

CHAPTER 5

GUIDEWAY TRAFFIC FLOW RELATIONSHIPS

5.1 Introduction

It is important to remember that the primary function of any transportation system is to provide a service that, from the engineer's point of view, must be measured in terms of such measures of effectiveness as safety, efficiency and impact on society and the environment. Socioeconomic impact was covered in Chapter 4. Environmental impact is beyond the scope of this research. Safety and efficiency are two sides of the same coin, involving the ability of a corridor, consisting of a guideway and freeway in this research, to accommodate vehicular traffic safely and efficiently. The basis for determining the functional effectiveness for such facilities is branch of traffic engineering called traffic flow analysis.

In undertaking such analysis, the various dimensions of traffic, such as quantity, type, speed and distribution over time, must be addressed in order that we may be able to: (1) give meaningful numerical value to the parameters and constants of appropriate traffic flow laws; (2) define and calculate the outputs of the transportation system; (3) develop system measure and indices which will permit comparisons between modes and network configurations; (4) influence geometric design; and (5) select operating and control alterations. Therefore, the provision of theoretically consistent quantitative techniques by which relevant aspects of vehicular traffic can be modeled forms the basis of traffic flow analysis. This chapter focuses on the presentation of some of the quantitative elements that are believed to be fundamental to AHS Maglev operational analysis.

5.2 The Macroscopic Approach

Traffic flow on a conventional roadway is a stochastic process, with random variations in vehicle and driver characteristics and their interactions. However, it is quite common to construct models of reality in which the effects of chance variations are ignored or averaged out, where any given input will produce an exactly predictable output. These models are deterministic.

The interaction between vehicles and their drivers, and also among vehicles, is a highly complex process. There are three main approaches to the understanding and quantification of traffic flow. The first is a macroscopic approach that looks at the flow in an aggregate sense. Based on such physical analogies as heat flow and fluid flow, the macroscopic approach is most appropriate for studying steady-state phenomena of flow and hence best describes the overall operational efficiency of the system. The second is a microscopic approach that considers the response of each individual vehicle in a disaggregate manner. Here the individual driver-vehicle combination is examined, such as car maneuvering. This approach is used extensively in highway safety work. The third approach is the human-factor approach. Basically, it seeks to define the mechanism by which an individual driver (and his or her vehicle) locates himself or herself with reference to other vehicles and to the highway/guidance system. Notice that the microscopic and the human factor approaches are closely related [69].

One way of combining all three approaches is to assume initially that a stream of traffic composed of identical vehicles and identical drivers, thus permitting easy integration of the various approaches. The simplest combination also assumes that the traffic moves at uniform speed and that the vehicle is dependent on speed. In other words, a vehicle's behavior is forced on it by other vehicles in traffic stream. Indeed, speed is assumed to be the only variable that influence flow. Naturally, there is one particular vehicle flow associated with a speed adopted by the traffic stream.

5.3 Platooning and Safe Following Distance

Consider the two-lane guideway shown in Figure 5.1. Both lanes contain two successive platoons of N vehicles. In the “through lane”, all vehicles are through traffic whereas only $(N - n)$ of the vehicles in the “weaving lane” are through traffic. In the weaving lane, n vehicles in each platoon are vehicles that have just entered the guideway or are about to exit the guideway. Assume that vehicles in a platoon are magnetically coupled so as to maintain headways of S feet. The platoons are moving at the cruise speed of v_0 ft/sec along the guideway at platoon headways of S_p feet (Fig 5.1a). Fig 5.1b shows the limiting acceptable conditions at the end of the stopping maneuver for the following platoon. The minimum platoon space headway S_p consists of three components: (1) the braking distance for the leading platoon, D_{i-1} ; (2) the stopping distance for the following platoon, D_i ; and (3) the vehicle space headway, S . Mathematically, as seen in Fig 5.1,

$$S_p = D_i - D_{i-1} + NS \quad (5.1)$$

The traffic concentration, k , for both lanes of the guideway is the number of vehicles ($2N$) divided by the distance S_p . Adjusting the units to give k in vehicles per mile, we have

$$k = \frac{5280(2N)}{S_p} \quad (5.2)$$

Multiplying both sides of Equation 5.2 by the cruise speed, V_0 (miles per hour), gives the guideway traffic volume, q (vehicles/hour):

$$q = \frac{5280(2N)V_0}{S_p} \quad (5.3)$$

The evaluation of Equation 5.3 depends on finding D_{i-1} and D_i based on the kinematics of braking and stopping shown in Fig 5.2. It follows that

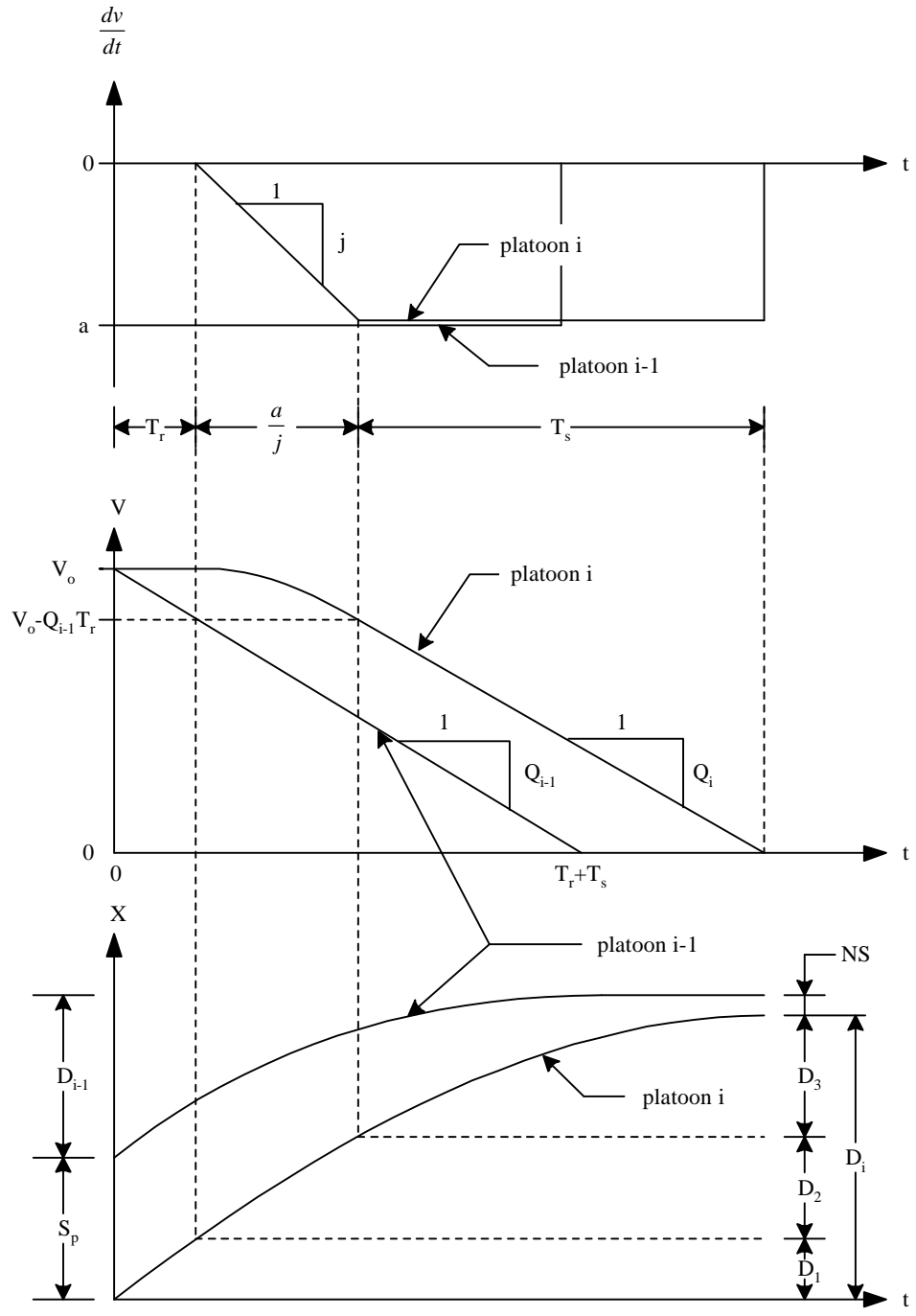


Figure 5.2 Dynamics of Platoon Flow

$$D_{i-1} = v_0(T_r + T_s) - a_{i-1}(T_r + T_s)^2 / 2 \quad (5.4)$$

$$D_i = v_0 T_r + v_0(a_i / j) - a_i(a_i / j)^2 / 6 + (v_0 - a_i T_r)T_s - a_i T_s^2 / 2 \quad (5.5)$$

where T_r is the technology response time (sec), T_s is the time that it takes a vehicle to decelerate from a speed of $v_0 - a_i T_r$ to zero, a_{i-1} and a_i are the decelerations of the $i-1$ and i platoons (ft/sec²), and j is jerk (ft/sec³)

The choice of specific values for the above variables has important implications with respect to the level of safety provided by the system's operation. Two braking or deceleration rates will be considered for the proposed AHS Maglev guideway operation: a_e , the emergency braking rate, assuming 32 ft/sec²; and a_s , the infinite braking rate (∞ ft/sec²) resulting in an instantaneous or so-called "stone-wall" stop, associated with a collision or sudden accident. Another decision variable is j which gives rise to nine possible combinations of a_{i-1} , a_i and j . Three important cases from the nine possible combinations will be described:

Case 1: $a_{i-1} = a_i = a_e$ and $0 < a_i/j \leq 1.0$

Case 2: $a_{i-1} = 0$, $a_i = a_e$ and $a_i/j = 0$

Case 3: $a_{i-1} = 0$, $a_i = a_e$ and $0 < a_i/j \leq 1.0$

The three cases are plotted in Fig 5.3 as the platoon space headway S_p (ft) is a function of the cruise speed V_0 (miles/hr), for number of vehicles in a platoon $N = 4$, intraplatoon headway $S = 15$ ft, a technology response time $T_r = 0.05$ sec and $a_i/j = 0.5$. The equations for the S_p , k and q relationships for the three cases are found by substituting the parameter values into Equations 5.6, 5.7 and 5.8 respectively:

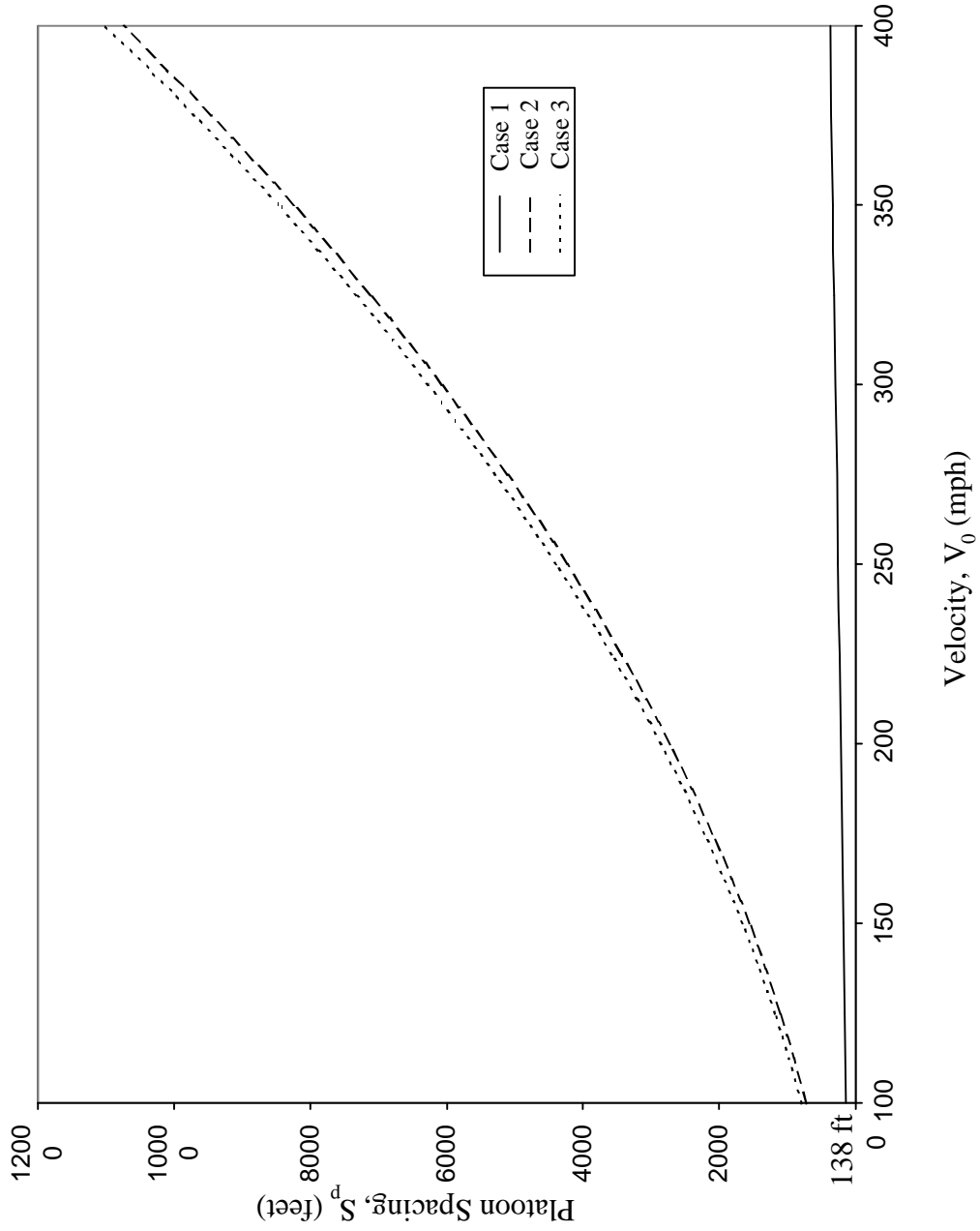


Figure 5.3 Relationship between Platoon spacing and velocity

Case 1:

$$S_p = 1.467V_0 \left(\frac{a_e}{j} \right) - \frac{a_e}{6} \left(\frac{a_e}{j} \right)^2 + \frac{a_e}{2} T_r^2 + NS \sim 0.783V_0 + NS \quad (5.6)$$

$$k = \frac{5280(2N)}{0.783V_0 + NS} \quad (5.7)$$

$$q = \frac{5280(2N)V_0}{0.783V_0 + NS} \quad (5.8)$$

Case 2:

$$S_p = \frac{v_0^2}{a_e} - v_0 T_r + a_e T_r^2 + NS \sim .067V_0^2 - 0.0734V_0 + NS \quad (5.9)$$

$$k = \frac{5280(2N)}{.067V_0^2 - 0.1467V_0 + NS} \quad (5.10)$$

$$q = \frac{5280(2N)V_0}{.067V_0^2 - 0.1467V_0 + NS} \quad (5.11)$$

Case 3:

$$S_p = v_0 \left(\frac{a_e}{j} \right) - \frac{a_e}{6} \left(\frac{a_e}{j} \right)^2 + \frac{v_0^2}{a_e} - v_0 T_r + a_e T_r^2 + NS$$

$$\sim .067V_0^2 + 0.587V_0 + NS \quad (5.12)$$

$$k = \frac{5280(2N)}{.067V_0^2 - 0.587V_0 + NS} \quad (5.13)$$

$$q = \frac{5280(2N)V_0}{.067V_0^2 - 0.587V_0 + NS} \quad (5.14)$$

For each case, the platoon size $N \leq S_p/2S$. It is interesting to note that if $N = S_p/2S$ for each case, then the densities for all three cases approach Equation 5.2

$$k = \frac{5280(S_p / S)}{S_p} = \frac{5280}{S} \quad (5.15)$$

Similarly, the volume for all three cases approach Equation 5.3

$$q = \frac{5280(S_p / S)V_0}{S_p} = \frac{5280V_0}{S} \quad (5.16)$$

The expression for concentration k and volume q are for a two-lane guideway. While the lane values are obviously half of these values, it is important to determine the through traffic values, k_T and q_T , and the weaving traffic value, k_W and q_W remembering that the “weaving lane” actually contains some through traffic. These expressions can be shown to be:

$$k_T = \frac{5280(2N - n)}{S_p} \quad (5.17)$$

$$q_T = \frac{5280(2N - n)V_0}{S_p} \quad (5.18)$$

$$k_W = \frac{5280n}{S_p} \quad (5.19)$$

$$q_W = \frac{5280nV_0}{S_p} \quad (5.20)$$

It is evident that the possible capacity of the guideway is independent of the case, or safety regime, chosen. The actual capacity realized, or the practical capacity, depends on the size of N in the equality

$$N \leq S_p / 2S \quad (5.21)$$

If the maximum platoon size, $N_{max} = S_p / 2S$, and N is the average platoon size, then the expression for practical capacities Q_T , Q_w and Q become:

$$Q_T = \mathbf{a} \left[5280(2 - \mathbf{b}) NV_{0,max} / S_p \right] \quad (5.22)$$

$$Q_w = \mathbf{a} \left[5280 \mathbf{b} NV_{0,max} / S_p \right] \quad (5.23)$$

$$Q = Q_T + Q_w \quad (5.24)$$

where $\mathbf{a} = N / N_{max}$ and $\mathbf{b} = n / N$. As $V_{0,max}$, the maximum operating speed on the guideway increases, the ability to form platoons will decrease, so $NV_{0,max} / S_p$ may be expected to remain fairly constant. In Fig 5.4, the practical capacities are plotted using $NV_{0,max} / S_p = 50$ for various values of \mathbf{a} and \mathbf{b} .

5.4 Network Assignment

In order to maintain the uninterrupted traffic stream, a vehicle has to be restricted to a minimum trip length on the guideway. Consider the possibilities of restricting the minimum trip length a links, where $a = 3, 4, 5,$ and 6 . Assume that interchanges along the AHS are equally spaced and $2Q_r$ vehicles trip per hour are generated at each direction – Q_r in each direction. In case $a = 3$ (Figure 5.5), a vehicle traveling from node 0 to node 1 has only one option which is to use freeway. A vehicles traveling from 0 to 2 has 2 options: traveling on freeway link 0-1 and 1-2, or traveling on guideway from 0-3 and take freeway back from 3-2. In such case, freeway link 3-2 would have had to bear freeway traffic from the opposite direction q_{31}^f and q_{32}^f , and guideway traffic q_{02}^g . Assuming guideway traffic volume is considerably large compared with that of freeway (which will be proved in the following paragraphs), this option is rather impossible due to relatively low capacity of the freeway. It is also true for all other values of a .

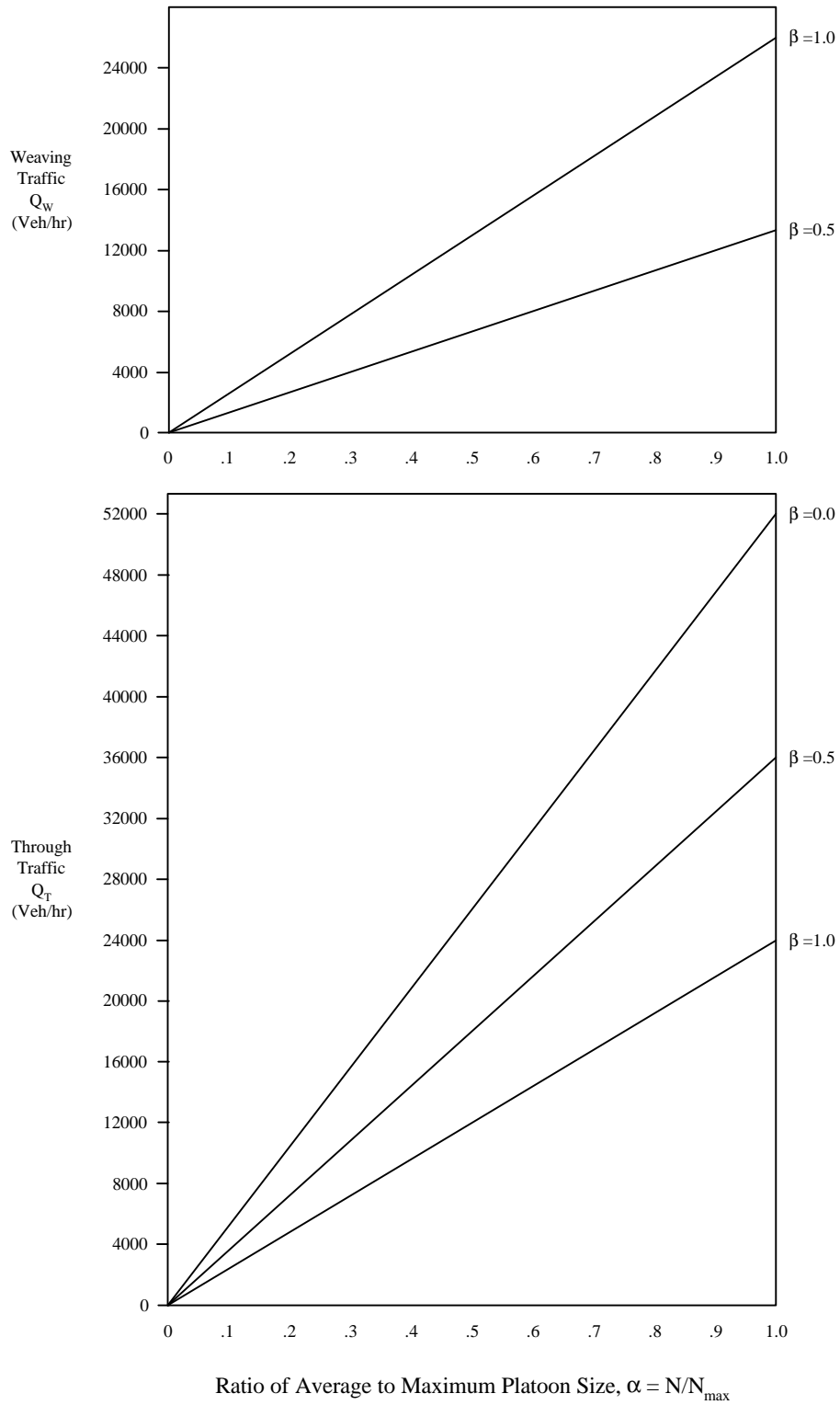


Figure 5.4 Guideway Practical Capacities

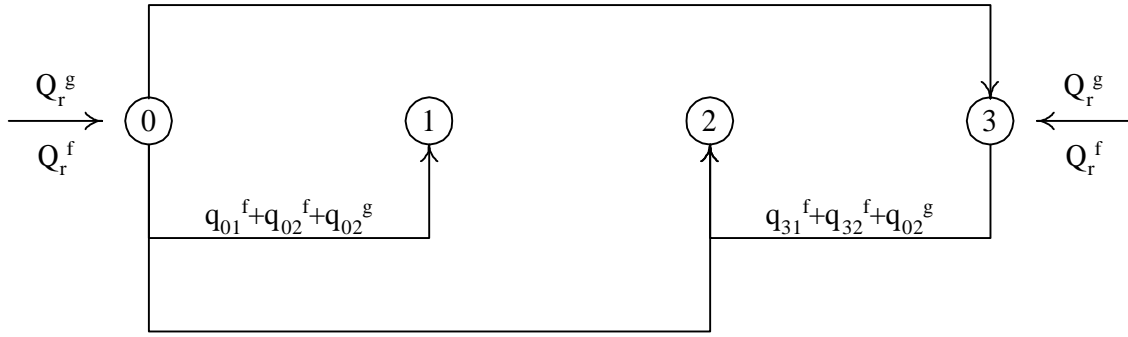


Figure 5.5 Maglev traffic network (a = 3)

It is assumed that traffic volume on freeway is much lower than that on the guideway. In this traffic assignment study, we consider only comparison between traffic traveling from beginning nodes to the end node.

Assuming travel time is a function of traffic volume, capacity, and traffic impedance. The travel time for a vehicle can be computed from

$$T = T_f \left(\frac{1 - (1 - j)(q/Q)}{1 - (q/Q)} \right) \quad (5.25)$$

where T_f is free flow travel time, q is traffic volume, Q is capacity, and j is traffic impedance.

Regulating the minimum number of links traveled on the guideway will evidently effect traffic impedance. Longer trips keep traffic flow smoothly while shorter trips cause frequent traffic turbulence due to speed changing while entering or exiting the guideway. Thus, parameter j changes inversely with a . In this study, we assume j linearly ranges from 0.2 for $a = 3$ to 0.1 for $a = 6$, and from 0.4 for $a = 3$ to 0.2 for $a = 6$ for trucks, as shown in Figure 5.6.

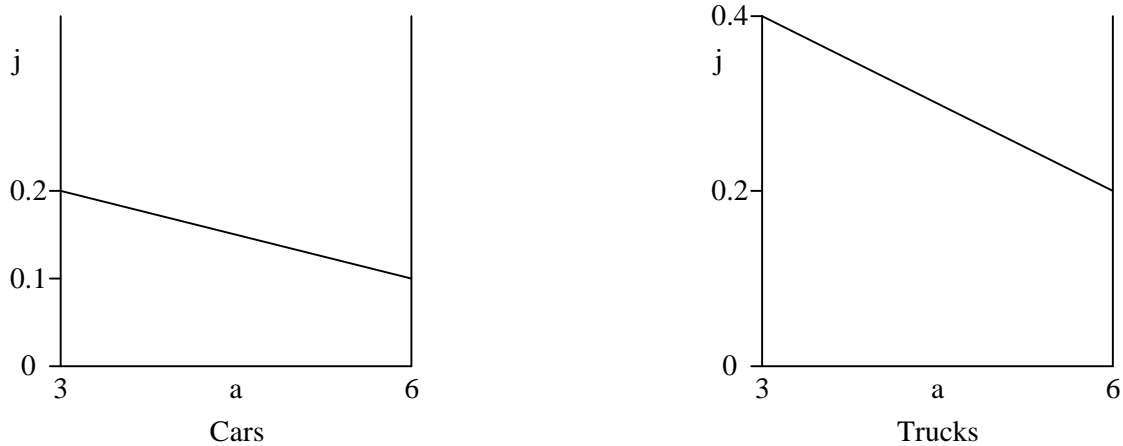


Figure 5.6 Traffic Impedance

Consider user equilibrium condition. Travel time on both guideway and freeway must be equal. Hence, the condition yields

$$T^g = T^f$$

$$T_f^g \left(\frac{1 - (1 - j^g)(q^g / Q^g)}{1 - (q^g / Q^g)} \right) = T_f^f \left(\frac{1 - (1 - j^f)(q^f / Q^f)}{1 - (q^f / Q^f)} \right) \quad (5.26)$$

where all notations are the same as those in equation 5.25, and superscripts g and f denote values for guideway and freeway respectively.

Assuming free flow travel time on a guideway link is 1.2 minutes for car, and 1.8 minute for trucks. Free flow travel time on a freeway link is 2 minutes for both cars and trucks.

Note that $q^f = q - q^g$ where q is total demand. To solve for q^g , Equation 5.26 is simplified to fit in form of quadratic equation $Aq^2 + Bq + C = 0$ where

$$A = -(1 - j^g) + \frac{T_f^f}{T_f^g} (1 - j^f) \quad (5.27)$$

$$B = Q^g + (1 - j^g)(q - Q^f) + \frac{T^f}{T^g}(Q^f - q(1 - j^f) - Q^g(1 - j^f)) \quad (5.28)$$

and

$$C = Q^g \cdot \left(Q^f \left(1 - \frac{T^f}{T^g} \right) - q \cdot \frac{T^f}{T^g} \cdot j^f \right) \quad (5.29)$$

A linear regression is utilized to search for more simple relationship between total traffic volume q and q^g . Thus, we obtain more simplified form of

$$q = a' + b'q^g \quad (5.30)$$

where a' and b' are constants over a particular traffic facility and minimum number of links that a vehicle has to travel, a .

A sample of calculation for cars, $a = 3$ is shown in Table 5.1. The complete results are shown in Table 5.2. Figure 5.7 and 5.8 illustrate the relationships between total volume q and volume on the guideway q^g with different values of a .

5.5 Car Maneuvering

Drew [69] emulated fluid-flow analogy to express car following behavior in terms of concentrations (or traffic densities) and concentration changes. The fluid equation of motion in cooperation with traffic equation of state (q-u-k relationship) was proved to have a mathematical bridge to the GM's final model as will be shown in the following paragraphs.

Returning to Section 2.6.1, treating the car following model of Equation 2.1 as a differential equation, steady state solutions can be obtained for various values of m and n . For example, if $n = 0$ and $m > 1$

$$a = (m-1)v_f k_j^{-(m-1)} \quad (5.31)$$

Table 5.1 Network Assignment: Sample of Calculation (Cars, a =3)

a =	3						
$T_{c_f}^g =$	3.6	min	$j_c^g =$	0.20	$Q_c^g =$	30000	veh/hr
$T_{c_f}^f =$	6.0	min	$j_c^f =$	0.25	$Q_c^f =$	2400	veh/hr
$T_{t_f}^g =$	5.4	min	$j_t^g =$	0.40	$Q_t^g =$	12000	veh/hr
$T_{t_f}^f =$	6.0	min	$j_t^f =$	0.50	$Q_t^f =$	1200	veh/hr

Cars

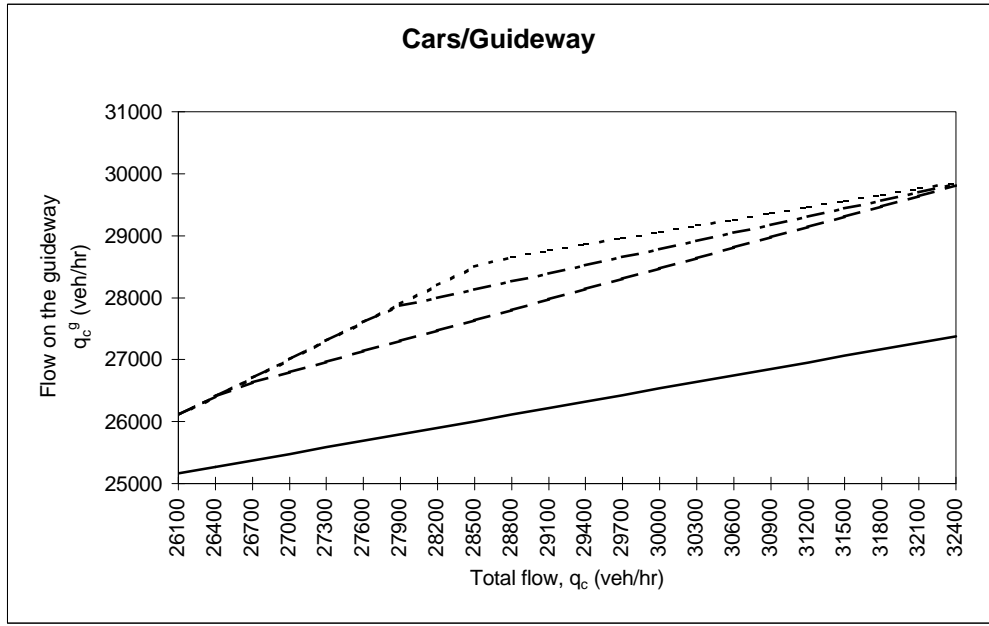
T =	1.667
$k^g =$	0.80
$k^f =$	0.75

q	A	B	C	q^g	q^f	qq^g	qq^f	q^2
0	0.450	-5420	-48000000	17978	-17978	0	0	0
2314	0.450	-6461	-30642857	18117	-15803	41928619.2	-36572701	5355918.37
4629	0.450	-7503	-13285714	18287	-13659	84644751.4	-63221078	21423673.5
6943	0.450	-8544	4071428.57	18498	-11555	128430314	-80227048	48203265.3
9257	0.450	-9586	21428571.4	18764	-9507	173698884	-88004190	85694693.9
11571	0.450	-10627	38785714.3	19104	-7533	221064018	-87166058	133897959
13886	0.450	-11669	56142857.1	19548	-5662	271434287	-78621226	192813061
16200	0.450	-12710	73500000	20131	-3931	326120066	-63680066	262440000
18514	0.450	-13751	90857142.9	20897	-2382	386887118	-44108342	342778776
20829	0.450	-14793	108214286	21885	-1056	455826666	-21997278	433829388
23143	0.450	-15834	125571429	23115	28	534954755	637082.001	535591837
25457	0.450	-16876	142928571	24579	878	625722526	22343596	648066122
27771	0.450	-17917	160285714	26243	1528	728810927	42441317.9	771252245
30086	0.450	-18959	177642857	28063	2022	844305655	60844549.2	905150204
32400	0.450	-20000	195000000	30000	2400	972000000	77760000	1049760000
243000				325210	-82210	5795828586	-359571443	5436257143

	guideway	freeway
a =	15983	-15983
b =	0.352	0.648

Table 5.2 Results of regression analysis for relationship between q and q^g

	CARS				TRUCKS					
		a = 3	a = 4	a = 5	a = 6		a = 3	a = 4	a = 5	a = 6
Guideway	a_c^g	15983	11745	15700	19063	a_t^g	289	1755	2690	3555
	b_c^g	0.352	0.558	0.436	0.333	b_t^g	0.889	0.776	0.708	0.645
Freeway	a_c^f	-15983	-11745	-15700	-19063	a_t^f	-289	-1755	-2690	-3555
	b_c^f	0.648	0.442	0.564	0.667	b_t^f	0.111	0.224	0.292	0.355



— $a = 3$ - - - $a = 4$ - · - · - $a = 5$ ····· $a = 6$

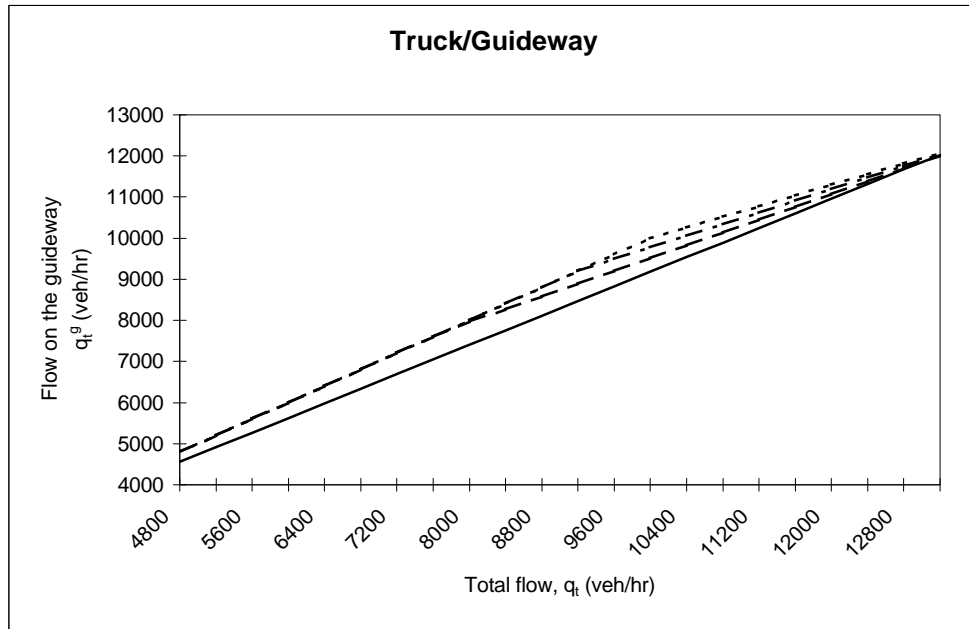
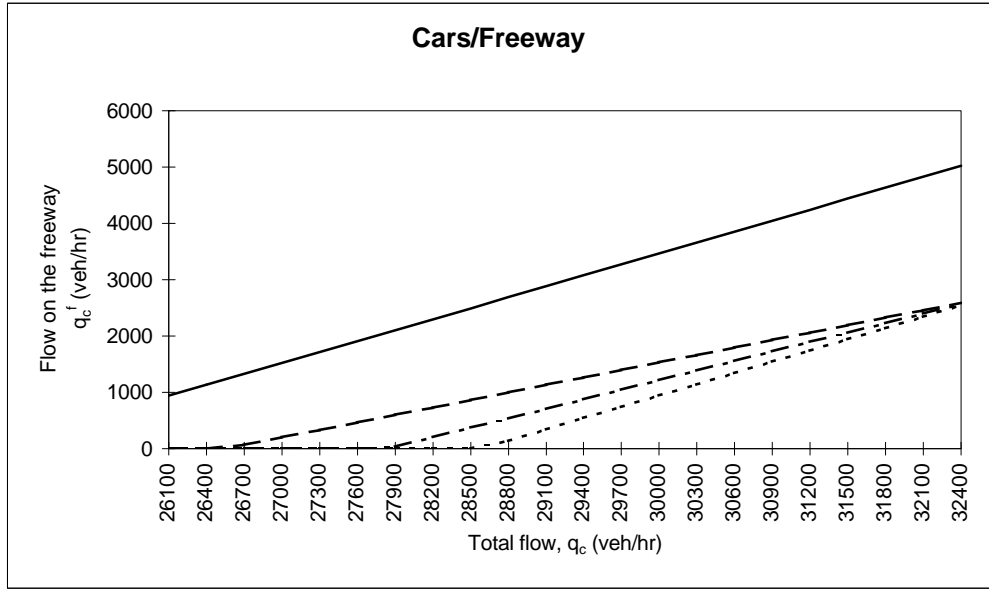


Figure 5.7 Relationship between total traffic volume and guideway volume



— $a = 3$ - - - $a = 4$ - · - · - $a = 5$ ····· $a = 6$

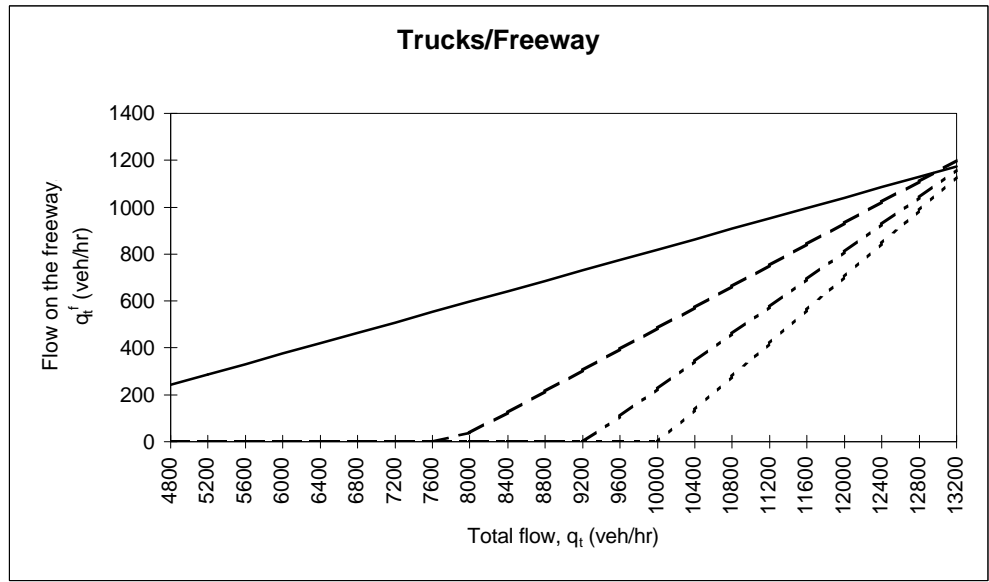


Figure 5.8 Relationship between total traffic volume and freeway volume

and

$$q = kv_f \left[1 - \left(\frac{k}{k_j} \right)^{m-1} \right] \quad (5.32)$$

where v_f is the free speed and k_j is the jam concentration. The macroscopic equivalent to Equation 2.1 with $n = 0$ and $m > 1$ is given by the differential equation of motion

$$\frac{dv}{dt} = -c^2 k^n \frac{\partial k}{\partial x} \quad (5.33)$$

When (5.33) is solved simultaneously with the equation of continuity of the traffic stream

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (5.34)$$

the equation of state obtained is

$$q = kv_f \left[1 - \left(\frac{k}{k_j} \right)^{(n+1)/2} \right], \quad n > -1 \quad (5.35)$$

where c in (5.33) is

$$c = \frac{(n-1)v_f}{2k_j^{(n+1)/2}}, \quad n > -1 \quad (5.36)$$

The relationship between m in (2.1) and n in (5.33) is seen to be

$$n = 2m - 3 \quad (5.37)$$

The similarities in the two approaches, the microscopic or car-following approach of (2.1) and the macroscopic or one-dimensional compressible fluid analogy of (5.33), provide the basis for developing a more realistic car-following model. The classical car-following models represented by (2.1) are highly simplified description of the responses to the world of stimuli that confronts a driver under manual control, or the computer under automatic control. Indeed the only stimulus is the relative speed between pair-wise

vehicles; the other terms on the right-hand side of (2.1) are sensitivity factors. Considerable more realism can be achieved by including several vehicles ahead, instead of just one, as well as the vehicle behind, the $i + 1$ vehicle.

It is evident that the verbal statements of the equations of motion in both the car-following model (Equation 2.1 with $n = 0$) and the fluid flow formulation (Equation 5.33) apply equally well to the following model:

$$\ddot{x} = -c^2 k_{i-j,i+p}(t-T)(\partial k / \partial x)_{i-j,i+p}(t-T) \quad (5.38)$$

where the subscripts $i-j$ and $i+p$ refers to the traffic between the $i-j$ vehicle and the $i+p$ vehicle. If $j = 1$ and $p = 0$ we have the usual car-following formulation. The geometric signification of (5.38) is given in Fig 5.9 for $j = 2$ and $p = 1$.

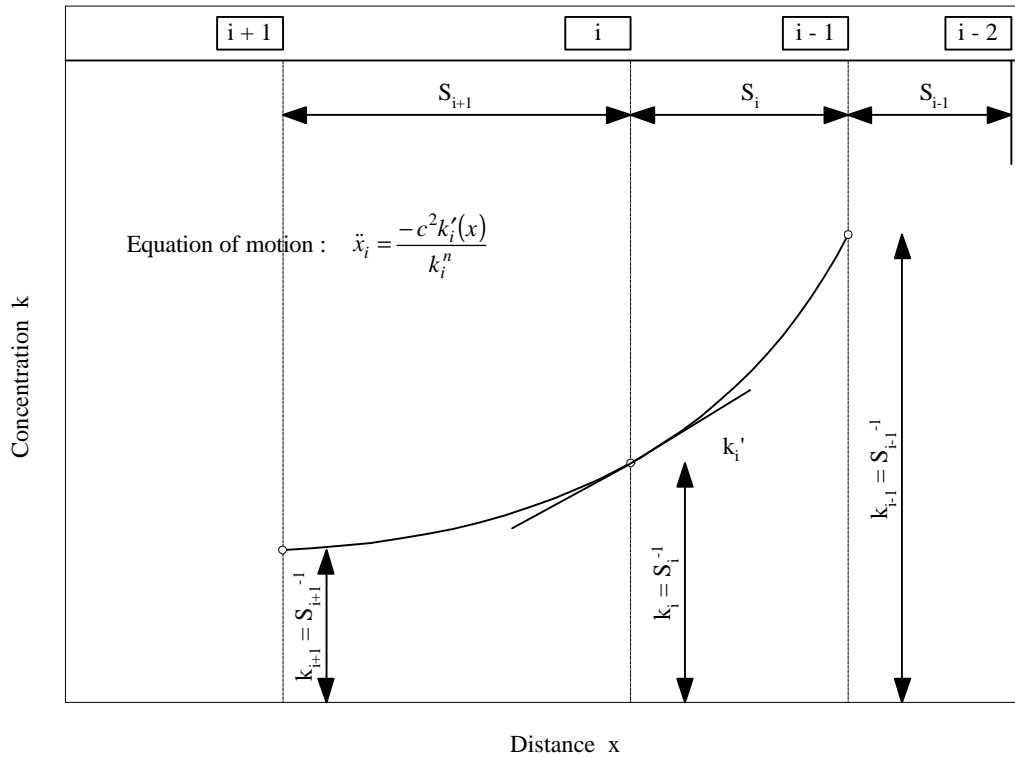


Figure 5.9 Determination of Variables for Car Maneuvering Theory

Referring to Fig 5.9, it is evident that before the slope $(\partial x/\partial k)_i$ of the $k = f(x)$ curve can be determined, the function $k = f(x)$ must be known. This can be resolved by fitting a polynomial to the points $(x_{i-1}, k_{i-1}; x_i, k_i; x_{i+1}, k_{i+1})$ and then evaluating the derivation at x_i, k_i .

Despite its capability to simulate traffic behavior to some extent, these models fail to explain driver reactions where traffic asymptotic instability exists, i.e. high reaction time, $\mathbf{D}t$, and high sensitivity response, \mathbf{a} [30, 70]. Addison and Low [71-73] suggested a spacing dependent term to smooth out the oscillations of traffic stream, based on the fact that drivers will attempt to achieve the desired spacing even though the difference in speed is zero. Thus, equation (5.30) or (5.33) can be extended to

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_{l,m} [\dot{x}_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] + \beta [x_n(t) - x_{n+1}(t) - \delta]^n \quad (5.39)$$

where b and n are constants, and \mathbf{d} refers to a desired spacing for the following vehicle.

Moving towards vehicle automation, Minderhoud et al. [74] proposed a simplified form of equation (5.39) and revealed a linear relationship between desired spacing and speed, and other constant parameters as follows:

$$\ddot{x}_{n+1}(t + \Delta t) = \alpha [v_n(t) - v_{n+1}(t)] + \beta [x_n(t) - x_{n+1}(t) - \delta] \quad (5.40)$$

and

$$\delta = L + b + t_s v_{n+1}(t) + \dots \quad (5.41)$$

where \mathbf{d} is the desired spacing which can be expressed in a linear or quadratic function of vehicle length, L , safety margin, b , setting time headway, t_s and speed of the following vehicle, v_{n+1} . The proposed longitudinal control scheme was experimented in incorporation of Autonomous Intelligent Cruise Control (AICC) and manual control mode. It was concluded that under a speed range of 90-110 km/hr, the simplified version of car-following control could well be applied to automatic control without resulting in collision.

Several studies pointed out that there should not be a global rule that governs vehicle behaviors [75-77]. On the contrary, vehicles will react under different controls due to several thresholds of headways, speeds, and accelerations realized. Fancher et al. [78] suggested switching between conventional cruise control (i.e. velocity control), and headway control modes, based on headways and headway changes. Drew et al. [79] demonstrated three thresholds for controlling maglev vehicle longitudinal headway according to their spacings as expressed by:

$$\ddot{x}_i(t) = \begin{cases} -c^2 k_{i-j,i+p}(t-T)(\partial k / \partial x)_{i-j,i+p}(t-T), & x_{i-1} - x_i \geq S_p \\ b[x_{i-1}(t-T) - x_i(t-T) - S_c]^l, & S_p \geq x_{i-1} - x_i \geq S_c \\ XAP(t) - XAD(t), & x_{i-1} - x_i \leq S_c \end{cases} \quad (5.42)$$

where the subscripts $i-j$, $i+p$ refers to the traffic between the $i-j$ vehicle and the $i+p$ vehicle, x is a position, k is density, T is the technology response time, b , c and l are constants, S_p is interplatoon spacing, and S_c is intraplatoon spacing.

5.6 Platooning

The feasibility of platooning in the operation of AHS was demonstrated in August 1997 on I-15 in San Diego. In this ISTEA sanctioned demonstration project, the University of California Program for Advanced Technology on the Highway (PATH) presented their “Platooning Scenario”. Eight automated cars were platooned with headways under 35 feet and were moved in a single-file guided by magnets embedded in the roadway. In addition to being able to accelerate, decelerate and stop as a platoon, the ability to split the platoon to allow for the entry of vehicles, and then to recreate a new platoon was demonstrated.

It is instructive to review how platoons form naturally on highways as a prelude to developing a strategy for forming platoons on an AHS Maglev Guideway. The two causal factors influencing platooning on conventional highways are traffic density and relative speed. Each will be considered.

As traffic densities and volumes increase, vehicles tend to form platoons or moving queues. The criteria for determining when two moving vehicles are platooned is arbitrary. For the purposes of this criterion, it will be assumed that two vehicles are platooned if they are close enough to be under each other's magnetic influence, a headway of S_c . The mathematical description of this aspect of platooning depends on performing a Bernoulli test with probability p and $1-p$ on each highway x so that

$$p = p(x < S_c) = \int_{S_c}^{\infty} f(x; k', a) dx \quad (5.43)$$

and

$$1 - p = p(x > S_c) = \int_0^{S_c} f(x; k', a) dx \quad (5.44)$$

where $f(x; k', a)$ is the natural distribution of headways in the traffic stream. The equation of the steady state probabilities is seen to be

$$p_n = p^{n-1} (1 - p) \quad n = 1, 2, \dots \quad (5.45)$$

with mean

$$N = (1 - p)^{-1} \quad (5.46)$$

If the two parameter distributions are Erlang so that

$$f(x; k', a) = \frac{(k'a)^a}{(a-1)!} x^{a-1} e^{-ak'x}; \quad (5.47)$$

then it follows that

$$N_{a=1} = e^{k'S_c} \quad (5.48)$$

$$N_{a=2} = e^{2k'S_c} / (2k'S_c + 1) \quad (5.49)$$

and

$$N_{a=3} = e^{3k'S_c} / [4.5(k'S_c)^2 + 3k'S_c + 1] \quad (5.50)$$

where

$$k' = k_T / 5280 \quad (5.51)$$

The critical headway S_c depends on the strength of the longitudinal magnetic force and as S_c increases the platoon forming capability of the guideway increases. The relationship between average platoon size N and S_c illustrated for the Erlang distribution ($a = 1$). Substituting (5.51) into (5.48), taking $N = S_p / 2S$ and assuming $\mathbf{b} = 1.0$ gives

$$N_{a=1} = e^{S_c/(2S)} \quad , \quad S_c / S \geq 1.0 \quad (5.52)$$

The relationship between N and S_c / S is plotted for $a = 1, 2, 3$ in Fig 5.10.

5.7 Merging and Weaving

Drew [69] suggested that the most desirable type of gap merge should be both optional and ideal, where the merging vehicle would be able to merge before running out of acceleration lane while not causing turbulence in the freeway stream. In order to achieve such a condition, a gap between two freeway vehicles has to be sufficiently large. Such a gap consists of three time components: (1) a safe time headway between the leading freeway vehicle and the merging vehicle T_r , (2) a time lost during the acceleration of the merging vehicle T_l , and (3) a safe time headway between the merging vehicle and the following freeway vehicle T_f .

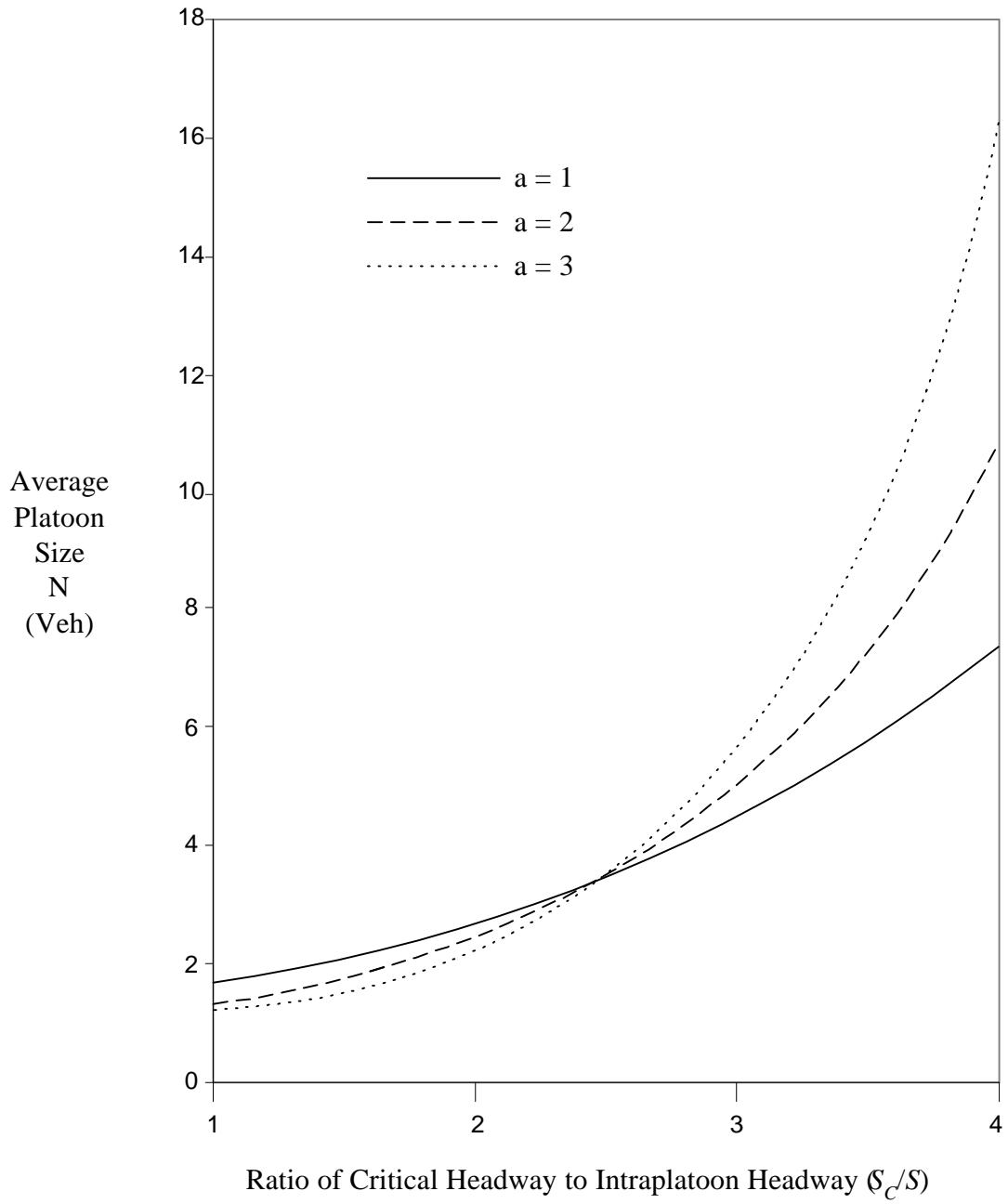


Figure 5.10 Relationship of Platoon Size to Longitudinal Control

5.7.1 Ideal Gap

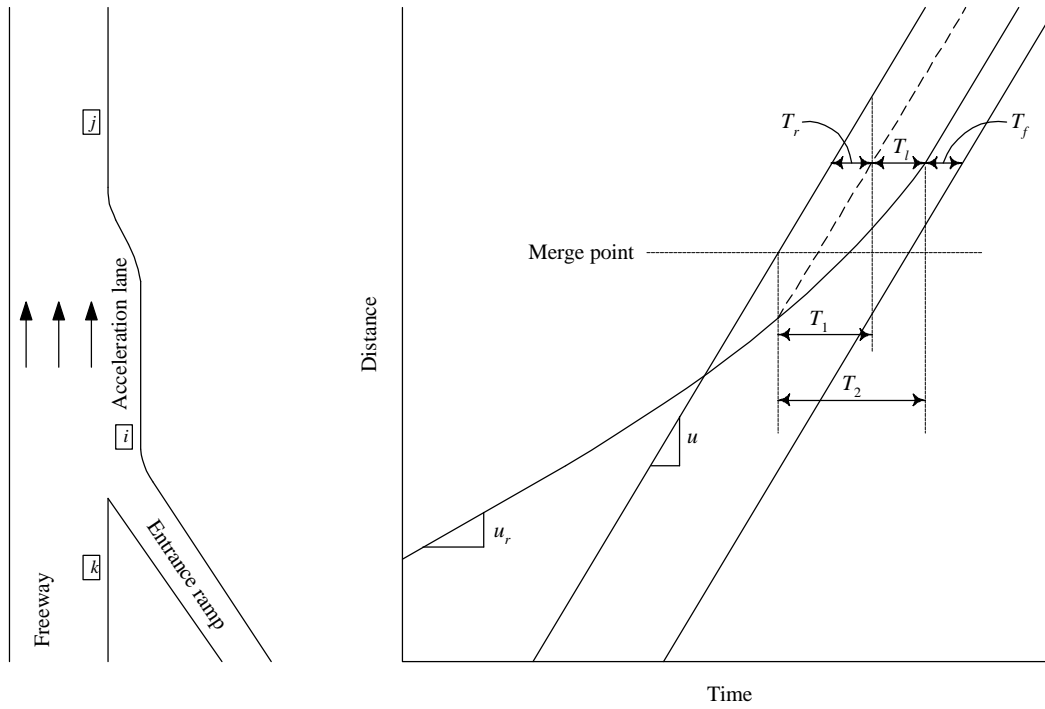


Figure 5.11 Time-space diagram of freeway merging maneuver

According to Figure 5.11, freeway vehicles travel at an identical speed u , and the merging vehicle travels at speed u_r . If we assume reaction time t , braking abilities and speed u are equal for all vehicles, the safe headway is $(L/u) + t$, where L is the length of vehicle in front. Assuming that the acceleration of ramp vehicle is inversely proportional of its speed:

$$\frac{du}{dt} = a - bu \tag{5.53}$$

Time needed for the ramp vehicle to accelerate from u_r to u , T_2 is solved to be:

$$T_2 = -\frac{1}{b} \ln \frac{a - bu}{a - bu_r} \tag{5.54}$$

Distance x_2 that is covered by the ramp vehicle during time T_2 is

$$x_2 = \frac{a}{b}t - \frac{a}{b^2}(1 - e^{-bt}) + \frac{u_r}{b}(1 - e^{-bt}) \quad (5.55)$$

Meanwhile time required for a freeway vehicle to cover the same distance is

$$T_1 = \frac{x_2}{u} = \frac{a}{bu}t - \frac{a}{b^2u}(1 - e^{-bt}) + \frac{u_r}{bu}(1 - e^{-bt}) \quad (5.56)$$

Thus, time gap required for an ideal and optional merge is

$$T = T_r + T_l + T_f = T_r + (T_2 - T_1) + T_f$$

$$T = \frac{L_f + L_r}{u} + 2\tau + \frac{u + u_r}{bu} + \frac{(a/b) - u}{bu} \ln \frac{a - bu}{a - bu_r} \quad (5.57)$$

The ramp capacity Q_r is restricted by the weaving capacity on the guideway Q_w as expressed

$$Q_r = Q_w - q \quad (5.58)$$

where q is guideway traffic volume.

5.7.2 Delay-Oriented Approach

The ramp capacity can also be determined on the concept of gap acceptance. As a vehicle only accepts time gap t that is greater than the critical value T , it may have to wait and reject several gaps before accepting one. The probability that a vehicle rejects n gap before accepting the $n+1$ corresponds to a geometric distribution:

$$P(n) = p^n(1 - p) \quad n = 1, 2, 3, \dots \quad (5.59)$$

where p is the probability of a vehicle rejecting a given gap and $1-p$ is the probability that a vehicle accepting a given gap.

The average number of gaps rejected is found to be

$$E(n) = \sum_{n=0}^{\infty} nP(n) = (1-p) \sum_{n=0}^{\infty} np^n$$

$$= (1-p)(p)(1+2p+3p^2+4p^3+\dots)$$

Recall that

$$(1+p+p^2+p^3+\dots) = \frac{1}{1-p}$$

$$(1+2p+3p^2+4p^3+\dots) = \frac{d}{dt} \left(\frac{1}{1-p} \right) = \frac{1}{(1-p)^2}$$

Thus,

$$E(n) = \frac{p}{1-p} = \frac{P(t < T)}{P(t > T)} = \frac{\int_0^T f(t) dt}{\int_T^{\infty} f(t) dt} \quad (5.60)$$

where $f(t)$ is a gap size distribution function.

The average time that a vehicle spend on waiting in one gap, D can be determined from total time spent in gaps smaller than T divided by total number of gaps smaller than T , or

$$D = \frac{q \int_0^T t f(t) dt}{q \int_0^T f(t) dt} \quad (5.61)$$

The delay or service time per ramp vehicle is

$$\mathbf{m} = E(n) \cdot D = \frac{\int_0^T t f(t) dt}{\int_T^{\infty} f(t) dt}$$

Taking $f(t)$ as a negative exponential distribution function,

$$f(t) = qe^{-qt}$$

the service time m becomes

$$m = \frac{q^{-1}(1 - qTe^{-qt} - e^{-qt})}{e^{-qt}}$$

$$m = \frac{e^{qt} - qT - 1}{q} \quad (5.62)$$

The ramp capacity is simply a reciprocal of m as

$$Q_r = \frac{q}{e^{qt} - qT - 1} \quad (5.63)$$

5.7.3 Multiple Merge Approach

A multiple merge occurs when more than one ramp vehicle accept the same gap. Consider an infinite ramp queue waiting to enter a freeway with equal headway between two successive vehicles, T' . If the guideway headway t is less than a minimum acceptable gap T , no ramp vehicle can merge; if t is between T and $T+T'$, one ramp vehicle can merge; if t is between $T+T'$ and $T+2T'$, two ramp vehicle can merge, and so forth. In general, when the guideway headway t is between $T+iT'$ and $T+(i+1)T'$, i vehicle can merge.

The ramp capacity Q_r can be determine from

$$Q_r = q \sum_{n=0}^{\infty} (n+1) \cdot P(T + nT' < t < T + (n+1)T')$$

Assuming that gap distribution conforms the negative exponential distribution, it follows that

$$\begin{aligned}
Q_r &= q \sum_{n=0}^{\infty} (n+1) \cdot \left(e^{-q(T+nT')} - e^{-q(T+(n+1)T')} \right) \\
Q_r &= q(0+1) \left(e^{-qT} - e^{-q(T+T')} \right) + q(1+1) \left(e^{-q(T+T')} - e^{-q(T+2T')} \right) \\
&\quad + q(2+1) \left(e^{-q(T+2T')} - e^{-q(T+3T')} \right) + \dots \\
&= q \left(e^{-qT} + e^{-q(T+T')} + e^{-q(T+2T')} + e^{-q(T+3T')} + \dots \right) \\
&= q e^{-qT} \left(1 + e^{-qT'} + e^{-2qT'} + e^{-3qT'} + \dots \right) \\
Q_r &= \frac{q e^{-qT}}{1 - e^{-qT'}} \tag{5.64}
\end{aligned}$$

5.7.4 Finite Capacity Queuing

In analyzing ramp queuing characteristics, one must first realize that there is a physical limitation to the number of vehicles that can be stored on the ramp. When the queue reaches a certain length, no vehicle will be allowed to enter on the ramp. This type of queue is referred to as finite capacity queuing.

The ramp queuing system can be taken as a first-come, first-serve, single server, random-arrival queuing system with finite queue capacity of M (M/M/1/M). Recognizing, infinite capacity queuing state, as long as queue length n is smaller than M , the probability of no vehicle on the ramp can be express using steady state solution of P_n ; viz.

$$P_n = \left(\frac{\mathbf{I}}{\mathbf{m}} \right)^n P_0 \tag{5.65}$$

and from a boundary condition $\sum_{n=0}^{\infty} P_n = 1$

$$1 = \sum_{n=0}^{\infty} \left(\frac{I}{m} \right)^n P_0$$

$$P_0 = \frac{1}{\sum_{n=0}^{\infty} \left(\frac{I}{m} \right)^n} = \frac{1}{\sum_{n=0}^{\infty} r^n}$$

$$P_0 = 1 - r \tag{5.66}$$

where r is the utilization factor, which equals to the ratio between arrival rate, I , and service rate, m . To obtain steady-state queuing, r must necessarily be less than 1.

From (5.66), the probability of n vehicles queuing on the ramp is

$$P_n = r^n P_0 = r^n (1 - r) \tag{5.67}$$

Since the ramp queuing system possesses infinite source or number of arrivals, equation (5.66) can be applied to the problem. However, when $n = M$, the equation (5.65) above is no longer valid. The state equation for ramp queuing problem can be derived from the boundary condition $\sum_{n=0}^M P_n = 1$. Then equation (5.66) can be rewritten as

$$P_0 = \frac{1}{\sum_{n=0}^M r^n} \tag{5.68}$$

The sum of the finite geometric series in (5.68) is obviously

$$\sum_{n=0}^M r^n = \frac{1 - r^{M+1}}{1 - r} \quad \text{where } r < 1$$

Thus we obtain,

$$P_0 = \frac{1 - r}{1 - r^{M+1}} \tag{5.69}$$

and
$$P_n = \frac{(1-r)r^n}{1-r^{M+1}} \quad n = 0, 1, \dots, M \quad (5.70)$$

Average number of vehicles on the ramp, L can be estimated from an expected value:

$$\begin{aligned} L &= \sum_{n=0}^M nP_n \\ &= \frac{(1-r)r}{1-r^{M+1}} \sum_{n=0}^M nr^{n-1} \\ &= \frac{(1-r)r}{1-r^{M+1}} \cdot \frac{1}{r} \left(\sum_{n=0}^M r^n \right) \\ &= \frac{(1-r)r}{1-r^{M+1}} \cdot \frac{1}{r} \left(\frac{1-r^{M+1}}{1-r} \right) \\ &= \frac{(1-r)r}{1-r^{M+1}} \cdot \frac{(r-1)(M+1)r^M + 1 - r^{M+1}}{(1-r)^2} \\ &= \frac{r[1 - (M+1)r^M + Mr^{M+1}]}{(1-r^{M+1})(1-r)} \end{aligned} \quad (5.71)$$

The expected number of vehicle queuing on the ramp is

$$L_q = L - (1 - P_0) = L - \frac{r(1-r)}{1-r^{M+1}} \quad (5.72)$$

The average waiting time on the ramp is intuitively obtained from $W = L/I_e$. Care should be taken as I_e in finite capacity queuing system only includes vehicles that actually enter the ramp. Given that the rate of arrival is I , then

$$I_e = I(1 - P_M) \quad (5.73)$$

where P_M is the probability that the ramp is fully loaded. Traffic volume needed to be diverted I_d is found from subtracting I_e out of λ or

$$I_d = IP_M \quad (5.74)$$

Consequently,

$$W = \frac{L}{I(1 - P_M)} \quad (5.75)$$

Similarly, the mean waiting time in the queue can be obtained from

$$W_q = \frac{L_q}{I(1 - P_M)} \quad (5.76)$$

5.7.5. State Dependent Queuing

A service rate may depend on the number of calling units waiting to be served. A ramp metering may give more green time when ramp queue is building up. The situation in which service rate depends on the number of calling unit in the queue is referred to as state-dependent service. On the contrary, when a calling unit or a freeway vehicle deciding to enter the guideway experiences a very long or overloaded queue, it may be diverted to another less congested ramp. This situation can be referred to as state-dependent arrivals.

In this study we assume that a vehicle is allowed to enter the ramp at a constant rate I_0 until the number of vehicles in the queue reaches j , then the arrival rate becomes a function of the queue length. The arrival rate can be mathematically expressed as

$$I_n = \begin{cases} I_0 & n \leq j-1 \\ \left(\frac{1}{n+1}\right)^b I_0 & n \geq j \end{cases} \quad (5.77)$$

We again assume that the service rate is constant until the queue length reaches j , then the service rate becomes a function of the queue length. The service rate can be mathematically expressed as

$$m_n = \begin{cases} m_0 & n \leq j-1 \\ n^a m_0 & n \geq j \end{cases} \quad (5.78)$$

where a and b in (5.77) and (5.78) are “pressure coefficients”.

Assuming finite capacity M , the probability of n vehicles queuing on the ramp is

$$P_n = \begin{cases} \left(\frac{\mathbf{I}_0}{\mathbf{m}_0} \right) P_0 & n \leq j-1 \\ \left(\frac{1}{(n+1)!n!} \right) P_0 & n \geq j \end{cases}$$

We evaluate P_0 from the boundary condition $\sum_{n=0}^M P_n = 1$, assuming $a = b = 1$.

$$1 = P_0 \left(\sum_{n=0}^{j-1} \left(\frac{\mathbf{I}_0}{\mathbf{m}_0} \right)^n + \sum_{n=j}^M \left(\frac{1}{(n+1)!n!} \right)^n \right)$$

$$P_0 = \left(\frac{1 + \mathbf{r}_0^j}{1 - \mathbf{r}_0} + \sum_{n=j}^M \left(\frac{1}{(n+1)!n!} \right)^n \right) \quad (5.79)$$

Other parameter including average queue length and average number of vehicles in the queue can be found from the same approaches as those of finite capacity queuing. However, these formulae usually vary upon the arrival and service rate functions.

5.8 Summary of Merging

Since the Maglev system is operated at very high speed, sudden changes in acceleration and velocity are not desirable. Merging and weaving maneuvers are necessarily conducted at ideal conditions, where such maneuvers do not cause traffic turbulence. Drew et al [79] establishes merging and weaving control schemes. Similar program is included in Appendix B.

Figure 5.12 illustrates a scenario where a set of vehicles is completing merging and weaving maneuvers. At initial condition, vehicle 1 and 2 are platooned and travel

along the outside lane; vehicle 3 is traveling on the inside lane; and vehicle 4 is a merging vehicle from the entrance ramp.

At the beginning phase, vehicle 4 accelerates from its initial velocity. It starts taking off at 150 mph and increases its speed close to velocity of the traffic stream before entering the guideway. Vehicle 4 enters the guideway right after the last vehicle, i.e. vehicle 3. After a short period, vehicle 4 enters the desired elevation. Sufficient longitudinal distance between platoons must be provided to allow possible lag of the merging vehicle.

From the graphs, one can notice that the maximum headway for vehicle 4 is less than 100 ft. Assuming the velocity on guideway traffic stream is 350 mph, the critical gap is 0.20 seconds. Taking a delay approach to estimate the ramp capacity:

$$Q_r = \frac{q}{e^{0.2q} - 0.2q - 1} \quad (5.80)$$

In case multiple merge approach is possible, ramp capacity can be estimated from

$$Q_r = \frac{qe^{-0.2q}}{1 - e^{-0.2q}} \quad (5.81)$$

The relationship between guideway flow and ramp capacity is shown in Figure 5.13 and 5.14 for the two approaches.

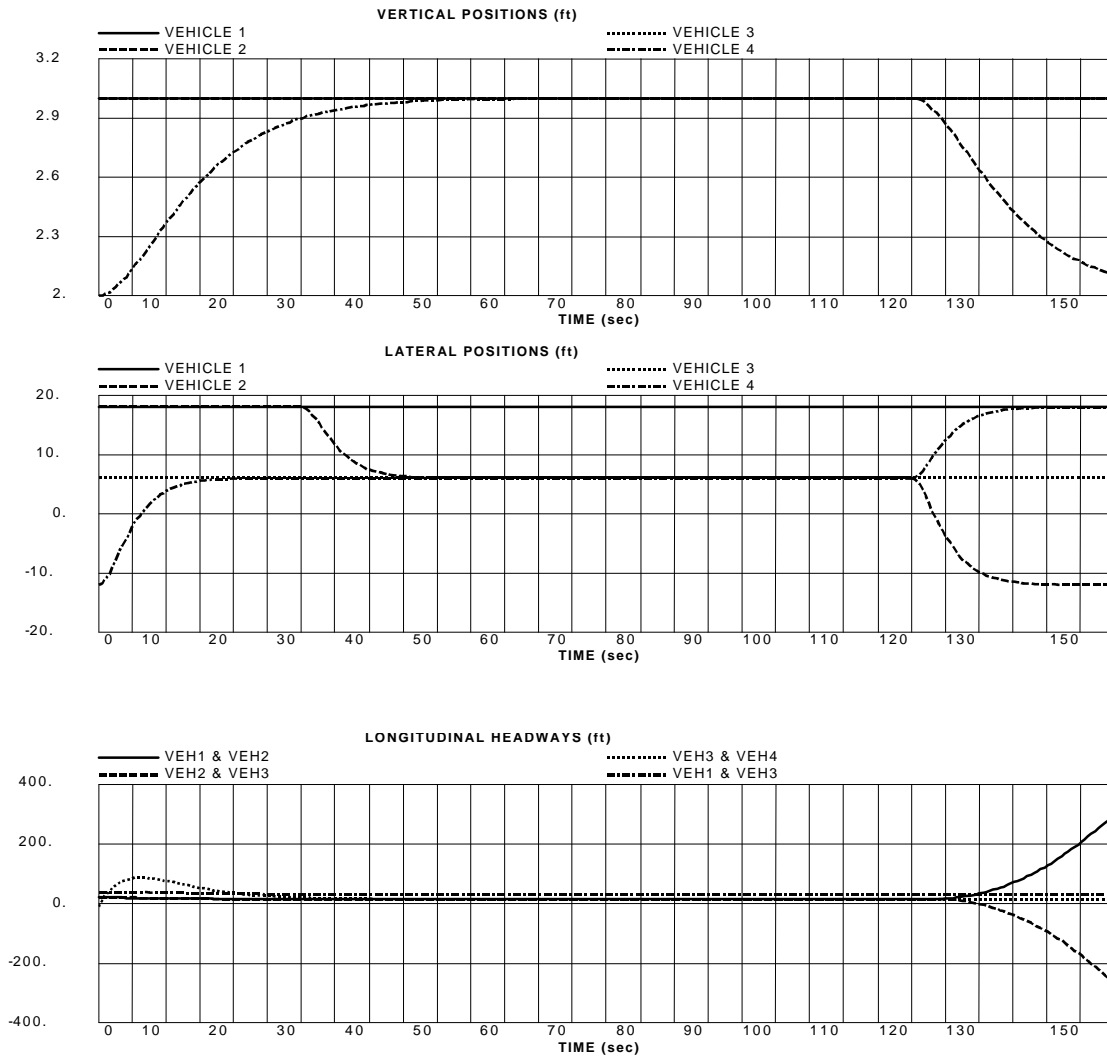
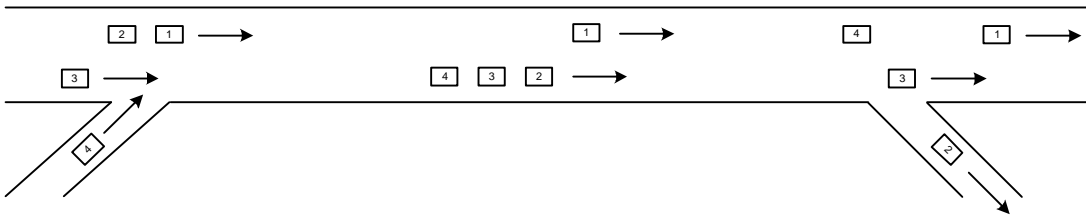


Figure 5.12 Merging Vehicles

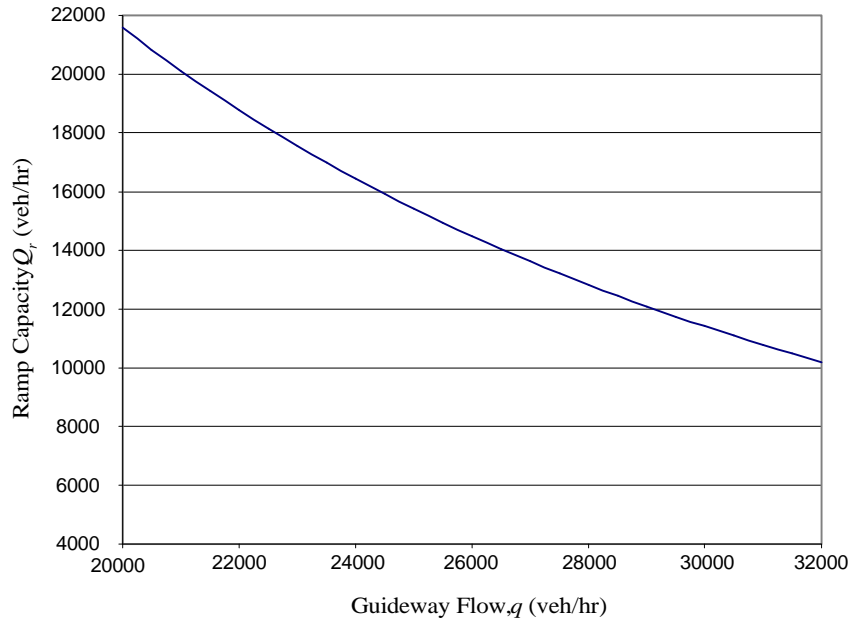


Figure 5.13 Relationship between guideway flow and ramp capacity by delay approach.

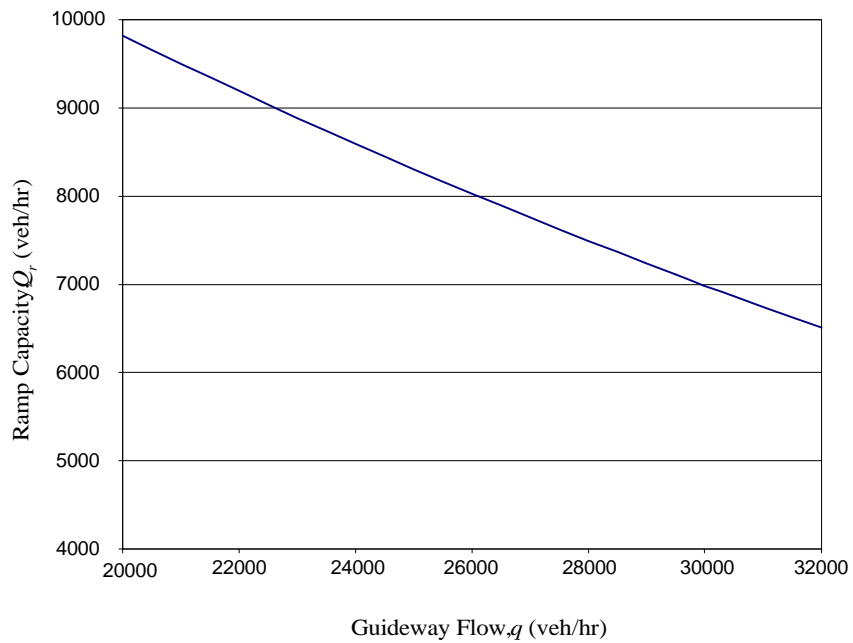


Figure 5.14 Relationship between guideway flow and ramp capacity by multiple merge approach.

Both approaches are based on the assumption that traffic arrival is absolutely random. Thus, they provide the lower bound of the ramp capacity, while in fact, the real capacity should be larger than computed since the arrival is approaching deterministic pattern. Nonetheless, each approach proves that small critical gap increases chances of merging. It can be assumed that magway ramps can sufficiently serve such high demand that traffic diversion can be neglected.

Ramp queuing system can be taken as a first-come, first-serve, single server, random arrival queuing system with a finite capacity. To be on the safe side, we assume high demand 3000 vehicles per hours on a ramp with low service rate 6000 vehicles per hour. Thus, the utilization factor r is 0.50. Therefore, from equation 5.70;

$$P_M = \frac{(1 - 0.5)0.5^M}{1 - 0.5^{M+1}} \quad (5.82)$$

One can visualize the possibility that the ramp is fully loaded P_M , which will force traffic diversion, versus the determination of ramp capacity M as shown in Figure 5.15.

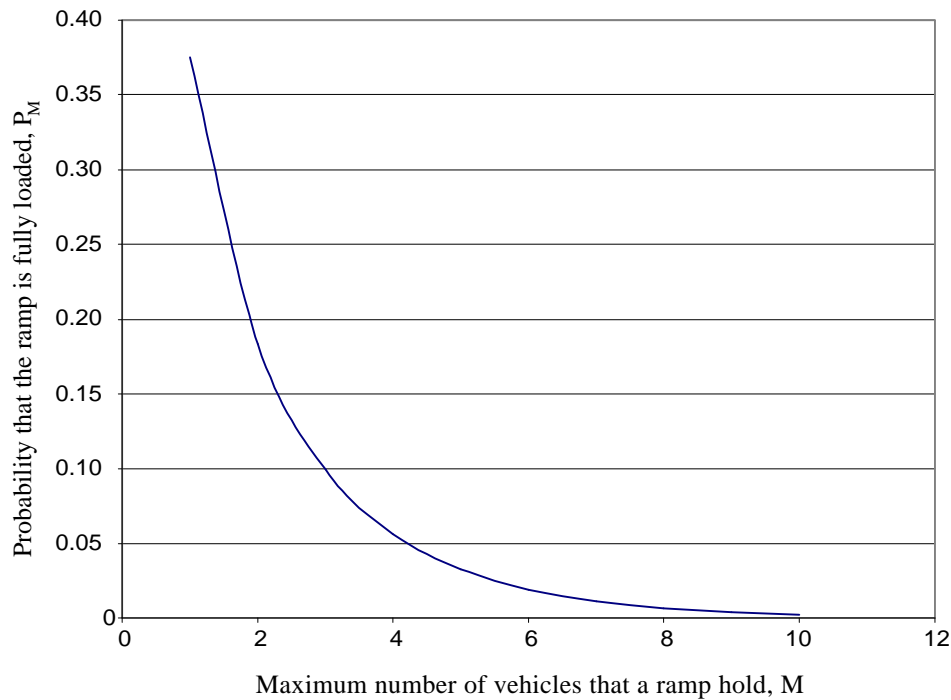


Figure 5.15 Probability that a ramp is fully loaded versus ramp capacity

It should be noted that in order to avoid lateral magnetic influence, no pair of vehicles from adjacent lanes should be at the same longitudinal position. When vehicle 2 begins merging, vehicle 3 must maintain a longitudinal distance at least $2S + L$ behind vehicle 1 at all time, where S is desired spacing and L is vehicle length. Vehicle 2 gradually navigates its lateral position to a new desired position to become the leader of a new platoon on the inside lane, while vehicle 3, not having to adjust its velocity, becomes the second vehicle of the platoon. Finally, at the last phase, vehicle 2 exits the entrance ramp, decelerates and lowers its elevation.