Chapter 5

Convex and Probabilistic Models for the Uncertainties in Geometric Imperfections of Stiffened Composite Panels

The objective of this chapter is to introduce a convex model for the uncertainties in the imperfect panel profile. In the previous chapter a manufacturing simulation model was introduced for the prediction of the cured profile of a specific panel. Uncertainties in the primitive variables were considered as the only source of imperfection in this model. In the suggested design scheme the manufacturing model is to be used in the generation of a large number of panels. These panels are then used in defining the family of panel profiles that can be expected from the specific manufacturing process when applied to the specific design in hand. The convex model presented in this chapter can determine the weakest panel profile in this family of panels providing that some assumptions regarding the sensitivity of the failure load with respect to the imperfection profile are satisfied. It will be shown that the convex model can be applied in this problem and that it allows us to achieve a great deal of savings when compared to the traditional probabilistic models. First a short revision of the most commonly used models for uncertainties is presented. Next, the assumptions used in the development of the suggested convex model are presented followed by its mathematical derivation. Validation of the convex model predictions when used in the problem in hand are presented, and comparisons with the Monte Carlo simulation for this problem are performed.

Engineers are accustomed to using imperical "knockdown factors" in order to accommodate the large discrepancy between theoretical and experimental values of buckling and elastic limit loads. The knockdown factor, when multiplied by the classical failure load for the perfect structure, yields an estimated lower bound of the failure load for the imperfect structure. Knockdown factors are often adopted as the lower bound of the failure loads obtained experimentally for a range of distinct structures, materials and manufacturing processes. Such an approach has several drawbacks. It would seem that the estimates should constantly be updated to include new experimental results. Furthermore, this approach mixes panels produced by rough manufacturing procedures (and therefore associated with a greater reduction of the buckling loads) with those produced by more refined techniques (and hence buckles at higher loads). This implies that the design of panels with low initial imperfections may be overly conservative.

A natural way to deal with uncertainty in the initial imperfections is to employ a stochastic approach. Apparently the first probabilistic analysis of initial imperfections, treated as random variables with specified joint distribution, was given by Bolotin [112]. The next step was treatment by Budiasky [113] of initial imperfections as random fields with given mean and autocorrelation function. These approaches have been bridged by Elishakoff [114] in the context of the Monte Carlo method. Probabilistic analysis treats the initial imperfections as random functions of the space coordinates x, y of the panel. Let $\eta(x, y)$ represents the deviation of the panel from its nominal shape at point x, y. If one knows an analytical relation between the failure load Λ and the initial imperfection function $\eta(x, y)$:

$$\Lambda = \Psi(\eta(x, y))$$

then one can relate the probabilistic characteristics of Λ with those of η, resulting in an expression for the probability density of the failure load. Except for the simplest cases, there is no analytical relation of this type available in the literature. Usually, the initial imperfection function is expanded in a Fourier series:

$$\eta(x, y) = \sum_{i,j} A_{ij} \, \varphi_{ij}(x, y)$$

where A_{ij} are the Fourier coefficients and φ_{ij} is a complete set of functions. Then, available computer codes yield relations of the type

$$\Lambda = \Psi(A_{ij})$$

The objective of this study is to exploit fragmentary information (which is usually all that is available) about the initial imperfection of stiffened panels, in order to determine the minimum failure load which may be expected. Explicitly, the minimum failure load will be determined as a function of parameters which characterize the range of possible initial imperfection profiles of the panel. Non-probabilistic convex models of uncertainty in the initial imperfections will be employed. The uncertainty in the initial imperfection profiles will be quantified in terms of the variability of the modal amplitudes of those profiles. The amplitudes of the first N most significant mode shapes are assumed to fall in an ellipsoidal set in N-dimensional Euclidean space. The minimum failure load is then evaluated as a function of the shape of the ellipsoid. The convex model of uncertainty was used by Linderberg [115] in the analysis of radial pulse buckling of shells. Ben-Haim and Elishakoff [116] used it in the analysis of static axial buckling of shells and in other applications [52].

Next, a short introduction to the nonlinear behavior of stiffened panels is given in order to describe the relationship between the geometric imperfections and the postbuckling response of these panels. The nonlinear behavior of stiffened panels can be classified into three general types: local postbuckling, Euler postbuckling, and an interaction between local and Euler buckling modes termed "modal interaction". Figure (5.1) shows the load versus end shortening response (normalized with respect to their critical values) for each type of behavior. The solid lines correspond to perfect panel behavior, while the dashed lines are for imperfect panels. For local postbuckling where the panel buckles into half-wave lengths, which are approximately equal to the width

between stiffeners, the panel can carry loads greater than its buckling load. For Euler (Global) postbuckling, the panel buckles into one half-wavelength along its length and its load-carrying capabilities remain essentially neutral after buckling. For the case of modal interaction, which is the case associated with optimum designs and where the local and Euler modes have critical loads of almost equal value, the panel is unable to carry loads greater than its buckling load. It is thus clear that designing the panel for local buckling leads to overly conservative designs since the postbuckling capabilities of the panel are ignored.   This suggests the use of postbuckling design techniques when designing these panels.
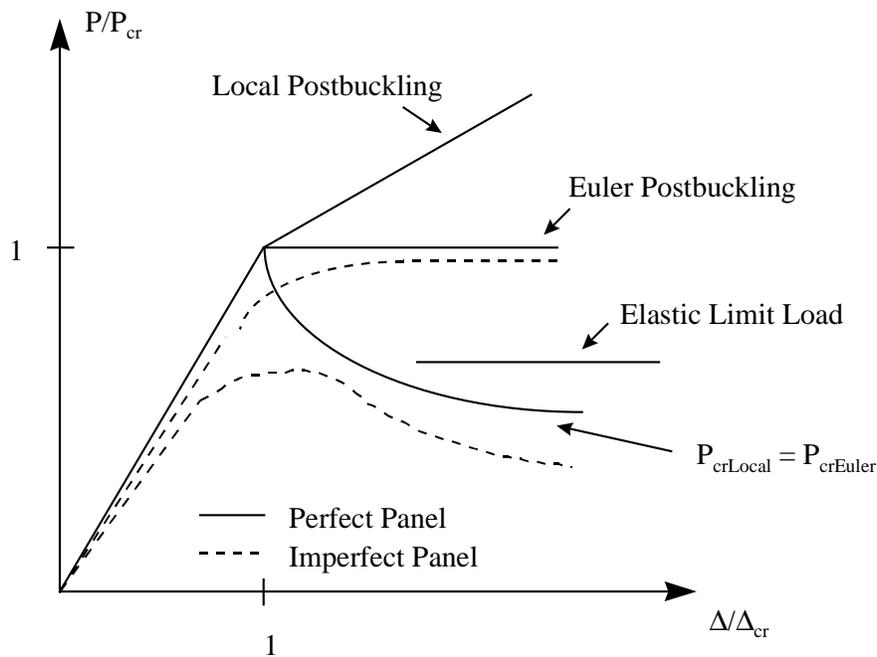
Figure (5.1) Nonlinear elastic behavior of stiffened panels

The finite element developed in Chapter 2 can (and will) be used in the nonlinear analysis and design of these stiffened panels. However, due to the large number of analysis required for the validation of the convex model presented in this chapter, and the relative simplicity of the example problem used in this validation, an approximate cost-efficient semi-analytical method will be used instead of the more expansive nonlinear finite element.

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An approximate semi-analytical method was recently developed for the cost-efficient, geometrically nonlinear analysis of thin-walled composite panels [2]. The analysis is capable of predicting the nonlinear postbuckling stresses and deformations, elastic limit points, and imperfection sensitivity of panels that are composed of linked prismatic plate strips (see Figure (5.2)) and subjected to a variety of load cases including uniaxial loads, combined inplane axial loads, pressure and temperature. The method was developed as an extension to the buckling analysis code VIPASA and uses buckling eigenfunctions, calculated by VIPASA [54] as the primary displacement shape functions for the nonlinear analysis.
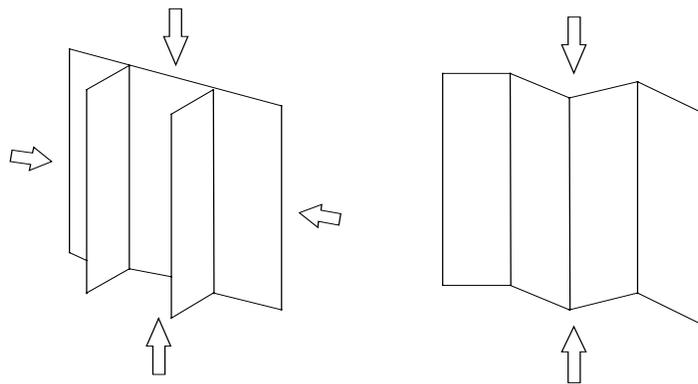
Figure (5.2) Typical compressively loaded prismatic linked-plate structures

In order to clarify the role of the suggested convex model in the new design loop, Section 5.2 of this chapter presents the traditional design-for-imperfection scheme currently used along with the suggested addition presented in this chapter. Section 4.3 presents the convex model employed in this study along with the mathematical analysis involved in predicting the weakest panel profile. Predictions of the convex model are compared with those obtained by direct minimization in Section 4.4. Finally, a probabilistic analysis of the problem is presented in Section 4.5 to demonstrate the validity of the results obtained by the convex model and the great simplicity and convenience compared to the traditional stochastic approach.

5.1 The Design Problem

The panel configuration used in this study is shown in Figure (5.3). The panel is composed of thin rectangular plate strips connected along their longitudinal sides. The panel model contains three plate strips, two internal node lines (2 and 3) and two boundary node lines (1 and 4). The boundary conditions at the longitudinal ends of the panel ($\bar{x}=0$ and $\bar{x}=L$) and the external boundary node lines ($\bar{y}=0$ and $\bar{y}=B$) are assumed to be simply supported. The panel is made of graphite/epoxy laminates with the following material properties

$$E_1 = 20.00 \times 10^6 \; psi$$
$$E_2 = 1.30 \times 10^6 \; psi$$
$$G_{12} = 1.03 \times 10^6 \; psi$$
$$\nu_{12} = 0.30$$

The panel dimensions are

Length	48.1 in
Width	6.28318 in
Blade Height	1.42189 in

The flange' laminate stacking sequence and thicknesses are
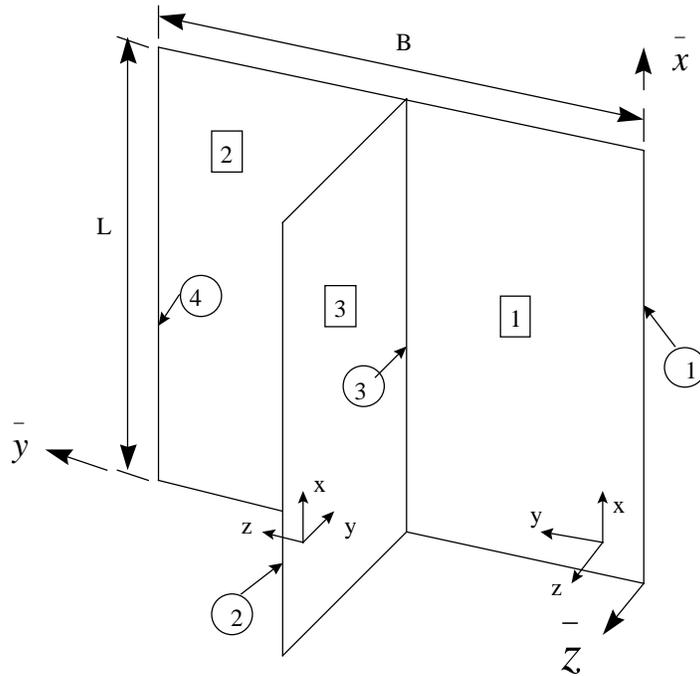
$$\theta = [45^o \; / \; -45^o \; / \; 90^o \; / \; 0^o]_{sym}$$

$$t = [0.006/0.006/0.012/0.006] \quad inches$$

The blade laminate stacking sequence and thicknesses are

$$\theta = [45^{o} / -45^{o} / 90^{o} / 0^{o}]_{sym}$$

$$t = [0.006/0.006/0.006/0.66] \quad \text{inches}$$

The panel is subjected to axial loading in the *x*-direction only, ($N_{x} = 1000 \, psi$) and neither out of plane pressure nor thermal loadings are applied.

Figure (5.3) Stiffened panel showing plate-strip and labeling conventions.

The analysis assumes that the panel displacements have the following form :

$$\{u\} = \lambda \{u_{L}\} + q_{i} \{u_{i}\} + q_{i}q_{j} \{u_{ij}\} \qquad i,j = 1,2,\ldots$$

where summation over *i* and *j* is implied, and the notation $\{u\} = [u \quad v \quad w]^{T}$ is used to refer to a set of compatible displacement fields. Displacements are represented as a sum of linear, unbuckled contributions, u_{L} , and a truncated perturbation expansion in terms

of modal amplitudes, q_i, which are the unknown amplitude multipliers for the buckling

mode shapes, u_i, that are used to represent the displacements in the nonlinear regime.

The magnitudes of q_i determine the amount of influence their respective buckling mode

has on the response of the panel. Displacement contributions of second-order in the

modal amplitudes are retained to achieve solutions of useful accuracy for a variety of

geometric contributions. For an unloaded panel, ($\lambda = 0$), the displacements degenerate to

the imperfection shape of the panel:

$$\{u\} = q_i^o \{u_i\} + q_i^o q_j^o \{u_{ij}\} \qquad i,j = 1,2,\ldots$$

where q_i^o are the modal imperfection amplitudes that generate imperfections in the shape

of their respective buckling mode.

The assumed form for the displacements and the imperfection shape is used in the

expressions for the midplane mechanical strains and curvatures. The governing equations

are then obtained by applying the principal of virtual work. Details for the methods of

determining the linear unbuckled, the buckling, and the second-order displacement shape

functions can be obtained in Stoll et al. [2].

The design problem is to minimize the weight/area of a linked-plate panel such

that it will sustain a specified design load N_D, without an elastic limit load, N_L, failure

or local material failure. In existing treatments of this problem [1], the geometric

imperfection is assumed to have a given shape, based on previous experience with the

manufacturing process outputs (See Figure (5.4)). However, due to the nature of the

manufacturing process, the actual shape of the geometric imperfection is different than

the assumed one and varies from panel to panel.

The objective of this chapter is to introduce a convex model to account for the

uncertainties in the imperfection profile of a manufactured panel. Thus, instead of using

an estimate for a nominal imperfection profile in the design problem, an estimate for a number of imperfection parameters is used. These imperfection parameters are chosen such as to represent a realistic ensemble of panels that can be expected from the manufacturing process in hand (These are obtained from the model presented in Chapter 4). The output of the convex model is the weakest panel profile which is then used in the design problem instead of the traditionally assumed nominal profile. A schematic of this process is shown in Figure (5.5).
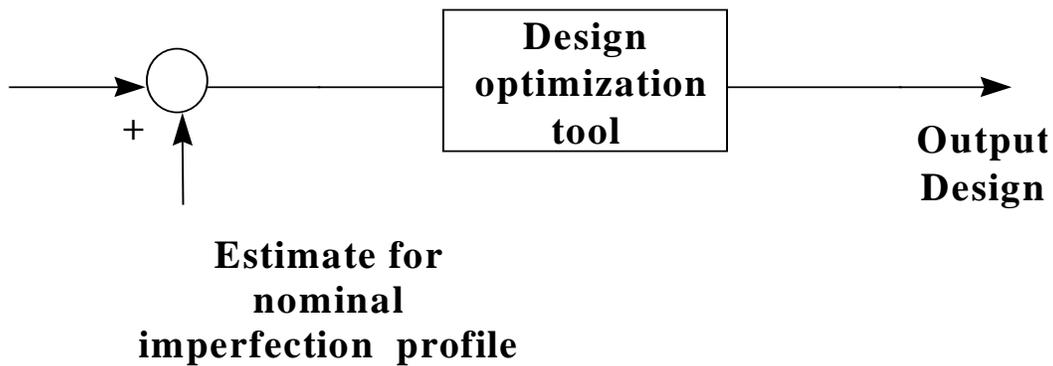
Figure (5.4) Existing geometric imperfections treatment
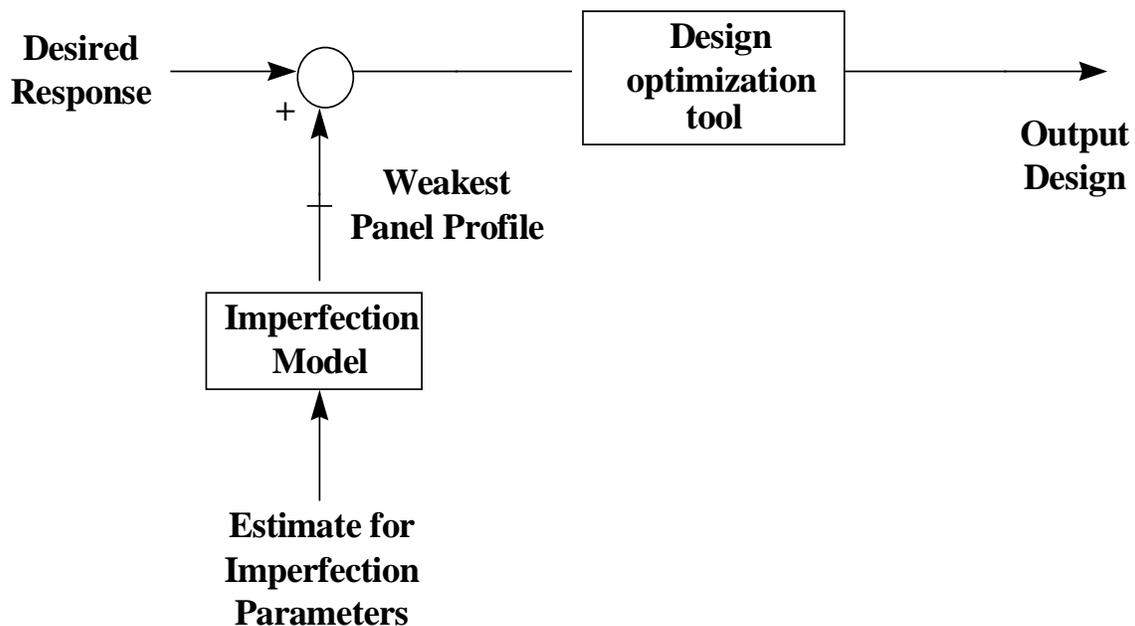
Figure (5.5) Design process with imperfection model for uncertainties

5.2 Convex Models of Uncertainty

The probabilistic approach to the modeling of uncertainty begins by defining a space of events and a probability measure on that space. The space is all-inclusive; everything that could occur, and also possibly events that cannot occur, are included. The probability measure contains all information concerning the relative frequency of different events.

The set-theoretic (convex-model) approach to the modeling of uncertainty is different. A space of conceivable events is defined, as in the probabilistic approach. However, no probability measure is defined. Rather, sets of allowed events are specified, and the structure of these sets is chosen to reflect available information on what events can and cannot occur. It is remarkable, and of considerable practical significance, that sets whose elements represent spatial or temporal uncertainty are often found to be convex [52]. A region is convex if the line segment joining any two points in the region is entirely in the region. Circles and triangles delimit convex regions whereas quadrilaterals may or may not, depending on whether their diagonals intersect within the region. In this study, convex models are used to represent the uncertainty of the initial imperfection profile of the stiffened panel.

Let $\vec{q}$ be a vector whose components are the N dominant mode shape amplitudes in the representation of the initial imperfection profile of the stiffened panel. Furthermore, let $\eta(\vec{q})$ be the elastic limit load for a panel with initial imperfection profile $\vec{q}$. Let $\vec{q}^{\,o}$ be a nominal average imperfection profile. Despite the uncertainties in many manufacturing parameters, panels which have been manufactured and handled under similar conditions have experienced forces which are likely to produce patterns of distortion common to all panels. Consequently, the average imperfection profile $\vec{q}^{\,o}$ is unlikely to be zero.

The elastic limit load for an arbitrary initial imperfection profile $\vec{q} = \vec{q}^o + \vec{\xi}$, where $\vec{\xi}$ is a small deviation from the nominal $\vec{q}^o$, is given in first order in $\vec{\xi}$ as

$$\eta(\vec{q}^o + \vec{\xi}) = \eta(\vec{q}^o) + \sum_{i=1}^{N} \frac{\partial \eta}{\partial q_i} \Big|_{\vec{q}^o} \cdot \xi_i \tag{5.1}$$

In the present work, the deviation $\vec{\xi}$ from the nominal initial imperfection $\vec{q}^o$ is assumed to vary on the following ellipsoidal set:

$$Z(\alpha, \vec{\omega}) = \left\{ \xi : \sum_{i=1}^{N} \frac{\xi_i^2}{\omega_i^2} \leq \alpha^2 \right\} \tag{5.2}$$

where the size parameter α and the semiaxes $\omega_1, \cdots, \omega_N$ are based on the experimental data available or results of a computer simulation for the manufacturing process (See Chapter 3). The components of the vector $\vec{\omega}$ are usually taken as the mean squared deviations from the average of the corresponding modal amplitude. Thus $Z(\alpha, \vec{\omega})$ can be chosen to represent a realistic ensemble of panels. For all the possible imperfection shapes that are contained in the convex set, we need to determine the weakest panel, i.e. the panel with the lowest elastic limit load. The lowest elastic limit which can be obtained in this ellipsoidal set Z can be expressed as:

$$\mu(\alpha, \vec{\omega}) = \min_{\xi \in Z(\alpha, \vec{\omega})} (\eta(\vec{q}^o) + \varphi^T \vec{\xi}) \tag{5.3}$$

where

$$\varphi^T = \left(\frac{\partial \eta}{\partial q_1} \Big|_{\vec{q}^o}, \cdots \cdots, \frac{\partial \eta}{\partial q_N} \Big|_{\vec{q}^o} \right)$$

Note that $\vec{\xi}$ could also be assumed to vary in a rectangular set defined by the upper and lower bounds of its components, instead of an ellipsoidal set. This would result in a weakest panel whose strength would be lower than that of the weakest panel obtained using an ellipsoidal set. Specifically, when using a rectangular set, we allow all the components of $\vec{\xi}$ to take their extreme values simultaneously (this event corresponds to one of the vertices of the rectangular set), whereas this event is considered impossible when using an ellipsoidal set. The above event is very unlikely to occur because the uncertain parameters are usually statistically independent and therefore the weakest panel is likely to be too conservative. Based on this consideration we chose an ellipsoid in the present work.

Equation (5.3) calls for finding the minimum of the linear functional $\varphi^T \vec{\xi}$ on the convex set $Z(\alpha, \vec{\omega})$. Based on inherent properties of a convex set, this extreme value will occur on the set of extreme points of the ensemble Z, i.e. the boundary of the ellipsoid, which is the collection of vectors $\vec{c} = (c_1, \cdots, c_N)$ in the following set:

$$C(\alpha, \vec{\omega}) = \left\{ c: \sum_{i=1}^{N} \frac{c_i^2}{\omega_i^2} = \alpha^2 \right\} \tag{5.4}$$

Thus the minimum elastic limit load in Equation (5.3) becomes

$$\mu(\alpha, \vec{\omega}) = \min_{c \in C(\alpha, \vec{\omega})} (\eta(\vec{q}^o) + \varphi^T \vec{c}) \tag{5.5}$$

Define Ω as an $N \times N$ diagonal matrix whose n^{th} diagonal element is $1/\omega_n^2$. Then, as seen from Equation (5.5), we must minimize $\varphi^T \vec{c}$ subject to the constraint:

$$f(\vec{c}) = \vec{c}^T \Omega \vec{c} - \alpha^2 = 0 \tag{5.6}$$

The method of Lagrangian multipliers is used here. Define the Hamiltonian as:

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$$H(\vec{c}) = \varphi^T \vec{c} + \gamma f(\vec{c}) \tag{5.7}$$

where γ is a constant multiplier whose value must be determined. For an extremum we require that the derivative of the Hamiltonian vanishes:

$$\frac{\partial H}{\partial \vec{c}} = \varphi + 2\gamma \Omega \vec{c} = 0 \tag{5.8}$$

Thus,

$$\vec{c} = -\frac{1}{2\gamma} \Omega^{-1} \varphi \tag{5.9}$$

Substituting this into the constraint, Equation (5.6) yields the following expression for the multiplier:

$$\gamma^2 = \frac{1}{4\alpha^2} \varphi^T \Omega^{-1} \varphi \tag{5.10}$$

Backsubstituting for γ in (5.9), we find that the extremum deviation vector $\vec{c}$ is:

$$\vec{c} = \pm \frac{\alpha}{\sqrt{\varphi^T \Omega^{-1} \varphi}} \Omega^{-1} \varphi \tag{5.11}$$

Thus, the minimum elastic limit load is given by:

$$\mu(\alpha, \vec{\omega}) = \eta(\vec{q}) - \alpha \sqrt{\varphi^T \Omega^{-1} \varphi} \tag{5.12}$$

To find the weakest panel profile, it is thus required to check only two panels given by:

$$\vec{q}_1^{\,*} = \vec{q}^{\,o} + \vec{c} \,, \qquad\qquad \vec{q}_2^{\,*} = \vec{q}^{\,o} - \vec{c} \qquad\qquad (5.13)$$

It is significant that the above analysis yields an explicit relationship between the minimum elastic limit load and the parameters defining the uncertainty in the initial imperfection α and $\omega_1, \cdots, \omega_N$.

In the next section, example problems are solved to validate the predictions of the convex model, and to show the great reduction in effort and cost achieved by using a convex model instead of the traditional probabilistic analysis techniques.

5.3 Validation of the Convex Model Predictions

In order to validate the convex model predictions for both the "weakest" panel profile and the minimum elastic limit load, a direct minimization problem is formulated as follows:

Objective Function: Minimize $\eta(\vec{q}) = \eta(\vec{q}^{\,o} + \vec{\xi})$

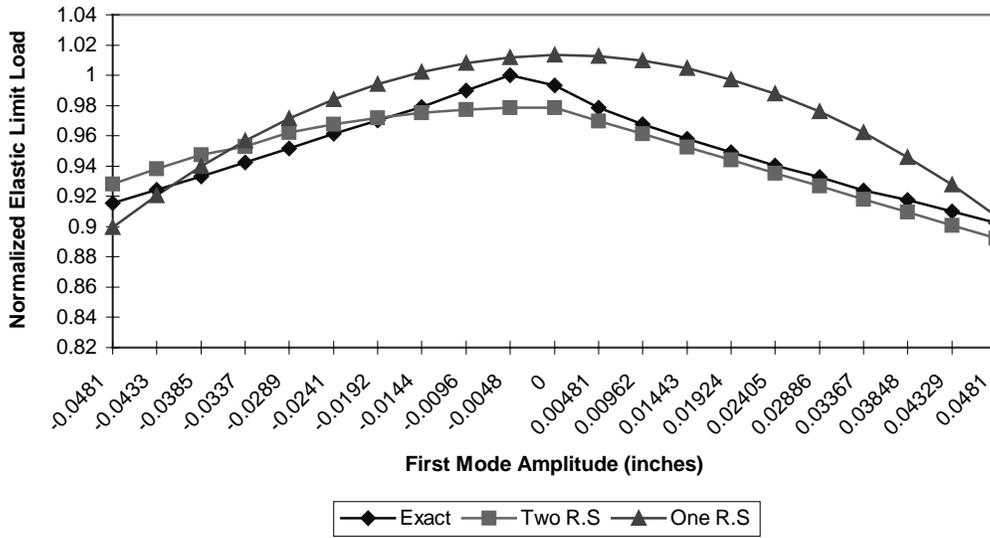Minimization Variables: $\vec{\xi}$

Constraint: $\displaystyle\sum_{n=1}^{N} \frac{\xi_n^2}{\omega_n^2} \leq \alpha^2$

Note that, as opposed to the convex model described in the previous section (which considers only the boundary of the ellipsoid), the complete interior of the convex set constitutes the design space in this formulation. The outcome of this minimization problem is the profile of the "weakest" panel $\vec{\xi}^{\,*}$ in the family of panels described by the

ellipsoidal constraint along with its corresponding minimum elastic limit load $\mu(\alpha, \vec{\omega})$. These are compared with the results of substituting for $\vec{q}^{o}, \alpha$, and $\vec{\omega}$ into Equations (5.12) and (5.13) of the convex model.

Before solving the direct minimization problem presented above, a quick look at the function to be minimized shows that the response is highly sensitive to noise in numerical calculations and thus using a continuous minimization algorithm (e.g. Conjugate Gradient) might not lead to the required minimum. In order to smooth out the response, it was suggested to employ a response surface approximation for the actual function. The quality of the model is measured using a quantity called R^2. This measures the portion of the variation in the actual response that the response surface model accounts for. A good model has typically an R^2 greater than 90% (see Ref. [117]). First, a single response surface was fitted around the point of zero imperfection using a Central Composite Design (CCD) with 324 function evaluations. However, low values of R^2 where obtained which means that the surface did not provide a good fit for the elastic limit load variation. This is mainly due to the sudden switch in behavior around the origin that the response surface was not able to model. Notice that the panel under consideration has stiffeners only on one side of the skin and, hence, is unsymmetric. Therefore, effects of the positive and negative imperfections on the response of the panel are quite different from one another. This suggested the use of two response surfaces instead of one. One of them for the region with negative q_1 (first mode amplitude) and the second for positive q_1 amplitudes. These surfaces were second order polynomials, each requiring 257 function evaluations (designs). Figure (5.6) shows the variation of the elastic limit load with the first mode's amplitude (q_1), as obtained from both the single and the two Response Surface approximations along with the exact variation. A good agreement is noticed, specially for values of q_1 away from zero. Based on the above, it was concluded that the Response Surface polynomials were acceptably accurate.

Figure (5.6) Elastic limit load variation with first mode shape amplitude

The direct minimization problem is now reduced to the simple minimization of the quadratic Response Surface functions subjected to the ellipsoidal constraints.

The comparison is now performed for the following case :

$$\vec{q}^{\,o} = [-0.0481, 0.02104, 0.018037, 0.015031,$$
$$0.01202, 0.009018, 0.006012, 0.003006]$$

$$\alpha^2 = 8$$

$$\vec{\omega} = [0.005879, 0.00514412, 0.00440925, 0.00367437,$$
$$0.0029395, 0.00220462, 0.00146975, 0.00073487]$$

A linear variation of the modal amplitudes was assumed, such that the first modal amplitude q_1 is set equal to its maximum value of $0.0481^{"}$ and the 9th mode shape amplitude is assumed to be zero. Values of $\vec{\omega}$ and α were obtained based on previous experience and general guidelines given by Ben-Haim and Elishakoff [51].

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In order to span the range of possible imperfection amplitudes, the nominal value of the first mode's amplitude is varied from -0.0481 to 0.0481 while the amplitudes of the remaining modes are fixed thus moving the center of the ellipsoid. The predictions of the convex model are compared to the minimum of the Response Surface approximation. Figure (5.7) shows the predictions for the minimum elastic limit as q_1^o is changed from -0.0481 to 0.0481; a very good agreement is obtained.
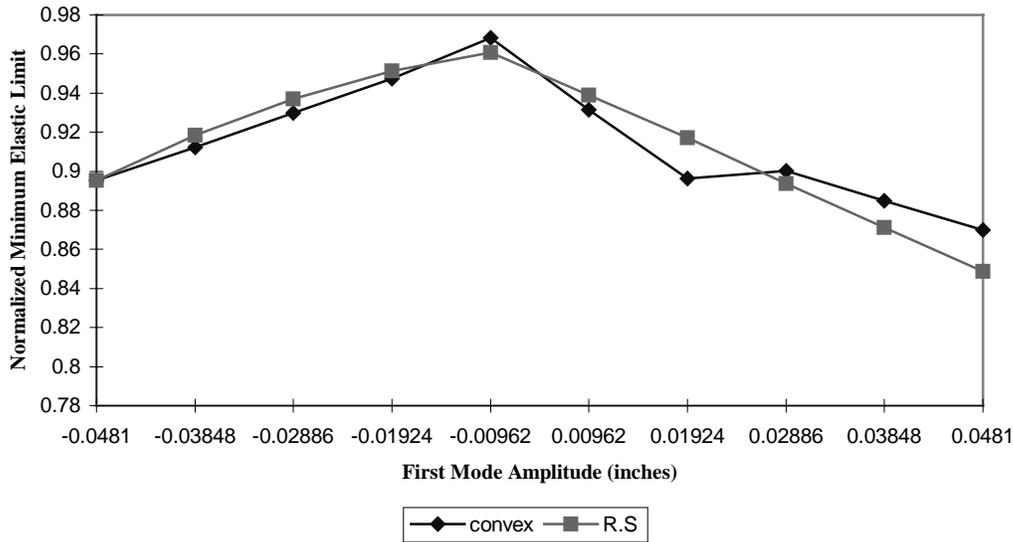
Figure (5.7) Predictions of the convex model for the minimum elastic limit load along with the minimum of the response surface

It is important to recall here that the convex model predictions are obtained by simple substitutions in Equations (5.12) and (5.13), while the construction of the Response Surface approximation required the analysis of more than 500 panels in order to get a good fit to the actual variation.

5.4 Probabilistic Models

In this section, the convex model is compared to a probabilistic model. There are three probabilistic approaches to a reliability assessment problem: 1) direct integration of the joint probability density function of the random variables over the failure region in the space of random variables, 2) second moment methods, and 3) Monte Carlo simulation.

Direct integration is too expensive for problems involving more than three variables because it involves a nested integration. Therefore it is not practical for most real life problems.

The key idea behind second moment methods is to approximate the performance function with a simple function, which allows us to find the probability of failure using a closed form, analytical expression [118]. The performance function is a function that is non negative if the structure survives, and negative if it fails. The approximating function can be a first degree or a second degree polynomials. The polynomial can be determined using Taylor series expansion of the performance function about a point in the space of the random variables. This point is called *most probable failure point* or *design point*. It is determined by transforming the random variables into standard independent Gaussian variables (Gaussian variables that have zero mean and unit standard deviation), and finding the point in the boundary between the survival and failure regions that is closer to the origin in the space of transformed variables. The origin in the space or the transformed variables corresponds to the mean values of the original random variables. Optimization is used to determine the most probable failure point. Second moment methods are far more efficient than direct integration and Monte-Carlo simulation, because they typically require less than ten function evaluations. An additional advantage is that they determine the *sensitivity factors*, which indicate the most important random variables. However, in few cases, the optimization algorithm used for finding the most probable failure point may not converge, or converge to a local instead of a global optimum.

Monte-Carlo simulation generates sample values of the random variables using a random number generator, calculates the performance function and checks if the structure fails. This procedure is repeated many times (from a few hundred to several thousand). The relative frequency of failure, i.e. the number of replications in which the structure has failed over the total number of replications, is an estimator of the failure probability. Monte Carlo simulation methods are easy to implement and robust but are also expensive. Therefore, they are used in cases where the performance function can be calculated rapidly. Some studies have approximated the performance function by a second degree polynomial and performed Monte-Carlo simulation using this polynomial instead of the performance function, which has reduced dramatically the computational cost [119].

In this study, the second degree polynomial approximation presented in Section 5.4 is used in the Monte Carlo simulation , instead of the numerical analysis based on NLPAN [2] for determining the panel elastic limit load, which reduced dramatically the computational cost of Monte Carlo simulation. Moreover, because the polynomial was found to approximate the actual elastic limit reasonably accurately, the estimated probability distribution of the elastic limit should also be accurate.

The coefficients that describe the deviations of the amplitudes of the modes from their nominal values were assumed to be independent and uniformly distributed random variables. They were assumed to vary in intervals defined by the axes of an ellipsoidal set (convex model). That is:

$$\xi_i \sim U(-\alpha\omega_i, +\alpha\omega) \quad i = 1,...,8 \qquad (5.14)$$

Where the values of α and ω_i are identical with those in Section 5.4. We considered ten cases where the nominal value of the first mode amplitude, q_1^o, was assumed to vary between -0.0481 to +0.0481. The remaining mode amplitudes were assumed to be equal to 0.0001 inches. Finally, the total number of panels generated was 100,000.

Figures (5.8a and b) compare the minimum elastic load found using the convex model and the 1% and 99% percentiles of the elastic load found using Monte Carlo simulation. The results of the convex and probabilistic models are in reasonably good agreement. Indeed, the minimum elastic load found from the convex model is quite close to the 1% percentile. However, the convex model predictions are not consistently below or above the 1% percentile. The probability of the elastic load predicted by the response surface polynomial being smaller than the elastic load predicted by the convex model ranged from zero, for $q_1^o = -0.02886$ to 11.6% for $q_1^o = 0.03848$. This variation in the non exceedence probability of the convex model prediction should be due to a) errors in the approximation of the elastic load by the second order response surface polynomial, and b) noise in the numerical model used to predict the elastic load of the panel. The discontinuity in the convex model prediction for $q_1^0 = 0.01924$ should also be because of numerical error.
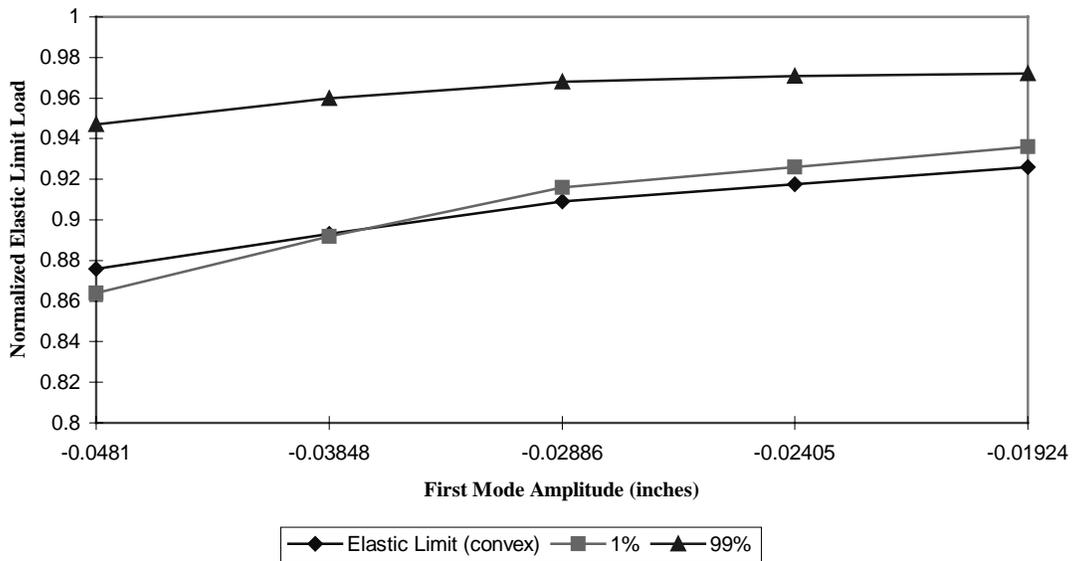
Figure (5.8a) Elastic limit load from the convex model and the probabilistic analysis for negative first mode amplitudes

For the panel under consideration, the convex model appears to be adequate. Since the convex model requires less information about uncertainties, and is considerably more efficient than probabilistic analysis, (it does not require to generate designs for fitting a response surface) it was decided to use it in the suggested design scheme.
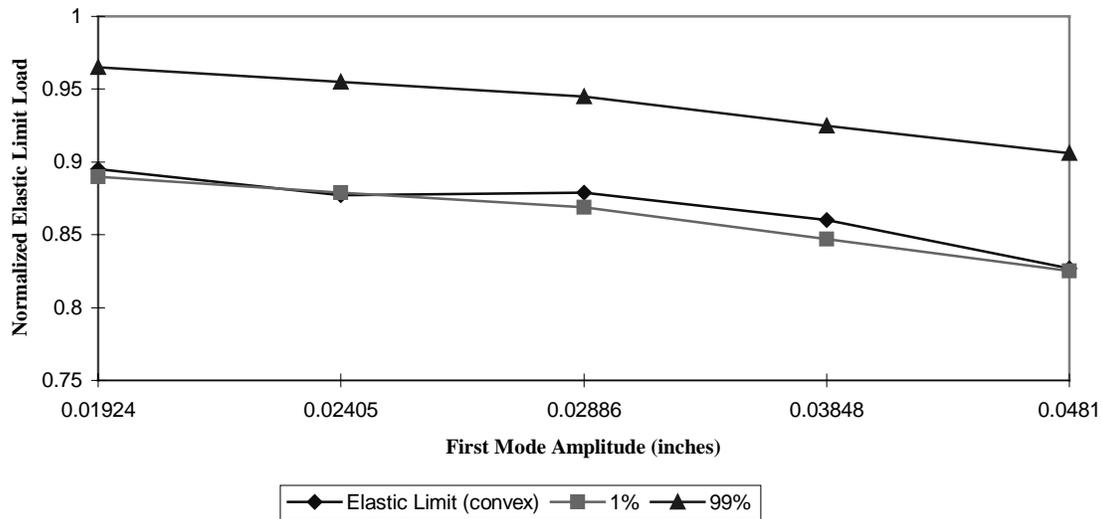
Figure (5.8b): Elastic limit load from the convex model and the probabilistic analysis for positive first mode amplitudes

In general, if little information about uncertainties is available, one should use a simple model that gives consistently conservative results. A convex model is likely to be better than a probabilistic model in these cases. However, a convex model that uses an ellipsoidal set to model uncertainties is not always conservative. For example, if some or all variables are strongly correlated the probability of all the variables taking their extreme values simultaneously can be significant. In this case, the convex model that employs an ellipsoidal set can yield an unreasonably high characteristic value (that is, there is a significant probability that a sample panel has lower strength than the characteristic value). Therefore, in problems where the correlation between the random variables is not known, it is better to assume that the uncertain variables vary in a rectangular instead of an ellipsoid.

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5.5 Concluding Remarks

The objective of this chapter was the development of a convex model for the uncertainties in the initial geometric imperfections of stiffened composite panels. We have demonstrated the suitability of the ellipsoidal linear convex model for the problem in hand. A great deal of cost and effort reductions have been achieved by the substitution of the more traditional probabilistic analysis by the convex model. The predictions of the convex model were compared to the results obtained by direct minimization. A good agreement was demonstrated for both the "weakest" panel profile and the minimum elastic limit load. Monte Carlo simulation has been performed and convex model predictions were compared to those of the probabilistic analysis. The predictions of the two models were found to be quite consistent.

The next step is the incorporation of the previously developed convex model for uncertainties together with the manufacturing model presented in Chapter 4 in the new design scheme. The output of the manufacturing model will be values for $\vec{\omega}$, $\vec{q}^{\,o}$ and α which together determine the family of panels that can be expected from the given manufacturing process. Feeding these parameters into the convex model yields the weakest panel shape in the ensemble. This is the shape of imperfection that will be used in the design instead of an assumed average shape.