48	8.46	7.57	4.56	100
Rural	0	48.85	24.03	16.93	5.11	3.80	1.28	100
	1	58.99	21.88	13.39	3.44	2.30	0.00	100
	2	60.40	20.67	12.33	3.49	2.86	0.25	100
	3	66.39	17.95	11.69	3.97	0.00	0.00	100
	4	61.99	30.41	4.09	3.51	0.00	0.00	100
	Total	54.48	22.78	14.72	4.38	2.92	0.71	100
Urban	0	23.17	17.01	23.36	11.70	14.27	10.48	100
	1	31.56	24.34	23.75	14.04	4.98	1.33	100
	2	32.73	25.66	24.67	11.68	4.61	0.66	100
	3	42.67	29.00	20.33	6.67	1.33	0.00	100
	4	57.81	22.92	14.58	2.60	2.08	0.00	100
	Total	26.93	19.50	23.19	11.64	11.19	7.55	100

Table 5.8 Frequency of poverty spells by education of head

		Household Head's gender		
	Years poor	Male	Female	Total
Total	0	92.18	7.82	100
	1	92.64	7.36	100
	2	93.06	6.94	100
	3	92.56	7.44	100
	4	87.64	12.36	100
	Total	92.26	7.74	100
Rural	0	94.54	5.46	100
	1	93.20	6.80	100
	2	92.29	7.71	100
	3	90.83	9.17	100
	4	82.56	17.44	100
	Total	93.28	6.72	100
Urban	0	90.77	9.23	100
	1	92.03	7.97	100
	2	94.08	5.92	100
	3	95.33	4.67	100
	4	92.19	7.81	100
	Total	91.46	8.54	100

Table 5.9 Frequency of poverty spells by gender of head

5.7.3 Chronic vs. Transitory Poverty

The evidence presented above indicate that (a) a large proportion of those who are poor in one year are not likely to be poor the next, and (b) the characteristics of those who are occasionally poor may differ from those who are frequently poor.

Table 5.10 shows that about 50 percent of poverty in Iran during the 1992-95 can be described as transient and the rest as chronic. The decomposition of poverty differs depending on region, education characteristics and gender of the household head (Tables 3.7-9). Although the shares of chronic and transitory poverty are similar when considered for all urban and rural population, deep differences emerge when I consider education of the household head.

		Poverty Decomposition				
		Chronic	Transient	Total (FGT)	Chronic share	Transient share
Rural						
	Illiterate	3.63	4.08	7.71	47.05	52.95
	Some primary	6.40	3.05	9.45	67.76	32.24
	Primary school	1.67	3.05	4.72	35.31	64.69
	Middle school	3.78	3.93	7.71	49.05	50.95
	High school	0.00	0.52	0.52	0.00	100.00
	Higher education	--	--	--	--	--
	<i>All education groups</i>	3.87	3.82	7.69	50.29	49.71
Urban						
	<i>Illiterate</i>	3.52	3.30	6.83	51.62	48.38
	Some primary	1.84	3.00	4.84	38.02	61.98
	Primary school	1.81	2.60	4.41	41.06	58.94
	Middle school	0.60	1.60	2.20	27.26	72.74
	High school	0.80	2.87	3.67	21.88	78.12
	Higher education	0.00	0.13	0.13	0.00	100.00
	<i>All education groups</i>	2.62	2.98	5.59	46.75	53.25
Total						
	<i>Illiterate</i>	3.59	3.77	7.35	48.77	51.23
	Some primary	4.03	3.02	7.06	57.16	42.84
	Primary school	1.77	2.72	4.49	39.37	60.63
	Middle school	1.66	2.38	4.04	41.13	58.87
	High school	0.64	2.40	3.04	21.12	78.87
	Higher education	0.00	0.13	0.13	0.00	100.00
	<i>All education groups</i>	3.26	3.41	6.67	48.85	51.15

Table 5.10 Decomposition of poverty by education of household head

In both rural and urban areas, chronic poverty declines with education (Table 5.10). However, whereas in urban areas for those in “illiterate households” the breakdown was 52 percent chronic and 48 percent transient, in illiterate rural households it was 47 vs. 53 percent. For those living in rural households with a head possessing a high school or higher education, poverty was almost all transient, whereas in urban households the chronic component exists.

The regional decomposition does not yield a noticeable pattern (Table 5.11). The Northwest and the Persian Gulf (which is more nomadic and less developed) regions, exhibit more transient poverty compared to the rest of the country both in rural and urban areas.

Relative to the total population, individuals living in female headed households are more likely to suffer from chronic than transient poverty. Whereas the share of chronic poverty for male-headed households is less than 50 percent in rural and urban areas, for female-headed families it is 54 and 57 percent, respectively (Table 5.12).

		Poverty Decomposition				
		Chronic	Transient	FGT	Chronic share	Transient share
<i>Rural</i>	Central	7.65	4.03	11.69	65.48	34.52
	Caspian	2.07	2.80	4.87	42.43	57.57
	Northwest	0.64	2.65	3.30	19.50	80.50
	West	7.13	3.98	11.11	64.16	35.84
	Gulf	0.89	3.71	4.60	19.27	80.73
	East	4.47	5.37	9.84	45.43	54.57
	Total	3.87	3.82	7.69	50.29	49.71
Urban	Central	1.73	2.51	4.24	40.80	59.20
	Caspian	4.72	3.49	8.21	57.49	42.51
	Northwest	0.00	1.11	1.11	0.00	100.00
	West	1.30	3.47	4.77	27.17	72.83
	Gulf	1.77	3.06	4.83	36.71	63.29
	East	4.91	3.71	8.62	56.91	43.09
	Total	2.62	2.98	5.59	46.75	53.25
Total	Central	3.36	2.93	6.28	53.39	46.61
	Caspian	2.89	3.02	5.91	48.94	51.06
	Northwest	0.59	2.52	3.11	18.89	81.11
	West	4.55	3.76	8.31	54.77	45.23
	Gulf	1.41	3.33	4.73	29.71	70.29
	East	4.65	4.67	9.32	49.94	50.06
	Total	3.26	3.41	6.67	48.85	51.15

Table 5.11 Decomposition of poverty by region

		Poverty Decomposition				
		Chronic	Transient	FGT	Chronic share	Transient share
Rural		3.87	3.82	7.69	50.29	49.71
	<i>Male</i>	3.59	3.62	7.21	49.75	50.25
	<i>Female</i>	7.10	6.13	13.23	53.69	46.31
Urban		2.62	2.98	5.59	46.75	53.25
	<i>Male</i>	2.43	2.89	5.32	45.62	54.38
	<i>Female</i>	5.76	4.40	10.16	56.71	43.29

Table 5.12 Decomposition of poverty by gender of household head

The decomposition according to size of households shows that the level (and share) of chronic poverty for urban households with fewer members is smaller than transient poverty (Table 5.13).

		Poverty Decomposition				
		Chronic	Transient	FGT	Chronic share	Transient share
Rural	Size<4	4.42	5.06	9.47	46.61	53.39
	4<Size<7	3.43	3.29	6.72	51.04	48.96
	Size>7	3.92	3.44	7.36	53.21	46.79
Urban	Size<4	2.41	4.95	7.36	32.80	67.20
	4<Size<7	1.98	2.21	4.19	47.25	52.75
	Size>7	3.83	3.09	6.91	55.35	44.65

Table 5.13 Decomposition of poverty by size category

As before, we note that being unemployed or out of the labor force, increase both chronic and transient poverty (Table 5.14). However, there is little variation in decomposition of poverty according to employment status.

		Poverty Decomposition				
		Chronic	Transient	FGT	Chronic share	Transient share
Rural	Employed	3.41	3.45	6.86	49.77	50.23
	Unemployed	4.84	3.86	8.70	55.66	44.34
	Out of labor force	5.90	5.86	11.76	50.17	49.83
Urban	Employed	2.42	2.74	5.17	46.91	53.09
	Unemployed	2.36	2.95	5.31	44.47	55.53
	Out of labor force	3.68	4.25	7.94	46.40	53.60

Table 5.14 Decomposition of poverty by employment category

For the instances studied in this section, the effects of considering household (this chapter) as opposed to individual (Salehi-Isfahani and Mohemkar-Kheirandish) are compatible with one another.

5.7.4 Determinants of Chronic and Transient Poverty

To combine these various characteristics in a regression framework I use the tobit method to explain chronic and transient poverty. The model used here is similar to the one used in Salehi-Isfahani and Mohemkar-Kheirandish (2002). As we will see, my results are stronger and more significant than theirs. There are several noteworthy results from the Tobit regressions (Table 5.15). The most remarkable is the role of gender, which confirms our previous finding that gender matters in both rural and urban areas. I also find that gender matters a lot more for chronic than transient poverty, and more in rural areas than urban areas. Living in a rural household headed by a female increases an individual's probability of chronic poverty to at least three times the probability of transient poverty. In other words, female headed households should be targeted by policy-makers who aim to combat poverty in general and permanent or chronic poverty in particular.

The age variable points to some interesting differences between chronic and transient poverty. Age seems to matter less for rural households, since the absolute values of coefficients are smaller in the rural areas. Compared to the young category (30 and younger), which is the omitted category, being in a rural or urban household headed by 50 to 65 years old *reduces* both chronic and transient poverty the most. Household head of age 30 to 50 are the second least vulnerable group. In urban areas older households are also subject to less chronic and transient poverty compared to very young ones (30 years old or less). In particular, the retired category showed less poverty than younger groups, with a greater effect on chronic than transient poverty (rural coefficients are not significant for retired group).

Independent Variables	Rural				Urban			
	Chronic		Transient		Chronic		Transient	
	Coefficient	Std. Err	Coefficient	Std. Err	Coefficient	Std. Err	Coefficient	Std. Err
Gender								
Female	5.18	1.29	1.65	0.38	1.84	1.32	0.65	0.31
Age								
30<Age<51	-2.23	1.15	-0.72	0.29	-3.80	1.18	-2.03	0.26
50<Age<66	-5.61	1.29	-1.38	0.32	-7.94	1.31	-3.48	0.29
65>Age	-1.14	1.41	-0.23	0.37	-6.18	1.47	-2.38	0.34
Education								
Some primary school	-0.27	1.07	-0.44	0.29	-5.19	0.95	-0.86	0.23
Primary school	-5.29	1.18	-1.31	0.28	-7.19	0.84	-2.28	0.20
Middle school	-4.25	1.90	-0.37	0.47	-10.82	1.26	-2.26	0.25
High school	-14.67	3.56	-2.77	0.60	-13.23	1.65	-4.53	0.31
Higher education	-12.61	7.15	-6.88	1.89	-70.65	.	-7.47	0.53
Household size								
4<Size<8	5.38	0.95	1.57	0.23	5.72	0.90	2.20	0.19
7<Size	9.21	1.05	2.81	0.26	13.05	1.07	3.67	0.24
Activity status								
Unemployed	7.46	2.25	1.62	0.68	4.01	2.01	1.21	0.55
Out of labor force	8.81	1.16	2.63	0.33	-0.41	0.98	-0.48	0.23
Region								
Caspian	0.65	1.06	2.91	0.28	6.35	0.96	1.67	0.25
Northwest	-9.19	1.57	0.84	0.33	-68.31	.	-4.64	0.41
West	4.63	0.99	4.94	0.27	3.09	0.85	2.12	0.20
Gulf	-10.23	1.79	-1.06	0.39	-4.70	1.33	-0.77	0.30
East	5.17	1.02	4.53	0.28	7.26	0.85	2.34	0.22
Constant	-21.93	1.59	-4.13	0.37	-15.94	1.44	-0.92	0.29
Number of observations	5890		5890		7560		7560	
Pseudo R-squared	0.0516		0.0407		0.1163		0.0793	

Table 5.15 Determinants of chronic and transient poverty (Tobit regressions)

For both rural and urban households, education appears to have a much larger impact in reducing chronic than transient poverty. Higher education plays a more important role in reduction of both kinds of poverty. This provides another important focus for policy-makers who are interested in reducing poverty.

Size of household plays a predictable role: The larger the size of household, the greater the probability of poverty (both chronic and transient). Also, the effect of size on chronic poverty is much higher than on transient poverty in both

rural and urban areas. This provides another criterion for policy-maker target poor families.

Being unemployed or out of the labor force in urban areas appears to matter for both transient poverty and chronic poverty. In urban areas, being unemployed increases poverty, whereas being out of the labor force reduces it. This observation could possibly be explained by the voluntary nature of being out of the labor force in urban areas. In rural areas, on the other hand, being unemployed or out of the labor force increases both kinds of poverty. Note that because of the agricultural nature of rural employment, being out of the labor force is involuntary so it increases poverty. In rural areas, the impact of unemployment is greater on chronic than transient poverty. Perhaps being unemployed in rural areas is more of a permanent condition than in urban areas.

The regional coefficients show a somewhat different picture than the simple cross-tabs discussed earlier, but remain difficult in yielding a general pattern. The only consistent reading is that the Caspian, West, and East appear to show more chronic and transient poverty, for both rural and urban populations compared to the Central regions. The Northwest and Gulf regions show significant decreases in chronic poverty relative to the (reference) prosperous Central regions, while the rural effect for transient poverty in the Northwest is positive.

5.8 Mobility

Despite the fact that the relative nature of transition matrices is a shortcoming for capturing the dynamics of poverty, they provide a good understanding of the severity of fluctuations in household expenditure. This was one of our original ideas of distinguishing between temporary and permanent poverty. If the expenditure changes a lot from year to year, then there could be lots of households under the poverty line that could escape from poverty the next year. Those that can not climb up by themselves should be the target of anti-poverty policies in long run.

5.8.1 Transition Matrix

In this section I examine the overall mobility of the population in terms of expenditure quintiles, using transition matrices. Note that here the grouping is based on a relative measure. Table 5.16 shows the high level of mobility experienced by the sample during 1992-95. In 1995 about 47 percent of the rural and 39 percent of the urban population who were in the poorest quintile in 1992 were still in that quintile, and about 28 percent moved up one quintile, and so on. These figures appear large but are within the range of similar data from other countries (See Fields et al, 2000, for data on Indonesia). Transitions during 1992-93 were slightly less pronounced but show the same pattern (Table 5.17).

One can look at moving in and out of poverty in terms of a binary transition matrix that is similar to these quintile transition matrices. Table 5.18 provides a transition matrix for falling below the poverty line or escaping out of poverty. For example in rural and urban areas, among those households who were poor in 1992, 40 percent stayed poor in 1993 and 60 percent escaped poverty. (see Baulch and McCulloch, 2000, for similar data on rural Pakistan)

		1995					
		1	2	3	4	5	Total
	<i>Rural</i>						
	1	47.11	27.80	13.36	7.40	4.33	100
	2	30.71	27.16	23.35	13.45	5.33	100
	3	25.37	23.53	22.06	18.01	11.03	100
	4	16.75	19.80	21.83	24.87	16.75	100
1	5	15.24	19.05	20.95	20.95	23.81	100
9	Total	32.85	25.23	19.12	14.06	8.74	100
9	<i>Urban</i>						
2	1	39.17	28.11	19.35	9.68	3.69	100
	2	17.50	25.31	29.38	18.44	9.38	100
	3	7.57	19.15	28.06	27.84	17.37	100
	4	4.17	13.52	18.89	33.40	30.02	100
	5	1.50	5.45	10.90	23.68	58.46	100
	Total	10.09	16.08	20.53	24.69	28.60	100

Table 5.16 1992-1995 transition matrix

		1993					
		1	2	3	4	5	Total
	<i>Rural</i>						
	1	53.51	28.59	12.94	2.88	2.08	100
	2	36.57	33.56	18.75	6.02	5.09	100
	3	30.69	25.86	24.48	13.45	5.52	100
	4	18.22	21.03	27.57	22.90	10.28	100
1	5	10.00	18.18	24.55	26.36	20.91	100
9	Total	37.80	27.75	19.08	9.63	5.74	100
9	<i>Urban</i>						
2	1	39.69	30.53	16.41	10.69	2.67	100
	2	15.38	32.69	25.24	19.47	7.21	100
	3	3.68	17.28	31.99	30.88	16.18	100
	4	2.30	7.38	20.49	37.05	32.79	100
	5	0.29	2.49	9.24	25.81	62.17	100
	Total	8.11	14.80	20.29	27.01	29.79	100

Table 5.17 1992-1993 transition matrix

		1993		
		Non-poor	Poor	Total
	<i>Rural</i>			
	Non-poor	81.0	19.0	100
1	Poor	59.5	40.5	100
9	Total	77.2	22.8	100
9	<i>Urban</i>			
2	Non-poor	92.6	7.4	100
	Poor	60.6	39.4	100
	Total	88.4	11.6	100

Table 5.18 1992-1993 poverty transition matrix

Table 5.19 shows a more detailed poverty transition matrix. It reveals, for those who escape poverty, how far they go, and for those who are poor, how far they are from the poverty line. For example, among those who had expenditure of less than 90 percent of poverty line in 1993, 35 percent come from the same status, 13 percent from expenditure between 90 and 110 percent of poverty line,

32 percent from expenditure between 110 and 200 percent of poverty line, and only 20 percent from expenditure above 200 percent of poverty line. Note that Tables 5.18 and 5.19 are based on an absolute measure (the poverty line), so these are better measures for “actual” dynamics of income and expenditure.

	Status in 1993				
Status in 1992	<=0.9 PL	0.9PL-1.1 PL	1.1PL-2.0PL	>2PL	Total
<=0.9PL	34.7	19.6	10.9	3.8	11.2
0.9PL—1.1PL	12.7	12.6	7.8	2.6	6.4
1.1PL—2PL	31.8	43.0	41.8	24.1	32.6
>2PL	20.8	24.8	39.8	69.6	49.9
Total	100.0	100.0	100.0	100.0	100.0

Table 5.19 Detailed 1992-1993 poverty transition matrix

5.8.2 Determinants of Growth

The transition matrices reported above illustrate the extent of mobility without saying anything about its determinants. In order to know the determinants of mobility, one can look at the effects of the household characteristics on the growth of expenditure. To do so, I take the difference of logarithm of real per capita expenditures in 1992 and 1995 and regress it on the characteristics of households (see Glewwe (1992) or Glewwe and Hall (1998)). The results in Table 5.20, suggest that gender matters in rural areas: Female-headed households experience more growth. The impact of age appears to be negative (and insignificant) with respect to the youngest group (30 years old and younger). Education has an insignificant effect. The effect of size is positive and significant most of the time. The employment status has different effects on growth in rural and urban areas. The coefficient for households with unemployed head is negative in rural areas while out of labor force heads in rural and urban areas and urban unemployed heads have a positive but insignificant effect. The impact of marital status is positive in both rural and urban areas but larger in rural areas. The Caspian and West rural and urban, and Northwest urban areas experience less

expenditure growth than the Central region (the omitted category), while other regions underwent growth higher than the Central region. The growth regression model does not yield significant results.

Independent Variables	Change in logarithm of real per capita expenditure			
	Rural		Urban	
	Coefficient	Std. Err	Coefficient	Std. Err
Gender				
Female	0.23	0.12	0.10	0.10
<u>Age</u>				
30<Age<51	-0.07	0.06	-0.04	0.05
50<Age<66	-0.01	0.07	-0.04	0.06
65>Age	-0.02	0.08	0.07	0.07
Education				
Some primary school	-0.05	0.05	-0.01	0.04
Primary school	-0.12	0.06	-0.03	0.05
Middle school	-0.08	0.10	-0.06	0.06
High school	-0.07	0.12	-0.07	0.06
Higher education	0.01	0.26	-0.08	0.07
Household size				
4<Size<7	0.07	0.05	0.17	0.04
7<Size	0.21	0.06	0.18	0.05
Activity status				
Unemployed	-0.14	0.14	0.14	0.12
Out of labor force	0.13	0.08	0.04	0.05
Region				
Caspian	-0.08	0.06	-0.10	0.06
Northwest	0.12	0.07	-0.02	0.05
West	-0.51	0.06	-0.31	0.04
Gulf	0.13	0.08	0.66	0.07
East	0.08	0.06	0.20	0.05
<u>Marital status</u>				
Married	0.10	0.10	0.07	0.08
Constant	-0.18	0.12	-0.28	0.10
Number of observations	1467		1875	
F-statistics	9.00		13.57	
R-squared	0.1057		0.1220	

Table 5.20 Determinants of logarithm of change in real expenditure during 1992-1995 (OLS regression)

6 Conclusions and Future Directions

In this dissertation, I introduced a two-type economy, and assumed a special kind of pure group externality for household formation as well as a specific consumption externality (Chapter three). This parsimonious model gave us some insight into whether each type ends up with a “better” (in a normative sense) outcome in a two-person household compared to remaining single. I observed that, if I do not allow any trade for individuals who divorce, the set of Pareto optimal outcomes which can be obtained as “stable” equilibrium allocations strictly contains the core of the economy, i.e., the extended core. On the other hand, if trade is allowed after divorce, a proper subset of the extended core can be supported as stable equilibrium allocations.

The main conclusion from the analysis of this model was that in the presence of externalities, household formation leaves the individuals with more choices. This is one of the main incentives for household formation.

In Chapter four, I extended the model to deal explicitly with labor and labor supply decisions of households. I showed how individual’s private good or leisure consumption decisions change with change in wages, price of private good, power in the household, relative importance of private consumption compared to leisure, and the level of altruism. The effects of relative price changes were also investigated. In the development and labor literature, where the features and trends of labor supply for women and men are comparatively discussed, the increase in the labor supply by women during recent years is viewed as a “better” outcome for the society. Spence’s signaling model that can be viewed as a special general equilibrium model helped us to reconsider this welfare conclusion. I used Spence’s model to explain the relationship between wage discrimination and education decision in Iran and how it is possible to have a sustainable discrimination.

For future research, I would like to explore if there could be a model under which a larger female labor supply would depress wages and even lower the level

of utility for women and households!²³ I would like to add production to my simple model of a two-type economy in a general equilibrium setting. I envision a production function with labor as the only factor (or, in a more general case, as a function of some other inputs, too), which assumes different productivity levels (high and low) for each type. I can investigate the general equilibrium outcome of this model. Furthermore, it could be interesting to look at the labor supply of each type of individuals. It may help explaining the different labor supply for each spouse. I am particularly interested in the Pareto optimal outcomes and Pareto improvements in such models. I can investigate the stability of such outcomes (in a positive sense), as well. For example, I can find the kind of situations which may encourage divorce.

Another way to enrich the current model is to treat benefits from household formation as random variables. I know that, in real life, one can not predict the gains of finding a mate a priori. More often than not, one will realize the true “benefit” or “loss” of household formation after the fact. I can use a two period model to investigate the possible outcomes of such scenarios. To do so, in the first period, I would assume that each type finds her or his mate based on some facts in the form of expectations about the benefits; it is only in the second period when she or he realizes the “true benefits”. Then, the main question is whether the individuals are willing to change their mates after the benefits are realized. Notice that the existence of individuals who want to break their current relationships does not necessarily mean that in the second round only “low quality” mates will be available. “High quality” mates, or the initiators of divorce, will also be in the market; this makes the problem more interesting.

Yet another way to improve the current research is to use the general framework to find out more specifically about the kind of externalities that can be assumed.

In Chapter five, I focused on decomposition of poverty into transient and chronic components. As I suspected, the determinants of permanent and temporary poverty are different and so for policy purposes it is important to

²³ It could be the other way around, too, but the point is that one should be careful about drawing general conclusions.

distinguish between permanent and temporary poverty. Long-run policy variables like education have a role in reducing permanent poverty, and will not have as big an effect on temporary poverty. The more educated the head of household the less likely the household is chronically poor. Further investigation shows that female-headed households are more likely to be permanently poor. Poverty rates and their decomposition are different in rural and urban areas. Households with bigger size are more likely to be permanently poor. Being literate, employed, and having a high school or higher degree reduces the probability of being permanently poor. In rural areas, kids play an important role in production, and literacy does not play an important role (compared to the urban areas). This further confirms the smaller effect of size and education on poverty in rural areas compared to urban areas. Quintile and poverty transition matrices both suggest a high mobility in Iran which indicates the importance of this research in targeting “actual” poor for policy-makers.

For future extensions, I suggest to introduce other variables like household investment into the model. Also, as newer panel data become available, the same analysis can be repeated for the new data. A more technical improvement to the current research could be achieved by finding a systematic remedy for attrition in the data to see if it is possible to take an unbalanced panel data and make it balanced by filling in the missed data by using other available data.

7 References

- [1] Alderman, Harold; Chiappori, Pierre-André; Haddad, Lawrence; Hoddionott, John and Kanbur, Ravi (1995), "Unitary Versus Collective Models of the Household: Is it Time to Shift the Burden of Proof?" *The World Bank Research Observer*, Washington DC, USA, 10(1), 1-12.
- [2] Atkinson, A. B. (1970), "On the Measurement of Inequality," *Journal of Economic Theory*, 2(3), 244-63.
- [3] Atkinson, A. B. (1995), *Incomes and the Welfare State: Essays on Britain and Europe*, Cambridge University Press, Cambridge, UK.
- [4] Becker, Gary S. (1973), "A Theory of Marriage: Part I", *Journal of Political Economy*, 81 (4), 813-846.
- [5] Becker, Gary S. (1978), *The Economic Approach to Human Behavior*, Paperback Edition, The University of Chicago Press, Chicago, IL.
- [6] Becker, Gary S. (2005), *A Treatise on the Family*, Enlarged Edition, Harvard University Press, Cambridge, MA.
- [7] Bane, Mary Jo, and David Ellwood (1986), "Slipping into and out of Poverty: The Dynamics of Spells." *The Journal of Human Resources*, 21(1), 1-23.
- [8] Basu, Kaushik (1999), "Child Labor: Cause, Consequence, and Cure, and Cure, with Remarks on International Labor Standards," *Journal of Economic Literature*, 37(3), 1083-1119.
- [9] Baulch, Bob and John Hoddinott (2000), "Economic Mobility and Poverty Dynamics in Developing Countries," *Journal of Development Studies*, 36(6), 1-24
- [10] Browning, Martin, Bourguignon, François, Chiappori, Pierre-André, and Lechene, Valérie (1994), "Income and Outcomes: A structural Model of Intrahousehold Allocation," *Journal of Political Economy*, 102(6), 1067-1096.

- [11] Browning, Martin and Chiappori, Pierre-André (1998), "Efficient Intra-Household Allocations: A General Characterization and Empirical Tests," *Econometrica*, 66(6), 1241-1278.
- [12] Carter, Michael R. and May, Julian (1999), "Getting Ahead or Falling Behind? The Dynamics of Poverty in Post-Apartheid South Africa", Working Paper.
- [13] Chiappori, Pierre-André (1988), "Rational Household Labor Supply," *Econometrica*, 56(1), 63-90.
- [14] Chiappori, Pierre-André (1992), "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100(3), 437-67.
- [15] Chiappori, Pierre-André (1999), "Labor Supply and Efficient Risk Sharing," Working Paper.
- [16] Chiappori, Pierre-André and Werning, Ivan (2002), "Comment on "Rotten Kids, Purity, and Perfection", *Journal of Political Economy*, 110(2), 475-480.
- [17] Cole, Harold L. and Prescott, Edward C. (1997), "Valuation Equilibrium with Clubs," *Journal of Economic Theory*, 74, 19-39.
- [18] Daly, Mary C., and Duncan, Greg J. (1997), "Earning Mobility and Instability, 1969-1995", *Working Papers in Applied Economic Theory 97-06*, Federal Reserve Bank of San Francisco..
- [19] Datt, Gaurav, and Ravallion, Martin (1992), "Growth and Redistribution components of changes in poverty measures," *Journal of Development Economics*, 38, 275-295.
- [20] Deaton, Angus (1997), *The Analysis of Household Surveys*, Johns Hopkins University Press, Baltimore and London.
- [21] Ellickson, Bryan (1979), "Competitive Equilibrium with Local Public Goods," *Journal of Economic Theory*, 21, 46-61.
- [22] Ellickson, Bryan, Grodal, Birgit, Scotchmer, Suzanne, and Zame, William R. (1999) "Clubs and the Market," *Econometrica*, 67, 1185-1218.
- [23] Ellickson, Bryan, Grodal, Birgit, Scotchmer, Suzanne, and Zame, William R. (2001), "Clubs and the Market: Large Finite Economies," *Journal of Economic Theory*, 101, 40-77.

- [24] Fields, Gary S. and OK, Efe A. (1996), "The Meaning and Measurement of Income Mobility", *Journal of Economic Theory*, 71, 349-377.
- [25] Fields, Gary S. and OK, Efe A. (1998), "The US Income Mobility in 80's and 90's" Working Paper.
- [26] Fields, Gary S., Cichell, Paul L., Freije, Samuel, Menendez, Marta, Newhouse, David (2000), "A four country story: Household Income dynamics in Indonesia, South Africa, Spain, and Venezuela, Working Paper.
- [27] Gale, David, Shapley, Lloyd (1962), "College admissions and the stability of marriage," *American Mathematical Monthly*, 92 (1), 9-15.
- [28] Gersbach, Hans and Haller, Hans (1999), "Allocation Among Multi-Member Households: Issues, Cores and Equilibria," in Alkan, A., Aliprantis, C. D. and Yannelis, N. C. (eds.), *Current Trends in Economics: Theory and Applications*, Springer-Verlag, Berlin.
- [29] Gersbach, Hans and Haller, Hans (2000a), "On the Power in Households with Free Exit," Working Paper.
- [30] Gersbach, Hans and Haller, Hans (2000b), "Intra-Household Bargaining Power and Equilibrium Consumption," Working Paper.
- [31] Gersbach, Hans and Haller, Hans (2000c), "The Affectionate Society: Does Competition for Partners Promote Friendliness?" Forthcoming in *Economic Theory*.
- [32] Gersbach, Hans and Haller, Hans (2001), "Collective Decision and Competitive Markets," *Review of Economic Studies*, 68, 347-368.
- [33] Gersbach, Hans and Haller, Hans (2003), "Competitive Markets, Collective Decisions, and Group Formations," Working Paper.
- [34] Gersbach, Hans and Haller, Hans (2005), "When inefficiency begets efficiency", *Economic Theory*, 25, 105-121.
- [35] Gersbach, Hans and Haller, Hans (2008), "Club Theory and Household Formations," Working Paper.
- [36] Gilles, Robert P. and Diamantaras, Dimitrios (1998), "Linear Cost Sharing in Economies with Non-Samuelsonian Public Goods: Core equivalence," *Social Choice and Welfare*, 15, 121-139.

- [37] Gilles, Robert P. and Suzanne Scotchmer (1997), "Decentralization in Replicated Club Economies with Multiple Private Goods," *Journal of Economic Theory*, 72, 363-387.
- [38] Glewwe, Paul (1992), "Targeting Assistance to the Poor: Efficient Allocation of Transfers When Household Income Is Not Observed," *Journal of Development Economics*, 38(2), 297-321
- [39] Glewwe, Paul and Hall, Gillette (1998) "Are some Groups More Vulnerable to Macroeconomic Shocks Than Others? Hypothesis Tests Based on Panel Data from Peru," *Journal of Development Studies*, 56(1), 181-206.
- [40] Hall, Philip (1935), "On Representatives of Subsets," *Journal of the London Mathematical Society*, 10, 26-30.
- [41] Haller, Hans (2000), "Household Decisions and Equilibrium Efficiency," *International Economic Review*, 41(4), 835-847.
- [42] Haller, Hans (2003), "Market and Matching," *IHS newsletter*, No. 1.
- [43] Jalan, Jyotsna and Ravallion, Martin (2000), "Is Transient Poverty Different? Evidence for Rural China," *Journal of Development Studies*, 36(6), 82-99.
- [44] Jenkins, Stephen P., Arie Kapteyn, and Bernard M.S. Van Praag (1995), *The Distribution of Welfare and Household Production: International Perspectives*, Cambridge University Press, Cambridge, UK.
- [45] Kanbur, Ravi and Haddad, Lawrence (1994), "Are Better off Households More Unequal or Less Unequal?" *Oxford Economic Papers*, 46 , 445-458.
- [46] Lillard, Lee and Robert Willis (1978), "Dynamic Aspects of Earning Mobility," *Econometrica*, 46(5), 985-1011.
- [47] Maak, Wilhelm (1936), "Eine neue Definition der fastperiodischen Funktionen," *Abhandlungen aus dem Mathematischen Seminar Hamburg*, 11, 240-244.
- [48] Maasoumi, Esfandiar and Zandvakili (1990), "Generalized Entropy Measures of Mobility for Different Sexes and Income Levels," *Journal of Econometrics*, 43, 121-133.

- [49] Maasoumi, Esfandiar (1986), "The Measurement and Decomposition of Multi-Dimensional Inequality," *Econometrica*, 54(4), 991-998.
- [50] Mazzocco, Maurizio (2004a), "Household Inter-temporal Behavior: A Collective Characterization and Empirical Tests," Working Paper, University of Wisconsin.
- [51] Mazzocco, Maurizio (2004b), "Intra-household Risk Sharing and Savings" Working Paper, University of Wisconsin.
- [52] Mazzocco, Maurizio (2004c), "Inter-temporal Behavior and Household Structure," Working Paper, University of Wisconsin.
- [53] Mazzocco, Maurizio (2004d), "Individual Euler Equations Rather Than Household Euler Equations," Working Paper, University of Wisconsin.
- [54] Mas-Colell, Andreu, Whinston, Michael D., and Green, Jerry R. (1995), *Microeconomic Theory*. Oxford University Press, New York, NY, USA.
- [55] McCulloch, Neil and Baluch, Bob (2000), "Simulating the Impact of policy upon chronic and Transitory Poverty in Rural Pakistan," *Journal of Development Studies*, 36(6), 100-130
- [56] McCulloch, Neil and Baluch, Bob (1999), "Distinguishing the Chronically from the Transitory Poor: Evidence from Rural Pakistan," Working Paper.
- [57] McCulloch, Neil and Baluch, Bob (1997), "Being Poor and Becoming Poor: Poverty Status and Poverty Transition in Rural Pakistan, I.D.S. Working Paper.
- [58] The World Bank MENA Development Report (2004), *Gender and Development in the Middle East and North Africa, Women in the Public Sphere*, The World Bank, Washington DC, USA.
- [59] Mohemkar-Kheirandish, Reza (2001), "Gains and Losses from Household Formation," Working Paper, Virginia Tech.
- [60] Mousavi, Shabnam and Mohemkar-Kheirandish, Reza (1997), "*Role of Rural Women in Development in Korea, Indonesia, and Bangladesh: A Comparative Study*," Papers and Proceedings (in Persian), Conference on Rural Women and Development, Tehran, Iran.

- [61] Okrasa, Wlodzimierz (1999), "Who Avoids and Who Escapes from Poverty During the Transition: Evidence from Polish Panel 1993-96", Working Paper, The World Bank.
- [62] Rendtel, Ulrich, Langeheine, Rolf, and Berntsen, Roland (1998), "The Estimation of Poverty Dynamics Using Different Measurements of Household Income." *Review of Income and Wealth*, 44(1), 81-98
- [63] Roth, Alvin E. and Sotomayor, Marilda A. Oliveira (1990), *Two Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, Cambridge University Press, Cambridge, UK.
- [64] Salehi-Isfahani, Djavad (1999), "Labor and the challenge of economic restructuring in Iran," *Middle East Report*, 29(210), 34-37.
- [65] Salehi-Isfahani, Djavad and Mohemkar-Kheirandish, Reza (2002), "Dynamics of poverty in Iran," A Report on Poverty Profile in Iran, Chapter 3, Prepared for The World Bank.
- [66] Salehi-Isfahani, Djavad (2008), "Poverty, inequality, and populist politics in Iran," *Journal of Economic Inequality*, Forthcoming.
- [67] Sen, C. T., "On Economic Inequality," Clarendon Press, Oxford.
- [68] Simon, Carl P., and Blume, Lawrence (1994), *Mathematics for Economists*, W. W. Norton & Company, Inc., New York, NY, USA.
- [69] Slesnick, Daniel T. (1993), "Gaining Ground: Poverty in the Postwar United States." *Journal of Political Economy*, 101(1), 1-38.
- [70] Shorocks, A. F., "The Measurement of Mobility," *Econometrica*, 46(5), 1013-1024.
- [71] Spanos, Aris (1986), *Statistical Foundations of Econometric Modelling*, Cambridge University Press, Cambridge, UK.
- [72] Spanos, Aris (1999), *Statistical Theory and Statistical Inference*, Cambridge University Press, Cambridge, UK.
- [73] Speder, Zsolt (1998), "Poverty Dynamics in Hungary during the Transformation." *Economics of Transition*, 6(1), 1-21.
- [74] Spence, Michael (1973), "Job Market Signaling," *Quarterly Journal of Economics*, 87(3), 355-374.

- [75] Spence, A. Michael (1974), *Market Signaling: Information Transfer in Hiring and Related Screening Processes*, Harvard University Press, Cambridge, MA, USA.
- [76] Spence, Michael (1976), "Competition in Salaries, Credentials and Signaling Prerequisites for Jobs," *Quarterly Journal of Economics*, 90(1), 51-74.
- [77] Spence, Michael (2002), "Signaling in Retrospect and the Informational Structure of Markets," *American Economic Journal*, 92(3), 434-459.
- [78] Spiegel, Murray R. and Liu, John (1999), *Mathematical Handbook of Formulas and Tables*, Schaum's Outline Series, Second Edition, McGraw-Hill, New York, NY, USA.
- [79] Stevens, Ann Huff (1995), "Climbing out of Poverty, falling Back in: Measuring the Persistence of Poverty over Multiple Spells," *NBER Working Paper Series*, Working Paper 5390.
- [80] Tabibian et al (1998). "A comparative study of poverty in Iranian households, 1996-1997," Working Paper, Institute for Research in Planning and Development, Tehran, Iran (in Persian).
- [81] Varian, Hal R. (1992), *Microeconomic Analysis*, W. W. Norton & Company, Third edition, New York, NY, USA.
- [82] Wooders, Myrna H. (1988), "Stability of Jurisdiction Structures in Economies with Local Public Goods," *Mathematical Social Sciences*, 15, 29-49.
- [83] Wooders, Myrna H. (1989), "A Tiebout Theorem," *Mathematical Social Sciences*, 18, 33-55.
- [84] Wooders, Myrna H. (1997), "Equivalence of Lindahl Equilibrium with Participation Prices and the Core," *Economic Theory*, 9, 115-127.